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Population and Employment Densities: Structure and Change

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Population and Employment Densities. Structure and Change*

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We examine spatial patterns and their changes during the 1970s for the Los Angeles region, by estimating monocentric and polycentric density functions for employment and population Downtown Los Angeles is clearly identified as the statistical monocentric center of the region, and it is the most consistently strong center in the polycentric patterns Polycentric models fit statistically better than monocentric models, and there was some shift in employment distribution toward a more polycentric pattern These findings verify the existence of polycentricity in Los Angeles and demonstrate for the first time that employment and especially population follow a polycentric pattern based on exogenously defined employment centers The results confirm that both employment and population became more dispersed during the 1970s © 1994 Academic Press, Inc

I INTRODUCTION

Geographical distributions of population and employment density are often used in analyses of urban structure While such analyses traditionally have assumed monocentricity [21, 23, 26, 29], recent studies have demonstrated the presence of employment subcenters in large American cities [4, 10, 13, 22] The theoretical basis for subcenters has also received attention [19, 30, 33 34, 37].

The polycentric nature of urban structure, however, has not been incorporated into the empirical work on density distributions Griffith [15, 16] and Gordon *et al* [11] are rare exceptions Unfortunately, Griffith was unable to detect any effects of secondary employment centers on

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population distribution in Toronto in 1971 or 1976, he concludes that their impacts "may not be appreciated unless an extremely large place, such as Los Angeles, Chicago or New York, is studied' [16, pp 308–309]

Gordon *et al.* [11] study the Los Angeles area in 1970 and 1980 using the polycentric model suggested by Griffith They find that for both employment and population, the polycentric model fits better than the monocentric model, with important subcenters exerting a marked influence overlaid on a general pattern of dispersion Their study, however, has several shortcomings First, the population distribution is based on endogenously determined population centers, whereas the theory relates it to employment centers Second, employment centers are defined using *ad hoc* criteria including the fit of the estimated density function, thereby precluding statistical inference concerning their effects on employment density. Third, polycentric density gradients for employment in 1970 are not estimated, so changes in employment distribution between 1970 and 1980 cannot be examined

In this paper, we estimate polycentric density functions for both employment and population, for 1970 and 1980, using small-zone data for the Los Angeles region. All density functions are based on employment centers, predefined using simple intuitive criteria on employment density and total employment In this way, we are able to perform rigorous hypothesis tests to verify the existence of polycentricity and to determine how its extent changes over time We also measure the impacts of employment subcenters on region-wide employment and population distributions, compare the polycentric and monocentric models, and examine changes over time in the overall degree of dispersion.

II DENSITY FUNCTIONS

Monocentric Model

The standard monocentric model assumes that an urban area has a single employment center Households trade off accessibility to this center against housing costs in order to maximize their utility As a result, residents are distributed in a circularly symmetric manner with density function $f(r)$, where r is the distance from the single center Employment is sometimes assumed to be located entirely in a central business district (CBD), but this is not at all necessary many writers instead assume that it has a distribution similar in shape to that of population but somewhat more centralized [17, 21, 26, 27, 38]. We follow these writers in using "monocentric" to mean any distribution which is circularly symmetric about a single center, we do not use the term in its more restricted meaning of all employment being in the CBD.

The negative exponential is the most commonly used density function in the monocentricity literature, and it is used in this paper In the case of population density, this specification can be derived theoretically either from a utility-maximizing model with a unitary value of the price elasticity of housing services [28, 29, 31] or from entropy maximization [3] In the case of employment density, the negative exponential form is derived by Mills [25] by assuming that the production functions for product and transportation have the Cobb–Douglas form, and that the demand for product has a constant price elasticity We give this negative exponential form a multiplicative error structure, which is supported (relative to an additive structure) by evidence reported by Greene and Barnbrock [14]

To summarize, the version of the monocentric model that we examine empirically is

$$D_m = Ae^{-br_m}e^{u_m}, \qquad m = 1, 2, \quad , M, \tag{1}$$

where D_m is the observed density of population or employment in zone m; M is the total number of zones in a metropolitan area, r_m is the distance of zone m to the CBD. e^{u_m} is a multiplicative error term associated with zone m, and A and b are parameters to be estimated

To allow for a possible crater at the center in the case of population, we also fit monocentric models with a quadratic distance term added to $-br_m$ Surprisingly, the results indicate no crater near the center for population, instead, a minimum density is predicted at a distance far away from the center (about 137 miles, which is well beyond the range covered by our data) We restrict subsequent discussion to the simple negative-exponential form (1)

Polycentric Model

The natural extension of the monocentric model is to assume that urban residents and firms value access to all employment centers, so that employment and population densities are functions of distances to all these centers As pointed out by Heikkila *et al.* [18], a polycentric density function could be postulated under several alternative assumptions If influences from different centers are perfectly substitutable, so that only the nearest center matters, then the polycentric function would be the *upper envelope* of functions applying to the various centers If those influences are complementary, so that some access to every center is necessary, then the polycentric density might be the *product* of such functions, as specified by McDonald and Prather [24] for Chicago [1] If the

[1]Note that the multiplicative density function does not have a separate intercept for each center, so there is no apparent way to take into account the variation in sizes of various centers Furthermore, it implies that adding a new center at one side of the region lowers densities far away on the opposite side

relationship among centers' influences is between these two extremes, then the *sum* of center-specific functions becomes a plausible specification We believe this last assumption is the most realistic. so we specify the polycentric density function to be additive, as do Griffith [15. 16] and Gordon *et al* [11]

Applying these ideas to the negative-exponential functional form leads to the following generalization of the monocentric model.

$$D_m = \sum_{n=1}^{N} A_n e^{-b_n r_{mn}} + \nu_m, \qquad m = 1, 2, \ldots, M, \tag{2}$$

where N is the number of employment centers; r_{mn} is the distance between zone m and center n, ν_m is the error term of density associated with zone m, and A_n, b_n are parameters to be estimate for each center n. The first term on the right side of the equation is a vertical sum of negative-exponential density functions, each reflecting the influence of one center on that location. The error term is specified to be additive in order to permit estimation by nonlinear least squares [2]

When the intercepts of all centers except one are zero, the polycentric form collapses to the monocentric form with an additive error Therefore, we can test statistically whether the polycentric model explains the actual distributions better than the monocentric model. We also can test the significance of center n in explaining the overall density pattern by means of an F test on its parameters A_n and b_n

In addition, we are able to measure the overall impact of each employment subcenter on region-wide employment and population distributions. Summing the estimated influence of center n on employment or population in all zones m in the region yields

$$\text{IMPACT}_n = \sum_{m=1}^{M} \left(\hat{A}_n e^{-\hat{b}_n r_{mn}}\right) S_m = \hat{A}_n \sum_{m=1}^{M} S_m e^{-\hat{b}_n r_{mn}}, \qquad n = 1, 2, \ldots, N, \tag{3}$$

where S_m is the area of zone m, and $\hat{A}_n$ and $\hat{b}_n$ are the estimated intercept and gradient for center n Clearly, the influence of center n is positively related to $\hat{A}_n$ and negatively related to $\hat{b}_n$ This method is more accurate than integrating over part of a circle, as in [11], because it accounts for the actual geography of the region

[2]We attempted to estimate a version of (2) with a multiplicative error term, by taking logarithms and applying nonlinear least squares, but we could not obtain convergence due to the extreme nonlinearity of the resulting equation for the sum of squared residuals

III STUDY AREA AND DATA

Our study area consists of the urbanized parts of five counties in the greater Los Angeles region namely, Los Angeles, Orange, Riverside, San Bernardino, and Ventura Counties. This region provides an ideal area for investigating the impacts of employment centers on spatial distributions of population and employment, because the presence of significant employment subcenters is well known [4, 10–12]

Data obtained from the California Department of Transportation (Caltrans) provide information on population and employment for 1135 "transportation analysis zones," as defined by the Southern California Association of Governments (SCAG) [3] The employment data are derived from records kept by the California Economic Development Department as part of its administration of workmen's compensation insurance [4] The matrix of road-network distances between zones, created as part of the Urban Transportation Planning Package (UTPP), was provided by SCAG. The zones cover a total land area of 3476 square miles, their population grew from 9 37 million in 1970 to 10 51 million in 1980, while employment grew from 3 95 million to 5 31 million.

IV IDENTIFICATION OF EMPLOYMENT CENTERS

Conceptually, an employment center for our purposes should be a dense concentration of employment large enough to have a potentially discernible effect on the metropolitan area. The empirical literature provides a variety of definitions For example, Dunphy [6] defines centers following a sequential process requiring specialized information from local sources McDonald [22] uses local peaks in gross employment density or in employment–population ratios Still other definitions are used by Greene [13] and Gordon *et al* [11]

[3]Like census tracts, these analysis zones are aggregates of census blocks, but they do not have roughly identical population, so "census-tract delineation bias" [7] is less likely to be a problem For simplicity, we have deleted 150 very low-density zones from SCAG's original 1285, all are remote from the highly developed parts of the region with the exception of 11 largely undeveloped zones in the Santa Monica mountains which separate the densely developed West Los Angeles corridor (roughly, Hollywood to Santa Monica) from the more suburban San Fernando Valley

[4]In other work on the Los Angeles region [10 35], we have instead used Census journey-to-work data defined for the same system of analysis zones (Of the 1146 zones used in [10], 11 are deleted here because they contain no employment in the Caltrans data set) Each data set has its own disadvantage for locating employment the Caltrans data suffer from underreporting of small employers, while the Census data suffer from inability to locate the reported workplace addresses of many respondents We believe the Caltrans data are superior for 1970-to-1980 comparisons, because SCAG used quite different procedures to allocate unknown addresses in the 1980 Census data than were available to us for 1970

TABLE 1
Employment Centers in 1970 and 1980

Rank and location	Total employment	Employment density (employees/acre)	Distance from CBD (miles)[a]
	1970		
1 Downtown L A	502,562	56 11	0 2
2 West L A	23,965	33 75	11 9
3 West Hollywood	38,097	26 77	8 4
4 L A Airport	33,500	24 15	17 1
5 Burbank Airport	30,800	33 37	16 6
6 UCLA/Santa Monica	24,199	39 87	15 4
7 Long Beach	23,529	32 19	25 8
	1980		
1 Downtown L A	524,700	46 82	0 1
2 West L A	119,900	27 65	8 9
3 West Hollywood	49,750	34 96	8 3
4 L A Airport	39,800	28 70	18 8
5 Burbank Airport	38,650	41 87	16 5
6 UCLA/Santa Monica	31,650	52 14	15 8
7 Long Beach	22,000	31 10	25 3
8 Downey	26 250	31 14	14 8
9 Santa Ana	21,574	22 81	32 9
10 Van Nuys	21,350	24 71	21 5

[a]Measured from zone of peak density in center to zone of peak density in downtown Los Angeles These zones, and hence the measured distances, vary from 1970 to 1980 especially for the West L A center

We use a version of McDonald's definition which is suggested by Giuliano and Small [10] An *employment center* is defined as a contiguous set of zones, each with employment density above some cutoff $\bar{D}$, that contains total employment above another cutoff $\bar{E}$ All contiguous zones meeting the density criterion are included in the center We refer to the highest-density zone as the *peak* In order to have a manageable number of centers in the density function estimation, we use cutoffs that are twice as high as those in [10] $\bar{D} = 20$ employees per acre and $\bar{E} = 20{,}000$

These criteria identify 7 centers for 1970 and 10 for 1980 They are ranked in order of total employment in Table 1 and are labeled by these ranks in Figure 1 The three largest, along with UCLA/Santa Monica, form a nearly contiguous corridor extending roughly along Wilshire Boulevard from downtown Los Angeles to the ocean The seven 1970 centers are distributed over the older developed part of Los Angeles County, from Burbank in the north (in eastern San Fernando Valley) to Long Beach in

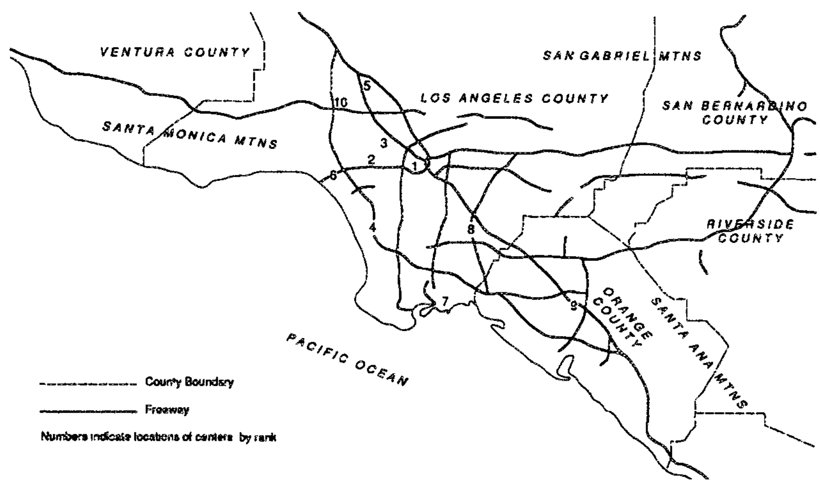

FIG 1 Location of 1980 employment centers

the south The three new centers that appear in 1980 are more peripheral, especially Santa Ana in Orange County (east of Long Beach) and Van Nuys in northwest Los Angeles County (central San Fernando Valley)

V ESTIMATION RESULTS MONOCENTRIC MODEL

We fit the monocentric density function (1) by ordinary least squares, after taking the natural logarithm of both sides and deleting any zero-density zones We first define the region's center as the zone of highest employment density in the region This zone, different for 1970 and 1980, is a part of the traditionally defined CBD of Los Angeles, which in turn is a (relatively small) part of the center which we have named downtown Los Angeles. Note that employment density D_m is the dependent variable in (1) and is also used in defining the point from which distance r_m is measured, in order to avoid the resulting correlation between D_m and r_m for the central zone, that zone is omitted in estimating the employment regression (1). As in Muth [29], the peak zone is also omitted in the population regression because land there is devoted almost entirely to employment use

Table 2 presents the results The gradient estimates ($\hat{b}$) show that the employment and population distributions were quite flat, with density declining by only four to six percent per mile This indicates a high degree of dispersion As expected, population was more dispersed than the employment

TABLE 2
Estimates of Monocentric Density Functions 1970 and 1980

1970				1980			
Intercept ($\hat{A}$)	Gradient ($\hat{b}$)	R^2	No observations	Intercept ($\hat{A}$)	Gradient ($\hat{b}$)	R^2	No observations
Employment							
8 548* (0 073)	0 0585* (0 0022)	0 395	1132	8 795* (0 069)	0 0542* (0 0020)	0 388	1130
Population							
9 442* (0 064)	0 0482* (0 0019)	0 362	1117	9 414* (0 057)	0 0411* (0 0017)	0 350	1123

Notes Dependent variable is the natural logarithm of employment or population density (persons per square mile) Standard errors are in parentheses Zones with zero density are omitted from the 1135 observations, as is the central zone of the Los Angeles CBD (from which distances are measured) R^2 is adjusted for degrees of freedom

*Estimate is statistically significant at the 0 05 level, one-sided test

We now examine several questions about the fitted monocentric models Does the degree of dispersion increase from 1970 to 1980? Does the monocentric model fit less well in the later year? And is the CBD really the region's center?

Have Monocentric Density Gradients Declined?

The estimated employment and population gradients both declined between 1970 and 1980, suggesting an increasing degree of dispersion. To test whether these declines are statistically significant, we perform a test on the difference of density gradients between the two years. This is done by estimating

$$\Delta(\ln D_m) = \alpha - \beta r_m + e_m,$$

where $\Delta(\ln D_m) = \log(D_m^{1980}) - \log(D_m^{1970})$, and $\beta = b^{1980} - b^{1970}$ The null hypothesis is $\beta = 0$, the alternative hypothesis is $\beta < 0$, implying that the distribution became flatter over the decade

The estimated values of β (with standard deviation in parentheses) are $-0\ 0067$ (0 0008) for population and $-0\ 0027$ (0 0008) for employment. In both cases, the estimated density function became flatter over the decade and the null hypothesis of no change is rejected at a 0 001 significance level Hence, the monocentric model provides clear evidence of increasing dispersion of population and employment between 1970 and 1980

Has the Monocentric Approximation Become Less Suitable?

Table 2 also shows that the monocentric density functions fit better in 1970 than in 1980, based on the coefficients of determination (R^2) This finding implies that the monocentric approximation has become less suitable in explaining employment and population distributions, it also possibly indicates a transformation from a monocentric to a more polycentric pattern during the 1970s. These statements are consistent with the results of Gordon *et al.* [11], and also with Mills [25, pp 247–249] who states explicitly:

> As time passes the urban area grows, and centers of economic activity other than the city center become more important, with the result that distance from the city center explains less of the variability in the land values

We cannot test this result for statistical significance because it is likely that error terms for a given zone are correlated across years

Where is the Real Center?

In the discussion above, we have assumed that the Los Angeles CBD is the monocentric center. However, it has been suggested by Gordon *et al* [11] that Los Angeles Airport acts more like a center to the region than does the CBD To test this, we reestimated the monocentric density functions, centering them at each of the four largest centers shown in Table 1, one of which is Los Angeles Airport (Each set of four estimations was performed on a common set of observations, which excludes the four center-peak zones) We adopt the reasoning of Alperovich [1] that the point best described as the real center is that which produces the highest R^2 (i e , lowest sum of squared residuals)

TABLE 3
Adjusted Coefficients of Determination (R^2) with Alternative Monocenter Locations

Location of monocenter	1970 Employment	1970 Population	1980 Employment	1980 Population
Downtown L A	0 394	0 365	0 387	0 355
West L A	0 357	0 338	0 351	0 323
West Hollywood	0 346	0 328	0 341	0 314
L A Airport	0 339	0 341	0 351	0 353
No observations	1129	1114	1127	1120

Note Dependent variable is the natural logarithm of employment or population density (persons per square mile) The four regressions shown in each column are for a common data set consisting of the full 1135 zones, less the highest-density zone of each of the four alternative monocenters, less all zones with zero density

Table 3 presents the resulting adjusted coefficients of determination (R^2) The fit is best with downtown Los Angeles as the center, in both years and for both employment and population distributions The R^2 values however, do not differ very much across center locations To test whether the fits using the downtown L A center are significantly different from those using other centers, we use the likelihood-ratio test for nonnested hypotheses developed by Vuong [36] This test computes the difference in fitted log-likelihood values between two models and compares it to a theoretical distribution that Vuong derives [5] We find that models centered in each of the three alternative locations are soundly rejected, at a significance level ranging from 0 01 to 0 0001 for both employment and population distributions in both years. We conclude that downtown Los Angeles is the strongest center in the region, for the rest of this paper, "the monocentric model" means the one centered there

VI ESTIMATION RESULTS POLYCENTRIC MODEL

We estimate the polycentric density function (2) for population and employment, each for 1970 and 1980 In order to reduce collinearity among the variables r_{mn} for different n, we omit the smaller of any two centers closer than five miles to each other, this criterion eliminates the UCLA/Santa Monica center. which is too close to West L A This leaves six centers in 1970 and nine in 1980 Hypothesis tests for monocentricity are carried out based on estimates with these centers For the same reasons as discussed with the monocentric model, we omit the highest-density zone of each center in carrying out polycentric estimations; we do not omit zero-density zones, however, because our dependent variable is now D instead of $\ln(D)$

These estimates reveal that one center, Burbank Airport, has a negative intercept in the population equation, it also has a large gradient ($\hat{b}_n = 2\ 45$)

[5]Applying Vuong's Theorem 5 1 [36, p 318], the ratio of the maximized likelihoods for models f (assuming one center) and g (assuming a different center) is asymptotically normal with variance ω^2 estimated by his Eq (4 2), p 314,

$$\hat{\omega}^2 = \frac{1}{M}\sum_{m=1}^{M}[\log(f_m/g_m)]^2 - \left[\frac{1}{M}\sum_{m=1}^{M}\log(f_m/g_m)\right]^2,$$

where f_m and g_m are the values taken by the corresponding probability densities for observation m, evaluated in each case at the corresponding maximum-likelihood parameter estimate We are ignoring the extremely remote possibility that the gradients of both distributions could be zero, doing so enables us to avoid the more cumbersome test applying to overlapping models

TABLE 4
Polycentric Employment Density Functions Five Centers

Center location	1970 Intercept	1970 Gradient	1970 Impact (1000s)	1980 Intercept	1980 Gradient	1980 Impact (1000s)
Downtown L A	572760*	1 5702*	515 4*	242640*	1 1179*	496 6*
	(41 11)	(46 94)	(25 52)	(9 61)	(14 98)	(14 91)
West L.A	44930	1 6058	38 9	46131	1 2740	62 9*
	(0 19)	(0 47)	(0 61)	(0 48)	(1 02)	(1 73)
West Hollywood	8438*	0 1722*	787 9	11989*	0 2238*	668 0*
	(2 13)	(1 86)	(1 36)	(2 86)	(2 86)	(2 01)
L A Airport	3201*	0 0240*	4022 6*	4316*	0 0205*	6190 7*
	(2 28)	(1 95)	(3 21)	(4 55)	(3 19)	(6 73)
Long Beach	23495	0 7450	84 3	30236	0 9237	68 8
	(0 51)	(0 85)	(1 07)	(0 49)	(0 93)	(1 47)
		$R^2 = 0\ 757$			$R^2 = 0\ 535$	

Note Estimated by nonlinear least squares t values are in parentheses Dependent variable is employment density (persons per square mile) Five center-peak zones are omitted from the 1135 observations R^2 is adjusted for degrees of freedom
*Estimate is statistically significant at the 0 05 level, one-sided test

in the employment equation, indicating a very localized influence.[6] For these reasons, we show in Tables 4–7 estimates with the Burbank Airport center excluded To facilitate comparison between the two years, the tables also show results for 1980 estimated with only the 1970 centers, and vice versa [7] Employment results are shown in Tables 4 and 5, and population in Tables 6 and 7

First, consider the employment densities estimated for each year using just those centers meeting our definitional criteria for the same year (left side of Table 4 and right side of Table 5) Three of these centers—downtown L A , West Hollywood, and L A Airport—have statistically signifi-

[6]For $\hat{b}_n = 2\ 45$, the influence from center n falls off 91 4% for a one-mile increase in distance since for any r,

$$\frac{\hat{A}_n e^{-\hat{b}_n(r+1)}}{\hat{A}_n e^{-\hat{b}_n r}} = e^{-2\ 45} = 0\ 086$$

[7]We also tried the multiplicative polycentric density function suggested by McDonald and Prather [24] Although it yielded plausible estimates with expected signs and mostly significant coefficients, we do not report it because of its theoretical weaknesses noted in Section II

TABLE 5
Polycentric Employment Density Functions Eight Centers

Center location	1970 Intercept	1970 Gradient	1970 Impact (1000s)	1980 Intercept	1980 Gradient	1980 Impact (1000s)
Downtown L A	318930*	1 1643*	592 7*	242240*	1 1166*	497 2*
	(10 52)	(17 18)	(16 91)	(9 63)	(14 96)	(14 83)
West L A	6572*	0 1204	997 5	44110	1 2470	62 8*
	(2 23)	(1 22)	(0 78)	(0 49)	(1 02)	(1 73)
West Hollywood	92488	1 6338	61 8	11674*	0 2026*	794 7*
	(0 34)	(0 87)	(1 36)	(3 01)	(2 72)	(1 89)
L A Airport	1259	0 0342	1107 5	4011*	0 0224*	5251 2*
	(0 46)	(0 25)	(0 26)	(3 78)	(2 90)	(4 59)
Long Beach	19059	0 6036	106 7	25531	0 8051	77 7
	(0 65)	(0 96)	(1 01)	(0 58)	(1 01)	(1 37)
Downey	21210	1 8428	11 2	108540	2 2056	38 4
	(0 02)	(0 06)	(0 04)	(0 04)	(0 17)	(0 09)
Santa Ana	1064	0 0097	2444 1	4272	0 2325	251 8
	(0 44)	(0 43)	(0 43)	(0 78)	(0 84)	(0 73)
Van Nuys	9854	0 8904	27 6	20448	0 9294	51 7
	(0 12)	(0 25)	(0 36)	(0 27)	(0 58)	(0 88)
		$R^2 = 0\ 499$			$R^2 = 0\ 537$	

Notes Estimated by nonlinear least squares t values are in parentheses Dependent variable is employment density (persons per square mile) Eight center-peak zones are omitted from the 1135 observations R^2 is adjusted for degrees of freedom For 1970, R^2 is smaller than in Table 4 because of the three additional omitted observations

*Estimate is statistically significant at the 0 05 level, one-sided test

cant coefficients both for intercept and gradient This is modest but not overwhelming evidence that overall employment patterns are influenced by subcenters Perhaps economies of agglomeration are relatively weak, so that only a few centers act as attractors to other firms

Population densities, by contrast, are strongly influenced by employment centers; six of the eight 1980 centers (right side of Table 7) have significant intercepts and gradients This finding is especially important because none of the previously published results such as [11] base population density on employment centers It suggests that the standard assumption of urban economics—that commuting plays a key role in household location—does retain explanatory power in a polycentric urban structure

We can make this discussion more precise by testing the validity of the monocentric model as a special case of the polycentric one The formal

TABLE 6
Polycentric Population Density Functions Five Centers

Center location	1970 Intercept	1970 Gradient	1970 Impact (1000s)	1980 Intercept	1980 Gradient	1980 Impact (1000s)
Downtown L A	7433*	0 0662*	3675 3*	6807*	0 0808*	2532 0*
	(8 61)	(5 33)	(3 81)	(7 57)	(4 88)	(4 60)
West L A	3646*	0 1911	237 4	3405*	0 1802	247 8
	(1 84)	(1 28)	(0 70)	(1 73)	(1 20)	(0 65)
West Hollywood	5485*	0 3081	155 3	3731	0 2995	112 4
	(1 92)	(1 28)	(0 70)	(1 32)	(0 86)	(0 47)
L A Airport	6981*	0 0230*	9105 9*	7717*	0 0184*	12012 6*
	(6 87)	(9 05)	(7 08)	(7 76)	(8 91)	(11 56)
Long Beach	51834*	0 9280*	116 8*	58673	1 0116*	110 1*
	(1 81)	(3 38)	(5 33)	(1 59)	(3 15)	(5 45)
		$R^2 = 0\ 523$			$R^2 = 0\ 480$	

Note Estimated by nonlinear least squares *t* values are in parentheses Dependent variable is population density (persons per square mile) Five center-peak zones are omitted from the 1135 observations R^2 is adjusted for degrees of freedom

*Estimate is statistically significant at the 0 05 level, one-sided test

test for monocentricity is based on the statistic

$$F = \frac{(\mathrm{SSR}^{\mathrm{r}} - \mathrm{SSR}^{\mathrm{u}})/q}{\mathrm{SSR}^{\mathrm{u}}/(M-p)},$$

where $\mathrm{SSR}^{\mathrm{r}}$ and $\mathrm{SSR}^{\mathrm{u}}$ are the restricted (monocentric) and unrestricted (polycentric) sums of squared residuals, M is the sample size; p is the number of parameters being estimated in the unrestricted estimate, and q is the number of restrictions on these parameters in the restricted estimate Under the null hypothesis, this statistic is distributed according to a central F distribution with degrees of freedom $(q, M-p)$, as shown by Gallant [8, pp 78–79] for the nonlinear-least-squares estimator used here In our case, letting N^{u} and N^{r} be the unrestricted and restricted number of centers, the degrees of freedom are calculated from $M = 1135 - N^{\mathrm{u}}$, $p = 2N^{\mathrm{u}}$, and $q = 2(N^{\mathrm{u}} - N^{\mathrm{r}})$.[8] The test results, shown in Table 8, indicate that the monocentric model is soundly rejected in every case

[8]The restricted estimate restricts two parameters ($A_n = b_n = 0$) for each center other than downtown Los Angeles If only $A_n = 0$ were imposed, b_n would be unidentified and the moment matrix ($\mathbf{F}'\mathbf{F}$ in Gallant's notation) would be singular, thereby invalidating the test, see Gallant [8, p 75]

TABLE 7
Polycentric Population Density Functions Eight Centers

Center location	1970 Intercept	1970 Gradient	1970 Impact (1000s)	1980 Intercept	1980 Gradient	1980 Impact (1000s)
Downtown L A	6308*	0 0957*	1811 3*	6046*	0 1188*	1219 7
	(4 78)	(2 73)	(2 04)	(5 23)	(2 38)	(1 52)
West L A	5607*	0 1356	689 0	5944*	0 1233*	865 2
	(2 89)	(1 54)	(0 79)	(3 03)	(1 75)	(0 98)
West Hollywood	4797*	0 2854	160 4	2892	0 3056	83 4
	(1 82)	(1 10)	(0 60)	(0 99)	(0 65)	(0 36)
L A Airport	5933*	0 0218*	8097 2*	5844*	0 0173*	9503 9*
	(4 85)	(6 80)	(5 95)	(4 57)	(5 73)	(5 68)
Long Beach	53124*	0 9292*	119 3*	54791*	0 9803*	115 3*
	(1 85)	(3 46)	(5 42)	(1 71)	(3 29)	(5 48)
Downey	3630*	0 0718*	1718 2	3582*	0 0592*	2184 9
	(3 03)	(1 89)	(1 38)	(2 95)	(1 98)	(1 49)
Santa Ana	13023*	0 4796*	170 8*	7682*	0 2809*	312 1*
	(1 65)	(2 30)	(2 13)	(2 27)	(2 37)	(1 89)
Van Nuys	5802	0 3603	117 1	6712	0 3570	127 2
	(1 13)	(1 40)	(1 36)	(1 30)	(1 60)	(1 35)
		$R^2 = 0539$			$R^2 = 0498$	

Note Estimated by nonlinear least squares *t* values are in parentheses Dependent variable is population density (persons per square mile) Eight center-peak zones are omitted from the 1135 observations R^2 is adjusted for degrees of freedom

*Estimate is statistically significant at the 0 05 level, one-sided test

Examining changes from 1970 to 1980, Tables 4 and 5 suggest a gradual transformation toward a more polycentric employment structure The *t* statistics relating to the downtown L A center decreased, while nearly all the others increased No such clearcut result is apparent for population, whose polycentric pattern was already solidly established in 1970.

We also test the 1980 equations for the hypothesis that of the eight centers used in deriving Tables 5 and 7, only the five largest (which were also centers in 1970) affect employment and population distributions The resulting test, shown in the last two rows of Table 8, indicates that the five-center model is not rejected for employment, but is for population. Hence the three new centers that appeared in 1980 contribute only marginally to explaining the overall distribution of employment, but significantly to explaining the overall distribution of population More sur-

TABLE 8
Tests for Smaller Numbers of Centers

	Null hypothesis	Alternative hypothesis	Degrees of freedom $(q, M - p)$	Value of test statistic $F(q, M - p)$	Level of significance (α)
			1970		
Employment	$N^r = 1$	$N^u = 6^a$	10,1117	8 99	< 0 0001
Population	$N^r = 1$	$N^u = 6^a$	10,1117	15 76	< 0 0001
Employment	$N^r = 5$	$N^u = 8$	6,1111	0 05	≈ 1 00
Population	$N^r = 5$	$N^u = 8$	6,1111	6 19	< 0 0001
			1980		
Employment	$N^r = 1$	$N^u = 6^a$	10,1117	23 61	< 0 0001
Population	$N^r = 1$	$N^u = 6^a$	10,1117	13 20	< 0 0001
Employment	$N^r = 5$	$N^u = 8$	6,1111	0 32	≈ 0 925
Population	$N^r = 5$	$N^u = 8$	6,1111	4 47	< 0 0001

[a]The Burbank Airport center is included in these estimates

prisingly, the same is true for explaining 1970 densities (third and fourth rows)· population, but not employment, seems to *anticipate* the rise of the new centers This may indicate that employment is attracted to preexisting population concentrations rather than vice versa, an idea consistent with the overall development history of the more outlying parts of the Los Angeles area, which began as bedroom suburbs and later attracted employment [9, p 307].

Data on industry mix, however, indicate that the three new centers are far from identical in their functions, so may have been attracted to population concentrations for different reasons. Santa Ana is predominantly a service, retail, and administrative center, with those sectors accounting for 82% and 84% of employment in 1970 and 1980, respectively, hence it probably grew to serve consumer markets, in keeping with its location near the heavily developed bedroom suburbs characterizing northern Orange County in 1970 Downey, on the other hand, emerged as a manufacturing center (accounting for 80% and 74% of its employment in these two years), so it more likely developed in response to labor supply Van Nuys contained a more balanced industrial mix, but experienced rapid growth in manufacturing, transportation, communication, utilities, and wholesale industries, whose combined share of employment rose from 44% in 1970 to 60% in 1980; so again labor supply seems a likely explanation for the timing of its emergence as a center.

These results suggest that existing employment centers became stronger during the 1970s, growing both in size and in their influence on the overall pattern of employment throughout the area At the same time, new employment centers emerged at locations already marking population concentration, but had little immediate effect on the overall employment pattern

Tables 4–7 also show the estimated total impact that each center had on the overall distributions of employment and population, as computed using Eq (3) Approximate t statistics are shown in parentheses [9] There is a wide range of total impacts, indicating that some centers, such as Los Angeles Airport and downtown Los Angeles, are much more influential than others with respect to the overall distributions of employment and population

Plots of residuals suggest that the variance of the error term in Eq (2) is higher where the predicted density is large In order to improve the efficiency of our estimators in light of this heteroscedasticity, we reestimated all equations using the "estimated generalized least squares" method of Judge *et al* [20, pp. 437–439], in which the variance of density is assumed proportional to some power p of fitted density $\hat{D}$ Following Judge *et al* but adapting it to nonlinear least squares, we estimated p for each of the models reported in Tables 4–7 by regressing $\log(\hat{v}^2)$ against $\log(\hat{D})$, where $\hat{v}$ is the residual and $\hat{D}$ the predicted density. We then reestimated the nonlinear regressions after dividing both dependent and independent variables by $\hat{D}^{p/2}$

The results for population densities were little different from those already presented, so are not shown But the heteroscedasticity-corrected results for employment densities, shown in Tables 9 and 10, differ somewhat from the uncorrected results shown earlier. In the 1980 regressions with eight centers, the statistically significant centers change West Hollywood and L A Airport drop out, and West L A. and Santa Ana become strong in their influence on surrounding densities Santa Ana now has a detectable influence even in 1970, despite its being too small to meet our criteria for inclusion as a center in that year

[9] IMPACT_n is a nonlinear function of two random variables, $\hat{A}_n$ and $\hat{b}_n$ Knowing their estimated variances and covariances, we can compute an approximate variance for IMPACT_n from the formula

$$\sigma^2_{\text{IMPACT}} = d'\Sigma d,$$

where Σ is the variance–covariance matrix of (A_n, b_n) and d is the vector of derivatives of Eq (3) with respect to A_n and b_n See Chow [5, pp 182–183] or Bacon [2] As it happens, our estimates of $\hat{A}_n$ and $\hat{b}_n$ are positively correlated, since A_n increases impact and b_n decreases it, the total impact is sometime estimated with greater precision (greater t values) than either $\hat{A}_n$ or $\hat{b}_n$ This is especially true for the Los Angeles Airport center

TABLE 9
Polycentric Employment Density Functions Heteroscedasticity Corrected

Center location	1970		1980	
	Intercept	Gradient	Intercept	Gradient
Downtown L A	571630*	1 5885*	298650*	1 2704*
	(2 91)	(8 27)	(3 15)	(7 13)
West L A	61356	1 7571	70775	1 5599
	(0 23)	(0 65)	(0 32)	(0 84)
West Hollywood	8033*	0 1398*	10712*	0 1782*
	(4 00)	(4 44)	(3 30)	(4 23)
L A Airport	2251*	0 0158*	4088*	0 0203*
	(4 97)	(4 76)	(7 63)	(7 78)
Long Beach	20263	0 6386*	28778	0 8837
	(0 99)	(1 93)	(0 63)	(1 35)

Note Estimated by method of "estimated generalized least squares (see text) t values are in parentheses (not corrected for endogeneity of the estimated heterscedasticity pattern) Dependent variable is employment density (persons per square mile) Five center-peak zones are omitted from the 1135 observations

*Estimate is statistically significant at the 0 05 level, one-sided test

The influence of Los Angeles Airport is of special interest because Gordon *et al* [11] have suggested that it is a more important center than downtown Los Angeles Our uncorrected estimates show the airport center to have a larger impact than downtown, due to its very small estimated gradients (about two percent per mile) [10] Furthermore, according to those estimates the influence of downtown Los Angeles declined over the decade, although its impact remains important (second or third in rank),[11] while that of the airport center increased over the decade However, the L A Airport center seems to lose all its explanatory power

[10]We thought the importance of L A Airport might be due to its location on the ocean shore, but its large impact remains even when we add a variable to the population density equation measuring distance to the ocean Furthermore, the Long Beach center, also located at the shore, does not have an especially large influence

[11]Heikkila *et al* [18] and Richardson *et al* [32] examine housing prices, finding a similar decline in the influence of downtown L A from one of statistical significance in 1970 to one of insignificance in 1980 However, we interpret our results as indicating that downtown L A is still important in explaining population densities in 1980 its coefficients are both statistically significant, and the estimated t statistic on IMPACT is borderline, given the approximation involved and the appropriateness of a one-sided test Hence downtown Los Angeles remained a potent if somewhat diminished influence on population in 1980, and we must look to counteracting influences, such as amenities, to explain its weak effects on housing prices

TABLE 10
Polycentric Employment Density Functions Heteroscedasticity Corrected

Center location	1970		1980	
	Intercept	Gradient	Intercept	Gradient
Downtown L A	414830*	1 3741*	269080*	1 1934*
	(3 02)	(7 83)	(3 21)	(7 20)
West L A	6877*	0 0894*	8124*	0 0753*
	(5 84)	(4 86)	(6 19)	(3 99)
West Hollywood	101680	1 7446	53698	1 1561
	(0 36)	(1 04)	(0 62)	(1 26)
L A Airport	890	0 0861	1107	0 0032
	(1 02)	(0 52)	(1 20)	(0 31)
Long Beach	18021	0 5717*	11492	0 4156*
	(1 13)	(2 11)	(1 22)	(1 79)
Downey	66137	2 6126	160130	2 3525
	(0 03)	(0 12)	(0 08)	(0 35)
Santa Ana	1216*	0 0100*	5011*	0 1913*
	(5 57)	(3 07)	(2 06)	(2 29)
Van Nuys	22926	1 4277	42175	1 3193
	(0 15)	(0 46)	(0 24)	(0 72)

Note Estimated by method of "estimated generalized least squares" (see text) t values are in parentheses, they are not corrected for endogeneity of the estimated heterscedasticity pattern, so are probably overestimates The dependent variable is employment density (persons per square mile) Eight center-peak zones are omitted from the 1135 observations

*Estimate is statistically significant at the 0 05 level, one-sided test

on employment when the heterscedasticity correction is applied Therefore, we remain dubious about suggestions that downtown has been eclipsed by the airport in its influence

VII GINI COEFFICIENTS AS MEASURES OF DISPERSION

With the polycentric density functions, we cannot determine whether population and employment became more dispersed unless all estimates of density gradients changed in the same direction The results presented in Tables 4–10, however, suggest that gradients increased for some centers and decreased for others

In order to examine the change in overall dispersion, therefore, we compute Gini coefficients of the distributions The Gini coefficient measures the degree of deviation of an actual density distribution from a uniform distribution, in this case "uniform" meaning evenly dispersed over space The coefficient is defined as the fraction by which the area under the Lorenz curve exceeds what it would be if the Lorenz curve were a straight line In our case, the Lorenz curve is obtained by ordering all our zones from lowest to highest density, and then plotting cumulative geographic area against cumulative population or employment This calculation uses the raw data, not the estimated density functions A smaller Gini coefficient indicates a more uniform distribution, i e , more dispersion.

The results in Table 11 show that the Gini coefficients for both the population and the employment distributions decreased between 1970 and 1980 This reinforces our earlier conclusion, based on monocentric density functions, that both population and employment became more dispersed.

The Gini coefficient also can be used to compare dispersion in subregions such as individual counties To avoid a possible bias caused by including different amounts of low-density outlying land in different counties, we use a density cutoff, excluding zones with density lower than the cutoff value The Gini coefficient, in this case, measures the degree of dispersion for the *well-developed* areas in these counties Our density cutoff is 1 5 persons per acre for employment and 3 0 persons per acre for population, which is approximately the highest density of any of the zones that we had already deleted from our data set (see Section III)

Table 11 shows such modified Gini coefficients for the region's two largest counties In each case the Gini coefficient is smaller in 1980, suggesting that the distributions became more dispersed. this confirms the finding of Gordon *et al* [11, p 168, Table 8] for population, and extends it to employment We also note that in each year employment was more dispersed in Orange County than in Los Angeles County The same is true for population in 1980, but 1970 population is surprisingly *less* dispersed in Orange County than in Los Angeles County. This probably reflects the high 1970 concentration of Orange County's population in bedroom suburbs at the northwest end of the County (bordering L A. County), and the

TABLE 11
Gini Coefficients for Population and Employment Distributions

	Region		Los Angeles County[a]		Orange County[a]	
	1970	1980	1970	1980	1970	1980
Employment	0 774	0 745	0 502	0 468	0 399	0 331
Population	0 658	0 615	0 318	0 293	0 336	0 226

[a]In the single-county calculations for employment or population distribution, zones with densities below 1 5 employees per acre or 3 0 residents per acre, respectively, are excluded

subsequent dispersal of population toward the southeast as employment grew there during the 1970s

VIII CONCLUSIONS

We have estimated monocentric and polycentric density functions for both employment and population, using 1970 and 1980 small-zone data for the Los Angeles region. The findings are:

(1) Downtown Los Angeles is identified as the monocentric center giving the best statistical fit, for both employment and population distribution in both 1970 and 1980 In the polycentric employment estimations, downtown Los Angeles is the most statistically reliable center and has by far the largest intercepts In the polycentric population estimations, downtown Los Angeles has no such clearcut dominance

(2) The estimated monocentric density gradients suggest that both employment and population became more dispersed during the 1970s Lorenz curves relating cumulative employment or population to cumulative area confirm this.

(3) The fit of the monocentric density functions became somewhat worse in 1980, indicating that the monocentric approximation became less suitable

(4) Additive polycentric density functions using predefined employment centers are practical to estimate using nonlinear least squares and are statistically superior to monocentric density functions on these data. In all cases, the monocentric model is soundly rejected in favor of the polycentric model using an F test. The polycentric estimates seem reasonably precise and robust in the case of population, but less so in the case of employment

(5) In the case of employment distribution, there was some shift toward a more polycentric pattern during 1970s For population, the polycentric distribution was already pronounced in 1970 and persisted in 1980

(6) Finally, the newly emerged centers in 1980 contribute only marginally to explaining the overall distribution of employment, but they contribute very significantly to explaining the population distribution, even in 1970, suggesting that these employment centers may have emerged because of preexisting population concentrations

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