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UNIVERSITY OF CALIFORNIA SAN DIEGO

Indeterminacy, Infinity, Ideality: Kant's Mathematical Antinomies

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy

in

Philosophy

by

Rosalind Kay Chaplin

Committee in charge:

Professor Eric Watkins, Chair Professor Lucy Allais Professor Samuel Buss Professor Donald Rutherford Professor Clinton Tolley

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The dissertation of Rosalind Kay Chaplin is approved, and it is accepted in quality and form for publication on microfilm and electronically.

University of California San Diego

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List of Abbreviations

Kant's Texts

С	Correspondence. Ed. and Tran. Arnulf Zweig. Cambridge University Press. 1999.
Critique	<i>Critique of Pure Reason</i> . Eds. and Trans. Paul Guyer and Allen W. Wood. Cambridge University Press. 1998.
KE	<i>The Kant-Eberhard Controversy</i> . Ed. and Tran. Henry Allison. Johns Hopkins University Press. 1973.
ID	Inaugural Dissertation of 1770 (On the Form and Principles of the Sensible and Intelligible World). In Theoretical Philosophy 1755-1770. Eds. and Trans. David Walford and Ralf Meerbote. Cambridge University Press. 2002.
Kästner	<i>On Kästner's Treatises</i> . Trans. Christian Onof and Dennis Schulting in <i>Kantian Review</i> 19.2: 305-313. 2014.
МН	Metaphysik Herder. Eds. and Trans. Karl Ameriks and Steve Naragon. In <i>Lectures on Metaphysics</i> . Cambridge University Press. 1997.
ML_1	<i>Metaphysik</i> L_1 . Eds. and Trans. Karl Ameriks and Steve Naragon. In <i>Lectures on Metaphysics</i> . Cambridge University Press. 1997.
MFNS	Metaphysical Foundations of Natural Science. Ed. and Tran. Michael Friedman. Cambridge University Press. 2004.
OPA	The Only Possible Argument in Support of a Demonstration of the Existence of God. In Theoretical Philosophy 1755-1770. Eds. and Trans. David Walford and Ralf Meerbote. Cambridge University Press. 2002.
Prol	Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science. Ed. and Tran. Gary Hatfield. Cambridge University Press. 1997.

Reference to the Critique of Pure Reason follow the standard A-edition/B-edition pagination.

References to all other texts follow the abbreviation conventions above, followed by the volume

and page numbers from the standard Akademie Ausgabe. Unless otherwise noted, I have

followed the translations provided in the Cambridge Edition texts.

Other Texts

A	Leibniz, G.W. Sammtliche Schriften und Briefe. Philosophische Schriften. Series 6, vols. 1-3. Berlin: Akademie Verlag. 1923-80.
AG	Leibniz. G.W. <i>Philosophical Essays</i> . Eds. and Trans. Roger Ariew and Daniel Garber. Hackett Publishing Company. 1989.
Metaphysics	Baumgarten, Alexander. <i>Metaphysics</i> . Eds. and Trans. Courtney D. Fugate and John Hymers. Bloomsbury. 2013.
Treatise	Hume, David. <i>A Treatise of Human Nature, Vol 1</i> . Eds. David Fate Norton and Mary J. Norton. Oxford University Press. 2007.

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Х

Vita

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Abstract of the Dissertation

Indeterminacy, Infinity, Ideality: Kant's Mathematical Antinomies

by

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Doctor of Philosophy in Philosophy

University of California San Diego, 2021

Professor Eric Watkins, Chair

This dissertation argues for a novel interpretation of the mathematical antinomies, which concern the cosmological questions of the world's extent in space and time and the divisibility of matter. In the resolution of the antinomies, Kant makes two striking claims. He claims (i) that the world is neither finite nor infinite in spatiotemporal extent and (ii) that spatiotemporal objects are composed neither from simples nor from infinitely many parts all of which are divided in turn. Against competing interpretations, I argue that these claims amount to a thesis of *metaphysical indeterminacy* for spatiotemporal phenomena. According to Kant, transcendental idealists alone can hold that spatiotemporal phenomena are metaphysically indeterminate in magnitude rather

than either finite or infinite, and this commitment to indeterminacy is what allows them to escape the antinomies. Reading Kant in this way provides an interpretation of the antinomies that is more charitable than many others on offer, and it explains how the antinomies can present a dialectically effective objection to Kant's rationalist interlocutors.

Chapter 1 summarizes the arguments of the mathematical antinomies and the main interpretive positions defended in the secondary literature. Chapter 2 explains how the notions of infinity and totality relate to one another and why Kant thinks denying finitude *and* infinitude for spatiotemporal phenomena also implies that spatiotemporal series of conditions cannot form unconditioned totalities of conditions. With these results in hand, chapter 3 argues that the solution to the mathematical antinomies must be metaphysical rather than purely epistemic. That is, Kant's solution is not *merely* that we cannot know or cognize whether spatiotemporal phenomena are finite or infinite; rather, he holds that spatiotemporal phenomena are *in fact* neither finite nor infinite. In chapter 4, I argue that a metaphysical indeterminacy reading is distinct from and more successful than a reading according to which the antinomies are resolved by appealing to the notion of potential infinity. Finally, in chapter 5, I show how interpreting transcendental idealism as a kind of intentional object phenomenalism can explain how metaphysical indeterminacy in spatiotemporal phenomena results from their mind-dependence.

Introduction

Historically, Kant scholarship has paid less attention to the Transcendental Dialectic than to other parts of the *Critique of Pure Reason*. Many scholars have thought that the important results of the *Critique* are secured in the Transcendental Aesthetic and Transcendental Analytic, and the role of the Dialectic is simply to illustrate how these results undermine the ambitious metaphysical programs of Kant's most prominent predecessors (such as Leibniz, Wolff, and Crusius, to name a few).¹ According to this reading, the Aesthetic and Analytic contain all that is required to show that traditional metaphysics is fundamentally flawed, for they suffice to show that metaphysics must stop at an account of the constitution of our experience. The objects treated in the traditional metaphysical disciplines of rational psychology, rational cosmology, and rational theology—namely, the soul, the world, and God—fall outside a proper "metaphysics of experience" and are thus entirely beyond our cognitive ken.²

In recent years, however, there has been a welcome upswell of work examining the positive arguments Kant makes in the Transcendental Dialectic.³ As scholars now acknowledge, part of Kant's aim in the Dialectic is to mount a comprehensive attack on traditional metaphysics, but he also introduces a number of novel arguments about the nature of human reason, the role of "transcendental illusion" in explaining why we so easily succumb to metaphysical error, and the way in which *ideas* of the soul, the world, and God can guide us in

¹ For example, Paton's classic, two-volume, *Kant's Metaphysic of Experience* (1936), simply ends its discussion after the Transcendental Analytic.

² In the German tradition in which Kant's thinking developed, rational psychology, rational cosmology, and rational theology made up the three disciplines of *special metaphysics* and treated the topics of the soul, the world, and God, respectively.

³ Representatives of this recent upswell include Ameriks (2003), de Boer (2020b), Grier (2004), Proops (forthcoming), Watkins (2019b) and Willaschek (2018).

inquiry even if we cannot have cognition of their objects.⁴ Nonetheless, most scholars continue to hold that Kant's account of the nature of empirical reality (i.e., spatiotemporal phenomena) is complete at the end of the Analytic. That is, most scholars hold that the Transcendental Dialectic does not make any further claims about the metaphysics of spatiotemporal reality and that understanding Kant's account of spatiotemporal phenomena requires only the Transcendental Assthetic and the Transcendental Analytic.⁵

In this dissertation, I attempt to correct this tendency to read too much metaphysical modesty into the Transcendental Dialectic. Taking the mathematical antinomies as my focus, I argue that Kant makes two metaphysically ambitious claims about spatiotemporal reality in the resolution of these two antinomies. First, he argues that the spatiotemporal world is *metaphysically indeterminate* in magnitude rather than either finite or infinite. Second, he argues that material objects are composed from parts that are likewise indeterminate in number rather than either finite or infinite—this too is a claim about what *exists* in spatiotemporal reality (rather than merely a claim about what we can represent) and is therefore an attribution of *metaphysical indeterminacy* to the spatiotemporal world.⁶

I also argue that Kant's treatment of the mathematical antinomies is driven by a controversial assumption about the nature of fundamental reality (that is, things in themselves). Namely, Kant assumes that fundamental reality is a *complete* explanatory order (in a sense to be further elucidated below). He constructs the antinomies on the assumption that his transcendental

⁴ This is Kant's claim that so-called "ideas of reason" have a therapeutic *regulative* use.

⁵ Exceptions to this include Americks (2003), Allais (2015), Jauernig (2021), Marschall (2019), Stratmann (2018), and Watkins (2005, 2019a, and 2019b). As Americks and Watkins show, there are further questions about whether Kant might *also* make metaphysical claims about objects which are *not* a part of spatiotemporal reality, viz., God and the soul. I put aside these questions in this project.

⁶ In chapter 3 below, I comment on how metaphysical indeterminacy differs from other, more familiar kinds of indeterminacy (e.g., epistemic and linguistic indeterminacy) and explain why, even in the context of transcendental idealism, it makes sense to say that spatiotemporal phenomena are "metaphysically" indeterminate.

realist interlocutors embrace this thesis of explanatory completeness for spatiotemporal phenomena (since they take spatiotemporal phenomena to be part of fundamental reality), and the assumption that fundamental reality is explanatorily complete explains why the antinomies are inescapable for transcendental realists. According to Kant, we cannot escape the mathematical antinomies unless we attribute metaphysical indeterminacy to the spatiotemporal world, but this indeterminacy contravenes the principle of explanatory completeness that holds for things in themselves. Therefore, since transcendental realists hold that spatiotemporal phenomena are things in themselves, the antinomies are inevitable for them.⁷

Thus, although it *is* one of Kant's central aims in the Dialectic to show that traditional metaphysicians overstep in claiming knowledge of objects that are in fact beyond our cognitive ken, the mathematical antinomies nonetheless fulfill an important metaphysical ambition for Kant. As Kant argues in their resolutions, the spatiotemporal world is *metaphysically indeterminate* in its magnitude properties, and this metaphysical indeterminacy accounts for *why* we cannot employ purely rational arguments to extend our cognition to the unconditioned objects treated in traditional metaphysics (or so I will argue). That is, on the interpretation I defend in what follows, Kant argues that pure reason cannot extend our cognition to the unconditioned objects treated in traditional metaphysics because the spatiotemporal world is metaphysically constituted so as to make pure reason's principles inapplicable to it. More specifically, its

⁷ Although commentators such as Ameriks (1992 and 2003), Allais (2015), Marschall (2019), Messina (2018), and Watkins (2005) have argued that claims concerning indeterminacy play some important role in Kant's arguments for idealism, I am not aware of any scholars who argue that Kant embraces metaphysically indeterminacy with respect to what *exists* in space and time. Notably, Marschall (2019) goes out of his way to say that Kant's solution to the second antinomy does *not* imply that it is indeterminate what *exists* in space and time. Recently, Jauernig (2021) has argued that the antinomies' resolutions entail the indeterminacy of spatiotemporal reality, but she does not connect these claims to Kant's views on the explanatory completeness of fundamental reality in the same way as do I.

metaphysical indeterminacy accounts for why purely rational arguments such as those presented in the mathematical antinomies fail.⁸

In the course of arguing that Kant makes these metaphysically ambitious claims in his treatment of the mathematical antinomies, I answer three key questions about the first and second antinomies and their role in the Transcendental Dialectic. 1) How should the antinomies inform our understanding of Kant's attack on traditional metaphysics? 2) What account of the spatiotemporal world results from Kant's resolution to the antinomies? 3) How should we understand Kant's claim that the mathematical antinomies provide an *indirect* argument for transcendental idealism?

Part of what makes the mathematical antinomies such fruitful territory for scholarly work is that they make use of three notions that are central to transcendental idealism and the correction it offers to traditional metaphysics. First, they concern Kant's notion of the unconditioned (*das Unbedingte*); in each of the mathematical antinomies, Kant presents arguments purporting to show that unconditioned objects exist in the spatiotemporal world. Second, both of the mathematical antinomies concern Kant's understanding of the infinite (*das Unendliche*) and the various ways in which spatiotemporal phenomena can or cannot be infinite. And finally, the antinomies inform us as to Kant's understanding of the role of the notions of determinacy and indeterminacy in transcendental realist and transcendental idealist systems, respectively.⁹ On the interpretation I advance, Kant argues that the infinite and finite alternatives

⁸ See especially chapters 3 and 4 for the details of this argument.

⁹ The German terms for determinacy (*Bestimmtheit*) and indeterminacy (*Unbestimmtheit*) are used less frequently by Kant than are the terms "infinite", "finite", "conditioned", and "unconditioned". However, they are employed at several key junctures in the antinomies' resolutions, e.g., at A526/B554, where Kant says that the multiplicity of parts in object is "absolutely indeterminate (*schlechthin unbestimmt*)", and at A518/B546, where he says that the world-series treated in the first antinomy cannot be a "determinate infinite (*bestimmtes Unendliches*)" or a "determinate finite (*bestimmtendliches*)". And regardless of Kant's use of terms, my contention will be that the *notions* of determinacy and indeterminacy are key to the antinomies and their resolutions.

presented as exhaustive in the antinomies turn out to be non-exhaustive once we allow for metaphysical indeterminacy in the spatiotemporal world, and it is in fact a consequence of transcendental idealism that indeterminacy of this sort obtains.

Although in what follows I focus on only the first and second antinomies (the antinomies Kant calls "mathematical"), my findings are significant for debates that extend beyond the antinomies and Kant interpretation.¹⁰ First, Kant's treatment of the antinomies expresses interesting and in many respects plausible views about the relationship between intelligibility and indeterminacy. According to Kant, the world could be fully intelligible to us only if complete explanations of its phenomena are possible, and indeterminacy with respect to what exists in space and time suggests that complete explanations of this sort are not available. To put the point in Kant's own terms, complete explanations require the existence of "unconditioned" conditions, and metaphysical indeterminacy suggests that no such unconditioned conditions exist in the world of appearances.¹¹

Second, the interpretation I defend also articulates a connection between indeterminacy and ideality that is worth taking seriously. As I argue, Kant believes we shouldn't immediately assume that everything indeterminate must be ideal, but we should demand an explanation for indeterminacy wherever it does occur. That is, we should require that indeterminacy not be left unexplained, and according to Kant, transcendental idealism is capable of explaining the specific kind of indeterminacy established in the first and second antinomies (namely, indeterminacy

¹⁰ Kant calls the first and second antinomies "mathematical" because the conditioned and is condition are homogenous and the series of conditions is "considered merely in its magnitude" (A528/B556). In contrast, the third and fourth antinomies are "dynamical" because the conditioned and its condition need not be homogenous—the conditioned could be something sensible, while its condition could be something merely intelligible (A530/B558).
¹¹ In the antinomies, Kant uses the terms "appearance" and "spatiotemporal object" interchangeably, and I adopt this same convention in this work. However, as Tolley (2017) shows, there may be good reasons in other contexts to distinguish more carefully between terms.

with respect to the size of the world and the compositional structure of objects). While we needn't follow Kant in thinking that the spatiotemporal world *is* indeterminate in precisely the ways he says it is, the ideas that some forms of indeterminacy require explanation and that mind-dependence (ideality) *could* explain indeterminate phenomena are plausible ones.

Third, Kant's treatment of the mathematical antinomies sheds light on a modest form of rationalism that is compatible with humility concerning our ability to cognize reality as it is in itself.¹² As I have indicated above (and as I will argue below), Kant's treatment of the antinomies proceeds from the assumption that fundamental reality must be explanatorily complete, which is to say that for anything requiring explanation in fundamental reality, its complete explanation is out there to be found (at least in principle). This is a kind of rationalist commitment, since it amounts to the claim that fundamental reality meets a certain intelligibility constraint. However, as Kant also argues, the guarantee that fundamental reality is intelligible in this way does not guarantee that metaphysical explanations are always accessible *to us*. For as Kant argues, knowing that fundamental reality meets an explanatory completeness constraint does not tell us anything about the particular kinds of explanations that are in fundamental reality, and it turns out we cannot have cognition of any things as they are in themselves.¹³ It also articulates only one dimension of intelligibility and leaves open the possibility that fundamental reality fails to

¹² Note: the distinction between cognition (*Erkenntnis*) and knowledge (*Wissen*) is an important one in Kant's theoretical philosophy, and there is evidence that Kant thinks both that cognition does not always amount to knowledge and that knowledge is possible is some cases where cognition is not. See Tolley (2017) and Watkins and Willaschek (2020) for further discussion. In chapter 3, I raise some issues about how attributing to Kant too narrow a conception of cognition may undermine the dialectical effectiveness of the antinomies as an indirect argument for idealism.

¹³ In fact, as I'll make clear below, the humility Kant endorses for things in themselves is even stronger than this. For Kant argues that we know that fundamental reality satisfies the intelligibility constraint according to which all metaphysical explanations are complete explanations (i.e., there are no partial metaphysical explanations in fundamental reality), but he does not rule out the possibility (at least on theoretical grounds) that there are only brute facts in fundamental reality (and so that there are no metaphysical explanations at all). See chapter 3, section 2 for further discussion.

meet other intelligibility constraints.¹⁴ For these reasons, the rationalist commitment driving Kant's treatment of the antinomies is *modest*.

For narrower questions of Kant interpretation, my arguments also have several important upshots. First, the interpretation I offer gives a more charitable reading of Kant's indirect argument for transcendental idealism than do many other available views. Many commentators are pessimistic about Kant's so-called "indirect" argument, arguing either that transcendental idealism is irrelevant to the resolution of the antinomies or that Kant begs the question against transcendental realists by tacitly assuming idealist principles in the antinomial arguments themselves.¹⁵ In contrast, I show that the mathematical antinomies do not presuppose transcendental idealism and that transcendental idealism does, in fact, play an ineliminable role in Kant's solution to the antinomies.¹⁶ Kant believes the conflicts of the antinomies follow from a commitment to complete metaphysical determinacy with respect how many things exists in the spatiotemporal world, and given the assumption that fundamental reality is explanatorily complete (an assumption Kant thinks all parties to the debate should make), transcendental realists must hold that the spatiotemporal world is determinate in this way.¹⁷ For Kant, transcendental idealism is required to explain how the spatiotemporal world could in fact be indeterminate, but it does not play a role in the antinomial arguments themselves.

¹⁵ For the former complaint, see Engelhard (2005, 307) and Strawson (1966, 193). For the latter worry, see Bennett (2016/1974, 120 and 126), Guyer (1987, 386 and 401-6), Smith (1918, 484-6), and Wood (2010, 259-60).
¹⁶ This said, I do not intend to argue that Kant's indirect argument for idealism is fully successful, for one may be able to escape the antinomies by resisting one or more of the steps in the thesis and antithesis arguments.

¹⁴ For instance, as noted in the footnote above, it leaves open the possibility of brute facts in fundamental reality. From this we can see that the Principle of Sufficient Reason (PSR) articulates a different intelligibility constraint than the principle of explanatory completeness driving the antinomies.

¹⁷ I take it almost all of Kant's interlocutors agree with him on this point and hold that fundamental reality must be fully determinate (at least putting aside the kinds of indeterminacy that might result from human freedom). Spinoza might be an exception to this, depending on how his views on the relationship between infinity and determinacy are to be interpreted. I put aside this question of Spinoza interpretation here. More generally, for reasons explained in footnote 56, I take it Kant is not intending to engage with a Spinozist view in the antinomies.

Second, I offer a new account of Kant's understanding of the notion of the unconditioned and its relationship to the notion of totality. In the Transcendental Dialectic, Kant claims that the mistakes of traditional metaphysics result from a misapplication of the notion of the unconditioned to the spatiotemporal world; according to Kant, that notion is in fact applicable only to things in themselves. Kant further indicates that the idea of the unconditioned is coextensive with the idea of a totality of conditions.¹⁸ I distinguish between two different notions of totality employed in Kant's texts and clarify which of these two notions is at the heart of his discussion of the unconditioned. As I argue, this allows us to see more clearly *why* Kant thinks indeterminacy is a barrier to the existence of the unconditioned. If it is indeterminate how many conditions there are for a given conditioned thing, Kant argues, the notion of totality relevant to the *unconditioned* cannot apply.

Third, I offer a correction to a common misconception of Kant's views concerning the infinite and the relationship between the infinite and the indeterminate. Whereas a traditional reading suggests that Kant understands infinite magnitudes as indeterminate, I show that Kant is instead committed to the view that infinite magnitudes are to be *contrasted* with indeterminate ones. This is not merely a rejection of the common view that Kant's core conception of infinity is a conception of potential infinity.¹⁹ Kant embraces a conception of actual infinity as his core conception of the infinite, and he holds that an object has a determinate magnitude if it is actually infinite.²⁰ Nonetheless, Kant does not think that the notion of a potentially infinite magnitude and

¹⁸ As Kant writes, "the **unconditioned** alone makes possible the totality of conditions, and conversely the totality of conditions is always itself unconditioned" (A322/B379).

¹⁹ For expressions of the common and yet mistaken view that Kant rejects actual infinities altogether, see Bennett (2016/1974), Kreis (2015), Radner (1998), Vanzo (2005), and Wartenberg (1992), among others.

²⁰ However, Kant does not allow for infinite *numbers* or cardinalities, so he holds that there is no number that describes the magnitude of an infinite quantity. Though this may sound strange to contemporary ears, there is nothing incoherent in a view like this. For Kant, the fact that there are no infinite numbers follows from his

the notion of an indeterminate magnitude are interchangeable. Instead, Kant holds that attributions of potential infinity and attributions of indeterminacy are compatible but not mutually entailed. This helps us see that even though Kant believes many spatiotemporal phenomena are potentially infinite, it is not the notion of potential infinity that explains how the antinomies are resolved. Rather, an appeal to indeterminacy is at the heart of the mathematical antinomies' resolutions.²¹

Finally, I offer a promising new direction for understanding the doctrine of transcendental idealism. If my interpretation is correct, then the reason Kant thinks the antinomies indirectly prove the ideality of appearances in space and time is that he thinks transcendental idealism (alone) can explain why spatiotemporal phenomena are metaphysically indeterminate. But given this, a constraint on a compelling interpretation of transcendental idealism is that it must explain *how* the ideality of spatiotemporal phenomena accounts for their indeterminacy. That is, beyond the fact that the non-fundamentality of appearances *allows* for their indeterminacy, we must also explain why their transcendental ideality *results* in their indeterminacy.²² And as I argue, Kant gives the following account. If transcendental idealism is true, then appearances exist only to the extent that they are *possibly* represented in experience (*Erfahrung*). But according to Kant, given the nature of our mental faculties, what is possibly represented in experience does not go beyond what is possibly discovered in a *successive* course

conception of number according to which a number must be reachable by counting. This finitistic conception of number does not prevent him from holding that there are infinite multiplicities (as in the case of the natural numbers) or from saying that being infinite and being finite are two ways of having a determinate magnitude. In fact, Kant defines a magnitude as infinite if its measure is strictly *greater* than any *number* of units (no matter what (finite) unit we choose).

 ²¹ Scholars who argue that the notion of potential infinity *is* the key to the antinomies' resolutions include Bennett (2016/1974), Boehm (2011), Chiba (2012), Engelhard (2005), Falkenburg (2000), and Holden (2004).
 ²² One way of thinking about this is that whereas indeterminacy violates the aforementioned requirement of *complete* explanation that Kant thinks must hold for things in themselves, he allows indeterminacy among appearances because it can be explained as a product of their ideality.

of inquiry. And as he argues, successively discovering the world's magnitude properties to be either finite or infinite is *metaphysically* impossible (for reasons to be explained below); hence, the world's magnitude properties must *be* indeterminate rather than either finite or infinite. So according to Kant, the succession-dependence of experience (per transcendental idealism) explains why the world's magnitude properties cannot be finite or infinite and hence why they must be indeterminate. I do not attempt to give conclusive arguments against other metaphysical readings of transcendental idealism, but I suggest that reading Kant as a kind of intentional object phenomenalist fits especially well with this picture.²³ That is, if transcendental idealism is a kind of intentional object phenomenalism, then we can understand why indeterminacy among spatiotemporal phenomena results from their succession-dependence (even when they do not present to us as successive).

The dissertation is structured as follows. In chapter 1, I introduce the mathematical antinomies, explain their role in the Dialectic, and summarize the central arguments of each of the thesis and antithesis positions. I then survey the extant literature on the antinomies' resolutions, distinguishing interpretive options by how they answer two main questions. First, what is the relationship between the notion of *totality* and the notion of *infinity* in the antinomies, and how does this figure in their resolutions? Second, does Kant make a semantic, an epistemic, or a metaphysical claim when he resolves the antinomial conflicts (or some combination of these options)? On the first question, interpreters generally defend one of three theses: (i) Kant argues that spatiotemporal phenomena *are* infinite and concludes that they therefore cannot be totalities; (ii) Kant denies that spatiotemporal phenomena are *actually infinite* on the grounds that they are

²³ At the most general level of description, this form of intentional object phenomenalism holds that appearances are the representational contents of possible experience. See chapter 5 for further discussion.

not totalities; and (iii) Kant denies that spatiotemporal phenomena form totalities for reasons unrelated to his conception of the infinite (or to any claims he makes about the infinite). On the second question, extant readings can be categorized as follows. Semantic readings are either *truth-theoretic* or *referential*, meaning they either understand Kant's solution to the antinomy as part of a broader account of empirical truth or as part of a broader account of what it is for a judgment to refer to an object. Epistemic readings likewise fall into two categories. Idealism as Epistemology readings interpret transcendental idealism as a view in epistemology (from which it follows that the solution to the antinomy is epistemic). Moderate Epistemic readings argue that transcendental idealism may include metaphysical commitments, but the solution to the antinomy nonetheless is furnished by Kant's account of cognition (*Erkenntnis*).²⁴ Finally, extant metaphysical readings interpret the resolution of the antinomy either as an affirmation of Metaphysical Anti-Realism (which rejects experience-transcendent existence in space and time) or as an affirmation of Actual State Phenomenalism (according to which spatiotemporal objects are constructions out of perceivers' actual states).²⁵

Chapter 2 explores in greater detail the question of the relationship between Kant's claims about *totality* and his claims about *infinity* in the antinomies and their resolutions. I begin by examining Kant's pre-Critical writings and argue that he consistently distinguishes between two different notions of totality throughout his career. One notion, which I call the notion of a *totality in the unity sense* (also a *unified totality*) is the notion of a plurality of items brought

²⁴ As readers familiar with Kant's account of cognition (*Erkenntnis*) will know, it is somewhat misleading to suggest that an account that puts claims about *cognition* at the heart of the antinomies' resolutions is *epistemic* rather than *semantic*. For although cognition on Kant's view is often a source of epistemic justification, it also requires *reference* to an object, and this is a semantic issue. Nonetheless, for ease of discussion, I retain the labels I have suggested here and simply grant that Moderate Epistemic readings count as "semantic" in some sense.
²⁵ In chapter 5, I take up the question to what extent Metaphysical Anti-Realism and Intentional Object Phenomenalism overlap (or are at least compatible with one another).

together via real relations of interconnection. Another notion, which I call the notion of a *totality in the completeness sense* (also a *complete totality*) is the notion of a plurality that is not a part of a greater plurality of its kind. Although the notion of a *unified totality* plays some role in the antinomies, I argue that Kant identifies the notion of a *complete totality* as the notion at the heart of the antinomies and their resolutions. That is, Kant holds that pure reason is in pursuit of *complete totalities* when it pursues the unconditioned, and it is the assumption that there are *complete totalities* of conditions among appearances that gives rise to the antinomies. I also argue that Kant has the following view on the relationship between *infinity* and *complete totalities*. According to Kant, complete totalities of conditions can be either finite *or* infinite as a general matter. That is, the fact that a plurality is infinite is not on its own a barrier to its being a complete totality (and likewise a plurality that is finite can be a complete totality). But a plurality that depends for its existence on *succession* cannot form a complete infinite totality.

With a clearer picture of the notion of totality at the heart of the antinomies in hand, I turn in chapter 3 to a discussion of the first antinomy and the reasons for favoring a Metaphysical Indeterminacy reading over a reading that takes the antinomy's resolution to be purely epistemic. I begin by presenting the core elements of a Metaphysical Indeterminacy approach and then discuss weaknesses in two prominent versions of merely epistemic readings. Idealism as Epistemology readings hold that transcendental idealism itself is a purely epistemic doctrine, and the resolution of the antinomy is simply an application of this doctrine to the case of rational cosmology. Moderate Epistemic readings allow that transcendental idealism may be a partly metaphysical doctrine but argue that the solution to the antinomies is nonetheless broadly epistemic (and follows from the account of cognition that Kant develops earlier in the *Critique*). I argue that Idealism as Epistemology readings fail to tell a compelling story about the

relationship between transcendental realism, on the one hand, and the principles of pure reason that contribute to the antinomy, on the other. And Moderate Epistemic readings do not provide compelling explanations of exactly where the thesis and antithesis proofs go wrong and exactly why pure reason cannot extend our cognition via such proofs. In contrast, a Metaphysical Indeterminacy account can explain how reason and transcendental realism jointly contribute to the antinomy, and it can do so in a way that points to a satisfying resolution to the antinomy. Kant resolves the antinomy by claiming that the spatiotemporal world is indeterminate in magnitude rather than either finite or infinite, and this claim concerning indeterminacy underwrites his claim that the thesis and antithesis proofs depend on a false premise. According to Kant, both the thesis and the antithesis proofs depend on the premise that the world in space and time is either finite or infinite, and this premise is in fact entailed by the combination of transcendental realism, according to which appearances are things in themselves, and the "Supreme Principle of Pure Reason," according to which fundamental reality must be a complete explanatory order (in a sense to be explained below). According to Kant, embracing transcendental idealism helps us escape the antinomy because it allows to say that appearances are not part of fundamental reality and hence need not meet the demand for explanatory completeness made in the Supreme Principle. That is, if transcendental idealism is true, then it is false that the spatiotemporal world must be either finite or infinite. And because the spatiotemporal world *does* turn out to be neither finite nor infinite, Kant argues, we can see why cognition of the unconditioned via purely rational proofs is impossible: pure reason infers from the existence of something conditioned to the existence of the *complete totality* of its conditions (and hence to the unconditioned), but because the spatiotemporal world is neither finite nor infinite, *complete totalities* of spatiotemporal conditions do not in fact exist.

In chapter 4, I consider whether a metaphysical reading that centers on the notion of *potential infinity* rather than *indeterminacy* can succeed. Taking the case of the second antinomy as my focus, I argue that a metaphysical indeterminacy reading is more successful than a potential infinity reading and that Kant resolves the second antinomy by endorsing a thesis of *compositional indeterminacy* rather than by claiming that objects' parts are potentially infinite. Once again, this indeterminacy underwrites Kant's assertion that objects' parts do not form *complete totalities* of conditions (and so do not form an *unconditioned* whole). I allow that Kant endorses the claim that objects' parts are potentially infinitely numerous, but as I argue, this claim does not do the important explanatory work in Kant's solution to the antinomy. For Kant, attributions of potential infinity and attributions of indeterminacy are distinct, and attributions of indeterminacy rather than attributions of potential infinity explain why the transcendental idealist (alone) can resolve the antinomy. In the course of arguing for these conclusions, I refute two different versions of a potential infinity approach: first, an approach inspired by an *intuitionist* conception of potential infinity and, second, an approach that takes potential infinity to be a purely modal notion (on which parts are potentially infinite just in case there always could be more). I argue that an intuitionist conception of potential infinity does not fit Kant's own account of the nature of spatiotemporal phenomena. And while a modal account is compatible with Kant's views on the nature of spatiotemporal objects (and may even be the correct articulation of the notion of potential infinity), it does not spell out a notion of potential infinity that can resolve the second antinomial conflict; in particular, it cannot underwrite Kant's claim that the thesis and antithesis statements are *both false*.

In chapter 5, I turn from the question why transcendental idealism *allows* for metaphysical indeterminacy in space and time (since it says appearances are not a part of

fundamental reality) to the question why indeterminacy results from the ideality of appearances. That is, given transcendental idealism, why does it make sense that spatiotemporal phenomena must *in fact* be metaphysically indeterminate in their magnitude properties? As I argue, Kant's own answer to this in the resolution of the antinomies appeals to the fact that appearances are succession-dependent, or, as Kant puts it, to the fact that they "exist in the successive regress" (A506/B534). I suggest we read Kant as follows. Given transcendental idealism, appearances are succession-dependent, but it turns out that both infinite and finite successive regresses are metaphysically *impossible*. While one might think that this forces us to embrace a kind of constructivist interpretation of transcendental idealism according to which the spatiotemporal world is literally constructed out of our mental states through time, I argue that a version of Intentional Object Phenomenalism (IOP) can explain how appearances are succession-dependent without resulting in an unappealing constructivism. Although I do not argue that other readings of transcendental idealism *cannot* explain the succession-dependence of appearances and avoid a radical constructivism concerning spatiotemporal reality, I do argue that IOP is especially wellsuited to explaining the importance of claims about succession in the antinomies' resolutions.

In a concluding chapter, I address two lingering worries that one might have about the specifics of my proposal. One is that there might be asymmetries between the first and second antinomies that my account cannot accommodate. In particular, Kant's discussion of the distinction between regresses that go *in indefinitum* and regresses that go *in infinitum* might undermine my reading. Another worry is that the reading I offer of the mathematical antinomies' resolutions might be incompatible with Kant's claim that the third and fourth antinomies (the dynamical antinomies) admit of a *both true* solution (i.e., that the thesis and antithesis statements are both true). I point towards promising solutions to both of these problems and suggest some

reasons for thinking that *any* reading of the antinomies' resolutions will encounter some version of these objections. I close by summarizing the main contributions my reading makes to Kant scholarship.

Chapter 1: Background and Literature Review

1. The Role of the Antinomies in the Critique of Pure Reason

Although the Antinomy of Pure Reason comes relatively late in the first *Critique*, it plays a crucial role in Kant's arguments for transcendental idealism, his criticisms of traditional metaphysics, and his development of an account of "transcendental illusion" to explain why it is that we are so prone to metaphysical error. Indeed, reflecting on the development of his Critical system in a 1798 letter to Christian Garve, Kant wrote that "[i]t was not the investigation of the existence of God, immortality, and so on, but rather the antinomy of pure reason — 'The world has a beginning, it has no beginning, and so on [...]' — that is what first aroused me from my dogmatic slumber and drove me to the critique of reason itself, in order to resolve the scandal of apparent contradiction of reason with itself" (C, 12:257-8).²⁶

Although I focus in this project on the first and second antinomies only, all four antinomies play a crucial role in Kant's Critical philosophy. They stand at the center of his critique of dogmatic metaphysics and explain the sense in which reason comes into conflict with itself when it is not properly constrained by Critical principles. They elucidate the nature of the "transcendental illusion" to which Kant says we are susceptible, given the nature of human reason. And they provide what Kant calls an "indirect" argument for transcendental idealism (A506/B534). According to Kant, transcendental realism *entails* the contradictions embodied in each of the antinomies, and since contradictions cannot be true, it follows that transcendental

²⁶ Translation modified. I have added an article to the sentence for ease of reading and use "apparent" rather than the Cambridge edition's "ostensible" for the German "*scheinbaren*".

realism must be false.²⁷ Since all other alternatives to transcendental idealism have been ruled out by the time of the antinomies, transcendental idealism is vindicated.²⁸

As to the content of the antinomies themselves, each antinomy concerns a question Kant thinks we are naturally driven to ask in the course of inquiring into the nature of the spatiotemporal world.²⁹ The first antinomy concerns the extent of the world: is the world finite or infinite in space and time? The second antinomy concerns the compositional structure of spatiotemporal objects: are spatiotemporal objects composed from finitely many simple parts, or are they composed of infinitely many parts, none of which are simple?³⁰ The third antinomy concerns the possibility of freedom: is there only natural causality in the world, or can non-natural causality coexist with natural causality? And finally, the fourth antinomy concerns the contingency of existence: is there anything in the world that exists necessarily, or is all existence contingent?

All four of the antinomies are also structured in a similar manner. Each presents two sets of arguments leading to conclusions that contradict one another (expressed in the thesis and

²⁷ Allison (2004, 384), Grier (2004, 181-2), and Wood (2010, 258) all say that only the mathematical antinomies supply an indirect proof of idealism. I see no reason why this should be true and no evidence that Kant subscribes to this position. If each antinomy embodies a contradiction, and if each antinomy follows from transcendental realism, each antinomy should be able to indirectly prove transcendental idealism.

²⁸ Note that there is some controversy with respect to whether transcendental idealism and transcendental realism are exhaustive options. Commentators such as Allision (2004) argue that they are, while commentators such as Jauernig (2021) and Willaschek (2018) argue that they are not. Here I do not want to argue for either position but rather want to point out that both "dogmatic idealism" and "problematic idealism" are ruled out prior to the antinomies, according to Kant (B274-5). So we can put aside whether or not those other forms of idealism are ultimately versions of transcendental realism.

²⁹ Kant claims that there must be exactly four antinomies because there are exactly four categories that give rise to the notion of a series of conditions (A415-16/B442-3). But the relationship between the table of categories and the antinomies is not completely clear, and it may be more plausible to think of the four questions treated in the antinomies as simply arising from our interactions with the world. That is, perhaps we simply encounter various features of the world that require explanation (conditioned things), and the four antinomies correspond to four questions our experiences of such things prompt to ask: How big is the world? Is matter infinitely divided? Is there freedom? And is there a necessary existent?

³⁰ Kant's reasons for ruling out the possibility that there are infinitely many simple parts are explained below.

antithesis statements, respectively), and in each case, Kant says that the arguments presented in the thesis and antithesis *would* be compelling if transcendental realism were true. That is, if transcendental realism were true, we would be forced to endorse four different contradictions, one corresponding to each of the four antinomies. We would conclude that the world is both finite and infinite, that objects both have and lack simple parts, that there is and isn't causality through freedom, and that something necessary both does and doesn't exist.

In this project I focus on the mathematical antinomies, which have special significance for understanding Kant's mature system for two reasons.³¹ First, although all four antinomies demonstrate that transcendental realism is contradictory (according to Kant), Kant's solution to the mathematical antinomies has especially puzzling consequences for his account of the empirical world. According to Kant, the way to resolve the first and second antinomies is to claim that the thesis and antithesis statements are both false (see A504-5/B532-3 and Prol 4:341). That is, on Kant's preferred solution to the mathematical antinomies, the world is neither finite nor infinite in spatiotemporal extent, and objects are composed of neither finitely many nor infinitely many parts. How could this be? What does it mean to say that the spatiotemporal world is neither finite nor infinite in extent, and what does it mean to say that objects have neither finitely many nor infinitely many parts? Given that the finite and infinite alternatives Kant presents in the antinomies *appear* exhaustive, it is not immediately clear what third alternative could remain. In contrast, the dynamical antinomies do not present this puzzle, since Kant says their resolutions consists in the assertion that the thesis and antithesis statements can both be *true* (once one is taken to hold for things in themselves and the other for appearances) (A531-2/B560-

³¹ Recall that the mathematical antinomies are ones in which "we ha[ve] no **object** other than one in appearance," whereas the dynamical antinomies "allow[] a further condition different in kind, one that is not a part of the series but, as merely intelligible, lies outside the series" (A529-31/B557-8).

1 and A537/B565). Among appearances, there is only natural causality and contingent existence, but at the level of things in themselves, both freedom and a necessary being are possible.³²

Second, the mathematical antinomies also have special significance insofar as their resolution supports Kant's claim that nothing unconditioned exists in the spatiotemporal world. For as Kant understands the positions presented in the thesis and antithesis statements of the first and second antinomies, each argument articulates a way in which the unconditioned might occur in empirical reality. In the thesis statements, the unconditioned occurs in the terminal member of a finite series of spatiotemporal conditions; in the antithesis statements, the unconditioned occurs in the entirety of an infinite series of such conditions. Kant's claim that the thesis and antithesis statements are both false then supports his claim that the unconditioned does not exist in either of these two ways among appearances. That is, in saying (a) that the world is neither finite nor infinite in spatiotemporal extent and (b) that objects are composed neither from finitely many simples nor from infinitely many composite parts, Kant rules out the possibility of things that are unconditioned (with respect to the conditioning relations treated in the first and second antinomies). This is significant, for given this, we can conclude that Kant's views concerning the conditioned and the unconditioned and his "both false" solution to the mathematical antinomies are closely linked; the "both false" solution supports his claim that nothing unconditioned exists in space and time, and his claim that nothing unconditioned exists in space and time requires the "both false" solution.³³

³² This is not to say that there are no puzzles concerning the resolutions of the third and fourth antinomies, but the puzzles are of a different kind. There are also puzzles concerning the relationship between the *both false* and *both true* solutions, which I address briefly in the dissertation's concluding chapter.

³³ Note: some commentators would object to my suggestion that the antinomies resolutions bear on what *exists* in space and time. I provide an argument for this aspect of my interpretation in the chapters that follow.

This point about the unconditioned raises one further issue that requires discussion before we can turn to the antinomial arguments themselves. What exactly is the notion of the *unconditioned (das Unbedingte)*, and what is it for something to be *conditioned (bedingt)*? The notions of the conditioned and the unconditioned are at the heart of the Transcendental Dialectic, so some brief explanatory remarks are in order.

As we have seen above, one of Kant's central aims in the Dialectic is to reveal the errors of traditional metaphysics, and on Kant's account, the source of many of these errors is a misuse of the faculty of *reason*. As Kant sees it, reason is a faculty that drives us to seek complete explanations, and the idea of an *unconditioned* object is the idea of an object that would satisfy these explanatory ambitions. That is, it is the idea of an object that explains others but requires no further explanation of its own.³⁴ For example, one might consider God an unconditioned object if one thought of God as explaining other things in the world but as not requiring any further explanation itself. And as a general matter, Kant wants to show throughout the Dialectic that we *too naively* assume that objects fulfilling reason's demand for complete explanations exist in the world and are accessible to us. This naïve assumption occasions error in metaphysics, and the Paralogisms, Antinomies, and Ideal each show how these errors play out in the three main disciplines of special metaphysics: rational psychology illicitly claims cognition of the soul, rational cosmology illicitly claims cognition of the world, and rational theology illicitly claims cognition of God.³⁵

³⁴ Exactly what it means to require explanation is a difficult issue. One might think, for instance, that an object requires explanation only if there is in fact something else in the world that explains it. But alternatively, one could argue that something can require an explanation even when there is no explanation out there to be found. Willaschek (2018, 213) advocates the latter view, whereas I advocate the former.

³⁵ See A334-5/B391-2 for Kant's explanation of the relation between the three parts of the Dialectic and psychology, cosmology, and theology.
Alongside this conception of the unconditioned and its relationship to the faculty of reason, Kant also develops an account of *conditioning*, which he divides into "logical" and "real" conditioning.³⁶ Both logical and real conditioning pertain to explanation, and at the highest level of generality, the terms "conditions" and "conditioned" refer to things that provide explanations and things that are explained, respectively.³⁷ The difference between logical and real conditioning relations is then accounted for by the difference between the kinds of entities that stand in explanatory relationships. *Logical* conditioning relations hold between premises and conclusions of arguments (or between *judgments*, broadly speaking). *Real* conditioning relations hold between *objects*, where "object" is loosely construed so as to include a variety of material and immaterial entities and even space and time themselves. Thus, the major premise of a syllogism is a logical condition of its conclusion, but material parts are real conditions of the objects they compose.

On this conception of conditions and conditioning, the mathematical antinomies concern real conditions and real conditioning relations. That is, they concern worldly entities and the relations of explanatory dependence that hold between them. The first antinomy inquires into the magnitude of the whole world by asking about how many spatiotemporal objects *bound* others

³⁶ In the *Critique*, the distinction between real and logical conditioning relations is given in Kant's discussion of the logical and real uses of reason in the introduction the Transcendental Dialectic (A303-9/B359-66). My understanding of these distinctions follows Watkins (2016, 2019a, and 2019b) and Willaschek (2018). ³⁷ Note: Willaschek (2018) understands the term 'conditioned' such that something could count as conditioned despite the fact that nothing actually explains it. This is possible, according to Willaschek, because something is conditioned just in case we can intelligibly *ask* for an explanation of it, and on his view, we can *always* intelligibly ask for further explanations of empirical things (regardless of whether those explanations are anywhere to be found) (213). In my view, however, an approach such as this gets us far too quickly to the conclusion that no objects of cognition are unconditioned. For if the conditioned is that of which we can intelligibly *ask* for an explanation, and if *everything* can be subject to this demand, then it follows trivially that nothing is unconditioned. As I see it, Kant owes transcendental realists a substantive argument for the conclusion that no objects of cognition are unconditioned, and so he cannot simply point to his conviction that it always makes sense to ask for further explanations of the things we encounter in experience.

(where the bounding relation determines the spatiotemporal extent of the world).³⁸ The second antinomy inquires into the compositional structure of matter by asking how many material parts compose individual objects in space and time. If objects are composed of only finitely many parts, then they are composed of simple parts; if they are composed of infinitely many parts such that each part is divided in turn, then no such simple parts exist in space and time.

Finally, it is important to stress that Kant sets up the antinomies by making a relatively novel claim (at least for his time) about the kinds of worldly scenarios that can satisfy reason's demand for complete explanations. As we have seen, Kant argues that *unconditioned* conditions provide complete explanations, but he does *not* argue that an explanatory series must be *finite* to provide a complete explanation. Instead, Kant says that complete explanations can occur in either finite or infinite explanatory series.³⁹ Consider Kant's explanation of the way in which reason's demand for complete explanations leads to the antinomies. First, Kant says that reason demands complete explanations according to the following principle (which Kant calls the "Supreme Principle of Pure Reason":

<u>Supreme Principle</u>: When the conditioned is given, then so is the whole series of conditions subordinated one to the other, which is itself unconditioned, also given (i.e., contained in the object and its connection). (A308-9/B364)

³⁸ In the case of the first antinomy, providing a precise characterization of the relevant conditioning relation is difficult. One problem is that the antinomy treats the *spatial* and *temporal* dimensions of the world's magnitude separately, and so there are really two conditioning relations at issue rather than just one. Another problem is that it is not obvious for either the spatial or the temporal bounding relation exactly how it should be characterized. For example, in the spatial case, is the series of conditions comprised of concentric spatial boundaries such that there is just one series of conditions? Or are there instead numerous series of spatially conditioned things, which together determine the spatial boundary of the world? See Stratmann (2018) and Wood (2010) for further discussion. Fortunately, these details do not affect the substance of my argument, for regardless of the precise nature of the conditioning relations, it is clear that Kant thinks the length of the series of conditions can be taken as a proxy for the magnitude of the whole world (such that the world is infinite in spatiotemporal extent if and only if the series of conditions is/are infinite).

³⁹ Bird (2006) seems to assume (mistakenly, in my view) that Kant thinks the Supreme Principle requires a finite series of conditions (676).

Second, Kant says that the entire Antinomy of Pure Reason depends on the application of this principle in the following dialectical argument: "If the conditioned is given, then the whole series of all conditions for it is also given; now objects of the senses are given as conditioned; consequently, etc." (A497/B525). Third, Kant says that reason can think of the unconditioned as existing *either* in a finite series of conditions, in which the ultimate member is unconditioned, *or* in an infinite series of conditions, in which only the whole series is unconditioned.

Kant puts this as follows in the first section of the antinomy, where he introduces the system of cosmological ideas. Because reason's ultimate aim is complete explanation, Kant argues, reason "has as its final intent the **unconditioned**" (A417/B445). But even if we assume that the world answers to reason's explanatory demands (i.e., even if we assume it answers to the above Supreme Principle), still this does not determine whether the explanatory series satisfying reason's needs will be finite or infinite. For according to Kant,

[O]ne can think of this unconditioned either as subsisting merely in the whole series, in which thus every member without exception is conditioned, and only their whole is absolutely unconditioned, and then the regress is called infinite; or else the absolutely unconditioned is only a part of the series, to which the remaining members of the series are subordinated but that itself stands under no other condition. In the first case the series is given *a parte priori* without bounds (without a beginning), i.e., it is given as infinite and at the same time whole, but the regress in it is never complete and can be called only *potentialiter* infinite. In the second case there is a first [member] in the series, which in regard to past time is called the **beginning of the world**, in regard to space the **boundary of the world**, in regard to the parts of a whole given in its bounds the **simple**, in regard to causes absolute **self-activity** (freedom), in regard to the existence of alterable things absolute natural necessity. (A417-18/B445-46)⁴⁰

In the first kind of case, the unconditioned occurs in only one individual (say, an unconditioned

first state of the world); in the second kind of case, the unconditioned occurs in the whole

⁴⁰ Translation slightly modified. The Cambridge edition simply omits the phrase "and then the regress is called infinite (*und dann heiβt der Regressus unendlich*)".

(infinite) series of conditions.⁴¹ And as Kant explains it in a footnote attached to the above paragraph, the reason an infinite series of conditions can itself count as unconditioned is as follows: "the absolute whole of the series of conditions for a given conditioned is always unconditioned, because outside it there are no more conditions regarding which it could be conditioned" (A417-18/B445 fn). That is to say, we can conceive of an infinite series of conditions as containing *all* the conditions of a given conditioned thing, which is to say that we can conceive of it as having left no conditions out of the series. But if there are no conditions left out of the series, then the whole series must be unconditioned.

Thus, against many in the philosophical tradition who argue that infinite series cannot furnish complete explanations, Kant holds that infinite series of conditions are not explanatorily unsatisfying, at least not as far as the faculty of reason is concerned.⁴² From the perspective of pure reason, an infinite series can provide a complete explanation just as well as can a finite one; so by knowing that a particular conditioned object is *completely* explained by the series of its conditions, we do not yet know whether its series of conditions is finite or infinite. In the case of the first and second antinomies, this means that assuming the spatiotemporal world contains *unconditioned* conditions does not yet settle whether the series of conditions we find in the spatiotemporal world are finite or infinite. If we assume the truth of the Supreme Principle for

⁴¹ For now, I remain neutral as to whether a *series of conditions* is anything over and above its members (though I will return to this issue in chapter 2 below).

⁴² Arguments against infinite explanatory regresses in the historical tradition can be found in al-Ghāzāli, Aquinas, Aristotle, Baumgarten, Crusius, and Leibniz, among others (though in some cases it is not clear that *all* infinite explanatory series are to be rejected). Note also that although Kant believes the faculty of *reason* recognizes finite *and* infinite series as capable of providing complete explanations, he may still think that infinite series are explanatorily deficient along other dimensions. For example, in section 3 of the antinomies, he suggests that a finite series satisfies a certain *practical* interest of reason, while an infinite series satisfies a certain *empiricist* interest. As I read Kant, however, he is distinguishing here between different kinds of explanatory needs and is not undermining his overarching claim that pure reason finds infinite and finite series equally capable of providing complete explanations.

the conditions treated in the first and second antinomies, we can infer that the relevant series of conditions must be either finite or infinite, but we cannot infer which.

2. The Arguments of the Mathematical Antinomies

With this setup in place, we are now in a position to present the particular arguments that make up the first and second antinomies. As stated above, Kant holds that the antinomies arise when we apply the Supreme Principle to the spatiotemporal world via a "dialectical" argument with the following general structure:

- 1) If the conditioned is given, then the whole series of its conditions and hence the unconditioned is also given.
- 2) Conditioned objects are given in the spatiotemporal world.
- Therefore, the whole series of their conditions and hence the unconditioned is also given in the spatiotemporal world.⁴³

As we have also seen, however, Kant holds that this argument leaves it open whether the unconditioned takes a finite or infinite form; it establishes that the series of conditions must be *either* finite *or* infinite (since the unconditioned can be conceived in only these two ways when the conditioned and its condition form a series), but it does not establish which in fact obtains. And in the mathematical antinomies, it turns out that the thesis and antithesis statements each advocate for one of these two alternatives. The thesis arguments endorse the finite alternatives, i.e., they claim that the unconditioned occurs in the terminal member of a finite series of

⁴³ Because Kant uses the term "given" both in presenting the Supreme Principle and in articulating the dialectical argument on which the Antinomy depends, I do the same. But we can understand "given" to have existential important in the context of the antinomies (where the disputes concern what there is in the spatiotemporal world and not, in the first instance, what can be presented to the mind). I address a reason for resisting this reading of the meaning of "given" in the antinomies in chapter 3 below.

conditions. The antithesis arguments endorse the infinite alternatives, i.e., they claim that unconditioned occurs in an infinite series of conditions.

The more specific positions in each of the antinomies then translate to the following. In the first antinomy, the thesis position argues that there is an unconditioned first state of the world and an unconditioned outmost spatial boundary; the antithesis position argues that the world is infinite with regard to both space and time (and so has no first state and no outermost boundary)—it is unconditioned only as a whole. In the second antinomy, the thesis position argues that objects are divisible into unconditioned simples through finitely many steps; the antithesis position argues that objects have infinitely many parts, and every part is divided into further ones—only the entire infinite series of its parts is unconditioned.⁴⁴ Thus, the antinomies together present the following conflicting claims concerning the features of the world that are unconditioned:

Antithesis: "The world has no beginning and		
no bounds in space, but is infinite with regard		
to both space and time" (A427/B455).		
What is unconditioned?		
The entire infinite series of past world-states		
and the entire infinite series of objects		
extending out in space.		

FIRST ANTINOMY

SECOND ANTINOMY

⁴⁴ Talk of a "series" of parts in the second antinomy may be somewhat unnatural, but Kant's general line of thinking is as follows. Take an object *o* and divide it into two parts. Then take one of those parts and make a further divide. Continue in this manner, either reaching simples or carrying on without end. The series of conditions is then the nested sequence of parts reached through this process of division. Further discussion of the second antinomy can be found in chapter 4 below.

<u>Thesis</u>: "Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of simples" (A434/B462).

<u>What is unconditioned?</u> The simple parts of objects. <u>Antithesis</u>: "No composite thing in the world consists of simple parts, and nowhere in it does there exist anything simple" (A435/B463).

<u>What is unconditioned</u>? The entire infinite series of parts into which an object is divided

As for the arguments that make up the proofs of the thesis and antithesis statements, each proceeds by combining a disjunctive syllogism with a reductio argument. As a first step, each argument assumes that the finite and infinite alternatives exhaust the options. As we have seen, this follows from assuming that the Supreme Principle is true for the spatiotemporal world and hence that either a finite or infinite series of conditions must exist for each conditioned spatiotemporal thing.⁴⁵ Then, the arguments assume (for reductio) that the disjunct embraced by the opponent's position obtains. And finally, they reason to an absurd or contradictory result from this assumption, thereby licensing the conclusions they set out to establish. In the first antinomy, this goes as follows:

<u>Thesis</u> : "The world has a beginning in time,	<u>Antithesis</u> : "The world has no beginning and	
and in space it is also enclosed in	no bounds in space, but is infinite with	
boundaries" (A426/B454).	regard to both space and time" (A427/B455).	
 Either the world is finite in space and	 Either the world is finite in space and	
time or it is infinite in space and time.	time or it is infinite in space and time.	
(DERIVED FROM THE SUPREME	(DERIVED FROM THE SUPREME	
PRINCIPLE)	PRINCIPLE)	

FIRST ANTINOMY

⁴⁵ One might wonder why the assumption that a series of conditions must be either finite or infinite needs any justification, since it looks like a trivial truth. As we will see, Kant's position is that a series of conditions could have an indeterminate magnitude and hence could be neither finite nor infinite. Hence, it is not a trivial truth after all.

- 2. The world is infinite in space and time. (ASSUMPTION FOR REDUCTIO)
- 3. If the world is infinite in space and time, then an infinite successive synthesis can be completed.
- 4. An infinite successive synthesis cannot be completed.
- 5. The world is not infinite in space and time (from 2-4).
- 6. Therefore, the world is finite in space and time (from 1 and 5).

- 2. The world is finite in space and time. (ASSUMPTION FOR REDUCTIO)
- 3. Then it must be possible for something to arise out of an empty time and stand in a relationship to an empty space.
- 4. It is impossible for anything to arise out of an empty time or stand in a relationship to an empty space.
- 5. The world is not finite in space and time (from 2-4).
- 6. Therefore, the world is infinite in space and time (from 1 and 5).

Two comments about these arguments are in order. First, notice that although the notion of a *series of conditions* does not appear explicitly in the reconstructions offered above, the antinomy *does* treat conditions that form series, and the arguments could be re-written (in a more cumbersome form) so as to make this explicit.⁴⁶ Consider the following intuitive way of understanding how the antinomy arises and the way in it concerns a series of conditions. In ordinary experience, we begin with the current state of the world, and we experience it as conditioned in two ways. First, we experience the current state as *temporally* conditioned by a previous state, which is temporally conditioned by a previous one before it, and so on. Thus, we are naturally led to ask how far back these states temporally conditioning one another go, i.e., how many past states are in the series of past world-states.⁴⁷ Second, we also experience the

⁴⁶ Indeed, a defining feature of the antinomies (in contrast to the paralogisms and the ideal of pure reason) is that they concern "the transcendental concept of absolute totality in the *series* of conditions for a given appearance" (A340/B398, my emphasis).

⁴⁷ Note that it seems to be assumed in the argument that the series of past states is discrete rather than continuous and that a series with infinitely many members would yield a temporally infinite world (i.e., one with an infinite past history). I propose that we simply accept this assumption here, since it seems to be a built-in feature of the temporal conditioning relation Kant is interested in. However, I'll note that this interacts in interesting ways with Kant's claim that space and time themselves are continuous magnitudes.

world in its current state as *spatially* conditioned in the sense that objects in our current location are bounded by further objects beyond them, which are bounded by even more remote objects, and so on. So, similarly, we are led to ask how far out in space these series of objects extend. If from our current location, there is a series of world-states extending infinitely into the past and a series of objects extending infinitely far into space, then the world is infinite. In contrast, if these series are instead finite, then the world has a first beginning and a spatial boundary (that is, it is as the thesis position describes it to be). So, although the reconstructions above do not explicitly refer to the notion of a *series of conditions*, the thesis argument in fact attempts to show that an infinite series is impossible (and so the relevant series must be finite), while the antithesis argument attempts to show that a finite series is impossible (and so the relevant series must be finite).⁴⁸

A second issue concerns the plausibility of the thesis and antithesis arguments themselves. Are they sufficiently compelling to support the indirect argument for transcendental idealism that Kant says the antinomy provides (A506/B534)? Here one might raise two separate worries. First, one might worry that the arguments smuggle in idealist assumptions such that transcendental realists can simply reject them out of hand. Second, one might worry that even if the arguments do not smuggle in transcendental idealism, still they can be resisted by transcendental realists. If either of these worries turns out to be well-founded, then the indirect

⁴⁸ As the thesis argument claims, an infinite series would require an impossible infinite successive synthesis. As the antithesis argument claims, a finite series would require an impossible first member—in the temporal case, a state that arises out of "an empty time" (A427/B455) and, in the spatial case, an object that is bounded by empty space and hence stands in a relation to something that "is nothing" (A429/B457).

argument for idealism fails, and transcendental realism is not provably contradictory after all (at least not in the first antinomy).⁴⁹

Although I think the arguments *can* in fact be resisted, there are two reasons for thinking that *if* they fail, it is not because they presuppose transcendental idealism.⁵⁰ First, on the thesis side, the main reason for thinking that the argument *does* presuppose idealism is the following. Premises 3 states that an infinite world would require an infinite successive synthesis, and premise 4 says that an infinite successive synthesis is impossible. But one might worry that "synthesis" must be understood here as a mental operation performed by finite minds, and transcendental realists would have no reason to say that the actual extent of the world depends on a synthesis of this sort.⁵¹ Fortunately, however, in the Dialectic, Kant use the term "synthesis" in both mentalized and non-mentalized ways. On a non-mentalized reading, "synthesis" can be used to denote any kind of combination of items or objects. And in the antinomy, this combination is brought about via the real conditioning relations under consideration, not via minds. Moreover, Kant indicates elsewhere in his theoretical writings that an infinite *successive* synthesis is *metaphysically* impossible and not simply impossible for finite minds to carry out; even God

⁴⁹ Commentators who claim that thesis argument of the first antinomy presupposes idealism include Bennett (2016/1974, 126), Guyer (1987, 386 and 401-3), and Smith (1918, 484-6), among others.

⁵⁰ Wood (2010) remarks that it is "probably asking too much" to ask that the indirect argument for idealism be fully successful, given the complexity of the issues tackled in each antinomy (246). I agree with Wood on this issue and am primarily interested in showing (a) that if they fail, it is not because they presuppose idealism and (b) that the thesis and antithesis statements really do exhaust the alternatives for transcendental realists.

⁵¹ See Smith (1918), who thinks "synthesis" must refer to a kind of "mental apprehension" (485). Similarly, Guyer (1987) says that "it is obvious that these arguments turn on purely epistemological conclusions, that is, on the claims that it is possible to *represent* or, by means of sense, *confirm* the existence of infinite past time or infinite space" (407). Russell (1914/2009) mounts a similar criticism (126).

could not completely combine infinitely many items via *successive* synthesis.⁵² Thus, the thesis argument does not presuppose transcendental idealism (initial appearances to the contrary).

The antithesis argument also does not presuppose transcendental idealism, even if it turns on premises that one could reasonably resist. The antithesis argument says that the thought that the world has a first state and outermost boundary leads to absurdity. According to the argument, a first state of the world would have to be preceded by an empty time, but an empty time contains nothing that could help explain why the first state arose when it did. And an outermost spatial boundary would imply a relationship between the world and empty space, which is impossible since empty space is not an object. Clearly, both of these claims are controversial—the argument concerning time seems to assume a Leibnizian Principle of Sufficient Reason, and the argument concerning space seems to assume the falsity of a Newtonian substantivalist position)—but neither presupposes transcendental idealism in an obvious way.⁵³

What about the second antinomy? How do its arguments proceed? We can summarize the thesis and antithesis arguments of the second antinomy as follows:

⁵² I discuss this point in greater detail in chapter 2 below. In the secondary literature, scholars who have criticized Kant's argument against the possibility of an infinite successive syntheses include Strawson (1966, 176-178), Bennett (2016/1974, 199) and, much earlier, Maimon (1794, 211-13). In my "Kant on the Possibility of Actually Infinite Aggregates" (ms), I argue that Kant's position is more compelling than many of these scholars take it to be, but at the same time, the thesis argument concerning the world's *spatial* extent is less successful than the argument concerning its temporal extent (since transcendental realists presumably have little reason to hold that *spatial* magnitude itself depends on *successive* synthesis).

⁵³ Al-Azm (1982) argues that the antithesis argument in the first antinomy is a broadly Leibnizian argument and that the thesis argument is a Newtonian one. Guyer (1987, 408) agrees that the antithesis position describes a Leibnizian view but worries that the arguments themselves depend on epistemological claims to which Kant is not in fact entitled. However, an alternative reading is to see the proponent of the thesis argument as making conceptual claims about relations in the spatial part of the argument and as relying on a version of the PSR that was widely accepted in the temporal part. It also bears emphasizing here that antithesis argument does not follow from the Supreme Principle alone (and that the PSR is not the same as the Supreme Principle). Premise 4 in the thesis argument can be read as a denial that an unconditioned condition in a finite series of conditions is acceptable (since it would leave the terminal, unconditioned condition unexplained), while the Supreme Principle allows for unconditioned conditions and does *not* express a general demand that everything be conditioned (or explained).

SECOND ANTINOMY

<u>Thesis</u> : "Every composite substance in the world consists of simple parts, and nothing exists anywhere except the simple or what is composed of simples" (A434/B462).		<u>Antithesis</u> : "No composite thing in the world consists of simple parts, and nowhere in it does there exist anything simple" (A435/B463).	
1.	Objects are composed either of finitely many parts or of infinitely many parts. (DERIVED FROM THE SUPREME PRINCIPLE)	1.	Objects are composed either of finitely many parts or of infinitely many parts. (DERIVED FROM THE SUPREME PRINCIPLE)
2.	Objects are not composed of finitely many parts. (ASSUMPTION FOR REDUCTIO)	2.	Objects are not composed of infinitely many parts. (ASSUMPTION FOR
3.	If objects are not composed of finitely many parts, then there are no simple parts.	3.	REDUCTIO) If objects are not composed of infinitely many parts, then they are
4.	Composition is a contingent relation, and so it is possible to remove (<i>aufheben</i>) the relation of composition from a	4.	composed of simple parts. If objects are composed of simple parts, then a simple part occupies a space.
5.	composite object. If objects are not composed of simple	5.	A simple part occupies a space. (from 2-4)
	parts, then when all composition is removed, nothing remains.	6.	Everything that occupies a space contains a manifold of elements
6.	If nothing remains when composition is removed from an object, then the object was composed from nothing.		external to one another (<i>ein außerhal</i> <i>einander befindliches Mannigfaltiges</i> <i>in sich faßt</i>) and so is composite.
7.	Objects are composed from nothing. (from 2-6)	7.	A simple part is composite. (from 5 and 6)
8.	An object cannot be composed from nothing.	8.	Objects are not composed of finitely many parts. (from 2-7)
9.	Objects are not composed of infinitely many parts. (from 2-9)	9.	9. Therefore, objects are composed of infinitely many parts; nowhere in the world does there exist anything simple. (from 1 and 8) ⁵⁴
10.	Therefore, objects are composed of finitely many parts; nothing exists anywhere except the simple or what is composed of simples. (from 1 and 9)		

⁵⁴ Note: the antithesis proof in fact contains a further argument, which apparently attempts to show that there also cannot be isolated simples that do *not* occupy a spatial extent (and so contain *no* "manifold of elements external to one another"). The argument turns on the claim that such a simple could never be given in experience, which obviously raises questions as to exactly which transcendental realists would endorse empiricist constraints of this sort.

As in the case of the first antinomy, several features of these arguments are worthy of comment. First, what is the relationship between questions about *simplicity* and questions about the *number* of parts objects have such that objects have simple parts only if their parts are finite in number? Second, are the more specific controversial premises (such as 5 and 6 in the thesis argument and 6 in the antithesis argument) justifiable from the point of view of transcendental realism?

Taking these issues in turn, notice that one could imagine an object composed of infinitely many simples; this possibility seems to undermine premise 3 of the thesis argument, which says that objects are composed of simple parts only if their parts are finite in number.⁵⁵ However, recall that Kant presents the antinomies as arising from an application of the Supreme Principle to the spatiotemporal world. In the case of the second antinomy, this occurs as follows. We encounter composite objects in experience, and these objects are given to us as compositionally *conditioned*; that is, they are given as depending on the parts that compose them.⁵⁶ As Kant describes the sense in which spatiotemporal objects are compositionally conditioned; that is, is likewise something conditioned, whose inner conditions are its parts, and the parts of those parts are the remote conditions, so that there occurs

⁵⁵ A related worry concerns Kant's claim that the second antinomy corresponds to the category of *quality* rather than to the category of *quantity* (where the latter is the category corresponding to the first antinomy). More specifically, Kant suggests that it pertains to *reality* in space. As one might argue, this must mean it pertains to *intensive* magnitude and so has nothing to do with how many discrete elements are contained in a series of conditions. However, there is no doubt that, on the whole, Kant thinks that both of the mathematical antinomies address questions that can be translated into questions about how many elements are in the relevant series of conditions. Indeed, as Kant writes in explaining why the mathematical antinomies must concern *homogenous* conditions, "The series of conditions are obviously all homogenous to the extent that one looks solely at how far they **reach**" (A530/B558). In other words, because in the mathematical antinomies one is concerned with how many conditions are in the series (i.e., how far the series reaches), the series must be composed of discrete conditions that are homogenous with one another.

⁵⁶ Another way of putting this is that the objects treated in the first and second antinomies are not holistically structured (as Spinoza would argue the world is). Note that it is also for this reason that I do not think the proponent of the antithesis position needs to show that isolated simples cannot exist; the conditioned object with which the antinomy begins is always *composite* (see footnote 54 above).

here a regressive synthesis, whose absolute totality reason demands" (A413/B440). If we assume that no object or part has infinitely many *non*-remote parts, i.e., that infinitely many parts would require an infinite "regressive synthesis", then it follows that the length of the regressive synthesis corresponds to the length of the series of compositional conditions for an object, and an object resolves into simples only if it has finitely many parts.⁵⁷

What about the other controversial premises in each argument? What justifies premises 5 and 6 in the thesis argument and premise 6 in the antithesis argument? The argument on the thesis side appears to be especially weak, for it seems that even if one grants that composition is a contingent relation, one need not admit that the absence of simples would mean that something is composed from *nothing* (which would be absurd). For from the assumption that composition is a contingent relation, it *does* follow that the parts of a composite object can exist without being parts of *that* object, but it does *not* follow that something must remain when all composition is removed; if objects' parts are composite all the way down, then nothing remains when all composition is removed, but still each individual part or object is composed from something (namely, its immediate parts, which are themselves composite). So, it seems the correct conclusion to draw is that although every part in every object *could* have failed to compose anything further (this is the contingency of composition), objects consisting exclusively of composite parts *can* in fact exist given that all composition is not in fact removed. Or to put the point in terms of the notion of a series of conditions, each condition could have failed to condition a further condition (the contingency of the conditioning relation), but there is nothing

⁵⁷ In fact, one must also assume here that every series of decomposition in an object is finite if any of them are so that, no matter what path one takes through the regressive synthesis, the length of the series will turn out the same. As to why Kant assumes that no objects are given as having infinitely many parts at the macro-level (or at any subsequent individual level), I leave this question open.

incoherent in the idea that the whole series exists only insofar as each condition is conditioned by something further.⁵⁸

Despite the problems with the thesis argument, notice that the issues discussed above do not stem from a tacit assumption of idealism, and the argument is plausibly one that transcendental realists of Kant's day would have accepted.⁵⁹ Similar points hold for the controversial claims in the antithesis argument. One might worry that a transcendental realist could reject premise 6 and say that something simple *can* occupy a space (e.g., via its field of force, as Kant himself argued in the *Physical Monadology* of 1756). But as Michael Radner (1998) has noted, most prominent metaphysicians in Kant's day held that the science of geometry reveals the properties of bodies such that if space is infinitely divisible then so too must be body.⁶⁰ Moreover, Kant himself provides an argument against the possibility of undivided spatial simples in the Metaphysical Foundations of Natural Science (1786), which attempts to show what was wrong with the position defended in his earlier *Physical* Monadology.⁶¹ And although Kant embraces transcendental idealism in the MFNS, the argument against the possibility of extended simples depends on general considerations about the nature of force and does not appeal to claims acceptable only to transcendental idealists. Thus, although both the thesis and antithesis arguments depend on assumptions that one might call into question,

⁵⁸ For a different articulation of the problem with the argument, see Van Cleve (1999), 63-4.

⁵⁹ Radner (1998) offers an especially charitable take on the second antinomy, arguing that both the thesis and antithesis arguments are in fact found in the works of Christian Wolff. Central to Radner's assessment of the thesis argument was that it assumes a substance ontology that had virtually no rivals in the 18th century. According to this ontology, relations must inhere in substances, from which it follows that if composition is a contingent relation, substances not depending on composition must exist (i.e., simples must exist). Although I do not think Radner is correct in claiming that this conclusion in fact follows, he makes a compelling case that this argument is found in Wolff.

⁶⁰ As Radner writes, "the antithesis operates on generally accepted eighteenth-century geometrical principles" (1998, 431).

⁶¹ See Proposition 4 in the Dynamics section.

neither is question-begging against transcendental realism as such, for neither makes assumptions that only transcendental idealists could accept.

On the whole then, although the thesis and antithesis arguments of both antinomies may not be *fully* compelling, we can see why Kant would have taken himself to be justified in claiming that the antinomies "indirectly" prove transcendental idealism (A506/B534). The arguments on each side are ones Kant believes transcendental realists must accept, and since they jointly entail two contradictions (corresponding to each of the antinomies), Kant concludes that transcendental realism must be false. And because transcendental *idealism* shows that the thesis and antithesis statements can both be *false* (or so he claims), transcendental idealism provides a way out of the antinomy.⁶² That is, if transcendental idealism is true, then it can be false that the world is finite *and* false that it is infinite; likewise, it can be false that objects are composed from finitely many simple parts *and* false that they have infinitely many parts all of which are composite. Given this, transcendental idealism shows that there is an alternative to the finitist and infinitist positions presented as exhaustive in the antinomies. In the section that follows, I turn to this alternative and survey available readings of the antinomies' resolutions in the secondary literature.

3. Interpretive Options in The Secondary Literature

In the secondary literature, readings of the mathematical antinomies' resolutions generally can be distinguished according to how they treat two separate issues. First, most commentators agree that the notions of *infinity* and *totality* are both important to the

⁶² As Kant puts it in the concluding remarks to the mathematical antinomies, on his solution "both opposed claims are declared as false (*beide entgegengesetzte Behauptungen für falsch erklärt wurden*)" (A528/B556).

mathematical antinomies, but there is disagreement as to exactly how Kant understands the relationship between these two notions.⁶³ For Kant claims that the antinomies can be resolved when we realize that the idea of totality is not "valid" of appearances (A506/B534), but he does not lay out exactly how this relates to his claims about infinity. Does he think the idea of totality is not valid of appearances because the idea of infinity *is* valid of them? Does he think that because the idea of totality is not valid of appearances, the idea of infinity also cannot be valid of them? Or does he think that whether or not the idea of totality is valid of appearances is independent of whether or not they are infinite? The second interpretive question dividing commentators is the following: what sort of philosophical move is Kant making in the resolution of the antinomes? Is he resolving them by making a purely semantic point about truth or reference? Is his solution primarily epistemic in the sense that it makes a claim about what we can know or cognize? Or is his solution instead metaphysical in the sense that it makes claim about how the world must be?⁶⁴

I organize the discussion that follows into two main parts. First, I give an overview of the three most prominent positions defended in the secondary literature on the relationship between infinity and totality in the antinomies. Second, I review the main interpretive options on the

⁶³ An exception to this is Bennett (2016/1974), who argues that the Kant's treatment of the antinomies really turns entirely on his treatment of infinity (290-1). I briefly discuss Bennett's reading in chapter 4 below.

⁶⁴ One might also phrase these questions as follows: when Kant claims that "absolute totality" is not "valid" of appearances, does he mean to be making a semantic, epistemic, or metaphysical point? As noted in the introduction above (in footnote 24), there is a sense in which it is misleading to call a reading turning on Kant's account of *cognition* "epistemic" rather than "semantic", since cognition is a partly semantic notion for Kant. But whereas the readings I call "semantic" say that Kant is offering a general theory of truth or reference in the antinomies' resolutions (depending on the view), the readings I call "epistemic" focus on Kant's account of a more specific kind of cognitive achievement (which may leave space for broader accounts of truth and reference that do not play a role in the antinomies' resolutions). In any case, I am happy to grant that "semantic" and "epistemic" readings in fact overlap in places.

meta-philosophical question as to whether Kant's solution to the antinomy is primarily semantic, epistemic, or metaphysical in nature.⁶⁵

3.1 The Relation Between Infinity and Totality

Few commentators doubt that the notions of infinity and totality both play important roles in the mathematical antinomies. As we have seen, the thesis and antithesis positions advocate finitistic and infinitistic pictures of the world, respectively, and Kant's solution to the mathematical antinomies implies (by his lights) that the thesis and antithesis positions are *both false* (A505/B533). That is, the spatiotemporal world is neither finite nor infinite in extent, and objects are composed neither from finitely many simples nor from infinitely many parts all of which have further parts within them. Kant also argues that the antinomies arise from the misapplication of the idea of *totality* to the spatiotemporal world. As he says, this notion is "valid" of things in themselves, but it should not be applied to appearances in space and time:

[T]he antinomy of pure reason in its cosmological ideas is removed by showing that it is merely dialectical and a conflict due to an illusion arising from the fact that one has applied the idea of absolute totality [*die Idee der absoluten Totalität*], which is valid [*gilt*] only as a condition of things in themselves, to appearances that exist only in representation, and that, if they constitute a series, exist in the successive regress but otherwise do not exist at all. (A506/B534)

In other words, Kant argues that the antinomy is resolved by denying that the notion of "absolute totality" applies to appearances, and since he also advocates a *both false* solution to the mathematical antinomies, the question naturally arises as to how these two aspects of his solution

⁶⁵ As is noted above and will become clear below, these are not meant to be exclusive options. One could argue, for example, that Kant's solution is both semantic and metaphysical, or that it is both epistemic and semantic (and so on). Nonetheless, most interpreters hold that just one kind of meta-philosophical move does the heavy lifting in resolving the antinomy. My aim will be to show that a metaphysical claim is at the heart of the mathematical antinomies' resolutions and that a satisfying solution cannot be *purely* semantic or *purely* epistemic.

fit together. What is the relationship between (a) denying that the notion of "absolute totality" is "valid" of appearances and (b) arguing that the thesis and antithesis statements of the mathematical antinomies are *both false*? In chapter 2 below, I examine more closely exactly what Kant means by "absolute totality" in the passage above (and the role of this notion in the antinomies), but we can briefly review three main ways in which Kant scholars have so far answered these questions.

3.1.1 Option 1: Infinity Affirmed and Totality Excluded

In the secondary literature, one answer to the aforementioned questions goes as follows. Kant means to justify his claim that the notion of totality is not applicable to spatiotemporal phenomena by asserting that spatiotemporal series of conditions are infinite. That is, according to one prominent reading of the antinomies, Kant thinks spatiotemporal phenomena *are* in fact infinite, and the reason the *antithesis* statements count as *false* is that the notion of totality applied in the antinomies fails to apply to infinite phenomena.

Consider Henry Allison's (2004) articulation of this view. According to Allison, Kant's point in the antinomies is to show that because spatiotemporal phenomena *are* infinite, the relevant notion of totality cannot apply to them. As Allison argues, Kant believes the cosmological notion of totality employed in the first and second antinomies is the notion of a *totum syntheticum*, which is just the notion of a whole that results from a combination or "synthesis" of pre-given parts.⁶⁶ Since such a whole can exist only if the combination of its parts is *complete*, Allison argues, and since Kant believes infinitely many items can never be

⁶⁶ As Allison puts it, "Not only does the concept of such a whole presuppose its distinct, pre-given parts, it is also conceived as the product of the collection (in Kant's term, 'synthesis') of these parts" (2004, 369).

completely combined (on Allison's reading of him), it follows that infinite *tota synthetica* are impossible.⁶⁷ That is, according to Allison, Kant resolves the mathematical antinomies by asserting the infinity of the series of conditions, and he infers from this infinity that the relevant series therefore cannot be *tota synthetica*. Consider how Allison puts this point:

[T]he assumption that the series is infinite entails not merely that it cannot be completed in a finite time but that it cannot be completed at all. Moreover, if this is the case, then it does not constitute a world (totum syntheticum). We thus have two alternatives: either (1) the series does not constitute a world, or (2) there is a first moment. The correct Kantian option is the first.⁶⁸

According to Allison, Kant's claim in the first antinomy that "[t]he true (transcendental) concept of infinity is that the successive synthesis of unity in the traversal of a quantum can never be completed" should be taken as evidence that he himself thinks infinite *tota synthetica* are a conceptual impossibility (A432/B460). And given this, Allison argues, it is reasonable to conclude that Kant means to deny the existence of *tota synthetica* without denying that spatiotemporal conditions are infinitely numerous. Since every series of conditions must be either finite or infinite, Allison reasons, Kant's claim that the notion of totality is not "valid" of appearances must be read as follows: because appearances *are* infinite, the notion of *totality* properly speaking cannot be applied to them.

3.1.2 Option 2: Actual Infinity Denied Because Totality Denied

Another interpretive option is to argue that the relevant notion of infinity is not applicable to spatiotemporal phenomena after all, and this is explained by the fact that the idea of totality is

⁶⁷ As indicated above, I think Allison is wrong to say that Kant believes the very concept of an infinite *totum syntheticum* is contradictory. Indeed, Kant's claim that we can *think* of the unconditioned as existing in a whole infinite series of conditions is conclusive textual evidence against Allison on this point (A417-18/B445-6). ⁶⁸ Allison (2004), 370.

not applicable to them. Views of this sort take the notion of infinity employed in the antinomies to be the notion of *actual infinity*, and they interpret the claim that the idea of totality is not applicable to appearances as Kant's *justification* for denying that the notion of infinity is applicable to them. As the argument goes, Kant's infers from the inapplicability of *totality* to appearances to the conclusion that appearances cannot be actually infinite, since the notion of actual infinity simply *is* the notion of an infinite totality. Thus, Kant's solution to the mathematical antinomies must be to say that the features of the world they concern are merely *potentially infinite*; for if the notion of potential infinity simply is the notion of an infinite antinomies are *totalities* and the assertion that they are *potentially infinite* go hand in hand.⁶⁹

Notably, there are two very different ways of motivating a view like this in the secondary literature. First, some interpreters have agreed with Allison that actually infinite *tota synthetica* are conceptually impossible but have then argued that the proper conclusion to draw is *not* that actually infinite series are not *tota synthetica* but rather that actually infinite series are impossible altogether. As the argument goes, once we see that the notion of actual infinity simply *is* the notion of a *complete infinity* or an *infinite totality*, we see that we cannot assert the actual infinity of a series of conditions without asserting its status as a totality. But if this is correct, then the correct conclusion is that spatiotemporal phenomena can be *potentially infinite* but not *actually infinite*. Thomas Holden (2004) advocates a line like this when he says that for Kant a "completely given, larger-than-finite collection of parts" is a contradiction in terms and "the only coherent notion of an infinite is that of the Aristotelian, ever-increasable *potential infinite*."⁷⁰

⁶⁹ See chapter 4 below for an in-depth discussion of potential infinity approaches.

⁷⁰ Holden (2004), 41.

Kristina Engelhard (2005) likewise argues in her discussion of the second antinomy that Kant's solution "consists in showing that the divisibility of matter must be understood as a potential infinity of material parts."⁷¹ Similar claims can be found in Vanzo (2005, 512 fn 17) and Falkenburg (2000, 23).

A second way of motivating a potential infinity view starts with a commitment to antirealism and then reasons to the conclusion that actual infinities are impossible. Consider Kiyoshi Chiba's (2012) metaphysical anti-realist reading, according to which Kant appeals to proto-Dummettian insights to show that the series of conditions treated in the antinomies can never be "absolute totalities".⁷² As Chiba argues, Kant believes each series of conditions *exists* only in the regress (i.e., in the process through which we come to encounter its members in experience), and Kant concludes from this that a series of conditions therefore cannot *be* a totality. And since it can never be a totality, Chiba argues, it also cannot be actually infinite—it can be only potentially infinite.⁷³

⁷¹ Engelhard (2005), 307. My translation. The original German reads: "Kants inhaltliche Lösung der Antinomie besteht von der Sache her zunächst darin, zu zeigen, daß die Teilbarkeit der Materie als ein potentiell Unendliches der Materieteile aufgefaßt werden müsse."

⁷² Chiba (2012), 96. According to Chiba, "Die absolute Totalität einer Bedingungsreihe ist das vollständige Ganze derselben, das bereits vor aller unserer Durchführung des Regressus aufseiten der Objekte komplett bestanden haben soll. [...] argumentiert Kant letztlich, dass es so etwas wie die absolute Totalität der Bedingungsreihe in Wahrheit nicht geben kann."

⁷³ Chiba (2012), 129. On the same page, Chiba writes: "Die Bedingungsreihen […] bestehen nur im Regressus, dieser kann aber weder als ein endliches noch als ein unendliches Ganzes gegeben werden. Daraus folgt, dass auch die Bedingungsreihen selbst weder endlich noch aktual-unendlich sein können." See also Chiba's claim "dass unter der Voraussetzung des Realismus raumzeitlicher Gegenstände die Möglichkeit ausgeschlossen wird, dass eine Bedingungsreihe bloß *potentiell*-unendlich ist, denn der Realismus muss ohnehin annehmen, dass jede Bedingungsreihe als ein vollständiges Ganzes existiert" (141, fn 208). Like Chiba, I think the successive-regress-dependence of appearances is an important part of Kant's idealism, but I do not think the notion of potential infinity is key to the antinomies' resolutions. See chapter 4 and 5 for further discussion.

3.1.3 Option 3: Infinity and Totality Issues are Independent

Finally, a third view in the secondary literature suggests that Kant's reasons for denying that the notion of absolute totality are "valid" of appearances are in fact independent of the arguments he makes concerning the notion of infinity. Consider Marcus Willaschek's (2018) interpretation, which argues that Kant considers a collection a totality if it meets two conditions. First, it must be complete in the sense implied by "true universal quantification"; as Willaschek puts it, "a set *S* is complete with respect to some property *F* iff it contains all *F*s."⁷⁴ Second, a collection must also be *considered as* containing all the items of the relevant type. As Willaschek writes, it must be "considered *as* a unity (i.e., *as* all the things [...])."⁷⁵ Thus, to be a *totality* of some object's conditions, a collection or series of conditions must contain *all* the object's conditions.

Willaschek also argues that we can understand this notion of totality as similar to the Cantorian conception of a *set*, and his discussion of how Kant can deny that appearances form such totalities reveals the extent to which he thinks Kant's views about infinity are independent of his views on the notion of totality. For as Willaschek argues, Cantor defines a set as "any collection into a whole M of well-defined objects of our intuition or thought," and Kant's commitment to regarding totalities as set-like reveals that he endorses something like the principle of comprehension from naïve set theory.⁷⁷ In set theory, the principle of comprehension states that for any predicate F, there is a set containing all the things to which the predicate F

⁷⁴ Willaschek (2018), 92.

⁷⁵ Willaschek (2018), 92.

⁷⁶ In chapter 2 below, I argue that this is not quite right as an understanding of Kant's notion of totality in the antinomies.

⁷⁷ Willaschek (2018), 95. The Cantor excerpt Willaschek cites goes as follows: "Unter einer 'Menge' verstehen wir jede Zusammenfassung M von bestimmten wohlunterscheidenen Objecten m unser Anschauung oder unseres Denkens (welche die 'Elemente' von M gennant werden) zu einem Ganzen" (Cantor 1985, 481).

applies (assuming F is in fact instantiated). And according to Willaschek, Kant's belief that the faculty of reason is inclined to accept a principle of comprehension can answer an important question: why does Kant think that reason is inclined to accept the Supreme Principle?⁷⁸

Recall that the Supreme Principle holds that the existence of something conditioned entails the existence of the totality of its conditions. As Willaschek argues, it can be difficult to see why a *totality* of conditions follows from the existence of one conditioned thing, but this "follows trivially [...] if we take Kant's definition of 'totality' to express the naïve principle of set formation (sometimes called the 'principle of comprehension')."⁷⁹ If one thing exists instantiating a certain kind of conditioning relation, then there is a totality containing *all* the things instantiating that kind of conditioning relation. Or so Willaschek argues. Moreover, if this is correct, then Kant's claim that there are *no* totalities of spatiotemporal conditions can also be given a ready rendering as follows: "Since Kant himself wants to *deny* that for empirical objects there is a totality of their conditions (A499/B527), Kant might be read as implicitly rejecting that principle [i.e., the principle of comprehension] for the domain of appearances."⁸⁰ That is, according to Willaschek, Kant holds that a principle of comprehension holds for things in themselves, but it does not hold for appearances in space and time.

Notice that this explanation of Kant's rejection of the applicability of the notion of totality to appearances does not rely on claims about infinity. Willaschek does not argue that the notion of totality does not apply to spatiotemporal conditions because they are infinite, and he

⁷⁸ Willaschek (2018), 95.

⁷⁹ Willaschek (2018), 95. Again, I will suggest in chapter 2 below that this is not the correct way to understand the notion of totality relevant to the antinomies.

⁸⁰ Willaschek (2018), 95 fn 46. In fact, Willaschek recognizes that Kant would not have explicitly conceived of things in terms of a comprehension principle (since set theory had not yet been developed), but he argues that Kant could have explained his commitments in this way (had he been aware of set theory).

also does not claim that spatiotemporal conditions cannot be (actually) infinite because the notion of totality is inapplicable to them. Rather, Willaschek argues that the denial of the principle of comprehension (and hence the unrestricted applicability of the notion of totality to appearances) is motivated by other means.⁸¹ Infinite sets are possible *in general*, on Willaschek's reading of Kant, and in resolving the antinomy he doesn't mean to be making a point about actual infinity.

In chapter 2 below, I present my own views on the relationship between the notions of *totality* and *infinity* in the antinomies and explain why Kant's discussion requires an alternative to all three of the positions just canvassed. In brief, I will argue that none of the extant views recognize that Kant distinguishes between two different notions of totality in his discussions of cosmological questions, and a correct account of his views on the relationship between infinity and totality in the antinomies requires an appropriate account of this distinction. In section 3.2 below, however, I turn to the question of Kant's meta-philosophical strategy in resolving the antinomies.

3.2 Is the Solution Semantic, Epistemic, or Metaphysical?

As noted above, we can understand readings of the antinomies' resolutions as falling into three main camps. Some commentators argue that the antinomies are resolved by adopting a

⁸¹ In fact, Willaschek thinks Kant lacks a fully satisfying answer to the question *why* the principle of comprehension should fail to hold for appearances. But as Willaschek insists, denying that the principle of comprehension holds for appearances remains Kant's best move: "I think that the philosophically most plausible way for Kant to resist the inference from the conditioned to the unconditioned totality of its conditions consists in denying the principle of comprehension, that is, the assumption that for every predicate there is the totality of objects of which it is true" (2018, 155). In the course of this discussion, Willaschek makes clear that he means to allow for the existence of infinitely many conditions and moreover does *not* mean to say that their infinity explains why the principle of comprehension does not hold for conditioned appearances.

particular account of truth or reference, depending on the view (semantic accounts). Others argue that the antinomies are resolved by appealing to a particular account of knowledge or cognition, again, depending on one's view (epistemic readings). And some commentators argue that the antinomies are resolved by making a metaphysical move, i.e., a claim about the nature of spatiotemporal objects (*metaphysical* readings). Notably, it is possible to adopt more than one of these approaches at once, and there are ways in which certain approaches are likely to overlap. For instance, epistemic approaches that say Kant appeals to his particular conception of *cognition* to resolve the antinomies are often partly semantic insofar as they say the special cognitive achievement that is cognition requires a special kind of reference to an object (namely, reference via *intuition*). Nonetheless, distinguishing somewhat artificially between semantic, epistemic, and metaphysical readings will be useful as a preliminary device for surveying the secondary literature. Ultimately, I will argue that a *metaphysical* reading of the antinomies' solutions is most compelling, but here I confine my attention to giving brief summaries of the three main kinds of approaches currently represented in the secondary literature (briefly noting objections to the extant views where relevant).

3.2.1 Option 1: Semantic Readings

Broadly speaking, semantic readings of the antinomies' resolutions come in two main forms. First, *truth-theoretic anti-realist accounts* suggest that Kant resolves the antinomies by proposing that a judgment about the empirical world cannot be *true* unless it is verifiable by some experience. Second, *referential accounts* suggest that a unique claim about what it is to *refer* to an object is at the center of the antinomies' resolutions; the concepts employed in the antinomies turn out not to be referring concepts, and this is what explains the illegitimacy of the antinomial arguments.

First consider truth-theoretic anti-realist accounts, whose most prominent defender in the secondary literature is Carl Posy (1983 and 2019).⁸² Taking Michael Dummett's anti-realism as his starting point, Posy argues that Kant's account of empirical *truth* is similar Dummett's account of *meaning*. According to Dummett, to understand the meaning of a sentence just is to understand the conditions under which one would be warranted in asserting it, and so the meaning of a statement is linked with our means of verifying it (or falsifying it). If there is no evidence that could verify (or falsify) a statement *even in principle*, Dummett argues, then the statement has no meaning at all. Few commentators argue that Kant endorses anti-realism concerning *meaning* (with the notable exception of Strawson, discussed below),⁸³ but according to Posy, Kant is a proto-Dummettian concerning *empirical truth*.⁸⁴ As Posy argues, Kant believes statements about the empirical world can be true only if they can in principle be verified in experience, and the thesis and the antithesis statements of the antinomies both turn out to fail this criterion of truth. Hence, once we accept anti-realism concerning empirical truth, the antinomies simply dissolve.⁸⁵

P.F. Strawson (1966) also suggests that Kant's solution to the antinomies turns on a kind of anti-realism about empirical truth. According to Strawson, Kant endorses the "principle of

⁸² For further discussion of anti-realism that do not take the antinomies as their focus, see Stevenson and Walker (1983) and Walker (1995).

⁸³ In fact, Posy (1983) argues that we *should* "view transcendental idealism as an evidential theory of meaning for empirical judgments" (83). But most commentators recognize that this cannot be correct, since (at minimum) important parts of Kant's practical philosophy require our being able to make *meaningful* statements about things in themselves.

⁸⁴ As Posy (2008) notes, Kant is arguably not a proto-Dummettian anti-realist about *mathematical* truth, since he is an "epistemic optimist" in mathematics and believes all mathematical problems are decidable (182).

⁸⁵ See Posy (1983), 91.

significance," according to which "there can be no legitimate, or even *meaningful*, employment of ideas or concepts which does not relate them to empirical or experiential conditions of their application."⁸⁶ This principle is at the heart of the antinomies' resolutions, Strawson argues, for Kant's solution turns on the observation that both the thesis and the antithesis arguments employ concepts that fail the test of the principle of significance; they employ the notion of "*the series* [of conditions] *as a whole*," but as Strawson argues, Kant thinks this notion is not one for which empirical conditions of application can be specified.⁸⁷ And if the statements made in the antinomies are not even *meaningful*, then certainly they also cannot be *true*.

How should we assess readings like Posy's and Strawson's? One might take them to be supported by Kant's famous claim that "thoughts without content are empty" (A51/B75), for as one might argue, if thoughts without content (i.e., intuitions) are empty, then, plausibly, thoughts in the absence of intuitions also cannot be true, false, or even meaningful. More generally, when Kant says that a judgment errs when it attempts to reach beyond the bounds of possible experience, one might take him to be making an anti-realist point: given that a judgment reaching beyond possible experience cannot be verified in intuition, it cannot have a truth value, and given that it cannot have a truth value, it should not be asserted.⁸⁸

One might also take the following line of reasoning to support a truth-theoretic antirealist interpretation. Recall from above that Kant affirms a *both false* solution to the mathematical antinomies. That is, he holds that on his solution, the thesis and antithesis

⁸⁶ Strawson (1966), 16, my emphasis. Note that Strawson does seem to commit here to anti-realism not only concerning *truth* but also concerning *meaning*. But as noted above, there is a near consensus in the secondary literature that Kant does not mean to endorse a semantic anti-realism of this sort.

⁸⁷ Strawson (1966), 158-9.

⁸⁸ E.g., see Kant's discussion at A409/B436, which suggests that the antinomies arise when reason tries to "**free a concept of the understanding** from the unavoidable limitations of a possible experience, and thus seek to extend it beyond the boundaries of the empirical..."

statements are both false such that the features of the world treated in the antinomies are neither finite nor infinite. Arguably, however, this means that the disjunction on which the antinomial arguments depend cannot properly be asserted. That is, we cannot properly assert that the world must be either finite or infinite, and likewise we cannot properly assert that objects must be composed either of finitely many parts (the ultimate parts of which are simple) or of infinitely many parts (all of which have further parts). But if Kant is meaning to propose anti-realism concerning truth as the solution to the antinomies, then we can readily see how these disjunctions might be avoided. As the arguments of the antinomies show (on the present interpretation), no possible experience could verify that the world is finite, and likewise no possible experience could verify that the world is infinite. Similarly, no possible experience could verify that objects have infinitely many parts. Hence, the disjunctions on which the antinomial arguments depend lack a proper warrant, and the arguments of the antinomies simply do not go through.⁸⁹

However, as already noted, Kant means to say more than that the disjunctions on which the antinomies depend cannot properly be asserted. As we have seen, he also means to say that the thesis and antithesis statements of both antinomies are *false*. According to truth-theoretic anti-realists, failing to be true and being false are different things, so truth-theoretic anti-realism requires a further explanation of how Kant's both false solution to the mathematical antinomies can be sustained. Indeed, it is a defining feature of anti-realism to say that a sentence *and* its negation may both fail to be true, so if falsity is the truth of negation, it is unclear how the anti-

⁸⁹ An obvious problem that arises on this solution is how to understand the "both true" solution of the third and fourth antinomies. As far as I am aware, Posy's defense of truth-theoretic anti-realism for the first and second antinomies does not answer this question. Notably, Chiba's Dummettian reading concedes that the anti-realist solution *does* imply a unified solution for both the mathematical and the dynamical antinomies (2012, 256 fn 367).

realist will explain the falsity of both the thesis and the antithesis statements. For the anti-realist, from the fact that *P* is not properly supported by empirical evidence it simply does not follow that $\neg P$ is properly supported by empirical evidence, and the statements made in the mathematical antinomies are paradigmatic cases in point.⁹⁰

Can truth-theoretic anti-realists solve this problem? One option would be to simply accept that the thesis and antithesis statements of the mathematical antinomies are not false in the strict sense of the term. However, this has an obvious textual cost. Alternatively, a different approach would be to argue that Kant had a more encompassing account of the truth of negation than do many anti-realists. Posy takes this route, arguing that a statement's negation is true for Kant just in case it meets two conditions. First, the statement must not currently be supported by empirical evidence. Second, there must be evidence that no imagined increase in information *would* support the statement in question.⁹¹ According to Posy, if we understand the truth of negation in this way, then Kant *can* show that the thesis and antithesis statements of the first and second antinomies are both false while at the same time adhering to the main principles of anti-realism. For, arguably, Kant believes that both the thesis and the antithesis statements of the first and second antinomies are not the sort of statements that *could* be supported by empirical or experiential evidence.⁹²

⁹⁰ Indeed, according to the anti-realist, it is one of the main faults of *realists* that they simply assume without warrant that the negation of a judgment is true if the judgment itself is not true.

⁹¹ Posy (1983), 84.

⁹² To take just one example, consider the first antinomy's thesis claim concerning time. The thesis statement asserts that the world has a beginning in time, and Kant would have considered it fairly uncontroversial that we do not *now* have experience of a first state of the world in time. But in addition, one could argue that Posy's second criterion is satisfied, for given that time itself is infinite, no matter what past state of the world we consider, there will also be the be the possibility that an earlier state of the world occurred. Indeed, there are infinitely many past times in which to look for an earlier world-state—so many that we cannot in principle get through them all in our stepwise investigative mode.

Even if this approach to explaining the "both false" solution succeeds, however, a further problem for truth-theoretic anti-realism is that it characterizes the resolution of the antinomies as incompatible with classical logic, whereas Kant insists that the antinomies' resolutions and classical logic are fully compatible. Note that in arguing that a statement and its negation may fail to be true, and in allowing that a statement may fail to be either true or false, truth-theoretic anti-realism rejects two important principles of classical logic. First, it rejects the law of the excluded middle, according to which " $P \vee \neg P$ " is tautologous; second, it rejects the principle of bivalence, according to which every statement has exactly one of two possible truth values, true or false. If Kant endorses anti-realism, then he too must reject classical logic, but there is strong evidence that he does not do this (and moreover that he does not see the dispute between realists and idealists as a dispute about whether or not the core principles of classical logic are true, even for empirical reality). In fact, Kant's explicitly states in the resolution of the antinomies that the law of excluded middle and bivalence *can* be assumed. As he writes, we *can* assume that the statement "the world is not infinite" is true if the statement "the world is infinite" is false (A503/B532). What *cannot* be assumed, according to Kant, is that the statement "the world is not infinite" is *equivalent* to the statement "the world is finite" (A503/B532), a point fully compatible with classical logic (given the right understanding of the substantive content of those two claims). Thus, although Kant holds that the disjunction "either the world is infinite or the world is finite" is false, he allows that the statement "either the world is infinite or the world is not infinite" is true by logic alone.93

⁹³ As Kant explains it, the *reason* the former disjunction is not true is that in asserting that the world is not infinite, "I would rule out only an infinite world, without positing another one, namely a finite one" (A504/B532).

Given this, we can safely conclude that Kant does not take the statement "the world is either finite or infinite" to be an instance of the law of excluded middle.⁹⁴ And since it is a crucial part of truth-theoretic anti-realist interpretations that they *do* see it that way (since this explains why transcendental realists, who are committed to classical logic, cannot escape the antinomies), we should rule out truth theoretic anti-realism as a correct interpretation of Kant's resolution to the antinomies. Contra truth-theoretic anti-realism, Kant takes classical logic for granted, and he instead resolves the antinomy by showing that the disjunction asserting that the world is either finite or infinite is not true *even by the standards of classical logic*.⁹⁵

Readings that focus on questions about *reference* form another distinct category of semantic interpretations.⁹⁶ Consider Arthur Melnick's (1989) interpretation of the antinomies, which argues that the antinomies are at the core of Kant's argument for a non-descriptivist theory of reference, which is the focus of the entire first *Critique*. According to Melnick, a number of Kant's contemporaries and predecessors (most notably, Leibniz) defended a version of descriptivism according to which reference is possible only insofar as representations are or contain definite descriptions standing in relations of isomorphism with their objects. Thus, a thought or concept of Caesar represents Caesar by containing a description of his properties. The

⁹⁴ For helpful discussion of the connection between this point and Kant's account of "infinite judgments," see Stang (2012). I return to some questions about the relationship between metaphysical indeterminacy and classical logic in chapter 4 below.

⁹⁵ One might worry here that these points are not compatible with Kant's claim that the statements of the thesis and antithesis must be "contradictory opposites" for transcendental realists. However, to say that to statements are contradictory opposites is just to say that they cannot both be true and they cannot both be false, and this will be *true* for transcendental realists (as my account in the chapters that follow explains). To get this result, we need not say that the two statements when put into a disjunction are an instance of the law of the excluded middle.

⁹⁶ Note: in counting *referential accounts* as "semantic" rather than "epistemic", I do not mean to suggest that questions about reference are not central to Kant's views on issues that might be understood as "epistemic" in a broad sense. For instance, intuition (*Anschauung*) clearly plays a role in securing reference to an object in *cognition* (*Erkenntnis*) on Kant's account of it. Here, however, here I use "referential accounts" to refer to views like Melnick's, which see the transcendental idealism as a theory of reference and the antinomies as one of Kant's central arguments for that theory. Broader issues about the role of reference in cognition (*Erkenntnis*) and knowledge (*Wissen*) fall under the heading of "epistemic" readings. See below for further discussion.

problem with an account of this sort, Melnick argues, is that definite descriptions fail to pick out actual as opposed to merely possible objects. For no matter how determinate a set of descriptive conditions may be, one can always come up with a fictional or merely possible entity that satisfies the description just as well as does an actual object. According to Melnick, this insight is at the heart of the first *Critique*, and Kant's chief point in the entire *Critique* is to show that definite descriptions pick out unique objects only to the extent that the right domain of candidate referents has already been specified.⁹⁷ In Kant's case, this domain-specification is achieved via his distinctive account of intuition (*Anschauung*). Since (empirical) intuitions secure a real relation to an object, Melnick argues, they are capable of explaining how representation can be "determinately fixed to actual objects" rather than to merely fictional or possible ones.⁹⁸

With this broad interpretive framework in mind, Melnick argues that the resolution of the antinomies should be understood as follows. Suppose we take on board the idea that intuitions involve a real relation to an object, and suppose we embrace the thought that reference or representation is not possible without this real relation. If we accept these points, then it follows that representation or reference is not possible in advance of intuition. But the positions advanced in the antinomies are precisely those positions that *do* take reference to objects to be legitimate *before* any intuitions take place, and they do this by way of their claims concerning

⁹⁷ As Melnick writes, "[D]efinite descriptions select or pick out an object as that unique entity satisfying the predicative conditions stipulated by the description, but they do so determinately only if a domain of <u>candidates</u> for satisfaction of the predicative conditions has been settled or fixed. It is only relative to a domain of entities that definite descriptions determinately refer by selecting one of 'those' entities as the entity uniquely satisfying its conceptual conditions. Thus, until the domain of actual entities has somehow been represented or fixed, no definite description refers to any of those actual entities and so may as well (as Kant says) be representing non-actual or merely possible entities" (1989, 2-3).

⁹⁸ Melnick (1989), 3. Note that important aspects of Allais's (2015) view also focus on the role of intuition in securing reference, but I read her as a proponent of a metaphysical rather than a semantic approach because she takes the solution to the antinomies to turn on the claim that spatiotemporal *existence* is not experience transcendent. In contrast, Melnick wants to avoid making metaphysical claims of this sort.

totalities. As Melnick argues, to consider the question whether the world is an infinite or a finite totality, we must think of objects in the world as existing prior to our experiences of them (i.e., prior to our intuiting them). If we did not, then given that we never experience the world *as a totality*, we would never represent it as such. Likewise, to ask whether objects have simple parts or infinitely many complex parts, we must attempt to represent a totality of parts in objects before intuition plays its reference-securing role.⁹⁹

In fact, according to Melnick, on a purely descriptivist account of reference, the conflicts of the antimonies are inevitable. For if we assume we can refer to objects prior to encountering them in the progress of experience, then (according to Melnick) we must assume that objects extend either infinitely far or finitely far in space and time.¹⁰⁰ The reason for this is that objects conceived of independently of the progress of intuition must be conceived of as a complete totality: if objects are there "to be met with" prior to the progress of intuition, Melnick argues, then they must form either an infinite or a finite whole. But once this is conceded, it follows that descriptivists must endorse one or the other of the thesis and antithesis positions (or so Melnick argues). The idealist, in contrast, can argue that both the thesis and antithesis statements are false, since she holds that no such total domain of objects can ever be specified for reference.¹⁰¹ That is, from a non-descriptivist theory of reference, the conclusion follows that we can never make legitimate claims about totalities of conditions. And once we cease making claims about such totalities, the conflicts of the antinomice simply fall away. Hence, according to Melnick, the

⁹⁹ See Melnick's chapter 2 for further discussion of his rationale here.

¹⁰⁰ Melnick (1989), 327.

¹⁰¹ The reason for this, according to Melnick, is that intuition always secures reference in a stepwise, ongoing process. Given that intuition functions in this way, a total domain of objects of reference is never possible.

antinomies succeed in showing that transcendental idealism is true, which on his view is just to say that they vindicate a non-descriptivist theory of reference.¹⁰²

Putting aside the question whether it is plausible to claim that transcendental realism and transcendental idealism are both primarily theories of reference, is Melnick right to say that a descriptivist theory of reference *would* legitimize the conclusion that the world is either a finite totality or an infinite totality? That is, is he right that we cannot escape the antinomies without adopting an account of reference like the one he attributes to Kant? As I hope to show in the chapters that follow, Melnick's claim here is not plausible. For as I will argue, Melnick's reading depends on the assumption that spatiotemporal reality in fact satisfies the conditions on *being* a totality, and this is precisely the assumption Kant is asking his readers to reconsider in the resolution of the antinomies. That is, Kant argues that we should not assume spatiotemporal conditions in fact form totalities (of the relevant type), and transcendental idealism helps us see why this is so.

3.2.2 Option 2: Epistemic Readings

Whereas semantic readings interpret the resolution of the mathematical antinomies as part of a general account of truth or reference (depending on the view), epistemic readings argue that Kant's solution to the antinomies turns on a claim about knowledge (*Wissen*) or cognition (*Erkenntnis*) (again, depending on the view). In this section, I give a brief overview of these

¹⁰² Melnick (1989, 366). Note, however, that Melnick thinks this is compatible with our saying, once we have intuited an object, that it was there before our intuition of it and that its existence in no way depends on human representation. The crucial point for Melnick is just that we cannot represent objects in any way in advance of encountering them in intuition. Once we have encountered objects in intuition, Melnick argues, it turns out the idealist can make almost all of the same metaphysical judgments that transcendental realists can make (1989, 373).

broadly epistemic readings. I chapter 3, I offer an extended criticism of epistemic readings in the course of a longer analysis of the first antinomy.

At the most general level, epistemic readings hold that the transcendental idealist escapes the antinomies by appealing to her unique account of knowledge or cognition. As the argument goes, transcendental *realists* must hold that the knowledge (or cognition) claimed in the thesis and antithesis arguments *is* possible for us, and this commitment makes the antinomies inescapable for her; in contrast, transcendental *idealists* hold that the knowledge (or cognition) claimed in the antinomies is *impossible* for us in principle, and the antinomy dissolves once this impossibility is acknowledged.

At one level of generality lower, we can divide epistemic readings of the antinomies' resolutions into two main categories. First, one category of commentators claims that transcendental idealism itself is a purely epistemological doctrine, and the antinomies' resolutions are simply an application of this epistemology to the case of rational cosmology (call these "Idealism as Epistemology" readings). Second, another category of commentators allows that transcendental idealism may be partly a metaphysical view (which makes a metaphysical distinction between appearances and things in themselves) and argues that the solution to the antinomy is nonetheless entirely non-metaphysical (call these "Moderate Epistemic" readings). Typically, readings of this latter kind focus on the significance of the Transcendental Analytic for the Transcendental Dialectic. Because Kant establishes in the Analytic that cognition requires an object that is both *given* in intuition and *thought* through concepts, the argument goes, he can conclude in the Dialectic that the cognition transcendental realists claim to have in the antinomies is impossible. The *whole world* is not an object that can be given to us in intuition, and likewise neither objects' simple parts nor their infinite complexity can be cognized by minds
like ours. Therefore, transcendental realists claim cognitive achievements in the antinomies that are in fact impossible for us, given the nature of mental faculties. And the antinomies are resolved by removing these epistemic errors.

It is difficult to overstate the influence of epistemic readings in the secondary literature. Almost every book-length treatment of the Dialectic opts for an epistemic reading of the antinomies' resolutions, even when they offer radically different accounts of Kant's doctrine of transcendental idealism. To take two prominent examples, consider Michelle Grier's (2004) Kant's Doctrine of Transcendental Illusion and Marcus Willaschek's (2018) Kant on the Rational Sources of Metaphysics. Grier endorses a methodological or epistemic reading of transcendental idealism (à la Allison) and argues that the antinomies arise from erroneously taking reason's purely *regulative* principles (especially the Supreme Principle) as *constitutive* principles from which one can legitimately "deduce knowledge about appearances (objects of experience)."¹⁰³ According to Grier, "once appearances are taken for things in themselves, it is assumed that the entire set of all appearances (the world) is an object about which we can acquire knowledge through reason alone."¹⁰⁴ Correspondingly, the solution to the antinomies is to remember the epistemological strictures recommended by transcendental idealism: because human knowledge is limited by the "conditions of sensibility," we cannot legitimately extend our knowledge through principles of pure reason as the proponents of the thesis and antithesis arguments think we can.¹⁰⁵

¹⁰³ Grier (2004), 178.

¹⁰⁴ Grier (2004), 178-9.

¹⁰⁵ As Grier writes, "Until, that is, one adopts transcendental idealism, once is left with the 'conflict' generated by the need to accommodate not only the principles enumerated in connection with sensibility but, simultaneously, those of reason" (2004, 193-4).

In contrast, Willaschek argues that transcendental idealism is a metaphysical doctrine (involving a metaphysical distinction between appearances and things in themselves), but the solution to the antinomies is nonetheless epistemic and follows from the results of the Analytic. Willaschek writes, "while according to the Transcendental Analytic there cannot be cognition from concepts alone, according to the Dialectic the cognitions of pure reason would have to be precisely that: purely discursive, cognitions from mere concepts."¹⁰⁶ As Willaschek sees it, the assumption that purely discursive cognition is possible manifests in our tendency to assume "that the rational principles that make us ask metaphysical questions (such as the Supreme Principle) are true of reality itself."¹⁰⁷ And given this, the proper solution to the antinomy is to abandon our commitment to the Supreme Principle as a principle that extends our knowledge or cognition. On this reading, we need not say that the spatiotemporal world is in fact neither finite nor infinite to resolve the antinomies; rather, we need only drop "the expectation that reality provides answers to our rational questions."¹⁰⁸ This is not to say that reality is *in fact* constituted so as to violate the Supreme Principle's demand for complete explanation, but it is to say that we lack a proper guarantee that arguments based on the Supreme Principle are sound.¹⁰⁹

Other interpreters who advance epistemic readings of the antinomies' resolutions in book-length projects include Sadik Al-Azm (1982) in *The Origins of Kant's Arguments in the*

¹⁰⁶ Willaschek (2018), 34.

¹⁰⁷ Willaschek (2018), 269.

¹⁰⁸ Willaschek (2018), 269.

¹⁰⁹ Admittedly, it is somewhat unclear whether Willaschek thinks that abandoning the principle of comprehension for appearances is a metaphysically neutral move (recall from 3.1.3 that he also sees this as part of the antinomy's resolution insofar as it is required to explain how the Supreme Principle could fail to be true for appearances). I put aside this question for now.

Antinomies,¹¹⁰ Brigette Falkenburg (2000) in *Kants Kosmologie*,¹¹¹ and Wolfgang Malzkorn (1999) in *Kants Kosmologie-Kritik: Eine Formale Analyse der Antinomienlehre*.¹¹² Henry Allison (2004), Graham Bird (2006), Karin de Boer (2020b), and Eric Watkins (2019b) also advance epistemic interpretations in the course of their broader interpretive projects. Allison and Bird argue that the antinomies arise from adopting the wrong account of human knowledge and that the correct account is provided by transcendental idealism.¹¹³ De Boer argues that the antinomial arguments rely on the assumption that cognition is possible through *unschematized* concepts, and Kant's earlier arguments in the *Critique* show that this is impossible.¹¹⁴ And, finally, Watkins writes in his discussion of the second antinomy: "Kant maintains that given our cognitive limitations, we cannot cognize whether the series of composition relations is finite or infinite," and so "we cannot cognize what is asserted by either the Thesis or the Antithesis, which resolves the conflict between them and prevents us from having the kind of metaphysical cognition that the traditional metaphysician claims to be able to support."¹¹⁵ Thus, according to Watkins, although Kant's idealism involves metaphysical commitments, these commitments are

¹¹⁰ Al-Azm writes that the "moral of the entire episode of the antinomy for the critical philosophy as a whole" is that "such rational principles, as the law of sufficient reason, are purely formal principles, from which nothing can be inferred about the nature of actuality" (1982, 35).

¹¹¹ Falkenburg argues that Kant aims to "convert (*umwandeln*)" cosmology from a metaphysical to an epistemic theory (2000, 23).

¹¹² According to Malzkorn, the solution to the antinomy is to avoid the mistaken assumption that the Supreme Principle is "objectively valid (*objectiv gültig*)"; and we achieve this when we realize that the entire series of an object's conditions is not necessary *given to cognition* with the given object (Malzkorn uses the phrase "*als Gegenstand der Erkenntnis an die Hand gibt*" to capture the notion of givenness *to cognition*) (1999, 56).

¹¹³ Sometimes Allison describes his solution as "methodological", but the relevant methodological recommendation is to avoid abstracting away from the "epistemic conditions" that make knowledge and cognition possible for the human mind, so it remains appropriate to call his reading "epistemic" (e.g., see Allison 2004, 11). ¹¹⁴ De Boer (2020b), 52.

¹¹⁵ Watkins (2019b), 6-7. As Watkins continues, "it does not follow from this argument that there cannot *be* simples, but only that if there are, we cannot cognize them" (7).

not immediately implicated in the solution to the antinomies—the way out of the antinomies instead consists in the broadly epistemic point that Kant makes about *cognition*.¹¹⁶

Given the importance of broadly epistemic readings in the secondary literature on Kant, I devote an entire chapter to explaining why they cannot provide a fully satisfying account of the antinomies' resolutions (see chapter 3 below). Now, I turn my attention to a brief presentation of the extant metaphysical readings of the mathematical antinomies' resolutions. Ultimately, I will develop a metaphysical reading that differs in important ways from the metaphysical readings currently on offer.

3.2.3 Option 3: Metaphysical Readings

At present, there are two main varieties of metaphysical readings of the antinomies' resolutions in the Kant scholarship. One variety is *metaphysical anti-realism* (to be distinguished from truth-theoretic anti-realism): according to these readings, Kant resolves the antinomies by denying the possibility of experience-transcendent *existence* in space and time. The other variety of metaphysical reading advances a form of phenomenalism as the solution to the mathematical antinomies: according to these readings (which attribute to Kant what I will call "*actual state phenomenalism*"), Kant resolves the antinomies by arguing that spatiotemporal objects are constructions out of perceivers' actual states.

¹¹⁶ Watkins has suggested to me in conversation that he ultimately does want to read metaphysical claims as playing a helping role in the antinomies' resolutions. Depending on the particulars of the view he accepts, it may be more or less compatible with the view I defend in the chapters that follow.

Recall that truth-theoretic anti-realism argues that Kant rejects bivalence and intends to be articulating a position in the philosophy of language, broadly construed.¹¹⁷ In contrast, metaphysical anti-realist interpreters notice that ontological claims do not seem to be a dispensable part of Kant's project in the first *Critique*. As they argue, however, one could develop anti-realism as a metaphysical rather than as a merely linguistic or semantic position, and this form of anti-realism plausibly *is* a thesis Kant meant to endorse. Consider Lucy Allais's (2003 and 2015) claim that Kant's idealism can be understood as a rejection of experience-transcendent *existence* for things in space and time.¹¹⁸ As Allais argues, to say that spatiotemporal existence does not transcend what we can in principle experience is a way of saying that spatiotemporal existence is mind-dependent, for experience is clearly a mind-dependent phenomeno.¹¹⁹

How does an anti-realism like Allais's resolve the antinomies? According to Allais

(2015), Kant resolves the antinomies by arguing that the world is neither finite nor infinite, and

he appeals to its mind-dependence to justify this claim. Allais writes:

[T]he kind of mind-dependence Kant appeals to here is the rejection of experience-transcendence. The world in space and time does not extend beyond the possibility of our cognizing it; the world as a determinate totality (whether an infinite or a finite totality) is not something we could cognize; therefore the world

¹¹⁷ Posy in fact admits that he is likely departing from Kant's original intentions in reading transcendental idealism in this way, but as he argues, a view that "reduces spatial objects to mere thought entities is scandalously counterintuitive" and should be avoided if at all possible (1983, 81). Interestingly, Walker (1995) gives an extended argument for the conclusion that semantic anti-realism (anti-realism concerning meaning) entails "ontological idealism"; presumably, he would also want to argue that truth-theoretic anti-realism entails ontological (or metaphysical) anti-realism.

¹¹⁸ The view is qualified so as to apply only to *spatiotemporal* existence because Kant clearly affirms the existence of things in themselves, which transcend experience. Allais writes: "Since transcendental idealism is not a theory of meaning, [Kant] does not think that statements which transcend the conditions of the possibility of experience have no meaning or have a third truth value, but he does think that the appearances of things do not transcend our possible experience of them, and that where a truth claim transcends the conditions of possible experience, it is not true with respect to appearances (2003, 386). See also Allais (2015), 212.

¹¹⁹ Allais (2003), 380.

in space and time does not exist as either a finite or an infinite totality.¹²⁰

Kiyoshi Chiba (2012) also attributes to Kant a form of metaphysical anti-realism and offers a

similar explanation of the solution to the antinomies (in his discussion of the first antinomy):

Kant's resolution to the antinomies demands that the thesis as well as the antithesis positions are false. [...] Kant's argument against the antithesis position is easy to understand: as anti-realism, transcendental idealism requires spatiotemporal objects to be experienceable for us in order to be actual at all. So if a series of conditions is to be actually infinite, it must be experienceable as an infinite totality, which is impossible for us. Hence, no series of conditions is actually infinite.¹²¹

Chiba then explains the falsity of the first antinomy's thesis position as follows. According to

Chiba, Kant's argument proceeds in five steps:

Step 1: For a series of conditions to be finite implies that it has an 'empirically absolutely unconditioned' member, which in that case must be bounded by nothing, or by the void.

Step 2: But *experience* of such a boundary is impossible.

Step 3: Therefore, it is not *experienceable* that a series of conditions is finite.

Step 4: Transcendental idealism as anti-realism requires that all spatiotemporal objects be experienceable for us to be actual at all. [The anti-realist argument]

Step 5: Consequently, it is *not the case* (not only: unexperienceable) that the series of conditions is finite.¹²²

¹²⁰ Allais (2015), 216.

¹²¹ Chiba (2012), 256. My translation. Note also that although this passage seems to reject truth-theoretic antirealism, elsewhere Chiba says that a "cognition-dependent conception of truth" *is* a defining feature of anti-realism: "Realism in the Dummettian sense is primarily a verification-independent truth-conception, i.e., the truth of statements is independent of the possibility of our verification of them. This leads to a cognition-independent *ontology* of objects. By contrast, anti-realism is a negation of realism and therefore leads to some version of a verification-dependent truth-conception and cognition-dependent ontology" (précis on Virtual Critique). So it is slightly unclear whether Chiba ultimately means to pair metaphysical anti-realism with truth-theoretic anti-realism or not.

¹²² Chiba (2012), 257. My translation.

Thus, both Chiba and Allais argue that metaphysical anti-realism justifies the conclusions that the thesis and antithesis statements of the antinomies are *both false*. Given this, they conclude that Kant's resolution to the antinomies simply is to embrace a form of anti-realism—this is available only to the transcendental idealist, and so it explains how Kant can take the antinomies to provide an indirect argument for idealism.

A different sort of metaphysical approach takes the solution to the mathematical antinomies to turn on Kant's putative commitment to a form of phenomenalism, construed as the thesis that spatiotemporal objects are constructions out of perceivers actual representational states (*actual state phenomenalism*).¹²³ James Van Cleve (1999) defends a classic account of this sort in *Problems From Kant*, where he interprets Kant's intended resolution to the second antinomy as follows. As we have seen above, Kant believes both of the mathematical antinomies follow from an argument of the following general form:

Major Premise: If something conditioned is given (exists), then the entire series of its conditions is given (exists)—that is, its own conditions exist, as well as any conditions of those conditions, and so on.

Minor Premise: Something conditioned exists.

Conclusion: The entire series of its conditions exists.¹²⁴

On Van Cleve's view, Kant responds to this argument in the resolution of the antinomies by

showing that the first premise is false on the presupposition of actual state phenomenalism:

[I]f matter exists merely as appearance, having no being apart from our perceptions, then the major premise is false. We cannot say that all the members of a given part series exist, for they come in to being only as we perceive them—

¹²³ Other interpreters who read Kant as intending to endorse a brand of phenomenalism in the antinomies include Guyer (1987) and Smith (1918). However, neither Guyer nor Smith think the antinomies actually require a metaphysical solution (instead, they suggest that a methodological or epistemic solution suffices).
¹²⁴ This is Van Cleve's paraphrase of the "dialectical argument" on which Kant says the entire antinomy depends in

the A497/B525 passage quoted above. See Van Cleve (1999), 69.

that is to say, successively, rather than all at once.¹²⁵

According to Van Cleve, this line of reasoning in facts *fails* to underwrite Kant's desired conclusion that the thesis and antithesis statements of the mathematical antinomies are both false, since a successive series of perceptions could ultimately turn out to be either finite or infinite. It could be infinite, Van Cleve argues, because if a perceiver starts perceiving now and never stops, "he *will have* infinitely many perceptions."¹²⁶ And it could be finite because, even if it is granted that there is no perception that "*must* be the last," this does not establish there is no perception that "*must* be the last," this does not establish there is no perception that is "*in fact* the last"—perhaps "the perceiver eventually turns his attention elsewhere."¹²⁷ Despite his pessimism concerning the success of Kant's solution to the antinomies, however, Van Cleve concludes that Kant clearly *intended* to resolve the antinomies by claiming that "material things are identical with certain of our perceptions."¹²⁸ That is, he intended to resolve the antinomies by embracing actual state phenomenalism.

4. Chapter Summary and Transition to Chapter 2

As indicated previously, my ultimate goal will be to vindicate a metaphysical reading of the antinomies' resolutions.¹²⁹ In particular, I will argue that Kant resolves the mathematical

¹²⁵ Van Cleve (1999), 69. Note that Van Cleve sometimes describes appearances as "virtual objects," which might lead one to believe that they are intentional objects and not constructions out of actual mental states (which would make his view much closer to the one I endorse in chapter 5). But Van Cleve's discussion of "virtual objects" makes clear that they are just shorthand for talk of sequences of (actual) representations (see especially 1999, 8-9). ¹²⁶ Van Cleve (1999), 70. As I explain in chapter 2 below, there is good evidence that Kant in fact considered a

successive infinity to be *metaphysically* impossible (and so the possibility to which Van Cleve points here is one that Kant rejects). ¹²⁷ Van Cleve (1999), 70.

¹²⁸ Van Cleve (1999), 70.

¹²⁰ van Cleve (1999), 71

¹²⁹ This said, I do not mean to suggest that Kant's metaphysical solution does not have semantic and epistemic upshots. Rather, I intend to show that a metaphysical claim about what spatiotemporal objects *exist* explains the both false solution, even if that claim has consequences for Kant's views on what we can know and cognize, what epistemological principles should guide us in inquiry, what objects we can genuinely refer to, and so on.

antinomies by affirming a thesis of *metaphysical indeterminacy* for the features of the spatiotemporal world treated in the first and second antinomies: according to Kant, the series of conditions at issue in the first and second antinomies do not form totalities because it is metaphysically indeterminate how many conditions are in them. This metaphysical indeterminacy explains why they are neither finite nor infinite and hence why the thesis and antithesis statements of the first and second antinomies are *both false*.

I also offer the following account of the indirect argument for idealism to go along with this metaphysical reading. According to Kant, transcendental realists are committed to the truth of the Supreme Principle for fundamental reality, and in virtue of this they must hold that every series of spatiotemporal conditions is either determinately finite or determinately infinite. Once this is granted, however, the arguments of theses and antitheses are up and running, and the antinomy is inevitable (or so Kant argues). In contrast, transcendental idealists deny that spatiotemporal phenomena are part of fundamental reality, and for this reason it is open to them to hold that series of spatiotemporal conditions are metaphysically indeterminate rather than either finite or infinite (since the Supreme Principle need not apply to them). Moreover, according to Kant, transcendental idealism provides a positive explanation of *how* the series of conditions relevant to the first and second antinomies in fact turn out to be neither finite nor infinite (an explanation I present in chapter 5).

Is this reading compatible with the anti-realist and actual state phenomenalist readings just reviewed? Unlike actual state phenomenalists such as Van Cleve, the metaphysical indeterminacy approach I defend holds that empirical reality extends well beyond our *actual* perceptions (or constructions out of them). For as I argue, Kant thinks facts about what cognitions are *possible* for minds like ours determine the scope of *actual* spatiotemporal reality,

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and one consequence of this is that spatiotemporal reality extends beyond anything that is represented in our *actual* perceptual states. As I suggest (in chapter 5 below), reading Kant as a kind of phenomenalist is not incorrect, but he is most plausibly read as an *intentional object* phenomenalist, i.e., as a phenomenalist who holds that appearances are the representational contents of experience (Erfahrung). Second, while my view is compatible with the metaphysical anti-realist's claim that spatiotemporal existence is not experience-transcendent, I assign to spatiotemporal objects (on Kant's view of them) a kind of mind-dependence that some antirealist interpreters would reject (e.g., Allais). However, as I see it, this form of mind-dependence is especially well-suited to capturing Kant's claim in the resolution of the antinomies that appearances depend for their existence on a successive regress (another claim I explain in greater detail in chapter 5). Finally, although my view has important points of overlap Chiba's (especially insofar as I argue that the impossibility of infinite *successive* syntheses plays an important role in Kant's overall picture), I reject Chiba's claim that the notion of *potential infinity* is at the heart of Kant's solution to the antinomies. As I argue (in chapter 4), to say that a magnitude is *actually* indeterminate is different from saying that it is *potentially infinite*, and Kant's solution to the antinomy turns on the notion of indeterminacy rather than potential infinity. Establishing this is the main aim of chapter 4.

I now turn to explaining Kant's views on the relationship between the notion of *totality* and the notion of *infinity* (i.e., the first interpretive question mentioned above). This will provide the basis for the metaphysical indeterminacy approach that I defend in the remaining chapters.

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Chapter 2: Totality and Infinity in Kant's Critiques of Rational Cosmology

The primary goal of this chapter is to explain Kant's views on the relationship between the notions of *totality* and *infinity* and the role these two notions play in his criticisms of rational cosmology. As we have seen above, Kant argues that the antinomies arise from a misapplication of the notion of "absolute totality" to appearances in space and time (A506/B534). He also presents the mathematical antinomies as conflicts between finitist and infinitist alternatives (A505/B533). This suggests that to understand the antinomies, we need to understand exactly how Kant conceives of the relationship between questions about *totalities*, on the one hand, and questions about the *magnitude* properties of the world, on the other. That is, how does Kant's claim that the infinite and finite alternatives are *both false* relate to his claim that the notion of "absolute totality" is not applicable to appearances? And when Kant refers to the notions of "absolute totality" and "infinity," what does he understand by them?

In section 3.1 of chapter 1 above, I identified three different answers to these questions that are currently represented in the secondary literature on Kant. First, some commentators hold that Kant thinks that whatever *is* infinite cannot be a totality. On this reading, Kant affirms the infinity of appearances and holds that the infinite alternatives in the mathematical antinomies are false because they assert that infinite series *can* be totalities (not because they falsely assert that there *are* infinite series of conditions). Second, some commentators hold that Kant *identifies* the notion of actual infinity with the notion of an infinite totality and therefore means to resolve the antinomies by claiming that spatiotemporal phenomena are merely potentially infinite. And finally, a third group of commentators holds that the notion of totality is applicable to appearances in space and time, but this denial neither assumes nor rules out the infinity of spatiotemporal series of conditions.

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In this chapter, I defend the following claim about Kant's own understanding of the notion of totality and its relationship to debates about the infinite. When Kant asserts that "absolute totality" is not applicable to appearances in space and time, he means to deny that the series of conditions treated in the first and second antinomies are either finite or infinite; however, he does not simply equate the notion of actual infinity with the notion of an infinite totality. Rather, on Kant's view, actual infinities may or may not be totalities in the sense intended in the term "absolute totality". I argue that a proper account of the antinomies requires acknowledging that Kant distinguishes between two different notions of totality throughout his career, and when he denies that the notion of "absolute totality" is applicable to appearances, he has just one of these notions in mind. Moreover, Kant believes that series of conditions treated in the first and second antinomie of infinite, so if the series of conditions treated in the first and second antinomies *do* turn out to be *neither* finite *nor* infinite, they cannot be "absolute totalities".¹³⁰

The chapter is divided into four main sections. In section 1, I show that a distinction between two different notions of totality first appears in Kant's pre-Critical account of the cosmological notion of a *world*. According to one of these notions, which I call a *totality in the unity sense* (also a *unified totality*), a multiplicity of items is a totality if its elements stand in real relations of interconnection—in virtue of these relations, the multiplicity is unified to form a

¹³⁰ This raises at least two further questions. One question is what alternative there is to finitude and infinitude. As I'll argue in chapter 3, Kant thinks a series that is indeterminate in magnitude is neither finite nor infinite. Another question is how transcendental idealism helps to explain the possibility of spatiotemporal series of conditions that are neither finite nor infinite. Here, there are two further points to be made. First, because spatiotemporal conditions are not a part of fundamental reality (according to transcendental idealism), the Supreme Principle need not hold for them, and hence they can be neither finite nor infinite (I explain this in the course of chapter 2's discussion). Second, given the way in which spatiotemporal conditions depend for their existence on the "successive regress" per transcendental idealism (A506/B534), it is intelligible how their indeterminacy *results from* their ideality (I explain this in chapter 5).

whole. According to the second notion, which I call a *totality in the completeness sense* (also a *complete totality*), a multiplicity is a totality if it is not a part of a greater multiplicity of the relevant kind; that is, a collection is a *complete totality* if it meets the mereological condition of not being a part or subset of a greater collection of the relevant kind. In the pre-Critical period, Kant argues that a *world* must be a totality in *both* senses, but unified totalities can in principle fail to be complete totalities, and complete totalities can in principle fail to be unified totalities.

In section 2, I present Kant's pre-Critical views on how we know that the world is a totality in both senses and explain how this bears on the questions about the possibility of infinite totalities. Focusing on the view expressed in the Inaugural Dissertation of 1770 (ID), I argue that Kant thinks the spatial and temporal interconnectedness of things proves the status of the world as a unified totality, and so our sensible faculty of representation provides us with adequate evidence that the world is a totality in the unity sense, regardless of whether or not it is infinite. In contrast, Kant holds that an argument of the *pure understanding* is required to establish that the world is a totality in the completeness sense. The reason for this is that sensibility represents the world as both infinite and successive, and according to Kant, a complete successive infinity is impossible. Importantly, however, Kant does not hold that all infinite multiplicities cannot be complete totalities; he allows that non-human minds can grasp infinite magnitudes in a non-successive manner, and he considers this to be evidence that infinite complete totalities are metaphysically possible.¹³¹ In this section, I also highlight a problematic feature of the account provided in the *Inaugural Dissertation*. In the *ID*, Kant argues that we should not infer from the fact that we must represent magnitudes successively (in intuitive

¹³¹ As will become clear below, Kant distinguishes between merely *thinking* (*denken*) of an object, which proves its *logical* possibility and grasping it in a richer way (in the *ID*, "distinctly apprehending" it), which proves its *real* or *metaphysical* possibility.

cognition) to the conclusion that actually infinite multiplicities are impossible altogether. However, he also characterizes the deliverances of *pure understanding* in the *ID* such that it is unclear how reality could turn out to be actually infinite after all. For according to Kant, sensibility *misleads* us into ruling out the actual infinity of the world, but a proof of the pure understanding nonetheless establishes that the world must be "limited" in magnitude. At least on the face of it, it is not clear how these two claims fit together as part of a coherent picture. How can we believe that the world's magnitude must be "limited" and at the same time hold that it might turn out to be actually infinite? In the *ID*, Kant leaves this problem unresolved.

In section 3, I show how the view Kant develops in the *Critique* resolves the aforementioned issue in the *ID*. Whereas in the *ID* Kant argues that the pure understanding must represent complete totalities as "limited" in magnitude, in the *Critique* he holds that *pure reason* is neutral as to whether a complete totality of conditions is finite or infinite. This goes hand in hand with Kant's Critical abandonment of the view that pure reason delivers substantive cognition of things in themselves. For although in the *Critique* Kant holds that purely rational considerations *can* guarantee us that there are *complete totalities* of conditions among things in themselves (assuming there are any conditioned things in themselves at all), he denies that this tells us anything in particular about how fundamental reality must be; since a complete totality can be either finite or infinite, Kant reasons, we cannot conclude from the fact that there are complete totalities of conditions among things in themselves that they are either finite or infinite or infinite, we cannot conclude that they are either finite or infinite, is magnitude is limited, we cannot conclude that there are simples, and so on.

Finally, in section 4, I return to the extant views on the relationship between notions of infinity and totality in Kant scholarship and explain how the claims defended in this chapter

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represent an improvement. As I argue, the extant views fail properly to distinguish between the notions of *unified totalities* and *complete totalities*, and as a result, they give incomplete and in some cases flawed accounts of the relationship between infinity and totality, as well as of Kant's claim that the notion of "absolute totality" is "valid" as a condition of things in themselves but not of appearances (A506/B534). Kant does not hold that infinite multiplicities cannot be totalities in either sense, and when he denies that the notion of "absolute totality" is "valid" as a condition of the condition of appearances, he means to deny *only* that appearances form *totalities in the completeness sense*.

1. Two Notions of Totality in the Pre-Critical Kant

Some of Kant's earliest documented remarks concerning the notion of totality occur in the metaphysics lectures recorded by Herder in the early 1760s. In discussing what is required for the existence of a *world*, Kant distinguishes in these lectures between two conditions that any multiplicity of elements must satisfy if it is to satisfy the definition of a world. First, a world must be a "*real whole <totum reale*>," which is to say that "all things in it stand in real connection" (*MH*, 28:39).¹³² Second, a world must be "a whole which is not a part of another <*totum quod non est parts alterius*>"; if it did not meet this mereological condition, Kant argues, then it "would be only a piece of the world" (ibid).¹³³ Putting these two conditions together, Kant

¹³² Here Kant cites §357 of Baumgarten's *Metaphysics*, which says that "In every world there are actual parts (§354, 155), each of which is connected with the whole (§14, 157), and hence each part is connected with every other (§33). Therefore, in every world there is a nexus of parts and a universal harmony (§48); i.e., *a world admits of no islands*."

¹³³ This condition is also endorsed by Baumgarten: "A world [...] is a series (multitude, whole) of actual and finite beings that is not a part of another" (*Metaphysics*, §354). It is worth noting, however, that Baumgarten builds finitude into the definition of the world, whereas Kant goes out of his way to say that the finitude of the world *cannot* be assumed. Commenting on Baumgarten's §354, Kant says, "It is not necessary that the finitude of the

defines a world as follows: "The world is therefore a (real) whole of actual things, which is not a part of another *<mundus ergo ist totum (reale) actualium, quod non est pars alterius>*" (ibid).

Kant retains this conception of a world in the 1770s, where he again claims in his lectures on metaphysics that a world is a (i) really connected whole that is (ii) not a part of another. He spells out the first of these criteria by saying that a world must be a "substantial composite" rather than a mere "aggregate"—a "substantial composite" is something whose elements are really reciprocally connected, and a mere "aggregate" is something in which "many things <plura> that stand in no reciprocal connection are thought" (*ML*₁, 28:196).¹³⁴ As in the earlier decade, Kant again articulates the second criterion in mereological terms:

The difference of the world from every other composite *<compositio>* is: that the world is a substantial whole which is not a part of another *<totum substantiale, quod non est pars alterius>*. - The plurality which is subordinate to none larger is the totality *<omnitudo*; G: *Allheit>*." (*ML*₁, 28:196)

Notice that in describing this mereological condition, Kant does not indicate that a multiplicity of items cannot be a "totality" (*omnitudo* or *Allheit*) unless it is also a substantial composite (*totum substantiale*). Instead, Kant says that a *plurality* (*plura*) is a totality if it is "subordinate to none larger". Thus, one can call a plurality a "totality" even if its members do not stand in real connection with one another, and a collection of items standing in relations of real connection may not be a "totality" (i.e., *omnitudo*). However, because a *world* must meet not only the mereological condition but also the condition of real interconnectedness, Kant defines a world as the conjunction of the two conditions: a world is a totality of reciprocally interconnected

world, which is yet to be proven, is brought into the definition" (MH, 28:39). In the same set of lectures, Kant indicates that we simply cannot know if the world is mathematically infinite, that is, if it is "in comparison with unity greater than every number" (MH, 28:40). "Is the world infinite in this way?" Kant asks, "Who is to say? – God can indeed imagine an infinite without number, like eternity, but this concept is still difficult to conceive" (ibid). ¹³⁴ That is, in a substantial composite, elements must be *really* connected with one another rather than connected only in *representation*.

substances, i.e., a (i) substantial whole that (ii) is not a part of another. Note that the two conditions are conceptually independent of one another despite being necessary requirements on a *world*: a plurality might in principle meet the mereological condition without meeting the real interconnectedness condition, and likewise a plurality could meet the real interconnectedness conditions without meeting the mereological condition.¹³⁵

Whereas in the metaphysics lectures, Kant identifies two main conditions in virtue of which a multiplicity counts as a world, in the *ID*, he argues that the conditions on worldhood are in fact threefold. First, a world must have *matter*, which provides the world with its parts and "which are here taken to be *substances*" (§2, 2:389). Second, a world must have *form*, "which consists in the *co-ordination*, not in the subordination, of substances" (§2, 2:390). And third, a world must be an *entirety*, or an "*absolute* totality of its component parts" (§2, 2:391).

How do these three conditions relate to the two conditions discussed in the metaphysics lectures? An examination of Kant's discussion in the *ID* makes clear that "*form*" and "*entirety*" correspond to the two conditions of worldhood discussed in the metaphysics lectures. The *form* of a world is that in virtue of which its parts count as really connected (rather than connected merely in representation). Kant explains this as follows:

[I]f there happened to be certain wholes consisting of substances, and if these wholes were not bound to one another by any connection, the bringing of these wholes together, a process by means of which the mind forces the multiplicity into an ideal unity, would signify nothing more than a plurality of worlds held

¹³⁵ As is well known, in holding that the elements of a world must be *really* reciprocally connected (rather than connected, say, only in *representation*), Kant takes himself to be spelling out an important alternative to the Leibnizian position, according to which substances comprise a world only in virtue of the pre-established harmony of their representations (where every substance is *causally* independent of all the others). See also Kant's claim that the coordination of items in a world must be "conceived of as *real* and objective, not as ideal and depending on the subject's power of choice, by means of which any multiplicity whatsoever may be fashioned into a whole by a process of adding together at will. For by taking several things together, you achieve without difficulty a *whole of representation* but you do not, in virtue of that, arrive at a *representation* of a *whole*" (*ID* §2, 2:390).

together in a single thought. (§2, 2:390)

And the condition of *entirety* guarantees that world is a whole that is not a part of another greater one. Kant writes:

[W]hen we consider some *given* compound, although that compound were still to be a part of another compound, there is always a certain *comparative* totality, namely, the totality of the parts which belong to that magnitude itself. But, in this present case [of a world], whatever things are related to one another as joint parts with respect to any whole *whatsoever*, are understood as posited together. (§2, 2:391)

Hence, *form* and *entirety* are the same as (i) the condition of real interconnectedness and (ii) the mereological condition, respectively.¹³⁶ Thus, in his metaphysics lectures and in his published pre-Critical writings, Kant holds that the elements of a world (i) must be *unified* so as to form a real whole rather than a merely representational whole and (ii) must include *all* of the really interconnected elements of the relevant type (such that the elements composing the world are not a mere part of a greater whole). In the *ID*, these two requirements come under the headings of *"form*" and *"entirety*" respectively.

Because Kant uses a variety of different terms to mark these requirements on worldhood in the pre-Critical period, I adopt the following terminology. A collection that is connected or unified in the way required by condition (i) above is a "*totality in the unity sense*". A collection that meets the mereological condition, i.e., condition (ii), is a "*totality in the completeness sense*". I will also call these "*unified totalities*" and "*complete totalities*", respectively.¹³⁷

¹³⁶ In fact, Kant's metaphysics lectures also identify a condition corresponding to the requirement of *matter*, for as he argues in ML_1 , a world must have *substances* as its matter, and "a whole of accidents is no world" (28:195). Thus, the difference between the three conditions of the *ID* and the two conditions of the metaphysics lectures is merely presentational.

¹³⁷ One might wonder why *unified totalities* deserve to be called "totalities" at all, since the notion pertains to the idea of collecting together (or unifying) elements and not to whether or not those elements are exhaustive in the sense of not leaving anything out. I'm sympathetic with this worry and think *unified totalities* could equally well be

Notice that although Kant employs these notions in the course of explaining what is required for the existence of a *world*, they are given general definitions such that they apply in broader contexts as well. For instance, suppose I want to know what it takes for a plurality of elements to constitute a further individual such that we must count it as a new item in our ontology. One possible view one could have on this issue is that a plurality of elements constitutes a further individual if it is a totality in the unity sense-that is, if it is a plurality of elements brought together by real relations of interconnection. This is not to say that one would have to say that unified totalities constitute further individuals (one could in principle hold that unified totalities are not new entities over and above their elements), but one *could* use the notion of a unified totality to specify what is required for some items to form a further individual. In fact, Kant himself seems to endorse a position of this sort when he says that the condition of real interconnectedness is a condition on any substantial compound whatsoever. As Kant writes, the reason a world requires "form" is that a mere collection of parts cannot account for the identity of a whole: "the identity of the *parts* is not sufficient for the identity of the whole; the identity of the whole requires an identity of characteristic *composition*" (ID §2, 2:390).¹³⁸ In making this point, I take Kant to be arguing that any collection of parts whatsoever can form a further individual only if they are "bound to one another" via real relations of interconnection (ibid).

called "unified wholes". But since Kant uses the term "totality" for both notions (despite their significant differences), I have chosen to disambiguate Kant's language with the terms "unified totalities" and "complete totalities". Note also that the distinction between unified and complete totalities as I describe it here is close to a distinction in Levey (2016) (though I intend to remain neutral on whether or not *unified totalities* are metaphysically distinct entities over and above their elements, and Levey arguably builds this into his version of the notion). ¹³⁸ Kant also thinks this shows that the relations in virtue of which the world is a unified individual must be understood as *possible* rather than *actual relations*. Because the *actual* relations between items in a world change while the identity of the world remains the same, Kant reasons, the form of the world must be constituted by

possible relations (which therefore can be essential to it) (ID §2, 2:390.)

That is, for Kant, real interconnectedness is a condition on *all* substantial compounds and not only on a world.

A second important point is that the notion of a *totality in the completeness sense* has special significance for questions in cosmology. For whereas the notion of a *totality in the unity sense* explains what the world has in common with other substantial compounds, the notion of a *totality in the completeness sense* lets us explain what makes the world different from all other substantial compounds. According to Kant, the world is unlike all other substantial compounds in that it is not only a unified totality but also a complete totality. That is, unlike all other unified totalities, the world is also a whole that is not a part of a greater whole, i.e., it is a *totality in the completeness sense*.

Finally, notice also that Kant's discussion of the world's status as a *complete totality* does not commit him to the view that absolutely everything that exists belongs to the world. In particular, because a world must be not only a complete totality but also a unified totality, and because the relations of real interconnection required for unified totalities are relations of *mutual* interaction in the *ID*, Kant is able to argue that God is not a part of the world. That is, because God does not stand in relations of mutual interaction with created things, God is outside the world rather than a part of it.¹³⁹ Thus, the world is properly speaking the complete totality *of all mutually related things* (rather than the complete totality of all things *simpliciter*). This is also an instance of a more general fact already observed: for Kant, some complete totalities are not unified totalities, and some unified totalities are not complete totalities.¹⁴⁰ The complete totality

 $^{^{139}}$ As Kant puts it, although God is the cause of the substances in the world, "the relation of caused to cause is not interaction but dependence" (*ID* §17, 2:407).

¹⁴⁰ Note that Kant in fact argues that God's independence guarantees that he cannot be a part of *any* other totality in the unity sense. But as I understand Kant, this does not establish that there cannot be any totalities *in the*

of all actual things is not a unified totality (because it includes God); the unified totality of a small substantial compound (such as an ordinary physical object) is not a complete totality of all substantial compounds because it excludes many other substantial compounds that are also parts of the world.¹⁴¹

1.1 Unified and Complete Totalities: Clarifications and Toy Cases

Before turning to Kant's understanding of the relationship between these two notions of totality and the notion of infinity, several clarifications are in order about how the notions of *unified totalities* and *complete totalities* should be understood. As a first point, recall that in explaining why God is not a part of the world, Kant appeals to the fact that the relations unifying the world are relations of *mutual* interaction (rather than one-way relations of connection). However, since Kant's discussion of unified totalities also emphasizes the importance of *real* connection (rather than of mere unity in *representation*), it is also natural to ask whether Kant might accept an expanded notion of a unified totality that requires real but not necessarily *mutual* relatedness. For instance, if elements are unified by one-way relations of real connection (such that they form, say, a series of conditions that are subordinated to one another), would Kant be willing to call these elements a unified totality (on an expanded understanding of the notion)? For now, I simply flag this issue, but below I will suggest that Kant *does* operate with an expanded notion of a unified totality in the *Critique* (and especially in his treatment of *series* of real conditions in the antinomics).

completeness sense that include God. For example, the complete totality of all actual things would include God (though it would not be a world because it would fail the unity condition). In general, being a member of a *complete totality* does not imply interdependence.

¹⁴¹ A further question I put aside here is whether a world could in principle contain only a single simple substance, given Kant's understanding of the notions of complete and unified totalities.

A second point of clarification concerns the notion of a *complete totality*. Consider the following toy case, which will help to bring the point to light:

<u>Garden Stones</u>: There are stones scattered throughout the soil in my garden. They make digging difficult, so I decide to collect them in a bucket. I collect a heavy bucket of stones on day 1, but when I return to my garden on day 2, I discover that I did not succeed in collecting all the stones—on day 2, I find some stones that my search on day 1 left out.

Now suppose that at the end of day 2, I want to know whether I have now succeeded in collecting all the stones in the garden. This is to ask the question whether the stones I have collected are a *totality in the completeness sense*. And since the notion of a complete totality is the notion of a plurality that is not a part of another (of the relevant kind), the answer to this question can be determined as follows. If the stones in my bucket are a mere subset of the stones that were in the garden, then I do not have the *complete totality* of stones; in contrast, if they are *not* a mere subset of the stones in the garden but instead include *all* the stones that were in the garden, then I do have the complete totality. But notice that the notion of a complete totality is employed here in a domain-relative way. If the stones in my bucket after day 2 should turn out not to be a complete totality, this would be because they are only a subset of the stones in the garden; it would not be because they are only a subset of the stones in my neighborhood, and it would also not be because they are only a subset of the stones in any other wider domain (e.g., the whole world). Rather, if the stones in my bucket should turn out not to be a complete totality, this would be because they are a mere subset of the stones relative to my domain of interestnamely, the domain defined by the predicate "things in my garden".¹⁴²

¹⁴² Are the stones in the bucket also a unified totality, by Kant's lights? According to Kant, they would be a "substantial whole" if they are unified via "real" rather than merely "ideal" (or representational) relations of interconnection, but perhaps surprisingly, it is somewhat unclear whether he would say that these conditions obtain (*ID* §2, 390-1). On the one hand, the stones in the bucket do not *seem* to be a "*substantial* whole," and it would seem

Interestingly, Kant recognizes this feature of the notion of a complete totality and notes that there is a sense in which *every* collection is a complete totality with respect to *some* domain. As Kant argues, one can specify one's domain of interest as narrowly as one wants, and on a certain narrow specification, every multiplicity of items possesses a certain "*comparative* totality". Kant writes, "when we consider some *given* compound, although that compound were still to be a part of another compound, there is always present a certain *comparative* totality, namely, the totality of the parts which belong to that magnitude itself" (*ID* §2, 2:391). Put differently, on Kant's view, any collection whatsoever is a complete totality relative to the domain specified by itself. Thus, in <u>Garden Stones</u>, the stones are (trivially) a complete totality with respect to the domain defined by the predicate "things in the bucket".

However, Kant also stresses that a *world* must be not only a "*comparative* totality" but also an "*absolute* totality". He writes: "in the present case [i.e., in the case of the *world*], whatever things are related to one another as joint parts with respect to any whole *whatsoever*, are understood as posited together" (*ID* §2, 2:391). Thus, we can say that in cosmology, we are interested in *complete totalities* relative to the widest possible domain. We are interested not just in whether some things constitute a complete totality relative to some narrow domain but also in whether they constitute a complete totality "with respect to any whole *whatsoever*" (to borrow Kant's language).¹⁴³

that Kant would want to be able to distinguish between a stones-in-bucket case and a case of items bound together more robust way. But on the other hand, Kant holds that all things have determinate spatial locations only insofar as they are in community, and this would seem to establish that any items in space *are* unified by real relations of interconnection. See Messina (2017) for further discussion of Kant's views on the relationship between spatial location and mutual interaction.

¹⁴³ To repeat, however, this does not include God in the world, since God is not a part of the world per the *unity* constraint. That is, the complete totality of all mutually interconnected things does not include God, even with respect to the widest possible domain.

This raises the following question: given that a collection is always a complete totality relative to the domain specified by itself, can we likewise say that it is trivial that there is a complete totality of all things relative to any domain whatsoever? That is, if it is trivial that everything is a complete totality relative to itself, is it likewise trivial that there is an "absolute totality" of things (to borrow the language from the *ID*)? On the one hand, one might reason that it *is* trivial as follows. Consider an unrestricted domain, D_U . This domain must be a complete totality relative to itself (per the above). But since there are no domains more inclusive than D_U , D_U must be an "absolute totality", i.e., a complete totality relative to any domain whatsoever.

On the other hand, however, one might also challenge this line of reasoning by pointing to a case like the following:

<u>Sets of Sets</u>: I wonder whether there is a set of all the sets that do not contain themselves. Reflecting on this, I realize there cannot be such a set. For no matter what set I consider, it cannot contain itself if it is a set of sets that do not contain themselves. But if it does not contain itself, then it does not contain *all* the sets that do not contain themselves—it leaves itself out.

As one might reason, if there is no complete totality of sets that do not contain themselves, then it is likewise conceivable that there is no complete totality of things in the world.¹⁴⁴ For although *things* and *sets* may have very different properties (which may require giving a very different argument for the conclusion that there is no complete totality of things), <u>Set of Sets</u> at least renders coherent the idea that the existence of a complete totality relative to any domain

¹⁴⁴ Note that if the line of reasoning articulated in <u>Set of Sets</u> is correct, then there is no set *whatsoever* that contains all the sets that do not contain themselves. Moreover, if we further assume that domains must be specified in terms of sets that do not contain themselves, then there also cannot be a domain of all the sets. Any domain leaves a set out, namely, itself. And so as one might conclude (via an admittedly controversial inference), if there is no domain containing all the sets, then there is no universal domain, and the argument rehearsed in the paragraph above cannot get off the ground. All this said, however, nothing in my argument turns on this line of reasoning being correct, and indeed one might point out that if domains can be defined by classes rather than by sets, then there might be a domain containing all the sets, and a complete totality of sets might exist after all (though it would have to exist as a class rather than as a set).

whatsoever sometimes is not trivial. Given <u>Set of Sets</u>, one can imagine the possibility that the notion of a complete totality of all things leads to absurdity; and in this case, we might conclude that since the notion of a *world* requires the existence of a complete totality of things, a world properly speaking does not exist.¹⁴⁵

What is Kant's view on this question? Is it trivial on Kant's view that the world exists as a complete totality "with respect to any whole *whatsoever*"? In the section that follows, I argue that Kant does not think this is a trivial truth and that seeing why this is so is a necessary prerequisite to understanding his views on the relationship between totality and infinity.

2. Successiveness and Completeness in the Inaugural Dissertation

For those interested in Kant's criticisms of metaphysics, the *Inaugural Dissertation* represents an important transitional document. On the one hand, Kant develops an account of metaphysical error in the *ID* that turns on his distinction between *sensible* and *intellectual* representation, a distinction that remains important in the *Critique of Pure Reason*. As Kant argues in the *ID*, when metaphysicians allow cognition via the pure intellect to be infected by sensible representation, they succumb to metaphysical error.¹⁴⁶ On the other hand, in the *ID*, Kant holds that the pure intellect *can* get us substantive knowledge of things as they are in themselves, and this is a claim Kant comes to abandon by the time of the publication of the first edition of the *Critique* in 1781 (at least for theoretical philosophy). In the *Critique*, Kant argues

¹⁴⁵ See Levey (2016) for a related argument concluding that there is no complete totality of all contingent truths. ¹⁴⁶ Not all commentators agree on how the main thrust of the *ID* relates to the main thrust of the first *Critique*. For example, Bird (2006) argues that metaphysical error in the *ID* boils down to taking *sensible* representations to be objective, whereas the *Critique* reverses this and says that metaphysical error boils down to taking *rational* representations to be objective (607-8). In contrast, De Boer (2020b) argues that in both the *ID* and the *Critique*, Kant holds that metaphysical error results from allowing intellectual representations to be infected by sensible ones.

that we are ignorant of the character of things in themselves *even if* pure reason can tell us minimal facts about fundamental reality (such as that conditioned things in themselves must form complete totalities *if* there are any conditioned things among things in themselves at all).¹⁴⁷

To see why Kant thinks it is *non-trivial* that the world exists as a complete totality, we should start with his discussion of the errors we make when we do not have a proper understanding of the differences between our sensible and intellectual faculties of representation. In the *ID*, Kant argues that our faculty of *sensibility* is governed by the laws of intuitive cognition and represents things "as they appear," while our faculty of intelligence is governed by the laws of "pure understanding" (or "reason") and represents things "as they are" (ID §4, 2:392). According to Kant, anything that "conflicts with the laws of the understanding and the laws of reason is undoubtedly impossible," but the laws of sensibility are not a completely trustworthy guide to reality in this way (ID §1, 2:389). As Kant writes, "whatever cannot be cognised by any intuition at all is simply not thinkable, and is, thus, impossible," but it is an error to assume that *our* kind of intuition is the only possible one (ID §25, 2:413). Because of this, we should not conclude that because we cannot represent something in intuition (i.e., sensibly), it cannot occur in reality; for Kant, it is only "the incautious" who err in "taking the limits, by which the human mind is circumscribed, for the limits within which the very essence of things is contained" (*ID* §1, 2:389).

With this distinction between the faculties of sensibility and intelligence (or pure understanding) in place, Kant makes two important points in the *ID* about how we know that the world exists as both a unified and a complete totality. First, he argues that *sensibility* gives us a representation adequate to justify the conclusion that the world is a *unified totality*. Although we

¹⁴⁷ See A498/B526.

do not represent the true natures of the world's relations of interconnectedness, our faculty of sensibility represents all things as related to one another in space and time, and Kant says space and time therefore "bear witness to some common principle constituting a universal connection" among things (*ID* §2, 2:391).¹⁴⁸ As Kant writes, space and time are not "primitive conditions which are already given in themselves, and, in virtue of which [...] it would not only be possible but also necessary that a number of actual things should be mutually related to one another as joint parts and should constitute a whole," but they nonetheless "bear witness to some common principle constituting a universal connection, though they do not expose it to view" (*ID* §2, 2:391). In other words, although our sensible representations of things as in space and time do not give us insight into the nature of the specific relations that make the world into a *unified totality*, they do give us evidence that the world *has* an underlying form and hence that it *is* a totality in the unity sense.

The second important point Kant makes in the *ID* concerns our knowledge of the world's status as a *complete totality*. Whereas Kant argues that sensibility gives us a representation adequate to justify the conclusion that the world is a totality in the unity sense, he claims that a proof of the *pure understanding* is required for us to know that the world is a totality in the completeness sense. According to Kant, the reason for this is that our sensible representations do not allow us to represent the world as a totality in the completeness sense; in fact, they suggest to us that the world *cannot* be a complete totality. Kant explains the problem with our sensible representations as follows:

[A]bsolute *totality* may present the aspect of an everyday and readily accessible concept [...] Yet, when we reflect on it more deeply, it is seen to present the

¹⁴⁸ According to Kant, the faculty of sensibility must represent things by means of "a certain law, which is inherent in the mind and by means of which it co-ordinates for itself that which is sensed from the presence of the object" (*ID* §4, 2:392). For minds like ours, this manifests as all things being represented as in space and time.

philosopher with a very serious problem. For it is hardly possible to conceive how the *never to be completed series* of the states of the universe, which succeed one another to *eternity*, can be reduced to a whole, which comprehends absolutely all its changes. Indeed, it necessarily follows from its very infinity that the series has no *limit*. Accordingly, there is no series of successive things except one which is part of another series. It follows that, for this same reason, comprehensive completeness or *absolute totality* seems to have been banished altogether here. [...] Let him who is to extricate himself from this thorny question note that neither the successive nor the simultaneous co-ordination of several things (since both co-ordinations depend on concepts of time) belongs to a concept of a whole which derives from the *understanding* but only to the conditions of *sensitive intuition*. (*ID* §2, 2:391)

In other words, Kant argues that because *sensibility* represents the world as series of *successive* states going to eternity, we cannot represent the world as a *complete totality* via our sensible faculties.

This raises two important questions. First, how should we understand Kant's claim that representing the world as *successive* is incompatible with representing it as a *complete totality*? Second, does Kant think that we should be agnostic as to whether there is a complete totality of things, given our inability to represent it sensibly? Or does he think we have some other way of representing its completeness (and knowing that a world in the true sense exists)?

Taking these questions in turn, notice that Kant goes out of his way to say that our inability to sensibly represent the world's completeness should *not* lead us to conclude that the world must be finite. In section §1 of the *ID*, he argues that "the concepts both of the *continuous* and of the *infinite* are frequently rejected" because "*according to the laws of intuitive cognition*, any representations of these concepts is absolutely impossible" (2:388). But as Kant stresses, it is an *error* to "reject the actual mathematical infinite" on these grounds (*ID* §1, 2:389 fn). While it is true that an "actual mathematical infinite" is not *measurable*, Kant argues that

measurability here only denotes relation to the unit adopted by the human

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understanding as a standard of measurement, and by means of which it is only possible to reach *the definite concept of a multiplicity* by successively adding one to one, and the *complete* concept, which is called a *number*, only by carrying out this progression in a finite time [...] [But] things which do not accord with a fixed law of a certain subject do not, for that reason, pass beyond all understanding. For there could be an understanding, though certainly not a human understanding, which might distinctly apprehend a multiplicity at a single glance, without the successive application of a measure. (*ID* §1, 2:389 fn)

Since a non-human understanding might "distinctly apprehend" an infinite multiplicity "at a single glance," Kant argues, actually infinite multiplicities are not objectively impossible after all.¹⁴⁹ Thus, although *we* cannot sensibly represent that world as both infinite and complete in our successive mode of representing magnitudes, we should not conclude that the world *as it really is* must therefore be finite.¹⁵⁰ We should allow for the infinity of the world and not succumb to the "subreptic axiom" according to which "*every actual multiplicity can be given numerically*, and thus every magnitude is finite" (*ID* §28, 2:415).¹⁵¹

Importantly, however, when Kant claims that there might be an understanding that can achieve a "definite concept" of an infinite multiplicity, he does not claim that this understanding could accomplish this via a successive mode of representation (if only its representational powers were not finite). Rather, Kant says that a mind unlike ours could "distinctly apprehend" an infinite multiplicity if it could do so in "a single glance, *without the successive application of a measure*" (*ID* §1, 2:389 fn, my emphasis). Since "*whatever cannot be cognised by any intuition*

¹⁴⁹ Recall his suggestion at *ID* §25 (2:413) that representability by *some* intuition is a reliable indicator of possibility. ¹⁵⁰ There is some controversy as to whether the *ID* understands the distinction between what sensibly appears to us and what we represent intellectually as a distinction between two kinds of objects or as a distinction between two ways in which we represent the same objects. However we should go on this issue, it is clear that Kant's discussion of magnitude in the *ID* is focused on warning us against thinking that things as they really are must be finite since sensibility cannot cognize things as infinite. See Carson (2004) for an especially helpful discussion of the nature of the distinction between things "as they appear" and things "as they are" in the *ID*. Guyer (1987) and Friedman (1992) offer competing takes.

¹⁵¹ See also Kant's discussion of our tendency to succumb to "the illusions of sensitive cognitions, which masquerade under the guise of cognitions of the understanding" (*ID* §26, 2:413).

at all is simply not thinkable, and is, thus, impossible," it is reasonable to conclude that Kant thought a complete *successive* infinity is impossible simpliciter. That is, for any mind whatsoever (even a divine mind) a complete successive infinity cannot be represented; hence, a complete successive infinity is impossible altogether (i.e., metaphysically impossible).¹⁵²

On the second issue (as to whether we should be agnostic as to the world's status as a complete totality), Kant's position in the *ID* is that our inability to represent the world as a complete totality in intuitive cognition should *not* lead us to conclude that no such "world" in the true sense of the term exists. Instead, Kant argues that "it can easily be shown by an argument, which is based on reasons deriving from the understanding, that both simples and a world are given" (*ID* §1, 2:389). Since a world cannot be given without being given as both a unified and a complete totality, it follows that our faculty of understanding furnishes a proof that the world as a complete totality exists. According to Kant, there is no conflict between this rational proof and the impossibility of a complete infinite succession because, as we have seen, the appearance of the world to us as successive is merely a representation of things *as they appear* and not *as they are (ID* §4, 2:392). As Kant writes, the limitations of sensible representation do not imply that a representation of the world as unified and complete "cease[s] to belong to the understanding. It is

¹⁵² But are Kant's views here on the impossibility of complete successive infinities compelling? One could imagine an objection against him that goes as follows. It is true that in an infinite successive series, every *finite* series is part of another series, but this simply does not establish that the infinite series itself is part of another. Kant simply has not provided an argument for this latter conclusion, and indeed it seems conceivable that an infinite successive series *could* be a series that is not a part of another. Moreover, Kant himself points to some considerations that seem to undermine his own argument. First, he says that although a complete "*simultaneous infinite*" would seem to be possible "because *simultaneity* seems expressly to declare that there is a combination *of all things at the same time*," we can in fact rule out complete simultaneous infinities because they imply successive infinities (*ID* §2, 2:391). "[I]f a simultaneous infinite were admitted," Kant writes, "one would also have to concede the totality of a successive infinite" (ibid). But as one might worry here, Kant has drawn exactly the wrong conclusion: instead, one should argue that because simultaneous infinities *can* be complete totalities, a complete successive infinities are plausible.

sufficient for this concept that co-ordinates should be given in some way or other, and that they should all be thought as constituting a unity" (*ID* §2, 2:392).

Given Kant's arguments in the *ID* about our tendency to be misled by our sensible representations, one might expect him to argue that we must remain neutral as to what kind of complete totality the world turns out to be. That is, one might expect Kant to say that the world may turn out to be an actually infinite complete totality, but it may also turn out to be a finite complete totality; given our representational limitations, one might think, we simply cannot determine which option in fact obtains.¹⁵³

However, Kant does not draw this conclusion in the *ID*. Instead, he says that the pure understanding (i.e., intelligence) gives us substantive knowledge of the way reality is, and in particular, we are in a position to draw three substantive conclusions concerning the cosmological questions ultimately treated in the first *Critique*'s first through third antinomies. First, Kant argues that "according to the laws of pure understanding," a series of causes and effects cannot form a limitless regress (the topic of the third antinomy) (*ID* §28, 2:415). Second, he argues that the pure understanding can prove that "if there is a substantial compound, then there are principles of composition, that is to say, simples" (the topic of the second antinomy). And finally, he argues that by the mere understanding we can also know with certainty "that the magnitude of the world is limited (not a maximum)" (the topic of the first antinomy) (*ibid*, 2:416).

¹⁵³ Does it make a difference here whether Kant understands the distinction between things "as they are" and things "as they appear" as a distinction between two kinds of objects or a distinction between how we represent one and the same set of objects? Although I'm sympathetic to the latter reading of the *ID*, Kant's warnings about drawing faulty inferences about things *as they are* end up the same: Kant is telling us not to use sensible representations to draw inferences about things as they are.

How can this be made compatible with the fact that Kant claims that we should not rule out the infinity of the world? Consider the following passage in which Kant suggests that the understanding's representation of the world's magnitude as "limited" does not establish that world is *mathematically* finite:

Accordingly, that the magnitude of the world is limited (not a maximum), that it acknowledges a principle of itself, that bodies consist of simples — these things can, indeed, be known under a certain sign of reason. But that the universe, in respect of its mass, is mathematically finite, that its past duration can be given according to a measure, that there is a definite number of simples constituting any body whatsoever — these are propositions which openly proclaim their origin in the nature of sensitive cognition. And, however much they may be true in other respects, they suffer nonetheless from the undoubted blemish of their origin. (*ID* $\S28, 2:416$)

In other words, Kant appears to hold that the understanding's proof that the world's magnitude is "limited" simply leaves untouched questions about the *mathematical* properties of the world. But how could this be? How could it be *both* that we know with certainty that the magnitude of the world is "limited" *and* that we nonetheless cannot conclude that it is mathematically finite? Kant indicates that mathematical properties are different from the worldly properties implicated in the pure understanding's proof, but he does not underwrite this claim with a further explanation of these two types of properties.¹⁵⁴ Thus, Kant's account in the *ID* remains incomplete and, to that extent, unsatisfying.

¹⁵⁴ Note: at one juncture, Kant suggests that the argument of the understanding makes a point about *dependence* rather than *measurability*, but still he does not explain *how* it can be established via this rational argument that "the magnitude of the world is limited (not a maximum)" or how this limitedness does not rule out an "*infinite series* of co-ordinates" (*ID* §28, 2:415-16). And more generally, even if Kant is right that a purely rational argument can establish that the world is a dependent being, it is not clear how this justifies the further conclusion that its *magnitude* is limited.

3. Critical Views on Totality and Infinity

How do Kant's views change between the time of the *ID* (1770) and the first edition of the *Critique* (1781)? One especially important change is that Kant gives up his earlier commitment to the pure understanding as a source of substantive cognition of the world as it is in itself. Related to this, he also modifies his account of the way in which our purely intellectual faculty represents complete totalities; whereas in the *ID* Kant holds that intelligence represents every complete totality as having a "limit", in the *Critique* he holds that pure reason represents complete totalities as *either* finite *or* infinite. As he argues in the *Critique*, a proof of pure reason can tell us that if there are any conditioned things in themselves, then there is also the complete totality of their conditions, but it cannot establish whether a complete series of conditions has a terminal, unconditioned member or whether it is instead infinite and unconditioned only as a whole.¹⁵⁵ Thus, pure reason cannot provide us with substantive cognition of reality as it is in itself, even if it can establish that complete totalities of conditions must exist among things in themselves if there are any conditioned things in themselves at all.¹⁵⁶

¹⁵⁵ Though I leave aside this issue in this chapter, pure reason also cannot tell us whether things in themselves are conditioned at all, at least not on the basis of its commitment to the Supreme Principle.

¹⁵⁶ The idea here is that substantive cognition would require being able to draw a more specific inference about whether the complete series of an object's conditions is infinite or finite. So for example, in the *ID* Kant argues that we are able to conclude that there are simples, and this counts as substantive cognition of things in themselves; in the *Critique* he argues that pure reason is neutral as to whether there are simples or whether everything is divided to infinity, and so pure reason simply gives us no substantive cognition of how things in themselves must be with respect to their magnitude properties. There are further questions about whether things in themselves *would* in fact be unconditioned if they do not stand in any conditioning relations at all (e.g., if there are only brute facts among things in themselves). Willaschek (2018) suggests that we should avoid understanding conditioning relations such that things that are not even *apt* to stand in a particular conditioned) (87, fn 34). If this is correct, then simply knowing that things in themselves are not conditioned would not tell us that they are unconditioned; we would need to know if they are apt to be conditioned (i.e., if there are *possible* objects that they *could* condition).

Recall from chapter 1 above that Kant believes the antinomies arise from a misapplication of the notion of "absolute totality" to appearances in space and time. As he writes,

[T]he antinomy of pure reason in its cosmological ideas is removed by showing that it is merely dialectical and a conflict due to an illusion arising from the fact that one has applied the idea of absolute totality, which is valid only as a condition of things in themselves, to appearances that exist only in representation, and that, if they constitute a series, exist in the successive regress but otherwise do not exist at all. (A506/B534)

He also argues that the "dialectical argument" underlying the antinomies involves an application of the Supreme Principle to conditioned appearances in space and time. Recall that the Supreme Principle says that "[w]hen the conditioned is given, then so is the whole series of conditions subordinated one to the other, which is itself unconditioned, also given" (A308-9/B364). So the dialectical argument driving the antinomies goes as follows: "If the conditioned is given, then the whole series of all conditions for it is also given; now objects of the senses are given as conditioned; consequently, etc." (A497/B525). Taking these claims together, we can also conclude that the "whole series" to which Kant refers in the Supreme Principle is an "absolute totality"; that is, the Supreme Principle employs the same notion of "absolute totality" that Kant says is responsible for the antinomies. Thus, applying the Supreme Principle to things in space and time is to apply the notion of "absolute totality" to appearances.¹⁵⁷

What is the relationship between this notion of "absolute totality" and the notion of *complete* and *unified* totalities discussed above? Kant's explanation of *why* the whole series of an object's conditions is always unconditioned makes clear that he has in mind the notion of a

¹⁵⁷ See also: "So the transcendental concept of reason is none other than that of the **totality of conditions** to that of a given conditioned thing. Now since the **unconditioned** alone makes possible the totality of conditions, and conversely the totality of conditions is always itself unconditioned, a pure concept of reason in general can be explained through the unconditioned..." (A322/B379).

totality in the completeness sense. Kant writes: "The absolute whole of the series of conditions for a given conditioned is always unconditioned, because outside it there are no more conditions regarding which it could be conditioned" (A417-18/B445 fn). In other words, if a series of conditions contains *all* of a conditioned object's conditions, Kant reasons, then it does not exclude any of the object's conditions. But if it does not exclude any of the object's conditions, then there can be nothing outside the series conditioning it, which is just to say that it is unconditioned. Notice that to say that the series does not exclude any of the object's conditions is to say that the series is not a mere subset or part of a greater collection of the object's conditions. Given this, we can conclude that Kant thinks the "absolute whole of the series of conditions for a given conditioned is always unconditioned" because it is a *totality in the completeness sense*.¹⁵⁸

How does this represent a change in Kant's views between 1770 and 1781? Notice that Kant employs the same notion of a *complete totality* in both decades, but recall that in the *Critique* (and not in the *ID*), he argues that pure reason is neutral as to whether the complete totality of an object's conditions occurs in a finite or in an infinite series of conditions. As Kant writes in introduction to the antinomies, the faculty of *reason* represents the unconditioned

either as subsisting merely in the whole series, in which thus every member without exception is conditioned, and only their whole is absolutely unconditioned, and then the regress is called infinite; or else the absolutely unconditioned is only a part of the series, to which the others are subordinated but that itself stands under no other condition. (A417/B445)

In the first case the series is "infinite and at the same time whole," while in the second case the series is finite. Either way, Kant argues, reason can represent the series as a *complete totality* of

¹⁵⁸ Note: this is not to say that it is not also a *totality in the unity sense*. But the reason it counts as *unconditioned* is not that it is a totality in the unity sense—it is that it is a totality in the completeness sense.
the object's conditions, i.e., as a collection of conditions that is not a part of a greater collection.¹⁵⁹

This point is important in the *Critique* because it contributes to Kant's explanation of how the antinomies arise from assuming that principles of pure reason can tell us about the nature of the spatiotemporal world. According to Kant, the antinomies are inescapable for transcendental realists but not for transcendental idealists because the assumption of transcendental realism forces one to apply the Supreme Principle to things in space and time. Why is this the case? According to Kant, the reason can be explained as follows. Transcendental realists *mistakenly* hold that appearances in space and time *are* things in themselves, but they also *correctly* hold that the Supreme Principle must be applicable to things in themselves. As Kant writes,

If the conditioned as well as its condition are things in themselves, then when the first is given not only is the regress to the second **given as a problem**, but the latter is thereby already **given** along with it; and, because this holds for all members of the series, then the complete series of conditions and hence the unconditioned is thereby simultaneously given, or rather it is presupposed by the fact that the conditioned, which is possible only through that series, is given. (A498/B526)

So according to Kant, the Supreme Principle must hold for things in themselves, which is to say that it *would* hold for spatiotemporal objects if they were things in themselves. But this just means that the inference from the conditioned to the unconditioned *would* hold for appearances

¹⁵⁹ Note here that Kant is siding against Leibniz, who argues that infinite wholes are impossible because they violate the whole-part axiom according to which the whole is always greater than the (proper) part (A, 6.3:549). In arguing that we can *think* of the unconditioned as occurring in an infinite whole series of conditions, Kant indicates that the idea of an infinite whole is at least not incoherent. This means, in addition, that Kant must either reject the wholepart axiom or reject the conception of size relied on in the Leibnizian proof that the whole is not always greater than the (proper) part in infinite quantities. My view is that Kant clearly takes the latter route, though a complete account of how he does this would require a paper of its own. For contemporary takes on this issue, see Mancosu's (2009) discussion of conceptions of size compatible with the whole-part axiom.

in space and time if transcendental realism were true. Importantly, the inference from the conditioned to the unconditioned in the Supreme Principle is neutral as to whether the complete totality of an object's conditions takes a *finite* or *infinite* form, so applying the Supreme Principle to the spatiotemporal world does not decide between the finite and infinite alternatives of the antinomies—it establishes only that one of them must be correct. Thus, by applying the notion of a *complete totality* to things in space and time via an application of the Supreme Principle to appearances, transcendental realists are forced to endorse one of the alternatives advocated in the thesis and antithesis statements, respectively. But it remains undecided which of the two must be correct, and this gives rise to the specific arguments in each of the antinomies.

Where does this leave us? We have seen that because reason's inference to the unconditioned is neutral as to whether the complete totality of an object's conditions is finite or infinite, in the *Critique* Kant holds that reason alone cannot decide whether the series of conditions for a given conditioned thing is finite or infinite. So even if the principles of pure reason *were* "valid" of things in space and time (something Kant ultimately denies), it would not follow that the spatiotemporal world must be finite, or that simples must exist.¹⁶⁰ This represents an important shift from the position advocated in the *ID*, where Kant held that "a certain sign of reason" *does* indicate that "the magnitude of the world is limited" and that "bodies consist of simples" (*ID* §28, 2:416).

¹⁶⁰ There are some places in the *Critique* where it looks like Kant *does* think a rational inference licenses the conclusion that simples exist if composites exist (e.g., A440/B468). Here, I see two interpretive options. One is to say that this is a holdover from the *ID*, which is not compatible with Kant's considered views about the Supreme Principle and the requirements on complete explanations. The second (likely more appealing) alternative is to say that *if* Kant is truly committed to the inference from the composite to the simple for things in themselves in the critical period, he does not see it as following from the Supreme Principle, which is neutral as to whether the series of conditions is finite or infinite.

Finally, notice that in the *Critique*, Kant also continues to hold that complete totalities of conditions can be actually infinite. For in saying that the faculty of reason can represent the unconditioned in the infinite case as "subsisting merely in the whole series, in which thus every member without exception is conditioned, and only their whole is absolutely unconditioned," Kant suggests that there is no barrier to thinking of an actually infinite series as one that is not a part of a greater series—i.e., there is no barrier to thinking of an actually infinite series as a totality in the completeness sense (A417/B445). This is confirmed when Kant says that we think of an infinite series as "infinite and at the same time whole" such that "outside it there are no more conditions regarding which it could be conditioned" (A417-18/B445 incl fn). It is also compatible with Kant's claim in the remark on the first antinomy that "[t]he true (transcendental) concept of infinity is that the successive synthesis of unity in the traversal of a quantum can never be completed" (A432/B460). If a series is actually infinite, Kant argues, then one cannot complete a *successive* process of counting or running through all of its members, but this is not to say that the series itself cannot be the complete totality of an object's conditions.¹⁶¹

Importantly, in the *Critique* Kant also is not simply defining the notion of an actually infinite series such that it is guaranteed to be a complete totality. According to Kant, a magnitude is actually infinite just in case its "relation to an arbitrarily assumed unit" is "greater than any number" (A432/B460).¹⁶² But a magnitude can satisfy this criterion without being a totality in the completeness sense. For consider a series of conditions in which every condition is conditioned by a further condition (such that the series is equinumerous with the natural numbers). If we exclude several members of this series (say, five of them) the remaining

¹⁶¹ Note that this latter point is of a piece with Kant's claim in the *ID* that a complete successive infinite is impossible.

¹⁶² Recall that Kant has a finitistic conception of number on which a number is always reachable by counting.

members of the series will still be infinitely numerous (by Kant's definition of actual infinity), but they will not be the complete totality of conditions (since they are a mere subset of the original series, which had five members that our collection of conditions now excludes). For Kant, an actually infinite series of conditions *can* be the complete totality of an object's conditions (if it includes *all* its conditions), but it is not in virtue of being actually infinite that the series is a complete totality; it is a complete totality if and only if it is not a mere part of another collection of the relevant kind.

Pulling the results of the foregoing sections together (and bracketing several complexities to be treated in the coming chapters), we can summarize the relationship between the finitude/infinitude distinction and the complete totality/unified totality distinction in a diagram as follows.¹⁶³ Just as there can be unified totalities that are not complete totalities (e.g., a single substantial compound meets the unity condition without being the complete totality of all substantial compounds), so too there can be complete totalities that are not unified totalities (the complete totality of actual things, which includes God but fails to meet the unity constraint as Kant articulates it in the *ID*). And just as there can be both finite and infinite unified totalities, so too there can be both finite and infinite complete totalities (putting aside the special case of successive infinities):

¹⁶³ Some of the complexities I put aside here are (a) where we should represent the distinction between determinate and indeterminate magnitudes in the diagram and (b) where potentially infinite phenomena belong. See the Appendix for the full picture, which requires arguments that have not yet been provided.



Figure 2.1 Totalities & Pluralities - Version 1

4. Comparison to Contemporary Scholarship

On the account developed in this chapter, Kant's discussion of the antinomies in the first *Critique* turns on his claims about *complete totalities*. When he says that the "**unconditioned** is always contained **in the absolute totality of the series**," he means that the unconditioned occurs wherever there is a *complete totality* of conditions (A416/B444). And when he says that the antinomies arise via an illegitimate application of the notion of "absolute totality" to appearances, he means that we must avoid the antinomies by denying that appearances are a *totality in the completeness sense* (A506/B534). Kant also holds that complete totalities of conditions can be either finite or infinite and that we can rule out infinite complete totalities only when those totalities depend on a *succession* (because a complete successive infinity is

metaphysically impossible on Kant's view).¹⁶⁴ In the final section of this chapter, I explain how this account compares to several other prominent accounts defended in the secondary literature.

Recall that there are currently three extant views on the relationship between infinity and totality in the resolution of the antinomies. First, some scholars hold that Kant argues that spatiotemporal series of conditions *cannot* be totalities because they *are* infinite. Second, other scholars hold that spatiotemporal series of conditions must be merely potentially infinite because they are not totalities. And finally, some scholars hold that series of spatiotemporal conditions cannot be totalities, but this is unrelated to whether or not they are infinite. With the distinction between *unified* and *complete* totalities in hand (and the understanding that the notion of "absolute totality" employed in the Supreme Principle is the notion of a *totality in the completeness sense*), we are now in a position to see where the views currently defended in the secondary literature go astray.

First, consider Allison's claim that series of spatiotemporal conditions cannot be totalities because they *are* infinite. Allison writes in his discussion of the first antinomy:

[T]he assumption that the series is infinite entails not merely that it cannot be completed in a finite time but that it cannot be completed at all. Moreover, if this is the case, then it does not constitute a world (totum syntheticum). We thus have two alternatives: either (1) the series does not constitute a world, or (2) there is a first moment. The correct Kantian option is the first.¹⁶⁵

Allison clarifies that he understands Kant's notion of a *world* such that a world must be a *totum syntheticum*, and the concept of a *totum syntheticum* just is the concept of an individual that is

¹⁶⁴ To repeat what was acknowledged in footnote 152 above, in the *ID* Kant seems to suggest that a successive infinity is possible if a simultaneous infinity is, and one might reasonably worry that he draws the wrong conclusion from this. That is, one might worry that he should conclude not that simultaneous infinities are impossible too but rather that successive infinities are possible after all. I set aside a discussion of the plausibility of Kant's position here, since my aim is to understand his views on the relationship between infinite and complete totalities rather than to assess whether or not he is right.

¹⁶⁵ Allison (2004), 370.

formed by a combination or bringing together of pre-given parts.¹⁶⁶ As the discussion above makes clear, however, the notion of a totality as a combination or bringing together of parts is the notion of a *totality in the unity sense*. So when Allison claims that Kant resolves the antinomy by denying that infinite series can form *tota synthetica*, he is claiming that Kant resolves the antinomy by denying that infinite series can form *unified totalities*.

What makes Allison's view interpretively less compelling than the one I have defended above? First, Allison's explanation of the solution to antinomy cannot account for Kant's discussion of *why* an "absolute totality" of conditions is always unconditioned. As we have seen, Kant argues that the absolute totality of the series of conditions for a given conditioned is always unconditioned "because outside it there are no more conditions regarding which it could be conditioned" (A417-18/B445). This is an explanation that appeals to the fact that the totality of the series is not a part of a greater series, which is to appeal to the status of the series as a *totality in the completeness sense*. But Allison instead argues that Kant's claims turn on his characterization of the world as a *totality in the unity sense* (in his claim that it is a *totum syntheticum*).

Second, contra Allison, Kant does not deny that actually infinite series can form unified totalities. For as we have seen at A417/B445, Kant holds that actually infinite multiplicities can be both *unified* and *complete* totalities of conditions. This follows from the claim that the unconditioned can occur in either infinite or finite series of conditions. If an infinite series of conditions is unconditioned, then it is a series outside of which there are no further conditions of the relevant type—i.e., it is a totality in the completeness sense. And if it is a *series* of conditions

¹⁶⁶ See Allison (2004), 369: "A totum syntheticum is a whole composed of parts that are given separately (at least in thought). Not only does the concept of such a whole presuppose its distinct, pre-given parts, it is also conceived as the product of collection (in Kant's term, 'synthesis') of these parts."

at all, then it is a collection of items unified by real conditioning relations—i.e., it is a totality in the unity sense. Or at the very least, assuming we allow a moderate expansion of the notion of a *unified totality* such that it includes whatever is unified via *any* kind of real conditioning relation and not only those things that are unified by relations of *reciprocal* interconnection (an expansion Kant endorses in the *Critique*), any *series* of conditions is a totality in the unity sense.¹⁶⁷ So, contra Allison, Kant clearly does not mean to deny that there are series of spatiotemporal conditions, and he would not hold that if something's conditions are infinite, then they cannot form either a unified or a complete totality.

Hence, we can allow that it is a necessary condition on the existence of a series that it be a totality in the unity sense, but this does not force us to say that Kant resolves the antinomy by denying that spatiotemporal conditions form unified totalities.¹⁶⁸ Kant's solution is to deny that spatiotemporal conditions form totalities in the completeness sense, and the claim that they are not complete totalities is what explains why the Supreme Principle is not "valid" of things in space and time.¹⁶⁹

¹⁶⁷ We should think the notion of a *unified totality* takes on this expanded meaning in the *Critique* because many of the conditioning relations Kant is most interested in are relations of "subordination" rather than of "coordination" (A409/B436). And despite the fact that "subordinating" conditioning relations form series of conditions in which conditioning goes in only one direction, there is a clear sense in which the conditioning relations *unify* the members of the series. So although Kant continues to think in the *Critique* that a *world* requires mutual interaction, there are many kinds of unified totalities that do not require reciprocal interaction (e.g., series of temporal conditioning, series of causal conditioning, and so on).

¹⁶⁸ Indeed, a unified totality *could* be a series conditioned from the outside by further conditions, since the unity condition does not establish that the totality in question is not a part of another. It is also worth emphasizing that neither a finite nor an infinite series *must* be a complete totality as a general matter. For if I start with an infinite series and consider just half of it, that half will be a mere part of the whole series, even though it is infinite. In general, we can imagine both infinite and finite series that leave some of the relevant conditions out.

¹⁶⁹ One might also criticize Allison as follows. On the one hand, Allison argues that Kant means to deny that series of conditions form *tota synthetica* in the resolution of the antinomy. On the other hand, however, in allowing that there are infinite *series* of conditions, Allison seems to allow that there are infinite *tota synthetica* after all. For on his own account of the notion of a *totum syntheticum*, a *totum syntheticum* is simply the product of a combination of pre-given parts (2005, 169). But a *series of conditions* satisfies this definition, since it is a collection of pre-given items (conditions) unified via conditioning relations to form a whole (i.e., the series itself).

These criticisms of Allison can also help us see the flaw in Allen Wood's (2010) attack on Kant's solution to the antinomy. Wood writes:

Kant's way of avoiding the contradiction [of the antinomies] comes down to the claim that the category of *totality* cannot be legitimately applied to "the world" (to the various series of conditions that generate the antinomies). But it is not clear how he can avoid applying the category of totality to the series, any more than he could avoid applying the categories of *unity* or *plurality* to it. For surely each series is *one* series that has *many* members – and if so, why is it not a *whole* series – whose magnitude, therefore, must be either finite or infinite?¹⁷⁰

Wood's assumption that Kant means to deny that the *category* of totality applies to spatiotemporal series of conditions is similar in important respects to Allison's claim that Kant's solution consists in denying that series of conditions are *tota synthetica*. For just as the concept of a *totum syntheticum* is the concept of a totality in the unity sense, so too the *category* of totality is most plausibly interpreted as the concept of a unified totality. Indeed, as Kant defines the category of totality, totality is "nothing other than plurality considered as a unity" (B111). The language of "considering as" raises the question whether Kant intends for the category of totality to apply to anything that can be *represented* as unified, or whether it requires the *real* unification of its elements, but as Wood's discussion indicates, a *series* of conditions satisfies the category of totality even on an ontologically more robust interpretation of its unity requirement.¹⁷¹ For a series of conditions is a *plurality* of things (the conditions) brought together to form a single thing (the series) via the conditioning relations that join them together. So the

¹⁷⁰ Wood (2010), 260.

¹⁷¹ As some commentators have pointed out, Kant's definition of the category of totality at B111 seems to concern the *representation* of a plurality of elements as unified rather than the conditions under which a plurality of items actually *is* unified. For to "consider" a plurality as a unity seems to leave open whether that plurality actually *is* a unity. With this in mind, Willaschek (2018) "tentatively conclude[s]" that Kant understands the notion of totality corresponding to the category of totality in a "notational" rather than an "ontologically robust way": "a totality is not an object over and above the members or elements it contains, but merely a way of considering them as a unity" (92). Though important for other questions in Kant interpretation, exactly how the category of totality should be interpreted can be put aside here.

category of totality applies to a series of conditions regardless of whether we understand its unity requirement as merely representational or as real, and regardless of how we interpret its unity requirement, it specifies a requirement of unity rather than a requirement of completeness. The category of totality therefore applies to a series of conditions regardless of whether or not that series is the *complete totality* of conditions for the conditioned thing in question. And given this, we can conclude that Wood's criticism of Kant's solution to the antinomy depends on the mistaken assumption that he intends to deny that spatiotemporal conditions form *totalities in the unity sense*. If we see that Kant instead means to deny that they form *complete totalities*, then Wood's criticism does not get a hold.

Now consider the view that Kant's claims about totality in the resolution of the antinomies are independent of his claims about infinity. Willaschek's (2018) discussion of the Supreme Principle represents a view of this sort, for according to Willaschek, Kant does not appeal to a claim about infinity in justifying his conclusion that appearances are not a totality of conditions, and in denying that they are a totality of conditions he also leaves open the possibility that they are infinitely numerous. We can get a better grip on Willaschek's view by examining his discussion of the Supreme Principle.

According to Willaschek, a proper understanding of the Supreme Principle requires noticing that it in fact makes two separate inferences. First, the principle says that if a conditioned object exists, then the totality of its conditions exists. Second, it says that if the totality of an object's conditions exists, then the unconditioned also exists. Willaschek recognizes that Kant himself explains the second inference in the A417-18/B445 footnote (discussed above): the totality of an object's conditions is unconditioned because there is nothing outside it that could condition it. But why should we think that if a conditioned object exists,

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then the totality of its conditions also exists? According to Willaschek, there is a genuine

problem here, for as he writes:

[T]here is no valid inference from

P1 There is something R-conditioned

and

P2 For everything R-conditioned there is at least one R-condition

to the conclusion that there is a *totality* of R-conditions [...], because it only takes us to the conclusion that either there is some unconditioned R-condition or the series of R-conditions is infinite.¹⁷²

How then can we draw the necessary inference to the totality of conditions? According to

Willaschek, the inference to the totality of conditions

follows trivially [...] if we take Kant's definition of 'totality' to express the naïve principle of set formation (sometimes called the 'principle of comprehension'). According to this principle, for every (instantiated) predicate (i.e., a predicate that applies to at least one object), there is a (non-empty) set of all objects that fall under it. Kant's definition of totality can be understood as saying just that: for every (actual or potential) plurality of objects that are F, there is the totality of Fs. For instance, if 'is red' is an instantiated predicate, there is a non-empty set of *all* red things – that is, *the totality* of red things. Equally, if 'is a condition of x' is an instantiated predicate, it follows that there is a *totality of conditions of x*.¹⁷³

Thus, Willaschek takes Kant's concept of a totality to be "roughly equivalent to Cantor's concept

of a set," and he thinks it is trivial that a totality of conditions (in this sense) exists if we assume

a principle of comprehension.¹⁷⁴

How should we assess Willaschek's proposal? If Willaschek is correct, then when Kant

denies (in the resolution of the Antinomies) that the notion of "absolute totality" is applicable to

¹⁷² Willaschek (2018), 95. Note that here the term "R-conditioned" just means "conditioned in the real conditioning relation, R".

¹⁷³ Willaschek (2018), 95.

¹⁷⁴ Willaschek (2018), 95 fn 46.

appearances in space and time, he does not mean to deny that objects have either a determinately finite or a determinately infinite plurality of conditions. For according to Willaschek, in denying that the Supreme Principle holds for appearances, Kant is denying *only* that appearances can always be unified or collected together to form a set. Indeed, this is clear from Willaschek's suggestion that it *does* follow from P1 and P2 (above) that the series of conditions for any R-conditioned object is either finite or infinite. Willaschek also confirms this in his discussion of Kant's *justification* for denying that the Supreme Principle holds for appearances. As Willaschek writes, "since Kant wants to *deny* that for empirical objects there is a totality of their conditions (A499/B527), Kant might be read as implicitly rejecting that principle [i.e., the principle of comprehension] for the domain of appearances."¹⁷⁵ And later, he adds:

Therefore, I think that the philosophically most plausibly way for Kant to resist the inference from the conditioned to the unconditioned totality of its conditions consists in denying the principle of comprehension, that is, the assumption that for every predicate there is the totality of objects of which it is true.¹⁷⁶

Thus, Willaschek thinks Kant can deny that appearances form totalities of conditions without that claim having any implications for whether series of spatiotemporal conditions are finite or infinite; on Willaschek's account, all Kant needs to do is deny the principle of comprehension for appearances—that is, all he needs to do is deny that appearances can always be collected together to form sets.¹⁷⁷

¹⁷⁵ Willaschek (2018), 95 fn 46.

¹⁷⁶ Willaschek (2018), 155.

¹⁷⁷ As indicated elsewhere, I think Willaschek is somewhat unclear as to whether denying the principle of comprehension is supposed to be an epistemic or metaphysical move. His overall view seems to be that the antinomy is resolved with the appropriate epistemic and methodological restrictions on the pretensions of pure reason, but his claim about the principle of comprehension seems at times to have metaphysical import and is not obviously related to the broader epistemological and methodological thrust of his reading (see Willaschek 2018, 155-6 for relevant discussion).

For reasons that should now be familiar, I do not think this proposal is satisfying. First of all, like Allison's and Wood's proposals, Willaschek's proposal makes the resolution of the antinomies turn on questions about whether spatiotemporal conditions can be collected together or *unified* rather than on questions about whether they are *complete totalities*. The unification involved in set formation is arguably very different from the unification involved in real conditioning, but proposing a restriction of the principle of comprehension nonetheless amounts to a proposal about the conditions under which things can be unified to form wholes. And as I have argued, Kant suggests that the notion of totality relevant to the antinomies' resolutions is the notion of a totality in the completeness sense, not the notion of a totality in the unity sense.

Second, Willaschek's proposal also leaves us with an unsatisfying explanatory lacuna. As we have seen, Kant does not think there is a general prohibition against *unified* infinite totalities of conditions, and since the principle of comprehension concerns the notion of a unified totality, Willaschek's proposal simply leaves it unexplained *why* the principle of comprehension would fail to hold for appearances.¹⁷⁸ In contrast, on the reading I will defend below, Kant holds that series of spatiotemporal conditions do not form *complete totalities* of conditions, and they do not form complete totalities of conditions must be either finite or infinite (per the conception of the unconditioned Kant articulates at A417-18/B445), a series of conditions that is neither finite nor infinite nor infinite nor infinite nor infinite cannot be a complete totality. As I'll argue below, appealing to this claim is in fact *how*

¹⁷⁸ Significantly, Willaschek acknowledges that transcendental idealism has no bearing on whether the principle of comprehension is true for spatiotemporal objects. However, Willaschek considers transcendental idealism an unappealing doctrine that *should* be separated from Kant's valuable discussion of the illusory nature of traditional metaphysics. Although I don't intend to argue that transcendental idealism *is* true, I think Willaschek's approach forecloses an important opportunity to see the connection for Kant between the ideality of the spatiotemporal world and its metaphysical indeterminacy (as will become clear in the chapters that follow).

Kant resolves the mathematical antinomies: he denies that spatiotemporal series of conditions form complete totalities *by denying* that they are either finite or infinite.

Finally, what about the view according to which Kant argues that because spatiotemporal series of conditions do not form totalities of conditions, they must be potentially rather than actually infinite? Chapter 4 is devoted to the question of the role of claims about potential infinity in the antinomies' resolutions, but one brief comment is in order here. As I have argued above, when Kant says that the notion of "absolute totality" is not "valid" of appearances in space and time, he has in mind the notion of a *totality in the completeness sense*. That is, he means to say that we cannot legitimately apply the notion of a *complete totality* to spatiotemporal series of conditions, given transcendental idealism. However, we have also seen that Kant does not simply equate the notion of an actually infinite collection of conditions with the notion of a *complete* infinite totality of conditions. According to Kant, a complete totality of conditions must be *either* finite or actually infinite, but an actually infinite collection of conditions is not necessarily a complete totality. For example, an actually infinite collection that is a subset of a more encompassing actually infinite collection is not a complete totality (as, for example, the even numbers are not the complete totality of all natural numbers). Hence, we can conclude that Kant does not simply assume that a series of conditions can fail to be a complete totality only by being potentially infinite.

5. Chapter Summary

The most important claim of this chapter has been that Kant identifies the notion of totality relevant to the resolution of the antinomies not with the notion of a totality in the unity sense but rather with the notion of a totality in the completeness sense. Thus, when Kant says

that the notion of "absolute totality" is not "valid" of appearances, he means to say that conditioned spatiotemporal objects and their spatiotemporal conditions never form a *complete totality* of conditions.

Related to this, the arguments above also show that in the Critical period, Kant abandons two claims endorsed in the *Inaugural Dissertation*. First, he abandons the claim that reason alone can establish the existence of any particular kind of object answering to the idea of a totality of conditions in the completeness sense (though he remains committed to the view that a totality of conditions would have to exist in *some* form among things in themselves if there is anything conditioned among them). Second, he abandons the view that a collection of conditions answering to the notion of a complete totality would have to be limited in magnitude. According to Kant in the Critical period, a totality of conditions in the completeness sense could be either a finite or an infinite multiplicity of conditions.

In this chapter, I have not yet explained how Kant intends to underwrite his claim that spatiotemporal series of conditions fail to be complete totalities. I turn to this task in the chapters that follow. As I argue, Kant underwrites his claim that spatiotemporal series of conditions are not complete totalities by appealing to their *metaphysical indeterminacy*. More specifically, since spatiotemporal series of conditions are metaphysically indeterminate in their magnitude properties, they are neither finite nor infinite and hence fail to be complete totalities. I sketch the beginnings of this view in chapter 3 below, where I also explain why Kant's solution to the antinomy requires a metaphysical claim about complete totalities and hence cannot be completely epistemic in nature. In chapter 4, I explain why a metaphysical indeterminacy reading is distinct from and more successful than a potential infinity approach. Finally, in chapter

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5, I explain why the ideality of appearances in space and time results in the particular kind of metaphysical indeterminacy to which Kant is committed in the resolution of the antinomies.

Chapter 3: Against Epistemic Solutions

In chapter 2 above, I drew the following main conclusions about Kant's solution to the first and second antinomies. First, when Kant says that the antinomies arise from a misapplication of the notion of "absolute totality" to the domain of appearances, he means to say that the notion of a *totality in the completeness sense* cannot legitimately be applied to appearances. For in the Critical period, Kant holds that the antinomies arise from reason's pursuit of the *unconditioned*, and as he sees it, a series of conditions contains something unconditioned just in case it contains the *complete totality* of an object's conditions. Second, Kant holds that a complete totality of conditions must be either finite or infinite, from which it follows that a series of conditions that is neither finite nor infinite cannot be a complete totality (and hence cannot present a case of the unconditioned).

In this chapter, I argue for the further claim that Kant's solution to the antinomies must be *metaphysical* in the following sense. When Kant says that the notion of "absolute totality" does not hold for appearances (and that the antinomies therefore admit of a resolution), he means to say not that we cannot *know* or *cognize* whether the series of conditions is finite or infinite for a given conditioned spatiotemporal object; rather, the series of spatiotemporal conditions cannot *be* either finite or infinite.¹⁷⁹ Thus, complete totalities of spatiotemporal conditions do not *exist* on Kant's considered resolution to the antinomy. Moreover, as I argue, Kant resolves the mathematical antinomies by appealing to the notion of *metaphysical indeterminacy*. On Kant's solution, the series of conditions treated in the first and second antinomies are in fact *indeterminate* in magnitude rather than either finite or infinite, and as Kant sees it, this is a claim about spatiotemporal phenomena that only transcendental idealists can accept. Among things in

 $^{^{179}}$ Interestingly, though we cannot cognize any series *as* either finite or infinite (given that no series *is* either finite or infinite), the view I defend suggests that we can *know* that no series is either finite or infinite once we have properly understood transcendental idealism.

themselves, Kant reasons, series of conditions must be determinate in their magnitude properties and hence must be either finite or infinite.

The chapter proceeds as follows. First, in section 1, I provide a brief overview of the Metaphysical Indeterminacy Account, explaining (a) how the notion of metaphysical indeterminacy differs from other notions of indeterminacy (e.g., epistemic and semantic indeterminacy) and (b) how a reading that appeals to the notion of metaphysical indeterminacy explains Kant's solution to the antinomy and the indirect argument for idealism it provides. For ease of discussion, I focus in this chapter on the first antinomy, but the arguments are applicable to the second antinomy as well.

In section 2, I address a question pertaining to Kant's justification for insisting that transcendental realists cannot endorse a metaphysical indeterminacy approach. Why does Kant think transcendental realists must deny that the extent of the world could be indeterminate and instead affirm that it is either determinately finite or infinite? As I argue, the answer to this is that Kant thinks it is simply *true* that the Supreme Principle must hold for things in themselves. Since transcendental realists hold that conditioned spatiotemporal objects are things in themselves, they must hold that the series of conditions for each conditioned spatiotemporal thing is either finite or infinite.

In section 3, I turn to the question whether a broadly epistemic approach to the antinomy's resolution can provide a less controversial and yet equally compelling reading. Two different varieties of epistemic approaches have prominent defenders in the secondary literature. First, proponents of Idealism as Epistemology readings argue that transcendental idealism is itself a purely epistemic doctrine, and so the resolution to the antinomies must likewise be epistemic; according to these readings, Kant's solution is to show that the claims made in the

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antinomies are ruled out by the epistemic strictures of transcendental idealism. Second, proponents of Moderate Epistemic readings hold that transcendental idealism may be a partly metaphysical doctrine, but the solution to the antinomy is nonetheless purely epistemic. As I argue, Idealism as Epistemology readings fail sufficiently to distinguish between endorsing the Supreme Principle and being a Transcendental Realist, a distinction Kant suggests is important. Moderate Epistemic readings fail to provide a compelling explanation of *why* the Supreme Principle fails to hold for spatiotemporal objects (and hence of why purely rational arguments cannot be used to extend our cognition to unconditioned objects).

Finally in section 4, I return to the Metaphysical Indeterminacy reading and explain the advantages it has over both of the epistemic approaches just described. As I argue, a Metaphysical Indeterminacy reading has three distinct advantages. First, it provides a satisfying explanation of *why* the Supreme Principle is inapplicable to conditioned things in space and time: series of spatiotemporal conditions are indeterminate in magnitude (rather than either finite or infinite), and hence the Supreme Principle is false for them. Second, it provides a reading of the antinomy's resolution that establishes common ground between Kant and his transcendental realist opponents, making the indirect argument for idealism dialectically more effective than merely epistemic readings can allow for. According to a metaphysical indeterminacy reading, both Kant and his transcendental realist interlocutors hold that things in themselves must satisfy the Supreme Principle, and this explains why the antinomial arguments are "well grounded" on the presupposition of transcendental realism (A507/B535). Finally, a metaphysical indeterminacy account can also explain how the antinomy supports Kant's claim that pure reason cannot extend our cognition to unconditioned objects. The purely rational principle driving the antinomies is the Supreme Principle, but because spatiotemporal series of conditions are

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metaphysically indeterminate in magnitude, we cannot infer from the existence of a conditioned spatiotemporal thing to the existence of the complete totality of its conditions. Hence, when the conditioned is something spatiotemporal, we cannot infer to the existence of anything unconditioned.¹⁸⁰

1. The Metaphysical Indeterminacy Reading: Preliminary Sketch

How should we understand the central commitments of a Metaphysical Indeterminacy reading? In this section, I provide a preliminary sketch of its two core elements. First, I explain the general sense in which it reads Kant's solution to the antinomies as a "metaphysical" one. Second, I explain the more specific notion of *metaphysical indeterminacy* and the way in which it differs from more commonly employed notions of indeterminacy (such as epistemic indeterminacy and semantic indeterminacy). With these two elements in hand, I then explain how a metaphysical indeterminacy reading interprets Kant's solution to the first antinomy.

What does it mean to say that Kant's solution to the antinomy is *metaphysical* rather than, say, epistemic? In brief, I take Kant's resolution to the antinomy to be metaphysical because it makes a claim about what *exists* in space and time. That is, on the reading I propose, Kant's solution to the antinomies does not consist *merely* in a claim about what we can know or cognize about spatiotemporal reality. Rather, his solution implies that spatiotemporal reality does not *have* the magnitude properties that either the thesis or antithesis arguments claim it has, and this

¹⁸⁰ One implication of this is that the conditioning relations treated in the first and second antinomies do not take us from the existence of conditioned things in space and time to the existence of any unconditioned things in themselves. But this is as it should be, given that the mathematical antinomies treat only sensible conditions (A530/B558). Moreover, it is not to say that Kant does not have other arguments that establish the existence of things in themselves.

is explained by the nature of spatiotemporal phenomena. Importantly, this is compatible with the claim that spatiotemporal objects do not exist in the same way as do mind-independent things in themselves. The core claim of metaphysical readings of the antinomies' resolutions is simply that spatiotemporal phenomena (whatever their metaphysical status may turn out to be) do not *have* the magnitude properties transcendental realists attribute to them in the thesis and antithesis arguments.¹⁸¹

What does it mean to say that Kant is committed to *metaphysical indeterminacy* in spatiotemporal reality? In brief, I hope to show that Kant resolves the antinomies by denying that the relevant series of spatiotemporal conditions are determinate with respect to their magnitude properties. According to Kant, being finite (the thesis position) and being infinite (the antithesis position) are two different ways of have *determinate* magnitude properties, and the thesis and antithesis positions are both false because spatiotemporal series of conditions are instead *indeterminate* in magnitude. As indicated above, this establishes that spatiotemporal series of conditions are not *complete totalities* of conditions and hence do not contain anything unconditioned.¹⁸²

But can we really make sense of the idea of *metaphysical* indeterminacy? The existence of epistemic and semantic indeterminacy is widely accepted, but in the history of philosophy as

¹⁸¹ Some commentators argue that the mind-dependence of spatiotemporal objects means they do not actually *exist* at all (see Jankowiak 2017). However, Kant himself talks freely of appearances as "existing" in the resolution of the antinomies, so I prefer to read him as a proponent of the view that appearances exist in a different way than do things in themselves (given their mind-dependence), while still existing. However, claims about what "exists" in space and time can also be reformulated into claims about what is *actual* (*wirklich*), and it is uncontroversial that Kant is committed to regarding spatiotemporal objects as *actual* (e.g., in the postulates of empirical thinking, A218/B265-6). So readers who think appearances do not *exist* can read "exists" as "is actual".

¹⁸² A further consequence of this is that a spatiotemporal *world* does not exist in the strict sense of the term (since a *world* on Kant's view must be both a *complete* and a *unified* totality). But I will sometimes refer loosely to the "spatiotemporal world" when alternative terminology is cumbersome. When I do so, I do not mean to suggest that spatiotemporal conditions in fact form *complete* totalities of conditions, as a true spatiotemporal "world" would require.

well as in current literature, there are a number of scholars who deny that metaphysical indeterminacy is a coherent notion at all. Russell (1923) voices this skeptical opinion as follows: "Vagueness and precision alike are characteristics which can only belong to a representation [...] *things* are what they are, and there is an end of it."¹⁸³ For scholars such as Russell, the idea of indeterminacy in reality is simply absurd, and talk of indeterminacy in *things* must therefore be reinterpreted.¹⁸⁴

When I say that it is indeterminate whether a certain particle has property X or property Y, I must really mean that we cannot *know* whether the particle has property X or Y.¹⁸⁵ Or when I say that it is indeterminate whether a certain man is bald, I must really mean that our linguistic rules do not specify whether the predicate "is bald" applies to the man in question.¹⁸⁶ In short, if philosophers like Russell are correct, *representations* and *meanings* can be indeterminate, but *things* cannot.

Against this opinion, however, other philosophers argue that there *could* be indeterminacy in reality, i.e., that it *can* be unsettled exactly how things are.¹⁸⁷ These proponents of metaphysical indeterminacy allow that *some* cases of indeterminacy are merely epistemic or semantic, but they argue that there are other cases in which indeterminacy may not just be a matter of how we represent things. That is, there may be some cases in which things in the world are themselves indeterminate. When I say that Kant resolves the mathematical antinomies by appealing to metaphysical indeterminacy in the spatiotemporal world, I mean to be attributing to

¹⁸³ Russell (1923), 85, my emphasis.

¹⁸⁴ Other prominent arguments against metaphysical indeterminacy in more recent literature can be found in Evans (1978), Heller (1996), and Sider (2003 and 2009).

¹⁸⁵ This would be to make a claim about *epistemic* indeterminacy.

¹⁸⁶ This is to make a claim about *linguistic* or *semantic* indeterminacy.

¹⁸⁷ Some proponents of metaphysical indeterminacy in the contemporary literature include Akiba (2004), Barnes

⁽²⁰¹⁰a, 2010b, and 2014), Barnes and Williams (2009 and 2011), Tye (1990), Williams (2008), and Wilson (2013).

him a view of this sort. According to Kant, it is not simply that we cannot *know* or *represent* whether spatiotemporal series of conditions are finite or infinite; instead, Kant denies that the series of conditions treated in the first and second antinomies *have* either determinate finite magnitudes or determinate infinite ones. And this means there is indeterminacy with respect to how many things *exist* in space and time. Importantly, Kant's idealism allows him to explain how this indeterminacy arises in ways that would not be open to contemporary metaphysicians who do not hold that spatiotemporal objects are mind-dependent, but the indeterminacy to which Kant is committed nonetheless is not metaphysically neutral in the way that epistemic and linguistic indeterminacy typically are understood to be. According to Kant, what *exists* in space and time is indeterminate because series of spatiotemporal conditions are in fact neither finite nor infinite.¹⁸⁸

To see how a Metaphysical Indeterminacy reading interprets the resolutions of the mathematical antinomies, we can look at the first antinomy in particular and examine the role of claims concerning metaphysical indeterminacy in its resolution. Recall that the first antinomy concerns the magnitude of the world in space and time, with the thesis position affirming the finitude of the world and the antithesis position affirming its infinity. Recall also that an application of the Supreme Principle to appearances gets the arguments up and running. The transcendental realist begins by assuming that for every conditioned spatiotemporal thing, the *complete totality* of its conditions must exist (and hence something unconditioned must exist). But since something unconditioned must be either the terminal member in finite series of

¹⁸⁸ I should acknowledge a point that will become clear in chapter 5, viz., that indeterminacy in spatiotemporal reality *is* a result of the fact that spatiotemporal objects exist only as *represented* objects (i.e., their existence is representation-dependent. But this does not make Kant's metaphysical indeterminacy reduce to epistemic or semantic indeterminacy in any ordinary sense, as I explain in chapter 5 below.

conditions or the entirety of an infinite series of conditions, it follows from an application of the Supreme Principle to appearances that the series of a spatiotemporal object's conditions must be either finite or infinite. In the context of the first antinomy, this licenses the assumption that the world is either finite or infinite in space and time.¹⁸⁹ The arguments of the thesis and antithesis then proceed as follows (for simplicity, I consider only the portions of the arguments concerning the world's *temporal* extent):

Thesis Argument	Antithesis Argument
1. Either the world is finite in time, or it is infinite in time. (DERIVED FROM THE SUPREME PRINCIPLE)	1. Either the world is finite in time, or it is infinite in time. (DERIVED FROM THE SUPREME PRINCIPLE)
2. The world is infinite in time. (ASSUMPTION FOR REDUCTIO)	2. The world is finite in time. (ASSUMPTION FOR REDUCTIO)
3. If the world is infinite in time, then an infinite successive synthesis can be completed.	3. If the world is finite in time, then it is possible for something to arise out of an empty time.
4. An infinite successive synthesis cannot be completed.	4. It is not possible for something to arise out of an empty time.
5. The world is not infinite in time (from 2-4).	5. The world is not finite in time (from 2-4).
6. Therefore, the world is finite in time (from 1 and 5).	6. Therefore, the world is infinite in time (from 1 and 5).

As we can see, the strategy of the thesis argument is to show that the infinite series an infinite world would require is impossible, from which it follows (if the Supreme Principle is true for

¹⁸⁹ As I emphasized in chapter 1 above, one might think that the premise that the world must be either finite or infinite is trivial and so requires no justification. Kant's view, however, is that it is not trivial, since denying that the world is infinite does not logically entail that it is finite, and denying that it is finite does not logically entail that it is infinite; it may instead be indeterminate. See A503-4/B531-2 and 20:291 for Kant's discussion of this point.

spatiotemporal objects) that the series must be finite. The strategy of the antithesis argument is to show that the terminal condition required by a finite world is impossible, from which it follows (assuming the validity of the Supreme Principle for appearances) that the series of conditions determining the world's magnitude is infinite.

Kant's solution to the antinomy on the present reading is to assert that the series of past world-states is metaphysically *indeterminate* in magnitude rather than either finite or infinite. That is, being finite and being infinite are two ways in which a series of conditions can have a *determinate* magnitude, and Kant's solution to the antinomy is to say that neither of these determinate states of affairs obtains. This means that premise 1 on each side of the argument is *false* for the transcendental idealist, and because it is false, the Supreme Principle is also false for spatiotemporal phenomena.

Notice that on this reading, Kant can *accept* the intermediary steps in the thesis and antithesis arguments, respectively. That is, he can grant that steps 2 through 5 in both arguments show that the spatiotemporal world is *not infinite* and *not finite*, respectively. What a metaphysical indeterminacy approach argues is that the further inference to the conclusion that the world is *both* finite *and* infinite is not valid. Because the series of past world-states may be *neither* finite *nor* infinite, we cannot infer from the dual conclusions that the world is *not infinite* and *not finite* to the further claim that it is *both* finite *and* infinite. As the transcendental idealist argues, the series of conditions is in fact *indeterminate* in magnitude, which is to say that it is neither finite nor infinite.

How should we understand the claim that the magnitude of a series of conditions can be indeterminate rather than either finite or infinite? That is, what does it mean to say that a series has an indeterminate length, and how is this different from the claim that it is infinite? In brief, I

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understand Kant's position here as follows. According to Kant, the magnitude of a series of conditions is finite just in case it is determinate that it has a last member, i.e., a condition that is not conditioned by anything further. And the magnitude of a series of conditions is infinite just in case it is determinate that every condition is conditioned by a further one. As Kant writes, an infinite unconditioned series of conditions is one in which "every member without exception is conditioned, and only their whole is absolutely unconditioned" (A417/B445). So a series of conditions is not finite and not infinite if it is not determinate that there is a last, unconditioned condition in the series *and* it is not determinate that every condition is conditioned by something further.¹⁹⁰ Put differently, it is *sufficient* for a series of conditions to be indeterminate in magnitude if it is neither finite nor infinite in either of these two senses. Notably, this leaves room for the series to have some magnitude properties (in virtue of which we may say its magnitude is partially determinate)—for instance, we might truly say that the series of past world-states has sufficiently many members to make the world at least 10 billion years old. The core claim of the Metaphysical Indeterminacy reading is simply that the series of past worldstates is not determinate in either of the two ways that the Supreme Principle requires: it is not determinate that there is an unconditioned first member, and it is not determinate that every condition is conditioned by a further one.

¹⁹⁰ Clearly this is not yet fully satisfying as an account of metaphysical indeterminacy, since I have not yet said what it means for it to be "not determinate that there is a last, unconditioned conditioned" and "not determinate that every condition is conditioned by something further." I hope chapter 5 will go some way towards answering these questions, but I also note here that it is not my aim to provide anything approaching a full semantics for claims employing the terms "determinate" and "indeterminate". I want to show only that Kant is committed to some kind of worldly indeterminacy (for spatiotemporal phenomena) and that this indeterminacy is what explains how the thesis and antithesis statements of the mathematical antinomies are both false.

2. The Supreme Principle and Things in Themselves

One of the key results of the Metaphysical Indeterminacy approach is that it explains why the Supreme Principle fails to hold for appearances (as Kant puts it, with conditioned appearances we can "by no means infer the absolute totality of the series of these conditions" A499/B527). As the argument goes, because spatiotemporal conditions form series that are neither finite nor infinite, they form series that are not *complete totalities* of conditions (and so do not contain anything unconditioned). Hence, the Supreme Principle turns out to be false for conditioned spatiotemporal things.

However, one might naturally ask at this point why Kant thinks transcendental realists cannot escape the antinomy in precisely the way that the Metaphysical Indeterminacy approach recommends? That is, why can't transcendental realists *also* deny that the Supreme Principle holds for conditioned things in space and time? If transcendental realists *can* make this claim, then they can join Kant in saying that the series of conditions determining the world's extent in space and time is neither finite nor infinite, and Kant's claim that the antinomy undermines transcendental realism loses all of its bite.

However, recall that Kant himself claims that the Supreme Principle must be true for things in themselves. He writes in section 7 of the antinomies:

If the conditioned as well as its condition are things in themselves, then when the first is given not only is the regress to the second **given as a problem**, but the latter is thereby really already **given** along with it; and, because this holds for all members of the series, then the complete series of conditions, and hence the unconditioned is thereby simultaneously given, or rather it is presupposed by the fact that the conditioned, which is possible only through that series, is given.

(A498/B526)¹⁹¹

Given the conviction that the Supreme Principle holds for things in themselves, we can understand Kant's assessment of the options open to transcendental realists as follows. Because transcendental realists assert that spatiotemporal objects *are* things in themselves, they cannot avoid applying the Supreme Principle to them. And if the Supreme Principle is applied to things in space and time, then it *does* follow that the extent of the world must be finite or infinite rather than indeterminate (because the Supreme Principle would entail the *complete totality* of conditions, which would entail a series of conditions that is either finite or infinite, per B445/A417-18). That is, Kant thinks it is simply correct to say that the Supreme Principle holds for things in themselves, so if one is committed to the view that appearances in space and time are things in themselves, one must be committed to the view that the Supreme Principle is legitimately applied to spatiotemporal phenomena. And as we have seen, this makes the antinomy inevitable (or so Kant argues).

Why would Kant make a substantive claim of this sort about things in themselves? That is, why would he assert that the Supreme Principle must be true for things in themselves? One might worry that this is exactly the sort of claim Kant wishes to avoid, given his commitment to a thesis of noumenal ignorance and the transcendental idealist's distinctive claim that we cannot cognize things in themselves. However, notice that a commitment to the truth of the Supreme Principle for things in themselves does not mean that we can know that there are conditioned things in themselves, and it also does not mean that if we *could* know that there are conditioned

¹⁹¹ Some proponents of epistemic readings interpret this passage very differently than do I. For example, Allison (2004) thinks Kant rejects all metaphysical claims about how things "really are" (i.e., about things in themselves). Willaschek (2018) thinks Kant's aim in the Dialectic is to deny the legitimacy of the assumption that fundamental reality corresponds to principles of pure reason. In section 3 below, I explain why I find these readings unconvincing.

things in themselves, then we could also know whether the series of their conditions are finite or infinite. Rather, the Supreme Principle when applied to things in themselves says only that *if* there are conditioned things in themselves, *then* the complete totality of their conditions must exist.

As I indicated earlier. I think it is fruitful to understand this as a kind of modest rationalist commitment concerning the intelligibility of fundamental reality. In holding that the Supreme Principle must be true for things in themselves, Kant is committing to the idea that fundamental reality meets a condition of explanatory completeness: whatever is metaphysically explained by anything else in fundamental reality exists with its complete metaphysical explanation. Notably, this does not say that we can know anything more specific about things in themselves, or even that any things in themselves are metaphysically explained by other things in themselves (i.e., are conditioned)—for all we know, things in themselves might not stand in any relations of metaphysical explanation at all. But Kant's commitment to the truth of the Supreme Principle for things in themselves does count as a kind of rationalism, since it asserts a fit between the Supreme Principle, which is a principle of *reason*, and fundamental reality. And it counts as a form of rationalism that is *modest* because it is compatible with the view that fundamental reality is radically cognitively inaccessible in many respects. If the Supreme Principle is true for fundamental reality, then we can know that fundamental reality does not contain merely partial or incomplete explanations, but we cannot draw much in the way of any substantive conclusions beyond that.¹⁹²

¹⁹² A competing way of understanding the Supreme Principle is that it expresses the following general claim about the nature of explanation: unless there are complete explanations, there are no explanations at all. However, I don't think this is how Kant intends the Supreme Principle. First, it does not seem plausible that there are no partial explanations. If we know a particular thing has one causal condition, then we have *some* causal explanation, and this

If all this is correct, then when Kant says transcendental idealists *alone* can escape the antinomy by endorsing the *both false* solution, he is operating on the background assumption that all parties to the debate should endorse the Supreme Principle for things in themselves. Given this, Kant reasons, the only way to reject premise 1 in the thesis and antithesis argument is to hold that spatiotemporal objects are not things in themselves after all. That is, if things in fundamental reality *must* satisfy the Supreme Principle, then it can be false that the spatiotemporal world is finite *and* false that it is infinite only if the spatiotemporal world is not a part of fundamental reality. In holding that spatiotemporal objects are things in themselves, the

3. Against Traditional Epistemic Readings

The metaphysical indeterminacy reading just sketched attributes to Kant a very controversial view of spatiotemporal phenomena. Not only are spatiotemporal phenomena not the things in themselves we naively take them to be, but they are indeterminate in ways that contravene one of the core explanatory demands of our faculty of reason (as Kant understands it): it is indeterminate how many conditions there are in the series of conditions that determine the spatiotemporal world's magnitude properties, which means complete explanations of the kinds of conditioned phenomena treated in the first and second antinomies are impossible in principle.

is not undermined if the series of its causes turns out to be incomplete. Second, Kant himself says that appearances condition one another despite the fact that we "can by no means infer the absolute totality of the series of these conditions" (A499/B527). Since all conditioning relations are explanatory for Kant, it follows from this that he does not endorse a general claim about the nature of explanation according to which there is no explanation without complete explanation. This would force him into the implausible position that there are no conditioning relations in space and time.

Can we avoid these controversial upshots by embracing an epistemic reading of the antinomies' resolutions rather than a metaphysical one? In this section, I turn to this question and consider two different ways in which one might motivate a purely epistemic reading of the mathematical antinomies' resolutions (i.e., a reading on which the resolution of the antinomies is neutral as to what magnitude properties spatiotemporal phenomena actually have). What these two broad strategies have in common is that they attempt to resolve the antinomies by properly restricting our cognition (*Erkenntnis*) or knowledge (*Wissen*) (depending on one's view) *without* making any controversial claims about how spatiotemporal phenomena actually are. In particular, they say that we can *cognize* (or *know*) only what can be given in intuition and thought through concepts, and since the claims made in the thesis and antithesis positions of the antinomies violate these restrictions, they must be rejected. And if they can be rejected on the grounds that they violate the Kantian account of cognition or knowledge (again, depending on one's view), then the antinomies can be resolved without appealing to any controversial claims about metaphysical indeterminacy.

However, as I hope to show in what follows, none of the epistemic readings currently on offer provide a fully satisfying reading of the antinomies' resolutions, and, moreover, there are principled reasons to think that a reading must appeal to some controversial metaphysical claims to explain how Kant intends to resolve the antinomies. I argue for these conclusions in two main subsections. First, in section 3.1, I consider readings according to which transcendental idealism itself is a purely epistemic doctrine, and the antinomies illustrate what happens when one fails to apply that epistemological position to the questions of cosmology. I call these readings Idealism as Epistemology readings and argue that they fail to distinguish sufficiently clearly between the Supreme Principle, on the one hand, and the doctrine of transcendental realism, on the other.

Second, in section 3.2, I consider readings that allow that transcendental idealism is a partly metaphysical doctrine but nonetheless maintain that the theory of cognition (*Erkenntnis*) associated with it suffices to resolve the antinomies. I call these readings Moderate Epistemic readings and argue that they fail to tell a satisfying story about *why* purely rational arguments cannot extend our cognition to things that cannot be given in experience.

3.1 Idealism as Epistemology Readings

In the secondary literature, the most prominent champions of Idealism as Epistemology readings are Henry Allison (1983, 2004), Michelle Grier (2004), and Graham Bird (2006).¹⁹³ Roughly speaking, these interpreters identify transcendental idealism with a theory of human knowledge and cognition and argue that the solution to the antinomies follows from applying idealism *qua* epistemological doctrine to the questions of traditional cosmology. Allison, for example, argues that transcendental idealism is the view that all human knowledge is necessarily standpoint-dependent, since it is limited to what can be represented in the spatial and temporal forms of *our* experience; once we accept this account of human knowledge, he argues, we can see that the claims to knowledge made in the antinomies violate transcendental idealism *qua* epistemic doctrine.¹⁹⁴ Grier likewise holds that transcendental idealism is an account of human knowledge according to which knowledge *for us* is constituted by the forms of intuition and the

¹⁹³ Note: there are other prominent interpreters who argue that transcendental idealism is an epistemic doctrine (e.g., Collins 1999 and Prauss 1974), but these are the three who extend an epistemic reading of transcendental idealism to an extensive discussion of the antinomies.

¹⁹⁴ Sometimes Allison describes the antinomial arguments as taking up a "God's eye" point of view and even *identifies* transcendental realism with this God's-eye perspective. For example, he writes, "transcendental realism is not itself a metaphysical position in the traditional sense; it is rather a standpoint, shared by proponents of diverse metaphysical views, from which both metaphysical and epistemological issues are approached. Specifically, it may be understood as the standpoint that approaches these issues (including the cosmological questions) from a God's-eye or theocentric perspective" (Allison 2004, 395).

categories, and as she argues, one of the main mistakes of transcendental realists is that they conflate subjective and objective principles of representation.¹⁹⁵ The antinomies are then resolved when we avoid the transcendental realist's "'mix-up' of sensibility and the understanding (i.e., of subjective and objective conditions of judgment)."¹⁹⁶ Finally, Bird also advocates reading transcendental idealism as a view in epistemology and argues that the antinomies arise only if we attempt to make claims that go beyond what is empirically decidable. Correspondingly, Bird argues, embracing the right epistemology (i.e., the one that rules out empirically undecidable claims) provides a way out of the antinomy.¹⁹⁷

As already indicated, my central complaint against Idealism as Epistemology readings is that they misrepresent the nature of the relationship between the Supreme Principle of pure reason, on the one hand, and the doctrine of transcendental realism, on the other.¹⁹⁸ In its broad strokes, this problem can be explained as follows. In the introduction to the Transcendental Dialectic, Kant introduces his doctrine of "transcendental illusion" and makes clear that it has some role to play in explaining how the antinomies arise (A295/B352). In particular, he suggests that transcendental illusion is what influences us to misuse the Supreme Principle and apply it to appearances in space and time (A497-507/B525-535). Especially when we have bought into

¹⁹⁵ Grier writes that "what Kant is offering us in the *Critique* is a methodological procedure for avoiding the errors stemming from the illicit extension of concepts and principles beyond the domain of their proper employment" (2004, 193).

¹⁹⁶ Grier (2004), 116.

¹⁹⁷ As Bird writes in summarizing his own view, "The account just offered turn on the notions of decidability or verifiability. Once the cosmological questions have made the transition from the empirical realm of appearances to the transcendent realm of things in themselves then the issues become undecidable for us. The proofs, according to Kant, already make that transition within the bogus topic of rational cosmology…" (2006, 680). As is clear from this passage, Bird thinks Kant rejects any form of cosmology that is not *empirical*. In contrast, the view I advocate below holds that Kant makes some claims about the spatiotemporal world that are not justified by empirical evidence. ¹⁹⁸ There are further problems pertaining to the plausibility of a reading of transcendental idealism that takes it to be a *purely* epistemological or methodological doctrine, but I put aside these worries at present and instead focus on a worry about Idealism as Epistemology readings that does not turn on such foundational questions about Kant interpretation.

transcendental realism, Kant argues, we cannot help but regard the Supreme Principle as a principle that is applicable to appearances. This might make one think that transcendental illusion simply *is* the belief in transcendental realism, but Kant stresses that this is not the case; transcendental illusion is inevitable, he argues, but transcendental realism is not. That is, although we cannot escape the *illusoriness* of transcendental illusion (i.e., we are all subject to the illusion), we can avoid thinking that transcendental realism is true (A298/B354-5).

Most proponents of Idealism as Epistemology readings attempt to uphold the distinction between transcendental illusion, on the one hand, and the errors that follow from it, on the other, but I think their account of transcendental idealism makes it difficult for them to uphold as clean a distinction as they want.¹⁹⁹ More specifically, because of how they treat the relationship between transcendental realism and the Supreme Principle, they cannot argue that transcendental illusion and transcendental realism make two fully distinct contributions to the antinomies. Recall that proponents of Idealism as Epistemology readings *identify* transcendental realism with the view that knowledge is possible beyond the bounds of experience (or some variant of this claim). And because the Supreme Principle expresses the conviction that one *can* infer from something given in experience to something unconditioned (which cannot be experienced), proponents of Idealism as Epistemology readings hold that the Supreme Principle is essentially a transcendental realist principle. But given that they also tend to *identify* transcendental illusion with the Supreme Principle (or with the assumption that it is true), transcendental illusion and the mistaken epistemology of transcendental realism end up collapsing into one another; the Supreme Principle ends up being an *instance* of mistaken transcendental realist epistemology. Thus, since transcendental illusion is unavoidable, and since it is (or results in) embracing the

¹⁹⁹ This is in fact a priority for both Grier and Allison (though it was not a priority for Allison in his early work).

Supreme Principle, Idealism as Epistemology readings suggest that we cannot help but take up the mistaken epistemological perspective of transcendental realism after all.

To see how this problem unfolds for each of Allison, Grier, and Bird, consider how they describe the relationship between transcendental realism and the assumption that the Supreme Principle holds for spatiotemporal objects. According to Allison, the Supreme Principle simply is "what Kant understands by transcendental illusion," and the principle is a distinctive principle of transcendental realism because it "involves a metaphysical assumption concerning the reality of a complete set of conditions for every conditioned, a set that must be considered as unconditioned, since, *ex hypothesi*, there can be nothing further by which it is itself conditioned."²⁰⁰ As Allison reads Kant, the *metaphysical* claim that there must exist a complete set of conditions for everything conditioned *already* violates the epistemic structures of transcendental idealism because any existence claims concerning unconditioned objects already assume the legitimacy of an epistemic perspective that is not conditioned by our forms of experience. Indeed, this is clear from Allison's claim that transcendental illusion and considering the spatiotemporal world as a thing in itself are identical in the antinomies. Allison writes: "the illusion in its cosmological guise just is that the totality of conditions (here appearances) exists in a timeless manner as a thing in itself."201 Elsewhere, Allison puts this point slightly differently when he says that transcendental realism can be *identified* with a "God's-eye or theocentric perspective, which systematically ignores the role of spatiotemporal conditions in the conception of how such totalities [of spatiotemporal objects] are 'given'."202 If transcendental realism just is

²⁰⁰ Allison (2004), 312. Later, Allison clarifies that "transcendental illusion consists in certain subjective principles of reason appearing to be objective" (328). It is clear that he takes the Supreme Principle to be the "objective" version of the subjective maxim to always *seek* further conditions for anything conditioned (329-30). ²⁰¹ Allison (2004), 385.

²⁰² Allison (2004), 395.
the perspective taken up in the Supreme Principle, and if the Supreme Principle *is* transcendental illusion (as Allison explicitly says it is), then transcendental realism and transcendental illusion are not separable after all.

Put differently, since on Allison's account the idea of the unconditioned itself "abstracts from the sensible conditions of empirical givenness," and since to make this abstraction is already to take up the point of view of the transcendental realist, Allison's account has the following upshot.²⁰³ To assume the truth of the Supreme Principle just is to be a transcendental realist, for cannot consider the Supreme Principle as true *without* taking up the perspective that abstracts from the conditions of possible experience and considers things from the "timeless," God's-eye perspective that Allison associates with transcendental realism. Thus, despite the fact that Allison *wants* to say that "[t]he antinomial conflict arises from neither [the Supreme Principle] nor transcendental realism considered in isolation but from their combination," his conception of transcendental idealism as identical with the epistemic perspective adopted in the Supreme Principle prevents him from doing so.²⁰⁴

²⁰³ As Allison writes in his discussion of the role of the Supreme Principle, the notion of "givenness" employed in the Supreme Principle is "givenness for a pure understanding or an intellectual synthesis, which abstracts from the sensible conditions of empirical givenness"—to take conditions to be *given* in this sense on his view simply is to take them as things in themselves (391). Here is a related objection against Allison that strikes me as compelling: if Allison is right, then it is a kind of category mistake to apply the Supreme Principle to appearances. But whereas Kant *does* think it is a mistake to apply the Supreme Principle to appearances, he clearly does not think it is a category mistake.

²⁰⁴ To this Allison may reply that the *apparent* truth of the Supreme Principle (and transcendental realism) and the *judgment* that transcendental realism is in fact true are two separate things. However, I take it Kant intends for a stronger distinction between the Supreme Principle and transcendental realism than even this allows for. Another problem with the Allisonian view, which Allison himself presumably would deny is a problem, is that it cannot accommodate Kant's claim at A498-9/B526-7 that the Supreme Principle is *true* for things in themselves. For on Allison's understanding of transcendental idealism, any claims to truths about things in themselves already adopt the illegitimate transcendental realist perspective. Thus, Allison must read Kant's claim at A498-9/B526-7 as a mere *description* of how we represent things when we (mistakenly) take the perspective of transcendental realism to be a legitimate one. On my view, we should take Kant at face value when he says that things in themselves *are* given with the complete totality of their conditions.

Grier's account is subject to a similar set of problems. According to Grier, Kant takes the Supreme Principle to be "rationally necessary but also illusory," and "the illusion [...] is said to be necessary and inevitable quite apart from the transcendental realist's conflation of appearances and things in themselves."²⁰⁵ The problem with the Supreme Principle, Grier argues, is that it asserts "an objective necessity," and in making the "assumption of an 'unconditioned' that holds of objects themselves," it illegitimately abstracts from "the conditions under which it could be applied to objects of experience."²⁰⁶ That is, the Supreme Principle employs the terms "conditioned" and "unconditioned" transcendentally, which on Grier's account means that it makes "an assertion that applies to objects considered in abstraction from the particular sensible conditions of our intuitions (i.e., to objects in general)."207 If this is correct, however, and if transcendental realism simply is the mistaken epistemological position that takes knowledge to be possible independently of the sensible conditions of experience (as Grier suggests it is), then the Supreme Principle must be understood as an expression of the transcendental realist's illegitimate epistemology.²⁰⁸ Indeed, Grier suggests as much when she writes: "Insofar as the principle is used without regard to (independently of) the conditions under which objects are given in experience, it is erroneously thought to be applicable to objects considered independently of these conditions (to things in themselves)."²⁰⁹ On her conception of transcendental realism, to think of objects in this way simply is to adopt the transcendental

²⁰⁵ Grier (2004), 187 and 181.

²⁰⁶ Grier (2004), 122.

²⁰⁷ Grier (2004), 175.

²⁰⁸ In fact, Grier typically glosses "transcendental realism" as "the confusion (or conflation) of appearances and things in themselves", which is compatible with both epistemic *and* metaphysical explanations of the nature of that conflation. However, Grier indicates in footnotes that she is "obviously indebted to Allison's entire account of the connection between the methodological standpoint of the transcendental realist and the conflation of appearances and things in themselves" (2004, 150 fn 15).

²⁰⁹ Grier (2004), 122-3.

realist's epistemology; hence, one cannot in fact hold that the Supreme Principle is "necessary and inevitable quite apart from the transcendental realist's conflation of appearances and things in themselves."²¹⁰ If transcendental illusion is the assumption of the Supreme Principle (as she says it is), and if transcendental realism is the position that erroneously makes claims about things in themselves (as she also asserts), then transcendental illusion and transcendental realism are inextricably bound together after all.²¹¹

Finally, consider Bird's treatment of the Supreme Principle. According to Bird, the notion of the unconditioned employed in the Supreme Principle "stands as a cipher for the inchoate wish on the part of reason for completeness in some final explanation of the world."²¹² The problem with this, on Bird's account, is that a notion of this sort violates the "principle of empiricism" to which Kant is (supposedly) committed.²¹³ As Bird argues, "empirical physical cosmology" is a legitimate discipline on Kant's account, but the Supreme Principle expresses the conviction that cosmological questions can be resolved through reason alone, and this violates the principles at the heart of transcendental idealism.²¹⁴ As Bird writes in his discussion of the first antinomy's thesis and antithesis positions:

Both sides, and the debate itself, can be rejected because they rest on the

²¹⁰ Grier (2004), 181.

²¹¹ A related worry for Grier is the following. Kant says the antinomial arguments "are not semblances but well grounded, that is, at least on the presupposition that appearances, or a world of sense comprehending all of them within itself, are things in themselves" (A507/B535). Grier must argue that here he is making the uninformative point that arguments made from the illegitimate epistemic perspective of transcendental realism would be legitimate if that perspective were itself legitimate. In contrast, my reading takes Kant to be acknowledging a more substantive point of agreement between himself and the transcendental realist. Namely, both Kant and the transcendental realist *agree* that the Supreme Principle holds for things in themselves. Similarly, when Kant says at A498/B526 that "If the conditioned as well as its condition are things in themselves, then when the first is given [...] the unconditioned is thereby simultaneously given," Grier must insist that he is simply *describing* the principle to which we *would* be committed if (*per impossibile*) transcendental idealism were false. In contrast, the reading I defend interprets Kant as making an important claim here, which establishes dialectical common ground between transcendental realists and transcendental idealists.

²¹² Bird (2006), 663.

²¹³ Bird (2006), 676.

²¹⁴ Bird (2006), 671.

questionable assumption that we can decide issues about the character of the physical universe by a priori reasoning alone. Both are consequently led to make claims about the universe as a thing in itself, and both are mistaken because we can strictly neither *affirm* nor *deny* such claims about things in themselves. For Kant Ideas of reason, used in a spurious transcendent way, have no possible connection to intuition, and their associated principles are consequently undecidable and unverifiable by us.²¹⁵

If transcendental idealism is a kind of empiricist epistemology, which rules out any role for purely rational arguments, then there is no daylight between the Supreme Principle and transcendental realism; the thesis and antithesis arguments violate the epistemic strictures of idealism by being based on the Supreme Principle and by employing ideas of unconditioned objects. In fact, on Bird's account, the details of the antinomial arguments do not matter once we realize all *a priori* arguments about the world violate the epistemological principles Kant intends to advance in defending transcendental idealism.²¹⁶ As Bird writes, "The real issues about the origins of the physical universe and the basic particles of matter belong to empirical science and cannot be resolved by an a priori, dogmatic metaphysics."²¹⁷ Given this, Bird cannot explain how on Kant's view the Supreme Principle and transcendental realism make *independent* contributions to the antinomies. The Supreme Principle is to be rejected because it attempts to extend our cognition beyond what can be verified in experience, and likewise transcendental

²¹⁵ Bird (2006), 674.

²¹⁶ As Bird puts it, "Since the basic point of his project is to resolve the apparent conflicts in the conclusions by claiming that the proofs are fundamentally flawed, it is less important to clarify them than to clarify why Kant thinks them illusory" (2006, 670). One page later, he writes, "The dialectical illusion arises because empirical physical cosmology is *misconstrued* as a pure rational discipline, so that its central issues can be resolved by pure reason alone in a rationalist metaphysics (B436)" (671). Another worry is that Bird's reading makes Kant's case against the rational cosmologist question-begging (since it rejects a priori rational proofs out of hand). I think Bird's reading *does* have this problem, and so the worries about question-beggingness I rehearse in section 3.2 below (in my discussion of Moderate Epistemic readings) apply to Bird's view too.

²¹⁷ Bird (2006), 666.

realism is to be rejected for the very same reason.²¹⁸ And when we embrace the "principle of empiricism" to which Bird thinks Kant is committed, none of the antinomial arguments can get off the ground.²¹⁹

To summarize, then, although Allison, Grier, and Bird offer readings that differ in several important ways, they share a common problem. While Kant presents the Supreme Principle and transcendental realism as making two separate contributions to the antinomial conflicts, Allison, Grier, and Bird characterize transcendental idealism and transcendental realism such that the Supreme Principle ends up reflecting the mistaken epistemology of transcendental realism. As they see it, transcendental realism is a set of mistaken epistemological commitments, and the Supreme Principle cannot be characterized independently of these commitments. Moreover, there is reason to think that all Idealism as Epistemology readings must encounter a similar problem. For if transcendental idealism is simply an epistemological doctrine, and if transcendental realism is likewise identified with the contrary epistemological doctrine, which gives rise to antinomies, then whatever role the Supreme Principle has in generating the antinomies must turn out to be a part of transcendental realism. I take it this conflicts with Kant's own suggestion that a misuse of *reason* (via an inappropriate application of the Supreme Principle) and transcendental realism are two separate things. And as should be clear, a metaphysical reading of the antinomy's solution can keep these two issues properly distinguished

²¹⁸ The following claim from Bird makes it clear that he thinks transcendental realism errs not in adopting a false metaphysics but rather in adopting a false (anti-empiricist) epistemology: "Kant accepts our knowledge of empirical things in themselves, inner *and* outer, but rejects knowledge of transcendental things in themselves" (2006, 184). As Bird argues, Kant believes empirical objects are metaphysically real (in the only sense that could matter to us), and the error made by transcendental realists is just that they reach for knowledge that goes beyond the empirical. ²¹⁹ As Bird writes of the Supreme Principle, "It is shown to be misleading and faulty because the latter objects [the unconditioned objects of reason referred to in the principle], accessible only through reason if the inference succeeds, fail to meet the legitimate requirements of the cited principle of empiricism at B496" (2006, 676). But it should be clear that the empiricist principle Kant discusses is *not* one that he himself endorses, since it instead describes the position of the *antithesis* arguments, which Kant himself rejects.

from one another: the Supreme Principle expresses the conviction that pure reason can extend our knowledge to the unconditioned, while in rejecting transcendental realism in the antinomy's resolution, Kant makes the metaphysical assertion that spatiotemporal objects are not mindindependent things in themselves.

3.2 Moderate Epistemic Readings

Whereas Idealism as Epistemology readings start with an interpretation of transcendental idealism as broadly epistemic in nature and conclude that the resolution of the antinomies must therefore also be epistemic, Moderate Epistemic readings allow that Kant makes some metaphysical claims about things in themselves and so do not insist on reading transcendental idealism as a completely non-metaphysical doctrine. Instead, Moderate Epistemic readings argue that Kant's solution to the antinomies follows from the account of cognition that is defended as a *part* of transcendental idealism (namely, as the non-metaphysical part). According to these readings, Kant's solution to the antinomies consists in his denial that we can cognize the objects of traditional metaphysics, and the proponents of the thesis and antithesis arguments err in thinking that such cognition *is* possible. As the argument goes in the case of the first antinomy, whereas the proponent of the thesis argument claims that we can cognize the world as finite, and the proponent of the antithesis argument claims that we can cognize it as infinite, Kant resolves the antinomy by claiming that the whole world is not a possible object of cognition for us at all. The world might turn out to be either finite or infinite, but there is no way for us to determine this in cognition, even in principle.

In the secondary literature, there are a wide variety of arguments for Moderate Epistemic readings, but an especially common argument goes as follows.²²⁰ In the Transcendental Analytic, Kant gives an account of the possibility of cognition that establishes two requirements for any possible cognition. First, cognition is possible only when an object is given to us in intuition; second, cognition also requires the application of a concept to that intuition in an act of thinking.²²¹ Given this, the argument goes, the claims made in the antinomies can be ruled out as illegitimate. For the thesis and antithesis positions make claims about objects of cosmological *ideas*, and *ideas* are representations of objects that can never be given in experience—i.e., they cannot meet the aforementioned requirements on *cognition* (A320/B377). From this it follows that the cognition transcendental realists claim to have in the antinomies is impossible. We cannot cognize the last member in a finite series of conditions, and we cannot cognize an infinite series of conditions; hence, we have no way to determine in cognition whether the series of conditions corresponding to the spatiotemporal extent of the world is finite or infinite, and neither the thesis nor the antithesis position can be sustained.

To be sure, some passages in the first *Critique* suggest that Kant does mean to resolve the antinomies by recommending his characteristic modesty concerning what we can cognize. In the section on Phenomena and Noumena, for example, he writes:

²²⁰ Proponents of Moderate Epistemic readings include Al-Azm (1982), de Boer (2020b), Malzkorn (1999), Watkins (2019b), Willaschek (2018), and Wood (2010). Other scholars who express sympathy for the reading include Engelhard (2005) and Smith (1918)—or at least they argue that *one* of Kant's solutions is purely epistemic. Falkenburg (2000) may also be counted as a proponent of a Moderate Epistemic Modesty readings, though she does not embrace a metaphysical reading of transcendental idealism (and so may ultimately be better characterized as a proponent of an Idealism as Epistemology reading). In addition, Guyer (1987) argues that the solution to the antinomies *should* have been wholly epistemic or methodological, but Kant mistakenly *thought* it required his controversial metaphysics.

²²¹ E.g., see A50/B74: "Intuition and concepts therefore constitute the elements of all our cognition, so that neither concepts without intuition corresponding to them in some way nor intuition without concepts can yield a **cognition**." In the Dialectic itself, Kant reiterates this, saying that "All our cognition starts from the senses, goes from there to the understanding, and ends with reason" (A298/B355).

The Transcendental Analytic accordingly has this important result: That the understanding can never accomplish *a priori* anything more than to anticipate the form of a possible experience in general, and, since that which is not appearance cannot be an object of experience, it can never overstep the limits of sensibility, within which alone objects are given to us. (A246-7/B303)²²²

Given that Kant defines *ideas of reason* as precisely those ideas that "go beyond the possibility of experience," it seems to follow straightforwardly that cognition of the *world* (the object of an idea of reason) is impossible (A320/B377). Moreover, in the appendix to the Transcendental Dialectic, Kant even goes as far as to assert that the antinomies are already resolved by the theory of cognition established in the earlier Transcendental Analytic:

The outcome of all dialectical attempts of pure reason not only confirms what we have already proved in the Transcendental Analytic, namely that all the inferences that would carry us out beyond the field of possible experience are deceptive and groundless, but it also simultaneously teaches us this particular lesson: that human reason has a natural propensity to overstep all these boundaries, and that transcendental ideas are just as natural to it as the categories are to the understanding... (A642/B670)²²³

This seems to suggest that the Analytic suffices to show that pure reason cannot extend our cognition beyond what can be given in experience. If this is correct, and if the arguments of the antinomies assume to the contrary that such an extension of our cognition via pure reason *is* possible, then it seems very plausible to conclude that Kant aims to resolve the antinomies by reaffirming the account of cognition defended in the earlier Analytic. That is, once we grant that cognition requires both intuitions and concepts, we will see that the cognition transcendental realists claim to have in the antinomies is spurious and that no such cognition of the world is

²²² For a similar claim, see A132/B171.

²²³ See also Kant's criticism of the first paralogism: "We have shown in the analytical part of the Transcendental Logic that pure categories [...] have in themselves no objective significance at all unless an intuition is subsumed under them" (A348-9). Later in the paralogisms, Kant again writes: "the above principles of the Analytic have sufficiently enjoined us to make none other than an experiential use of the categories (such as substance)" (B417-18 fn).

possible. And if this is correct, then the antinomies can be resolved without making any metaphysical claims at all about the nature of the spatiotemporal world. The problem with the arguments of the antinomies is simply that they assume cognition is possible beyond the boundaries of possible experience, and Kant's solution is to limit the cognition that is in fact possible for us (per the Analytic's guidance).

It is difficult to overstate the influence of Moderate Epistemic readings in the secondary literature on the antinomies. For a brief sampling of scholars who endorse these readings, consider the following statements from de Boer (2020b), Willaschek (2018), Malzkorn (1999), and Watkins (2019b), all of whose work on the Transcendental Dialectic has been influential. De Boer writes that Kant's goal in the Dialectic is to show that "synthetic *a priori* cognition of objects *must* rely on schematized concepts [...] but that metaphysics *cannot* rely on such concepts if it is to establish itself as a purely intellectual discipline."²²⁴ The account of schematized concepts establishes that "synthetic *a priori* cognitions are warranted if they pertain to appearances, but not if they pertain to purely intellectual concepts such as the world, the soul, and God," and since the antinomies purport to establish synthetic *a priori* cognition of the *world* (for which there is no schematized concept), we can safely conclude that their arguments are faulty.²²⁵

Willaschek also claims that the Transcendental Analytic contains the key to the antinomies' resolutions. As he writes:

[I]n the Transcendental Analytic Kant had argued that synthetic cognition cannot be purely discursive, but always requires some intuitive element (minimally, a relation to *possible* experience). Reason, by contrast, is a purely discursive faculty for Kant, which means that rational insight into first principles cannot be based on

²²⁴ De Boer (2020b), 52.

²²⁵ De Boer (2020b), 47-8.

anything intuitive (which in human beings is always sensible), but only on logical reasoning and the discursive principles and concepts that come with it. Already here, at the very beginning of Kant's investigation into the real use of reason in the Transcendental Dialectic, we can therefore foresee that this story will not end well: while according to the Transcendental Analytic there cannot be cognition from concepts alone, according to the Dialectic the cognitions of pure reason would have to be precisely that: purely discursive, cognitions from mere concepts.²²⁶

In short, according to Willaschek, Kant considers it a settled matter at the start of the

Transcendental Dialectic that cognition of objects such as God, the soul, and the whole world is

impossible. As he puts it later in his discussion,

But since the specific objects of speculative metaphysics — the supersensible in general and the unconditioned in particular — are not empirical objects and cannot be given in space and time, it follows from the general restriction of human cognition [...] that any attempt to gain metaphysical cognition of the soul, the world and large, and God must fail.²²⁷

So understood, the antinomies' resolutions do not require anything more than a return to the earlier parts of the *Critique*. We need to exercise the appropriate epistemic modesty, but we do *not* need to assert any controversial claims about the nature of spatiotemporal reality or the magnitude properties spatiotemporal phenomena in fact have.²²⁸

Finally, consider the way in which Malzkorn and Watkins argue for readings like this in the course of their discussions of the Supreme Principle. According to Malzkorn, the problem with using the Supreme Principle to derive conclusions about the world as a whole is that the term "given" in the Supreme Principle in fact means "available to the mind for cognition". Thus, the Supreme Principle in fact expresses reason's *mistaken* assumption that whatever exists can

²²⁶ Willaschek (2018), 34.

²²⁷ Willaschek (2018), 252.

²²⁸ As noted above, I am unsure of how Willaschek understands the relationship between this part of his reading and his remarks on restricting the principle of comprehension for appearances.

also be given to the mind, and according to Malzkorn, Kant's goal in the antinomies is to reveal the error in this approach. Malzkorn glosses the Supreme Principle as follows:

(SP): $\forall x(x \text{ is conditioned } \land x \text{ is given} \rightarrow \forall y(y \text{ is a condition of } x \rightarrow y \text{ is given}))^{229}$

And as Malzkorn explains it, the problem with this principle is that it is *false* that the whole series of an object's conditions is *given to the mind* whenever an object is given.²³⁰ Indeed, we very often have cognitive access to an object without having cognitive access to all the objects that explain it.

Watkins offers a similar assessment of the Supreme Principle in his discussion of its role in the antinomial arguments. According to Watkins, it is *true* that all of an object's conditions must *exist* if the conditioned object exists, but it is *not* true that all these conditions can be cognized. So although the Supreme Principle might be a legitimate basis for concluding that something unconditioned *exists*, it is not a legitimate basis for concluding that we can *cognize* the unconditioned. As Watkins puts it, "Kant argues that despite the indispensability of the concept of the unconditioned to our cognitive practices, the limitations of our cognitive faculties entail that we cannot in fact have cognition of the unconditioned, even if we are committed to the existence of what is unconditioned."²³¹ As is clear, Watkins's reading is metaphysically more robust than many of the other readings canvased above, since he allows for proofs of the *existence* of unconditioned objects. Nonetheless, Watkins has a broadly epistemic reading in the sense that he interprets the antinomial arguments as making claims to *cognition*, and

²²⁹ Malzkorn (1999), 55. The original German reads: "(GV): $\forall x(x \text{ ist bedingt } \land x \text{ ist gegeben} \rightarrow \forall y(y \text{ ist Bedingung von } x \rightarrow y \text{ ist gegeben})$." (Where 'GV' stands for 'Grundsatz der Vernunft').

²³⁰ As Malzkorn writes, "Die synthestischen Sätze, von denen hier die Rede ist, ergeben sich (wiederum durch unterschiedliche Spezifikationen der bedingungrelation), wenn man (irrtümlich) davon ausgeht, daß der Grundsatz (GV) objective gültig ist und tatsächlich 'die ganze Reihe einander untergeordneter Bedinungen, die mithin selbst unbedingt ist', als Gegenstand der Erkenntnis an die Hand gibt" (1999, 56).

²³¹ Watkins (2019b), 13.

correspondingly, he argues that the solution to the antinomies is to abandon the view that we can cognize unconditioned objects via the purely rational proofs offered in the antinomies. So even if the Supreme Principle has some metaphysical upshots, Watkins argues, the antinomies are resolved by making a broadly epistemological move.²³²

How should we assess Moderate Epistemic readings? In my view, there are both textual and philosophical reasons to resist them. First, consider the textual reasons. Although it is true that some texts suggest the Transcendental Analytic *already* establishes that cognition of objects answering to the cosmological ideas is impossible, other textual evidence suggest that Kant intends for it to be an open question at the start of the Transcendental Dialectic whether pure reason might extend our cognition. That is, other texts suggest Kant means for us to take seriously the possibility that we can *begin* with a cognition that satisfies the requirements of the Analytic and then use principles of pure reason to extend our cognition beyond what can be met with in experience. For instance, in the introduction to the Transcendental Dialectic, Kant indicates that the Dialectic will set out to answer the following question:

Can we isolate reason, and is it then a genuine (*eigener*) source of concepts and judgments that arise solely from it and through which it relates to objects [...] In a word, the question is: Does reason in itself, i.e., pure reason, contain a priori synthetic principles and rules, and in what might such principles consist? (A305-6/B362-3)²³³

In asking whether reason is a genuine source of concepts and judgments through which it can relate to objects (and in asking whether pure reason contains *a priori* synthetic principles), I take

²³² This said, one difference between Watkins and some of the other proponents of Moderate Epistemic readings discussed above is that he does not think the Analytic conclusively proves that reason *cannot* extend our cognition—the Dialectic is required for that. Watkins may also include metaphysically more robust claims in his reading of the antinomy's solution (in forthcoming work), and depending on how the details go, his view may not be subject to the criticisms I raise against other Moderate Epistemic readings here.

²³³ Cambridge edition translation slightly modified to clarify the referent of "it" in line 2.

it Kant is asking whether pure reason might be a source of new cognition for us.²³⁴ That is, this passage suggests that at the start of the Dialectic, Kant intends for us to be open to the possibility that principles like the Supreme Principle can extend our cognition from what is given in experience as conditioned to objects that are unconditioned and are not given in experience.

More importantly, however, consider the following philosophical argument against the Moderate Epistemic reading. Suppose Kant did intend to say that the Transcendental Analytic already rules out cognition of the objects of traditional cosmology. Or to return to the case of the first antinomy, suppose he did mean to say that cognition of the whole world or its magnitude properties is in principle impossible, given the results of the Transcendental Analytic. If this is the case, then Kant's transcendental realist opponents can present him with the following dilemma. On the one hand, if he means to deny that the antinomial arguments deliver cognition on the grounds that they deal with objects that are not experienced, then the transcendental realist will object that she did not mean to claim cognition in this experiential sense.²³⁵ On the other hand, if Kant allows that cognition need not be experiential, then the transcendental realist will argue that he is simply wrong to say that cognition of the unconditioned objects treated in the antinomies is impossible. After all, Kant himself says that the antinomial arguments are "well grounded, at least on the presupposition that appearances, or a world of sense comprehending all of them within itself, are things in themselves" (A507/B535). Presumably, he means to say here

²³⁴ To preempt a misunderstanding, when I say that Kant's solution to the antinomy is metaphysical rather than epistemic in this chapter, I do not mean to deny that he is interested in whether or not pure reason can extend our cognition beyond experience in the Dialectic (he clearly is). Rather, I mean that in resolving the antinomy, Kant *justifies* his conclusion that pure reason cannot extend our cognition by making a claim about the metaphysics of the spatiotemporal world, viz., the spatiotemporal world is metaphysically indeterminate in its magnitude properties. So against proponents of epistemic readings, I claim that he does not *justify* his conclusion by appealing to the idea that objects represented by reason might not be properly intuited or thought (the Analytic's requirements).
²³⁵ As she will say, it is precisely in contexts where experiential cognition is impossible that reductio-style arguments like the ones she uses in the antinomies are so useful.

that the arguments are both *valid* and *sound* on the assumption that transcendental realism is true. But if the arguments are both valid and sound on the assumption of transcendental realism, then Kant must explain exactly *how* his solution to the antinomy shows that the arguments are either not valid or not sound (given that transcendental realism is false). And on the face of it, it is not clear how the account of cognition defended in the Analytic could furnish such an explanation.²³⁶

Let us take a closer look at this problem by returning to the particular case of the first antinomy (again narrowing our focus to the temporal part of the argument). Recall that the transcendental realist proponents of the thesis and antithesis arguments begin with the fact that the current state of the world is given to us in experience as temporally conditioned by a previous state. They then infer via an application of the Supreme Principle to the conclusion that the *complete totality* of states leading up to the present state must have existed, and hence the complete series of past world-states must be either finite or infinite. The arguments then proceed as follows:

Thesis Argument	Antithesis Argument
1. Either the world is finite in time, or it is infinite in time. (DERIVED FROM THE SUPREME PRINCIPLE)	1. Either the world is finite in time, or it is infinite in time. (DERIVED FROM THE SUPREME PRINCIPLE)
2. The world is infinite in time. (ASSUMPTION FOR REDUCTIO)	2. The world is finite in time. (ASSUMPTION FOR REDUCTIO)

²³⁶ I will admit that part of what is motivating me here (and in what follows) is a fairly minimal conception of transcendental idealism as the doctrine according to which space and time are mere forms of intuition rather than mind-independent features of things in themselves, and according to which spatial and temporal objects are therefore also mind-dependent appearances (and not things in themselves). I take Kant at face value when he says that transcendental idealism is proved in the Aesthetic and that it is simply the doctrine according to which all spatiotemporal objects are "nothing but appearances, i.e., mere representations" (A490-1/B518-19; see also A369). On this reading, Kant's restriction of cognition to appearances is a core *consequence* of transcendental idealism but not, strictly speaking, part of the definition of transcendental idealism. Correspondingly, to claim cognition of things in themselves is not *by definition* to be a transcendental realist (on this point I diverge widely from Willaschek 2018).

- 3. If the world is infinite in time, then an infinite successive synthesis can be completed.
- 4. An infinite successive synthesis cannot be completed.
- 5. The world is not infinite in time (from 2-4).
- 6. Therefore, the world is finite in time (from 1 and 5).

- 3. If the world is finite in time, then it is possible for something to arise out of an empty time.
- 4. It is not possible for something to arise out of an empty time.
- The world is not finite in time (from 2-4).
- 6. Therefore, the world is infinite in time (from 1 and 5).

Can Moderate Epistemic Readings explain exactly where these arguments go wrong? Recall again that Kant suggests these arguments are both valid and sound on the assumption of transcendental realism—as he puts it, they are "well-grounded". But given this, a resolution of the antinomy requires either an explanation of why they are not, in fact, valid or an explanation of why they are not, in fact, sound. I think neither explanation can be provided if Kant's solution to the antinomy is merely epistemic; and if one were to insist that no such explanation is required, then we are forced to conclude that Kant's solution simply fails.

To see why this is so, first consider the possibility that the thesis and antithesis arguments *are* both valid and sound, and Kant's claim is only that valid and sound arguments do not always deliver *cognition*. As one might argue, what we care about is *cognition*, and the kinds of reductio proofs given in the antinomies simply are not cognition-securing arguments. But notice that this line of reasoning generates a version of the dilemma described above. On the one hand, if Kant insists on a conception of cognition according to which all possible objects of cognition are possible objects of *experience*, then proponents of the thesis and antithesis arguments should object that "cognition" in that sense is not the only valuable mental state they care about. After all, they might argue, if an argument is valid and does not have false premises, then it takes us to

a new *truth*, and *truth* is often what we are after. Indeed, it would seem significant to have a proof of the truth of the claim the world is finite (or, alternatively, infinite), even if we cannot achieve "cognition" (in the experiential sense) of its finitude (or infinitude). Worse still, if Kant's strategy *is* to argue that the arguments deliver truths but not cognitions, then an antinomy remains. If the thesis proof establishes that it is true that the world is finite, and the antithesis argument establishes that it is true that the world is infinite, then they jointly establish a problematic contradiction: it is true that the world is finite and true that it is infinite.

On the other hand, if Kant allows that we should count as "cognition" whatever substantive knowledge about the world we can achieve (regardless of whether or not the acquired cognitive state is *experiential* or even *possibly* experiential), then Kant again owes proponents of the thesis and antithesis arguments an explanation of *why* the proofs they offer do not deliver the cognition they think they do. And this explanation *cannot* appeal to the fact that the unconditioned objects whose existence they assert are not objects we can *intuit* (or *experientially* cognize). For presumably, the proponents of the thesis and antithesis positions consider the reductio arguments they employ adequate to establish what experience cannot. One argument uses reductio and disjunctive syllogism to establish that the world is finite; the other argument uses the same strategy to establish that it is infinite. But if this is correct, then an antinomy again remains. The thesis argument proves (non-experiential) cognition that the world is finite; the antithesis proves (non-experiential) cognition that the world is infinite, and together they entail an unacceptable contradiction.

This means proponents of Moderate Epistemic readings must show that Kant's account of cognition either renders the arguments not valid or not sound. Which strategy has the greatest likelihood of success? Consider some scholars' efforts to show that key premises in the thesis

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and antithesis arguments actually *do* bake in epistemological assumptions that Kant's account of cognition rules out. Al-Azm (1982), for example, writes that "The moral of the entire episode of the antinomy for the critical philosophy as a whole" is that "such rational principles, as the law of sufficient reason, are purely formal principles from which nothing can be inferred about the nature of actuality."²³⁷ So when the proponent of the antithesis argument claims that *it is not possible for something to arise out of an empty time*, she in fact presupposes (erroneously) that rationalist principles such as the Principle of Sufficient Reason can tell us about the nature of reality. Or consider the thesis argument's claim that *if the world is infinite in time, then an infinite successive synthesis can be completed*. Perhaps "synthesis" is here understood as a mental act such that the premises asserts that if the world is infinite in past time, then we can completely run through all its past states in a successive representational process. One might think this suggestion especially plausible because Kant often complains that transcendental realists fail to distinguish properly between what we can represent and what can occur in reality.²³⁸

But does Kant actually mean to say that the key premises in the thesis and antithesis arguments bake in false epistemological claims (i.e., claims that are undermined by Kant's own account of cognition)? One reason to think that he does not is that he explicitly *endorses* the key claims just discussed in the remarks on each argument. For instance, in the remarks on the thesis

²³⁷ Al-Azm (1982), 35.

²³⁸ De Boer (2020b) reads Kant in this way. According to de Boer, Kant's strategy for resolving the antinomies in the *Critique* is roughly the same as his strategy for avoiding metaphysical error in the *Inaugural Dissertation*. Namely, he intends to show in both works that we should not let our sensible representations infect our purely intellectual representations, and when we take experiential cognition to give us access to things as they are in themselves, we do just that. Guyer (1987) likewise argues that Kant *should* have endorsed a non-metaphysical solution to the antinomies, though he *actually* endorsed a brand of metaphysical phenomenalism. As Guyer suggests, the most plausible solution to the antinomies available to Kant is to use them to "warn against the careless assumption that reason's speculations can necessarily be confirmed by sense" (404). But clearly this solution succeeds only if the antinomial arguments in fact assume that what reason represents *can* be sensibly represented.

argument, Kant writes that he employs a correct conception of infinity in the thesis argument and that it follows with "complete certainty" from this conception that the world cannot have a temporally infinite past. As Kant writes, "I could have given a plausible proof of the thesis by presupposing a defective concept of the infinity of a given magnitude," according to which a magnitude is infinite "if none greater than it (i.e., greater than the multiple of a give unit contained in it) is possible" (A430/B458). But Kant explains that this notion would have yielded a flawed proof, since "no multiplicity is the greatest" (A430/B458). Thus, to avoid begging the question against the proponent of an infinite world, the thesis argument employs a correct conception of infinity:

[The correct concept of infinity] is not the concept of a **maximum**; rather, it thinks only of the relation to an arbitrarily assumed unit, in respect of which it [the infinite magnitude] is greater than any number. According as the unit is assumed to be greater or smaller, this infinity would be greater or smaller; yet infinity, since it consists merely in the relation to this given unit, would always remain the same, even though in this way the absolute magnitude of the whole would obviously not be cognized at all, which is not here at issue. (A430-2/B458-60)

Kant then continues in the next paragraph:

The true (transcendental) concept of infinity is that the successive synthesis of unity in the traversal of a quantum can never be completed. From this it follows with complete certainty that an eternity of actual states, each following upon another up to a given point in time (the present), cannot have passed away. (A432/B460)

As these passages show, Kant does not mean to reject the part of the thesis argument that rules out the possibility of the world's infinite past history. Hence, it cannot be the case that he thinks his own theory of cognition undermines this part of the argument.

Kant's own explanation of the role of the term "synthesis" in the "true" concept of infinity also suggests that his concern is with what can occur in the world and not, in the first

instance, with what we can represent. As he suggests, an infinite multiplicity is one whose parts cannot be completely traversed in a successive process, but this is a point that both transcendental idealists and transcendental realists should accept and does not hinge on claims about our representational limitations.²³⁹ As he writes in a footnote attached to the above passage, *if* a magnitude is one whose units can never be completely traversed in a successive process, then it is a quantity that "contains a multiplicity (of given units) that is greater than any number, and that is the mathematical concept of the infinite" (A432/B460). But this is a criterion articulating what it is for something to *be* infinite, and whether the "absolute magnitude" of the world would be "cognized" "is not here at issue" (A432/B460).²⁴⁰

The suggestion that Kant's own account of cognition falsifies key claims in the antithesis argument also is not convincing. For in the remark on the antithesis argument, Kant again indicates agreement with its main claim that a finite world bounded by empty time and empty space is impossible. He writes, "it is [...] uncontroversial that one surely would have to assume these two non-entities, empty space outside the world and empty time before it, if one assumes a boundary to the world, whether in space or in time" (A433/B461). And as he goes on to say (again speaking in his own voice), "A space, therefore (whether it is full or empty), may well be bounded by appearances, but appearances cannot be bounded **by an empty space** outside themselves. The same also holds for time" (A432/B459). In other words, Kant *affirms* the antithesis argument's claim that empty space and time "cannot [...] determine the reality of

²³⁹ Note that if supertasks are possible (and especially *really possible* rather than merely *logically possible*), then Kant's conception of the infinite might be undermined. However, I am aware of no arguments that show super tasks are more than merely logically possible.

 $^{^{240}}$ Thus, we can understand the term "synthesis" in the thesis argument to refer to the combining of conditions *via* the conditioning relation relevant to the antinomy. This fits well with the fact that the argument concerns the possibility of the *elapsing* of world states. Or as Kant puts it, the argument shows that an "infinitely elapsed world-series is impossible" (A426/B454).

things in regard to their magnitude and shape" (A432/B459). Given this, he cannot mean to resolve the antinomy by claiming that his own account of cognition falsifies these key claims.²⁴¹

Thus, although if we as contemporary readers wanted to escape the antinomies, it may in fact be our best strategy to argue that premises 3 and 4 in the reconstructions above are false (or to argue that at least one of them is false). But this is clearly not Kant's strategy, for he suggests that premises 3 and 4 on each side are convincing simpliciter and that neither transcendental realists nor transcendental idealists should deny them. This means that the antinomy's resolution cannot consist in showing that the intermediary premises in the thesis and antithesis arguments are falsified by Kant's own account of cognition.

What about the possibility that Kant's account of cognition undermines the *validity* of the arguments? This might be plausible if Kant intended to adopt a non-classical logic for judgments about the spatiotemporal world (per Posy's recommendation), but as I have argued in chapter 1, Kant stresses that the core principles of classical logic are compatible with his solution (A503-4/B531-2). Moreover, the arguments (as I have reconstructed them above) *are* valid on classical principles.²⁴²

If all this is correct, then we are left with the following. Kant must resolve the antinomy by saying that premise 1 in the reconstructions above is false. That is, the disjunction that the world is either finite or infinite in past time must be false. Is this compatible with the Moderate Epistemic reading? I have already explained why we cannot simply reconstrue this as a claim about what we can cognize and say that premise 1 is the claim that we can cognize either the

²⁴¹ Admittedly, there is some question as to whether Kant's reasoning in the remark on the antithesis argument presupposes transcendental idealism, but even if it does, this does nothing to undermine the claim that Kant does not himself intend to deny that an arising out of empty time is impossible (and so he does not think his account of cognition falsifies it).

²⁴² After all, each argument is of this basic form: Either X or Y. Assume X. If X, then Z. But not-Z. Therefore, Y.

world's finitude or its infinity. As I have argued, if "cognize" is taken to denote an experiential mental state, then the transcendental realist will deny that she intends to make a claim about "cognition" in this sense. But if "cognition" is interpreted more broadly, then the transcendental realist will demand an explanation of the possibility that premise 1 is false, and that explanation cannot be that the spatiotemporal world is in fact neither finite nor infinite (since that would be to abandon the metaphysical neutrality characteristic of Moderate Epistemic readings). Thus, Moderate Epistemic readings cannot provide a satisfying explanation of the antinomy's resolution.

In contrast, if Kant means to say that premise 1 is false because the series of past world states is in fact neither finite nor infinite, and if he means this as a metaphysical claim about the magnitude properties that series of spatiotemporal conditions have, then it is clear how Kant's solution resolves the antinomy. Because it is false that the series of past world states must be either finite or infinite, the thesis's proof that the world is finite and the antithesis's proof that the world is infinite do not go through. It may be impossible for the world to have a first, unconditioned state (per the relevant portion of the antithesis argument) and likewise impossible for it to have an actually infinite series of past states (per the relevant portion of the thesis argument), but since the world may be neither finite nor infinite, the further inference to the conclusion that it is *both* finite *and* infinite does not go through. Consequently, the contradiction embodied in the antinomy is removed.

Notice also that this coheres with Kant's own description of the antinomy's resolution. Kant writes in the resolution of the antinomies that "if [...] the world were **not** given **at all as a thing in itself**," then it would be given "as regards its magnitude, neither as infinite nor as finite" (A504/B532). This seems to suggest that premise 1 is false because the spatiotemporal world is

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neither finite nor infinite in magnitude, a point that is further supported by Kant's claim that he endorses a *both false* solution for the mathematical antinomies (A503-4/B531-2). Kant also claims in the resolution of the antinomies that the "world-series" can be neither "a determinate infinite, nor yet something determinately finite," and that "it is clear from this that we can assume the magnitude of the world to be neither finite nor infinite" (A518/B546 fn). I take this as further confirmation that his solution to the antinomy is to deny premise 1 on the grounds that the spatiotemporal world is indeterminate in magnitude rather than either finite or infinite.

Finally, before turning to a discussion of the relative advantages of a Metaphysical Indeterminacy reading, let us consider one final way in which one might propose to resolve the first antinomy without making any positive metaphysical claims about the magnitude properties that spatiotemporal series of conditions in fact have. Namely, suppose one held that premise 1 is false not because the spatiotemporal world has a magnitude that is neither finite nor infinite but rather because the very notion of a *spatiotemporal world* is contradictory. A number of commentators express sympathy for this view (including commentators like Grier and Allison), and Kant himself seems to suggest it in the Prolegomena, where he says that talk of the "spatiotemporal world" is like talk of a "rectangular circle" (4:341). As he explains in the Prolegomena, one can perhaps say that it is false that a rectangular circle is round because it is rectangular, and likewise one can say that it is false that it is not round because it is a circle; but in fact, the real reason both propositions are false is that the concept underlying them is contradictory. Similarly, Kant suggests, the concept 'sensible world' is contradictory, and because of this, the thesis and antithesis statements and any premises employing the notion of the sensible world (or the spatiotemporal world) are all false (Prol, 4:342).

While there is certainly some textual evidence for a reading of this sort, I think it

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provides a spurious solution to the antinomy for the following reason. Although the thesis and antithesis arguments employ the term "spatiotemporal world", talk of the magnitude of the "spatiotemporal world" is merely a stand-in for talk of the length of the relevant series of conditions. And importantly, Kant does *not* hold that the notion of a "series of spatiotemporal conditions" is contradictory. On the contrary, Kant speaks of series of spatiotemporal conditions in the course of spelling out his *own* views in the resolution of the antinomy, and he suggests that the notion of a series of spatiotemporal conditions is entirely unproblematic. So if the proposed solution is just that we cannot employ the notion of a "spatiotemporal world" without contradiction, the antinomy is not resolved: the thesis and antithesis arguments can simply be reformulated by reference to the notion of a series of spatiotemporal conditions.

At this point, one might object that Kant *does* say that a series of spatiotemporal conditions cannot be a *complete totality*, so perhaps the notion of a series of spatiotemporal conditions that is complete totality *is* contradictory. As I see it, this may be true, but it does not help to vindicate a non-metaphysical reading of the antinomy's resolution. For even if the notion of complete totality of spatiotemporal conditions is contradictory, Kant still needs an explanation of what it means to say that a series of spatiotemporal conditions is *not* a complete totality, and of *why* the notion of a complete totality of spatiotemporal conditions is contradictory. As we have seen (in chapter 2), Kant holds that both finite and infinite series can be complete totalities as a general matter, so it cannot be a general conceptual point about the concepts 'infinite complete totality' or 'finite complete totality'.²⁴⁴ But then the likeliest explanation is that the sub-arguments in steps 2 through 5 of the thesis and antithesis positions show why complete

²⁴³ In fact, Kant himself uses the language of a "series of states of things" in presenting the thesis argument (A426/B454).

²⁴⁴ To reiterate, here I oppose a line made popular by Allison (2004).

totalities of spatiotemporal conditions are contradictory. But by Kant's own lights, these subarguments show that the series of conditions treated are neither finite nor infinite. So the explanation of *why* spatiotemporal series of conditions cannot be complete totalities (on pain of contradiction) does not leave intact the assumption that series of spatiotemporal conditions must be either finite or infinite. Rather the claim that spatiotemporal series of conditions cannot be complete totalities goes hand in hand with the claim that spatiotemporal series of conditions are neither finite nor infinite.

4. Advantages of the Metaphysical Indeterminacy Account

In the section above, I argued that attempts to undermine the thesis and antithesis arguments *without* making any substantive metaphysical claims about the character of the spatiotemporal phenomena are unsuccessful. In this section, I explain why a Metaphysical Indeterminacy account provides a more satisfying reading of the antinomy. As I argue, if the spatiotemporal world is neither finite nor infinite in extent but rather has an *indeterminate* magnitude, then the thesis and antithesis arguments of the first antinomy turn out to be *unsound*; they depend on the false premise that the spatiotemporal world must be either finite or infinite in extent, and without this premise, no contradiction can be extracted from the conjunction of the thesis and antithesis arguments. It can be established that the spatiotemporal world is *neither* finite *nor* infinite but not that it is *both* finite *and* infinite. Moreover, because the false premise on which the arguments rely is entailed by an application of the Supreme Principle to conditioned things in space and time, a metaphysical indeterminacy approach can also explain *why* the Supreme Principle fails to hold for appearances. It fails to hold for appearances because

the spatiotemporal series of conditions treated in the mathematical antinomies are neither finite nor infinite.²⁴⁵

What are the main advantages of a Metaphysical Indeterminacy approach vis-à-vis the broadly epistemic readings discussed above? And what exactly does it mean to say that the series of spatiotemporal conditions treated in the first antinomy are indeterminate in magnitude rather than either finite or infinite? First, consider the question what it means to say that a series of conditions is indeterminate in magnitude rather than either finite or infinite. I have suggested in section 1 above that a series of an object's conditions is finite on Kant's account if it is determinate that there is a last condition in the series, i.e., a condition that is not conditioned by anything else (in the relevant conditioning relation). And a series is infinite if it is determinate that every condition is conditioned by a further one (again, in the relevant conditioning relation). At minimum, therefore, Kant is committed to the claim that neither of these determinate states of affairs obtains for the series of conditions treated in the first antinomy. As one might note, however, this still leaves open a number of questions about exactly how we should understand the resulting indeterminacy. Do series of conditions whose magnitude properties are indeterminate properly speaking have any magnitude properties at all? Is indeterminacy in magnitude similar to other types of indeterminacy with which we might be more familiar? And can we explain the indeterminacy to which Kant is committed by appealing to other elements in his Critical metaphysics?

²⁴⁵ In "How Many there Are Isn't," Goldwater (2020) argues for "count-indeterminacy" as a distinct kind of metaphysical indeterminacy. While Goldwater's understanding of metaphysical indeterminacy diverges from Kant's in various important respects, the term "count-indeterminacy" is useful insofar as it emphasizes that the relevant indeterminacy concerns (in the first instance) *how many* things there are and not the qualities or properties that things have.

In chapter 5, I answer several of these questions, arguing that a form of intentional object phenomenalism can explain why spatiotemporal phenomena turn out to be indeterminate in exactly the ways Kant suggests they are in the resolution of the antinomies. However, here I offer the following brief remarks to explain the scope of the indeterminacy to which I take Kant to be committed. Suppose we use " ∇ " to mean "it is indeterminate whether", suppose "*Cxy*" means "*x* is a condition of *y*", and suppose "*o*" denotes an object. Using this terminology, we might then distinguish between two different ways of articulating the claim that a series of conditions is indeterminate in magnitude rather than either finite or infinite. First, we might hold that for some conditioned object *o*, the series of *o*'s conditions is neither finite nor infinite because there is some condition of *o* for which it is indeterminate whether a further condition exists:

Option 1:
$$\exists x (Cxo \land \nabla \exists y Cyx)$$

Or second, we could articulate a weaker claim as follows:

Option 2:
$$\nabla \forall x(Cxo \rightarrow \exists y \ Cyx) \land \nabla \exists x (Cxo \land \neg \exists y \ Cyx)$$

That is, we could say that it is indeterminate whether all of *o*'s conditions have further conditions, *and* it is indeterminate whether *o* has some conditions that do not have any further conditions. This second option does not commit Kant to the existence of a particular last condition that is determinately conditioned (and beyond which it is indeterminate whether there are any further conditions).²⁴⁶

²⁴⁶ Note: clearly this does not amount to an interpretation of the indeterminacy operator " ∇ ", and I do not mean to suggest that Kant had in mind any formalizations of this sort. Nonetheless, using an uninterpreted operator can be helpful insofar as it allows for easy articulations of distinctions such as the one made here between Option 1 and Option 2.

At present, I will simply assume the weaker Option 2, which does not commit Kant to the existence of a last determinately conditioned condition in every series of spatiotemporal conditions. Even assuming this, however, a pressing issue is the following. Why should we think that the thesis and antithesis statements are true only if determinate states of affairs obtain and are false otherwise? That is, supposing Kant endorses Option 2, why should we think that the thesis and antithesis statements are both *false*? One could imagine arguing that their truth values must be *indeterminate* rather than false, given that the corresponding states of affairs in spatiotemporal reality are indeterminate.

However, recall that indeterminacy in the magnitude of the spatiotemporal series of conditions provides a rationale for Kant's claim that the notion of "absolute totality" employed in the Supreme Principle is not "valid" of appearances (A506/B534). And recall also that the Supreme Principle is first and foremost a principle expressing a demand for a certain kind of complete explanation. It says that if something is metaphysically explained by something else (i.e., if it is conditioned), then there must exist everything that is required for that thing's complete explanation (i.e., the *complete totality* of its conditions must exist, which is unconditioned). As we have seen, this is to say that where there are *any* metaphysical explanations, there are *complete* metaphysical explanations in the sense of explanations that do not leave any explanans out. But given that the Supreme Principle is linked to explanation in this way, it makes sense that Kant would want to say that a series of conditions satisfies the Supreme Principle *only if* reality is determinate in the respects ruled out by Option 2. That is, it makes sense that Kant should want to say the Supreme Principle is true only if it is either (a) determinate that the series has a last condition or (b) determinate that every item in the series is conditioned by a further one. For if neither of these determinate states of affairs obtain, then no

matter what conditions in a series we consider, we cannot know that those conditions do not leave some condition out (because it is not determinate that no further conditions exist). And if we cannot know (even in principle) that the conditions do not leave some relevant condition out (because it is indeterminate *how many* conditions there are), we do not have the kind of explanation the Supreme Principle demands.²⁴⁷

Clearly, the connections between explanation and determinacy are complex, and a complete exploration of these issues would take us far afield from Kant's specific arguments in the antinomies. Nonetheless, I hope the above remarks serve to shed some light on why Kant would have thought the thesis and antithesis statements are true only if the relevant series of conditions have determinate magnitudes and are false otherwise.

Turning now to the question of the relative advantages a Metaphysical Indeterminacy approach, how does it improve on the purely epistemic readings examined previously? First, notice that if Kant holds that the spatiotemporal world is *metaphysically* indeterminate in extent (rather than either finite or infinite), then we can explain why *pure reason* cannot extend our cognition to the unconditioned. Pure reason cannot extend our cognition to the unconditioned because the Supreme Principle is in fact *false* for spatiotemporal reality. Since series of spatiotemporal conditions are metaphysically indeterminate in magnitude rather than either finite or infinite (i.e., it is indeterminate *how many* spatiotemporal conditions exist), it is not true that each conditioned spatiotemporal thing exists with the *complete totality* of its conditions. For as

²⁴⁷ One could insist here that it is only *indeterminate* whether we have the kind of explanation the Supreme Principle demands, and perhaps it would be fair to challenge Kant in this way. But I think that although Kant allows for complete explanatory series that are both finite *and* infinite, he does not allow for complete explanatory series in which it is indeterminate exactly how many explaining items there are. At least intuitively, this strikes me as a plausible position. It strikes me as plausible that a complete explanation should include a kind of "and that's all" clause that guarantees no explanans have been excluded—indeterminacy in how many explanans there are undermines this.

we have seen, a *complete totality* of conditions on Kant's view must be either finite or infinite; if series of spatiotemporal conditions are neither finite nor infinite, they cannot contain anything unconditioned. For a similar reason, the specific arguments of the thesis and antithesis positions are unsuccessful: they depend on the premise that the series of conditions must be *either* finite *or* infinite, but it turns out spatiotemporal series of conditions are *neither*. That is, it is false that the series of conditions is finite and false that it is infinite, and for this reason the thesis and antithesis and antithesis and antithesis and for this reason the thesis and antithesis and antithesis arguments fail to establish their desired conclusions.²⁴⁸

Notice that this can help us see with greater clarity what is unsatisfying in a proposal like Willaschek's, which argues that Kant means to remain agnostic as to the actual length of series of spatiotemporal conditions and argue only that we should abandon the confidence in *reason* that the Supreme Principle expresses.²⁴⁹ According to Willaschek, the lesson of the antinomies is that we should not assume the spatiotemporal world is the kind of rational order our faculty of reason wants it to be, and part of this anti-rationalist modesty is agnosticism concerning the true answers to the questions of the world's extent, the divisibility of objects, and so on. In fact, according to Willaschek, transcendental realism simply *is* the thesis that "nature is a rational order that necessarily conforms to the principles of reason."²⁵⁰

²⁴⁸ As noted already above, my reading here turns on the fact that the conditioning relations treated in the mathematical antinomies always take *spatiotemporal* conditions as their relata. In contrast, the series of conditions treated in the dynamical antinomies are not subject to this constraint. To take the case of the third antinomy as an example, a casually conditioned thing in space and time *could* have a thing in itself as one of its causal conditions, and this helps to explain how the "both true" solution to the dynamical antinomies can be sustained alongside the "both false" solution to the mathematical ones. See the concluding chapter for further discussion.

²⁴⁹ As Willaschek argues, the antinomies arise when we falsely assume "that the rational principles that make us ask metaphysical questions (such as the Supreme Principle) are true of reality itself" (2018, 269).

²⁵⁰ Willaschek (2018), 238. Part of what is unusual about Willaschek's reading is that he understands transcendental realism and transcendental idealism such that one could be a transcendental idealist (affirm the mind-dependence of spatiotemporal phenomena) *and* be a transcendental realist (affirm that things in themselves conform to the Supreme Principle). In fact, the reading I am proposing makes Kant into a transcendental realist on Willaschek's' unconventional account of what transcendental realism is (see also 244 and 248). Needless to say, I do not think affirming the truth of the Supreme Principle for things in themselves makes one into a transcendental realist; rather,

But as Willaschek himself acknowledges, this sort of philosophical modesty is not fully satisfying, for this does not yet establish that the Supreme Principle is not *true* for appearances. And it would seem that the antinomial arguments retain their force unless this can be denied. Willaschek's own answer to this problem is to say that we must reach beyond the resources available to Kant to explain why the Supreme Principle does not hold for appearances. As we have seen, he suggests rejecting the principle of comprehension:

I think that the philosophically most plausible way for Kant to resist the inference from the conditioned to the unconditioned totality of its conditions consists in denying the principle of comprehension, that is, the assumption that for every predicate there is the totality of objects of which it is true [...] From our current perspective, this is a plausible move since we know that the principle of comprehension must be restricted anyway (although in a way that has nothing to do with the distinction between appearances and things in themselves) in order to avoid Russell-style antinomies (e.g. about the set of all sets that do not contain themselves). But of course this cannot be the reason for Kant's rejection of the principle of comprehension. Therefore, we will have to leave the question of how Kant can deny that the Supreme Principle holds for appearances unanswered. But note that such an answer is not required for the purposes of this book, because rejecting the Supreme Principle for appearances is part of Kant's *critique* of speculative metaphysics, not part of his account of why we must ask, and think we can answer, metaphysical questions.²⁵¹

However, if my arguments above have been compelling, then there is a more serious problem here than Willaschek acknowledges. For as I have argued, Kant's solution to the antinomy in fact *requires* an explanation of how the Supreme Principle could be false for appearances. Moreover, given the results of chapter 2, this explanation cannot be that the principle of comprehension is restricted or false. For the principle of comprehension pertains to what can be collected together to form sets (or, in our case, *series*), and as we have seen, Kant does not mean to deny that finite

I think transcendental realism is the doctrine that says spatiotemporal objects are mind-independent things in themselves (and that space and time are features of things in themselves rather than mere forms of intuition). ²⁵¹ Willaschek (2018), 155-6.

or infinite collections of conditions can form *series*. Rather, he means to deny that spatiotemporal series of conditions are *complete totalities*. Put differently, restricting the principle of comprehension may show that conditions cannot always form *unified totalities*, but Kant requires a reason for denying that spatiotemporal conditions form *totalities in the completeness sense*—this is the notion of totality Kant denies is "valid" of appearances when he denies that the Supreme Principle holds for them. A Metaphysical Indeterminacy reading furnishes the explanation a reading like Willaschek's lacks: the series of conditions treated in the first antinomy are neither finite nor infinite, and for this reason the Supreme Principle is false for appearances.²⁵² (For a diagram illustrating the relationship between the unified/complete totality distinction and the finite/infinite/indeterminate distinction, see the Appendix.)

Related to this, a second advantage of the Metaphysical Indeterminacy Reading is that it acknowledges substantive common ground between Kant and his transcendental realist opponents. This is important insofar as it helps to explain why Kant would have taken the antinomies to be a compelling "indirect" proof of transcendental idealism (A506/B534). For consider the following. According to the reading I have been recommending, Kant thinks the Supreme Principle is false for appearances because they are indeterminate in number (following Goldwater (2020), we might say "count-indeterminate"), and he thinks this indeterminacy is made possible by the ideality of appearances.²⁵³ Because appearances are mind-dependent and

²⁵² These observations might lead one to ask what other metaphysical commitments might violate the Supreme Principle. For instance, if Hume is right that appearances do not stand in real conditioning relations at all, would that violate the Supreme Principle for appearances? I take it this would not, since the Supreme Principle says only that where an object is conditioned by at least one thing, the *complete totality* of its conditions must exist. It does not say that everything (or even anything) *is* conditioned, and hence it does not rule out brute facts or things that stand in no conditioning relations at all.

²⁵³ See footnote 245 above for a brief remark on Goldwater (2020).

do not belong to fundamental reality, it is possible for them to violate the Supreme Principle.²⁵⁴ However, alongside this Kant also argues that transcendental realists are *right* to say that the Supreme Principle holds for things in themselves. Thus, he can say that the transcendental realist is right to apply the Supreme Principle to appearances *insofar as* she takes appearances for things in themselves. And she is right about what *would* follow from an application of the Supreme Principle to appearances: it would follow that the spatiotemporal world is finite (per the thesis argument) and infinite (per the antithesis argument). In this sense, a Metaphysical Indeterminacy reading makes room for Kant to say that the transcendental realist is reasoning well apart from her mistaken belief that spatiotemporal objects belong to fundamental reality. In contrast, purely epistemic readings of the resolutions must argue that transcendental realists are reasoning poorly in employing the Supreme Principle simpliciter. For the Supreme Principle either reflects a false conception of human knowledge (per Idealism as Epistemology readings) or expresses the unjustified assumption that pure reason has the power to deliver cognition (per Moderate Epistemic readings). In my view, a reading that allows for substantive common ground between Kant and his transcendental realist opponents is preferable insofar as it yields a dialectically more effective argument for transcendental idealism in the antinomies.

Finally, it is also worth emphasizing why a metaphysical *indeterminacy* reading is superior to a different sort of metaphysical reading one might adopt. Some commentators have suggested that the thesis and antithesis statements of the first antinomy are both false for the simple reason that the spatiotemporal world does not exist—or at least it does not exist *as a thing in itself*, as the thesis and antithesis positions take it to. But although an analysis of this sort can

²⁵⁴ Note, however, that there is more to say about how the ideality of appearances *results in* their indeterminacy. See chapter 5.

sometimes be made to look appealing by Kant's own remarks, a point similar to one made earlier serves as a response to this suggestion.²⁵⁵ Namely, Kant clearly is committed to the existence of spatiotemporal objects, which (even if they are mind-dependent) form series of spatiotemporal conditions. And as we have seen, the arguments of the thesis and antithesis can be re-articulated by appeal to the notion of a series of spatiotemporal conditions alone: the argument of the thesis shows that an infinite series is impossible, while the argument of the antithesis shows that a series terminating in an unconditioned first condition is impossible. Nothing in the arguments turns on the assertion that these objects are things in themselves, except insofar as that assertion justifies taking the Supreme Principle to hold for them. But this takes us back to the need for an explanation of *why* the Supreme Principle is false for spatiotemporal series of conditions. And as I have argued, unless the series of conditions is *neither* finite *nor* infinite (per a metaphysical indeterminacy approach), it follows (from the thesis and antithesis arguments) that the series is *both* finite *and* infinite, which is impossible.²⁵⁶

Thus, the advantage of a Metaphysical Indeterminacy reading is that it can explain exactly where the antinomial arguments falter and exactly why pure reason cannot extend our cognition beyond the limits of possible experience. The rational principle that *would* allow for this extension is the Supreme Principle, but spatiotemporal phenomena turn out to be metaphysically constituted such that the Supreme Principle does not hold for them. Hence, although we *are* given conditioned spatiotemporal things in experience, we cannot use the

²⁵⁵ For example, at A503/B531, Kant claims that two mutually opposed judgments may both be false if they both depend on an "inadmissible condition," and one might think the inadmissible condition is precisely the condition that a world of things in space and time exists. Alternatively, one might also take the inadmissible condition to be the condition that the world of things in space and time is a thing in itself (e.g., depending on whether one wants to allow for the possibility of a *world* that is not a thing in itself).

²⁵⁶ Note: I have not yet explained why an alternative solution is not to say that the series of conditions are *potentially infinite*. This is the topic of chapter 4 below.

Supreme Principle to infer to the complete totality of their conditions. Hence, we cannot use the Supreme Principle or arguments like the ones given in the first antinomy to conclude that anything unconditioned exists in space and time.²⁵⁷

5. Chapter Summary

Although Kant's solution to the first antinomy establishes that we cannot cognize the spatiotemporal world either as finite or as infinite, it does so *via* a metaphysical claim about spatiotemporal reality: the series of spatiotemporal conditions that determine the extent of the spatiotemporal world are neither finite nor infinite *in actuality*. I have argued that a Metaphysical Indeterminacy reading provides a compelling account of Kant's solution to the antinomy and that it tells a dialectically compelling story about *why* the Supreme Principle fails to hold for appearances in space and time. It also provides a clear account of Kant's claim that the thesis and antithesis statements of the first antinomy are both false. The thesis statement is false because it is not determinate that the series of conditions is finite; the antithesis statement is false because it as common ground between himself and his transcendental realist interlocutors that the Supreme Principle is true for fundamental reality (i.e., for things in themselves), Kant's reasons for thinking that the antinomy indirectly proves transcendental idealism are clear. Since the thesis

²⁵⁷ Note: in arguing that an exclusively epistemic reading of the mathematical antinomies' solutions is not compelling, I do not mean to rule out the possibility that Kant's solutions to other dialectical arguments in the Transcendental Dialectic are broadly epistemic (e.g., the Paralogisms or the Transcendental Ideal). But determining his views here would require a close analysis of these other arguments, which I cannot provide at present. That said, here I will note that the antinomies are somewhat unique in starting with conditioned spatiotemporal objects that *are* given in experience. Given this starting point, it is especially pressing that Kant explain why we cannot use the Supreme Principle to infer that the complete totality of their spatiotemporal conditions exists, thereby extending our cognition to the unconditioned. And as I have argued, Kant's explanation appeals to the fact that spatiotemporal series of conditions are metaphysically indeterminate rather than either finite or infinite.

and antithesis arguments effectively demonstrate that the spatiotemporal world is neither infinite nor finite, spatiotemporal series of conditions violate the Supreme Principle. But this possible only if they are not things in themselves, i.e., only if transcendental idealism is true.

I have also offered specific criticisms of two different versions of broadly epistemic readings prominent in the secondary literature. Against Idealism as Epistemology readings, I have argued that identifying transcendental idealism with an epistemological doctrine leaves too little space between the contribution transcendental realism makes to the antinomy, on the one hand, and the contribution the Supreme Principle makes to it, on the other. Against Moderate Epistemic readings, I have argued that a solution with no metaphysical upshots fails to provide compelling explanations of *why* the Supreme Principle is false for appearances and of where the proponents of the thesis and antithesis positions go wrong in laying out their proofs.

Chapter 4: Against Potential Infinity Solutions
In the previous chapter, I argued that a metaphysical reading of the antinomies' resolutions is more compelling than a merely epistemic one and highlighted the relative advantages of a *metaphysical indeterminacy* approach in particular. In this chapter, I turn to a question I left unanswered in the previous chapter. Namely, can the antinomies be resolved by appealing to the notion of *potential infinity* rather than indeterminacy? That is, can the solution consist in the claim that the spatiotemporal phenomena treated in the mathematical antinomies are merely *potentially infinite* rather than either strictly finite (the thesis position) or actually infinite (the antithesis position)? Given that the notion of potential infinity has a long historical pedigree, one might consider a solution of this sort more appealing than a reading that appeals to the less familiar notion of metaphysical indeterminacy.

However, in this chapter, as before, I argue that a metaphysical indeterminacy approach satisfies interpretive desiderata that other readings cannot. In particular, now taking the second antinomy as my focus, I argue that a *metaphysical compositional indeterminacy* reading provides a more satisfying reading of the second antinomy than does a *potential infinity approach*.²⁵⁸ Kant resolves the antinomy by claiming that it is metaphysically indeterminate whether or not all the parts of spatiotemporal objects have further actual parts within them, and this claim explains both why the thesis and antithesis statements are *both false* and why the antinomy indirectly proves the doctrine of transcendental idealism.²⁵⁹ As Kant argues, only transcendental idealists can appeal to the notion of compositional indeterminacy, and if spatiotemporal objects are

²⁵⁸ As explained in chapter 3 above, this notion of compositional indeterminacy counts as "metaphysical" on my view because it pertains to what *exists* or is *actual* in space and time. And this is fully compatible with the claim that indeterminacy with respect to what exists in space and time is ultimately a result of facts about our representations, given Kant's idealism (see chapter 5).
²⁵⁹ As I clarify below, this is not to say that *no* material parts determinately have further parts. Rather, it is to say that

²⁵⁹ As I clarify below, this is not to say that *no* material parts determinately have further parts. Rather, it is to say that it is not determinate that *all* do. See section 3 below for a more precise articulation of the sense in which objects are compositionally indeterminate on the reading I defend.

compositionally indeterminate, then the thesis and antithesis arguments of the second antinomy do not go through. In contrast, although a potential infinity reading is appealing at first blush, once we consider more carefully what it might mean to say that an object's parts are merely potentially infinite, we see that the notion of potential infinity as such cannot be at the heart of the antinomy's resolution. For depending on how we articulate the notion of potential infinity, the potential infinity view either (a) saddles Kant with a constructivist account of spatiotemporal objects that conflicts with core aspects of his Critical metaphysics of nature or (b) cannot provide a satisfying account of Kant's claim that the thesis and antithesis statements of the second antinomy are *both false*.

The plan for the chapter is as follows. In section 1, I review the conflict of the second antinomy and the so-called "indirect" argument for idealism that Kant claims it provides. According to Kant, transcendental realists identify spatiotemporal objects with things in themselves, and it follows from this that either the thesis or the antithesis statement of the second antinomy must be correct. That is, if transcendental realism is true, spatiotemporal objects must have either finitely many or infinitely many material parts. However, as Kant argues, this assumption yields a contradiction, since it can be proved from this assumption *both* that all spatiotemporal objects have finitely many parts (the thesis argument) *and* that they all have infinitely many parts (the antithesis argument). In contrast, transcendental idealism allows us to escape this contradiction by holding that the thesis and antithesis positions are *both false*. Kant therefore concludes that we can "draw from the antinomy true utility [...]: namely, that of thereby proving indirectly the transcendental ideality of appearances" (A506/B534).

With this understanding of the indirect argument for idealism in place, in section 2 I ask how we should understand the account of objects' compositional structures that results from

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Kant's solution and whether a potential infinity approach provides a compelling account of the antinomy's solution.²⁶⁰ According to proponents of *potential infinity approaches*, Kant resolves the second antinomy by claiming that objects have a potential infinity of parts rather than a strictly finite number or an actually infinite multiplicity of parts, and this appeal to potential infinity explains both (a) why the thesis and antithesis statements are *both false* and (b) why Kant's solution is not available to transcendental realists. As the argument goes, only transcendental idealists can attribute mere potential infinity to the multiplicity of material parts in objects, and so only idealists can escape the antinomy. I consider two initially plausible ways of fleshing out the details of this view and argue that neither is satisfying. An *intuitionist* approach to potential infinity saddles Kant with a form of ontological constructivism that he rejects (section 2.1), while a *modal* version of a potential infinity approach cannot tell a compelling story about the sense in which the thesis and antithesis statements of the second antinomy are *both false* (section 2.2).

In section 3, I turn to an articulation and defense of a *metaphysical compositional indeterminacy* reading. I argue that attributions of metaphysical compositional indeterminacy are compatible with but not equivalent to attributions of potential infinity, and the notion of metaphysical indeterminacy rather than potential infinity is at the heart of Kant's solution to the second antinomy. Not only does metaphysical compositional indeterminacy explain why the thesis and antithesis statements of the second antinomy are both false, but it also explains why Kant's solution is not open to transcendental realists.²⁶¹ The thesis and antithesis statements are

²⁶⁰ For example, see Bennett (2016/1974), Chiba (2012), Engelhard (2005), Holden (2004), and Falkenburg (2000), among others.

²⁶¹ This said, I do not defend the claim that the indirect argument for idealism is *fully* successful, for this would require defending every inference made in the antinomial arguments themselves, a task well beyond the scope of this project. See chapter 1 above for further discussion.

both false because there is metaphysical indeterminacy with respect to how many *actual* parts spatiotemporal objects have. And transcendental realists (even those who accept potential infinity) cannot endorse this solution because they are committed to a principle of complete metaphysical determinacy for what actually exists in space and time, a commitment that follows from their identifying spatiotemporal objects with mind-independent things in themselves.

1. The Second Antinomy and the Indirect Argument for Idealism

As we have seen, the topic of the second antinomy is the compositional structure of spatiotemporal objects: if we divide a spatiotemporal object into its material parts, the antinomy asks, do we find that objects resolve into simple material parts after a finite number of divisions, or do we find that every part contains further material parts to infinity? According to the thesis argument, the former position is correct: objects are composed of finitely many material parts the ultimate parts of which are simple. According to the antithesis argument, the latter position is correct: objects are composed of infinitely many material parts all of which are composite.

As in the case of the first antinomy, Kant argues that the second antinomy results from an improper application of the Supreme Principle to conditioned things in space and time, and each side of the antinomy exemplifies one of two ways in which unconditioned conditions could be thought to occur in the spatiotemporal world. According to the thesis argument, the unconditioned occurs in the simple parts of objects from which all spatiotemporal things are composed. According to the antithesis argument, the unconditioned occurs in the entire infinite

series of parts into which an object can be divided.²⁶² And as in the case of the first antinomy, each of the arguments proceeds by a combination of disjunctive syllogism and reductio. The thesis argument claims that objects are divided into either finitely many or infinitely many parts, and since they cannot be infinitely divided, the number of their parts must be finite. The antithesis argument claims that objects are divided into either finitely many or infinitely many parts, and the finite scenario is impossible; hence, the infinite alternative must obtain. Thus, the thesis argument concludes that objects' simple parts are the last members in a finite series of conditions; the antithesis argument concludes that the series of compositional conditions is actually infinite. And per Kant's conception of the unconditioned articulated at A417-18/B445-6, both alternatives give us *complete* explanations of compositionally conditioned objects (i.e., both articulate ways in which the explanatory demand made in the Supreme Principle can be satisfied for the conditioning relation of composition).²⁶³

Kant then argues that the antinomy "indirectly" proves transcendental idealism as follows (A506/B534). If transcendental realism is true, Kant argues, then *both* the thesis *and* the antithesis arguments are compelling. That is, if transcendental realism is true, then it is true *both*

²⁶² When Kant says that the parts of an object form a "series" (*Reihe*) of its conditions, we can take him to be referring to the collections of parts that compose objects, where these parts are ordered via conditioning relations of composition (larger parts being made possible by smaller ones, which they are next to in the series, and those smaller parts being made possible by still smaller ones, and so on).

²⁶³ Why couldn't objects be composed from infinitely many simples? I take it Kant's reasoning here goes roughly as follows. Imagine an object O, which is divided into parts a and b at the macro-level. Suppose further that a is divided into a_1 and a_2 at the next level down, a_1 is divided into a_1 and a_2 at the next level down, a_1 is divided into a_1 and a_2 at the next level down, a_1 is divided into a_1 and a_2 at the next level down, a_1 is divided into a_1 and a_2 at the next level down, a_1 is divided into a_1 and a_1 at the next level down after that, and so on. (We can likewise assume that b and a_2 have further parts of their own.) According to Kant, O, a, a_1 , and a_1 form (at least part of) a "series of decomposition," and if every series of decomposition in an object is finite, then the object has simple parts—in this case, the last part in the series of decomposition would be an unconditioned simple. In contrast, if each of the resulting series of decomposition is infinite, then the object does not have simple parts but rather is composed of composite parts to infinity. As I understand Kant, he simply assumes that no object has infinitely many parts at the macro level and that we do not suddenly encounter infinitely many divisions all at once at some subsequent level in the division. From these assumptions it follows that objects have simple parts if each series of decomposition is finite, and objects do not have any simple parts if every series of decomposition is infinite.

that every series of decomposition terminates in an unconditioned simple *and* that every series of decomposition is infinite and unconditioned in its entirety (A501-5/B529-33; A507/B535). But since a contradiction cannot be true, transcendental realism must be false. And because transcendental idealism can vindicate a view that "declar[es] both of the opposed assertions to be false," the antinomy and its resolution constitute an indirect proof of the transcendental ideality of spatiotemporal objects (A528/B556). As Kant writes, the antinomy proves that "appearances in general are nothing outside our representations, which is just what we mean by their transcendental ideality" (A507/B535).

Putting aside the question whether the thesis and antithesis arguments are compelling in their details (see chapter 1 above for critical discussion), we can then draw the following conclusions about the interpretive constraints a satisfying reading of the second antinomy must take. First, it must account for Kant's claim that his considered views on the compositional structure of spatiotemporal objects show that the thesis and antithesis statements of the second antinomy are *both false*. Second, it must explain why Kant's solution to the antinomy is tenable for transcendental idealists but not for transcendental realists. That is, it must show that transcendental realists cannot embrace the same "both false" solution to the antinomy that Kant embraces. And third, it must cohere with Kant's own claims about the picture of spatiotemporal reality that results from his resolution. I turn now to an explanation of why the two most prominent versions of a potential infinity approach cannot meet these constraints.

2. Against Two Versions of a Potential Infinity Approach

On the face of it, Kant's explanation of the indirect argument for idealism makes it plausible to conclude that the second antinomy is resolved by appealing to the notion of *potential*

*infinity.*²⁶⁴ As one might reason, the thesis statement asserts that objects have a strictly finite number of parts, while the antithesis statement asserts that objects have an actually infinite multiplicity of parts; given this, it is natural to conclude that Kant believes objects' parts are merely *potentially infinitely* numerous, and his solution to the antinomy consists in this claim.²⁶⁵ Indeed, this reading is made all the more plausible by the fact that Kant endorses the claim that spatiotemporal objects are "divisible to infinity (*ins Unendliche teilbar*)" (A525/B533). If objects are *divisible* to infinity, one might argue, then their parts must be potentially infinite, and if this is what's distinctive about Kant's position, it must also constitute his solution to the antinomy. According to a potential infinity approach, therefore, transcendental idealism alone makes it possible to hold that objects' parts are potentially infinite rather than either strictly finite or actually infinite, and Kant's solution to the antinomy consists in this claim.²⁶⁶

In the last half-century, commentators have championed two broader interpretive lines that might be taken to provide additional support for a potential infinity view. First, some interpreters have argued that the antinomies are part and parcel of a general attack on the notion

²⁶⁴ Commentators who explicitly endorse a potential infinity approach include Bennett (2016/1974), Holden (2004), Falkenburg (2000), Chiba (2012), and Engelhard (2005). As these scholars do not all understand the notion of potential infinity in the same way (and in some cases do not explain in detail how they understand potential infinity), one of my aims in in what follows is to get clear on how the notion *might* be understood and to show that two especially prominent options do not define a notion of potential infinity that Kant could accept *and* that can do the important explanatory work in Kant's solution to the antinomy.

 $^{^{265}}$ Although it is not controversial to say that the thesis statement asserts the strict finitude of objects' parts, one might wonder why we should think the antithesis statements assert their *actual infinity*. However, Kant explicitly says at A521/B549 that the antithesis position of the first antinomy asserts the "actual infinity" (*wirkliche Unendlichkeit*) of the spatiotemporal world; parity of reasoning would suggest that he intends for the second antinomy's antithesis statement to be read in a similar manner. It is also not unlikely that Kant had Leibniz in mind in composing the antithesis argument, for Leibniz held that matter is not only infinitely divisible but also actually infinitely divided (Letter to de Volder, January 19, 1706 in *AG*, 185).

²⁶⁶ As will be made clear below, different versions of the potential infinity approach must spell out the connection between transcendental idealism and potential infinity differently. Intuitionist approaches typically link potentially infinite phenomena to *constructed* phenomena (and hold that idealists assert that the spatiotemporal world is a construction). Modal accounts vary, but see Holden (2004) for one suggestion according to which Kant's rivals were committed to an "actual parts metaphysic," which rules out a potential infinity view.

of actual infinity. Jonathan Bennett (2016/1974), for example, has argued that Kant intends to reject the notion of actual infinity *altogether* on the grounds that it (a) violates phenomenalist constraints on legitimate concepts²⁶⁷ and (b) undermines a properly finitistic conception of number.²⁶⁸ In fact, according to Bennett, the antinomies do not turn on claims about a notion of totality peculiar to *reason* at all; rather, they depend entirely on Kant's phenomenalism and his "difficulties with the concept of infinity."²⁶⁹ As Bennett writes in his discussion of the first antinomy (which applies to the second antinomy as well):

Kant thinks he can show that the world is not infinite, without having to allow that it is only finite. He fails, of course, because he is wrong about the concept of infinity. His argument rests upon his false assumption that the statement 'Every finite set of Fs excludes some Fs', because it does not represent the Fs as a 'totality', does not involve outright infinity but only more-than-finiteness or 'potential infinity'.²⁷⁰

As Bennett argues, Kant's resolution to the antinomy fails because it rests on false claims about the infinite and, in particular, on "the mistaken view that 'whole' or a relative of it is required for 'infinite' in its proper sense."²⁷¹ That is, Bennett thinks Kant rejects the notion of actual infinity (albeit for bad reasons) and resolves the antinomy by saying that the relevant phenomena are merely potentially infinite.

²⁶⁷ As Bennett argues, Kant's starting point is a "mild" form of phenomenalism according to which claims about infinite must be cashed out in terms of some experience I could have or some action I could possibly perform (2016, 124). Once this form of phenomenalism is granted, Bennett argues, it follows that the claim that the world is actually infinite or that objects can be divided into actually infinite multiplicities of parts has no legitimate justification.
²⁶⁸ According to this reading, Kant begins with a finitistic conception of number according to which a number must always be reachable by counting (at least in principle), and he then infers from this conception of number to the conclusion that actual infinities are impossible simpliciter. See also Sutherland (2017), Kreis (2015, 80) and Vanzo (2005, 512 fn 17).

²⁶⁹ Bennett (2016), 291.

²⁷⁰ Bennett (2016), 138.

²⁷¹ Ameriks (2003) mounts a similar complaint against Kant, claiming that Kant attacks a straw man version of transcendental realism in the antinomies. As Ameriks writes, Kant resolves the antinomy by rejecting the notion of a "determinate infinite" to which he believes transcendental realists are committed, but "it is questionable whether Kant's notion of a 'determinate infinite' is more than a straw man; therefore, it is not clear that his solution (that we can go without end in experience) must be incompatible with transcendental realism and can fit only (let alone provide an independent basis for) his own idealism" (117 fn 5).

Second, even amid growing consensus that Kant does *not* intend to reject the notion of actual infinity *tout court*,²⁷² many commentators continue to hold that the antinomies are part of a more restricted attack on the actually infinite.²⁷³ According to these interpretations, Kant believes *space and time* are actually infinite, but he holds that their actual infinity is made possible by their unique mereological structure—viz., the whole is prior to the parts in the magnitudes of space and time. In contrast, these readings argue, Kant believes actually infinite magnitudes are *impossible* when the parts precede the whole. Since the composite objects treated in the second antinomy are magnitudes of precisely this sort, it follows that composite spatiotemporal objects *cannot* have actually infinite multiplicities of parts. And as some commentators have concluded, this can be taken to lend support to the view that Kant intends to resolve the antinomy by claiming that objects' parts are merely potentially infinite.

However, as we have seen in chapter 2, Kant does not endorse a general prohibition on actually infinite totalities composed from parts. He argues in the *Critique* that we can *think* of an actually infinite whole composed from parts (in the case of an infinite series of conditions), and in the *ID*, he suggests that an understanding unlike ours might also grasp an infinite multiplicity in a "definite concept"—this is because it might grasp such a magnitude "at a single glance"

²⁷² An influential exchange between Emily Carson (1997) and Michael Friedman (2000 and 2012) played an important role in this development. Friedman's and Carson's exchange brought attention to Kant's 1790 comments on the mathematical treatises of Abraham Kästner, where Kant articulated a distinction between *metaphysical space*, which is given, and *geometrical space*, which is constructed, and argued that the potential infinity of geometrical space presupposes the actual infinity of metaphysical space. Moreover, as we have seen above, there are a number of places in which Kant signals that he does not mean to reject the notion of actual infinity altogether. Recall, for instance, the *ID*'s defense of the "actual mathematical infinite" (*ID 2*:389) and the *Critique* discussion of "defective concepts" and the first antinomy's reliance on a non-defective one (which Kant later identifies as a concept of actual infinity (A430/B438 and A521/B549). See also *Käster* (1790). For further recent works supporting the view that Kant does not intend to reject actual infinities altogether, see Friedman (2015), Guyer (2018), and Tolley (2016), among others. It may be worth noting that some commentators (e.g., Boehm 2011 and Chiba 2012) call things "potentially infinite" that actually meet Kant's definition of actual infinity (and that Kant would therefore call "actually infinite"). I think this kind of confusion about Kant's conception of actual infinity is part of why so many commentators ascribe to him a potential infinity approach.

²⁷³ See especially Boehm (2011), Engelhard (2005), Falkenburg (2000), and Holden (2004).

rather than via the "successive application of a measure" (*ID* §1, 2:389 fn). Given this, Kant must not believe there is a general argument against the conceptual impossibility of infinite composition (or even against their real possibility).²⁷⁴ Nonetheless, this does not establish that a potential infinity approach is not correct, for Kant might have other reasons for thinking that the second antinomy can be resolved only if spatiotemporal objects' parts are merely potentially infinitely numerous (and he might have thought this is an alternative open only to transcendental realists). Thus, I now consider two different versions of a potential infinity approach, which are distinguished by how they understand the notion of potential infinity: the first is an *intuitionistic* approach to potential infinity; the second is a *modal* approach.

2.1 An Intuitionist Potential Infinity Approach

To assess the plausibility of a potential infinity approach, we must consider more carefully exactly what it can mean to say that objects are composed from a potentially infinite multiplicity of parts. One possible way of elaborating a potential infinity approach appeals to an *intuitionist* conception of potential infinity. As a general matter, intuitionist accounts of potential infinity begin with an account of truth and meaning for statements about the infinite, and in light of this, it is not immediately obvious how an intuitionist approach could ground a *metaphysical* reading of the antinomy's solution. However, we can begin by explaining what motivates a purely truth-theoretic or semantic approach to intuitionism, review the reasons for thinking that such an approach to potential infinity cannot be Kant's, and then consider whether a

²⁷⁴ Here I challenge Allison's claim that actually infinite aggregates can be ruled out on purely conceptual grounds on Kant's view (2004, 370).

metaphysically more robust adaptation of intuitionism might be more fitting and supply the solution to the antinomy that Kant requires.

Most intuitionistic approaches to infinity reject the notion of actual infinity altogether and arrive at this conclusion by reasoning from philosophical convictions about the meanings of mathematical statements.²⁷⁵ Roughly speaking, intuitionists hold that mathematical statements are *about* mathematical constructions, and the truth of a mathematical statement consists in the means we would have for proving it to be true. Dummett (1977) describes the philosophical convictions at the foundations of intuitionistic mathematics as follows:

From an intuitionistic standpoint, [...] an understanding of a mathematical statement consists in the capacity to recognize a proof of it when presented with one; and the truth of such a statement can consist only in the existence of such a proof. From a classical or platonistic standpoint, the understanding of a mathematical statement consists in a grasp of what it is for that statement to be true, where truth may attach to it even when we have no means of recognizing the fact [...] Hence the platonistic picture is of a realm of mathematical reality, existing objectively and independently of our knowledge, which renders our statements true or false. On an intuitionistic view, on the other hand, the only thing which can make a mathematical statement true is a proof of the kind we can give: not, indeed, a proof in a formal system, but an intuitively acceptable proof, that is, a certain kind of *mental* construction.²⁷⁶

One we accept this account of the meaning and truth of mathematical statements, several key results follow. First, since there are cases in which we lack a constructive proof both of a statement and of its negation, the law of excluded middle (LEM) is not true. Second, we also cannot employ reductio proofs like those given in the antinomies. For even if we assume that the

²⁷⁵ Dummett (1977) nicely captures the importance of the intuitionist's starting philosophical commitments when he writes: "Intuitionism is a scandal to those who think that philosophy is of no importance, or that it cannot affect anything outside itself, or at least that there are some things which are sacrosanct and beyond the reach of philosophy to meddle with, and that among them are the accepted practices of mathematicians. Intuitionists are engaged in the wholesale reconstruction of mathematics, not to accord with empirical discoveries, nor to obtain more fruitful applications, but solely on the basis of philosophical views concerning what mathematical statements are about and what they mean" (ix).

²⁷⁶ Dummett (1977), 4-5.

disjunction "the series of condition is either finite or infinite" is an instance of LEM,

intuitionistic logic says we cannot infer from a proof that the series is *not* finite to the conclusion that it *is* infinite (and vice versa).²⁷⁷ And finally, the convictions articulated above also yield a particular approach to the infinite: intuitionists reject actual infinity and argue that all infinity is potential infinity.²⁷⁸

How exactly should we understand the intuitionist's conception of potential infinity? Dummett adopts the language of "complete infinity" and "incomplete infinity" to characterize actual infinity and potential infinity and explains the difference between these two notions as follows:

[T]he thesis that there is no completed infinity means, simply, that to grasp an infinite structure is to grasp the process which generates it, that to refer to such a structure is to refer to that process, and that to recognize the structure as being infinite is to recognize that the process will not terminate. [...] It is, however, quite integral to classical mathematics to treat infinite structures as if they could be completed and then surveyed in their totality, as if we could be presented with the entire output of an infinite process. [...] [T]he platonist destroys the whole essence of infinity, which lies in the conception of a structure which is always in growth, precisely because the process of construction is never completed.²⁷⁹

Exactly how to understand this metaphor of growth is a topic of much dispute in the literature on intuitionism, but a classic approach is to understand the intuitionistic construction and the "process of growth" in explicitly temporal terms.²⁸⁰ Mathematical constructions take place in time, and potentially infinite structures are therefore ones that will continue to grow in a literal, temporal sense.

²⁷⁷ Indeed, even if we derive a contradiction from "not-P" and allow that we can therefore infer "not-not-P", we still cannot infer "P" (since to show that there can be no proof of "not-P" is not to show that there *is* a proof of "P"). ²⁷⁸ As Dummett writes, "In intuitionist mathematics, all infinity is potential infinity: there is no completed infinite" (1977, 40).

²⁷⁹ Dummett (1977), 40-1.

²⁸⁰ See Brouwer (1975) and Heyting (1956).

Above, I remarked that the intuitionist's preoccupation with the notions of meaning and truth might lead one to think that an intuitionistic approach can ground a non-metaphysical, purely semantic reading of the antinomy's resolution. For as one might argue, perhaps Kant means to link empirical truth with intuitionistic provability *without* arguing that the spatiotemporal world is itself in growth in the way intuitionistic constructions are.²⁸¹ Then, the claim that objects' parts are potentially infinite might be a claim about the *logic* appropriate to judgments about spatiotemporal objects and *not* a claim about the ontological structure of matter. However, recall (from chapter 1) that Kant explicitly rejects the key logical commitments of intuitionism in his discussion of the antinomies' resolutions. In his discussion of his solution to the first antinomy, for example, he writes:

Accordingly, if I say that as regards space either the world is infinite or it is not infinite (*non est infinitus*), then if the first proposition is false, its contradictory opposite, 'the world is not infinite,' must be true. Through it I would rule out only an infinite world, without positing another one, namely a finite one. But if it is said that the world is either infinite or finite (not-infinite), then both propositions could be false (A503-4/B531-2).²⁸²

Kant then goes on to say that "what has been said here about the first cosmological idea, namely the absolute totality of magnitude in appearance, holds also for the others" (A505/B533). This is a clear indication that Kant does not intend to reject LEM as part of his solution to the antinomy. As Kant argues, the world might be neither finite nor infinite, but this does nothing to undermine the classical logical truth that if the statement "the world is infinite" is false, then the statement "the world is not infinite" is true.²⁸³ Indeed, as Kant sees it, the *both false* solution to the

²⁸¹ Posy (1983 and 2019) adopts this approach.

²⁸² Compare to Posy, who argues that the disjunction "Thesis v Antithesis" in both antinomies *is* a classical logical truth, and transcendental idealism saves the day by showing that it is *not* a logical truth once we accept intuitionism (2019, 5). I think it is sufficient to refute Posy's view to note that Kant does *not* see his solution to the antinomy as entailing the falsity of LEM.

²⁸³ For a helpful discussion of Kant's account of "infinite judgments" in relation to this passage, see Stang (2012).

mathematical antinomies does not undermine the core principles of classical logic because infinitude and finitude are not contradictories after all. Kant believes the disjunction of the thesis and antithesis statements is false, but he does not believe this disjunction is an instance of LEM.

If this is correct, then the intuitionist's approach to potential infinity must be adapted to provide a solution to the antinomy that Kant would plausibly accept. Can this be done? One option is to give the intuitionist's approach to potential infinity a metaphysical rendering and jettison its core logical commitments. One can imagine doing this by proposing a reading that goes as follows. Whereas transcendental realists assume that objects must be composed from either a strictly finite multiplicity of parts (the thesis position) or a *complete* infinity of parts (the antithesis position), Kant resolves the antinomy by asserting that objects' parts in fact constitute an *incomplete* infinity. Moreover, we can flesh out the relation of this solution to Kant's idealism as follows. Whereas transcendental realists argue that spatiotemporal objects simply have all the parts they will ever have, Kant's idealism allows him to embrace a kind of constructivism according to which objects literally come into being through the "regress of the decomposing synthesis," which the intuitionist interprets as a mental process taking place as we continue to carry out our investigations into the world around us (A505/B533).²⁸⁴

Moreover, we could flesh out the details of the solution more precisely as follows. Whereas transcendental realists assert that objects' parts are complete rather than in growth, Kant resolves the antinomy by asserting that objects' parts are incomplete and always changing. The difference between the thesis position, the antithesis position, and Kant's position can then be articulated as follows. Given an object *O*, *O*'s parts count as complete and finite just in case there

²⁸⁴ As the citation indicates, there is some textual evidence for this position. For instance, at A524/B552, Kant writes that the division of an object into parts "consists only in the progressive decomposition, or in the regress itself, which first makes the series actual."

is a time *t* at which *O* has a finite number of parts *and* that finite multiplicity of parts is all the parts *O* will ever have. An object *O*'s parts count as complete and infinite just in case there is a time *t* at which *all* of *O*'s parts contain further parts *and O* does not have parts at future times that it does not already have at *t*. And finally, *O*'s parts count as incomplete and infinite (that is, potentially infinite) just in case at every time *t*, *O* has smallest parts within which there are no further parts *at t*, but for every *t*, there is also a t' > t at which the smallest parts of *O* at *t* will come to have further parts. This provides a conception of completeness and incompleteness that delivers the right verdicts about the thesis and antithesis positions of the second antinomy (namely, that they articulate two ways in which objects' parts might be *complete*), and it provides a clear way of articulating the intuitionist's growth metaphor: an object's parts count as incomplete (and hence "in growth") when the collection of its parts at earlier times is a proper part of the collection of its parts at later times. There is also nothing in this characterization that mandates ruling out principles of classical logic such as LEM.

However, this interpretation of objects' parts as literally coming into being through time saddles Kant with a form of constructivism that is both unappealing in its own right and in tension with other important claims that he wants to make about the actuality of phenomena we have not yet reached in the course of our inquiry. For instance, in the Postulates of Empirical Thinking, Kant writes that there are *actual* (*wirklich*) things in space and time that have not yet been perceived or experienced, and their actuality consists in their connectedness with things that *have* been perceived. As he writes in the second postulate, "That which is connected with the material conditions of experience (of sensation) is **actual**" (A218/B266). And a few paragraphs later:

The postulate for cognizing the actuality of things requires perception, thus

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sensation of which one is conscious – not immediate perception of the object itself the existence of which is to be cognized, but still its connection with some actual perception in accordance with the analogies of experience, which exhibit all real connection in an experience in general. (A225/B272)

This suggests that there are the things we have perceived, on the one hand, and the things we have not yet perceived, on the other, and both count as *actual*. In particular, things we have not yet perceived (such as remote objects in space and tiny parts in objects) count as actual because they are lawfully connected with things we *have* already perceived.²⁸⁵ Kant confirms this when he writes that the "magnetic matter penetrating all bodies" is actual not because we can immediately perceive it but rather because we can perceive the iron filing it attracts (A226/B273). In fact, Kant writes, "an immediate perception of this matter is impossible for us given the constitution of our organs" (A226/B273). But this does not undermine our reasons for thinking that there is such matter, for we have evidence of it through the immediate perception of the attracted iron filings, and "in accordance with the laws of sensibility and the context of our perceptions [*Kontext unserer Wahrnehmungen*] we could also happen upon the immediate empirical intuition of it in an experience if our senses, the crudeness of which does not affect the form of possible experience in general, were finer" (A226/B273).

We can articulate the problem this presents for a metaphysical rendering of the intuitionist's potential infinity approach as follows.²⁸⁶ If objects' parts are literally coming into

²⁸⁵ Further discussion of this criterion of actuality can be found in chapter 5 below.

²⁸⁶ In *Kants Ontologie der raumzeitlichen Wirklichkeit*, Chiba (2012) defends a reading that he describes as a metaphysical adaptation of Dummettian anti-realism. However, Chiba attempts to articulate a "time-neutral" version of potential infinity, which (as far as I can tell) does not succeed in distinguishing potentially infinite series of conditions from actually infinite ones (supposing we adopt Kant's conception of actual infinity). Chiba says that a series of conditions has as many members as *can* be cognized, and the reason the series is not actually infinite is that it is not cognizable *as a totality* (2012, 295-300). But Kant allows that a series is actually infinite if it is one in which every member is conditioned by a further one, and he defines a magnitude as actually infinite if it is strictly greater than any finite magnitude (i.e., greater than any finite number of units one could measure or pick out in it). But given this (and contra Chiba), Kant must not mean to say that the series *is infinite* and deny only that it is an infinite

being as we meet with them in the course of inquiry, then they do not exist prior to our actual perception of them (contra the Postulates). That is, on the intuitionistic proposal, parts are actualized *via* empirical discovery and not in virtue of standing in lawful connections with other parts, which *are* immediately perceived. So if the metaphysical rendering of the intuitionist's version of a potential infinity approach is correct, there are no parts beyond those we have actually discovered. This incompatible with the postulate of actuality and with Kant's broader commitments in the *Critique*, which often make use of the idea that there are existing objects we have not yet experienced.²⁸⁷

Thus, although an intuitionist's version of a potential infinity approach *can* underwrite the claim that the thesis and antithesis statements of the second antinomy are both false, the intuitionist's articulation of a potential infinity approach is not plausibly attributed to Kant. A semantic or truth-theoretic articulation of the intuitionist's approach is not compatible with the logical principles Kant explicitly endorses, and a metaphysical rendering of its conception of potential infinity attributes to Kant a form of constructivism he rejects.²⁸⁸

totality. See Chiba's claim that "Was Kant ablehnt, ist nicht die Unendlichkeit schlechthin, sondern die Unendlichkeit *einer Totalität*, nämlich die aktuale Unendlichkeit" (2012, 295). Thus, although I am sympathetic with Chiba's claim that Kant appeals to *possible experience* to determine the length of series of conditions (see chapter 5), I think Chiba misunderstands Kant's views on the infinite, and in virtue of this, his argument that Kant resolves the antinomy by appealing to potential infinity is unsuccessful.

²⁸⁷ In chapter 5, I will suggest that we instead interpret the "regress of the decomposing synthesis" as a *possible* empirical regress (rather than an actual one). As I will argue, a reading like this can bolster a metaphysical indeterminacy approach and it also fits better with Kant's claim in the postulates that things count as actual when "we can get from our actual perceptions to the thing in the series of possible perceptions" (A225-6/B273).
²⁸⁸ Note that there are other, non-intuitionistic approaches to potential infinity that we might also call "incompletist" but that *do* allow for actual infinities (and that also do not give *time* such a central role in the articulation of incompleteness). For example, some have argued that an incompletist's conception of potential infinity can be articulated by reference to the notion of *indefinite extensibility*. Suppose a concept defines a potentially infinite multiplicity just in case the phenomena falling under the extension of that concept are indefinitely extensible. To take an example, consider the concept *set that does not contain itself*. This concept is indefinitely extensible because any set of such sets does not contain itself and so omits a set of the relevant type. That is, any attempt to collect *all* the sets satisfying the description "set that does not contain itself" generates a new set that both (a) satisfies the description and (b) was not included in the initial collection. Given this, however, it is impossible to form a set of all the sets that do not contain themselves. And as the argument goes, this provides a way of articulating the claim that

2.2 A Modal Potential Infinity Approach

Given that an intuitionist version of a potential infinity fails, suppose we instead take seriously the modal language in the term "potential infinity" and hold that an object has a mere potential infinity of parts just in case it *could* have infinitely many parts (but does not *actually* have infinitely many parts). Can a potential infinity view based on this *modal* conception of potential infinity succeed? As I argue below, it cannot provide a compelling account of Kant's *both false* solution to the second antinomy (even if the modal conception of potential infinity is not itself objectionable on Kantian grounds).

To begin, consider the following modal articulation of the claim that objects have merely potentially infinitely many parts.²⁸⁹ Suppose "*Pxy*" means "*x* is a proper part of *y*," and suppose "*o*" denotes our object. Then, if *o* has a mere potential infinity of parts, the following holds for *o*:

Mere Potential Infinity: $\forall x (Pxo \rightarrow \Diamond \exists y Pyx) \land \neg \forall x (Pxo \rightarrow \exists y Pyx)$

the entire universe of sets is *potentially* rather than actually infinite. Every actually infinite multiplicity can form a set containing *all* of its members, but potentially infinite multiplicities are generative in a way that makes a complete collection (or set) of their members impossible. Moreover, in virtue of the special generative nature of indefinitely extensible (or potentially infinite) phenomena, proponents of this view conclude, we are also justified in saying that potentially infinite multiplicities are strictly *greater than* actually infinite multiplicities; whereas potentially infinite multiplicities are *too large* to form sets, actually infinite phenomena are not. As should be clear, however, this way of thinking of potential infinity cannot be Kant's, for Kant in his pre-Cantorian approach to the infinite would not have held that potentially infinite phenomena are strictly greater than actually infinite phenomena. This said, see Shapiro and Wright (2006) for an interesting and helpful discussion of the relationship between potential infinity, actual infinity, and the notion of indefinite extensibility. Levey (2016) also includes useful discussion of the notion of indefinite extensibility.

²⁸⁹ The proposal that follows is inspired by parts of Linnebo's and Shapiro's (2017) discussion. However, whereas Linnebo and Shapiro use a modal account of potential infinity to explicate the notion of *construction* that might be at play in discussions of indefinite extensibility, I simply mean to show how the notion of potential infinity might be articulated in modal terms *without* weighing in on any debates concerning mathematical construction, indefinite extensibility, or the semantics appropriate to a mathematical account of potential infinity fleshed out by means of modal operators. I am happy to admit that if an intuitionistic approach *is* properly articulated by appealing to a modal notion of construction rather than to an explicitly temporal one, then there may not be a clean distinction between intuitionist and modal versions of a potential infinity approach.

Or in plain English: Necessarily, for all x, if x is a proper part of o, then it is possible that x has a proper part, *and* it is not the case that every proper part of o actually has a further proper part.²⁹⁰

Recall from above that part of what is required in an adequate reading of Kant's solution to the antinomy is an explanation of why his solution rules out both the thesis and the antithesis positions—this is required by his claim that his solution shows that the thesis and antithesis statements are *both false*. The notion of potential infinity just articulated can clearly satisfy this requirement when it comes to the *antithesis* statement, for the antithesis's claim that objects have an actually infinite multiplicity of parts could be articulated as follows:

Actual Infinity: $\forall x (Pxo \rightarrow \exists y Pyx)$

Or in plain English: For all x, if x is a proper part of o, then there is a y such that y is a proper part of x.

Actual Infinity is clearly incompatible with Mere Potential Infinity, for the former says that each part of each object *actually* contains further parts, whereas the latter denies that this is true.

However, can Mere Potential Infinity likewise explain why the *thesis* statement of the second antinomy is false? On its face, Mere Potential Infinity is not incompatible with the claim that objects have multiplicities of parts that are actually *finite*, for one could very well hold *both* that every part of an object possibly has further parts *and* that the very same object resolves into parts which are not *actually* composed of anything further. Thus, a modal version of a potential

²⁹⁰ Note that one might also formulate Mere Potential Infinity in a slightly stronger way as follows: $\forall x(Pxo \rightarrow \forall x(Pxo \rightarrow \exists y Pyx)) \land \neg \diamond \forall x(Pxo \rightarrow \exists y Pyx)$. By adding a possibility operator in the second conjunct, this version of Mere Potential Infinity says that every part possibly has a further part, but it is not even *possible* that all the parts actually have further parts. Linnebo and Shapiro opt for this formulation, since their aim is to describe phenomena in mathematics that *could not* be actually infinite (corresponding to the indefinitely extensible).

infinity approach (as it has so far been articulated) cannot explain why the thesis statement of the second antinomy is false.

Perhaps we could improve the view as follows. Suppose we emphasized that the thesis statement requires not only that objects resolve into finitely many actual parts but also that their ultimate parts are *simple*. And as one might argue, perhaps Kant means to adopt a modal conception of simplicity on which the thesis statement is true only if objects resolve into finitely many *indivisible* parts. If this is how we should understand simplicity, then Mere Potential Infinity *is* incompatible with the thesis position, for the thesis position would then say that objects have an actually finite number of parts, the smallest parts of which *cannot* be further divided, and in endorsing Mere Potential Infinity, Kant would say that objects consist of an actually finite number of parts the smallest parts of which *can* be further divided and hence are not simple.

Is a modal potential infinity approach compelling when it is wedded in this way to a modal account of simplicity? Two considerations suggest that it is not. First, if we define simplicity modally, then there is at least prima facie pressure to adopt a modalized conception of what it is to be composite. For if we assume that an object's parts are either determinately simple or determinately composite, then we should be able to infer from the fact that an object lacks simple parts to the conclusion that all its parts are composite.²⁹¹ However, on the modal conception of simplicity currently under consideration, this means that a composite part is anything that *could* contain further parts. That is, suppose "*p*" denotes a part, and suppose "*Pxp*"

²⁹¹ This inference will turn out not to be a good one if we embrace compositional indeterminacy (as I argue we should below). For if objects are compositionally indeterminate, then it can be indeterminate whether some parts are actually simple or actually composite. However, the potential infinity view insofar as it is an *alternative* to a compositional indeterminacy view assumes that all parts are either actually simple or actually composite.

means "*x* is a part of *p*". If *p* is simple if and only if $\neg \Diamond \exists x(Pxp)$, then *p* must be composite if and only if $\Diamond \exists x(Pxp)$. But then Kant must embrace the awkward conclusion that a part lacking *actual* parts can count as *actually* composite.²⁹²

Second, and more importantly, adopting a modal conception of simplicity also forces us to abandon Kant's longstanding commitment to the view that questions about whether or not there are *simples* resolve into questions about whether or not the series of decomposition is *finite*. As early as the *Physical Monadology* (1756), Kant held that objects resolve into simples only if their parts are finite in number. As Kant put it then, a body that consists of simples must consist of a "determinate number" of parts, and since a *number* is always finite (on Kant's view), it follows that bodies consist of simples only if they have a finite number of parts (1:479).²⁹³ In the *Critique*, Kant also held that objects have simple parts only if their parts are finite in number. Indeed, this is built into his approach to thinking about the unconditioned, for as we have seen, Kant holds that the unconditioned can occur in only two ways (when the conditioned and its condition form a series). Either it must occur in a terminal member of a finite series of conditions, or it must occur in the entirety of an infinite series (A417-18/B445-6). This translates to a view on which an unconditioned simple is a last member in a finite series of decomposition.²⁹⁴ Given this, we can explain the problem with a modal conception of simplicity

²⁹² A much more natural position would be to hold that parts without *actual* parts are merely *possible* aggregates (rather than actual aggregates).

²⁹³ For Kant's finitistic conception of number, see B111, A142-3/B181-2, *ID* 2:389 fn. See also Sutherland (2017). Note also that readers may wonder here on what grounds the Critical Kant rules out the position he endorsed in the *Physical Monadology* according to which *space* is infinitely divisible, but *bodies* nonetheless reduce to finitely many simples. Kant's most focused argument against this position is given in Proposition 4 of the Dynamics section of the *Metaphysical Foundations of Natural Science*, but I put aside an assessment of the plausibility of this argument here.

²⁹⁴ See footnote 263 for an explanation of why Kant thinks about things in this way. Note also: if one is tempted to object here that it is a failure on Kant's part that he does not rule out the Leibnizian position on which infinitely many *spatial* parts are the well-founded phenomena resulting from the representations of *non-spatial* simple

as follows. Although Kant is committed to saying that simple parts exist just in case the series of decomposition is actually finite, a modal conception of simplicity denies this and says that an actually finite series of decomposition might not terminate in simples if it *could* have had more members.²⁹⁵

We can draw out a further aspect of the problem with adopting a modalized conception of simplicity by noting that it forces us to adopt an asymmetrical account of Kant's "both false" solution to the antinomy. Suppose for the sake of argument that Kant *does* adopt a modal conception of simples and composites such that a part *p* is simple if and only if $\neg \Diamond \exists x(Pxp)$ and composite if and only if $\Diamond \exists x(Pxp)$. If this is the case, Kant simply has no disagreement with the proponent of the *antithesis* position as to whether objects are composed of parts all of which are composite. Both Kant and the antithesis realist agree that objects are composed of parts all of which are composite statisfy $\Diamond \exists x(Pxp)$.²⁹⁶ But this means that the potential infinity approach currently under consideration must appeal to two very different kinds of considerations to explain why the thesis and antithesis statements are both false. It must say that Kant explains the falsity of the *thesis*

monads, we can respond on behalf of Kant as follows. Leibnizian monads do not *compose* spatial objects, and so it is simply a mistake to say that Leibnizian monads *condition* spatial objects in the same conditioning relations as do their spatial parts. Thus, a Leibnizian monad could never be a member of a spatial object's series of decomposition, since it is not a *spatial* condition at all (and objects in a series of decomposition are united by the same *kinds* of conditioning relations). Hence, it is a mistake to see a Leibnizian view as challenging Kant's solution to the antinomy. See A442/B470 for confirmation of this point.

²⁹⁵ A possible objection to this line of reasoning goes as follows. Kant sometimes speaks as if a simple part *is* indivisible (e.g., A523/B551). If Kant did mean to conceive of simplicity modally in this way, then he *could* align a modal conception of simplicity with the claim that simple parts exist only if an objects' parts are finitely numerous by assuming that all possible divisions are actualized. For then a series of decomposition would have finitely many parts only if it contained some indivisible part. However, this seems to me in tension with Kant's claim that the infinite divisibility of space renders spatial objects infinitely divisible. For if objects are infinitely divisible, and all possible divisions are actualized, then objects are actually infinitely divided. And Kant explicitly denies that objects are actually infinitely divided in the discussion at A525-28/B553-56. Hence, we should conclude that Kant had a non-modal conception of simplicity on which a part is simply just in case it is not actually divided (even if it could be divided).

²⁹⁶ Recall that Kant allows that all parts of objects are *divisible*, since the decomposition of an object into its parts "goes to infinity" (A513/B531).

position by appealing to the fact that all parts are *divisible*, but he explains the falsity of the *antithesis* position by appealing to something different—viz., his denial that every part *actually* contains further parts. Thus, on this reading, to explain why the thesis position is false, Kant must appeal to the *modal-mereological* fact that objects have *indivisible* parts, but to explain why the antithesis is false, he must appeal to the purely *quantitative* fact that objects do not contain a finite number of *actual* parts. This does not sit well with Kant's own claim that the thesis and antithesis are both false because "the multiplicity of parts in a given appearance is in *itself neither finite nor infinite*" (A505/B533, my emphasis). This claim suggests (as indicated above) that Kant intends to answer the question whether spatiotemporal objects have simple parts by way of answering the question whether their *actual* parts are finite in number.

To summarize then, a modal version of a potential infinity approach faces a dilemma. If it adopts a non-modal conception of simples and composites, then the claim that objects' parts are potentially infinite is compatible with the truth of the antinomy's thesis statement. But if it adopts a modal conception of simples and composites, then it must explain the falsity of the thesis statement by appealing to the mereological fact that objects do not have simples, whereas it must explain the falsity of the antithesis statement by appealing to the quantitative fact that objects' actual parts are not infinite. This is in tension with Kant's own suggestion that a simple exists only if the series of decomposition is finite and that all parts are composite only if the series of decomposition if infinite.

3. The Compositional Indeterminacy View

If the arguments above are correct, then neither of two initially plausible versions of the potential infinity view give a satisfying account of Kant's solution to the second antinomy. An

intuitionistic approach either attributes to Kant logical principles he would or ascribes to him a form of constructivism about empirical objects that is incompatible with his discussion of actual, unperceived objects. A modal approach does not give a satisfying account of Kant's *both false* solution to the antinomy (even if the notion of potential infinity it articulates is not itself objectionable).

I turn now to an articulation and defense of a compositional indeterminacy reading (the form a metaphysical indeterminacy reading takes in the resolution of the second antinomy). According to a compositional indeterminacy approach, whereas the proponent of the thesis position holds that it is metaphysically determinate that objects have smallest parts within which no further actual parts exist, and whereas the proponent of the antithesis position holds that it is metaphysically determinate that all parts contain further actual parts within them, Kant resolves the antinomy by claiming that it is metaphysically *indeterminate* whether or not all parts have further *actual* parts. Thus, whereas the thesis statement claims that objects have a determinate finite number of actual parts, and the antithesis statements claims that objects' actual parts are determinately infinite, Kant claims that it is metaphysically *indeterminate* how many actual parts objects have.

As before, in describing compositional indeterminacy as a form of "metaphysical" indeterminacy, I mean to highlight the fact that it does not consist *merely* an indeterminacy with respect to what we represent.²⁹⁷ Rather, on Kant's view, the character of the spatiotemporal world itself leaves open exactly how many material parts objects have—it is not determinate that

²⁹⁷ This said, given Kant's idealism, indeterminacy in the spatiotemporal world may ultimately be explained by facts about our representations and representational capacities (as I suggest it is below).

objects' *actual* parts are finite in number, but it also is not determinate that they are infinitely numerous.

Note also that compositional indeterminacy so construed is not equivalent to potential infinity. Kant *does* hold that objects are *divisible* to infinity (a point he thinks follows from the infinite divisibility of space), and this *does* have the implication that objects have potentially infinite parts *on a modal conception of potential infinity*. However, this claim concerning objects' infinite *divisibility* is not what does the important work in resolving the antinomy. Rather, Kant's claim that objects are compositionally indeterminate is what (a) explains why the thesis and antithesis statements are both false and (b) explains why Kant's solution to the antinomy is not open to transcendental realists.

To see that this is so, we can begin by summarizing the account of Kant's indirect argument for idealism that a compositional indeterminacy reading provides. According to a compositional indeterminacy reading, transcendental realists are committed to the thesis that spatiotemporal objects are completely compositionally determinate because they identify spatiotemporal objects with things in themselves. Given that the Supreme Principle holds for things in themselves, it follows from the assumption of transcendental realism that every compositionally conditioned object in space and time has a finite or infinite series of conditions. That is, one of two scenarios must obtain. Either it is determinate that spatiotemporal objects resolve into a finite number of simples (where the last member in the finite series of conditions is an unconditioned simple), or it is determinate that objects have infinitely many material parts (where the series of conditions is infinite and unconditioned as a whole). The transcendental idealist escapes the antinomy by arguing that spatiotemporal objects are not compositionally determinate, and she is in a position to do this because she holds that spatiotemporal objects are not things in themselves and hence need not answer to the demand for explanatory completeness made in the Supreme Principle. As the transcendental idealist argues, the plurality of parts in a spatiotemporal object is not finite because it is indeterminate whether the object has smallest parts within which no further parts exist. Likewise, the plurality of parts is not infinite because it is not determinate that *all* its parts contain further parts. Thus, the thesis and antithesis statements are *both false* because the number of *actual* parts composing objects is neither finite nor infinite.²⁹⁸

Three further merits of a compositional indeterminacy reading are deserving of emphasis. First, a compositional indeterminacy reading can provide a compelling account of Kant's claim that we can never achieve cognition of the unconditioned in the spatiotemporal world. As we have seen, Kant holds that the unconditioned can occur only in (i) a terminal member of a finite series of conditions or (ii) the entirety of an infinite series of conditions. But since a series of decomposition is neither finite nor infinite on Kant's solution to the second antinomy, neither kind of series obtains. That is, because objects are compositionally indeterminate, nothing compositionally unconditioned exists in space and time, and so nothing compositionally unconditioned can be cognized by us in the spatiotemporal world.²⁹⁹

²⁹⁸ One might object here that this implies that some parts are not determinately extended (given the Axioms of Intuition and some of Kant's claims in the *Metaphysical Foundations of Natural Science*). However, Kant can hold that all parts are extended in virtue of filling space without holding that everything extended has further parts. In particular, he can distinguish between things that are extended but have an *indeterminate* spatial magnitude (because it is not determinate whether they have further actual parts) and things that are extended and have a *determinate* spatial magnitude (i.e., things that determinately *do* have further parts). Note also that Kant himself uses the language of indeterminacy to describe his solution when he says that in every spatiotemporal object, "a multiplicity of parts is given which is in itself absolutely indeterminate (*eine an sich schlechthin unbestimmte Menge von Teilen gegeben ist*)" (A526/B554).

²⁹⁹ Here one might raise two separate concerns. First, one might wonder why we should not say that neither the thesis nor the antithesis is true but refrain from holding that both are false. Second, one might worry that my proposal cannot explain why Kant believes we should always keep looking for further conditions (an important part of his view in the Dialectic). As one might argue, if it is not determinate that all parts are conditioned, why does it make sense for reason to demand that we search for further conditions for all parts? On my understanding of the

Second, a compositional indeterminacy reading can also accommodate Kant's claim that objects are *divisible* to infinity without making the mistake of suggesting that this claim concerning infinite divisibility is doing the important explanatory work in the "both false" solution to the antinomy. Suppose we again use " $\nabla \Phi$ " to mean "it is indeterminate whether Φ ", and as before, suppose "*Pxy*" means "*x* is a proper part of *y*" and "*o*" denotes our object. What I have expressed only informally above can now be put as follows. Objects are compositionally indeterminate when the following obtains:

Compositional Indeterminacy: $\nabla \forall x (Pxo \rightarrow \exists y Pyx) \land \nabla \exists x (Pxo \land \neg \exists y Pyx)^{300}$

Or in plain English: It is indeterminate whether for all x, if x is a part of o, then x has further parts, *and* it is indeterminate whether o has some part that does not have any further parts of its own.

As emphasized above, Compositional Indeterminacy is meant to be read as a claim about what is *actual* and not as a claim about what parts objects *possibly* have. This is important because claims concerning objects' *possible* parts cannot adequately explain why the thesis and antithesis statements are both false (as shown in section 2.2 above). In contrast, Compositional Indeterminacy says the thesis and antithesis statements are both mistaken about what is *actual*, but this is compatible with Kant's claim that objects' parts are *divisible* to infinity (and hence are

faculty of reason, however, reason looks for further conditions whenever it doesn't have a complete explanation, and it has a complete explanation only when it is determinate that its explanation terminates in something unconditioned. As noted in chapter 3 above, this also helps to explain why indeterminacy *falsifies* the thesis and antithesis statements of the antinomy, for given that reason seeks complete explanations in searching for the unconditioned, and given that complete explanations in the relevant sense cannot occur when it is indeterminate how many conditions exist, the thesis and antithesis positions assert that a count-determinate state of affairs obtains and are false just in case no such determinate state of affairs is actual.

³⁰⁰ Note that Compositional Indeterminacy as described here does not cover the case where it is indeterminate whether *o* has any parts at all. For simplicity's sake, I do not attempt to revise the formulation so as to account for this, but I take it to be Kant's view that all ordinary spatiotemporal objects in fact determinately have at least *some* parts. Compositional Indeterminacy as described here also leaves it open how much indeterminacy there is.

potentially infinitely numerous on a modal conception of potential infinity). We can say *both* that objects' parts are potentially infinitely numerous because $\forall x(Pxo \rightarrow \diamond \exists y Pyx)$ and that the thesis and antithesis statements are both false because Compositional Indeterminacy obtains. These two claims are fully compatible with one another.

Third, a Compositional Indeterminacy reading also fits comfortably with the non-modal account of simplicity that Kant endorses. According to a compositional indeterminacy reading, no parts are simple because it is not determinate that any parts lack further actual parts. This allows Kant to deny that unconditioned simples exist in space and time, an important part of his Critical view.³⁰¹ But note that even commentators who believe Kant *does* adopt a modal approach to simplicity can endorse a compositional indeterminacy reading. For we can flesh out the relationship between claims concerning compositional indeterminacy and *modal* compositional facts as follows. If it is indeterminate whether a part contains further parts, then it is *possible* that that part contains further parts but *not necessary* that it contains further parts.³⁰² Correspondingly, if it is *not possible* that a part contains any further parts, then it also is *not indeterminate* whether it does. Importantly, however, attributions of indeterminacy cannot just be replaced by modal claims, since the converse entailments do *not* hold. If it is possible that a part contains further parts, it could be *either* determinate *or*

³⁰¹ See A483/B511 and A508/B536.

³⁰² Note: although I talk here as if Kant believes there are specific parts for which it is indeterminate whether they have further parts, I don't mean to commit him to this (per the discussion in chapter 3 above). Here, I adopt this way of talking only for ease of expression in explaining the relationship between claims about possibility and claims about determinacy and indeterminacy.

indeterminate whether that part *actually* contains further parts (likewise for the other converse entailments above).³⁰³

Thus, although on my view the weight of the evidence suggests that Kant endorses a nonmodal account of simplicity according to which a part is simple just in case it does not have any further actual parts, one could also marry a compositional indeterminacy reading to an interpretation that attributes to Kant a modal conception of simplicity. One can say that a part is simple if and only if it is *indivisible* ($\neg \diamond \exists x(Pxp)$), and Compositional Indeterminacy shows that no such parts exist (because if it is *indeterminate* that Φ , then it is *possible* that Φ). But even if we adopt a modal approach to simplicity, Compositional Indeterminacy shows that we cannot make the further inference to the conclusion that all parts are composite. For even if all parts are *divisible*, it may still be indeterminate whether or not those parts (or some of those parts) *actually* contain further parts. Hence, once we allow for compositional indeterminacy, we simply cannot infer from the fact that something is not simple to the conclusion that it is composite.³⁰⁴

To summarize, then, a compositional indeterminacy reading gives the following overall interpretation of Kant's solution to the second antinomy. The thesis statement's claim that an object has a finite number of parts the ultimate parts of which are simple can be articulated like this: for every decomposing series of parts, $\exists x (Pxo \land \neg \exists y Pyx)$. And the antithesis statement's claim that an object has an actually infinite multiplicity of parts can be articulated like this: $\forall x(Pxo \rightarrow \exists y Pyx)$. Since statements lacking indeterminacy operators specify determinate states

³⁰³ In chapter 5, I explore these relations in greater detail (and their relationship to Kant's claim some conception of *possible experience* defines the scope of empirical reality).

³⁰⁴ Thus, proponents of a modal conception of *simplicity* should see it as a further advantage of a compositional indeterminacy reading that allows them to resist the inference from the claim that a part is not simple to the conclusion that it is actually composite. Proponents of a modal concept of simplicity who reject compositional indeterminacy cannot resist this inference (as seen in section 2.2 above).

of affairs, it follows that Compositional Indeterminacy is incompatible with both the thesis and the antithesis statements of the second antinomy.³⁰⁵ The thesis statement is true if and only if it is determinate that each regress of decomposition terminates in a part that does not actually have further parts, and the antithesis statement is true if and only if it is determinate that every part has further actual parts.³⁰⁶ In contrast, Compositional Indeterminacy denies that either of these states of affairs obtains. Thus, since transcendental realists are committed to complete compositional determinacy (given their commitment to the Supreme Principle), Compositional Indeterminacy is incompatible with transcendental realism.

Finally, as recognized in chapter 3 above (and as I will explain in greater detail in chapter 5 below), note that we can tell a plausible story about the relationship between transcendental idealism and compositional indeterminacy. One of the characteristic claims of transcendental idealism is that spatiotemporal objects are mind-dependent, which is to say that some of their features depend on our representations and representational capacities. But this makes room for an argument of the following sort: because our representations and representational capacities do not always metaphysically determine spatiotemporal objects to be one way rather than another, compositional indeterminacy obtains. That is, given transcendental idealism and the nature of our

³⁰⁵ In fact, we must I assume both that statements lacking indeterminacy operators assert determinate states of affairs and that these statements are *false* when those determinate states of affairs fail to obtain. These assumptions could be contested, but I think these are natural assumptions to make in the Kantian context and given his views on the relationship between conditioning and explanation (see chapter 3 above).

³⁰⁶ As noted earlier, it should be clear that I have not attempted to provide anything approaching a complete semantics for an indeterminacy operator. What route we take here is not inconsequential, for it will affect such issues as how we understand the basic logical connectives. Nonetheless, Kant's texts do not seem to me to determine a single correct answer to these all these formal questions (though they do constrain us to make the semantics consistent with classical logical principles), and the apparatus of an indeterminacy operator is convenient insofar as it allows us to clearly distinguish and compare claims concerning potential infinity and claims concerning indeterminacy (and also to note a few entailments that intuitively hold between them).

representational capacities, we can explain why spatiotemporal objects are composed from neither finitely many nor infinitely many material parts.

4. An Objection: From Indeterminacy to a Rejection of LEM?

In my arguments against the intuitionist's conception of potential infinity above, I emphasized that Kant intends for his solution to the mathematical antinomies to be compatible with core principles of classical logic (and LEM in particular). But one might object that if Kant is committed to metaphysical indeterminacy, then violations of LEM are inevitable, and a compositional indeterminacy reading must therefore be rejected for the same reasons as must an intuitionist's account.

Is there a compelling argument to be made that metaphysical indeterminacy undermines the law of excluded middle? Although I think the objection ultimately does not succeed, we can see its initial plausibility by considering an argument Kant himself endorses in *The Only Possible Argument in Support of a Demonstration of the Existence of God* (1763). Here, Kant argues that neither possible nor actual things can be indeterminate with respect to what predicates they have, since this would violate the law of excluded middle. Kant writes:

[T]he proposition that a possible thing, regarded as such, is indeterminate with respect to many of its predicates, could, if taken literally, lead to serious error. For such indeterminacy is already forbidden by the law of excluded middle which maintains that there is no intermediate between two predicates which contradict each other. It is for example impossible that a man should not have a certain stature, position in time, age, location in space, and so forth. (*OPA*, 2:76)

As one might argue, here Kant seems to suggest that any argument for indeterminacy is simultaneously an argument against LEM. And if this is correct, then since Kant *is* committed to

LEM, he cannot mean to resolve the second antinomy by embracing a thesis of compositional indeterminacy.

However, this objection can be answered with two points. First, Kant's claim in the *Only Possible Argument* is that there is no intermediary between two predicates that contradict one another, *not* that everything is completely determinate in all respects. That is, he says that between any two predicates P and $\neg P$, one of these must apply to every object. But as we have already seen, Kant goes out of his way to argue that the predicates "finite" and "infinite" do not relate to one another as P and $\neg P$ relate to one another (they are not contradictories). As he argues in section 7 of the antinomies, if it is false that the world is infinite, then it must be true that it is *not* infinite, and this is not the same as saying that it is finite (A503-4/B531-2). Given this, LEM does not establish that a series of conditions must be either finite or infinite—it establishes only that if it false that it is infinite, then it is not infinite (and likewise that if it is false that it is finite, it is true that it is not finite). Hence, the claim that a series of conditions is indeterminate in magnitude rather than either finite or infinite does not violate LEM.

But second, a further point is that in the *Critique*, Kant suggests that the so-called "Principle of Thoroughgoing Determination" (PTD) in fact is *synthetic* and is not true by logic alone. According to the PTD, "among **all possible** predicates of **things**, insofar as they are compared with their opposites, one must apply to it," and as Kant writes, this principle "does not rest merely on the principle of contradiction" (A571-2/B599-600). Instead, according to Kant, "The **determinability** of every single **concept** is the **universality** (*universalitas*) of the principle of excluded middle between two opposed predicates; but the **determination** of a **thing** is subordinated to the **allness** (*universitas*) or the sum total of all possible predicates" (A572/B600

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fn). Putting aside exactly how this passage should be interpreted, it is clear that Kant does not take the PTD to follow from LEM or to be entailed by merely logical principles. Thus, even if it should turn out that compositional indeterminacy *does* undermine the PTD, this would not threaten Kant's commitment to principles of classical logic.³⁰⁷ The claim that every thing is fully determinate with respect to all possible predicate pairs is not, for Kant, a claim that can be proved by logic alone, and its denial does not undermine LEM.³⁰⁸

5. Chapter Summary

I have argued in this chapter that a compositional indeterminacy reading provides an overall compelling account of the resolution to the second antinomy. It explains why the thesis and antithesis statements are both false: *how many* parts objects have is metaphysically indeterminate. It also explains why Kant thinks transcendental idealism is necessary for a solution to the antinomy: transcendental realists are committed to a principle of compositional determinacy, given their commitment to the Supreme Principle, and so transcendental realists must say that objects have either a determinate finite number of parts or a determinate infinity of parts. Once this is granted, the antinomy is up and running. In contrast, transcendental idealists

³⁰⁷ Is Kant wrong that metaphysical indeterminacy and classical logic can be united in this way? Barnes and Williams (2011) provide one good example of how metaphysical indeterminacy might be embraced alongside classical logic.

³⁰⁸ Here, I do not take a stand on whether or not the PTD is in fact undermined by Kant's solution to the second antinomy. I think this would depend on whether indeterminacy with respect to *how many* things exist also yields indeterminacy in the properties of individual things, as well as on what Kant means by "**all possible** predicates of **things**" in the PTD (and on whether this is understood as equivalent to all possible predicates whatsoever or a more limited set of predicates, whether appearances count as *things* in the relevant sense, and so on.). I also do not take a stand on whether or not Kant intends to embrace the PTD in any way at all in the Transcendental Ideal. For a commentator who argues that he does, see Rohs (1978). For a commentator who argues that Kant denies the PTD for appearances, see Van Cleve (1999, 49-50). For a commentator who thinks it is in fact unclear whether or not embraces the PTD, see Stang (2012). Stang also provides a helpful explanation of *why* Kant may have thought the PTD does not follow from LEM.

hold that spatiotemporal objects are *indeterminate* with respect to how many parts they have; by allowing for indeterminacy with respect to what exists in space and time, transcendental idealists can embrace a "both false" solution to the antinomy.

Admittedly, the reading I have advanced attributes to Kant a belief in metaphysical indeterminacy that some would regard as controversial (at best) and highly counterintuitive (at worst). How could it be indeterminate what there is in the spatiotemporal world, one might ask? However, the very fact that claims to metaphysical indeterminacy strike us as both puzzling and explanatorily unsatisfying is a point in favor of the view I have defended above. For it is precisely one of Kant's most interesting insights that transcendental realism and its associated assumption of metaphysical determinacy is the default position we take up in our reasoning about the world. As Kant argues, part of the reason the antinomies are so difficult to avoid is that we cannot help but find the perspective of transcendental realism appealing.³⁰⁹ We naively approach the spatiotemporal world as if it were a thing in itself, and part of what this means for us is that we assume complete determinacy with respect to what exists in space and time (and hence assume a kind of determinacy with respect to the explanations of things that we can in principle attain). As we have seen, this is rooted for Kant in our (justified) belief that things in themselves must satisfy the Supreme Principle. Thus, insofar as we find claims to metaphysical indeterminacy puzzling, it is plausible to think that we do this *because* we take it for granted that the world is fully metaphysically determinate—and we do this because we naively assume that the spatiotemporal world is a thing in itself that must answer to the Supreme Principle. That is,

³⁰⁹ For example, Kant says that all the antinomies rest on a "illusion" that "put[s] reason at odds with itself" (A516/B544). This illusion is "an entirely natural mistake of common reason" (A500/B528) and applies ideas and principles which are "valid only as a condition of things in themselves, to appearances that exist only in representation" (A506/B534).

an assumption of metaphysical determinacy is a part of the naïve, transcendental realist perspective we default to in the course of inquiry about the world.

This said, none of this is to say that we must *now* take evidence of compositional indeterminacy (or any other kind of metaphysical indeterminacy) as evidence of transcendental idealism. For instance, we might now want to abandon the assumption that fundamental reality must answer to the Supreme Principle such that mind-independent reality *could* be compositionally indeterminate. Or we might be able to show that the thesis and antithesis arguments of the second antinomy simply are not sound, but for reasons having nothing to do with transcendental realism or transcendental idealism (recall some of the problems with the arguments' key premises discussed in chapter 1 above). This would show that the arguments do not jointly entail a contradiction, and the reductio against transcendental realism fails.

However, these qualifications aside, many philosophers in Kant's day *did* affirm a principle of complete metaphysical determinacy for spatiotemporal existence, and it is at least intuitively plausible to think that fundamental reality must be metaphysically determinate.³¹⁰ Hence, if spatiotemporal objects are things in themselves, it is intuitively plausible to think that they must be compositionally determinate. Moreover, as we have seen, Kant thinks a commitment to the applicability of the Supreme Principle to fundamental reality *entails*

³¹⁰ For instance, Baumgarten claimed that an "actual being" is "determined with regard to all the affections that are [...] compossible in it" (*Metaphysics*, §§53-4). Hume claimed that "'[t]is a principle generally receiv'd in philosophy, that every thing in nature is individual, and that 'tis utterly absurd to suppose a triangle really existent, which has no precise proportion of sides and angles [...] 'tis impossible to form an idea of an object, that is possest of quantity and quality, and yet is possest of no precise degree of either" (*Treatise*, 1.1.7). And Leibniz extended this to the case of composition in particular, writing that "in actual things nothing is indefinite, indeed, in them every division that can be made has been made" (Letter to de Volder, January 19, 1706 in *AG*, 282). Not all of these claims are about the same kind of determinacy, but they are representative of a general belief in the 17th and 18th centuries that what really exists must be fully metaphysically determinate. While this leaves open the possibility that ideal entities (such as mathematical objects) are indeterminate even for Kant's competitors, this does not undermine the claim that transcendental *realists* would take spatiotemporal reality to be determinate (since it is not ideal on their view).

compositional determinacy for what belongs to fundamental reality, so to the extent that transcendental realists regard spatiotemporal objects as things in themselves, they must commit to compositional determinacy. For Kant, therefore, the identification of appearances with things in themselves forces the disjunction that objects' parts must be either determinately finite or determinately infinite, and with this disjunction in place, the thesis and antithesis proofs against the infinite and finite alternatives (respectively) show that spatiotemporal objects cannot be a part of fundamental reality. Thus, a commitment to compositional indeterminacy entails the falsity of transcendental realism, on Kant's view, and makes possible a *both false* solution to the second antinomy.
Chapter 5: Transcendental Idealism and Metaphysical Indeterminacy

In the chapters above, I have argued that the resolution of the mathematical antinomies is *metaphysical* in nature and, more specifically, that Kant resolves the first and second antinomies by asserting that the relevant series of spatiotemporal conditions are *metaphysically indeterminate* in magnitude rather than either finite or infinite. That is, it is indeterminate *how many* conditions exist in the series of conditions for the kinds of conditioning that are treated in the first and second antinomies. As I have argued, this is a solution that Kant thinks is available to transcendental idealists *alone*, since transcendental realists hold that spatiotemporal conditions are things in themselves, and the Supreme Principle must hold for things in themselves (by Kant's lights). Given the assumption that spatiotemporal conditions are things in themselves, kant argues, it follows that the series of conditions treated in the mathematical antinomies must be either determinately finite or determinately infinite. And assuming the particular arguments of the theses and antitheses are compelling (as Kant thinks they are), transcendental realism.

In this chapter, I turn to a different question about the nature of the relationship between transcendental idealism, on the one hand, and metaphysical indeterminacy, on the other. Namely, how does Kant understand the doctrine of transcendental idealism such that indeterminacy in spatiotemporal reality *results from* the transcendental ideality of appearances? It is one thing to say that transcendental idealism makes metaphysical indeterminacy in spatiotemporal reality *possible* (because the Supreme Principle can be false for things that are not part of fundamental reality). But it is another thing to explain why the particular *way* in which appearances are mind-dependent also accounts for the particular kinds of metaphysical indeterminacy that Kant affirms in the resolution of the antinomies. And as one might point out, if we are sympathetic with the idea that metaphysical indeterminacy must be explained in a way that renders it *intelligible* (and

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not just in a way that shows it to be non-contradictory), we might think that an explanation of this sort is important.³¹¹ That is, we might conclude that we need to understand not only how it can be *coherent* to say that there is Supreme Principle-violating indeterminacy among spatiotemporal conditions, but also how that indeterminacy turns out to be *intelligible* in light of its source in the mind-dependence of appearances.³¹²

In what follows, I argue that reading Kant as a kind of intentional object phenomenalist can help to explain why the transcendental ideality of appearances results in their indeterminacy (that is, how transcendental ideality metaphysically explains why series of spatiotemporal conditions end up being indeterminate rather than either finite or infinite). While I do not aim to make a definitive case against other interpretations of transcendental idealism, I do aim to show that Intentional Object Phenomenalism (IOP) has an advantage when it comes to explaining why spatiotemporal series of conditions must be indeterminate in magnitude, given that they are transcendentally ideal.

I begin in section 1 by explaining what IOP is and how it is able to account for Kant's claims about the role of *successive syntheses* and *successive regresses* in accounting for the indeterminacy of things in space and time. As I argue, Kant argues from (i) the claim that appearances are successive-synthesis-dependent and (ii) the claim that an infinite successive synthesis is metaphysically impossible to establish the conclusion (iii) that spatiotemporal series of conditions must be indeterminate in magnitude rather than either finite or infinite. I also argue

³¹¹ Note: I think Kant does think that indeterminacy requires explanation as a general matter, for wherever he allows for indeterminacy, he also furnishes an explanation of how it arises. In the antinomies, he says that indeterminacy arises from the way in which appearances "exist only in representation" and only "in the successive regress" (to be discussed below). He appeals to the free will of noumenal agents to explain how indeterminacy with respect to what an agent will do is possible (or at least this is arguably what he does). And so on.

³¹² As noted elsewhere in the chapters above, I follow Kant in this chapter in using the terms "appearance" and "spatiotemporal object" interchangeably. In the antinomies at least, Kant uses the terms as equivalents.

that for Kant this is not an argument that turns on claims about our human finitude; rather, Kant thinks an infinite successive synthesis is *metaphysically* impossible, and this metaphysical impossibility is what explains how the indeterminate magnitude of spatiotemporal series of conditions arises from transcendental ideality.³¹³

In section 2, I explain how an IOP reading avoids the conclusion that spatiotemporal reality is a growing construction of objects that literally come into being as inquiry proceeds. That is, I explain how IOP readings avoid some of the pitfalls of metaphysical intuitionist proposals (seen in chapter 4 above), as well as some of the pitfalls of actual state phenomenalism (mentioned in chapter 1). As I argue, IOP can identify the scope of *actual* empirical reality with the scope of *possible* experience, and on such a reading, spatiotemporal reality extends well beyond what we have actually experienced *without* being either determinately infinite or determinately finite. A further advantage of a reading like this is that it can accommodate Kant's claim (in the Postulates of Empirical Thinking) that there are actual objects that we have not yet actually perceived. Rather than being in tension with the account of actuality Kant gives in the antinomies' resolutions, the Postulates are of a piece with IOP as it is endorsed in the antinomies. Because the successive regress on which appearances depend is *possible* rather than actual, Kant's claims in the Postulates are compatible with his claim in the antinomies that appearances "exist in the successive regress but otherwise do not exist at all" (A506/B534).

In section 3, I address two objections one might make against the proposal of the previous two sections. First, one might object that IOP makes metaphysical indeterminacy in spatiotemporal reality into a kind of representational indeterminacy, and so calling Kant's

³¹³ That is, no possible increase in our mental powers could make it possible for us to complete an infinite successive synthesis. Compare to Jauernig (2019), who argues that claims about human finitude are central to Kant's idealism.

solution to the antinomies a "metaphysical" indeterminacy solution is a misnomer. Second, one might object that the version of IOP that I articulate undermines the distinction (emphasized in chapter 4) between the non-modal claim that it is indeterminate how many conditions *actually* exist in each spatiotemporal series of conditions and the modal claim that spatiotemporal series of conditions are *potentially* infinite (on a modal conception of potential infinity). I argue that both of these objections can be dealt with in a satisfactory way.

Finally, in section 4, I consider whether other metaphysical readings of transcendental idealism can explain the role of *successive* synthesis in the resolutions of the antinomies just as effectively as can an IOP reading. In particular, can a form of metaphysical anti-realism that does not embrace phenomenalism succeed? Without attempting to make a definitive case against antiphenomenalist metaphysical anti-realist readings, I suggest that such readings at least face a prima facie problem in explaining how spatiotemporal objects end up being *successive*-regress-dependent.

1. Intentional Object Phenomenalism and Successive Synthesis

When Kant explains why transcendental idealism results in series of spatiotemporal conditions that are neither finite nor infinite, he suggests that the *successive synthesis*-dependence or *successive regresses*-dependence of spatiotemporal conditions is at the heart of the issue. A characteristic passage expressing this point is the following:

Accordingly, the antinomy of pure reason in its cosmological ideas is removed by showing that it is merely dialectical and a conflict due to an illusion arising from the fact that one has applied the idea of absolute totality, which is valid only as a condition of things in themselves, to appearances that exist only in representation, and that, if they constitute a series, exist in the successive regress but otherwise do not exist at all. (A506/B534)

Several paragraphs earlier, Kant makes a similar point in explaining why the Supreme Principle does not hold for appearances:

[T]the synthesis of conditions in appearance [...] is necessarily given successively and is given only in time, one member after another; consequently, here I could not presuppose the absolute totality of synthesis and the series represented by it, as I could in the previous case [i.e., in the case of things in themselves], because there all members of the series are given in themselves (without time-condition), but here they are possible only through the successive regress... (A500-1/B528-9)

These passages suggest that transcendental idealism renders intelligible how appearances could fail to be both finite and infinite by showing that (a) appearances exist only in representation and (b) in being representation-dependent, appearances depend for their existence on something *successive* (here Kant refers to the "successive regress", but sometimes he refers to a "successive synthesis").³¹⁴

Why should successiveness help to explain why appearances form series that are indeterminate in magnitude rather than finite or infinite? One way of answering this question starts by reading Kant's idealism as a kind of intentional object phenomenalism about appearances. Here is how this reading goes. First, we can define intentional object phenomenalism (IOP) as follows. According to IOP, appearances are the representational contents of mental states and exist only insofar as they are represented in these states. That is, appearances are the *representeds* of mental states (they are the *intentional objects* of mental states and not the states themselves), and they count as mind-dependent entities because their existence depends on being represented. Transcendental realists deny that appearances

³¹⁴ See A417/B444-5 and A486/B514.

(spatiotemporal objects) are representation-dependent, which is to say that they reject IOP.³¹⁵ But further, if we properly specify the *kind* of representational state that has appearances as its contents, then we can also explain why appearances end up being successive-regress-dependent. And this, in turn, will allow us to see why they cannot form either finite or infinite series of conditions (but rather must be indeterminate). Or so I will argue.

Taking our cue from scholars like Jankowiak (2017 and ms), Jauernig (2021), and Stang (2021 and ms), suppose the particular kind of mental state in which appearances are represented is the representational state Kant calls *experience* (*Erfahrung*).³¹⁶ Experience on Kant's account is a more robust state than sensation (*Empfindung*), intuition (*Anschauung*), or even perception (*Wahrnehmung*), for something represented in *experience* must also be thought (*gedacht*) through concepts so as to be an "empirical cognition" (*empirische Erkenntnis*) (A93-4/B125-6).³¹⁷

Again following scholars like Jankowiak and Stang, now suppose in addition that "experience" in the sense relevant to IOP is not the private representations of individuals but rather an idealized, public representation that unifies the empirical cognitions we all can have. As Stang (2021) writes, suppose "experience" is "universal experience," understood as follows:

Universal experience is the maximally unified and lawful representation of objects in space and time that is compatible with the *a priori* forms of experience

³¹⁵ A number of commentators read Kant as a kind of intentional object phenomenalist, though among these scholars there is disagreement about exactly how his intentional object phenomenalism should be understood. For a sampling of the views, see Aquila (1981), Chignell (forthcoming), Jankowiak (2017 and ms), Jauernig (2021), Stang (2021 and forthcoming), and Tolley (forthcoming). Note also that whereas some people think intentional objects do not exist at all (and have *no* ontological status), many of the scholars just cited offer good reasons for thinking it makes sense to assign to intentional objects a kind of existence (albeit a mind-dependent kind of existence).

³¹⁶ Textual support for a reading like this can be found in passages like the following: "objects of experience are **never** given **in themselves**, but only in experience, and they do not exist at all outside it" (A492/B521). But this claim is somewhat controversial, even among phenomenalist interpreters of Kant. For instance, Van Cleve suggests that appearances are intuition-dependent (though as I read him he is an actual state phenomenalist rather than an intentional object phenomenalist) (1999, 9).

³¹⁷ See Tolley (2017) for helpful related discussion.

and justified by the totality of subjects' perceptual states, or the conjunction of such representations if there is no unique such representation.³¹⁸

That is, suppose appearances are the intentional objects of experience understood in this

encompassing sense. Call this "Experience", for short.³¹⁹

Note that Kant himself suggests that he endorses a conception of empirical reality as that

which is represented in Experience in passages like the following:

In space and time, however, the empirical truth of appearances is satisfactorily secured, and sufficiently distinguished from its kinship with dreams, if both are correctly and thoroughly connected up according to empirical laws in one experience. [...] That there could be inhabitants of the moon, even though no human being has ever perceived them, must of course be admitted; but this means only that in the possible progress of experience we could encounter them; for everything is actual that stands in one context with a perception in accordance with the laws of the empirical progression. (A492-3/B520-21)

And:

There is only one experience, in which all perceptions are represented as in thoroughgoing and lawlike connection, just as there is only one space and time, in which all forms of appearance and all relation of being or non-being take place. If one speaks of different experiences, they are only so many perceptions insofar as they belong to one and the same universal experience. (A110)

In these passages, Kant says that everything that is actual (wirklich) is connected with possible

perceptions in one experience, and he says that experience in this sense represents things as

³¹⁸ Stang (2021).

³¹⁹ Note that IOP is compatible with Kant's claim that representation does not "produce [*hervorbringt*] its object as far as its **existence** is concerned (*dem Dasein nach*)" (A92/B125). For according to Kant, we cannot represent spatiotemporal objects in experience without *sensation* (*Empfindung*), and sensations are occasioned (in ways not comprehensible to us) by things in themselves. So when Kant says that representation does not produce its object as far as its existence is concerned, he is indicating that experience requires things in themselves to occasion sensations in us. This does not undermine the claim that, as intentional objects of experience, appearances exist only insofar as they are represented, for plausibly, the only appearances that *can* be represented in experience are those that are grounded in the appropriate way in things in themselves. See Stang (forthcoming) for further helpful discussion of the sense in which appearances may partly depend on things in themselves as the intentional objects of experience.

lawfully connected with one another in a coherent picture of the world. These are good indications that Kant thinks Experience determines the scope of empirical reality.

But in what sense is Experience *successive*-synthesis-dependent, and why does this help to explain why spatiotemporal series of conditions are neither finite nor infinite? First, note that for Kant, Experience is a representation of collective *empirical* cognition and so depends on what we can (at least in principle) discover through a course of empirical inquiry. And as Kant suggests in the antinomies, these processes of empirical inquiry for us are necessarily successive. As he writes, appearances are given "through the successive synthesis of the manifold of intuition" (A417/B444), and "the synthesis of conditions in appearance [...] is necessarily given successively and is given only in time, one member after another" (A500/B528). But this means that if something *cannot* be discovered via a successive regress, then it cannot be represented in Experience and hence cannot be a part of empirical reality.

Second, recall that Kant argues that a complete infinite succession is metaphysically impossible (see chapter 2 above). That is, it is impossible for an infinite successive synthesis to be completed, and this impossibility is a metaphysical fact about succession rather than a fact about the limitation of our finite minds: no increase in our representational powers could allow us to complete an infinite successive synthesis because complete infinite successions are metaphysically impossible on Kant's view. But given this, Experience also cannot include anything that would require an infinite successive synthesis or an infinite successive regress. So since infinite *successive* regresses are metaphysically impossible and because appearances depend on successive regresses (per transcendental idealism), spatiotemporal series of conditions cannot *be* infinite.

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Can successive-regress-dependence also explain why *finite* series of conditions are not represented in Experience? Here is a suggestion that builds off of some of Kant's own claims. According to IOP, what exists in space and time is what is represented in Experience, and what is represented in Experience is what we could discover through "successive syntheses" in "empirical regresses". But drawing on what Kant says, there are reasons to think that discovering a last member in a series of conditions via a successive regress actually requires an infinite regress. Consider what Kant says about the possibility of discovering that the world in space and time is finite. Kant writes:

[I]n the empirical regress there can be encountered **no experience of an absolute boundary**, and hence no experience of a condition as one that is **absolutely unconditioned empirically**. The reason for this, however, is that such an experience would have to contain in itself a bounding of appearance by nothing, or by the void, which the regress, carried on far enough, would have to encounter by means of a perception – which is impossible. (A517/B546).

This line of reasoning does not yet appeal to facts about the successiveness of inquiry, and so one might worry that successive-regress-dependence in fact plays no rule in ruling out the finite alternative. But imagine reasoning as follows. There are two ways in which we might discover that the world has an outermost boundary in a successive regress. First, we might reach an object that appears to us *as* an outermost object. That is, we might experience an outermost boundary *as an outermost boundary*. As Kant argues in the passage above, this is impossible. But second, even if we cannot experience an outermost by confirming that there are no further objects beyond it. That is, we might *indirectly* discover that a terminal condition in a finite series of conditions as unconditioned by ruling out the possibility that it is conditioned by any further conditions. But given that space itself *is* infinite, this would require searching through *all* of

infinite space. And since our manner of searching is always successive (according to Kant), this would require the completion of an infinite successive synthesis, which is impossible (again, according to Kant).

So given that we cannot immediately experience an object *as unconditioned* (or as outermost), and given that experiencing it as unconditioned (or outermost) by ruling out the existence of any further conditions in infinite space would require an impossible infinite succession, we can conclude that cannot empirically discover an unconditioned outermost condition at all. One can also imagine generalizing this argument to justify Kant's conclusion that we cannot experience a terminal condition in *any* series of spatiotemporal conditions *as unconditioned*. We can have "no experience of a condition as one that is **absolutely unconditioned empirically**" because this would require either a direct experience of it *as unconditioned* (which is impossible) or empirical confirmation that there are no further conditions of the relevant kind anywhere (which is also impossible, given that it would require a complete infinite succession) (A517/B546).³²⁰ In this way, we can argue that successiveness plays a role in explaining why Experience cannot represent *either* infinite series of conditions *or* finite series of conditions. And if Experience represents series of conditions that are neither finite nor infinite, spatiotemporal series of conditions must be indeterminate in magnitude.³²¹

³²⁰ Here I simply bracket the worry that there might be some kind of *non-successive* empirical proof that shows that a finite series of conditions does not exclude any conditions (and so is unconditioned).

³²¹ Jauernig (2021) tells a somewhat similar story about the impossibility of the infinite alternative, but she rules out the possibility of finite regresses differently. According to Jauernig, it is in the nature of the understanding and the forms of intuition to represent *every* spatiotemporal condition *as conditioned*, from which it follows (given her account of transcendental idealism) that every condition *is* conditioned by something further (see 91 and 93). In contrast, I think it is important to avoid saying that it is determinate that every spatiotemporal object is conditioned by something further, since this entails that the series of conditions is infinite.

2. The Possible Regress and Avoiding Constructivism

Recall from chapter 4 above that a constraint on a plausible reading of the antinomies' resolutions is that it must avoid a constructivist account of the empirical world according to which spatiotemporal objects are literally coming into being as our empirical inquiries proceed. For Kant makes clear in the Postulates that there are *actual* objects we have not *actually* discovered, and a plausible reading should accommodate this. In this section, I elaborate on how an IOP reading can accommodate the anti-constructivist thrust of Kant's thinking without abandoning his claim that appearances "exist in the successive regress but otherwise do not exist at all" (A506/B534). In particular, I argue that the successive regresses whose length determines the magnitude properties of spatiotemporal phenomena are *possible* rather than *actual* regresses.

It is uncontroversial that Kant is committed to some version of the view that what is not an object of *possible* experience also cannot *be* a spatiotemporal object. For example, at A158/B197, Kant writes that "[t]he conditions of the **possibility of experience** in general are at the same time conditions of the **possibility of the objects of experience**." On a straightforward reading, this means that what cannot be experienced also cannot *exist* as an object in space and time. However, some proponents of IOP have argued that the "successive regresses" to which Kant appeals in the resolution of the antinomies should nonetheless be construed as *actual* regresses. For example, Jauernig writes that "the regress in which appearances and the empirical world exists must be an actual regress," since only an *actual* regress is something we can "set for ourselves as a task" (a claim Kant makes).³²²

³²² Jauernig (2021), 101. Jauernig makes a number of arguments for this conclusion, one of which is that "[s]aying that I set a *possible* regress as a task for myself is hardly intelligible" (ibid).

But consider the following reasons for thinking that we should understand the successive regress as *possible* such that Experience represents everything that *could* be reached in an empirical course of discovery.³²³ First, suppose Kant *does* mean to say that the successive regress is a regress of *actual* empirical discoveries. Then, he must choose between two options, both of which have unappealing consequences. First, he could say that experience represents what our actual course of discovery has uncovered up to the current time. Or second, he could say that experience represents what our actual course of discovery yields at *any* time, including future times. On the first option, the extent of spatiotemporal reality literally grows larger as time progresses (assuming empirical discoveries continue such that Experience continues to have more and more content)—this entails the kind of constructivism we have already said Kant wants to avoid. But if Kant thinks spatiotemporal reality includes what is represented at any actual stage in the regress, including future stages, we again get implausible results. For even if this does not yield a growing world (since what exists *now* could include what is discovered in the future), it does suggest that whatever is never in fact empirically discovered is not a part of spatiotemporal reality. To borrow an example from Jauernig, if we had gone extinct before the development of paleontology, it follows on the present reading that there would have been no dinosaurs (since dinosaurs would have never been actually represented). This would not be to say that dinosaurs exist only *in* the times in which they *are* represented (rather, they would exist as paleontology represents them if paleontology develops at all, i.e., they would date back

³²³ Note: I take it the direct textual evidence is inclusive. Kant says in one passage that the successive regress is "given only through one's actually completing it," which suggests that Experience includes only what is actually reached through the regress (A500-1/B528-9). But he says in another passage that the series of spatiotemporal conditions can be "neither bigger nor smaller than the possible empirical regress" (A518/B546 fn), which suggests that Experience includes everything we *could* reach in a successive regress (where the sense of 'could' remains to be articulated).

millions of years). But it *is* to say that objects exist *at all* only on the condition that they are discovered in an actual successive regress at some time. Had humans never represented dinosaurs (say, because we were annihilated before the development of paleontology), dinosaurs would never have existed.³²⁴ This is a very counterintuitive consequence, to say the least.³²⁵

Beyond the counterintuitive consequences, however, an actual regress reading also sits uncomfortably with Kant's second postulate of empirical thinking. As we have seen, the second postulate says that everything "connected with the material conditions of experience (of sensation) is **actual**" (A218/B266). And in the ensuing discussion, Kant clarifies that the view is one on which actual things include *both* things that are immediately perceived *and* things that are causally connected with things that are immediately perceived (according to the analogies of experience) (A225/B272).³²⁶ But now suppose some things are represented in experience because they are discovered in a future actual discovery rather than in a current one. Then, some objects exist *now* because they *will be* represented in the future. But then what exists now is not causally connected to our current perceptions (since future things do not causally interact with present ones). At least on the face of it, this is incompatible with Kant's discussion of the second postulate.

³²⁴ Jauernig in fact embraces these consequences and endorses an actual regress reading (2021, 101 fn 192). ³²⁵ Note: a "possible regress reading" does not avoid all counterintuitive consequences, but it has those that I think an idealist account should have. For example, even if spatiotemporal reality extends as far as *possible regresses*, so long as "possible regresses" are understood as representing what *actual* representers can discover, it follows that there are no spatiotemporal phenomena at all in the absence of human minds. This is a counterintuitive result but appropriate, given Kant's idealism.

³²⁶ There is some controversy in the literature as to whether the postulate spells out a metaphysical or an epistemic criterion of actuality. That is, does it say what it is to be actual or how it is that we know which things are actual? Here, I assume that it provides a metaphysical criterion or definition of actuality. This is consistent with Kant's claim that "the principles of modality are also nothing further than definitions of the concepts of possibility, actuality, and necessity in their empirical use" (A219/B266). Here, I follow Jankowiak (ms) and Stang (2021).

Could Kant instead mean to say in the second postulate that the objects of future empirical discoveries are actual *now* because they *will be* causally connected with something immediately perceived in the future? This may be a viable option, but it represents a significant departure from Kant's original formulation of the second postulate. For the claim that something is actual now because it is causally connected with our perceptions is substantively different from the claim that it is actual now because it will be causally connected with our perceptions (but is not currently). So when Kant says that an unperceived actual thing is one whose "existence [...] is still connected with our perceptions in a possible experience, and with the guidance of the analogies we can get from our actual perceptions to the thing in the series of possible perceptions," I think it is most plausible to read him as saying that all actual things are currently actually connected with our current perceptions (via the causal laws articulated in the analogies of experience) (A225-6/B272-3). This fits well with a possible regress reading. For according to a possible regress reading, what is actual is what is causally connected to what we currently perceive, and in virtue of this connection it is reachable in a regress we *could* complete. As Kant himself puts it in discussing the resolution to the first antinomy,

This world-series can thus be neither bigger nor smaller than *the possible empirical regress*, on which alone its concept rests. And since this cannot yield a determinate infinite, nor yet something determinately finite (something absolutely bounded), it is clear from this that we can assume the magnitude of the world to be neither finite nor infinite, since the regress (through which this magnitude is represented) admits of neither of the two. (A518/B546 fn, my emphasis) In other words, the actual magnitude of the world depends on how it is represented in a "possible empirical regress." This is not an account of actuality on which actuality depends on being *actually* discovered or perceived at any time, contra the actual regress approach.³²⁷

Finally, two clarifications about the IOP reading I have been recommending are in order. First, although I have suggested adopting elements of Stang's and Jankowiak's accounts, which conceive of Experience in an idealized, encompassing way, there is a least one sense in which Experience fails to be a representation of an *ideal* end of inquiry on the account I endorse. Namely, Experience fails to be the kind of end of inquiry that *reason* would accept as completely fulfilling its explanatory needs. For as we have seen, reason demands *complete totalities* of conditions, and Experience represents only those things that it is *possible* for us to discover via empirical regresses; *possible* regresses can be neither determinately finite nor determinately infinite (as argued above), so the content of Experience is not a *complete totality* of appearances.

Second, notice on the reading recommended here, we may not be able to *know* what is represented in Experience.³²⁸ We can know that everything we *could* discover (and so everything that *actually* exists) must conform to the formal conditions of experience (i.e., the forms of intuition and the principles of the pure understanding), but it may not be possible for us to know exactly which so-far-unexperienced things exist in the empirical world. Likewise, we can know that things in themselves must constrain empirical reality in *some* way (even if we cannot know exactly how they do so), and we can know that empirical reality *cannot* include determinately

³²⁷ So, what makes it the case that, say, unicorns are not a part of the empirical world but an undiscovered dinosaur fossil might be? On the present interpretation, unicorns cannot be "correctly and thoroughly connected up" with the representations of all other things in experience (A492-3/B520-21), whereas the undiscovered dinosaur fossil can be (as far as we know). Or put differently, unicorns are not represented in Experience, whereas dinosaur fossils might turn out to be. Hence, a *possible* empirical regress could discover the dinosaur fossils (or might be able to as far as we know), whereas no possible empirical regress could discover unicorns.

³²⁸ See Stephenson (2015), for a point similar to this.

finite or actually infinite series of conditions; but we cannot know exactly which scientific discoveries we will make in the future. However, this is a feature rather than a bug of IOP and is plausibly a consequence of the fact that things in themselves play an important role in determining the content of Experience.

3. Two Objections Addressed

Before considering metaphysical readings of transcendental idealism that do not endorse IOP (in section 4 below), I now address two worries one might have about the fit between an IOP reading of transcendental idealism, on the one hand, and the metaphysical indeterminacy account I have defended in the chapters above, on the other. The first worry concerns the relationship between metaphysical indeterminacy in spatiotemporal reality and indeterminacy in what is represented in Experience: does metaphysical indeterminacy reduce to epistemic or representational indeterminacy after all? The second worry concerns the relationship between modal facts about possible regresses and indeterminacy with respect to what is *actual* in space and time: does my account of the relationship between possible regresses and actual indeterminacy actually vindicate a potential infinity approach (against the arguments of chapter 4)?

Taking these worries in order, first recall that the central claim of IOP is that *existence* in space and time is a matter of being *represented* in Experience. Given this, one might naturally raise the following question: Does *metaphysical* indeterminacy in space and time therefore reduce to a kind of representational indeterminacy? And if it does, can we still say that the antinomies are resolved by an appeal to "metaphysical indeterminacy"?

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I think the right thing to say here is the following. As we have seen in chapter 3, a metaphysical indeterminacy approach counts as "metaphysical" because it says that what *exists* in space and time is indeterminate. IOP says that spatiotemporal objects are intentional objects, but it is compatible with this to say that spatiotemporal objects (appearances) *exist as* intentional objects. Moreover, the *reason* spatiotemporal series of conditions must be indeterminate rather than either finite or infinite is only partly representational, on the version of IOP articulated above. For while it is true that spatiotemporal objects exist only as representations according to IOP, facts about the *metaphysical* impossibility of infinite successive regresses help to explain why the content of Experience must be indeterminate. So a representational story plays an important part in explaining why the spatiotemporal world is indeterminate, but IOP nonetheless does not imply that the indeterminacy to which Kant appeals in the resolution of the antinomies is *merely* representational.

Turning to the second worry, is it a problem for a metaphysical indeterminacy approach that facts about *possible* regresses explain what is *actual* in space and time (by explaining what is represented in Experience)? One might articulate a worry here as follows. If the arguments above are correct, then Kant holds that the scope of the *actual* empirical world is determined by the content of *possible* experience, which is neither determinately finite nor determinately infinite. But as I have argued in chapter 4, one of the appealing features of a metaphysical indeterminacy approach is that it allows us to distinguish between Kant's claim that spatiotemporal series of conditions are *potentially infinite* and his claim that they are *actually* neither finite nor infinite. This is important, I argued, since Kant *does* want to say that spatiotemporal phenomena are potentially infinite, and there is good evidence that this potential infinity cannot explain why the thesis and antithesis statements of the mathematical antinomies are *both false*. But now it might look like a *modal* notion of potential infinity *does* explain the both false solution. For as I have argued, indeterminacy arises from the fact that empirical regresses are not *possibly* finite or infinite.³²⁹

However, recall that one way of articulating the claim that every series of conditions is *potentially* infinite is as follows: *each* condition in every series is possibly conditioned by a further condition. Or more formally, where "*o*" denotes a conditioned object and "*Cxy*" means "*x* is a condition of *y*", the series of an object's conditions is potentially infinite when $\forall x(Cxo \rightarrow \forall \exists y Cyx)$. And even if we assume (per chapter 4) that claims about indeterminacy and claims about modality are related such that it is not necessary that Φ if it is indeterminate whether Φ (i.e., if $\nabla \Phi$, then $\neg \Phi$), still it does not follow that metaphysical indeterminacy rules out potential infinity. For potential infinity in the aforementioned sense may obtain even if it is indeterminate whether every condition is conditioned by something further ($\forall x(Cxo \rightarrow \exists y Cyx)$)) and indeterminate whether there is a condition that is not conditioned by anything further ($\nabla \exists x$ ($Cxo \land \neg \exists y Cyx$)). So at the least, explaining *actual* indeterminacy by appealing to *possible* regresses does not rule out holding that every regress is one in which each condition is *possibly* conditioned by something further.

But one might object that these observations do not fully solve the problem. For as one might argue, the real problem is not that claims about potential infinity cannot be combined with claims about actual indeterminacy at all. Rather, the real problem is that Kant's discussion of possible regresses seems to suggest that both infinite and finite series of conditions are *impossible*, and, as I have claimed previously, if it is *impossible* that Φ , it also cannot be

³²⁹ Putting the worry in a slightly different way, it might look like since the scope of the empirically possible *just is* the scope of the empirically actual, things cannot be *potentially* infinite after all.

indeterminate whether Φ . So it seems that if it is *impossible* for a successive regress to be infinite (i.e., to be one in which there is a further condition in the regress for every condition), then it cannot be *indeterminate* whether the series of conditions is infinite. And similarly, if it is impossible for the regress to be finite, it cannot be indeterminate whether the series of conditions has a last, unconditioned conditioned. Thus, it seems that once we try to explain *actual* indeterminacy by appealing to facts about *possible* experience, the claims I have made about metaphysical indeterminacy in chapters 3 and 4 are undermined.

At present, I see two possible ways of dealing with this problem. One option is to abandon the general claim that for any Φ , if it is indeterminate whether Φ , then it is possible that Φ . After all, at least intuitively, if it is impossible for a series to be finite *and* impossible for it to be infinite, but it nonetheless has *some* conditions, it must be indeterminate how many conditions it actually has. So perhaps we should conclude that the earlier formalizations of indeterminacy claims (in chapters 3 and 4) and their relations to modal claims need to be partially revised; this would not undermine the main claim I have been defending, which is that spatiotemporal series of conditions are actually indeterminate in magnitude rather than either finite or infinite.

Second, another alternative is to draw a distinction between (i) the modal claims we make in transcendental philosophy when we explain why Experience does not contain finite or infinite series of conditions and (b) the modal claims we make in the course of actual empirical inquiry, where we do not take objects to be actually successive-regress-dependent. As we might argue, in science and in ordinary experience, we take the existence of spatiotemporal objects to be *independent* of the possibility of our successively discovering them, and Kant's position is that we are *correct* in doing this: from within empirical reality (and in ordinary life and science), we should not treat all spatiotemporal phenomena as succession-dependent. (After all, Experience does not represent all spatiotemporal phenomena *as* succession-dependent.) But if this correct, then perhaps we can solve the above problem as follows. Although transcendental philosophy uncovers modal facts about *possible* experience, which explain why spatiotemporal series of conditions represented in Experience *cannot* be finite and also *cannot* be infinite, from within the disciplines that do *not* treat existence as necessarily successive-synthesis dependent (e.g., the empirical sciences), we *can* appropriately hold that phenomena are possibly infinite (in the sense of it being possible for all conditions to have further conditions). So just as science and ordinary experience does not represent them as being that way), so too science and ordinary experience should not take actually infinite series of conditions to be metaphysically impossible (except in those specific cases where it represents them as succession-independent). Similar claims could explain why we can say in ordinary life that the spatiotemporal world is possibly *finite*.

In short, although more needs to be said about how modal facts about *possible* successive syntheses determine the scope of *actual* empirical reality, there are avenues open for spelling out Kant's views in a coherent way such that he can sustain the claims he wants to make about the *potential* infinity of spatiotemporal phenomena. Given this, I think we should take the objection articulated above as an invitation to say more about the complex role that different kinds of modal claims play in Kant's idealist metaphysics rather than as a reason to think that claims about metaphysical indeterminacy in *actual* spatiotemporal existence are indistinguishable from claims about what *possibly* exists in space and time.³³⁰

³³⁰ For some sense of just how complex Kant's views about modality are, see Stang (2016b).

4. Alternatives to Intentional Object Phenomenalism?

Thus far in this chapter, I have suggested that interpreting transcendental idealism as a form of IOP can help us explain how metaphysical indeterminacy results from the ideality of appearances in space and time. More specifically, I have argued that we can understand the role of successive syntheses in explaining the indeterminacy of the spatiotemporal world if appearances are the intentional objects represented in Experience, which includes everything we can empirically discover in a *possible* regress. For this allows us to see how the mind-dependence of appearances results in their indeterminacy without entailing the kind of constructivism that Kant seems to be rejecting in the Postulates of Empirical Thinking.

Can other metaphysical readings of transcendental idealism do equally well at explaining how transcendental ideality results in indeterminacy in the scope of the spatiotemporal world and the compositional structures of objects? Recall that in chapter 1 above, I distinguished between two different metaphysical readings of the antinomies' resolutions currently defended in the secondary literature. Actual state phenomenalism holds that Kant resolves the antinomies by appealing to a conception of appearances as constructions of out perceivers actual mental states. Metaphysical anti-realism holds that Kant resolves the antinomies by denying experiencetranscendent *existence* in space and time. I have already explained why we should reject actual state phenomenalism: insofar as it regards the successive regress as *actual* rather than possible, it results in a kind of constructivism that Kant wants to reject. But what about metaphysical antirealist readings? At least on the face of it, they are compatible with IOP, since they say that spatiotemporal existence does not transcend the possibility of experience. Thus, I first consider

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what distinguishes metaphysical anti-realism from IOP such that it is not just a version of the view that appearances are the intentional objects of Experience.³³¹

One strategy for distinguishing metaphysical anti-realism from phenomenalism is to argue the very same objects that appear to us in experience also have mind-independent natures as things in themselves. This is to embrace a so called "one-object" or "metaphysical dual-aspect" interpretation of the distinction between appearances and things in themselves.³³² One prominent interpreter who can be read in this way is Allais (2015), who describes metaphysical anti-realism as follows:

Kant thinks that appearances are not just perceptible things (as a realist might think); they are *essentially* perceptible or *essentially* manifestable. Here, we have a kind of idealism or anti-realism which holds that spatio-temporal reality does not transcend what is essentially manifestable to finite receptive creatures like us. Empirical reality is restricted to what can be presented to consciousnesses like ours, but what can be presented to consciousness is not something which exists merely in the mind.³³³

On the view Allais endorses here, appearances do not exist *merely* in virtue of being represented by minds like ours, for the very things that appear to us also have mind-independent existence as things in themselves. So whereas IOP says that the things that appear to us (appearances) cannot exist without being represented in Experience, Allais's metaphysical anti-realism suggests that

³³¹ Note: it may be that metaphysical anti-realist readings and phenomenalist readings are not, in fact, distinct. E.g., Chiba (2012) characterizes anti-realism and phenomenalism such that they appear compatible (he also embraces a "two-world" reading of the distinction between appearances and things in themselves, 89). Stephenson (2015) also argues that "it is difficult to deny that Kant espouses *some* form of anti-realism," given that Kant "certainly seems to define *some* class of truth in terms of possible experience" (18-19). So if Stephenson is right, then all interpretations of Kant should classify him as *some* kind of anti-realist.

³³² Note that Allais in fact explicitly disavows talk of "one-world" or "one-object" and "two-world" or "two-object" readings, since the distinctions have been subject to so much dispute and confusion in the literature. Instead, Allais describes her position as "a moderate metaphysical interpretation which sees Kant as holding that the things of which we have knowledge have a way they are in themselves that is not cognizable by us, and that the appearances of these things are genuinely mind-dependent, while not existing merely in the mind" (2015, 8). But there is broad consensus in the literature that Allais belongs to the "one-world" camp, at least to the extent that those labels can be applied at all.

³³³ Allais (2015), 13.

the things that appear to us also exist independently of being represented. Only their minddependent (*essentially manifest*) qualities count as part of spatiotemporal reality, but as objects with mind-independent qualities (as things in themselves), they are not wholly representationdependent.³³⁴

Here are two related issues that I think are difficult for a non-phenomenalist anti-realist reading to tackle. First, recall that Kant holds that the Supreme Principle is true for things in themselves but *false* for appearances. At least on the face of it, this seems to mandate that appearances and things in themselves are two different classes of objects. For it is not clear how appearances could fail to form *complete totalities* of conditions, while those very same things could form complete totalities of conditions as things in themselves. This is especially so if it should turn out to be essential to appearances that the Supreme Principle does not hold for them (which is arguably the case, since it follows from their mind-dependence, which is essential to them even on Allais's account). Generally speaking, if an object x has the essential property p_{i} , and an object y has the property not-p, x and y cannot be numerically identical to one another. I don't take this to be a conclusive argument against so-called "one-object" or "one-world" readings, and proponents of such readings are aware of issues of the sort I have just raised, but I do think something more needs to be said about how the Supreme Principle can be false for appearances but true for things in themselves if appearances are just the mind- or representationdependent properties of mind-independent things in themselves.

A second issue concerns the way in which non-phenomenalist metaphysical anti-realist readings explain the relationship between the ideality of appearances and their metaphysical

³³⁴ This said, both Stang (forthcoming) and Tolley (forthcoming) argue there are varieties of intentional object phenomenalism that can accommodate the idea that appearances are appearances *of* things in themselves. And for this reason, appearances may not be *wholly* representation dependent.

indeterminacy. As we have seen, IOP readings can hold that appearances depend for their existence on successive syntheses, since they exist only insofar as they are represented in a possible regress, and all possible regresses for us take place successively. Because of this succession-dependence, IOP argues, appearances cannot form either determinately infinite or determinately finite series of conditions. In contrast, metaphysical anti-realist interpreters such as Allais argue that the very objects that appear to us also exist "in themselves" and therefore are not entirely succession-dependent. But this raises the question why their transcendental ideality explains their indeterminacy. Here is what Allais says on this issue:

However, once we realize that the world of which we have experience does not exist independently of our being able to cognize it, we can explain how it could be that it does not exist as a determinate totality. We cannot cognize it as a determinate totality (either a finite or an infinite totality) and therefore it does not exist as one.³³⁵

What does it mean to say that we cannot cognize it as either a finite or an infinite totality?

According to Allais, this means that it cannot be given to us as either finite or infinite in

intuition. She writes:

[Kant's] objection to the existence of appearances as a complete totality is not that a complete totality could not be *constructed* but that it could not be *given*. [...] A sophisticated phenomenalist allows as empirically real what can be constructed out of possible experience; this kind of idealism would not do the work Kant requires his idealism to do in the resolution of the antinomies, since he does not provide reasons why the size and extent of the world could not be theoretically constructed. Rather, he appeals to what can be *given*: what can possibly be manifest to us. Empirical reality is limited to what can be given to our presented to us, to what we can have acquaintance with; neither a boundary in space nor the world existing as an infinite totality, can be presented to us. They are therefore not empirically real.³³⁶

³³⁵ Allais (2015), 94.

³³⁶ Allais (2015), 217.

As seen above, I think it is correct to say that empirical reality is not a construction, and IOP can accommodate this. But I do not think that when Kant says appearances exist only in the successive regress, he means to say that they are not complete totalities because such totalities cannot be given in *intuition*. Rather, he means to say that they cannot be represented in *Experience* because representing them in *Experience* would require the *possibility* of a complete infinite succession. Consider the following passage from section 4 of the antinomy, which suggests that things *can* be given to us in intuition independently of succession:

Assume that nature were completely exposed to you; that nothing were hidden from your senses and to the consciousness of everything laid before your intuition: even then you still could not, through any experience, cognize *in concreto* the object of your ideas (for besides this complete intuition, a completed synthesis and the consciousness of its absolute totality would be required, but that is not possible through any empirical cognition [...] With all possible perceptions, you always remain caught up among **conditions**, whether in space or in time, and you never get to the unconditioned, so as to make out whether this unconditioned is to be posited in an absolute beginning of the synthesis or in the absolute totality of the series without a beginning. (A482-3/B510-11)

In this passage, Kant suggests that a "complete intuition" would not give us "experience" or "empirical cognition" of an "absolute totality" because experience of an absolute totality would require a "completed synthesis," which is impossible. But as we have seen, Kant thinks these syntheses are always successive *for us* (A417/B444). So consciousness of a complete totality is impossible not because it cannot be "laid before [our] intuition" but rather because it cannot be synthesized in experience. This suggests that the succession-dependence of possible experience is what explains why appearances are indeterminate. To the extent that metaphysical anti-realists deny that appearances depend for their existence on succession (arguing instead that they depend on being given in intuition, which does not require successive-synthesis), this speaks in favor of IOP as I have articulated it above. Once again, I do not think this is conclusive evidence against

metaphysical anti-realism such as Allais endorses, but I do think it presents a prima facie problem for the reading.³³⁷

To summarize, then, I take it that part of what motivates anti-realists such as Allais to focus on appearances as given in *intuition* rather than as represented downstream in *experience* is that they want to maintain a kind of "one-object" understanding of the relationship between appearances and things in themselves.³³⁸ And if appearances depend for their existence on successive synthesis, it is hard to see how their *existence* (as appearances) could consist primarily in being given in *intuition.*³³⁹ One of the advantages of IOP is that it can easily accommodate the claim that appearances "exist in the successive regress but otherwise do not exist at all" (A506/B534), and in accommodating this claim, it can explain why the succession-dependence of appearances results in metaphysical indeterminacy in the spatiotemporal world. Given the succession-dependence of all appearances, proponents of IOP can argue, it follows that spatiotemporal series of conditions are indeterminate in magnitude rather than either finite or infinite.

³³⁷ Perhaps non-phenomenalist anti-realists like Allais can grant that appearances depend for their existence on succession. If they can embrace a picture of this sort, then I think they are as well-equipped to embrace a metaphysical indeterminacy reading of the antinomy's solution as is IOP, and other considerations would have to decide between the two readings.

³³⁸ Interestingly, Chignell (forthcoming) formulates a "one-world" version of phenomenalism, so it may that considerations favorable to one-world readings do not even recommend anti-phenomenalism.

³³⁹ Note also that Kant thinks things can be given in intuition that we do not *consciously* represent (see especially Kant's claim that Newton's lamellae "really are represented in our empirical intuition" if they exist, even if they are not "consciously apprehended", *KE*, 122). So what is *given in intuition* does not necessarily change or evolve as conscious empirical discoveries proceed.

5. Chapter Summary

In this chapter, I have argued that Kant appeals to the metaphysical impossibility of an infinite succession to show that appearances form series of conditions that are *actually* indeterminate in magnitude rather than either finite or infinite. On this account, the succession-dependence of appearances does not underwrite a view according to which spatiotemporal reality is literally being constructed through time; rather, it explains why *possible* empirical regresses cannot be either finite or infinite and hence why Experience does not actually represent spatiotemporal series of conditions as either determinately finite or determinately infinite. As I have argued, this recommends a version of IOP that is compatible with Kant's account of actuality in the Postulates of Empirical Thinking, for it allows for the existence of spatiotemporal objects that we have not actually perceived and may never actually perceive.

I have also addressed several objections to my proposal that we marry an IOP reading of transcendental idealism to a metaphysical indeterminacy account of the antinomies' resolutions. One of these objections was that IOP reduces metaphysical indeterminacy to representational indeterminacy; as I have argued, even according to IOP, there is a meaningful sense in which indeterminacy among appearances is "metaphysical" and not merely representational. Another objection was that putatively non-modal facts about indeterminacy in the scope of *actual* empirical reality reduce to facts about what *possibly* exists on the non-constructivist version of IOP I have endorsed. Here too, I have suggested that this problem can be given a satisfying solution and that there remains a meaningful difference between modal claims about the *potential* infinity of spatiotemporal series of conditions and non-modal claims about the actually indeterminate magnitudes of these series.

Finally, I have raised some problems for other metaphysical readings of transcendental idealism, and specifically those readings that are anti-realist and anti-phenomenalist. While my arguments were not intended to be conclusive, I have suggested that any account of transcendental idealism that also gives a satisfying reading of the antinomies' resolutions will need to explain the sense in which appearances are succession-dependent. Insofar as anti-realist readings put claims about what can be given in *intuition* (which does not require synthesis) at the heart the antinomies' resolutions, they face a *prima facie* problem. However, this is not to say there is no way for them to accommodate Kant's claims about successive-regress-dependence; so while I think an IOP reading of transcendental idealism is most compelling (and best fits Kant's claims about the successive-regress-dependence of appearances), other metaphysical readings of transcendental idealism may also be able to accommodate a metaphysical indeterminacy reading of the mathematical antinomies' resolutions.

Conclusion

Not all commentators see the mathematical antinomies as a compelling part of the first *Critique* or as important to understanding Kant's considered account of spatiotemporal reality and the doctrine of transcendental idealism. In contrast, I hope to have shown that the mathematical antinomies are one of the richest parts of the *Critique*, that their arguments are neither question-begging nor wholly implausible, and that without studying the mathematical antinomies, we cannot fully appreciate the commitments that led Kant to see transcendental realism as a contradictory position and the radical doctrine of transcendental idealism as the only coherent position.

However, given the complexity of the issues treated in the antinomies, the arguments I have given above are necessarily incomplete in some respects. I cannot fill all these gaps in these final remarks, but I do want to close by addressing two loose ends that might seem especially salient to readers familiar with the Transcendental Dialectic and the antinomies. The first is whether the account I have endorsed above can account for Kant's remarks on the ways in which the resolutions of the first and second antinomies are asymmetrical. The second is whether my proposal spells trouble for the resolutions of the third and fourth antinomies, i.e., the dynamical antinomies. After addressing these issues in sections 1 and 2, respectively, I close by reviewing the main contributions I take myself to have made to Kant scholarship (in section 3).

1. Asymmetries Between the First and Second Antinomies

On the view I have defended above, there is a fundamental symmetry between the resolutions of the first and second antinomies. In both cases, I have argued, Kant claims that the

relevant series of spatiotemporal conditions are indeterminate rather than either finite or infinite, and this explains why the thesis and antithesis statements of the antinomies are both false. I have also argued (in chapter 5 above) that Kant's claims concerning the metaphysical impossibility of infinite successive syntheses help to explain why no series of spatiotemporal conditions can be either determinately finite or determinately infinite. However, as one might point out, Kant also claims that the first and second antinomies are to be treated *asymmetrically* in the following respect: while the first antinomy involves a regress *in infinitum* (A512-14/B540-42). Can my account explain what Kant means by this? And can it accommodate his distinction?

As a first step towards answering these questions, consider Kant's own characterization of the distinction between a regress *in indefinitum* and a regress *in infinitum*. He first introduces the distinction in section 8 of the antinomy, which concerns "The regulative principle of pure reason in regard to the cosmological ideas" (A508/B536). And according to Kant, the difference between the two kinds of regresses is a difference in "how far we are to continue" the empirical regress and in "the rule to be followed in this progress" (A514/B542). Kant writes:

If the whole has been empirically given, then it is **possible** to go back **to infinity** in the series of its inner conditions. But if that whole is not given, but rather is first to be given only through an empirical regress, then I can say only that it is **possible** to progress to still higher conditions in the series **to infinity**. In the first case I could say: There are always more members there, and empirically given, than I could reach through the regress (of decomposition); but in the second case I can say only: I can also go still further in the regress, because no member is empirically given as absolutely unconditioned, and thus a higher may be admitted as possible and hence the inquiry after it may be admitted as necessary. In the former case it was necessary to **encounter** more members of the series, but in the latter case it is always necessary to **inquire** after more of them, because no experience is bounded absolutely. (A514/B542) One point worth noting is that there are ways of reading this passage that make Kant's remarks in the antinomies' resolutions look contradictory. For instance, if he really means that in a series of decomposition there are more members there than one could reach through the regress, then it cannot also be the case that the parts "exist in the successive regress but otherwise do not exist at all" (A506/B534).³⁴⁰

However, I think we can avoid readings of this sort and can interpret Kant such that his remarks are not inconsistent. For instance, suppose Kant is making the following relatively intuitive point. When we are dealing with a spatial object and the parts that compose it, we see (because the whole object is already before us) that we will *succeed* in finding further parts for whatever parts we choose to investigate. In fact, since we know (according to Kant) that the divisibility of space guarantees the divisibility of the object, we *know* that finding further parts is always imminently possible (A525/B553). In contrast, when we are investigating the kinds of conditions treated in the first antinomy, we do not have the same guarantee that we will succeed in finding further conditions as soon as we look for them. As Kant puts it, we have only the *rule* "that however far I may have come in the ascending series, I must always inquire after a higher member of the series, whether or not this member may come to be known to me through experience" (A518/B546). So, we can say that although both the antinomies concern series that

³⁴⁰ Here is another problematic passage from the resolution of the second antinomy: "For though all the parts are contained in the intuition of the whole, the **whole division** is **not** contained in it; this division consists only in the progressive decomposition, or in the regress itself, which first makes the series actual. Now since this regress is infinite, all its members (parts) to which it has attained are of course contained in the whole as an **aggregate**, but the whole **series** of the **division** is not, since it is infinite successively and never is **as a whole**; consequently, the regress cannot exhibit any infinite multiplicity or the taking together of this multiplicity into one whole" (A524/B552). As I see it, Kant should not suggest (as he seems to suggest here) that objects *have* infinitely many parts and that they simply cannot be *unified* ("taken together into a whole"). After all, if an object has any *parts*, then those parts are already unified (and form a series) in virtue of standing in conditioning relations of composition. So I think Kant's considered position cannot be what this passage seems to suggest. However, see Marschall (2019) for an alternative take on these issues.

are neither finite nor infinite, given transcendental idealism (and the successive regressdependence of both kinds of conditions), the two antinomies are nonetheless different in the following way: we have a stronger guarantee of *actually* finding further conditions in the case of the second antinomy than in the case of the first antinomy.

So understood, we might even say that the distinction between regresses that go *in indefinitum* and regresses that go *in infinitum* is a distinction between two types of *potential infinity* (which characterizes the *possibility* of finding further conditions in first antinomy-type cases and second antinomy-type cases, respectively). This would not undermine the claim that both kinds of series are actually indeterminate in magnitude, for in both cases, both strictly finite and actually infinite regresses are impossible. Indeed, as Kant stresses in his discussion of the second antinomy, even though the regress in the second antinomy is a regress *in infinitum*, "it is by no means permitted to say of such a whole, which is divisible to infinity, that **it consists of infinitely many parts**" (A524/B552). As Kant explains this:

[T]o assume that the whole is articulated (*gegliedert*) to infinity – this is something that cannot be thought at all, even though the parts of matter, reached by its decomposition to infinity, could be articulated. For the infinity of the division of a given appearance in space is grounded solely on the fact that through this infinity merely its divisibility, i.e., a multiplicity of parts, which is in itself absolutely indeterminate, is given, but the parts themselves are given and determined only through the subdivision... (A526/B554)

I take passages like this as good evidence that Kant does not, in fact, mean to deny (as passages such as the A514/B542 passage seem to suggest) that empirical objects have parts that do not depend for their existence on the regress of decomposition. So, ultimately, Kant offers a unified explanation of why the series of conditions treated in *both* the first *and* the second antinomies must be indeterminate rather than either finite or infinite, an explanation that appeals to the successive regress-dependence of both types of series. The distinction between regresses *in*

indefinitum and regresses *in infinitum* is therefore best understood *not* as marking a difference in the *actual* magnitudes of regresses; rather, roughly speaking, it is best understood as marking a difference in the ways in which the regresses associated with the first and second antinomies appear to us as potentially infinite.

2. The Both True Solution of the Dynamical Antinomies

A different kind of objection that one might make to my proposal is that it undermines Kant's solution to the third and fourth antinomies, where transcendental idealism is supposed to explain how the thesis and antithesis statements can be *both true* (A532/B560). Taking the case of the third antinomy as an example, we can articulate the worry as follows. The third antinomy's thesis statement claims that "Causality in accordance with laws of nature is not the only one from which all the appearances of the world can be derived. It is also necessary to assume another causality through freedom in order to explain them" (A444/B472). In contrast, the antithesis statement asserts that "There is no freedom, but everything in the world happens solely in accordance with laws of nature" (A445/B473). And according to the standard reading, Kant's *both true* solution to this antinomy is to be understood as follows. Once we distinguish appropriately between appearances and things in themselves, we can hold that the thesis statement may be true for things in themselves, while the antithesis statement is true for appearances. That is to say, causality through freedom is *possible* for things in themselves, while among appearances there can be no kind of causality other than natural causality.³⁴¹

³⁴¹ For helpful discussion, see Watkins (2005), ch 5.

But as the objection goes, if it is an upshot of the mathematical antinomies that every series of spatiotemporal conditions is indeterminate in magnitude rather than either finite or infinite, then it is not clear how the antithesis statement of the third antinomy *could* be true for appearances. For one might think that if everything that happens in space and time must follow from what happened previous according to a causal law, then the series of natural causes in the world must be actually infinite. And so the spatiotemporal world must be actually infinite to sustain this causal chain.

Notably, some commentators have argued that the dynamical antinomies *do* in fact admit of a *both false* solution. Smith (1918), for instance, claims that Kant's distinction between the mathematical and dynamical antinomies is "artificial" and that this "becomes clear when we recognize that the opposed solutions, which he gives of the two sets of antinomies, can be mutually interchanged."³⁴² Chiba (2012) also argues that Kant's solution to the first and second antinomies is in fact a *general* solution that holds for the dynamical as well as for the mathematical antinomies (and according to which all four sets of thesis and antithesis statements are *false*).³⁴³

However, I think Kant's own remarks on the sense in which the third antinomy's antithesis statement is *true* for appearances can help us understand why a metaphysical indeterminacy reading does not undermine his solution to the dynamical antinomies. Kant writes:

[T]he dynamical concepts of reason [...] have the peculiarity that since they do not consider their object as a magnitude but have to do only with its **existence**, one can thus abstract from the magnitude of the series of conditions, and with them it is merely a matter of the dynamical relation of conditioned to condition;

³⁴² Smith (1918), 511.

³⁴³ As Chiba writes, "Wie ich in 4.2.3 (S. 124 f.) zeigte, bietet Kant zwei distinkte Auflösungen der Antinomien an. Die eine ist die generelle Auflösung, die sowohl auf die mathematischen als auch die dynamischen Antinomien unterschiedslos angewandt wird, und ihr gemäß sind die Thesen sowie die Antithesen *falsch*. Die andere hingegen richtet sich nur auf die dynamischen Antinomien..." (2012, 254-5).

thus the difficulty we encounter in the question about nature and freedom is only whether freedom is possible anywhere at all [...] The correctness of the principle of the thoroughgoing connection of all occurrences in the world of sense according to invariable natural laws is already confirmed as a principle of the transcendental analytic and will suffer no violation. (A535-6/B563-4)

In saying that one can abstract from the magnitude of the series of conditions in the dynamical antinomies, I take Kant to be arguing that the issues treated in the dynamical antinomies do not turn on whether the relevant series of conditions have finitely many or infinitely many members. Rather, they turn on the kind of conditioning that is possible between the conditioned and its condition. As Kant puts it, they turn on the "dynamical relation of conditioned to condition." So the reason the *antithesis* statement of the third antinomy is true for appearances is that there can be no *kind* of causality among appearances other than natural causality. This means that for anything that is caused in space and time, its cause must be natural (and, in particular, it must follow from a previous state according to a natural law). But importantly, this does not entail that there is an actually infinite series of causes and effects in the natural world. In particular, it might be false that *every* cause is also the effect of something else. What matters is just that there can be no *kind* of causality among appearances other than natural causality.³⁴⁴

There are of course other questions we might ask about how Kant's *both true* solution to the dynamical antinomies should be understood, but I hope these comments suffice to show that a metaphysical indeterminacy reading of the mathematical antinomies' resolutions does not immediately undermine Kant's efforts to resolve the dynamical antinomies with a *both true* solution. If he is right that questions about the "dynamical relation of conditioned to condition"

³⁴⁴ Similarly, we can say that Kant's solution to the mathematical antinomies does not undermine the Second Analogy of Experience, which says that "All alternations occur in accordance with the law of the connection of cause and effect" (A189/B232). The Second Analogy asserts that every alteration is caused, but it does not assert that absolutely everything that causes an alteration is itself an alteration (A189/B232).
"abstract from the magnitude of the series," then it does not matter to the solution of the dynamical antinomies that the relevant series of conditions are *mathematically* neither finite nor infinite.³⁴⁵ Moreover, note that even if this response to the worry articulated at the outset of this section is unsuccessful, this is a difficulty that any reading of the antinomies' resolutions must tackle. For regardless of how one explains why the thesis and antithesis statements of the mathematical antinomies are *both false*, the question arises why those same considerations do not show that the dynamical antinomies also admit of a both false solution.

3. Main Results Reviewed

The mathematical antinomies serve no fewer than four important purposes in the *Critique* of *Pure Reason*. First, they help to vindicate Kant's claim that transcendental realism is a contradictory position, in light of which Kant argues that we must embrace the doctrine of transcendental idealism. Second, they help to show why *pure reason* is not a faculty that can extend our cognition to the unconditioned objects treated in the traditional metaphysics. That is, they are an important part of Kant's attack on traditional rational cosmology. Third, they show that spatiotemporal reality does not meet our naïve expectations about the magnitude properties it will have: the extent of the world in space and time is neither finite nor infinite, and material

³⁴⁵ Note: this is further supported by Kant's claim that on his solution to the dynamical antinomies, "while the dialectical arguments that seek unconditioned totality on the one side or the other collapse, the rational propositions, on the contrary, taken in such a corrected significance, may **both be true**; which could never have occurred with the cosmological ideas dealing merely with mathematically unconditioned unity, because with them there is no condition of the series of appearances that is not itself also appearance" (A531-2/B559-60). In saying that the dialectical arguments that seek unconditioned totality collapse, I take him to be saying that *truth* of the thesis and antithesis statements (for things in themselves and appearances, respectively) do not require the existence of a complete totality of conditions. This is compatible with the claim that causality through freedom posits a causally unconditioned cause, since its status as unconditioned in that case consists in its being "the faculty of beginning a state **from itself**" (A533/B561).

objects are composed neither from finitely many simples nor from infinitely many parts all of which have further parts within them. And fourth, the antinomies help explain why it is so easy for us to fall into error in the discipline of metaphysics. Because we naively take spatiotemporal reality to answer to the Supreme Principle, Kant argues, we easily find ourselves concluding that unconditioned things exist in space and time.

The reading I have defended makes sense of how all these points fit together, and it does so in a way that allows us to draw four additional (often underappreciated) conclusions about Kant's position. First, Kant is a proponent of what we would now call "metaphysical indeterminacy" in the spatiotemporal world. More specifically, he believes it is metaphysically indeterminate *how many* things exist in the series of spatiotemporal conditions that determine the extent of the world and the compositional structure of objects. This makes Kant into a surprisingly early proponent of the view that there is indeterminacy in reality that is not merely epistemic, semantic, or representational.

Second, Kant's understanding of the relationship between the notions of *infinity* and *totality* are more nuanced and more plausible than is often recognized. For as I have argued, in the course of articulating what it is for something to be *unconditioned*, Kant distinguishes between two different notions of totality and suggests (against many of his contemporaries and predecessors) that there is nothing incoherent in thinking that infinite totalities of both kinds exist. Only in certain cases—namely, when *succession* is involved—does Kant say there is anything problematic about the notion of an infinite totality. Moreover, Kant's position also allows us to see important differences between claims about indeterminacy and claims about potential infinity. Whereas a traditional reading suggests that Kant understands indeterminate magnitudes as potentially infinite, I have argued that attributions of indeterminacy and

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attributions of potential infinity are different for Kant and that indeterminacy rather than potential infinity is at the heart of the antinomies' resolutions.

Third, Kant's account of the role of indeterminacy in explaining why the Supreme Principle is false for appearances points to a provocative (and in my view plausible) claim about the relationship between metaphysical determinacy and metaphysical indeterminacy, on the one hand, and broadly rationalist demands that reality contain complete explanations, on the other. For if Kant is right about the indeterminacy that occurs in spatiotemporal reality, then a certain kind of complete explanation is inaccessible to us principle—namely, for things that are conditioned in space and time, we cannot access an explanation that *exhaustively* explains them (in the sense of leaving no conditions out). Moreover, if we join Kant in his conviction that the Supreme Principle must be true for fundamental reality, we can see how one might articulate a kind of rationalist commitment about fundamental reality without committing to any substantive cognition of things as they are in themselves. That is, we might articulate a rationalist commitment that is compatible with epistemic humility. For given that the Supreme Principle demands explanatory completeness that can be satisfied in *both* finite *and* infinite scenarios, knowing that the Supreme Principle is true for fundamental reality does not allow us to determine whether things in themselves are infinite or finite, and in fact it does not even rule out the possibility that all facts in fundamental reality are brute.

Fourth, and finally, I have also argued that Kant's claims about the metaphysical impossibility of infinite *successive* syntheses point to a promising new direction for settling disputes about the nature of Kant's idealism. On the view I have defended, Kant thinks the features of the world treated in the antinomies are indeterminate because of the way in which they depend on *successive* regresses for their existence, and so any plausible account of

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transcendental idealism must explain the relationship between ideality and successive-regressdependence. Most views in the secondary literature either (a) acknowledge this and paint Kant as a kind of constructivist about empirical reality or (b) deemphasize the importance of succession and read the indeterminacy of spatiotemporal reality as following from facts about what can be given in intuition. But as I have argued, an intentional object phenomenalist interpretation of idealism allows us to accommodate Kant's claims about successive regress-dependence *and* avoid the kind of constructivism Kant denies in various parts of the *Critique*.

Appendix: Final Distinctions Diagram

Put into a diagram, the interrelations between the core distinctions (finitude/infinitude, complete/unified totality, determinate/indeterminate) can be mapped as follows. Note that things anywhere on the diagram can be potentially infinite (in the modal sense).



Figure A.1 Final Distinctions Diagram

Recall:

- A Complete Totality is a plurality that is not a part or subset of a greater plurality (of the relevant kind).
- A Unified Totality is a plurality whose elements are unified or brough together via real relations of connection or dependence*

*Note: In chapter 2, I considered whether the notion of a unified totality might be expanded such that *any* plurality is a unified totality if it is "considered as" a unity (per a literal interpretation of Kant's discussion of the *category* of totality at B111). If we understand unified totalities in this way, then the ellipse representing unified totalities above might expand to include all complete totalities as well as finite, infinite, and indeterminate pluralities. However, since the antinomies concern *series* of conditions whose members are brought together via *real* conditioning relations, I have mapped the distinctions by assuming a more restrictive notion of a unified totality.

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