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UNIVERSITY OF CALIFORNIA
RIVERSIDE

Visuospatial Cognition, Movement, and the Mathematic Achievement of Students

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Education

by

Courtney M. Hilton

June 2018

Dissertation Committee:

Dr. Keith Widaman, Chairperson

Dr. Rollanda O'Connor

Dr. Jan Blacher

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2018

The Dissertation of Courtney M. Hilton is approved:

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University of California, Riverside

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DEDICATION

I would like to dedicate this dissertation to my dear mother, a teacher, and my first educator. I would also like to dedicate this work to the young girl who often took the long way home, down University Avenue, just to get a glimpse of college life. She snuck into the jams to hear the bands. She took long peeks at the professors loading into fancy cars and thought she wanted to be one. She chatted with the girls in front of the freshman dorm. She knew she loved the university.

And to the universe, a constant friend.

ABSTRACT OF THE DISSERTATION

Visuospatial Cognition, Movement and the Mathematic Achievement of Students

by

Courtney M. Hilton

Doctor of Philosophy, Graduate Program in Education

University of California, Riverside, June 2018

Dr. Keith Widaman, Chairperson

The study investigated the predictive relations of language and cognitive variables and three domains of mathematics achievement in grade levels spanning first to undergraduate students. The research is particularly interested in the moderating effects of grade level on the relations of visuospatial working memory and mathematics. The research is guided by Baddeley's model of working memory (Baddeley and Hitch, 1974, 1986, 1996, 2000), Logie's two-part visuo-sketchpad (1995), and the growing body of research investigating the dissociative properties of working memory. A sample of 2,375 participants was drawn from the Woodcock Johnson-Fourth Edition standardization sample. The participants were administered assessments in the areas of oral language, mathematics, verbal working memory ability, and visuospatial ability. A theoretical visuospatial working memory variable was created from the visuospatial ability assessments. The results of four hierarchical regression analyses revealed oral language, verbal working memory, and visual working memory are significant predictors of mathematics achievement. The tests of moderating effects results showed grade level

moderated the relations between visuospatial working memory and mathematics achievement.

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Chapter 1: Introduction

Mathematics is a prominent topic in academic communities, although this is a relatively new distinction. The literature often tracks increased mathematics discourse to periods surrounding the launch of Sputnik and the innovative publications emerging from that era. The said publications detailed the need to focus on individual differences and ways to nurture scientific and technical potential in U.S. students (Newcombe et al., 2009; Wai, Lubinski, & Benbow, 2009). Researchers highlighted the need for improved skills in science, technology, engineering, and mathematics (STEM) domains.

Several studies have compared the math achievement of U.S. students to the achievement of students in other nations. The results often reveal U.S. students fall below their counterparts in other countries. For example, the Organization for Economic Cooperation and Development's Program for International Student Assessments reported that 15-year-old students in the United States ranked 24th out of 29 countries in problem solving and mathematics literacy (Lemke et al., 2004). Cross-cultural studies with a comparison of U.S. students to East Asian students revealed East Asian students consistently outperform U.S. students in almost every area of mathematical knowledge (Geary, Fan, & Bow-Thomas, 1992; Lemke et al., 2004; Lim & Son, 2013). The prior findings increased national interest in improved mathematic outcomes and particularly led to robust interest in a variety of factors related to math learning (e.g., instruction, intervention, learning, and cognition).

The present study pertained to cognitive factors related to mathematic achievement. The collective body of research regarding cognition and mathematic

achievement has demonstrated several cognitive factors contribute to mathematic performance. For example, working memory (Chong & Siegel, 2008; Geary, 2011; Mazzocco & Myers, 2003; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2014), phonological or language processing (Swanson & Beebe-Frankenberger, 2004; Szucs et al., 2014), and attentional systems (Fuchs et al., 2006; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014; Geary, Hoard, Nugent, & Bailey, 2012) have substantial support in the literature as cognitive predictors of mathematic achievement. The present research was a quantitative analysis of the predictive power of several cognitive processes related to mathematic achievement in children and adults in elementary grades through college. Of primary interest was the predictive relations of visual-spatial working memory, grade level, and multiple mathematic domains.

The first section of this chapter is an overview of the developments in mathematic research and the changes to content standards for mathematics. The second section provides a review of the literature on three subdomains of mathematics (arithmetic, algebra, and geometry). The individual differences associated with mathematic performance are also discussed. The final section includes a review of the component processes underlying mathematic competence (e.g., working memory, spatial perception, and attention).

Developments in State Content Standards for Mathematics

The lagging mathematic performance of U.S. students has led to increased attention from national committees, policy-makers, and councils. The extensive interest in improving math outcomes led to a significant shift in mathematics instruction in public

classrooms. The Common Core State Standards for Mathematics unveiled in 2010 were part of the Common Core State Standards Initiative, and revealed drastic changes to content goals, placement of topics in grades, and attention to mathematic practices, such as problem solving, reasoning, and modeling (Teuscher, Tran, & Reys, 2015). One example of the sweeping change involves classroom instruction in mathematic problem solving. Common Core State Standards for Mathematics and Common Core State Standards Initiative transformed the role of problem solving by defining it as central to all mathematic content areas and advocated for its instruction to be connected to multiple processes and integrated into student experiences, rather than taught as an isolated unit or lesson. Another example is algebra and geometry concepts previously thought to be too challenging for young children; however, these concepts were recommended for early learning with the unveiling of Common Core State Standards for Mathematics.

Algebra was once thought to be developmentally inappropriate for young children (Sfard, 1991). The terms *cognitive gap* and *interference* were used in the literature to explain the difficulty young children encountered in developing algebraic competence (Linchevski & Herscovics, 1996; Sfard, 1991). Recent researchers suggested arithmetic principles involve generalizations that are algebraic in nature; therefore, algebra warrants a prominent role in early instruction (Blanton & Kaput, 2001; Carraher & Schliemann, 2002; Fuchs et al., 2012). The National Council of Teachers of Mathematics (2000) identified algebra as one of the content standards for students in kindergarten through Grade 12. With Common Core State Standards (2010), algebraic reasoning became a Common Core domain area in elementary school, a part of several domain areas in

middle school curriculum, and is one of six important areas of study for high school students (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010; Powell & Fuchs, 2014).

Geometry also received attention from national committees and councils. Geometry standards from Common Core State Standards and the National Council for Teachers of Mathematics in 2000 called for effective geometry instructional programs for children in kindergarten through Grade 12 as a response to international research showing that the geometry problem solving skills of U.S. students had fallen behind the skills of peers in other countries (Bybee & Stage, 2005; Lemke & Gonzales, 2006; Zhang, Ding, Stegall, & Mo, 2012). Teuscher et al. (2015) analyzed the content of geometry taught in the middle grades and reported dramatic changes to the instruction of geometry. For example, after reviewing the geometry content of the Common Core State Standards for mathematics, Teuscher et al. determined more than 50% of what should be taught to meet the new standards will be new information to the respective grade levels in six of the eight states reviewed. Furthermore, according to the Common Core State Standards 2010, students should begin learning geometry in kindergarten and progress to using more precise definitions and developing proofs while in elementary and secondary schools. For example, kindergarteners should be able to identify shapes (squares, circles, triangles, rectangles, hexagons, cubes, and spheres), and by third grade, students should be able to reason with shapes and their attributes (Zhang et al., 2012).

As expectations in higher-order math domains increase for U.S. students, natural concern arises regarding the mastery of concepts. Only 26% of 12th-grade students

performed at or above the proficient level in math on the National Assessment of Educational Progress (NAEP; National Center for Educational Statistics, 2010). Students with learning disabilities are especially at-risk for learning challenges in higher order math domains. Dobbin, Gagnon, and Ulrich (2014) reported only 6% of students with learning disabilities scored at or higher than proficient on 12th-grade NAEP tests in the math domain. In this study, the researcher investigated the relations between mathematic achievement and cognitive variables across multiple grade levels to determine whether different variables have stronger associations with achievement at different times in a student's advancement through the grades.

State of Research on Mathematic Competence

Math difficulties have been researched far less than reading difficulties (Mazzocco & Myers, 2003). One explanation for the lack of research on mathematic difficulties is because poor math skills have long been considered more socially acceptable than poor literacy skills (O'Hare, Brown, & Aitken, 1991). It is therefore plausible that the limited number of studies on math performance is because of the amount of resources devoted to reading difficulties in the last several decades. Another explanation for the neglectful trends of the past with respect to mathematic competence may be attributed to the persistent belief that mathematic difficulties are based in linguistic competencies (Rourke & Conway, 1997). Reading development has benefited much from this belief. Empirical research devoted to reading development has proven beneficial to students. From this body of research came the identification of core cognitive deficits: phonological processing in early identified reading disabilities and oral

language deficits in late-identified disabilities (Catts, Compton, Tomblin, & Bridges, 2012; Mazzocco & Myers, 2003). Furthermore, recommendations for evidence-based instruction and intervention resulted from the reading research.

The mathematics performance of U.S. students has not received the same attention as reading performance. Currently, there is no consensus on the definition of *math disability*. Mazzocco and Myers (2003) reported it is unlikely that poor mathematic performance is linked to a core deficit. Geary (2011) referred to early mathematic cognition as a “suite” of basic quantitative competencies that include understanding of numeric magnitude and quantities, principles of counting, and mastery of addition and subtraction concepts. Vukovic and Siegel (2010) found the most persistent types of math problems are associated with deficits in calculation, practical problem solving, number facts, and reading.

Despite the lack of empirical mathematic research spanning several decades, evidence of important outcomes exists from researchers who have studied math. The literature includes several cognitive deficits and fundamental arithmetic difficulties attributed to math disability (Fuchs & Fuchs, 2006). It is important to continue expanding math research, as mathematic difficulty can be a persistent and pervasive problem for students who struggle in mathematic domains (Simos et al., 2008). Furthermore, mathematics competence accounts for significant variance in employment, wages, and productivity (Rivera-Batiz, 1992). Poor mathematics skills limit access to advanced mathematic concepts and preclude many employment opportunities (McCloskey, 2007). Nearly 60% of North American adults have lower levels of mathematics competence than

what is considered necessary for coping with everyday life and work in advanced societies (Statistics Canada and Organization for Economic Cooperation and Development, 2005; Vukovic & Siegel, 2010).

The current state of math research suggests the neglectful trends in mathematics research are a thing of the past. Researchers are conducting more empirical studies to investigate math difficulties and learning disabilities; for example, a search of peer-reviewed articles on ProQuest databases using the term *math achievement* from the years 1986–2016 (30 years) yielded 13,579 studies. Another search using the term *math disability* yielded 3,369 articles on mathematic disabilities. When the same topics were searched between the years of 1956–1986 (the preceding 30-year period), the results were 8,290 and 478 citations, respectively. The numbers from the years 1986–2016 were substantially higher than for the years 1956–1986.

Another concern with the previous state of mathematic research is the observation that earlier investigators disproportionately focused on the acquisition of basic facts. When compared to basic math skills, studies of mathematic competence with algorithms and other higher- order skill are few (Fuchs & Fuchs, 2006). A search of peer-reviewed articles investigating algebra and math disabilities between 1986 and 2016 revealed 188 studies. A search using the keywords *geometry* and *math disabilities* within the same time frame yielded 140 peer-reviewed studies. Between the years 1956 and 1986, the number of peer-reviewed studies in algebra and geometry paired with math disabilities were 11 and 8, respectively. Again, the numbers were remarkably higher for the years 1986–2016.

The present study contributes to growing research on mathematics competence and cognition, across multiple mathematics domains and grade levels.

Subdomains of Mathematics Competence: Arithmetic

Researchers do not fully understand all factors that influence children's mathematical learning or the sources of individual differences in mathematics competence, but progress has occurred (Geary, 1994). For example, Duncan et al. (2007) reported children who begin school with less understanding of number counting and simple arithmetic than their peers are at risk of falling behind throughout their schooling. Many researchers of empirical studies investigated math competence in terms of skill development. Geary (1993) reported children who have trouble with math tend to use immature problem-solving strategies, have long solution times, have atypical long-term memory representations of basic addition facts, and frequently commit computational and memory retrieval errors. These difficulties may be because of poor procedural (i.e., steps and procedures) and conceptual knowledge (e.g., logical relations, working memory resources, attention, memory retrieval, speed of processing, especially counting speed, and counting concepts; Fleischner, Garnett, & Shepherd, 1982; Geary, Bow-Thomas, & Yao, 1992; Geary, Brown, & Samaranayake, 1991; Geary & Widaman, 1987; Goldman, Pellegrino, & Mertz, 1988).

Geary et al. (2012) noted that children with math disabilities have severe deficits in the ability to develop long-term memory representations of basic arithmetic facts and retrieve learned arithmetic facts. Geary et al. (1991) found children without math disabilities rely more on memory retrieval, have fewer errors, and rely less on counting to

solve addition problems; moreover, the speed of executing both computational and retrieval strategies increases across time for students without math disabilities. In contrast, children with math disability showed no change in their use of strategies or their rate of fact retrieval. For many children with math disability, this pattern does not appear to change substantially across the elementary school years, even with extensive drilling (Fleischner et al., 1982; Howell, Sidorenko, & Jurica, 1987). The findings of these studies suggest a retrieval developmental delay that many children may not outgrow (Geary, 1993; Goldman et al., 1988).

Geary, Bow-Thomas, et al. (1992) found first graders with math disability are developmentally delayed when compared to nondisabled peers in their understanding of counting concepts, which contributes to poor computational and arithmetic skills later. Children with math disability have difficulty acquiring more complex mathematical skills because of a fundamental deficit in basic arithmetic and number sense. Based on early findings of retrieval and computational deficits in children with a math disability, considerable research exists to expand these investigations. Children with math disabilities have significant deficits in procedural skill (e.g., ability to use algorithms to solve simple and complex calculations), and the most consistent computational deficit is poor fact retrieval (Chong & Siegel, 2008; Geary, 1990). Deficiencies in fact mastery and calculation fluency may be persistent and defining features of math disabilities (Andersson, 2008; Chong & Siegel, 2008; Jordan, Kaplan, & Hanich, 2002; Jordan, Hanich, & Kaplan, 2003a, 2003b).

Mathematic research has primarily pertained to whole numbers, but other types of numbers deserve the attention of researchers. Many children struggle to acquire fraction knowledge (Siegler & Pyke, 2013). Fractions represent a mathematic skill area that presents persistent difficulty for children with mathematic difficulties (Algozzine, O'Shea, Crews, & Stoddard, 1987; Hecht & Vagi, 2010; Hecht, Vagi, & Torgesen, 2007; National Math Advisory Panel [NMAP], 2008a). A recent NAEP report revealed that 50% of eighth-grade students could not order a set of three fractions from smallest to largest correctly and these difficulties continue into high school. Another NAEP report revealed less than 30% of 11th graders were able to translate .029 into the correct fraction (Siegler & Pyke, 2013). Poor mastery of fractions has long-term effects. Siegler et al. analyzed (2012) two longitudinal data sets and the results revealed fifth-grade students' knowledge of fractions predicted success with algebra and overall math achievement 5 to 6 years later. Therefore, fractions are a gateway to advanced mathematic concepts.

Children who have difficulty with both the conceptual and procedural aspects of fractions are at a higher risk for obstacles that prevent moving beyond basic math to advanced topics in mathematics (Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2010; Heller, Post, Behr, & Lesh, 1990; Loveless, 2003; NMAP, 2008a). For example, conceptual fraction knowledge may include knowing fractions are numbers that can range from negative to positive, knowing numerator and denominator relations, and understanding magnitude (Siegler & Pyke, 2013). Procedural knowledge of fractions may include addition and subtractions of fractions, common denominators, and multiplication and division of fractions. Individual differences in conceptual and procedural knowledge

of fractions explain the variability in success with fraction computation and fraction word problems, set-up, and accuracy (Siegler & Pyke, 2013). Conceptual knowledge uniquely explains individual differences in fraction estimation skills (Hecht, 1998).

Problem solving is an area of mathematic skill that presents considerable difficulty to children struggling with mathematic competence. Difficulty with word problems may arise because of the requirement to use multiple steps and skills to solve the problems (Parmar, Cawley, & Frazita, 1996; Powell & Fuchs, 2014). Difficulty may also arise from language-based challenges as word problems are embedded within a linguistic context and many students experience challenges with language (Fuchs, Seethaler, et al., 2008; Powell & Fuchs, 2014). To correctly solve a word problem, students must use the language of the problem narrative to develop a problem model, identify the missing information, generate a number sentence that represents the problem model and incorporates the missing information, and derive the calculation problem for finding the missing information (Powell & Fuchs, 2014). Using narrative to extract pertinent information for developing a number sentence from a word problem can be difficult for many students (Carpenter, Moser, & Bebout, 1988; Herscovics & Kieran, 1980; Powell & Fuchs, 2014). Math education reform during the last few decades has prompted schools to emphasize the development of a more complex problem-solving capacity in children (Fuchs & Fuchs, 2006; Resnick & Resnick, 1992) with a measurement focus on performance with real-world problem solving and solutions that involve applied mathematical skills (Fuchs & Fuchs, 2006).

Andersson (2008) extended the literature on mathematics competence by looking beyond acquisition of basic math skills (e.g., number sense, basic calculations) and investigated math performance in children who struggle mathematically and children who struggle with reading, across a variety of math tasks. Andersson found that children with math difficulties experienced substantial difficulty with mathematics word problem solving. The challenges with problem solving were attributed to several processes: poor skills in multidigit calculation, poor arithmetic fact retrieval, poor understanding of calculation principles, and deficits related to specific problem-solving processes (e.g., establishing a problem representation and developing a solution plan). Fuchs et al. (2009) reported mathematic problem solving conceptually is a transfer task that requires students to apply skills, knowledge, and strategies to novel problems. Furthermore, this form of transfer can be highly difficult for younger children and children with disabilities who have difficulties generalizing.

The literature on mathematics competence with problem solving also suggests working memory and listening comprehension are predictors of children's skill with solving word problems and pre-algebraic concepts (Fuchs, Powell, et al., 2014). Explicit word-problem instruction has been effective in helping students who struggle with word problems (Kroesbergen, Van Luit, & Maas, 2004; Powell & Fuchs, 2014). In recent work, researchers have focused on schemas, with which students are taught to recognize problems as belonging to problem types and to apply solutions that match the accompanying schemas (Fuchs et al., 2004; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007). Developing schemas for categorizing word problems helps students

understand that problems can belong to familiar categories (Cooper & Sweller, 1987). In addition, developing schemas accommodates working memory deficits that may exist, which are linked to difficulty with word problems (Swanson & Beebe-Frankenberger, 2004).

Subdomains of Mathematics Competence: Algebra

Landy, Brookes, and Smout (2014) defined *algebra* as the structural representation of numerical relations. Moreover, algebraic equations are described as one of the most powerful mathematical representations for expressing quantitative relations. Algebraic statements in which variables represent relations among unknown quantities express some of the most abstract assertions many people will ever consider. Algebra is qualitatively different from the thinking involved in arithmetic (Tolar, Lederberg, & Fletcher, 2009). For example, arithmetic problems are typically set on the page and require minimal structural changes or movements, thus allowing students to concentrate easily on computational demands. One may even argue that algebra is quantitatively different from arithmetic because of an increased number of operations and increased demands on cognition. Algebra requires students to manage structural, procedural, and numerical relations simultaneously. Additionally, formal notations found in algebra typically make extensive use of physical proximity and spatial relations (Landy et al., 2014).

Theories of mathematics achievement suggest algebra is cognitively demanding because students must flexibly switch between operational and structural views of mathematical expressions to be proficient (Mason, 1989; Sfard & Linchevski, 1994;

Tolar et al., 2009). Algebra achievement is not only a function of acquired arithmetical skill, but it is also a function of basic cognitive resources that may have direct effects on algebra achievement. Tolar et al. (2009) reported three-dimensional visualization as one of three cognitive resources related to algebra achievement. The other two cognitive resources are working memory and computational fluency (Engle, Tuholski, Laughlin, & Conway, 1999; Geary, Saults, Liu, & Hoard, 2000; Reuhkala, 2001). Working memory may be related to algebra achievement because of the requirement to actively maintain multiple conceptions of mathematic expressions (e.g., objects with features as a set of procedures) while solving algebraic problems and switching between them as appropriate (Tolar et al., 2009). In addition to working memory, visual-spatial tasks that include mental manipulation of three-dimensional objects correlate with math assessments involving higher-order math skill in samples of adolescents and adults (Tolar et al., 2009).

Subdomains of Mathematics Competence: Geometry

Geometry requires strong visual imagery and higher-order cognitive skills, such as metacognitive skills, prediction, planning, monitoring, and evaluation of math information (Dobbins et al., 2014; Zhang et al., 2012). Children may be especially challenged by geometric problem solving because the comprehension of geometric problems is much more complex than that of other math domains, such as calculation (Wong, Hsu, Wu, Lee, & Hsu, 2007). Researchers of geometry reported problems in geometric competence stem from lacking the prerequisite skills (e.g., calculation skills,

graphing, working with lines, reasoning with geometric ideas) for geometry (Carroll, 1998; Mistretta, 2000).

Many students who struggle with learning geometry have visual-spatial learning deficits. Geometry competence depends on having the ability to spatially represent mathematic relations (Zhang et al., 2012). McLean and Hitch (1999) reported children with mathematic difficulties performed at a lower level in spatial working memory tasks than did their peers. Students who have high levels of competence with geometry demonstrate the ability to manipulate and hold the complex spatial information required to solve the geometric problems (Zhang et al., 2012).

New developments in research on visual-spatial skills include increased interest in the relations between spatial reasoning and geometry (Mulligan, 2015). Relative to other mathematics domains, geometry is intuitively and mathematically spatial because of the relations between points or collections of points, such as lines. The fact that a point is at a certain distance from a second point is a geometric property and a spatial property (Cheng, Huttenlocher, & Newcombe, 2013). Studies of mathematics learning highlight the role of spatial ability in the development of skills involved with interpreting patterns (Clements & Sarama, 2011; Papic, Mulligan, & Mitchelmore, 2011). The implications of these studies for teaching, learning, and professional practice make these studies different from traditional educational views that ignored visual-spatial development. This difference results from design of spatial reasoning tasks and instruments, attention to children's growth of broader mathematical conceptual ideas, and application in practice. The results of these studies raised important questions regarding how to differentiate

instruction, assessment, curriculum, and intervention programs for learners who display differences in spatial ability and mathematics learning (Mulligan, 2015).

Individual Differences in Mathematics Competence

Landy et al. (2014), described mathematic ability as the ability to interpret abstract relations among abstract entities. In another definition, math is described as a language (Terao et al., 2004). According to Terao et al. (2004), mathematics is a language of formal symbols that describe the phenomenon being viewed. Many theorists argue math is fundamentally visual. Dehaene, Spelke, Pinel, Stanescu, and Tsivkin (1999) suggested mathematical thinking emerges from an interplay between the symbolic areas of the brain and the visual-spatial systems.

Whether one advocates language-based learning of mathematics, visual learning, or the combination of factors, one of the promising aspects of math research is the consistency among the relations of cognitive variables (Mazzocco & Myers, 2003). Per Mazzocco and Myers (2003), consistency across reports shows that both reading-related skills (e.g., phonological processing) and executive skills (e.g., working memory, inhibition) are associated with math achievement. Still, researchers have not identified the extent to which these cognitive correlates underlie one or more specific types of math difficulties or disabilities. This finding highlights the need for additional research.

Mathematics builds on several cognitive abilities (Chong & Siegel, 2008; Geary, 2011; Mazzocco & Myers, 2003; Szucs et al., 2014). Researchers suggest the individual differences of mathematics learning during elementary school depend not only on the school's instructional program, but also on the children's early numerical competencies,

such as understanding of magnitude as well as general cognitive abilities (Fuchs, Geary, et al., 2014; Swanson & Beebe-Frankenberger, 2004). Chong and Siegel (2008) identified working memory as the strongest cognitive deficit linked to math difficulties based on prior research (Bull, Espy, & Wiebe, 2008; Passolunghi, Mammarella, & Altoe, 2008; Swanson, 2011). However, other researchers have hypothesized that sets of cognitive variables associated with aspects of math development may differ as a function of the math domain (e.g., working memory supports fact fluency; phonological processing supports computation; Fuchs et al., 2005; Geary et al., 1991; Wilson & Swanson, 2001). Reading and phonological processing have been tied to mathematic competence for some time. Phonological processing is associated with computations in Grades 2–5 (Hecht, Torgesen, Wagner, & Rashotte, 2001). Swanson and Beebe-Frankenberger (2004) and Swanson, Jerman, and Zheng (2008) found phonological processing is a predictor of performance on word problems in elementary children.

Solutions to multiplication and addition problems are likely to be verbally encoded and retrieved from long-term phonological memory, rather than computed on demand (Ashcraft, 2002; Ashcraft & Bataglia, 1978; Ashcraft & Stazyk, 1981; Szucs et al., 2014). However, early models of memory advocate for fact retrieval from declarative memory or semantic memory (Cohen & Squire, 1980; Tulving, 1987). Leather and Henry (1994) reported strong correlations between phonological awareness measures and arithmetic test scores in 7-year-olds. Hecht et al. (2001) found phonological memory, the rate of access to phonological codes in long-term memory (rapid naming of letters and numbers), and phonological processing were strongly associated with computational

ability; additionally, overall phonological skill nearly completely explained the relation between reading and computational ability in elementary students. Swanson and Beebe-Frankenberger (2004) and Simmons, Singleton, and Horne (2008) reported similar findings in young children.

According to Szucs et al. (2014), a large number of researchers have reported strong relations of short-term memory and working memory with mathematic achievement (Gathercole, Pickering, Knight, & Stegmann, 2004; Passolunghi & Mammarella, 2010; Passolunghi & Siegel, 2001; Raghubar, Barnes, & Hecht, 2010; Simmons, Willis, & Adams, 2012; Swanson & Jerman, 2006). Poor working memory resources can influence mathematic skills development (Geary, 1993; Hecht & Vagi, 2010). For example, Geary et al. (1991) found a numerical memory span advantage of about 1 digit for students without math learning disabilities when compared to students with math disabilities; moreover, the shorter the memory span, the more frequent the computational errors (Geary, 1993). Swanson (2011) reported working memory as explaining a significant portion of the variance in children's solution accuracy with word problems. Wilson and Swanson (2001) reported that students with poor mathematic competence have more problems with visual working memory than their typical peers. The current study will extend the literature by investigating visual-spatial working memory's relation with multiple mathematic domains.

Another important consideration in the working memory and math achievement literature is what Swanson (2011) reported. According to Swanson, the body of working memory research does not consistently identify working memory as having a significant

relation with mathematic problem solving. Other studies report reading proficiency mediates the role between working memory and problem solving (Lee, Swee-Fong, Ee-Lynn, & Zee-Ying, 2004); additionally, Lee et al. reported high intelligence and vocabulary skills led to better performance on mathematic word problems. Fuchs et al. (2006) also did not find a significant relation between working memory and mathematic problem solving. Furthermore, nonverbal problem solving, concept formation, sight word efficiency, and vocabulary were identified as predictor variables.

Attention is also identified as a robust cognitive predictor of mathematics competence in the literature (Fuchs, Compton, et al., 2014; Fuchs et al., 2005, 2006; Geary et al., 2012). William James, the 19th century psychologist, described attention as the “taking of the mind” out of several possible objects or trains of thought, to effectively deal with one of them. Posner (1994) added the element of “selection” to the definition of attention. Several models of memory emphasize the role of attentional processes. Attention is involved in all aspects of memory (e.g., selecting stimuli for further processing, encoding, storage, and retrieval; Muzzio, Kentros, & Kandel, 2009; Posner, 1994). To maintain information in short term memory, one must focus one’s attention in the present (Anderson, 2005; Greenstein, Blachstein, & Vakil, 2009). Active rehearsal and retrieval of information requires attention (Cowan, Nugent, Elliott, Ponomarev, & Sauls, 1999; Greenstein et al., 2009). Attention is considered to be a part of the central executive of Baddeley’s (2002) model of working memory. The central executive is believed to control the flow of information: switching between tasks, selection of stimuli, and inhibition of irrelevant ones, which are functions mainly attributed to attention.

Children with math difficulties tend to engage in less attending behavior during math instruction (Fuchs et al., 2005; McKinney & Speece, 1986; Passolunghi, Cornoldi, & De Liberto, 1999). Bull, Johnston, and Roy (1999) and Miyake, Friedman, Rettinger, Shah, and Hegarty (2001) found that executive skills (a subsystem of attentional systems) contribute to math performance. Swanson (2011) reported inattention as a co-morbid condition in children with working memory deficits. Fuchs et al. (2006), using path analysis, found teacher ratings of inattentive behavior significantly predict mathematic problem solving.

Fennema (1979) suggested that all mathematical tasks require spatial thinking, particularly as the mathematic concepts become more complex (van Garderen, 2006). Visual-spatial reasoning predicts many forms of mathematics learning such as arithmetic and word-problem solving (Fuchs et al., 2014). Spatial skills can improve children's development of numerical knowledge by helping them to acquire a linear spatial representation of numbers (Gunderson, Ramirez, Beilock, & Levine, 2012). Visual-spatial deficits can disrupt performance in arithmetic (Geary, 1993). Son and Meisels (2006) found that early kindergarten visual-motor skills added a small, but unique amount of variance to prediction of achievement in math at the end of first grade. Fuchs et al. (2014) identified visual-spatial reasoning as one of four general cognitive abilities involved in individual differences in numeration (collections constitutes magnitude), along with working memory, listening comprehension, and attentive behavior.

In adolescence, spatial-mechanical reasoning is correlated with performance on math tests measuring fractions, number sense, measurement, geometry, and data

representation (Casey, Nuttall, & Pezaris, 2001; Gunderson et al., 2012); moreover, mental rotation ability among college students and high-ability middle school students predicts performance on the math portion of the SAT-M (Casey, Nuttall, Pezaris, & Benbow, 1995; Gunderson et al., 2012).

Hegarty and Kozhevnikov (1999) discussed the relation between two types of visual-spatial representations and mathematic problem solving. The types of representations discussed were schematic representations that encode spatial relations described in problems and pictorial representations that encode the visual appearance of the objects described in the problems. The results of analysis by Hegarty and Kozhevnikov revealed that the use of schematic representations was more closely associated with mathematic performance than was pictorial representations.

Math Difficulties and Disability

Like reading difficulty, having challenges in math is a substantial obstacle to academic achievement (Mazzocco & Myers, 2003). The acquisition and application of mathematical skills, such as counting and subtraction, are significant because of demands of formal schooling, daily living activities, and employment (Floyd, Evans, & McGrew, 2003; Rivera-Batiz, 1992; Rourke & Conway, 1997). Math disability in children is not unusual (Badian, 1983; Geary, 1993). Badian (1983) reported that 6.4% of elementary and junior high school students have a math disability when compared to 4.9% who showed a form of reading disability. Fuchs and Fuchs et al. (2005) reported similar findings, showing 5% of the school-age population experiences some form of math disability. Moreover, it is possible that many additional children struggle with low

mathematic performance without a formal diagnosis of math disability (Shalev, Auerbach, Manor, & Gross-Tsur, 2000).

Geary (1993) suggested three subtypes of math disability: semantic memory, procedural, and visual-spatial. The semantic subtype is associated with reading disability and is characterized by poor fact retrieval and variable response time. The procedural subtype is characterized by immature strategies, errors in execution, and conceptual delays. The visual-spatial subtype involves misalignment of numeric information, sign confusion, number omission or rotation, and general misinterpretation of spatially relevant information. Of the three math disability subtypes, the visual-spatial subtype is the least understood because of the lack of empirical studies (Mazzocco & Myers, 2003). However, critical gaps in understanding of math disability overall exist, such as how mathematics deficits occur in children over time and within developmental levels (Mazzocco & Myers, 2003). Further research is necessary to have a better understanding of the definition of math disability and the manifestation of underlying deficits.

Geary (2006) reported mathematic disabilities theoretically can result from deficits in the ability to “represent” or process information in one or all of the many areas of mathematics. Therefore, math difficulties may logically stem from a vast number of mathematic domains (e.g., arithmetic, geometry, graphing, theorems; Geary, 2006, p. 199). Mazzocco and Myers (2003) stated math disabilities may vary in type and features as a function of development. Not enough is known for most mathematical domains (e.g., geometry, algebra); however, theory and experimental methods are well developed in the

areas of counting and simple arithmetic (Briars & Siegler, 1984; Geary, 1994, 2006; Gelman & Meck, 1983; Siegler & Shrager, 1984).

No current consensus exists regarding the definition of math disability; furthermore, there is no agreement in how math disability is diagnosed. This deficit exists despite the recent emergence of topics on the identification of children with math disability. In the schools, practitioners use discrepancy models or a student's response to mathematic intervention over time as tools to help educational teams determine whether a student has a learning disability (Individuals with Disabilities Education Act, 2004). Clinical practitioners rely on the use of the *Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition* (DSM-V), which defines a math learning disability as a neurodevelopmental disorder of biological origin manifested in learning difficulties and problems in acquiring academic skills and markedly below age-level performance. The symptoms must manifest in early school years, last for at least six months, and must not be attributed to intellectual disabilities, developmental disorder, or neurological or motor disorders (DSM-V; American Psychiatric Association, 2013). Most researchers have relied on standardized tests of achievement and intellectual functioning (IQ) to determine whether a student is mathematically disabled, whereas, scores lower than the 20th or 25th percentile on a math achievement test combined with low average or higher than average IQ scores constitute a math learning disability (Geary, 2006; Geary, Hamson, & Hoard, 2000; Gross-Tsur, Manor, & Shalev, 1996).

The problem with current methods of identifying mathematics disabilities is the potential to have over- and under-identification of students with learning disabilities.

Geary (1990) and Geary et al. (1991) found that use of scores below the 20th and 25th percentile applied in a single academic year led to many false positives and that children typically improved in later grades. Furthermore, the 25th percentile cut-off does not correspond to the popular estimation of math disability affecting 5–8% of the population (Geary, 2006). The discrepancy model has long been criticized for its inaccuracy in identifying learning disabilities because of its formula seemingly overlooking those children that struggle academically despite not having a discrepancy. Recent critics of response to intervention (RTI) describe RTI as a way of identifying the lowest 10% of the learning-disabled distribution while neglecting more competent students who may be performing markedly below their potential. The present researcher was concerned with identifying predictors of math competence across grade levels to advance research on improving lagging math performance in school-aged children.

Component Processes Underlying Competence in Mathematics

Working memory. Working memory is a cognitive system with limited capacity and is essential to encoding, storage, and retrieval of information being processed on any cognitive task (Atkinson & Shiffrin, 1971; Baddeley, 2003; Engle, Kane, & Tuholski, 1999). Working memory is therefore a robust predictor of performance on a range of cognitive abilities such as language functioning, problem-solving, and reasoning (Towse & Cowan, 2005). The working memory system maintains information in an active state while that or other information is being processed (Baddeley & Logie, 1999). Short-term memory requires temporary retention of information, whereas working memory requires an additional processing component (Szucs et al., 2014).

Baddeley and Hitch (1974) proposed a model of working memory that consists of three components: the central executive, the phonological loop, and the visuospatial sketchpad. The central executive is a system for controlling attention, directing the resources of working memory, and ensuring these resources are used appropriately to achieve the goals that have been set. The model also included two temporary storage systems: the phonological loop and the visuospatial sketchpad (Baddeley & Hitch, 1974). The phonological loop is responsible for holding speech-based information. The second storage system is for holding visual and spatial information and is known as the visuospatial sketchpad. The phonological loop and the visuospatial sketchpad storage mechanisms hold information briefly. The central executive is not an area of storage; rather, it is the active portion of working memory. Baddeley and Hitch's model of working memory has undergone several revisions with the most current version adding a component called the *episodic buffer* (Baddeley, 2000). The episodic buffer is a link to long-term memory, additional storage beyond the sensory perception and sensory stores, and an integrator of information from the other systems.

Many researchers associate working memory with mathematic competence. Leikin, Paz-Baruch, and Leikin (2014) reported working memory storage is critical to solving multistep arithmetical problems (Hitch & Baddeley, 1976; Hoard, Geary, Byrd-Craven, & Nugent, 2008) and in solving single-digit addition problems (Barrouillet & Lépine, 2005). Dark and Benbow (1991) revealed individual differences in working memory span are related to mathematic intellectual giftedness. Hoard et al. (2008) found intellectually gifted individuals have an advantage in visual-spatial working memory.

In most studies regarding working memory, researchers have relied on Baddeley's model of working memory and have assumed the "additional processing" component that differentiates working memory from short-term memory relies on the domain-general central executive function. This assumption resulted from studies involving only verbal tasks to test working memory. Evidence suggests verbal and visual working memory function can be distinguished (Jarvis & Gathercole, 2003; Klauer & Zhao, 2004; Shah & Miyake, 1996; Szucs et al., 2014) and may differently relate to mathematical competence. For example, several researchers testing both verbal and visual memory found that only visual, but not verbal, working memory performance discriminates children with poor and typical mathematical achievement (Andersson & Östergren, 2012; Kyttälä & Lehto, 2008; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2013; Szucs et al., 2014; White, Moffitt, & Silva, 1992). The results of prior research on visual-spatial working memory revealed a need to research both verbal and visual working memory separately.

Following the publication by Baddeley and Hitch (1974), researchers analyzing the visuospatial sketchpad sought to explain its role in mental imagery. The visuospatial sketchpad was thought to offer a medium within which to generate and manipulate images (Logie & Pearson, 1997). Recent findings suggest the visuospatial sketchpad is less a model for mental imagery and more an online "cache" for visual or spatial information (Logie, 1995; Logie, Zucco, & Baddeley, 1990; Pearson, Logie, & Green, 1996; Salway & Logie, 1995). Moreover, recent research suggests there are two components of the visuospatial sketchpad. The idea of a two-part visuospatial working memory evolved from observations by Goldman-Rakic (1987), who described the

organization of visual working memory as mirroring the what and where organization of the visual system (Postle, Idzikowski, Della Sala, Logie, & Baddeley, 2006).

According to Logie (1995), within the two-part visual-spatial working memory system, one system is for the retention of recently presented visual forms, and another system is for the retention of movement sequences. Logie argued for a form of partnership in working memory between a “visual cache” and a spatially oriented “inner scribe.” The visual cache theoretically stores information about visual form (color, shape), is a passive memory store of static visual patterns, and is closely tied to visual perception (Pearson, 2001; Rudkin, Pearson, & Logie, 2007; Salway & Logie, 1995). The inner scribe retains information about sequential movements and is closely linked to planning and executing movements. Therefore, a fractionation occurs within the visuospatial sketchpad. A large body of evidence supports the notion that visuospatial working memory is not a unitary system; rather, visuospatial working memory consists of at least two separate subcomponents (Pearson, 2001; Salway & Logie, 1995).

Researchers have investigated the distinction between the subcomponents of the visuospatial sketchpad using Corsi Blocks and Matrix patterns (Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999; Logie & Pearson, 1997; Salame, Danion, Peretti, & Cuervo, 1998). The Corsi Block task is assumed to rely on spatial working memory because it requires memory of movement sequences (Rudkin et al., 2007). For example, in Della Sala et al. (1999), the Corsi Block task consisted of nine wooden blocks unevenly distributed across (and fixed upon) a flat board. The experimenter tapped sequences of blocks at the rate of one block per second, and the subject was asked to tap

out the same sequences. The sequences were randomly presented, and the difficulty level increased progressively. In contrast, matrix patterns are usually visual images that are still on the page. They are considered to be a purer form of visual memory than the Corsi Block Test, which also taps memory for positional sequences (Della Sala et al., 1999).

Numerous researchers have investigated the relation between spatial working memory and execution of planning of movement (Pearson & Sahraie, 2003; Quinn, 1988, 1991; Quinn & Ralston, 1986; Smyth, Pearson, & Pendleton, 1988). This research has revealed a specialized system that controls and monitors movements specific to spatial locations. This system appears to be related to the system employed during the encoding of an imaged matrix pattern and covert movements around the image (Quinn & Ralston, 1986). Shifts in spatial attention are related to this system and spatial attention shifts can disrupt recall of movement sequences (Smyth & Scholey, 1994). Della Sala et al. (1999) investigated the two subsystems of visuospatial working memory and found evidence for distinct components of visual short-term memory and visual and spatial and sequential memory. Moreover, Della Sala et al. reported a double association in the pattern of interference produced by visual and spatial secondary tasks. In other words, the concurrent presentation of irrelevant pictures disrupted matrix span recall to a significantly higher degree than the Corsi span, while the opposite pattern of results occurred for concurrent tapping. The tapping movements involved in the Corsi task movements disrupted the Corsi task to a more significant extent.

The present study pertained to the visual-spatial components of working memory and their relations with mathematic competence. The researcher extended previous research on the dissociative properties of working memory by examining language cognition, verbal and visual working memory, and their relations to mathematic achievement across grade levels and mathematic domains. This study is the first of its type to include an investigation of the dissociative properties of working memory and its relation to grade level.

Spatial visualization. Lohman (1994) adequately defined *spatial ability* as the ability to generate, retain, retrieve, and transform well-structured visual images. Research has shown that visual-spatial ability can account for a significant amount of unique variance associated with general intelligence and academic achievement (Johnson & Bouchard, 2005; Rohde & Thompson, 2007) and that students with more highly developed abilities of spatial perception perform better in mathematics, science, and other areas requiring spatial skills (Leikin et al., 2014). Van Garderen and Montague (2003) reported gifted students used significantly more visual-spatial representations than average students and students with learning disabilities. Moreover, successful mathematical problem solving was positively correlated with use of schematic representations. In another study, Stavridou and Kakana (2008) investigated how graphic abilities in adolescents relate with performance in several academic areas. The research findings suggested a correlation between the level of graphic abilities and student performance in mathematics and science (Stavridou & Kakana, 2008).

Recent analyses have shown that spatial abilities uniquely predict STEM achievement and attainment even while holding constant other variables, such as math and verbal skills (Humphreys, Lubinski, & Yao, 1993; Shea, Lubinski, & Benbow, 2001; Wai et al., 2009). Given the recent focus on developing STEM domains and improving student performance, spatial visualization is an important skill that deserves instructional attention. In the domain of mathematic competence, Sherman (1979) reported spatial ability as one of the main factors affecting mathematic performance. According to Chavez, Reys, and Jones (2005), time spent helping students develop their spatial visualization skills benefits students' mathematical growth and improves test performance. Battista (1998) documented specific problems that middle school students have in developing spatial structuring (e.g., building prisms of same volume but different forms and turning containers filled with liquid to observe changes). Spatial visualization together with logical reasoning call on higher-level cognitive processes and can improve with practice (Chavez et al., 2005). Uttal et al. (2013) found spatial skills of children and adult participants are malleable (i.e., capable of being shaped or trained) and gains in spatial skills training can transfer to other spatial tasks. The teaching of spatial thinking has gained such momentum that in 2006, the National Research Council published a report solely on the topic of teaching students to think spatially across the K–12 years of schooling.

Mental imagery. The notion of human beings experiencing mental representations is fraught with debate. The concept of mental representation is abstract in nature and is therefore controversial. Many theorists imply it is nearly impossible to

experimentally discriminate between what is a visual representation and what is a language-based mental representation, because it is probable the same representation can be explained both ways. Logie, Pernet, Buonocore, and Della Sala, (2011, pp. 3071) described the debate in terms of propositional knowledge versus mental images, where propositional knowledge is described as “being enough” to support performance and visual images play a key functional role. What is accepted is that mental representations exist. Recent theorists of mental representation embrace the idea that multiple representations are necessary to capture the flexibility of cognitive performances, such as thinking, reasoning, and comprehension and individuals may differ in their capacity for, or experience of, mental imagery (Fleming, Ball, Ormerod, & Collins, 2011; Logie et al., 2011).

Movement and Visual-Spatial Working Memory

Baddeley (1986) briefly discussed eye movements and visuospatial working memory. Early studies by Baddeley et al. explained possible functional links between visuospatial working memory and motor systems. Early among these were demonstrations that concurrent visual tracking of a pursuit rotor and an overhead swinging pendulum interfered with spatial versions of the Brooks memory span task, which led to a consideration of the role of eye movements in establishing and maintaining a spatial frame of reference. Hebb (1968) found eye movements, or their control system, may play an important intermediate role in imagery. In the late 1970s and early 1980s, Baddeley et al. conducted a series of experiments to investigate the role of eye movements in visual working memory. Baddeley (1986) referenced these three

experiments on eye movements. Postle et al. (2006) presented the studies in full, along with a fourth experiment. In Experiment 4 by Postle et al., participants were markedly less able to produce perfect sequences on a memory task in one of three conditions where the eyes were moving. The reaction time data from Experiment 4 showed a highly significant main effect for eye movement when the background changed. Overall, making voluntary eye movements significantly impairs performance on a mental task involving imagery; therefore, the system involved in control of voluntary eye movements must share capacity with the encoding of images and the retrieval or scanning of images. Postle et al. demonstrated several facts about the sensitivity of visuospatial working memory to eye movements; it is eye movement control, not the movement per se, that produces disruptive effects. These effects are limited to location (spatial information) and do not generalize to working memory for shapes (visual forms). The researchers found saccadic distraction to disrupt spatial working memory, but not performance on a comparable nonspatial task (Postle et al., 2006).

Brooks (1967) developed a pair of tasks pertinent to developing the concept of a special visuospatial resource in working memory. These tasks required study participants to generate and retain a mental image of a matrix pattern and concurrent verbal sequences. This particular task has had widespread use in the working memory literature and has given it the status of an almost definitive task for visuospatial working memory (Baddeley & Lieberman, 1980; Logie et al, 1990; Quinn & Ralston, 1986; Salway & Logie, 1995). The task asked participants to imagine a square matrix pattern. The subjects were given oral instructions (sentences) or were asked to listen to the oral instructions

and read them simultaneously. The instructions asked the participants to place consecutive numbers in particular squares, starting in the second row. The subjects repeated the sequence aloud using their generated image as a mnemonic aid for recall. Brooks found the additional requirement to read the sentences resulted in poorer recall than listening alone, and similar results were found with other forms of mental imagery. From this apparent interference, Brooks stated the visual input systems used in reading required some of the same cognitive resources used for generating the mental image (Salway & Logie, 1995). Brooks tested the interference using an equivalent verbal task in which the words *up*, *down*, *left*, and *right*, were replaced by the words *good*, *bad*, *slow*, and *quick*. This created a set of nonsense sentences that could not easily be retained by a generated image and relied on verbal rehearsal. This type of task was recalled better when subjects read and listened to the sentences than when they listened alone. The visual requirement to read did not interfere with the verbal task. Therefore, some specialized visual imagery resources overlapped with mechanisms of visual perception, and some specialized verbal rehearsal mechanisms rely on some of the speech input channels.

Other researchers have conducted follow-up studies to Brooks (1967) with similar findings. Baddeley and Lieberman (1980) conducted a follow-up study using the Brooks tasks and had similar findings. Placing spatial or movement demands on the participants interfered with the matrix task, but the spatial and movement demands did not interfere with the verbal version of Brooks's task (Baddeley & Lieberman, 1980; Salway & Logie, 1995). Quinn and Ralston (1986) asked subjects to perform the Brooks matrix task while

moving their hands, with their hands hidden from view. The movements were described as either incompatible or compatible with Brooks' task and were passive or active. Compatible hand movements were hand movements in the same direction as the Brooks matrix instructions (Quinn & Ralston, 1986). Incompatible movements were not compatible with the matrix instructions. Active hand movements were movements initiated and performed by the subjects. Passive hand movements involved the experimenter holding the arms of subjects and moving the hands for them.

Quinn and Ralston (1986) found that incompatible movements interrupted performance on recall of the matrix instructions and the disruption was observed when the movements were passive. In a follow-up study, Quinn (1994) found that only passive predictable movements disrupt performance; only when subjects could anticipate the movement of their arms or executed the movement themselves was there evidence for competing cognitive demands. The results of Quinn and Ralston (1986) and Quinn (1994) suggest mechanisms involved in performance on the Brooks matrix overlap with cognitive mechanisms for planning movements and not just movement executions. Postle et al.'s (2006) unpublished study provided further evidence for a link between movement control and the visuospatial sketchpad, as discussed by Baddeley (1986). Idzikowski et al. showed that concurrent intentional eye movements disrupted performance on the Brooks matrix task while passive eye movements (eye movements resulting from being spun in a chair) did not.

Logie and Marchetti (1991) demonstrated the cognitive demands of arm movements may also affect the retention of a spatial or movement sequence. Johnson

(1982) indicated a complementary finding in which visual input of moving patterns appeared to disrupt the ability to imagine and to plan movement sequences. Other researchers have failed to show disruptions (Quinn, 1991). However, the data from studies involving concurrent dual-task performance strongly support an overlap between the construction of images of spatial information and the planning and production of movement (Salway & Logie, 1995).

Verschaffel, De Corte, and Pauwels (1992), by tracking eye movements, found students make more comprehension errors when word problem terms are not consistent with a preferred order (e.g., when the problem includes the word more, but is a subtraction problem). Saccadic eye movements refer to rapid, jerk-like movement of the eyeball that subserve vision by redirecting the visual axis to a new location. Saccades can be voluntary or reflexive. People make rapid, saccadic eye movements to change the locus of fixation when they read, view pictures, or explore the world around them. Abnormalities in saccadic eye movement (SEM) are seen in a variety of disorders; therefore, SEM is a sensitive instrument for analyzing some psychopathologies. Researchers consider SEM a cognitive parameter to evaluate attention.

Bittencourt et al. (2013) found abnormal SEMs are heavily involved in several psychiatric disorders: schizophrenia, ADHD, Anxiety Disorder, Bipolar Disorder. If abnormalities in saccadic eye movements are found in several disorders, such as ADHD, it is plausible that those with learning challenges may experience abnormal patterns in SEMs because of strong correlations between attention and learning. Researchers now explored the possible relation between attention and oculomotor control through

microsaccades, which are tiny eye movements that persist during fixation (Martinez-Conde, Macknik, & Hubel, 2004).

Microsaccades may also affect visual-spatial working memory. Voluntary saccades during retention decrease spatial span to a greater degree than attention shifts (Lawrence, Myerson, & Abrams, 2004; Pearson & Sahraie, 2003). During saccadic eye movement, visual input is reduced such that visual perception is confined to fixations, which is a phenomenon called *saccadic suppression* (Irwin & Brockmole, 2004).

According to Irwin and Brockmole (2004), several recent studies have shown that certain cognitive processes are suspended during saccadic eye movements (e.g., memory scanning, stimulus encoding, counting tasks, and mental rotation; Irwin & Brockmole, 2000) and other processes are not suspended. Irwin and Brockmole (2004) suggested the distinction between what is suppressed and what is not suppressed during saccades is as simple as *what* (e.g., object identification) and *where* (e.g., object location/orientation). *Where* processes were found to be suppressed during saccades and *what* processes were not. Irwin and Brockmole (2000) found saccadic movements suppress mental rotation efforts.

Mental Rotation

Recently, a debate has emerged regarding the role of motor processes in mental rotation. In early neuroimaging studies, researchers found activity during mental rotation tasks in areas in the posterior frontal cortex are associated with motor planning and execution (Cohen & Bookheimer, 1994; Zacks, 2008). Larsen (2014) defined mental rotation as the ability to determine that objects have the same shape despite differences in

orientation or size. Based on definitions of mental rotation and the components of the visuospatial sketchpad, mental rotation is theoretically a task of the inner scribe components of the visuospatial sketchpad (Hyun & Luck, 2007; Salway & Logie, 1995). Mental rotation involves passive storage of visual information but also its active manipulation (Albers, Kok, Toni, Dijkerman, & de Lange, 2013). First, an object is encoded into a mental representation. Second, this encoded representation is manipulated in a rotation-like manner. An example of a passive working memory task that does not involve active manipulation is the change-detection task (Liesefeld, Liesefeld, & Zimmer, 2014; Luck & Vogel, 1997).

According to Liesefeld and Zimmer (2013), inherent in the conceptualization of mental rotation (e.g., introspection, theories, and interpretations) is a certain type of mental representation; namely, visual mental images are implied. The results of their research revealed contrary but compelling results, suggesting mental representation cannot be accurately described as purely visual (in the sense of visual form). Furthermore, during the process of mental rotation, orientation-dependent spatial information is extracted from the visually complex information. In other words, Liesefeld and Zimmer (2013) found “in an early time window, the observed working memory load-dependent slow potentials were sensitive to the stimuli’s visual complexity; however, as the cognitive load visual stimuli are mentally rotated, orientation-dependent information is contained in the rotated representation” (Liesefeld & Zimmer, 2013). Something within human cognition separates spatially-oriented information from visual information in

cognitive systems. This finding supports the notion of two separate visual systems in working memory.

While investigating studies of motion aftereffect, Larsen (2014) reported findings of a close relation between transformation of visual images in mental rotation and visual motion perception. According to Larsen, motion aftereffect is found to interfere in tasks of mental rotation (Corballis & McLaren, 1984; Seurinck, de Lange, Achten, & Vingerhoets, 2011). A meta-analysis of 32 investigations of brain activations during mental rotation was conducted and the findings revealed “all experiments using transformation specific contrasts (i.e., within-task comparisons of effects of mental rotation, comparing large rotations with small rotations) have found activations located about ($-47.5, -59.5, -10.0$, in Talairach space) that corresponds to the visual motion area (V5/MT)” (Larsen, 2014; Zacks, 2008). This finding provides further evidence of an area within the brain for processing visual movement and motion.

Larsen (2014) found a linear relation between the number of eye movement switches between and stimuli as a function of angular difference in orientation. The initial processing time of subjects was almost constant, until the first switch between stimulus objects. The duration of the remaining trials increased linearly as a function of discrepant angles (Larsen, 2014). The linear increase resulted from the number of saccades and the number and duration of fixations. This information supports the notion that a mental rotation task produces increased demands on saccadic movements because of the angular discrepancies involved in the task. In this study, the researcher investigated whether a student’s ability on a visuospatial working memory task involving mental imagery and

mental rotation predicted math performance. Moreover, the researcher investigated whether the relations between the components of working memory vary by grade level.

Chapter 2: The Present Study

Rationale

According to Lim and Son (2013), cross-cultural studies comparing the U.S. students to East Asian students typically find that East Asian students consistently outperform U.S. students in almost every area of mathematical knowledge (Geary et al., 1992; Lemke et al., 2004). It has been said that the difference in the success of students in high-performing countries when compared to the United States may be attributed in part to the teacher's knowledge of student needs and the teacher's ability to craft instruction at the student's proximal zones of development (Vygotsky, 1978). Moreover, teaching in high-performing countries is a highly-esteemed profession, in contrast to the U.S. Educational entities in other countries often recruit the most talented professionals and teachers are well compensated.

Terao et al. (2004), in a grant proposal to the National Science Foundation, stated that "Instruction is effective to the degree that it is sensitive to the individual student." It is therefore plausible to assume that U.S. teachers are having difficulty with implementing effective instruction, in part, due to lacking the tools and resources necessary to adequately address individual differences among students. If one is to ever solve the problem of poor mathematics achievement in America, one should have individualized understanding of the unique differences contributing to poor mathematic performance. U.S. teachers may not be fully aware of the individual needs of students. Fuchs et al. (2012) reported that little is known about individual differences in the

development of competence with algebra, and Fuchs and Fuchs (2006) stated that little is known about several other math domains beyond computation.

According to Leikin et al. (2014), working memory is thought to be critical to many aspects of mathematical learning (Meyer, Salimpoor, Wu, Geary, & Menon, 2010). Many early studies of working memory and learning used only verbal tasks to test working memory. Evidence is now accumulating that verbal and visual working memory functions can be dissociated (Jarvis & Gathercole, 2003; Klauer & Zhao, 2004; Shah & Miyake, 1996; Szucs et al., 2014) and may relate differentially to mathematical competence. Several studies testing both verbal and visual memory found that only visual but not verbal working memory performance discriminates children with poor and typical mathematical achievement (Andersson & Östergren, 2012; Kytälä & Lehto, 2008; Szucs et al., 2013; Szucs et al., 2014; White et al., 1992).

According to Logie (1995), visual-spatial working memory is a two-part system. One system is for the retention of recently presented visual forms and another system is for the retention of movement sequences. Logie (1995) argues for a form of partnership in working memory between a “visual cache” and a spatially oriented “inner scribe.” The visual cache theoretically stores information about visual form (color, shape), is a passive memory store of static visual patterns, and is closely tied to visual perception (Pearson, 2001; Rudkin et al., 2007; Salway & Logie, 1995). The inner scribe retains information about sequential movements and is closely linked to planning and executing movements. Therefore, a fractionation occurs within the visuospatial sketchpad. A large body of evidence supports the notion that visuospatial working memory is not a unitary system;

rather, visuospatial working memory consists of at least two separate subcomponents (Pearson, 2001; Salway & Logie, 1995). Growing evidence that visual-spatial skills are related to mathematic competence, especially in higher-order domains (Fennema, 1979; Humphreys et al., 1993; Shea et al., 2001; Wai et al., 2009).

As discussed earlier, Geary (1993) suggested three distinct subtypes of math disability: semantic memory, procedural, and visual-spatial. The semantic subtype is associated with reading disability and is characterized by poor fact retrieval and variable response time. The procedural subtype is characterized by immature strategies, errors in execution and conceptual delays. The visual-spatial subtype involves misalignment of numeric information, sign confusion, number omission or rotation, and general misinterpretation of spatially relevant information. Of the three types of well-established patterns of math difficulty and disability, the visuospatial subtype is the least understood and has had the least amount of empirical research conducted on its attributes (Mazzocco & Myers, 2003). This is somewhat surprising considering the growing body of research that suggests visual-spatial skills correlate with several mathematic skillsets and domains. In the present study, I will investigate mathematics performance and cognitive variables across multiple grade levels in order to analyze whether differences exist as a function of grade level. In other words, do verbal working memory variables or language variables have more robust relation with math achievement in younger students than in older students? And does visual working memory have a more robust relation with the mathematic performance of older students? The present study is interested in determining whether children scoring lowest on cognitive measures that tap into visual-spatial

working memory are more likely to have trouble with specific mathematic domains (e.g., calculation, problem solving, fact retrieval) and in what grade level these relations are most prominent. Additionally, the present research is interested in knowing the amount of variance in various areas of math achievement is explained by the visual-spatial components of working memory. The data for this research was obtained from the Standardization data from the *Woodcock-Johnson® IV (WJ IV®)*. Copyright © by The Riverside Publishing Company. All rights reserved. Used with permission of the publisher.

Research Questions

The research questions in the present study are:

1. Do visuospatial working memory, verbal working memory, and language ability predict mathematic achievement as measured by the Woodcock Johnson-IV? This has been indicated in many studies, although the relations are not consistent across studies.
2. Does grade level moderate the relations between visuospatial working memory, verbal working memory, and language ability with mathematics achievement as measured by the Woodcock Johnson-IV? It has been suggested that, as math becomes more complex, spatial skills become better predictors of mathematic performance. However, prior research has not investigated this question by analyzing grade levels.

Research Question 1 in many ways will likely replicate previous research findings regarding the cognitive correlates of mathematic achievement in various domains. It extends the literature by being the first of its kind to investigate the partitioning of working memory and its relation to grade level.

The second aspect of my research question related to visuospatial working memory and movements was developed out of my own experiences with mathematic difficulties. Although numeric and arithmetic competence was not a significant issue, I experienced difficulty with advanced concepts due to managing the conceptual, spatial, and structural demands. I noticed the movements of teachers as they worked problems from left to right and top to bottom of the board/page interfered with my cognition. As soon as numbers were “represented” by letters and they started to “move”, I experienced conceptual difficulty. Even as I entered advanced statistics, I noticed the movements of the professor around an algebra matrix hampered my ability to follow the lecture, and I required extended practice with matrix solving, while my peers had done the problems during class.

This discovery led into research question two, which looks at the relations of cognitive variables across grade levels. Previous research has found arithmetic and early mathematic competence is related to verbal, cognitive and academic variables (phonological processing, vocabulary, oral language, verbal working memory, reading). Although there are recent studies identifying visuospatial relations and early math competence, (Fuchs et al., 2014; Geary, 1998a; Gunderson et al., 2012; Son & Meisels, 2006), the present research seeks to identify whether these variables have more

prominence at certain grade levels. Moreover, the present study seeks knowledge about whether the relations between visuospatial skills and mathematics competence is moderated by grade level.

Mazzocco and Myers (2003) referred to an important aspect in the identification of math disabilities: emerging evidence supports the need to define math disability because approximately one-third of individuals who meet low achievement criteria for math disability at any one time do not maintain low achievement over time (Mazzocco & Myers, 2003). Unfortunately, most of the research on math disability has been informed by studies that have not investigated the transient nature of math disability (Vukovic & Siegel, 2010). The purpose of this study was to examine the academic and cognitive characteristics of students across grade levels to provide insight into which cognitive variables are related to math achievement as mathematic concepts become progressively complex. Many experts have argued that mathematics relies more strongly on visuospatial skills as the math concepts become more advanced (Casey et al., 1995; Casey et al., 2001; Fennema, 1979; Gunderson et al., 2012; Hegarty & Kozhevnikov, 1999). Such an analysis is necessary in understanding the development of mathematic competence.

Variables

Visuospatial working memory. Visuospatial working memory will consist of two variables. These include the visualization and picture recognition subtests of the Woodcock Johnson-IV.

Verbal working memory. Verbal working memory will consist of three variables. These include the verbal attention, numbers reversed, and memory for words subtests of the Woodcock Johnson-IV. The scores from these subtests will be averaged to create a single composite score for verbal working memory.

Language ability. Language ability will consist of two variables. These include listening comprehension and oral expression subtests of the Woodcock Johnson-IV. The scores from these subtests will be averaged to create a single composite score for language ability.

Mathematic achievement. Mathematic achievement will consist of three variables. These include the applied problems, calculation, and math facts fluency subtests of the Woodcock Johnson-IV. The scores from these subtests will be averaged to create a single composite score for mathematic achievement.

Demographic variables. The demographic variables of interest in this study will be grade level and age. Grade level will be represented as a continuous variable.

Hypotheses

H1_a: Visuospatial working memory, verbal working memory, and language ability significantly predict mathematic achievement as measured by the Woodcock Johnson-IV. The visuospatial working memory variable will add unique variance the model.

H2_a: Grade level significantly moderates the relations between visuospatial working memory, verbal working memory, language ability, and mathematic achievement as measured by the Woodcock Johnson-IV.

If it is determined that visuospatial working memory is significantly linked to math competencies, and grade level is found to moderate the relation, visuospatial working memory interventions that provide extended practice with the skill necessary to store mathematic information necessary to solve equations, interpret visual stimuli, and solve multi-step problems images can be developed. Students may develop more cognitive space for abstract mathematic concepts if they are not lost in the demands for managing memory and structural/visual-spatial aspects of the task. If it is determined that certain components of working memory have stronger relations at certain grade levels, it will inform educators of the important cognitive variables at play as students develop.

Chapter 3: Methods

Participants

The national standardization sample of the Woodcock Johnson-IV was comprised of more than 7,000 individuals aged 2 to 90. It is important to note that the Woodcock Johnson norms are based on a single sample. The cognitive, oral language, and achievement test data were collected at the same time on the same participants. (Mather & Wendling, 2014a,b,c). The sample for the present study included undergraduate college students, with community characteristics and demographics closely resembling the general United States population. This present study sampled participants from the larger standardization sample from first, third, fifth, seventh, ninth, and eleventh grades, and undergraduates for the purpose of comparison across school-aged children and undergraduates.

A power analysis was conducted using G*Power 3.1.9.2 (Faul, Erdfelder, Buchner, & Lang, 2014) to determine the minimum sample size necessary to obtain statistically valid results. The power analysis was based on a hierarchical regression with nine predictors (four terms for the independent variables, one term for the continuous grade level variable, and four interaction terms), a medium effect size of 0.15 for the overall significance of the model, a power level of .80, and a significance level of .05. The results of the power analysis revealed that the minimum sample size required for this study is 114 total cases across all grade levels.

Description of Instrument

The data analyzed in the present study are the normative Woodcock Johnson Fourth Edition (WJ-IV) test of achievement, cognition, and oral language data already published by The Riverside Publishing Company (2014). This gives me the unique opportunity to study relations between broad measures of academic, language, and cognitive performance. The Woodcock Johnson-IV is a refined, revised version of previous versions of the instrument.

The Woodcock Johnson-IV is administered individually; therefore, individuals are typically tested in an area with no other persons, other than the examiner or clinician administering the assessment. According to the WJ-IV examiner's manual, the testing room is a place that is quiet, comfortable, and has adequate ventilation and lighting. The room should have all of the necessary equipment for test administration (e.g., table, chairs, test books, audio equipment, response booklets, writing utensils). The Woodcock Johnson-IV's examiner's manual lists several standardized procedures for test administration and recommends adherence to these procedures (e.g., rapport, order of administration, timing, basals, ceilings, and allowed accommodations).

According to the Woodcock Johnson-IV examiner's manual, the Cognitive portion of the Woodcock Johnson-IV battery includes eighteen subtests comprising a standard and extended battery. The WJ-IV cognitive assessment is designed to be on the cutting edge of practice and expands on its previous basis in CHC theory to focus on the most important broad and narrow CHC abilities. The broad CHC abilities include comprehension and knowledge, fluid reasoning, long-term retrieval, visual processing,

auditory processing, processing speed, short-term working memory, quantitative knowledge, and the narrow CHC abilities include perceptual speed, quantitative reasoning, auditory memory span, number facility, vocabulary, and cognitive efficiency. The differentiation of broad and narrow abilities is based in the original work of Cattell (1941, 1943, 1950), Horn (1965, 1998; Horn & Blankson, 2005), and Carroll (1993, 1997, 2005) (Mather & Wendling, 2014a).

The Achievement portion of the WJ-IV assessment has a total of twenty subtests which create the standard and extended batteries. The WJ-IV Achievement battery is a comprehensive battery of achievement domains including basic reading, reading comprehension, calculation, math applications, written expression, spelling, history, and science. The Oral Language battery of the WJ-IV is comprised of twelve subtests of phonological and language-based skills (Mather & Wendling, 2014b). The standard scores of the participants analyzed in the present study were derived from the following subtest tests.

Cognitive subtests and composites.

Verbal attention. Verbal attention is a test of verbal working memory. Verbal attention evaluates one's ability in tasks of attentional control or controlled executive function. The task requires the examinee to listen to an intermingled list of animals and digits, before answering a specific question about the sequence. For example, "Say the animal that came before the 5" (Mather & Wendling, 2014a).

Numbers reversed. The numbers reversed subtest is a test of short-term working memory and is delivered orally. It requires examinees to listen to increasing spans of numbers, before repeating them in reversed order (Mather & Wendling, 2014a).

Memory for words. The Memory for Words subtest is a test of auditory working memory. The participants are asked to repeat a list of unrelated words in correct sequence (Mather & Wendling, 2014a).

Visualization. The visualization composite consists of two subtests: Spatial Relations and Block Rotation. The Spatial Relations subtest requires the examinee to identify the two or three pieces that form a complete target shape. The target shapes become increasingly difficult and complex. The block rotation task requires the examinee to identify two block patterns that match a target pattern (Mather & Wendling, 2014a). The present study's theoretical perspective considers these subtests' tasks processed in the visuospatial sketchpad, specifically the visual cache and the inner scribe respectively.

Picture recognition. The picture recognition measures visual memory for objects or pictures. The examinee is shown a subset of pictures within a field of distracting patterns, and after a five-second delay, they are asked to select pictures through recognition (Mather & Wendling, 2014a).

Achievement subtests.

Applied problems. Applied problems is a test measuring the participant's skills with constructing mental models via language comprehension, and application of calculation, reasoning, and insight.

Calculation. Calculation is a test measuring the participant's access and application of knowledge of numbers and procedures, including verbal associations between numbers represented as a string of words.

Math facts fluency. The math facts fluency test measures speed and access with application of digit-symbol arithmetic procedures.

Oral language subtests.

Listening comprehension. Listening comprehension is a measure of listening ability and verbal comprehension. This cluster is composed of *Understanding Directions* (pointing to objects in a picture after given oral directions) and *Oral Comprehension* (listening to a passage and providing the final word to complete the passage).

Oral expression. Oral expression is a measure of lexical knowledge, language development, and syntactic knowledge. It is a combination of *Picture Vocabulary* (identifying names of specific pictures) and *Sentence Repetition* (remember and repeat single words, phrases and sentences).

Analytic Approach

Prior to analysis, the data will be checked for missing scores and the presence of outliers. It is important to note that missing cases are unlikely to occur in the data as the dataset has been taken from a normative sample. In the event of missing scores, cases with missing scores on the variables of interest will be excluded from the analysis. The presence of outliers will be examined using standardized values. Tabachnick and Fidell (2012) suggest that scores with standardized values greater than 3.29 or less than -3.29 should be considered outliers and removed prior to analysis.

An exploratory factor analysis was conducted prior to the main analysis in order to determine the most appropriate composite groupings for items on the WJ-IV subtests that were described previously. For the factor analysis, the unweighted least squares method of extraction and a Harris-Kaiser rotation were used. The analysis was specified to retain four factors, which corresponded to visuospatial working memory, verbal working memory, language ability, and mathematic achievement. The full results and statistics for the exploratory factor analysis will be presented in the results chapter.

In order to address Research Questions 1 and 2 of this study, a hierarchical linear regression analysis will be conducted. A hierarchical linear regression is an appropriate statistical analysis when the goal of the research is to investigate the strength and direction of relations between a continuous dependent (outcome) variable and multiple independent (predictor) variables (Stevens, 2009; Tabachnick & Fidell, 2012). The dependent variable in this analysis will be the mathematic achievement composite variable (consisting of applied problems, calculation, and math facts fluency) measured by the Woodcock Johnson-IV. The independent variables in this analysis will be visuospatial working memory, verbal working memory, language ability, age, and grade level. The regressions will include an analysis in which visuospatial working memory is represented by one composite variable. To aid in interpretation, all independent variables will be mean-centered.

The assumptions of hierarchical linear regression will be tested prior to analysis. These assumptions include normality, homoscedasticity, and absence of multicollinearity. The assumption of normality states that the regression residuals must be normally

distributed. This will be tested by examination of a normal P-P plot. The assumption of homoscedasticity states that the scores must be equally distributed around the regression line. This will be tested by examination of a scatterplot. Finally, the absence of multicollinearity means that the independent variables are not too highly correlated with each other. This will be tested using Variance Inflation Factors (VIF). Stevens (2009) suggests that VIF values greater than 10 indicate the presence of multicollinearity.

For each regression model, the variables will be entered in steps. In order to address Research Question 1, the first step of each model will include visuospatial working memory, verbal working memory, and language ability. In order to address Research Question 2, grade level will be added in Step 2, and interaction terms for visuospatial working memory x grade level, verbal working memory x grade level, and language ability x grade level will be entered into each model at Step Three. The interaction terms will assess the moderating effect of grade level on the relations between visuospatial working memory, verbal working memory, and language ability with mathematic achievement. A significant interaction term indicates the presence of a moderating effect (Baron & Kenny, 1986). The overall model significance at each step will be tested using the F-test at a significance level of .05. Additionally, R-squared will provide a measure of the proportion of variability in the dependent variable that the independent variables explain at each step. If the overall model is significant, the significance of individual predictors will be tested using t-tests at a significance level of .05. Squared semi-partial correlations will be calculated to determine the amount of variability explained by each predictor.

Limitations

There are several limitations inherent to the present study design. First, although a quantitative method is able to answer the specific research hypotheses, it is unable to examine the depth, underlying details, and subjective experiences that students have in regard to mathematic achievement (Mitchell & Jolley, 2001). Second, although the present study can examine the relations among the independent and dependent variables, it cannot determine the causal nature of the relations. This study is not able to determine if having better visuospatial working memory causes better mathematical achievement. Only an experimental design can produce causal conclusions about the relations among variables.

Chapter 4: Results

The purpose of this study is to examine cognitive processing and mathematic achievement in children and college students. This chapter contains the results of the data analysis conducted to address the research questions. First, the data cleaning procedures will be described, followed by descriptive statistics for the variables of interest. Then the results of the data analyses will be presented, including an exploratory factor analysis and a series of regressions. Finally, this chapter will conclude with a summary.

Pre-Analysis Data Cleaning

A total of 2417 cases were received in the initial dataset. Prior to analysis, the data were screened for missing data and outliers. Four cases had missing scores for the variables of interest, so these cases were excluded from the analysis. Outliers were examined using standardized values. Tabachnick and Fidell (2012) suggested that scores with standardized values greater than 3.29 or less than -3.29 should be considered outliers. Thirty-eight cases contained outliers on the variables of interest, so these cases were excluded from analyses. A final total of 2375 cases were included in analyses.

Descriptive Statistics

Table 1 displays descriptive statistics for the demographic and research variables. The age range of the sample was 5 to 61 years ($M = 13.90$, $SD = 5.73$). Grade levels included in the sample were 1, 3, 5, 7, 9, 11, and undergraduate (13-16). The average standardized scores on the composite variables and subtests (i.e., visualization and picture recognition) used in the analysis ranged from 101.01 to 101.21 with standard deviations ranging from 11.91 to 13.81.

Table 1

Descriptive Statistics for Demographic and Research Variables

Variable	Min	Max	<i>M</i>	<i>SD</i>
Age (in years)	5.00	61.00	13.90	5.73
Grade level	1.00	16.00	7.85	4.72
Mathematic achievement	55.27	142.56	101.02	13.30
Visuospatial working memory	54.96	149.55	101.04	12.89
Visualization	49.11	152.76	101.01	15.61
Picture recognition	55.28	150.43	101.07	15.15
Verbal working memory	61.63	139.22	101.06	11.91
Language ability	53.81	141.10	101.21	13.81

Exploratory Factor Analysis

An exploratory factor analysis was conducted prior to the main analysis in order to determine the most appropriate composite groupings for items on the WJ-IV subtests. The exploratory factor analysis was conducted using the unweighted least squares method of extraction and a Harris-Kaiser rotation (with power set to 0), which allowed factors to be correlated. The analysis was specified to retain four factors, and Table 2 displays the eigenvalues for the unrotated factors from the reduced correlation matrix. Correlations among the rotated factors are shown in Table 3, and these correlations ranged from .51 to .66.

Table 2

Eigenvalues for Reduced Correlation Matrix for Exploratory Factor Analysis

Factor	Eigenvalue	Eigenvalue Difference	Proportion of Variance	Cumulative Proportion of Variance
1	4.32	3.40	0.68	0.68
2	0.92	0.34	0.14	0.83
3	0.57	0.06	0.09	0.92
4	0.51	0.36	0.08	1.00

Table 3

Inter-Factor Correlations for Exploratory Factor Analysis

Factor	1	2	3
1. Mathematic Achievement	-		
2. Language Ability	.58	-	
3. Verbal Working Memory	.54	.66	-
4. Visuospatial Working Memory	.51	.55	.51

Table 4 displays the rotated factor loadings for each of the subtests included in the exploratory factor analysis. Applied problems, calculation, and math facts fluency all loaded strongly (i.e., loading greater than 0.40) on Factor 1 (Mathematic Achievement). Oral expression and listening comprehension loaded strongly on Factor 2 (Language Ability). Verbal attention, numbers reversed, and memory for words all loaded strongly on Factor 3 (Verbal Working Memory). Finally, visualization and picture recognition loaded strongly on Factor 4 (Visuospatial Working Memory). The scores on the subtests loading on each factor were averaged to create composite scores for mathematic

achievement, language ability, verbal working memory, and visuospatial working memory (see Table 1).

Table 4

Rotated Factor Loadings for Exploratory Factor Analysis

Subtest	Factor			
	1. Mathematic Achievement	2. Language Ability	3. Verbal Working Memory	4. Visuospatial Working Memory
Verbal attention	.16	.17	.64	-.26
Numbers reversed	.26	-.12	.46	.11
Memory for words	-.16	-.13	.91	.10
Applied problems	.67	.12	-.01	.18
Calculation	1.03	-.11	-.05	-.01
Math facts fluency	.73	.09	-.06	-.05
Visualization	.04	-.05	.04	.85
Picture recognition	-.17	.29	-.01	.42
Oral expression	.07	.49	.29	-.03
Listening comprehension	-.03	1.07	-.07	-.02

Note. Boldfaced loadings are salient loading > 0.40.

Linear Regression Analyses

Regression 1: Predicting math achievement composite scores. In order to address Research Questions 1 and 2 of this study, eight regression analyses were conducted. For Regression 1, the dependent variable was the mathematic achievement composite variable. The independent variables in this analysis were visuospatial working memory, verbal working memory, and language ability. Additionally, age was included as a covariate variable. The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested prior to the analysis. Normality was tested by examination of a normal P-P plot (see Figure 1). The data did not deviate from the normal (diagonal)

line, so this assumption was met. Homoscedasticity was tested by examination of a scatterplot (see Figure 2). The data were equally distributed around zero, so this assumption was also met. Finally, multicollinearity was tested using Variance Inflation Factors (VIF). All VIF values were below 10 (see Table 5), indicating that multicollinearity was not a problem.

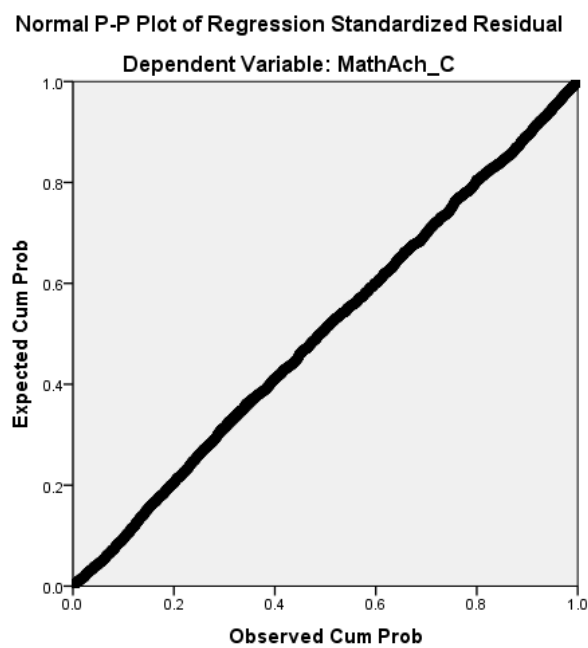


Figure 1. Normal P-P plot for Regression 1.

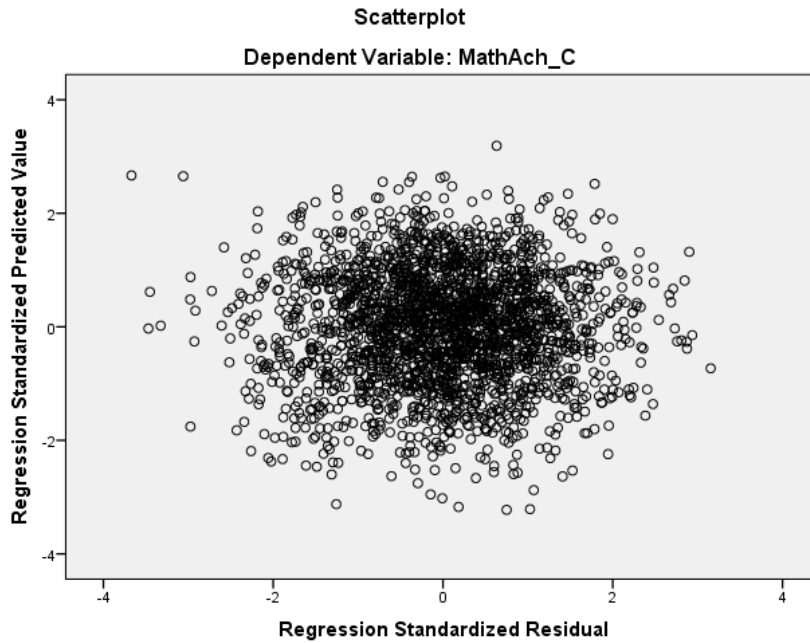


Figure 2. Scatterplot for Regression 1.

Table 5

Regression 1: Predicting Mathematic Achievement Composite Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Visuospatial working memory	0.17 (0.02)	8.38	< .001	0.03	1.34
Verbal working memory	0.23 (0.02)	10.14	< .001	0.04	1.53
Language ability	0.33 (0.02)	15.74	< .001	0.09	1.67
Age	0.12 (0.04)	3.09	.002	< 0.01	1.01

Note. $F(4, 2370) = 315.01, p < .001, R^2 = .35$.

The results of the regression were significant, $F(4, 2370) = 315.01, p < .001, R^2 = .35$, which indicates that, collectively, visuospatial working memory, verbal working memory, language ability, and age significantly predicted mathematic achievement. The R^2 value indicates that these variables accounted for 35% of the variability in mathematic

achievement. The full results of the regression are presented in Table 5. Visuospatial working memory ($B = 0.17, p < .001$), verbal working memory ($B = 0.23, p < .001$), language ability ($B = 0.33, p < .001$), and age ($B = 0.12, p = .002$) were all individually significant positive predictors. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher mathematic achievement scores, after controlling for age.

Regression 2: Predicting applied problems subtest scores. For Regression 2, the dependent variable was the applied problems subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, and language ability. The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested prior to the analysis. Normality was tested by examination of a normal P-P plot (see Figure 3). The data did not deviate from the normal (diagonal) line, so this assumption was met. Homoscedasticity was tested by examination of a scatterplot (see Figure 4). The data were equally distributed around zero, so this assumption was also met. Finally, multicollinearity was tested using Variance Inflation Factors (VIF). All VIF values were below 10 (see Table 6), indicating that multicollinearity was not a problem.

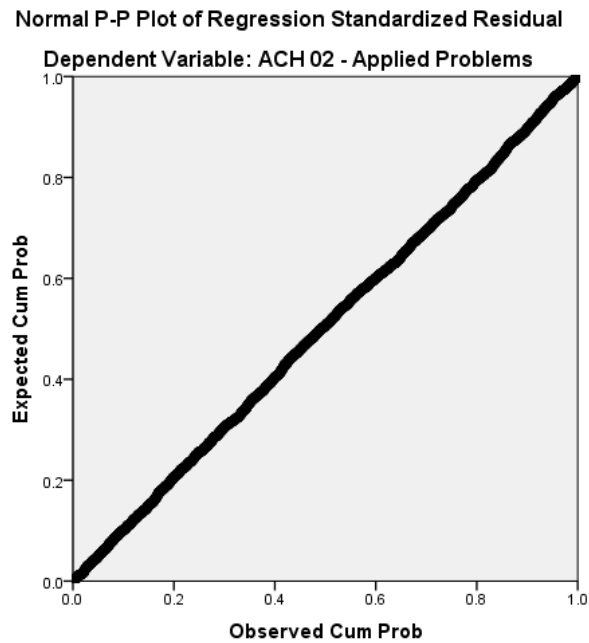


Figure 3. Normal P-P plot for Regression 2.

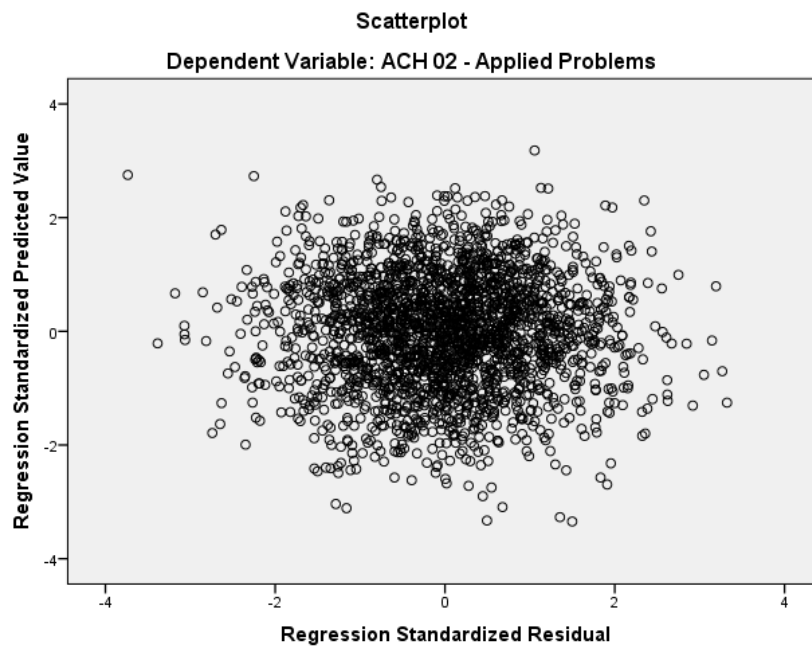


Figure 4. Scatterplot for Regression 2.

Table 6

Regression 2: Predicting Applied Problems Subtest Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Visuospatial working memory	0.23 (0.02)	10.56	< .001	0.04	1.34
Verbal working memory	0.23 (0.03)	9.20	< .001	0.03	1.52
Language ability	0.42 (0.02)	18.72	< .001	0.13	1.67

Note. $F(3, 2371) = 522.40, p < .001, R^2 = .40$.

The results of the regression were significant, $F(3, 2371) = 522.40, p < .001, R^2 = .40$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted applied problems subtest scores. The R^2 value indicates that these variables accounted for 40% of the variability in applied problems. The full results of the regression are presented in Table 6. Visuospatial working memory ($B = 0.23, p < .001$), verbal working memory ($B = 0.23, p < .001$), and language ability ($B = 0.42, p < .001$) were all individually significant positive predictors. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher applied problems scores.

Regression 3: Predicting calculation subtest scores. For Regression 3, the dependent variable was the calculation subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, and language ability. The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested prior to the analysis. Normality was tested by examination of a normal P-P plot (see Figure 5). The data did not deviate from the normal (diagonal) line, so this assumption was met. Homoscedasticity was tested by

examination of a scatterplot (see Figure 6). The data were equally distributed around zero, so this assumption was also met. Finally, multicollinearity was tested using Variance Inflation Factors (VIF). All VIF values were below 10 (see Table 7), indicating that multicollinearity was not a problem.

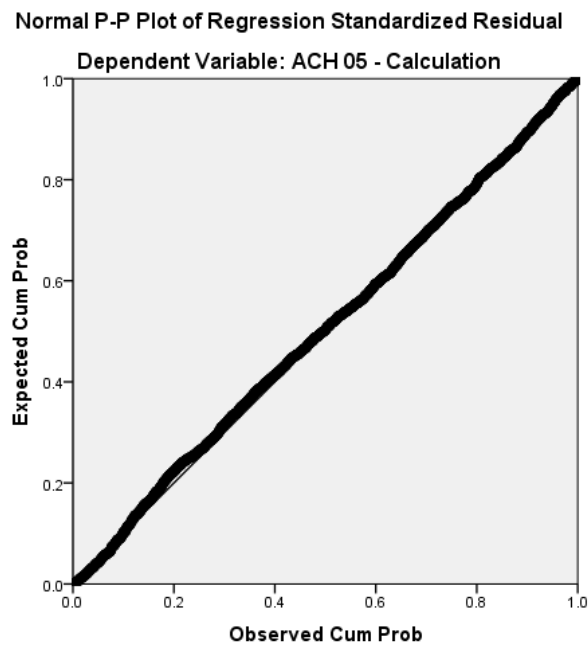


Figure 5. Normal P-P plot for Regression 3.

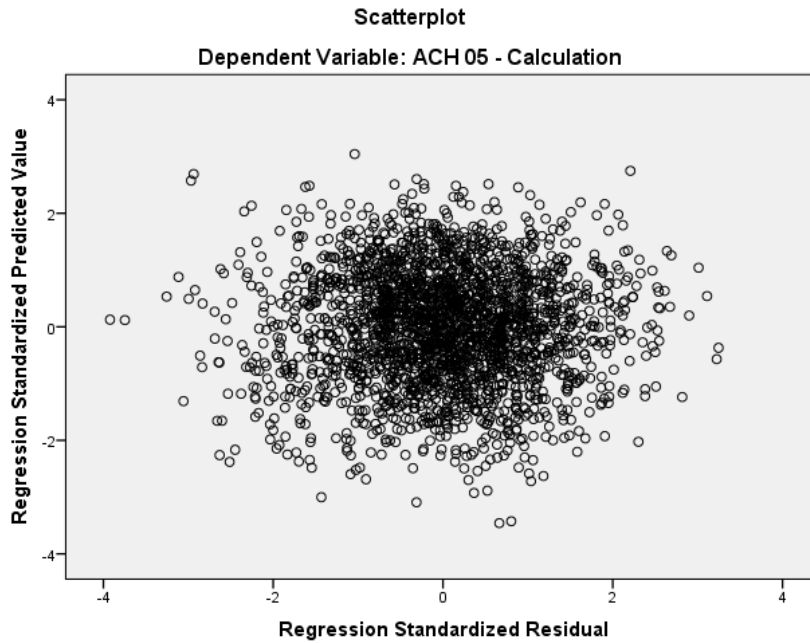


Figure 6. Scatterplot for Regression 3.

Table 7

Regression 3: Predicting Calculation Subtest Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Visuospatial working memory	0.12 (0.02)	5.04	< .001	0.01	1.34
Verbal working memory	0.27 (0.03)	9.71	< .001	0.04	1.52
Language ability	0.29 (0.03)	11.44	< .001	0.05	1.67

Note. $F(3, 2371) = 245.06, p < .001, R^2 = .24$.

The results of the regression were significant, $F(3, 2371) = 245.06, p < .001, R^2 = .24$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted calculation. The R^2 value indicates that these variables accounted for 24% of the variability in calculation. The full results of the regression are presented in Table 7. Visuospatial working memory ($B = 0.12, p <$

.001), verbal working memory ($B = 0.27, p < .001$), and language ability ($B = 0.29, p < .001$) were all individually significant positive predictors. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher calculation scores.

Regression 4: Predicting math facts fluency subtest scores. For Regression 4, the dependent variable was the math facts fluency subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, and language ability. The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested prior to the analysis. Normality was tested by examination of a normal P-P plot (see Figure 7). The data did not deviate from the normal (diagonal) line, so this assumption was met. Homoscedasticity was tested by examination of a scatterplot (see Figure 8). The data were equally distributed around zero, so this assumption was also met. Finally, multicollinearity was tested using Variance Inflation Factors (VIF). All VIF values were below 10 (see Table 8), indicating that multicollinearity was not a problem.

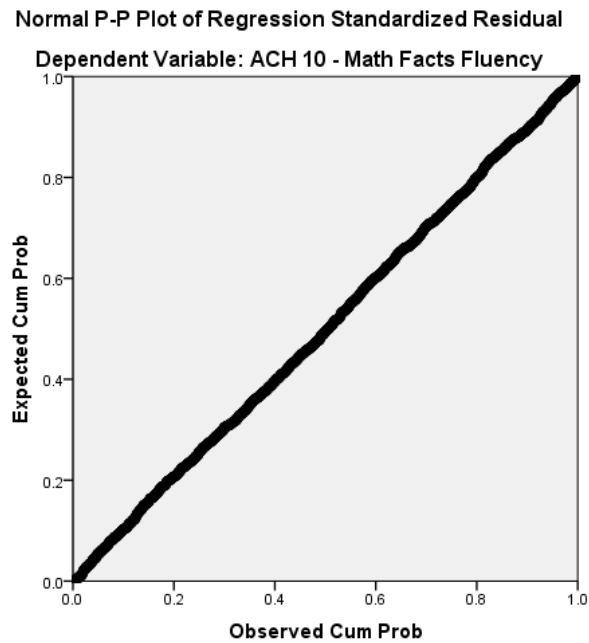


Figure 7. Normal P-P plot for Regression 4.

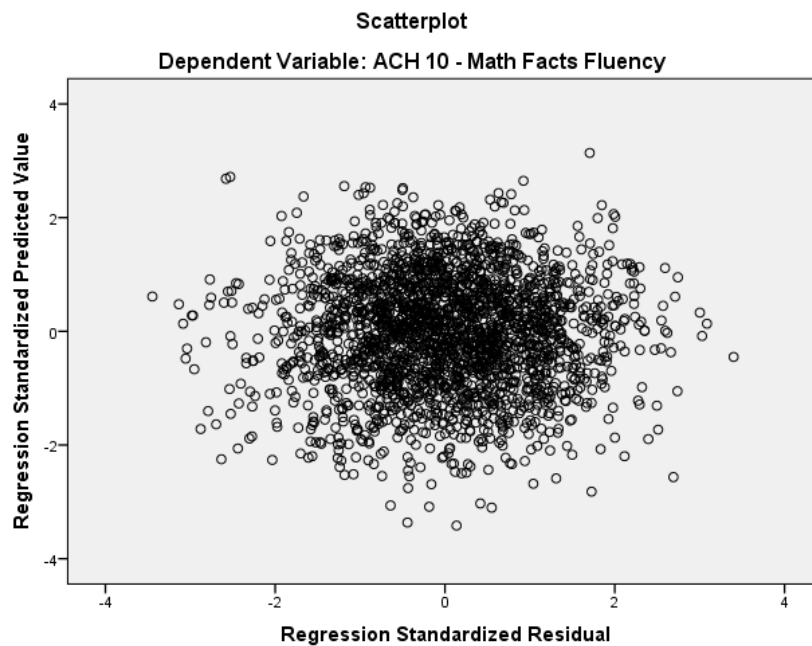


Figure 8. Scatterplot for Regression 4.

Table 8

Regression 4: Predicting Math Facts Fluency Subtest Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Visuospatial working memory	0.14 (0.03)	5.62	< .001	0.01	1.34
Verbal working memory	0.20 (0.03)	6.95	< .001	0.02	1.52
Language ability	0.27 (0.03)	10.21	< .001	0.04	1.67

Note. $F(3, 2371) = 181.52, p < .001, R^2 = .19$.

The results of the regression were significant, $F(3, 2371) = 181.52, p < .001, R^2 = .19$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted math facts fluency. The R^2 value indicates that these variables accounted for 19% of the variability in math facts fluency scores. The full results of the regression are presented in Table 8. Visuospatial working memory ($B = 0.14, p < .001$), verbal working memory ($B = 0.20, p < .001$), and language ability ($B = 0.27, p < .001$) were all individually significant positive predictors. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher math facts fluency scores.

Regression 5: Testing whether grade moderates prediction of math achievement composite scores. For Regression 5, the dependent variable was the mathematic achievement composite variable. The independent variables in this analysis were visuospatial working memory, verbal working memory, language ability, and grade. To aid in interpretation, all independent variables were mean-centered (before computing the interaction terms), and the variables were entered in steps. The first step of the model included visuospatial working memory, verbal working memory, and language ability. In

the second step, grade level was entered into the model. In the third step, interaction terms for visuospatial working memory x grade level, verbal working memory x grade level, and language ability x grade level were entered into the model. The interaction terms were used to assess the moderating effect of grade level on the relations between visuospatial working memory, verbal working memory, language ability, and mathematic achievement.

The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested in the same manner as the previous regressions. The normal P-P plot (see Figure 9) showed that the data did not deviate from the normal line, so the normality assumption was met. Figure 10 shows that the data were equally distributed around zero, so the assumption of homoscedasticity was also met. Finally, all VIF values were below 10 (see Table 9), indicating that multicollinearity was not a problem.

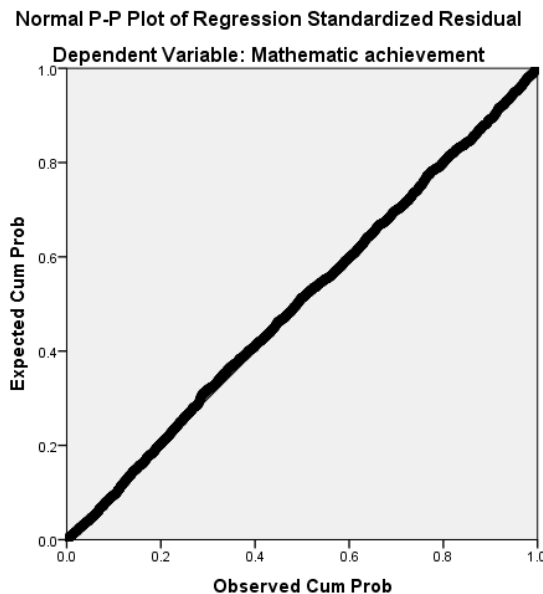


Figure 9. Normal P-P plot for Regression 5.

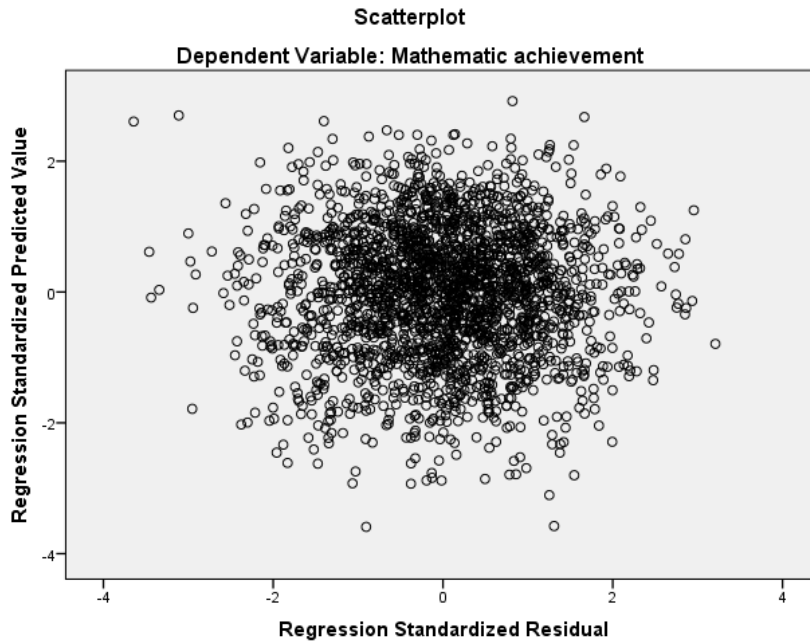


Figure 10. Scatterplot for Regression 5.

The results of the regression at Step 1 were significant, $F(3, 2371) = 415.34, p < .001, R^2 = .34$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted mathematic achievement at Step 1. The R^2 value indicates that these variables accounted for 34% of the variability in mathematic achievement.

The results of Step 2 of the regression were also significant, $F(4, 2370) = 317.42, p < .001, R^2 = .35$, which indicates that the set of independent variables and grade level significantly predicted mathematic achievement at Step 2. The R^2 value indicates that these variables accounted for 35% of the variability in mathematic achievement, or 1% more than visuospatial working memory, verbal working memory, and language ability alone. The R^2 change from Step 1 to Step 2 was significant, $F(1, 2370) = 15.86, p < .001$,

indicating that the addition of grade level at Step 2 accounted for significantly more variance in mathematic achievement compared to Step 1, $\Delta R^2 = .01$.

Table 9

Regression 5: Testing Main and Moderating Effects of Grade on Mathematic Achievement Composite Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Step 1					
Visuospatial working memory	0.16 (0.02)	8.28	< .001	0.03	1.34
Verbal working memory	0.24 (0.02)	10.26	< .001	0.04	1.52
Language ability	0.33 (0.02)	15.81	< .001	0.10	1.67
Step 2					
Visuospatial working memory	0.17 (0.02)	8.37	< .001	0.03	1.34
Verbal working memory	0.23 (0.02)	10.09	< .001	0.04	1.53
Language ability	0.32 (0.02)	15.67	< .001	0.09	1.67
Grade level	0.19 (0.05)	3.98	< .001	0.01	1.01
Step 3					
Visuospatial working memory	0.16 (0.02)	8.29	< .001	0.03	1.34
Verbal working memory	0.23 (0.02)	10.03	< .001	0.04	1.54
Language ability	0.32 (0.02)	15.64	< .001	0.09	1.68
Grade level	0.20 (0.05)	4.14	< .001	0.01	1.02
Visuospatial working memory x Grade	-0.01 (0.00)	-2.01	.045	0.00	1.38
Verbal working memory x Grade	0.00 (0.01)	-0.89	.375	0.00	1.49
Language ability x Grade	0.00 (0.00)	0.17	.864	0.00	1.66

Note. Step 1: $F(3, 2371) = 415.34, p < .001, R^2 = .34$. Step 2: $F(4, 2370) = 317.42, p < .001, R^2 = .35$. Step 3: $F(7, 2367) = 182.92, p < .001, R^2 = .35$.

The results of Step 3 of the regression were also significant, $F(7, 2367) = 182.92, p < .001, R^2 = .35$, which indicates that the set of independent variables and interaction

terms significantly predicted mathematic achievement at Step 3. The R^2 value indicates that these variables accounted for 35% of the variability in mathematic achievement, or less than 1% more than the independent variables at Step 2. The R^2 change from Step 2 to Step 3 was significant, $F(3, 2367) = 2.68, p = .045$, indicating that the addition of the interaction terms at Step 3 accounted for significantly more variance in mathematic achievement compared to Step 2, $\Delta R^2 < .01$.

The full results of the regression are presented in Table 9. Visuospatial working memory ($B = 0.16, p < .001$), verbal working memory ($B = 0.23, p < .001$), and language ability ($B = 0.32, p < .001$) were all individually significant positive predictors in the final model. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher mathematic achievement scores. Grade level was also a significant positive predictor in the final model ($B = 0.20, p < .001$). The visuospatial working memory x grade interaction was significant ($B = -0.01, p = .045$), indicating that grade level significantly moderated the relation between visuospatial working memory and mathematic achievement.

Specifically, among students with low visuospatial working memory scores, students at higher grade levels tended to have higher mathematic achievement scores compared to students at lower grade levels. Among students with high visuospatial working memory scores, students at higher grade levels tended to have similar mathematic achievement scores compared to students at lower grade levels (see Figure 11). In other words, in the lower grades, visuospatial working memory is more strongly related to math

achievement, but as grade level increases, the relations between visuospatial working memory relates and mathematic achievement weakens.

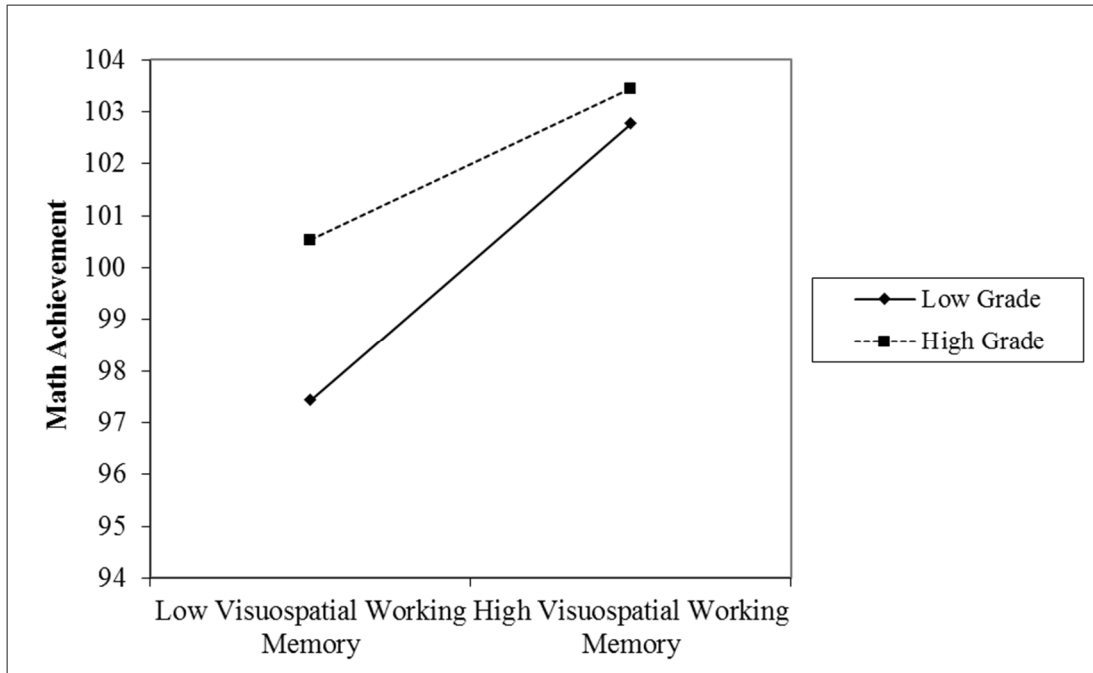


Figure 11. Interaction of visuospatial working memory and grade for Regression 5.

Regression 6: Testing whether grade moderates prediction of applied problems subtest scores. For Regression 6, the dependent variable was the applied problems subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, language ability, and grade. To aid in interpretation, all independent variables were mean-centered (before computing the interaction terms), and the variables were entered in steps. The first step of the model included visuospatial working memory, verbal working memory, and language ability. In the second step, grade level was entered into the model. In the third step, interaction terms for visuospatial working memory x grade level, verbal working memory x grade level, and language ability x grade level were entered into the model. The interaction

terms were used to assess the moderating effect of grade level on the relations between visuospatial working memory, verbal working memory, language ability, and applied problems.

The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested in the same manner as the previous regressions. The normal P-P plot (see Figure 12) showed that the data did not deviate from the normal line, so the normality assumption was met. Figure 13 shows that the data were equally distributed around zero, so the assumption of homoscedasticity was also met. Finally, all VIF values were below 10 (see Table 10), indicating that multicollinearity was not a problem.

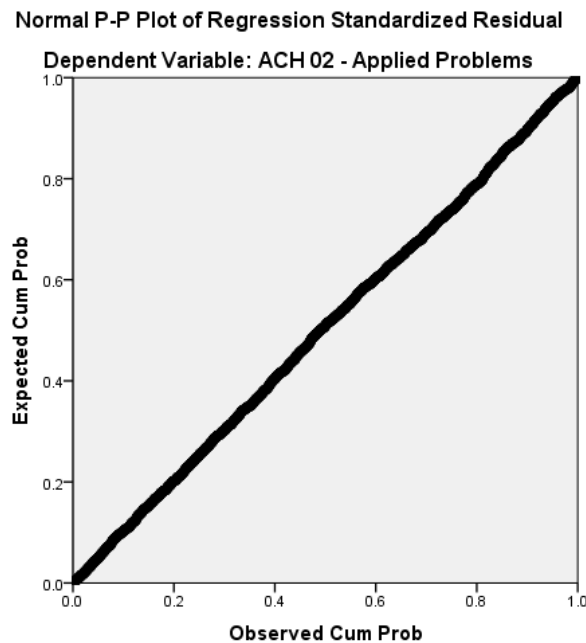


Figure 12. Normal P-P plot for Regression 6.

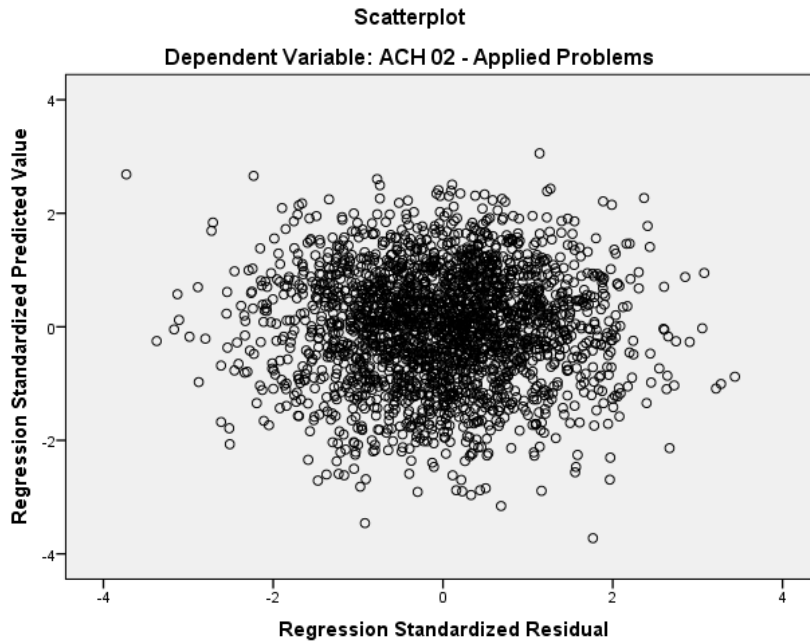


Figure 13. Scatterplot for Regression 6.

The results of the regression at Step 1 were significant, $F(3, 2371) = 522.40, p < .001, R^2 = .40$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted applied problems at Step 1. The R^2 value indicates that these variables accounted for 40% of the variability in applied problems.

The R^2 change from Step 1 to Step 2 was 1%, which was statistically significant $F(1, 2370) = 28.62, p < .001$, indicating that the addition of grade level at Step 2 accounted for significantly more variance in applied problems compared to Step 1, $\Delta R^2 = .01$. The R^2 change from Step 2 to Step 3 was less than 1% which was not statistically significant $F(3, 2367) = 1.77, p = .151$, indicating that the addition of the interaction terms at Step 3 did not account for significantly more variance in applied problems compared to Step 2, $\Delta R^2 < .01$.

Table 10

*Regression 6: Testing Main and Moderating Effects of Grade on Applied Problems**Scores*

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Step 1					
Visuospatial working memory	0.23 (0.02)	10.56	< .001	0.04	1.34
Verbal working memory	0.23 (0.03)	9.20	< .001	0.03	1.52
Language ability	0.42 (0.02)	18.72	< .001	0.13	1.67
Step 2					
Visuospatial working memory	0.23 (0.02)	10.71	< .001	0.05	1.34
Verbal working memory	0.22 (0.03)	8.98	< .001	0.03	1.53
Language ability	0.42 (0.02)	18.57	< .001	0.13	1.67
Grade level	0.27 (0.05)	5.35	< .001	0.01	1.01
Step 3					
Visuospatial working memory	0.23 (0.02)	10.68	< .001	0.05	1.34
Verbal working memory	0.22 (0.03)	8.91	< .001	0.03	1.54
Language ability	0.41 (0.02)	18.49	< .001	0.13	1.68
Grade level	0.28 (0.05)	5.45	< .001	0.01	1.02
Visuospatial working memory x Grade	-0.01 (0.01)	-1.80	.071	0.00	1.38
Verbal working memory x Grade	0.00 (0.01)	0.13	.899	0.00	1.49
Language ability x Grade	0.00 (0.01)	-0.35	.728	0.00	1.66

Note. Step 1: $F(3, 2371) = 522.40, p < .001, R^2 = .40$. Step 2: $F(4, 2370) = 403.52, p < .001, R^2 = .41$. Step 3: $F(7, 2367) = 231.56, p < .001, R^2 = .41$.

The full results of the regression are presented in Table 9. Visuospatial working memory ($B = 0.23, p < .001$), verbal working memory ($B = 0.22, p < .001$), and language ability ($B = 0.41, p < .001$) were all individually significant positive predictors in the final

model. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher applied problems scores. Grade level was also a significant positive predictor in the final model ($B = 0.28$, $p < .001$). None of the interaction terms was significant (all p -values $> .05$).

Regression 7: Testing whether grade moderates prediction of calculation subtest scores. For Regression 7, the dependent variable was the calculation subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, language ability, and grade. To aid in interpretation, all independent variables were mean-centered (before computing the interaction terms), and the variables were entered in steps. The first step of the model included visuospatial working memory, verbal working memory, and language ability. In the second step, grade level was entered into the model. In the third step, interaction terms for visuospatial working memory x grade level, verbal working memory x grade level, and language ability x grade level were entered into the model. The interaction terms were used to assess the moderating effect of grade level on the relations between visuospatial working memory, verbal working memory, language ability, and calculation.

The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested in the same manner as the previous regressions. The normal P-P plot (see Figure 14) showed that the data did not deviate from the normal line, so the normality assumption was met. Figure 15 shows that the data were equally distributed around zero, so the assumption of homoscedasticity was also met. Finally, all VIF values were below 10 (see Table 11), indicating that multicollinearity was not a problem.

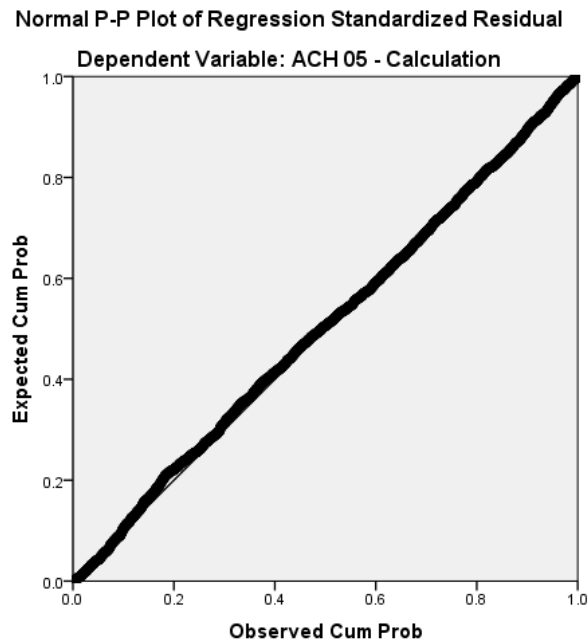


Figure 14. Normal P-P plot for Regression 7.

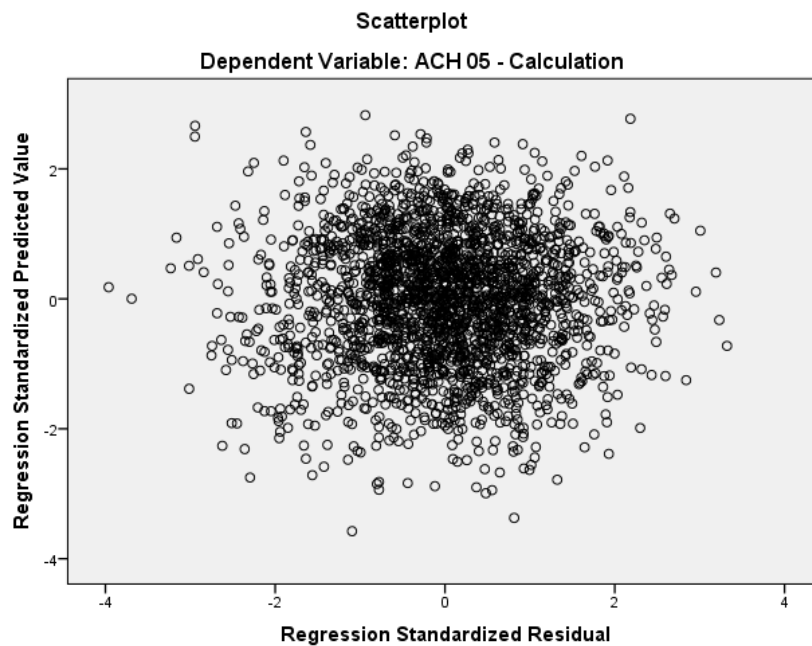


Figure 15. Scatterplot for Regression 7.

Table 11

Regression 7: Testing Main and Moderating Effects of Grade on Calculation Scores

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Step 1					
Visuospatial working memory	0.12 (0.02)	5.04	< .001	0.01	1.34
Verbal working memory	0.27 (0.03)	9.71	< .001	0.04	1.52
Language ability	0.29 (0.03)	11.44	< .001	0.05	1.67
Step 2					
Visuospatial working memory	0.12 (0.02)	5.10	< .001	0.01	1.34
Verbal working memory	0.27 (0.03)	9.56	< .001	0.04	1.53
Language ability	0.29 (0.03)	11.32	< .001	0.05	1.67
Grade level	0.18 (0.06)	3.19	.001	0.00	1.01
Step 3					
Visuospatial working memory	0.12 (0.02)	5.05	< .001	0.01	1.34
Verbal working memory	0.27 (0.03)	9.49	< .001	0.04	1.54
Language ability	0.29 (0.03)	11.29	< .001	0.05	1.68
Grade level	0.19 (0.06)	3.30	.001	0.00	1.02
Visuospatial working memory x Grade	-0.01 (0.01)	-1.73	.084	0.00	1.38
Verbal working memory x Grade	0.00 (0.01)	-0.66	.509	0.00	1.49
Language ability x Grade	0.00 (0.01)	0.28	.778	0.00	1.66

Note. Step 1: $F(3, 2371) = 245.06, p < .001, R^2 = .24$. Step 2: $F(4, 2370) = 187.06, p < .001, R^2 = .24$. Step 3: $F(7, 2367) = 107.73, p < .001, R^2 = .24$.

The results of the regression at Step 1 were significant, $F(3, 2371) = 245.06, p < .001, R^2 = .24$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted calculation at Step 1. The R^2 value indicates that these variables accounted for 24% of the variability in calculation.

The R^2 change from Step 1 to Step 2 was less than 1%, but was statistically significant $F(1, 2370) = 10.19, p = .001$, indicating that the addition of grade level at Step 2 accounted for significantly more variance in calculation compared to Step 1, $\Delta R^2 < .01$. The R^2 change from Step 2 to Step 3 was less than 1% which was not statistically significant $F(3, 2367) = 1.73, p = .160$, indicating that the addition of the interaction terms at Step 3 did not account for significantly more variance in calculation compared to Step 2, $\Delta R^2 < .01$.

The full results of the regression are presented in Table 11. Visuospatial working memory ($B = 0.12, p < .001$), verbal working memory ($B = 0.27, p < .001$), and language ability ($B = 0.29, p < .001$) were all individually significant positive predictors in the final model. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher calculation scores. Grade level was also a significant positive predictor in the final model ($B = 0.19, p = .001$). None of the interaction terms was significant (all p -values $> .05$).

Regression 8: Testing whether grade moderates prediction of math facts fluency subtest scores. For Regression 8, the dependent variable was the math facts fluency subtest of mathematic achievement. The independent variables in this analysis were visuospatial working memory, verbal working memory, language ability, and grade. To aid in interpretation, all independent variables were mean-centered (before computing the interaction terms), and the variables were entered in steps. The first step of the model included visuospatial working memory, verbal working memory, and language ability. In the second step, grade level was entered into the model. In the third step, interaction

terms for visuospatial working memory x grade level, verbal working memory x grade level, and language ability x grade level were entered into the model. The interaction terms were used to assess the moderating effect of grade level on the relations between visuospatial working memory, verbal working memory, language ability, and math facts fluency.

The assumptions of normality, homoscedasticity, and absence of multicollinearity were tested in the same manner as the previous regressions. The normal P-P plot (see Figure 16) showed that the data did not deviate from the normal line, so the normality assumption was met. Figure 17 shows that the data were equally distributed around zero, so the assumption of homoscedasticity was also met. Finally, all VIF values were below 10 (see Table 12), indicating that multicollinearity was not a problem.

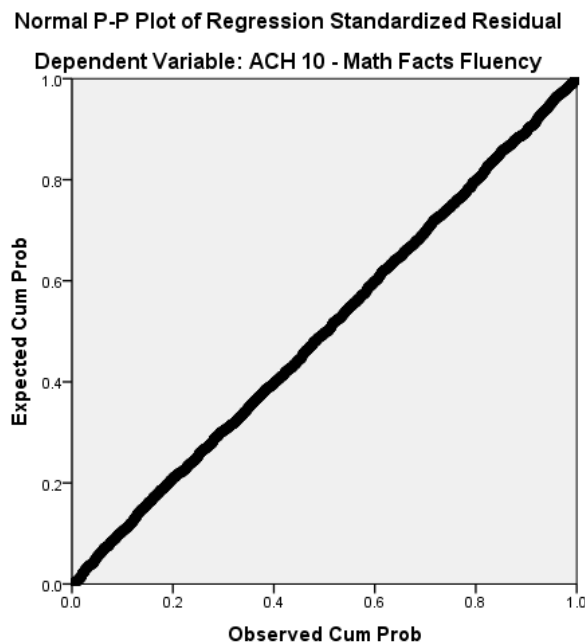


Figure 16. Normal P-P plot for Regression 8.

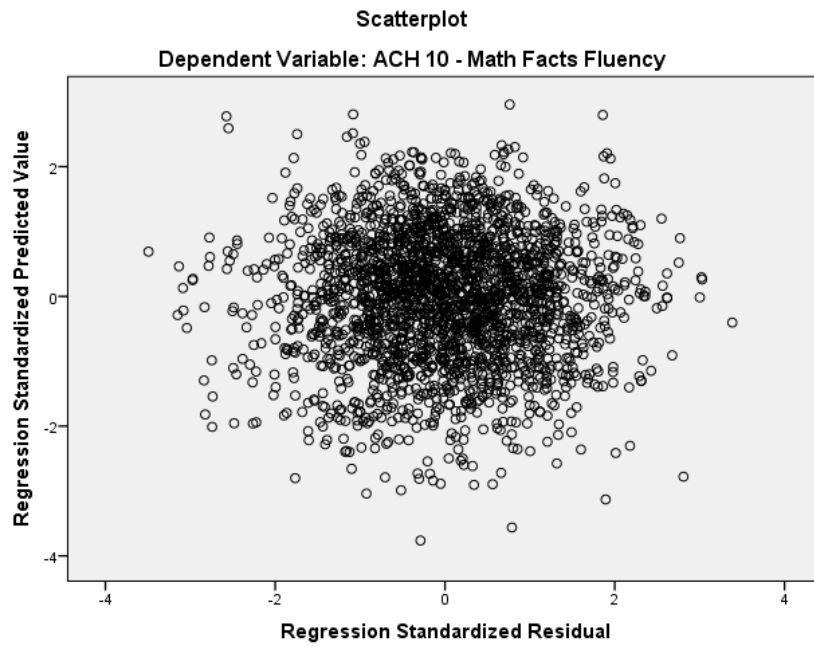


Figure 17. Scatterplot for Regression 8.

Table 12

*Regression 8: Testing Main and Moderating Effects of Grade on Math Facts Fluency**Scores*

Variable	<i>B</i> (<i>SE</i>)	<i>t</i>	Sig.	Squared Partial Correlation	VIF
Step 1					
Visuospatial working memory	0.14 (0.03)	5.62	< .001	0.01	1.34
Verbal working memory	0.20 (0.03)	6.95	< .001	0.02	1.52
Language ability	0.27 (0.03)	10.21	< .001	0.04	1.67
Step 2					
Visuospatial working memory	0.14 (0.03)	5.65	< .001	0.01	1.34
Verbal working memory	0.20 (0.03)	6.86	< .001	0.02	1.53
Language ability	0.27 (0.03)	10.13	< .001	0.04	1.67
Grade level	0.10 (0.06)	1.74	.082	0.00	1.01
Step 3					
Visuospatial working memory	0.14 (0.03)	5.54	< .001	0.01	1.34
Verbal working memory	0.20 (0.03)	6.83	< .001	0.02	1.54
Language ability	0.27 (0.03)	10.14	< .001	0.04	1.68
Grade level	0.12 (0.06)	1.92	.055	0.00	1.02
Visuospatial working memory x Grade	-0.01 (0.01)	-1.51	.131	0.00	1.38
Verbal working memory x Grade	-0.01 (0.01)	-1.55	.121	0.00	1.49
Language ability x Grade	0.00 (0.01)	0.43	.671	0.00	1.66

Note. Step 1: $F(3, 2371) = 181.52, p < .001, R^2 = .43$. Step 2: $F(4, 2370) = 137.02, p < .001, R^2 = .43$. Step 3: $F(7, 2367) = 79.54, p < .001, R^2 = .44$.

The results of the regression at Step 1 were significant, $F(3, 2371) = 181.52, p < .001, R^2 = .43$, which indicates that, collectively, visuospatial working memory, verbal working memory, and language ability significantly predicted math facts fluency at Step

1. The R^2 value indicates that these variables accounted for 43% of the variability in math facts fluency.

The R^2 change from Step 1 to Step 2 was less than 1% which was not statistically significant $F(1, 2370) = 3.03, p = .082$, indicating that the addition of grade level at Step 2 did not account for significantly more variance in math facts fluency compared to Step 1, $\Delta R^2 < .01$. The R^2 change from Step 2 to Step 3 was less than 1% which was not statistically significant $F(3, 2367) = 2.55, p = .054$, indicating that the addition of the interaction terms at Step 3 did not account for significantly more variance in math facts fluency compared to Step 2, $\Delta R^2 < .01$.

The full results of the regression are presented in Table 11. Visuospatial working memory ($B = 0.14, p < .001$), verbal working memory ($B = 0.20, p < .001$), and language ability ($B = 0.27, p < .001$) were all individually significant positive predictors in the final model. This means that students with higher visuospatial working memory, verbal working memory, and language ability scores tended to have higher math facts fluency scores. None of the other predictors was significant (all p -values $> .05$).

Summary

Eight regressions were conducted to address the research questions. The results showed that visuospatial working memory, verbal working memory, and language ability significantly predicted the mathematics achievement composite score and all three subtest scores. Therefore, $H1_a$ was supported. The results also showed that grade level significantly moderated the relation between visuospatial working memory and mathematics achievement in Regression 5. However, grade level did not moderate the

relation between language ability and mathematic achievement. Therefore, H2_a received only mixed support. The next chapter will contain a discussion of these results, as well as directions for future research.

Chapter 5: Discussion

The present study was guided by Baddeley's model of working memory, Logie's two-part model of the visual-spatial sketchpad, and recent studies that highlight potential dissociative properties of verbal and visual working memory (Baddeley, 1986, 2000; Logie, 1995; Szucs et al., 2014). The purpose of this study was to examine the relations among math achievement, student grade level, and multiple cognitive variables (visual-spatial working memory, verbal working memory, and language ability) to determine whether grade level moderated the relation between the cognitive variables and math achievement. This research found that the cognitive variables (visual-spatial working memory, verbal working memory, and language ability) are predictors of mathematic performance. Additionally, grade level enhances the relation between visual-spatial working memory and mathematic achievement. The findings of the present study confirm, as children progress through the grades, visual-spatial cognition is an important predictor of mathematic competence but becomes a weaker predictor of mathematics competence at higher grade levels.

The language composite of the Woodcock Johnson-IV included two variables: listening comprehension and oral expression. Visual-spatial working memory consisted of two variables: visualization and picture recognition. Verbal working memory consisted of three variables: verbal attention, numbers reversed, and memory for words. Mathematic achievement consisted of three variables: applied problems (word problems), calculation, and math facts (timed-test of simple math facts). The scores from these

subtests were averaged to create a single composite score for mathematic achievement.

The additional variables were demographic: age and grade level.

The cognitive variables were expected to predict performance on the math subtests and composite scores on the Woodcock Johnson Test of Achievement. Additionally, grade level was expected to moderate the relations between the cognitive variables and mathematic achievement. In this chapter, the findings, theoretical implications, and educational implications of the study are discussed. Directions for future research and limitations are also examined.

Findings from this research revealed that the cognitive variables of language, verbal working memory, and visual-spatial working memory all predict a child's performance on multiple mathematic domains. Additionally, age and grade are predictors of math performance. Grade level proved to have a moderating effect on the relation between the mathematic achievement composite and visual-spatial working memory, with a stronger relation between visual-spatial working memory and math achievement among students in lower grades. This is an important finding because of the limited research detailing the relations of visual-spatial ability and mathematic achievement as math topics become more complex. Grade level did not moderate the relation between verbal working memory, language ability, and the composite score for mathematic achievement. Moreover, the analyses of the relations between the individual domains of mathematic achievement (applied problems, calculation, and math facts fluency) and cognitive variables (visual-spatial working memory, verbal working memory, and language ability) were not found to be moderated by grade level.

These findings affirm the first hypothesis that multiple areas of cognition (visual-spatial working memory, verbal working memory, and language ability) were predictors of mathematic achievement. As previously stated, grade level moderated the relation between visual-spatial working memory and the mathematic achievement composite. There was no moderating effect for the subdomains of mathematics. Therefore, the results revealed a partial confirmation for the second hypothesis that estimated grade level moderates the relation between mathematic achievement and the cognitive variables. These findings only partially supported the second hypothesis as there was no significant interaction for grade level and the subdomains of mathematic achievement (applied problems, calculation, and facts fluency). Additionally, there was no significant interaction for grade level with either verbal working memory or language ability on mathematic achievement.

Cognitive Variables and Mathematic Achievement

In the present study, the research addressed whether the cognitive variables of visual-spatial working memory, verbal working memory, and language ability as defined by the Woodcock Johnson Test of Cognitive Ability-Fourth Edition, would predict math performance on a mathematic composite of achievement and individual subtests of mathematic achievement (applied problems, calculation, and facts fluency). Cognitive variables have much support in the literature as predictors of mathematic achievement. For example, working memory (Chong & Siegel, 2008; Geary, 2011; Mazzocco & Myers, 2003; Szucs et al., 2014), phonological/language processing (Henry, 1994; Swanson & Beebe-Frankenberger, 2004; Szucs et al., 2014), and attentional systems

(Fuchs et al., 2005, 2006; Fuchs et al., 2014; Geary et al., 2012) have substantial support in the literature as predictors of mathematic achievement. As previously discussed, prior research does not consistently list the said variables as predictors (Fuchs et al., 2006; Lee et al., 2004; Swanson, 2011). The current research supports visual-spatial working memory, verbal working memory, and language ability as unique predictors of mathematic achievement.

A significant finding from the present study was that visual-spatial working memory ability did not explain most of the variance in any of the regression models. Recent research has placed emphasis on the importance of visual-spatial and working memory skills in relation to mathematic development; however, language processing consistently emerged as the variable that explained the largest amount of variance in each regression model, often followed closely by verbal working memory. Visual-spatial working memory explained the least amount of variance in all but one regression (Regression 2, in which applied problems was the dependent variable), which suggested that it was the least important predictor among those included in the present research. This finding was consistent across the dependent variables, except for the applied problems variable.

The finding that visual-spatial working memory, in general, explained the least amount of variance in the model does not eliminate the importance of visual-spatial working memory in the acquisition of mathematic competence. This finding merely puts the relation in perspective. For the sampled students, poorly developed visual-spatial working memory was associated with low mathematic achievement, particularly among

students in lower grades. Visual-spatial working memory positively related to mathematic achievement in all regression equations. Thus, as visual-spatial working memory ability increases, so does performance on measures of mathematic achievement.

For many decades, researchers and educators assumed that poor mathematic performance was based in linguistic competencies (Rourke & Conway, 1997). The present research appears to support previous notions that language competence is relatively strongly related to math achievement; however, the present research moves beyond this notion and provides support that visual-spatial abilities are also significantly related to mathematic competence, but to a lesser degree. Therefore, it is safe to assume that to have whole mathematic learners, educators need to know whether poor visual-spatial working memory deficits are contributing to a student's weakness in mathematic performance. Some students may need instruction or strategies that assist in compensating for or overcoming visual-spatial deficits to achieve optimal levels of mathematic performance (Gade, Zoelch, & Seitz-Stein, 2017; National Research Council, 2006; Uttal et al., 2013).

Consider the early task of “adding on” in which students are required to hold a numerical representation in memory, while adding a second quantity. Another example is early word problems in which children must create numerical representations from text and transform both into mathematic solutions. Visuospatial working memory may significantly affect these early skills. The same consideration could be given to the task of single-digit addition and subtraction using a number line. Students who are adept at perceiving visual stimuli may find this exercise relatively easy to achieve, but a student

with visual-spatial deficits or delays may struggle with managing the conceptual aspects of simple calculation and the structural aspects of moving back and forth on a number line. The student may require additional support and practice to master this exercise, not because he or she does not understand the linguistic concepts of simple calculations, but because the added demand of visual-spatial perception via the number line places additional demands on the task beyond the cognitive resources available. Similarly, early concepts of magnitude, such as putting numbers in order from least to greatest and greatest to least; use of the greater than, less than, and equal to symbols in early mathematic expressions; or managing the directional aspects of rounding up and down can be adversely affected by visuospatial perception. Difficulty visualizing the number line may contribute to difficulty with perceiving magnitude in young children—a concept strongly correlated with math competence (Geary, Hoard, et al., 2008; Fuchs et al., 2014).

Readers and future researchers should also consider the findings of this study related to grade level. The researcher expected grade level to moderate the relation between mathematic achievement and visual-spatial working memory. The findings of the present study supported this expectation (review the results of Regression 5 for a full summary). The results of Regression 5, which regressed math achievement composite scores on predictors, revealed a significant finding: being in a higher grade level related to a weakened relation between visual-spatial skills and mathematic achievement. In other words, children in lower grade levels who had high visual-spatial working memory experienced correspondingly high mathematic competence, whereas students in higher grade levels' math achievement did not increase as sharply in relation to their visual-

spatial working memory, even if those students had similarly high visual-spatial skills. Although this finding gives little insight into why, previous researchers asserted that visual-spatial ability becomes more important as students age (Casey et al., 1995; Casey et al., 2001; Fennema, 1979; Gunderson et al., 2012). The result of the present research presents a quandary because the results indicate visual-spatial working memory is more important for mathematic achievement in the early grades and, as students age, they are able to achieve similar mathematic achievement with less contribution by visual-spatial working memory ability.

Perhaps the reason that visual-spatial working memory became less strongly related to math competence with increasing grade was the math achievement measures available in this study, which tested ability to deal with a wide array of problems but did not teach any new math skills. With increasing age, various forms of math become more automatized and rely more on memory retrieval, reducing the need to involve current solution processing with visual-spatial skills. Another way of thinking about the interaction observed in the present research is the relation between visuospatial skills and the acquisition of new math skill. Visuospatial cognition may be rather important during the early learning of many math concepts. For example, a student who has relatively weak visuospatial skills may initially struggle with new math lessons, relative to a student with more advanced visuospatial skills. The result is a stronger relation between visuospatial skills and math competence during earlier stages of learning in a new domain of math. However, with increasing age or grade, students become increasingly more familiar with many kinds of mathematics material. When this occurs, solution algorithms,

heuristics, and math facts tend to be recalled from memory, decreasing the influence of visuospatial skills on successful solution of math problems and thereby decreasing the relation between individual differences in visuospatial skill and individual differences in math competence. It is also possible that older students have developed resilience and have learned to compensate for weaknesses in visuospatial ability by utilizing previously learned strategies or habits.

The results of the present research contribute to the body of literature by indicating a difference or a shift in the importance of visual-spatial ability, wherein older students' visual-spatial abilities were not as important to math achievement when compared to younger students. The present research findings show that strong capabilities in visual-spatial working memory are positively associated with mathematic competence. The present research provides insight into the potentially transient nature of math learning disability as reported by Vukovic and Siegel (2010): whereas younger students may be at-risk for learning challenges if visual-spatial deficits are not addressed, older students may rely less on visual-spatial ability for mathematic competence. Additionally, the present research may have developmental implications as previous researchers indicated working memory continues to develop through adolescence (Isbell, Fukuda, Neville, & Vogel, 2015).

Working Memory

The present research was guided by the literature on working memory and learning. Researchers have highlighted working memory as having a primary role in learning (Baddeley, 1986; Baddeley & Hitch, 1974; Chong & Siegel, 2008; Fuchs et al.,

2010; Geary, 2011; Mazzocco & Myers, 2003; Szucs et al., 2014). Working memory research has grown considerably in recent decades because of the abundance of studies supporting an active state of memory (working memory) that is directly linked to learning. Many researchers have been invested in exploring working memory in hopes of improving outcomes for learners.

The present study's findings did not support the expected finding of working memory variables explaining the largest amount of variance in math achievement scores. The results revealed working memory variables did not explain the largest amount of variance in any model. This result may have been different if the verbal and visual-spatial working memory variables were combined. Furthermore, the present study was primarily interested in the recent studies highlighting the dissociative aspects of visual-spatial and verbal working memory, where the subcomponents relate differently to tasks.

A growing body of research suggests that multiple aspects of working memory can be distinguished or dissociated and relate differently to mathematics (Jarvis & Gathercole, 2003; Klauer & Zhao, 2004; Logie, 1995; Shah & Miyake, 1996; Szucs et al., 2014). The results of the present study revealed the visual and verbal subcomponents of working memory show considerable differences in their predictive capacity on the selected mathematic tasks (see Tables 5–11). Verbal working memory consistently exceeded visual-spatial working memory in explaining variance in all areas of mathematic competence except for one area: applied problems (see Table 5–11). The visual and verbal subcomponents of working memory explained equal amounts of variance in the applied problems model (see Table 2).

Limitations of the Study

The present study had limitations that warrant consideration. First, although the methods used in the present study allowed the researcher to answer the research questions, the researcher was not able to examine the underlying details, latent variables, and the subjective experiences of students. Although the relation among the independent and dependent variables was examined, the researcher could not determine the causal nature of the relation. Only an experimental design can produce causal conclusions about the relation among variables. The predictor variables explained significant amounts of variance in the models; however, more than 50% of the variance was not explained by the predictor variables.

The second limitation was construct validity. The researcher created a visual-spatial working memory variable using two subtests from the Woodcock Johnson-IV, Test of Cognitive Abilities. The authors of the Woodcock Johnson-IV did not identify the visual-spatial subtests as tests of visual-spatial working memory. The researcher accepted the visual subtests as tests of visual-spatial working memory because the structures are well supported by neurological research (Engle & Kane, 2004) and they fit widely accepted definitions of visuospatial working memory, in that they required short-term retention of visual stimuli as well as place additional processing demands on the subjects, creating an active state of memory (Baddeley & Hitch, 1974; Jarvis & Gathercole, 2003; Klauer & Zhao, 2004; Logie, 1995; Shah & Miyake, 1996; Szucs et al., 2014). Moreover, the factor analysis results in the current study revealed the visual subtests loaded on the same factor, providing further support for a visual-spatial working memory variable.

Formal standardized tests that specifically measure visual-spatial working memory are relatively few and are have yet to be published in the United States. A search for a standardized, widely used measure of visual-spatial working memory revealed zero tests meeting these criteria in the United States.

Many definitions of working memory exist (Baddeley & Hitch, 1974; Cowan, 1995; Engle & Kane, 2004; Voyer, Voyer, & Saint-Aubin, 2017). The third limitation is related to the multiple definitions of working memory. A growing body of evidence suggests attention and working memory's shared capacity is the overarching, predictive variable in complex learning, and it is the shared capacity of working memory that holds the true predictive power (Cowan, 1995; Engle et al., 1999; Engle & Kane, 2004; Kyllonen & Christal, 1990; Süß, Oberauer, Wittmann, Wilhelm, & Schulze, 2002). Adding a shared working memory capacity variable to the regressions may have provided additional insight into the relations between working memory and mathematic achievement. However, much like widely accepted standardized measures of visual-spatial working memory, standardized measures of shared working memory capacity are scant. A review of recent literature revealed studies of shared working memory capacity primarily involved measures of attention and executive function, span tasks, and measures of fluid intelligence to evaluate working memory capacity (Daneman & Carpenter, 1980; Kane & Engle, 2002).

A fourth limitation of the present research related to the varying literature on what constitutes a visual-spatial working memory task. A review of visual-spatial working memory research revealed use of a wide array of visual-spatial tasks (i.e., Corsi Blocks,

rotation tasks, locations tasks) used as measures visual-spatial working memory. The present researcher elected to use a large standardization sample from a widely used national assessment of cognition, language, and academic achievement that included comparable tasks, although not exactly identical tasks. The lack of consensus in the research community regarding the definitions of working memory and the appropriate instruments used to measure it posed limitations. The research was concerned with the most widely accepted and researched definition of working memory: Baddeley and Hitch's (1974) compartmental model and fractionation for the visuospatial sketchpad (Logie, 1995). The compartmental definitions of working memory suited the present research, not because other definitions of working memory were not appropriate, but rather to satisfy the interests of the present study. The present researcher focused on the individual components of visual and verbal working memory (Baddeley & Hitch, 1974; Logie, 1995) and the additional processing demands that theoretically access the central executive component (Baddeley & Hitch, 1974; Engle, 2002; Engle et al., 1999).

Implications for Future Research

The findings from the present study have implications for theory and educational practice. The contributions of visual-spatial working memory to mathematic competence are poorly understood. It is well accepted that visual-spatial skills are associated with mathematic competence in older children, but little is known in addition to this relation. Moreover, the results of the current study were inconsistent with prior research, as visual-spatial ability was more strongly associated with math competence in younger children. The body of research on visual-spatial working memory is expanding and the literature

on the connection between mathematic achievement and visual-spatial cognition creates fertile ground for educators who may benefit from strategies and techniques that advance mathematic learning.

The present research adds to the discussion on the importance of assessing visual-spatial ability in young children. The results revealed visual-spatial working memory is a significant predictor of mathematic achievement across the mathematic domains of applied problems, calculation, and facts fluency. However, increased grade level reduced the relation between visual-spatial working memory and mathematics competence.

Future research is necessary to determine what concepts in early grades are linked to visual-spatial working memory. Mix et al. (2016) determined mental rotation is the best predictor of mathematics in kindergarten and visual working memory is the best predictor of mathematics in sixth grade. Future researchers should seek to discover both basic and advanced mathematic concepts based on visual-spatial working memory capacity for learning. If the unique components of basic and advanced math topics that place high demand on visual-spatial working memory resources are properly identified, it will assist educators in providing support to children who are weak in select mathematic topics. By identifying the concepts that are highly correlated to visual-spatial working memory ability, educators are given the footing to match learners who potentially have delayed development of visual-spatial abilities to compensatory strategies and interventions. Current researchers are proving that visual-spatial skills are malleable (Gade et al., 2017; Uttal et al., 2013); however, more research is needed to establish practical strategies for educators and parents to help children who experience delays or

weaknesses in visual-spatial cognition. If mathematics topics associated with visual-spatial working memory are identified, educators and families can address the delays in younger students and help them achieve adequate performance in advanced mathematic topics.

Future research is needed to establish a firm definition of visuo-spatial working memory. Currently, three themes in the working memory literature define working memory: compartment-based literature, attention-based literature, and shared capacity literature. The research may soon establish a consensus on how to integrate the models or determine the most efficient manner of defining working memory across human development. Additionally, future researchers should seek to understand whether different definitions of visual-spatial working memory explain the differences observed as students develop. For example, it is unclear if compartmental approaches to working memory (Baddeley & Hitch, Logie) better explain working memory relations in younger students and attentional or capacity models (Cowan, Engle, Kane) better explain relations for complex mathematic topics. Additionally, research into defining a consistent set of tasks used to evaluate visual-spatial working memory is needed.

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