UNIVERSITY OF CALIFORNIA

Los Angeles

Scalar Modification and Pointwise Exhaustification

A thesis submitted in partial satisfaction of the
requirements for the degree Master of Arts
in Linguistics

by

Philippe Côté-Boucher

2018

ABSTRACT OF THE THESIS

Scalar Modification and

Pointwise Exhaustification

by

Philippe Côté-Boucher

Master of Arts in Linguistics

University of California, Los Angeles, 2018

Professor Yael Sharvit, Chair

This thesis investigates the interaction of exclusive particle 'only' with modified degrees 'at least *n*' and 'at most *n*.' Following Alonso-Ovalle (2006, 2008)'s treatment of disjunction, it is proposed that expressions containing a modified degree denote a set of Hamblin alternatives, and that the semantic contribution of 'only' applies pointwise to each member of that set.

The thesis of Philippe Côté-Boucher is approved.

Edward L. Keenan

Timothy L. Stowell

Yael Sharvit, Committee Chair

University of California, Los Angeles

2018

TABLE OF CONTENTS

ACKNOWLEDGMENTS

This thesis was originally written in 2013–2014. I would like to thank everyone who provided feedback during that period. Special special thanks to Benjamin Spector, who stepped in at a key moment to validate the project.

1 Introduction

This paper is concerned with the comparative distribution of the exclusive particle 'only' and scalar implicature. As shown in (1), the meaning contribution of these two linguistic devices are closely related.[1]

(1) a. Miles swam 5 laps.
$\rightsquigarrow \neg$(Miles swam 6 laps.)
b. Miles only swam 5 laps.
$\rightsquigarrow \neg$(Miles swam 6 laps.)

This has led many to model them in such a way as to reflect this similarity. A so-called "grammatical view" (Chierchia, Fox & Spector 2008) is adopted here, where scalar implicature is equated to an exhaustification operator named Exh. Under any version of this view, 'only' and Exh are quasi-synonyms, and 'only' differs minimally in projecting presuppositional content. A minimalist representation of the meaning of 'only' is given in (2). I adopt the lexical entry in (3) for the exhaustivity operator Exh, noting that nothing crucial hinges on this decision.

(2) $[\![\text{only}]\!] = \lambda p.\lambda w : p(w).[\![\text{Exh}]\!](p)(w)$

(3) $[\![\text{Exh}]\!] = \lambda p.\lambda w.p(w) \wedge \forall p' \in Alt(p), p'(w) \rightarrow p \subseteq p'$
(Chierchia, Fox & Spector 2008)

This makes the strong prediction that 'only P' should be felicitous in a proper subset of the cases where P is felicitous on its exhaustive reading.

(4) *Distributional implication of (2)*
$\boxed{\text{'Only' environments}} \subset \boxed{\text{Exh environments}}$

[1] In the paper, $\rightsquigarrow$ is used to introduce an inference, whatever its type.

The distributional statement in (4) has been suggested to be empirically borne out, and called the "*Only* Implicature Generalization" (Fox & Hackl 2006, Fox 2007).[2]

The following data present an apparent counterexample. First, (5) shows an observation due to Krifka (1999): modified degrees appear not to be subject to scalar implicature. This suggests that the sentences in (5) are not Exh environments.

(5) a. Miles swam at least 5 laps.
$\not\rightsquigarrow$ ¬(Miles swam at least 6 laps.)
b. Miles swam at most 5 laps.
$\not\rightsquigarrow$ ¬(Miles swam at most 4 laps.)

However, looking at (6), we see that 'at least N' and 'at most N' differ in their distribution. While the former is not compatible with 'only,' as predicted by (4), the latter is.

(6) a. *Miles only swam at least 5 laps.
b. Miles only swam at most 5 laps.

(6)b is admittedly quite marked, and at the margins of what counts as plausible language use, but this can be made sense of by realizing that any context rich enough to support such a sentence needs to jointly satisfy the contextual requirements of 'only' *and* 'at most.' Conventionally, 'at most' in (6)b indicates epistemic uncertainty regarding the exact number of laps swum, while 'only' expresses that the number of laps swum is low relative to an expectation or standard. Pretheoretically, we can give for (6)b the paraphrase in (7)b, and, by symmetry, for (6)a the one in (7)a. Since the state of affairs (7)b describes can obtain in a context, this paraphrase predicts (6)b to be a possible sentence. I provide such a context in (8).

(7) a. The number of laps swum by Miles is i) equal to n for some $n \geq 5$, and ii) low

[2] The exact formulation of this generalization is rather vague and non-committal: "Utterance of a sentence, S, as a default, licenses the inference/implicature that (the speaker believes) only S′, where S′ is S with focus on scalar items."

relative to an expectation.

b. The number of laps swum by Miles is i) equal to n for some $n \leq 5$, and ii) low relative to an expectation.

(8) *Context for (6)b:*

Miles and Bertha each swam laps. Between the two of them, they swam exactly 10 laps. A says: "I think Miles swam 6 laps." B knows that Bertha swam 5 laps, and maybe more. So, B is entitled to retort, "Miles only swam at most 5 laps."

The problem with this naïve paraphrase is that a context homologous to (8) can be devised to verify (7)a, wrongly suggesting that (6)a is a possible sentence. Another intuitive paraphrase stands a better chance at fitting the observed pattern:

(9) a. The number of laps swum by Miles is i) equal to n for some $n \geq 5$, and ii) *as such,* low relative to an expectation.

b. The number of laps swum by Miles is i) equal to n for some $n \leq 5$, and ii) *as such,* low relative to an expectation.

Put it another way, the paraphrase in (9)b says that any number of laps not exceeding 5 is low relative to an expectation. This is most certainly a coherent statement. Now consider the case of 'at least 5,' which denotes a range of degrees that's not upper-bounded. (9)a says that any number of laps greater than or equal to 5 is low relative to an expectation, which suggests an infinitely strong—hence intuitively incoherent—expectation. The paraphrases in (9)—which will serve as the guiding intuition behind the analysis presented in §3—predict a complete distributional gap for (6)a, but allow (6)b up to the satisfaction of rather stringent contextual requirements.

A Google search for "only at least" (2,660,000 hits) and "only at most" (31,200,000 hits) at first glance seems to support those predictions. Upon careful analysis, the hits for "only at least" turn out to be spurious, while we find plenty of genuine "only at most" tokens, some of which are presented in (10). The best explanation for such a strong (nearly 12:1) disparity in number of search results is that the string "only at least" is categorically ruled out as a consequence of how the meanings of 'only' and 'at least' interact.

(10) a. However, since the Pentium had a superscalar pipeline, it could often perform 2 instructions per cycle, where the 486 did only one at most.
`http://scalibq.wordpress.com/2012/06/01/multi-core-and-multi-threading/`

b. Outside my family, at the age of 33, I have only at most 3 close friends.
`http://www.boards.ie/vbulletin/showthread.php?p=73087959`

c. It's only at most 60 degrees out and all these girls are wearing dresses and skirts. Can't wait to see what they'll wear when it's 80.
`http://twitter.com/mahneeka/status/454581074883268609/`

There are at least two plausible answers to the puzzle posed by (6)b. One option is to accept the counterexample as such, and to conclude that the *Only* Implicature Generalization is essentially incorrect. A fortiori, this means that a lexical entry like (2) for 'only' is untenable, and that 'only' and scalar implicature have less in common than originally believed. Specifically, 'only' cannot have an exhaustification component, if that is indeed the case. A second option is to maintain that the generalization is correct, and that (6)b does not constitute a true counterexample.[3] This is the strategy taken up here. I argue that Exh is in fact present in all sentences involving modified degrees like (5)a and (5)b, but that its effect is imperceptible. Such an analysis is made possible by a new theory of scalar modifiers couched in a Hamblin semantics.

The paper is organized as follows. In §2, the empirical facts regarding the interaction

[3]Yet another idea, brought to my attention by Tim Stowell, is that 'at most' takes covert scope above 'only.' Something that could lend support to this idea is the fact that (i)a is hard to distinguish in meaning from (i)b.

(i) a. At (the) most, Miles only swam 5 laps.
b. Miles only swam at most 5 laps.

The analysis I present predicts this equivalence or near-equivalence, and also predicts additional readings (which are verified in some dialects) for the configuration 'at the least ... only N.'

of scalar implicature and modified degrees are introduced, and an existing theory is criticized. The core theoretical proposal, laid out in §3, is meant as an alternative to this theory. The proposal is augmented and extended in the two sections that follow. In §4, I treat the interaction between 'only' and modified degrees. In §5, I present and discuss residual data from comparative scalar modifiers. This data, too, is covered under the new analysis. The conclusion is found in §6.

2 Modified degrees

In this section, an existing account of the facts in (5) is presented—which I dub the Büring–Sauerland–Fox theory. Three criticisms are raised, which set the stage for the alternative proposal in §3. But first, some background on scalar modifiers.

2.1 Empirical landscape

Scalar modifiers include, but are not limited to, the expressions found in (11). Functionally speaking, these expressions are used to denote a non-exact degree, or a range of degrees. At a first approximation, they are means made available by natural language to express the relations $\leq$, $\geq$, $<$ and $>$. Because of their morphological makeup, 'at least' and 'at most' are commonly called the superlative modifiers, and 'more than' and 'less than' the comparative modifiers. To simplify exposition, the discussion initially focuses on superlative modified degrees.

(11) $\leq$: at least N $\quad$ $<$: more than N

$\geq$: at most N $\quad$ $>$: less/fewer than N

There exists a long and fruitful tradition to approach these with the tools of generalized quantifier theory. In this framework, the meaning of sentences with modified cardinals is represented as in (12).

(12) Bertha read at least 5 books.

$|\{x : x \text{ is a book}\} \cap \{x : \text{Bertha read } x\}| \geq 5$

However, Krifka (1999) exposes some shortcomings of this approach. First, scalar modifiers can be used even in the absence of a cardinal expression. For instance, we see in (13) that it can associate with the non-cardinal NP 'Moby Dick.'

(13) Bertha read at least Moby Dick.

Second, unlike quantifiers and bare cardinals, which are subject to scalar implicature (SI), (14), modified cardinals consistently resist it, (5), repeated in (15).

(14) a. Bertha read most books.

$\rightsquigarrow$ ¬(Bertha read every book.)

b. Miles swam 5 laps.

$\rightsquigarrow$ ¬(Miles swam 6 laps.)

(15) a. Miles swam at least 5 laps.

$\not\rightsquigarrow$ ¬(Miles swam at least 6 laps.)

b. Miles swam at most 5 laps.

$\not\rightsquigarrow$ ¬(Miles swam at most 4 laps.)

This is especially puzzling given that the scale used to compute implicatures for bare cardinals in (14)b should arguably be available to put modified cardinals on a scale as well. What is more, the meaning of 'five' and 'at least five' cannot easily be distinguished if we treat them as generalized quantifiers, because they are both represented using the *less or equal* relation.

An important qualification to the observation that modified degrees resist SI is that certain grammatical environments have the power to cancel the effect (Fox & Hackl 2006, Spector 2006, Geurts & Nouwen 2007, Büring 2007). When a modified degree is found under the scope of a universal quantifier, we find that scalar implicatures arise once again.[4] The phenomenon is exemplified in (16).

(16) a. You must swim at least 5 laps.

$\rightsquigarrow$ ¬(You must swim at least 6 laps.)

b. Everyone swam at least 5 laps.

$\rightsquigarrow$ ¬(Everyone swam at least 6 laps.)

c. I always swim at least 5 laps.

$\rightsquigarrow$ ¬(I always swim at least 6 laps.)

2.2 The Büring–Sauerland–Fox theory

Büring (2007) offers a explanation for the lack of SI with modified degrees. He is only concerned with 'at least' and 'at most,' from which it follows that the lack of SI with comparative modifiers has to be due to something else.[5] Summarizing, his proposal is that (15)a is a morphologically opaque version of (17), and is therefore semantically and pragmatically equivalent to it.

(17) Miles swam exactly 5 or more than 5 laps.

Following Sauerland (2004), it's common to present the alternative set for (17) (whose meaning I abbreviate as $= 5 \vee > 5$) ordered by the entailment relation, as in (18). This set is obtained via the following procedure: i) any proposition resulting from the pruining of a disjunct is an alternative (giving us $= 5$ and > 5); ii) any proposition resulting from the substitution of an n for 5 is an alternative (giving us, for all n, $= n \vee > n$, $= n$ and $> n$).[6]

[4]Though the phenomenon has mostly been discussed strictly from the point of view of modal expressions, it's important to note that it is rather general, and evidently not tied to modality.

[5]That these two should receive different explanations is more or less the current received view. Geurts & Nouwen (2007) also develop an account of the lack of SI with superlative modifiers, which makes use of explicit modals. I focus on Büring's proposal here because it is conceptually closer to my own.

[6]The alternatives for $p \vee q$ usually also include $p \wedge q$. We can safely ignore the conjunctive alternatives in this case, since they are contradictions.

(18)

$$\begin{tikzcd}[column sep=small]
=n \arrow[dr, dash] & & >n \arrow[dl, dash] \\
& =n\vee>n \arrow[dr, dash] & \\
=6 \arrow[dr, dash] & & >6 \arrow[dl, dash] \\
& =6\vee>6 \arrow[dr, dash] & \\
=5 \arrow[dr, dash] & & >5 \arrow[dl, dash] \\
& \boxed{=5\vee>5} &
\end{tikzcd}$$

Given (18), a satisfactory theory of SI computation should derive none here. One such theory is Fox (2007)'s Innocent Exclusion algorithm. For an alternative p' to the uttered sentence's meaning p to be excludable, Innocent Exclusion says that its negation taken in conjunction with p must not entail an alternative p'' at least as strong as p. It's easy to see that this obtains for none of the alternatives in (18), due to the clashing monotonicity of the alternatives found in the set. This is demonstrated in (19). This means that no alternative is excludable, and the lack of SI is predicted.

(19) Potential exclusions: a. $\neg(=5)$, b. $\neg(>5)$, c. $\neg(=6)$, d. $\neg(>6)$[7]

a. $\neg(=5) \wedge (=5 \vee >5) \quad \Rightarrow \quad >5$ (found in (18))

b. $\neg(>5) \wedge (=5 \vee >5) \quad \Rightarrow \quad =5$ (found in (18))

c. $\neg(=6) \wedge (=5 \vee >5) \quad \Rightarrow \quad =5 \vee >6$ (entails $=5 \vee >5$)

d. $\neg(>6) \wedge (=5 \vee >5) \quad \Rightarrow \quad =5 \vee =6$ (entails $=5 \vee >5$)

$\rightsquigarrow$ $*\neg(=5)$, $*\neg(>5)$, $*\neg(=6)$, $*\neg(>6)$, and by induction,

$*\neg q$ for all alternatives q in (19)

This analysis also predicts that the scope of universal modals should make superlative modified degrees amenable to SI once again. To see that this is so, consider the minimally different sentence (16)a. The situation is illustrated in (20); we derive—in accordance with the intuitive meaning of "You must swim at least 5 laps"—the implicatures in (20) that

[7]We can ignore the potential exclusion $\neg(=6 \vee >6)$, since it entails $\neg(=6)$ and $\neg(>6)$.

there is no n such that you must swim exactly n laps, which means ultimately that you are allowed, for any $n \geq 5$, to swim n laps.

(20) Potential exclusions: a. $\neg\Box[=5]$, b. $\neg\Box[=6 \vee >6]$, c. $\neg\Box[=7 \vee >7]$

a. $\neg\Box[=5] \wedge \Box[=5 \vee >5] \quad \Rightarrow \quad \Diamond[>5]$ (entails no alt.)

b. $\neg\Box[=6 \vee >6] \wedge \Box[=5 \vee >5] \quad \Rightarrow \quad \Diamond[=5]$ (entails no alt.)

c. $\neg\Box[=7 \vee >7] \wedge \Box[=5 \vee >5] \quad \Rightarrow \quad \Diamond[=5 \vee =6]$ (entails no alt.)

$\rightsquigarrow \neg\Box[=5], \neg\Box[=6 \vee >6], \neg\Box[=7 \vee >7]$, and by induction,

$\neg\Box[=n \vee >n]$ for any $n \neq 5$

2.3 Problems with the Büring–Sauerland–Fox theory

The theory presented in §2.2 correctly derives appropriate SIs for modified degrees embedded under universal modal operators. As I've shown in (16), however, these implicatures are not particular to modals, and occur under the scope of any universal quantifier. The theory faces a problem with these other contexts. In the case of modal necessity sentence (16)a, the inference that you are allowed to swim n laps for any $n \geq 5$ is warranted. However, consider first the sentence (16)b, 'Everyone swam at least 5 laps.' By the above logic, we derive the inference that, for any $n \geq 5$, someone swam n laps. This is clearly not a desirable consequence. The same obtains for (16)c, 'I always swim at least 5 laps,' which we predict to license the inference that for any $n \geq 5$, I sometimes swim n laps. This is a hard problem, which the account I'm about to present is designed to solve.

There is, in addition, what I consider to be a compositional problem with the Büring–Sauerland–Fox analysis. The decomposition of 'at least n' as 'exactly n or more than n' is obviously inspired by a paraphrase, but qua paraphrase, its linguistic status is dubious.[8] The most troublesome aspect of this is that the meaning of the disjuncts in the paraphrase is merely taken for granted, and never explicitly derived. Once we try to flesh out these

[8] A similar objection is raised by Coppock & Brochhagen (2013). They go on to propose a theory of scalar modification that shares a lot conceptually with the account presented in this paper.

meanings, problems arise. The technical option of making 'exactly N' and 'more than N' generalized quantifiers would be open, if it weren't for Krifka's objections that I raised earlier, and the possibility of sentences like (13). Barring this, the only other option I'm aware of is to assume that, one way or another, local exhaustification has applied to the first disjunct. In any case, the derivation of an exact degree disjunct is crucial, and needs to be motivated.

2.4 Comparative modifiers

Questions emerge when we extend our scope of inquiry to comparative scalar modifiers. Geurts & Nouwen (2007) conclusively show that there are irreducible differences between the behavior of comparatives and superlatives, even with otherwise truth-conditionally equivalent degrees like 'more than 3' and 'at least 4.' Assuming that the speaker has the knowledge that pentagons have exactly 5 sides, (21)b is infelicitous, presumably because (21)b obligatorily gives rise to an inference that pentagons may not in fact have exactly 5 sides.

(21) a. A pentagon has more than 3 sides.

b. ??A pentagon has at least 4 sides.

$\rightsquigarrow \Diamond\neg$(A pentagon has exactly 5 sides.)

A satisfactory theory should take this difference between comparative and superlative modifiers seriously, but it should not lose sight of their similarities. When it comes to the data points at hand here, the two classes behave identically. It is implicit in the Büring–Sauerland–Fox theory that the comparative modifier facts should receive an entirely different explanation. I want to point out that this lack of connection between explanations is not ideal: on grounds of simplicity alone, it would be preferable to attribute the lack of SI with comparative and superlative modifiers to a single underlying cause. Clearly, Büring's account could not be extended to comparative modifiers, because his meaning of superlatives piggybacks on a comparative meaning in its second disjunct.[9]

This invites a fundamental question: is the presence of the offending inference in (21)b a lexical accident, or does it arise as a consequence of the meaning of 'at least'? Consider a simple classification of modifiers based on whether they do give rise to this obligatory inference or not. And consider the four inequality relations $\geq, >, \leq, <$. With those two factors, we obtain the eight possible modifiers in (22).

(22)

	$\geq$	$>$	$\leq$	$<$
Inference	at least N	(?)	at most N	(?)
No inference	(?)	more than N	(?)	fewer than N

Are the empty cells in (22) accidental paradigm gaps, or is there a deep-seated reason for them? The Büring–Sauerland–Fox theory says that it is the latter. As will become clear in §3.4, this is because those inferences are always triggered by disjunction, and disjunction is used in the paraphrase. In contrast, the theory I present says that the gaps in (22) are purely accidental. I return to comparative modifiers in §5.

3 Generalized scalar modification

In this section, I develop an alternative to the Büring–Sauerland–Fox theory of scalar modifiers. Two crucial aspects of Büring's paraphrase make their way into this proposal. First, the idea that modified degrees are underlyingly disjunctive translates into the fact that modified degrees and disjunction are related linguistic devices: they both introduce complex Hamblin denotations. Second, the idea that this disjunction involves an exact degree is also preserved; exact degrees are derived via local exhaustification. In addition to paving the way to an explanation of the 'only' facts shown in (6), this proposal seeks

[9] One explanation for the lack of SI with comparative modifiers is due to Fox & Hackl (2006). Consistent with the idea that the account of superlatives and comparatives should be kept separate, their proposal can only account to comparatives.

to remedy the two problems laid out in §2.3: it extends to comparative modifiers, and constitutes a more compositionally-conscious attempt at making Büring's paraphrase obtain.

In the spirit of Krifka (1999), I compositionally derive the meaning of modified cardinals from the meaning of bare cardinals. Let us assume that the meaning of a cardinal expression like 'five' is predicative: it is the set of pluralities made up of (exactly) 5 atoms. Some composition rules and apparata that will be assumed are predicate modification and existential type shifting.

(23) a. $[\![5]\!] = \lambda x.|x| = 5$

b. $[\![5 \text{ books}]\!] = [\![5]\!] \cap [\![\text{books}]\!]$ (predicate modification)

$= \lambda x.|x| = 5 \wedge book'(x)$

c. $[\![\text{read } 5 \text{ books}]\!] = \lambda y.\lambda w.\exists x, |x| = 5 \wedge book'(x) \wedge read'(y)(x)(w)$

(existential type shifting)

3.1 Hamblin semantics

In a Hamblin semantics (Hamblin 1973, Kratzer & Shimoyama 2012), the standard denotation a of a linguistic expression A is lifted to a singleton α containing this denotation, $\alpha = \{a\}$.[10] Regular composition rules are applied pointwise following the schema in (24).

(24) *Pointwise composition*

$\alpha \times \beta = \{a \circ b : a \in \alpha \wedge b \in \beta\}$, where $\circ$ stands for any composition rule, e.g. functional application, or the rules in (23)

Application of this schema in most cases results in a normal derivation "trapped in curly brackets." (25)a-d are an example of this. When two singleton sets compose pointwise, the result is also a singleton set. A Hamblin denotation can at any point be "flattened" to

[10]Notationally, this will be the format followed in the paper: capital non-italicized roman characters (A, B, ...) are reserved for natural language expressions, italicized roman characters ($a, b, A, B, ...$) for standard meanings, and greek characters ($\alpha, \beta, ...$) for Hamblin denotations (sets of standard meanings).

its standard meaning equivalent using existential closure. We can assume that it is this operation that generates a standard propositional meaning at the end of the derivation in step (25)e, only *after* exhaustification has taken place.

(25) a. $[\![5]\!] = \{\lambda x.|x| = 5\}$

b. $[\![\text{swam}]\!] = \{\lambda x.\lambda w.swam'(x)(w)\}$

c. $[\![5]\!] \times [\![\text{swam}]\!] =_{\exists\text{ shift}} \{\lambda w.\exists x, |x| = 5 \wedge swam'(x)(w)\}$

d. $[\![\text{Exh}]\!] \times [\![\text{5 swam}]\!] =_{\text{f. appl.}} \{exh(\lambda w.\exists x, |x| = 5 \wedge swam'(x)(w))\}$

e. $\bigcup$(25)d $= exh(\lambda w.\exists x, |x| = 5 \wedge swam'(x)(w))$

But some expressions have the power to build denotations that are non-singleton sets. Alonso-Ovalle (2006, 2008) uses such a semantics to account for disjunction. Disjunction is interpreted according to the rule in (26). One advantage of this framework is that it provides an elegant compositional way to derive propositional disjunction from constituent disjunction.

(26) *The* or *rule* (Alonso-Ovalle 2008)

Where $[\![A]\!], [\![B]\!] \in D_\sigma, [\![A \text{ or } B]\!] \in D_\sigma = [\![A]\!] \cup [\![B]\!]$

(27) $[\![\text{Miles or Bertha}]\!] = \{m\} \cup \{b\} = \{m, b\}$

I propose that scalar modifiers are similar in the respect that they build non-singleton Hamblin denotations made up of cardinality predicates. Set-theoretically speaking, the denotations of any two numerical expressions N and M are disjoint ($[\![N]\!] \cap [\![M]\!] = \emptyset$), so we need a notion of strength with which such predicates can be related. Any two members of these predicates are naturally related on the part-whole individual relation, so that we can determine the strength of predicates recursively from the strength of their members, following the definition in (28). In everything that follows, the cases discussed call only for this natural notion of strength, although any pragmatic scale can be substituted, as needed.

(28) a. If P and Q are two individuals, Q is at least as strong as P, $P \leq Q$ iff P is a mereological part of Q.

b. If P and Q are two predicates in $D_{<\sigma,t>}$, $P \leq Q$ iff for all $p \in P$, there exists a $q \in Q$ such that $p \leq q$.

With this in mind, we can discuss the meaning of scalar modifiers. These expressions are focus-sensitive; they find the focal alternatives (Rooth 1986, 1992) of their complement and "cannibalize" a subset of them into the Hamblin denotation. An example is given in (29) with 'at least.' 'At least' cannibalizes any alternative at least as strong as its prejacent P.

(29) a. $[\![\text{at least}]\!] = \{\lambda P.\lambda Q \in Alt(P).P \leq Q\}$

b. $Alt(\lambda x.|x| = 5) = \{\lambda x.|x| = n : n \text{ is a number}\}$

c. $[\![\text{at least}]\!] \times [\![5]\!] = \{\lambda x.|x| = n : n \geq 5\}$

Thus, while 'five' denotes a singleton, 'at least 5' denotes an infinite set of cardinality predicates. We allow (29)c to compose up to the propositional level. Existential closure in (30)c gets us our propositional meaning.

(30) a. (29)c $\times [\![\text{swam}]\!] = \{\lambda w.\exists x, |x| = n \wedge swam'(x)(w) : n \geq 5\}$

b. $[\![\text{Exh}]\!] \times$ (30)a $= \{exh(\lambda w.\exists x, |x| = n \wedge swam'(x)(w)) : n \geq 5\}$

c. $\bigcup$(30)b $= \lambda w.\exists n \geq 5, exh(\lambda w.\exists x, |x| = n \wedge swam'(x)(w))$

Which gives us the following paraphrases:

(31) a. $[\![\text{5 swam}]\!]$ = (25)e $\approx$ Exactly 5 swam.

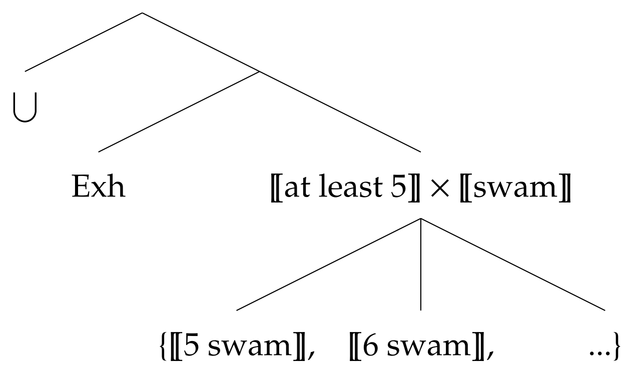
b. $[\![\text{At least 5 swam}]\!]$ = (30)c $\approx$ Exactly n swam, for some $n \geq 5$.

These meanings are in accord with our intuitions. This shows that, with the right set of assumptions, exhaustification can take place without yielding the expected global effect. The key is that this exhaustification is the result of a low instance of Exh—below the existential closure operator—which composes pointwise, and therefore operates over bare degree meanings. As a result, no complicated stipulations need to be made regarding the computation of alternatives: the only alternatives this system feeds on are the alternatives to bare degrees, whose computation is uncontroversial. As another advantage, the meaning of cardinal expressions is kept constant, and the meaning of modified cardinals is derived compositionally—without recourse to any covert disjunction—by the cannibalizing of focal alternatives described above.

Here we now also see why Büring's disjunctive paraphrase works. This theory of modified degrees establishes a structural similarity between modified degrees and disjunction (adopting Alonso-Ovalle's proposal), as attested by (32) and (33).

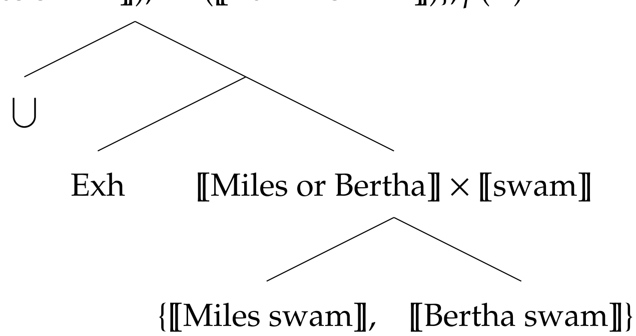
(32) At least 5 swam.

$\lambda w.\exists p \in \{exh(⟦\text{5 swam}⟧), exh(⟦\text{6 swam}⟧), ...\}, p(w)$



(33) Miles or Bertha swam.

$\lambda w.\exists p \in \{exh(⟦\text{Miles swam}⟧), exh(⟦\text{Bertha swam}⟧)\}, p(w)$



3.2 Obligatory exhaustification

As we've seen, pointwise exhaustification can apply to 'at least N' meanings without yielding a problematic global implicature. In this sense, exhaustification is vacuous with 'at least N.' Paraphrasing, 'exactly m for some $m \geq n$' is indistinguishable from 'at least m for some $m \geq n$.' In the case of 'at most N' meanings, however, exhaustification is necessary to derive to correct interpretation, since 'at least m for some $m \leq n$' is not an attested reading of 'at most N.'[11] What seems to be needed is a device to enforce the insertion of Exh in this case. Notice that disjunctive degrees call for exactly the same kind of device. The sentence in (34) seems to only have an interpretation paraphrasable as a disjunction of exact degrees.

(34) Miles swam 5 or 6 laps.

$\rightsquigarrow$ Miles swam exactly 5 or exactly 6 laps.

$\not\rightsquigarrow$ Miles swam at least 5 laps.

On closer inspection, the disjunction in (34) presents a potential violation of Hurford's Constraint (HC). As noted by Hurford (1974), disjunctions where one disjunct entails the other are ill-formed. Typical cases are shown in (35).

(35) a. *Miles is an American or a Californian.

b. *Bertha wants a pet or a dog.

Authors like Chierchia, Fox & Spector (2008) have used a certain class of apparent violations as evidence for the existence of local implicatures. Unlike the violations in (35), these acceptable "violations" (shown in (36)) usually involve an explicitly stronger disjunct associated with a traditional Horn scale. If the logical form of these sentences involved disjunction scoping over exhaustified meanings, no HC violation would occur.[12]

[11] It is this problematic reading that leads Krifka (1999) down the path of viewing 'at most' as imposing falsity—rather than truth—conditions.

[12] Chierchia, Fox & Spector (2008) actually argue for the local insertion of Exh on any disjunct *that is not the strongest*. Examples such as the following suggest otherwise:

(i) Bertha read some or most of the books. (#In fact, she read all of them.)

(36) a. Bertha read Moby Dick, Crime & Punishment, or both.

b. Some or all of the suspects were arrested.

Disjunctive degrees create a comparable configuration. The disjuncts are not logically independent, but up to their individual exhaustification, they are. In a Hamblin semantics, pointwise exhaustification serves the conceptual role of local exhaustification. I advance that the insertion of Exh is obligatory with disjunctive degrees because it rescues a derivation that would otherwise incur a HC violation. Note that the exact same reasoning obtains for modified degrees. What we are forced to conclude is that both (15)a and (15)b, as a matter of necessity, include Exh to avoid a HC violation.

3.3 Behavior under quantification

So far the discussion has focused on cases of unembedded modified degrees. But recall that the same degrees, when found under universal quantification, do display a global implicature, contrary to Krifka's observation. The relevant contrast is between (15) and (16), and is repeated here as (37) and (38).

(37) Miles swam at least 5 laps.

$\not\rightsquigarrow$ ¬(Miles swam at least 6 laps.)

(38) Everyone swam at least 5 laps.

$\rightsquigarrow$ ¬(Everyone swam at least 6 laps.)

$\rightsquigarrow$ ¬(Everyone swam exactly 5 laps.)

The proposal so far was one designed to explain the lack of global implicature in (37), and it would seem like the way this was achieved would preclude there to be anything to say about (38). However, this bears on another issue: given the semantics of Exh, which takes a propositional (type $< s, t >$ argument), there is nothing to prevent it to be inserted at the very end of the syntactic derivation to a flattened Hamblin denotation. In fact, we predict the two syntactic configurations in (39) to be possible, unless blocked by some extraneous factor. I should note that this prediction comes with a minor caveat: in the Hamblin semantics, i.e. below sentential closure, Exh is inserted in its lifted form, as a singleton, $[\![\text{Exh}]\!] = \{exh\}$. Above sentential closure, Exh needs to inserted in its classical form, $[\![\text{Exh}]\!] = exh$.

(39) a. Low insertion: $\bigcup > \text{Exh} > \phi$

b. Root insertion: $\text{Exh} > \bigcup > \text{Exh} > \phi$

Since Rooth's (1985, 1992) theory guarantees that every node in a well-formed tree should have a focus value—that is, there is no point in the derivation at which the composition of focus stops its course—a set of focal alternatives is straightforwardly generated for $\bigcup \phi$. It's easy to see that, if Exh is allowed to be inserted high in the case of (37), and to feed on those alternatives, we once again predict a spurious 'exactly *n*' reading of 'at least *n*.' Let us verify this claim. Because flattened Hamblin denotations are the same as their classical semantics equivalent, we derive the alternative set in (40). (To simplify exposition, I write the denotations as paraphrases.) Because this set does contain strictly stronger alternatives, exhaustification has the effect of negating them.

(40) Alternatives to 'Miles swam at least 5 laps.'

(≈ *Miles swam exactly n laps, for some n* ≥ 5)

$$\left\{\begin{array}{c} \ldots \\ \textit{Miles swam exactly n laps, for some } n \geq 4 \\ \textit{Miles swam exactly n laps, for some } n \geq 5 \\ \textit{Miles swam exactly n laps, for some } n \geq 6 \\ \ldots \end{array}\right\}$$

Hence, the configuration (39)b, which is not ruled out under our system, gives us the wrong result for (37). On the other hand, root insertion of Exh could be exactly what we need to generate the implicatures in (38). What we need, then, is a way to allow root insertion only in the case where a modified degree in the structure is found in the scope of a universal quantifier.

3.4 Modal inferences

Gazdar (1979) first discusses "possibility inferences" in relation to the use of disjunction. A disjunction 'P or Q,' $p \vee q$, quite generally licenses $\Diamond p$, and $\Diamond q$. In most work following Sauerland (2004), possibility inferences are the result of non-necessity inferences taken together with the assertion of the disjunction: $\Box(p \vee q)$ in conjunction with $\neg\Box p$ and $\neg\Box q$ implies $\Diamond p$ and $\Diamond q$. For Sauerland, these non-necessity inferences are called "primary implicatures," and SIs as we've been discussing them so far (of the type generated by Exh) are called "secondary implicatures."

In Sauerland's system, primary implicatures play an important role in limiting the number of warranted secondary implicatures. Specifically, the epistemic step whereby a primary implicature $\neg\Box q$ is turned into a secondary one $\Box\neg q$ is not warranted whenever $\Box\neg q$ taken in conjunction with $\Box p$ and p's other primary implicatures yields a contradiction. To take a simple case, consider what happens with an assertion of 'P or Q' in (41). Since the negation of either disjunct in (41) yield contradictions, neither inference is allowed as a secondary implicature.

(41) Asserted: $\Box(p \vee q)$; Primary implicatures: a. $\neg\Box p$, b. $\neg\Box q$

Potential secondary implicatures: a. $\Box\neg p$, b. $\Box\neg q$

a. $\Box\neg p \wedge \Box(p \vee q) \wedge \neg\Box q \quad \Rightarrow \quad \bot$

b. $\Box\neg q \wedge \Box(p \vee q) \wedge \neg\Box p \quad \Rightarrow \quad \bot$

$\rightsquigarrow$ $^*\Box\neg p$, $^*\Box\neg q$

I propose to augment the system developed in this section by introducing non-necessity inferences.[13] The idea can be traced back to this simple intuition: disjunction and scalar

[13]Whether we cash these inferences as possibility inferences or non-necessity inferences makes little

modification can be seen as not only building propositional content with which a context can be updated, but also as conventionally imposing restrictions on the range of possible contexts that can result from an update with this content. Since Veltman (1990)'s work on epistemic 'might,' meanings belonging to this dimension have been known as 'tests.' Presuppositions are a good example of tests: they are restrictions imposed on the contents of the input context.[14] In contrast, the kind of non-necessity inference called for here I'll call a postsupposition, in Brasoveanu & Szabolcsi (2013)'s sense: "a test imposed on the sentence-final output context."[15] The source of those postsuppositions can be traced back to the $\bigcup$ operator itself. Imagine that this operator triggers the obligatory inference in (42).

(42) *(Non-)necessity inference of closure operator*

$\bigcup \phi$ triggers the inference that, for all $\phi' \subseteq \phi$, $\Box \bigcup \phi' \leftrightarrow \phi' = \phi$

When a Hamblin denotation is a singleton, there is only one (non-empty) subset of ϕ, ϕ, and (42) is just as good as an assertion of its single member. But when ϕ has more than one member, non-necessity inferences $\neg\Box\phi'$ for every proper subset ϕ' of ϕ are triggered. It is those inferences—Sauerland's primary implicatures—that I call postsuppositions. We can view those inferences as obligatory and recast them as definedness conditions, making an explicit mention of input and output context. In (43), C' is the output context, the result of updating C with the asserted content of $\bigcup \phi$.[16] We can see that, like presuppositions, postsuppositions ultimately impose restrictions on admissible input contexts, the difference being that those restrictions are evaluated in terms of the output context; the test

difference. I follow Sauerland and others in going for the latter.

[14]See Beaver (1992)'s attempt to connect Veltman's notion of test to presupposition theory.

[15]Postsuppositions have been put to work in the context of free choice and exotic indefinites, cf. Farkas (2002), Lauer (2009).

[16]Thus, writing $\lambda w : \forall w' \in C, q(w').p(w)$ for the proposition asserting p and presupposing q is equivalent to the more common notation $\lambda w : q(w).p(w)$. Postsuppositions, however, cannot be expressed using the standard notation. The proposition asserting p and postsupposing q can be expressed with the following formula: $\lambda w : \forall w' \in C', q(w').p(w)$.

takes place after—instead of prior to—the update. Finally, the definedness conditions are inserted right in the lexical entry of $\bigcup$ in (44).

(43) *Definedness conditions of closure operator*

$\bigcup \phi$ is defined iff, for all $\phi' \subseteq \phi$, $(\forall w \in C', \bigcup \phi'(w)) \leftrightarrow \phi' = \phi$

(44) *Existential closure operator*

$[\![\bigcup]\!]^C = \lambda\phi_{<st,t>} : \forall\phi' \subseteq \phi, (\forall w \in C', \bigcup \phi'(w)) \leftrightarrow \phi' = \phi.\lambda w.\exists p \in \phi, p(w)$

Here it's useful to point out that (43) does some further work. Namely, it derives Hurford's Constraint. When a Hamblin denotation ϕ contains members that are not logically independent, i.e. violates HC, it can be proven that ϕ violates (43). This enforces the insertion of Exh below the existential closure operator. (45) exposes the fact that (43) entails HC.[17]

(45) *Theorem* $\bigcup$HC: The definedness conditions of $\bigcup$ entail HC, i.e. Hamblin denotations which violate HC immediately fail to satisfy (43).

(46) *Proof:* Assume a HC violation. Then there is in ϕ a p, p' such that $p \subseteq p'$ (by the definition of HC). Then $\bigcup \phi = \bigcup(\phi - \{p\})$. By (43), $\Box \bigcup \phi$, so $\Box \bigcup(\phi - \{p\})$. But $(\phi - \{p\}) \subset \phi$, which means that (43) is not satisfied. ■

Taking stock, I've argued that non-necessity inferences—Sauerland's primary implicatures —are postsuppositions, in the sense that they are a test on output contexts. Just like presuppositions, they constitute *conventional* content. In this particular case, the trigger is the sentential closure operator, and I make the strong prediction that these postsuppositions will be found with disjunction, scalar modifiers, indefinites, and any phenomenon one decides to treat under a Hamblin semantics. There are two main conceptual differences with Sauerland. First, unlike primary implicatures, these inferences are obligatory—though like any suppositional phenomenon potentially subject to accommodation. Second, where

[17]The idea that Hurford's Constraint is linked to non-necessity inferences, and the result of some sentential operator has come up at least one other time in the literature, cf. Meyer's (2014) K.

Sauerland invokes this inference as a precursor to SIs of all kinds (e.g., 'some' → ¬'all'), the present framework says that it is particular to disjunction and the class of phenomena that involve Hamblin alternatives. The proposal is therefore in line with Gazdar (1979) in viewing such inferences as divorced from the phenomenon of implicature at large. (Gazdar called these "clausal quantity implicatures.")

3.5 Selective blocking of root Exh

Now consider our sentence (37) again. By paraphrase, this sentence asserts that Miles swam exactly n laps for some $n \geq 5$, (47)a. By (44), we now also get an obligatory non-necessity inference: we don't allow the output context, for any $n \geq 5$, to be made up exclusively of worlds where Miles swam exactly n laps, (47)b. Put the two together, and we get the possibility inference in (47)c.

(47) a. *Miles swam exactly n laps for some* $n \geq 5$.

b. $\rightsquigarrow \forall n \geq 5, \neg\forall w \in C'$, *Miles swam exactly n laps in w.*

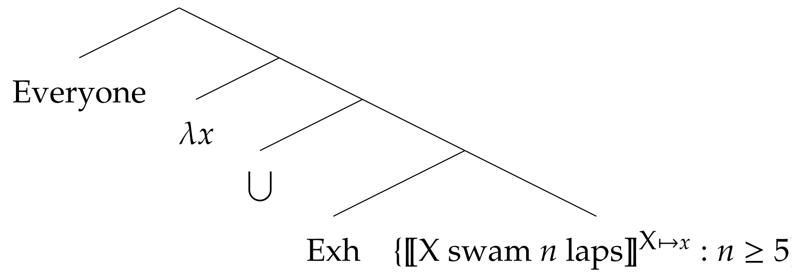
c. $\rightsquigarrow \forall n \geq 5, \exists w \in C'$, *Miles swam exactly n laps in w.*

We now need to see if the configuration (39) does allow the stronger alternatives in (40) to be ruled out. But it's easy to see that the negation of the first stronger alternative (*Miles swam exactly n laps, for some* $n \geq 6$) clashes with (47). The same obtains for any stronger alternative. This, in turn, is what I argue blocks Exh from being inserted high—post-existential closure—in this instance.

What of the quantificational case in (38)? First, a quantifier-raised structure like the one in (48) is needed to get an interpretable proposition. This gives us a meaning paraphrasable as (49)a. By reasoning in a way parallel to (47), we obtain the postsupposition that for every x and every $n \geq 5$, the output context contains at least one world where x swam exactly n laps, (49)c.

(48) Everyone swam at least 5 laps.

$$\lambda w.\forall x, \exists p \in \{exh([\![x \text{ swam 5 laps}]\!]), exh([\![x \text{ swam 6 laps}]\!]), ...\}, p(w)$$



(49) a. *For every x, x swam exactly n laps for some n* $\geq$ 5.

b. $\rightsquigarrow \forall x, \forall n \geq 5, \neg\forall w \in C'$, *x swam exactly n laps in w.*

c. $\rightsquigarrow \forall x, \forall n \geq 5, \exists w \in C'$, *x swam exactly n laps in w.*

Negating the first stronger alternative (*For every x, x swam exactly n laps for some n* $\geq$ 6) does not contradict this requirement, nor does negating *any* alternative. Thus, we predict that root insertion of Exh is possible in exactly the cases where the alternative-introducing expression (i.e., the scalar modifier) is found under the scope of a universal quantifier. In other words, global implicatures are possible when a quantifier scopes over the sentential existential closure operator $\bigcup$.

4 'Only'

At this point, we are finally ready to revisit the 'only' data, repeated here in (50).

(50) a. *Only at least 5 swam.

b. Only at most 5 swam.

Remember the key technical proposal of this paper. Modified degrees are sets of degree predicates, and global implicatures do not arise in unembedded contexts because exhaustification is allowed to apply pointwise to each of these degree predicates. Pointwise exhaustification here then serves the conceptual role of local exhaustification in a non-Hamblin framework. This partially solves our original puzzle: it's no longer surprising that 'only' can associate with modified degrees, given that Exh also associates with these degrees. What remains to be explained is the asymmetry between lower-bounded and upper-bounded modified degrees with respect to this fact. As is apparent in (50), it's still the case that 'only at least N' is categorically ill-formed.

I appeal to the intuition, first discussed in §1, that (50)a and (50)b receive the paraphrases (51)a and (51)b, respectively. These paraphrases are both plausible and make the problem with (50)a apparent.

(51) a. The number of laps swum by Miles is i) equal to n for some $n \geq 5$, and ii) *as such,* low relative to an expectation. [☠]

b. The number of laps swum by Miles is i) equal to n for some $n \leq 5$, and ii) *as such,* low relative to an expectation. [✔]

The key to accounting for the contrast will be to say that 'only,' in a way completely parallel to Exh, can be inserted below sentential closure to compose pointwise with Hamblin alternatives. We can make the simplest assumption about the meaning of 'only,' and view it as a presuppositional version of Exh, (52). The nature of the relevant presupposition has been the subject of debate in the literature. Although I take 'only P' to presuppose P, nothing in the present proposal makes it incompatible with alternative views regarding the particle's presupposition.

(52) $[\![\text{only}]\!] = \lambda p : \forall w' \in C, p(w').\lambda w.exh(p)(w)$

With this in mind, we are in position to see what happens when we substitute 'only' for Exh in the derivation of our sentences. Starting with grammatical (50)b:

(53) a. $[\![\text{at most 5 swam}]\!] = \{\lambda w.\exists x, |x| = n \wedge swam'(x)(w) : n \leq 5\}$

b. $[\![\text{only}]\!] \times$ (53)a $= \{\lambda w : \exists x, |x| = n \wedge swam'(x)(w).$
$exh(\lambda w.\exists x, |x| = n \wedge swam'(x)(w)) : n \leq 5\}$

We have now generated a set containing a presuppositional proposition for every $n \leq 5$.

What is supposed to happen beyond that is largely dependent on the theory of presupposition projection we adopt. Judgements regarding projection remain murky in many ways, and I shall not discuss them in detail. However, we can assess the predictions made by our existential closure operator $\bigcup$. First, notice that the asserted content of $\bigcup$(53)b, in and of itself, fails to pass on any presupposition. This is shown in (54); (54) denotes a complete function over the domain of worlds.

(54) *Asserted content of* $\bigcup$(53)b

$\bigcup(53)\text{b} = \lambda w.\exists p \in (53)\text{b}, p(w)$

However, recall that we have built in some definedness conditions for the existential closure operator in §3.4. This non-asserted content can be broken down into two conditional statements. The first statement makes a necessity claim for $\bigcup$(53)b, (55). We get (56) as a corollary, by the following reasoning: for any presuppositional proposition p to be entailed by the output context C', its presupposition q must also be entailed by C'. q, in turn, must therefore be entailed either by the input context C, or the assertion p such that $C \cap p = C'$. We can rule out the latter case. Which leaves us with $C \subseteq q$. By assumption, $q(w) \leftrightarrow w \in Dom(p)$, so $\forall w \in C, w \in Dom(p)$.

(55) *One definedness condition of* $\bigcup$(53)b

$\bigcup$(53)b defined only if $\forall w \in C', \exists p \in (53)\text{b}, p(w)$

(56) *Corollary of (55)*

$\forall w \in C, \exists p \in (53)\text{b}, w \in Dom(p)$

(56) is sufficient to enforce existential presupposition projection. It ensures that $\bigcup$(53)b will be defined only when (53)b contains at least one defined proposition. This corresponds to the behavior of disjunction in a strong Kleene trivalent semantics. So $\bigcup$(53)b ends up presupposing that someone swam, and asserting that exactly n swam for some $n \leq 5$, an

informative update.[18]

Let us see what happens with ill-formed (50)a.

(57) a. ⟦at least 5 swam⟧ $= \{\lambda w.\exists x, |x| = n \wedge swam'(x)(w) : n \geq 5\}$

b. ⟦only⟧× (57)a $= \{\lambda w : \exists x, |x| = n \wedge swam'(x)(w).$

$exh(\lambda w.\exists x, |x| = n \wedge swam'(x)(w)) : n \geq 5\}$

By (56), $\bigcup$(57)b presupposes that at least 5 swam, and it asserts that exactly n swam for some $n \geq 5$. But this is clearly a contextual tautology, since what is asserted is exactly was is presupposed. I argue that this is the cause of (50)a's ill-formedness.

(58) a. *Only at least n*: presupposes m for some $m \geq n$, asserts exactly m for some $m \geq n$ [☠]

b. *Only at most n*: presupposes m for some $m \leq n$, asserts exactly m for some $m \leq n$ [✔]

5 Comparative modifiers

The empirical picture for superlative scalar modifiers, summarized in (59) has now been fully accounted for.

(59)

	Low Exh $\bigcup >$ Exh	Low 'only' $\bigcup >$ only	Root Exh Exh> $\bigcup >$ Exh	
			$\exists/\emptyset$	$\forall$
'at least'	✓	*	*	✓
'at most'	✓	✓	*	✓

In this last section, I discuss the data from comparative modifiers. These data, which I

[18]One problem with this story is that 'at most 5' is clearly compatible with a situation where no one swam. This problem, noted by many authors (cf. Krifka 1999) can be avoided if we allow a metaphysics where entities can have the cardinality zero.

present below in §5.1, diverge from (59), and so at first glance look like they're muddying the water. Nevertheless, I propose that comparative and superlative modifiers can be treated in a very similar fashion. In particular, I claim that they all involve the alternative cannibalizing mechanism introduced in §3.1, and that the full paradigm for scalar modifiers is as in (60).

(60) a. $[\![\text{at least}]\!] = \{\lambda P.\lambda Q \in Alt(P).P \leq Q\}$

b. $[\![\text{at most}]\!] = \{\lambda P.\lambda Q \in Alt(P).P \geq Q\}$

c. $[\![\text{more than}]\!] = \{\lambda P.\lambda Q \in Alt(P).P < Q\}$

d. $[\![\text{less/fewer than}]\!] = \{\lambda P.\lambda Q \in Alt(P).P > Q\}$

This goes against the received wisdom (Geurts & Nouwen 2007, Büring 2007, Fox & Hackl 2006), which says that the two classes of modifiers should receive radically different analyses with respect to their behavior with SI.

5.1 Empirical landscape II

The most striking difference between superlative and comparative modifiers, as noted by Geurts & Nouwen, is that in unembedded contexts, the former *obligatorily* imply ignorance/epistemic uncertainty, while the latter seem to only imply it *optionally*. Assuming that the speaker knows that pentagons have exactly 5 sides as a matter of definition, (61)b sounds weird, while (61)a can still be uttered felicitously.

(61) a. A pentagon has more than 3 sides.

b. ??A pentagon has at least 4 sides.

$\rightsquigarrow \Diamond\neg$(A pentagon has exactly 5 sides.)

At a first approximation, this looks like something we could attribute to the difference between conventional meaning on the one hand, and pragmatic meaning on the other. The account that was developed in §3 builds those inferences right in the meaning of the sentential closure operator, as definedness conditions. They are therefore conventional meanings predicted to arise whenever $\bigcup$'s argument is a non-singleton Hamblin denotation.[19] Accounting for (61), and for all other data points I'm about to discuss, will be a matter of exempting comparative modified degrees from these non-necessity inferences.

5.1.1 Scalar implicature

The second observation is the following. Although comparative modified degrees tend to resist SI, it is possible, modulo a discourse rich enough to provide discrete alternatives, for unembedded comparative modified degrees—unlike superlative modified degrees—to be the target of SI.

(62) a. A: If you swim 5 laps or less, you're a poor swimmer, and if you swim 10, congratulations, you're with the average.
B: Miles swam more than 5 laps. $\rightsquigarrow$ $\neg$(Miles swam 10 laps.)
b. A: If you swim less than 5 laps, you're a poor swimmer, and if you swim 10, congratulations, you're with the average.
B: Miles swam at least 5 laps. $\not\rightsquigarrow$ $\neg$(Miles swam 10 laps.)

This is shown perhaps more naturally when the discrete degree alternatives are tied to human referents. Thus, if Miles swam 5 laps, and Bertha swam 10:

(63) a. Sid swam more laps than Miles.
$\rightsquigarrow$ $\neg$(Sid swam more than Bertha.)
b. Sid swam at least as many laps as Miles.
$\not\rightsquigarrow$ $\neg$(Sid swam at least as many as Bertha.)

[19] But recall that superlative modified degrees don't always come with an ignorance inference. Precisely in those grammatical environments where they are subject to SI, i.e. under universal quantification, no ignorance inference need arise. The account just developed gets this observation right, since the modal inferences are not intrinsically tied to epistemic modality.

(i) a. Any such shape has more than 3 sides.
b. Any such shape has at least 4 sides.

5.1.2 'Only'

Behavior with 'only' is also different. First, as shown in (64), 'only' is compatible with 'more than N' modified degrees, which lack an upper bound. Again, if Miles swam 5 laps, and Bertha swam 10, then (64) is possible.[20] This reading requires prosodic prominence on the degree focus 'Miles,' which I mark by capitalization.

(64) Sid only swam more laps than MÍLES. $\rightsquigarrow$ ¬(Sid swam more than Bertha.)

Second, 'only fewer than N' is also well-formed, and is in fact ambiguous. Under one reading, (65)a, we get the global implicature unavailable with 'only at most N.' (65)a shows scale reversal: the standard/expectation is that Sid should/would swim *even fewer* laps than he did. This reading, which I call the "reverse scale" reading, is realized with prosodic prominence on the degree focus. This is to be contrasted with the "congruent" reading in (65)b, whose prominence pattern is noticeably neutral in comparison.[21] The congruent reading should be familiar by now. I argue that it is essentially the reading available with 'only at most N.' Here the standard/expectation is that Sid should/would swim *more* laps than he did.

(65) a. A: Who swam more laps than Sid? [reverse scale reading]
B: Sid only swam fewer laps than BÉRTHA.
$\rightsquigarrow$ Sid did better than everyone but Bertha.
b. A: Miles swam 5 laps, and Bertha swam 10. [congruent reading]
But what about Sid?
B: Sid only swam fewer laps than Miles.

---

[20]Even without knowing who swam how many laps, (64) provides enough information to deduce that Sid finished second to last.

[21]Tentatively, this could be indicative of a difference in F-marking:

(i) a. reverse scale reading: only$_i$ less than$_j$ $[[\mathrm{N}]_{F_j}]_{F_i}$
b. congruent reading: only$_i$ [less than$_j$ $[\mathrm{N}]_{F_j}]_{F_i}$

$\rightsquigarrow$ Sid did poorly—even more so than Miles.

Notice that this could only be so, if this standard is to be equated with 'Miles' (5 laps) since 5 laps itself is the lowest contextually salient degree available. Picking an unreasonable standard for 'only' usually achieves a sarcastic effect (Klinedinst 2005), e.g. when I tell you that I "only" spent ten dollars on my cup of coffee. The possibility of sarcastic readings is intrinsic to the congruent reading. I mark this reading with ± in (66)a. On the reverse scale reading, however, picking a standard on the wrong end of the scale is downright impossible, as shown in (66)b.

(66) Context: Miles swam 5 laps, and Bertha swam 10.

a. Sid only swam fewer laps than {BÉRTHA/±Bertha}.

✓ reverse scale; ± congruent (sarcastic)

b. Sid only swam fewer laps than {*MÍLES/Miles}.

* reverse scale; ✓ congruent

I propose the following: just like Exh, 'only' can be inserted at two different syntactic levels, namely below or above the sentential closure operator (at the root). With upper-bounded 'less/fewer than N,' low insertion results in the congruent reading, while root insertion results in the reverse scale reading. With 'more than N,' only root insertion is possible. Simply put, I argue for the picture in (67).

(67)

	Low Exh $\bigcup$ > Exh	Low 'only' $\bigcup$ > only	Root Exh Exh> $\bigcup$ > Exh $\exists/\emptyset$	 $\forall$	Root 'only' only > $\bigcup$ > Exh
'at least'	✓	*	*	✓	*
'at most'	✓	✓	*	✓	*
'more than'	✓	*	✓	✓	✓
'less than'	✓	✓	✓	✓	✓

5.2 Why comparatives are more versatile

The phenomenon of scale inversion now gets a natural explanation. Quite expectedly, we get the effect whenever exhaustification is allowed to operate over flat upper-bounded denotations, where the degree focus is in a monotone decreasing environment. In effect, the readings we get with root exhaustification are essentially the ones we initially set out to prevent, to honor Krifka (1999)'s original observation.

But what is it that makes superlative and comparative modified degrees different? Remember the key discovery of §5.1, summarized in (67): sentences with comparative modified degrees possess all the readings of their superlative counterparts, but have the *additional* readings generated by root exhaustification. The question we then need to ask is why comparative modifiers allow root exhaustification where superlative ones don't. We already know what blocks root insertion of Exh in the general case: the definedness conditions of $\bigcup$ as I've implemented them in §3. These definedness conditions are non-necessity postsuppositions, restrictions on the output context. If comparative modified degrees were exempt from these restrictions, this would give Exh and 'only' exactly the distribution that we find in (67).

The proposal is the following. First, there is, available in English, a second sentential closure operator that's minimally different from $\bigcup$ in *not* imposing a non-necessity requirement. This new, laxer version of the operator I'll call $\bigcup^{\mathsf{K}}$.[22] The two operators are contrasted in (68)

(68) *Typology of declarative sentential closure operators*

a. $[\![\bigcup]\!] \;=\; \lambda\phi : \forall\phi' \subseteq \phi, (\forall w \in C', \bigcup\phi'(w)) \leftrightarrow \phi' = \phi.\lambda w.\exists p \in \phi, p(w)$

b. $[\![\bigcup^{\mathsf{K}}]\!] \;=\; \lambda\phi : \forall w \in C', \bigcup\phi(w).\lambda w.\exists p \in \phi, p(w)$

Second, superlative modifiers have the syntactic property that they must be found under

[22]This is a reference to Hintikka (1962)'s epistemic certainty operator K. The idea is that $\bigcup^{\mathsf{K}}$ allows the speaker to use degree modification even with a precise degree in mind.

the scope of $\bigcup$, while comparative modifiers are indifferent as to the type of sentential closure operator used. This type of proposal is not new in the Hamblin semantics literature. Because the quantificational force of expressions that introduce Hamblin alternatives is not local, it's a consequence of this view that the connection between the morphological form of these expressions and the source of the quantification (i.e. the operator) needs to be mediated somehow. Already in Kratzer & Shimoyama (2002), the use of uninterpretable syntactic features was suggested to account for the Latvian indefinite series, and German 'irgen-' series. Yanovich (2005), who develops an account of the Russian indefinite series, adopts such a strategy.[23]

Another area where syntactic selection could be needed is in the context of disjunctive questions. 'P or Q?' has two interpretations, differentiated by intonation: as a polar question (where the expected answer is yes or no), and as an alternative question (where the expected answer is one of P or Q). It's natural and desirable not to attribute the source of this interpretative difference to the proper meaning of the coordinator 'or,' but to some other mechanism. Some authors have identified this mechanism with covert scoping (Larson 1985), and ellipsis (Pruitt & Roelofsen 2013). Another option is that each interpretation reflects the presence of a distinct question operator at the root. Some languages like Basque and Arabic morphologically track the distinction between polar and alternative questions (Haspelmath 2007). What is perhaps surprising is that these languages choose to morphologize this contrast locally on the coordinator, and not at the level at which we understand the quantification to take place. A syntactic story here is natural: we can say that the question operator is a silent probe, regulating the flavors of 'or' allowed to appear in its scope.

To sum up, the idea is that the superlative modifiers' limited distribution is due to the fact that they can only appear under the scope of $\bigcup$, while comparative modifiers are free

[23]Note also the standard analysis of Negative Concord, and Beghelli & Stowell (1997)'s approach to distributivity.

to appear under $\bigcup$ or $\bigcup^{\mathsf{K}}$. In principle, any Hamblin alternative-introducing expression could pattern in one way or the other. This proposal thus has leverage when it comes to accounting for potential cases of variation, and in fact predicts such variation to be possible. In other words, the cells in (69) are empty in English, but only accidentally so. Because the difference between superlative and comparative modifiers comes down to a syntactic feature, a simple change could make 'more than N' pattern like 'at least N,' and conversely. This kind of variation is completely in line with the diachrony of other expressions sensitive to their "semantic" environment, like negative polarity items.

(69)

	$\geq$	$>$	$\leq$	$<$
Inference	at least N		at most N	
No inference		more than N		fewer than N

Quite generally, we predict there to exist, for any Hamblin alternative-introducing expression, what we could call a free-choice version and a regular version of this expression. This opens up the way to a new research program: to identify such (intra- and cross-linguistic) pairs of expressions. This, like the idea of an exhaustivity operator Exh, constitutes a partial formalization of the Maxim of Quantity, because the modal inferences are elevated to the status of conventional content. One methodological consequence of this view is that even meanings that look obvious on the surface like 'or' could turn out to differ on this simple dimension. Take the case of disjunction, which in English and a lot of familiar languages is inherently a free-choice expression. (Under the present system, this means that it needs to appear under $\bigcup$.) Now imagine being faced with a language whose word for 'or' differs minimally in not being a free-choice expression (which means its host clause can be closed with $\bigcup^{\mathsf{K}}$). In this language, the range of uses for 'or' will necessarily be wider, and include uses which will—to the untrained eye—appear like violations of the Cooperative Principle. The present view invites a (re)analysis of Quantity anomalies. We no longer have to call into question the universality of Gricean reasoning (pace Keenan 1976), or blame a difference in cultural norms. At the very least, we have reclaimed a corner of the pragmatics wastebasket, something I take to be a major advantage for the view.

5.3 Taking stock

The goal of this paper was to develop a theory of the distribution of scalar implicature and 'only' in sentences that include modified degrees. In §3 and §4, I achieved this for superlative modified degrees. In this last chapter, it was shown that comparative modified degrees, which have a wider distribution, can be reconciled with the theory, by appealing to syntax to constrain distribution. At last, the information contained in (70) is sufficient to capture the behavior of every major scalar modifier with respect to its interaction with SI and 'only.' Ultimately, the picture we have is one where syntax and semantics conspire to determine the nature of this interaction.

(70) *The meaning and syntactic distribution of scalar modifiers*

	Scope of $\bigcup$	Scope of $\bigcup^{K}$	Meaning
'at least'	✓	*	$\{\lambda P.\lambda Q \in Alt(P).P \leq Q\}$
'at most'	✓	*	$\{\lambda P.\lambda Q \in Alt(P).P \geq Q\}$
'more than'	✓	✓	$\{\lambda P.\lambda Q \in Alt(P).P < Q\}$
'less than'	✓	✓	$\{\lambda P.\lambda Q \in Alt(P).P > Q\}$

6 Conclusion

This paper was concerned with the distribution of 'only' and scalar implicature in sentences with modified degrees. We started with a distributional puzzle. Under any grammatical theory of SI, the semantics of 'only' and the exhaustivity operator Exh gets us the distributional implication in (71), something that's been called the "*Only* Implicature Generalization." Yet, it appears on first pass that 'only' has a wider distribution, since it is compatible with upper-bounded modified degrees like 'at most N.'

(71) $\boxed{\text{'Only' environments}} \subset \boxed{\text{Exh environments}}$

I proposed to solve this puzzle by saying that exhaustification and 'only' in a sense apply "locally," which means that no global implicature arises. This effect is elegantly achieved once we adopt a Hamblin framework, and assume that 'only' and Exh can appear below sentential closure to compose via pointwise functional application. A novel theory of the meaning of scalar modifiers was deployed to do just this. Under this new theory, scalar modifiers cannibalize the focal alternatives of their argument into the main denotation. This proposal establishes a strong structural connection between disjunction and scalar modification, and derives the equivalence of modified and disjunctive degrees. Because of this, we can say that it preserves the spirit of the Büring–Sauerland–Fox theory, which relies on a disjunctive paraphrase for modified degrees. Definedness conditions were built in the meaning of the sentential closure operator $\bigcup$, so as to block the insertion of Exh and 'only' *above* it (at the root level) in the majority of cases. This blocking is selective, and the definedness conditions still allow for root exhaustification when the modified degree is found under the scope of universal quantification, correctly deriving a global implicature in this case.

Finally, it was argued that a second declarative sentential closure operator $\bigcup^{\mathsf{K}}$ is available, but that only comparative—and not superlative—scalar modifiers can be found in its scope, as a matter of syntactic selection. This operator is minimally different in its definedness conditions, and imposes no non-necessity requirement, which in turn allows root exhaustification across the board. This grants comparative modifiers their wider distribution, and in particular makes them a viable target to SI. Under the Büring–Sauerland–Fox analysis, the very reason superlative modifiers mean what they mean at the core ($\geq$, $\leq$) is the reason why they come with their familiar modal inference. In contrast, the stance taken in this paper entails that this particular juxtaposition of meanings is subject to syntactic arbitrariness. The framework developed says that any alternative-introducing expression—like disjunctive coordinators, scalar modifiers, and indefinites—

can be lexicalized with, or without a free-choice component. This generalized notion of free-choice provides well-needed relief for a tired Maxim of Quantity, and invites a new research program: can we find more manifestations this type of variation, both from an intra- and cross-linguistic point of view?

7 References

Alonso-Ovalle, Luis. 2006. Disjunction in Alternative Semantics. Dissertation, University of Massachusetts, Amherst.

Alonso-Ovalle, Luis. 2008. "Innocent Exclusion in an Alternative Semantics." Natural Language Semantics 16(2).

Beaver, David. 1992. "The kinematics of presupposition. In P. Dekker & M. Stokhof, eds., Proceedings of the 8th Amsterdam Colloquium." ILLC.

Beghelli, Felipe, and Timothy Stowell. 1997. "Distributivity and negation: The syntax of each and every." In A. Szabolcsi, ed., Ways of Scope Taking. Kluwer.

Brasoveanu, Andreas, and Anna Szabolcsi. 2013. "Presuppositional too, postsuppositional too." In M. Aloni, M. Franke & F. Roelofsen, eds., The dynamic, inquisitive, and visionary life of ϕ, ?ϕ, and $\Diamond\phi$. A festschrift for Jeroen Groenendijk, Martin Stokhof, and Frank Veltman.

Büring, Daniel. 2007. "The least at least can do." West Coast Conference on Formal Linguistics (WCCFL) 26.

Chierchia, Gennaro, Danny Fox, and Benjamin Spector. 2008. "The grammatical view of scalar implicatures and the relationship between semantics and pragmatics." In P. Portner, C. Maienborn & K. von Heusinger, eds., Handbook of Semantics. Mouton de Gruyter.

Coppock, Elizabeth, and Thomas Brochhagen. 2013. "Raising and resolving issues with scalar modifiers." Semantics and Pragmatics 6.

Farkas, Donka. 2002. "Varieties of indefinites." In B. Jackson, ed., Proceedings of SALT 12. Cornell University.

Fox, Danny. 2007. "Free choice and the theory of scalar implicatures." In U Sauerland & P. Stateva, eds., Presupposition and Implicature in Compositional Semantics. Palgrave.

Fox, Danny, and Martin Hackl. 2006. "The universal density of measurement." Linguistics and Philosophy 29.

Gazdar, Gerald. 1979. Pragmatics: Implicature, Presupposition, and Logical Form. Academic Press.

Geurts, Bart, and Rick Nouwen. 2007. “At least et al. The semantics of scalar modifiers.” Language 83(3).

Haspelmath, Martin. 2007. “Coordination.” In T. Shopen, ed., Language typology and syntactic description vol. II: Complex constructions. Cambridge University Press.

Hamblin, C. L. 1973. “Questions in Montague English.” Foundations of Language 10.

Hintikka, Jaakko. 1962. Knowledge and Belief. Cornell University Press.

Hurford, James. 1974. “Exclusive or Inclusive Disjunction.” Foundations of Language 11.

Keenan, E. O. 1974. “The Universality of Conversational Postulates.” In R. W. Fasold & R. W. Shuy, eds., Studies in Linguistic Variation. Georgetown University Press.

Klinedinst, Nathan. 2005. Scales and Only. MA Thesis, UCLA.

Kratzer, Angelika, and Junko Shimoyama. 2002. “Indeterminate pronouns: The view from Japanese.” In Yukio Otsu, ed., Proceedings of the third Tokyo conference on psycholinguistics. Hituzi Syobo.

Krifka, Manfred. 1999. “At least some Determiners aren’t Determiners.” In Current Research in the Semantics/Pragmatics Interface 1.

Lauer, Sven. 2009. Free relatives with -ever. Ms.

Larson, Richard K. 1985. “On the Syntax of Disjunction Scope.” Natural Language and Linguistic Theory 3.

Meyer, Marie-Christine. 2014. “Grammatical Uncertainty Implicatures and Hurford’s Constraint.” Handout. SALT 2014. New York University.

Pruitt, Kathryn, and Floris Roelofsen. 2013. “The interpretation of prosody in disjunctive questions.” Linguistic Inquiry 44(4).

Rooth, Mats. 1985. Association with focus. Dissertation, University of Massachusetts, Amherst.

Rooth, Mats. 1992. “A theory of focus interpretation.” Natural Language Semantics 1(1).

Sauerland, Uli. 2004. “Scalar implicatures in complex sentences.” Linguistics and Philosophy 27.

Spector, Benjamin. 2006. Aspects de la pragmatique des opérateurs logiques. Dissertation, Paris 7 - Denis Diderot.

Veltman, Frank. 1990. “Defaults in Update Semantics.” In H. Kamp, ed., Conditionals, Defaults and Belief Revision, DYANA Report R2.5.A. Centre for Cognitive Science, University of Edinburgh.

Yanovich, Igor. 2006. “Choice-functional series of indefinites and Hamblin semantics.” In E. Georgala & J. Howell, eds., Proceedings of Semantics and Linguistic Theory (SALT) 15. Cornell Linguistics Publications.