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How MPT Works in Reality?

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How MPT Works in Reality?

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by

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ABSTRACT OF THE THESIS

How MPT Works in Reality?

by

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Master of Science in Statistics

University of California, Los Angeles, 2013

Professor Rick Paik Schoenberg, Chair

The major goal of this thesis is to discuss and test some of the models and fundamental elements of the Modern Portfolio Theory in order to learn about whether the optimal portfolios constructed on the theory formulate the best asset allocation in reality. In an effort to solve this question, three different models are tested by three fundamental elements of the theory respectively. The outcomes for each model and element are discussed and evaluated.

The empirical result shows that the models are inconsistent, which leads to the conclusion that optimal portfolios based on Modern Portfolio Theory are not the best asset allocation strategies in the real investment world. The main reason can be concluded that the models based on the theory make oversimplified approximations of the reality.

The thesis of Han Bai is approved.

Ying Nian Wu

Nicolas Christou

Rick Paik Schoenberg, Committee Chair

University of California, Los Angeles

2013

How MPT works in Reality?

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1 Introduction

One of the key issues facing an individual is how to allocate wealth among alternative assets. The most important theory of financial assets allocation is Modern Portfolio Theory whose fundamental objective is to predict expected returns and optimal allocations. Modern Portfolio Theory emerged from the mathematics of chance and statistics and has been for years the foundation of financial management despite its criticism (Mandelbrot B., 2004). It is the methods of Modern Portfolio Theory (MPT) that will be the issue in this thesis.

For decades, scholars and investors have had their doubts about whether predictions of expected returns, which is the most important part of MPT, are accurate or even possible. In fact, the general consensus has been that expected returns are notoriously difficult to predict. This leads me to think whether MPT works perfectly in reality. What investors care most is to find a realistic way to allocate their money to a stock portfolio. Can this be achieved under MPT? The kernel of my thesis is to explore the practicality of MPT by taking the accuracy of expected returns, optimal allocations and future monthly returns into account.

This thesis is divided into 5 sections. Section 2 presents a basic historical review of MPT, the major models used for estimating, and its limitations. Section 3 discusses my research design which explains the data details and the process of testing MPT models. In section 4, empirical test is conducted to solve the major question: Are the optimal portfolios suggested by the Modern Portfolio Theory the realistic asset allocation strategy in investment world? In an effort to answer how realistic it is, the following questions are tested in section 4:

1. Do the models predict accurate future returns, namely expected returns? If the models work, predicted future returns should be the same as the real future returns.
2. Are the optimal portfolio weights realistic for investors? For example, if a stock's weight is 8, or -8(when [1]*short selling allowed), then it is impossible for investors to raise the amount of money. An investor's capital is 1.
3. When applying optimal allocations to real returns, what is the monthly return? It is supposed to be the higher the better.

The last section of the thesis summarizes the main results of the empirical test and answers the above center question which is how practical the theory is.

2 Literature Review

2.1 History of Modern Portfolio Theory

In 1952, Harry Markowitz(1952,1959) set a mile stone in financial theory. He is the father of modern portfolio theory and also the first person to quantify the relationship between assets. He proved the fundamental theorem of mean-variance portfolio theory, namely holding constant variance, maximize expected return, and holding constant expected return minimize variance. Mean-variance portfolio theory is developed to establish the optimal portfolio concerning return distributions over a single period. An investor is assumed to estimate the mean

[1]* short selling: In finance short selling is the practice of selling securities or other financial instruments that are not currently owned, with the intention of subsequently repurchasing them ("covering") at a lower price.

return and variance of return for each asset. These estimates are derived from historical data. Based on the theory, an efficient frontier can be formulated for investors to choose his or her preferred portfolio, depending on personal risk return preference. MPT delivered an important message that assets cannot be selected merely on characteristics, but also each security's co-movement with all other securities. Portfolios that neglect these co-movements is less efficient compared to those that take the interaction between securities into account(Edwin,1997).

Though the theory is profound, its simple calculation may lead some researcher to think there might be a more complicated and better model. For example, researchers (Lee, 1977; Kraus and Litzenberger, 1976) stated alternative portfolio theories that had more moments such as skewness or researchers (Fama, 1965; Elton and Gruber, 1974) indicated alternative portfolio theories for more realistic descriptions of the distribution of return. However, mean-variance theory has remained the keystone of MPT regardless of those alternatives. No evidence shows that adding additional concerns to the theory improves the performance (Edwin,1997). Even 60 years later, Markowitz's model is still widely used among private and professional investors, despite its shortcomings.

2.2 Models for Modern Portfolio Theory

After mean-variance portfolio theory was first developed, estimating inputs becomes a crucial and necessary task. While Modern Portfolio Theory is an important theoretical advance, its application has universally encountered a problem: it is difficult to come up with reasonable estimates of expected returns.

Models mainly focus on how to estimate returns accurately.

The principal tool developed for estimating covariance was index models. The earliest index model that received wide attention was the Single Index Model. Markowitz discussed this first, but was developed and popularized by Sharpe (1967). Shortly after the model was developed, a number of researchers started to explore whether Multi Group Models better explained reality. These two models estimate expected returns and covariances using historical return. Composing a portfolio based only upon historical statistical measures may yield simplistic results. Black–Litterman overcame this problem by not requiring the user to input estimates of expected return. Instead it assumes that the initial expected returns are whatever is required so that the equilibrium asset allocation is equal to what we observe in the markets.

2.21 Single Index Model:

SIM is one of the earliest models that aroused wide attention. One variant of SIM was especially drawing attention. It is the market model. The SIM states that

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

where R_{it} is the return of stock i in period t, α_i is the unique expected return of security i, β_i is the sensitivity of stock i to market movements, R_{mt} is the return on the market in period t, and e_{it} is the unique risky return of security i in period t and has a mean of zero and variance σ_{it}^2. Therefore,

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m$$

The significance of the market model is that it reduces the number of estimates required, makes it easier for analyst to understand the type of inputs needed and increases the accuracy of portfolio optimization. The accuracy of the market model in estimating covariances is obviously higher than direct estimation.

The above is the traditional SIM. But nowadays, people use a lot of adjusted betas. One the most efficient techniques is Vasicek.'s technique.

$$\beta_{i2} = \frac{\sigma^2_{\beta_{i1}}}{\sigma^2_{\bar{\beta}_1} + \sigma^2_{\beta_{i1}}} \bar{\beta}_1 + \frac{\sigma^2_{\bar{\beta}_1}}{\sigma^2_{\bar{\beta}_1} + \sigma^2_{\beta_{i1}}} \beta_{i1}$$

where $\bar{\beta}$ is the average beta for the sample of stocks in the historical period and $\sigma^2_{\bar{\beta}_1}$ is the variance of the betas for the sample of these stocks. β_{i1} is the beta of stock i in the historical period and $\sigma^2_{\beta_{i1}}$ be the variance of β_{i1}. Vasicek's technique will be applied to the empirical parts as well.

2.22 Multi Group Model:

After the market model was developed, a number of researchers started to explore whether Multi Group Model better explained reality. The stocks are grouped by industry and correlations between industries are added to the model. The most important assumption is that the correlation coefficients between any firms in one group and all other firms are identical for members of the same group. Let's define the elements that will be used in the model.

ρ_{kk} = the correlation coefficient between members of group k

ρ_{kt} = the correlation coefficient between members of group k and t

σ_i =the standard deviation of security i

σ_{ij} =the covariance between security i and security j

$\bar{R}_i$ =the expected return on security i

R_f =the risk-free rate of interest

$\bar{R}_p$ =the expected rate of return on the optimal portfolio

σ_p =the standard deviation of the optimal portfolio

N_k = the number of securities in group k

X_k =the set of stocks in group k

P =the number of groups

M_i =the fraction of funds invested in security i

The Multi Group Model states

$$\bar{R}_i - R_f = z_i\sigma_i^2 + \sum_{\substack{j=1 \\ j\neq 1}}^{N} z_j\,\sigma_{ij} \qquad (1)$$

where $z_i = M_i(\bar{R}_p - R_f)/\sigma_p^2$

For a security i, which is a member of group k, equation (1) can be written as

$$\bar{R}_i - R_f = z_i\sigma_i^2(1-\rho_{kk}) + \sigma_i \sum_{g=1}^{p} \rho_{kg}\,\phi_g$$

where $\phi_g = \sum_{j\in X_g} \sigma_j\, z_j$

2.23 The Black-Litterman Model:

The Black- Litterman model is also an asset allocation model which provides a tool for investors to calculate the optimal portfolio weights under specified parameters. It was first developed in 1990 by Fischer Black and Robert Litterman at Goldman Sachs. This model combines ideas from the Capital Asset Pricing Model (CAPM) and the Markowitz's mean-variance optimization model.

Before the Black-Litterman came out, investors used historical returns as estimates. However, the complex model often returns weights that does not make sense. The great thing about the Black-Litterman is that it produces relatively neutral weights for the investors and investors are able to add their personal opinions into the model by translating their own options into matrices.

There are two main assumptions behind the model. First, the model assumes that all asset returns follow the same probability distribution (usually normal distribution is selected, but investors can choose any distribution that seems fit). Second, variance of the prior and the conditional distribution about the true means of the assets and investor views are unknown.

Assuming there are N-assets in the portfolio, Π is implied returns which is a Nx1 vector. The model states,

$$\Pi = \delta\Sigma\omega$$

δ= Risk aversion coefficient. It can either be an arbitrary assumption or can be given by $\delta = (E(R) - R_f)/\sigma^2$

E(R) = Return of the market portfolio (a portfolio that includes all the assets in the market or any other index benchmark that the investor decide to choose)

R_f = Risk free market rate

σ^2 = Variance of the market portfolio

Σ = A covariance matrix of the assets (NxN matrix)

ω= Weights of assets according to their market capitalization.

If the investor is happy about this market assessment he can stop right there. But if not, the investor's opinion can be incorporated under Black-Litterman. The approach produces optimal portfolios that start at a set of neutral weights and then the views of the investor are tilt in. The investor can control how strongly his or hers views influence the portfolio weights and also which views are expressed most strongly in the portfolio (Black & Litterman, 1992). However, this thesis is not going to implement the part of adding investor's opinion. Otherwise, it depends to much on my personal views which does not conform to the goal of this thesis.

2.3 Extension of MPT and Estimation Problems

Although MPT has been widely used over 60 years, it has retained theoretical problems on which a large amount of work has been done. One of the major problems is how the single-period problem should be modified if the investor's true problem is multi-period in nature. Fama (1970), Hakansson (1970, 1974) have issued papers solving this problem by discussing it under various assumptions. A second major problem is that concerns about the appropriate length of a single period is still vague and has never been defined.

Little research has been done on the issues of length. A third major problem involves two types of theoretical research which are the analysis of portfolio problems in continuous time and intentions to understand how current holding and transaction costs affect portfolio rebalancing. Those two have received substantial attention in the literature but have not had a major impact on the implementation of portfolio management. The final area that has received great attention is the accuracy of estimation of the inputs to the portfolio selection. As we come closer to the present date, the emphasis of the critique has increasingly been on the error in estimating means, variances and covariances in security returns. Frankfurter, Phillips and Seagle (1971) argued that their experiment on the matter showed that the impact of estimation error was so strong as to bring into question the usefulness of mean variance approaches to portfolio selection.

3 Research Design

The main goal of this research is to see if MPT is realistic by solving the three questions brought up in the introduction. They are

1. Do the models predict accurate future returns, namely expected returns?
2. Are the optimal portfolio weights realistic for investors?
3. When applying optimal allocations to real returns, what is the monthly return?

The first part of the research concerns stock selections. Since this paper is not mainly focusing on how to choose the best stocks, I will simply use P/E ratios as a standard to filter 6 sectors and 30 stocks out of the market. P/E ratio is developed to analyze the market's stock valuation of a company and its shares relative to the income the company is actually generating. It is regarded as a good guide to use. Then the efficiency of the portfolio has to be assured. In other words, the portfolio should consist of 30 normal stocks, not a set of stocks with enormous returns or losses. This can be assured by testing alpha of each stock. Once finished with choosing stocks, the next step is to answer the questions.

3.1 Expected Returns Vs. Actual Returns:

In order to answer the first question, historical prices from these stocks are collected and returns are calculated. The data is applied to the three models which are SIM, Multi Group Model, and Black-Litterman. The application gives us the predictions of expected returns so that we can compare expected returns with actual returns. T-statistic, p values and 95% confidence interval will be used as tool to analyze. The significance level for p is 1%. Historical prices are collected from two 5 years' periods, Jan 2001-Jan 2006 and Jan 2007 to Jan 2012.

3.2 Optimal Portfolio Allocations:

First of all, data is applied to the three models to generate three different optimal weights of the portfolio under short sales allowed, and then the absolute values of these weights are added up to see if their sum is too large. If it is, then it is not realistic for investors to implement because raising fund is difficult. For example, if the sum is three, it means investing with short sales allowed needs a fund three times larger than it would be when investing without short sales. The capital is always considered to be one.

3.3 Optimal Portfolio Monthly Returns

In order to get monthly return, I apply the optimal portfolio allocations got from 3.2 to real return. The multiplication of these two 30×1 matrices will give us monthly return. If monthly returns are positive then it is realistic to use, and larger monthly returns are better.

The three means will be applied to each model separately, so it can be identified which model performs best.

4 Empirical Results

Models will be discussed one by one. Two periods' historical prices will be used, and each period's data is a five years' monthly return.

Period 1: 2001-1-1 to 2006-1-1

Period 2: 2007-1-1 to 2012-1-1

Portfolio Efficiency

Before getting to the actual tests, question may be raised if those stocks in the portfolio perform normally? Is that possible that stocks in the portfolio have abnormal returns which may cause the models unable to function properly? This can be solved by finding each stock's alpha. Statistical tests can be conducted to see if each alpha is equal to zero or not. Also, normal QQ plots and residuals versus fits plots are drawn to see how each stock fitted.

Alpha, or Jensen's alpha (1968), in connection to constructing optimal portfolio is a risk adjusted performance measure that adjusts expected or average returns for beta risk (Nielsen & Vassalou, 2004). Alpha in the regression equation is, put in simple terms, a return a portfolio is attaining over a comparing investment, an index, taking risk also into consideration. Alpha is the active components of an investment and typically represents either market timing or security selection (Scott, 2009). Alpha of a security is therefore the component of a securities return that is independent of the market's performance, or a random variable. In other words it represents that component of return insensitive to the return on the market (Elton, Gruber, Brown, & Goetzmann, 2007).

In order to see if all the alphas are zero, the following hypothesis is tested:

H0: True value of alpha (α) = 0

H1: True value of alpha (α) ≠ 0

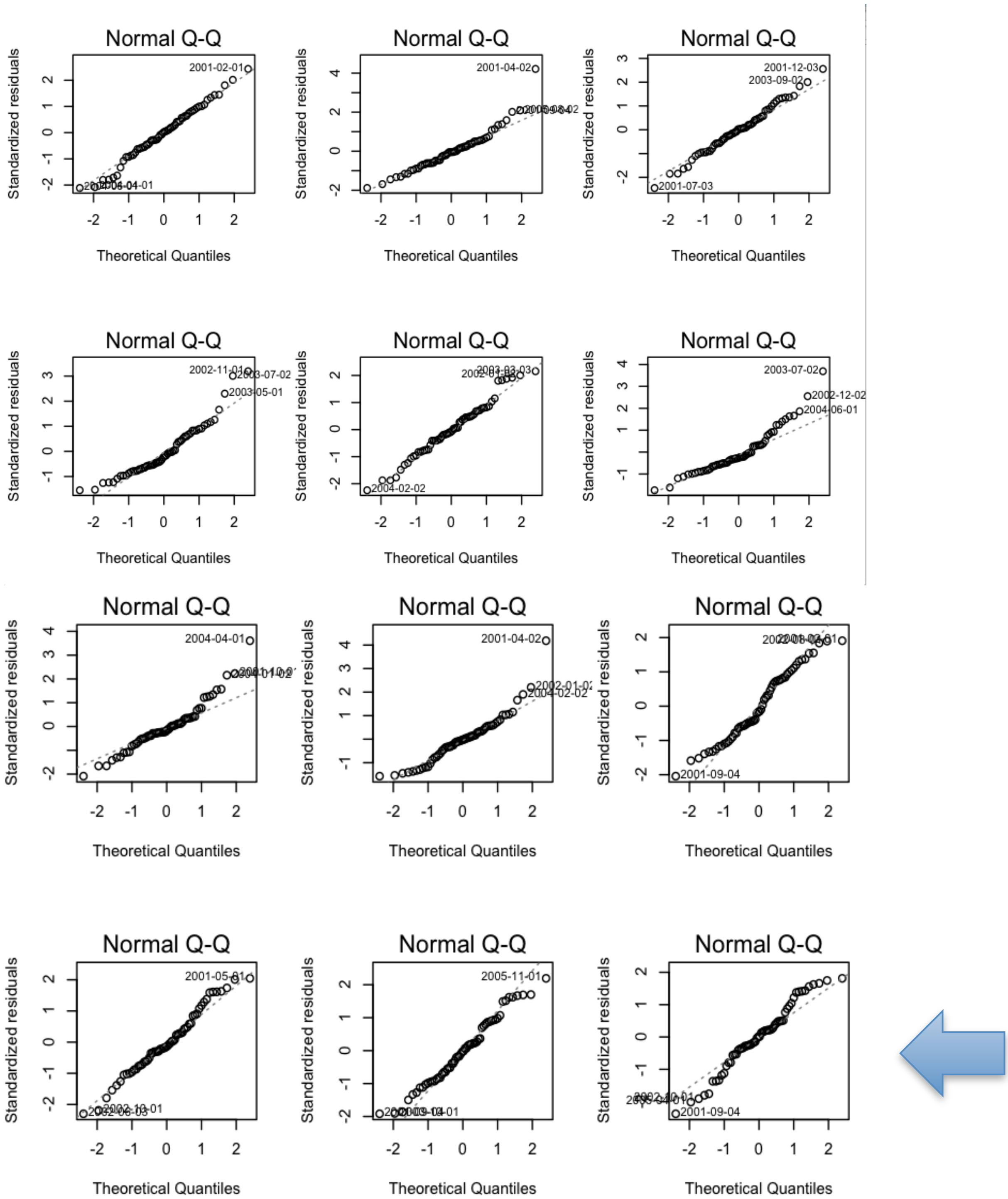
Alphas and tests for period 1:

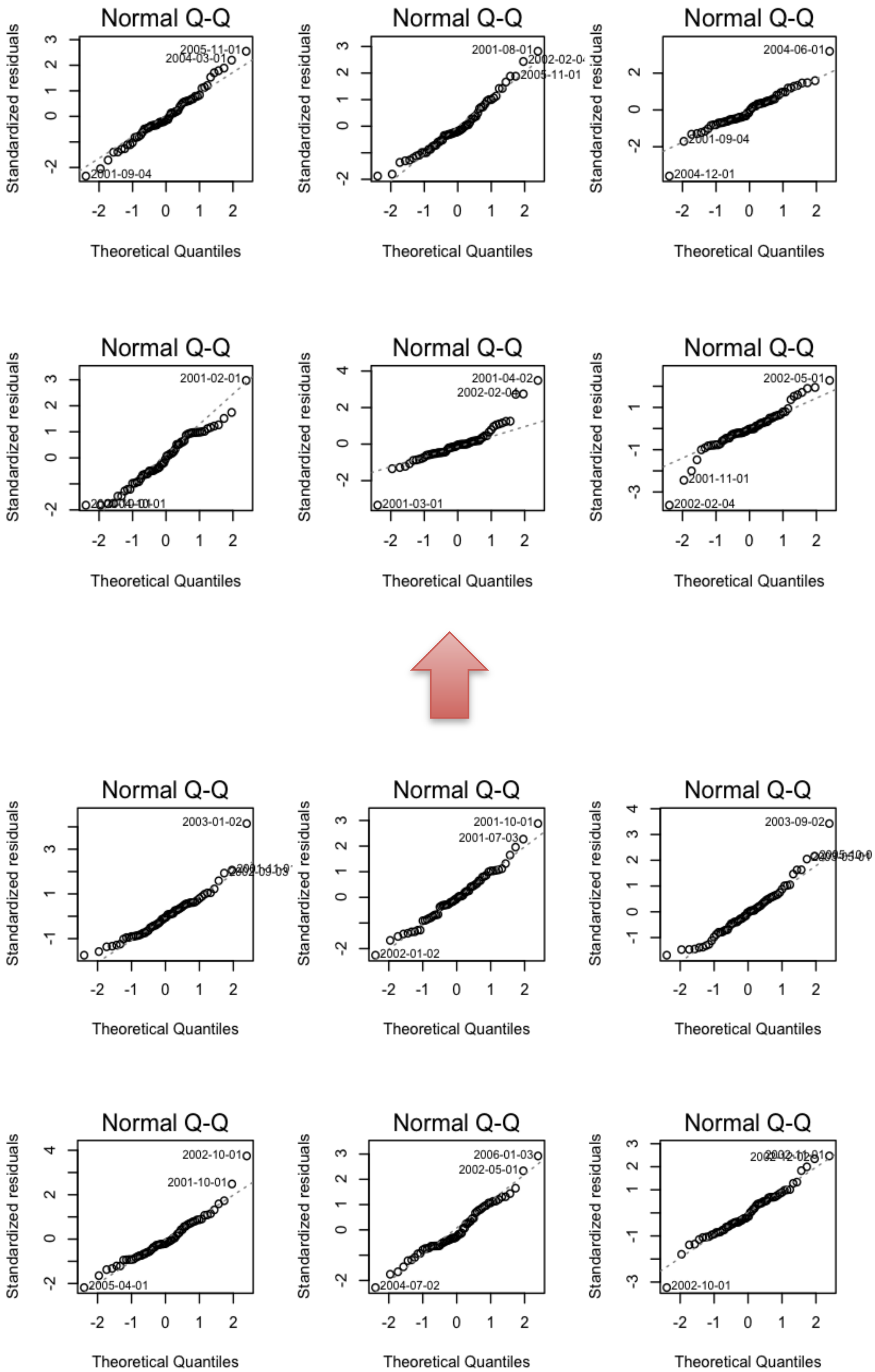
Stocks	Alpha	p-value	95% C.I.	mean
#1 The Ultimate Software Group, In [ULTI]	0.0389	0.33	(-0.02192135, 0.06123169)	0.02
#2 CGI Group, Inc. Common Stock [GIB]	0.0105	0.06	(-0.001060855, 0.069743861)	0.03
#3 Keynote Systems, Inc. [KEYN]	0.0005	0.19	(-0.05729284, 0.25827869)	0.10
#4 Sina Corporation [SINA]	0.0532	0.74	(-0.2642588, 0.1918148)	-0.04
#5 Eastern Virginia Bankshares, In [EVBS]	0.0109	0.38	(-0.0535918, 0.1310123)	0.04
#6 MidSouth Bancorp Common Stock [MSL]	0.0295	0.84	(-0.07797702, 0.06455211)	-0.01
#7 ROEBLING FIN CP NEW [RBLG]	0.0202	0.25	(-0.03284307, 0.11638938)	0.04
#8 Crescent Financial Bancshares, [CRFN]	0.0279	0.14	(-0.01739619, 0.11528662)	0.05
#9 BARRATT DEV PLC [BDEV.L]	0.0281	0.08	(-0.01359092, 0.24103824)	0.11
#10 HENRY BOOT [BHY.L]	0.0251	0.85	(-0.1526568, 0.1269883)	-0.01
#11 BOVIS HOMES GROUP [BVS.L]	0.0181	0.09	(-0.02110115, 0.26537289)	0.12
#12 REDROW [RDW.L]	0.0202	0.04	(0.005636362, 0.186165598)	0.10
#13 PERSIMMON PLC [PSN.L]	0.0326	0.03	(0.01684226, 0.22431526)	0.12
#14 BELLWAY [BWY.L]	0.0243	0.10	(-0.02053632, 0.19913276)	0.09
#15 BERKELEY GRP [BKG.L]	0.0129	0.58	(-0.3799410, 0.2215237)	-0.08
#16 TAYLOR WIMPEY [TW.L]	0.0209	0.14	(-0.0404299, 0.2627955)	0.11
#17 GENUS [GNS.L]	0.0335	0.66	(-0.06988985, 0.10655407)	0.02
#18 SHIRE [SHP.L]	0.0000	0.09	(-0.007423066, 0.100838749)	0.05
#19 Heska Corporation [HSKA]	0.0294	0.14	(-0.1055864, 0.6491936)	0.27
#20 Neogen Corporation [NEOG]	0.0193	0.79	(-0.10604074, 0.08188898)	-0.01
#21 ABAXIS, Inc. [ABAX]	0.0325	0.30	(-0.0934946, 0.2808018)	0.09
#22 International Business Machines [IBM]	-0.0005	0.70	(-0.02254974, 0.03264746)	0.01
#23 Cray Inc [CRAY]	0.0215	0.35	(-0.1473147, 0.3900820)	0.12
#24 AVEVA GROUP [AVV.L]	0.0167	0.67	(-0.07121139, 0.04744711)	-0.01
#25 CGI GROUP INC CL A SV [GIB-A.TO]	0.0046	0.03	(0.003513459, 0.081645112)	0.04
#26 MARKS & SPENCER [MKS.L]	0.0130	0.42	(-0.06580619, 0.14849093)	0.04
#27 DairyFarm 900 US$ [D01.SI]	0.0370	0.77	(-0.1679706, 0.1275630)	-0.02
#28 BRIT LAND CO REIT [BLND.L]	0.0176	0.19	(-0.01158030, 0.05347369)	0.02
#29 HAMMERSON REIT [HMSO.L]	0.0178	0.25	(-0.01229115, 0.04426101)	0.02
#30 Equity One, Inc. Common Stock [EQY]	0.0211	0.48	(-0.02074538, 0.04206827)	0.01

There are three stocks whose alphas, being zero, does not fall into the 95% confident intervals. However, their p values are not too small and they are all above .1, which means all 30 stocks are basically normal.

Alphas and tests for period 2 appears to be very similar, only two stocks whose alphas being zero do not fall into the 95% confident intervals. Also, p values are not too small. These numbers indicate the portfolio is on the right track.

Normal QQ plots for Period 1:




Normal Q-Q
Standardized residuals
Theoretical Quantiles
2005-11-01
2004-03-01
2001-09-04
2001-08-01
2005-11-01
2004-06-01
2001-09-04
2004-12-01
2001-02-01
2001-04-02
2001-03-01
2002-05-01
2001-11-01
2002-02-04
2003-01-02
2001-10-01
2001-07-03
2002-01-02
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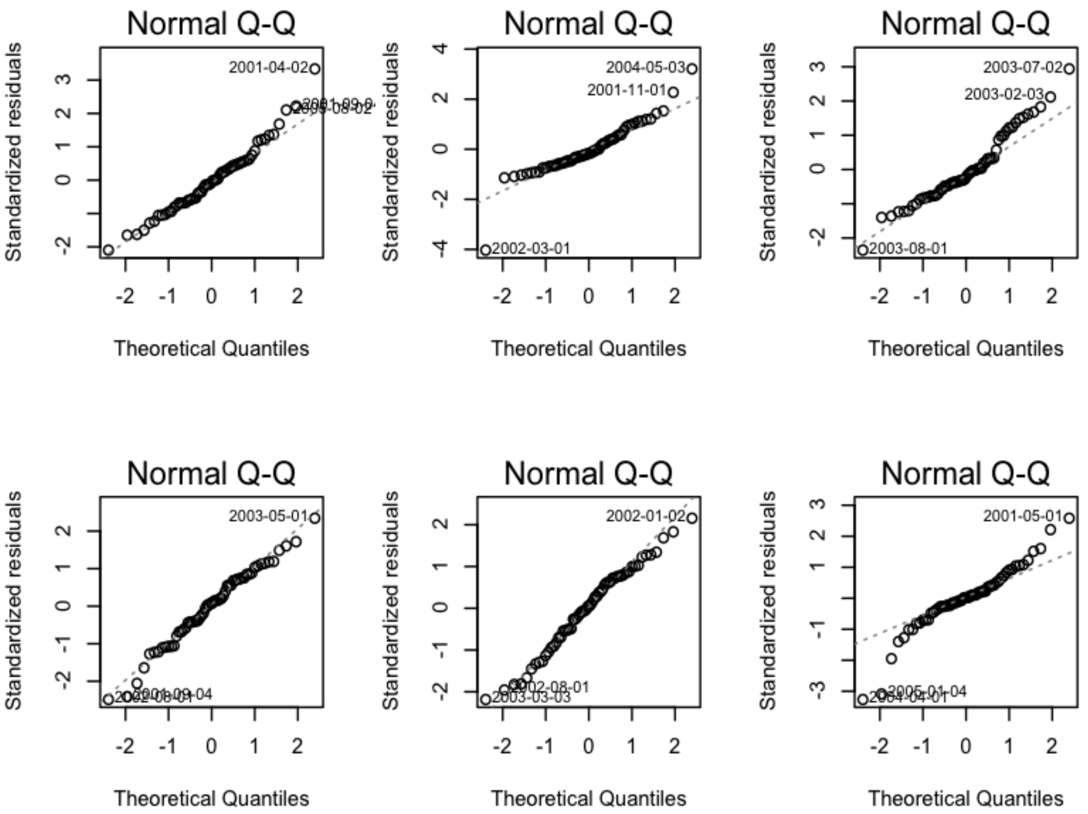
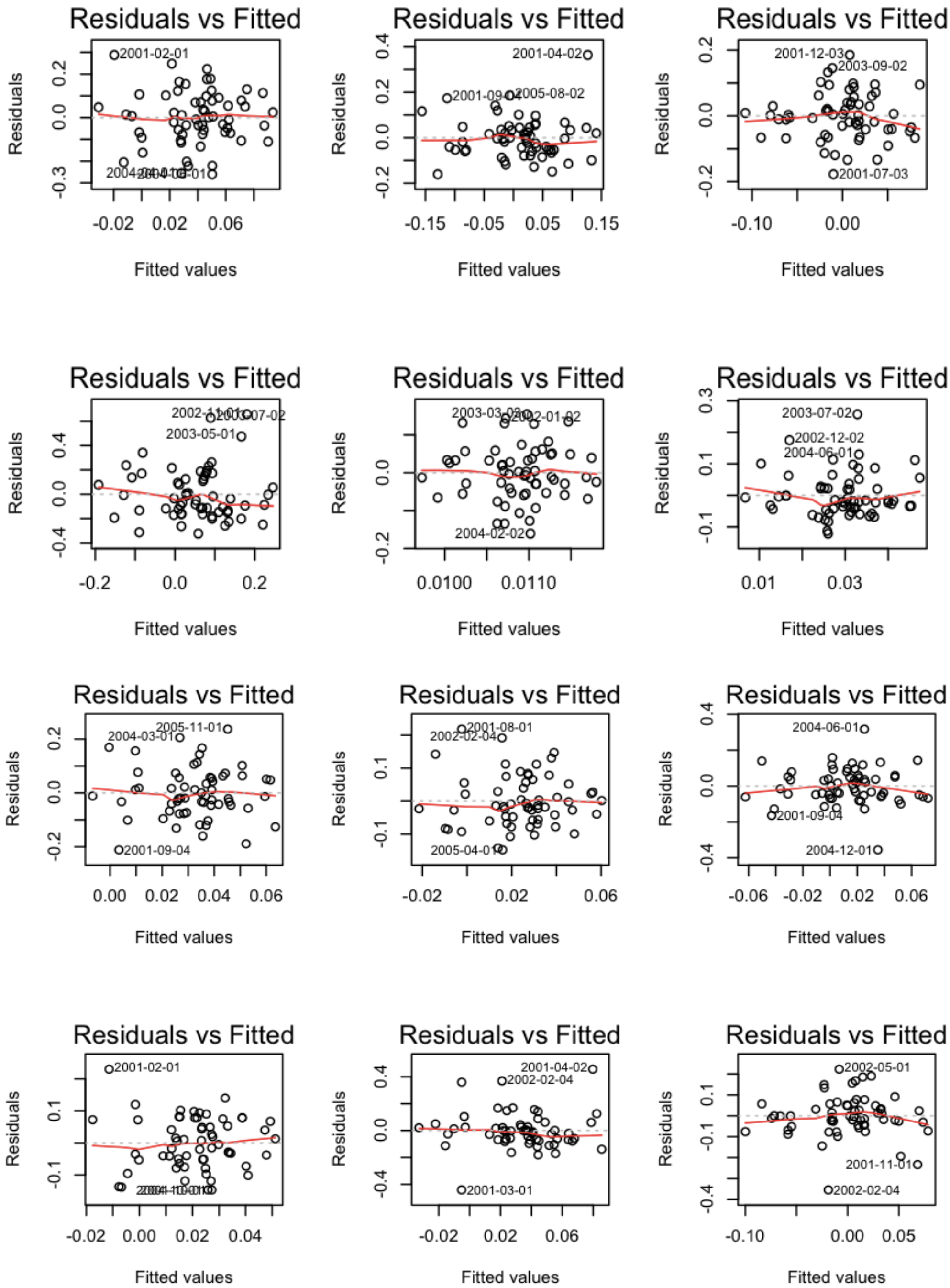
2004-07-02
2002-10-01



All the plots are normal with most points lying on the center straight line. The 12th plot(which has a blue arrow pointing to it) may be a little off the line but it is still approximately normal. The 17th plot(which has a red arrow pointing to it) has one or two large outliers, but it is still basically normal. The graphs for period 2 are very similar. They have few outliers but approximately normal. All the stocks are normal under normal qq plots.

Residuals Vs. Fits Plot for Period 1


Residuals vs Fitted
Residuals
Fitted values
2001-02-01
2001-04-02
2001-12-03
2003-09-02
2001-07-03
2003-05-01
2004-02-02
2003-07-02
2002-12-02
2005-11-01
2004-03-01
2001-09-04
2001-08-01
2004-06-01
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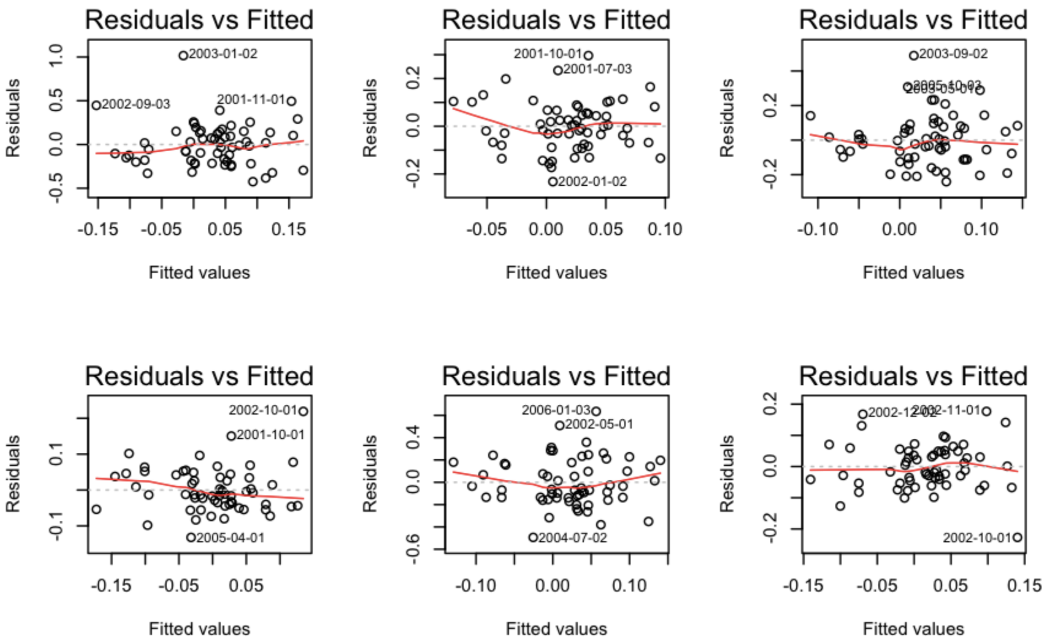
2005-08-02
2002-01-02
2002-02-04
2001-03-01
2002-05-01
2001-11-01


Residuals vs Fitted
Residuals
Fitted values
2003-01-02
2002-09-03
2001-11-01
2001-10-01
2001-07-03
2002-01-02
2003-09-02
2002-10-01
2001-10-01
2005-04-01
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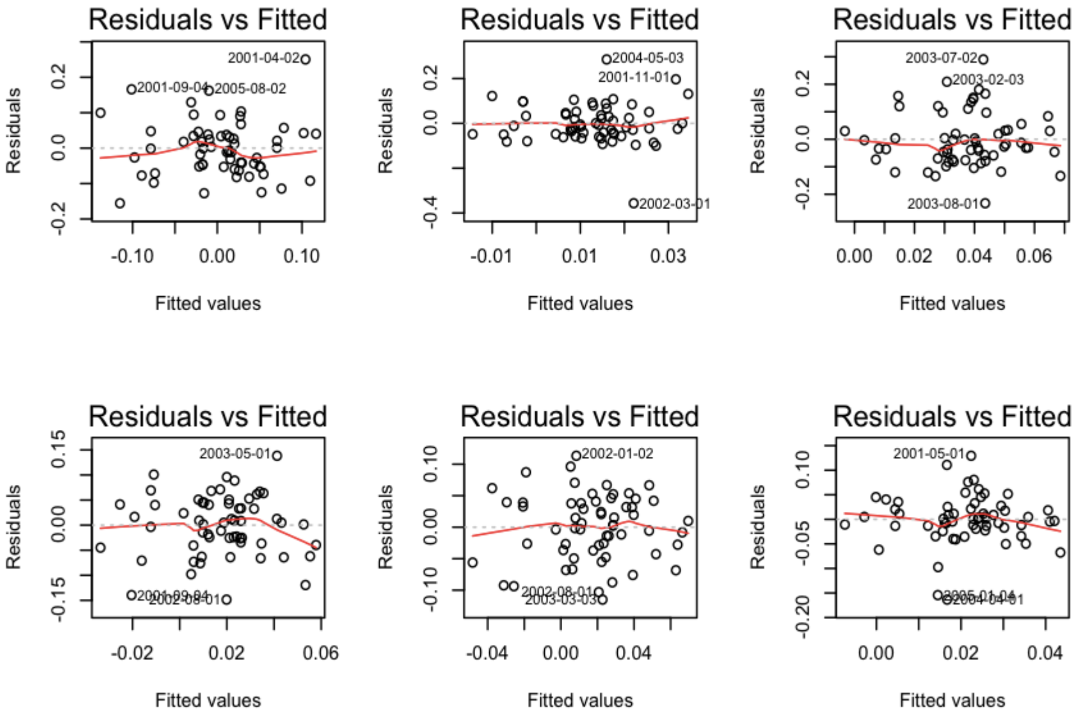
2004-07-02
2002-11-01
2002-10-01


Residuals vs Fitted
Residuals
Fitted values
2001-04-02
2001-09-04
2005-08-02
2004-05-03
2001-11-01
2002-03-01
2003-07-02
2003-02-03
2003-08-01
2003-05-01
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2002-01-02
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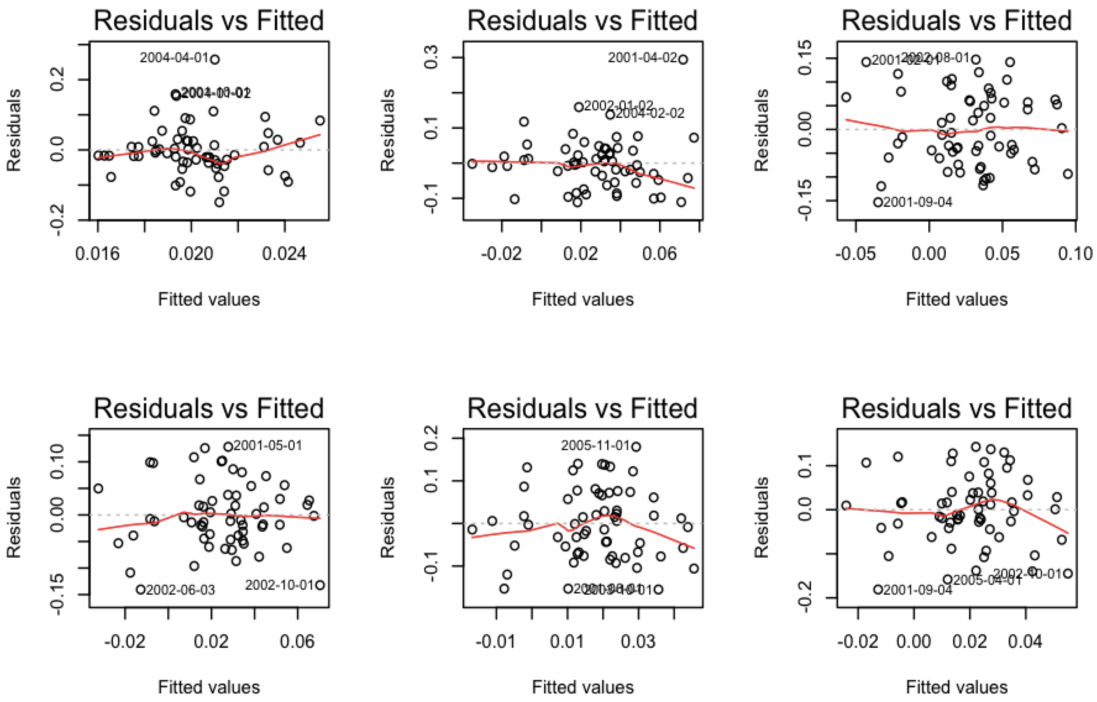
2001-05-01
2004-04-01



The plots are normal though we cannot say they are perfect. A good sign is that there exists no trend of increasing residuals in any of the plots. All of them are approximately normal. The graphs for period 2 are very similar, which also indicating all the stocks perform normally.

Combining the results from alpha test, normal qq plot and residual versus fits plot, we can conclude that all those stocks in the portfolio are normal.

4.1 Single Index Model:

4.11 Expected returns Vs. Actual returns

First of all, period 1's data is applied to the Single Index Model and expected returns for the next period are calculated. A following period after historical period could be the following month or the following year. I think using the following month is more proper because the historical returns are based on monthly returns. But still I do both to see if any one works.

For Period 1:

From the table below we can see the expected returns versus real returns. The second column “Rbar” is the expected return obtained using historical period’s data, and the third column “Return” is the actual return for the following period from 2006-01-01 to 2006-02-1. If the model works, they should match.

	stock	Rbar	Return
[1,]	1	0.0388024638	0.0173913043
[2,]	2	0.0102596436	-0.0025740026
[3,]	3	0.0003643798	-0.0728033473
[4,]	4	0.0528124708	-0.0322303395
[5,]	5	0.0108876960	-0.0306748466
[6,]	6	0.0294505146	0.0005402485
[7,]	7	0.0202024789	0.0000000000
[8,]	8	0.0277847314	-0.0032653061
[9,]	9	0.0279464115	0.0546312896
[10,]	10	0.0250493247	0.0000000000
[11,]	11	0.0180818230	0.0762976384
[12,]	12	0.0200945071	0.0622532645
[13,]	13	0.0325691522	0.1065995066
[14,]	14	0.0242226650	0.0653344378
[15,]	15	0.0128074176	0.0406129710
[16,]	16	0.0208787276	0.0647532155
[17,]	17	0.0333702098	0.0960947539
[18,]	18	-0.0001706517	-0.0182828525
[19,]	19	0.0291164474	0.0000000000
[20,]	20	0.0191177486	-0.0246791708
[21,]	21	0.0323137409	0.0659793814
[22,]	22	-0.0007961612	-0.0047202554
[23,]	23	0.0213011796	-0.0577777778
[24,]	24	0.0164251585	0.0038549793
[25,]	25	0.0044007775	-0.0235955056
[26,]	26	0.0129227817	0.0432157700
[27,]	27	0.0369533036	-0.0157728707
[28,]	28	0.0175298626	0.0309043287
[29,]	29	0.0176817403	0.0363971610
[30,]	30	0.0210285596	0.0012828736

Obviously, they do not look alike. However, a statistical test is used to make sure. Let’s assume

H0: Difference in mean return = 0

H1: Difference in mean return ≠ 0

Doing t –test on their differences gives us the following:

t = -5.4698, df = 29, p-value = 6.88e-06, 95 percent confidence interval:

(-4.177094e-18, -1.903473e-18)

The extremely small p value suggests strong rejection to the null hypothesis, which is not a good sign. Vasicek's technique is developed to make an more accurate prediction by using adjusted betas. The following statistics is generated under Vasicek's technique.

t = 3.3581, df = 29, p-value = 0.002208

95 percent confidence interval:

0.01083223 0.04458149

P value goes up compared to original SIM's, but it still does not reach 1% significance level.

For Period 2:

The following is the expected returns and actual returns on the second historical period.

	stock	Rbar	Return
[1,]	1	0.024941859	0.034637877
[2,]	2	0.018838542	0.036174430
[3,]	3	0.015816496	0.025065274
[4,]	4	0.022500083	-0.036003985
[5,]	5	-0.027013942	0.166666667
[6,]	6	-0.005101985	0.053948397
[7,]	7	-0.013534025	0.022857143
[8,]	8	-0.013445565	0.411575563
[9,]	9	-0.007811541	0.190476190
[10,]	10	0.041199772	-0.020565355
[11,]	11	-0.006459705	0.156897613
[12,]	12	-0.015323714	0.051094891
[13,]	13	-0.004734522	0.156712550
[14,]	14	-0.002361089	0.109905359
[15,]	15	0.001356292	0.076555176
[16,]	16	-0.005261776	0.149773432
[17,]	17	0.013173301	0.088322196
[18,]	18	0.014441417	0.069326469
[19,]	19	0.003269183	0.070221066
[20,]	20	0.024399315	0.089346024
[21,]	21	0.011095582	-0.031095942
[22,]	22	0.014241387	0.008164564
[23,]	23	0.010513833	0.063087248
[24,]	24	0.017012354	0.065312205
[25,]	25	0.015541195	0.028641975
[26,]	26	-0.004011186	0.093042669
[27,]	27	0.020744218	0.047866805
[28,]	28	-0.011903338	-0.009635074
[29,]	29	-0.012446700	0.065238799
[30,]	30	0.002161833	0.050969529

Making similar assumptions as period 1, we got the t-test result as follows:

t = -4.1361, df = 29, p-value = 0.0002761

95 percent confidence interval:

-0.10674296 -0.03610652

The small p value suggests the high possibility of rejecting the null hypothesis. The t-test result also indicates their differences are significant. 0 does not fall into the 95 percent confidence interval. Adjusted betas under Vasicek's technique's also do not indicate good stats.

Below are the results from changing the following period from month to year.

	stock	Rbar	Return
[1,]	1	0.0388024638	0.0117516997
[2,]	2	0.0102596436	-0.0066166917
[3,]	3	0.0003643798	-0.0125524931
[4,]	4	0.0528124708	0.0384697707
[5,]	5	0.0108876960	0.0025304242
[6,]	6	0.0294505146	0.0253288292
[7,]	7	0.0202024789	0.0210338438
[8,]	8	0.0277847314	0.0035697033
[9,]	9	0.0279464115	0.0185964334
[10,]	10	0.0250493247	0.0358739836
[11,]	11	0.0180818230	0.0320756560
[12,]	12	0.0200945071	0.0221821979
[13,]	13	0.0325691522	0.0145859496
[14,]	14	0.0242226650	0.0250319389
[15,]	15	0.0128076732	0.0290563588
[16,]	16	0.0208787276	0.0103326135
[17,]	17	0.0333702098	0.0382360596
[18,]	18	-0.0001706517	0.0184701490
[19,]	19	0.0291164474	0.0202981653
[20,]	20	0.0191177486	-0.0009747788
[21,]	21	0.0323137409	0.0040040298
[22,]	22	-0.0007961612	0.0161008416
[23,]	23	0.0213011796	0.0397884900
[24,]	24	0.0164251585	0.2501630820
[25,]	25	0.0044007775	-0.0053509305
[26,]	26	0.0129227817	0.0308756691
[27,]	27	0.0369533036	0.0013490807
[28,]	28	0.0175297413	0.0277697033
[29,]	29	0.0176817403	0.0322125711
[30,]	30	0.0210285596	0.0209636400

	stock	Rbar	Return
[1,]	0	0.024941859	0.031175799
[2,]	0	0.018838542	0.018266910
[3,]	0	0.015816496	-0.011789316
[4,]	0	0.022500083	-0.011629924
[5,]	0	-0.027013942	0.065955399
[6,]	0	-0.005101985	0.026843033
[7,]	0	-0.013534025	0.086898645
[8,]	0	-0.013445565	0.045781751
[9,]	0	-0.007811541	0.070909215
[10,]	0	0.041199772	0.018765149
[11,]	0	-0.006459705	0.032467410
[12,]	0	-0.015323714	0.042731134
[13,]	0	-0.004734522	0.049547492
[14,]	0	-0.002361089	0.040868353
[15,]	0	0.001356314	0.033248132
[16,]	0	-0.005261776	0.054015016
[17,]	0	0.013173301	0.036496332
[18,]	0	0.014441417	0.001268390
[19,]	0	0.003269183	0.014561235
[20,]	0	0.024399315	0.032872162
[21,]	0	0.011095582	0.036442996
[22,]	0	0.014241387	0.002921423
[23,]	0	0.010513833	0.089225379
[24,]	0	0.017012354	0.028186563
[25,]	0	0.015541195	0.017475526
[26,]	0	-0.004011186	0.014970054
[27,]	0	0.020744218	0.019499690
[28,]	0	-0.011903362	0.016651307
[29,]	0	-0.012446700	0.024786780
[30,]	0	0.002161833	0.017850225

Period 1:

t = -0.5223, df = 29, p-value = 0.6054

95 percent confidence interval:

-0.02158787 0.01280476

Period 2:

t = -4.2116, df = 29, p-value = 0.0002246

95 percent confidence interval:

-0.03988524 -0.01380973

It seems the prediction for the first period is fine, but the small p value and t-test for the second period indicate the predictions are negative.

Only the prediction of yearly return for the first period works among the four. These results indicate that SIM does not work properly on predicting next period's returns.

4.12 Optimal Portfolio Allocations

Period 1:

The following is the portfolio allocations obtained from period 1. For the factors to be realistic, we may expect the sum of absolute values to not be too large.

```
> x_short

 [1]  0.042174668  0.118214643  0.071325584  0.069011603  0.148798686  0.043708389
0.053060612  0.064563089

 [9]  0.043032507  0.030555316  0.024812963  0.050531474  0.083674321  0.099539551
0.067128451  0.064485398

[17]  0.074514973  0.016819485  0.012437527  0.015078383  0.007555466  0.002840237
0.001592218  -0.009149037

[25] -0.026397726 -0.046728697 -0.053708719 -0.127004980 -0.028554833  0.086088443
```

The factors look good. The sum of absolute values is under 2. It is good for investing. I also did the following 4 months' assets allocations, and they all look normal. It seems that the SIM worked out very well for this part.

Period 2:

x_short

[1] 0.498328854 0.377603480 0.486980521 0.044682346 0.289366396 0.370547038 0.591352334 0.794891569

[9] 0.313107494 0.129857848 0.145181030 0.169114490 0.053234752 0.005884417 -0.051011541 -0.294640510

[17] -0.045602667 -0.054924122 -0.099836926 -0.065210338 -0.226204490 -0.104756179 -0.184983813 -0.175642674

[25] -0.466185981 -0.177104819 -0.499783394 -0.171019045 -0.236897287

-0.416328782

The sum of absolute values for period 2 is over 6, which makes it very expensive and unrealistic for investors to deal with. I conduct a few more months and all ends up with sums over 5. This indicates the factors are not realistic for using. It seems SIM is not very steady for generating optimal weights.

4.13 Optimal Portfolio Returns

Period 1:

monthly return when short sales allowed: 0.03879937

monthly return when short sales not allowed: 0.02541904

	2/1/06	3/1/06	4/1/06	5/1/06	6/1/06
Shorts	0.0388	0.0379081	0.0025381	-0.0532001	-0.0015289
No shorts	0.02542	0.0336944	-0.0009133	-0.0503337	-0.0195242
GSPC	0.00045	0.011095841	0.01215566	-0.0309169	-0.01401

Unfortunately, it seems that the returns are going down month by month. After April, it has even negative returns. If an investor actually had used SIM in 2006, he or she would be losing money. But a fact I may need to disclose is that the whole market was not doing very well during those months in 2006. The portfolio without short sales does not perform well possibly due to a bad market, and it could have nothing to do with the system of SIM. If every stock is going down, how can you expect to make money? However, we should expect the portfolio with short sales allowed still performs good. As a matter of fact, it does not, which points out that SIM may not to be a good choice for this part.

Period 2:

The following is the monthly return for 1-1-2012 to 2-1-2012 by using the the assets allocations calculated from period 2 under SIM.

Monthly returns for Short sales allowed: 0.03325059

Monthly returns for Short sales not allowed: 0.02850477

The returns appear to be good. Here are a few more months.

	2/1/12	3/1/12	4/1/12	5/1/12	6/1/12
Shorts	0.033251	-0.06941917	-0.1055408	-0.1957767	0.00845925
No shorts	0.028505	0.01854021	-0.0169288	-0.0404443	0.0273504
GSPC	0.040589	0.031332377	-0.0074975	-0.0626507	0.03461723

If investors used SIM to make predictions, then unfortunately they would lose money. For 4/1/12 and 5/1/12, the whole stock market is negative which can explain that it is very hard to make money without allowing short sales. But for the other time, if SIM work properly, investors should be making money which is not the case here.

Single Index Model does not predict the accurate returns, and it creates very large weights and negative monthly returns. Combining all these numbers, SIM appears to not be functioning properly.

4.2 Multi-group Model

The same analysis conducted above will be applied to the Multi Group Model. The only difference is that short sales are always allowed in the Multi Group Model.

4.21 Expected Returns VS. Real Returns

The following table combines the expected returns obtained from the model and the following period's real returns.

	Expected Returns	Real Returns
[1,]	0.03900	0.070938215
[2,]	0.01000	-0.033462033
[3,]	0.00036	-0.110460251
[4,]	0.05300	-0.038676407
[5,]	0.01100	-0.037321063
[6,]	0.02900	-0.004862237
[7,]	0.02000	0.040456432
[8,]	0.02800	-0.018775510
[9,]	0.02800	0.035580256
[10,]	0.02500	0.000000000
[11,]	0.01800	0.060386437
[12,]	0.02000	0.053576851
[13,]	0.03300	0.110571172
[14,]	0.02400	0.020904949
[15,]	0.01300	0.005414358
[16,]	0.02100	0.066653995
[17,]	0.03300	0.226297559
[18,]	-0.00017	-0.012738634
[19,]	0.02900	0.000000000
[20,]	0.01900	-0.003948667
[21,]	0.03200	0.103608247
[22,]	-0.00080	-0.010551159
[23,]	0.02100	-0.128888889
[24,]	0.01600	-0.012482790
[25,]	0.00440	-0.041573034
[26,]	0.01300	0.066887566
[27,]	0.03700	-0.015772871
[28,]	0.01800	0.053096724
[29,]	0.01800	0.091952717
[30,]	0.02100	-0.060936498

Our goal is to see if the predictions are accurate. Let's assume

H0: Difference in mean return = 0

H1: Difference in mean return ≠ 0

Furthermore, we will do a t-test. The following is the result:

t = -0.4291, df = 29, p-value = 0.671

95 percent confidence interval:

-0.03015852 0.01969754

The t-test and the large p-value tell us that these two lists of numbers are very similar which means Multi Group Model predicts the accurate returns for the next period. Here are the results for the second period.

Period 2:

	Expected Returns	Real Returns
[1,]	0.024941859	0.046933573
[2,]	0.018838542	0.062438057
[3,]	0.015816496	0.020887728
[4,]	0.022500083	-0.031450121
[5,]	-0.027013942	0.100000000
[6,]	-0.005101985	-0.014073495
[7,]	-0.013534025	0.022857143
[8,]	-0.013445565	0.292604502
[9,]	-0.007811541	0.347985348
[10,]	0.041199772	0.007394285
[11,]	-0.006459705	0.137747076
[12,]	-0.015323714	0.025952960
[13,]	-0.004734522	0.283015608
[14,]	-0.002361089	0.107185479
[15,]	0.001356354	0.095501460
[16,]	-0.005261776	0.171953255
[17,]	0.013173301	0.342491424
[18,]	0.014441485	0.043681673
[19,]	0.003269183	0.232769831
[20,]	0.024399315	0.065397605
[21,]	0.011095582	-0.019340159
[22,]	0.014241387	0.025394974
[23,]	0.010513833	0.069798658
[24,]	0.017012354	0.086239345
[25,]	0.015541195	0.045925926
[26,]	-0.004011186	0.110797188
[27,]	0.020744218	0.045785640
[28,]	-0.011903330	-0.037453668
[29,]	-0.012446700	0.038979156
[30,]	0.002161833	0.009418283

t = 4.2032, df = 29, p-value = 0.0002298

95 percent confidence interval:

0.04440953 0.12858920

Obviously, the result is negative. The model does not make good predictions for period 2. Combining the results from period 1, we may conclude that the Multi Group Model sometimes works perfectly in predicting returns but not all the time.

4.22 Optimal Portfolio Allocations

For short sales' weights to be realistic, the sum of their absolute values is supposed to be small as is previously explained. This table below shows the optimal portfolio's weights based on the data from period 1:

```
          weights
 [1,]  0.082491819
 [2,] -0.010722527
 [3,] -0.071606882
 [4,]  0.027246414
 [5,]  0.072976705
 [6,]  0.227589035
 [7,]  0.186212959
 [8,]  0.213477613
 [9,]  0.061804303
[10,]  0.135926237
[11,] -0.055947182
[12,] -0.023113880
[13,]  0.066022553
[14,]  0.030348708
[15,] -0.121933940
[16,] -0.017281136
[17,]  0.046708920
[18,] -0.031894222
[19,]  0.004839531
[20,]  0.031475441
[21,]  0.032239148
[22,] -0.105987445
[23,] -0.019800634
[24,]  0.014514415
[25,] -0.057707698
[26,]  0.069411039
[27,]  0.147535487
[28,] -0.032446603
[29,] -0.011425195
[30,]  0.109047019
```

The sum of the weights' absolute values is 2.1 which is acceptable. It is possible for investors to use the weights for investing.

Period 2:

The following is the second period's optimal portfolio's allocation.

```
          weights
 [1,]  0.060897566
 [2,]  0.231913582
 [3,] -0.112533947
 [4,] -0.064757778
 [5,] -0.737595107
 [6,] -0.445779120
 [7,] -0.237681589
 [8,] -0.133721209
 [9,] -0.024014320
[10,]  0.125136300
[11,] -0.121638449
[12,] -0.214443345
[13,] -0.046081861
[14,] -0.011812499
[15,]  0.113748440
[16,] -0.007116936
[17,]  0.285752485
[18,]  0.382344179
[19,] -0.065517968
[20,]  0.401469866
[21,] -0.023880931
[22,]  1.117733090
[23,]  0.078225223
[24,]  0.304959473
[25,]  0.773919144
[26,] -0.467916415
[27,]  0.294931780
[28,] -0.540953586
[29,] -0.302883340
[30,]  0.387297271
```

The sum of the weights' absolute values is 8.116657. This is too large and would cost investors too much money to implement.

4.23 Optimal Portfolio Returns

Applying calculated optimal portfolio allocations to both periods following 6 months, we got 6 monthly returns. GSPS is the market index.

6 months after Period 1							
	2/1/06	3/1/06	4/1/06	5/1/06	6/1/06	7/1/06	Total
Monthly Returns	0.0294	0.0245	0.0604	-0.0307	-0.0088	0.0636	0.1385
GSPC	0.0005	0.0111	0.0122	-0.0309	-0.0140	0.0051	-0.0161

6 months after Period 2							
	2/1/12	3/1/12	4/1/12	5/1/12	6/1/12	7/1/12	Total
Monthly Returns	0.0660	-0.1138	-0.0223	-0.0413	0.1664	-0.1982	-0.1431
GSPC	0.0406	0.0313	-0.0075	-0.0627	0.0346	0.0126	0.0490

We have the conclusion that the model makes good predictions for period one. As a result, the model performs well for the six months after period one. If an investor used the multi-group model in 2006, then he or she would make a good amount of money. A 13.85% return for a half of a year is a very good return. Unfortunately, if an investor used this model in 2012, then he or she would lose money. All in all, the Multi Group model is not steady, namely not realistic. Investors cannot rely solely on this model.

4.3 Black-Litterman Model

The same analysis conducted for the previously two models will be applied to Black-Litterman Model as well.

4.31 Expected returns Vs. Actual returns

The following table combines the real returns and implied returns for the period 1/1/2006 -2/1/2006. If the model works, they are statistically the same.

Period 1: 1/1/2001 – 1/1/2006

	RealReturns	ImpliedReturns
[1,]	0.0173913043	-0.0022695512
[2,]	-0.0025740026	-0.0062516995
[3,]	-0.0728033473	-0.0043856120
[4,]	-0.0322303395	-0.0085404457
[5,]	-0.0306748466	0.0004572246
[6,]	0.0005402485	-0.0008656703
[7,]	0.0000000000	0.0004250575
[8,]	-0.0032653061	-0.0018757944
[9,]	0.0546312896	-0.0023877241
[10,]	0.0000000000	-0.0001202709
[11,]	0.0762976384	-0.0003163813
[12,]	0.0622532645	-0.0013539894
[13,]	0.1065995066	-0.0010449949
[14,]	0.0653344378	-0.0011876936
[15,]	0.0406161573	-0.0007308699
[16,]	0.0647532155	-0.0004169908
[17,]	0.0960947539	-0.0006647032
[18,]	-0.0182747530	-0.0033989087
[19,]	0.0000000000	-0.0047621662
[20,]	-0.0246791708	-0.0033028290
[21,]	0.0659793814	-0.0032502978
[22,]	-0.0577777778	-0.0080380517
[23,]	0.0038549793	-0.0047713286
[24,]	-0.0235955056	-0.0035563318
[25,]	0.0432157700	-0.0055939779
[26,]	-0.0157728707	-0.0016316999
[27,]	0.0308974644	0.0002556419
[28,]	0.0363971610	-0.0003496745
[29,]	0.0012828736	-0.0011402579
[30,]	0.0173913043	-0.0006066205

Now, let's do a t-test to see if the mean of their differences equal to zero.

t = 2.4314, df = 29, p-value = 0.02145

95 percent confidence interval:

0.003036391 0.035200906

Since we set the significance level to be 1%, we don't reject the null hypothesis here. So far, Black-Litterman functions great. Its implied returns are almost the same as the following period's return.

For Period 2,

	RealReturns	ImpliedReturns
ULTI	0.097325103	-0.0006923944
GIB	0.033748702	-0.0005779726
KEYN	0.117960877	-0.0008927066
SINA	0.003167155	-0.0007822844
EVBS	-0.195789474	-0.0003722678
MSL	0.060559006	-0.0003722406
RBLG	0.092342342	-0.0003439895
CRFN	-0.061135371	-0.0004021426
BDEV.L	0.082474227	-0.0015209275
BHY.L	0.061603716	-0.0010782257
BVS.L	0.008001123	-0.0005422688
RDW.L	0.095197978	-0.0002770182
PSN.L	0.076853367	-0.0008552401
BWY.L	0.031834474	-0.0005256669
BKG.L	0.117020185	-0.0002857930
TW.L	0.098862642	-0.0014085986
GNS.L	-0.009003615	-0.0003036310
SHP.L	0.052824715	-0.0003986741
HSKA	0.199575372	-0.0011068169
NEOG	0.029477197	-0.0004192084
ABAX	0.007404521	-0.0004961181
IBM	0.021601179	-0.0007347084
CRAY	0.008021390	-0.0007008113
AVV.L	0.031477944	-0.0009460146
GIB-A.TO	0.016614746	-0.0002498239
MKS.L	0.002805006	-0.0003519629
D01.SI	-0.095953757	-0.0004029095
BLND.L	0.086785304	-0.0005296560
HMSO.L	0.042628467	-0.0006495512
EQY	0.022981732	-0.0005839876

t = 4.9684, df = 29, p-value = 2.769e-05

95 percent confidence interval:

0.04583731 0.10997780

The extremely small p value leads us to reject the null hypothesis. These are two different set of numbers. It seems Black-Litterman is not steady as well. Nevertheless, investors can have their financial views incorporated in the model, which may lead to a more accurate result.

4.32 Optimal Portfolio Allocations

Period 1 and Period 2:

	weights
ULTI	0.042624562
GIB	-0.585687077
KEYN	0.155992392
SINA	-0.080870150
EVBS	0.314786668
MSL	-0.010116348
RBLG	0.134135564
CRFN	-0.122185158
BDEV.L	-0.296860428
BHY.L	-0.023395403
BVS.L	0.217756566
RDW.L	0.036864190
PSN.L	-0.067672047
BWY.L	-0.104800020
BKG.L	0.110007808
TW.L	0.152201444
GNS.L	0.106142956
SHP.L	0.018370979
HSKA	-0.046694075
NEOG	0.120271631
ABAX	-0.057957713
IBM	0.080792880
CRAY	0.004819015
AVV.L	-0.005929628
GIB-A.TO	0.816518653
MKS.L	0.039866258
D01.SI	-0.024663228
BLND.L	0.014480165
HMSO.L	0.076611305
EQY	0.002276040

	weights
ULTI	-0.107837277
GIB	-0.568361391
KEYN	-0.039163546
SINA	-0.068573573
EVBS	0.075798660
MSL	0.104088759
RBLG	0.044076891
CRFN	0.100648625
BDEV.L	-0.073273209
BHY.L	-0.000256569
BVS.L	-0.023609438
RDW.L	0.010341756
PSN.L	0.049235975
BWY.L	-0.023013799
BKG.L	0.109443479
TW.L	-0.022706456
GNS.L	0.108410331
SHP.L	-0.035892959
HSKA	-0.049608204
NEOG	-0.021710114
ABAX	0.059742637
IBM	0.310927047
CRAY	-0.030540377
AVV.L	0.015896216
GIB-A.TO	0.643858030
MKS.L	0.050592129
D01.SI	0.180163101
BLND.L	0.060371611
HMSO.L	0.047729552
EQY	0.087321724

For period 1 and 2, the sums of their absolute values are 3.8 and 3 which are fairly large. But compared to the results in SIM, they are still smaller.

4.33 Optimal Portfolio Returns

1 year after Period 1

Monthly Returns	2/1/06	3/1/06	4/1/06	5/1/06	6/1/06	7/1/06	8/1/06	9/1/06	10/1/06	11/1/06	12/1/06	1/1/07	Total
Shorts	-0.023	0.030	-0.043	-0.044	-0.040	-0.019	0.042	0.032	0.069	0.048	-0.024	0.029	0.176

1 year after Period 1

	2/1/12	3/1/12	4/1/12	5/1/12	6/1/12	7/1/12	8/1/12	9/1/12	10/1/12	11/1/12	12/1/12	1/1/13	
Shorts	-0.004	-0.001	0.001	0.003	-0.003	0.025	-0.020	0.047	0.068	0.036	0.040	-0.018	0.174

If you are using Black-Litterman during 2006 and 2012, then you have made a considerable amount of money. This is a positive sign.

5 Conclusions

I would say none of these models are realistic, but there must be a reason that Markowizts's Modern Portfolio Theory has been popular for over 60 years. These models do work sometimes, but not all the time. From my research, Single Index Model appears to be the last one you would choose. It does not make accurate predictions of return for both periods. Undoubtedly, the most important function of those models is to predict future returns because optimal portfolio allocation is calculated based on the model's prediction of returns. Everything can possibly be wrong if prediction of returns goes wrong. SIM also has more negative monthly returns than positive ones. It may be due to that SIM is the 'oldest" one and has least considerations. For example, Multi Group Model takes correlations between sectors into account but SIM does not. The result of applying data to Multi Group Model shows it has a good forecast during the first period. It has an impeccable prediction of returns which is proved by the large p value, realistic allocations, and more positive monthly returns than negative ones'. It is perfect if we stop right there. However, the forecast for period two appears to be a totally opposite side of period one's. The result is not exceptional at all. In terms of performance, Black-Litterman has similar results compared to Multi Group Model. The first period's prediction works excellently. It foresees the future returns, makes good portfolio allocation, and has much more positive monthly returns than negative ones'. Nevertheless, same thing happens to Black-Litterman Model as well. It has terrible predictions in period two, such as not accurate return predictions and negative monthly returns.

However, being realistic, it must have good predictions most of the time since we are dealing with money. Clearly, none of these models achieves it when looking at my research. But, I have to admit my dataset is quite limited and lacks of historical periods and stocks. If conditions allowed, the results generating from doing 100 stocks and 50 historical periods instead of 30 stocks and 2 periods will be more convinced. Based on my research, the answer to the thesis' main question is clearly negative. I believe the main reason leading to the negative answer is that those models are oversimplified approximations of reality. Possibly, they would be more accurate by adding more complex calculations and considerations. Also, using historical returns to predict future returns is not reliable. Especially for relatively small companies which are the majorities of my portfolio, their stock price goes up and down all the time. It is very difficult to say how the return is going to be for the next month.

I would not suggest any one to use those models as tools for making money. But when investors have ideas in mind, it is a smart choice to use these models including graphs for testing and decision.

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