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# The polysemy of measurement ${ }^{\text {™ }}$ 

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#### Abstract

The first goal of this paper is to argue that a number of independently treated phenomena - the 'measure' interpretation of pseudopartitives (Landman, 2004), amount relatives (Heim, 1987; Grosu and Landman, 1998), the how many ambiguity - are different instantiations of the same phenomenon, the general ability for DPs to denote an individual or a degree corresponding to the measure of that individual. I refer to this as 'individual/degree polysemy'. I show that a particular semantic restriction on the degree interpretations of DPs indicates that the degree interpretation is derived from the individual interpretation (not vice-versa). And I argue that this pervasive polysemy is a natural consequence of degree semantic theories that postulate a null measure operator to measure, when appropriate, individuals, events or degrees. The second goal of this paper is to tie the behavior of this null measurement operator to the similar behavior of quantity adjectives like many and much, giving further support to the claim that quantity adjectives measure sets of degrees (Rett, 2007, 2008). © 2014 Elsevier B.V. All rights reserved.


Keywords: Polysemy; Degree semantics; Monotonicity; Pseudopartitives; Quantity adjectives; Amount relatives

## 1. Introduction

The idea that some DPs can denote degrees instead of individuals is a familiar one. Landman (2004) and Rothstein (2009) observe that pseudopartitives can come to denote a measure instead of an individual with a particular measure, and Heim (1987) and Grosu and Landman (1998) observe that a particular sort of relative clause ('amount relatives') can do the same. The broader empirical goal of this paper is to argue that these prima facie independent phenomena are in fact different manifestations of a general individual/degree polysemy, illustrated in (1) and (2).
a. Four pizzas are vegetarian.
individual
b. Four pizzas is more than we need.
degree
(2) a. Many guests are drunk.
b. Many guests is several more than Bill anticipated.
individual degree

I will give evidence for this claim, and discuss its connection to previous work, in section 2 . In section 3 , l'll argue that the ability of DPs to denote degrees is semantically constrained in a particular way: the relevant dimension of measurement must be monotonic on the part-whole structure of the individual, a restriction independently observed in pseudopartitives (Schwarzschild, 2006b).

[^0]The goal of the analysis in section 4 is to provide a semantics of how DPs come to denote degrees, and to explain why these interpretations are subject to a monotonicity restriction. Schwarzschild (2006b) attributed monotonicity in pseudopartitives to a syntactic linking rule; I instead argue that individual/degree polysemy - and the monotonicity restriction - is a natural consequence of the way measurement is treated in natural language. The thesis, developed from the proposal of measurement in Cresswell, 1976, is that natural language employs a null measurement operator, or something equivalent, that can optionally map an entity to its measure provided that the two are related in an informative way. I highlight parallels with the phenomenon of deferred reference, which is similarly restricted in terms of informativity.

The analysis has clear consequences for theories of polysemy as well as degree-semantic theories of measurement. In section 5 I argue that it also provides interesting support for the characterization of quantity adjectives like many and much as overt instantiations of this measurement operator, in particular as degree modifiers, denoting a relation between a set of degrees and its measure (Rett, 2007, 2008). The result is a relatively uniform degree-semantic treatment of a variety of apparently diverse constructions that involve measurement, including pseudopartitives, relative clauses, constructions with many and much, and the data in (1) and (2).

## 2. The degree interpretations of DPs

I'll begin by motivating my claim that this individual/degree polysemy is pervasive.

### 2.1. Previous discussion of individual/degree polysemy

A word or phrase is polysemous if it receives two or more interpretations that are related to one another in a systematic way (Apresjan, 1974; Pelletier, 1975; Krifka, 1998). (3) and (4) illustrate two canonical types of polysemy.
(3) a. The window is cracked. figure
b. Mary crawled through the window. ground
(4) a. John smashed the bottle of wine. container
b. John drank the bottle of wine. substance

In (3), the window can denote either a pane of glass (the figure) or the frame around the window or where the window generally is (the ground). In (4) the DP the bottle of wine exhibits 'container polysemy': it can denote a container or the substance it contains. l'll refer to these DPs as pseudopartitives; they're formed with a measure noun ('MN,' e.g. bottle) and the partitive morpheme (of). (Partitives are like pseudopartitives except in partitives, the second DP is definite, e.g. bottle of the wine.)

The two distinct readings are generally diagnosed by the selectional restrictions of the relevant predicates: panes of glass (but not open spaces) can be cracked while open spaces (but not panes of glass) can be crawled through. And so forth for wine and bottles of it. Depending on the language and the type of entities denoted, polysemy can also be detected via agreement, as Stavrou (2003) has shown for Greek.
(5) a. Iparhun /iparhi mis sira diavathmisis.
is-3PL .is-3sG a range-sG gradations-PL
'There is/are a range of gradations.'
b. Ena buketo luludja itan pesmen-o/ -a sto patoma. a bunch flowers PST is-3sG /is-3pL thrown on.the.floor
'A bunch of flowers was/were thrown on the floor.'
Depending on the number agreement of the predicate, Stavrou argues, the DP in (5a) can denote either a (single) range of gradations or the plurality of gradations composing that range; and the DP in (5b) can denote either a (single) bunch of flowers or the flowers themselves.

However, some predicates can occur with either polysemous interpretation, making the conditioning of polysemous phrases seem more contextually influenced. (6) is an example of this with standard (individual/individual) polysemy, and (7) is an example with individual/degree polysemy.
(6) John likes that bottle of wine.
(7) John ordered three pizzas.
individual/amount
It's this apparent influence of context that led Nunberg $(1979,1995)$ to argue that polysemy is a pragmatic, rather than semantic, phenomenon (likening it to metaphorical word use). Nunberg suggests that a DP is semantically associated
with an individual $x$, and that it can be pragmatically associated with a distinct but related individual $y$ as long as the relationship between $x$ and $y$ is obvious and noteworthy. He refers to this as a 'transfer of meaning' and considers it widely available.

Chierchia (1998), in contrast, treats the container/substance polysemy as a semantic phenomenon. He proposes that the substance meaning is derived from the container meaning via a type-shifter, illustrated in (4) (and repeated in (8)).
(8) a. John smashed the bottle of wine.
container
b. John drank the bottle of wine.
substance
Chierchia says, "Often the objects associated with [measure nouns] display the behavior of "containers" and are used to refer to their content, via either type-shifting devices or, possibly, lexical entailments of the relevant predicates" (p73). He proposes a null type-shifter $C$ from a container $x$ to its contained substance $y$ :
(9) a. John smoked two packs of cigarettes.
b. $[\lambda x($ smoke $(J o h n, C(x)))](2(p L($ pack $(\iota$ cigarettes $))))$, where ' $C$ ' maps a "container" into its "content"

The truth conditions in (9b) can be read informally as 'John smoked the contents of two packs of cigarettes'.
While I won't endorse any particular account of the container/substance polysemy, I am sympathetic with the assumption, advocated in Pustejovsky (1995) and Pustejovsky and Bouillon (1995), that a polysemy which exhibits syntactic or semantic restrictions should be analyzed as semantic. l'll discuss this more in what follows. For now, l'll introduce the empirical focus of the paper, individual/degree polysemy.

Landman (2004) and Rothstein (2009) observe a peculiar correlate of the container/substance polysemy, demonstrated in (8). When context and the selecting predicate forces a pseudopartitive to receive a container interpretation, as in (10), the DP triggers plural agreement. In contrast, when the pseudopartitive receives a substance interpretation, it can trigger either plural or singular agreement, as in (11).
(10) a. There are two cups of wine on this tray.
b. \#There is two cups of wine on this tray.
c. The two cups of wine on this tray are blue.
a. There are two cups of wine in this soup.
b. There is two cups of wine in this soup.
c. \#The two cups of wine in this soup are blue.

Landman concludes from the ability of the pseudopartitive in (11) to receive singular or plural number agreement that the substance interpretation of pseudopartitives is itself polysemous between the substance interpretation and a 'measure' interpretation. The true substance interpretation is available in (11a), signified by a combination of the predicate be in the soup and its plural agreement, while the 'measure' interpretation is available in (11b), signified by a combination of the predicate and its singular agreement. ((11c) shows that the pseudopartitive, in this context, cannot receive a container interpretation.)

Landman attributes the container/substance polysemy to the claim that measure nouns like cup or glass are lexically ambiguous. In one denotation, cup is a relational noun, denoting a relation between a substance $y$ and its container $x$, as in (12a) (see also Stavrou, 2003). In the second denotation, cup relates a quantity $n$ to an abstract subcomponent $x$ of a property $P$ (as in (12b)).

$$
\begin{array}{lrr}
\text { a. } & \llbracket \operatorname{cup} \rrbracket=\lambda y \lambda x \cdot \operatorname{cup}(x) \wedge \operatorname{contain}(x, y) & \text { container }  \tag{12}\\
\text { b. } & \llbracket \operatorname{cup} \rrbracket=\lambda n \lambda P \lambda x . P(x) \wedge \operatorname{mEAS}(x)=\langle n, \text { cupful }\rangle & \text { substance }
\end{array}
$$

Landman proposes that the substance interpretation has the constituency in (13a), while the degree interpretation has the constituency in (13b).
a. [DP three $i\left[\right.$ NumP $t_{i}[\mathrm{NP}$ glasses of [DP wine] $]$ ] $]$
individual
b. [DP [NP [Measp [Num three] [meas ${ }^{\circ}$ glasses off] [ N wine]]]
degree
Rothstein (2009) amends this analysis slightly based on some syntactic and semantic properties of measure phrase constructions in Modern Hebrew.

This is a semantic (as opposed to pragmatic) account of the ability of pseudopartitives to receive a degree interpretation. But it is a construction-specific one; the availability of the substance and measure interpretations is tied to the ambiguity of the measure noun and the syntax of the phrase.

However, the ability of a DP to denote a degree or measure has been observed elsewhere, in a certain subtype of relative clause (Carlson, 1977; Heim, 1987). An 'amount relative' can denote either an individual or a degree, depending on context and the selecting predicate, as in (14).
a. John drank [DP the champagne they bought that evening].
indiv.
b. It will take us the rest of our lives to drink [DP the champagne that they spilled that evening]. degree

While the relative clause in (14a) denotes an individual - champagne, a substance reading - the relative clause in (14b) doesn't; it is, our world knowledge tells us, unfeasible to drink spilled wine. The intuition is that the relative clause in (14b) instead denotes the quantity of wine they spilled. Grosu and Landman (1998) offer a syntactic and semantic account of these 'amount relatives' based in part on their similarity to other non-degree-denoting relative clauses. This account is similarly construction-specific.

Recently, Grosu (2009) has argued that degree-denoting relative clauses should receive an analysis independent from individual-denoting ones. He is interested in part in Romanian relative clauses, exemplified in (15).
(Cele) nouă kilograme cât cântăre ste bagajul tău de mână nu te vor (the) nine kilos how-much weighs luggage-the your of hand not you will-pL împiedica să te urci in avion. prevent sUBJ REFL climb-1.SG in plane
'(The) nine kilos that your hand luggage weighs won't prevent you from boarding the plane.
Grosu says of (15) (and similar examples) that the gap associated with the wh-phrase cât "is the internal argument of a predicate that selects degrees (on a scale that the predicate specifies)". His empirical interest is centered on the difference between definite DP relative clauses and bare nominal relative clauses. In Romanian, both can be degreedenoting, while in Hebrew in English, only the definite relative can occur with a degree predicate (cf. the English *(The) nine kilos your bag weighs...).

Grosu's analysis of the individual/degree polysemy of relative clauses involves a combination of the 'restricted degree' approach pursued in Grosu and Landman (1998) and something like a maximality operator. A more detailed version of this analysis can be found in Kotek (2011). Kotek assumes gradable predicates like weigh denote relations between degrees ordered along a scale and individuals, and in which measure phrases (MPs) like 9 kg denote sets of degrees equal to that measure (as in (16)).

## $\llbracket 9 \mathrm{~kg} \rrbracket=\lambda d . d=9 \mathrm{~kg}$

The denotation of the relative clause that your bag weighs - type $\langle d, t\rangle$ - combines with the denotation of the MP via set intersection (a degree version of predicate modification). This gives the denotation in (17).
(17) $\llbracket 9 \mathrm{~kg}$ that your bag weighs $\rrbracket=\lambda d$. weigh(your-bag, $d) \geq d \wedge d=9 \mathrm{~kg}$

When the relative clause does not occur with a definite determiner, Kotek assumes a process of existential closure, resulting in an indefinite interpretation ('there exists a degree such that...'). When it does occur with a definite determiner, Kotek assumes that the definite is interpreted as in (18).
(18) $\llbracket$ the $\rrbracket_{\langle\langle d, t\rangle, d\rangle}=\lambda f_{\langle d, t\rangle}$ : there is exactly one contextually salient $d$ : $f(d)=1$. the unique $d$ in the context such that $f(d)=1$.

This results in the denotation in (19), given the satisfaction of the uniqueness presupposition represented explicitly in (18).
(19) 【The 9 kg that your bag weighs】 = the unique $d$ in the context such that weigh(your-bag, $d) \geq d \wedge d=9 \mathrm{~kg}$

Kotek proposes that definite relative clauses can also denote a substance via the general process of deferred reference. She concludes that Romanian degree relatives and their English counterparts "can involve a range of references - to entities, degrees, paths, processes, facts, durations, achievements, etc. - distinguishable from the
"ordinary" amount reading of the degree relatives". She is agnostic about whether deferred reference should be treated as a semantic or pragmatic phenomenon.

The analysis in section 4 will share several of these assumptions: that predicates like that your bag weighs $t_{d}$ are predicates of degree; that individual/degree polysemy requires a mechanism like existential closure (see also Rett, 2008), and that the degree interpretations of DPs can involve degrees of measures other than amounts. It will differ in that it will cover much more empirical ground. While Grosu (1994) and Kotek (2011) attribute the degree interpretation of these amount relatives to the presence of MPs like 9 kg (which triggers the degree interpretation of the definite determiner), the analysis below attributes individual/degree polysemy to the way in which natural language treats measurement generally.

To summarize this section: there have, historically, been two distinct types of DPs that have been alleged to denote an individual or a degree, depending on the (linguistic) context: pseudopartitives like bottle of wine and amount relatives like the champagne they bought that evening. Each has been analyzed independently, and their formal treatment has been relatively distinct, with proposals involving lexical ambiguity, structural ambiguity, and/or coercion.

These theories all assume that the two readings of these constructions are differentiatable based on number/gender agreement and on the selectional properties of the predicate. In section 2.2 , I use these assumptions to argue that individual/degree polysemy is more general than these accounts have assumed. I begin the discussion by addressing more directly what constitutes a degree interpretation of a DP (and why).

### 2.2. Individual/degree polysemy generally

I consider the examples in (20) and (21), repeated from (1) and (2), to be canonical examples of individual/degree polysemy.
(20) a. Four pizzas are vegetarian/were eaten by the senators.
individual
b. Four pizzas is enough/is more than Bill asked for. degree
(21) a. Many guests are drunk/were arrested after the party.
individual
b. Many guests is more than Bill had anticipated.
degree
In addition to number agreement on and selectional restrictions of the predicate, we can differentiate between the two readings in terms of the pronominal anaphors they license (this test is adapted from Rothstein, 2009).

John bought four pizzas.
a. They were delicious. individual
b. It was more than we needed.
degree
I take the plausible selectional restrictions on these predicates, in combination (where possible) with verbal number agreement, to be evidence for the interpretation of these DPs. Importantly, I assume that the predicates used to bring out the degree interpretations - be too much, be enough and be more than - can function as predicates of degrees, reminiscent of Davidson's (1967) event predicates. ${ }^{1}$

While this move is precedented - see, for instance, Grosu and Landman (1998) and Grosu (2009) - l'll say a little more to motivate the idea that be more than Bill had asked for and be enough/sufficient are degree predicates in these constructions. First, they are not behaving like individual predicates; morphologically plural count nouns generally cannot trigger singular agreement on individual predicates (e.g. *Four pizzas is delicious).

Second, there is independent reason to think that, in these constructions, these predicates select for degrees: they can have numeral or MP subjects, as (23) shows (cf. *4 is delicious).

[^1]a. 6 is enough (friends) to get the group discount.
b. $60 z$ is enough wine for me!
c. (I know you have a lot of lemons from your garden, but) 10lbs is more than Bill asked for.
d. (For a healthy adult), 2,000 a day is a sufficient amount of calories.

I'll assume that a predicate compatible with an MP subject can select for degrees. Because, when it does so, the predicate receives singular agreement, and because the interpretations of (20b) and (21b) can be paraphrased in particular contexts with MP subjects, I assume that (20b) and (21b) are instances in which the DP denotes a degree of measurement. ${ }^{2}$

It's important to keep in mind that predicates like be enough and be sufficient invoke a standard of comparison that is in many cases covert (for extensive discussion and analysis, see Meier, 2003). A sentence like That couch is enough could, depending on the context of utterance, mean something like 'long enough for my brother to sleep on'; 'expensive enough to be a good wedding present'; 'big enough to fill the room'; or 'tacky enough to scare off the guests'. Each of these interpretations invoke different dimensions of measurement - length, price, area, and tackiness, respectively and the dimension of measurement a particular sentence invokes will be very important for the discussion to follow. I will assume that while an enough construction is in principle compatible with a number of different dimensions, only one dimension is salient in a given context. In what follows I will be as explicit as possible about which dimension I consider to be salient.

It also seems as though be enough can receive plural agreement; this suggests that it is ambiguous between an individual and a degree predicate, depending on the salient standard of comparison. The predicates in (24a) and (24b) are arguably agreeing with two different entities, but the sentences are nevertheless equivalent (presumably due to the one-to-one mapping from a plurality of drinks to a quantity of alcohol, which l'll discuss in more detail in section 3). ${ }^{3}$
a. Four drinks are enough (drinks) to get me drunk.
b. Four drinks is enough (alcohol) to get me drunk.

In addition to DPs formed with numerals or quantity adjectives, like the ones in (20) and (21), individual/degree polysemy extends to (pseudo-)partitives formed with MPs, as in (25). Notice that the degree interpretation isn't reliant on the presence of the definite article, in contrast to the English data discussed in Kotek, 2011.
a. Four feet of (the) plywood are warped. individual
b. Four feet of (the) plywood is more than three/than Bill asked for.
degree

The degree interpretation also isn't confined to DPs in subject position, as (26) demonstrates. However, in these examples, we can no longer rely on subject agreement to determine when a DP receives a degree interpretation. In (26), I rely on the to argument to make explicit the object's alternative.
(26) a. John prefers three square pizzas to four round ones.
b. John prefers three (square) pizzas to two.

Note that individual/degree polysemy is a distinct semantic phenomenon from specificity. That is, the sentence in (26a) can only receive an individual interpretation, and yet is ambiguous between a specific and a non-specific interpretation. It's compatible with there being four particular square pizzas John prefers to round ones - the specific interpretation - or it could be that he prefers four square pizzas, any four wheat pizzas, to round ones, the non-specific interpretation. In (26b), on the other hand, the quantity four is the object of John's desire or preference.

[^2](27) and (28) show that DPs with some overt determiners can be individual/degree polysemous:
(27) a. The three paintings he salvaged were damaged by the smoke.
individual
b. The three paintings he salvaged was enough to save the collection.
degree
(28) a. Sue accepted money from gun lobbies, like so many politicians before her.
individual
b. Sue will get in trouble for bringing so many politicians to the party. degree

But the sentences in (29) and (30) show that determiners that inflect for number constrain the possible interpretations of the DPs, with plural determiners (these) being compatible with only a plural individual interpretation, and singular determiners (this) only with a degree interpretation.
(29) a. These many children were advanced a grade.
individual
b. *These many children was more than expected.
\#degree
(30) a. These many guests asked for their coats at the same time. They had arranged for several cabs.
individual
b. This many guests asked for their coats at the same time. *They had arranged for several cabs.
\#individual
The degree interpretation seems to be available to quantificational DPs, as well, although the degree readings of some of these quantifiers seem more acceptable when they head a partitive construction.
a. We didn't manage to bring all of the cookies, but some (of the) cookies is more than they deserve.
b. We brought all of the cookies, but even all ??(of the) cookies is less than we need.

Even bare DPs can receive a degree interpretation, provided that context provides a salient dimension of measurement they can associate with. ${ }^{4}$
(32) a. Cheese doodles were eaten by the senators.
individual
b. Cheese doodles is not enough. The senators will need protein.
degree
Finally, individual/degree polysemy is also a property of wh-phrases, in relative clauses (as discussed) or in constituent questions.
a. How many books are on the table?
individual
b. How many books is too many? degree

This polysemy is arguably related to a phenomenon referred to as 'how many ambiguity' (Longobardi, 1987; Cresti, 1995; Romero, 1998), exemplified by (34).
(34) How many books must John read?

One reading of (34) (the object reading) corresponds to a situation in which John was told he has to read Book A, Book B and Book C. This reading of the question can be paraphrased as 'What is the number $d$ of books $x$ such that John must read $x$ ?'. The amount reading of (34), on the other hand, corresponds to a situation in which John was told that he has to read three books, any three books (imagine he's taking a speed-reading course). This reading of the question can be paraphrased as 'What is the number $d$ of books such that the numeral of books John must read is $d$ ?'.

The readings correspond to the two answers in (35), which can be seen as corresponding to an individual and degree interpretation of the wh-phrase, respectively.
(35) a. The books John must read are $A, B$ and $C$.
b. The number of books John must read is 3 .

[^3]It's been assumed that the two meanings arise from some component of how many books scoping either above (the object reading) or below (the amount reading) the modal must. Romero (1998), based on earlier analyses in e.g. Cresti (1995), accounts for the readings in terms of the scope of the modal must and the individual existential quantifier associated with the plurality books. In her formal account, the existential quantifier is contributed by the quantity adjective many; while it raises overtly with the wh-phrase in English, it can be reconstructed semantically. In the larger context of individual/degree polysemy - including the data in (35b) - these readings call for a less restrictive analysis.

To sum up: DPs formed with plural count nouns can nevertheless receive singular number agreement with certain predicates. These predicates - be enough, be sufficient, be more than Bill asked for - are, in certain contexts, degree predicates, selecting for degrees of measurement instead of atomic individuals. In the next section, l'll introduce a semantic restriction on the availability of the degree reading that will form the basis for the semantic account of individual/ degree polysemy presented in section 4.

## 3. Polysemy and monotonicity

There is a particular semantic restriction on the degree interpretation of DPs. The result is that any DP can denote a degree, but only in certain contexts and with certain interpretations. An account of this the restriction (and the corresponding asymmetry between the individual and degree interpretations) is a clear goal for any theory of individual/ degree polysemy.

The relevant semantic restriction is monotonicity on the part-whole structure. I'll first introduce the concept of monotonicity using Schwarzschild's (2006b) observations about pseudopartitives; I'll then show how it plays a role in individual/degree polysemy.

### 3.1. Monotonicity and MPs

Schwarzschild (2002, 2006b) observes an interpretational difference between measure phrases (MPs) used attributively and those in pseudopartitives:
a. three gallons of water
b. *three-gallon water
partitive attributive
a. *18 karats of gold partitive
b. 18-karat gold attributive
a. two inches of cable
partitive; length
b. two-inch cable attributive; diameter

Schwarzschild argues that the (un)grammaticality of the constructions in (36) and (37) has the same source as the interpretational restrictions on the same sorts of constructions demonstrated in (38). In particular, he argues that the pseudopartitive configuration requires that its dimension of measurement be monotonic on the (salient) part-whole structure of the individual it is measuring, while the attributive configuration requires that the dimension of measurement be non-monotonic. He describes monotonicity as follows (p.73):
"If you have a pile of cherries, it has a certain weight. Take some of the cherries away, the weight goes down; add some cherries to the pile and the weight goes up. By contrast, you can add cherries without changing their temperature, their weight per unit, or their color. Weight tracks the part-whole relation among groups of cherries. In a similar way, volume tracks part-whole relations in a domain consisting of portions of wine. [...] When one ordering tracks another ordering, it is said to be monotonic on that ordering. [...] The discussion of cherries and wine shows that weight and volume are monotonic on the relevant part-whole relations. However, temperature and color (or hue) are not monotonic on those part-whole relations."

In this respect, monotonicity is very similar to Krifka's (1989, 1990, 1992) notions of additivity or cumulativity of reference. Schwarzschild's formal definition of non-monotonicity on the part-whole relation is reproduced (from p. 77) in (39). ${ }^{5}$

[^4](39) A dimension Dım is non-monotonic iff $\forall x, y\left[x \leq{ }_{\text {part }} y \rightarrow x=\operatorname{Dim} y\right]$ ("all parts of $y$ have the dimension to the same extent as $x$ ")

In (36), the measure noun gallon encodes the dimension of measurement 'volume'; volume is monotonic on the partwhole relation, assuming that the relevant part of a mass of water is a smaller sub-part of that water. Since attributive MPs require a measurement dimension that is non-monotonic, (36b) is ungrammatical (but the pseudopartitive in (36a) is grammatical).

Conversely, in (37), the measure noun karat encodes the dimension 'purity'; purity is non-monotonic on the part-whole relation, assuming again that the relevant part of a mass of gold is a smaller sub-part of that gold. Since partitive MPs require a monotonic dimension, (37a) is ungrammatical, while (37b) is grammatical.

Schwarzschild (2006b) demonstrates that these restrictions are at least somewhat crosslinguistically robust. While he focuses on the attributive/pseudopartitive distinction, his account of these constructions draw from a broader empirical base (p.83-4), as shown in (40) and (41).
(40) non-monotonic dimensions
a. six-pound cherries

English attributive MP
b. desiati-gradus-naja voda

10-degree-ADJ water
'10-degree water' Russian adjectival affix MP
c. foif-gred-igs Wasser

5-degree-ADJ water '5-degree water'

Swiss German adjectival affix MP
(41)
monotonic dimensions
a. six pounds of cherries English MP pseudopartitive
b. kilogramm jablok kilogram.nом apple.gen.PL
'a kilo of apples'
Russian case marking MP
c. 2 Meter Kábel
'2 meters of cable' (length) German, no case marker
Schwarzschild treats the difference in monotonicity between (40) and (41) syntactically, in terms of the difference between "weakly lexical syntax" with zero-level projections (the non-monotonic constructions; Sadler and Arnold, 1994) and constructions with higher projections (the monotonic constructions). Specifically, he proposes that the monotonic constructions in (41) are monotonic because they project a higher phrase MonP whose head, Mon ${ }^{\circ}$, can be realized in English as the linker of and which "encodes the linking requirement" (p.94). These configurations are illustrated in (42) and (43), respectively.

The relevant generalization is that monotonic constructions require some higher projection that encodes a montonicity restriction, while non-monotonic constructions are relatively syntactically impoverished.
non-monotonic constructions

monotonic constructions


Schwarzschild is clear that, while his proposal accounts for the difference in monotonicity between these constructions, it is not explanatory. It specifies that the linker Mon ${ }^{\circ}$ requires monotonicity, but he does not explain why. He says (p.80): "It seems therefore unlikely that the rules as we have them are a true reflection of the grammar as opposed to mere corollaries of the actual rules, rules that match fundamental semantic notions with correspondingly fundamental syntactic ones."

Brasoveanu (2009) observes that Romanian pseudopartitives behave similarly to those in English. He argues that the monotonicity restriction on pseudopartitives can be derived from the semantic notion of "individuation of measure," or "equality of measure". The idea is that the measure of an individual has the potential to be informative about the individual itself, but only in those cases in which the measure is of a monotonic dimension. If what we know about the water is that there is two liters of it, then we can infer something interesting about the substance itself (namely, its size). In contrast, if what we know about the water is that it is 40 degrees, then we cannot reliably infer something relevantly interesting about the substance itself.

In section 4, I will depend heavily on the notion of equality of measure, i.e. the idea that measures along only monotonic dimensions carry information about the individual being measured. But while I assume that the degree interpretation of a DP is derived from its individual denotation, and thus use equality of measure to constrain the availability of the degree interpretation, Brasoveanu treats the degree interpretation as primary (and uses it to constrain the availability of the individual interpretation). He says (p.147):
"[T]he nominalization of a measure expression is the degree-to-individual polysemic shift that applies to a domain of individuals and its associated part-whole structure and yields a sub-domain and a sub-structure that materially partition the input domain. The output individuals measure exactly one unit according to the measure function that is part of the basic degree-based denotation of the measure expression."

Effectively, Brasoveanu proposes a semantic account of individual/degree polysemy in pseudopartitives, whereas Landman's proposal of the same phenomenon was syntactic in nature. The contrast between these two analyses will be useful for what follows. While Landman's proposal is very much tied to the precise configuration of pseudopartitives - that they involve measure nouns and their particular syntax - Brasoveanu's analysis has the potential to be more general. And, it has the potential to explain the monotonicity restriction, which is important for individual/degree polysemy generally.

### 3.2. Monotonicity and polysemy

(44) repeats the empirical focus of Schwarzschild's monotonicity account: MPs in pseudopartitives can only correspond to dimensions of measurement (e.g. length) that are monotonic on the part-whole relation, while attributive MPs can only correspond to non-monotonic dimensions of measurement (e.g. diameter, or width).

> a. two inches of cable
> b. two-inch cable
partitive; length
attributive; diameter
I will argue here that the degree interpretations of DPs demonstrated in the previous section are also subject to a 'monotonicity restriction': a DP can only receive a degree interpretation when the degree it denotes is associated with a monotonic dimension of measurement.

The effect of monotonicity on individual/degree polysemy in pseudopartitives is visible in the contrast in (45).
(45) a. Four inches of cable are warped/is more than Bill asked for.
b. Four-inch cables are warped/\#is more than Bill asked for.

The pseudopartitive in (45a) can receive an individual reading (brought out by the predicate are warped) or a degree interpretation (brought out by the predicate is more than Bill asked for). In the degree reading, as Schwarzschild predicts, the pseudopartitive can denote a degree of length - which is monotonic on the part-whole relation - but not of width. The degree interpretation of (45a) cannot mean 'Bill asked for three-inch-wide cables, and four-inch-wide cables are wider than that.'

The surprising observation, given Schwarzschild's account, is (45b). Because it involves an attributive MP, Schwarzschild predicts the DP can only measure a non-monotonic dimension of measurement, e.g. width, as in (38b). And while the individual interpretation is possible, the DP cannot receive a degree interpretation in which it denotes the width of the cables. It cannot receive the interpretation 'Bill asked for three-inch-wide cables, and four-inch-wide cables are wider than that.', despite independent evidence that the pseudopartitive is compatible with contexts in which the relevant dimension of measurement is width.
(45) demonstrates the monotonicity restriction on the ability of DPs to denote a degree: a DP can only denote a degree if that degree represents a monotonic dimension of measurement of the individual. While Schwarzschild (2006b) (and Brasoveanu, 2009) accounted for the observation that the interpretation of MPs in pseudopartitives must involve monotonic dimensions, the claim here is different: only DPs whose salient dimension of measurement is monotonic on the part-whole structure of the individual can receive degree interpretations. This restriction corresponds in the predictable way with attributive and predicative uses of pseudopartitives.

Importantly, the dimension of measurement invoked by a construction is semantically constrained - by elements like the measure noun in an MP or the relevant predicate - but it is often determined at least in part by context. Schwarzschild (2002) describes this context-sensitivity of measurement as follows (p.232): "The choice of function will be constrained by the meanings of the measure phrase and the noun phrase but it won't always be determined by them". So the measure noun meter and the noun building in (46a) will ensure that the salient dimension of measurement isn't e.g. beauty, but we still need context to determine if the meters are measures of length, width, height, surface area, etc.
a. a 300-meter building
dimension: height
b. a 300-meter traffic jam dimension: length

In (45b), the most salient measure (width) is not monotonic on the part-whole structure, and so the DP cannot receive a degree interpretation. But it's possible for the DP to receive a degree interpretation in sentences in which a different, monotonic dimension of measure - quantity, in this case - is salient. The plural marking on the noun cables seems to make salient that there is a quantity of four-inch cables greater than one, which seems to allow for the reading, 'More than one four-inch cable is more than Bill asked for'. But the DP cannot denote a degree corresponding to a width. ${ }^{6}$

This demonstrates the context-sensitivity of dimensions of measurement and (correspondingly) monotonicity: whether a DP denotes a degree corresponding to a monotonic dimension of measurement depends on the measure noun, the predicate, and context. The claim here about the relationship between monotonicity and polysemy is not that specific DPs can never receive a degree interpretation, it's that no DP can receive a degree interpretation when the salient dimension of measurement is non-montonic.
(48) is repeated from (20). In both cases, the dimension of measurement invoked by four pizzas is quantity.
(48) a. Four pizzas are vegetarian/were eaten by the senators.
individual
b. Four pizzas is enough/is more than Bill asked for. degree

The dimension of measurement 'quantity of pizzas' is monotonic on the relevant part-whole structure: as we subtract parts from the whole (individual pizzas from the plurality), the quantity of the plurality changes in a predictable way. And, as we expect, the DP four pizzas can receive a degree interpretation in which it denotes a quantity of pizzas.

But the monotonicity restriction predicts that four pizzas cannot receive a degree interpretation in a context in which some other, non-monotonic dimension of measurement is the most salient. In a context in which the four pizzas are all 16 inches in diameter - a dimension of measurement, interpreted distributively, that does not change when we subtract individuals from the whole - the DP four pizzas in (48b) cannot denote the diameter of the pizzas; i.e. that (48b) cannot be paraphrased as, "Four sixteen-inch pizzas are larger than what Bill asked for." And this is in fact the case.

It might seem as though the interpretational restriction on four pizzas might have more to do with some affinity between numerals and quantity than the monotonicity requirement l'm proposing. But this restriction extends to DPs without numerals or measure nouns, as (49) shows.
a. Heavy barbells are on that side of the gym.
individual
b. \#Heavy barbells is more (heavy) than Bill had asked for.
degree

[^5]When it's a prenominal modifier, heavy encodes a dimension of measurement, weight, that is interpreted distributively on a plurality (requiring that each barbell be heavy; see Schwarzschild, 2011, for discussion). This makes (distributive) weight a salient dimension of measurement for the sentence in (49b). And distributive weight is non-monotonic on the partwhole structure of the plurality: subtracting an atom (an individual barbell) from the plurality of barbells does not change the fact that the remaining atomic elements in the plurality of barbells each count as heavy.

In accordance with the monotonicity restriction, however, (49b) cannot mean that Bill asked for barbells that were lighter than whatever weight counts as heavy for a barbell in that context. In a context in which a heavy barbell is one that weighs more than 30lbs., for instance, (49b) cannot be interpreted as describing that Bill asked for barbells lighter than 30lbs.

However, like the 'cheese doodles' sentence in (32b), the DP in (49b) can receive a degree interpretation if the contextually salient dimension of measurement is monotonic. A reviewer offers the discourse in (50) as an example.
(50) Heavy barbells is not enough - to get in shape, you'll need to do some cardio too.

In this example, the DP heavy barbells can be thought of as denoting a degree of exertion or something similar. In this sense, exertion is monotonic on the part-whole structure of an exercise routine; cutting out a chunk of an exercise routine amounts to a reduction in exertion. So when context specifies a monotonic dimension of measurement, in this case exertion, the DP heavy barbells can receive a degree interpretation.

The example in (49b) also demonstrates that whether or not a dimension of measurement is monotonic on the partwhole structure of the relevant individual depends critically on the type of individual (mass/count), whether it is plural or atomic, and whether the relevant gradable predicate is interpreted collectively or distributively (as discussed in Schwarzschild, 2011). In section 4, I will argue that the corollaries of monotonicity can be used to explain some other restrictions on the degree interpretations of DPs.

To review the discussion so far: it's been observed that constructions like pseudopartitives and (certain) relative clauses can denote degrees as well as individuals. The accounts of these data are construction-specific by design: they attribute the degree interpretation to some lexical ambiguity (Landman, 2004; Rothstein, 2009) or to some syntactic configuration (Grosu and Landman, 1998; Grosu, 1994). The proposal in Kotek (2011) is slightly less constructionspecific, but predicts a degree interpretation only in relative clauses formed with MPs or numerals. But the data here suggest the need for an even broader account, one that allows a DP without an MP or numeral to be associated with a degree (e.g. heavy barbells), and one that can account for the fact that DPs can only come to be associated with degrees when the salient dimension of measurement is monotonic on the part-whole relation.

On the flip side of the coin are the accounts in Schwarzschild (2006b) and Brasoveanu (2009), which observe a monotonicity restriction on pseudopartitive constructions in English and other languages. Schwarzschild attributes the restriction to a particular syntactic configuration but stops short of explaining why the presence of a functional projection is correlated with a monotonicity requirement (as opposed to some other - or no - requirement).

The phenomenon of individual/degree polysemy gives empirical motivation for a unified theory of these constructions and their semantic restrictions. I'll argue in the next section that such a theory is a relatively natural consequence of existing accounts of how measurement is dealt with in natural language semantics. l'll also demonstrate how such an approach can explain Schwarzschild's observations about the relationship between syntax (of pseudopartitives and related constructions) and semantics (i.e. the monotonicity restriction).

## 4. A type-flexible semantics of measurement

A core aspect of this proposal is motivated by the apparent asymmetry between the individual and degree interpretations of DPs. The latter is semantically restricted, while the former is not. Consequently, I assume that the degree interpretation of a DP is primary, derived from its individual denotation. The utility of this approach is underscored by the intuitive appeal of a semantic theory in which a DP like cheese doodles denotes a plurality of cheese doodles.

In particular, I propose that the degree interpretation of DPs is possible due to a general semantic process of measurement. A semantic theory must account for the ability of individuals, events, etc., to be associated with a measure, and it must allow for the fact that the measurement operation is sometimes lexicalized (by the quantity adjective in e.g. John has that * (many) dogs, which I return to in section 5) and sometimes not (as in John has 5 (*many) dogs). I will argue that individual/degree polysemy, and the monotonicity restriction, are a natural consequence of this assumption.

I will present a particular version of this proposal, in which the measurement operation, when not lexicalized, is encoded in a null semantic operator. In this proposal, the degree interpretation of a DP is derived by this operator in the compositional semantics. This is consistent with previous approaches to the semantics of measurement, and provides an easy explanation for how the difference in reference can condition a difference in number agreement (and satisfy selectional restrictions on predicates).

But it is possible to render the account proposed here pragmatically, given some theory of how a pragmatically conditioned interpretation could effect morphosyntactic processes like agreement (e.g. 'enriched pragmatics,' Jackendoff, 2002). Given such a framework, the generalizations made here about monotonicity and equality of measurement need not be encoded semantically to appropriately account for individual/degree polysemy and the monotonicity restriction.

There is a clear parallel in the phenomenon of deferred reference, as Kotek (2011) suggests. As detailed in Ward (2004), there is no consensus on whether deferred reference should be treated as a semantic or pragmatic phenomenon. The sentences in (51) are canonical instances of deferred reference; in (51a) the DP ham sandwich is interpreted as denoting the individual who ordered the ham sandwich, and in (51b) the DP John is interpreted as denoting the car John owns or drove.
a. The ham sandwich is at Table 7.
b. John is parked across the street.

The parallels between individual/degree polysemy and deferred reference are even more striking; Nunberg (1995) offers (52) as evidence that deferred reference can condition number agreement.

## (52) The/That french fries is getting impatient.

And standard instances of deferred reference are subject to some semantic restrictions that are, arguably, parallel to the monotonicity restriction observed for individual/degree polysemy. While the primary interpretation of the DP John is relatively unrestricted, the name is only felicitous in a sentence like (51b) in a context in which John is affiliated with one and only one salient car. This constraint on deferred reference will be very similar to the account l'll provide of the monotonicity restriction.

I'll begin the account by introducing a relatively standard degree-semantic framework in which I will couch the analysis.

### 4.1. A degree-semantic foundation

I will assume a semantic ontology that includes degrees, type $d$, as a primitive. Degree semantic theories are useful for, among other things, attributing the difference between gradable and non-gradable adjectives (as in (53)) to a difference in valency (as in (54)).
a. John is very tall.
b. \#That frog is very amphibian.
a. $\quad \llbracket \operatorname{tall} \rrbracket=\lambda d \lambda x . \operatorname{tall}(x, d)$
b. $\quad \llbracket$ amphibian $\rrbracket=\lambda x . \operatorname{amphibian}(x)$

In these approaches, measure phrases like six feet denote degrees, type $d$, that serve as the degree argument of gradable adjectives, as in $(55) .{ }^{7}$ The same is assumed to be true of numerals like six (see Kennedy, 2013 for a very nice discussion of how this approach can be reconciled with considerations involving scalar implicature, etc.).
(55) John is 6 ft tall.
a. $\quad \llbracket 6 \mathrm{ft} \rrbracket=6 \mathrm{ft}$
b. $\llbracket \operatorname{tall}(6 \mathrm{ft}) \rrbracket=\lambda x$. tall $(x, 6 \mathrm{ft})$
c. $\quad$ tall( 6 ft$)(\mathrm{John}) \rrbracket=$ tall $(\mathrm{j}, 6 \mathrm{ft})$

I will adopt from Bartsch and Vennemann (1972) and Bierwisch (1989) the assumption that sets of degrees (i.e. scales) are triples $\left\langle D,<_{O}, \psi\right\rangle$ with $D$ a set of points, $>_{O}$ a total ordering on $D$, and $\psi$ a dimension (e.g. 'height'). Consequently, a degree $d$ is shorthand for a triple $\left\langle\mathrm{d},>_{O}, \psi\right\rangle$, with d a point on a scale $D_{,}>_{O}$ a total ordering on $D$, and $\psi$ a dimension of measurement.

[^6]The ability of common nouns to be modified by numerals, as in (56), has created tension for degree-semantic theories in which MPs denote entities of type $d$.
(56) John has five cats.

Instead of assuming that these nouns are lexically specified as some higher type - $\langle d,\langle e, t\rangle\rangle$, as the analysis for gradable adjectives in (54a) - semanticists like Bartsch and Vennemann (1972) and Cresswell (1976) proposed covert measure operators to function as type-shifters between a set of individuals and the quantity of that set.

Cresswell's account involved two "quantity operators" to associate plural and mass individuals respectively with a degree that corresponds to the quantity of that individual. He analyzes the plural count noun men as at times denoting a one-place predicate of the form " $x$ is a set of men" and at times denoting, with the aid of a quantity operator, a two-place predicate of the form " $x$ is a $y$-membered set of men", where $y$ ranges over degrees. The concept has been widely adopted and further motivated (e.g. Villalba, 2003; Abeillè et al., 2006; Kayne, 2007). It's also been extended to dimensions of measurement other than quantity (Parsons, 1990; Higginbotham, 1994; Nerbonne, 1995; Schwarzschild, 2002, 2005; Kennedy and Svenonius, 2006; Rett, 2007, 2008; Champollion, 2010).

I'll refer to this null measure operator as M-Op; its definition, based on proposals in Schwarzschild (2002, 2006b) and Nakanishi (2007a,b), is in (57).

$$
\begin{array}{llr}
\text { a. } & \mathrm{M}-\mathrm{OP} \rightarrow \lambda P \lambda d \lambda x . P(x) \wedge \mu(x)=d & \text { attributive }  \tag{57}\\
\text { b. } & \mathrm{M}-\mathrm{OP} \rightarrow \lambda d \lambda x . \mu(x)=d ; & \text { predicative } \\
& \text { with } \mu \text { a variable over dimensions of measure, valued contextually } &
\end{array}
$$

I assume that the two versions are related just as predicative and attributive versions of adjectives are, via a type-shifter or something equivalent. ${ }^{8}$
(58) provides a derivation of the compositional semantics of a sentence with an individually-interpreted numeral DP.
(58) Four M-Op pizzas are vegetarian.
a. $\quad \llbracket$ M-Op pizzas $\rrbracket=\lambda d \lambda x . \operatorname{pizzas}(x) \wedge \mu(x)=d$
b. $\llbracket$ four M-Op pizzas $\rrbracket=\lambda x$. $\operatorname{pizzas}(x) \wedge \mu(x)=4$
c. 【four M-Op pizzas are vegetarian】
$=\lambda x$. vegetarian $(x) \wedge \operatorname{pizzas}(x) \wedge \mu(x)=4$
d. $\quad={ }_{E C} \exists x[\operatorname{veg}$ getarian $(x) \wedge \operatorname{pizzas}(x) \wedge \mu(x)=4]$

This derivation makes use of existential closure (EC) to existentially bind the individual argument. Encoding this existential quantifier lexically, for instance in the numeral itself (as has been done in GQT accounts; Barwise and Cooper, 1981; Keenan and Stavi, 1986) incorrectly predicts that numeral DPs cannot occur with overt determiners (cf. The four pizzas are vegetarian). Encoding the existential quantifier in the meaning of the number would also make the compositional semantics of a numeral DP significantly more inflexible in a way that presents a challenge to an account of individual/degree polysemy.

What individual/degree polysemy appears to require from a compositional semantic theory is a certain amount of flexibility in terms of how the components of the sentence are composed. This is the subject of section 4.3. Before presenting the analysis, l'll first introduce the notion of equality of measure.

### 4.2. Monotonicity and individual-degree homomorphisms

In this section, I lay the foundation for an analysis of the semantic coercion from individuals to degrees by presenting an explanation of the monotonicity restriction on the degree intepretation of DPs. I argue that individual/degree polysemy and its monotonicity restriction are natural consequences of a semantic theory in which a null measurement operator can measure sets of degrees as well as individuals (see also Rett, 2007, 2008).

[^7]
### 4.2.1. Measuring degrees

There's independent evidence that natural language employs a null measurement operator to measure entities other than individuals; in particular, events (Nakanishi, 2007a,b; Doetjes, 2007; Burnett, 2012). We can capture this flexibility the ability of $\mu$ to have individuals or events in its domain - by assuming that the domain of $\mu$ is semantically underspecified (and that the dimension of measurement varies with the domain of the function as well as contextually).

If this is right, we can imagine a version of M-Op that measures the size of a set of degrees, as in (59).

$$
\begin{equation*}
\mathrm{M}-\mathrm{OP}_{d} \rightarrow \lambda D \lambda d^{\prime \prime} \cdot \mu(D)=d^{\prime} \tag{59}
\end{equation*}
$$

When the set of degrees $D$ is non-dense, $\mu(D)$ counts the number of degrees in the set. When $D$ is dense, $\mu(D)$ will be $d_{l}-d_{u}$, for $d_{l}$ the lower bound and $d_{u}$ the upper bound (see Chandalia, 2007, for an easily accessible mathematics of $\mu$ within Measure Theory).

This is, effectively, a type-raised degree version of the individual version of $\mathrm{M}-\mathrm{Op}$ in (57), which I will refer to as $\mathrm{M}-\mathrm{Op}_{\mathrm{e}}$. Instead of measuring plural entities (as $\mathrm{M}-\mathrm{Op}_{e}$ does), it measures a set of entities. It is possible to characterize $\mathrm{M}-\mathrm{Op}_{d}$ in terms of plural degrees, which would make more transparently related to $\mathrm{M}-\mathrm{Op}_{\mathrm{e}}$. But doing so would require a formal proof that plural degrees are equivalent to sets of degrees as well as a move from standard degree semantics to an interval based one, like the one in Schwarzschild and Wilkinson (2002) (which Heim, 2006, argues is formally equivalent to a setbased semantics).

I choose instead, for the sake of succinctness, to type-raise the degree version of M-Op to make it more in line with standard degree-semantic theories. The relevant prediction is that the domain of $\mathrm{M}-\mathrm{Op}$ is a plural entity: a plural individual, event, or degree (modeled here as a set of degrees). In many of the cases discussed here, this means that $\mathrm{M}-\mathrm{Op}_{d}$ can only apply after $\mathrm{M}-\mathrm{Op}_{e}$. But there are other empirical phenomena for which the application of $\mathrm{M}-\mathrm{Op}_{d}$ does not require a prior application of $\mathrm{M}_{-\mathrm{Op}_{e}}$, for instance, in differential comparatives like John is much taller than Bill, which l'll discuss more in section 5.

I've assumed that degrees $d$ are shorthand for triples that provide information about the scale on which the point lies, including the scale's ordering $<_{O}$ and the relevant dimension of measurement. Measures of individuals, calculated using $\mathrm{M}-\mathrm{OP}_{e}$, can represent the individual's height or width, two different dimensions, along two different orderings (tracking the difference between tall and short, or between wide and narrow). There is similar contextually conditioned variabilty in measures of degrees, calculated using $\mathrm{M}-\mathrm{Op}_{d}$. I will assume that a measure $d$ of a set of degrees $D$ tracks, in its dimension $\psi$, the dimension of measurement of its input, a sort of meta-dimension of measurement. This allows $\mathrm{M}-\mathrm{Op}_{d}$ to track the difference between e.g. the size of a set of heights and the size of the set of widths.

In what follows, I'll argue that the ability of M-Op to measure sets of degrees accounts for the degree interpretations of DPs as well as the restriction on their monotonicity.

### 4.2.2. Equality of measure

In Rett, 2007, 2008, I argued that $\mathrm{M}-\mathrm{Op}_{d}$ is overtly encoded in Balkan quantity adjectives like the Romanian mult. (l'll return to this suggestion in section 5.) The relevant observation is that, for certain dimensions of measurement, the number of measures of a plural individual is equal to the maximum measure of that plurality. I'll refer to this property as 'equality of measure'.
(60) A dimension of measurement $\omega$ demonstrates equality of measure iff, for all $x, \mu_{d}(\omega(x))=\operatorname{MAx}(\omega(x))$
(61) demonstrates that the dimension of quantity displays equality of measurement for a plural individual $a \oplus b \oplus c$.
a. $\quad \mu_{\text {quantity }}(a \oplus b \oplus c)=\{1,2,3\}$
b. $\quad \mu_{d}(\{1,2,3\})=3$

Although the maximum plurality in (61) numbers 3 , there is also a member of the plurality (e.g. $a \oplus b$ ) that numbers 2 , and another member (e.g. a) that numbers 1 . So the set of quantity measures of the plurality $a \oplus b \oplus c$ is $\{1,2,3\}$.

An intuitive way of thinking about this equality, borrowed from Schwarzschild (2006b), is as follows: if a ruler begins at zero, and if the points on the ruler are equidistant from each other, then the measurement of that ruler (e.g. 12 inches) will be equivalent to the highest point on that ruler. This means that equality of measure is a property only of downward-monotonic dimensions, those for which we can infer from the measure $d$ to the measure $d-1$. This predicts that non-monotonic dimensions - e.g. temperature or purity - and upward-monotonic dimensions do not display equality of measure.

The exclusion of upward- and non-monotonic dimensions of measurement is the primary difference between an analysis like this one, which uses $\mu_{d}$ to derive a degree set's maximum, and an analysis that uses a maximality operator to do so (Grosu, 2009; Kotek, 2011). Any set of degrees has a maximum, but only some demonstrate equality of measure, the
property important for individual/degree polysemy. (The accounts also differ in how they are constrained; here, I advocate the free distribution of M-Op, while the distribution of Max must be restricted somehow to specific syntactic or semantic configurations.)

The result is that the property in (60) is a proxy for Schwarzschild's notion of monotonicity on the part-whole structure, allowing (if necessary) some additional flexibility (see fn. 7 and Champollion, 2010). Among the contributing factors to the equivalence demonstrated in (61) is the fact that each sub-part of the maximal individual differs in the relevant measure (quantity). So, $\mu_{\text {quantity }}(a \oplus b)=\{1,2\}$ while $\mu_{\text {quantity }}(a \oplus b \oplus c \oplus d)=\{1,2,3,4\}$.

Imagine now a dimension of measurement that is not monotonic on the part-whole structure, e.g. purity, in a context in which there is one lump of gold, a, whose purity is 18 karats.

$$
\begin{array}{ll}
\text { a. } & \mu_{\text {purity }}(\mathrm{a})=\{18\}  \tag{62}\\
\text { b. } & \mu_{d}(\{18\})=1
\end{array}
$$

The set of degrees corresponding to measurements of purity of individuals in the domain satisfying the predicate gold is a singleton set because purity is non-monotonic on the part-whole relation; there is a single lump of gold a and every subpart of a has the same purity as a itself. The result is that the measure $\mu$ of the set of degrees of purity is 1 , which counts as an instance in which we cannot infer from the measure of a set of measures to the set of measures itself. The measure of a set of degrees representing a non-monotonic measure is uninformative in every case; it doesn't vary across contexts in which we vary the individual itself.
$\mu_{d}$ is strictly speaking a total function; $\mathrm{M}_{-\mathrm{OP}_{d}}$ is defined in (59) over sets of degrees representing non-monotonic measurements. But individual/degree polysemy gives evidence that it is, in fact, unacceptable in these cases. It seems plausible that this derived degree interpretation is licensed if and only if there is a contextually invariant mutual entailment between the two interpretations.

Instead of encoding this requirement in a presupposition (thereby making $\mu_{d}$ a partial function), I will treat it as a more general, pragmatic prohibition against uninformative mappings, akin to the unacceptability of deferred reference in cases in which there is more than one salient relation (e.g. the unacceptability of (51b) John is parked across the street in contexts in which John is affiliated with one salient car).

### 4.2.3. The relationship between $M-O P_{d}$ and $M^{\circ}{ }^{\circ}$

Before I present a compositional semantics of the degree interpretation of DPs using M-Op ${ }_{d}$, l'll discuss an interesting connection between the $\mathrm{M}-\mathrm{Op}_{d}$ proposal and the account in Schwarzschild (2005).

Recall that Schwarzschild (2005) was interested in particular syntactic configurations that seemed to regulate the (non)monotonicity of the salient dimension of measurement. He concluded that monotonic constructions require some higher projection that plausibly encodes a montonicity restriction, while non-monotonic constructions are relatively syntactically impoverished. He labels the functional projection 'MonP' and described its head, Mon ${ }^{\circ}$, responsible for imposing the monotonicity restriction, as (p.96) "a copula in a very general sense."
non-monotonic constructions

(64)
monotonic constructions


I'll also follow Schwarzschild in adopting the assumption (from Corver, 1998) that MPs, in pseudopartitives, occupy the specifier of MonP, as in (64). It requires a type-raised version of MPs, as in (65), where the MP six pounds denotes a generalized degree quantifier (type $\langle\langle d, t\rangle, t\rangle$ ) instead of a degree (type $d$ ). ${ }^{9}$
(65) $[[$ six pounds]] $=\lambda D . D(6 \mathrm{lbs})$.
type-raised version
Schwarzschild proposes that Mon ${ }^{\circ}$ encodes the monotonicity restriction, but he did not provide an explanation of why. I argue instead that $\mathrm{M}-\mathrm{OP}_{d}$ is responsible for the monotonicity restriction. If we assume that its presence requires a functional projection, such an account can be seen as compatible with Schwarzschild's syntactic proposal. I will not have anything to say about the semantics of the linker of, but I will (in section 5) argue that quantity adjectives are overt instances of $\mathrm{M}-\mathrm{OP}_{d}$.

### 4.3. A semantics for the degree interpretation

I've argued that the monotonicity restriction can plausibly be attributed to the relatively broad distribution of a null measurement operator, whose semantic contribution is itself constrained by considerations of informativity. This predicts that the degree interpretation of a DP involves an extra layer of meaning than the individual interpretation, which l've argued is compatible with Schwarzschild's independent syntactic claims about monotonic constructions like the pseudopartitive. In this section, I present the details of a compositional account and discuss the consequences for different types of DPs (and in different languages).

### 4.3.1. A compositional account

The proposal, then, is that the degree interpretation of a DP is available as the result of a general process of measurement available in natural language semantics. That the degree interpretations of DPs are restricted to dimensions of measurement that are monotonic on the part-whole structure of the denoted individual is a natural consequence of the equality of measure: that the size $d$ of a set of degrees $D$ is meaningful only when $D$ contains monotonic measures.
(66) shows a compositional semantics for the degree interpretation of the DP in Four pizzas is enough (pizzas to feed the group). It involves an application of $\mathrm{M}-\mathrm{OP}_{e}$ (whose domain is a set of individuals) and existential closure of the noun's individual argument followed by an application of $\mathrm{M}-\mathrm{Op}_{d}$. I assume that existential closure can apply to either the individual or the degree argument, depending on the construction and context; here, be enough is analyzed as a degree predicate, so the individual argument undergoes existential closure. ${ }^{10}$ Because it is in a specifier position, I follow Schwarzschild (2006b) in treating the numeral as denoting a type-raised general degree quantifier, as in (66f).
(66) [ four [[ M-Op ${ }_{d} M-O p_{e}$ pizzas ] [ is enough ]]]
a. $\quad \llbracket \mathrm{M}-\mathrm{Op}_{e} \operatorname{pizzas} \rrbracket=\lambda d \lambda x \cdot \operatorname{pizzas}(x) \wedge \mu_{\text {quantity }}(x)=d$
b. $\quad={ }_{E C} \lambda d \exists x\left[\operatorname{pizzas}(x) \wedge \mu_{\text {quantity }}(x)=d\right]$
c. $\llbracket \mathrm{M}-\mathrm{OP}_{d} \mathrm{M}-\mathrm{OP}_{e}$ pizzas $\rrbracket=$
$\lambda d^{\prime \prime} . \mu_{d}\left(\lambda d \exists x\left[\operatorname{pizzas}(x) \wedge \mu_{\text {quantity }}(x)=d\right]\right)=d^{\prime}$
d. $\quad \llbracket i s$ enough $\rrbracket=\lambda d$. enough $(d)$
e. $\quad \llbracket \mathrm{M}-\mathrm{Op}_{d} \mathrm{M}-\mathrm{Op}_{e}$ pizzas is enough $\rrbracket=$
$\lambda d^{\prime \prime} . \mu_{d}\left(\lambda d \exists x\left[\operatorname{pizzas}(x) \wedge \mu_{\text {quantity }}(x)=d\right]\right)=d^{\prime} \wedge \operatorname{enough}\left(d^{\prime \prime}\right)$
f. $\quad \llbracket \mathrm{four} \rrbracket=\lambda D . D(4)$
g. 【four pizzas is enough】=
$\mu_{d}\left(\lambda d \exists x\left[\operatorname{pizzas}(x) \wedge \mu_{\text {quantity }}(x)=d\right]\right)=4 \wedge$ enough $(4)$
The derivation involves an instance of degree predicate modification in (66e) and a relatively early application of existential closure (in (66b)), which I assume to be freely available to all types of entities. Semantic treatments of

[^8]polysemy, or at least of cross-domain polysemy, seems to require this sort of quantificational flexibility. The truth conditions in $(66 \mathrm{~g})$ predict that this sentence will be true in a context in which there is a plurality of pizzas $x$, the set of degrees $d$ corresponding to the quantities of pizzas in the context measures 4 , and 4 counts as enough in the context. ${ }^{11}$

In section 2, I argued that the individual and degree interpretations can be differentiated by the selectional restrictions of the predicates they combine with, as well as the number agreement inflected on these predicates. An individual interpretation of a DP may trigger singular or plural agreement, depending on whether or not the individual is atomic. But a degree interpretation triggers singular agreement in every case; this is consistent with the account above, in which the degree argument of degree predicates is always atomic.
(66) is one of presumably several ways to represent a sentence involving a degree interpretation of an individual/ degree polysemous DP. Instead of postulating a process of semantic coercion in which the monotonicity constraint on polysemous DPs is stipulated, this proposal takes advantage of the wide distribution of a null measurement operator $\mathrm{M}-\mathrm{Op}$ to predict the existence of and restrictions on individual/degree polysemy. When a set of measurements $\{d: d\}$ and its measure $d^{\prime \prime}$ are related in a particular way - specifically, when the nature of one uniquely determines the nature of the other - a degree interpretation of a measure DP is possible, and we get truth conditions like the ones in (66). When $\{d: d\}$ and $d^{\prime \prime}$ are not so related, an application of $\mathrm{M}-\mathrm{Op}_{d}$ effects the truth conditions, so the semantic coercion is not possible.

Recall that a degree $d$ is shorthand for a triple $\left.\langle\mathrm{d},\rangle_{o}, \psi\right\rangle$, with $\psi$ a dimension of measurement valued contextually. I assumed that a degree representing a measure of an individual is associated with dimensions like quantity (of pizzas) or height (of linguists), while a degree representing a measure of a set of degrees is associated with a dimension like 'amount of quantities of pizzas' or 'size of heights of linguists'. In this respect, $\psi$ can be seen as a contextually valued pragmatic variable, responsible in this account, as a reviewer points out, for predicting that the two sentences in (67) are not synonymous.
(67) a. Four pizzas is too many.
b. Four eggs is too many.

That this approach involves a sort of semantic coercion, as opposed to analyzing the DP as ambiguous or semantically underspecified, fixes the denotation of the DPs in a particular context. Among other things, this predicts the DPs can't be predicated of a conjoined individual and degree property, as seems to be the case in (68). ${ }^{12}$
(68) a. *Four pizzas is enough and (are) delicious.
b. *Four feet of plywood are warped and (is) more than Bill asked for.
(69) (repeated from (32)) shows that DPs that don't involve measure morphology can nevertheless display individual/ degree polysemy.
(69) a. Cheese doodles are delicious/were eaten by the senators. individual b. Cheese doodles is not enough. The senators will need protein. degree

The truth conditions of these sentences are derived in parallel to that of four pizzas, as shown in (70) and (71).
(70) a. $\quad$ cheese doodles $\rrbracket=\lambda x$. cheese-doodles $(x)$
b. $\quad$ are delicious $\rrbracket=\lambda x$. delicious $(x)$
c. $\quad \llbracket$ Cheese doodles are delicious $\rrbracket=\lambda x$. cheese-doodles $(x) \wedge \operatorname{delicious}(x)$

[^9]a．$\llbracket \mathrm{M}-\mathrm{Op}_{e}$ cheese doodles $\rrbracket=\lambda d \lambda x$ ．cheese－doodles $(x) \wedge \mu_{\text {nutrition }}(x)=d$
b．$={ }_{E C} \lambda d \exists x\left[\right.$ cheese－doodles $\left.(x) \wedge \mu_{\text {nutrition }}(x)=d\right]$
c．$\llbracket \mathrm{M}-\mathrm{OP}_{d} \mathrm{M}-\mathrm{OP}_{e}$ cheese doodles $\rrbracket=\lambda d^{\prime \prime} . \mu_{d}\left(\lambda d \exists x\left[\right.\right.$ cheese－doodles $\left.\left.(x) \wedge \mu_{\text {nutrition }}(x)=d\right]\right)=d^{\prime}$
d．$\llbracket i s ~ e n o u g h \rrbracket=\lambda d$ ．enough $(d)$
e．$\llbracket$ Cheese doodles is enough $\rrbracket=\lambda d^{\prime} . \mu_{d}\left(\lambda d \exists x\left[\right.\right.$ cheese－doodles $\left.\left.(x) \wedge \mu_{\text {nutrition }}(x)=d\right]\right)=d^{\prime} \wedge$ enough $\left(d^{\prime}\right)$ $={ }_{E C} \exists d^{\prime}\left[\mu_{d}\left(\lambda d \exists x\left[\right.\right.\right.$ cheese－doodles $\left.\left.(x) \wedge \mu_{\text {nutrition }}(x)=d\right]\right)=d^{\prime} \wedge$ enough $\left.\left(d^{\prime}\right)\right]$

Recall that the sentence in（69b）is acceptable in a context in which nutrition is the salient dimension of measurement． A natural context for（69b）is one in which the relevant whole is a collection of possible snacks；for nutrition to be monotonic on the part－whole structure in this context is for the nutrition of the group of snacks to decrease with the subtraction of its parts．This is similar to the barbell example in（50），repeated in（72）．
（72）Heavy barbells is not enough－to get in shape，you＇ll need to do some cardio too．
In（72），the relevant dimension of measurement is exertion or something similar，and the relevant whole is an entire exercise routine．The prediction is that the DP heavy barbells in（72）can denote a degree in such a context，provided that the degree of exertion decreases with the subtraction of parts of the exercise routine．In contrast，the same DP in（73） （repeated from（49b））cannot receive a degree interpretation because the salient dimension of measurement，distributive heaviness，is not monotonic on the part－whole structure（here，the plurality of barbells）．${ }^{13}$
（73）\＃Heavy barbells is more（heavy）than Bill had asked for．
a．$\quad$ heavy $\rrbracket=\lambda P \lambda d \lambda x \forall y$ ᄃ $x[\operatorname{heavy}(y, d) \wedge P(y)]$
b．【heavy barbells】 $=\lambda d \lambda x \forall y$ ᄃ $x[\operatorname{heavy}(y, d) \wedge \operatorname{barbell}(y)]$
c．$\quad={ }_{E C} \lambda d \exists x[\forall y \sqsubset x[\operatorname{heavy}(y, d) \wedge \operatorname{barbell}(y)]]$
d．$\quad \mathrm{M}-\mathrm{Op}_{d}$ heavy barbells】
$=\lambda d^{\prime \prime} . \mu_{d}(\lambda d \exists x[\forall y \sqsubset x[\operatorname{heavy}(y, d) \wedge \operatorname{barbell}(y)]])=d^{\prime}$
e．$\quad$ be more heavy than Bill had asked for $\rrbracket=\lambda D . \operatorname{Max}(D)>\operatorname{Max}(\lambda d$ ．asked－for $(b, d))$
f．【Heavy barbells is more than Bill had asked for】
$=\operatorname{Max}\left(\lambda d^{\prime \prime} . \mu_{d}(\lambda d \exists x[\forall y \sqsubset x[\operatorname{heavy}(y, d) \wedge \operatorname{barbell}(y)]])=d^{\prime}\right)>\operatorname{Max}(\lambda d . \operatorname{asked}-$ for $(b, d))$
The truth conditions in（73f）represent a conversion from an individual denotation（a plurality of heavy barbells）to a degree denotation involving a non－monotonic dimension of measurement（distributive，rather than collective， heaviness）．However，given the restriction from section 4．2．2 that $\mu_{d}$ is defined only in cases of equality of measure， and given that heaviness is non－monotonic on the part－whole structure，this sentence is predicted to lack a truth value．

The account proposed here can also be extended to the＇how many ambiguity＇phenomenon discussed in section 2.1 and illustrated below（repeated from（33））．
（74）a．How many books are on the table？individual
b．How many books is too many？degree
The two readings will parallel the derivations in（70）and（71），assuming（with Groenendijk and Stokhof，1989）that wh－ clauses denote properties（in this case，degree properties）that undergo existential closure．

To sum up，the account of individual／degree polysemy presented here involves an extension of the degree－ semantic proposal that measurement in natural language semantics is enabled by the fairly general distribution of a null measurement operator，M－Op．M－Op was initially defined as denoting a relation between plural individuals and degrees；I argue here that it can also relate plural degrees（or sets of degrees）and degrees．If this is right，we have a natural account of individual／degree polysemy：an individual－denoting DP can denote a degree corresponding to the measure of its measures，but only in cases where the measure is informative，i．e．if it is monotonic on the part－whole structure．

A quick side note：despite the fact that Brasoveanu＇s account is centered around monotonicity，its extension outside of pseudopartitives doesn＇t correctly predict the monotonicity facts．In particular，it correctly predicts that DPs can only denote degrees reflecting monotonic dimensions of measurement，but it incorrectly predicts that they can

[^10]only denote individuals reflecting monotonic dimensions of measurement as well. This is a direct result of the assumption that the DPs denote (monotonic) degrees - motivated by Schwarzschild's observations about pseudopartitives - and that the individual interpretation comes about via a homomorphism from these degrees. This appears to be the wrong way to characterize DPs like heavy barbells, which can refer to a set of barbells that are distributively heavy.

### 4.3.2. Other constraints on individual/degree polysemy

While Schwarzschild (2006b) attributes the monotonicty restriction on pseudopartitives to Mon ${ }^{\circ}$, he suggests that the requirement that attributive MPs invoke a non-monotonic dimension of measurement is a semantic one. He observes that attributive MPs in English only modify singular count nouns, as in six-pound cherry or three-inch rope, which denote atomic individuals. The only relevant part-whole relationship of an atomic individual is one in which the individual is divided up into parts, which Schwarzschild claims is incompatible with a presupposition, carried by singular count nouns, that (p.107) "the elements of its extension do not overlap with each other". The result, Schwarzschild argues, is that the combination of an MP and a singular count noun can only invoke dimensions of measurement that are not monotonic on the part-whole structure of the denoted individual.

This is a significant point for the treatment of any phenomenon that is sensitive to monotonicity. If a construction requires monotonicity, and if DPs differ in the extent to which they're compatible with monotonic dimensions of measurement, then we would expect that not all DPs are equally acceptable in that construction. And this seems to be true for the degree interpretations of DPs.

I'll mention a few relevant corollaries. First, as demonstrated in section 4.2.2, the computation of monotonicity on a partwhole structure presupposes a fully determined whole. I illustrated equality of measure on a specific plural individual, $a \oplus b \oplus c$, in (61) (repeated below).

$$
\begin{array}{ll}
\text { a. } & \mu_{\text {quantity }}(a \oplus b \oplus c)=\{1,2,3\}  \tag{75}\\
\text { b. } & \mu_{d}(\{1,2,3\})=3
\end{array}
$$

This suggests that while a DP with a specific interpretation can receive either an individual or a degree interpretation, as discussed in section 2 , specificity of reference is a precondition on monotonicity. In other words, the calculation of whether a dimension of measurement is monotonic on the part-whole structure presupposes that there is one (and only one) salient whole.

There is some evidence that this is in fact the case; the data in (76), repeated below, suggest that quantified DPs receive a degree interpretation much more naturally if they occur in the partitive construction, which marks specificity of some sort (Enç, 1991).
(76) a. We didn't manage to bring all of the cookies, but some (of the) cookies is more than they deserve. b. We brought all of the cookies, but even all ??(of the) cookies is less than we need.

It's possible that this precondition on monotonicity can be used to explain the observations in Grosu (1994) and Kotek (2011) that, in English and Hebrew, only relative clauses headed by definites can function as amount relatives, as in (77).
(77) a. *(The) nine kilos your bag weighs won't prevent you from boarding.
b. *(The) $\$ 20$ this book costs won't upset your students.

They associate definiteness with the ability to denote a degree. If this were the case, we would expect a general restriction that only definite DPs can denote degrees. But we've seen (for instance, in the four pizzas and cheese doodles examples) that this is not the case.

It seems reasonable, then, to look for a construction-specific explanation for these data. One observation is that the MPs in these constructions, as well as the MPs in degree-denoting MPs, receive a strong, 'exactly' interpretation (in contrast to a weak, 'at least' interpretation; see Koenig, 1991; Kennedy, 2013). This, too, seems to be a precondition on monotonicity; while the measurement 'weighs exactly 9 kilos' is monotonic on the part-whole structure of the bag, 'weighs at least 9 kilos' isn't. This suggests that there is an indirect relationship between 'exactly' interpretations, definiteness, and monotonicity.

Finally, it is relatively natural to assume some amount of crosslinguistic variation in DP meaning, especially in terms of subtle semantic properties like specificity and definiteness. Given Schwarzschild's point about singular count nouns, and the suggestion above that specificity might play a role in monotonicity, there is clear room in the proposal here for variation with respect to which DPs can denote degrees and which cannot. This analysis does however make some strong
predictions: no DP can denote a degree but not an individual; and no DP can denote a degree corresponding to a non-monotonic dimension of measurement. But there is, presumably, some linguistic variation with respect to which DPs can denote degrees at all.

In the next section, I conclude by arguing that the proposed treatment of individual/degree polysemy can provide independent motivation for recent semantic proposals of quantity adjectives like many and much.

## 5. Measurement and quantity adjectives

In his discussion of measure phrases, Schwarzschild (2006b) observes that DPs containing the quantity adjectives (QAs) many, few, much or little always invoke dimensions of measurement that are monotonic on the part-whole structure of the individual. This is more surprising for the QAs much and little than it is for many and few, which seem to only measure quantity. Schwarzshild (p.106) attributes the monotonicity of QAs to their syntactic position in a higher functional projection:
"I left open what forces $Q$ adjectives to have monotonic interpretations. At this point, we have an answer. Their interpretations as predicates of scale parts makes them unfit to be (intersective?) attributive modifiers. The only way they can enter the nominal projection is by being in Spec,MonP - but that entails a monotonic dimension."

As before, I will adopt Schwarzschild's syntactic theory, but supplement it with a semantic proposal. In this case, I will introduce data related to the individual/degree polysemy to support previous proposals that QAs denote degree modifiers, functions from a set of degrees to the singleton set containing its measure (Rett, 2007, 2008). Specifically, that QAs, at least English QAs, denote degree measurement operators.

It's been proposed that QAs are overt versions of individual M-Op beginning, I believe, with Cresswell (1976) (p. 266), who says: "In fact I am going to regard the word much as a surface marker that signals the underlying degree variable (Bresnan, 1973, p. 277f)."

Schwarzschild (2005), Brasoveanu (2009), Solt $(2006,2008,2009,2010)$ have argued that quantity adjectives can in some contexts mean something like (78) based on their ability to modify e.g. comparative differentials.

In these proposals, QAs can be thought of as denoting the version of M-Op with an unspecified domain: it can either measure individuals, events or degrees. ${ }^{14}$

$$
\begin{equation*}
\llbracket \operatorname{many} \rrbracket=\lambda D \lambda d^{\prime} . \mu(D)=d^{\prime \prime} \tag{78}
\end{equation*}
$$

Rett (2007, 2008) and, more recently Solt (2013), make a slightly stronger argument: QAs denote M-OP ${ }_{d}$. That is, whenever a sentence contains a QA, the truth conditions involve a measure of a measure of an individual (or event). (79) and (80) present truth conditions for an individual and degree interpretation of a QA respectively given this assumption. ${ }^{15}$

$$
\begin{align*}
& \llbracket \text { Many M-Op guests are drunk } \rrbracket=^{\exists d^{\prime}\left[\mu_{d}\left(\lambda d \exists x\left[g u e s t s(x) \wedge \operatorname{drunk}(x) \wedge \mu_{\text {quantity }}(x)=d\right)=d^{\prime} \wedge d^{\prime}>s\right]\right]} \quad \text { individual } \tag{79}
\end{align*}
$$

(80) $\quad$ Many M-Op guests is enough $\rrbracket$ = degree
$\exists d^{\prime}\left[\mu_{d}\left(\lambda d . \exists x\left[\right.\right.\right.$ guests $\left.(x) \wedge \mu_{\text {quantity }}(x)=d\right] \wedge$ be - enough $\left.\left.(d)\right)=d^{\prime} \wedge d^{\prime \prime}>s\right]$
(79) imposes truth conditions along the lines of "There is a plurality of individuals who are drunk guests; the measure of their cardinality is high relative to some relevant contextual standard." (80) imposes truth conditions along the lines of "There is a plurality of individuals who are guests and whose cardinality counts as enough. The measure of that cardinality is high relative to some contextually valued standard."

In Rett, 2007, 2008 I presented evidence from how many questions in Balkan languages like Romanian to support this claim. But, based on the discussion above, we can evaluate this proposal based on the monotonicity of these constructions. Recall that an application of $\mathrm{M}-\mathrm{OP}_{d}$ - unlike the application of just an individual $\mathrm{M}-\mathrm{Op}$ - requires equality of

[^11]measure, which means that it returns a value only for monotonic dimensions of measurement. If QAs are overt versions of $\mathrm{M}-\mathrm{OP}_{d}$, then we predict QA constructions to be subject to a monotonicity restriction. And this is in fact what we see.

Of course, many and few - by virtue of the fact that they modify count nouns - are restricted to cardinality measures (81).
(81) a. Many pizzas is enough.
b. Few pizzas is not enough.

But the QAs much and little - as well as degree quantifiers formed from QAs like more and less - can be associated with dimensions of measurement other than quantity (and many languages, like Romanian, employ a single QA for many/ much on the one hand and few/little on the other). As Schwarzschild (2005) observes (p. 89), however, these can only be monotonic dimensions. This is illustrated in his example (82) for the DP too much gold.
(82) He put too much gold in the ring.
a. possible dimensions: weight, volume
b. prohibited dimensions: purity, color

The observation is that the sentence above can be used in situations in which the ring contains too much gold by weight or volume but not in situations in which the gold in the ring is too high in e.g. purity.

Interestingly, QAs are monotonic in a way that doesn't seem tied to their syntax, which makes them distinct from numerals and MPs in Schwarzschild's system. Recall Schwarzschild's claim that attributive measure words and phrases require a non-monotonic dimension of measurement, while those in the partitive required dimensions which are monotonic on the part-whole relation. Attributive QAs - to the extent to which they're acceptable in English - are unlike attributive MPs in that they require a monotonic dimension. ${ }^{16}$
(83) Much rice was consumed that day.
a. possible dimensions: volume, weight
b. prohibited dimensions: stickiness, expense-per-unit
(84) John has as much cheese as he does beer.
a. possible dimensions: volume, weight
b. prohibited dimensions: age; acidity

Schwarzschild addresses this difference between QAs and other measure DPs by suggesting that QAs are "high" in the functional projection (his MonP) relative to other attributive adjectives. I believe that another, more explanatory suggestion is that QAs are overt instances of $\mathrm{M}-\mathrm{Op}_{d}$, measuring sets of degrees and, as a result, requiring that the dimension of measurement be monotonic on the part/whole structure. This is consistent with the independently motivated conclusions in Rett, 2007, 2008.

There are some interesting facts about the distribution of QAs which can be taken as evidence for this claim. Examination of individual/degree polysemy demonstrated that measure DPs can in context denote a measure; I can, for instance, point at a table full of pizzas and say That's not what we ordered at last year's party, referring to the quantity of pizzas. But this isn't always possible; when a DP is used as a measure anaphor, it must include a QA (examples from Corver, 1997).
(85) a. John is fond of Sue. Maybe he is too *(much) so.
b. John is fond of Sue. But not that *(much) so.

It even seems to be the case that the demonstrative that cannot receive a degree interpretation when used on its own, for instance, as the answer to a quantity question:
(86) A: How many pizzas did you order?

B: (pointing to pizzas on the table) That *(many).

[^12]Corver used the data in (85) to argue that much is a functional head deleted in front of adjectives (*much tall) but pronounced in front of proadjectives (i.e. so), much like do-support requires pronunciation of the auxiliary do in cases of ellipsis. But the data in (86) suggest that much is required in a superset of those syntactic contexts.

So what explains the presence and obligatoriness of the quantity adjective in these examples? Below is a minimal pair that suggests that monotonicity on the part-whole relation plays a role.
a. John is [MP 3 feet] tall and Bill is [mp that (*many/much)] wide.
b. John is [mp 3 feet] tall and Bill is [mp that *(many) inches] wide.

In (87a) the degree anaphor corefers with an entire MP, while in (87b) the degree anaphor corefers with the numeral within that same MP. This difference in referent makes all the difference for the distribution of much: when an anaphor corefers with an MP measuring the height of an atomic individual, it cannot involve a QA. When an anaphor corefers with a numeral measuring the cardinality of a unit, it must involve a QA, even if that unit is itself measuring the height of an atomic individual.

This contrast between (87a) and (87b) can be explained in terms of monotonicity. Consider (87b), in which the MP 3 feet measures an atomic individual. Because atomic individuals lack a part-whole structure in natural contexts, the measure of an atomic individual fails to be monotonic on the part-whole structure. This accounts for why the degree anaphor in (87b) cannot include a QA. In contrast, the anaphora in (87a) is to a numeral, not an MP. Numbers track cardinality and the cardinality of a plurality - even a plurality of units of measure like feet and inches - is monotonic on the part-whole structure of the plurality. This accounts for why the degree anaphor in (87a) can involve a QA. (An explanation for why this QA is obligatory is less straightforward. It seems plausible that QAs are made compulsory in contexts in which their presence effects anaphora resolution.)

Of course, I haven't presented a complete semantics of QAs, including their use in differentials or their unavailability as adjectival modifiers (*John is much tall, see Doetjes, 1997). These issues are addressed in Rett, 2008; Solt, 2014, and references therein. I do hope to have given suggestive evidence that the current account of individual/degree polysemy is on the right track: it accounts for the monotonicity restriction using a broader theory of measurement, and there's independent reason to think that other monotonicity restrictions should be dealt with along the same lines.

## 6. Conclusion

I have shown that individual-denoting DPs can also denote degrees, but only if the salient dimension of measurement is monotonic on the part-whole structure. I've argued that this observation extends to several previously studied phenomena, including amount relatives, pseudopartitives, and the apparent ambiguity of how many questions.
l've argued based on this monotonicity restriction that the degree interpretation is secondary, derived under a certain condition from the individual interpretation, a condition I dubbed "equality of measure". I've argued that this monotonicity restriction is a natural consequence of the assumption that natural language employs a null measurement operator that can measure different types of entities. I've shown that this claim is consistent with previous work on monotonicity in other phenomena (Schwarzschild, 2006b). Additional support for the connection between polysemy, measurement and monotonicity comes from some new observations about the distribution of quantity adjectives like much in English.

While my proposal is a semantic one, I've suggested that there are parallels with other cases of deferred reference, and that the true source of the polysemy may be pragmatic. I have tried to argue, however, that any account of individual/ degree polysemy should treat the individual interpretation as primary and should derive the degree interpretation using the same mechanism of measurement employed elsewhere in natural language.

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[^1]:    ${ }^{1}$ These are not the first observed instances of 'cross-domain' polysemy; Doetjes $(1997,2007)$ suggests that VPs modified by adverbials like $t$ wice can denote either a plural event or a degree corresponding to a quantity of events. This is illustrated below; see also Nakanishi (2007a) and Burnett (2012).
    (i) a. John rode the roller coaster five times, (all of) which were fun. event
    b. John rode the roller coaster five times, which was enough. degree

    In (a), the VP rode the roller coaster $n$ times denotes a plurality of roller-coaster-riding events, which were scary or fun (and which trigger plural agreement on the predicate). In (b), the VP denotes an amount of riding events, tiggering singular agreement on the predicate. Ideally, given a semantics that treats individuals on par with events, the discussion here of individual/degree polysemy can extend to event/degree polysemy.

[^2]:    ${ }^{2}$ Brasoveanu (2009) suggests that pseudopartitives like a two-liter water receives singular agreement as the result of the "coercion of a plural entity into a singular group/amount/kind" (a liter, in this case). He draws an explicit comparison to the sentence in (i).
    (i) Scrambled eggs and bacon is your favorite food.

    But this explanation can't extend to the data above. It's clear that the predicate be your favorite food is not a degree predicate, so it seems wrong to analyze the DP in (i) as denoting a degree. It instead seems to denote a kind - plausibly the sort of thing be your favorite food selects for - and can arguably do so via semantic coercion or the process behind deferred reference. In contrast, predicates like be too many eggs for one robin to lay don't select for kinds or groups, but rather degrees, as demonstrated by its selectional restrictions on other subjects.
    (ii) a. Seven eggs is too many for one robin to lay.
    b. \#Blue eggs is too many for one robin to lay.
    ${ }^{3}$ Thanks to an anonymous reviewer for this point.

[^3]:    ${ }^{4}$ Thanks to Roger Schwarzschild (p.c.) for this example.

[^4]:    ${ }^{5}$ This notion of degrees and dimensions of measurement follows Bartsch and Vennemann (1972) in taking a degree to be a point on a scale with a) a particular ordering (cf. tall and short) and b) a dimension of measurement. In this respect, Schwarzschild's use of degree semantics which I adopt explicitly in section 4 - can be seen as a version of Grosu and Landman's (1998) 'thick degrees,' degrees with a sortal description.

[^5]:    ${ }^{6}$ In some constructions, the context dependence of the dimension of measurement is confounded by the context sensitivity of the standard of comparison invoked by be enough and similar predicates. A reviewer offers the example in (i).
    (i) 9-karat gold is enough to make the ring valuable.

    Again, we see that what counts as 'enough' depends on what the relevant standard of comparison is (see section 2). If the standard were to do with purity, a non-monotonic measure, we would expect (47) to be ungrammatical. It's likely that the standard invoked by enough in (47) is something else: the value of the ring, making the gold's purity relevant and monotonic in a complicated way. See Champollion (2010) for similar observations that Schwarzschild's notion of monotonicity is oversimplistic..

[^6]:    ${ }^{7}$ Just as Partee and Rooth (1983) and Partee (1987) found it useful to treat proper names as having a type-variable denotation, degree semanticists have found it useful to treat numerals and MPs (qua degree proper names) as having a type-variable denotation. I will additionally assume, following Schwarzschild, 2005, 2006a, that degrees can be type-raised to a generalized quantifier type $\langle\langle d, t\rangle, t\rangle$. So John can denote an individual of type $e$ or a generalized quantifier $\lambda P . P(\mathrm{j})$, and 6 ft can denote a degree of type $d$ or a generalized quantifier $\lambda D . D(6 \mathrm{ft})$.

[^7]:    ${ }^{8}$ Numeral DPs like five cats - the phenomenon used to originally motivate null measurement operators - suggest a semantic theory in which MOp is selected for by the numeral or an MP. But other data suggest the need for M-Op to have a distribution that allows it to be independent of numerals, e.g. exclamatives like What children you have! (Rett, 2011).

[^8]:    ${ }^{9}$ A similar meaning is assumed to be primary in Schwarzschild (2005), who uses it to explain the incompatibility of MPs and negative antonyms, as in *4ft short.
    ${ }^{10}$ The denotation of be enough is oversimplified here; as mentioned earlier, and as discussed in Meier (2003), be enough has other (implicit) arguments.

[^9]:    ${ }^{11}$ A reviewer points out that these truth conditions for the degree interpretation predict that the sentence presupposes the existence of pizzas (in the world of evaluation). I believe that this prediction is correct; it seems odd to say There are no pizzas, and four pizzas is enough to feed the group, although perfectly acceptable to say There are no pizzas, but four pizzas would be enough to feed the group.
    ${ }^{12}$ These data present a possible contrast with deferred reference, which arguably does allow for disjoint anaphora, as in Hey, Shirley, Ive got a filet mignon at Table 7 that says it's the best steak he's ever eaten (Ward, 2004). But the situation is not entirely clear in either case. A reviewer observes the contrast in (i).
    (i) a. John bought four pizzas. They were delicious, but it was more than we needed.
    b. John bought four pizzas. They were delicious, but *it was too many.

    It's possible that it denotes the event of John buying four pizzas, as it does in the sentence Mary read War and Peace. It took longer than she expected. But the contrast in (i) suggests the need for caution in examining the data.

[^10]:    ${ }^{13}$ See Schwarzschild（2011）for an attempt to explain why heavy must receive a distributive interpretation in this and similar constructions；l＇ve stipulated it in（73）by incorporating a distributivity operator into the meaning of heavy so that it operates on elements $y$ of the plurality $x$ ．

[^11]:    ${ }^{14}$ See Tanaka (2006) for evidence that Japanese has two different (positive-antonym) QAs: one (takusan) corresponding to the individual M-Op, and another (ooku) corresponding to degree M-Op. Both can modify nouns, but only the latter can modify comparative differentials.
    ${ }^{15}$ Because these QAs are not modified or bound overtly, these sentences count as instances of the positive construction, e.g. John is tall. I follow the compositional semantics in Rett, 2008 for attributing to such constructions the semantic property of evaluativity, the restriction that the degree in question exceed a contextually valued standard $s$ (i.e. $\wedge d^{\prime}>s$ ).

[^12]:    ${ }^{16}$ A reviewer wonders about differential QAs, as in That pool has too much warm water. Here, too, much is subject to the monotonicity restriction: the sentence can be used to describe a situation in which the pool has more warm water than it should. But it cannot be used to describe a situation in which the pool's water is warmer than it should be.

