

## **UC Riverside**

### **UC Riverside Electronic Theses and Dissertations**

**Title**

Dynamic Container Leasing and in Intermodal Freight Transportation

**Permalink**

<https://escholarship.org/uc/item/8wc00018>

**Author**

Luo, Ting

**Publication Date**

2012

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA  
RIVERSIDE

Dynamic Container Leasing and Load Acceptance Management in Intermodal Freight  
Transportation

A Thesis submitted in partial satisfaction  
of the requirements for the degree of

Master of Business Administration

in

Management

by

Ting Luo

September 2012

Thesis Committee:

Professor Long Gao, Chairperson  
Professor Mohsen El Halfsi  
Professor Yunzeng Wang

Copyright by  
Ting Luo  
2012

The Thesis of Ting Luo is approved:

---

---

---

Committee Chairperson

University of California, Riverside

## Acknowledgments

I want to first thank my parents. They are my first and most important tutors in life, and have ignited my aspirations not only for intellectual exploration, but also for the virtues that the human kind seeks: integrity, tolerance, courage and faith. I wish to express my most genuine thanks to my advisor, Dr. Long Gao, for his patient and careful directions in my research. Without his help and encouragement, I would not have been here. I also give my sincere thanks to my committee members: Dr. Mohsen El Halfsi and Dr. Yunzeng Wang.

## ABSTRACT OF THE THESIS

Dynamic Container Leasing and Load Acceptance Management in Intermodal Freight Transportation

by

Ting Luo

Master of Business Administration, Graduate Program in Management  
University of California, Riverside, September 2012  
Professor Long Gao, Chairperson

We study the multi-period container leasing and load acceptance problem in intermodal freight transportation, where the container capacity could be replenished before allocated to load orders with different profit levels. The container leasing decision is exogenously determined by the initial container inventory left unused from last period and the returned containers in the current period. The load acceptance decision involves trade-offs between accepting the order now or reserve the container for potential high profit level order in the future. Our objective is to characterize the optimal policies in order to maximize the total profit. The two-stage sequential decision problem is formulated as a stochastic dynamic programming problem. We show the optimal leasing quantity follows a base-stock policy and the optimal allocation for different class of demand follows a rationing policy. We further study the impact of demand variability on the optimal policy. Given stochastically higher demand in the future, both total profit and marginal value increase. Such change in demand also results in a higher leasing threshold and a higher rationing level for each demand class.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Literature Review</b>	<b>5</b>
<b>3</b>	<b>Model Formulation</b>	<b>8</b>
<b>4</b>	<b>The Structure of the Optimal Policy</b>	<b>12</b>
<b>5</b>	<b>The Impact of Demand Variability</b>	<b>25</b>
<b>6</b>	<b>Conclusions</b>	<b>33</b>
	<b>Bibliography</b>	<b>35</b>

# Chapter 1

## Introduction

Dated back to the 18th century and first used for coals shipment, intermodal freight transportation has long been considered efficient and cost-saving. In 1980s, the railway deregulation allowed a substantial rise in its productivity and a dramatic fall in the freight cost. Stimulated by the tide of imports on the west coast, and the shortage of highway lorry drivers, intermodal freight transportation has been the largest as well as the fastest-growing segment in US rail freight over the past 20 years, accounting for 22% of the rail revenues (American Association of Railroads (2008)). The Economist (2010) estimated that the current capacity of the intermodal shipments has to rise 90% in order to meet the forecasted demand by 2035. Due to this rapid growth, better planning strategies towards higher profitability with limited supply of capacity are highly expected in both practice and theoretical study.

Intermodal freight transportation employs multiple modes of transportation without directly handling of the freight when changing modes. Usually the cargos are containerized and transported by trucking for local pickup and delivery, and by rail for the long haul portion of the shipment. The intermodal freight transportation service



provider is called an Intermodal Marketing Company (IMC), who arranges the intermodal transportation on behalf of its customers while maintaining good relationships with the trucking companies and railways. It coordinates the supply and demands between container providers and its customers to maximize its own economic advantage.

In practice, very few IMCs ever exercise sophisticated revenue management techniques. Instead, they pick up some simple management rules such as increasing rate for “busy season” and lowering rates for “back haul lanes”, not the location specific and on-the-fly revenue management based on short term capacity conditions. Most have no short term forecast of demand and a soft idea of available supply of other capacities. Many take the first-come-first-serve (FCFS) approach, so that a load is accepted regardless of the available capacity, thus subsequent high-profit-level service request suffers. In case of turning down a load due to capacity shortage, that “lost order” is not usually tracked, so the opportunity cost of capacity shortfalls can’t be determined.

The aforementioned load acceptance process is also called load tendering, which starts with the customers sending out a service request with information such as the origin and destination of the shipment, and the date of shipment. Then the IMC has to respond immediately or in a very short term to whether accept or reject the tendered load. The delay in response may result in customers switching to other competitors. Typically, the request for service is delivered within the next week, and most often the service is initiated in the next 72 hours.

Our study was motivated by the following two improvements in the operation of intermodal freight transportation.

First, the change of ownership of the containers. In the past, IMCs were non-asset based service providers who use containers owned by the railroads. Now they tend

to keep a private container fleet and borrow from the railroads only when necessary. Such lease is often on a transaction-by-transaction basis and is more costly and requires early planning. The IMCs need to avoid its own containers idle time and assure sufficient total container capacity at the same time. They are also in charge of repositioning containers returned from customers after service.

Second, the change of traditional load tendering policy. Historically, the industry norm for load tendering is based on FCFS policy, in which each customer is typically given a contract of specified rates for each origin and destination pair, thus not all shipments are equally profitable to the IMCs. In order to avoid the running out of capacity before the demand is completely met and the foregone of more profitable loads in favor of less profitable ones, IMCs need to implement dynamic allocation practice that best matches the available containers with the load demands.

We focus on the short term container leasing and load tendering process of an IMC, who uses its private containers fleet and borrowed containers at higher cost from local railroads to satisfy the multi-profit-level service demands. The execution cycle is divided into multiple small-enough periods, so that all demands come continuously throughout the time. In each period, the decision of container leasing and load acceptance are made sequentially: first, whether to fulfill service requests with available containers or expand the current capacity by leasing from local railroads given the initial inventory is predetermined from last period and the number of returned container is random variable follows certain distribution; second, how and whether to accept or reject service demands for each class. The leasing containers (incurs leasing cost in this period) lies on the available capacity consisted of last period leftover inventory (incurs holding cost in last period) and the returned containers from previous service. The order is lost (incurs lost sale cost) if it can't be accepted in the current period. The accepted

order that can't be fulfilled in the current period will be backlogged (incurs backlog cost in this period).

The remainder of the our work is organized as follows. We begin in §2 to briefly review the related literature. In §3, we set up the value function that captures the dynamic nature of the problem by Markov dynamic process. We show that the optimal policy could be sought though a two-stage sequential process. In §4 we analyze the optimal structure of container replenishment policy and load acceptance policy respectively. The container capacity should be replenished up to a critical level, while the load orders are accepted in the order of descending profit levels and down to a critical container capacity level for each demand class. We also derive managerial insights for each result as well. In §5 we study how to cope with the impact of demand variability. It is known that there are identifiable factors that determine the demand environment. The change of these factors over time results in demand fluctuation. Therefore we update the parameter of the optimal policy to maximize the total profit. Finally we make concluding remarks in §6.

## Chapter 2

# Literature Review

We start from literature on periodic review inventory models with multiple demand classes. Similarly, we use the rationing lever to accept incoming load orders in each period. Topkis (1968) considers the inventory problems associated with multi-class demands for stocks of varying importance and identifies the optimal procurement policy as a set of critical rationing levels. Frank et al. (2003) characterize the optimal policy for a two-class inventory system, with a rationing level to avoid setup cost. Gupta and Wang (2007) consider a two-class allocation problems for perishable manufacturing capacity and characterize the policy by a state dependent rationing level. Shumsky and Zhang (2008) examine multi-period capacity allocation problem for fixed capacity. The lead time in multi-period inventory system is generally intractable, so most two-class models are based on the assumption of zero lead time. In our model, we also consider the multi-class demands problem with zero lead time. Unlike the work of Li and Zheng (2006) on joint inventory replenishment and pricing control, where they analyze the single-item, periodic-review model with random demands and yield, our work focus on sequential decisions on replenishment and capacity allocation rather than pricing control. We also

consider the inclusion of volatile or unpredictable demands due to uncertain factors that determine the demand levels. It is similarly to the inventory models with demand variability. Song and Zipkin (1993) derive some basic characteristics of optimal policies and certain monotonicity patterns in the parameter are reflected in the optimal policy. Yang et al. (2006) consider a stochastic inventory control problem with Markovian capacity and option of order rejection and show its optimal policy as the combination of a modified base-stock production policy and critical point order acceptance policy, where the policy parameters change over the current capacity level in an intuitive way.

Revenue management (RM) is also close to our topic on intermodal freight transportation, though the related literature mainly focuses on airline passengers, e.g., see Lautenbacher and Stidham (1999), Karaesmen and van Ryzin (2004), Zhang and Cooper (2005) and revenue management by Talluri and van Ryzin (2004) for details. The literature on freight revenue management is very recent, and yet airline based, e.g., see Amaruchkul, Cooper and Gupta (2007); Bartodziej, Derigs and Zils (2007). Unlike airlines, where generally lower profit order goes first, we have interspersed, simultaneous, customer orders from various profit segments common to freight transportation. Also note that the capacity can be inventoried and carried to the next period if not used now, not like the perishable capacity of airlines, where seats are not available after the plane takes off. We make use of a single supply source (origin) for multiple customer products (origin-destination), while the nature of airlines dedicates supply to a single product in the short run. Recent literature has compared and contrasted cargo and passenger yield management, see Freeland (2007); Sabdu and Klabjan (2006). The very few rail revenue management research does not focus on the container unit of capacity (equivalent to a seat on an aircraft) as we use in our model, but rather on the whole train (equivalent to a flight).

Most literature in intermodal transportation focus on long-term strategic or mid-term tactical decisions. Gorman (2001, 2002) describes an intermodal pricing optimization application focus on pricing as a mechanism for balancing cost. Li and Tayur (2005) study the medium-term pricing and operation planning and solve for optimal train frequencies. Adelman (2007) investigates a fleet management problem on a logistics network employing an internal pricing mechanism. Gorman (2010) develops an integrated production decision support system to improve the revenue management and container allocation for Hub Group, the largest IMC in North America. In contrast, our problem involves real-time execution strategy that lead to load acceptance decision, rather than one of setting optimal price levels for various segments in previous intermodal transportation literature. Literature in freight revenue management has been theoretical or strategic in nature; this research proposes tactical and real time execution strategy for freight revenue management.

## Chapter 3

# Model Formulation

The IMC institutes a two-level hierarchical decision making mechanism: The central management controls the balance of network supply and demand over execution cycles by setting *strategic inventory target*  $S$  for each location; each local manager maximizes his expected total profit of each execution cycle by discretionally accepting orders and dynamically adjusting supply through leasing. We focus on a local manager's decision-making process over the execution cycle  $\mathcal{T}$  with  $T$  periods. For the manager, the inventory target  $S$  from the central management and scheduled arrivals  $\{R_t : t \in \mathcal{T}\}$  from other locations are exogenously given input parameters.

The manager faces a multiperiod, multiclass, container leasing and load acceptance problem. Before the start of the cycle, the manager receives from the central management the strategic inventory target  $S$  for the current cycle. The purpose of such target is to direct local decisions so as to balance supply-demand over the network. At the end of the cycle, based on his inventory relative to target  $S$ , the manager is charged the mismatch cost for repositioning to or expediting containers from other locations: the excess containers incur  $\alpha$  unit cost while the shortages pay unit deficit cost  $\beta$ .

During the cycle, each period consists of two stages—container leasing and load acceptance. Time is indexed backward with  $t = T$  as the first period. Let  $I_t$  denote the *net inventory* at the beginning of period  $t$ . There are three sources of container supply in each period  $t$ . First, leftover inventory (if  $I_t > 0$ ) from previous period is available at the beginning of the period. Second, the scheduled arrivals and returns from other locations provide additional random  $R_t$  containers. Third, the manager can also lease  $\ell_t$  containers at unit leasing cost  $c$  from the local railway with realized random capacity  $Q_t$ . Because of the proximity, the leasing lead time is negligible.

The random demand for shipping services come from  $J$  classes of customers. The demand classes, indexed by  $j \in \mathcal{J}$ , are differentiated by service priorities and profit margins. After receiving demand  $\mathbf{D}_t = (D_t^1, \dots, D_t^J)$ , the manager needs to make acceptance decision  $\mathbf{x}_t = (x_t^1, \dots, x_t^J)$  on how many orders to accept from each class. Each accepted order from class  $j$  brings  $\tilde{r}^j$  profit whereas each rejected order incurs  $b^j$  penalty. Thus total revenue from accepting  $\mathbf{x}_t$  orders given  $\mathbf{D}_t$  is  $\mathbf{r} \cdot \mathbf{x}_t - \mathbf{b} \cdot (\mathbf{D}_t - \mathbf{x}_t) = \sum_j (\tilde{r}^j + b^j) \cdot x_t^j - \sum_j b^j D_t^j$ . Without loss of generality, we define for each class  $j \in \mathcal{J}$  the profit margin  $r^j \equiv \tilde{r}^j + b^j$  and rank the classes such that  $r^1 > r^2 > \dots > r^J$ . The accepted orders are fulfilled on a first-come-first-served basis using *post-leasing inventory*  $I_t + R_t + \ell_t$ . Unfilled accepted orders are backlogged while leftover inventory is carried forward to the next period. Both backorders and leftover inventory at the end of the period are charged a convex penalty cost  $h(I_t) \equiv h(I_t^-) + h(I_t^+)$ , where  $h(I_t^-)$  is the backorder cost for  $I_t < 0$  and  $h(I_t^+)$  is the holding cost for  $I_t \geq 0$ .<sup>1</sup>

Let  $\|\mathbf{x}_t\| \equiv \sum_{j=1}^J |x_t^j|$  be the  $\ell_1$ -norm of vector  $\mathbf{x}_t$ . The sequence of events of the cycle unfolds as follows (see Figure 3.1).

- In period  $t = T$ , receive strategic target  $S$  for the current cycle

---

<sup>1</sup> $I_t^+ = \max(I_t, 0)$  and  $I_t^- = \min(I_t, 0)$ .



- In period  $t < T$ :
  - Observe initial net inventory  $I_t$  and receive random arrival containers  $R_t$ , resulting in post-arrival inventory level  $J_t = I_t + R_t$
  - After observing railway capacity  $Q_t$ , lease quantity  $\ell_t \leq Q_t$  and pay leasing cost  $c\ell_t$ ; inventory level raises to post-leasing level  $K_t = I_t + R_t + \ell_t$
  - Demand  $\mathbf{D}_t$  materializes
  - Accept loads  $\mathbf{x}_t \leq \mathbf{D}_t$  and collect profit  $\mathbf{r} \cdot \mathbf{x}_t$ ; inventory level decreases to  $I_{t-1} = I_t + R_t + \ell_t - \|\mathbf{x}_t\|$
  - Deliver orders and assess inventory cost  $h(I_t + R_t + \ell_t - \|\mathbf{x}_t\|)$
- In period  $t = 0$ , evaluate strategic mismatch cost relative to the target  $S$

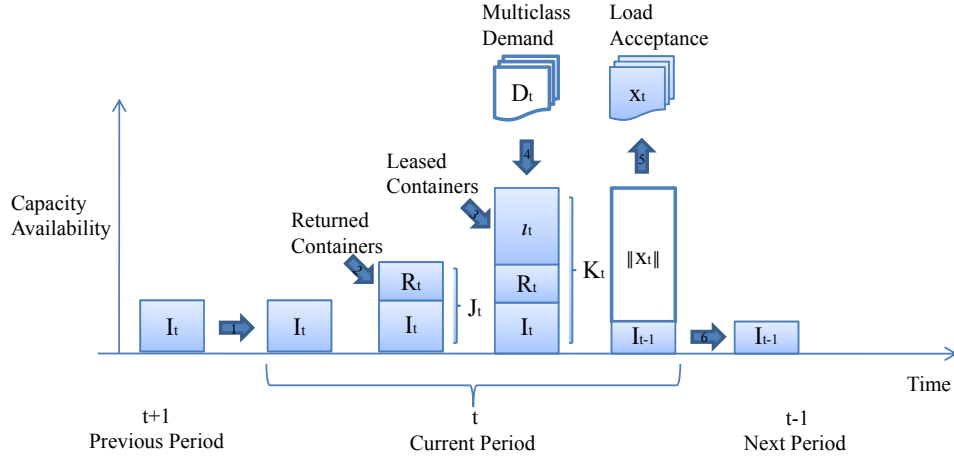


Figure 3.1: Container Leasing and Load Acceptance Process in Intermodal Freight Transportation

The problem can be formulated as a Markov Decision Process (MDP). Each

period  $t \in \mathcal{T}$  has two stages. The optimality equations for container leasing stage are

$$V_t(I_t, R_t, Q_t) = \max_{0 \leq \ell_t \leq Q_t} H_t(I_t + R_t, \ell_t), \quad (3.1)$$

$$H_t(J_t, \ell_t) = -c\ell_t + \mathbb{E}_{\mathbf{D}_t}[W_t(J_t + \ell_t, \mathbf{D}_t)], \quad (3.2)$$

where  $V_t(I_t, R_t, Q_t)$  is the value function for leasing in state  $(I_t, R_t, Q_t)$ ,  $H_t$  is the objective function,  $W_t(J_t + \ell_t, \mathbf{D}_t)$  is the value function for load acceptance, and  $\mathbb{E}_{\mathbf{D}_t}[\cdot]$  is the expectation operator relative to random demand vector  $\mathbf{D}_t$ .

The optimality equations for order acceptance stage are

$$W_t(K_t, \mathbf{D}_t) = \max_{0 \leq \mathbf{x}_t \leq \mathbf{D}_t} G_t(K_t, \mathbf{x}_t), \quad (3.3)$$

$$G_t(K_t, \mathbf{x}_t) = \mathbf{r} \cdot \mathbf{x}_t - h(K_t - \|\mathbf{x}_t\|) + \mathbb{E}_{R_{t-1}, Q_{t-1}}[V_{t-1}(K_t - \|\mathbf{x}_t\|, R_{t-1}, Q_{t-1})], \quad (3.4)$$

where  $W_t$  and  $G_t$  are the value and objective functions for load acceptance after observing post-leasing inventory level  $K_t = I_t + R_t + \ell_t$  and demand realization  $\mathbf{D}_t$ . The objective function  $H_t$  includes the revenue  $\mathbf{r} \cdot \mathbf{x}_t$  from acceptance, the inventory holding and backorder cost  $h(K_t - \|\mathbf{x}_t\|)$  and the expected profit-to-go function

$$V_{t-1}(I_{t-1}) \equiv \mathbb{E}_{R_{t-1}, Q_{t-1}}[V_{t-1}(K_t - \|\mathbf{x}_t\|, R_{t-1}, Q_{t-1})]$$

before observing random vector  $(R_{t-1}, Q_{t-1})$ . The next period initial inventory is given by

$$I_{t-1} = I_t + R_t + \ell_t - \|\mathbf{x}_t\|. \quad (3.5)$$

The boundary condition is

$$V_0(I_0) = -\alpha(S - I_0)^+ - \beta(I_0 - S)^+, \quad (3.6)$$

where the first and the second terms are the overage and shortage costs for missing the strategic target  $S$ .

## Chapter 4

# The Structure of the Optimal Policy

In this section, we characterize the properties of the optimal leasing and acceptance policies, and provide their managerial implications. Before that, we list a few useful properties of concavity and submodularity in Lemma 1. The proof can be found in Boyd and Vandenberghe (2004), and Topkis (1998).

**Lemma 1.** (i) Define  $h \circ g(x) = h(g_1(x), \dots, g_m(x))$ , with  $h: \mathbb{R}^m \rightarrow \mathbb{R}$ ,  $g_i: \mathbb{R}^n \rightarrow \mathbb{R}$ ,

$i = 1, \dots, m$ . Then  $h \circ g(x)$  is concave if  $h$  is concave and nondecreasing in each argument, and  $g_i$  is concave for each  $i$ .

(ii) If  $h: \mathbb{R}^m \rightarrow \mathbb{R}$  is a concave function, then  $h(Ax + b)$  is also a concave function of  $x$ , where  $A \in \mathbb{R}^m \times \mathbb{R}^n$ ,  $x \in \mathbb{R}^n$ , and  $b \in \mathbb{R}^m$ .

(iii) Assume that for any  $x \in \mathbb{R}^n$ , there is an associated convex set  $C(x) \subset \mathbb{R}^m$  and  $\{(x, y) : y \in C(x), x \in \mathbb{R}^n\}$  is a convex set. If  $h(x, y)$  is concave and the function  $g(x) \equiv \sup_{y \in C(x)} h(x, y)$  is well defined, then  $g(x)$  is concave over  $\mathbb{R}^n$ .

- (iv) If  $f(x)$  and  $g(x)$  are concave (supermodular) on  $X$  and  $\alpha, \beta > 0$ , then  $\alpha f(x) + \beta g(x)$  is concave (supermodular) on  $X$ .
- (v) Assume that  $F(y)$  is a distribution function on  $Y$ . Assume also that  $f(x, y)$  is concave (supermodular) in  $x$  on a lattice  $X$  for each  $y \in Y$ , and integrable with respect to  $F(y)$  for each  $x \in X$ . Then  $g(x) \equiv \int_Y f(x, y) dF(y)$  is concave (supermodular) in  $x$  on  $X$ .
- (vi) If  $X$  and  $Y$  are lattices,  $S$  is a sublattice of  $X \times Y$ ,  $S_y$  is the section of  $S$  at  $y$  in  $Y$ , and  $f(x, y)$  is supermodular in  $(x, y)$  on  $S$ , then  $\operatorname{argmax}_{x \in S_y} f(x, y)$  is increasing in  $y$  on  $\{y \in Y : \operatorname{argmax}_{x \in S_y} f(x, y) \neq \emptyset\}$ .
- (vii) Suppose that  $Y$  is a convex subset of  $\mathbb{R}^1$ ,  $X$  is a sublattice of  $\mathbb{R}^n$ ,  $a_i \in \mathbb{R}^1$  for  $i = 1, \dots, n$ ,  $\sum_{i=1}^n a_i x_i \in Y$  for  $x \in X$ . If  $a_i > 0 \forall i$ , and  $g(y)$  is concave in  $y$  on  $Y$ , then  $f(x) \equiv g(\sum_{i=1}^n a_i x_i)$  is submodular in  $x$  on  $X$ .

We first establish the concavity of our value functions and objective functions in the following theorem.

**Theorem 2.** (i)  $H_t(J_t, \ell_t)$  is concave in  $(J_t, \ell_t)$ , and  $G_t(K_t, \mathbf{x}_t)$  is concave in  $(K_t, \mathbf{x}_t)$ .

(ii)  $V_t(I_t)$  is concave in  $I_t$ ,  $V_t(I_t, R_t, Q_t)$  is concave in  $(I_t, R_t, Q_t)$ , and  $W_t(K_t, \mathbf{D}_t)$  is concave in  $(K_t, \mathbf{D}_t)$ .

*Proof.* Since functions  $H_t$ ,  $G_t$ ,  $V_t$  and  $W_t$  are defined recursively, we show their concavity together by induction on  $t$ . For  $t = 0$ , by the boundary condition (3.6), the conclusion hold trivially.

Suppose that the concavity holds for  $t - 1$ . We shall show it also holds for  $t$ . First consider  $G_t(K_t, \mathbf{x}_t)$ : The first term  $\mathbf{r}_t \cdot \mathbf{x}_t$  is the linear combination of  $x_t^j$  and hence concave in  $(K_t, \mathbf{x}_t)$  by (iv) of Lemma 1; the second term  $-h(K_t - \|\mathbf{x}_t\|)$  is the

composition of a concave function  $-h(\cdot)$  and an affine function  $(K_t - \|\mathbf{x}_t\|)$ , thus concave in  $(K_t, \mathbf{x}_t)$  by (ii) of Lemma 1; by (ii) of Lemma 1 and the hypothesis (ii), the last term is also concave in  $(J_t, \mathbf{x}_t)$ . Because summation preserves concavity (part (iv) of Lemma 1), we conclude that  $G_t(K_t, \mathbf{x}_t)$  is concave in  $(K_t, \mathbf{x}_t)$  and thus in  $(K_t, \mathbf{x}_t, \mathbf{D}_t)$ .

Upon taking maximization of  $G_t(K_t, \mathbf{x}_t)$ ,  $W_t(K_t, \mathbf{D}_t)$  is concave in  $(K_t, \mathbf{D}_t)$  by (iii) of Lemma 1.

Applying the similar argument, the concavity of  $H_t(J_t, \ell_t)$  and  $V_t(I_t, R_t, Q_t)$  is readily established. Finally, by (v) of Lemma 1,  $V_t(I_t)$  is concave. This completes the induction step and thus the proof.  $\square$

The concavity of the objective functions  $H_t$  and  $G_t$  implies that both the leasing and acceptance problems are well-behaved. Consequently the optimal constrained solutions  $\ell^*(J_t, Q_t)$  and  $\mathbf{x}_t^*(\mathbf{K}_t, \mathbf{D}_t)$  are well-defined and can be characterized through the first order condition approach. However, such approach results in state-dependent characterization and offers little insights into the interplay of policy behavior and the the system parameters. The state-dependence for acceptance decision is especially problematic for computation as  $(\mathbf{K}_t, \mathbf{D}_t)$  is multi-dimensional. We therefore seek to identify easily computable, state-independent policy parameters on which we anchor our characterization.

We first characterize the optimal leasing policy  $\ell_t^*(J_t, Q_t)$ . Let  $\partial_y f(x, y) \equiv f(x, y) - f(x, y - 1)$  denote the difference function of  $f$  relative to  $y$ . We define threshold  $C_t$  for leasing by

$$C_t \equiv \min \{K_t \in \mathbb{Z} : -c + \partial_{K_t} \mathbb{E}W_t(K_t, \mathbf{D}_t) \leq 0\}, \quad (4.1)$$

where  $C_t = -\infty$  if no such  $K_t$  exists. The concavity of  $W_t$  ensures that  $C_t$  is well-defined. Intuitively,  $C_t$  is the *base stock* level at which the marginal value of net inventory equals

the marginal leasing cost  $c$ . When  $C_t \leq 0$ , the location will be over-supplied based on the forecast of future supply  $\{R_\tau : \tau \leq t\}$  and demand  $\{\mathbf{D}_\tau : \tau \leq t\}$ . Therefore aggressive order acceptance and possible outbound repositioning are preferred.

Based on state-independent threshold  $C_t$ , the following theorem characterizes the optimal leasing policy.

**Theorem 3.** *In each period  $t$ , given post-arrival inventory  $J_t$  and railway capacity  $Q_t$ , the optimal leasing policy is*

$$\ell_t^*(J_t, Q_t) = (C_t - J_t)^+ \wedge Q_t, \quad (4.2)$$

where the threshold  $C_t$  is defined by (4.1).

*Proof.* For expositional convenience, we treat  $J_t$ ,  $Q_t$  and  $\ell_t$  as continuous variables; define  $C_t$  as the stationary point such that  $-c + \partial_{K_t} \mathbb{E}W_t(K_t, \mathbf{D}_t) = 0$ .

To obtain optimal leasing quantity  $\ell_t^*$ , we solve  $\max_{0 \leq \ell_t \leq Q_t} H_t(J_t, \ell_t)$ . Since all constraints are affine, linear constraints qualification holds. Also, since this is a concave math program, the Karush-Kuhn-Tucker (KKT) conditions are both necessary and sufficient for characterizing  $\ell_t^*$  as both the local and global optimizer.

$$\text{Lagrangian function: } \mathcal{L}(\ell_t, \lambda_1, \lambda_2) = H_t(J_t, \ell_t) + \lambda_1(-\ell_t + Q_t) + \lambda_2(\ell_t)$$

Its local maximizer  $(\ell_t^*, \lambda_1^*, \lambda_2^*)$  satisfies the following KKT conditions:

$$\text{Stationarity: } \frac{\partial \mathcal{L}(\ell_t^*, \lambda_1^*, \lambda_2^*)}{\partial \ell_t} = \frac{\partial H_t(J_t, \ell_t^*)}{\partial \ell_t} - \lambda_1^* + \lambda_2^* = 0;$$

$$\text{Primary feasibility: } -\ell_t^* + Q_t \leq 0, \ell_t^* \geq 0;$$

$$\text{Dual feasibility: } \lambda_1^* \geq 0, \lambda_2^* \geq 0;$$

$$\text{Complementary slackness: } \lambda_1^*(-\ell_t^* + Q_t) = 0, \lambda_2^*(\ell_t^*) = 0;$$

We now show (4.2) holds for the following four mutually exclusive and collectively exhaustive cases.

Case 1:  $\lambda_1^* = 0, \lambda_2^* = 0$ . This implies  $\partial H_t(J_t, \ell_t^*)/\partial \ell_t = 0$  by Stationarity. By the definition of  $C_t$ , we have

$$-c + \partial_{K_t} \mathbb{E}W_t(J_t + \ell_t^*, \mathbf{D}_t) = -c + \partial_{K_t} \mathbb{E}W_t(C_t, \mathbf{D}_t) = 0,$$

which shows by the concavity of  $W_t$  that  $J_t + \ell_t^* = C_t$ , i.e., (4.2) holds.

Case 2:  $\lambda_1^* = 0, \lambda_2^* > 0$ . Then the complementary slackness implies that  $\ell_t^* = 0$ .

Moreover,

$$0 > \partial H_t(J_t, \ell_t^*)/\partial \ell_t = -c + \partial_{K_t} W_t(J_t + 0, \mathbf{D}_t),$$

which implies that  $C_t < J_t$  and thus  $\ell_t^* = (C_t - J_t)^+ = 0$ , i.e., (4.2) holds.

Case 3:  $\lambda_1^* > 0, \lambda_2^* = 0$ . Then KKT implies that  $\ell_t^* = Q_t$  and  $\partial H_t(J_t, \ell_t^*)/\partial \ell_t >$

0. Thus,

$$0 < -c + \partial_{K_t} \mathbb{E}W_t(J_t + Q_t, \mathbf{D}_t),$$

which implies  $J_t + Q_t > C_t$ . Therefore,  $\ell_t^* = (C_t - J_t)^+ \wedge Q_t = Q_t$  so (4.2) holds.

Case 4:  $\lambda_1^* > 0, \lambda_2^* > 0$ . KKT implies  $\ell_t^* = 0$  and  $\ell_t^* = Q_t$ , which is infeasible.

This completes the proof. □

In each period  $t$  after observing  $(J_t, Q_t)$ , the manager makes no lease if  $J_t \geq C_t$ ; otherwise leases up to  $C_t$  or uses up railway capacity  $Q_t$ , whichever occurs first. Compared with traditional revenue management such as airline or hotel industries where capacity is fixed in advance, leasing provides additional lever to match supply with demand. The efficacy of leasing lever, however, is limited by the random availability from the railway. The capacity problem is especially acute in busy season when shortage is an industry wise phenomenon. This calls for careful advance planning as well as cooperation with local railway in order to secure sufficient container availability.

Since initial inventory is critical for the leasing decision, we characterize its impact in the following theorem.

**Theorem 4.** (i) For each  $t \in \mathcal{T}$ ,  $H_t(J_t, \ell_t)$  is submodular in  $(J_t, \ell_t)$ .

(ii) The optimal leasing quantity  $\ell_t^*(J_t, Q_t)$ , defined in (4.2), is decreasing in post-arrival inventory  $J_t$  for each capacity limit  $Q_t$ . Moreover,

$$\ell_t^*(J_t, Q_t) \geq \ell_t^*(J_t + 1, Q_t) \geq \ell_t^*(J_t, Q_t) - 1. \quad (4.3)$$

*Proof.* (i) Consider  $H_t(J_t, \ell_t)$ : the first term  $-c\ell_t$  is independent of  $J_t$  and thus submodular in  $(J_t, \ell_t)$ ; the second term is submodular in  $(J_t, \ell_t)$  by the concavity of  $W_t$ , parts (v) and (vii) of Lemma 1; and thus  $H_t$  is submodular in  $(J_t, \ell_t)$  by part (iv) of Lemma 1.

(ii) From the submodularity of  $H_t$  in  $(J_t, \ell_t)$  and (a variant of) part (vi) of Lemma 1, it immediately follows that  $\ell_t^*(J_t, \ell_t) \geq \ell_t^*(J_t + 1, \ell_t)$ . This proves the first part of (4.3).

Next, we show the second part of (4.3) holds, i.e.,  $\ell_t^*(J_t + 1, Q_t) \geq \ell_t^*(J_t, Q_t) - 1$ . First, observe that  $a^+ \geq (a - 1)^+ \geq a^+ - 1$ , for  $a \in \mathbb{Z}$ . Depending on  $Q$ , we have two mutually exclusive and collectively exhaustive cases.

If  $Q \geq a^+$ , we have

$$Q \wedge a^+ = a^+ \geq Q \wedge (a - 1)^+ = (a - 1)^+ \geq Q \wedge (a^+ - 1) = a^+ - 1 \geq Q \wedge a^+ - 1.$$

If  $Q \leq a^+ - 1$ , we have

$$Q = Q \wedge a^+ = Q \wedge (a - 1)^+ > Q \wedge a^+ - 1 = Q - 1.$$

Therefore, for each  $Q \in \mathbb{Z}$ , we have

$$Q \wedge a^+ \geq Q \wedge (a - 1)^+ \geq Q \wedge a^+ - 1. \quad (4.4)$$



Now by letting  $Q = Q_t$  and  $a = C_t - J_t$  in (4.4), and by the definition of  $\ell_t^*(J_t, Q_t)$  in (4.2), we have

$$\begin{aligned}\ell_t^*(J_t, Q_t) &= Q_t \wedge (C_t - J_t)^+ \\ &\geq \ell_t^*(J_t + 1, Q_t) = Q_t \wedge (C_t - J_t - 1)^+ \\ &\geq \ell_t^*(J_t, Q_t) - 1 = Q_t \wedge (C_t - J_t)^+ - 1,\end{aligned}$$

which establishes (4.3). □

Theorem 4 states that the marginal value of leasing increases as internal container supply decreases. This again underscores the importance of securing external supply in time of inventory scarcity. Moreover, the leasing level is a contraction function of inventory: a unit increase in inventory reduces leasing level by at most one unit.

We now characterize the optimal load acceptance decision  $\mathbf{x}_t^*(K_t, \mathbf{D}_t)$  by solving the second stage problem  $W_t(K_t, \mathbf{D}_t) = \max_{0 \leq \mathbf{x}_t \leq \mathbf{D}_t} G_t(K_t, \mathbf{x}_t)$  as defined in (3.3) and (3.4). The primary challenge is the multi-dimensionality problem, especially for computation. The main idea to overcome this difficulty is to identify possible state-invariant policy parameters for characterization. We implement this idea in two steps. The first step specifies the sequence of load acceptance over classes, while the second step determines actual acceptance quantity for each class. Let  $\mathbf{e}_i$  denote the  $i$ th unit vector with 1 for the  $i$ th coordinate and 0 otherwise.

**Theorem 5.** *For each  $t \in \mathcal{T}$ ,  $G_t(K_t, \mathbf{x}_t + \mathbf{e}_i - \mathbf{e}_j) > G_t(K_t, \mathbf{x}_t)$ , for  $i < j$ ,  $i, j \in \mathcal{J}$ .*

*Proof.* The change from  $(K_t, \mathbf{x}_t)$  to  $G(K_t, \mathbf{x}_t + \mathbf{e}_i - \mathbf{e}_j)$  represents saving one more request from class  $j$  to fulfill one from a more profitable class  $i < j$ . Since the total acceptance

quantity  $\|\mathbf{x}_t\|$  remains unchanged, we have  $G_t(K_t, \mathbf{x}_t + \mathbf{e}_i - \mathbf{e}_j) - G_t(K_t, \mathbf{x}_t) = r^i - r^j > 0$ , and thus  $G_t(K_t, \mathbf{x}_t + \mathbf{e}_i - \mathbf{e}_j) > G_t(K_t, \mathbf{x}_t)$ .  $\square$

As the first step in characterizing  $\mathbf{x}_t^*(K_t, \mathbf{D}_t)$ , Theorem 5 establishes the priority sequence of load acceptance over classes. It states that higher (profit) classes should always have priority over lower classes in accessing containers. Because the acceptance  $\mathbf{x}_t$  is bounded by realized demand  $\mathbf{D}_t$ , the priority sequence and a simple exchange argument implies the following *priority structure*

$$\mathbf{x}_t^*(K_t, \mathbf{D}_t) = (D_t^1, D_t^2, \dots, D_t^{j-1}, x_t^{j*}, 0, 0, \dots, 0), \quad (4.5)$$

where  $j$  is the marginal (last) class with positive share,  $x_t^{j*} > 0$ . Consequently, as long as total acceptance  $\|\mathbf{x}_t^*\|$  is determined, individual acceptance  $x_t^{j*}$  can be readily derived from structure (4.5). Moreover, by the linear system dynamics  $I_{t-1}^* = K_t - \|\mathbf{x}_t^*\|$ , deciding total acceptance  $\|\mathbf{x}_t^*\|$  is equivalent to decide ending inventory  $I_{t-1}^*$ . For each class  $i \in \mathcal{J}$ , define

$$s_t^i \equiv \min \{ I_{t-1} \in \mathbb{Z} : -\partial h(I_{t-1}) + \partial V_{t-1}(I_{t-1}) \leq r^i \}, \quad (4.6)$$

where  $s_t^i = -\infty$  if no such  $I_{t-1}$  exists. Intuitively,  $s_t^i$  is the *rationing level* for class  $i$  at which marginal value of reserving the inventory for the future equals selling it to class  $i$ . Therefore,  $s_t^i$  is the target initial inventory level for class  $i$  in the next period. Based on state-invariant parameter  $s_t^i$  and the priority structure in (4.5), we completely characterize the optimal load acceptance policy as follows. For vector  $(D_t^1, \dots, D_t^J)$ , let  $\bar{D}_t^i \equiv \sum_{j=1}^i D_t^j$  denote its cumulative sum for  $i \leq J$ .

**Theorem 6.** (i) *The vector of rationing levels  $\mathbf{s}_t = (s_t^1, \dots, s_t^J)$  is state independent and increasing in  $j \in \mathcal{J}$ , i.e.,*

$$s_t^1 \leq s_t^2 \leq \dots \leq s_t^J. \quad (4.7)$$

(ii) For each period  $t \in \mathcal{T}$  and class  $i \in \mathcal{J}$ , the optimal acceptance decision, given post-leasing inventory  $K_t$  and demand  $\mathbf{D}_t$ , is

$$x_t^{i*}(K_t, \mathbf{D}_t) = D_t^i \wedge (K_t - \bar{D}_t^{i-1} - s_t^i)^+. \quad (4.8)$$

*Proof.* (i) From  $s_t^i$ 's definition in (4.6) it is immediate that  $s_t^i$  is independent of state  $(K_t, \mathbf{D}_t)$ . Moreover, since  $-\partial h(I) + \partial V_{t-1}(I)$  is decreasing in  $I$  and  $r^i$  is decreasing in  $i$ , the monotonicity of (4.7) follows immediately from (4.6).

(ii) As discussed earlier, to prove (4.8), we first determine the optimal total acceptance  $\|\mathbf{x}_t^*\|$ , then using (4.5) to derive individual acceptance  $x_t^{j*}$ .

Given  $(K_t, \mathbf{D}_t)$ , the priority structure (4.5) yields class  $i$  as the last class with positive acceptance. From the linear system dynamics  $I_{t-1}^* = K_t - \|\mathbf{x}_t^*\|$ , deciding total acceptance  $\|\mathbf{x}_t^*\|$  is equivalent to decide ending inventory  $I_{t-1}^*$ . Thus, given  $(K_t, \mathbf{D}_t)$  and the last acceptance class  $i$ , the objective function  $G_t(K_t, \mathbf{x}_t)$ , after changing the decision variable from  $\mathbf{x}_t$  to  $I_{t-1}$ , yields

$$G_t^i(K_t, \mathbf{D}_t, I_{t-1}) = \sum_{j=1}^{i-1} r^j D_t^j + r^i (I_{t-1} - \bar{D}_t^{i-1}) - h(I_{t-1}) + V_{t-1}(I_{t-1}),$$

where the first two terms are the total revenue under (4.5). Clearly  $G_t^i$  is concave in  $I_{t-1}$  and thus FOC gives interior optimal solution  $I_{t-1}^* = K_t - \|\mathbf{x}_t^*\| = s_t^i$ , where rationing level  $s_t^i$  is defined in (4.6). Therefore,  $\|\mathbf{x}_t^*\| = K_t - s_t^i$  when the stopping class is  $i$ , and thus by (4.5) and the monotonicity in part (i)

$$\mathbf{x}_t^* = (D_t^1, D_t^2, \dots, D_t^{i-1}, K_t - s_t^i - \bar{D}_t^{i-1}, 0, \dots, 0),$$

which proves (4.8) in part (ii) when imposing affine constraints  $0 \leq \mathbf{x}_t^* \leq \mathbf{D}_t$ . □

Theorem 7 makes three contributions. First, they provide the local manager with easy-to-implement optimal acceptance policy: he should always accept from class 1

onwards; for each class  $i$ , accept until either all loads  $D_t^i$  is admitted the rationing level  $s_t^i$  is reached, whichever occurs first; once a rationing level is reached, all rest classes are rejected.

Second, the monotonicity of rationing levels in (4.7) and simplicity of allocation in (4.8) significantly simplify the computation. Instead of conventional MDP value- or policy-iteration methods, which are often crippled by the *curse of dimensionality*, our state-independent characterization facilitates the development of efficient polynomial-time algorithms.

Third, the simple structure in (4.8) reveals, in an explicit expression, the dependence of the load acceptance decision on demand, supply and the rationing levels. The explicit expression enables the detailed sensitivity analysis of these factors in the following theorem.

**Theorem 7.** (i) For each  $t \in \mathcal{T}$ ,  $G_t(K_t, \mathbf{x}_t)$  is supermodular in  $(K_t, x_t^j)$ ,  $\forall j \in \mathcal{J}$ .

(ii) The optimal acceptance  $x_t^{j*}(K_t, \mathbf{D}_t)$  is increasing in  $K_t$  for each  $\mathbf{D}_t$ . Moreover,

$$x_t^{j*}(K_t, \mathbf{D}_t) \leq x_t^{j*}(K_t + 1, \mathbf{D}_t) \leq x_t^{j*}(K_t, \mathbf{D}_t) + 1, \quad j \in \mathcal{J}. \quad (4.9)$$

$$\|\mathbf{x}_t^*(K_t, \mathbf{D}_t)\| \leq \|\mathbf{x}_t^*(K_t + 1, \mathbf{D}_t)\| \leq \|\mathbf{x}_t^*(K_t, \mathbf{D}_t)\| + 1. \quad (4.10)$$

*Proof.* (i) It is equivalent to show  $\partial_{x_t^j} G_t(K_t + 1, \mathbf{x}_t) \geq \partial_{x_t^j} G_t(K_t, \mathbf{x}_t)$ , and we have

$$\begin{aligned} \partial_{x_t^j} G_t(K_t + 1, \mathbf{x}_t) &= G_t(K_t + 1, \mathbf{x}_t) - G_t(K_t + 1, \mathbf{x}_t - \mathbf{e}^j) \\ &= r^j + \partial_{x_t^j} h(K_t + 1 - \|\mathbf{x}_t\|) - \partial_{x_t^j} V_{t-1}(K_t + 1 - \|\mathbf{x}_t\|) \\ &\geq r^j + \partial_{x_t^j} h(K_t - \|\mathbf{x}_t\|) - \partial_{x_t^j} V_{t-1}(K_t - \|\mathbf{x}_t\|) \\ &= G_t(K_t, \mathbf{x}_t) - G_t(K_t, \mathbf{x}_t - \mathbf{e}^j) = \partial_{x_t^j} G_t(K_t, \mathbf{x}_t), \end{aligned}$$

where the inequality follows from the concavity of  $-h(\cdot)$  and  $V_{t-1}(\cdot)$ .

(ii) From the supermodularity of  $G_t$  in  $(K_t, x_t^j)$  and (a variant of) part (vi) of Lemma 1, it immediately follows that  $x_t^{j*}(K_t + 1, \mathbf{D}_t) \geq x_t^{j*}(K_t, \mathbf{D}_t)$ . This proves the first inequality of (4.9).

Now we show the second inequality of (4.9) holds, i.e.,  $x_t^{j*}(K_t + 1, \mathbf{D}_t) \leq x_t^{j*}(K_t, \mathbf{D}_t) + 1$ . First, observe that  $a^+ \leq (a + 1)^+ \leq a^+ + 1$ , for  $a \in \mathbb{Z}$ . Depending on  $D$ , we have two mutually exclusive and collectively exhaustive cases.

If  $D \geq a^+ + 1$ , we have

$$D \wedge (a^+ + 1) = (a^+ + 1) \geq D \wedge (a + 1)^+ = (a + 1)^+ \geq D \wedge a^+ = a^+.$$

If  $D \leq a^+$ , we have

$$D = D \wedge a^+ = D \wedge (a + 1)^+ \leq D \wedge a^+ + 1 = D + 1.$$

Therefore, for  $D \in \mathbb{Z}$ , we have

$$D \wedge a^+ \leq D \wedge (a + 1)^+ \leq D \wedge a^+ + 1.$$

Now by letting  $D = D_t^j$  and  $a = K_t - \bar{D}_t^{i-1} - s_t^j$ , and by the definition of  $x_t^{j*}(K_t, \mathbf{D}_t)$ , we have

$$\begin{aligned} x_t^{j*}(K_t, \mathbf{D}_t) &= D_t^j \wedge (K_t - \bar{D}_t^{i-1} - s_t^j)^+ \\ &\leq x_t^{j*}(K_t + 1, \mathbf{D}_t) = D_t^j \wedge (K_t + 1 - \bar{D}_t^{i-1} - s_t^j)^+ \\ &\leq x_t^{j*}(K_t, \mathbf{D}_t) + 1 = D_t^j \wedge (K_t - \bar{D}_t^{i-1} - s_t^j)^+ + 1, \end{aligned}$$

which establishes (4.9).

Since  $x_t^{j*}(K_t, \mathbf{D}_t) \leq x_t^{j*}(K_t + 1, \mathbf{D}_t)$  for each  $j \in \mathcal{J}$ , we have

$$\|\mathbf{x}_t^*(K_t, \mathbf{D}_t)\| = \sum_j x_t^{j*}(K_t, \mathbf{D}_t) \leq \sum_j x_t^{j*}(K_t + 1, \mathbf{D}_t) = \|\mathbf{x}_t^*(K_t + 1, \mathbf{D}_t)\|,$$

which proves the first part of (4.10).

Now we show the second part of (4.10). For expositional simplicity, assume  $D_t^j > 0$  for each  $j \in \mathcal{J}$ . Given  $(K_t, \mathbf{D}_t)$ , let  $i$  be the last class with positive acceptance, i.e,  $x_t^{i*} > 0$  and  $K_t - \bar{D}_t^i - s_t^{i+1} \leq 0$ . We have two cases:

Case 1:  $x_t^{i*}(K_t, \mathbf{D}_t) = D_t^i$ ,  $D_t^i \leq K_t - \bar{D}_t^{i-1} - s_t^i$ . This is the case where the last class  $i$  is bounded by the demand  $D_t^i$  and the remaining inventory does not warrant any acceptance from class  $i + 1$  onwards. By priority structure (4.5), Theorem 7 and inequalities (4.9), we have

$$\begin{aligned}
& \| \mathbf{x}_t^*(K_t + 1, \mathbf{D}_t) \| - \| \mathbf{x}_t^*(K_t, \mathbf{D}_t) \| \\
&= \left( \bar{D}_t^{i-1} + D_t^i + x_t^{(i+1)*}(K_t + 1, \mathbf{D}_t) \right) - \left( \bar{D}_t^{i-1} + D_t^i + 0 \right) \\
&= x_t^{(i+1)*}(K_t + 1, \mathbf{D}_t) - x_t^{(i+1)*}(K_t, \mathbf{D}_t) \\
&\leq 1,
\end{aligned}$$

where  $x_t^{(i+1)*}(K_t, \mathbf{D}_t) = 0$ .

Case 2:  $x_t^{i*}(K_t, \mathbf{D}_t) = K_t - \bar{D}_t^{i-1} - s_t^i$ ,  $D_t^i > K_t - \bar{D}_t^{i-1} - s_t^i$ . This is the case where the last class  $i$  is bounded by the supply and the solution is interior. By priority structure (4.5) and Theorem 7, we have

$$\begin{aligned}
& \| \mathbf{x}_t^*(K_t + 1, \mathbf{D}_t) \| - \| \mathbf{x}_t^*(K_t, \mathbf{D}_t) \| \\
&= \left( \bar{D}_t^{i-1} + x_t^{i*}(K_t + 1, \mathbf{D}_t) \right) \left( \bar{D}_t^{i-1} + x_t^{i*}(K_t, \mathbf{D}_t) \right) \\
&= \left( K_t + 1 - \bar{D}_t^{i-1} - s_t^i \right) - \left( K_t - \bar{D}_t^{i-1} - s_t^i \right) \\
&= 1,
\end{aligned}$$

where establishes the second part of (4.10). □

Part (i) states that higher inventory encourages aggressive load acceptance for

all classes. Part (ii) further specifies the magnitude of such changes for both individual and aggregate load acceptance.

## Chapter 5

# The Impact of Demand

## Variability

In this section, we study the impact of demand variability on optimal policies. First, we need some concepts in stochastic comparison theory. We say random variable  $X$  is smaller than random variable  $\hat{X}$  in *stochastic order*, written as  $X_s \leq_{st} \hat{X}_s$ , if  $\mathbb{E}f(X) \leq \mathbb{E}f(\hat{X})$  for any increasing function  $f$ . We say random variable  $X$  is smaller than random variable  $\hat{X}$  in *stochastic increasing concave order*, written as  $X_s \leq_{icv} \hat{X}_s$ , if  $\mathbb{E}f(X) \leq \mathbb{E}f(\hat{X})$  for any increasing concave function  $f$ . We denote the systems with demand streams  $\{\mathbf{D}_t\}$  and  $\{\hat{\mathbf{D}}_t\}$  by System- $\mathbf{D}$  and System- $\hat{\mathbf{D}}$ ; denote the variables in each system correspondingly.

Let  $V_{t-1}(I_{t-1}) \equiv \mathbb{E}_{R_{t-1}, Q_{t-1}}[V_{t-1}(I_{t-1}, R_{t-1}, Q_{t-1})]$  and  $W_t(K_t) \equiv \mathbb{E}_{\mathbf{D}_t}[W_t(K_t, \mathbf{D}_t)]$ .

The following theorem characterizes the impact of demand variability.

**Theorem 8.** *Assume  $\mathbf{D}_t \leq_{st} \hat{\mathbf{D}}_t$  for each  $t \in \mathcal{T}$ .*

(i)  $W_t(K_t, \mathbf{D}_t)$  is supermodular in  $(K_t, \mathbf{D}_t)$ .

(ii)  $W_t(K_t) \leq \hat{W}_t(K_t)$ .



(iii)

$$\partial_{I_t} V_t(I_t, R_t, Q_t) \leq \partial_{I_t} \hat{V}_t(I_t, R_t, Q_t), \quad (5.1)$$

$$\partial_{K_t} W_t(K_t) \leq \partial_{K_t} \hat{W}_t(K_t), \quad (5.2)$$

$$\partial_{I_{t-1}} V_{t-1}(I_{t-1}) \leq \partial_{I_{t-1}} \hat{V}_{t-1}(I_{t-1}). \quad (5.3)$$

(iv)

$$C_t \leq \hat{C}_t, \quad (5.4)$$

$$\ell_t^* \leq \hat{\ell}_t^*, \quad (5.5)$$

$$s_t^i \leq \hat{s}_t^i, \quad i \in \mathcal{J}, \quad (5.6)$$

$$x_t^{i*} \geq \hat{x}_t^{i*}, \quad i \in \mathcal{J}. \quad (5.7)$$

*Proof.* (i) It suffices to show  $W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t + 1, \mathbf{D}_t) \geq W_t(K_t, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t, \mathbf{D}_t)$ .

Let  $\mathbf{x}^*$  be the optimal solution to state  $(K_t, \mathbf{D}_t)$  in period  $t$ ; let  $x^* = \|\mathbf{x}^*\|$  be the total acceptance. We prove for the following three mutually exclusive cases.

For expositional brevity, we assume  $D_t^j > 0$  for all  $j \in \mathcal{J}$ . For state  $(K_t, \mathbf{D}_t)$  let  $i$  be the last class with positive acceptance, i.e.,  $x_t^{i*}(K_t, \mathbf{D}_t) = D_t^i \wedge (K_t - \bar{D}_t^{i-1} - s_t^i) > 0$ , and  $K_t - \bar{D}_t^i \leq s_t^{i+1}$ . Depending on  $D_t^i$ , we have two cases.

Case 1:  $0 < K_t - \bar{D}_t^{i-1} - s_t^i < D_t^i$ . In this case,  $x_{0,0} = K_t - s_t^i$  is interior.

If  $j \leq i$ , then extra order  $\mathbf{e}^j$  is no less valuable than  $\mathbf{e}^i$ . By concavity of  $G_t$ , the extra  $\mathbf{e}^j$  should be filled before  $\mathbf{e}^i$ .

In state  $(K_t + 1, \mathbf{D}_t + \mathbf{e}^j)$ , the extra inventory is used to fill the extra order  $\mathbf{e}^j$  so that  $x_{1,1} = x_{0,0} + 1$ , and  $W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) = r^j + W_t(K_t, \mathbf{D}_t)$ .

In state  $(K_t + 1, \mathbf{D}_t)$ , the extra inventory is used to fill addition  $\mathbf{e}^i$  so that  $x_{1,0} = x_{0,0} + 1$  and  $W_t(K_t + 1, \mathbf{D}_t) = r^i + W_t(K_t, \mathbf{D}_t)$ .

In state  $(K_t, \mathbf{D}_t + \mathbf{e}^j)$ , we accept one more  $\mathbf{e}^j$  order and one less  $\mathbf{e}^i$ , so that  $x_{0,1} = x_{0,0}$  and  $W_t(K_t + 1, \mathbf{D}_t) = r^j - r^i + W_t(K_t, \mathbf{D}_t)$ .

Therefore, we have

$$\begin{aligned} & W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t + 1, \mathbf{D}_t) \\ &= r^j - r^i \\ &= W_t(K_t, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t, \mathbf{D}_t), \end{aligned}$$

and thus part (i) holds.

If  $j > i$ , then order  $\mathbf{e}^j$  is less valuable than order  $\mathbf{e}^i$ . By Theorem 7, it should not be accepted.

In states  $(K_t + 1, \mathbf{D}_t + \mathbf{e}^j)$  and  $(K_t + 1, \mathbf{D}_t)$ , all  $j$  orders are rejected and one more class  $i$  order is accepted, thus  $x_{1,1} = x_{1,0} = x_{0,0}$  and two corresponding value functions are equal. In states  $(K_t, \mathbf{D}_t + \mathbf{e}^j)$  and  $(K_t, \mathbf{D}_t)$ , all class  $j$  orders are rejected, thus  $x_{0,1} = x_{0,0}$  and two value value functions are equal. Therefore, we have

$$W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t + 1, \mathbf{D}_t) = W_t(K_t, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t, \mathbf{D}_t) = 0,$$

and thus part (i) holds with equality.

Case 2:  $D_t^i < K_t - \bar{D}_t^{i-1} - s_t^i$ . In this case,  $\mathbf{x}_{0,0} = \sum_{j=1}^i D_t^j \mathbf{e}^j$  is a boundary solution.

If  $j \leq i$ , order  $\mathbf{e}^j$  is no less valuable than  $\mathbf{e}^i$  and thus has priority over  $\mathbf{e}^i$ . By (4.8), we have  $\mathbf{x}_t^*(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) = \mathbf{x}^* + \mathbf{e}^j$ ,  $\mathbf{x}_t^*(K_t + 1, \mathbf{D}_t) = \mathbf{x}^*$  if  $K_t - \bar{D}_t^i < s_t^{i+1}$  and

$\mathbf{x}^* + \mathbf{e}^{i+1}$  otherwise, and  $\mathbf{x}_t^*(K_t, \mathbf{D}_t + \mathbf{e}^j) = \mathbf{x}^* + \mathbf{e}^j$ . Thus,

$$\begin{aligned} & W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t + 1, \mathbf{D}_t) \\ &= \begin{cases} r^j - [-\partial h(K_t + 1 - x^*) + \partial V_{t-1}(K_t + 1 - x^*)], & \text{if } K_t - \bar{D}_t^i < s_t^{i+1}; \\ r^j - r^{i+1}, & \text{otherwise;} \end{cases} \end{aligned}$$

by optimality equation

$$\geq r^j - [-\partial h(K_t - x^*) + \partial V_{t-1}(K_t - x^*)], \quad \text{by convexity of } -h \text{ and } V_{t-1} \text{ and (4.6).}$$

$$= W_t(K_t, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t, \mathbf{D}_t). \quad \text{by optimality equation.}$$

If  $j > i$ , the order  $\mathbf{e}^j$  is less valuable than  $\mathbf{e}^i$  and should be rejected. By (4.8), we have  $\mathbf{x}_t^*(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) = \mathbf{x}_t^*(K_t + 1, \mathbf{D}_t) = \mathbf{x}^*$  if  $K_t - \bar{D}_t^i < s_t^{i+1}$  and  $\mathbf{x}^* + \mathbf{e}^{i+1}$  otherwise, and  $\mathbf{x}_t^*(K_t, \mathbf{D}_t + \mathbf{e}^j) = \mathbf{x}^*$ . Thus,

$$\begin{aligned} & W_t(K_t + 1, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t + 1, \mathbf{D}_t) \\ &= 0 \quad \text{by optimality equation.} \\ &= W_t(K_t, \mathbf{D}_t + \mathbf{e}^j) - W_t(K_t, \mathbf{D}_t). \end{aligned}$$

(ii) Our proof is by induction on  $t$ . The case  $t = 0$  is trivially true and assume the case  $t - 1$  is also true,  $W_{t-1}(\cdot) \leq \hat{W}_{t-1}(\cdot)$  and  $V_{t-1}(\cdot) \leq \hat{V}_{t-1}(\cdot)$ . Define  $i(x_t) = \min\{i \in J : \bar{D}_t^i \geq x_t\}$ . Let  $x_t$  and  $\hat{x}_t$  be the optimal acceptance in state  $(K_t, \mathbf{D}_t)$  in System- $\mathbf{D}$  and System- $\hat{\mathbf{D}}$ , then we have

$$\begin{aligned} W_t(K_t, \mathbf{D}_t) &= r^{i(x_t)} - h(K_t - x_t) + V_{t-1}(K_t - x_t) \\ &\leq r^{i(x_t)} - h(K_t - x_t) + \hat{V}_{t-1}(K_t - x_t) \\ &\leq r^{i(\hat{x}_t)} - h(K_t - \hat{x}_t) + \hat{V}_{t-1}(K_t - \hat{x}_t) = \hat{W}_t(K_t, \mathbf{D}_t), \end{aligned}$$

where the first inequality holds for the induction hypothesis and the second

holds for the optimal solution of  $\hat{x}_t$  in System- $\hat{\mathbf{D}}$ . So we have shown that  $W_t(K_t, \mathbf{D}_t) \leq \hat{W}_t(K_t, \hat{\mathbf{D}}_t)$  for any realization  $\mathbf{D}_t$  and  $\hat{\mathbf{D}}_t$ .

Upon take expectations, we have  $\mathbb{E}_{\mathbf{D}_t}[W_t(K_t, \mathbf{D}_t)] \leq \mathbb{E}_{\mathbf{D}_t}[\hat{W}_t(K_t, \hat{\mathbf{D}}_t)]$ . It is easy to see that  $W_t(K_t, \mathbf{D}_t)$  is an increasing function of confirmed orders  $\mathbf{D}_t$ , so  $\mathbb{E}_{\mathbf{D}_t}[W_t(K_t, \mathbf{D}_t)] \leq \mathbb{E}_{\mathbf{D}_t}[\hat{W}_t(K_t, \hat{\mathbf{D}}_t)]$ . Therefore, we have

$$W_t(K_t) = \mathbb{E}_{\mathbf{D}_t}[W_t(K_t, \mathbf{D}_t)] \leq \mathbb{E}_{\mathbf{D}_t}[\hat{W}_t(K_t, \hat{\mathbf{D}}_t)] \leq \mathbb{E}_{\mathbf{D}_t}[\hat{W}_t(K_t, \hat{\mathbf{D}}_t)] = \hat{W}_t(K_t).$$

(iii) and (iv) The two parts are proved together by induction on  $t$ . The statement holds trivially true for  $t = 0$  and we hypothesize that  $\partial_{I_{t-2}}V_{t-2}(I_{t-2}, R_{t-2}, Q_{t-2}) \leq \partial_{I_{t-2}}\hat{V}_{t-2}(I_{t-2}, R_{t-2}, Q_{t-2})$ ,  $\partial_{K_{t-2}}W_{t-2}(K_{t-2}) \leq \partial_{K_{t-2}}\hat{W}_{t-2}(K_{t-2})$ ,  $\partial_{I_{t-1}}V_{t-2}(I_{t-1}) \leq \partial_{I_{t-1}}\hat{V}_{t-2}(I_{t-1})$ ,  $C_{t-1} \leq \hat{C}_{t-1}$ ,  $\ell_{t-1}^* \leq \hat{\ell}_{t-1}^*$ ,  $s_{t-1}^i \leq \hat{s}_{t-1}^i$  and  $x_{t-1}^{i*} \geq \hat{x}_{t-1}^{i*}$ .

Since  $\mathbf{D}_{t-1} \leq_{st} \hat{\mathbf{D}}_{t-1}$ , we may assume  $\mathbf{D}_{t-1} \leq \hat{\mathbf{D}}_{t-1}$  with probability 1. This implies that for any realization  $\mathbf{D}_{t-1}$  and  $\hat{\mathbf{D}}_{t-1}$ , we have  $\mathbf{D}_{t-1} \leq \hat{\mathbf{D}}_{t-1}$ . Therefore, we need to prove  $\partial_{K_{t-1}}W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}}\hat{W}_{t-1}(K_{t-1}, \hat{\mathbf{D}}_{t-1})$ . From (i), we know  $\hat{W}_{t-1}(K_{t-1}, \hat{\mathbf{D}}_{t-1})$  is supermodular in  $(K_{t-1}, \hat{\mathbf{D}}_{t-1})$ , so  $\partial_{K_{t-1}}\hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}}\hat{W}_{t-1}(K_{t-1}, \hat{\mathbf{D}}_{t-1})$ , thus we only need to prove  $\partial_{K_{t-1}}W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}}\hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1})$ .

Let  $x_{t-1}$  and  $x'_{t-1}$  be the optimal acceptance level in state  $(K_{t-1}, \mathbf{D}_{t-1})$  and  $(K_{t-1} - 1, \mathbf{D}_{t-1})$  in system- $\mathbf{D}$ ; similarly  $\hat{x}_{t-1}$  and  $\hat{x}'_{t-1}$  be the optimal acceptance level in state  $(K_{t-1}, \mathbf{D}_{t-1})$  and  $(K_{t-1} - 1, \mathbf{D}_{t-1})$  in system- $\hat{\mathbf{D}}$ . We must have either  $x'_{t-1} = x_{t-1}$  or  $x'_{t-1} = x_{t-1} - 1$ . From the hypothesis,  $x_{t-1} \geq \hat{x}_{t-1}$  and  $x'_{t-1} \geq \hat{x}'_{t-1}$ . We prove for the following four mutually exclusively cases.

**Case 1.**  $x'_{t-1} = x_{t-1} - 1$  and  $\hat{x}'_{t-1} = \hat{x}_{t-1} - 1$

$$\partial_{K_{t-1}}W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) - W_{t-1}(K_{t-1} - 1, \mathbf{D}_{t-1}) = r^{i(x_{t-1})}$$

$$\partial_{K_{t-1}}\hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) - \hat{W}_{t-1}(K_{t-1} - 1, \mathbf{D}_{t-1}) = r^{i(\hat{x}_{t-1})}$$

Since  $x_{t-1} \geq \hat{x}_{t-1}$  and  $r^{i(\cdot)}$  is decreasing, we have

$$\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}).$$

**Case 2.**  $x'_{t-1} = x_{t-1} - 1$  and  $\hat{x}'_{t-1} = \hat{x}_{t-1}$

We take suboptimal solution  $\hat{x}_{t-1} + 1$  in state  $(K_{t-1} - 1, \mathbf{D}_{t-1})$  in System- $\hat{\mathbf{D}}$ , then we have

$$\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = r^{i(x_{t-1})}$$

$$\partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \geq r^{i(\hat{x}_{t-1}+1)}$$

Since  $x_{t-1} = x'_{t-1} \geq \hat{x}'_{t-1} + 1 = \hat{x}'_{t-1} + 1$  and  $r^{i(\cdot)}$  is decreasing, thus  $\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1})$ .

**Case 3.**  $x'_{t-1} = x_{t-1}$  and  $\hat{x}'_t = \hat{x}_{t-1} - 1$

We take suboptimal solution  $\hat{x}_{t-1} - 1$  in state  $(K_{t-1} - 1, \mathbf{D}_{t-1})$  in System- $\mathbf{D}$ , then we have

$$\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq r^{i(x_{t-1})}$$

$$\partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = r^{i(\hat{x}_{t-1})}$$

Since  $x_{t-1} \geq \hat{x}_{t-1}$  and  $r^{i(\cdot)}$  is decreasing, we have

$$\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}).$$

**Case 4.**  $x'_{t-1} = x_{t-1}$  and  $\hat{x}'_{t-1} = \hat{x}_{t-1}$

$$\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = -\partial h(K_{t-1} - x_{t-1}) + \partial V_{t-2}(K_{t-1} - x_{t-1})$$

$$\partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) = -\partial h(K_{t-1} - \hat{x}_{t-1}) + \partial \hat{V}_{t-2}(K_{t-1} - \hat{x}_{t-1})$$

Since both  $-\partial h(\cdot)$  and  $\partial V_{t-2}(\cdot)$  are decreasing and  $\partial V_{t-2}(\cdot) \leq \partial \hat{V}_{t-2}(\cdot)$  by hypothesis, then we have  $\partial_{K_{t-1}} W_{t-1}(K_{t-1}, \mathbf{D}_{t-1}) \leq \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}, \mathbf{D}_{t-1})$ .

Now we prove the first inequality based on the above result. Let  $\ell_{t-1}^*$  and  $\ell_{t-1}^{*'}$  be the optimal leasing quantity in state  $(K_{t-1}, \mathbf{D}_{t-1})$  and  $(K_{t-1} - 1, \mathbf{D}_{t-1})$  in system- $\mathbf{D}$ ; similarly  $\hat{\ell}_{t-1}^*$  and  $\hat{\ell}_{t-1}^{*'}$  be the optimal leasing quantity in state  $(K_{t-1}, \mathbf{D}_{t-1})$  and

$(K_{t-1} - 1, \mathbf{D}_{t-1})$  in system- $\hat{\mathbf{D}}$ . From the hypothesis,  $\ell_{t-1} \geq \hat{\ell}_{t-1}^*$  and  $\ell_{t-1}^{*'} \geq \hat{\ell}_{t-1}^{*'}$ . We need to prove  $\partial_{J_{t-1}} H_{t-1}(J_{t-1}, \ell_{t-1}) \leq \partial_{J_{t-1}} \hat{H}_{t-1}(J_{t-1}, \ell_{t-1})$ .

$$\partial_{J_{t-1}} H_{t-1}(J_{t-1}, \ell_{t-1}) = -c(\ell_{t-1}^* - \ell_{t-1}^{*'}) + \partial_{K_{t-1}} W_{t-1}(K_{t-1}),$$

$$\partial_{J_{t-1}} \hat{H}_{t-1}(J_{t-1}, \ell_{t-1}) = -c(\hat{\ell}_{t-1}^* - \hat{\ell}_{t-1}^{*'}) + \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}),$$

We have proved  $\partial_{K_{t-1}} W_{t-1}(K_{t-1}) \leq \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1})$  and we also have either  $\ell_{t-1}^{*'} = \ell_{t-1}^*$  or  $\ell_{t-1}^{*'} = \ell_{t-1}^* + 1$ . It is easily established for the other three combination except when  $\ell_{t-1}^{*'} = \ell_{t-1}^* + 1$  and  $\hat{\ell}_{t-1}^{*' } = \hat{\ell}_{t-1}^*$ . We take suboptimal policy  $\hat{\ell}_{t-1}^{*' } = \hat{\ell}_{t-1}^* + 1$  in System- $\hat{\mathbf{D}}$ , then

$$\partial_{J_{t-1}} \hat{H}_{t-1}(J_{t-1}, \ell_{t-1}) \geq c + \partial_{K_{t-1}} \hat{W}_{t-1}(K_{t-1}) \geq c + \partial_{K_{t-1}} W_{t-1}(K_{t-1}) = \partial_{J_{t-1}} H_{t-1}(J_{t-1}, \ell_{t-1}),$$

Since  $V_{t-1}(I_{t-1}, R_{t-1}, Q_{t-1}) \equiv \mathbb{E}_{Q_{t-1}} H_{t-1}(J_{t-1}, \ell_{t-1})$ , the first inequality could be established for  $t - 1$ . This completes the induction steps for part (iii).

For part (iv), we have

$$C_t = \min\{K_t : c \geq \partial_{K_t} W_t(K_t, \mathbf{D}_t)\} \leq \min\{K_t : c \geq \partial_{K_t} \hat{W}_t(K_t, \mathbf{D}_t)\} = \hat{C}_t,$$

$$\ell_t^* = Q_t \wedge (C_t - J_t)^+ \leq Q_t \wedge (\hat{C}_t - J_t)^+ = \hat{\ell}_t^*,$$

$$s_t^i = \min\{I_{t-1} : r^i \geq -\partial h(I_{t-1}) + \partial V_{t-1}(I_{t-1})\} \leq \min\{I_{t-1} : r^i \geq -\partial h(I_{t-1}) +$$

$$\partial \hat{V}_{t-1}(I_{t-1})\} = \hat{s}_t^i,$$

$$x_t^{i*} = D_t^i \wedge (K_t - \bar{D}_t^{i-1} - s_t^i)^+ \geq D_t^i \wedge (K_t - \bar{D}_t^{i-1} - \hat{s}_t^i)^+ = \hat{x}_t^{i*}.$$

So  $x_t = \sum_i x_t^i \geq \sum_i \hat{x}_t^i = \hat{x}_t$ , that is, more orders are accepted in System- $\mathbf{D}$  than in System- $\hat{\mathbf{D}}$ . This completes the induction steps for part (iv). □

Part(i) states the marginal value of available inventory is more valuable when the confirm order is higher, or the marginal value of confirmed orders is more valuable

when the inventory level is higher. Part (ii) establishes the *first order stochastic property* for the value function  $W_t(K_t)$  in the concave order sense, where the higher future demands evoke higher profit. Part (iii) establishes the *second order stochastic property* for the value function  $V_t(I_t)$  and  $W_t(K_t)$ . It states the marginal value of inventory is more valuable when there is a higher future demands in the stochastic order sense during the decision epochs. Part (iv) shows the impact of future larger demands on the leasing policy parameters, such as the optimal leasing threshold and leasing quantity. Both optimal leasing threshold and optimal leasing quantity increase for the future higher demands, which implies stock up for more higher class orders in the future. It also shows the impact of future larger demands on the order acceptance policy parameters, such as the optimal rationing level and order acceptance for each demand class. The optimal rationing levels for each demand class increases with higher future demands and less orders are accepted for each demand class, which implies more capacity reservation for the future higher profit level demands.

## Chapter 6

# Conclusions

Combined container leasing and multi-profit-level load acceptance problem in intermodal freight transportation has been studied in this work. We show the optimal leasing policy is similar to the stock-up-to policy, where the containers inventory should be leased up to a critical level. Note that less containers are borrowed as more available containers are presented and at most one container needs to be borrowed given one more extra container inventory. We also show the optimal load acceptance policy follows allocation in the order of decreasing profits together with a rationing policy for each demand class. So the available container inventory could be allocated to a certain class until reaching its corresponding rationing level. More allocation is rendered when coming across more available containers and one more available containers could result in at most one more allocation. When the demand increases in the sense of stochastic order, we find that both the container leasing threshold and the rationing level increase. In practice, it means more containers are supposed to be borrowed and less orders are accepted, that is, stock up in hope of more future high profit level orders. We provide the IMC managers an applicable solution and an easy execution when they face the daily



problem of container capacity management and demand fulfillment. We also stress the importance of effective coordination between the IMC and its container suppliers and the feasibility of demand forecasting. To the best of our knowledge, there is no similar work address the same issue.

There are several aspects to extend our work. First, to allow more suppliers in the model. We only consider one specific container supply, while many IMCs have access to several containers sources with different capacity and at different cost. Second, to add lead time to tally with practice. We didn't consider the lead time for receiving the borrowed containers or the lead time for delivery of the accepted orders. Third, to consider supply uncertainty. We have illustrated the variability in demands in our work and similar idea could also be applied to containers supply. So far our work has provided a decent start for these future research directions.

# Bibliography

- [1] Amaruchkul, K., W. Cooper, D. Gupta. (2007). Single-Leg Air-Cargo Revenue Management. *Transportation Science*, 41(4), 457-469.
- [2] American Association of Railroads. (2008). Overview of America's Freight Railroads. [Http://www.aar.org/IndustryInformation/AboutTheIndustry/IndustryInfo.aspx](http://www.aar.org/IndustryInformation/AboutTheIndustry/IndustryInfo.aspx).
- [3] Bartodziej, P., U. Derigs, M. Zils. (2007). O&D Revenue Management in Cargo Airlines-a Mathematical Programming Approach. *OR Spectrum*, 29(1), 105-121.
- [4] Cooper, W. L., T. Homem-de-Mello. (2007). Some Decomposition Methods for Revenue Management, *Transportation Science*, 4(3), 332-353.
- [5] Frank, K.C., R.Q. Zhang, I. Duenyas. (2003). Optimal Policies for Inventory Systems with Priority Demand Classes. *Operations Research*, 51(6), 993-1002.
- [6] Freeland, L.C. (2007). Adoption of Customer-centric Cargo Revenue Management: Brief History of Cargo Revenue Management vs Passenger Revenue Management. *Journal of Revenue & Pricing Management*, 6(4), 284-286.
- [7] Gao, L., S. Xu. (2008). An Available-to-Promise Inventory-Production System with Pseudo Orders. Working paper.
- [8] Gorman, M.F. (2001). Intermodal Pricing Model Creates a Network Pricing Perspective at BNSF. *Interfaces*, 31(4), 37-49.
- [9] Gorman, M.F. (2002). Pricing and Product Mix Optimization in Freight Transportation. *Transportation Quarterly*, 56(1), 135-148.
- [10] Gorman, M.F. (2010). Hub Group Implements a Suite of OR Tools to Improve Its Operation. *Interfaces*, 40(5), 368-384.
- [11] Gupta, D., L. Wang. (2007). Capacity Management for Contract Manufacturing. *Operations Research*, 55(2), 367-377.
- [12] Karaesmen, I., G.J. van Ryzin. (2004). Overbooking with Substitutable Inventory Classes. *Operations Research*, 52(1), 83-104.
- [13] Kraft, E.R., B.N. Srikar, R.L. Phillips. (2000). Revenue Management in Railroad Applications. *Transportation Quarterly*, 54(1), 157-176.

- [14] Lautenbacher, C. J., S. Stidham. (1999). The Underlying Markov Decision Process in the Single-leg Airline Yield-management Problem. *Transportation Science*, 33(2), 136-146.
- [15] Li, L., S. Tayur. (2005). Medium-Term Pricing and Operations Planning in Inter-modal Transportation. *Transportation Science*, 39(1), 73-86.
- [16] Li, Q., S. Zheng. (2006). Joint Inventory Replenishment and Pricing Control for Systems with Uncertain yield and Demand. *Operation Research*, 54(4), 696-705.
- [17] Sabdu, R., D. Klabjan. (2006). Fleeting with Passenger and Cargo Origin-Destination Booking Control. *Transportation Science*, 40(4), 517-528.
- [18] Shumsky, RA, F. Zhang. (2008). Dynamic Capacity Management with Substitution. *Operations Research*, forthcoming.
- [19] Song, J.S., P. Zipkin. (1993). Inventory Control in a Fluctuating Demand Environment. *Operations Research*, 41, 351-370.
- [20] Talluri, K.T., G.J. van Ryzin. (2004). *The Theory and Practice of Revenue Management*. Springer.
- [21] Topkis, D.M. (1968). Optimal Ordering and Rationing Policies in a Nonstationary Dynamic Inventory Model with n Demand Classes. *Management Science*, 15(8), 160-176.
- [22] The Economist. (2010). American Railway: High-speed Railroading. [Http://www.economist.com/node/16636101](http://www.economist.com/node/16636101)
- [23] Yang, J., X. Qi, Y. Xia, G. Yu. (2006). Inventory Control with Markovian Capacity and the Option of Order Rejection. *European Journal of Operational Research*, 174, 622-645.