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Learning to Redefine "Good at Math": Tensions and Possibilities in Equity-Oriented Mathematics Teachers' Everyday Practice

By

Nicole L. Louie

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

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in the

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University of California, Berkeley

Committee in charge:

Professor Alan H. Schoenfeld, Chair Professor Judith W. Little Professor Na'ilah Suad Nasir Professor Raka Ray

Spring 2015

Learning to Redefine "Good at Math": Tensions and Possibilities in Equity-Oriented Mathematics Teachers' Everyday Practice

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Abstract

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University of California, Berkeley

Professor Alan Schoenfeld, Chair

What does it mean to be "good at math"? Traditionally, schools have valued getting the right answer quickly—a perspective that excludes important aspects of mathematics, as well as many students. This multi-site case study investigates how teachers work together to redefine mathematics and mathematical competence. The study involved more than a year of ethnographic observations and interviews at two diverse urban high schools on the West Coast of the United States, where teachers expressed strong commitments to serving *all* students, especially students from non-dominant backgrounds.

The dissertation tells a complex story of teacher learning, as viewed through the lenses of classroom instruction (Chapter 2), collegial conversation (Chapter 3), and the organization of teachers' professional support networks (Chapter 4). Drawing on scholarship that takes learning as a negotiation of meaning through engagement in social practices (Vygotsky, 1986; Wenger, 1998; Saxe, 2012), the dissertation examines the relationships between extra-local systems of meaning and moment-to-moment interactions. Extending beyond prior work, the dissertation elucidates the negotiation of intensely conflicting meanings—namely, culturally *dominant* definitions of mathematics as a discipline, students as learners, and teachers as professionals, and *non-dominant* definitions that attempt to expand both teachers' and students' opportunities to engage with rich, challenging, and rewarding learning experiences.

In each of the contexts studied, navigating tensions between dominant, restrictive meanings and non-dominant, expansive meanings was a challenge for all of the teachers. Dominant discourses frame mathematical activity as consisting primarily of computation and memorization; mathematical ability as innate, fixed, and distributed along a bell-shaped curve; and the work of teaching as private, autonomous, and grounded in personal style and preference. In contrast, equity- and reform-oriented discourses frame mathematical activity as inclusive of a wide variety of skills and practices; position *all* students as capable learners; and position teachers as learners who benefit from ongoing collaboration and support. Dominant discourses are *restrictive:* they limit students' opportunities to learn rich mathematics and teachers' opportunities to negotiate equity- and reform-oriented shifts in their practice. But as teachers engage with non-dominant

meanings that potentially *expand* learning opportunities, commonsense meanings do not simply disappear. Rather, they interact with non-dominant meanings in messy and complex ways that require careful study in order to understand how and what teachers learn.

The theme of negotiating meaning is laid out in Chapter 1, with a discussion of the dissertation's underlying theoretical perspective. Research sites are introduced; the dissertation's structure is presented; and major findings and contributions are highlighted.

Chapter 2, "(Re)Framing Mathematical Competence in Everyday Instruction: Struggles and Successes of Equity-Oriented Teachers," examines tensions and contradictions in teachers' classroom practice. It shows that despite the best intentions of the teachers in this study, many of their efforts to support all students position some students as capable of engaging with challenging mathematics and others as just the opposite. Conversely, teacher moves that are counterintuitive within dominant frames of teaching, employed by two of the teachers in the study, are shown to expand students' opportunities to develop positive mathematical identities. The chapter thus contributes to conversations about what equitable mathematics instruction looks like, while illuminating obstacles—cultural as well as technical—that teachers face as they attempt to enact classroom practices that support all students.

Chapter 3, "Tensions in Equity- and Reform-Oriented Learning in Teachers' Collaborative Conversations," examines how collaborative conversations open up and close down opportunities for teachers to navigate the tensions between restrictive and expansive discourses of mathematical competence, through close analysis of a 9½-minute segment of a routine meeting of mathematics teachers. Although the group appeared to be an ideal professional learning community in many ways, and the focal interaction and others like it were generative in a number of respects, teacher talk enacted both restrictive and expansive discourses. The existence of tensions between these discourses presented opportunities for the teachers to negotiate non-dominant meanings for themselves, i.e., to learn; but the ways that teachers framed their own collaborative work interfered with these opportunities. By highlighting conversational norms that impede collaborative learning, the chapter contributes to the field's understanding of the challenges of equity- and reform-oriented learning in teachers' professional communities.

Ways of supporting teachers to negotiate expansive meanings are examined in Chapter 4, "Supporting Teachers' Equity-Oriented Learning and Identities: A Resource-Centered Perspective." The chapter investigates two cases of ongoing teacher engagement with nondominant practice and two cases of relative disengagement, illustrating how various resources come together to support teachers' learning and identity development (or not). Four types of resources are found to be critical, and learning and identity processes are shown to intertwine in mutually informing ways as teachers interact with these different resources.

In elucidating both challenges and supports associated with making sense of non-dominant meanings, this dissertation contributes to the field's understanding of equity- and reform-oriented teacher learning and why it is so difficult. It also points to ways that the contexts in which teachers work might be constructed to support their engagement with non-dominant, expansive meanings, so that they can support all of their students to engage with rich, challenging mathematics and to develop identities as powerful learners and doers of mathematics.

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Chapter 1

What Does It Mean to be Good at Math?

At the end of the 2012-2013 school year, I worked with teachers to administer a survey asking students about their attitudes toward mathematics. One of the prompts read, "Describe people who are good at math. What are they like? What do they do in math class?" Many students responded with stereotypes, conjuring images of glasses-wearing "geniuses" who "know a lot of equations" and "solve problems easily." Only a few wrote that everyone can be good at math. One student made this point especially poignantly, saying, "It's funny when I think of people who are good at math because then I realize it can be anybody. I learned this from my classmates." Under these words, the student drew a table juxtaposing contrasting characteristics:





For this student, seeing "anybody" as good at math was not abstract or purely aspirational. It was something "learned ... from my classmates," through classroom activity that had supported students with diverse ways of knowing and being to make meaningful and authentic contributions to their mathematics learning community.

This dissertation is animated by an interest in how people—especially math teachers develop conceptions of mathematical competence that create opportunities for *all* learners to take up identities as powerful mathematical thinkers and to engage in rich, rigorous mathematics learning. Dominant discourse *restricts* such opportunities, framing mathematical competence like intelligence more generally—as an innate, fixed, and quantifiable asset that is not only distributed unequally amongst individuals but also distributed unequally amongst racialized, gendered, and classed groups, reifying and legimating existing social hierarchies (Martin, 2009; Oakes, Wells, Jones, & Datnow, 1997; Parks, 2010). Despite reformers' calls for "mathematics for all" (e.g., National Council of Teachers of Mathematics, 2000) mathematical ability is still popularly understood as the property of Asians, Whites, boys, and men, to the exclusion of Blacks, Latinos, girls, and women (Boaler, 2007; Shah, 2013).

A unifying issue across the three papers that comprise this dissertation concerns relationships between extra-local systems of meaning—such as culturally dominant discourses about mathematical competence—and the ways that teachers take up and also resist these systems of meaning in their interactions with their students and with one another. The contributions that the dissertation makes toward understanding these relationships are of particular interest because all of the teachers involved in the project had expressed a firm commitment to serving students from traditionally marginalized backgrounds, and they were all supported by an intensive professional development program (modeled after Jilk & O'Connell, 2014) that aimed to expand teachers' and students' notions of what it means to be mathematically "smart." Thus, the teachers' resources for negotiating non-dominant, *expansive* conceptions of mathematical competence were unusually rich, making theirs a potential best-case scenario. The learning in which they engaged during the period of the study was nonetheless "messy" and fraught with contradictions (cf. Parks, 2008; this finding is also consistent with the literature on teacher beliefs and their resistance to change, e.g., Haberman & Post, 1992; Oliveira & Hannula, 2008).

Put another way, this dissertation investigates the learning that equity-oriented teachers do as they work to redefine mathematical competence in expansive and inclusive ways, so that they can enact increasingly equitable instruction with their students. While professional learning is typically conceptualized as an induction into well-established practice (Lave & Wenger, 1991; Wenger, 1998), the learning at the heart of this dissertation entails the active, ongoing negotiation of tensions between expansive, inclusive definitions of mathematical competence and dominant discourses about intelligence and mathematics. It involves the transformation of the practice of teaching itself, as well as the transformation of individual practitioners.

The chapters that follow this introduction are written as articles. Taken together, the articles tell a complex story of teacher learning, seen from the perspective of classroom instruction, collegial conversation, and the organization of teachers' professional support networks. Each article also stands on its own, with its own sections on theoretical background, prior research, methods (including data sources and analysis techniques, which vary from paper to paper), findings, and implications. Here, I briefly summarize the aspects of my theoretical perspective that cut across the chapters; describe the entire body of data that I collected; outline key findings from each chapter; and step back to assess the contributions of the dissertation as a whole.

Theoretical Perspective

Teachers' practice, in the classroom and out, is at the heart of my analyses. I view practice as fundamentally social. As Wenger (1998) writes,

The concept of practice connotes doing, but not just doing in and of itself. It is doing in a historical and social context that gives structure and meaning to what we do. In this sense, practice is always social practice. (p. 47).

The social negotiation of meaning and the "work[ing] out [of] common sense through mutual engagement" (p. 47) is central to Wenger's conception of practice and to this dissertation. I investigate how teachers make sense of their practice, both shaping and being shaped by their participation in social networks, with consequences for their practice itself and for their identities in relation to their practice. The aspect of teacher learning that this dissertation examines is thus less about technical dimensions of practice (though it addresses these, too) than it is about the meanings of their practice that teachers negotiate, both consciously and unconsciously, through their interactions with others.

In taking the process of learning as the negotiation of meaning, I draw on a scholarly tradition that addresses the interplay of cultural structures writ large and individuals' moment-tomoment interactions with one another and with their local contexts (Vygotsky, 1986; Wenger, 1998; Saxe, 2012). This work highlights the constancy of meaning-making in everyday interaction, directing attention to the ways in which people simultaneously shape and are shaped by systems of meaning that extend far beyond them. Drawing on this perspective, this dissertation privileges neither individual knowledge, beliefs, and agency nor cultural roles and discourses but the dynamic relationships between them, as these relationships develop over two time scales: in moment-to-moment interaction, and over the course of individual teachers' development as professionals (what Saxe terms *microgenesis* and *ontogenesis*).

The dissertation also departs from previous work by examining the negotiation of intensely conflicting meanings, as people make deliberate efforts to transform existing practice. In Wenger's (1998) work, practice is treated as more or less stable, and the learning that individuals accomplish is treated as movement along a trajectory that leads from peripheral participation to increasingly central participation in the community of practice. As they progress, individuals make their own sense of the practice, but in ways that change the practice itself only incidentally. Changes in collective practice are of primary interest in Saxe's (2012) work; but the changes that he examines, like those to which Wenger alludes, occur gradually and unintentionally. In contrast, this dissertation examines purposeful attempts to accomplish shifts in meaning, as these attempts run up against—and sometimes, unintentionally reproduce— entrenched meanings. Its chapters analyze the complexities of working toward meanings that are somewhat abstract and idealized, while other, more restrictive meanings remain dominant. As such, the dissertation adds to the field's understanding of the interplay of social and individual contributions to the making of meaning.

Methods of Data Collection

Research sites and participants

This dissertation is based on a year of ethnographic observations and interviews in two high schools in an urban school district on the West Coast of the United States. I call the schools Union and Boxer (all school, teacher, and student names are pseudonyms). Demographic information for the student body at each school is shown in Table 1.

Table 1

Student demographic characteristics at Union and Boxer High Schools.

Category	Union	Boxer
# of students	800-850	550-600
% African American	10-20	10-20
% American Indian	< 5	< 5
% Asian	20-30	10-20
% Filipino	< 5	< 10
% Hispanic or Latino	40-50	60-70
% Pacific Islander	< 5	< 5
% White (Not Hispanic)	< 10	< 5
% Multiple or No Response	< 5	< 5

% Socioeconomically disadvantaged	70-80	70-80
% English learners	50-60	50-60
% Students with Disabilities	10-20	10-20
% Proficient/Advanced, STAR math	< 10	< 10

Note. Data were drawn from publicly available district reports. The categories used are the district's. Ranges are provided (rather than precise figures) to protect participants' anonymity.

Union and Boxer were both racially, socioeconomically, and linguistically diverse. Both schools were also underperforming, as reflected by the percentage of students who were "proficient" or "advanced" on the state standardized test (below 10% at both schools). The mathematics teachers at both sites had chosen these schools, and the schools had chosen them, because of expressed commitments to serving all students, especially students from historically marginalized groups. Furthermore, all 13 math teachers at Union and all 5 at Boxer were participants in an equity-oriented professional development (PD) program offered by their district. The program was grounded in Complex Instruction (CI; see Cohen & Lotan, 1997; Nasir, Cabana, Shreve, Woodbury, & Louie, 2014), a pedagogical approach that posits that all students have important intellectual contributions to make to their classroom learning communities, and that teachers are responsible for drawing out every student's "smartnesses" while also helping every student develop new strengths and abilities. The PD targeted these beliefs as well as instructional strategies for "equalizing status"—i.e., for leveling hierarchies of perceived ability and worth—such as the use of complex, open-ended tasks that require students to pool their various skills and work together. The teachers participated in periodic CI trainings and coaching sessions. In addition, the PD emphasized the development of robust, departmentbased professional support networks, and teachers at Union and Boxer had dedicated time each week to collaborate around mathematics instruction. This array of resources made Union and Boxer promising contexts for examining how teachers learn to redefine what it means to be "good at math."

I selected focal course teams at each school: the Geometry Team at Union and the Algebra Team at Boxer. Both teams had primary responsibility for teaching ninth graders at their school and, therefore, for socializing the newest students into their school's way of teaching and learning mathematics. From each team, I selected focal teachers (four at Union, two at Boxer), aiming to capture range along two dimensions: years of experience teaching, and leadership roles in the district CI community.

Procedures

I conducted observations in a number of settings, beginning with CI training and collaborative planning in the summer of 2012 and ending at the end of the 2012-2013 school year. I observed routine meetings of the math department and of focal teams at Union and Boxer; instruction in focal teachers' classrooms (between 4 and 8 times per teacher); and CI professional development sessions. Table 2 provides an overview of these observations. Midway through the year, I elected to focus more intensively on Union, since teachers there collaborated more regularly than did teachers at Boxer. I continued to observe sporadically at Boxer, however, and

my analyses make use of data from both sites. Audio recordings, field notes, and photographs of whiteboard inscriptions, worksheets, and other artifacts were produced for each observation. All audio recordings were transcribed.

Table 2

Number (and length in hours) of observations at Union and Boxer High Schools.

Observation type	<u>Union High</u>	Boxer High	Total
Department meetings	10 (12.5)	4 (4)	14 (16.5)
Course team meetings	18 (16)	4 (4.5)	22 (20.5)
Classroom lessons	24 (30)	13 (17)	37 (47)
Professional development sessions			20 (85)
Total	52 (58.5)	21 (25.5)	93 (169)

Note: Professional development sessions were concentrated in the summer; 8 of the 20 sessions were 6-hour workshop days that took place before schools opened to students.

I also conducted semi-structured interviews with focal teachers in the spring of 2013. (The protocol is included as Appendix A.) Less formal interviews were conducted throughout the year, in conjunction with observations (e.g., in the casual time before meetings officially began, or at the end of class when students were gone).

Chapter Overview

Each of the chapters that follows addresses a different aspect of teachers' negotiation of meaning, taking a distinct angle on the ways in which teachers reproduce and resist culturally dominant discourses about mathematical competence. Chapters 2 and 3 investigate this negotiation of meaning in teachers' classroom practice and collegial conversation, respectively. Both chapters illustrate how teachers' best efforts to move away from restrictive discourses may yet be enmeshed in these same discourses. Chapter 4 moves beyond the school site to examine how teachers' professional networks may support their engagement with non-dominant discourses and practices. In summary, I find that despite their common commitments (as expressed to colleagues and to me) to equity-oriented reforms and their membership in the same communities of practice, the resources that individual teachers found to support their own learning varied widely, as did the opportunities to learn that they created for their students.

Chapter 2: (Re)Framing Mathematical Competence in Everyday Instruction: Struggles and Successes of Equity-Oriented Teachers

Chapter 2, "(Re)Framing Mathematical Competence in Everyday Instruction: Struggles and Successes of Equity-Oriented Teachers," examines tensions and contradictions in teachers' classroom practice. The analysis explores how instruction afforded opportunities for students to develop a sense of agency, authority, and mathematical competence, quite apart from teachers' intentions, by looking at the ways that teachers framed a) mathematics as a discipline and b) students as learners—*epistemological* and *positional framing*, respectively, in Greeno's (2009) terms. The teachers in the study employed frames that expanded students' opportunities to develop identities as powerful learners and doers of mathematics, but they also employed frames that restricted these opportunities, as Table 3 shows.

Table 3

Ways teachers framed mathematical competence.

	Restrictive (Dominant)	Expansive (Non-dominant)
Positional	<i>The school math frame</i> . Mathematics is composed of discrete chunks of well-established knowledge and skill. These chunks build sequentially, from the basic to the complex.	<i>The multidimensional math frame.</i> Mathematics involves a wide variety of complementary skills, many of which have not traditionally been valued. These include communication, creativity, and making connections.
Epistemological	<i>The high and low frame</i> . Some students are smart, gifted, and high-level; others are struggling, weak, and low-level. <i>The expert-dependent frame</i> . Students need external authorities to explain what	<i>The multi-ability frame.</i> All students have intellectual strengths that enable them to learn and to contribute to others' learning, though these strengths may not be immediately obvious.
	they should do.	<i>The students-as-resources frame</i> . Students are capable of tackling and surmounting challenges by pooling their many strengths and working together.

To illustrate how these frames were enacted in day-to-day instruction, the chapter examines excerpts of classroom activity surrounding two instructional practices: adapting and assigning tasks, and responding to students' struggles. I find that teachers' best efforts to support students-efforts that are consistent with our cultural commonsense about "good teaching"-can limit these opportunities. For example, providing students with assignments of varying degrees of "challenge" is often considered an effective means of differentiating instruction for students with different needs. Yet this practice frames mathematics as linear and unidimensional (consistent with the school math frame) and positions some students as less capable than others (consistent with the high and low frame). Conversely, teacher moves that expand students' opportunities to develop identities as powerful learners and doers of mathematics may be counterintuitive within dominant frames of teaching. For example, posing a complex mathematical challenge to a group of students, none of whom have all the skills necessary to solve it, goes against commonly held ideas about what teachers should do. However, two teachers in the study routinely did just that, supporting students' learning and identity development by explicitly highlighting the *multidimensional* nature of mathematical activity and the various intellectual contributions that each student could contribute to the group's collective work (consistent with the *multi-ability* and *students-as-resources* frames).

Chapter 2 thus contributes to conversations about what equitable mathematics instruction looks and sounds like, while elucidating the obstacles—cultural as well as technical—that teachers face as they attempt to enact classroom practices that support all students.

Chapter 3: Tensions in Equity- and Reform-Oriented Learning in Teachers' Collaborative Conversations

Chapter 3, "Tensions in Equity- and Reform-Oriented Learning in Teachers' Collaborative Conversations," generates related insights in the context of teacher-to-teacher talk, examining how collaborative conversations open up and close down opportunities for teachers to navigate the tensions between restrictive and expansive discourses of mathematical competence. A 9¹/₂-minute episode in a routine meeting of Geometry teachers at Union serves as the focal point for the analysis. The episode revolved around a problem posed by one of the teachers, Eliza, who was having trouble managing student presentations and asked her colleagues to share their strategies for supporting students to "make their understanding public without having to stand up and present." Close analysis of the ensuing conversation shows that like many other conversations in the data corpus, it supported teacher learning in a number of ways. Most salient to the teachers themselves (judging by their statements in meetings and interviews), it gave them access to a variety of practical ideas and immediately usable techniques. The conversation also gave teachers opportunities to think expansively about mathematical activity and student competence. Teachers framed communication and explanation as key components of mathematical activity, and they highlighted their own agency over and responsibility for student learning in ways that challenged deficit perspectives of students.

But the conversation also reproduced dominant frames of mathematical competence. Although communicating, explaining, and sharing understanding were emphasized in the abstract, descriptions of *what* students should communicate and share alluded to answers and procedures (rather than, for example, students' reasoning, connections to concepts, or connections between different strategies). And though teachers countered deficit perspectives on students, they did so in ways that were obscured by expressions of sympathy and agreement with deficit-oriented statements. Thus, despite the ways in which it was supportive of teachers, the conversation did not generate robust opportunities for participants to learn to redefine mathematical competence.

The chapter highlights the tenacity of dominant, restrictive frames of mathematical competence and the tensions between these frames and expansive, inclusive ones. It also traces the difficulty of navigating these tensions to dominant framings of *professional interaction* itself. The teachers gathered regularly to collaborate, but consistent with a longstanding tradition of professional privacy and autonomy (Little, 1990; Lortie, 1975), they framed the purpose of their collaboration as sharing/hearing a variety of experiences and strategies and framed their professional knowledge as personal and idiosyncratic, subject to differences in personality and circumstance. These framings were linked to the framing of appropriate participation in ways that constrained teachers' opportunities to learn together, for example by obviating any need for connections to general principles of teaching and learning and by discouraging disagreement, since each person had autonomy to make whatever decisions s/he wished.

Chapter 3 thus demonstrates three things. It shows how equity-oriented teachers negotiate the meaning of mathematical competence in moment-to-moment interactions with one another, drawing on dominant, restrictive frames as well as expansive and inclusive ones. It shows how

this negotiation is shaped by dominant framings of teachers' collaborative work. And it shows that professional communities may defy simple categorization as "learning communities" or "traditional communities" by simultaneously supporting teachers to engage with innovative ideas and practices (such as "making student understanding public") and reinforcing traditional meanings (e.g., a focus on correct answers).

Chapter 4: Supporting Teachers' Equity-Oriented Learning and Identities: A Resource-Centered Approach

The fourth and final chapter, "Supporting Teachers' Equity-Oriented Learning and Identities: A Resource-Centered Perspective," looks beyond teachers' school-based communities of practice. Four kinds of resources are shown to be instrumental to teachers' professional learning: *orienting resources*, which support teachers to envision their ideal practice; *technical resources*, which support teachers with the "nuts and bolts" of enactment; *relational resources*, which support teachers is ense of connection to their work; and *positional resources*, which support teachers to take up identities as competent and valued members of their profession. I make use of Wenger's (1998) and Nasir and Cooks's (2009) insights into the relationships between learning, identity, and practice, showing how resources of different types come together to support learning and identity processes, as the two intertwine. At the same time, I find that too narrow a focus on local communities of practice à la Wenger misses the fact that the resources for professional learning that are most critical to an individual may lie outside the workplace, particularly when the learning at issue runs counter to dominant discourse and established practice.

The chapter investigates four teachers' engagement with equity- and reform-oriented practice and the ways that the resources provided by their professional networks shaped this engagement. Two of the teachers, Ryan Sower and Amanda Pepper, were deeply engaged with non-dominant ideas and teaching practices, as indicated by both their classroom practice and their participation in communities of non-dominant practice. Both committed significant time and energy to organizing and maintaining large professional networks that provided a diverse array of resources. For Ryan, strong orienting and technical resources were especially important. He felt that colleagues at his school had the same fundamental values that he did, and they provided a sense of community (relational resources) in which he was positioned as a central member (positional resources). But he also felt that they had fewer resources than he did for envisioning and enacting those values in the classroom, and he relied heavily on images of and strategies for teaching (orienting and technical resources) from an extended Complex Instruction network that included teachers at other schools and his mentor, a nationally renowned Complex Instruction teacher with whom he had spent a year as a student teacher. For Amanda, who had weaker orienting resources, relational and positional resources were critical. Her department was less unified than Ryan's, and she described "question[ing] what I'm doing every day." In this context, having outsiders from her district's Complex Instruction network validate both her general approach (e.g., seeing every student as "smart") and her personal enactment of it was critical in sustaining her engagement with her non-traditional practice.

Two other teachers, Luke McCormick and William Barrett, were relatively disengaged with non-dominant ideas and teaching practices. Both were teachers at Union, like Ryan, and both described feeling supported by a sense of mission-driven solidarity and camaraderie at their school (relational resources) as well as a variety of technical resources. However, neither had

strong orienting or positional resources. Without a clear vision to orient toward, it was difficult for Luke and William to make use of the technical resources they had. And without a sense of their own competence and capacity (positional resources), it was difficult for them to persist. Luke adopted an identity as a "struggling" and "lazy" teacher and subsequently left the profession. While William maintained an identity as a skilled and improvement-oriented teacher, he felt "disheartened" when it came to Complex Instruction and the goal of supporting and challenging each student in heterogeneous classrooms. Importantly, William recognized that he could have had more support if he were willing to reach out more, for example by attending meetings and workshops after school and on weekends, as Ryan and Amanda did. However, he felt that it was necessary for the sustainability of his work and the sake of his family to "set up boundaries."

The chapter shows that it is possible to support teachers to engage meaningfully with non-dominant ideas and practices, and it shows how resources for learning and identity development may come together to provide this support. It also raises the crucial question of how more *kinds* of resources, and not just more resources, can be made accessible to more teachers in order to improve teacher learning and ultimately, to make richer learning experiences available to all students.

Contributions

Horn and Little (2010) describe a teacher community that they studied as one in which teachers "explicitly relieved one another from blame for problems of practice, while still signaling that they were collectively responsible for student learning and conveying the expectation that they as teachers would continue to learn in and from their teaching practice" (p. 201). This balance of relief from blame and assignment of shared responsibility supported the teachers—and the students they taught—to engage in rich, ongoing, and deeply humanizing learning (Nasir et al., 2014). This dissertation endeavors to treat teachers with a similar respect, relieving them from blame for the shortcomings of their practice while also illuminating these shortcomings so that teachers—and those who shape the contexts within which teachers work—can collectively take on responsibility for continuing to learn and improve their practice.

A crucial part of this project is illustrating—as this dissertation does—the dominance of narrow, restrictive discourses and their impact on teachers' efforts to think and teach more equitably. Some readers might find it strange that I refer to equity- and reform-oriented ideas about mathematical competence as non-dominant; after all, equity-oriented reforms have acquired mainstream status, reflected in the surge in publications, conference sessions, and even newspaper headlines drawing attention to equity issues (Gutiérrez, 2013a). However, as Gutiérrez notes, "the theoretical framings of equity in mainstream mathematics education tend to reflect equality rather than justice, static identities of teachers and students rather than multiple or contradictory ones, and schooling rather than education" (p. 38). That is, popular conceptions of equity often reinforce aspects of traditional discourses of mathematical competence. "Achievement gap" discourse, for example, frames mathematical proficiency in terms of standardized test scores while reifying images of some students (indeed, entire racial and socioeconomic groups) as intrinsically less capable than others (Gutiérrez, 2008, 2013a), to the point where teachers can be found using the categories "far below basic," "below basic," "basic," "proficient," and "advanced" to describe students, not test performances (K. Seashore, personal communication, 2009). Thus, some version of "equity" may be "in," but framing mathematical

competence expansively—in ways that create opportunities for every student to engage with rich, challenging mathematics learning experiences and to develop an identity as a powerful learner and doer of mathematics—involves both practices and ways of thinking that are very much non-dominant.

Understanding how dominant discourse continues to function is therefore key for understanding what makes non-dominant practice so difficult for teachers to learn to enact—and what kinds of support teachers might need to sustain non-dominant learning. Previous work has highlighted the political nature of equity- and reform-oriented teacher learning and the ways in which professional development must prepare teachers to combat traditional conceptions of students, knowledge, and teaching (especially at the preservice level; see, e.g., Cochran-Smith, 2010; Gutiérrez, 2013b). This dissertation draws on this perspective to break new ground, bringing it to bear on the empirical study of teacher learning, in the everyday contexts in which teaching and learning take place.

This work advances the field's understanding of equity- and reform-oriented teacher learning, revealing such learning as a socially and culturally situated negotiation of competing meanings. It demonstrates how this negotiation occurs through interaction, as a collective work in progress; it shows that individual teachers' knowledge, beliefs, and identities are not achieved and then fixed but fluid and dynamic, constantly shaping and being shaped by social and cultural systems of meaning.

In elucidating both challenges and supports associated with making sense of nondominant meanings, the dissertation also points to ways in which teachers' contexts can be constructed to support engagement with non-dominant, expansive meanings. It is my hope that this research will enable scholars, policy makers, and educators to more effectively develop professional communities in which teachers can be both recipients and contributors of various kinds of resources, so that they can support all of their students to engage with rich, challenging mathematics and to develop identities as powerful learners and doers of mathematics.

Chapter 2

(Re)Framing Mathematical Competence in Everyday Instruction: Struggles and Successes of Equity-Oriented Teachers

Abstract

Drawing on a year of observations in two urban high schools, this chapter investigates the tensions that mathematics teachers face as they attempt to move away from teacher-centered to more student-centered pedagogies. Using frame analysis, I examine the opportunities that teachers created for their students to develop a sense of mathematical agency, authority, and competence. I find that despite the best intentions of the teachers in this study, many of their efforts to support all students position some students as capable of engaging with challenging mathematics and others as just the opposite. Teacher moves that expand students' opportunities to learn and develop identities as powerful mathematical thinkers are also analyzed, showing how instruction can frame mathematics and students in more equitable ways. The chapter enriches the field's understanding of how (in)equity is organized in everyday instruction and develops frame analysis as a tool for studying both instruction itself and the tensions that teachers navigate as they work to transform their practice.

Introduction

Educators, scholars, and policy makers have said for decades that when it comes to mathematics learning, "everybody counts" (National Research Council, 1989). As the National Council of Teachers of Mathematics (2000) states, "mathematics can and must be learned by all students" (p. 13, emphasis original; see also the National Commission on Excellence in Education, 1983; National Research Council, 2001; Common Core State Standards Initiative, 2010). That is, all students should be supported to develop mathematical knowledge, skills, and habits of mind. All students should also be supported to develop a sense of their own agency, authority, and competence, such that they understand themselves as sense-makers with both the right and the responsibility to contribute to the mathematical work of their classroom communities (Boaler & Greeno, 2000; Cobb, Gresalfi, & Hodge, 2009; Schoenfeld, Floden, & the Algebra Teaching Study and Mathematics Assessment Project, 2014). This perspective goes beyond closing achievement gaps or getting all students "up to standard." It entails positioning all students as valuable contributors and building on what each student brings to the complex task of learning mathematics. Such identity work supports students' content learning (e.g., through motivation and persistence; see Boaler & Greeno, 2000; Dweck, 2006). Positive identities are also worth supporting in and of themselves, as a crucial component of respectful and humane mathematics learning experiences (Boaler, 2008a).

This chapter investigates teachers' efforts to reframe mathematics so that all students, especially those who have previously been framed as unsuccessful, can experience themselves as powerful learners and doers of mathematics. This deliberate attention to *all* students points to the fact that classroom norms—a typical and in many ways, useful mechanism for studying agency, authority, and competence—do not guarantee opportunities to develop a sense of agency, authority, and competence to every child. Even in a classroom where authority rests with students in general (as opposed to being held primarily by the teacher), some students are still likely to be positioned (by the teacher, by peers, by themselves) as less competent than others. These students participate less and learn less, and everyone's learning opportunities are less rich as a result (E. G. Cohen & Lotan, 1997).

Creating equitable opportunities for students to develop a sense of agency, authority, and competence is extremely difficult. Drawing on data from a year of ethnographic observations in two diverse California high schools, I illustrate the tensions that teachers face as they attempt to simultaneously support content learning and positive identity development amongst their students. I argue that despite the potential for content learning and identity development to go hand in hand, teachers' actual efforts to support the former can sometimes undermine the latter—and even efforts to foster positive identities can undermine students' sense of their own agency, authority, and competence, depending on how content learning and student ability are framed. A focus on how instructional moves frame mathematical competence therefore affords an important lens on practices that may look equitable at first glance. In addition, I take seriously the reasons that teachers might have for backgrounding agency, authority, and competence in favor of other concerns at any given moment. This is an important step toward supporting teachers to meet the complex demands of teaching equitably, and too rarely done.

This chapter also enriches our understanding of what it takes to frame students as competent and valued contributors to mathematics classrooms by presenting detailed images and microanalyses of classroom interactions in which teachers frame ability expansively. Such images are important for developing a common vision across the field of mathematics education of the kind of teaching that is both desirable and possible.

Prior Research and Theoretical Framework

Some 25 years ago, Schoenfeld (1988) discussed "the disasters of 'well-taught' mathematics courses." He found that a geometry class that ran smoothly and that supported students to perform well on standardized tests was nonetheless problematic: students developed the belief that "mathematics is studied passively, with students accepting what is passed down 'from above' without the expectation that they can make sense of it for themselves," and that "only geniuses" can develop any kind of mathematical authority (p. 151). For the non-geniuses, competence was restricted to "performing the tasks, to the letter, as described by the teacher" (p. 151).

Since then, a body of research addressing the development of students' mathematical agency and authority has come into being. This work has theorized the kinds of agency that classrooms may make available; how norms related to agency, authority, and competence come about through classroom activity; and how such norms affect students' learning and identities (e.g., Boaler & Greeno, 2000; Cobb et al., 2009; Gresalfi, Martin, Hand, & Greeno, 2009; Staples, 2008). For example, Boaler and Greeno compared the identities that were fostered by classrooms that featured "didactic" teaching and those that featured "discussion-based" teaching. In the latter environment, mathematics was framed as social and "relational." Students were correspondingly framed as "constructive knowers" and "co-authors" of mathematical understanding. They were expected to work together to develop strategies and solutions, and to explain their ideas to one another-in other words, to exercise what Pickering (1995) has called conceptual agency. High percentages of the students in discussion-based classrooms reported enjoying mathematics and planning to pursue further mathematics courses (94% and 80%, respectively). In contrast, didactic instruction was characterized by teachers "presenting procedures that students were supposed to learn to perform" (p. 176), emphasizing knowledge of pre-established procedures and formulas (see Pickering's [1995] notion of *disciplinary agency*). Students were positioned as "received knowers"-an identity that many students found alienating. Much lower percentages of students reported enjoying mathematics and planning to pursue further mathematics courses (56% and 47%, respectively).

Similarly, studies of authority have analyzed different types of authority that students may experience and the implications for their learning (Amit & Fried, 2005; Engle, 2011). Amit and Fried, for example, distinguish *expert authority*, "a source of information and guidance [to which] one turns ... for instructions, not, by contrast, for a discussion" (p. 148) from *anthropogogical* or *shared authority*, which requires "obedien[ce] to the community of practitioners ... [of which one] is fully a part" (pp. 150-151). In other words, expert authority is intrinsically hierarchical, where shared authority is not. Amit and Fried found that although the teacher's expert authority in their work together. These students were able to "think and interact meaningfully" with one another and with mathematical ideas (p. 160), while their peers accepted others' claims with "no true dialogue" (p. 161).

These studies illustrate how different kinds of instruction can support or undermine the development of students' identities as agents, authors, and competent learners and doers of mathematics. Their findings have important implications for practice, highlighting the strengths

of some teacher moves and the problems with others. They also have important theoretical implications, demonstrating that what it means to be an agent, an authority, and a competent learner is socially constructed. Likewise, whether or not a particular individual has agency, authority, or competence is not a static property of that individual but a social construction; thus, students who appear incompetent in one setting might look quite competent in another (E. G. Cohen & Lotan, 1997; Gresalfi et al., 2009; McDermott, 1996). For example, in one classroom, a student's mistake might be framed as a flaw to be corrected, while in another, the same mistake might be framed as a valuable resource to explore and build upon. In other words, the same behaviors may have vastly different meanings in different contexts. These meanings are themselves dynamic and constantly negotiated in interaction (Gresalfi et al., 2009; Hand, 2010).

As the primary organizers of students' mathematics learning opportunities and the typical holders of classroom authority, teachers play pivotal roles in shaping the forms and degrees of agency, authority, and competence that are available to students. But a significant body of research documents the persistence of traditional practices and conceptions of students that limit students' opportunities to develop a sense of their own agency, authority, and competence (Borko et al., 1992; D. K. Cohen, 1990; Parks, 2008). Some studies highlight technical barriers to change, for example, weaknesses in teachers' pedagogical content knowledge (Borko et al., 1992; Hill et al., 2008). But knowledge is clearly not the whole picture; teachers must also navigate discourses and ideologies that reinforce traditional practice, whether or not they endorse them (Gutiérrez, 2013; Oakes, Wells, Jones, & Datnow, 1997; Parks, 2010).

This chapter uses *framing* as a way of making the social construction of agency, authority, and competence—and the embedded negotiation of dominant and alternative discourses—visible and available for examination (Hand, Penuel, & Gutiérrez, 2012). Framing creates an interpretive context that communicates to participants an answer (or answers) to the question, "What is it that is going on here?" (Goffman, 1974). This involves frames that describe the nature of the activity in which participants are engaged (for example, as completing worksheets or as an opportunity for sense-making; see Hammer, Elby, Scherr, & Redish, 2005), as well as frames that position participants (for example, as knowledgeable speakers or as novices obligated to listen). As the gerund form of the word implies, *framing* is an active process—albeit one in which participants often engage without conscious effort or intention.

Greeno (2009) elaborates two aspects of framing with particular relevance to education, which he terms *epistemological framing* and *positional framing*. Epistemological framing "refers to the kind of task that a participant or participants understand themselves to be engaged in, especially regarding the kind(s) of knowledge that are relevant to and expected to be constructed in order to succeed in the task" (p. 271; see also Hammer et al., 2005). Positional framing "refers to ways in which an individual is entitled, expected, or perhaps obligated to participate in interactions of an activity system, such as a classroom" (p. 272).¹ These two aspects interact,

¹ This term combines two ideas—framing and positioning—in ways that are both useful and potentially confusing. In this chapter, I use *positioning* to talk about the identities and positions that are made available to *particular* individuals from moment to moment, whereas I use *positional framing* to talk about the range of positions that are made available to a *class* of individuals (e.g., students, teachers, police officers, etc.). Positioning and positional framing are mutually constituting but distinct. For example, within the positional frame of students *as a group* as empty vessels, a *particular* student may be positioned as good at acquiring new information (competent or smart) or as bad at the same (incompetent, not smart).

often in mutually reinforcing ways. For example, the common epistemological framing of mathematics learning as a task that is principally about acquiring and applying established rules works in concert with the equally common positional framing of students as empty vessels, whose principal obligations are to listen to the teacher, write notes on the teacher's lectures, and solve problems using the methods the teacher has demonstrated (cf. Boaler & Greeno, 2000; Cobb et al., 2009).

Frame analysis is a powerful tool for studying the construction of agency, authority, and competence for many reasons. First, frame analysis captures the tacit, taken-for-granted "metamessages" (Bateson, 1972) through which what counts as competent behavior and who is positioned with authority are often organized in moment-to-moment activity. Methods that rely on explicit articulations of these messages inevitably miss the action, so to speak. A second advantage of frame analysis is that it does not need to assume individual participants' intentions in order to interrogate the functions of the frames they employ. This is significant both because it can be quite a challenge to know what an individual's intentions are or were, and because an utterance may invoke a frame quite independent of the speaker's intention, with non-trivial consequences for participants. For example, teachers rarely if ever intend to communicate that students' ideas about mathematics do not matter, yet they frame such ideas as worthless when, day in and day out, they stand at the front of the room delivering lectures, asking questions that elicit only one- or two-word answers from students.

The ability of frame analysis to make the tacit visible is especially important when social hierarchies that are widely understood as unacceptable but which nonetheless persist in everyday life are at issue (e.g., racism, sexism, and hierarchies of intelligence and ability). Actors can easily position themselves and others as inferior or superior without meaning to do so, through inadvertent references to personal histories or broader cultural frames (Davies & Harré, 1990). For example, a teacher might praise Amy for performing a simple calculation correctly while praising Ben for creating a novel problem-solving strategy. The teacher might intend to encourage both students, but the nature of her encouragement could position Ben as having greater competence and authority than Amy, especially if Amy has a history of low achievement and Ben has a history of high achievement, or if Ben and Amy have other "status characteristics" (E. G. Cohen & Lotan, 1997) that are stereotypically associated with high and low mathematical competence, for example, if Ben is a boy and Amy is a girl, or if Ben is White and Amy is African American. Neither Ben nor Amy has to accept the ways that others frame or position them in order for the act of framing to affect their identities, their learning, or their achievement, especially as particular ways of framing are repeated over time-for example, by limiting their access to rigorous coursework (Darling-Hammond, 2010; Tyson, 2006) and by imposing burdens associated with both consciously and unconsciously navigating stereotypes (Horvat & O'Connor, 2006; Nasir & Shah, 2011; Steele & Aronson, 1995).

Frame analysis is also useful because it can illuminate deliberate efforts to disrupt dominant frames by replacing them with more equitable frames (Hand et al., 2012). These efforts may be significant even if they are not taken up by all participants to a given situation in a given moment, because reframing competence in systems as familiar (and familiarly hierarchical) as mathematics classrooms typically requires a great deal of persistence—as anyone who has ever tried to convince "dumb" students that they are smart will know.

Methods

Research setting

The data for this study were collected at two high schools, which I call Union and Boxer (all school, teacher, and student names are pseudonyms). The schools were part of the same school district in a mid-size city in California. Both schools were racially diverse, with significant Latino, Asian, and African American populations but few White students (under 10%). At each school, more than half of the students were classified as English learners, and almost three quarters were classified as socioeconomically disadvantaged. Both schools struggled with the state standardized mathematics exam, with fewer than 20% of students meeting the "proficient" standard in 2012.

I selected these schools because of their mathematics teachers' common commitment to a professional development (PD) program offered by their school district, called Complex Instruction in Secondary Mathematics (CI; see Boaler & Staples, 2008; E. G. Cohen & Lotan, 1997; Nasir, Cabana, Shreve, Woodbury, & Louie, 2014). CI posits that all students have important intellectual contributions to make to their classroom learning communities, and that teachers are responsible for drawing out every student's strengths while also helping every student develop new strengths. CI PD targets these beliefs as well as instructional strategies for "equalizing status"—i.e., for leveling hierarchies of perceived ability and worth—such as the use of complex, open-ended tasks that require students to pool their various skills and work together. CI PD also emphasizes the development of robust, department-based professional support networks, and teachers at Union and Boxer had dedicated time each week to collaborate around mathematics instruction. In addition, they participated in periodic CI trainings and coaching sessions. This array of resources made Union and Boxer rich contexts in which to examine teachers' ideas about what it means to be "good at math" and their efforts to enact these ideas in their classroom instruction.

Data collection

From the 13 mathematics teachers at Union and 5 at Boxer, I recruited 6 focal teachers: 4 from Union (Ryan Sower, William Barrett, Cyril Nazemi, and Luke Farber) and 2 from Boxer (Amanda Pepper and Rob Daly). Teachers were selected based on factors that appeared to be connected to the depth of their engagement with CI in my early observations. Specifically, I selected for range along two dimensions: years of experience teaching, and leadership roles in the district CI community. But, as described above, all teachers were invested in CI to some degree; they all attended a week-long CI summer training course and follow-up sessions throughout the year, and they all received support from their district's CI coaches. In addition, during interviews all six teachers reported that fostering students' sense of competence and ownership of mathematics was an important personal goal.

I observed each teacher's classroom instruction 4 to 8 times over the course of the 2012-2013 school year. I also observed routine meetings of teachers' department and course teams (30 hours of meetings at Union, and 11 at Boxer) and CI PD sessions (70 hours). Midway through the data collection, I elected to focus more intensively on Union, since teachers there collaborated more regularly than the teachers at Boxer did. The data for this chapter are drawn primarily from Union. I continued to observe sporadically at Boxer, however, to enrich my perspective on the goings on at Union, and this is also reflected in my use of data here. Audio recordings, field notes, and photographs of whiteboard inscriptions, worksheets, and other artifacts were produced for each observation. All audio recordings were transcribed.

Data analysis

I developed a coding scheme to capture teachers' framings of mathematics and of students, using initial coding procedures (Charmaz, 2006) on a subset of routine teacher meetings (see Chapter 3). This process coordinated a "bottom up" approach grounded in the data with a "top down" approach drawing on frames suggested by the literature—i.e., those provided by traditional ability discourse as well as alternative framings presented by Complex Instruction. I paid particular attention to keywords that denoted or implied comparisons between students (e.g., "high," "low," "percentile," "smart"), using these terms to identify passages for further analysis. The coding scheme was subsequently refined and validated through application to the full corpus of teacher meetings and interviews. I then applied the codes to classroom instruction to see how the framings that arose in teachers' conversations with each other appeared in teachers' work with students.

This chapter includes extended transcripts of groupwork episodes in which contrasting frames are evident. (A glossary of transcript symbols appears as Table 4.) These transcripts give readers detailed images of practice, providing rich resources for fine-grained analysis and for developing a holistic vision or "feel" for the interactions they depict.

Table 4

Symbol	Description
_	Interruption, including self-interruption
(.), (5s)	Very slight pause, 5-second pause
]	Beginning of overlapping utterances
?	High rise in intonation
	Low fall in intonation
(())	Unclear utterance; any words in parentheses are a best guess
<i></i>	Reading aloud
underline	Marks non-English words
italics	Marks comments such as translations and descriptions of gestures and other non-verbal actions

A glossary of transcript symbols.

Findings

I begin this section with a summary of the epistemological and positional frames that the teachers at Union and Boxer invoked in their work with students, connecting these frames to students' opportunities to develop a sense of mathematical agency, authority, and competence. I

then show how these frames came to life—intertwining with one another in ways that were sometimes mutually reinforcing and sometimes contradictory—in classroom instruction.

Epistemological framing

Recall Greeno's (2009) definition: "Epistemological framing refers to the kind of task that a participant or participants understand themselves to be engaged in, especially regarding the kind(s) of knowledge that are relevant to and expected to be constructed in order to succeed in the task" (p. 271). As this definition implies, any epistemological frame is composed of many sub-frames (e.g., what kind of knowledge is relevant to the task, what kind of knowledge participants should aim to construct, how this knowledge should be displayed, etc.). I found that clusters of sub-frames hung together in classroom activity, gaining coherence from being used in concert. I group these sub-frames under the headings *restrictive* and *expansive* to highlight their discursive functions. *Restrictive* frames limit students' opportunities to develop a sense of agency, authority, and competence. In the process, they exclude many students. In contrast, *expansive* frames afford a wide variety of opportunities for students—with diverse strengths, interests, and needs—to develop a sense of their own agency, authority, and competence.

Restrictive framing. Mathematics teaching at Union and Boxer often reified the *school math frame* (cf. the "doing school" frame in Hand et al., 2012). The curriculum generally treated mathematics as composed of discrete chunks of knowledge and skill, proceeding from the basic to the sophisticated—a framing that is typical in American mathematics instruction (Ernest, 1991; Parks, 2010). As William Barrett said in a planning meeting at the beginning of the school year, "[S]tart with something simple, and build on it and build on it and get more complex. … That's how I think about building a unit, is like, a very linear kind of thinking." Assignments granted students a narrow, constrained form of agency, authorizing them to use the procedures presented by their teachers and textbooks (evident in teachers' invocations to "use your notes"; cf. Pickering's [1995] notion of disciplinary agency). The main evidence that teachers gathered to assess students' learning and understanding took the form of written tests and quizzes, with problems like those used in teacher demonstrations and textbook examples.

Expansive framing. What I call the *multidimensional math frame* (cf. Boaler & Staples, 2008) appeared in my data as an alternative to the school math frame. This frame casts mathematics as inherently complex and multidimensional. Within it, doing math involves the use of familiar procedures, as in the school math frame. But equally if not more important are skills that are not traditionally valued in mathematics classrooms, such as organizing collaborative problem solving, making appropriate use of technology, connecting across different mathematical representations (e.g., equations, tables, diagrams, and graphs), and especially communicating one's ideas and explaining one's reasoning. This frame affords students disciplinary agency and much more, with a wide variety of ways for them to contribute to their classroom communities and to experience themselves as competent. Students are expected to make their own sense of mathematics and to express their understandings in a variety of ways: orally, in writing, following conventions, and according to their own thinking. In the same meeting referenced above, William mused about structuring the curriculum in a less linear way in order to foster multidimensionality, saying, "I wonder like, can we start with something more challenging [instead of starting with the basics], and see what kind of smartness comes out from the students."

Positional framing

Positional framing "refers to ways in which an individual is entitled, expected, or perhaps obligated to participate in interactions of an activity system, such as a classroom" (Greeno, 2009, p. 272). As with epistemological framing, many sub-frames can come together to create a positional framing. I found that in practice, sub-frames that were ideologically consistent with one another were often deployed independently. I nonetheless group them under *restrictive* and *expansive* headings, again to highlight how these frames function to limit or expand students' opportunities to develop a sense of agency, authority, and competence.

Restrictive framing. Teachers framed students and their abilities restrictively (i.e., in ways that limited students' opportunities to develop a sense of agency, authority, and competence) using two sub-frames: the *high-low frame* and the *expert-dependent frame*. The high-low frame positioned some students as competent, "strong," or "high-level" and others as "struggling," "weak," or "low." These labels appeared regularly in teachers' routine meetings, both reflecting and re-enacting culturally dominant frames of intelligence and ability (Oakes et al., 1997). Though teachers did not use these words with their students, their instruction sometimes positioned students in relation to ability hierarchies nonetheless, as when they kept calling on the same few students for correct answers to their questions, or when they always began groupwork by checking in with the same few students to make sure they knew what to do while allowing other students to proceed on their own. This latter behavior also invokes the expert-dependent frame, which positions (some) students as dependent on the mathematical authority of the teacher, or sometimes of more expert peers, to explain how to solve problems of a particular type, to provide help if students got stuck, and to determine the validity of answers. Notably, students often positioned themselves as expert-dependent, telling the teacher, "I need help," or asking, "Is this right?" Teachers could choose to accept and reinforce this frame, or they could reframe students expansively, as discussed below.

Expansive framing. Two expansive sub-frames appeared in my data, roughly paralleling the restrictive sub-frames—probably not by coincidence, as expansive framing was often a deliberate response to the restrictive framing that dominated both teachers' and students' experiences of learning math. The *multi-ability frame*, in contrast to the *high-low frame*, described all students as having intellectual, mathematically relevant strengths. Teachers sometimes invoked the multi-ability frame with general statements, such as Ryan Sower's comment during a lesson launch: "You guys have a lot of different ways you're smart, just in your team. I want you to have a chance to show it." Other times, teachers were more specific. It is worth quoting Amanda Pepper at length, to illustrate the variety of strengths that can be (but generally are not) made visible and valorized in mathematics classrooms. Amanda ended a lesson by listing specific ways that students could have been "smart in math" that day and asking them to reflect on their own smartness, saying:

[I want] you to tell me, by writing one or two things, what you felt like you were smart in math today doing. Okay? For those in the back, I'm going to read them one time through so you can see. *Referring to a list that is being projected at the front of the room, she says:* One, can you recognize patterns? Two, did you show patterns using multiple representations such as tables, or graphs, or rules? Did you use technology to help you solve the problem? I saw a couple graphing

calculators out. That's a great way to be smart in math. Did you persist, did you keep trying even if you were stuck? Did you predict something that you cannot see? Can we see the a hundredth pattern? ... Did you make sure that everyone could understand and explain the problem? Did you ask questions? And did you organize in a clear way so that other people could understand your work. ... Pick as many as you want, because that's smart in math. Okay? And I'm gonna walk around and see, how did you think you were smart today in math.

The multi-ability frame implies the *students-as-resources frame*, the expansive counterpart to the *expert-dependent frame*. In the students-as-resources frame, students are positioned as capable of tackling and surmounting challenges by pooling their many strengths and working together. No student is positioned as an expert; rather, all students are positioned as contributors who must both share their resources and depend on the contributions of their peers. Together, they have not just the authority but also the responsibility to generate their own knowledge and strategies and to make their own sense of mathematics. A common use of the students-as-resources frame in my data was a response teachers sometimes gave to students' requests for help: instead of stepping in to solve problems for students, they would turn the problem back to the group. When done well, this move supported students to view themselves and each other as competent and mutually dependent equals.

Frames in classroom activity

To illustrate more fully how the teachers in my study framed mathematics ability in their classroom instruction, I examine two instructional practices: adapting and assigning tasks, and responding to students' struggles. Although the teaching profession lacks an agreed upon set of component practices (Grossman & McDonald, 2008), I submit that these two practices are essential for successful teaching and also common across virtually all instances of teaching, regardless of differences in individual teachers' styles and approaches. Certainly, they were part of everyday instruction for each of the six teachers in this study. There was also substantial variation in how teachers enacted the practices. These practices therefore warrant close analysis to illuminate the opportunities for students to develop a sense of mathematical agency, authority, and competence that are created by different frames, and to show teachers' struggles and successes in organizing such opportunities through expansive framing.

Adapting and assigning tasks. Especially at the beginning of the school year, the teachers at Union talked frequently and explicitly about how to "meet the needs of every student" in their heterogeneous, untracked classes. They wanted to make sure that *all* students— not just those "in the middle"—received instruction that challenged them to learn and grow, and they designed what they called *menus* with this goal in mind. But these menus tended to frame mathematics and students in ways that *restricted* students' opportunities to develop a sense of agency, authority, and competence. As I show, the school math frame and the high-low frame were fundamentally embedded in the way the teachers conceptualized menus.

The Union menus were multi-page tasks that began with "appetizers," simple practice problems that all students were expected to complete. For example, members of the Geometry Team used a menu for their trigonometry unit that began with the problems shown in *Figure 2*. As the directions at the top of this particular menu explained: "First, you will complete the problems on this page, then check your answers on the top of the back side. You can then choose

the problems that you need to practice, always checking your answers as you go." Based on the correctness of their work on the appetizers, then, some students would continue working on similar problems; the next eight problems on the menu were nearly identical to the three shown in Figure 1, with only the numbers changed. In the meantime, students who answered all of the appetizer problems correctly would go on to more complex problems, such as those in *Figure 3*.

Solve for the variables below. Label and show all your work.



Figure 2. "Appetizer" problems at the beginning of a menu used by Union High School's Geometry Team.

1. Given: DE = 4 in. Find the length of EF.



2. Mr. S is a frequent jay walker. Because of his geometry knowledge, he always crosses the intersection of 17th St and D St. at a 34° angle diagonal right through the center of the intersection. When the police stop him one day all he says is, "But the hypotenuse is soooooooo much shorter than the two legs combined." If the distance to walk down 17th St and legally cross D Street (one leg) is 50 ft, how long is his diagonal walk? What distance (in ft) did Mr. S save by walking the diagonal, rather than using the crosswalk like a normal law-abiding citizen. [*sic*]

Figure 3. "Trig Challenges" and "Trig Situations" near the end of the menu.

Like all three of the menus I saw at Union (one for each unit in the spring semester except for the last unit, which was cut short to accommodate the standardized testing schedule), this assignment embodies the school math frame. It presents mathematics as a subject that must be mastered in small increments, building from discrete skills that the teacher has demonstrated with several examples isomorphic to the problems students are to solve (e.g., to solve for an unknown side length in a right triangle using tangent, sine, or cosine). The "dessert" problems use these same basic skills but also require students to remember content from other units (e.g., what angles are formed when parallel lines are crossed by a transversal), to interpret more complex diagrams and stories, and to coordinate these various resources in multi-step solution processes. These demands begin to layer the multidimensional math frame over the school math frame, but for many students, multidimensionality remains invisible; in actual use, only one or two students per class reached the challenge problems for any given menu, while the rest spent the entire period on basic practice.

Union's menus also limited students' opportunities to develop a sense of agency, authority, and competence by framing students in restrictive, hierarchical ways. The very structure of the menus positioned some students as high—competent and capable of tackling challenges—and others as low, incapable of tackling challenges. Indeed, this was how the teachers talked about students in the meetings where they discussed menus. The idea of using menus arose out of their concern that students at the "high end" were "bored," and they talked about challenge problems as being for the fastest kids, "your 100% students." All students theoretically had access to these problems, but it was not expected that all students would actually work on them—much less that students who worked slowly or experienced obstacles would.

Importantly, teachers themselves viewed the menus as a mechanism for *supporting all students*, academically as well as emotionally. Many of them saw menus as providing students with opportunities to work on the mathematics that was appropriate for them, thereby promoting their content learning while also giving them a chance to feel successful—something that would not happen if "low" students were assigned problems that were too difficult, or if "strong" students were assigned problems that were too easy. As Emilio Gonzalez told his colleagues, "[K]ids want to be pushed ... so I think, that's where differentiation comes in. Being able to push all students so that they all feel like they're being pushed, but that they can meet those demands." William Barrett aimed to empower students with menus in an even broader sense. Speaking specifically about the answer-checking feature, which he hoped would promote students' "metacognition," he said, "[I]f we help them learn how to reflect and really know what they don't know and what they do know, then I think they can make good choices for themselves." In his eyes, then, the menus he used were a means of supporting students to take control over their own learning.

Menus were thus a caring and sensible response on teachers' part to apparent disparities in their students' skills and ability levels. But this response was enmeshed in restrictive framings of mathematics as linear and two-dimensional and of mathematical competence as hierarchical. Teachers did not invent these frames or even consciously enact them, but through menus and other means, their teaching still functioned at times to give some students opportunities to learn rich mathematics and to develop a sense of agency, authority, and competence, while denying these opportunities to others.

An alternative practice. The approach to challenging all students described above is so commonplace as to seem natural. In contrast, Complex Instruction presented teachers with the idea of the "groupworthy problem," a task so rich and complex that no one student can complete it alone, but every student has something to contribute (E. G. Cohen, 1994; Horn, 2005). Embedded in the idea of the groupworthy problem, then, are expansive framings of mathematics as multidimensional, requiring many different kinds of strengths; of every student as a thinker with strengths that are valuable and necessary; and of students as key resources for each other, in relations of mutual dependence.

The teachers at Union and Boxer used the word "groupworthy" to describe multi-day explorations with strong hands-on components. The "roof problem" was one example: students were asked to work with their tablemates to construct a model of a 30-foot antenna in the center

of a rectangular roof with given dimensions, then calculate the amount of wire that would be required to anchor the antenna to the corners of the roof. Teachers appreciated the levels of student engagement that such activities could generate, yet they were wary of using them too often because they required a significant investment of class time without necessarily producing a big "pay off" in terms of mathematics learning. However, Ryan Sower (at Union) and Amanda Pepper (at Boxer) often structured students' work on relatively routine tasks in ways that made them more complex and groupworthy. For example, Ryan assigned a "group challenge" with the problems shown in *Figure 4*, which do not appear particularly groupworthy or multidimensional. Indeed, his colleagues assigned the same problems as individual work, in the form of a menu.

Group Challenge #2 (excerpt)

Find the area and perimeter of the polygons below.



Group Challenge #4

Directions: Shade in B and label H. Then find the Surface Area and Volume of the polyhedron below. Make sure to show all your work and include units.



Figure 4. Parts of a "group challenge" in Ryan Sower's Geometry class.

For students working on the problems as a menu, the primary task was to recall (or look up) the correct formulas to use, then apply those formulas accurately. But Ryan's instructions made his students' primary task communication and reasoning. Twice, he reminded students that they would be graded on both the content of their mathematical thinking and on "how you are working together as a team." He emphasized sharing ideas and asking each other questions:

I want to hear lots of teams asking questions. What kinds of questions should you be asking? Well. Here's a couple of good ones, right? *He gestures toward posters on the wall above the board*. I want to be hearing these after teams are finished, or if you're stuck, here's one you could definitely be asking, right. "What should we

do next?" *A handful of students practice pronouncing "should.*" I like that, thank you. I think we can find the base face area first. Or let's try to figure out the height. Right. This is also something that I want to be hearing, right? "Do we agree? Do we agree that the answer is correct?" And this might be the most important, which is why I put it by itself. "How did you do that? How did you get that answer?" That's a question I want to hear, if you're not sure especially.

Structured this way, this assignment frames mathematics as multidimensional, highlighting social aspects—asking questions, giving explanations, and managing group dynamics—as well as more traditional content-focused ones. This expansive framing of mathematics in turn creates space to frame students expansively. Students who have not yet memorized the correct formula or figured out how to apply it can learn this content while simultaneously contributing meaningfully to their group's work. And indeed, there were many instances in this lesson in which students' interactions with each other forced them to think more deeply—about why a given number should be plugged into a formula while another number should be ignored, about why a particular formula works in the first place, or about how to articulate understandings that they had not previously put into words. Importantly, Ryan did not frame working together as an activity in which students who "get it" need to help students who don't, something that often occurred during group work when teachers did not explicitly structure their group tasks as "group challenges." This supported him to position all students as competent doers of mathematics with important contributions to make to their groups in his subsequent interactions with students as they worked.

Thus, the requirements of "group challenges" gave students expansive opportunities to develop a sense of agency, authority, and competence by expanding what counted as a competent contribution to the mathematical work of the class. This gave students a range of opportunities to see themselves as smart, even if they had not been successful in math class before. In addition, by calling the entire assignment a "group challenge" for all students to work on with their teammates rather than as a menu with challenge problems at the end, this task structure frames all students as capable of engaging productively with difficult mathematics.

Responding to students' struggles. Struggle is an important part of learning, not only in terms of the content at hand but also in terms of its potential to support learners to come to see themselves as capable of changing, growing, and surmounting obstacles (Dweck, 2006). But struggle can also be counterproductive, discouraging students and leading them to see themselves as incompetent and incapable. Managing struggle is therefore important for the development of students' sense of agency, authority, and competence; well-managed struggle can foster such development, while poorly managed struggle can hinder it.

A common strategy for managing students' struggles amongst the math teachers at Union and Boxer was to recruit students to help each other, as in the following episode. The episode takes place in William Barrett's "newcomer" Geometry class, a class for students who are recent immigrants to the U.S. and who are formally classified as English Language Learners. In the episode, Assad, Efrain, Juan, and Hakim are seated together at a table (see *Figure 5*). Assad and Hakim speak Arabic as their first language; Efrain and Juan are both native Spanish speakers. Hakim and Efrain speak English hesitantly but proficiently. The episode begins with the teacher's arrival at their table. He looks at their work and notices that Juan and Hakim have made similar sketches (see *Figure 6*) but arrived at different answers. In the first part of the episode (1a), William makes an extended effort to help students explain their thinking to each other. He thus frames both mathematics and students expansively, highlighting communication and explanation as important mathematical practices and students as mathematical thinkers with valuable ideas that deserve their peers' attention.



Figure 5. Seating arrangement in Episode 1.



Figure 6. The task students are working on in Episode 1. Given information is shown in black. Juan and Hakim's common inscriptions are shown in green.

Episode 1a. "Look here for a minute."

1 2 3 4 5 6 7	Teacher:	Hey, can you guys look here for a minute? Uh, Assad and Efrain. I want you to see something. Both Juan and Hakim? Used the area, right? Used the pictures of squares, to set up their problem. And they both have 100 here and 9 here, right? Assad can you sit up? Can you can see this? I want—okay, so they set it up, 100 and 9, 100 and 9. Because 10 times 10 is 100, 3 times 3 is 9. They both agree, right? Now, here, Juan has 109, (()) but Hakim, you don't think so.
8	Hakim:	10 times 10. When I do minus, 100—100 minus 9, 100 minus 9 is 91.
9 10	Teacher:	Why, Hakim? (2s) I agree that 100 minus 9 is 91. Why do you think this is going to give you the right answer? (4s)
11	Hakim:	I don't know how to say it really.
12	Teacher:	Can you write it down? Show me what you're thinking, using the numbers?
13 14		Hakim shows work on the paper in front of him but doesn't say anything. (11s)
15 16 17 18	Teacher:	Efrain, I want you to listen to this too, and Assad. I want you to listen. Because this is an important, this is an important thing that they both are— they're solving different ways, okay? I'm gonna close the door. Keep talking. (9s)
19	Hakim:	Because we need to find, ((the answer though)).
20 21	Teacher:	Okay. (3s) And, okay. Juan, why did you put 109? (2s) I want you to explain, to everyone.
22 23		<i>Efrain puts his paper in the middle of the table and says something in Spanish to Juan.</i>
24 25	Teacher:	Hold on let's look at Juan, and I wanna see if you agree with one of them, or $(())$.
26	Efrain:	But I haven't done it.
27 28		Efrain and Juan talk about the problem in Spanish. After 30 seconds of this, the teacher stops them.
29 30	Teacher:	Juan and Efrain, hold on. I want you guys to try in English, okay. I want you to try in English because Hakim cannot speak very much Spanish. Okay.
31	Efrain:	To Hakim. You speak Spanish.
32	Hakim:	No.
33	Efrain:	Yes.
34 35 36	Teacher:	Uh, about as much as me. So. Hakim, Hakim, you know, described a little bit why he put 91. I want Juan to say why he put 109. I want you to try in English.
37	Juan:	(())

Teacher:	The area?
Juan:	Mmhm.
Teacher:	(2s) But it's different, because—what did you do to these two numbers? Did you add or subtract?
Juan:	(())
Teacher:	Can you show, show, show your group on your paper. Hakim, Hakim said subtract, right. Hakim said 100 minus 9, right? Equals 91, which is how he got this, right. What did, what did you do, Juan?
Juan:	(())
Teacher:	Okay show your group on your paper.
Juan:	I don't understand the uh— <u>no no no sé, espérate</u> . (" <i>No no I don't know, wait</i> .")
	Teacher: Juan: Teacher: Juan: Teacher: Juan: Teacher: Juan:

William's efforts to frame students as resources for each other and to frame doing mathematics as more than answer-getting are evident in this episode. Nine times in 4 minutes and 8 seconds—nearly every time he speaks—he prompts students to share and explain their methods (lines 9-10, 12, 17, 20-21, 29-30, 34-36, 40-41, 43-45, and 47). And when Hakim and Juan struggle to articulate their thinking, he doesn't immediately provide his own explanation, but instead attempts to support them, suggesting that they show their thinking in writing if they can't communicate it orally. As the episode continues, however, William steps in with more directed questions and his own reasoning, layering restrictive frames over the expansive ones he invoked earlier in the episode.

Episode 1b. "Juan is correct."

50 51 52	Teacher:	So what, what did you do to the two areas? Did you add or subtract? (6s) Look. <i>He points at the vocabulary words on the wall. Juan looks up.</i> Turn the other way. Turn your head the other way. Did you add or subtract?
53	Juan:	Oh I, subtract.
54	Teacher:	You subtracted? This is subtract?
55	Juan:	Oh, is the—I don't know. Subtract is the—
56	Teacher:	You see the blue sign?
57	Juan:	Uh. It's uh, it's plus.
58 59 60 61	Teacher:	Plus. Plus is the name of the symbol, the verb is? (2s) The verb is to add, okay? So Juan added, and Hakim subtracted. Okay. Which is correct? (6s) Let's think. Which, which one is the biggest area? Which one is the biggest area, the legs or the hypotenuse.
62	Hakim:	The hypotenuse.
63	Teacher:	The hypotenuse is the biggest. So is this ok?

64	Juan:	No.
65	Teacher:	Is this the hypotenuse, [squared?
66	Juan:	[Yeah.
67 68	Teacher:	Yes, this is the hypotenuse squared, because here is the right angle, right, so that's okay. What about—Hakim, where is your hypotenuse?
69	Hakim:	It's right here. He points to the hypotenuse.
70	Teacher:	Here. So is this the biggest area? Indicating 91, Hakim's answer.
71	Hakim:	(1s) Yeah.
72	Teacher:	It's, it's bigger than 100?
73	Hakim:	No.
74 75 76	Teacher:	No. So is this possible? (5s) It's not, right, because this is the hypotenuse, right? So this must be the biggest side and the biggest area, right? So you do add, you add—these are the two leg areas, right, so you do add. So Juan—
77	Hakim:	Juan is good?
78	Teacher:	Juan is correct. Because you add the two leg areas to get the hypotenuse area

From line 50 onward (1 minute, 42 seconds), William engages the group in a series of initiation-reply-evaluation sequences (Mehan, 1978). He initiates by asking closed-ended questions to which Juan and Hakim give 1- to 3-word answers, which he then evaluates, repeating the answer if it is correct or repeating his question if it is not. He ends his interaction with the group by giving his own explanation of how to solve the problem at hand, and by making the correct solution explicit. When he leaves, just under 6 minutes after arriving, Juan and Hakim have hardly explained their work, and the expectation that either of them do so has been buried under the teacher's explanation. Opportunities to position Assad or Efrain as resources for the group's learning have similarly been lost.

William's help in this episode effectively layers frames that afford fewer opportunities for students to develop a sense of agency, authority, and competence over frames that afford more. By the episode's conclusion, what has surfaced as mathematically important is knowing how to apply the Pythagorean Theorem (more specifically, when to add and when to subtract) to solve for a missing length—more in line with the school math frame than the multidimensional math frame. Students themselves, initially positioned as owners of important ideas worth sharing with others, end up positioned as dependent on the teacher to provide reasoning and to evaluate answers.

Although it is impossible to know exactly why William shifted frames in the middle of his interaction with Juan, Hakim, Assad, and Efrain, there are many plausible reasons. One is an unwillingness to leave students stranded to sort through points of confusion without guidance. Concern for content learning, narrowly construed, might contribute to this unwillingness; in intervening as he did, William ensured that students heard correct reasoning and found out what the correct answer was. He might have considered this key for mastering the skills at hand and been uncertain that it would happen if he left the students to their own devices. William might also have felt pressure to prioritize correctness because of the intense urgency of raising
students' scores on the state standardized test, where correctness is all that counts (Union stood to lose a significant portion of its funding if scores did not go up). Relatedly, William might have been worried that there was insufficient time for students to continue to ruminate over explanations; in meetings, he frequently expressed anxiety about the amount of material that he needed to teach, and more than once, he explicitly acknowledged the tension between "depth"— allowing students time to explore and make their own sense of mathematical ideas—and "breadth," or coverage over a wide variety of topics, all of which were part of the state standards for Geometry.

It might also have been the case that William was attending to students' emotional wellbeing. He might have experienced students' struggles in this episode as mathematically unproductive and even damaging to their identities as competent thinkers. From such a standpoint, his intervention helped them save face; in asking questions that required short, clearcut answers, he gave them opportunities to act and feel as though they did know something about mathematics.

This array of possible reasons for acting as William did is important to attend to because it points to the complexity of teaching in a way that supports students to develop a sense of agency, authority, and competence. Without question, William intended to do what was best for his students, and within a restrictive frame of reference—one that frames mathematics as a collection of discrete skills and students as teacher-dependent—he was a very effective teacher. All this notwithstanding, from the perspective of an expansive frame of reference, his responses to students' struggles in the episode above failed to afford them identities as powerful learners and doers of mathematics.

Another way that teachers in this study responded to students' struggles was to deliberately withhold help, framing students as capable of independent problem solving. For example, a teacher had this exchange with a student who was stuck on the "dessert" portion of a menu:

Teacher:	You get it?
Matthew:	No.
Teacher:	Keep working on it. It's a tough one. (.) Break it up into shapes.
Matthew:	I already did though.
Teacher:	Keep going. It's a good challenge for you, Matthew.

This encouragement may or may not have supported Matthew to persist with his work and to see himself as competent. Either way, however, it was problematic, because the teacher did not position all of his students this way—with consequences (albeit unintended ones) along gendered and racialized lines. He spent most of the class period giving students hints on basic "appetizer" problems, pointing out their errors, telling them which formulas to use, and at one point taking the pencil from a brown-skinned girl in a wheelchair and saying, "Let's fix that. I'll fix it for you, okay?" The contrast between these responses and his response to Matthew—one of three Asian boys in the class, and the only student to reach the "dessert," as far as I observed—invokes the high-low frame on top of the expert-dependent frame. Thus, efforts to give each student what he or she needs, whether part of the task structure (e.g., Union's menus) or part of teachers' interactions with students, can function in ways that teachers do not intend, dramatically constraining opportunities for many students to develop a sense of agency, authority, and competence and reinforcing stereotypes about who is "good at math" and who is not.

An alternative practice. Knowing that Hakim and Juan are English language learners, and seeing the ways that they struggled in Episode 1, it can be difficult to imagine how else William could have supported their opportunities to learn the lesson's mathematical content. One might reasonably ask if it is possible to simultaneously provide such opportunities and support students' sense of agency, authority, and competence in the context of these students' needs.

Episode 2, taken from Ryan Sower's Geometry class, provides a useful contrast. As in Episode 1, the students are English Language Learners in a newcomer class, and they struggle somewhat to express themselves in English. But the teacher leaves significant mathematical work to them, walking away well before they have solved their problem. He provides a high level of support, but for the most part, this support takes the form of encouragement and scaffolding for their interactions with each other, rather than scaffolding that reduces the problem solving demands of the mathematical task.



Teacher Dashiin



The episode begins with Ryan's arrival at Ana, Carmen, and Dashiin's table (see *Figure* 7). Ana and Carmen are native Spanish speakers; Dashiin immigrated from Mongolia less than two months prior to the episode (and he is the only Mongolian speaker in the room). The class has been practicing finding missing angle measures and side lengths of right triangles using tangent ratios. They have just begun to consider other trigonometric ratios (i.e., sine and cosine). They are working to find the lengths of the legs in triangles where the length of the hypotenuse and the measure of one angle are labeled. Ana has called the teacher over to answer a seemingly simple question: they want to know which side of a particular triangle is opposite the reference angle. As Ryan responds, he positions the students as sense-makers with important resources to offer one another. Instead of answering the question himself, he draws each of the students into a conversation and focuses on helping them to make their own sense of the words "opposite" and "adjacent" (cf. Moschkovich, 2013).

Episode 2a. Making sense of "opposite."

1	Teacher:	Okay. Ana, what was the question? You had a team question?
2 3	Ana:	Um, the question, we are not sure, if this is the reference angle, is this the opposite or is the num—is the letter B?
4	Teacher:	Great question. What have you decided so far?

5	Carmen:	[We don't know.
6	Teacher:	[So who—so [Dashiin, what do you think?
7	Ana:	[He has opposite—
8	Teacher:	Ana's question is, if this is the reference angle, which, which part is opposite.
9	Dashiin:	This opposite, like this. Like this. He points with his pencil.
10	Teacher:	Why?
11	Dashiin:	He pauses and smiles. (6s) Reference angle, this is.
12	Teacher:	Hm?
13	Dashiin:	This is the reference angle.
14 15	Teacher:	That's the reference angle, good. (2s) What does it mean if the side is opposite?
16	Dashiin:	Um. (4s)
17	Teacher:	You guys can help, you can help Dashiin, cuz I know it's hard,
18	Ana:	[I don't know!
19 20	Teacher:	[with the English. (1s) What does opposite mean, just in general? Not in this context, but.
21	Ana:	Opposite.
22		Ana and the teacher both laugh.
23	Teacher:	Opposite means opposite?
24	Ana:	I don't, like—
25 26	Carmen:	I have an [idea. Opposite here, and opposite here. She makes some inscriptions on the triangle on her paper.
27 28	Ana:	[The other, like, I'm opposite of you. She gestures at herself and Dashiin, who is sitting across from her.
29 30	Teacher:	Yeah, like you and I are on opposite sides of the table, right? We're across from each other.
31	Ana:	Uh huh.
32 33	Teacher:	Dashiin and I are adjacent. We are on the same side of the table, right. Dashiin is next to me.
34	Carmen:	So—
35	Ana:	Uh huh.
36	Teacher:	So, if opposite means across from, which side is across from 76 degrees?
37 38	Carmen:	Points at something on Ana's paper with her pencil; in the video, it isn't clear what.
39	Ana:	B.

40 Teacher: Yeah. Exactly, yeah. But good. I really like the way that you're trying hard to41 like think about what makes sense.

Throughout this exchange (1 minute, 50 seconds), Ryan positions students as resources for each other. To see this, consider what he does not do as well as what he does. He could have ended the entire interaction with a single syllable in line 4: "B." Doing so would have freed him to move on. It also would have positioned the students as teacher-dependent, albeit subtly. Instead, Ryan elicits what they know, asking Dashiin to answer Ana's question (line 6). When Dashiin gives the correct answer, Ryan again could have stepped away. But this, too, would have framed Ana and Carmen as dependent on expertise external to themselves. Instead, Ryan presses Dashiin for a justification (line 10), and when Dashiin struggles to give one, Ryan recruits Ana and Carmen to help him—positioning *all* of the students as resources for their peers. None are singled out as smarter or more knowledgeable, or dumber or less knowledgeable, than any of the others.

Ryan's framing of students as resources is interwoven with and bolstered by his framing of mathematics as multidimensional, in particular as having space for (and indeed, requiring) students' own sense-making and interpretations. In supporting the group to explain why side B is opposite the reference angle, he does not direct their attention to formal definitions in their textbook or in their notes. Rather, he asks the whole group to draw on their everyday meanings for "opposite" (lines 19-20). He jokes with Ana about her struggle to put her understanding into words (lines 22-23), but he takes up her gestured explanation quite seriously, revoicing and extending it (lines 29-33). He also publicly praises "the way that [she is] trying hard to think about what makes sense" (lines 40-41). Thus, he frames mathematics as a sense-making activity while simultaneously positioning Ana as a valuable contributor to the work her group is doing. The fact that she did not know which side was opposite the reference angle at the start of the episode is irrelevant.

As the episode continues, Ana asks another question about the problem shown in *Figure* δ (they are to solve for x and y). Ryan restates the given information without suggesting what they should do with it. Of his own accord, Dashiin offers a way to set up an equation to solve for x. Ryan then does two things: he encourages the group, telling them to "keep going" because they are "on the right track," and he gives them specific information about what each of them has to offer the others, positioning them as equally important contributors to the group's work despite some apparent asymmetry in what the students know and can do with trigonometric ratios. Then he walks away, leaving the students to solve the problem themselves.



Figure 8. The diagram Ana, Carmen, Dashiin, and their teacher are examining in Episode 2b.

Episode 2b. "I'm gonna step away"

42	Ana:	And—and the other question was, [uh.
43	Carmen:	[This is adjacent. (3s)
44	Teacher:	Hm?
45 46 47	Ana:	The other question was for the first one? Uh, if, if we know the adjacent and the opposite and the hypotenuse, how we, how do we know if we have to use the sine, <u>para</u> . (" <i>for</i> ")
48 49	Teacher:	Good question. Good question. So, um. Do we know—here, we know what number the hypotenuse is, right?
50	Ana:	Uh huh.
51	Teacher:	What do we not know so far? Dashiin, can I have you looking here too?
52		Dashiin has been working on the next problem. He stops.
53 54	Teacher:	So what—we don't know the x, the opposite side, and we don't know the y, [the adjacent side.
55	Ana:	[Uh-uh.
56 57	Dashiin:	Uh. Sine—sine is (2s) uh 14—24. <i>He points at the triangle on Ana's paper and looks at the teacher.</i>
58	Teacher:	Good.
59	Dashiin:	Equal (2s), x, x— <i>He draws a horizontal line in the air with his pencil.</i>
60	Ana:	X.
61	Teacher:	Over.
62	Dashiin:	Over (1s), 14.
63 64 65 66 67 68 69 70 71 72 73	Teacher:	Okay. So I, I think you're on the right track Dashiin. I want you to think—I want you to make sure that that makes sense to everyone at your table. Cuz you've got an idea. And one thing, Dashiin, that I want you to practice: Ana and Carmen are excellent at showing their work, like they're showing it really clearly, and right now, some of the work that I see on here? <i>Pointing at Dashiin's paper</i> . Is missing some steps. So I want you to think, how could you show your work, really, really clearly. And part of that? Is if you do a good job of explaining, your idea that you just said, then Ana will be able to help you, figure out how to show the work. A little better. Okay? So keep going. I think you guys are on the right track. I'm gonna step away though, and uh, check in with the other teams. <i>He walks away</i> .

As in Episode 2a, Ryan draws the whole group's attention to Ana's question, recruiting her peers as resources not just for her, but also for each other in the collective project of making sense of mathematics. He highlights sense-making explicitly, in line 64. He also emphasizes

showing work clearly, highlighting this as a specific strength that Ana and Carmen have to offer Dashiin and thereby reinforcing the *students-as-resources frame* with the *multi-ability frame*. In many classrooms, Dashiin would simply have been positioned as the "smart" one, because he seems able to fluently set up equations and solve for the desired values. It was not uncommon in the classroom observations I conducted for teachers to position some students this way, as experts who ought to help their less able peers. Such a setup might support students' content learning, giving students in Ana's and Carmen's positions more access to explanations and giving students in Dashiin's position opportunities to practice explaining. But it is clearly distinct from the teacher's approach in this episode in terms of the opportunities for all students to develop a sense of mathematical agency, authority, and competence.

Ryan's decision to step away from the group does additional work to frame students as capable mathematical thinkers who can solve problems by working together. In Episode 1, William initially positioned students this way but stayed at their table to monitor and direct their interaction, suggesting that the students could not actually succeed in the task he had set without his presence. In leaving, Ryan communicates his confidence in the group and removes relying on him as an option. Importantly, turning the problem back to students and then stepping away was not a teacher move that worked in isolation. Rather, it was accompanied by careful listening and specific and genuine encouragement from Ryan, centering on the resources the students brought to the task at hand. It was also part of a larger pedagogical approach that characterized Ryan's (and also Amanda's) teaching. Throughout the year, Ryan (and Amanda) regularly employed an array of instructional routines to foster students' faith in themselves as problem solvers and establish student independence from the teacher and mutual dependence on each other as a classroom norm. Leaving students to resolve their own questions-even in situations in which they were not obviously "on the right track"—was just one of these routines. Holding students accountable to norms of mutual dependence by grading students' participation, as in the "group challenge" described above, was another.

In any given instance, there is no guarantee that students will generate sound mathematical ideas if their teachers do not guide them closely. But over time, when teachers routinely position students as teacher-dependent, they deprive both students and themselves the chance to find out—and as Ryan put it, to "be surprised" by—just how much students can do when granted the agency and authority to take on challenges and the sense of competence to convince them that their struggles are worthwhile.

Discussion and Conclusion

This chapter has demonstrated the difficulty of creating opportunities for students to develop a sense of agency, authority, and competence—a sense at the heart of meaningful learning—in mathematics classrooms. Using frame analysis, I have unpacked some of the limitations of teacher moves that represent reasonable, well-intentioned efforts to meet a range of content-focused and socioemotional needs. I have argued that viewing these moves through the lens of students' opportunities to develop a sense of agency, authority, and competence provides a perspective that is essential for understanding how equity and inequity are organized in everyday instruction.

The counterexamples in this chapter also demonstrate that it is possible to foster students' sense of agency, authority, and competence—but that doing so requires both epistemological reframing of mathematics as a discipline and positional reframing of students as learners and

doers of mathematics. When mathematics is understood as a rule-following activity, students are likely to receive instruction that sends some the message that they are less than competent, capable, and smart, even as it attempts to meet their needs by providing challenges and scaffolds at the "appropriate level." Teachers must therefore step outside of dominant frames that exclude important dimensions of mathematical practice and many students, seeing mathematics and students expansively and adapting their instruction accordingly—no small task in an educational system that is structured to assess, sort, and rank students based on their degree of mathematical competence.

Because of the enormity of this work, teachers need structures and systems that support the reframing of mathematical competence. Parents, administrators, curriculum writers, researchers, policy makers, and others must also participate in moving away from a discourse that has long framed mathematics and students in restrictive and hierarchical terms, toward an expansive discourse that affords all students opportunities to develop a sense of agency, authority, and competence through the learning and doing of rich, rigorous mathematics. But as the primary organizers of students' opportunities to learn mathematics, teachers do have a critical role to play. An important next step for research is to further investigate how teachers make sense of the work of reframing mathematics and mathematics ability, and how they can be supported to engage with this work and the challenges it presents.

Chapter 3

Tensions in Equity- and Reform-Oriented Learning in Teachers' Collaborative Conversations

Abstract

This chapter explores how teachers' collaborative conversations afford and constrain their opportunities to learn, through the close analysis of an interaction between mathematics teachers who were attempting to learn together. In that interaction, drawn from a routine meeting of geometry teachers at a diverse, urban high school, a teacher asked her colleagues for help in supporting students to "make their understanding public." This chapter unpacks teachers' participation in the ensuing conversation in fine detail, connecting it to socially, culturally, and historically constructed meanings using techniques from conversation analysis and frame analysis. The analysis reveals that although the group appeared to be an ideal professional learning community in many ways, and although the focal interaction and others like it were generative in a number of respects, the teachers were frequently caught in tensions between narrow, restrictive discourses about mathematics and mathematical competence and more inclusive but culturally non-dominant discourses. The existence of these tensions presented opportunities for the teachers to negotiate non-dominant meanings for themselves, i.e., to learn; but the ways that teachers framed their own collaborative work stifled these opportunities. By highlighting conversational norms that impede collaborative learning, this chapter contributes to the field's understanding of the challenges of equity- and reform-oriented learning in teachers' professional communities.

Introduction

A "research consensus" holds collaboration between teachers as one of a handful of "characteristics of professional development that are critical to increasing teacher knowledge and skills and improving their practice" (Desimone, 2009, p. 183; see also Loucks-Horsley & Matsumoto, 1999; Stoll, Bolam, McMahon, Wallace, & Thomas, 2006). Opportunities for collaboration appear to be widely valued by teachers themselves, and there is some evidence that participation in a professional learning community supports improvements in professional culture, classroom practice, and student achievement (Louis & Marks, 1998; Stoll et al., 2006; Vescio, Ross, & Adams, 2008). Research has identified a number of features that distinguish professional learning communities from teacher groups that reinforce traditions,² such as a shared orientation toward all students as learners and a shared vision of instructional improvement (Bolam, McMahon, Stoll, Thomas, & Wallace, 2005; Little, 1982; Louis, Kruse, & Marks, 1996; McLaughlin & Talbert, 2001).

This article contributes to a growing literature that investigates how teachers' professional communities may support their learning in moment-to-moment interaction, asking *how teachers' collaborative conversations afford and constrain teachers' opportunities to learn.* Following previous research on teachers' opportunities to learn, I do not make claims here about effects on individuals' actual learning (Dobie & Anderson, 2015; Horn & Kane, in press; Little, 2003). Rather, I examine conversational affordances for teacher learning, in particular, opportunities for teachers to participate in the negotiation of equity- and reform-oriented understandings of mathematics, students, and their own collective work. (Everyday conversation also provides abundant opportunities to learn and re-learn traditional understandings. Such opportunities are not the focus of this chapter.) Of course, an opportunity is not a guarantee of learning. It is, however, a prerequisite.

A 9¹/₂-minute interaction between mathematics teachers who were attempting to learn together serves as a focal point for my analysis. The interaction is drawn from a data corpus that encompasses a year of ethnographic observations in routine teacher meetings at a diverse, urban high school. Audiorecordings, fieldnotes, and transcripts for all 28 meetings in the corpus were analyzed, showing that the school's mathematics department had many of the features of effective professional learning communities that the literature identifies. That is, they dedicated time to frequent, instruction-focused collaboration; they welcomed innovation and strove for improvement; their explicit mission was to serve all students; and they displayed an ethos of mutual support, trust, and respect. Yet in their conversations with each other, teachers casually (if only occasionally) reproduced hierarchical and exclusive conceptions of mathematical ability, describing "high-level" and "low-level" students, some of whom "can" (think conceptually, act right, etc.) and some who "just can't." In classrooms, the same few students always seemed to have the right answers, while others (typically not so few) always seemed to be struggling.

² The literature on professional learning communities defines teacher learning (often implicitly) as learning that is directed toward generating high achievement and powerful learning experiences for *all* students (e.g., Benitez, Davidson, & Flaxman, 2009; Louis, Kruse, & Marks, 1996; McLaughlin & Talbert, 2001). Learning that is tangential to or in conflict with such instruction is also possible and may actually be more common than learning that supports ambitious instruction. However, following the literature, this chapter uses "teacher learning" and "opportunities to learn" to refer to equity- and reform-oriented learning unless otherwise noted.

The chapter contributes to the literature on teacher learning in communities of practice in two ways. First, it challenges the dichotomy of "learning communities" and "traditional communities," showing that a community may support some kinds of equity- and reformoriented teacher learning while being unable to support others. Second, in examining moment-tomoment interaction, it demonstrates how consequential teachers' framing of their own collaboration is in supporting (or failing to support) teachers to depart from restrictive, traditional discourses about mathematics and mathematical competence.

Theoretical Perspective: Teacher Learning as a Negotiation of Meaning

A conception of learning as a negotiation of meaning is central to this chapter (Vygotsky, 1986; Wenger, 1998). I am particularly concerned with how teachers negotiate the tensions between two kinds of meanings: traditional meanings-of mathematics as a discipline, students as learners, and the work of teaching itself—and equity- and reform-oriented meanings. I term the latter meanings non-dominant as well as equity- and reform-oriented in order to highlight their opposition to the meanings that contemporary American society makes commonsensicali.e., to dominant discourses and ideologies. Whereas reform discourses frame mathematical activity as inclusive of a variety of skills and practices, such as making sense of problems, reasoning quantitatively, and constructing arguments (e.g., Common Core State Standards Initiative, 2010), dominant discourse frames mathematical activity as consisting primarily of memorization and computation (Ball, 1988; Grossman & Stodolsky, 1995; Stigler & Hiebert, 1998). Whereas reform discourses position all students as capable learners (e.g., National Council of Teachers of Mathematics, 2000), dominant discourse frames intellectual and especially mathematical ability as innate, fixed, and distributed along a bell-shaped curve (Oakes, Wells, Jones, & Datnow, 1997; Parks, 2010; Ruthven, 1987). And whereas reform discourses position *teachers* as learners who benefit from ongoing collaboration and support (e.g., Benitez, Davidson, & Flaxman, 2009), dominant discourse frames the work of teaching as private, autonomous, and grounded in personal style and preference (Little, 1990; Lortie, 1975).³ Dominant discourses are thus restrictive: they limit students' opportunities to learn rich mathematics and teachers' opportunities to negotiate equity- and reform-oriented shifts in their practice. As teachers engage with non-dominant meanings that potentially expand learning opportunities, commonsense meanings do not simply disappear. Rather, they interact with nondominant meanings in messy and complex ways that require careful study in order to understand how and what teachers learn.

Opportunities for the negotiation of meaning arise not only in settings that are formally organized for learning but also in everyday interactions. As Wenger writes, "negotiating meaning" connotes "an accomplishment that requires sustained attention and readjustment, as in 'negotiating a sharp curve'" (p. 53). Even tasks that appear routine or thoroughly prescribed entail negotiation, because everyday life presents complexities that must be managed. Managing

³ Some might argue that the meanings I term non-dominant are more mainstream than those I term dominant. Certainly, equity-oriented reforms have prompted many to reject the latter and adopt some of the former, at least rhetorically. But enactments of equity- and reform-oriented ideals remain quite rare in schools and classrooms. Negotiating equity- and reform-oriented meanings therefore continues to present challenges for teachers that negotiating more dominant meanings does not.

complexity in turn requires the constant making of meaning—and it creates opportunities to make new meanings, i.e., to learn.

Of course, individuals do not learn in a vacuum. Negotiating meaning involves coordinating history, the present, and the hoped-for future, working with "a world of both resistance and malleability" (Wenger, 1998, p. 53). Of particular relevance to this chapter is Wenger's characterization of all meaning-making as engaging "a multiplicity of factors and perspectives"—e.g., restrictive discourses as well as more expansive ones—such that people must continually produce "new resolution[s] to the convergence of these factors and perspectives." These resolutions are necessarily incomplete or "partial, tentative, ephemeral, and specific to a situation" and therefore require ongoing renegotiation and re-resolution (p. 53).

For teachers, local communities of practice provide key sites for learning. Groups such as subject-matter departments or school-based teams may provide teachers with opportunities for interaction that are focused on their practice, which may not be available elsewhere. In addition, teachers' communities of practice have their own local histories, which give rise to systems of meaning and naturalized ways of "doing things" that make particular meanings more or less available to teachers. Thus, in one professional community, teachers might be supported to talk about and view students who complete their work quickly as "fast learners" who need to be exposed to more content than their slower peers, while in another, speed might be understood as a barrier to complex thinking (Horn, 2007).

Extra-local systems of meaning also shape teachers' opportunities to learn and negotiate meanings. Categories like "fast" and "slow" may be locally negotiated, but they are part of broader discourses about what a good student is and does. Although such discourses do not fully determine the meanings that teachers construct in their interactions with each other, they do make some meanings more sensible and more automatic than others. Because of this, extra-local systems of meaning may make *shifts in meaning* difficult to accomplish from moment to moment. For example, in order to come to view a "slow" student as a competent learner and contributor, a teacher must learn to see ways of contributing that are not generally visible, much less valued, in American schools. She must also resist dominant interpretations of speed that continue to assert themselves (e.g., through lengthy lists of standards outlining what should be learned in a given year, through pressures to "accelerate" students who are "gifted," through ways that colleagues and students talk about competence, and through her own socially and culturally constructed intuitions about intelligence).

This chapter investigates teachers' opportunities to learn in everyday interactions in their communities of practice, as they take up, enact, and renegotiate local and extra-local meaning in moment-to-moment activity. I find that teachers encounter persistent tensions between restrictive and expansive discourses about mathematics, students, and teaching. The literature on teachers' professional communities suggests that equity- and reform-oriented communities support teachers to shift toward more inclusive, student-centered instruction, largely ignoring these tensions. I find that in the context of these tensions, how teachers frame their own collaborative conversations—in particular, the kinds of knowledge that are relevant and the ways of participating that are appropriate—may render even an equity- and reform-oriented professional learning community unable to support robust opportunities for teachers to learn.

Analytic Tools

The interplay of dominant and non-dominant, restrictive and expansive meanings frequently produces tensions and contradictions. Learning non-dominant ways of understanding

mathematical competence therefore differs significantly from the learning that much of the research on professional learning has studied and theorized, in which novices are inducted into a more or less established practice (Goodwin, 1994; Stevens & Hall, 1998; Wenger, 1998). In contrast to such settings, this chapter examines teacher work groups composed of peers, none of whom have mastered the practice that their learning is directed toward. Capturing the interaction of extra-local systems of meaning, local communities of practice, and moment-to-moment interaction in this context is thus a somewhat novel task for research, requiring strategic selection and adaption of analytic tools.

Frames and frame analysis

Frame analysis provides one set of tools for revealing and understanding the interaction between dominant and non-dominant, restrictive and expansive discourses in moment-tomoment interaction. As Goffman's (1974) seminal work describes, framing creates an interpretive context that communicates to participants an answer (or answers) to the question, "What is it that is going on here?" Frame analysis has been taken up in a variety of ways, in a variety of fields.⁴ Here I borrow a perspective from the learning sciences, articulated by Greeno (2009). Greeno delineates two kinds of framing with particular relevance to settings that are organized (at least ostensibly) for learning: *epistemological framing* and *positional framing*. Epistemological framing "refers to the kind of task that a participant or participants understand themselves to be engaged in, especially regarding the kind(s) of knowledge that are relevant to and expected to be constructed in order to succeed in the task" (p. 271; see also Hammer et al., 2005). Positional framing "refers to ways in which an individual is entitled, expected, or perhaps obligated to participate in interactions of an activity system, such as a classroom" (p. 272). Both epistemological and positional framing shape the opportunities to learn that are available to, for example, students in mathematics classrooms and teachers in meetings with their colleagues.

Frame analysis has several affordances for studying the kind of teacher learning at the center of this study (as I have described elsewhere; see Chapter 2). First, frame analysis captures the tacit, taken-for-granted "metamessages" (Bateson, 1972) through which what counts as "mathematics," "mathematically smart," and "teaching" are often organized in moment-to-moment activity. Methods that rely on explicit articulations of these messages inevitably miss the action, so to speak. A second advantage of frame analysis is that it does not need to assume individual participants' intentions in order to interrogate the functions of the frames they employ. This is significant both because it can be quite a challenge to know what an individual's intentions are or were, and because an utterance may invoke a frame quite independent of the speaker's intention, with non-trivial effects on the learning opportunities that are made available for participants.

The ability of frame analysis to make the tacit visible, without assuming individuals' intentions, is especially important for investigating discourses and ideologies that are widely regarded as unacceptable but which nonetheless persist. It seems unlikely that the mathematics teachers in this study ever meant to frame mathematical activity as memorizing formulas and

⁴ Perhaps most prominent in current scholarship, social movement theorists have adapted frame analysis to study activists' deliberate efforts to frame social issues in ways that motivate particular solutions (Benford & Snow, 2000). Coburn's (2006) work on problem framing and policy implementation is an example of this usage of frame analysis in the field of education.

following set algorithms; indeed, they stated explicit opposition to such framing on several occasions. Similarly, it seems unlikely that they ever intended to frame their students as mathematically or otherwise incapable. Nonetheless, the ways that they talked about their work sometimes reproduced these unproductive framings of mathematics and students. Whatever their intentions, moments in which restrictive frames were reproduced provided teachers with opportunities to learn—through the explicit juxtaposition and discussion of dominant and alternative frames (learning that challenges restrictive frames), or through the absence of such contestation (learning that reinforces restrictive frames—still learning, though not the focus of this chapter).

Frame analysis is also useful because it can illuminate deliberate efforts to disrupt dominant frames (Hand, Penuel, & Gutiérrez, 2012). Efforts to replace restrictive frames with more expansive and equitable ones in moment-to-moment conversation need to be made visible in order to understand how opportunities for teachers to learn are created and closed down in interaction, even when such opportunities are not taken up by all participants to a given situation.

Conversation analysis

As I use it, frame analysis is centrally concerned with the relationships between locally constructed meanings and extra-local structures and systems of meaning. Conversation analysis (Jefferson, 2004; Ochs, 1979) complements this approach with its focus on locally constructed structures and meanings, revealing the work that participants do to manage their interactions. Of particular relevance to this chapter, a few studies have used conversation analysis to examine relational work as it shapes the learning opportunities that arise in face-to-face interaction (e.g., Dobie & Anderson, 2015; Engle & Greeno, 1994; Little, 2002). For example, through turn-byturn analysis of facilitated teacher meetings in a video club setting, Dobie and Anderson found three forms of "expressions of contrasting ideas": open discussion, implicit critique, and serial turns. All three appeared responsive to interpersonal concerns, but only open discussion-in which participants were responsive to each other's ideas and did not avoid disagreementfostered rich opportunities for teacher learning. Similarly, Engle and Greeno used conversation analysis to show how participants in their study (student math teachers who were asked to work with a partner to generate written explanations for two geometry problems) worked to simultaneously manage "interpersonal concerns regarding the maintenance of a good relationship with one's partner and *conceptual concerns* gravitating around the desire ... to produce a high quality explanation that both participants agree to" (p. 266, emphasis added). By examining the structure of talk turns in which disagreement was expressed, they illustrate the integration of these two concerns in participants' methods of "managing disagreement." They found that their participants were much less averse to disagreement than the literature predicts, leading them to suggest that "intellectual conversations" like the ones they studied make conceptual concerns more salient than the interpersonal concerns that are critical in "everyday conversation."

These studies and the larger body of research on constructs such as "facework" (Goffman, 1955; Arundale, 2006) suggest that attending to the ways in which interlocutors manage interpersonal concerns is an important task for research that aims to understand the development of ideas and of opportunities to learn in conversation. Conversation analysis provides a means of accomplishing this task.

This chapter endeavors to combine elements of conversation analysis and frame analysis to ask what sort of situation mathematics teachers engaged in collaborative conversations might understand themselves to be engaged in, how that understanding both enacts and re-produces extra-local meanings, and how it affects their moment-to-moment participation in order to open up or close down opportunities to learn new ways of understanding their work.

Prior Research

In his classic study on the working lives of schoolteachers, Lortie (1975) observed that teachers typically labor in isolation, for various reasons: the physical, "egg crate" arrangement of classrooms and schools; a professional culture of individualism and independence; the absence of a "shared technical culture"; and the great demands placed on teachers' time, which center around interactions with students rather than colleagues. Since then, the potential of professional communities to help schools and teachers meet increasingly diverse student needs has made these communities a "hot topic" in the United States and elsewhere (Stoll et al., 2006, p. 221). Opportunities for teachers to participate in collaborative professional communities are often regarded as essential for effective professional development, especially professional development targeting student-centered reforms (Desimone, 2009; Jilk & O'Connell, 2014; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2009). Research suggests that the complexity and dynamism of ambitious, student-centered instruction require teachers to engage in learning is unending and often challenging; in this context, a professional community that supports learning-for all teachers, not only those who are new or inexperienced with reform-is crucial (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009; Franke, Carpenter, Levi, & Fennema, 2001).

Of course, all communities are not created equal. Decades of research have worked to characterize communities that support teacher learning (in particular, learning toward ambitious instruction; see, e.g., Lampert, Boerst, & Graziani, 2011) and to determine what distinguishes them from communities that do not (Bolam et al., 2005; Little, 1987; Vescio et al., 2008). In an early study of workplace culture in schools, Little (1982) found that norms in "unsuccessful" schools discouraged interaction between teachers. In contrast, schools that were "conducive to continued 'learning on the job'" and to high student achievement maintained norms of frequent interaction, in particular, interactions that supported collective experimentation. McLaughlin and Talbert (2001) later showed that even "strong" communities of practice, in which teachers collaborate on instruction, may be fundamentally conservative, maintaining traditional routines and attributing poor student performance to deficiencies that are rooted in students (e.g., lack of motivation, inadequate support at home, or gaps in background knowledge). And even when they are convened with the express purpose of supporting equity- and reform-oriented teacher learning, norms of privacy and autonomy often persist in teacher communities. As Little and Horn's (Little, 1990, 2003; Horn, 2007; Horn & Little, 2010) analyses of collegial conversations have demonstrated, many work groups "close down" opportunities for learning by dismissing problems of practice as normal or inevitable, or by rendering problems and their solutions in generic terms that leave actual classroom practice opaque. A small number of groups instead "open up" problems of practice for collective interpretation and inquiry.

This chapter contributes to the literature on teachers' professional learning communities in two ways. First, it adds to the small but growing number of studies that look closely at interactions between teachers (e.g., Dobie & Anderson, 2015; Horn & Kane, in press; Horn, 2007; Little, 2002, 2003) for a greater degree of specificity about what occurs within teacher learning communities than research on their structural features has afforded. This specificity allows for greater nuance than do the categories "learning community" and "traditional community," illustrating how features that support and hamper learning may coexist within a single teacher work group. Second, it turns attention to ways in which behavior that hampers learning may make sense for teachers by examining the frames of collaboration within which such behavior is situated. This approach extends research that examines how teachers frame mathematics and students' mathematical ability (e.g., Horn, 2007; Jackson, Gibbons, & Dunlap, in press; Parks, 2010) by investigating how teachers frame their work together and the influence that such framing may have on teachers' opportunities to learn to frame mathematics and students in more equitable ways.

Methods

Research site and participants

At the outset of this study, I asked people whom I considered "insiders" in the local mathematics education community (including district personnel, university-based researchers, and teacher educators) to nominate schools in which math teachers were committed to learning how to redefine mathematics instruction together. I was especially interested in teacher groups that were collaborating to expand what it meant to be "good at math" in order to give *all* students access to rich, rigorous, and empowering mathematics learning experiences. Everyone I talked to mentioned Union High School (a pseudonym; all teacher names are also pseudonyms).

Union was a large and racially, linguistically, and socioeconomically diverse school in an urban school district on the West Coast of the United States. In many respects, the mathematics department at Union was an exemplary teacher work group, with many of the characteristics of effective professional learning communities identified by research (e.g., Bolam et al., 2005; Louis et al., 1996; McLaughlin & Talbert, 2001). The teachers dedicated several hours each week to collaboration, engaged in joint planning and curricular innovation, and enjoyed a supportive administration. They prided themselves on their cohesion around a view of all students as learners and of themselves as agents of change, and they deliberately hired new teachers who shared this perspective. All of the teachers in the department were also participants in a professional development program grounded in Complex Instruction (CI; see Cohen & Lotan, 1997; Nasir, Cabana, Shreve, Woodbury, & Louie, 2014). As a pedagogical approach, CI posits that all students are smart; that issues of *status*—who is perceived as smart and who is not-interfere with students' participation and learning; and that it is teachers' responsibility to provide all students with opportunities to reveal how they are smart and to develop new ways of being smart, by engaging students in authentic collaboration around rich and challenging problems. CI thus provides an alternative to culturally dominant conceptions of the work of teaching and learning, in that it highlights student-centered practices and the need for teachers to disrupt traditional expectations for student competence. All of the mathematics teachers at Union had attended a CI course and follow-up workshops offered by their district. Within the district, they were positioned as CI leaders.

When the study began, Union was in its first year of "geometry for all," an initiative that placed all ninth graders in Geometry, regardless of whether they had passed an algebra course. The department had urged the school administration to make this change because of teachers' sense that placing some freshmen in Geometry and others in Algebra (a policy that remains the norm in schools throughout the country) created status issues and reinforced the idea that some

students are smart (the ones in Geometry) and others are not (the ones in Algebra). During the data collection period, all ninth graders and most tenth graders were taking Geometry, so the faculty Geometry Team was large and included a diverse range of perspectives on mathematics teaching and the challenges and opportunities presented by teaching heterogeneous groups, making it a rich site for data collection.

Data sources

This chapter focuses on data drawn from a year of ethnographic observations of routine meetings of the mathematics teachers at Union, including Geometry Team meetings (held every Tuesday and some Thursdays) as well as meetings of the full math department (held on most Thursdays). Students were dismissed early on both Tuesdays and Thursdays, so that teachers could collaborate during the workday. Several of the Geometry teachers were still unable to attend Tuesday team meetings on a regular basis because they were working on other courses (Advanced Algebra, etc.). The core Geometry group included 5 teachers who were almost always present (William, Eliza, Margaret, Luke, and Cyril), and 5 more who rotated in and out.

I attended 18 Geometry Team meetings and 11 department meetings. With a few exceptions, I attended the Tuesday Geometry Team meeting each week until mid-February, when teachers' energy for instruction-based collaboration seemed to wane (teacher attendance dropped, and meetings were shorter and more narrowly focused on preparation for the state standardized test). I increased my attendance at department meetings to find out how teachers talked about students and mathematics as a departmental unit. During meetings, I took field notes that focused on how teachers framed the nature of mathematics and students' mathematical competence. I also recorded meetings using a digital voice recorder, collected handouts (e.g., meeting agendas and proposed student activities), and took photographs of teachers' public inscriptions (e.g., notes on the whiteboard).

Data analysis

I transcribed all audio recordings, adding details from field notes and creating content logs for each meeting. Initial coding (Charmaz, 2006) of transcripts was used to begin to identify ways of framing mathematical activity and student competence. It quickly became evident that these framings were linked to how the meetings themselves were framed. Initial coding continued, examining three types of framing: framing mathematical activity, framing student competence, and framing teachers' collaborative conversations. Three corresponding coding schemes were generated and applied to all of the meeting transcripts using ATLAS.ti (a computer-assisted qualitative data analysis program). The software then generated reports showing all of the quotations associated with each code.

To investigate how different kinds of frames were connected to each other and to teachers' opportunities to learn, I selected a focal episode for close analysis. As Little (2002) writes, "To get at professional community as a practical accomplishment in day-to-day work will require looking closely and systematically at the ongoing exchanges that teachers have with one another" (p. 920). Thus, while coding provides an overview of the frames employed across the data corpus—an overview that supplies useful context for small slices of interaction—close analysis of *conversation* (as opposed to the isolated utterances that were the unit of analysis for coding) was necessary to get at the dynamics of "professional community as a practical

accomplishment," worked out through routine interaction. Close analysis of a focal *segment* of conversation, coordinated with constant references to content logs and code reports for the entire data set, "offer[ed] the virtue of manageability while ... satisfy[ing] the demands of analytic accountability" (p. 920).

The focal episode that I selected came from a meeting of the Geometry Team that took place two months into the school year. By this time, the team had developed certain patterns: they almost always met in William's classroom, and Eliza almost always brought snacks and printed copies of the agenda, which she compiled based on email and other communications with her colleagues (having been appointed to "facilitate" meetings by William). Teachers almost invariably spent the bulk of their meeting time sharing ideas about general instructional strategies (e.g., how to grade students' test corrections or manage "homework checks") or discussing the lessons that would be taught in the coming days. Conversations about upcoming lessons were especially important because half of the teachers on the team were teaching geometry for the first time, and the team as a whole was constantly engaged in rewriting curriculum. Teachers took turns leading curriculum development and, correspondingly, leading discussions in which they shared what they were planning.

The focal episode is representative of the broader data corpus in terms of the structure of teacher participation (sharing ideas) and the ways in which mathematics and students were represented. That is, the tensions between restrictive, hierarchical ways of framing mathematical competence and expansive, inclusive frames that appear in the episode are characteristic of teacher-to-teacher talk across the meetings I observed. The episode is also atypical in that there are more representations of students than usual, and more teachers than usual offer their ideas. (Talk tended to focus on teacher moves, and there was often a clear leader/primary sharer based on who was in charge of curriculum development for the current unit.) This atypicality renders the episode particularly rich for analysis, but not in ways that make the patterns of interaction that appear in the episode unrepresentative. That is, both of these differences support the study of how teachers co-construct opportunities to learn expansive ways of framing mathematics, students, and their own collaborative work; and despite these differences, the episode reflects the dynamics of the group without diverging in any ways that the teachers themselves seem to notice or mark.

The focal episode was re-transcribed, annotating non-verbal features of the conversation (e.g., pace, emphasis, overlapping speech) using transcript symbols found in the work of Jefferson (2004) and Ochs (1979). I then examined teachers' epistemological and positional framings (Greeno, 2009) of two kinds of activity: classroom mathematics learning, and teacher collaboration itself (see Table 5). That is, I analyzed how the conversation framed what should be "going on" in classrooms, in particular with regard to a) the nature of *mathematical activity* and the kinds of knowledge and skill that are relevant, and b) the ways in which students are expected to participate, especially the kinds of *competence* that students are (or are not) expected to display. (For analysis of teachers' framing of classroom activity in classroom instruction, see Chapter 2.) I also examined how the conversation framed what should be going on in collaborative conversations themselves, in particular a) the nature of the tasks that collaboration ought to address and the kinds of *professional knowledge* that are relevant, and b) the kinds of *teacher participation* that are expected or appropriate in collegial conversation.

Table 5

Epistemological and positional frames of two kinds of activity: classroom mathematics learning and teachers' collaborative conversations.

	Classroom mathematics learning	Teachers' collaborative conversations
Epistemological framing	The nature of mathematical activity and the knowledge and skills that are relevant	The nature of collegial collaboration and the knowledge and skills that are relevant
Positional framing	The ways in which students are expected to participate in mathematics classrooms	The ways in which teachers are expected to participate in collegial conversations

I created a series of tables to reflect different kinds of framing. The first table parsed the conversation into chunks based on shifts in speaker, with a row for each chunk displaying the relevant line numbers and excerpts of transcript that captured the flow of the conversation (see Table 6, below). For each of the four kinds of framing represented in Table 5 (epistemological framing of classroom mathematics learning, positional framing of classroom mathematics learning, epistemological framing of teachers' collaborative conversation, and positional framing of teachers' collaborative conversation), I created a new table, substituting the excerpts in Table 6 with all of the lines of transcript that were relevant to that strand but keeping the parsing into chunks shown in Table 6. The distinctness of each table allowed me to analyze each strand on its own terms, while the organization by chunks allowed me to see interactions with the other strands as the conversation progressed through time.

Throughout my analysis of the focal episode, I was attentive to the ways in which the episode both reflected and re-created the history of the group (insofar as I had access to that history) and of broader cultural narratives about mathematics, ability, and the work of teaching.

Researcher positionality

During the time of the study, I was an insider in communities that the mathematics teachers at Union had immediate contact with and which commanded their respect. I had been a classroom teacher, trained in a locally known and well-regarded credentialing program; the district had hired me part-time as a Complex Instruction coach (though not at Union); the leaders of the CI professional development program were friends of mine; and I was pursuing a Ph.D. at a prestigious university. Thus, it may have been that my presence triggered certain associations for teachers and influenced their behavior. I worked to minimize this by positioning them (not myself) as the experts on their practice, and I believe that they saw my work as aligned with theirs and therefore accorded me a level of trust that allowed them to focus on the complexities of their work without attending to what I would think. But even if my presence did prompt the teachers to attempt to align themselves with CI or other reforms more closely than they otherwise would have, their ways of negotiating CI and its meanings remain very much their own.

Tensions Between Dominant and Non-Dominant Discourses in a Focal Episode: "Making Understanding Public"

The teachers in this study framed mathematical competence in restrictive, narrow ways *and* in expansive, inclusive ways, at turns reifying and challenging traditional framings of school mathematics and deficit perspectives on students. These different frames co-existed in teacher talk, creating tensions that, if explored, could have created rich opportunities for teachers to learn non-dominant ways of understanding mathematical competence. However, teachers' framing of their collaborative conversations was a significant obstacle to the realization of these learning opportunities. While the conversations were generative in certain respects, they were enacted within frames of collaboration that highlighted sharing (rather than inquiring) and interpersonal concerns (with less attention to conceptual ones). Instead of opening opportunities to negotiate non-dominant meanings, these frames of collaboration were barriers to resolving tensions between restrictive and expansive frames of students and mathematics.

A 9½-minute episode from a Union Geometry Team meeting illustrates this phenomenon. In the episode, Eliza shared a routine problem of practice, explaining that it was important to her to support her students to "make their understanding public" and describing some of the difficulty she was having in accomplishing this goal. The following analysis explores the learning opportunities that the ensuing conversation afforded—and foreclosed—for participants. Drawing on ethnographic conversation analysis and frame analysis, I unpack teachers' moment-to-moment participation in this episode, connecting it to socially, culturally, and historically constructed systems of meaning.

I begin with a brief description of the episode to introduce readers to its basic contours: who spoke and when, and the essence of what was said from a literal perspective. I then characterize the ways in which the interactions in the episode reflected and reinforced frames for professional interaction that are *generative* of opportunities to learn. This focus on the productivity of teachers' interactions serves two purposes. First, it challenges conceptualizations of "effective" professional learning communities, suggesting that current definitions miss facets of such communities that are critical for supporting equity- and reform-oriented learning. Second, it is necessary for understanding why the teachers did what they did. Their conversations were problematic, but they were not *only* problematic, and although they were not completely transformative, they did challenge aspects of restrictive discourses about the nature of teachers' work as well as provide support for their participants.

The next two sections analyze the ways in which mathematical activity and student competence were framed in the focal episode. Again, I highlight ways in which teacher talk challenged restrictive discourses while illustrating the limits of those challenges and the persistence of these discourses. The final section returns to analyzing frames of professional interaction in the focal episode, showing how frames that were productive and useful for teachers were also limited in the extent to which they supported teachers to negotiate tensions between restrictive and expansive ways of framing mathematical activity and student competence.

A brief description of the episode

It was the end of October, approximately two months into the school year. Present were William, who was hosting the meeting in his classroom (as usual); Eliza, who was in charge of compiling the meeting agenda each week; Margaret, one of the two co-chairs of the math department; Luke; Cyril; and Emilio. William and Cyril were veteran teachers and the only two members of the team who had taught Geometry at Union before. Eliza and Margaret were also experienced math teachers, but this was their first time teaching Geometry at Union. Luke and Emilio were both in their second year teaching. Four other teachers (Ryan, Meriwa, Samila, and Jenny) also taught Geometry at Union but were not present at the meeting because of other school-related obligations.

Twenty-three minutes into the hour-long meeting, Luke looked at the next item on the agenda Eliza had prepared—"making understanding public"—and asked, "What is that?" (line 438). Eliza proceeded to articulate a problem of practice, describing difficulties that she and Ryan had been discussing regarding the management of student presentations. She asked her colleagues for strategies that would support students to "make their understanding public without having to stand up and present" (lines 454-455).

Luke, Margaret, and William took turns sharing strategies that they had used and found effective. Luke's strategy provided a structure for students to share their thinking with a partner (lines 479-532). Margaret then described how she had been managing student presentations in front of the class, emphasizing her efforts to "get [students] to converse with each other" instead of talking to her and highlighting the improvement she had seen over time (lines 533-578). Emilio chimed in to say that "it just takes practice" and to offer an interpretation of students' difficulties that centered on their feelings of anxiety surrounding anything unfamiliar (lines 580-600). William picked up on Emilio's interpretation, sharing a strategy he had used to create a "comfortable environment" for student presentations (lines 601-619). Margaret re-entered the conversation to share two short "thing[s] that I learned" about managing student presentations (lines 620-635). After a pause and some shuffling of papers, Eliza concluded the 9½-minute episode by announcing that she had "got[ten] some ideas" (line 639).

Table 6

Lines	Description	Excerpt
438-	Luke introduces Eliza's issue	LUKE: What is that? What is "making understanding public"?
439	into the conversation	
440- 478	Eliza describes a problem of practice that she has been experiencing and solicits her colleagues' help	ELIZA: They can't, (.) um, they just can't <u>listen</u> to each other present, or <u>they</u> can't present?, um. So we were just thinking about, can we build in some ways of making understanding public without [students] having to stand up and, present.
		ELIZA: they need to be able to explain their thinking. So I said I'll put it on the agenda and see if anybody else has other ideas of good ways for um, kids to share their understanding.
479- 532	Luke shares an instructional strategy	LUKE: [There's] this really structured technique that I like [that] gets them talking. [It's called] SOLAR. It's like, it's an acronym
533- 578	Margaret describes how she manages Eliza's issue	MARGARET: We do a lot of presenting And in the beginning, it was awful, and I can definitely see them getting much better now.
		MARGARET: [one of the group members] talks about, we did this, we did this, and does everybody understand. Take questions from the class I say don't tell me, tell them I'm

A summary of the focal episode with excerpts of conversation.

		trying to get them to converse with each other.
580- 600	Emilio gives another interpretation of the challenge Eliza has described	EMILIO: I think you made an important distinction that it just takes practice they just need to, do it more to become comfortable with actually going up and presenting.
601- 619	William shares how he has managed the challenge Emilio described	WILLIAM: [yesterday] I wanted it to be so <u>quiet</u> in the audience that whoever came up could <u>try</u> to answer [the problem], without being interrupted, like to have some time and some space to do that [I] wanted them to have enough, like enough of a comfortable environment where they could see, or like try if they knew it or not.
620- 635	Margaret adds to her previous comment	MARGARET: And, and one thing that I learned —if a group has not done the homework, they can still come up [and present]. Oh and if you've got two groups doing one question, you can make the other one, group call a friend, you know.
636- 639	The conversation concludes	ELIZA: Got some ideas.

Note. See Appendix B for a glossary of transcription symbols and a full transcript of the episode.

Generating opportunities for learning

In this section, I analyze the ways in which teachers opened up opportunities for their own learning in the focal episode, focusing on how their conversation fostered productive orientations to problems of practice and to their professional community as a problem-solving unit.

Reinforcing productive group norms. In sharing an instructional challenge that she was experiencing with the group (lines 440-478), Eliza reinforced a Geometry Team norm of exposing one's practice as problematic. Such exposures had been routine since the start of the school year, and the fact that Eliza put her issue on the official agenda signaled the legitimacy of problems of practice as items of discussion (alongside, for example, planning instruction and taking care of administrative issues) as well as signaling their importance not just for the individual teachers who raised them but also for the group as a whole. This behavior ran counter to the privacy that has historically characterized teachers' work (Lortie, 1975). It represented a critical step toward creating opportunities for teachers to learn in and from their practice; practice that is not viewed as problematic (or only viewed as such in ways that colleagues do not acknowledge to each other) is not likely to give rise to collaborative inquiry and work toward improvement.

In discussing Eliza's problem as they did, the rest of the team reinforced their orientation to collective improvement. They took her issue seriously, treating it as actionable and offering their ideas without positioning her as somehow deficient for experiencing trouble. Thus, their responses were neither dismissive nor limited to sympathy and commiseration, which might support teachers to take on the vulnerability that sharing problems of one's own practice requires without supporting learning (Horn & Little, 2010).

Providing a variety of ideas. The responses that Eliza's colleagues gave also supported the group's learning by providing a variety of ideas. None of the teachers had access to all of

these ideas when the meeting began; at its conclusion, they all did (though their resources for making sense of them most likely varied). These ideas included *strategies* for "making [student] understanding public," conveyed with some detail to support Eliza and others to develop a mental picture of how they might work. The group successfully pressed for such detail on a number of occasions. For example, Margaret asked Luke to clarify his strategy, and he responded with a physical demonstration of how students should "square off" to listen to one other (lines 493-501). Later in the conversation, William asked Margaret "what type of interaction" she wanted students to have with her and their peers, and she responded with a "rehearsal" (Horn, 2005) of what she would actually say to students and expect them to say in response (lines 555-566).

In addition to sharing instructional strategies, the teachers created opportunities for learning by sharing a range of *interpretations* of Eliza's issue. While Eliza initially indicated that "having [students] stand up and present" (line 455) was off the table because her students "can't present" (line 451), Margaret put presentations back on the table. She described her success with student presentations with an emphasis on her role as the teacher, for example her provision of a clear structure:

557	MARGARET:	So when they go up I say to them, don't assume that everybody
558		knows what the question is, because you, you've been studying this.
559		So the first thing I want one of the group members to do is, is sort of
560		summarize the question. So we were asked in this question to draw a
561		histogram of this, this, you know. And then somebody else puts the,
562		puts their work on the ELMO, and talks about, we did this, we did
563		this, and does everybody understand. Take questions from the class.

Margaret also highlighted her efforts to "get [students] to converse with each other" instead of focusing their attention on her:

564	MARGARET:	And if they look at me, I tell them not to look at me. That's not okay,
565		don't talk to me, I know the answer. Or if somebody says that's
566		wrong and looks at me I say don't tell me, tell them. That kind of
567		thing

Thus, whereas Eliza interpreted her difficulties with student presentations as rooted in students' lack of ability, Margaret suggested that teachers are capable of and responsible for supporting students to develop presentation and discussion skills.

When Margaret hedged her interpretation, saying that student presentations were "obviously easier to do in Stats," an upper-level course, Emilio reiterated that student presentations can be successful when the teacher provides support, emphasizing opportunities for students to "practice" (lines 581). He also introduced a new interpretive lens for the group to consider, namely students' feelings, saying that students "just need to, do it more to become comfortable with actually going up and presenting" and to overcome their anxiety with being in front of the class (lines 584-585).

William and Margaret also shared alternative interpretations of what a student presentation *is*. Whereas Eliza's articulation of her problem framed presentations as opportunities for students to communicate what they already know, William and Margaret each

indicated that presentations can be opportunities for students to ask for help (lines 607-609 and 621-624, respectively). No one elaborated on this conception of presentations, but it had the potential to support them to think about presentations as tools not only for assessment or getting the correct answers "out there," but also for fostering a classroom culture of mutual support and supporting deeper interaction between students than purely evaluative exchanges.

Brief commentary. The conversation in the focal episode provided teachers with opportunities to learn about the challenges of student presentations and how to manage them, while also supporting them to take up productive orientations to their practice. The generativity of this interaction is especially evident when the Geometry Team is compared to other teacher work groups. For example, Tsu (1998) "barely knew the names of the other teachers" in her department, and the chair "did not insist upon holding regular meetings" (p. 12), while many of the groups McLaughlin and Talbert (2001) studied reinforced deficit orientations toward students that did not motivate professional learning or improvements in practice.

The Union Geometry Team appeared to embody many qualities of effective professional learning communities identified by the literature. The teachers took collective responsibility for their work and shared the goal of supporting *all* students. They were open to innovation and professional learning, made time for collaboration, and displayed an ethos of trust, respect, and mutual support (Bolam et al., 2005; Louis & Marks, 1998; McLaughlin & Talbert, 2001). Thanks to the efforts of their department co-chairs, who dealt with many administrative issues outside of team meeting time, only 6 to 12 percent of the time in each Geometry Team meeting was spent on issues like how to check out textbooks, what software was available in the computer lab, and so on. The great majority of teachers' meeting time could therefore be devoted to discussing teaching and learning, in the productive ways described here.

Despite the ways in which the episode was generative of teacher learning, however, the conversation also evidenced tensions between restrictive and expansive framings of the nature of mathematical activity and of student competence. These tensions are at the core of equity-oriented reforms, and as the next two sections describe, the conversation provided teachers with few conceptual or practical tools for resolving them.

Framing mathematical activity

In the focal episode, teachers' epistemological framing of mathematical activity repeatedly invoked expansive definitions of school mathematics. The conversation centered on Eliza's wish to support students to "make their understanding public," and the formulas and algorithms that are typically the focus of school math were notably absent from her characterization of what she wanted her students to know and be able to do. Indeed, she explicitly said,

461	ELIZA:	if you can't
462		communicate what you know then, this is one of the important (.) job
463		skills, that people need, right. Um. They don't need an-triangle
464		congruence, but they need to be able to explain their thinking

That is, instead of absorbing an established body of knowledge, Eliza wanted her students to be able to "communicate," "explain their thinking," and "share their understanding." In responding to Eliza's request for ideas, her colleagues validated these priorities.

Yet there was a tension in the conversation between this expansive framing of what counts as mathematical activity and a more traditional framing, centering on answers and right/wrong binaries. The conversation focused on "making understanding public" in a general way: references to specific topics were limited to brief mentions of "triangle congruence" (Eliza, lines 463-464), "angle relationships" (Luke, line 489), and "histogram[s]" (Margaret, line 561). But "understanding" and "thinking" were underspecified, and without deliberate attention to what students should be thinking about, what they should understand, or what they should share when they present, "answers" seemed to become the default. Eliza said very little to clarify what she meant by "understanding"; Luke described the value of his strategy by saying that it "gets [students] talking" (lines 481-482), without saying anything about what they talk about; and the detail that Margaret provided about the content of her students' presentations revolved around answers and procedures:

549 550	MARGARET:	\dots I try to be quiet, very hard, and sit at the back and let them present the answers, and.=
552	MARGARET:	=And let the others agree or disagree, that kind of thing
559 560 561 562 563 564 565 566	MARGARET:	the <u>first thing I want one of the group members to do is, is sort of summarize the question. So we were asked in this question to <u>draw a histogram of this, this, you know.</u> And then somebody else puts the, puts their work on the ELMO, and talks about, we did this, we did this, and does everybody understand. Take questions from the class. <u>And if they look at me, I tell them not to look at me. That's not okay, don't talk to me, I know the answer. Or if somebody says that's wrong and looks at me I say don't tell me, tell them. That kind of</u></u>
567		thing. I'm trying to get them to converse with each other

Margaret was drawing attention to important aspects of the work that she was doing to support conversations between students. As she explained, she would sit at the back of the classroom to physically remove herself from the spotlight and insist that students talk to one another instead of to her. But what she described students talking *about* is limited to procedures ("we did this, we did this"), answers, and their correctness ("agree or disagree"). Absent were links to important mathematical concepts, discussions about reasoning or strategy, connections to other students' ideas, or other forms of mathematically rich exchange. A similar phenomenon occurred when William described how he structured his students' presentations on a review problem:

604	WILLIAM:	I
605		wanted it to be so <u>quiet</u> in the audience that whoever came up could
606		try to answer it, without being interrupted, like to have some time
607		and some space to do that
610		I also wanted them to (.) have
611		enough, like enough of a comfortable environment where they could
612		(.) see, or like <u>try</u> if they knew it or not.

Like Margaret, William brought his colleagues' attention to classroom norms and the work that they had to do as teachers to create "a comfortable environment" that would support students to

take the risks associated with presenting in front of the class. That is, William focused on the social dynamics in his classroom. And like Margaret's, his descriptions of the mathematical work of his class were both peripheral to his point and based on the right answers and procedures, whether students "knew [them] or not."

Thus, the conversation at times represented mathematical activity as sharing "understanding" and at times as sharing and evaluating answers. These competing ideas about the goals of instruction twisted together into a tangled ball that their conversation did not support the teachers to unravel. This was true not only in the focal episode, but also across the rest of the data set. There were allusions to reform-oriented practices such as communicating, reasoning mathematically, and developing conceptual understanding in almost every meeting, but the relationships between these practices and the content that students were to learn were consistently left vague and unexplored. The instances in which teachers came closest to articulating such relationships were still abstract, without concrete representations of what students would say, do, or think:

I thought the MIRA [transparent mirror] was great last time, last year in terms of getting students to have a good sense of, like, getting to understand reflection in particular, and, um, it was really helpful in that respect, but. I don't know, I mean it didn't necessarily seem like it helped them get more concrete in terms of making transformations on the grid. ((Ryan, 09.04.2012))

Remember before the break, we brought those squares ... to introduce the whole concept of Pythagorean Theorem through conceptual, geometric understanding. This is the geometric. ((Cyril, 01.29.2013))

At other times, "understanding" was conflated with getting correct answers, as in Meriwa's summary of an assignment that began with three "basic practice" problems: "If they can get all these three *correct*, and they *understand* it, then they can move on to—right, because this page is just extra practice, so then they would move here, with the situations" (02.07.2013, emphasis added). In the absence of deliberate attention to how understanding would develop or what it would mean to understand, dominant framings of mathematical activity, with their focus on answers and procedures, seemed to come automatically and without conscious thought.

Framing student competence

A tension between restrictive and expansive positional frames characterized teacher talk about student competence, in much the same way that it characterized their talk about mathematics. In discussing how students could be supported to "make their understanding public," teachers implicitly placed value on students' ideas and knowledge. Similarly, in emphasizing that students need to talk to each other and not only to the teacher, the teachers suggested that making understanding public was not only a matter of assessment but also one of building a student-centered classroom culture, in which students are positioned as contributors to one another's learning.

Yet in talking about what their students were actually capable of, teachers also reproduced ability hierarchies. For example, Eliza juxtaposed students who "just can't" with those who can:

448	ELIZA:	$\dots >$ I was able to complete it in one of my classes=the other
449		two—< <u>could</u> n't do it. They can't, (.) um, they just can't <u>lis</u> ten to
450		each other pre <u>sent</u> , or <u>they</u> can't present?, um
456 457	ELIZA:	the Pass the Pen one kid at a time <u>only</u> worked in my third period class. There was <u>no</u> way,
458	():	[((quiet chuckle))
459	ELIZA:	[my other kids—my seventh period class could do anything like that.

It might have been that for Eliza, the observation that students "can't listen" and "can't present" did not imply that they could not understand and generate powerful mathematical ideas. It may even have been that she believed her students could *learn* to listen and present (though she asked for "other ideas of good ways for kids to share their understanding" "without having to stand up and present," rather than asking for help figuring out how to support such learning). But whatever she believed, her portrayal of her students framed them in terms of deficits that are in turn framed as immutable. Similarly, this exchange between Margaret and Eliza highlighted Geometry students' deficiencies in comparison to students in Stats (a mathematics elective):

567 568	MARGARET:	I'm trying to get them to converse with each other. It's obviously easier to do in Stats,=
569	ELIZA:	[Yeah.
570 571	MARGARET:	=[because they <u>fee</u> l a responsibility to think at a (.) higher level, I suppose so.
572	ELIZA:	And more of them do the homework.
574	MARGARET:	Exactly. Yeah, exactly.

Whether or not the teachers *intended* to frame mathematics ability as hierarchical (versus multidimensional) and students in terms of deficits, these statements *did not support* them or their colleagues to learn to redefine mathematical competence. Nor were these statements countered by comments that re-focused the conversation on students' strengths or ways of being mathematically "smart."

Thus, tensions between an abstract belief in all students and observations of some students as more able than others go unexplored. Their conversation failed to provide teachers with conceptual or practical tools (cf. orienting and technical resources, Chapter 4) for resolving these tensions, leaving a void for culturally dominant views of mathematical ability to fill. That these views persisted at Union despite teachers' expressed commitments to Complex Instruction and to serving all students was evident in the abundance of deficit- and disparity-oriented utterances throughout the year (I coded 96 instances in 28 hours of meetings), the distribution of these utterances across teachers, and the unambiguous character of many of them, such as:

They're just lazy, you know what I mean. And so then, you can see that. If they're lazy, and they're not producing, and they're just sitting around, then that's them, like, you know. ((Cyril, 10.16.2012))

I tend to target my lessons towards the like, if I have to do percentiles, like tenth percentile to seventy, seventy-fifth percentile. Like not the one or two kids that just cannot follow what I'm doing, and not the six, seven, five kids at the high end that are fine without me but, you know, are on the verge of being bored at times. ((William, 11.15.2012))

I think probably 80% of my students are solid right now, on everything? Maybe there's another 10% who are pretty good, and then you know, there's just those usual few who aren't understanding anything. ((Luke, 03.12. 2013))

Framing collaboration as opening and closing opportunities to learn

The analysis of this episode could end here, with the documentation of recurrent tensions between expansive and restrictive ways of framing mathematical activity and student competence and some inferences about teachers' problematic knowledge or beliefs about mathematics and students. Such an analysis might contribute to the field's understanding of just how difficult it is for teachers to shift away from dominant definitions of mathematical competence, even when they express commitments to non-dominant definitions and are supported by professional development opportunities. An implication of such research might be that professional development should more intensively target changes in teachers' knowledge and beliefs about mathematical competence.

But further analysis of the episode shows that the ways in which teachers framed their own collaborative conversations were at least as problematic as the ways that they framed mathematical activity and student competence, in terms of supporting teachers to generate and take up opportunities to learn. What they said, how they said it, and what they did not say framed the purpose of their collaboration as the generation of a variety of strategies for solving problems of practice, while the relevant knowledge was framed as idiosyncratic and rooted in personal experience. Sharing strategies in ways that avoided disagreement was framed as an appropriate, normal, and expected form of participation. These intertwined epistemological and positional framings of their collaborative conversations supported teachers in important ways while simultaneously limiting their opportunities to collectively investigate their practice and to learn.

The purposes of teacher collaboration. Both explicitly and implicitly, the Union math teachers framed their collaboration as an opportunity to address problems of practice by sharing a variety of experiences and instructional strategies. Eliza launched the conversation about "making understanding public" by asking "if anybody else has other ideas of good ways for um, kids to share their understanding" (lines 467-468), inviting her colleagues to share techniques that she (and others) could immediately use in her own classroom. On average, 80% of each Geometry Team meeting was spent sharing in "serial turns," with one teacher after another presenting his or her ideas—without responding to prior speakers' ideas (Dobie & Anderson, 2015; cf. Katz, Earl, & Ben Jafar, as cited in Earl & Timperley, 2008, p. 2) or giving any particular justification beyond "It worked well in my class."

Sharing ideas and strategies came out of a productive focus on instructional improvement. As Margaret said to her colleagues in a November department meeting:

We're all trying our best, we're looking for what's not working and see if anybody else here has the specific strategy that would solve it. ... I want to leave [this meeting] with something, [so I can say,] this is what I'm going to do to improve it. ((11.15.2012))

The search for "the specific strategy that would solve" teachers' problems of practice was thus a means of taking responsibility for and expressing agency to improve practice.

Yet the ways in which the teachers shared with each other served to enact and reproduce a culturally dominant framing of teacher knowledge as idiosyncratic and personal (Lortie, 1975; Hiebert, Gallimore, & Stigler, 2002). This framing was evident in teacher conversations at Union in the lack of connections in teacher talk-not only to ideas that had previously been shared but also to evidence of student thinking and to explicit principles about teaching and learning (e.g., in Luke's, Margaret's, and William's sharing of strategies in the "making understanding public" episode). Teachers found the quick sharing of ideas and strategies helpful (as they said to each other and to me in interviews, and as the frequency with which they did it might indicate); it provided them with a variety of tools that they could adapt for their own immediate or nearimmediate use. But it left them on their own to decide which strategies to use, based on personal preference and what they felt would "work" for them and their students rather than on data (including detailed anecdotes about student understanding and student work samples as well as grades and test scores) or theories of learning (including practical principles like "people learn through hands-on experiences" as well as the kinds of theories that are produced in the academy). Teachers sometimes connected their autonomy to a sense of "professionalism," as in a comment Cyril made in one of the first Geometry Team meetings of the year:

You can use the lab if you want. If there's activities that you create, or people that create, use it. If it's getting to the goal that you, you're trying to address, by all means, go for it, do it. Um. But if you feel like there's another activity that's going to address the goal that you're trying to get to, then use that. That's where our professionalism comes in, right? ((09.11.2012))

But framing professional knowledge as personal and collaboration as sharing limited teachers' opportunities to learn by preventing them from interrogating their practice more deeply and making connections between strategies, student learning, and notions of mathematical activity and competence. These habits could have supported them to collectively navigate the complexities of framing mathematical activity and competence that surfaced in their conversations.

Appropriate participation in collaborative conversations. Consistent with their frequent use of sharing, teachers' interaction framed the management of interpersonal concerns—in particular, the management of disagreement—as central to appropriate participation in their collaborative conversations. As Engle and Greeno's (1994) work demonstrates, such a framing is characteristic of "everyday" (as opposed to "intellectual") conversation. One of its markers is an avoidance of disagreement, evident in the focal episode in the ways that teachers chained disconnected strategies together without responding directly to others' ideas, and in their coupling of disagreement with expressions of agreement, empathy, and

affirmation. Such expressions tended to obscure opportunities to learn in some instances and foreclose them in others.

The ways the teachers in the focal episode responded to restrictive framings of students illustrate their avoidance of disagreement and the effects on their opportunities to learn. Margaret's description of the thought and effort that she put into making student presentations successful is in some sense a rebuttal to Eliza's assertion that her students "just can't." Without stating that she was doing so, Margaret *reframed* Eliza's problem, attributing it not to students but to teaching practices that Eliza can use more effectively. Yet Margaret did not explicitly question Eliza's interpretation; to the contrary, she qualified her own alternative by saying, "it's obviously easier to do in Stats." This nod to Eliza's deficit-oriented framing at the end of Margaret's lengthy talk turn closed down the opportunities that the rest of her turn seemed to open for *all* of the teachers—including Margaret herself—to learn to see all students as competent.

Emilio responded by asserting his own alternative interpretation—but like Margaret's, his delivery no sooner created opportunities to learn than it closed them down again. He began by referencing what Margaret had said: "you made an important distinction that, it just takes practice" (lines 580-581). Emilio's interpretation did echo aspects of Margaret's framing of students as capable of learning to give good presentations, but Margaret had not exactly made the point that "it *just* takes practice" (emphasis added); rather, she had analyzed the supports she provided to students in order to foster improvement in their presentations. In couching his statement as an agreement with Margaret, Emilio limited the extent to which the substantial differences in their statements could provoke collective inquiry and learning.

Like their expressions of agreement, teachers' communication of empathy and affirmation worked to manage their interpersonal relations while closing off opportunities for them to learn. For example, even as he proposed affective concerns as an important factor in explaining students' difficulties with presentations, Emilio forestalled further conversation of this interpretation by framing Eliza's problem as one that would go away with time:

584	EMILIO:	they just need to, do it more to become
585		comfortable with actually going up and presenting.=
587	EMILIO:	=So, it's going to take some time. And that could be frustrating but.
588		(You know.) Work your way through it and know that there's going
589		to be a little light at the end of the tunnel.

This compact statement did a great deal of work. It proposed an alternative understanding of student struggles, centered on "becom[ing] comfortable"; it acknowledged the negative emotions that students' struggles may generate for teachers ("that could be frustrating"); and it provided encouragement ("know that there's going to be a little light at the end of the tunnel"). It also removed the need for collective inquiry by suggesting that what Eliza needed was simply persistence.

Discussion

Despite their expressed commitments to innovative, student-centered mathematics instruction and their participation in a reform- and equity-oriented professional development program, the teachers in this study were frequently caught in tensions between restrictive and expansive ways of framing mathematical competence. They framed mathematical activity in

terms of communication, explanation, and understanding but also in terms of getting the right answer; they positioned students as potential contributors to mathematical discussions but also as incapable or inferior. The teachers did not invent either the expansive or the restrictive ways of framing mathematical competence. Rather, they drew on dominant discourses that frame mathematical competence as unequally distributed and more or less fixed, marked by getting the correct answers. They also drew on alternative, non-dominant discourses with roots in Complex Instruction and other reform-oriented strains of thought that frame mathematical competence as something that all students have, in a great diversity of ways. Both discourses provided resources for teachers to make sense of their work. Navigating the conflicts between these discourses was an important part of their professional learning.

Meta-discourses concerning the nature of their collaborative conversations shaped the ways that teachers navigated tensions between discourses and thereby their opportunities to learn. Teachers' efforts to frame mathematical competence in non-dominant and expansive ways were undermined by their management of interpersonal concerns and, relatedly, their avoidance of inquiry and disagreement in favor of more superficial sharing. These activities both reflect and re-enact anew dominant framings of professional knowledge as personal (Lortie, 1975), in which it is sensible for individual teachers to determine for themselves which ideas and instructional strategies suit their styles and preferences without reference to evidence of student learning or to shared principles of teaching and learning. The preservation of this autonomy goes hand in hand with a vagueness about content and about students; such vagueness respects boundaries between individual teachers' personal practice. But it provides few opportunities to reframe mathematical activity, student ability, and mathematical competence. Thus, the teachers in this study rarely talked explicitly about students. When they did, it was often through brief allusions rather than extended discussions in which students were at the center. Teachers' treatment of mathematics was typically shallow as well, referencing topics without getting into much detail about what understandings they were aiming to develop, how ideas connected to one another, or other potentially problematic or difficult aspects of teaching mathematics content. That is, teachers' collaborative conversations generally avoided *explicitly* reproducing restrictive, culturally dominant frames of students and mathematics, but in ways that also avoided the rich learning opportunities that deeper interrogation of their practice could have created.

Implications and Conclusion

The foregoing analysis highlights the ongoing meaning-making that occurs through teachers' collegial interactions. In showing how culturally constructed frames are both enacted and re-produced anew as teachers negotiate meaning in moment-to-moment interaction, this analysis extends the field's understanding of the tensions and challenges that teachers face as they attempt to negotiate non-dominant meanings. Learning to frame mathematics and students in expansive ways is less a matter of internalizing "correct" ideas once and for all than an ongoing negotiation of competing meanings and competing demands, some of which make dominant meanings more sensible or more functional than non-dominant ones. Because of this, professional communities may simultaneously support teachers to engage with innovative ideas and practices (such as "making student understanding public") and reinforce traditional meanings (e.g., a focus on correct answers).

Consistent with prior research (Horn & Little, 2010; Little, 1982; McLaughlin & Talbert, 2001), this chapter has also shown how teachers' opportunities to learn alternative ways of framing mathematics and students are shaped by the ways in which they frame collegial

conversation and professional knowledge. This chapter extends this finding by showing the limitations of practices that might be presumed to support teacher learning—namely, sharing ideas and strategies and carefully managing interpersonal relationships. Increased attention to the ways that teachers define their own work could support the field to better understand processes of teacher learning and to better support it. For example, the finding that framing collaboration as sharing interferes with teachers' opportunities to learn suggests that making other frames salient (e.g., of collaboration as intellectual inquiry) might support teachers to engage in the unending work of negotiating meanings that are inclusive of all students, in mathematically rich ways.

Additionally, though the foregoing analysis has highlighted tensions and limitations in collegial conversations amongst the mathematics teachers at Union, the extent to which their interactions supported them to conceptualize their practice in increasingly equitable and studentcentered ways should not be overlooked. The Union teachers' discourse departed in non-trivial ways from traditional discourses about school mathematics, student competence, and the work of teachers. This study captured but a year in their practice, during which time the ways that teachers framed mathematical activity, student competence, and their own work together were fairly stable. Future research might fruitfully investigate how teachers' professional learning communities develop over more extended periods. It is possible that the group described here is on a productive trajectory, with strengths that could be built upon over the course of many years in order to better support teachers' learning. More generally, future research might continue to investigate supports for collaborative conversations amongst teachers, in order to better understand how these supports function-in particular, how they build teachers' capacity to transform a professional culture that places high value on privacy and non-interference. Further developing these understandings not only at a structural level but also at the level of moment-tomoment interaction could contribute to the creation of more contexts that enable teachers to interrogate problems of practice and to generate profoundly transformative teaching and learning, for both themselves and their students.

Chapter 4

Supporting Teachers' Equity-Oriented Learning and Identities: A Resource-Centered Perspective

Abstract

This chapter examines four high school mathematics teachers' equity-oriented learning. Drawing on a year of ethnographic observations in classrooms, routine teacher meetings, and professional development settings as well as teacher interviews, the analysis highlights four types of resources that appear critical for supporting teachers' engagement with forms of practice that are non-dominant but essential for advancing equity in and through mathematics instruction. The chapter investigates two cases of ongoing teacher engagement with non-dominant practice and two cases of relative disengagement, illustrating how resources of different types come together to support—or fail to support—teachers' learning and identity development. Learning and identity processes are shown to intertwine in mutually informing ways as various resources are made available to and taken up by teachers. Implications for future research and practice in support of teacher learning are discussed.

Introduction

Teachers, like most of us, often talk about students as "high" and "low," "bright" and "slow." These labels reflect an "ideology of intelligence" that describes individuals' intellectual ability as innate, fixed, and quantifiable on a linear scale (Oakes, Wells, Jones, & Datnow, 1997). Many scholars have documented the particular dominance of this ideology in popular conceptions of *mathematics* ability (Ernest, 1991; Parks, 2010; Ruthven, 1987), noting, for example, the common belief that only "geniuses" and "nerds" can be good at math (Boaler, 2008b; Schoenfeld, 1988)—and the negative impact this belief may have on students' mathematics learning and identity development (Boaler & Greeno, 2000).

Redefining mathematical competence in more inclusive ways is thus an important aspect of advancing equity in and through mathematics education. Yet the teachers in the study on which this chapter reports frequently (though often subtly and unintentionally) reified hierarchies of mathematical ability, positioning some students as smart and others as slow—despite their commitments to equity-oriented shifts in instruction and their participation in a professional development program that specifically aimed to expand teachers' and students' notions of who could be mathematically "smart." In this respect, their cases mirror others; the tenacity of the traditional "grammar" of schooling (Tyack & Cuban, 1995; D. K. Cohen, 1990) and of hierarchical and exclusive ideologies of intelligence (Oakes et al., 1997) in the face of reform is well-documented. In previous chapters, I have analyzed the persistence of dominant discourses and ideologies in the daily work of the teachers in this study, examining how they come to life in teachers' classroom instruction (Chapter 2) and collegial conversations (Chapter 3).

Two of the teachers in this study—Amanda Pepper and Ryan Sower⁵—were unusual in that they consistently framed mathematical competence in culturally *non-dominant* ways, as something that *all* students possessed in a variety of forms. For example, launching a group task early in the year, Ryan used a "multiple-ability orientation" (as it is called in Complex Instruction parlance; see Tsu, Lotan, & Cossey, 2014):

We're gonna do a task today where you're gonna need all the different smartnesses of your group. You need people who are good at estimating, measuring, making conjectures, and seeing patterns ((writing each "smartness" on the whiteboard as he says it)). I know every single one of you is good at at least one of these. So everyone has something to offer your group.

Through statements such as this and a number of other instructional strategies, Ryan and Amanda disrupted dominant understandings of mathematical "smartness" in ways that created opportunities for each of their students—including students would not have been viewed as "good at math" in most classrooms—to engage with rigorous, challenging mathematics and to develop a sense of agency, authority, and competence (see Chapter 2).

Images of teachers in popular culture suggest that Ryan and Amanda are heroes, "islands of hope" (Gutiérrez, 1999) in a sea of dysfunction (think of the films *Stand and Deliver, Dangerous Minds*, and *Freedom Writers*, to name a few). Walking into either of their classrooms on any given day and observing their unusual instruction, it would be easy to conclude that they were reinventing their practice more or less alone. Following them out of their classrooms, however, it is obvious that though they deserve a great deal of credit, they organized support

⁵ All names are pseudonyms.

networks that provided them with a rich mixture of resources to foster their ongoing engagement with non-dominant practice. To better understand how these resources supported the teachers, this chapter addresses three questions:

- 1. *Where* do teachers encounter resources that support their engagement with non-dominant teaching practice?
- 2. *What kinds* of resources support teachers' engagement with non-dominant teaching practice?
- 3. *How* do different kinds of resources come together to support teachers' patterns of engagement with communities of non-dominant teaching practice?

That is, the chapter examines the sources of support that the teachers in this study experienced, the kinds of resources that these sources provided, and the ways that these resources interacted to support or undermine teachers' engagement with non-dominant teaching practice.⁶

Ryan and Amanda's cases show that it is possible for teachers to accomplish the kind of learning that the task of reframing mathematical competence requires. This chapter analyzes how such learning can be supported, finding that it requires a diverse array of resources—including resources that directly support teachers' learning as well as resources that indirectly support learning by fostering teachers' *identity* development. The ways in which learning and identity processes shape and inform each other, as different kinds of resources are made available to, taken up by, and sought out by teachers, are illustrated using the contrasting cases of four teachers (Ryan, Amanda, and two others). This analysis also highlights the interplay of environmental factors and teacher agency in seeking out and advocating for resources. Thus, it addresses a limitation of existing research on teacher learning communities, showing that important resources for learning may come from sources outside teachers' school-based communities of practice. This may be especially relevant when the learning at issue involves not only the transformation of the individual practioner but also the transformation of the practice itself. The chapter therefore highlights the importance of building not only more resources but also more kinds of resources into teachers' working lives, to better support ongoing engagement with equity- and reform-oriented learning.

Theoretical Perspective

The idea that identity processes play a fundamental role in both mediating and constituting learning is a central premise of this chapter. From this perspective, teacher learning cannot be understood in isolation from identity, because the two are dynamically intertwined. That is, both learning and identity are constantly taking shape, as opposed to being achieved and then fixed, and they take shape together, in mutually informing and mutually constituting ways.

⁶ I use the term "non-dominant" to describe teaching practice that aims to redefine mathematical competence, highlighting the direct opposition of such practice with culturally dominant views of mathematical ability as something that is distributed across populations in grossly unequal ways, leaving some people intrinsically "good at math" and others intrinsically "bad at math." The non-dominant aspect of the practice I describe is particularly significant in the context of this chapter because of the challenges that navigating dominance creates for teachers as they negotiate shifts in their practice.

Identity is both a cause and an effect of learning; learning is both a cause and an effect of identity. Yet the two are distinct; a person may "appropriate cultural tools (an aspect of learning) without taking them on internally (an aspect of identity)" (Nasir & Cooks, 2009, p. 44). For example, one teacher may learn to use instructional strategies associated with a particular reform and develop an identity as a reformer, while another may learn the same strategies but maintain an identity as someone whose work is peripheral to the reform.

This chapter's investigation of the relationships between teacher learning and identity processes is heavily informed by theories of learning in *communities of practice* (Wenger, 1998). A significant limitation of work on communities of practice, however, is its focus on the relatively strong relationships that arise through mutual engagement in a joint enterprise. I draw on theories of social networks, in particular Granovetter's (1973) perspective on "the strength of weak ties," to address this limitation and explore the resources that "weak ties" may provide for teacher learning and identity development.

Learning and identity in communities of practice

As Wenger (1998) defines them, communities of practice are organized around mutual engagement in a joint enterprise, with a shared repertoire. Through everyday participation in communities of practice, people negotiate the nature of the enterprise and the shared repertoire that defines it, drawing on—and constrained by—the history of the practice but also inevitably making it their own. This negotiation, Wenger argues, is simultaneously a process of learning and a process of becoming: our participation in social practices, he writes, "shapes not just what we do, but also who we are and how we interpret what we do" (p. 4).

Nasir and her colleagues (2012; Nasir & Cooks, 2009) build on Wenger's analysis to reveal how learning and identity processes are linked to each other and to social practice through interactions around three types of "identity resources": material, relational, and ideational. Nasir (2012) defines these resources as follows.

By *material resources*, I mean the ways that the physical environment, its organization, and the artifacts in it support one's sense of connection to a practice. *Relational resources* refer to the way in which positive relationships with others in the context can increase one's connection to a practice. *Ideational resources* refer to the ideas about oneself and one's relationship to and place in a practice and the world, as well as ideas about what is valued and what is good. (p. 110, emphasis original).

These resources work together to foster trajectories of participation in particular practices and communities of practice. Thus, an instructional strategy (say, the multiple-ability orientation described above) might be a material resource that supports a teacher (such as Ryan) to participate in non-dominant teaching practice. The strategy might also serve as an ideational resource, shaping his ideas about what constitutes "good teaching" to include responsibilities that he had not previously considered (e.g., expanding students' ideas about "smartness"). Other ideational resources (e.g., a general commitment to supporting struggling students), relational resources (e.g., connections to more experienced practitioners who provide advice and encouragement), and material resources (e.g., a curriculum that presents mathematics as multidimensional) might simultaneously bolster his efforts to learn and develop his practice in equity-oriented, non-dominant directions. In other words, these resources might support an

inbound trajectory (Nasir & Cooks, 2009; Wenger, 1998) of deepening engagement with communities of non-dominant teaching practice. On the other hand, insufficient resources might contribute to an *outbound trajectory*, in which the teacher leaves the profession because of a lack of support for learning and identity development.

I follow Nasir and Cooks (2009) in conceptualizing learning as "shifts in use of artifacts (both cultural and cognitive) for problem solving, sense making, or performance" (p. 44). When I talk about "teacher learning" in this chapter, I am typically describing shifts in teachers' use of artifacts that are directed at making sense of and solving the problems of equity-oriented, non-dominant teaching practice. But it is worth noting that teacher learning (e.g., learning to use traditional teaching methods such as lecturing) may lead them away from rather than toward more equitable teaching practice, though such learning is not the focus of this chapter.

I conceptualize identity development in terms of two kinds of shifts surrounding participation in a particular practice: shifts in how important a person's participation in the practice is to who she is and wants to become (i.e., shifts in what Nasir and Hand (2008) call "practice-linked identities"), and shifts in how the person perceives her own competence or value as a participant. As she participates in non-dominant teaching practice, for example, this practice might gradually become more central to a teacher's sense of herself as an educator, even though she continues to view herself as an unskilled novice with respect to it.

Social network theory

This chapter draws on Wenger's (1998) and Nasir's (2012; Nasir & Cooks, 2009) work to investigate the ways that teachers' learning and identity processes relate to and potentially support one another, through teachers' everyday interactions with the resources their communities of practice make available. I also make use of an insight from social network theory: that weak ties (e.g., acquaintances) provide people with access to resources that are both critical and not provided by strong ties (e.g., close friends-or members of one's community of practice). Thus, in his seminal article on "the strength of weak ties," Granovetter (1973) reported that recent job changers were more likely to have received information about their new jobs from contacts with whom they interacted occasionally or rarely than from people they saw often. Building on this perspective to study teacher development, Anderson (2010) found that one teacher's network included not only teachers and staff members at her school but also teachers at other schools, researchers, and local business owners. To think of local business owners as engaged in the same enterprise as classroom teachers risks stretching the concept of mutual engagement beyond all usefulness (Wenger, 1998). But as Anderson describes, they provided crucial support for the focal teacher's professional development and for her sense of herself as an effective educator.

In recent years, interest in and use of social network analysis has grown tremendously (Daly, 2010). Two strengths of social network analysis are its ability to illuminate the structure of people's networks in ways that are not confined to strong ties, and its ability to represent this structure in increasingly sophisticated ways. (A number of software programs have been designed to aid in the generation of representations; see, e.g., Daly, Moolenaar, Bolivar, & Burke, 2010). However, the mechanisms through which networks support change—i.e., what kinds of resources flow across ties, and how—have received somewhat superficial attention.

This chapter therefore takes up Wenger's (1998) notion of learning and identity as intertwined through the negotiation of meaning and shows how it applies to networks that extend beyond the boundaries of communities of practice as they have traditionally been conceived. I
continue to use the term "community of practice," following Wenger's definition, to refer to groups (such as subject-matter departments) that overlap but are not synonymous with the teachers' professional support networks.

Prior Research

Much of the literature on teacher learning focuses on the privileged moments created by pre-service teacher education programs or professional development workshops. While these moments may certainly be formative, research has also begun to attend more closely to the day-to-day learning that practicing teachers accomplish in their communities of practice. A major contribution of this research has been the identification of community and network characteristics that support teacher learning. With respect to reform-oriented learning in particular, scholars have used social network analysis to show that networks with high-depth interaction, strong ties, and high expertise support teachers to both adopt and sustain ambitious instructional reforms (Coburn, Russell, Kaufman, & Stein, 2012; Daly et al., 2010; Penuel, Riel, Krause, & Frank, 2009). Research on professional learning communities (PLCs) has linked other kinds of characteristics—e.g., shared values, collaboration around student learning, accountability structures, openness, and trust—to support for teacher learning (Bolam, McMahon, Stoll, Thomas, & Wallace, 2005; DuFour, DuFour, & Eaker, 2008; Louis, Kruse, & Marks, 1996; McLaughlin & Talbert, 2001).

Fine-grained analyses of interactions between teachers illuminate processes of learning in professional learning communities. For example, Horn and her colleagues (2005; Horn & Little, 2010; Horn & Kane, in press) have conducted detailed examinations of episodes of teachers' "pedagogical reasoning," finding that more opportunities for teacher learning than are typical are present in collegial conversations in which teachers focus their attention on problems of practice, coordinate general ideas about teaching with specific instances of practice, and use multivocal "replays" of classroom activity to render practice transparent.

This chapter extends this work in three ways. First, it explores the mechanisms through which teachers' participation in communities and networks support their learning, taking resources for learning and identity as its unit of analysis. This approach bridges work at the level of moment-to-moment interaction (e.g., discourse analysis) and work at the level of policy and organizations (e.g., studies of social networks and PLC characteristics). Second, the chapter links identity and learning. This makes visible kinds of resources that are often missed in studies of teacher learning, which tend to focus on conceptual resources such as material artifacts and access to technical expertise (e.g., Coburn, Choi, & Mata, 2010; Lampert, Boerst, & Graziani, 2011). It also makes possible the examination of identity not just as a cause or an effect of learning (as is common in research on teacher identity; see, e.g., Foote, Smith, & Gillert, 2011; Hodgen & Askew, 2007) but as a process that both shapes and is shaped by learning in dynamic ways through everyday interaction. Third, the chapter looks beyond the traditional boundaries of school-based networks and professional learning communities to locate critical but frequently overlooked sources of support and to illustrate teachers' agency in organizing supportive networks, thereby contributing to the field's understanding of the ways in which teachers and their social contexts-both within and beyond their school walls-co-construct equity-oriented learning and growth.

Methods

Research sites and participants

This study was initially organized around the question: How do teachers' professional communities support them to redefine mathematical competence? The literature on professional learning communities (e.g., Gutiérrez, 1999; Horn & Little, 2010; Horn, 2005; Little, 2002; McLaughlin & Talbert, 2001) led me to expect that teachers' school-based communities, especially their departments and course teams (e.g., the Algebra Team and the Geometry Team), would be critical sources of support. I therefore sought to recruit mathematics departments that were collectively committed to redefining mathematical competence.

Based on nominations from "insiders" in the local mathematics education community (including university-based researchers, teacher educators, and school district personnel), I selected two diverse schools in an urban district in Northern California for participation in the study: Union High and Boxer High. All of the mathematics teachers at both schools expressed a commitment to supporting all students, especially students who had not been academically successful in the past (in their conversations with each other and in research interviews). To that end, all of the teachers participated in an equity-oriented professional development program offered by their district. The program centered on Complex Instruction (CI; see Boaler & Staples, 2008; E. G. Cohen & Lotan, 1997). CI posits that all students have important intellectual contributions to make to their classroom learning communities, and that teachers are responsible for drawing out every student's strengths while also helping every student develop new strengths. The professional development program in which the teachers were participants targets these beliefs as well as instructional strategies for "equalizing status"—i.e., for leveling hierarchies of perceived ability and worth-such as the use of complex, open-ended tasks that require students to pool their various skills and work together. The PD also emphasized the development of robust, department-based professional communities, and teachers at Union and Boxer had dedicated time each week to collaborate around mathematics instruction. In addition, they participated in periodic CI trainings and coaching sessions. This array of resources made Union and Boxer rich contexts in which to examine supports for teacher learning around nondominant practice.

From the thirteen mathematics teachers at Union and five at Boxer, I recruited six focal teachers: four from Union (Ryan Sower, William Barrett, Cyril Nazemi, and Luke Farber) and two from Boxer (Amanda Pepper and Rob Daly). Teachers were selected based on factors that appeared to be connected to the depth of their engagement with CI in my early observations. Specifically, I selected for range along two dimensions: years of experience teaching, and leadership roles in the district CI community.

Data collection

I observed routine meetings of teachers' department and course teams (30 hours of meetings at Union, and 11 at Boxer) and Complex Instruction professional development sessions (70 hours) throughout the 2012-2013 academic year. I also conducted 4-8 classroom observations for each focal teacher. Audio recordings, field notes, and photographs of whiteboard inscriptions, worksheets, and other artifacts were produced for each observation. Informal interviews often accompanied observations. In addition, focal teachers were formally interviewed at the end of the school year following a semi-structured protocol that focused on

teachers' goals and the challenges and supports that they experienced in relation to their goals. Formal interviews were audio recorded. I transcribed all audio recordings.

Analysis techniques

Locating sources of support. Recall the first research question guiding this study: *Where* do teachers encounter resources that support their engagement with non-dominant teaching practice? To address this question, I analyzed interview transcripts with a focus on the ways that teachers themselves reported feeling supported by their colleagues and others. Responses to three interview questions were especially relevant:

- Are there any experiences or people who you would point to as being especially important in your development as a math teacher? (How?)
- Are there ways that your colleagues—here at [school name] or elsewhere support your development as a teacher?
- Are there ways that your colleagues—here at [school name] or elsewhere support you to manage the challenges and dilemmas you've described?

Interview coding was not limited to these three questions, however; all talk about people and experiences that shaped or supported teachers' thinking about their work was considered.

Teacher reports were used to generate egocentric network diagrams (Anderson, 2010; Borgatti & Ofem, 2010). These diagrams (essentially a black-and-white version of *Figure 9*) were a first step in visualizing each teacher's network and the supports it provided. The focal teacher was placed at the center of the diagram and connected to each of his or her supporters with a line segment. The length of each line segment was used to show the frequency of teaching-related interaction between parties (based on teachers' reports and on my observations). The shortest segments represent daily interaction; longer segments represent 1 or 2 times per week, 1 or 2 times per month, less than once a month, and never. In addition, supporters were clustered together based on setting (e.g., colleagues at the focal teacher's school were placed near one another). I did not use social network analysis tools such as standardized survey instruments and network analysis software; rather, the network analysis that I conducted developed organically from interviews and observations.

Identifying forms of support. To address the second question in the study—What kinds of resources support teachers' engagement with non-dominant teaching practice?—I used open coding (Emerson, Fretz, & Shaw, 1995) to analyze interview transcripts, again with a focus on teachers' descriptions of how they had been supported by their colleagues and others. Emergent themes were coordinated with the literature on learning, identity, and meaning-making, in particular, with Wenger's (1998) and Nasir and Cooks's (2009) frameworks. This coordination of research questions, data, and the literature led to four categories of support for teachers' equity-oriented learning and identity development. These four categories abstracted the phenomena that open coding had initially revealed, moving toward theorizing the relationships between each category and processes of learning and identity development (e.g., "emotional support" became "relational resources," and "support for nuts and bolts" became "technical resources"). The four categories were then used to systematically code interview transcripts. I also examined field notes and transcripts from my observations to more fully understand the

functions that various supports played, seeking both different perspectives on the supports that teachers themselves described and instances of the four categories that teachers did not mention. I paid particular attention to positional resources, reasoning that whereas all of the teachers alluded to the other types of resources in their interviews, the novelty of positioning as a concept may have made it difficult for them to notice or name this as a resource.

Network diagrams were then color-coded to show which resources flowed through each relationship, with a different color for each type of resource (as shown in *Figure 9*).

Characterizing teachers' patterns of engagement. As a prelude to linking different constellations of resources to teachers' patterns of engagement with non-dominant teaching practice, I characterized the latter across two settings: their classrooms and their professional communities.

To characterize *classroom engagement* with non-dominant teaching practice, I used both qualitative and quantitative analyses. In the hours I had spent in teachers' classrooms, differences in instruction were almost tangible. I selected episod2es from each teacher's classroom for close analysis as a means of both clarifying what those differences were and searching for disconfirming evidence (i.e., of similarities). Episodes were selected to capture teachers' practices around 1) adapting and assigning tasks and 2) responding to students' struggles. The analysis examined how instruction framed mathematics as a discipline and students as learners of mathematics (as exemplified in Chapter 2). I also sought to quantify differences in classroom practice. Taking the extent to which teachers' instruction was student-centered (versus teachercentered) as a measure of their engagement with Complex Instruction and related non-dominant ideas, I coded time-stamped segments of classroom activity as teacher-led, including teacher presentations or class discussions in which student contributions were limited to brief responses to close-ended questions; student-led, including student presentations or work time in which students were presented with opportunities for problem solving; other work time, including work time in which students were presented with exercises identical to ones the teacher had demonstrated how to solve earlier in the lesson; and non-mathematical, including time spent on announcements and transitions between activities. I then calculated the percentage of class time each teacher spent in teacher-led versus student-led mode. A higher percentage of the latter and a lower percentage of the former indicate deeper engagement on the teacher's part with Complex Instruction. For each teacher, I also calculated the median number of words in each student talk turn during whole-class discussions, associating a higher median with deeper engagement (arithmetic averages were also calculated, but median was selected as a more accurate measure of central tendency because averages were prone to distortion based on one or two unusually long talk turns).

To characterize teachers' engagement in *communities of non-dominant teaching practice,* I examined the frequency with which teachers attended meetings of their district's Complex Instruction network (which I attended) as well as teachers' own reports of interaction with and connection to other educators who were linked to the regional Complex Instruction network (e.g., colleagues in other school districts who had attended CI-focused teacher preparation programs in the area).

Linking patterns of engagement to network resources. To address the third research question—How do different kinds of resources come together to support teachers' patterns of engagement with communities of non-dominant teaching practice?—I sought to understand the

interplay between different kinds of resources in support of teachers' learning and identity development. The network diagrams made a number of patterns visible, in particular, connections between the density of teachers' networks and the depth of their engagement with non-dominant practice, and differences in the distribution of each type of resource in different teachers' networks. These patterns were then explored in narratives that I wrote to describe each teacher's engagement with non-dominant teaching practice and the resources that supported that engagement.

Member checking. This chapter focuses on four of the six focal teachers in the study: Ryan Sower, Luke McCormick, and William Barrett at Union, and Amanda Pepper at Boxer. Data collected with all six teachers were analyzed to generate findings. The four cases presented here have been selected to showcase differences in teachers' networks and trajectories of engagement with non-dominant practice and radical change.

Completed narratives were shared with three of the four teachers featured in this chapter (Luke McCormick had since left the profession, and I was unable to reach him). Comments provided by the teachers have been incorporated.

Learning and Identity Resources in Teachers' Professional Networks

For the six focal teachers in this study, finding ways to reach students who had previously been unsuccessful in school was an explicit goal. For them, this entailed giving students access to mathematics content. But it also meant shaping students' ideas about "smartness" and "success" so that all students could take ownership over their own learning and see themselves as competent. Three of the six focal teachers (Ryan, Luke, and Cyril) said that one of their goals was to make their students "feel smart," using that exact phrase. Others said that they were "trying to redefine what success is" (William), to "rethink … what's it mean to be smart at math" (Rob), and to "change [students'] status of what they [think] about themselves" as mathematics learners (Amanda). Yet their engagement with non-dominant teaching practice—as evidenced by both their classroom instruction and their work with their colleagues—varied widely.

A comparison of teachers' professional support networks shows that their patterns of engagement are related to the distribution of four types of resources in their networks: *orienting resources*, which support teachers to envision the kind of practice they want to achieve; *technical resources*, which support teachers with the "nuts and bolts" of enactment; *relational resources*, which support teachers' sense of belonging and identification with their practice; and *positional resources*, which support teachers' sense of worth and competence as professionals (cf. Nasir & Cooks, 2009). This chapter proceeds with an elaboration of each of these definitions, followed by a discussion of the contrasting cases of four teachers. These cases are used to illustrate how orienting, technical, relational, and positional resources came together to produce four distinct patterns of engagement with the work of redefining mathematical competence: two patterns of ongoing engagement; and a pattern of disengagement with teaching practice. The cases also show the importance of resources that lie outside teachers' local, school-based communities of practice.

Orienting resources

Orienting resources provide teachers with a vision of "good teaching," presenting images of practice that inform teachers' ideas of what their classrooms should look and sound like, what their roles should be, and whether or not they are successful. That is, they support teachers to orient to their work in particular ways. Some orienting resources function by explicitly focusing teachers' attention on fundamental principles and big ideas about teaching. For example, CI training attempted to orient teachers to the ways that status problems (perceived rather than actual differences in competence and worth) affect students' participation and learning. Other orienting resources structure teachers' thinking more tacitly, providing models to emulate and lenses through which to interpret their work. The "apprenticeship of observation" (Lortie, 1975), i.e. teachers' own experiences as elementary and secondary school students, was thus an important source of orienting resources for many of the teachers in this study.

Orienting resources shaped teachers' learning in important ways, defining what it was that they should know and be able to do and providing lenses through which to understand the materials and strategies that were available for enacting their practice. Orienting resources could also be resources for learning insofar as they provided teachers with concrete examples to draw from and work toward (though they were not always specific enough to fill this function). And orienting resources shaped teachers' professional identities, setting standards against which teachers could measure themselves (discussed in more detail below) and supporting a sense of belonging to both real and imagined communities of educators with a shared, non-dominant vision.

Technical resources

Teachers' environments are full of technical resources—the materials and strategies that support them to enact their practice on a daily basis. Participating in the exchange of materials (e.g., worksheets) and strategies (e.g., methods for managing small group interactions) is a way of garnering immediately usable information and learning "tricks of the trade." It is also a way for teachers to identify themselves as part of a community of practice, as a contributor or as an accepting recipient. In other words, teachers develop new ways to participate in their practice i.e., learn—as they interact with these resources. They also develop identities as professionals who are interested in or competent at (or both, or neither) using particular materials and strategies.

Relational resources

Relational resources are those connections with others that support a sense of belonging to and identification with a practice (Nasir & Cooks, 2009). Connections to colleagues, students, and others make teachers' work livable and enjoyable, as teachers not only give but also receive the "caring" that some have characterized as central to their profession (Noddings, 2003). For teachers working to "teach against the grain" (Cochran-Smith, 1991), relationships with others with a shared mission can also bring a sense of solidarity that supports their experience of connection to their work. In addition, by connecting teachers to others, relational resources provide channels through which orienting and technical resources can flow. There is no guarantee that these resources will support teachers to engage in *change-oriented* learning and

identity development, however; some relational resources may feel supportive to teachers while supporting them to reproduce traditional practice.

Positional resources

Related to the caring embedded in teachers' relational resources are *positional resources* that support teachers' sense of their own worth and competence as professionals. Being included in a professional community and being recognized as a competent member of it do not always go together. In addition, teachers are part of multiple professional communities, with norms and standards of competence that are sometimes at odds. Consequently, they are sometimes positioned as highly skilled in one community of practice but not in another. For example, the veteran teachers in this study were positioned as experts at their school sites and as learners with respect to the district's Complex Instruction community.

Positional resources can both support and undermine teachers' learning and identity development. Being positioned as an expert, for example, may suggest to a teacher that her own learning is unnecessary. However, this positioning can support her to take up an identity as a competent professional, thereby supporting her continued engagement with her practice. On the other hand, being positioned as a novice can nudge teachers to pursue opportunities to learn while fostering feelings of incompetence that promote their disengagement with their practice. Or novice identities may be framed in terms of promise and potential in ways that support ongoing engagement. The cases that follow illustrate these various ways that positional resources are offered to and taken up by teachers.

Resources for learning, resources for identity

Orienting, technical, relational, and positional resources all support both learning and identity, but in more and less direct ways. Orienting and technical resources work in mutually reinforcing and interdependent ways to *directly* support teachers to learn and develop their practice by providing ideas (orienting resources) and tools for enacting those ideas (technical resources). Technical resources support orienting resources by making the technologies that operate within any expansive vision transparent and concrete; without them, orienting resources may prove too abstract to support shifts in teachers' practice. At the same time, technical resources are rendered sensible by their connections to the "big picture" that orienting resources can provide.

Relational and positional resources, in comparison, support teachers' learning by fostering *identities* that motivate and encourage learning. Learning—especially non-dominant learning—is hard work. Most of the teachers in this study experienced setbacks and doubts about whether they should continue to pursue non-dominant learning over the course of the year. Teachers' capacity to understand themselves as part of something bigger than themselves and as capable of continued learning, growth, and success supported them to persist through challenges, advocate for additional resources, and find support in their professional networks.

Of course, to delineate resources for learning and resources for identity so cleanly is an oversimplification. In practice, learning and identity development imply and depend on each other. Thus, learning to use a new instructional strategy (say, a multiple-ability orientation) is mediated by a teacher's ideas about who she is as a professional and who she wants to become. At the same time, the teacher's ideas about her identity may shift as she engages with the strategy and the field of possibilities that it brings into view. What matters is not drawing

boundaries between learning and identity but understanding the interplay between them as teachers engage with their practice.

Patterns of Engagement with Non-Dominant Practice

Constellations of orienting, technical, relational, and positional resources work together to support or undermine teachers' engagement with non-dominant teaching practice inside and outside of their classrooms—in other words, to produce *patterns of engagement*. I use the term "pattern of engagement" instead of "trajectory" because my data capture not so much change over time (which I did not see over the year of the study) as teachers' closeness to equityoriented practice and their apparent direction (whether inbound, outbound, or stable) at a particular moment or set of moments. This perspective is especially important in light of questions about sustainability; a teacher who appears to be on an inbound trajectory of increasingly engagement with her practice one year might "burn out" the next. Thus, the concept of a pattern of engagement is not meant to predict where someone is headed; rather, its utility lies in its ability to help us understand the kinds of supports people have and might need in order to move in particular ways.

Two of the teachers in this study, Ryan Sower and Amanda Pepper, maintained active engagement with non-dominant teaching practice throughout the period of the study. Their classroom instruction consistently framed mathematical competence in ways that expanded students' opportunities to develop identities as powerful learners and doers of mathematics (see Chapter 2). They were also core members of communities of practice that were organized around non-dominant teaching practice, including recognizable ones like the district's Complex Instruction community as well as more figurative ones (cf. Holland, Lachicotte, Skinner, & Cain, 1998) that were joined (at least in their imaginations) by shared values around teaching and learning. The networks that Ryan and Amanda organized to support their work were both densely populated and rich in a variety of types of resources—sourced from a variety of places (see Figure 9[a] and [b]). But the particular arrays of orienting, technical, relational, and positional resources that each of them took up were different in important ways, showing that there are multiple ways to support teachers' ongoing engagement with reform.

In comparison to Ryan and Amanda, William Barrett and Luke McCormick had few resources—especially for engaging with non-dominant teaching practice—at the time of the study (see Figure 9[c] and [d]). It was rare for either of them to attend district-sponsored CI gatherings or to take up other opportunities for learning and collaboration outside the school day. William's resources were nonetheless sufficient to sustain his student-centered teaching style and his sense of connection to his work—but they were inadequate to support him to actively engage with reframing mathematical competence. This work remained peripheral to his classroom instruction, and he himself remained at the periphery of the communities of non-dominant teaching practice in which Ryan and Amanda were central. For his part, Luke did not have the resources to support continued engagement with either non-dominant teaching practices or with mathematics teaching more generally, and the year after the study concluded, he left the profession.

The character of teachers' classroom instruction was part and parcel of their participation in communities of non-dominant teaching practice. The resources that were available to and taken up by the four teachers in the study are thus reflected by their work with students. Table 7 gives a flavor of the differences between Ryan, Amanda, William, and Luke's instruction. (For detailed analysis of classroom episodes, see Chapter 2.) The cases of these four teachers demonstrate the ways that learning and identity processes intertwine as orienting, technical, relational, and positional resources are offered to and taken up by teachers. In particular, they show how these four types of resources may support teachers to engage in different patterns of participation in the work of reframing mathematical competence.



Figure 9. Egocentric social network diagrams depicting the professional support networks of (a) Ryan Sower, (b) Amanda Pepper, (c) Luke McCormick, and (d) William Barrett. Shorter lines indicate more frequent interaction. Darker shading represents overlapping membership in multiple communities (from the focal teacher's perspective).

Table 7

	More engaged		Less engaged	
	Ryan	Amanda	William	Luke
% of class time spent on teacher-led presentations	10%	12%	22%	27%
% of class time spent on student-led work ^a	51%	40%	12%	14%
Median # of words per student talk turn ^b	5	6	1	2

Indicators of teachers' classroom engagement with non-dominant teaching practice.

Note. ^aStudent-led work includes student presentations as well as group problem solving. Time spent having students work routine exercises (either in groups or individually) is not represented in this table. ^bOnly data from whole-class discussions were used for this analysis.

Ryan Sower: A pattern of ongoing engagement

Throughout the year of the study, Ryan Sower was an active participant in a variety of communities that supported his engagement with non-dominant teaching practice. These communities included the mathematics department at his school, his district's CI network, and a group of friends from the teacher education program he attended. His classroom practice reflected this engagement; he often used strategies associated with Complex Instruction, and the idea that mathematics classrooms should be intellectual communities in which all students are supported to contribute and learn permeated his instruction. Ryan himself was supported by a variety of orienting, technical, relational, and positional resources. His case illustrates how relatively abundant resources can come together to support mutually supportive learning and identity processes, producing a pattern of ongoing engagement with non-dominant teaching practice and with reframing competence specifically.

Orienting resources. Ryan drew on his memories of Tom and Ina, two of his first teachers in elementary school, to orient toward a vision of a teacher as someone who made every student feel loved, accepted, and treated as though he or she had "something to contribute." But like the other teachers in this study, Ryan characterized the mathematics instruction he received as a student as "traditional," emphasizing "discrete skills" and "mathematical rules" with little place for making sense of ideas and connections, especially in high school. Before he began his training as a teacher, this instruction was his primary resource for imagining his own practice; as he said, "I was envisioning myself teaching much the same way that I'd been taught." But through his teacher education program and the networks that the program opened up for him, Ryan gained access to orienting resources that supported him to learn new ways of conceptualizing the work of teaching secondary mathematics.

The teacher education program that Ryan attended was grounded in Complex Instruction, and it drew Ryan's attention to issues of power and status in mathematics classrooms. His math methods instructor, Ruth, introduced him to a pedagogy that was designed to address these

issues, linking somewhat abstract ideas he had had about accepting and valuing students to a concrete vision of mathematics teaching and learning. Ruth took her students "seriously," "really honoring all the different things that [they brought] to the table." She assigned his cohort complex mathematics problems and centered her instruction on students' ideas instead of her own lectures, framing mathematics not as a fixed body of knowledge but as a rich terrain of big, interconnected ideas to be explored. Reflecting on his own mathematics learning through the lens of Ruth's example, Ryan found that he had been poorly served by instruction that focused on following procedures without helping students to "understand" mathematics (even though he had earned good grades and been seen as a strong mathematics student in school). Thus, Ruth's instruction oriented him firmly away from "traditional" teaching practice and toward an alternative in which redefining mathematics and mathematical competence were central.

Ryan's learning around reframing mathematics and "honoring" all students continued throughout his year-long placement as a student teacher at Railside High School (the pseudonym given by Boaler & Staples, 2008). At Railside, he and a fellow student, Jasmine, were apprenticed to Guillermo Reyes, a nationally-known leader in Complex Instruction (see, e.g., Horn & Little, 2010; Horn, 2007). Guillermo's class provided a compelling image for Ryan: "I just knew right away that I wanted to create that kind of classroom. I wanted to create that kind of space for kids to explore math, and feel smart, and be excited about working with their peers and their teacher."

At the time of the study, Ryan was in his second year at Union, which was also his second year as a full-time teacher. He appreciated many things about his colleagues, and they supported him in ways that will be described below, but he encountered limitations in their capacity to share—let alone enrich—his perspective on what ideal mathematics instruction should look and sound like. In his view, this was not because they didn't share his fundamental values; on the contrary, it was his feeling that "everybody [at Union] wants to get better at [CI] and we're all really committed to it." But, he said, "I just don't think we've had enough of a vision for what a CI department is." In this context, the images of good teaching that he drew from Ruth, Guillermo, Tom, and Ina continued to be important orienting resources for Ryan, supporting him to maintain his engagement with the work of reframing mathematical competence and to persist in striving to improve his own practice along those lines.

Technical resources. Ryan's work with Ruth and especially Guillermo also supported his practice by providing him with technical resources for its enactment. Through his year of student teaching, he learned to use curricular materials and instructional strategies in ways that became integral to his practice. His colleagues at Union also had access to these materials and strategies, but they were less able to make sense of them. For example, they had all experienced "participation quizzes" in the district's summer CI course, working together on math problems as the instructors circulated, listening and taking public notes on the helpful, productive, and smart things that they did (see Staples [2008] for a detailed description). For many teachers, this was a powerful example of a strategy for developing and enforcing particular norms for groupwork. But it was only one example. In contrast, as an intern in Guillermo's classroom, Ryan witnessed dozens of participation quizzes and led many himself, with feedback and support from Guillermo and Jasmine. In my year of observations at Union, Ryan was the only teacher I saw lead a participation quiz.

The broader meaning of teaching for Ryan—developed through his array of orienting resources—clearly informed his use of his technical resources. For example, supporting students

to "feel smart" by "honoring" their contributions to their own and their peers' intellectual growth played a central role in the participation quiz that I saw him lead. As he enacted the debrief portion of the quiz, for instance, he reported to the class that at Group 1,

there were some really good questions, a lot of people asking like, Is this line the base or the height, [and] how do you know? The part that's not on the bottom is the height, or is the base. They had a little bit of an argument but then they figured it out. And they did a really great job of making sure the new student got caught up right away, which was excellent. So like I would definitely give them an A+ right now if I gave them a grade.

As he continued to debrief, Ryan similarly highlighted specific actions and utterances that positioned each group as mathematically (and socially) competent, orienting toward the goal of reframing competence and disrupting hierarchies. At the same time, his use of technical resources like participation quizzes and other CI strategies provided Ryan with additional orienting resources. Employing these strategies gave him opportunities to look for and see the ways that his students were capable, smart contributors to each other's learning, reinforcing his vision of good teaching as teaching that reframes mathematical competence. Strategies that were common in other teachers' practice did not always make such opportunities available (for example, when teachers had students work in groups but felt compelled to jump from one group to another offering assistance).

But technical resources were few and far between for Ryan by his second year at Union. In his first year, colleagues in the department shared their lesson plans, responded quickly to his questions and concerns about his students, and welcomed him into their classrooms to observe. William, who was his officially his coach, was "a huge support," collaborating with him to plan lessons and even co-teaching with him occasionally. At the end of that year, however, several experienced teachers left the school (including Guillermo, who was there for Ryan's first year), and Ryan was "slotted in" to leadership roles. He spearheaded the math department's effort to rewrite the Advanced Algebra curriculum (in an effort to bring it into closer alignment with the state standardized test, under intense pressure to raise test scores) and took on more responsibilities on his grade-level team as well, knowing that if he didn't, his colleagues would be in untenable positions. For another teacher, or for Ryan at another time, these roles might have fostered learning and growth. But in this instance, he felt that they stymied it:

I was really hoping that my second year, I'd be able to just focus on one or two aspects of my teaching, and really kind of dig into those. And I don't feel like I've had the time or space to do that.

Thus, with more technical resources, Ryan's engagement with equity-oriented practice might have deepened instead of staying constant. He might have learned to use Complex Instruction more effectively, for example, and found more space to nurture his identity as a competent member of a community of learners (versus as an expert practitioner, an identity that made him uncomfortable, as discussed below). The fact that he was able to maintain his engagement with the work of reframing mathematical competence despite limited technical resources suggests the power of other types of resources for processes of teacher learning and identification. **Relational resources.** Abundant relational resources supported Ryan to develop and maintain an identity as a member of a community of non-dominant teaching practice. He had—and took advantage of—membership in many different professional communities. The math department at his school was one of the most important, not least because the department constituted his immediate working environment, day in and day out. His colleagues in the department valued CI, as he did, and their solidarity on this point was an important relational resource that fostered Ryan's sense of belonging in the department and enriched his connection to non-dominant aspects of his practice. The district CI community, which met once or twice a month (with Ryan almost always in attendance), provided similar relational resources. Getting to know "other teachers throughout the district who are also thinking really hard about this work" supported Ryan's sense that he was part of something worthwhile, something bigger than himself.

His teacher preparation program also gave Ryan an enduring professional community that affirmed his identity as an equity-oriented teacher. During the period of the study (his second year teaching), he continued to consult with his mentors and meet with his peers from the program a few times a year. Though it was hard for him to articulate exactly how this community helped him, its value can be understood in terms of the relational resources and sense of connection to a particular brand of practice that it provided. As Ryan said, "I don't know that I get anything concrete out of it, you know. I don't get new ideas about teaching or anything. It's just like, a support network, where we talk about our work, and we just laugh and drink and do whatever. So that's been a big resource."

In addition to giving him a sense of connection to his teaching practice that fostered his ongoing engagement, Ryan's relational resources linked him to other types of resources. Some of his relational resources doubled as orienting resources by reifying certain ideals; for example, seeing friends from his teacher preparation program was a reminder of shared experiences and ideas learned from Ruth and Guillermo, whether those two were present or not. And some relational resources doubled as positional resources, as described in the next section.

Positional resources. An important way that his multiple communities of practice supported Ryan's engagement with non-dominant teaching practice was by supporting him to take up an identity as a skilled and valued member. At Union, he felt as soon as he was hired that there were a lot of people "kind of looking out for me ... [who] believed in my potential," perhaps partly because of his connections to Guillermo and Railside. And throughout his first two years, several of his colleagues were "a big support in terms of just, kind of assigning me competence." In one meeting, for example, a department co-chair reported (with light-hearted self-deprecation) that she had finished an observation in Ryan's class "depressed cuz he's so good. It's a great class, isn't it? He's great." Though there were drawbacks to this kind of positioning (discussed below), it supported Ryan's sense of connection to his practice.

Members of the district CI network also positioned Ryan as a high-status insider. For example, the CI coordinator chose to feature footage from Ryan's classroom in the district-wide Video Club, which was structured to support strengths-based collective inquiry. The Video Club afforded technical resources for all of the teachers involved, as they worked to name strengths of Ryan's and his students that they all could build upon in their own classrooms. It also afforded additional positional resources for Ryan, reinforcing his reputation as a "great" CI teacher. In this and other ways, Ryan's positional resources created a snowball effect, amplifying Ryan's opportunities to learn and develop an identity as a worthy member of the CI community. Yet Ryan himself was keenly aware that he still needed more support to grow as a teacher, and while he appreciated his colleagues' recognition, he objected to being positioned as better than anyone else. He saw himself as a learner, first and foremost. In an interview, he said:

I sometimes feel like people are, some people are looking at me as someone who knows how to do CI, and I'm like, yeah I don't know shit about how to make groupworthy tasks! Like I, all the groupworthy tasks I do come from Guillermo. I don't know how to do this stuff either.

Being positioned as an expert may not have been the greatest barrier to Ryan's learning; the paucity of expertise in non-dominant teaching (in terms of both orienting and technical resources) amongst his colleagues at Union—the members of his support network that he had the most frequent contact with—was certainly quite significant as well. But it is worth keeping in mind that the positional resource of high status was potentially a double-edged sword for Ryan. It could clear a path to learning opportunities, but only in communities of practice in which high status was associated with continued learning.

Summary. Ryan's case illustrates that when many types of resources are available, these resources can work in concert, not only reinforcing but also amplifying each other in ways that support teachers to develop practices and identities in which reframing mathematical competence plays a central role. It also demonstrates that important resources for teacher learning may come from outside their school-based communities of practice, for teachers in strong school-based communities as much as for those who are not. At the same time, Ryan's case shows that even a teacher who is embedded in multiple reform-oriented communities, who has strong connections to nationally recognized teachers, and who devotes an extraordinary amount of time to collaboration and professional development may still need additional support in order to do more than simply maintain his engagement with his practice—to continue to learn and grow. (And even simple maintenance might require more resources over time.)

Amanda Pepper: Ongoing engagement supported by strong identity resources

Like Ryan Sower, Amanda Pepper had a rich array of resources, sourced from a large and diverse support network. Asked to name people who had influenced her development as a teacher, she said, "So it's been a village. There's been a lot of people that've helped me." Because her math department was both smaller and less unified than Ryan's, ties to people outside her school were especially important for Amanda. In addition, the forms that her resources took were somewhat different from Ryan's, illustrating that there are multiple constellations of resources that can support the kind of teaching that both she and Ryan enacted. In particular, Amanda had fewer orienting resources, but she coordinated these resources with strong relational and positional resources to support her practice and her identity as a CI teacher.

Orienting resources. Amanda got into teaching through Teach for America (TfA). As a TfA corps member, she did not receive much training. Her first experiences of deep engagement with equity-oriented approaches to mathematics instruction came from two workshops that she attended the summer after her first year. One was a training in College Preparatory Mathematics (CPM; Sallee, Kysh, Kasimatis, & Hoey, 2000), a standards-based reform curriculum that her school had adopted. The other was the Complex Instruction summer course. The workshops

"opened [her] mind to a different way of teaching," and she learned to re-orient to her practice with a focus on student thinking, strengths, and successes. Following the workshops, Amanda actively sought out additional orienting resources. For example, she arranged to visit her CPM instructor's classroom to see her teach; she was a regular attendee at district CI events; and she read books like Carol Dweck's (2006) *Mindset*. Although the orienting resources that these experiences provided were shallow compared to Ryan's year of student teaching, Amanda stretched them by consciously working to imagine what her mentors and role models would say and do. She described asking herself in a moment of frustration, "What would Lee [a CI coach] say in this situation. What would Jessica [another CI coach] say in this situation. How can we find some smart things that are happening?" The images of teaching that she developed through CI and CPM training were thus constant resources for her practice—in particular, for her efforts reframe mathematical competence by seeing and naming "smart things."

Her day-to-day work environment seemed oriented against Amanda's efforts to reframe mathematical competence, however. At Boxer, she found that many of her colleagues "believe [in] one way of smartness, and it's an easy thing to believe because it's very easy to measure." She described her own doubts regarding the approach to teaching that she was pursuing, especially when it came to reframing competence:

I'm constantly up against this traditional view of what smart looks like. And I think I still have it in my head. ... I question what I'm doing every day. Which is good, but also can drive me crazy, and make me have little confidence.

In this context, Amanda's relational and positional resources were especially important.

Relational resources. Challenges to her approach from members of her own department and others highlighted the non-dominant nature of her practice for Amanda. Describing two of her colleagues, she said:

They were very questioning of CI. They still are. And so, sometimes I'm like, is it because I'm not critically thinking about it that I like CI, and I'm just going and running with it? Or, like do I need to be thinking more critically about it? And I think that also gets back to my smartness. Like am I really, are [my students] really smart? Or am I just trying to compensate ...

An important role that members of her support network played was therefore to provide relational resources in the form of reassurance. She relied especially heavily on Lee and Jessica to "validate" her vision of all students as smart:

I'd like to see [my view of smartness] validated beyond my opinion, and I don't see it validated. Beyond my opinion of it. Except when Lee and Jessica come. Then I'm like they're gurus, they've got it. Okay, now I can go back to believing it again. Like I need them to be confident for me.

Thus, Amanda's relational resources affirmed for her that she was not alone, wrong, and crazy but a member of a larger community, organized around a practice that was important precisely because it was not the norm.

Positional resources. In addition to reinforcing her confidence in her perspective on all students as smart, members of her support network helped Amanda feel personally validated and to develop an identity as a good CI teacher. Amanda's classroom (like Ryan's) was featured in the district's CI Video Club, where her colleagues took her students and her teaching as a model of CI in action. The monthly meetings for CI teacher-leaders across the district were held after school in Amanda's classroom, too, allowing her to be present even though she was not officially a teacher-leader and subtly underscoring her centrality in the CI community. Amanda recognized that she had "been given a lot of status" and that this supported her teaching practice, saying, "[I] have been told I'm good at [CI], in different ways, which makes me want to continue to do it."

Technical resources. Amanda put a great deal of effort into finding technical resources. After school, on weekends, and during the summer, she spent her time attending workshops, meeting with other teachers, reading books about intelligence, and so on. She leaned heavily on her textbook, which she trusted to align with her vision of good teaching, and on Dana, a teacher at another school who had attended the same CPM and CI workshops Amanda had. The two met regularly on Saturdays to plan lessons, and Amanda learned to use new materials and instructional strategies through their collaboration. Amanda was also bold about learning from her own practice, adopting materials and strategies for her own use with very little guidance. For example, she adopted the language of "growth" versus "fixed mindset" after reading *Mindset*, largely on her own, and after observing Lee shift a typically disengaged student's participation by publicly recognizing her as a "table master" (in response to the student's effective use of a table of values), Amanda started to identify all her students as "masters" of something mathematical.

Summary. Despite her substantial efforts to garner resources, Amanda still felt like she was "riding on the seat of [her] pants most days," pointing to an ongoing need for further support. Like Ryan, she maintained her engagement with non-dominant teaching practice, both inside her classroom and out. But, also like Ryan, she wished she had more resources not just to maintain her practice but also to "push" her and "challenge" her to grow. Such resources might have made a difference for Amanda's engagement with equity-oriented mathematics teaching in the long run; as of this writing (two years after formal data collection ended), Amanda is planning to leave her position as a mathematics teacher, citing family reasons and an interest in environmental education.

Luke McCormick: A pattern of disengagement

Like Ryan, Luke encountered Complex Instruction in teacher training. But whereas Ryan's program provided rich orienting, technical, and relational resources that paved the way to strong positional resources, Luke's appeared to leave him with fewer enduring ties and much thinner resources of all kinds. Luke took up an identity as a "struggling" and "lazy" teacher (to use his words), and instead of pursuing resources that would have supported his learning and shifts in his identity, he disengaged, taking as many vacation days as he could and questioning whether teaching was the right profession for him. In the middle of his third year teaching (the year following the study), he resigned from his position at Union and soon after took a job in another field. His case illustrates how teachers' matrix of resources may support a *non-teacher identity*, even as individual colleagues attempt to be supportive.

Orienting and technical resources. Luke's career as a teacher grew out of his tutoring experiences in college. He had always "really enjoyed math" and been a successful math student, and he liked helping others, so tutoring and then teaching math seemed natural. As a student teacher, what stood out to Luke was his cooperating teacher's strict discipline and focus on "math, math all the time." Luke oriented toward his cooperating teacher's example while struggling to find his own style, "without being something that I'm not." Luke had a gentle, relaxed way of interacting with his students, which supported them to take ownership over their work. But he also had trouble controlling his classroom, and student learning was often compromised by a lack of focus. For Luke, Complex Instruction hinted at how he might support students' engagement with mathematics, but he found that it had "the potential to be so bad," too, because students would "take advantage" of the opportunity to talk to each other and "get off task easier."

In the absence of a clear alternative, Luke turned to the images and techniques of the teachers he had looked up to as a high school student. On some level, he recognized that the orienting and technical resources that these role models provided were at odds with the student-centered instruction that he wanted to achieve. But at times, he wasn't sure what else to do. He described reading through the student-centered curricular resources that his colleagues at Union had developed and thinking,

I'm not sure how to run it, or I'm not sure what questions to ask to get them to be successful. And I felt like well, I could either do [the group activities] and be totally unsure that they'll get what they need? Or I can do [a lecture], and at least I told them what they need [to know]. ... As the year went on, I found myself more and more like doing that, just like lecture, and then I'd [catch myself] talking for thirty minutes and be like, holy shit.

Thus, in the face of a blurred vision of how to "run" groupwork, Luke turned to lecturing as an instructional strategy more often than he would've liked. He also "did more and more practice problems," drawing on his textbook because he felt that practice was missing from the resources that his colleagues shared. Luke's example demonstrates a common phenomenon: teachers' own schooling often supplies them with tools for enacting their daily instruction that function as technical resources for reproducing the status quo. Importantly, in turning away from student-centered instruction, Luke did not give up on the CI vision of all students as competent. He just didn't know how to "break down" students' perceptions of themselves while supporting their learning. As he said,

I sort of refuse to believe that any, like there's this one student who's just like not capable, of doing [math]. It's just other things that I think, and so, my, like I feel challenged in breaking that stuff down.

Luke occasionally saw CI-based instructional strategies modeled, and that modeling revealed glimpses of equity-oriented teaching practice. Instead of supporting his practice, however, this potential technical resource interacted with his positional resources to discourage him from engaging with non-dominant teaching practice. **Positional resources.** It was my observation that Luke's colleagues treated him as a smart and promising new teacher. On the Geometry Team, he had a reputation as a "math whiz"; during one meeting, when the group was working on a geometry puzzle together, Margaret looked at his solution and said, "You're just like the brain box in the group, I can tell there Luke, with that." His ideas about teaching were also met with glowing (if vague) praise in the rare times that he shared them. For example, when it was his turn to take the lead on writing curriculum, he presented his outline for the unit and his colleagues called it "fantastic" and "awesome." Margaret added, "I'm going to need you on my team next year, no matter what I'm teaching, that's—so you can do more of the same."

But Luke himself felt that he was "too new." In interviews, he said, "I don't feel like my opinion has any value yet ... [my colleagues] could've done it a million times, they know it doesn't work—for example if I try to suggest a solution to something." He also interpreted his struggles in the classroom as indications that teaching was the wrong profession for him. For instance, he described his reaction to seeing one of his students presenting to the class after a brief interaction with Jessica, his CI coach:

[This] one student had always told me he's not very comfortable, like he didn't want to come up. I'd ask him, I was like yeah you want to do it? Like yeah, you got the right answer, whatever. Um. And so I sort of took him to be a very shy student ... but Jessica said something to him, and he was up at the board, and it was like, night and day. Like the way that he acted with me was like totally totally different. Which was cool, but then it puts me in a spot where I'm just thinking like what, like. You know, I'm just not, something's not going right.

Similarly, he described the frustration he encountered when he tried to use participation quizzes (an instructional strategy from Complex Instruction, described above):

I think I did maybe in the first two grading periods, I did a couple participation quizzes, and then that was it. You know? And I saw the benefits of it, but I just, I couldn't run it like I've seen other people run it, and that frustrated me and so instead of, you know, trying, I sort of gave up on it.

Thus, some of the opportunities that he had to watch other educators at work—which could have served as orienting and technical resources in a different context—for Luke fit into a narrative about himself as struggling. Sometimes, he did find them helpful, as when William (who was officially assigned to provide support to new math teachers at Union) visited his classroom and asked questions or gave a mini-lecture in a different way than Luke would have. But often, instead of taking up learning opportunities or figuring out how to build a more supportive network, Luke understood his struggles as reflections of his personality. "I'm not necessarily the type of person to ask for a lot of help, like maybe when I need it," he said, and he repeatedly called himself "lazy."

Luke's perception of himself as "lazy" was reinforced by structures and norms at Union. For example, he knew that it was "beneficial" to have time for faculty collaboration, and he appreciated that it was a priority at Union (where teachers spent several hours each week meeting with department and grade-level teams). But he "almost kind of env[ied]" a colleague who taught at a school where meetings were sporadic and brief, because "my personality just doesn't want it. Just doesn't want to be here, doing those types of things for that long." In much the same way, he felt that achieving the kind of focused student engagement that his cooperating teacher had would require him to be someone that he wasn't. There were several ways, then, that Luke had difficulty bringing his views of himself into alignment with the expectations he perceived for teachers at Union.

Relational resources. There were ways that Luke's colleagues succeeded in communicating that he was one of them, despite his overall assessment that he did not belong. In particular, he appreciated their responses to his struggles, saying,

They've totally been supportive. And they're willing to sort of back up what they say, and come in and help out, or give advice, and just spend the time. ... It was like, yeah, we all sort of have been through this, and still go through it sometimes, and let me help you out. You know, not like, you can't do your job.

But this kind of solidarity and camaraderie were insufficient to support Luke either to learn or to develop a strong connection to his practice.

Summary. Luke's case highlights the importance of positional resources to support teachers' learning and identity development. Whereas Amanda's strong positional resources encouraged her to seek out resources that were not immediately accessible and to persist in the face of challenges, Luke interpreted his struggles not as natural components of his professional learning or as indicative of a need for more support, but as intrinsic to his person—resulting in a non-teacher identity. He was not supported to develop a vision of teaching that built on his strengths, needs, and ideas about himself, and this limited his capacity to learn and identify as a teacher at all, let alone a non-dominant one.

William Barrett: Limited resources, limited engagement

William Barrett was a caring and reflective educator, well respected by his colleagues at Union and at other schools. He played a central role in the Union math department, especially on the Geometry Team. He constantly worked to improve his and his colleagues' practice, experimenting with new activities in his classroom and raising important questions in meetings that pressed his colleagues to think deeply about how to meet their students' needs. But William also set deliberate boundaries around his work, which kept him at the periphery of the district CI community—and kept the work of reframing mathematical competence peripheral to his practice. His case raises questions about how to provide resources for teacher learning in ways that respect teachers' accomplishments and are sustainable for them as professionals.

Orienting and technical resources. William came to teaching after a brief career in human resources. Part of the meaning that he found in his work as a teacher was connected to "making a difference" in an unjust world. He got his first teaching job at Union, and ten years in, he was still proud to teach at a school "where every kid in the city is welcome ... [and] there is this explicit expectation that if a kid is on your roster, you teach them. And you find a way to reach them." Yet this goal was moderated by his orientation toward "realistic" expectations for success, inspired by a mentor (Hiro) who he remembered saying, "If you devote yourself fully for a whole year, and if you make a difference for just one or two kids, it's been a good year." By

carefully circumscribing his ambitions and "getting that save-the-world sort of idealism out of my system," William hoped to sustain his participation in the profession for many years to come:

I would like to do this until I retire. And, I think without hopefully settling or diminishing my hopes and dreams for my impact, too much. [So] I also am careful about not overextending myself or even, just putting the bar so high up there that I can never reach it. And then just, being disappointed all the time, bummed out, and [feeling that it's] time to move on.

Thus, William oriented toward a vision of good teaching in which working steadily at incremental change—and being careful not to overextend and burn out—were central. This vision may have been effective in supporting his longevity in the teaching profession, but it also functioned to restrict his access to learning and identity resources that would have engaged him more centrally with non-dominant teaching practice.

For example, Complex Instruction provided William with orienting resources, presenting a more transformative view of teaching than he had previously encountered. But he took up the CI vision of equitable instruction in quite limited ways. The "chills" he occasionally felt when he saw CI working were accompanied by challenges that he found "disheartening":

[M]ore times than not what happens [when I assign a group task] is, the kids who have, you know, better access to it love it and go for it, and the kids that don't tend to do a lot of copying, and I don't know how engaged they are. ... so I'm sort of jaded, or a little disheartened.

William explained the challenges he faced in using CI in terms of his technical resources and "skills":

A lot of it—I think a big part of it is my skill level. ... I do believe every kid is smart. I totally believe in multiple intelligences and all sort—like I really really believe that is true. I think it's, again it's a skill thing.

But whereas Amanda put significant effort into garnering resources that were not readily available to her, William did not. He was a willing learner, but as a veteran with 10 years' experience, he was no longer "on the receiving end" when it came to support for learning at Union. Indeed, William periodically raised questions and dilemmas with his colleagues, but as a group, their vision of CI seemed too "abstract" or "idealized" (to use William's words) to drive their practice toward reframing mathematical competence. And William's prioritization of sensible limits meant that he rarely accessed resources beyond his immediate circle at Union. Doing so would have required an investment of time and energy outside of the regular school day, and he had "childcare issues" and other concerns, including protecting himself from burning out. "I can't complain so much about supports," he said, because

there are ample opportunities to collaborate, with all these people [across the district]. This is more just my decision to set up boundaries. ... For the sake of making it at least another ten years, I say no a lot. And that's a good thing for me.

Thus, William deliberately decided to limit the energy he put into cultivating his professional network, and the decision made sense given the context of his life and his goals for himself as a teacher. But he had fewer resources for developing his practice and his identity as an equity-oriented teacher as a result.

Relational resources. William's professional network may have been small, but in terms of relational resources, he felt "really well cared for" by his colleagues at Union. The math department's solidarity around core values was especially important to him:

It's not just CI, the practice of CI. It's the values behind CI. Like how do you view—do you view every student as a contributor to the learning environment. I mean. So I, I do think that everybody here believes that. And at, I mean that's kind of a baseline. But that's pretty huge. ... We have some differing opinions, I think, around [how we carry it out]. But I really trust the intentions of the teachers in the department? And I think, overall, we have each other's backs.

William also highlighted enduring connections with a few students from each class, saying halfjokingly that staying in touch with them and "keep[ing] involved in their lives" was his "plan to save the world."

The relational resources provided by his rapport with his colleagues and his students supported William's sense of connection to Union and his sense of identification with his practice more generally. In the presence of richer orienting and technical resources, William's relational resources could have provided a foundation with the potential to support his ongoing learning.

Positional resources. Despite his low engagement with out-of-school networks, William was highly involved in Union's math department. He hosted and often facilitated the Geometry Team's weekly meetings; he wrote most of the Geometry homework assignments (classwork was divided between team members); he coached new teachers; and in part because his classes were generally a few days ahead of his colleagues', he often gave advice and fielded questions about upcoming lessons during team meetings. He saw himself as "the veteran" with "a fair number of tools in my toolbox," and he accepted leadership as his duty. He saw his colleagues as responsive to his influence (a view that Ryan's and Luke's descriptions of his support for their work confirmed). Thus, he took up a position at the core of his department but on the margins of broader CI networks. This position supported his identity as a competent professional but afforded limited opportunities for him to learn.

Summary. William's case highlights the tension that teachers face, especially in underresourced and high-need schools, between preserving their own lives outside of work and participating in their practice in ways that support them to learn. William was a thoughtful and reflective person, and over the course of ten years, he had developed enough skill to be regarded as a leader in his department and to feel confident about many aspects of his practice while remaining open to continued improvement. But none of these factors were enough to support radical shifts in his practice in the absence of readily accessible resources for developing a clear vision of an alternative to traditional practice (orienting resources), for enacting such a vision (technical resources), and for seeing himself as a competent and valued member of communities of non-dominant teaching practice (positional resources). This is not to say that William was not learning or developing his practice in any ways that were significant for him or for his students, just that the extent to which he engaged in reframing mathematical competence was limited. Nor is it to say that his engagement with non-dominant practice *cannot* deepen. But he would require more resources (like the ones that Ryan and Amanda got from attending Video Club, from observing and co-teaching with teachers who were centrally engaged with CI, and from coaching that highlighted their strengths) in order to support such deepening.

Implications

This article began by asserting the force of narrow, exclusive ways of understanding what it means to be good at math, which continue to pervade American classrooms and American culture more broadly. The four teachers described in this study all cared deeply about their students, and they shared a commitment to the idea that all students are mathematically capable. Yet "traditional view[s] of what smart looks like" were still present "in [their] head[s]" and in their practice (to repeat Amanda's phrasing).

Nonetheless, the cases of Ryan Sower and Amanda Pepper show that it is possible for teachers to engage in meaningful ways with the work of redefining mathematical competence and through such engagement, to learn to enact forms of mathematics instruction that expand students' opportunities to see themselves as powerful mathematical thinkers. This finding suggests that such learning is not a magical process that only superheroes can perform but a challenging and counter-cultural task that requires many kinds of support. The importance of four particular resources—orienting, technical, relational, and positional—in turn illustrates the ways that learning and identity development inform and constitute each other in teachers' engagement with non-normative shifts. For Ryan and Amanda, learning to enact new practices was enmeshed with becoming a new kind of teacher and coming to belong to communities of non-dominant teaching practice. In contrast, for Luke and William, unsuccessful attempts at learning were both a cause and a result of identities as peripheral members of communities of non-dominant practice, in a cycle of inadequate resources and disengagement with the work of redefining mathematical competence.

In focusing on the resources that are provided to, sought out by, and taken up by teachers, this analysis situates teachers' work—both when it successfully reframes mathematical competence and when it reproduces hierarchies—in social contexts (e.g., networks) that support or fail to support their engagement with non-dominant practice. At the same time, it illustrates some of the ways that teachers participate in shaping these contexts, pursuing some supports but not others and making their own sense of varied and conflicting resources.

A resource-focused analysis has implications at many levels. For individual teachers who are interested in sustaining or deepening their engagement with non-dominant practice, a typology of resources may be a useful pointer to supports that might be worth seeking out. For policy makers and teacher educators, the analysis suggests that whereas practice (like research) has often attended exclusively to resources that can be characterized as orienting or technical, more attention ought to be given to the ways that teachers' access to and pursuit of these resources is mediated by relational and positional resources, as learning and ongoing identity work intertwine. In other words, those who are interested in supporting teacher learning ought to think creatively not only about how to provide teachers with more resources, but also about how to provide them with more *kinds* of resources.

Further investigation of the resources that support teacher learning and identity is warranted to address questions such as: Are orienting, technical, relational, and positional resources all equally important? In what contexts, and for whom? Are other kinds of resources also necessary? Under what circumstances are the resources provided by ties outside one's local community of practice worth the time and energy that those ties consume? And perhaps most importantly, how can we design environments that make the resources teachers need more accessible, in order to improve teacher learning and ultimately, to make richer learning experiences available to all students?

Limitations

The foregoing analysis highlights aspects of what we can learn from the data described in this study. What follows is a discussion of three limitations of the data and the implications of these limitations for future studies.

First, a year is a short period for capturing changes that may occur very slowly. A longitudinal study designed to follow teachers for several years would no doubt generate important insights into how resources function that were not possible to develop through this study. A second and related limitation concerns kinds of data that were not collected for this study. Missed were informal hallway meetings between teachers and their colleagues, formative experiences from teachers' pasts (e.g., Ryan's teacher education program, or Luke's apprenticeship with Cyril), and interactions between teachers and their family and friends, which several teachers described as important sources of support. It is possible to imagine a study more thoroughly grounded in observation of the full range of teachers' professional lives. This was not that study—but it might provide some clues as to what such a study might look for and how it might go about it.

A third concern is that the data might obscure some resources that are potentially quite important, not because of flaws or limitations in the study design but because of the range of phenomena that exist to be studied. For example, their content knowledge did not seem to support the teachers in this study to redefine mathematical competence. In fact, the teacher with the highest mathematics degree (Rob) seemed most rigid in his views of both the discipline and his students' abilities, whereas Ryan and Amanda seemed to work more from a general orientation to students as smart than from content knowledge that was remarkable in any way. But it seems reasonable to expect that *some* form of content knowledge would support teachers to develop their practice in ways that would support the redefinition of mathematical competence. Thus, the hypothesis that other types of resources (or sub-types of orienting, technical, relational, and positional resources) matter for supporting teachers' engagement with non-dominant practice warrants further investigation.

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Appendix A Teacher Interview Protocol

Consent to participate in research

Background

- Tell me about how you decided to become a math teacher. Was it something you always knew you wanted to do? [probe for math specifically if necessary]
- Are there any experiences or people who you would point to as being especially important in your development as a math teacher? (How?)
- How long have you been teaching now?
- Have you been at [school] that entire time?
- What drew you to [school]?

Goals

• How would you describe your goals as a teacher? At the end of the day, what makes you feel, or would make you feel, like you've been successful?

Challenges

- What challenges do you feel like you're currently dealing with as far as making your goals a reality?
- Can you think of a particular moment when ...?
 - External pressures like covering standards, prepping kids for standardized tests?
 - Heterogeneity? I know this an issue for a lot of teachers and I'm wondering if it's something you also struggle with.
 - How would you characterize the differences between kids who are doing well in your classes and kids who aren't doing so well?
- Can you talk about Complex Instruction a little bit? Do you feel like it helps you manage the heterogeneity in your classes at all?

Supports

- Do you think your colleagues in the math department here experience the same challenges you do?
- Are there ways that your colleagues—here at [school] or elsewhere—support you to manage the challenges and dilemmas you've described?
- More generally speaking, are there ways that your colleagues—here at [school] or elsewhere—support your development as a teacher?
- Can you think of a particular conversation ...?
- Are there ways you wish you got more support?
- Do you feel like you're able to influence the way your colleagues make sense of the challenges you all face as teachers?

Is there anything you'd like to add?

Appendix **B**

Transcription symbols and transcript for the "making understanding public" episode

Transcription symbols (adapted from Jefferson, 2004; Ochs, 1979):

,	marks low rise
?	marks high rise
?,	marks medium rise
	marks low fall
	An em dash indicates self-interruption
[A left bracket indicates the point at which a current speaker's talk is overlapped by the talk of another.
]	A right bracket indicates the point at which two overlapping utterances end.
=	Equals signs indicate the absence of a break or gap. A single equals sign indicates no break in an ongoing piece of talk, where one might otherwise expect such a break (e.g., after a completed phrase or sentence). A pair of equals signs can indicate the absence of a break between different speakers' talk turns. It can also signal instances in which a single speaker's talk is broken up in the transcript (e.g., to accommodate overlapping talk by another speaker) but is actually continuously produced by its speaker.
(2)	Numbers in parentheses indicates elapsed time in seconds.
(.)	A dot in parentheses indicates a brief interval (less than two seconds) within or between utterances.
	Underscoring indicates some form of stress, via pitch and/or amplitude. A short underscore indicates lighter stress than a longer underscore.
::	Colons indicate prolongation of the immediately prior sound. More colons indicate longer prolongation.
0	Text between degree symbols was uttered with lower intonation than surrounding text (e.g., whispering).
><	Right/left carats bracketing talk indicate that the bracketed material is sped up or compressed, compared to surrounding talk.

- (word) Parenthesized words and speaker designations indicate the transcriber's uncertainty about what was said or who said it. Empty parentheses indicate that the transcriber was unable to hazard even a guess at what was said or who said it.
- ((word)) Double parentheses contain the transcriber's insertions, e.g., descriptions of sounds such as laughter and gestures.

Meeting transcript (excerpt):

438 439	LUKE:	What is that? ((reading the next item on the printed meeting agenda)) What is making understanding public?
440 441 442 443 444	ELIZA:	Uh, that's something that, um, Ryan and I have been talking about, um. Kind of CI-ish, um. It's also, it's sort of very ff—fundamental to the Algebra Project, um, is that, you know you, your understand— >you can understand it, but if you,< if you're not able to share it, it doesn't mean,
445	LUKE:	Mm. Right.
446	ELIZA:	too much? And, like the Pass the Pen activity?
447	WILLIAM:	°Mmhm.°
448 449 450 451 452 453 454 455 456 457	ELIZA:	Um, which he did today cuz his kids needed a review?, and they struggled. >I was able to complete it in one of my classes=the other two—< $\underline{couldn't}$ do it. They can't, (.) um, they just can't <u>listen</u> to each other present, or they can't present?, um. So we were just thinking about, can we build in some ways of making understanding public=I did do, um, last week, I had them make <u>pos</u> ters and do a <u>Gal</u> lery Walk. So that was a way for each group to make their understanding public without having to stand up and, present. But the, the Pass the Pen one kid at a time <u>only</u> worked in my third period class. There was <u>no way</u> ,
458	():	[((quiet chuckle))
459 460 461 462 463 464	ELIZA:	[my other kids—my seventh period class could do anything like that. So. Um. Anyway we've just been trying to talk about thinking about (.) ways that we can make that <u>happen</u> , um. (.) Because if you can't communicate what you know then, this is one of the important (.) job skills, that people need, right. Um. They don't need an—triangle congruence, but they need to be able to explain their thinking.=
465	MARGARET:	°Hm.°

466 467 468	ELIZA:	=So. Anyway we've just been talking about it and I said well I'll put it on the agenda and see if anybody else has other ideas of good ways for um, kids to share their understanding.
469	WILLIAM:	When—when you say (.) public, do you mean whole class public?
470 471 472 473 474	ELIZA:	It ca:n be, or it could be, even >if it's like an individual assignment public< within the group? Um, for instance cuz sometimes there's a worksheet that's really <u>not</u> a groupworthy thing? >(And) my kids said to me yesterday=they were doing something and they were like,< is this <u>groupwork</u> ?=
475		((several people laugh))
476 477 478	ELIZA:	=And I said well, it's <u>not</u> actually but there's nn, you can certainly talk to each other, but, uh. (.) Individual presentations seem to be really hard, um. (.) So, other things we can $[().$
479 480 481 482 483 484 485 486 487 488 489 490 491 492	LUKE:	[An—Andy uses this really structured technique that I like, that I use in my physics class sometimes? It's only, like with a partner?, but at least it gets them talking?, and he calls it SOLAR? It's like, it's an acronym=it's like square off, open up, listen, um, affirm, and, respond? And so, I mean if you—like (.) I haven't even thought about using it for my math class, but now that you're talking about that it might be interesting to like, build that if everybody wanted to do it, or if you wanted to just sort of, start scaffolding it in, like have certain problems that they do and then say, now we're gonna do SOLAR with your partner, like have one person do it on angle relationships and the other person do it on, I don't know, if you want to do it on review or whatever. But it's, it's in such a way where, like the kids think it's goofy?, but they'll totally, like do it? [Um.
493	MARGARET:	[Okay so sorry, square uh, square ah-
494	LUKE:	Square off.
495	MARGARET:	What does that mean?
496 497 498	LUKE:	It means like, so I'm, you're not over there talking and I'm listening like this ((turning his body away from Margaret)). It means they actually have to like turn their chairs=
499	MARGARET:	[Okay.
500 501 502	LUKE:	=[and they like go like this ((turning toward her)) and then they have their (papers) in front of them. Um, open up, right, means that you're—

503	MARGARET:	Open to hearing? Like,
504 505	LUKE:	Yeah, [like, not like—I don't know, a kid wouldn't do <u>th</u> at but ((laughs))
506	MARGARET:	[Yeah.
507	WILLIAM:	Or like, you know, reading their paper, [or, (for example).
508	LUKE:	[Right, exactly.
509	ELIZA:	Take your earphones out of your ears!
510	WILLIAM:	[Uh huh.
511	LUKE:	[Yeah.
512		(8)
513 514	LUKE:	I just, yeah. Like I said it already, but I think that like the structure of it, is like, [I don't know, don't just share=
515	ELIZA:	[It's sort of a pair share.
516	LUKE:	=but—yeah, but.
517	ELIZA:	Yeah.
518	LUKE:	Do it like this.
519	MARGARET:	And what's the, R, the last one, sorry.
520	LUKE:	Respond.
521	MARGARET:	(.) This is great.
522	():	Yeah.
523	MARGARET:	The last thing we need is another acronym, right?
524		((several people [laugh))
525 526	ELIZA:	[But I know I have some kids who are in your physics class. (.) So they know it already.
527 528	LUKE:	They'll know it. (.) And, Andy's kids will know it. (.) If he's doing it. I haven't checked in °with him in a while.°
529	ELIZA:	°That's a good idea.°

530		(7)
531	MARGARET:	°That's great.°
532	ELIZA:	°Yeah.°
533 534 535 536 537 538	MARGARET:	We do a lot of presenting. In Stats. (.) And, I just feel like it's very easy for me not to do it, you know, cuz sometimes I think oh I want to move <u>on</u> , I don't want to spend <u>another</u> day and go back and have them present these posters but I'm <u>making</u> myself now=I'm just scheduling in, we've <u>go</u> ne on to a new topic and they have these posters that they did, >which are not even that good,<
539	():	((quiet chuckle))
540 541 542 543 544 545 546 547 548 549 550	MARGARET:	but I just figure, two a day. Two groups a day, it's going to take me <u>six</u> minutes, three minutes each, something like that, and at <u>least</u> they're getting better all the time. And, uh, also with the <u>homework?</u> , I found, if I just say there's <u>four</u> questions, I say um, everybody talk about the homework. Okay then now, and then, <u>five</u> minutes before I want them to present I'll say, Group 1 and 2, you guys are in charge of the first question. 3 and 4 you're in charge of the next question. So I assign them a question each, so that gives them five minutes to sort of >figure it out.< Then I roll a die or <u>what</u> ever that I pick which group comes up. And then I try to be quiet, very hard, and sit at the back and let them pre <u>sent</u> the answers, and.
551	ELIZA:	Mmhm.
552 553 554	MARGARET:	And let the others agree or disagree, that kind of thing. And in the beginning it was <u>awful</u> , and I can <u>definitely</u> see them getting much better now.
555 556	WILLIAM:	(.) Is your, uh. >Yeah what type of interaction,< do you, do you want them to have with yourse—with you, or with the other students?
557 558 559 560 561 562 563 564 565 566 567 568	MARGARET:	So when they go up I say to them, don't assume that everybody knows what the question is, because you, you've been studying this. So the first thing I want one of the group members to do is, is sort of summarize the question. So we were <u>asked</u> in this question to <u>dr</u> aw a histogram of <u>th</u> is, this, you know. And then somebody else puts the, puts their work on the ELMO, and talks about, we did this, we did this, and does everybody understand. Take questions from the class. <u>And</u> if they look at me, I tell them not to look at me. That's not okay, don't talk to me, I know the answer. Or if somebody says that's wrong and looks at me I say don't tell me, tell them. That kind of thing. I'm trying to get them to converse with each other. It's obviously easier to do in Stats,=
569	ELIZA:	[Yeah.
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570 571	MARGARET:	=[because they <u>fee</u> l a responsibility to think at a (.) higher level, I suppose so.
572	ELIZA:	And more of them do the homework.
573	LUKE:	((chuckle))
574 575	MARGARET:	Exactly. Yeah, exactly. You could do it with classwork in, in Geo, I guess.
576	ELIZA:	Yeah. (.) °That's a good idea.°
577		(12)
578	ELIZA:	°Thank you.°
579		(4)
580 581 582	EMILIO:	I just think (whatev—) whatever we do, I—like you made an important distinction that, it just takes practice. So like, w—whether it's=
583	(WILLIAM):	[Mmhm.
584 585	EMILIO:	=[Pass the Pen or anything, they just need to, do it more to become comfortable with actually going up and presenting.=
586	WILLIAM:	Right.
587 588 589	EMILIO:	=So, it's going to take some time. And that could be frustrating but. (You know.) Work your way through it and know that there's going to be a little light at the end of the tunnel.
590	():	Mmhm.
591		(4)
592 593	EMILIO:	So when we did Pass the Pen, it was—(.) I think one class went really well, the other class, not so well, so.
594	():	Yeah.
595 596	EMILIO:	And a lot of it I thi—I just think, the kids were an—really anxious about it. Like,=
597	ELIZA:	[Mm.

598	EMILIO:	=[they just <u>do</u> n't—they don't have the skill yet, so.		
599		(4)		
600	EMILIO:	Of either speaking or listening.		
601 602 603 604 605 606 607 608 609 610 611 612 613 614 615	WILLIAM:	(.) Yeah I was trying to be, I did on, yesterday, as a quiz review?, I actually took one of the versions of the quiz and I just was like, we've gotta go through this quiz problem together. Questions. So, um. (.) I was <u>trying</u> to be really clear that I <u>wanted</u> whoever came up? Like I wanted it to be so <u>quiet</u> in the audience that whoever came up could <u>try</u> to answer it, without being interrupted, like to have some time and some space to do that. And <u>then</u> , if they needed help, they got stuck, they could look up and they could <u>ask</u> , either generally, or could ask a specific person for help. Um. Cuz I didn't want them to feel like totally paralyzed and on the spot? But I also wanted them to (.) have enough, like enough of a comfortable environment where they could (.) see, or like <u>try</u> if they knew it or not. Um. Cuz you never know what part of the problem you're going to get the pen for, you know. They might've known the three previous steps? And they get to the fourth step, and like, cra:p, <u>now</u> I get the pencil?=		
616	():	((quiet chuckle))		
617 618 619	WILLIAM:	=You know, or whatever. But uh. I think that helped, but I, I, yeah I, I totally hear what you're saying. (.) Like, it's about being consistent, and. Hopefully it'll pay off in February or March, and yeah.		
620 621 622 623 624 625 626 627 628	MARGARET:	And, and one thing that I learned somewhere along the line which maybe we learned in CI, >I don't know, but.< That if a kid—if a group has not done the homework, they can still come up. Cuz you can read that question and tell us what the question is and then get the rest of the class to help you. And so, uh I used to think, that you bring up the kids who can do it the best, to show everybody else, but it's actually fine to bring up the other kids, and I think that >works pretty well.< Oh and that other thing=if you've two groups doing one question you can make the other one, group call a friend, you know.		
629	LUKE:	[Mm.		
630	ELIZA:	[Oh yeah.		
631 632	MARGARET:	So if you're stuck, they should know the answer as well [cuz they were assigned that, yeah.		
633 634	ELIZA:	[Give them a		

635	WILLIAM:	Hm.
636		(6)
637	():	°Okay.°
638		((shuffling of papers))
639	ELIZA:	°Got some ideas.°