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On Separating Equilibria with Positive Excess Demand for a Monopolistic Market^{*}

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Abstract

This paper offers an explanation for the rationality of why successful businesses sometimes maintain excess demand. The explanation is motivated from the signaling role of excess demand for firms. Considering a monopolistic market for a good whose quality is not known to the consumers, a firm may use excess demand as a signal by strategically cutting back its supply. The paper establishes conditions with which there exists a separating equilibrium with positive excess demand. It also demonstrates that it is more profitable to signal quality by excess demand than by (traditional) advertising under certain conditions. This in turn shows that restraining supply is another way to advertise.

Keywords: Advertising, Queuing, Signaling, Perfect Bayesian Equilibrium. (JEL C72, L15.)

1 Introduction

Why don't some successful restaurants, plays, sporting events, and other activities raise prices even with persistent excess demand? When information is complete,

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the sales of a monopolist or a monopolistically competitive firm is not affected by small changes in price in the presence of excess demand if demand functions are continuous; and so, raising prices a little can increase its profit. In other words, in case of complete information the presence of excess demand and profit maximization are not compatible if demand functions are continuous. This paper is motivated by the question of how to rationalize such pricing behavior on the part of the firm.

In the literature, the problem has been analyzed with specially structured demand functions. Basu (1987) considers a monopolistic firm facing a market demand which he assumes to depend positively on excess demand. Since excess demand in turn depends on demand by each individual consumer as well as on the supply by the firm, the “effective” market demand the firm actually faces is obtained by assuming that the consumers all correctly expect the exact size of excess demand; the exact size of excess demand they expected when making their individual demands coincides with the actual size in the end.

Becker (1991) on the other hand includes market demand and the gap between market demand and market supply (i.e. the excess demand) as separate arguments in consumers’ demand function to explain why supply and price are not increased. He recognizes that the consumption of certain goods has some dimension of social activities. In that case people consume a product or service together, most possibly in public. This led Becker to assume that demand by a typical consumer depends positively on the aggregate demand.

Becker’s answer to why supply is not increased is that “market demand depends not only on price and market demand itself but also positively on the gap between market demand and supply ... greater supply might not pay because that lowers the gap and, hence, the optimal price available to a producer.” Becker (1991, p. 1115) then notes that “entering the gap into the demand function to explain why supply does not increase appears to be an ad hoc invention of a “good” to solve a puzzle.

In contrast, while Basu (1987) and Becker (1991) explain the pricing behavior as resulting from profit maximization with specially structured demand functions under complete information, we consider the pricing behavior as an outcome of profit maximization with usual demand functions under asymmetric information. In this respect, we consider a model in which a monopolistically competitive firm is endowed with one of the two technologies. One technology produces the good with high quality at a higher marginal cost and another produces the good with lower quality at a lower marginal cost. The firm knows its technology type but the consumers do not. As a business strategy, the firm may produce less than market demand to induce queuing that signals quality. Evidence of this business strategy

is abundant. For example, Phil Patton (1999) reported that Luxury brands such as Ferrari, Patek Phillipe, Montblanc sometimes place limited editions on the market. K-Paul's in New Orleans is famous for long queues of customers outside it.

We assume that the market for the high quality good is weak relative to the consumers' prior beliefs, in the sense that the high quality firm incurs a loss by operating in the absence of updating in the consumers' beliefs. This is the "lemon problem" as in Akerlof (1970) for a divisible good. Our paper establishes conditions for the existence of a separating equilibrium with positive excess demand.¹ Our paper also establishes conditions for when excess demand is a more profitable signal of quality than advertising. These results justify in part the use of excess demand as a signal of quality and provide an explanation for why a monopolistic firm sometimes maintain excess demand. In particular, the paper shows that the strategy to restraining supply is an alternative to advertisement.

The rest of the paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 consider queuing via excess demand and advertising as signals of quality, respectively. Section 5 compares queuing with advertising as signals of quality. All the proofs are provided in section 6.

2 Model

Consider a situation in which one good or service is produced and supplied by a monopolistically competitive firm. There are, however, two types of technology. The type h technology produces the good with high quality and the type l technology produces the good with low quality. Both types of technology are of constant returns to scale. Denote by c_t the constant per unit cost of production for technology t . Fixed cost is zero. Technology is predetermined. However, information is asymmetric; the firm knows its technology type but the customers do not.

Consumers are price-taking and equally informed. Their demand for the good depends on both the price and their beliefs about the quality of the good. We denote by $D(p, \lambda)$ the total quantity demanded of the good when the price is p and the probability with which the consumers all believe that the good is of high quality. For convenience, we write $D(p, 0)$ as $D_l(p)$ when consumers are completely informed that the good is of low quality; $D(p, 1)$ as $D_h(p)$ when they are completely informed that the good is of high quality. Prior beliefs are denoted by $\bar{\lambda}$, which

¹When excess demand is present, demand must be rationed at the price being charged and a queue is therefore generated. To simplify the discussion, we assume, as in Becker (1991), that the method used to ration demand is costless so that the money price is the full cost to the consumers.

is the probability with which consumers initially believe that the firm supplies the high quality good (i.e. the firm is endowed with technology h).

Fix excess demand $z \geq 0$. Consider the following profit maximization problem:

$$\max_{p \geq 0} [p - c_t][D_t(p) - z] \quad \text{subject to} \quad D_t(p) \geq z. \quad (1)$$

Denote the solution for (1) by $p_t(z)$. The price $p_t(z)$ is the complete information monopoly price given technology type t and excess demand z that the firm wishes to cut back its supply. We use the upper case $P_t(Q)$ to denote the inverse of $D_t(p)$ for $t = h, l$. An illustration of the price $p_t(z)$ is given in Figure 1 below.

— Figure 1 —

Given excess demand z , the quantity the firm sells are in excess of quantity z . It follows that the marginal revenue curve $MR_t(Q; z)$ from the inverse demand $P_t(Q)$ subject to excess demand z begins at quantity z as shown in Figure 1. Consequently, the maximum profit with excess demand z is given by the area of the rectangle with corners a, b, c, and d in Figure 1.

We denote by $\Pi_t(p) = [p - c_t]D_t(p)$ the type t firm's complete information profit function and by $\Pi_t^m(z) = [p_t(z) - c_t]D_t(p_t(z))$ its complete information monopoly profit with excess demand z . We consider the following properties for the demand function $D(p, \lambda)$:

A1: For $\lambda \in [0, 1]$, there exists a finite price $\bar{p}_\lambda > 0$ such that $D(p, \lambda) = 0$ for $p \geq \bar{p}_\lambda$. $D(p, \lambda)$ is continuous and differentiable at $(p, \lambda) \in (0, \bar{p}_\lambda) \times (0, 1)$ with $\partial D(p, \lambda)/\partial p < 0$ and $\partial D(p, \lambda)/\partial \lambda > 0$.

A2: $\Pi_h^m(0) > \Pi_l^m(0) > 0$.

A3: $c_h \geq \bar{p}_\lambda$.

Property A1 is standard. The condition that $\partial D(p, \lambda)/\partial \lambda > 0$ implies that given price p , the more likely the consumers believe that the good is of high quality, the more of the good they will demand. Property A2 makes the signaling problem nontrivial. It implies that when information is complete, the type h 's monopoly profit is higher than the type l 's monopoly profit. Since the type l 's production cost is lower than the type h 's, the type l firm would have an incentive to mislead the consumers into believing that it is of type h , provided it is not too costly to do so. Notice that with $\bar{\lambda} > 0$, A1 and A2 imply $0 = D(\bar{p}_0, 0) < D(\bar{p}_0, \bar{\lambda})$. It follows that

$\bar{p}_\lambda > \bar{p}_0 > p_l(0)$. Consequently, A3 together with A1 and A2 implies $c_h > p_l(0)$. The high-quality firm will be driven out of the market if there is no update in the consumers' prior beliefs. This resembles the lemon problem as in Akerlof (1970).

Example 1: Consumers' utility functions depend on consumption good (x), income (I), and the quality of the good. Their aggregate utility level at (x, I) with $x \geq 0$ and with the consumption good having quality t is given by

$$u_t(x, I) = \frac{a_t}{b_t}x - \frac{1}{2b_t}x^2 + I$$

where a_t and b_t are parameters. Assume total endowment of income is $\bar{I} > 0$ and that of the consumption good is 0. Assume further a_h, b_h, a_l, b_l , the prior belief $\bar{\lambda}$, and the marginal costs c_h, c_l are all positive such that ²

- (i) $a_h > a_l > 0, 0 < b_h < b_l$,
- (ii) $0 < c_h < \frac{a_h}{b_h}, 0 \leq c_l < \frac{a_l}{b_l}$,
- (iii) $\bar{\lambda}\frac{a_h}{b_h} + (1 - \bar{\lambda})\frac{a_l}{b_l} \leq c_h$,
- (iv) $b_l(a_h - b_h c_h)^2 > b_h(a_l - b_l c_l)^2$.

Given price p and probability λ with which the consumers believe that the good is of high quality, total demand for the good is derived by solving

$$\max_x \{ \lambda u_h(x, I) + (1 - \lambda)u_l(x, I) + \bar{I} - px \} \text{ subject to } x \geq 0.$$

Simple calculation shows that total demand for the consumption good is $D(p, \lambda) = A_\lambda - B_\lambda p$ when $p < A_\lambda/B_\lambda$ and $D(p, \lambda) = 0$ when $p \geq A_\lambda/B_\lambda$, where

$$A_\lambda = \frac{\lambda a_h b_l + (1 - \lambda) a_l b_h}{\lambda b_l + (1 - \lambda) b_h} > 0$$

and

$$B_\lambda = \frac{b_h b_l}{\lambda b_l + (1 - \lambda) b_h} > 0.$$

Notice that $\partial A_\lambda / \partial \lambda = (a_h - a_l) b_h b_l / (\lambda b_l + (1 - \lambda) b_h)^2$ and $\partial B_\lambda / \partial \lambda = -b_h b_l (b_l - b_h) / (\lambda b_l + (1 - \lambda) b_h)^2$. By (i), $\partial A_\lambda / \partial \lambda > 0$ and $\partial B_\lambda / \partial \lambda < 0$ which implies

²As a numerical example satisfying conditions (i)-(iv), set $a_h = 4, a_l = 2, b_h = 1, b_l = 2, c_h = 2, c_l = 0$, and $\bar{\lambda} \leq 1/3$.

$\partial D(p, \lambda)/\partial \lambda > 0$. Since $B_\lambda > 0$, $\partial D(p, \lambda)/\partial p = -B_\lambda < 0$. Notice also $D(p, \lambda) = 0$ when $p_\lambda = A_\lambda/B_\lambda$. This shows that the example satisfies A1 with

$$\bar{p}_\lambda = \bar{\lambda} \frac{a_h}{b_h} + (1 - \bar{\lambda}) \frac{a_l}{b_l}.$$

Next, when $\lambda = 0$, $A_0 = a_l$ and $B_0 = b_l$; when $\lambda = 1$, $A_1 = a_h$ and $B_1 = b_h$. It follows from the linearity in both $D_h(p)$ and $D_l(p)$ that $p_h(0) = (a_h + b_h c_h)/2b_h$ and $p_l(0) = (a_l + b_l c_l)/2b_l$. Thus, $D_h(p_h(0)) = (a_h - b_h c_h)/2$ and $D_l(p_l(0)) = (a_l - b_l c_l)/2$. It follows

$$\Pi_h^m(0) = \frac{(a_h - b_h c_h)^2}{4b_h} \quad \text{and} \quad \Pi_l^m(0) = \frac{(a_l - b_l c_l)^2}{4b_l}.$$

Finally, $A_{\bar{\lambda}}/B_{\bar{\lambda}} = \bar{\lambda} \frac{a_h}{b_h} + (1 - \bar{\lambda}) \frac{a_l}{b_l}$. By (ii) - (iv), the example also satisfies (A2) - (A3).

In the rest of the paper we consider both the effectiveness of queuing and advertising as signals of quality, and whether one dominates the other as a signal of high quality from the firm's perspective. We study the effectiveness of queuing via excess demand and advertising as signals of quality by establishing the existence of a separating *perfect Bayesian equilibrium* (henceforth PBE). We then compare the separating PBEs for the dominance of one signal over the other.

A PBE is characterized by two properties: (a) Strategies are optimal given beliefs and (b) Beliefs are obtained from strategies and observed actions via Bayes' rule. The reader is referred to Fudenberg and Tirole (1992) for a systematic introduction to the concept of a PBE.

3 Queuing as a Signal of Quality

Consider the possibility for the firm to signal its technology type (hence the quality of the good) by cutting back its supply, in the sense that the firm chooses to sell less than what is demanded for at the price the firm charges. We assume that the firm can commit to supply less by reducing its capacity to supply or to produce the good.

We consider a signaling game between the firm and consumers, in which the firm chooses an action in the form of a pair (p, z) with price p and excess demand z and subsequently the consumers observe the firm's action and decide how much of the good to buy. A strategy for the firm specifies an action for each type. Thus, a strategy for the firm can be represented by $((p_h, z_h), (p_l, z_l))$. A strategy $((p_h, z_h), (p_l, z_l))$

is *feasible* if

$$0 \leq z_t \leq D_t(p_t), \quad t = h, l.$$

The following lemma shows the existence of an excess demand for the type h firm, that makes it indifferent for the type l between misleading the consumers into believing that it is of type h by incurring that excess demand or revealing its true type by not incurring any excess demand.

Lemma 1: *Suppose the demand function satisfies (A1)-(A3). Then, there exists an excess demand $0 < z < D_h(p_h(z))$ such that*

$$[p_h(z) - c_l][D_h(p_h(z)) - z] = \Pi_l^m(0). \quad (2)$$

PROOF: See Section 6.

In a separating PBE, it must be that the type l firm chooses excess demand equal to zero. Indeed, we now show in Lemma 2 below that the larger the excess demand the firm chooses the smaller its complete information monopoly profit will be. Since the type h has no incentive to mimic type l 's action, choosing a positive excess demand yields less profit to type l firm than its complete information monopoly profit $\Pi_l^m(0)$ in separating PBE. It follows that type l must take action $(p_l^*, z_l^*) = (p_l(0), 0)$ in separating PBE.

Lemma 2: *Suppose the demand function satisfies (A1). Then for $t = h, l$, the monopoly profit*

$$\Pi_t^m(z) = [p_t(z) - c_t][D_t(p_t(z)) - z]$$

is decreasing in z whenever $\Pi_t^m(z)$ is well-defined at z .

PROOF: See Section 6.

Although choosing a positive excess demand does not convey information directly, but the consumers can observe a proxy of the excess demand, say as reflected by the corresponding queue. It is therefore possible to have an equilibrium in which the consumers rationally expect the firm with different technologies to choose different excess demands. That is, there could be an equilibrium in which type h and the type l choose different excess demands, thereby their types are completely revealed via choices of these excess demands.

Theorem 1: *Suppose the demand function satisfies A1-A3. Then there exists a separating PBE with the high quality type incurring positive excess demand.*

PROOF: See Section 6.³

Notice that multiple separating equilibria are possible. Indeed, given the assumptions, there may be multiple excess demands z^* that satisfy (7) in the proof of Theorem 1 in section 6. Any such excess demand is supportable as type h 's excess demand in a separating PBE. Thus it is sensible to consider the “least-cost” separating equilibria for type h firm. Figure 2 provides an illustration for a separating equilibrium.

— Figure 2 —

In a separating equilibrium with actions $(p_h(z^*), z^*)$ and $(p_l(0), 0)$, the type l enjoys its monopoly profit $\Pi_l^m(0)$ given by the area of the rectangle with corners G, H, c_l , and $p_l(0)$ in Figure 2. However, if it takes action $(p_h(z^*), z^*)$ instead, it will mislead the consumers into believing that it sells the high quality good, in which case it earns a profit equal to the area of the rectangle with corners A, B, E, and F which is smaller.

Assume the complete information demand function $D_h(p)$ for the high quality good is concave and differentiable. Then it can be derived from the first order condition for problem (1) that $p'_h(z) < 0$ and $D'_h(p_h(z))p'_h(z) < 1$ whenever $p_h(z)$ is well-defined. In this case, the derivative of $[p_h(z) - c_l][D_h(p_h(z)) - z]$ is negative. Consequently, the excess demand that satisfies (7) with equality is the smallest of those satisfying that condition. Since by Lemma 2 type h 's complete information monopoly profit $\Pi_h^m(z)$ is decreasing in z , we conclude that for type h firm, the least-cost separating equilibrium is the one whose associated excess demand z^* satisfies (7) with equality and yields

$$\Pi_h^m(z^*) = \Pi_l^m(0) - [c_h - c_l][D_h(p_h(z^*)) - z^*]. \quad (3)$$

³In contrast, Basu (1987) establishes equilibrium outcomes involving excess demand by directly assuming market demand depends on excess demand. He demonstrates that the dependence of market demand on excess demand may result in discontinuity in the effective market demand. This discontinuity leads to the existence of positive excess demand when the firm maximizes its profit.

Similarly, Becker (1991) establishes equilibrium outcomes involving excess demand by directly assuming that market demand depends on itself. He demonstrates that the “self dependence” of market demand may result in discontinuity, which leads to the existence of positive excess demand when the firm maximizes its profit.

to type h firm.

Thus in the least-cost separating equilibrium, type h 's profit is less than type l 's by exactly $[c_h - c_l][D_h(p_h(z^*)) - z^*]$ amount. This result is not as counter intuitive as it may appear to be. Recall that the situation is that though the type h firm does make more profit than the type l when information is complete, consumers' prior beliefs about the quality being high are not strong enough so that if consumers do not update their prior beliefs, type h firm will have to exit the market with a profit of 0, while the type l firm can profitably stay. This is the lemon problem as discussed in Akerlof (1970). The information asymmetry puts the type h in a rather disadvantageous position. Given that the technology is prefixed, it is worth doing for the type h if positive profit can be made after incurring a cost for signaling its type. Of course, in case the consumers purchase the good repeatedly, type h would be more profitable than type l whenever it is more profitable when information is complete and the number of repetition is large enough.

4 Advertising as a Signal of Quality

Sellers of high-quality products want to convey that fact to consumers, but of course any seller can make such a claim. As argued in Nelson (1974, 1975), advertising is a signal. Even though advertising may convey no information directly, but consumers can observe the total amount of money or a proxy of it that the firm is spending on advertising. It follows that as with queuing it is possible to have an equilibrium in which the consumers rationally expect the firm with different technologies to spend different amounts on advertising. That is, there could be an equilibrium in which the firm can signal its type via advertising.

An action for a firm in this case is a pair (p, E) with a price p and an advertising expenditure E . That is, an action for the firm now is a simultaneous choice of how much to spend on advertising and how much to charge the consumers to pay. A strategy for the firm now specifies what action it will take for each type. Thus, a generic strategy can be written as $((p_h, E_h), (p_l, E_l))$, where (p_t, E_t) is the action the firm will take if its type is $t = h, l$. Advertising is dissipative in the sense that it is only a signal that the firm is able to spend a lot of money, such as sponsoring sporting events. Thus, in a separating PBE, the type l firm's action must be $(p_l(0), 0)$ as in the case with queuing. We now have:

Theorem 2: *Suppose that the demand function satisfies A2-A3 and*

$$A4: D_h(p_h(0)) < \Pi_l^m(0)/(c_h - c_l).$$

Then, there exists a separating PBE with the high quality type advertising.

PROOF: See Section 6.

Notice that as with queuing, multiple separating equilibria with advertising are possible. Indeed, the assumptions guarantee that expenditures satisfying

$$[p_h(0) - c_l]D_h(p_h(0)) - \Pi_l^m(0) \leq E < \Pi_h^m(0)$$

exist and any such expenditure yields a separating equilibrium.⁴ The higher the expenditure, the lower the type h firm's profit will be. Thus, it is sensible to consider the “least-cost” separating equilibrium with advertising. The least-cost separating equilibrium with advertising is the one with expenditure E where

$$E = [p_h(0) - c_l]D_h(p_h(0)) - \Pi_l^m(0). \quad (4)$$

In a separating equilibrium with prices $p_h(0)$, $p_l(0)$, and expenditure E , type h 's profit is $\Pi_h^m(0) - E$ and that of type l is $\Pi_l^m(0)$. By (4), in the least-cost separating equilibrium,

$$\Pi_h^m(0) - E = \Pi_l^m(0) - [c_h - c_l]D_h(p_h(0)). \quad (5)$$

Thus, in the least-cost separating equilibrium with advertising, type h 's profit is less than type l 's with the difference in profits being $[c_h - c_l]D_h(p_h(0))$ instead of $[c_h - c_l][D_h(p_h(z^*)) - z^*]$ as in the least-cost separating equilibrium with queuing. The reason that type h makes less than type l does in a separating equilibrium can be similarly explained as with queuing.

5 Queuing versus Advertising as a Signal of Quality

Equations (3) and (5) together make it possible to specify conditions with which it is more profitable for the type h to use queuing to signal its type. Indeed, with A1-A4 it follows from (3) and (5) that queuing is more profitable for type h than advertising if and only if

$$[c_h - c_l][D_h(p_h(z^*)) - z^*] < [c_h - c_l]D_h(p_h(0)),$$

where z^* satisfies (7) with equality. Since $c_h > c_l$, the above condition is satisfied when

⁴See the derivation for the consistency of (11) in the proof of Theorem 2.

A5: $D_h(p_h(z)) < D_h(p_h(0)) + z$ whenever $p_h(z) > c_h$.

Figure 3 below provides a graphical illustration of A5.

— Figure 3 —

Given excess demand z , $P_h^z(Q) = P_h(Q - z)$ is the inverse demand function which is obtained by shifting $P_h(Q)$ to the right in parallel fashion by the amount of z . The marginal revenue curve $MR_h^z(Q)$ from inverse demand $P_h^z(Q)$ meets marginal cost c_h at quantity $D_h(p_h(0)) + z$. On the other hand, the marginal revenue curve $MR(Q; z)$ from the inverse demand $P_h(Q)$ subject to excess demand z meets marginal cost curve c_h at quantity $D_h(p_h(z)) < D_h(p_h(0)) + z$.

The comparison result shows that the signal the high-quality firm will adopt depends on the nature of the market or the demand condition. In some markets, queuing is a more profitable signal. It may appear that a firm could set a higher price, however, is charging a lower price. This seemingly loss in revenue due to queuing can be viewed as a “shadow” (*de facto*) cost of advertising in the pursuit of profit maximization. It is all about providing signals about the quality of the good. In a town or a particular section of a town with a large floating tourist population, such as Manhattan in New York City or the Latin Quarter in New Orleans, the queuing strategy may be more effective for high quality restaurants since customers are getting the signals right at the time of having a meal than in a town where most customers are known to the restaurants.

6 Proofs of Results

Proof of Lemma 1: Define $\phi : [0, D_h(c_h)] \longrightarrow \Re$ by

$$\phi(z) = [p_h(z) - c_l][D_h(p_h(z)) - z] - \Pi_l^m(0).$$

Since D_h is continuous on $[0, \bar{p}_1]$, the correspondence $F : [0, D_h(c_h)] \longrightarrow [0, \bar{p}_1]$ with $F(z) = \{p \in [0, \bar{p}_1] \mid D_h(p) \geq z\}$ is continuous.⁵ Hence by A1 and by the Maximum Theorem, as a solution of problem (1) $p_h(z)$ is continuous over $[0, D_h(c_h)]$. It follows that ϕ is also continuous over $[0, D_h(c_h)]$. By A2, $\phi(0) > 0$. Since $\Pi_l^m(0) > 0$, we also have $\phi(z) < 0$ for z close to $D_h(c_h)$. Hence, by the Intermediate Value Theorem,

⁵Recall that the complete information demand D_h for the high-quality good correspond to our general specification of the demand function $D(p, \lambda)$ with $\lambda = 1$.

there is $0 < \bar{z} < D_h(c_h)$ such that $\phi(\bar{z}) = 0$ or equivalently, $\bar{z}$ satisfies (2). Since $p_h(\bar{z}) \geq c_h$, $D_h(p_h(\bar{z})) \geq \bar{z}$, and since $\Pi_l^m(0) > 0$, it must be $0 < \bar{z} < D_h(p_h(\bar{z}))$ to satisfies (2). $\blacksquare$

Proof of Lemma 2: By A1 and by the Implicit Function Theorem, $\Pi_t^m(z)$ is differentiable at z whenever $\Pi_t^m(z)$ is well-defined at z . At such excess demand,

$$\begin{aligned} \frac{d\Pi_t^m(z)}{dz} &= p'_t(z)[D_t(p_t(z)) - z] + [p_t(z) - c_t][D'_t(p_t(z))p'_t(z) - 1] \\ &= p'_t(z)[D_t(p_t(z)) - z] + [p_t(z) - c_t]D'_t(p_h(z))p'_t(z) - [p_t(z) - c_t]. \end{aligned} \quad (6)$$

From the first order condition for problem (1) it follows $[p_t(z) - c_t]D'_t(p_t(z)) = -[D_t(p_t(z)) - z]$. This equality together with equation (6) implies $d\Pi_t^m(z)/dz = -[p_t(z) - c_t] < 0$. $\blacksquare$

Proof of Theorem 1: Consider actions $(p_h^*, z^*) = (p_h(z^*), z^*)$, $(p_l^*, z_l^*) = (p_l(0), 0)$, and beliefs $\mu^*(t|p, z)$, where $z^* \leq D_h(p_h(z^*))$,

$$[p_h(z^*) - c_l][D_h(p_h(z^*)) - z^*] \leq \Pi_l^m(0), \quad (7)$$

and

$$\mu^*(h|p, z) = \begin{cases} 1 & \text{if } z \geq z^* \text{ and } p \geq p_h(z^*), \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

By Lemma 1, (7) is consistent. Given beliefs in (8), type h firm's profit with action $(p_h(z^*), z^*)$ is $\Pi_h^m(z^*) > 0$. As argued before, A3 implies $c_h > p_l(0)$. It follows that it does not pay for the type h firm to take any action that will mislead the consumers into believing that it is of type l , because these actions all make it profit equal to 0. By (8), they are the actions (p, z) with either $p_h < p_h(z^*)$ or $z < z^*$. On the other hand, the type h firm's profit at actions (p, z) with $p \geq p_h(z^*)$ and $z^* \leq z \leq D_h(p^*)$ will be $[p - c_h][D_h(p) - z] \leq \Pi_h^m(z)$. By Lemma 2, $\Pi_h^m(z)$ is decreasing in z , and hence type h firm cannot make more profit than $\Pi_h^m(z^*)$ by deviating from action $(p_h(z^*), z^*)$ to any action (p, z) with $p \geq p_h(z^*)$ and $z^* \leq z \leq D_h(p)$.

Since action $(p_l(0), 0)$ already yields the complete information monopoly profit $\Pi_l^m(0)$ to the type l firm, it follows from beliefs in (8) that it suffices to show that type l does not have any incentive to deviate to actions that will mislead the consumers into believing that it is of type h . These are actions (p, z) with both $p \geq p_h(z^*)$ and $z \geq z^*$. To check the profitability of such actions, notice first

$$[p - c_l][D_h(p) - z] = [p - c_h][D_h(p) - z] + [c_h - c_l][D_h(p) - z]. \quad (9)$$

Since $c_h > c_l$, increasing either p or z or increasing both decreases the second term on the right-hand-side of equation (9). On the other hand, as shown in the previous paragraph, with beliefs in (8) no actions (p, z) with $p \geq p_h(z^*)$ and $z \geq z^*$ can yield profits bigger than $\Pi_h^m(z^*) = [p_h(z^*) - c_h][D_h(p_h(z^*)) - z^*]$. Thus, by (9), the highest profit type l can get by actions that will mislead the consumers into believing that it is of type h is $[p_h(z^*) - c_l][D_h(p_h(z^*)) - z^*]$. However, since z^* satisfies (7), this profit is almost equal to type l 's complete information monopoly profit $\Pi_l^m(0)$. This shows that type l firm does not have any incentive to deviate from its action $(p_l^*, z_l^*) = (p_l(0), 0)$ to any action (p, z) with $p \geq p_h(z^*)$ and $z^* \leq z \leq D_h(p)$.

In summary, we have shown that neither type have any incentive to change their actions given beliefs in (8). On the other hand, beliefs in (8) are updated via the Bayes rule from the firm's strategy and the observed action. This concludes that $((p_h^*, z_h^*), (p_l^*, z_l^*), \mu^*)$ is a separating PBE with excess demand z^* . ■

Proof of Theorem 2: Notice first that A4 implies

$$[p_h(0) - c_l]D_h(p_h(0)) - \Pi_l^m(0) < \Pi_h^m(0). \quad (10)$$

Since $c_l < c_h$, by A2 the left-hand-side of (10) is positive. Now consider actions (p_h^*, E_h^*) and (p_l^*, E_l^*) , where $p_l^* = p_l(0)$, $p_h^* = p_h(0)$, $E_l^* = 0$, and $E_h^* = E^*$ satisfies

$$[p_h(0) - c_l]D_h(p_h(0)) - \Pi_l^m(0) \leq E^* < \Pi_h^m(0). \quad (11)$$

By (10), (11) is consistent and consequently actions (p_h^*, E_h^*) and (p_l^*, E_l^*) are well-defined. Consider next consumers' updated beliefs $\mu^*(\cdot | (p, E))$, where

$$\mu^*(h | p, E) = \begin{cases} 1 & \text{if } E \geq E^*, p \geq p_h(0), \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Given beliefs in (12), type l 's profit with (p_l^*, E_l^*) is $\Pi_l^m(0)$ whereas its profit from action (p, E) with either $E < E^*$ or $p < p_h(0)$ is $[p - c_l]D_l(p) - E$. Since $\Pi_l^m(0)$ is type l 's complete information monopoly profit, we have $\Pi_l^m(0) \geq [p - c_l]D_l(p) - E$. This shows that the type l firm does not have any incentive to deviate from action $(p_l(0), 0)$ to any action (p, E) with $E < E^*$ or $p < p_h(0)$.

Now consider actions (p, E) with $E \geq E^*$ and $p \geq p_h(0)$. From the beliefs in (12), such an action would lead the consumers to believe that the type l sells the high-quality good. Thus, the type l 's profit at such an action will be $[p - c_l]D_h(p) - E$. Notice that $[p - c_l]D_h(p) = \Pi_h(p) + [c_h - c_l]D_h(p)$. Since $\Pi_h(p) \leq \Pi_h^m(0)$ and since $p \geq p_h(0)$ implies $[c_h - c_l]D_h(p)$ is decreasing in p , it follows that

$$[p - c_l]D_h(p) - E \leq [p_h(0) - c_l]D_h(p_h(0)) - E. \quad (13)$$

Together with $E \geq E^*$, (11) and (13) imply $[p - c_l]D_h(p) - E \leq \Pi_l^m(0)$ at any action (p, E) with $p \geq p_h(0)$ and $E \geq E^*$. This shows that the type l firm does not have any incentive to deviate from action $(p_l(0), 0)$ to any action (p, E) with $p \geq p_h(0)$ and $E \geq E^*$.

For the type h firm, its profit at action $(p_h(0), E^*)$ and beliefs μ^* in (12) is $\Pi_h^m(0) - E^*$. Actions (p, E) with $p \geq p_h(0)$ and $E \geq E^*$ would enable the type h firm to sell quantity $D_h(p)$. Hence, its profit at such an action will be $\Pi_h(p) - E$. Since $\Pi_h(p) \leq \Pi_h^m(0)$ and $E \geq E^*$, we have $\Pi_h(p) - E \leq \Pi_h^m(0) - E^*$. This shows that the type h firm does not have any incentive to deviate from action $(p_h(0), E^*)$ to (p, E) with $p \geq p_h(0)$, $E \geq E^*$, and $(p, E) \neq (p_h(0), E^*)$. Now consider actions (p, E) with either $p < p_h(0)$ or $E < E^*$. In either case, beliefs in (12) would lead the consumers to believe the type h sells the low-quality good. Correspondingly, the type h firm's profit from such an action will be $[p - c_h]D_l(p) - E$. By A1 and A3, $D_l(c_h) = 0$. we have $[p - c_h]D_l(p) - E \leq -E$. Since $\Pi_h^m(0) - E^* > 0$, we conclude that the type h firm does not have any incentive to deviate from action $(p_h(0), E^*)$ to action (p, E) with either $p < p_h(0)$ or $E < E^*$.

To summarize, we have shown that neither type have any incentive to change their actions given beliefs in (12). On the other hand, beliefs in (12) are updated via the Bayes rule from the firm's strategy and the observed action. We can therefore conclude that the actions (p_h^*, E_h^*) , (p^*, E_l^*) , and the belief system μ^* together form a separating PBE. ■

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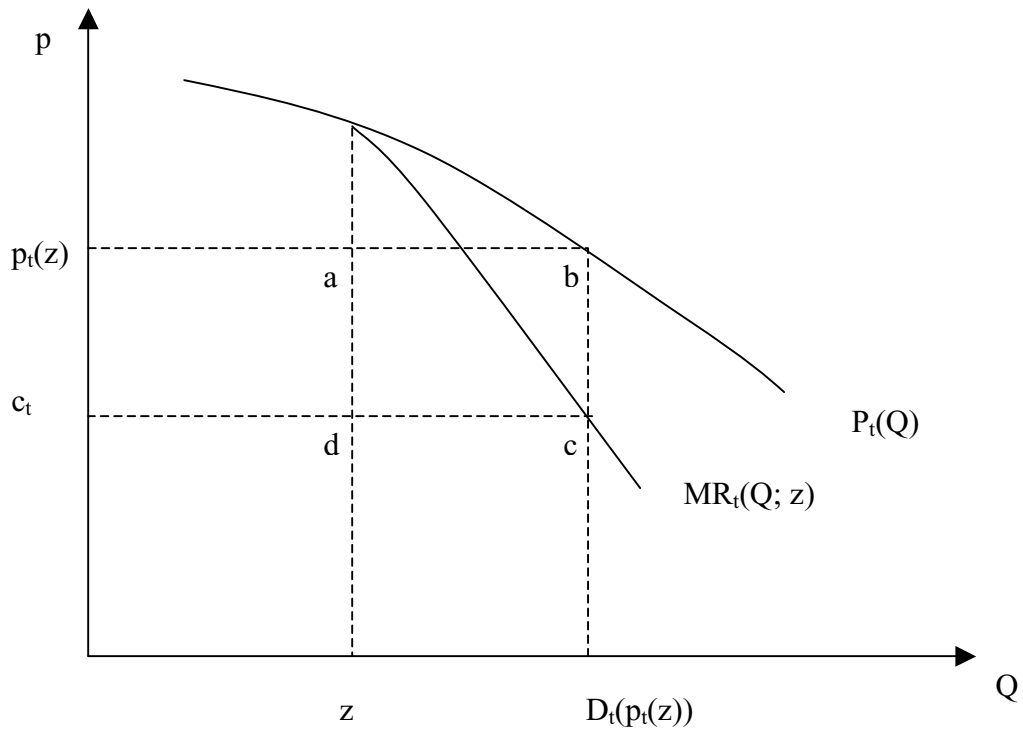


Figure 1: $MR_t(Q; z)$ is the Marginal Revenue Curve from the Inverse Demand $P_t(Q)$ subject to Excess Demand z . Type t 's Profit-Maximizing Price $p_t(z)$ given Excess Demand z Yielding Profit Equal to the Area of Rectangle with Corners a , b , c , and d .

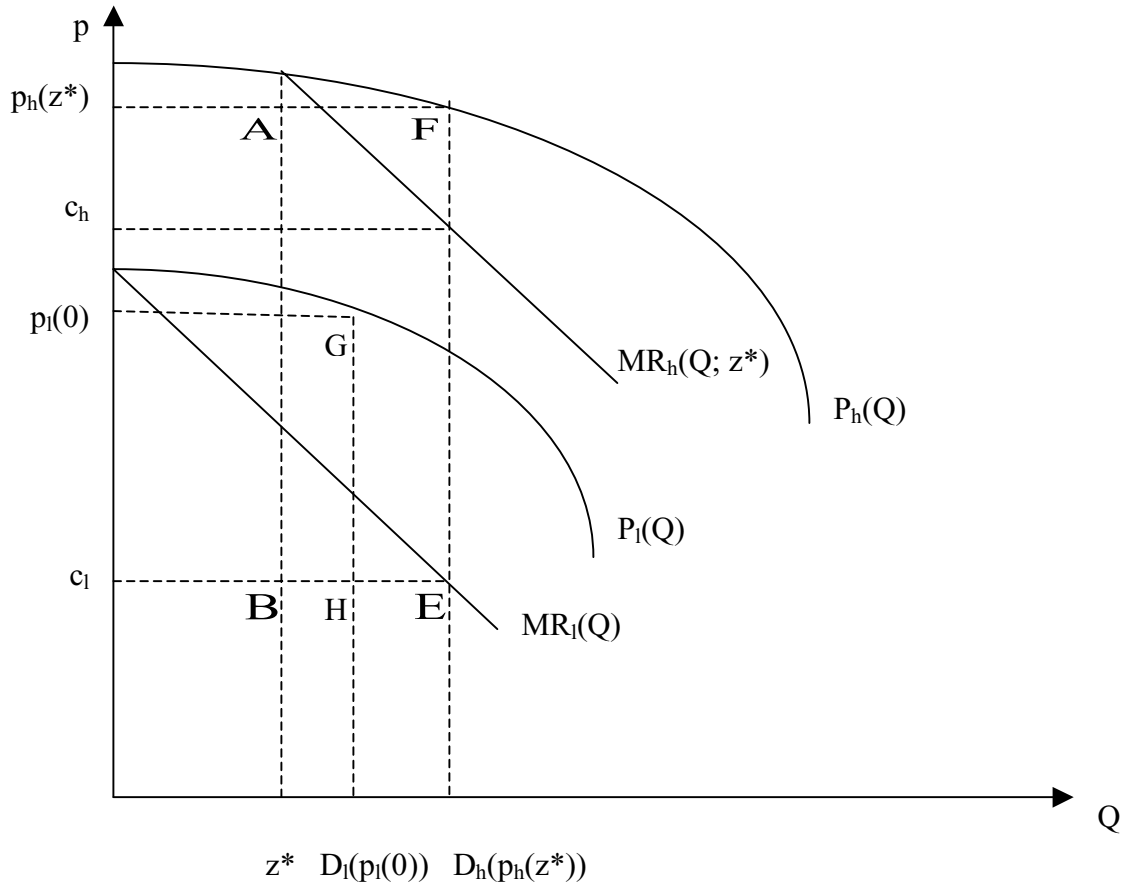


Figure 2: $MR_h(Q; z^*)$ is the Marginal Revenue Curve from the Inverse Demand $P_h(Q)$ subject to Excess Demand z^* . Type I's Profit for Mimicking Type h's Strategy $(z^*, p_h(z^*))$ Equals the Area of the Rectangle with Corners A, B, E, and F; Its Monopoly Profit Equals the Area of the Rectangle with Corners $p_l(0)$, G, H, and c_l in a Separating Equilibrium.

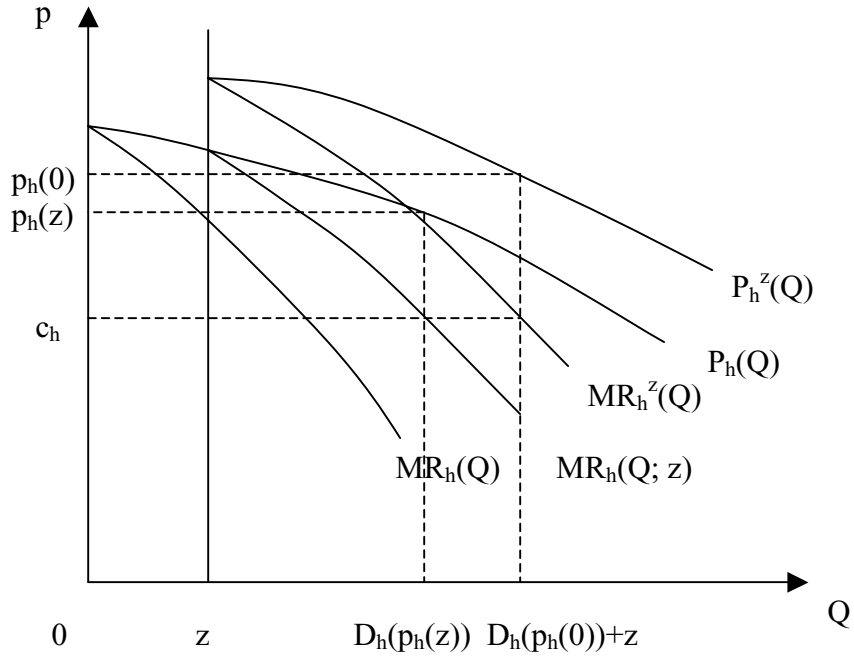


Figure 3: An Illustration of a Demand Curve for Type h Satisfying A5. $P_h^z(Q)$ Is Obtained by a Parallel Shift of $P_h(Q)$ to the Right by the Amount of z so that $P_h^z(Q) = P_h(Q - z)$. $MR_h^z(Q)$ Is the Marginal Revenue Curve from $P_h^z(Q)$. $MR_h(Q; z)$ is the Marginal Revenue Curve from $P_h(Q)$ subject to Excess Demand z .