Title
MEASURING THE INEQUALITY NATURE OF EUROPEAN MICRO INCOME DATA

Authors
Villas-Boas, Sofia B
Fu, Jenny
Judge, George

Publication Date
2016-12-21

MEASURING THE INEQUALITY NATURE OF EUROPEAN MICRO INCOME DATA

SOFIA B. VILLAS-BOAS, QIUZI FU, AND GEORGE JUDGE

December 21, y

Abstract: In this paper we use an information theoretic behavioral approach to recover country-based income probability density-distribution functions from samples of time ordered European micro household income data. To recover the country based income probability density functions and to order and quantify income inequality, we use a member the Cressie-Read flexible family of entropy based information divergence measures. A maximum entropy based measure, is used to order and group the country based probability density functions-income distributions. Finally we investigate the heterogeneity and dynamics of European income inequality.

JEL: D31, E21, C1, C10, C24

Keywords: Income inequality, Micro Income Data, Probability Distribution Function, Information theoretic methods, Cressie-Read Divergence, Entropy Maximization.

Corresponding author: Sofia Villas-Boas, Professor of ARE, University of California, Berkeley, CA, 94720 (sbert@berkeley.edu), Qiuzi Fu, National School of Development, Peking University, Beijing, China, 100871 (jennyfuberkeley@gmail.com), George G. Judge, Professor of the Graduate School and Giannini Fondation, 207 Giannini Hall, University of California Berkeley, Berkeley, CA, 94720 (e-mail:gjudge@berkeley.edu).
We thank Emma Aisbett, S. Gold, Bill Griffiths, Emmanuel Saez, T. Squartini, Itai Trilnick, Sammy Gold, Andrew Stevens, and Kenneth Train for comments. We thank the Giannini Foundation for support.

I. INTRODUCTION

In this paper we use an information theoretic behavioral approach to recover country-based income probability density-distribution functions from samples of time ordered European micro household income data. Country based income density-distributions functions evolve from complex, uncertain, and volatile economic behavioral systems that are seldom in equilibrium. In economics, where everything is connected to everything else, these connections and the resulting networks turn a complicated behavioral income system into a complex one. This means that economic behavioral system outcomes may no longer be analyzed in terms of averages and must be viewed in a probability context. Thus at an economic unit-country level the probability density function-distribution of income is significant because it contains information on how the market is functioning, how the allocation and distribution system is performing, and in terms of dynamics, how the economic system has and is changing over time. Therefore, if we can recover a country's probability density-income distribution function, we have the groundwork for drawing inferences as to how an economy is functioning and a basis for providing a precise definition of equality-inequality along with a unique choice of metric-measure. In this paper, we obtain this type of information using an Informational Theoretic entropy based econometric method and annual samples of European micro income data as a basis for estimating, ordering and determining the informational content of countrywide income probability density functions-distributions. A unique criterion related entropy measure of income equality-inequality is provided for each probability density-distribution-country.

The study of income distributions has a long history. Pareto (1896) originally used the income distribution to describe the allocation of wealth among individuals and demonstrate that a larger portion of the wealth of any society is owned by a small percentage of the people. Pareto's description of the nature of the income distribution is sometimes expressed more simply as the Pareto fat tail principle or the "80-20 rule", which says that 20% of the population controls 80% of the wealth. Many possible measures, such as the Gini concentration ratio, the Lorenz curve (see for example Theil, 1967 and Adamou and Peters, 2016) and the exponential distribution (Drăgulescu, and Yakovenko, 2001 and Cho, 2014), have been proposed to measure income equality-inequality and reflect the nature of the distribution of income. In addition the dynamic evolution of income inequality (for a survey see Piketty and Saez, 2014) has been the focus of a growing empirical literature in the context of developed countries (for example, in the US, see Saez and Zucman, 2016; Piketty and Saez, 2003; and Alvaredo et al, 2013; and in Europe, see Roine and Waldenstroem, 2015).

Building on the productive efforts noted above, we contribute to this literature by presenting a new uniform entropy-based behavior-related method that simultaneously recovers country based income probability density-distribution functions and the corresponding income equality-inequality measure, from samples of micro income data. We extend this literature by recovering country based probability density-income distribution functions that serves as a basis for inference and a measure of income equality-inequality. Traditionally, there is no commonly accepted definition of inequality and the basis for determining the underlying income distribution is unrelated to the equality measure. Thus, there are different interpretations of the inequality concept as well as existing questions about the underlying dynamics of income inequality and the way in which income distributions and income equality-inequality changes over time. The Eurostat data series, of about two million household-country-year observations, permits us to pursue some of these questions and to use a new conceptual framework and information recovery method to study income distributions and income inequality in a European country based context.

In the sections to follow, using some of the tools associated with information theory and statistical physics, we argue in section II that the probability distribution of income is given by the possible ways in which a collection of non interacting micro incomes may occupy a set of discrete outcome states. Consistent with the data generation process, we use an information theoretic entropy based method (see Judge and Mittelhammer, 2012) and micro sample data to recover the unknown probability density-distribution function. Using European country level micro income data described in Section III, in section IV we investigate whether economic behavior and the nature of income equality-inequality is captured by this entropy based distribution and inequality measure. In this information recovery process, we recognize the connection between adaptive intelligent behavior, causal entropy maximization, and self-organized equilibrium seeking behavior in an open economic behavioral system and use an information theoretic method to determine the nature of the distribution of European country based micro income data for the sample of years 2008 to 2013. In Section V we investigate the dynamics of income inequality for countries featuring the longest available income series. Finally, in section VI we discuss the implications of the methods and the sample results.

II. AN INFORMATION RECOVERY FRAMEWORK

As we focus on the recovery of the underlying probability density functions (PDFs) from micro income data, we recognize and emphasize the behavior related nature of the observations. This means we recognize that like prices, incomes do not behave, but that people behave. Thus an income distribution is one way to exhibit and summarize economic behavior and performance. In 1948, Shannon looked at a communication system and saw information in the context of a dynamic entropic system. In a similar way, recognizing the potential use of information theory in economics leads us to the question: does the distribution of samples of the micro income data provide a basis for recovering information regarding the unknown parameters of a micro behavioral income distribution system? In seeking an answer to this question we follow Wissner-Gross and Freer (2013) and recognize the connection between adaptive intelligent behavior, causal entropy maximization (AIB-CEM), and self-organized equilibrium seeking behavior in an open dynamic economic system. Under this optimizing criterion each microstate can be seen as a causal consequence of the macro income state to which it belongs. This connection between causal adaptive behavior and entropy maximization, which is based on a causal generalization of entropic forces, suggests that economic social systems do not evolve in a deterministic or a random way, but tend to adapt behavior in line with an optimizing principle. In the sections ahead, we exhibit new ways to think about income information recovery and the causal adaptive behavior of large complex micro economic systems, and use entropy as the systems status equality-inequality measure.

II.A. Problem Formulation and Solution

In this section we discuss how an adaptive intelligent behavior causal entropy maximization framework and information theoretic methods may be used to establish a data based link to recover income probability density functions-distributions. It seems reasonable that the resulting distribution of a sequence of positive real numbers from a sample of income data should vary over countries and economic systems. In this context information theoretic methods offer a natural way to capture the inequality nature of income distributions in the form of a probability density function (PDF).

In recovering the income probability distribution-density function from a sample of N positive real numbers, we assume the income probability to be represented by partitions-boxplots–shares that span the income sample space. Samples from these box plots yield histogram outcomes of the discrete random income variable d_j, for $j=1,2,\ldots,n$, and under repeated observation, one of n digits is observed with probability p_j. Further suppose after N trials, we have first-moment sample information in the form of the mean value of a country's income:

$$\sum_{j=1}^{n} d_j p_j = \bar{d}. \tag{1}$$

Given this first-moment sample nformation and the inverse problem of identifying an income distribution from the sample income data, we seek the best predictions of the unknown probabilities $p_1, p_2, \ldots, p_n$. It is readily apparent that there is one data point $\bar{d}$, and n unknown p_i. From an information recovery standpoint there are an infinite number of possible discrete probability distributions with $\bar{d} \in [1,n]$. Based only on the *information* $\sum_{j=1}^{n} d_j p_j = \bar{d}$, $\sum_{j=1}^{n} p_j = 1$, and $0 \le p_j \le 1$, the problem cannot be solved for a unique solution. Thus, a function must be inferred from insufficient information when only a feasible set of solutions is specified. In such a situation it is useful to have an approach that allows the investigator to use sample based information recovery methods without having to choose a parametric family of probability densities on which to base the income function. This is the topic to which we now turn.

II.B. The Information Theoretic Family

Making use of the connection between adaptive intelligent behavior and causal entropy maximization as an optimizing criterion-status measure, in this section we discuss how information theoretic entropy based methods provide a natural basis for establishing a causal influence-economic-econometric-inferential link to the data and solving the resulting ill posed stochastic inverse problem. Thus we face a system which may have more than one solution, or in which the solutions depend discontinuously upon the initial data. This type of uncertainty regarding the statistical model and associated estimating equations and the data sampling-probability distribution function create unsolved problems as they relate to information recovery. Although likelihood is a common loss function used in fitting statistical models, the optimality of a given likelihood method is fragile inference-wise under model uncertainty. In addition the precise functional representation of the data sampling process cannot usually be justified from physical or behavioral theory. Given this situation, a natural solution is to use estimation and inference methods that are designed to deal with systems that are fundamentally ill posed and stochastic and where uncertainty and random behavior are basic to information recovery. To identify estimation and inference measures that represent a way to link the model of the process to a family of possible likelihood functions associated with the income data, we use the Cressie and Read (1984) and Read and Cressie (1988) single parameter CR family of entropic functional-power divergence measures given by

$$I(\mathbf{p},\mathbf{q},\gamma)=\frac{1}{\gamma(\gamma+1)}\sum_{i=1}^{n}p_i\left[\left(\frac{p_i}{q_i}\right)^{\gamma}-1\right]. \tag{2}$$

In (2), g is a parameter that indexes members of the CR-entropy family of distributions, p_i's represent the subject probabilities and the q_i's are interpreted as reference probabilities. Being probabilities, the usual probability distribution characteristics of $p_i, q_i \in [0,1]\ \forall i$, $\sum_{i=1}^{n} p_i = 1$, and $\sum_{i=1}^{n} q_i = 1$ are assumed to hold. In (2), as γ varies, the resulting CR-entropy family of estimators that minimize power divergence, exhibit qualitatively different sampling behavior that includes Shannon's entropy, the Kullback-Leibler measure and in general includes a range of independent (additive) and correlated systems (see Gorban, et al., (2010), Judge and Mittelhammer, (2012a) and (2012b)). In identifying the probability space, the CR family of power divergences is defined through a class of additive convex functions and the CR power divergence measure leads to *a broad family of likelihood functions* and test statistics. The CR measure exhibits proper convexity in **p**, for all values of γ and **q**, and embodies the required probability system characteristics, such as additivity and invariance with respect to a monotonic transformations of the divergence measures (see Gorban, et al., 2010).

In the context of extremum metrics, the general CR family of power divergence statistics represents a flexible family of pseudo-distance measures from which to recover the joint distribution probabilities and encompasses a wide array of empirical goodness-of-fit and information recovery criteria. As γ varies power law behavior is efficiently described and the resulting estimators that minimize power divergence exhibit qualitatively different sampling behavior. To place the CR family of power divergence statistics in an entropy perspective, we note, following Gorban, et al. (2010), that there are corresponding families of entropy functionals–divergence measures. Over defined ranges of the divergence measures, the CR and entropy families are equivalent.

If in the CR family of entropy functionals $\gamma \rightarrow 0$, the solution of the first-order condition leads to Shannon entropy and the logistic expression for the conditional probabilities. Alternatively, if in the family of CR entropy functionals $\gamma \rightarrow -1$, the solution of the first-order condition leads to the maximum empirical likelihood distribution for the conditional probabilities. These criterion based entropy functionals permit us to use the information content of the structure of the income-probability density function distribution as a measure of inequality (Thiel, 1967 and Colwell, 2003). To obtain the probability density function income distribution the principle is to maximize the CR-entropy subject to constraints. Recovering the income distribution from a sample of positive real numbers, through the use of the CR-entropy criterion (2), suggests we seek, a solution to the following extremum problem:

$$\hat{p} = \underset{p}{argmin} \left[I(p,q,\gamma) \vee \sum_{j=1}^{n} p_j d_j = \bar{d}, \sum_{j=1}^{n} p_j = 1, p_j \geq 0 \right]. \qquad (3)$$

Solving this optimization problem provides a solution to the income probability distribution function and to the entropy inequality measure. In general, these solutions to this extremum problem do not have a closed-form expression and the optimal values of the unknown network parameters must be numerically determined.

III. THE EUROSTAT MICRO INCOME DATA

Eurostat is a Directorate-General of the European Commission. Its main responsibilities are to provide statistical information to the institutions of the European Union (EU). Considering data availability and country characteristics, we use income household data for the following 19 countries listed in Table I.

Table I

COUNTRIES AND ABBREVIATIONS AND NUMBER OF OBSERVATIONS

Source: EUROSTAT.

Abbreviation	Country	Number Observations
AT	Austria	58,669
BE	Belgium	59,053
CH	Switzerland	44,035
CZ	Czech republic	77,726
DE	Germany	119,883
DK	Denmark	57,735
EL	Greece	45,245
ES	Spain	102,211
FI	Finland	106,547
FR	France	77,053

IE	Ireland	51,895
IT	Italy	138,943
NL	Netherlands	89,436
NO	Norway	56,708
PL	Poland	124,487
PT	Portugal	37,395
SE	Sweden	67,536
SK	Slovakia	47,166
UK	United Kingdom	83,797
Total in 19 country Sample		1,445,520
Total negative		804
Total missing		59
Percent Data used from Raw data		99.94%

Among all income related variables in the Eurostat's micro survey database, we use the variable titled "HY010: Total household gross income" to measure the income level. It is measured in Euros without inflation factor. If we take a closer look at this variable, it measures the sum for all household members of gross personal income components (gross employee cash or near cash income; gross non-cash employee income; employers' social insurance contributions; gross cash benefits or losses from self-employment (including royalties). So it is a comprehensive and well-defined variable for the study, consistent with income measures used in previous studies such as Piketty and Saez (2003, 2014), who focus on the long-run evolution of the inequality of gross income, i.e., before taxes and government transfers.

Not all countries report data for all the years. In Table II we specify which countries have data available for each of the years from 2004 to 2013.

Table II

AVAILABLE YEARLY INCOME DATA SAMPLES BY COUNTRY

Years	Countries
2004	AT,BE,DK,IE,NO,FI,SE
2005	AT,BE,CZ,DE,DK,FI,IE,NL,NO,PL,SE,SK,UK

2006	AT,BE,CZ,DE,DK,ES,FI,HU,IE,NL,NO,PL,SE,SK,UK
2007	AT,BE,CZ,DE,DK,ES,FI,FR,GR,IE,IT,NL,NO,PL
2008	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK
2009	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK
2010	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK
2011	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK
2012	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK
2013	AT,BE,CH,CZ,DE,DK,EL,ES,FI,FR,IE,IT,NL,NO,PL,PT,SE,SK,UK

Source: EUROSTAT.

The household level income data are in Euros for all of the 19 countries. Summary statistics of the household income data by country and by year are reported in the appendix in Table A.1, along with the percent negative income and percent missing income data by country and year. There are a total number of 1,446,383 observations in the data. As shown in the bottom of Table I, 59 of the observations for income are missing and 804 of which are negative. Table I also contains the total number of non-missing and non-negative household income observations by country used in the analysis. In the end we use a total of about 1.6 million observations of yearly household income data without sample weights. As can be seen in the appendix, once broken up by country year, less than 0.82% of country year income observations are negative. Furthermore, 0.46% of the observations are missing for Spain 2006 and 0.05% for Norway in 2006. By country-year almost all data are quite complete and clean.[1] After removing the negative and missing entries we keep 99.94% of the original household sample data and have a total 1,445,520 household level income observations by country year.

In Figure I, we depict the household data for Sweden for each available year featuring two standard deviation box plots around the average over the years. Similar box plots can be found in the Appendix, for all the remaining 18 countries in the data set.

1 See the data appendix for details on the percentage by country of missing and negative data as well as for yearly total number of observations in the raw and final dataset used in this analysis.

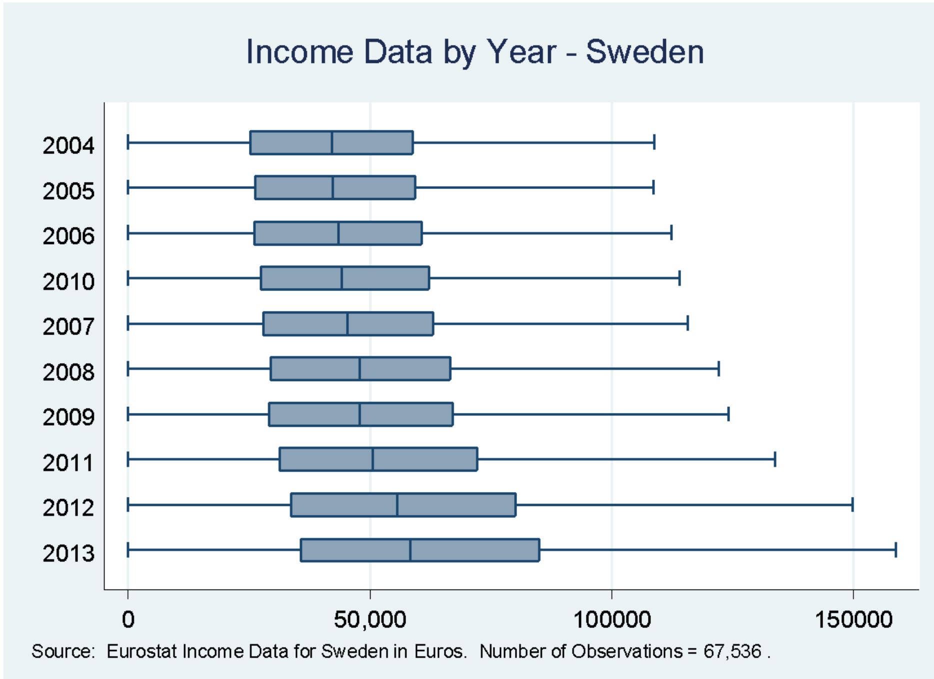

FIGURE I

SWEDEN INCOME HOUSEHOLD DATA BY YEAR

There are a total of 67,536 household year observations for Sweden and each year we have a sample around 6 to 7 seven thousand households. I: In Figure I, we see that although the average household income has increased over the years, so has the income standard deviation.[2] We use the complete sample of income data for all the countries to estimate the income distributions by year and investigate the evolution of income inequality using the proposed estimated resulting entropy measure.

IV. AN INFORMATION THEORETIC DATA INCOME ANALYSIS

In this section, we focus on the analysis of samples of micro income data from nineteen European countries that range over the years 2004 to 2013. In the analysis of the income data, we make use of the information theoretic methods of Section II as a basis

2 While we highlight the data for Sweden, we refer the reader to the Appendix where country specific yearly patterns are depicted in 21 additional box plots of the household income data for the remaining countries.

for summarizing the income data in the form of a probability density function, and use entropy as an income distribution measure of equality or inequality.

As background information, we illustrate in Figure II, for each country, the average or mean gross household's income in Euros over the period of 2009 to 2013. In this figure, the countries appear in increasing order of mean income. The mean income differences of Eastern Europe countries (Poland, Slovakia, and Czech Republic) are as expected, significantly different from Central and northern European countries that are located on the right of Figure II.

FIGURE II

AVERAGE INCOME BY COUNTRY IN THE EUROSTAT MICRO DATA

While showing the cross country income average is informative, we also investigate the patterns of country income distributions using twelve income level-histograms that span the micro sample space and use an entropy measure of income inequality to compare income equality-inequality across countries and time.

While showing the cross country income averages is informative, using twelve income levels-histograms that span the micro sample space, we investigate the patterns of country income distributions and also make cross country and over time comparisons in terms of income equality-inequality using an entropy measure.

IV.A. Maximum Exponential Empirical Likelihood (MEEL) Formulation

Two information-theoretic variants of the CR-entropy $I(p,q,\gamma)$ discrepancy-distance measure are prominent in the literature. Consideration of the choice of g implies that as a measure of uncertainty about the realization of the micro sample data, that $\gamma \rightarrow 0$ is the most appropriate measure of effective support size (Grendar, 2006). Twelve income –histogram levels are used to span the range of the micro income data space. Consequently in analyzing the Eurostat samples of income data we use the CR-MEEL criterion, $\gamma \rightarrow 0$, a uniform reference distribution q ($q_j = 1/n, \forall j$), twelve income-histogram levels, and first-moment information $\sum_{j=1}^{12} d_j p_j = \bar{d}$, as a basis for recovering discrete income probability density function-distributions such that the probabilities and $\sum_{j=1}^{12} pj$ = 1. Under this specification, when $\gamma \rightarrow 0$, the CR $I(p,q,\gamma)$ converges to an estimation criterion equivalent to the maximum exponential empirical likelihood (MEEL) metric $H(p) = -\sum_{j=1}^{12} p_j \ln ¿ (¿ p_j)$. Our extremum problem likelihood-entropy function may then be formulated as

$$\max_p \left[-\sum_{j=1}^{12} p_j \ln p_j \vee \sum_{j=1}^{12} p_j d_j = \bar{d}, \sum_{j=1}^{12} p_j = 1, p > 0 \right]. \qquad (3)$$

The corresponding Lagrange function-extremum problem is

$$(\text{¿}\, p_j) + \lambda\left(\bar{d} - \sum_{j=1}^{12} p_j d_j\right) + \eta\left(1 - \sum_{j=1}^{12} p_j\right).$$

$$L(p, \eta, \lambda) \equiv -\sum_{j=1}^{12} p_j \ln \text{¿} \tag{4}$$

Solving the first-order conditions yields the exponential result

$$\begin{array}{l} \text{¿} \\ \exp(\text{¿} - d_j \hat{\lambda}) \\ \sum_{j=1}^{12} \text{¿} \\ \exp \dfrac{(\text{¿} - d_j \hat{\lambda})}{\text{¿}} \\ \hat{p}_j = \text{¿} \end{array} \tag{5}$$

for the j th income outcome and the mean-related income distribution. As the mean of income varies over a range of micro data sets, an exponential family of distributions results. In equation (5), $\hat{p}_j$ is a function of $\hat{\lambda}$, the Lagrange multiplier for constraint (4). This information may be used as a basis for modifying the distribution of income probabilities. The CR-MEEL-entropy criterion, provides an empirical representation of the joint income probability distribution function, where the p_j are chosen to assign the maximum joint probability among all of the possible probability assignments. Using the CR ($\gamma \rightarrow 0$) formulation and the mean of a countries income data, we can then calculate the resulting probability density income distribution.

IV.B. Entropy Measure of Income Equality

In order to provide the information that is needed to group and compare the income distributions of the 19 European countries, we make use of the entropy measure

$E = \gamma \rightarrow 0 = \sum_{j=1}^{12} p_j \ln p_j$. In the entropy criterion-measure we seek a probability density function solution for each country over the years 2008-2013, that is as close to a uniform PDF-distribution of income (an equal a distribution with the least inequality), as the sample data will permit.

Making use of this entropy measure for the years 2008-2013 leads us to the ranking of the 19 European countries in Figure III. Again, the Central and Northern Europe countries shown on the half right half of Figure III, display a significant difference in the equality of income when compared to the Eastern European countries on the left half of Figure III. A high entropy measure means lower inequality in the income distribution. For instance, Norway's high entropy measure means that Norway has the lowest level of inequality of the European countries studied. On the other hand, Poland's low entropy measure indicates Poland has the highest level of inequality out of the European countries studied.

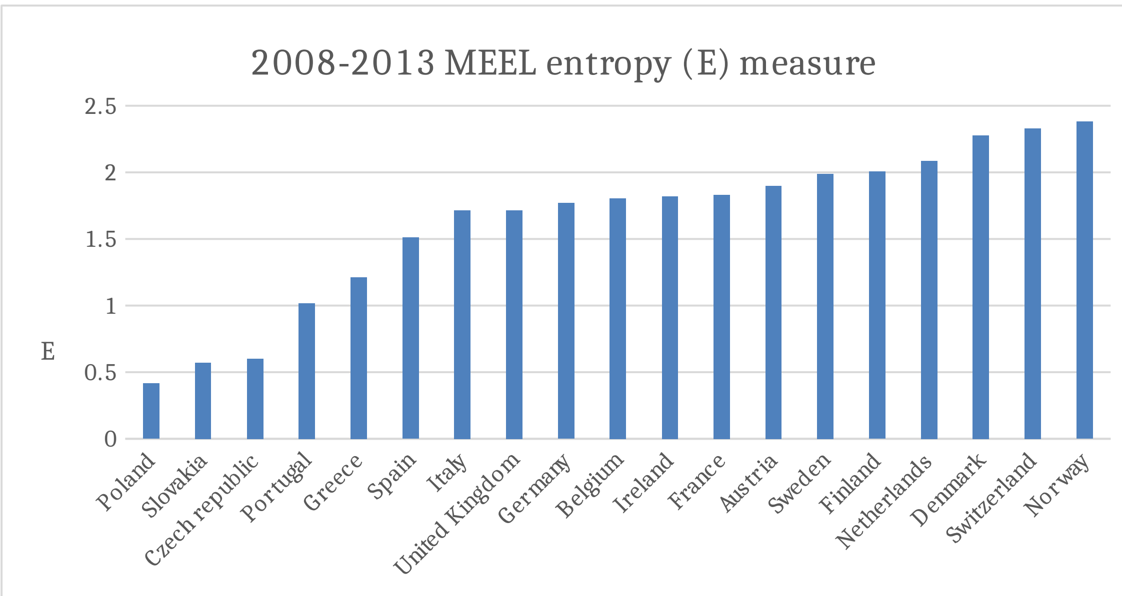

FIGURE III ENTROPY MEASURE OF INCOME INEQUALITY BY COUNTRY

Given the summarized illustration of heterogeneity in terms of income equality in Europe, displayed in Figure III, the nature of the income inequalities is dramatically demonstrated by the income probability density functions, of the next sections.

IV.C. Income Probability Density Functions

To develop income probability density functions for each of the 19 European countries, we use a 2008-2013 data sample, and in line with the information recovery method of section IV.A, divide the income data for each country into 12 box plots-histogram categories. After computing and displaying the income frequencies for the years 2008-2013 for each of the 19 countries, we note that in line with the entropy measures in Figure III, they fall broadly into the following four main types of income distributions: the Non Euro Zone, Central Europe, Portugal and Greece, and Eastern Europe

Three Low Inequality Non Euro Zone countries

In Figure IV we display the probability density-income distributions for NO-Norway, CH-Switzerland and DK-Denmark. This group represents two non-European Union (EU) members (NO and CH), and Denmark. They have in common not having adopted the Euro as their common currency and sole legal tender. These three countries, using statistical measures, conform to the exponential income distribution form, and have a high equality entropy measure value, but vary in terms of their probabilities over the income levels. Interestingly, the income distribution for CH-Switzerland lies between the other two income distributions as did its entropy based inequality measure as shown in Figure III. As the entropy measure suggests, NO-Norway has the most uniform probability income distribution with an entropy measure approaching 2.5 and thus displays lower probabilities of being in the lower income levels and higher probability of being in the higher income levels. It would be interesting to comment on their economic and social systems, but that is another topic.

Figure IV

INCOME DISTRIBUTIONS FOR LOWEST INEQUALITY COUNTRIES

Central Europe with Entropy Measures between 1.5 and 2

In Figure V we jointly display, the income PDFs for the next block-group of countries with inequality measures between 1.5 and 2. This group consists of 10 European Union and one non-EU country: AT-Austria, BE-Belgium, FI-Finland, DE-Germany, FR-France, IE-Ireland, IT-Italy, NL-Netherlands, ES-Spain, SE-Sweden (not in EU), and the UK-United Kingdom (not in the Eurozone). All these countries have the exponential distribution shape and share a similar feature relative to their probability density-income distribution functions and entropy measures as seen in Figure III. We note that Spain has higher probabilities of lower income levels and lower probabilities of being in the higher income levels

Figure V

INCOME DISTRIBUTIONS FOR CENTRAL EUROPEAN COUNTRIES

Greece and Portugal

In Figure VI we display the income distributions for EL-Greece and PT- Portugal. Their income probability distribution functions differ sharply from those of the previous two block groups, in terms of having high probabilities for the low-income segments and almost uniform probabilities for the higher income segments. The income probabilities for Greece are slightly superior to Portugal. The entropy equality measures of around 1 for these two countries, as shown in Figure III, reflect the income inequality nature of these distributions.

Figure VI

INCOME DISTRIBUTIONS FOR PORTUGAL AND GREECE

East European Countries with the Lowest Entropy Measures

In Figure VII we display the probability density functions-income distributions for CZ-Czech Republic, PL-Poland, and SK-Slovakia. The income probabilities-distribution for these countries differ sharply from those of Northern and Central Europe. All of these East European countries indicate high probabilities for the lower third of the income levels and almost uniform-equal probabilities for the upper three fourths of the income scale. These emerging countries still show the impact of being under another type of economic-choice system and they display the highest levels of income inequality given the entropy measures that all fall below the level of 1.

Figure VII

INCOME DISTRIBUTIONS FOR EASTERN EUROPEAN COUNTRIES

To make clear the implications of the entropy measure, we include in Figure VIII the probability density functions for Portugal and Greece with the probability density functions for the Eastern European countries. In this comparison, PT-Portugal and EL-Greece have lower probabilities for the lowest income levels and higher probabilities for the immediately next income levels than the Eastern European group. For this reason, the entropy measure for PT and EL is higher than the Eastern European countries' entropy equality measure.

Figure VIII

INCOME DISTRIBUTIONS FOR EASTERN EUROPEAN COUNTRIES AND PORTUGAL AND GREECE

V. THE DYNAMICS OF INCOME EQUALITY-INEQUALITY

Using the fact that we observe micro income data as a yearly ordered time series, we can estimate the distribution functions for each year for each country and the corresponding entropy measure to investigate the time evolution of connected markets and the dynamics of country specific income inequality. Essentially we do this by comparing the probability density function of the first and the last available yearly income micro data. Four countries (Austria, Belgium, Finland and Ireland) are in the earliest available 2004 data set and the last available data set 2013. In Figure IX we focus on Austria and Belgium, and note the time shift in the 2004 and the 2013 probability distributions.

Figure IX

2004 & 2013 INCOME DISTRIBUTIONS - AUSTRIA AND BELGIUM

From the patterns of evolution of the 2004 to the 2013 probability density function (pdf) for both countries, there is a shift towards a flatter PDF, which ultimately means a higher level of entropy and thus lower inequality. [3]

Alternatively, looking at the two countries Germany (DE) and United Kingdom (UK) that lie right in the middle of Figure III in terms of income entropy measures, we

3 Finland and Ireland also display similar patterns and results are available upon request.

note an interesting and quite different income dynamic in terms of shifts in the pdfs and resulting entropy measure of income inequality. In Figure X, Germany, has the opposite shift or rotation in the pdf than the UK. The UK 2005-PDF places a higher probability on lower income levels which then gets shifted to the next levels of income in the 2013 pdf, a repetition of the dynamic pattern previously discussed in Figure IX. However, Germany has the opposite pattern. The lowest income levels have a higher probability in 2013 than in 2005, at the expense of the next levels of income. We can thus infer that income inequality has increased in Germany from 2005 to 2013.

FIGURE X

2005-2013 INCOME PDF'S- GERMANY AN UNITED KINGDOM

It is interesting to be able to observe the time ordered country impact of connected markets and networks of credit, investment and information. This would seem to suggest the danger of country based generalizations concerning the impact of new technology and networks of trade finance and information. In addition, many economic studies in measuring income inequality use a ratio of the highest and lowest quintiles (for example see Dorling, 2016). These studies are strangely silent in terms of what happens over of the remainder of the distribution. As a take away, these illustrations point out the *importance* of looking at individual countries and considering the *entire probability density -income distribution function*, when drawing static and dynamic inferences concerning income inequality.

V.A. Illustration of Income distribution Dynamics for Sweden

In this subsection we focus on the income dynamics for SE-Sweden for the decade 2004-2013. As denoted in Figure XI, in Sweden from 2004 to 2013, there has been an important shift toward income equality. The dynamics of this change is displayed further in the yearly entropy measures of Figure XII and in Figure XIII where the complete set of income PDFs for the years 2004-2013 are displayed.

FIGURE XI

2004-2013 INCOME PROBABILITY DENSITY FOR SWEDEN

In the entropy measures of Figure XII, except for the financial crash years of 2009 and 2010, the higher entropy values indicate there has been a steady march over time toward income equality in Sweden.

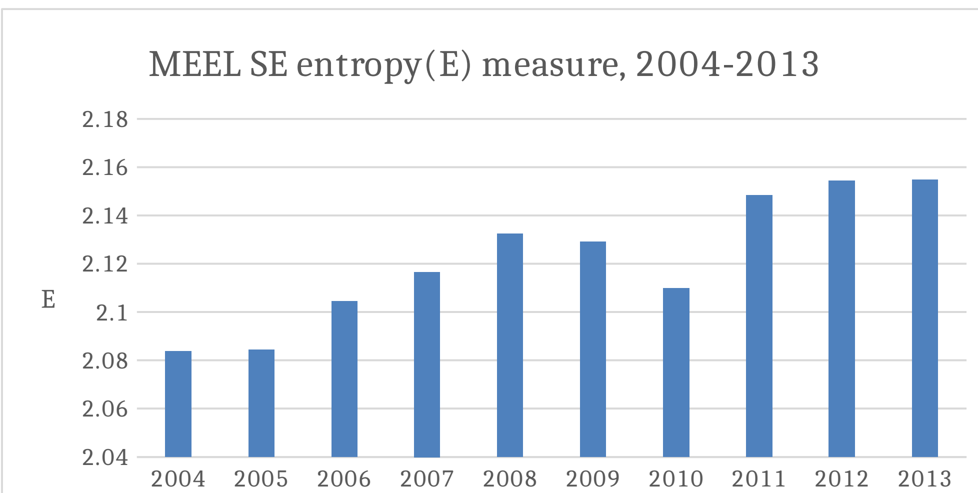

FIGURE XII - ENTROPY MEASURES BY YEAR FOR SWEDEN

Finally, Figure XIII shows the same pattern over time, with the more recent income PDFs flattening out relative to the early 2000 years. This dynamic pattern is consistent with a shift towards more inequality and a more uniform income distributions in Sweden.

FIGURE XIII

INCOME PDF'S FOR SWEDEN OVER THE YEARS

Taken together, displaying the yearly entropy measure in Figure XII and the income PDFs-distributions in Figure XIII, gives a new information recovery tool for measuring and presenting the dynamics of income equality-inequality.

VI. CONCLUSIONS AND IMPLICATIONS

In this paper we have presented a new uniform information theoretic behavior related methodological basis for recovering country based income probability density-distribution functions from samples of time ordered European micro income data, and measuring income inequality. Information theoretic methods and the Cressie-Read family of entropy based information divergence measures have been used to provide a flexible family of functions to recover the unknown country based probability density functions-income distributions and yield a measure of income equality-inequality. A 2008-2013 data sample was used to develop the probability density income function for 19 European countries.

Using an entropy measure of income inequality the countries were then divided into four groups and graphically displayed and analyzed. Using yearly data from 2004 to 2013 for five countries in central Europe the dynamics of income inequality is discussed. The importance of using the entire income distribution when measuring income inequality is emphasized. We find that there is heterogeneity across countries in terms of income inequality in Europe, with Portugal and Greece and three Eastern European countries being the ones with the highest levels of inequality measured by the entropy measure, while Northern European countries outside the European Union and the euro zone have the lowest inequality in their income distributions. Given that most countries have household micro income survey data sets collected over time, the information theoretic method provides a general approach for this type of information recovery.

When time ordered samples of income data are used in terms of information recovery, the problems of estimation and inference and the measure of inequality, should be analyzed as one- joint behavior related problem. We have investigated the dynamic shift in the income distributions in selected European countries and found there to be, once again, quite heterogeneous paths in terms of the evolution of the income inequality measure and income distributions. Given this base, analysis of the possible reasons behind such different income patterns, is an important future question to be investigated.

The dynamics of income equality-inequality is a very important economic and econometric problem (see Gabaix, et al., 2016). To our knowledge, a satisfactory way of handling income inequality dynamics does not currently exist. The information theoretic framework used in this paper provides one basis for moving from static analyses and incorporating dynamics into the inequality discussion. In an information theoretic context one possible way to introduce dynamics is to use the income distribution in time T as the reference-prior distribution in time T+1. Evaluating these and other alternatives are major topics of future research.

REFERENCES

Adamou, A. and O. Peters, 2016, Dynamics of Inequality, Significance, Royal Statistical Society, June, 32-35.

Alvaredo, F., Atkinson, A. B., T. Piketty, and E. Saez, 2013. “The Top 1 Percent in International and Historical Perspective,” *The Journal of Economic Perspectives*, Volume 27 (3), Summer: 3-20(18).

Cho, A., 2014, Physicists Say Its Simple, Science, The Science of Inequality, May 2014,Vol. 344,Issue 6186:828.

Colwell, F., 2003, Thiel and the Structure of Income Distribution, Memorial Conference, London School of Economics.

Cressie, N. and T. Read, 1984. Multinomial Goodness of Fit Tests. *Journal of Royal Statistical Society of Series* B 46, 440-464.

Deininger, K. and L. Squire, 1996. “A New Data Set Measuring Income Inequality,” *The World Bank Economic Review*, 10 (3): 565-591.

Dorling, D., 2016, Dynamics of Inequality: In Response, Significance, Royal Statistical Society, June, 36-37.

Gabaix,X., J. Lasry, P. Lions and B. Nolli, 2016, The Dynamics of Inequality, Econometrica, 84: 2071-2111.

Gorban, A., P., Gorban, and G. Judge, 2010, Entropy: The Markov Ordering Approach. *Entropy*, 12, 1145-1193.

Grendar, M.,2006, Entropy and Effective Support Size, Entropy, 8:169-174.

Judge, G.G. and R.C Mittelhammer, 2012a, An Information Theoretic Approach To Econometrics, Cambridge University Press.

Judge, G.G. and R.C. Mittelhammer, 2012b, Implications of the Cressie-Read Family of Additive Divergences for Information Recovery. *Entropy* 14, 2427-2438.

Pareto, V., 1896, Cours d’Economique Politique, Library Droz, Reprinted 1964

Piketty, T. and E. Saez, 2014, Inequality in the Long Run, *Science* 344 (6186), 838-843.

Piketty, T. and E. Saez, 2003, Income Inequality in the United States, 1913-1998, *Quarterly Journal of Economics*, 118:1-39.

Ravallion, M. 2014. “Income Inequality in the Developing World,” *Science*, 344 (6186), 851-855.

Read, T.R. and N.A. Cressie, 1988, Goodness of Fit Statistics for Discrete Multivariate Data, New York: Springer Verlag.

Roine, J., and D. Waldenstroem, ‘‘Long-run Trends in the Distribution of Income and Wealth,’’ in Handbook of Income Distribution, Vol. 2, Anthony B. Atkinson and Francois Bourguignon, ed. (Amsterdam and New York: Elsevier/North Holland, 2015).

Saez, E. and G. Zucman, 2016, Income Inequality in the United States Since 1913, *Quarterly Journal of Economics*, 131:519-578.

Shannon, C., 1948, The mathematical Theory of Communications, Bell Syst. Techn. Jour, 27:379-423.

Thiel, H., 1967, Economics and Information Theory, pages 121-128, Amsterdam: North Holland.

Wissner-Gross, A. and C.E. Freer, 2013, Causal Entropic Forces. *Physical Review Letters* PRL 110, 168702-1-5.