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Social Responsibility in Supply Chains in the Context of Emerging Economies

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Social Responsibility in Supply Chains in the Context of Emerging Economies

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in Management

by

Prashant Chintapalli

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ABSTRACT OF THE DISSERTATION

Social Responsibility in Supply Chains in the Context of Emerging Economies

by

Prashant Chintapalli

Doctor of Philosophy in Management

University of California, Los Angeles, 2018

Professor Christopher Siu Tang, Chair

In this dissertation, we focus on three different and important operational issues that arise primarily in the context of emerging economies.

In the first chapter, we discuss three audit mechanisms that buyers can adopt to ensure supplier compliance in a multi-buyer-single-supplier supply chain. When suppliers (i.e., contract manufacturers) fail to comply with health and safety regulations, buyers (retailers) are compelled to improve supplier compliance by conducting audits and imposing penalties. We discuss three audit mechanisms – *independent*, *joint*, and *shared* – and evaluate their performance. We show that the damage costs of the buyers and the compliance cost of the supplier play a crucial role in the choice of the audit mechanism that improves channel profits.

In the second chapter, we focus on a single-buyer-single-supplier supply chain, not necessarily in the context of emerging economies, and discuss two contracts that can coordinate the supply chain when advance-orders are cheaper to manufacture than rush orders. We show that advance-order discount, when combined with minimum-order-quantity or with inventory-delegation, coordinates the supply chain.

In the third chapter, we focus on the role of crop minimum support prices (MSPs) in the context of emerging economies in which farming communities largely comprise of small farmers. We show that MSPs, when not chosen properly, can backfire by hurting farmers' profits.

The dissertation of Prashant Chintapalli is approved.

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Kevin F. McCardle

Rakesh K. Sarin

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University of California, Los Angeles

2018

DEDICATION

Mātṛ dēvō bhava

Pitṛ dēvō bhava

Ācārya dēvō bhava

– Taittirīyōpaniṣat 6th Century BCE

Revere your mother as God.

Revere your father as God.

Revere your teacher as God.

To my parents, Smt. C. Padma and Dr. Ch. Mohan Rao, and to my teachers.

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Acknowledgements

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>
> – Alfred North Whitehead (1861-1947)

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Publications

Felipe Caro, **Prashant Chintapalli**, Kumar Rajaram and Christopher S. Tang, "Improving supplier compliance through joint and shared audits." *Manufacturing & Service Operations Management* (2018). Forthcoming.

Prashant Chintapalli, Stephen M. Disney, and Christopher S. Tang. "Coordinating supply chains via advance-order discounts, minimum order quantities, and delegations." *Production and Operations Management* 26.12 (2017): 2175-2186.

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Felipe Caro, **Prashant Chintapalli**, Kumar Rajaram and Christopher S. Tang, "Can Buyer Consortiums Improve Supplier Compliance?" *Revisiting Supply Chain Risk Springer Series in Supply Chain Management*, co-edited by George A. Zsidisin and Michael Henke, forthcoming, 2018.

Chapter 1 Improving Supplier Compliance Through Joint and Shared Audits with Collective Penalty

Abstract

When suppliers (i.e., contract manufacturers) fail to comply with health and safety regulations, buyers (retailers) are compelled to improve supplier compliance by conducting audits and imposing penalties. As a benchmark, we first consider the ***independent*** audit-penalty mechanism in which the buyers conduct their respective audits and impose penalties independently. We then examine the implications of two new audit-penalty mechanisms that entail a collective penalty. The first is the ***joint*** mechanism under which buyers conduct audits jointly, share the total audit cost incurred, and impose a collective penalty if the supplier fails their joint audit. The second is the ***shared*** mechanism in which each buyer conducts audits independently, shares its audit reports with the other buyers, and imposes a collective penalty if the supplier fails any one of the audits. Using a simultaneous move game-theoretic model with 2 buyers and 1 supplier, our analysis reveals that both the joint and the shared mechanisms are beneficial in several ways. First, when the wholesale price is exogenously given, we establish the following analytical results for the joint mechanism in comparison to the independent mechanism: (a) the supplier's compliance level is higher; (b) the supplier's profit is lower while the buyers' profits are higher; and (c) when the buyers' damage cost is high, the joint audit mechanism creates supply chain value so the buyers can offer an appropriate

transfer-payment to make the supplier better off. Second, for the shared audit mechanism we establish similar results but under more restrictive conditions. Finally, when the wholesale price is endogenously determined by the buyers, our numerical analysis shows that the above key results continue to hold.

Keywords

Supply Chain Risk, Supplier Compliance, Audits, Collective Penalty, Socially Responsible Operations

1.1 Introduction

Low labor costs in the East have encouraged many firms to source their products from countries like Bangladesh, China, Indonesia, and Vietnam. However, without strong commitment from buyers and consistent law enforcement by governments, some suppliers (i.e., contract manufacturers) ignore basic health and safety standards at their factories. Over the past decade, Bangladesh has been a popular low cost country for many western companies (e.g., Walmart, H&M, Mango, and Adidas) to source apparel products. However, the tragic collapse of the Rana Plaza building in 2013, which occurred due to the negligence of a supplier, has raised serious concerns about worker-safety standards in supply chains. Donaldson (2014) commented that there is a perception that 20% of the factories in Bangladesh are unsafe in terms of building structure safety, fire safety, electrical safety, and the like. Besides Bangladesh, developing countries such as China, Cambodia, and Vietnam are facing similar challenges from non-compliant suppliers with unsafe factories (Fuller and Bradsher, 2013; Demick, 2013; Wong and Fung, 2015).

While the international brands are not directly and legally responsible for the safety standards employed in their suppliers' factories, they face a "sourcing dilemma". If they do not source from these countries, millions of poor workers will go unemployed because garment exports constitute a substantial portion of the countries' exports in many developing countries such as Bangladesh (Tang, 2013). On the other hand, if they continue to source from these countries, the international brands are under public pressure to improve worker-safety standards at their suppliers' factories. To address these challenges, many companies often adopt an ***independent*** audit-penalty mechanism in which they independently conduct audits of their suppliers' factories and impose individual penalties when non-compliance is detected. For example, PVH Corp. (the parent company of brands such as Calvin Klein and Tommy Hilfiger) increased its efforts in auditing its supplier factories. Since 2012, PVH audited 84% of its tier-1 suppliers at least once per year and reported the non-compliant health and safety issues on its website (`www.pvhcsr.com`). Despite its prevalence, the independent mechanism has two drawbacks: (a) the penalty imposed by a single buyer may not be severe enough to ensure that the supplier complies with the required safety standards, especially when the supplier has many buyers, and (b) the audit process can be costly and time consuming.

In this paper, we consider two new audit-penalty mechanisms: joint and shared. These audit-penalty mechanisms are based on a collective penalty and can potentially reduce the drawbacks mentioned above using different auditing procedures. Specifically, the ***joint*** mechanism is conducted by a "consortium" of buyers who share the total audit cost, and the supplier is subjected to a collective penalty if it fails the joint audit. In contrast, the ***shared*** mechanism consists of audits conducted independently by buyers who then share their findings among themselves. In doing so, a supplier's non-compliance is exposed to all the buyers when the supplier fails even one audit, and the supplier will then be subjected to a collective penalty. The collective penalty under both these mechanisms can be more severe than the penalty imposed by each buyer independently and this mitigates the first drawback. Furthermore, the buyers gain savings in the joint and shared mechanisms. In the joint mechanism they gain savings through sharing the audit cost, whereas in the shared mechanism, given the advantages of information sharing, the buyers save on auditing by lowering their individual audit levels. This mitigates the second drawback.

We present a unified framework to analyze the independent, joint, and shared mechanisms. Such analysis provides a better understanding of the approaches recently employed by retailers to improve supplier compliance in their supply chain. Two well publicized approaches are the Accord on Fire and Building Safety in Bangladesh (`bangladeshaccord.org`) instituted by the European retailers and the Alliance for Bangladesh Work Safety (`bangladeshworkersafety.org`) set up by the North American retailers.[1] More details and discussion on the differences between these initiatives can be found in Greenhouse and Clifford (2013), Economist (2013), and Jacobs and Singhal (2015). From our perspective, the joint audit mechanism captures two key aspects of these initiatives: (i) instituting common work place safety standards through a joint audit, and (ii) imposing a collective penalty on a non-compliant supplier. Thus our framework provides a basis to develop a better understanding of the Accord and the Alliance. Furthermore, since these initiatives have affirmed to share information about suppliers and impose collective penalties on non-compliant suppliers, their interactions can be analyzed by the shared mechanism.

Figure 1.1 summarizes the three audit-penalty mechanisms. As shown in the figure, while the joint and shared mechanisms impose the same collective penalty, they differ in terms of the auditing process: joint versus independent audits. On the other hand, the independent and shared mechanisms use the same audit process but they differ in terms of the penalty they impose: individual versus collective penalty. Therefore, it is unclear which mechanism is more effective from the buyers' perspective. This serves as the motivation to examine the following three key questions

[1]The Accord is a legally binding agreement signed in May 2013 by 166 apparel corporations from 20 countries in Europe, North America, Asia and Australia, along with numerous Bangladeshi unions and NGOs (e.g., Workers Rights Consortium, International Labor Organization). The goal of the Accord is to improve workplace safety of over 2 million workers at 1,800 factories (Kapner and Banjo, 2013). To reduce the exposure to broad legal liability, U.S. retailers formed the Alliance in 2013, a non-legally binding, five-year commitment to improve safety in Bangladeshi ready-made garment factories. The Accord is committed to provide funds to improve building safety whereas the Alliance is not committed to finance safety improvements.

in this paper:

1. Which of the three mechanisms results in a higher supplier compliance?

2. Which mechanism results in a higher payoff to the supplier?

3. Which mechanism is the most effective from the buyers' perspective?

To study these questions, we develop a simultaneous move game-theoretic model with 3 players (2 buyers and 1 supplier) to capture the essence of the independent, joint, and shared mechanisms. For each of these mechanisms, the buyers select their audit levels and the supplier selects its compliance level simultaneously.

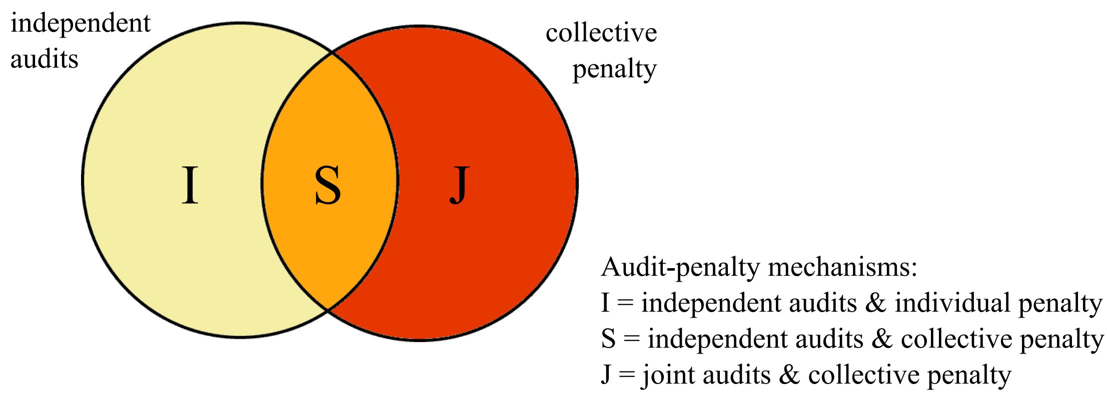

Figure 1.1: The independent (I), shared (S), and joint (J) audit-penalty mechanisms.

When the wholesale price is exogenously given and remains the same across all three mechanisms, our key findings are as follows. First, the joint mechanism improves supplier's compliance. Second, compared to the independent mechanism, the joint mechanism yields a higher profit to the buyer but a lower profit to the supplier. Third, when the buyers' damage cost is higher than the supplier's compliance cost, the supplier can always be made better off under the joint mechanism through a transfer-payment by the buyers. We establish similar results (with smaller impact) for the shared mechanism under more restrictive conditions. Therefore, when a collective penalty is combined with joint audits, the joint mechanism (instead of shared mechanism) offers more opportunities to create supply chain value.

Likewise, when the wholesale price is *endogenously* determined by the buyers, our numerical results show that most of the key structural results derived in the exogenous wholesale price model continue to hold. In particular, we find that, relative to the independent mechanism, the joint mechanism can be Pareto improving so that both the buyers and the supplier are better off. Additionally, we find that the joint mechanism dominates the other two mechanisms in terms of supplier's compliance level and buyers' profits. By combining our analytical and numerical results, we conclude that the joint mechanism is an effective mechanism for improving supplier's compliance level and the buyers' profits. This result provides a more formal justification for the value of the Accord and the Alliance that are designed to make suppliers increase their compliance levels.

Our paper belongs to a new research stream in supply chain risk management that examines three types of supply chain disruptions (Sodhi et al., 2012). The first type is due to disruptions caused by natural disasters (e.g., Japan's Tōhoku earthquake and tsunami, Thailand's major flood, etc.) and human induced disasters (e.g., the terrorist attacks on 9/11). Sodhi and Tang (2012) provide a comprehensive discussion on this type of supply chain disruptions. The second type of disruption is caused by major financial crises (e.g., Asian currency devaluations in 1997, the sub-prime financial crisis in 2008) that can disrupt supplier's operations (Babich et al., 2007). Our paper deals with the third type of supply chain disruptions that are caused by an "intentional act" committed by the supplier. Well-publicized examples include Mattel's lead tainted toys in 2007, melamine tainted milk in 2008, and Baxter's adulterated Heperin in 2008. The research in this area examines issues of product adulteration that occur when suppliers use unsafe materials to produce products that can cause physical harm to consumers (Babich and Tang, 2012; Rui and Lai, 2015).

Such supplier non-compliance issues have forced many western firms to take action to improve supplier compliance. In this setting, Plambeck and Taylor (2015) use a game-theoretic model with a single buyer and a single supplier to explore the interactions between the buyer's audit level and the supplier's compliance and deception effort. By examining the equilibrium outcomes (supplier's compliance level, supplier's deception effort, and buyer's audit level) they show that when a supplier deceives the auditors by hiding certain critical information, the buyer's actions could motivate the supplier to cause more harm.

In the context of environmental violations, Kim (2015) examines the interactions between a regulator's inspection policy and a firm's non-compliance disclosure timing decisions. By considering the case when environmental violations are stochastic, this work shows that there are conditions under which periodic inspections can be more effective than random inspections. Orsdemir et al. (2015) investigate how vertical integration can be used as a strategy to ensure compliance. They examine the scenario of two supply chains, one of which is vertically integrated, and highlight that the presence of a supply chain partnership plays a key role in determining supplier compliance. They argue that, in the absence of a partnership, overly tight scrutiny of violations can backfire and degrade compliance when negative reporting externalities are high. However, tighter scrutiny encourages compliance in the presence of partnership. Moreover, if the positive externalities are high, the integrated and compliant firm will cease to share responsibly sourced components with its competitors thus hurting the industry-wide compliance. More recently, Fang and Cho (2015) consider a setting with joint and shared audits in which multiple buyers engage in a cooperative game in the presence of externalities by which the violation of one buyer can affect the profit of other buyers.

While our paper also deals with the issue of supplier compliance, it is fundamentally different from the existing literature on supply chain risk management in three ways. First, the papers listed above primarily focus on the strategic interaction between one buyer and one supplier. Instead, we examine and compare three different mechanisms (independent, joint, and shared) by capturing the strategic interactions among two buyers and one potentially non-compliant supplier. Second, we consider the issue of a non-compliant supplier and employ the notion of "collective penalty" imposed by both buyers when such a non-compliant supplier fails the joint audit under the joint mechanism, or one of the audits under the shared mechanism. Our contribution is to examine the implications of a collective penalty facilitated by the joint and shared mechanisms. Third, in comparison to Fang and Cho (2015), our paper has a different motivation. Our work is geared towards comparing three audit-penalty mechanisms and understanding when they can increase supplier compliance and supply chain profits in a non-cooperative setting. In particular, our model and results emphasize the tension between buyers and the supplier, whereas Fang and Cho (2015) mostly study the cooperation among buyers when the supplier is indifferent between auditing schemes. Though our research is motivated by workplace safety, it also applies to other regulations that require auditing to verify compliance.

This paper is organized as follows. In Section 3.3 we present our modeling framework and the resulting equilibrium outcomes, and in Section 1.3 we compare the results across all the three mechanisms. In Section 1.4, we extend our analysis to the case when the wholesale price is endogenously determined by the buyers. In Section 1.5 we discuss implications for the the Alliance and the Accord. We present our conclusions in Section 1.6. All proofs are provided in the Appendix A.4.

1.2 The Model

Consider a supply chain comprising of two buyers ($i = 1, 2$) and one supplier s. For ease of exposition, we focus our analysis on the case when the buyers are identical so that buyer i sells one unit of its product at price p and pays the supplier a wholesale price w. We denote the supplier's unit cost by c. Since our focus is on the audit-penalty mechanism, we consider p, w and c to be exogenous so that the values of these parameters do not depend on the mechanism adopted by the buyers. In other words, the strategic intent of different mechanisms is to encourage the supplier to improve its compliance level, but not to increase selling prices, or reduce wholesale prices (e.g., Van Mieghem, 1999), or do both. This seems reasonable in the context of outsourcing agreements between western firms and suppliers located in developing countries because reducing the wholesale price would create public concern about the firm's moral and ethical standards. However, in Section 1.4 we extend our analysis to the case when the wholesale price is endogenously determined by the buyers under each mechanism.

We use a simultaneous move game to model the dynamics between the buyers and the supplier for all the three mechanisms. Specifically, each buyer i simultaneously selects its audit level z_i, $i = 1, 2$, and incurs an audit cost of αz_i^2, where $\alpha > 0$ and $z_i \in [0, 1]$ (in the joint mechanism the buyers choose z_i but reach a joint audit level z through a process that will be explained later). Here, z_i represents the probability that buyer i's audit will be effective in detecting non-compliance (if it exists). This notion of audit probability is commonly used in the literature (e.g., Babich and Tang, 2012; Orsdemir et al., 2015). While the buyers select their audit levels, the supplier simultaneously selects its compliance level x and incurs a compliance cost γx^2, where $\gamma > 0$ and $x \in [0, 1]$. Here, x represents the probability that the supplier complies with the workplace safety regulations. In practice, the supplier might face other decisions besides compliance. However, we focus exclusively on the compliance decision in order to have a parsimonious model that serves our research goal. Incorporating other decisions is left for future work.

The simultaneous move framework is justifiable when the supplier cannot observe the buyer's audit level. However, if this is observable, then a sequential move framework would be the more appropriate in which the buyers will first select their audit levels simultaneously in each mechanism. By anticipating the buyers' audit levels, the supplier selects its compliance level. For completeness, we also analyzed the sequential game model and found that the key results are consistent with those in the simultaneous game model. We refer the interested reader to Caro et al. (2015).

To facilitate analytical comparisons, we assume that the audit cost α remains the same across all the three mechanisms, even though the same approach can be applied to examine the case when audit cost depends on the audit mechanism chosen. We also assume a convex auditing cost αz_i^2 since one would expect the buyers to prioritize the most cost-effective activities. Moreover, this assumption is quite standard whenever each marginal increase in effort is more costly, e.g., see Plambeck and Taylor (2015).

Regardless of the mechanism adopted by the buyers, all parties face the following risks. First, if a non-compliant supplier is detected by buyer i, the buyer will reject the unit product without payment, and the supplier will incur a goodwill cost g associated with the contract termination imposed by buyer i. Second, if a non-compliant supplier is not detected by buyer i, the buyer will accept the unit product and pays the supplier the wholesale price w. However, there is a chance that this non-compliance will be exposed to the public. In that case, buyer i will incur an expected "collateral damage" d due to the spillover effect of the non-compliant supplier. Throughout this paper, we assume that the collateral damage d is severe enough so that there is an incentive for the buyer to audit its supplier. For this reason, we make the following two assumptions that provide motivation for the supplier to care about compliance and for the buyer to care about auditing:

Assumption 1. *The supplier's goodwill cost g associated with contract termination imposed by buyer i, $i = 1, 2$, is higher than the supplier's profit margin (i.e., $g > w - c$).*

Assumption 2. *The damage cost d of buyer i, $i = 1, 2$, due to a non-compliant supplier is higher than the buyer's profit margin (i.e., $d > p - w \equiv m$).*

After all players have made their (audit or compliance) decisions, the sequence of events is as follows: (i) the supplier produces the product and incurs the production cost c; (ii) the buyers inspect for non-compliance; (iii) trade occurs only if non-compliance is not detected by the buyers; otherwise, g is incurred by the supplier; (iv) the public finds out about any possible non-compliance in which case the buyers incur d and the supplier incurs a discounted penalty ηg, with $0 \leq \eta \leq 1$. For ease of exposition, we analyze the non-cooperative simultaneous game for the case when $\eta = 0$. The analysis associated with the case when $\eta > 0$ is omitted because the results change in the expected direction (i.e., the supplier complies more and the buyers audit less compared to when $\eta = 0$).

1.2.1 Independent Mechanism (I)

Under the independent mechanism, buyer i selects its audit probability z_i and the supplier selects its compliance level x. Figure 1.2 depicts the extensive form of the simultaneous game under the independent mechanism. We follow the convention that the dashed line represents information imperfection in the game tree. We begin our analysis with the supplier's problem. From the figure we observe that the supplier will fail buyer i's audit with probability $z_i(1-x)$. By considering the wholesale price w, the goodwill cost g, and the compliance cost γx^2, the supplier's problem for any given audit levels z_1 and z_2 is given by:

$$\begin{aligned}\pi_s(z_1, z_2) &= \max_{x\in[0,1]} \sum_{i=1}^{2} \left[w(1 - z_i(1-x)) - g z_i(1-x) - c\right] - \gamma x^2 \\ &= \max_{x\in[0,1]} 2(w-c) - (w+g)(1-x)\cdot \sum_{i=1}^{2} z_i - \gamma x^2. \end{aligned} \tag{1.1}$$

To ensure that the supplier has incentive to fully comply, we assume that the supplier's profit margin is high enough so that the supplier's expected profit is non-negative under full compliance (i.e., when $x = 1$). By considering the objective function given in (1.1), this assumption can be

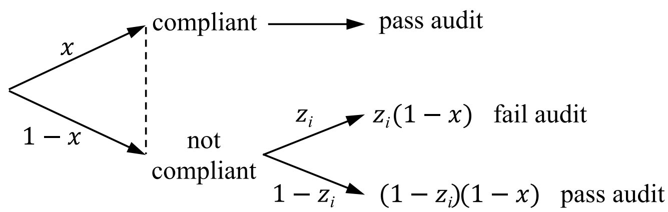

Figure 1.2: Independent mechanism extensive-form game: supplier's compliance level x and buyer's audit level z_i.

stated as:

Assumption 3. *The supplier's total profit margin is higher than its full compliance cost so that* $2(w - c) \geqslant \gamma$.

Before determining the supplier's best-response, observe that $\frac{\partial \pi_s}{\partial x}$ evaluated at (1,1) is equal to $2(w+g) - 2\gamma x$. Hence, we can interpret the term $r \equiv \frac{w+g}{2\gamma}$ as the supplier's "rate of return on compliance per buyer." By applying Assumptions 4 and 6, it is easy to check that $2g > 2(w-c) \geqslant \gamma$ so that $2w > \gamma$. Thus, we conclude that $r > \frac{1}{2}$. As we shall see later, r will be used in proving and interpreting our results. By considering the first order condition associated with (1.1), the supplier's best response for any given buyers' audit levels z_1 and z_2 is given by $x^I(z_1, z_2) = \min\{1, r(z_1 + z_2)\}$. (Throughout this paper, we use superscripts I, J and S to denote the outcomes associated with the independent, joint, and shared mechanisms respectively.)

Next, we determine buyer i's best response $z_i(x, z_j)$ for a given supplier compliance level x and buyer j's audit level z_j. We assume that the general public is not aware that the buyers have a common supplier, so the two buyers are treated independently by the public. Following Figure 1.2 and considering the profit margin $m \equiv (p - w)$, the damage cost d, and the audit cost αz_i^2, the profit of buyer i is given by:

$$\Pi_i(z_i; x, z_j) = m(1 - z_i(1-x)) - d(1 - z_i)(1 - x) - \alpha z_i^2. \qquad (1.2)$$

From the first order condition we obtain buyer i's best response to be $z_i^I(x, z_j) = \min\{\frac{d-m}{2\alpha}(1-x), 1\}$

for $i = 1, 2$. By considering the supplier's best response $x^I(z_1, z_2)$ and buyer i's best response $z_i^I(x, z_j)$ simultaneously, it can be easily established that the equilibrium compliance and audit levels are given by

$$x^I = \frac{r(d-m)}{\alpha + r(d-m)} \quad \text{and} \quad z^I = \frac{d-m}{2(\alpha + r(d-m))}. \tag{1.3}$$

Note that $x^I < 1$ and $z^I < 1$ because $r \equiv \frac{w+g}{2\gamma} > \frac{1}{2}$ so we are guaranteed to obtain an interior solution. The characteristics of the equilibrium in Equation (1.3) are described in the following lemma:

Lemma 1. *Under the independent mechanism I, the buyer's audit level z^I and the supplier's compliance level x^I given in* (1.3) *possess the following properties:*

(i) The supplier's compliance level is always higher than the buyer's audit level (i.e., $x^I = 2rz^I > z^I$).

(ii) Both supplier's compliance level x^I and the buyer's audit level z^I are increasing in the buyer's damage cost d, and decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^I is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^I is increasing in γ.

(iv) The supplier's compliance level x^I is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^I is decreasing in g.

(v) The supplier's compliance level x^I is increasing in the wholesale price w. However, the buyer's audit level z^I is increasing in w if, and only if, $w < \sqrt{2\alpha\gamma} - (d-p)$.

Lemma 1 has the following implications. The first statement reveals that the buyer's audit has an "amplifying" effect as it makes the supplier to increase its compliance level by the factor of

$2r(> 1)$ (i.e., twice the rate of return on compliance). Consequently, the first statement implies that the buyer can encourage the supplier to comply fully (i.e., $x = 1$) without conducting full audits (i.e., $z_i < 1$). The second statement is intuitive. A higher damage cost d will force the buyers to increase their audit levels that, in turn, will cause the supplier to increase its compliance level. In the same vein, the audit cost has a dampening effect. A higher audit cost will force the buyers to reduce their audit levels that, in turn, leads to a lower compliance of the supplier. The third statement shows the opposite effect of the supplier's compliance cost γ. When the supplier's compliance cost γ increases (i.e., as r decreases), the supplier will lower its compliance level x^I. On anticipating this, the buyer will increase its audit level z^I. To interpret the last statement, it is intuitive that the supplier would increase its compliance level when the buyer offers a higher wholesale price. However, to explain the characteristics of buyer's audit level, we consider the case when w is low so that the supplier's compliance level is low. When this is the case, a buyer can easily expose the supplier's non-compliance without needing to exert a high audit level. However, when w gets larger, the compliance increases and the buyer needs to exert a higher audit level to detect the residual level of non-compliance by the supplier.

By substituting z^I and x^I given in (1.3) into (1.1) and (1.2), and by noting that $x^I = 2rz^I$, the buyer's profit $\Pi^I(z^I)$ and the supplier's profit $\pi_s^I(z^I)$ at equilibrium are given by:

$$\Pi^I(z^I) = m(1 - z^I(1 - 2rz^I)) - d(1 - z^I)(1 - 2rz^I) - \alpha {z^I}^2, \tag{1.4}$$

$$\pi_s^I(z^I) = 2(w - c) - \gamma + \gamma(1 - 2rz^I)^2 = 2(w - c) - \gamma + \gamma(1 - x^I)^2. \tag{1.5}$$

1.2.2 Joint Mechanism (J)

Next, we analyze the simultaneous game for the joint mechanism. For any given joint audit level z selected by the consortium (i.e., both the buyers), the supplier will fail the joint audit with a probability of $z(1-x)$. Upon failing the joint audit, the supplier receives no payment and it will be subject to the collective penalty $2g$ imposed by both the buyers. Hence, the supplier's problem can be written as:

$$\pi_s(z) \quad = \quad \max_{x \in [0,1]} \{[2w(1 - z(1-x)) - 2gz(1-x) - 2c] - \gamma x^2\}. \tag{1.6}$$

Using the first-order condition, the supplier's best response $x^J(z)$ is obtained as:

$$x^J(z) \quad = \quad \min\{2rz, 1\}. \tag{1.7}$$

Identifying the buyers' best response requires specifying how the joint audit level is selected and how the audit cost is shared. For that, consider buyer i's profit when the joint audit level is z and buyer i pays a proportion θ_i of the auditing cost:

$$\Pi_i(\theta_i; z, x) = m(1 - z(1-x)) - d(1-z)(1-x) - \theta_i \alpha z^2. \tag{1.8}$$

Suppose for a moment that buyer i is able to unilaterally select the joint audit level. Clearly, in that case buyer i would want z to maximize the profit above. From the first order condition, buyer i would want the joint audit level z to be:

$$z = z_i(\theta_i) \equiv \frac{(d-m)(1-x)}{2\alpha\theta_i}. \tag{1.9}$$

Note that if $\theta_i = \frac{1}{2}$ for $i = 1, 2$, then both buyers would want the joint audit level to be $\frac{(d-m)(1-x)}{\alpha}$, and therefore they would reach consensus automatically. With that in mind, in what follows we assume that the buyers a priori agree to evenly share the audit cost. We make this assumption for ease of exposition. However, in the Appendix A.2 we formally show that $\theta_1 = \theta_2 = \frac{1}{2}$ is indeed the outcome of a non-cooperative game between the two buyers.

Given $\theta_i = \frac{1}{2}$, we can derive buyer i's best response from (1.9), and together with the supplier's best response in (1.7) we can solve the simultaneous equilibrium as:

$$x^J = \frac{2r(d-m)}{\alpha + 2r(d-m)} \quad \text{and} \quad z^J = \frac{d-m}{\alpha + 2r(d-m)}. \tag{1.10}$$

An interior solution is guaranteed since $x^J < 1$ and $2r > 1$ implies that $z^J < 1$. Lemma 6 in Appendix A.1 is analogous to Lemma 1 and shows that the joint mechanism equilibrium in Equation (1.10) exhibits the same characteristics as stated in the independent mechanism equilibrium given in Lemma 1 (i.e., Equation (1.3)).

By using (1.6), (1.7), (1.8) and (1.10) along with $\theta_1 = \theta_2 = \frac{1}{2}$, the equilibrium profits of the buyers and supplier under the joint mechanism can be written as:

$$\Pi^J(z^J) = m(1 - z^J(1 - 2rz^J)) - d(1 - z^J)(1 - 2rz^J) - \frac{1}{2}\alpha\, {z^J}^2, \tag{1.11}$$

$$\pi_s^J(z^J) = 2(w-c) - \gamma + \gamma(1 - 2rz^J)^2 = 2(w-c) - \gamma + \gamma(1 - x^J)^2. \tag{1.12}$$

1.2.3 Shared Mechanism (S)

In this section, we analyze a simultaneous game to examine the third mechanism: the shared mechanism. In this mechanism, each buyer conducts its own audit independently, but shares its findings with the other buyer so that a non-compliant supplier will be exposed to both buyers if it fails either of the buyers' audits. Figure 1.3 provides the extensive-form game of the shared mechanism. For any given audit levels z_1 and z_2, the supplier with compliance level x will fail buyer i's audit with probability $[z_i(1-x) + z_j(1-z_i)(1-x)]$ for $i = 1, 2$, and $j \neq i$. By noting that the supplier will fail buyer i's audit with probability $z_i(1-x)$ under the independent mechanism

(Figure 1.2), we can conclude that, through sharing audit reports, the shared mechanism enables buyer i to identify a non-compliant supplier with an "additional probability" of $z_j(1-z_i)(1-x)$. This additional probability plays an important role in analyzing the shared mechanism.

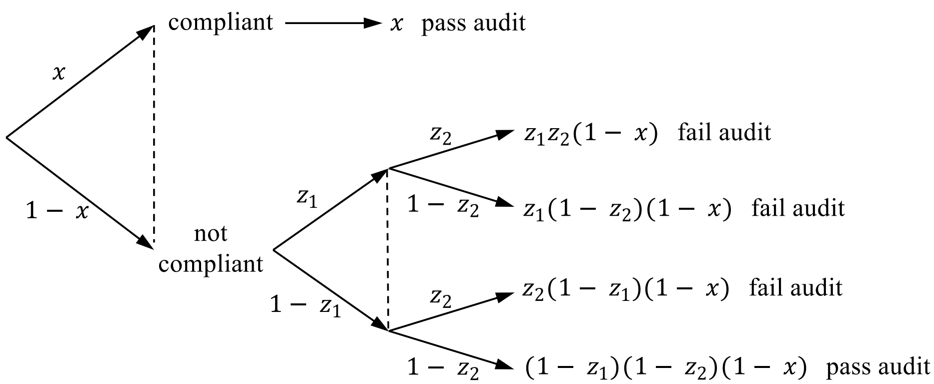

Figure 1.3: Shared mechanism extensive-form game: buyer i's audit level z_i $(i=1,2)$ and supplier's compliance level x.

Under the shared mechanism, supplier's profit can be written as

$$\pi_s(x;z_1,z_2) \quad = \quad 2(w-c)-2(g+w)(z_1+z_2-z_1z_2)(1-x)-\gamma x^2 \tag{1.13}$$

and buyer i's $(i=1,2)$ profit can be written as

$$\Pi_i(z_1;z_2,x) \quad = \quad m\left[1-(z_1+z_2-z_1z_2)(1-x)\right]-d(1-z_1)(1-z_2)(1-x)-\alpha z_i^2. \tag{1.14}$$

The best responses of the supplier and the buyers are given by:

$$x(z_1,z_2)=2r(z_1+z_2-z_1z_2) \quad \text{and} \quad z_i(x,z_j)=\frac{(d-m)}{2\alpha}(1-z_j)(1-x), \tag{1.15}$$

where $i=1,2$ and $i\neq j$. By solving the above three equations simultaneously, we characterize the equilibrium in Lemma 2.

Lemma 2. *Under the shared mechanism S, the buyer's audit level z^S and the supplier's compliance* *level x^S can be characterized as follows:*

(i) The buyer's audit level z^S is the unique root $z \in (0, 1-\sqrt{\frac{2r-1}{2r}})$ of the following cubic equation:

$$V(z) \equiv 2rz^3 - 6rz^2 + \left(1 + 4r + \frac{2\alpha}{d-m}\right) z - 1 = 0. \tag{1.16}$$

(ii) The supplier's compliance level is $x^S = 2rz^S(2 - z^S)$ and $x^S \in (0, 1)$.

Lemma 7 in Appendix A.1 shows that the shared mechanism equilibrium (as implicitly defined in Lemma 2) exhibits the same characteristics as stated in Lemma 1. Finally, the supplier and the buyer profits under the shared mechanism are given by:

$$\Pi^S(z^S) = m\left[1 - (2z^S - (z^S)^2)(1 - x^S)\right] - d(1 - z^S)^2(1 - x^S) - \alpha {z^S}^2, \tag{1.17}$$

$$\pi_s^S(z^S) = 2(w - c) - \gamma + \gamma(1 - x^S)^2, \tag{1.18}$$

where z^S and x^S are the equilibrium audit and compliance levels as given in Lemma 2.

1.3 Comparison of Equilibrium Outcomes Across Mechanisms

To gain a deeper understanding about the results derived in the last section, we now compare the equilibrium decisions across all three audit-penalty mechanisms. Then we compare the buyers' and the supplier's profits across the mechanisms.

1.3.1 Comparison of buyers' audit and supplier's compliance levels

We compare the equilibrium decisions across the three mechanisms in Proposition 1.

Proposition 1. *Across all three mechanisms, the buyers' audit levels satisfy: $z^S < z^I < z^J$. Additionally, the supplier's compliance levels satisfy the following:*

(i) $x^J > x^I$ *and* $x^J > x^S$.

(ii) $x^S > x^I$ *if and only if* $\alpha \geq \widetilde{\alpha} \equiv \max\{(d-m)(\widetilde{r}-r), 0\}$, *where* $\widetilde{r} \equiv \frac{1}{\sqrt{5}-1} (\approx 0.81)$.

Proposition 1 has the following implications. First, relative to the independent mechanism, the buyer can afford to audit less under the shared mechanism because all the audit findings are shared. On the other hand, relative to the independent mechanism, the buyer can afford to increase their joint audit level under the joint mechanism because the joint audit cost is shared by the two buyers. This explains the first statement.

Statement (i) in the second statement indicates that because the joint audit level is higher (i.e., $z^J > z^I$), the supplier must commit to a higher compliance level under the joint mechanism in response to the increased audit level and the higher (collective) penalty for non-compliance. Hence, $x^J > x^I$. Next, while both the joint and shared mechanisms impose the same collective penalty, the buyers in the consortium maintain a higher audit level under the joint mechanism. In response, the supplier must commit to a higher compliance level under the joint mechanism. Thus, $x^J > x^S$.

Statement (ii) is noteworthy because it shows that, relative to the independent mechanism, the shared mechanism can make the supplier to comply more and yet the buyer to audit less. When rate of return on compliance r is high ($r \geq \widetilde{r} \Leftrightarrow \widetilde{\alpha} = 0$ by definition), the supplier will comply more under the shared mechanism because of the collective penalty. However, when the rate of return on compliance is low ($r < \widetilde{r} \Leftrightarrow \widetilde{\alpha} > 0$), the compliance level is driven by the audit cost α of the buyers. If $\alpha < \widetilde{\alpha}$, then the buyers become complacent and try to delegate the responsibility of auditing to each other because the cost of auditing is low. The supplier takes advantage of this behavior and complies less under the shared mechanism. However, when $\alpha \geq \widetilde{\alpha}$, each buyer, realizing that the other buyer alone cannot audit at a greater level due to the high audit cost, seriously takes up the responsibility to audit and this makes the supplier to comply more. Figures 1.4 and 1.5 illustrate the results stated in Proposition 1. For all the plots in Section 1.3 we use the following parameter values: $d = g = 1000, c = 0, p = 1800$, and $w = 900$. (In Appendix A.3, we provide different plots for the case when $d = 2g = 2000$.)

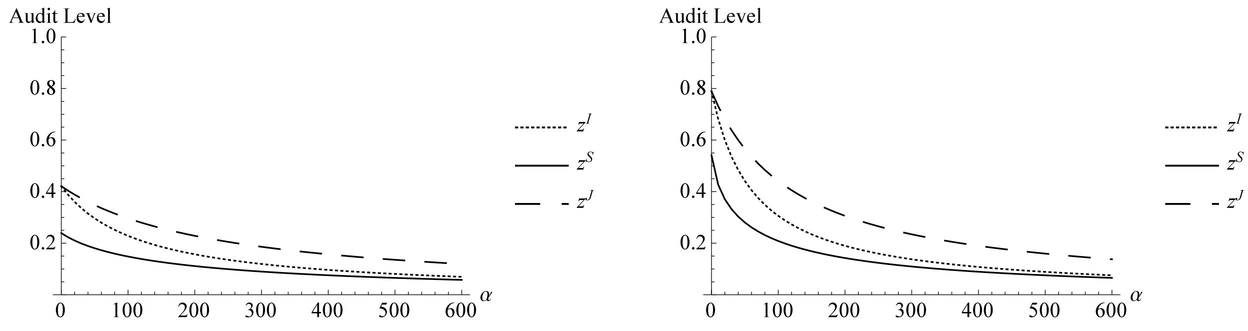

Figure 1.4: Audit levels for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

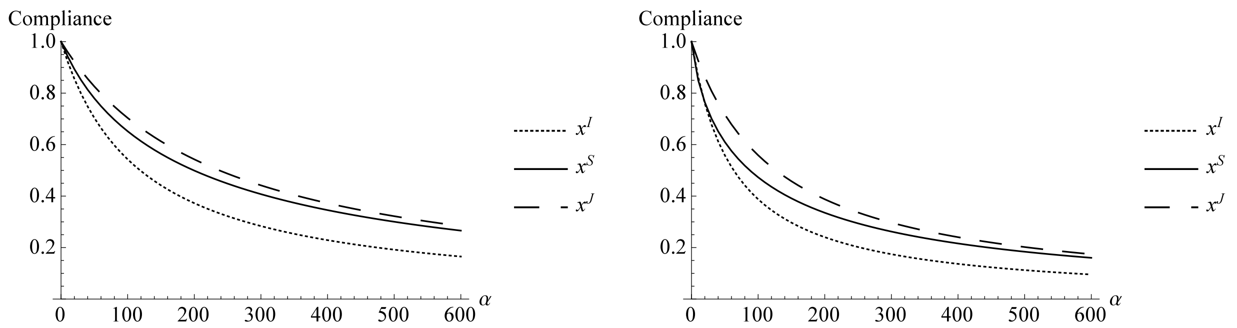

Figure 1.5: Compliance levels for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

1.3.2 Comparison of supplier's profits

Using the equilibrium profits of the supplier as given in (1.5), (1.12), and (1.18), we establish the following result that compares supplier's profits across different mechanisms.

Proposition 2. *The supplier's profit possesses the following properties:*

(i) $\pi_s^J(z^J) \leqslant \pi_s^I(z^I)$ *and* $\pi_s^J(z^J) \leqslant \pi_s^S(z^S)$.

(ii) $\pi_s^S(z^S) \leqslant \pi_s^I(z^I)$ *if and only if* $\alpha \geq \widetilde{\alpha}$, *where* $\widetilde{\alpha}$ *is defined as in Proposition 1.*

Because the supplier's profit is driven by the compliance level, the results as stated in Proposition 2 are congruent with Proposition 1. In particular, the supplier has the lowest profit in the joint mechanism due to the collective penalty and the higher compliance level (statements (i) and (ii) in Proposition 1). Figure 1.6 illustrates the findings of Proposition 2. Here $\widetilde{\alpha} = 0$ for $\gamma = 800$ and $\widetilde{\alpha} = 17.6$ for $\gamma = 1500$ so we observe $\pi_s^S(z^S) \leqslant \pi_s^I(z^I)$ for most values of α.

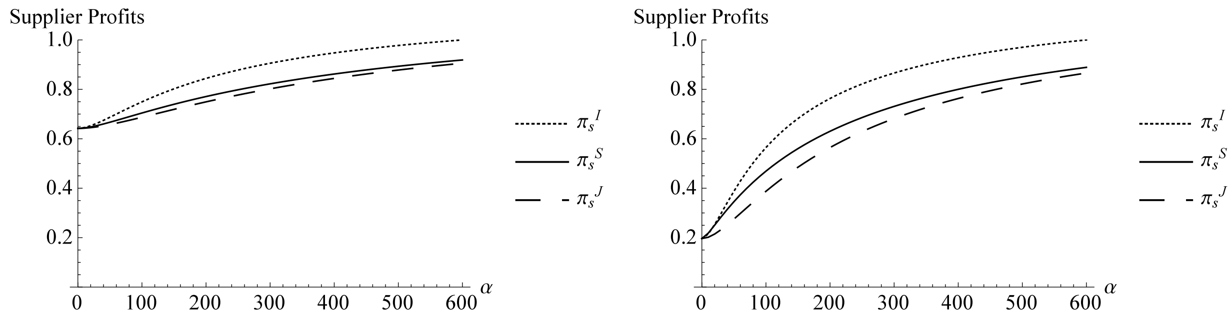

Figure 1.6: Supplier's profits (normalized) for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

1.3.3 Comparison of buyers' profits

The following result compares the buyers' profits across the different mechanisms.

Proposition 3. *The buyers' profits possess the following properties:*

(i) $\Pi^J(z^J) \geqslant \Pi^I(z^I)$.

(ii) $\Pi^S(z^S) \geqslant \Pi^I(z^I)$ *if and only if* $\alpha \geq \widetilde{\alpha}$, *where* $\widetilde{\alpha}$ *is defined as in Proposition 1.*

Proposition 3 has the following implications. The first statement illustrates that each buyer can obtain a higher profit under the joint mechanism than under the independent mechanism because the buyers share the total audit cost incurred by the consortium while forcing the supplier to comply more. Further, one would intuitively think that the buyers' profits would improve if they can attain higher supplier compliance through lower audit levels. This is the finding in the second statement of the above proposition: when α is large, as shown in Proposition 1, the supplier complies more ($x^S > x^I$) while the buyers audit less ($z^S < z^I$), and therefore they make higher profits under the shared mechanism compared to the independent mechanism.

Proposition 3 does not provide a comparison of the buyers' profit between the joint and shared mechanisms. Our numerical results indicate that $\Pi^J(z^J) \geqslant \Pi^S(z^S)$ as it can be seen in Figure 1.7. It seems intuitive that the buyers would be better off in the joint mechanism since they can save on the auditing cost while inducing the highest compliance. For a few limiting cases (e.g., $r \to \frac{1}{2}$ and $\alpha \to 0$) one can indeed show analytically that $\Pi^J(z^J) \geqslant \Pi^S(z^S)$, which provides partial support for our numerical observation.

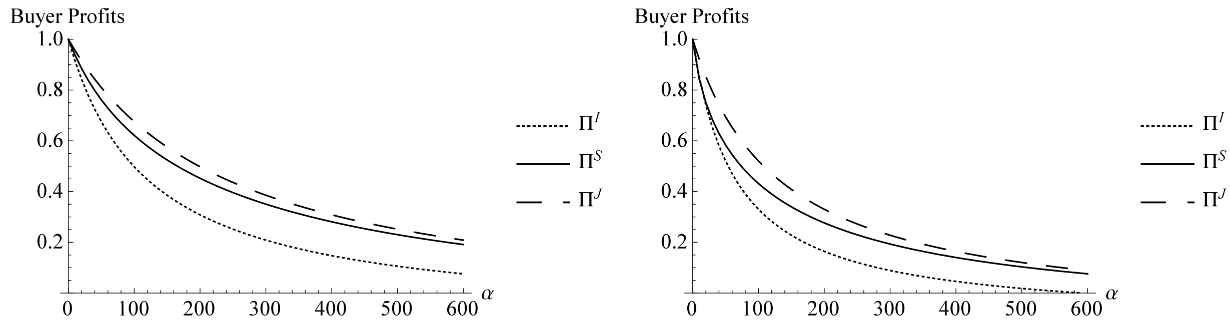

Figure 1.7: Buyers' profits (normalized) for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

1.3.4 Comparison of supply chain profits

From Propositions 2 and 3 we observe that buyers are better off but the supplier is worse off when there is a collective penalty under the joint mechanism. In the context of emerging economies such as Bangladesh, making the supplier substantially worse off could be perceived as being socially unfair and the buyers may face adverse publicity. Therefore, we now examine if the buyers can offer transfer-payments to the supplier so that both the buyers and the supplier are better off.

Consider for instance the joint mechanism versus the independent mechanism. When each buyer i offers a transfer-payment $T(> 0)$ to the supplier, all parties will be better off if $\Pi^J - T \geqslant \Pi^I$ for each buyer and $\pi_s^J + 2T \geqslant \pi_s^I$ for the supplier. That is, there exists a transfer-payment T that is Pareto improving if, and only if, the supply chain profit is higher (i.e., $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$). Such Pareto-improving transfer-payment will make the joint mechanism acceptable to both the buyers and the supplier. By considering the buyer's profit given in (1.4) and (1.11) and the supplier's profit given in (1.5) and (1.12) we obtain the following results:

Proposition 4. *The total supply chain profit under the joint mechanism is higher than that under the independent mechanism if* <u>*any*</u> *of the following conditions hold:*

(i) The audit cost α is sufficiently low.

(ii) The damage costs of each buyer is larger than the compliance cost of the supplier (i.e., $d > \gamma$).

(iii) The total damage cost incurred by the buyers is greater than the compliance cost of the supplier (i.e., $2d > \gamma$) and the cost of non-compliance for each buyer is greater than the cost of non-compliance for the supplier (i.e. $d - m > g + w$).

Proposition 4 provides a set of sufficient conditions ensuring the existence of a transfer-payment $T > 0$ such that the joint mechanism creates supply chain value compared to the independent mechanism. Part (*i*) in Proposition 4 states that, regardless of the other parameter values, if the audit cost α is low enough, then the savings from the joint audit will outweigh the decrease in the supplier's profit. To see this, note that x^I and x^J tend to one when the audit cost α approaches zero. Since $\frac{z^I}{x^I} = \frac{z^J}{x^J} = \frac{1}{2r}$, it follows that the audit level in the joint and independent mechanisms are equal to $\frac{1}{2r}$ when $\alpha \to 0$. This can be confirmed in Figures 1.4 and 1.5. Hence, when α is close to zero, $x^I \approx x^J$ and $z^I \approx z^J$, so $\pi_s^I \approx \pi_s^J$, but $\Pi_i^I < \Pi_i^J$ because the joint mechanism has an audit cost saving of $\frac{\alpha}{2}$ compared to the independent mechanism. By continuity, there must exist a range $(0, \alpha')$, with $0 < \alpha' \leq \infty$, such that $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$, which is statement (*i*) in Proposition 4.

Parts (*ii*) and (*iii*) in Proposition 4 are conditions to ensure that the supply chain will earn net positive savings through compliance. In contrast, if there is a net loss through compliance, then the joint mechanism might lead to lower supply chain profits compared to the independent mechanism.

This can only happen when α is large so the audit cost advantage of the joint mechanism has less impact – see the discussion of part (i) of Proposition 4 in the previous paragraph.

The shared mechanism is harder to analyze because we only have an implicit characterization of the audit level z^S as stated in Lemma 2. Our best attempt is summarized in Proposition 14 in Appendix A.1, which is similar to part (iii) of Proposition 4. Nevertheless, in our extensive numerical study we observed that the results in Proposition 4 – in particular, parts (i) and (ii) – also held true for the shared mechanism as shown, for instance, in Figure 1.8.

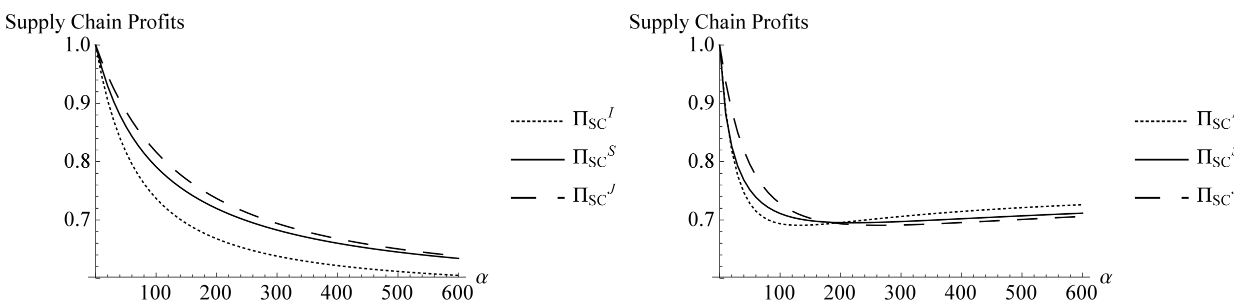

Figure 1.8: Supply chain profits (normalized) for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

The left plot in Figure 1.8 has $d > \gamma$, so $2\,\Pi^J + \pi_s^J \geqslant 2\,\Pi^I + \pi_s^I$ for all α per part (ii) of Proposition 4. We observe the same for the shared mechanism. The right plot in Figure 1.8 has $d < \gamma < 2d$ and $(d-m) < 2(d-m) < g+w$, so neither parts (ii) or (iii) of Proposition 4 apply (and Proposition 14 for the shared mechanism does not apply either). Hence, in the right plot of Figure 1.8 only part (i) of Proposition 4 applies and the joint mechanism yields a higher supply chain profit for lower values of α (here $\alpha' = 190.83$), but for larger values the independent mechanism is better from a channel perspective. The same can be said for the shared mechanism. Overall, as consumers become more aware of compliance issues, one would expect the collateral damage d to become high enough such that $d > \gamma$, which would ensure that the joint (or shared) mechanism yields a higher profit for any audit cost α.

1.3.5 Comparison of consumer surplus

The profit comparisons are crucial from the supply chain perspective. However, from the social responsibility perspective, one needs to consider the impact of the mechanisms on consumer surplus.

Consider a typical end consumer who derives an intrinsic utility V from the product and hence, gains a surplus of $V - p$ when consuming one unit of the product. The expected utility of a representative consumer is thus given by $(V - p) \cdot \Pr(Sale)$, where $\Pr(Sale)$ is the probability that trade occurs in equilibrium. We assume that $V > p$, else the consumer would not purchase the product. Thus, to compare the consumer surplus under different audit-penalty mechanisms, it suffices to observe the sale probability $\Pr(Sale)$ under each of the mechanisms.[2] The sale probabilities under the independent, joint and shared mechanisms are given as: $S^I \equiv 1 - z^I(1 - x^I)$, $S^J \equiv 1 - z^J(1 - x^J)$, and $S^S \equiv 1 - z^S(2 - z^S)(1 - x^S)$. With these definitions we have the following result:

Lemma 3. *1. $S^J > S^I$ if and only if $\sqrt{2}\, r(d - m) > \alpha$.*

2. There exists a threshold value α_J such that $S^J > S^S$ if and only if $\alpha < \alpha_J$.

3. There exists a value α_I such that $S^I < S^S$ if and only if $\min\{\alpha_I, \widetilde{\alpha}\} < \alpha < \max\{\alpha_I, \widetilde{\alpha}\}$, where $\widetilde{\alpha}$ is defined as in Proposition 1.

Lemma 3 shows that the independent mechanism has a higher sale probability than the joint mechanism when the audit cost α is large. The sale probability is better in the joint mechanism when the compliance level is relatively high – in fact, much greater than 0.5 – but such high compliance level can only be attained if auditing is not too costly, as shown in Figure 1.5. Hence, consumer surplus is lower under the joint mechanism when α is large. Note however, that the parameter d implicitly captures how much society values compliance. As d increases, it follows

[2] Our model assumes that p is exogenous. In practice, some consumer might be willing to pay a premium for responsible sourcing practices.

from Lemma 3 that there is a wider range of α for which the sale probability is higher in the joint mechanism than in the independent case. Similar observations can be made for the shared mechanism.

1.4 Endogenous Wholesale Price

In this section, we extend our model to the case when the wholesale price w_i and the audit level z_i are endogenously determined by buyer i and when the compliance level x is endogenously determined by the supplier. Since the game for each of the three mechanisms involves 5 different decisions, i.e., $(w_1, z_1; w_2, z_2; x)$, selected by 3 players (2 buyers and 1 supplier), the analysis is complex and the analytical comparisons across all the three mechanisms are no longer tractable. Therefore, we make these comparisons through numerical analysis. To facilitate such analysis, we solve a two-stage game: in the first stage the buyers simultaneously choose the wholesale prices and then the second stage corresponds to the simultaneous game analyzed in Section 1.3. Note that the Alliance for Bangladesh does not include any provisions for the garment prices, whereas the Accord only states that prices should ensure financial feasibility (see Table 1 in Jacobs and Singhal, 2015). In other words, these consortiums do not address pricing and auditing simultaneously, which is consistent with our sequential approach.

To incorporate the issue of endogenous wholesale price to be determined by each buyer, we define two additional terms: (a) buyer i's profit margin $m_i \equiv p - w_i$, $i = 1, 2$; and (b) the supplier's "rate of return on compliance to buyer i's audit" $r_i \equiv \frac{g+w_i}{2\gamma}$. Notice that both terms depend on the wholesale price w_i to be determined by buyer i. In what follows, we first describe how we determine the best-response functions (i.e., the supplier's compliance level and the buyers' audit level) for any given wholesale price vector (w_1, w_2) under each of the three mechanisms. We then explain how we compute the wholesale price and the corresponding profits in equilibrium.

1.4.1 Independent Mechanism I

By using the same approach presented in Section 1.2.1, it is easy to check that, for any given wholesale price vector (w_1, w_2), the supplier's profit and the buyers' profit can be written as

$$\begin{aligned}
\pi_s(x; z_1, z_2, w_1, w_2) &= \sum_{i=1}^{2}[w_i(1 - z_i(1-x)) - g z_i(1-x) - c] - \gamma x^2 \\
&= \sum_{i=1}^{2}(w_i - c) - (w_i + g)(1-x) \cdot \sum_{i=1}^{2} z_i - \gamma x^2, \qquad (1.19) \\
\Pi_i(z_i, w_i; x) &= m_i(1 - z_i(1-x)) - \alpha z_i^2 - d(1-z_i)(1-x),\ i = 1, 2. \qquad (1.20)
\end{aligned}$$

On solving the simultaneous game between the supplier and the buyers for a given wholesale price vector (w_1, w_2), we obtain the equilibrium audit and compliance decisions as below:

$$\begin{aligned}
z_i^I(w_1, w_2) &= \frac{(d - m_i)}{2\alpha + r_1(d - m_1) + r_2(d - m_2)},\ i = 1, 2, \qquad (1.21) \\
x^I(w_1, w_2) &= \frac{r_1(d - m_1) + r_2(d - m_2)}{2\alpha + r_1(d - m_1) + r_2(d - m_2)}. \qquad (1.22)
\end{aligned}$$

By substituting the above equilibrium into (1.19) and (1.20), we obtain the profits of the supplier and the buyers, which we denote by $\pi_s^I(w_1, w_2)$ and $\Pi_i^I(w_1, w_2), i = 1, 2$, respectively.

By using $\pi_s^I(w_1, w_2)$ and $\Pi_i^I(w_1, w_2)$ and by inducting backward we obtain the equilibrium wholesale prices w_1^I and w_2^I by solving a non-cooperative game between the two buyers as follows. First, we consider the bounds imposed on wholesale prices by Assumptions 1 and 2 (i.e., $\max\{0, p - d\} \leqslant w_i \leqslant \min\{p, g + c\}$) and by Assumption 3 (i.e., $w_1 + w_2 - 2c \geqslant \gamma$). We then compute the best-response function of buyer i (i.e., $w_i^*(w_j)$) numerically by solving the following problem of buyer i for different values of w_j:

$$\mathbf{PI}: \quad \max_{w_i} \Pi_i^I(w_i, w_j)$$

$$\text{subject to } (1.21), (1.22),$$

$$\max\{0, p-d\} \leqslant w_i \leqslant \min\{p, g+c\} \text{ for } i = 1, 2,$$

$$w_1 + w_2 - 2c \geqslant \gamma,$$

$$\Pi_i^I(w_i, w_j) \geqslant 0, \text{for } i = 1, 2.$$

In this problem, the last two constraints correspond to the individually rational constraints associated with the supplier and buyers, respectively. Next, we determine the equilibrium wholesale price w_1^I and w_2^I as the point of intersection of the above derived best-response functions. As the buyers are identical, we observe that $w_1^I = w_2^I \equiv w^{I*}$. Finally, we retrieve the corresponding equilibrium outcomes $(z^{I*}, x^{I*}, \pi_s^{I*}, \Pi^{I*})$ through substitution.

1.4.2 Joint Mechanism J

For any given wholesale price w_1 and w_2, we can use the same approach as presented in Section 2.2 to determine the supplier's profit as:

$$\pi_s(z) = \max_{x\in[0,1]} \{(w_1 + w_2)(1 - z(1-x)) - 2gz(1-x) - 2c - \gamma x^2\} \tag{1.23}$$

where z is the joint audit level adopted by the consortium. The best response of the supplier is obtained as $x^J(z) = \min\{(r_1 + r_2)z, 1\}$.

Now suppose buyer i is able to select unilaterally the joint audit level z. Then, buyer i would choose a joint audit level of

$$z = z_i(\theta_i) \equiv \frac{(d - m_i)(1 - x)}{2\alpha\theta_i} \tag{1.24}$$

that maximizes its profit

$$\Pi_i(\theta_i; z, x) = m_i(1 - z(1 - x)) - d(1 - z)(1 - x) - \theta_i \alpha z^2. \tag{1.25}$$

Thus, if $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j}$ then both buyers choose the same joint audit level and hence would automatically reach a consensus. Using this fact, we assume that buyer i and buyer j agree a priori to share the audit cost in the ratio $\frac{\theta_i}{\theta_j} = \frac{d-m_i}{d-m_j}$. As before, we make this assumption for ease of exposition and in Appendix A.2 we formally show that $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} = \frac{1}{2d-m_1-m_2}$ is the outcome of a non-cooperative game. By using these proportions θ_1 and θ_2, we can determine the equilibrium audit and compliance levels as:

$$z^J \equiv z^J(w_1, w_2) = \frac{(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)} \tag{1.26}$$

$$x^J \equiv x^J(w_1, w_2) = \frac{(r_1 + r_2)(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)}. \tag{1.27}$$

By substituting the equilibrium above into (1.23) and (1.25), we can express the supplier's and buyer i's profits as $\pi_s^J(w_1, w_2)$ and $\Pi_i^J(w_1, w_2)$; respectively. We then induct backwards to obtain the equilibrium wholesale prices w_1^J and w_2^J by solving a non-cooperative game between the two buyers. We obtain the best-response function of buyer i by solving the problem **PJ**, which is the same as problem **PI** except that the profit function $\Pi_i^J(w_i, w_j)$ is based on the equilibrium expressions (1.26) and (1.27) (instead of (1.21), (1.22)). The ensuing procedure to obtain the equilibrium outcomes $(z^{J*}, x^{J*}, \pi_s^{J*}, \Pi^{J*})$ is the same as in explained in Section 4.1.

1.4.3 Shared Mechanism S

Akin to (1.13) and (1.14), we obtain the supplier's and the buyers' profits as:

$$\pi_s(x; z_1, z_2, w_1, w_2) = \sum_{i=1}^{2}\{(w_i - c) - (w_i + g)(1 - x) \cdot (z_i + z_j + z_i z_j)\} - \gamma x^2, \tag{1.28}$$

$$\Pi_i(z_i, w_i; z_j, x) = m_i(1 - (z_i + z_j + z_i z_j)(1 - x)) - \alpha z_i^2 - d(1 - z_i)(1 - z_j)(1 - x), \tag{1.29}$$

so that the best-response functions of the players for any given wholesale price vector (w_1, w_2) are

$$z_i^S = \frac{(d - m_i)(1 - z_j^S)(1 - x^S)}{2\alpha}, \; i = 1, 2, \; i \neq j, \tag{1.30}$$

$$x^S = (r_1 + r_2)(z_1^S + z_2^S - z_1^S z_2^S), \tag{1.31}$$

where for notational convenience we suppress the arguments (w_1, w_2) of z_i^S and x^S. As before, we obtain the equilibrium wholesale prices by solving the best-response functions of the two buyers simultaneously. The best-response function of buyer i is obtained by solving the problem **PS**, which is analogous to **PI** and **PJ**. The remaining steps to obtain the equilibrium outcomes $(z^{S*}, x^{S*}, \pi_s^{S*}, \Pi^{S*})$ are the same as in the independent and joint mechanisms.

1.4.4 Numerical Analysis

In this section, we use the approach outlined in sections 1.4.1, 1.4.2 and 1.4.3 to compute the equilibrium outcomes (i.e., $w^{k*}, z^{k*}, x^{k*}, \pi_s^{k*}, \Pi^{k*}$) associated with mechanism k, where $k = I, J, S$. Also we used the same parameter values as in Section 1.3 (except the fact that the wholesale price w_i is now computed instead of exogenously given). The following figures summarize our results.

First, since the buyers impose a collective penalty under the joint and shared mechanisms, one would expect the buyers to offer a higher wholesale price under these mechanisms than under the independent mechanism to incentivize the supplier. This intuition is confirmed in Figure 1.9, but only when the buyer's audit cost α is sufficiently high. This is because when audit costs are low, the buyers can afford to audit at a higher level, which in turn increases supplier's compliance without the need to offer higher wholesale prices.

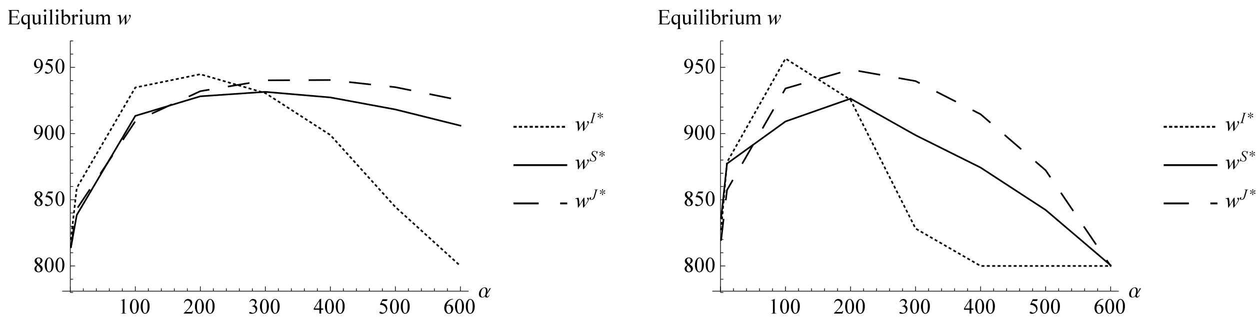

Figure 1.9: Equilibrium wholesale price w for I, S and J mechanisms with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

Second, when the wholesale price is endogenously determined by the buyers, Figures 1.10 and 1.11 indicate that the results stated in Proposition 1 continue to hold for the case when the buyer's audit cost α is low. More importantly, we confirm that the joint and the shared mechanisms can make the supplier more compliant. However, contrary to the finding made in Proposition 1, when α is high and the wholesale prices are endogenous, we notice that the buyers audit more under the shared mechanism than what they would otherwise do under the independent mechanism. Additionally, as depicted in Figure 1.9, when α is high, the buyers also offer a higher wholesale price to encourage a higher supplier compliance under the shared mechanism. Thus, the buyers use higher audit levels and higher wholesale prices as two levers to increase supplier's compliance under the shared mechanism when the wholesale price is endogenously determined.

Third, Figures 1.12 and 1.13 indicate that, among all three mechanisms, the buyers earn the most and the supplier earns the least under the joint mechanism. This finding is consistent with Propositions 2 and 3. Hence, from the buyer's perspective, the joint mechanism still dominates the other two mechanisms. Note from Figure 1.12 that the supplier always makes a positive profit

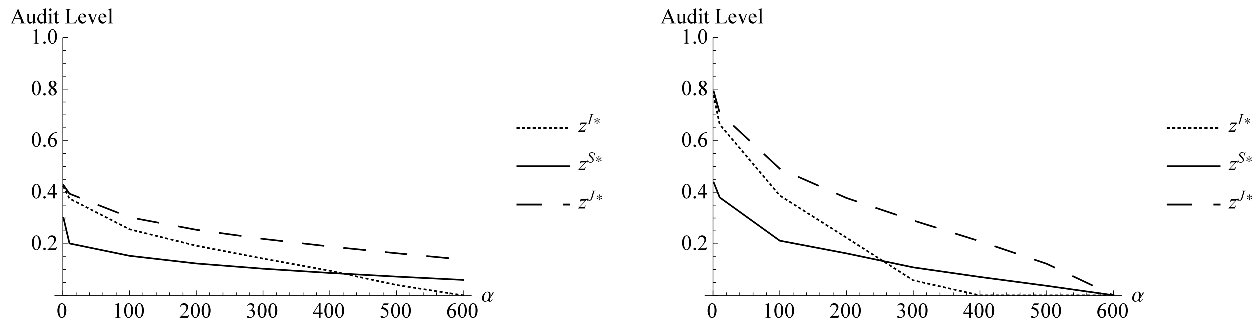

Figure 1.10: Audit levels when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

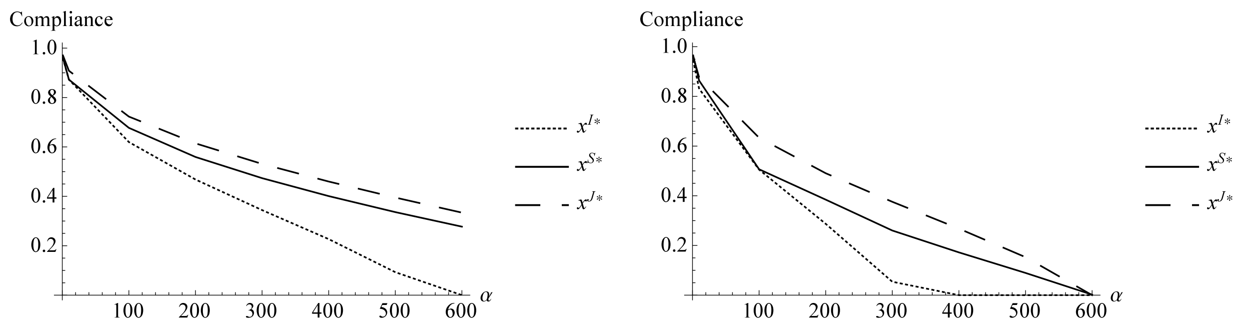

Figure 1.11: Compliance levels when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

when $\alpha > 0$ under all three mechanisms. In contrast, Figure 1.13 shows that the buyers' profit vanishes when the audit cost α is significantly high, and this happens sooner than with exogenous w because the competitive pressure makes the buyers' profit decrease faster.

Finally, Figure 1.14 is the counterpart of Figure 1.8 when the wholesale prices are endogenous. We observe the same results as in Proposition 4. In particular, when the buyers' damage costs is higher than the supplier's cost of compliance ($d > \gamma$), the joint and shared mechanisms create supply chain value compared to the independent mechanism for all values of α. This allows for a transfer-payment to compensate the supplier for its higher compliance. In general, a Pareto-improving transfer-payment is always possible when the audit cost α is low enough, as seen in the right plot of Figure 1.14.

Thus, as demonstrated by the numerical analysis in this section, the key analytical results that we obtained with an exogenous wholesale price in Section 1.3 continue to hold when the wholesale

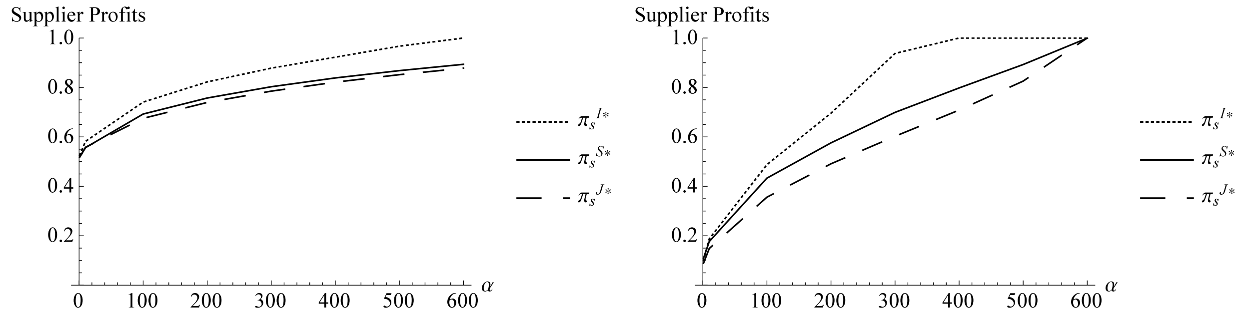

Figure 1.12: Supplier's profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

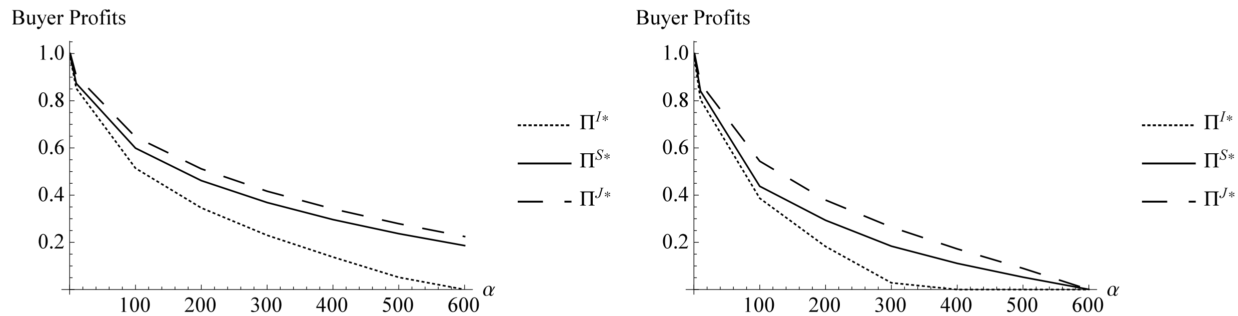

Figure 1.13: Buyers' profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

price is endogenous.

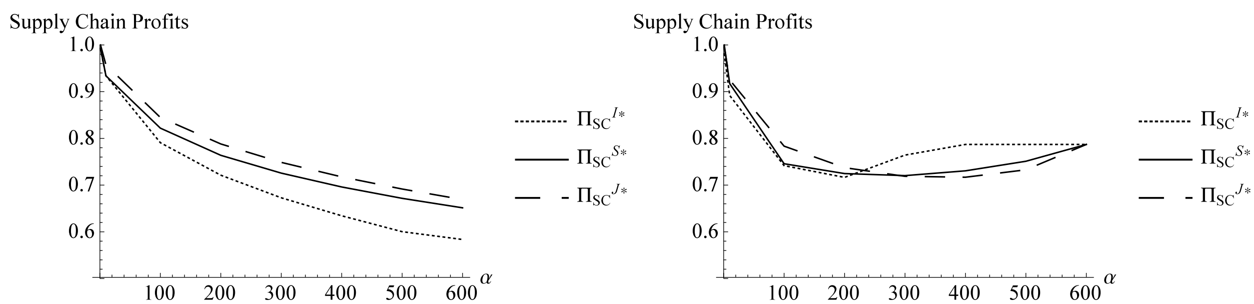

Figure 1.14: Supply chain profits when w is endogenous with $\gamma = 800$ (left) and $\gamma = 1500$ (right)

1.5 Discussion

In this section we discuss some of our model implications in relation to the Alliance and the Accord. It should be noted that our model by no means fully represents these agreements; instead, it captures various salient features especially the audit-penalty mechanism. Nevertheless, our findings can be relevant for the design of future consortia.

The Alliance and the Accord are fundamentally similar in many aspects (Labowitz and Baumann-Pauly 2014) and both advocate joint audits. However, one important difference is that the Accord is legally binding whereas the Alliance is not (Economist 2013). Specifically, under the Accord, factory workers can take legal action if they believe that the Accord fails to "follow through on their commitment".[3] This can be incorporated in our model through the damage cost d. Assuming a higher damage cost d for the Accord would be consistent with the additional legal costs faced by the Accord when its auditing effort fails to detect non-compliance. A higher damage cost implies higher audit and compliance levels (per Lemma 6), but it also implies lower profits for the buyers. So this would indicate that the Accord might ensure safer factories compared to the Alliance, but at the expense of lower profits due to a higher liability.

The Accord stipulates that a non-compliant factory that fails to eliminate safety hazards must be terminated. This commitment is also legally binding.[4] In contrast, the Alliance is not legally bound to terminate a non-compliant factory. In other words, there is a positive chance that the buyers might continue to do business with a factory that failed the audit. This can be incorporated in our model through the goodwill cost g.[5] Assuming a lower goodwill cost g for the Alliance would be consistent with the fact that the supplier is less likely to be terminated when non-compliance is detected. If g is lower, then the rate of return on compliance r is lower, and per Equation (1.10) the audit and compliance levels will decrease.

Aside from being legally binding or not, both agreements stipulate contributions from the buyers toward helping the supplier's compliance. This can be incorporated into the model by assuming that the buyers incur a certain portion δ of the compliance cost γx^2. It can be shown that when

[3] `http://www.cleanclothes.org/resources/background/comparison-safety-accord-and-the-gap-walmart-scheme`

[4] `http://www.just-style.com/news/bangladesh-accord-cuts-ties-with-four-more-factories_id127323.aspx`

[5] Alternatively, one can include an expected payment from the buyer to the supplier that is proportional to $z(1-x)$.

$\delta > 0$, for all three mechanisms (I, J, and S): the compliance level is higher, the audit level is lower, and the supplier's profit increases. In contrast, the buyers' profit increases only when the audit cost α is high.[6] Hence, as expected, providing financial assistance benefits the supplier but might not be in the best interest of the buyers.

Finally, we have shown that the joint mechanism effectively increases compliance, so both the Alliance and the Accord should be able to achieve their primary goal. If these consortiums also want to ensure that the suppliers are better off (or at least not worse off), then our results show that some form of transfer-payment is needed.

1.6 Conclusions and Future Work

In this paper, we presented a unified framework of three different audit-penalty mechanisms (independent, joint, and shared) for improving supplier's compliance in supply chains. By considering a simultaneous move game involving 2 buyers and 1 supplier, we analyzed and compared the equilibrium outcomes (the supplier's compliance level, the buyer's audit level, the supplier's profit, the buyers' profits and the supply chain profit) across all three mechanisms for the case when the wholesale price is exogenously given. We also extended our analysis to the case when the wholesale price associated with each mechanism is endogenously determined by the buyers. We show that the joint mechanism dominates in terms of supplier compliance and the buyers' profit. Moreover, in our numerical analysis we observe that the key structural findings that we made for the case of exogenous wholesale price continued to hold even when the wholesale price is endogenously determined by the buyers.

Overall, we can summarize the key findings for the joint mechanism as follows:

1. The supplier's compliance always improves, and it always results in higher buyer profit under

[6] The details of this analysis are skipped here for brevity and are available from the authors upon request.

the joint mechanism.

2. The supplier, however, earns the lowest profit under the joint mechanism and earns the highest profit under the independent mechanism.

3. The buyers have to offer a Pareto-improving transfer-payment to the supplier to make the latter better off under the joint mechanism.

4. Such transfer-payment is possible when the audit cost is low or when the buyers' damage cost is higher than the supplier's cost of compliance. When these conditions hold, the supply chain profit under the joint mechanism is higher than the profit under the independent mechanism and this enables the buyers to provide the Pareto-improving transfer-payment.

We find similar results for the shared mechanism, which shows that it is also a viable mechanism to create supply chain value through collective penalty.

Overall, our results enable us to gain a better understanding about the dynamic interactions among the buyers and the supplier under independent, joint and shared mechanisms. Since the joint mechanism captures two salient features (collective penalty and joint audits), our results provide additional justification for the implementation of the Accord and the Alliance in Bangladesh.

Future research could consider alternative audit-penalty mechanisms and settings where our modeling assumptions do not apply. These include settings in which the buyers are non-identical (different price/cost structure, different bargaining power, etc.), scenarios with incomplete information on costs, or an extension in which the retail price p is endogenous. All of this could potentially affect the ordering of the three mechanisms. Given the current concerns over supplier compliance, addressing these questions could be worthwhile avenues for future research.

Chapter 2 Coordinating Supply Chains via Advance-Order Discounts, Minimum Order Quantities, and Delegations

Abstract

To avoid inventory risks, manufacturers often place rush orders with suppliers only after they receive firm orders from their customers (retailers). Rush orders are costly to both parties because the supplier incurs higher production costs. We consider a situation where the supplier's production cost is reduced if the manufacturer can place some of its order in advance. In addition to the rush order contract with a pre-established price, we examine whether the supplier should offer advance-order discounts to encourage the manufacturer to place a portion of its order in advance, even though the manufacturer incurs some inventory risk. While the advance-order discount contract is Pareto-improving, our analysis shows that the discount contract cannot coordinate the supply chain. However, if the supplier imposes a pre-specified *minimum order quantity requirement* as a qualifier for the manufacturer to receive the advance-order discount, then such a *combined* contract can coordinate the supply chain. Furthermore, the combined contract enables the supplier to attain the first-best solution. We also explore a *delegation* contract that either party could propose. Under this contract, the manufacturer delegates the ordering and salvaging activities to the supplier in return for a discounted price on all units procured. We find the delegation contract coordinates the supply chain and is Pareto-improving. We extend our analysis to a setting where the suppliers capacity is

limited for advance production but unlimited for rush orders. Our structural results obtained for the one-supplier-one-manufacturer case continue to hold when we have two manufacturers.

Keywords

Advance-Order, Minimum Order Quantity, Delegation, Supply Chain Contracts

2.1 Introduction

Grocery retailers often sell *private labels* of holiday celebration products (eg. moon cakes, pumpkin pies, etc.) with a single selling season. Well before the selling season starts, the grocer's food technology team, the supply chain department, and the marketing department work together to develop recipes, design packaging, and select contract food manufacturers. After completing the selection, the retailer will place a firm order to the contract food manufacturer, who will, in turn, place a firm order to its packaging material supplier. Because the food product is perishable and the packaging is specifically customized to the retailer, neither the manufacturer nor the supplier will produce the corresponding items in advance. Consequently, all orders along the supply chain are rush orders that are costly to fill.

The above business context motivates us to consider a situation when both the manufacturer and the supplier have to deal with rush orders of highly customized products. Due to the high production cost for rush orders, the supplier charges the manufacturer a high, pre-established contract price. Though the supplier's production costs could be substantially lowered if the orders are placed in advance, the manufacturer refrains from doing so given its apprehension of overstocking the customized material[1]. Hence, a discounted wholesale price that a supplier may offer, could

[1]While the retailer-specific packaging materials can be inventoried before the selling season, these materials have only recycling value after the selling season especially when the packaging design changes every year or when the manufacturer may not win the contract during the following year.

encourage the manufacturer to place an advance-order[2]

Based on our work with a grocery retailer in the UK, we learned that food contract manufacturers and packaging material suppliers are aware of the trade-off between the benefits of advance-order discounts and the (imputed) costs of the leftover packaging materials. This trade-off motivated us to examine the following research questions for the case when the original rush order contract price is already established[3]:

1. Should the supplier offer advance-order discounts?

2. Can the advance-order discount contract coordinate a decentralized supply chain? If not, how about a variant that combines the advance-order discount contract with other commonly observed terms such as minimum order quantity and/or inventory management delegation?

We use a two-echelon supply chain model in a two-period setting to explore the above research questions. We first show that a simple discount contract does not coordinate the supply chain. Then we explore a variant of the contract that "combines" the advance-order discounts with a pre-specified minimum advance-order quantity. The combined contract enables the supplier to coordinate the supply chain. Under this combined contract, the supplier extracts the entire surplus of the manufacturer, while offering the manufacturer a discounted wholesale price. This finding provides a good rationale (in addition to the economies of batch production processes) for the omnipresent industry practice of minimum order quantities[4].

[2]We are also aware of a case in the commemorative medal industry. Here, the manufacturer makes medals to celebrate special events (sporting events, royal weddings, special anniversaries etc.). The commemorative medals are made out of high value metals (gold, silver, platinum etc.) and are sold through retailers such as Harrods, the Post Office and the company's own web-site. After receiving the orders from the retailers, the medals are manufactured in a single production run. The medals are sold in presentation boxes, often hand-crafted from mahogany or walnut by a supplier. This case also fits our modelling assumptions.

[3]As it turns out, our result remains the same even when the supplier can determine both the contract price and the advance-order discount factor.

[4]We note that minimum order quantities are almost always imposed by packaging suppliers posted on www.alibaba.com.

Besides minimum order quantities, we consider the case wherein the manufacturer (or the supplier) proposes that the ordering decisions and the salvaging activities are *delegated to the supplier* in exchange for a new discounted price for all the units procured by the manufacturer[5]. Such a contract is akin to the vendor managed inventory (VMI) setup in supply chains. We demonstrate that when price discount is coupled with delegation, this combined contract not only coordinates the supply chain but is also Pareto-improving under mild conditions on the demand process.

Our analysis generates the following three key insights:

1. Though advance-purchase discount contracts by themselves do not coordinate a supply chain, they do coordinate the supply chain when coupled with either (a) a minimum-order quantity requirement, or (b) an inventory management delegation contract.

2. Combining advance-purchase discount and minimum-order-quantity can always coordinate a supply chain.

3. There exists a necessary and sufficient condition for the existence of an advance-purchase discount and inventory management delegation contract that coordinates the supply chain.

To our knowledge, the existing literature does not examine the role of minimum order quantities and inventory management delegations in combination with advance-purchase discounts. The insights we draw provide additional reasons for suppliers to offer minimum order quantity contracts and VMI-like services in decentralized supply chains. In this paper, we first prove our results for the case of a one-supplier-one-manufacturer supply chain. Then, we discuss how our model can be extended to the case of two buyers.

Our paper is organized as follows. We perform a brief literature survey in Section 2. In Section

[5]This setting is plausible when the supplier is in a better position to salvage or recycle the leftover packaging materials.

3, we present our supply chain model with uncertain demand and we establish two benchmarks. In section 4, we show that the advance-order discount contract cannot coordinate the supply chain, and in Section 5 we show that a combination of the advance-order discount contract and a minimum order quantity can coordinate the supply chain. In Section 6, we consider the situation when the manufacturer delegates the responsibility of managing the inventory decisions to the supplier in exchange for a discounted wholesale price. We extend our analysis to the case of one-supplier-two-manufacturers supply chain in Section 7. We conclude the paper in Section 8 and provide the proofs in the Appendix B.1.

2.2 Literature Review

As one of the first articles that examine minimum order quantity contracts, Chow et al. (2012) consider a minimum order quantity contract in a *quick response* context where the manufacturer can *postpone* its single order decision until he obtains updated demand information[6]. They find that if the supplier can postpone the specification of the minimum order quantity till some information about demand is observed, then such an MOQ contract can coordinate the supply chain. In general, a manufacturer may be reluctant to participate in such a contract when the supplier cannot commit to the contractual terms (i.e., the minimum order quantity) in advance. In this paper, we show that, by combining the advance-order discounts with minimum order quantity contract, the supplier can commit to the contractual terms in advance and coordinate the supply chain at the same time.

Our model differs from Chow et al. (2012) in three important aspects. First, Chow et al. (2012) consider a setting in which the manufacturer orders exactly once (one decision), while we consider a different setting in which the manufacturer can place *two orders* (i.e., two decisions): (a) an advance-order that is subject to a minimum order quantity, and (b) a top-up order after

[6]We thank the anonymous reviewer who brought this paper to our attention.

the demand is realized. Second, we consider a situation when the discount factor for the advance order is *endogenously determined* by the supplier, while Chow et al. (2012) assume this factor is *exogenously given*. Third, our analysis is based on a *general demand distribution* that possesses Increasing Generalized Failure Rate (IGFR) properties, while Chow et al. (2012) assume that the demand is normally distributed (which is a special case of the IGFR distributions).

Our research is also related to the advance purchasing literature (eg. Xie and Shugan (2001), Tang et al. (2004) etc.). In the field of advance-order discounts arising from supply chain management, our base model that deals with advance order discount is closely related to Cachon (2004) and Özer et al. (2007). Cachon (2004) shows that advance-purchase discounts can coordinate a manufacturer-retailer supply chain when the manufacturer can set both the advance-purchase discount and the regular wholesale price. Our base model contrasts with Cachon (2004) in two respects. First, in the initial model presented in Cachon (2004), the manufacturer's production cost is the same for both advance and regular purchase and there is only one production opportunity. Later, as an extension, Cachon (2004) incorporates a positive shipping cost for rush orders. The shipping cost is incurred by the supplier and hence this cost can eventually be treated as an increase in the supplier's unit production cost for rush orders. Even in our setting, the supplier's production cost is lower for advance-orders, and higher for rush-orders. However, our setting accounts for additional flexibility in production because we consider the supplier to have two production opportunities – one for advance-orders and one for rush-orders – which facilitate more informed production decisions. While our preliminary analysis about the Pareto improving nature of advance-order contract concurs with the findings of Cachon (2004), our main contribution lies in modifying the traditional advance-order discount contract to ensure supply chain coordination in a Pareto improving manner. Second, while Cachon (2004) assumes that the manufacturer can set the purchase price for both advance and regular orders, our base model can be viewed as a special case when the rush-order price has been established in advance, and the supplier can only offer an advance-order discount on the rush-order price.

More recently, Özer et al. (2007) examine the optimal ordering policy with demand forecast updating when the supplier can set its price *before* and *after* the manufacturer updates its demand forecast. They determine the conditions under which the supplier should offer advance-order discount. That is, when the price in the first period should be strictly less than the price in the second period. They also show that the optimal contract is Pareto-improving. Our context is different from that considered by Özer et al. (2007) in two ways. First, while Özer et al. (2007) consider a generic setting in which the demand forecast is updated after one period, our base model can be viewed as a special case of their model by assuming that the demand is realized after one period. Second, in addition to the Pareto-improvement that was shown by Özer et al. (2007), we show in the analysis of our base model that the advance-order discount contract cannot coordinate the decentralized supply chain. More importantly, we show that, by combining advance-order discount with minimum order quantity, or with inventory management delegation, the two combined contracts can coordinate the supply chain in a Pareto improving way.

Thus, although our advance-order discount base model is directly related to Cachon (2004) and Özer et al. (2007), we leverage our base model analysis to examine two new *combined* contracts that occur in practice but have not been examined in the literature hitherto. In particular, while it is known that advance-order discount contract cannot coordinate the decentralized supply chain (Özer et al., 2007), we show that the supplier can coordinate the supply chain and achieve the first-best solution if it combines advance-order discount with either minimum advance-order quantities or by delegating inventory management decisions to the manufacturer.

To our knowledge, ours is the first paper to investigate the impact of (i) the combination of advance order discounts and the minimum-order-quantity contract, and (ii) the combination of advance order discounts and inventory management delegation contract, on supply chain coordination.

2.3 The Model

Consider a two-level supply chain comprising of a supplier and a manufacturer. The manufacturer sells its product to retailers at the wholesale price p. While p is set beforehand, the underlying product demand D, from all the retailers over a single selling season is uncertain. We assume that D follows a probability distribution $F(\cdot)$ with density function $f(\cdot)$ that satisfies the IGFR property (i.e., the function $\frac{xf(x)}{1-F(x)}$ is increasing in x)[7].

To avoid obsolescence, the manufacturer places rush orders with the supplier only after receiving firm orders from the retailers. On the other hand, without a quantity commitment from the manufacturer, the supplier is reluctant to produce in advance, especially when the product is specifically customized for the retailers. Hence, the supplier has to expedite its production process in order to deliver the (rush) order on time. As a result of the expedited production process, the supplier incurs an inflated unit production cost, which we denote by e. Let r denote the regular contract price that the supplier quotes to the manufacturer. We assume $p \geqslant r > e$. Therefore, for any exogenous regular price r established in advance, the ex-ante expected profits for the manufacturer and the supplier for a rush order, which we denote by Π_m^o and Π_s^o respectively, are given by[8]:

$$\Pi_m^o = (p - r)E(D), \text{ and} \tag{2.1}$$

[7]As noted in Cachon (2004) and Lariviere (2006), IGFR distributions are fairly general because they include common distributions like the Uniform, the Normal, the Exponential, the Gamma, and the Weibull distributions. Furthermore, the IGFR distributions ensure that the supplier's profit function (in a newsvendor setting) is unimodal.

[8]We use the sub/superscript o to denote the base case.

$$\Pi_s^o = (r - e)E(D). \tag{2.2}$$

Also, the ex-ante expected total supply chain profit is $\Pi^o = (p - e)E(D)$[9].

2.3.1 The Advance-Order Discount

Consider the case when the food retailer has specified the recipe, selected the manufacturer, and approved the packaging design in period 0. The price r remains the same for the rush-order when the manufacturer delays its order until a firm order is received from the retailer at the beginning of the second period. However, the supplier realizes that it can lower its unit production cost from e to c if it can begin the production in period 1 and deliver the order in period 2. The supplier has to decide if it has to offer a discounted price of δr (where δ is a decision variable in $(0, 1)$) in order to encourage the manufacturer to place an advance-order in the first period that will eventually be delivered at the beginning of the selling season[10]. That is, both the advance-order (placed at the beginning of period 1) and the rush-order (placed at the beginning of period 2) will be delivered before the end of the second period.

Figure 1 depicts the setting of the advance-order discount contract, which includes the rush-order case (i.e., without discount when $\delta = 1$) as a special case. For exposition, we shall assume that $\delta \in (\frac{c}{r}, 1)$ so that the supplier will not offer the advance-ordering discount at a loss (i.e., $\delta r \geqslant c$)[11].

[9] Clearly, if the supplier aims to maximize its profit subject to the manufacturer's participation constraint, the supplier's problem in the wholesale contract can be formulated as: $\max_{r \geqslant e} \Pi_s^o(r)$, subject to $\Pi_m^o(r) \geqslant 0$. (For the ease of exposition, we scale the value of the manufacturer's outside option to zero.) The supplier can extract the entire surplus from the manufacturer by setting $r = p$ under this setting. Rather than setting $r = p$, we assume r, the pre-established contract price, is an exogenous variable, and focus on the issue of advance-order discount and other such factors (i.e., minimum order quantities and delegations).

[10] Note that the discount is actually $(1 - \delta)$, but congruent with the established literature, we shall refer to δ simply as *the discount*.

[11] We later show that it is not optimal for the supplier to set $\delta \leqslant \frac{c}{r}$.

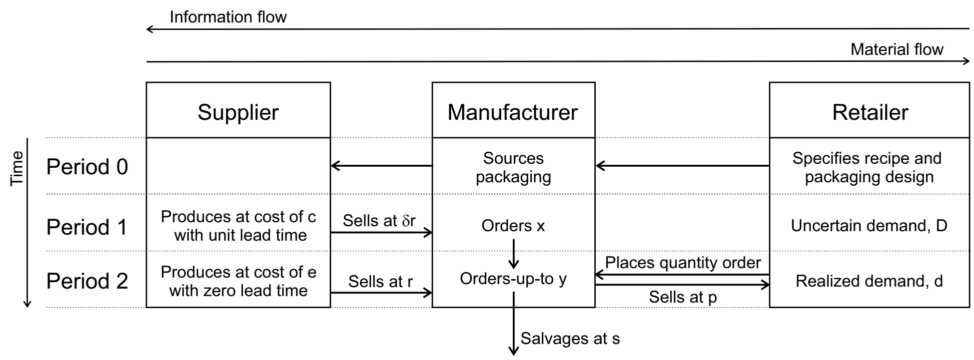

Figure 2.1: Schematic of our advance-order setting.

Under the advance-order discount contract, the supplier offers an advance-order price of δr per unit and the manufacturer must place an advance-order of x (> 0) in the first period before the demand D is realized, in order to avail the discounted price. Later on, once the demand D is realized, the manufacturer *orders-up-to* quantity y ($\geqslant x$) in the second period. Thus, the *effective* order quantity in the second period (i.e., the top-up order quantity) is $[y - x]^+$. If $y > d$, the manufacturer salvages the over-stocked $[y - d]^+$ units for a unit salvage price s. We assume that $s \geqslant 0$ and that s is net of any costs involved in salvaging the overstocked units.

We model the strategic interaction between the supplier and the manufacturer as a (two-period) Stackelberg game in which the supplier acts as the leader who sets the advance-order discount δ and the manufacturer acts as the follower who chooses the order quantities x and y[12]. We use backward induction to determine the optimal advance-order quantity x and the optimal order-up-to quantity y for a fixed discount δ. Then, we examine the optimal discount contract under different settings.

2.3.2 First-Best Solution: The Centralized Case

Before we analyze the decentralized supply chain for the case when the supplier offers an advance-order discount contract δ, we analyze the centralized system in which the supplier and the man-

[12]In many instances, packaging suppliers in the food industry are often large multi-national companies, whereas contract food manufacturers are typically smaller national companies that often focus on specialized niche items. Thus, it is reasonable to assume that the supplier is the Stackelberg leader in our game.

ufacturer operate under a central planner, to establish a benchmark. Given the advance-order x placed in period 1 and a realization d of the demand, the central planner determines its order-up-to level y by solving the following problem at the beginning of the second period:[13] $\Pi_2^c(x,d) = \max_{y\geqslant x}\{p\cdot\min\{d,y\}-e[y-x]^+ + s[y-d]^+\}$, where the addends in the objective function denote the sales revenue, the production cost in period 2, and the salvage value, in that order. It is easy to see that the optimal order-up-to level is $y^* = \max\{x,d\}$. Hence, $\Pi_2^c(x,d)$ can be simplified to $\Pi_2^c(x,d) = pd - e[d-x]^+ + s[x-d]^+$. Thus, the expected profit of the central planner in the first period can be written as: $\Pi_1^c = \max_{x\geqslant 0}\{-cx + E(\Pi_2^c(x,D))\} = \max_{x\geqslant 0}\{(p-c)E(D)-(e-c)E[D-x]^+ - (c-s)E[x-D]^+\}$. Observe that Π_1^c resembles the expected profit function of a newsvendor problem with $(e-c)$ as the unit shortage cost and $(c-s)$ as the unit over-ordering cost. From the first-order condition, the optimal initial order quantity x^* is given by:

$$x_c^* = F^{-1}\left(\frac{e-c}{e-s}\right), \tag{2.3}$$

where $F^{-1}(\cdot)$ is the inverse of the probability distribution of the demand D. By substituting x_c^* into Π_1^c, the *first-best* supply chain profit can be obtained as:

$$\begin{aligned}\Pi^c &= (p-c)E(D) - (e-c)E[D-x_c^*]^+ - (c-s)E[x_c^*-D]^+ \\ &= -cx_c^* + pE(D) - eE[D-x_c^*]^+ + sE[x_c^*-D]^+.\end{aligned} \tag{2.4}$$

Observe that it is always feasible for the central planner to set $x=0$ and $y=d$ so that the supply chain profit is equal to $\Pi^o \equiv (p-e)E(D)$, which is the profit in the base case. Hence, when the central planner optimizes its profit jointly over x and y, the optimal supply chain profit is at least as much as that in the base case, that is, $\Pi^c \geqslant \Pi^o$.

[13]We use the sub/superscript c to denote the centralized case.

2.4 Optimal Advance Order Discount Contract: The Decentralized Case

We now examine the advance-order discount contract in a decentralized system. Consider a decentralized system in which the supplier determines the discount (δ), and the manufacturer chooses its advance-order quantity (x) and its order-up-to quantity (y). For a given advance-order x and a realized demand d, the manufacturer needs to determine the *order-up-to* quantity y during the second period by solving the following problem:[14]

$$\Pi_2^d(x,d) = \max_{y \geqslant x}\{p\min\{d,y\} - r[y-x]^+ + s[y-d]^+\},$$

where r is the regular unit procurement cost and s is the unit salvage value (refer to Figure 1). It is easy to verify that the optimal order-up-to quantity for any given x is $y^* = \max\{x,d\}$. Hence, $\Pi_2^d(x,d)$ reduces to:

$$\Pi_2^d(x,d) = pd - r[d-x]^+ + s[x-d]^+.$$

Using the above optimal profit $\Pi_2^d(x,d)$ in the second period, the manufacturer needs to determine its optimal first period order quantity x, ordered at the discounted unit price δr, by solving the following problem:

$$\begin{aligned}\Pi_1^d(\delta) &= \max_{x \geqslant 0}\{-\delta r x + E\left[\Pi_2^d(x,D)\right]\} \\ &= \max_{x \geqslant 0}\{(p-\delta r)E(D) - r(1-\delta)E[D-x]^+ - (\delta r - s)E[x-D]^+\}. \qquad (2.5)\end{aligned}$$

[14]Here we use the sub/superscript d to denote the decentralized case.

By using the first order condition, the optimal initial order quantity is obtained as:

$$x_d^* = F^{-1}\left(\frac{(1-\delta)r}{r-s}\right). \tag{2.6}$$

On substituting x_d^* into the objective function, the manufacturer's profit associated with a given discount δ is obtained to be:

$$\Pi_1^d(\delta) = (p-\delta r)E(D) - r(1-\delta)E[D-x_d^*]^+ - (\delta r - s)E[x_d^* - D]^+. \tag{2.7}$$

Similarly, by noting that $y^* = \max\{x_d^*, d\}$, we obtain the supplier's expected profit, for any given δ, as:

$$\Pi_s^d(\delta) = (\delta r - c)x_d^* + (r-e)E[y^* - x_d^*]^+ = (\delta r - c)x_d^* + (r-e)E[D - x_d^*]^+, \tag{2.8}$$

where $(\delta r - c)$ and $(r-e)$ represent the supplier's profit margins in periods 1 and 2, respectively.

Over-production by the supplier in the first period

It is plausible that, by taking advantage of the lower production cost c in the first period, the supplier may be willing to risk over-producing in the first period (i.e., produce z units in the first period, where z is larger than the advance-order quantity x_d^* placed by the manufacturer during the first period)[15]. When the supplier over-produces, the supplier's profit given in (2.8) can be modified as:

$$\Pi_s^d(z,\delta) = \delta r x_d^* - cz + rE[y^* - x_d^*]^+ - eE\left[\left[[D-x_d^*]^+ - (z - x_d^*)\right]^+\right], \tag{2.9}$$

[15] We thank one anonymous reviewer for suggesting us to examine whether the combination of over-production and advance-order discount can coordinate a decentralized supply chain.

where the supplier makes two decisions: (a) $z(\geqslant x_d^*)$, the production quantity of the supplier during the first period, and (b) δ, the discount offered for the advance-orders.

However, in many practical instances, such an over-production strategy is seldom employed by a supplier for the following reason. First, suppliers have finite production and inventory holding capacities and they transact with multiple manufacturers (in addition to the one we capture in our model). As such, suppliers would prefer to use the capacity to produce for other manufacturers with firm orders rather than taking the risk to over-produce. Second, besides the underlying risk of over-production, that is, the supplier ends up with certain *unwanted units* with virtually zero salvage value (due to they being customized products), the opportunity cost incurred by the supplier who uses its capacity to over-produce (i.e., produce more than the advance-order placed by a manufacturer) is considerably high compared to the benefits that it gains from such an overproduction strategy. Nevertheless, for the sake of completion, we analyze the case when the supplier over-produces during the first period in Appendix B. We show that, even with over-production by the supplier in the first period, an advance-order discount contract cannot coordinate the supply chain.

In the remaining portion of this paper, for the ease of exposition and for tractability we shall focus on the scenario when the supplier produces the exact quantity ordered by the manufacturer during the first period.

2.4.1 Optimal Advance-Order Discount Contract

By considering the profit functions given in (2.7) and (2.8), along with the manufacturer's participation constraint, the supplier's problem can be formulated as:

$$\begin{aligned} \max_{\delta \in [0,1]} \Pi_s^d(\delta) &= \max_{\delta \in [0,1]} (\delta r - c) x_d^* + (r-e) E[D - x_d^*]^+ \\ \text{subject to } \ \Pi_1^d(\delta) &\geqslant \Pi_m^o \equiv (p-r)E(D), \end{aligned} \tag{2.10}$$

where x_d^* is given in (2.6). The following proposition characterizes the supplier's optimal discount $\hat{\delta}$ that solves the supplier's problem given in (2.10).

Proposition 5. *Let $\delta^* \equiv 1 - \left(\frac{r-s}{r}\right)\left(\frac{e-c}{e-s}\right)$ (note that $\delta^* > \frac{c}{r}$). In a decentralized system, the supplier's optimal discount $\hat{\delta}$ possesses the following properties:*

1. *The optimal discount $\hat{\delta} \in (\delta^*, 1)$.*

2. *The supplier's profit function $\Pi_s^d(\delta)$ is unimodal in δ in the interval $[\delta^*, 1]$ so that the optimal discount $\hat{\delta}$ is the unique solution of the first order condition $\frac{d\Pi_s^d}{d\delta} = 0$. Furthermore, for the case when $D \sim N(\mu, \sigma^2)$, the supplier's optimal discount $\hat{\delta}$ is decreasing in σ.*

3. *The optimal discount contract $\hat{\delta}$ is Pareto-improving. That is, both the supplier and the manufacturer can obtain a higher profit than the base case (i.e., $\Pi_s^d(\hat{\delta}) \geqslant \Pi_s^o \equiv (r-e)E(D)$ and $\Pi_1^d(\hat{\delta}) \geqslant \Pi_m^o \equiv (p-r)E(D)$).*

Proposition 5 has the following implications[16]. The first statement shows that $\hat{\delta} < 1$ so that it is beneficial for the supplier to offer a strictly positive advance-order discount. Also, observe that $\hat{\delta} r > c$, which indicates that it is not required for the supplier to offer such a deep discount that it incurs loss in the first period. The second statement of the proposition implies that when

[16] The proof of Proposition 5 is given in the Appendix. Note that Proposition 5 requires access to Lemma 12, which is also given in Appendix A.

demand becomes more uncertain, it is optimal for the supplier to offer a larger discount. The third statement of Proposition 5 resembles a more general result stated in Theorem 7 of Özer et al. (2007). It illustrates that the optimal discount contract $\hat{\delta}$ is Pareto-improving; that is, both the supplier and the manufacturer can obtain higher profits relative to the base case associated with rush orders.

However, it remains to determine if the advance-order discount contract can coordinate the supply chain. To address this issue, observe from (2.7) and (2.8) that the *decentralized* supply chain profit can be written as:

$$\Pi^d(\delta) = \Pi_s^d(\delta) + \Pi_1^d(\delta) = -cx_d^* + pE(D) - eE[D - x_d^*]^+ + sE[x_d^* - D]^+, \tag{2.11}$$

where x_d^* is given in (2.6). By comparing (2.11) and (2.4), and by setting $x_d^* = x_c^*$, it is easy to check that a discount contract that has $\delta = \delta^* \equiv 1 - \left(\frac{r-s}{r}\right)\left(\frac{e-c}{e-s}\right)$ can coordinate a decentralized supply chain. However, from the first statement of Proposition 5 we can conclude that the supplier will never set $\hat{\delta} = \delta^*$. In the following proposition we claim that the supplier optimal discount contract can never coordinate the supply chain.

Proposition 6. *In a decentralized system, the optimal advance-order discount contract $\hat{\delta}$ can never coordinate the supply chain. Specifically, the supplier's optimal discount factor $\hat{\delta} > \delta^*$.*

Though Proposition 6 shows that the optimal discount contract $\hat{\delta}$ alone can never coordinate the supply chain, the coordination will be possible if the supplier makes a transfer payment of $S = \Pi_1^d(\delta^*) - \Pi_m^o$ to the manufacturer. When such a payment is made, the manufacturer is no worse off than the base case and the supplier achieves the highest possible profit because,

$$S + \Pi_m^o + \Pi_s^d(\delta^*) = \Pi_1^d(\delta^*) + \Pi_s^d(\delta^*) \ = \Pi^c.$$

Even though the supplier can combine the optimal discount contract $\hat{\delta}$ with a transfer payment to coordinate the supply chain, such mechanism requires a change to the pre-existing pricing structure. This gives rise to the following question: *Without changing the existing price structure by introducing a transfer payment as discussed above, can the supplier leverage the advance-order discount contract to coordinate the supply chain* <u>*and*</u> *extract the entire surplus from the manufacturer?* If such a mechanism exists, it is the *optimal* contract among all possible contracts because it enables the supplier to attain the highest possible profit in a decentralized system. We shall examine such a contract in the next section.

2.5 Advance-Order Discount Contract with a Minimum Order Quantity

Consider the scenario in which the supplier imposes a minimum advance-order quantity q as a *qualifier* for the manufacturer to receive a discount δ. That is, in order to benefit from a discounted advance-order, the manufacturer has to order at least q units in the advance-order. We shall refer to such a contract as a *combined contract* because it combines an advance-order discount with a minimum advance-order quantity. For the combined contract (δ, q) that the supplier quotes, it is easy to check that the manufacturer's optimal order-up-to quantity remains the same, $y^* = \max\{x, d\}$, as described in Section 2.3.2. It also follows from (2.5), and the fact that the manufacturer will receive the discount δ only when its advance-order quantity x is as much as q, the manufacturer's problem in the first period can be formulated as:[17]

$$\begin{aligned}\Pi_1^q(\delta, q) &= \max_{x \geqslant q}\{(p - \delta r)E(D) - r(1-\delta)E[D-x]^+ - (\delta r - s)E[x-D]^+\} \\ &= \max_{x \geqslant q}\{pE(D) - rE[D-x]^+ + sE[x-D]^+ - \delta r x\}.\end{aligned} \tag{2.12}$$

[17]We use the sub/superscript q to denote the advance-order discount with a minimum order quantity contract.

By considering the first order condition along with the constraint $x \geqslant q$, and noting that the objective function is strictly unimodal in x, it is easy to show that the optimal initial order quantity is:

$$x_q^*(\delta, q) = \max\left\{F^{-1}\left(\frac{(1-\delta)r}{r-s}\right),\ q\right\}. \tag{2.13}$$

By incorporating the manufacturer's best response function $x_q^*(\delta, q)$ in (2.8) we can write the supplier's profit function as:

$$\Pi_s^q(\delta, q) = (\delta r - c)x_q^*(\delta, q) + (r-e)E[D - x_q^*(\delta, q)]^+, \tag{2.14}$$

and formulate the supplier's optimization problem as follows:

$$\begin{aligned} &\max_{q \geqslant 0} \max_{\delta \in [0,1]} \{(\delta r - c)x_q^*(\delta, q) + (r-e)E[D - x_q^*(\delta, q)]^+\} \\ &\text{subject to } x_q^*(\delta, q) = \max\{F^{-1}\left(\frac{(1-\delta)r}{r-s}\right),\ q\}, \quad \text{and} \\ &pE(D) - rE[D - x_q^*(\delta, q)]^+ + sE[x_q^*(\delta, q) - D]^+ - \delta r x_q^*(\delta, q) \geqslant (p-r)E(D). \end{aligned} \tag{2.15}$$

By analyzing the supplier's problem (2.15) for the case when $x_q^*(\delta, q) = F^{-1}\left(\frac{(1-\delta)r}{r-s}\right)$ and for the case when $x_q^*(\delta, q) = q$ individually, and by comparing the supplier's optimal profit associated with these two cases, we obtain Proposition 7.

Proposition 7. *The optimal combined contract* $(\tilde{\delta}, \tilde{q})$ *is given by*

$$\tilde{\delta} = \frac{r[E(D) - E[D-\tilde{q}]^+] + sE[\tilde{q} - D]^+}{r\tilde{q}}, \ \textit{and} \tag{2.16}$$

$$\tilde{q} = F^{-1}\left(\frac{e-c}{e-s}\right). \tag{2.17}$$

Also, the optimal combined contract $(\tilde{\delta}, \tilde{q})$ *has the following properties:*

1. Relative to the coordinated discount contract δ^*, it offers a smaller discount (i.e., $1 > \tilde{\delta} > \delta^* > \frac{c}{r}$).

2. It induces the manufacturer to set its initial order quantity as in the centralized case (i.e., $x_q^*(\delta, q) = \tilde{q} = x_c^*$).

3. It enables the supplier to extract the entire surplus from the manufacturer (i.e., $\Pi_1^q(\tilde{\delta}, \tilde{q}) = \Pi_m^o$).

4. It coordinates the supply chain (i.e., $\Pi_s^q(\tilde{\delta}, \tilde{q}) + \Pi_1^q(\tilde{\delta}, \tilde{q}) = \Pi^c$).

We draw the following insights from Proposition 7. The first two statements of the proposition quantify the optimal combined contract $(\tilde{\delta}, \tilde{q})$. The third and the fourth statements imply that the optimal combined contract $(\tilde{\delta}, \tilde{q})$ can both coordinate the supply chain and enable the supplier to extract the entire surplus from the manufacturer. Hence, the supplier achieves its highest possible profit under the optimal combined contract. Under the optimal contract the manufacturer is made to order $\tilde{q} = x_c^*$ in the advance order. Further, it enables the supplier to gain a higher profit as $(\tilde{\delta}r - c) \geqslant (\delta^* r - c)$. In summary, Proposition 7 demonstrates the superior performance of the combined contract that involves minimum order quantities. The proposition offers a plausible explanation to why the minimum-order-quantity contracts are widely observed in practice; the contract enables the supplier to attain the first-best solution (i.e., the highest profit) by coordinating the supply chain.

Proposition 7 shows that the optimal combined contract $(\tilde{\delta}, \tilde{q})$ can coordinate the decentralized supply chain when the demand follows a general IGFR probability distribution. To examine the impact of demand uncertainty on the optimal combined contract $(\tilde{\delta}, \tilde{q})$ further, we consider the case when the demand is normally distributed, which is a member of IGFR distributions. By using the properties of the standard normal distribution, we establish the following corollary:

Corollary 1. *When $D \sim N(\mu, \sigma^2)$, the optimal combined contract $(\tilde{\delta}, \tilde{q})$ given in (2.16) and (2.17) can be simplified to:*

$$\begin{aligned} \tilde{\delta} &= 1 - \frac{(r-s)[\phi(k) + k\Phi(k)]\sigma}{r(\mu + k\sigma)} \text{ and} && (2.18) \\ \tilde{q} &= \mu + k\sigma, \text{ where} && (2.19) \\ k &= \Phi^{-1}\left(\frac{e-c}{e-s}\right). \end{aligned}$$

Both the optimal discount $\tilde{\delta}$ and the optimal minimum order quantity $\tilde{q}$ are increasing in μ. Furthermore,

1. *If $e > 2c - s$ (i.e., when $k > 0$), then the optimal minimum order quantity $\tilde{q}$ is linearly increasing in σ and the optimal discount $\tilde{\delta}$ is decreasing and convex in σ.*

2. *If $e = 2c - s$ (i.e., when $k = 0$), then the optimal minimum order quantity $\tilde{q}$ is independent of σ and the optimal discount $\tilde{\delta}$ decreases linearly in σ.*

3. *If $e < 2c - s$ (i.e., when $k < 0$), then the optimal minimum order quantity $\tilde{q}$ is linearly decreasing in σ. Also, the optimal discount $\tilde{\delta}$ is decreasing and concave in σ if $\phi(\Phi^{-1}\left(\frac{e-c}{e-s}\right)) + \left(\frac{e-c}{e-s}\right)\Phi^{-1}\left(\frac{e-c}{e-s}\right) > 0$.*

Corollary 1 has the following implications. First, as the mean demand increases, it is always beneficial for the supplier to set a higher minimum order quantity (i.e., increase $\tilde{q}$) and to discount less (i.e., increase $\tilde{\delta}$). Second, when σ increases, it is optimal to set a higher minimum order quantity $\tilde{q}$ (see, (2.19)) and to discount more (i.e., to set $\tilde{\delta}$ smaller) if the expedited production cost is sufficiently high (i.e., $e > 2c - s$). We can interpret the other statements in the same manner and so omit the details.

2.6 Discount Contracts with Delegations

In Proposition 7 we argued that the combined contract (that combines the advance-order discount and the minimum order quantity initiated by the supplier) can enable the supplier to achieve the first-best solution by coordinating the supply chain. By noting that the aforementioned combined contract is initiated by the supplier, we want to verify if there is a similar contract, that when initiated by the manufacturer, can coordinate the supply chain. In this section of the paper, we explore and analyze such a contract.

Consider a contract in which the manufacturer can delegate its inventory decisions (i.e., order placement and salvage decisions) to the supplier, who can lower its unit production cost from e to c when the production is undertaken early. We term this contract as the *delegation contract*. In exchange to this delegation contract, the manufacturer requires that the supplier should satisfy the realized demand (by using either the advance production in period 1 or the expedited production in period 2) and offer a discounted price θr, where $\theta < 1$, on all the units. Now, it is not clear if the supplier should accept such a delegation contract offered by the manufacturer[18].

2.6.1 Supplier's Problem under the Delegation Contract

In the event that the supplier rejects the delegation contract offered by the manufacturer, the manufacturer's expected profit is $\Pi_m^o = (p - r)E(D)$ and the supplier's expected profit is $\Pi_s^o = (r - e)E(D)$, see (2.1) and (2.2). On the other hand, should the supplier accept the delegation contract, then the manufacturer is passive (because the manufacturer delegates all the ordering decisions and the salvage operations to the supplier). In such a case, the manufacturer's expected

[18] Our delegation contract is akin to the vendor managed inventory (VMI) agreement under which the manufacturer manages the replenishments on behalf the retailer (for example see, Lee et al. (1997) and Aviv and Federgruen (1998)). To execute this delegation contract, the supplier needs to observe the realized demand as in most VMI contracts (Çetinkaya and Lee, 2000; Disney and Towill, 2003).

profit becomes $\Pi^g_m(\theta) = (p - \theta r)E(D)$, with θr as the discounted purchase price[19]. Clearly, the manufacturer is better off under the delegation contract because $\theta < 1$. It remains to check if the supplier is not worse off under the delegation contract so that it may participate in the contract.

Under the delegation contract, the supplier (and not the manufacturer) has to determine its advance production quantity x in the first period, and its produce-up-to level y in the second period. Additionally, the supplier should also account for the salvage operations after the selling season. For a given advance-production quantity x in the first period and a realization d of the demand, the supplier determines its produce-up-to level y by solving the following problem:

$$\Pi^g_{s,2}(x, d; \theta) = \max_{y \geqslant x}\{\theta r d - e[y - x]^+ + s[y - d]^+\},$$

where the addends in the right hand side denote the revenue from the manufacturer based on the realized demand d, the expedited production cost in the second period, and the salvage income, in that order. It is easy to see that the optimal produce-up-to quantity is $y^* = \max\{x, d\}$, and so $\Pi^g_{s,2}(x, d; \theta)$ can be simplified as:

$$\Pi^g_{s,2}(x, d; \theta) = \theta r d - e[d - x]^+ + s[x - d]^+. \tag{2.20}$$

Using a dynamic program the supplier's problem in the first period can be formulated:

$$\begin{aligned} \Pi^g_{s,1}(\theta) &= \max_{x \geqslant 0}\{-cx + E\left[\Pi^g_{s,2}(x, D; \theta)\right]\} \\ &= \max_{x \geqslant 0}\{(\theta r - c)E(D) - (e - c)E[D - x]^+ - (c - s)E[x - D]^+\}. \end{aligned} \tag{2.21}$$

From the first-order condition, the optimal advance production quantity x^* that is to be produced

[19]We use the sub/superscript g to denote the delegation contract.

in the first period can be shown to be:

$$x_g^* = F^{-1}\left(\frac{e-c}{e-s}\right). \tag{2.22}$$

Observe that the advance production quantity x_g^* given in (2.22) is identical to the optimal initial order quantity x_c^* given in (2.3) under the centralized case. By substituting x_g^* in (2.21), we can write the supplier's optimal profit under the delegation contract as:

$$\Pi_{s,1}^g(\theta) = (\theta r - c)E(D) - (e-c)E[D - x_g^*]^+ - (c-s)E[x_g^* - D]^+. \tag{2.23}$$

By using the fact that $\Pi_m^g(\theta) = (p - \theta r)E(D)$, we can show that the total supply chain profit under the delegation contract is:

$$\Pi_{s,1}^g(\theta) + \Pi_m^g(\theta) = (p-c)E(D) - (e-c)E[D - x_g^*]^+ - (c-s)E[x_g^* - D]^+.$$

Then, because $x_g^* = x_c^*$, we have $\Pi_{s,1}^g(\theta) + \Pi_m^g(\theta) = \Pi^c$, where Π^c is the optimal centrally controlled supply chain profit that is given in (2.4). Therefore, we conclude that the delegation contract coordinates the supply chain.

2.6.2 Discount Factor

In this section, we examine the existence of a discount factor θ that can facilitate a discount contract with delegation to coordinate the supply chain. The crux of the discount factor selection hinges on the following two factors. First, observe from (2.22) that x_g^* is independent of the discount factor θ. This indicates that the supplier's profit $\Pi_{s,1}^g(\theta)$ given in (2.23) is linearly increasing in θ and the supplier will not be worse off as long as $\Pi_{s,1}^g(\theta) \geqslant (r-e)E(D)$. Second, observe that the manufacturer's profit $\Pi_m^g(\theta) = (p - \theta r)E(D)$ is linearly decreasing in θ. Hence, the manufacturer will be better off if $\theta < 1$.

The above two factors imply that the existence of such a delegation contract (in combination with advance order discount) hinges on the existence of a nonempty region that satisfies these two conditions: (a) $\theta < 1$ (i.e., manufacturer's participation constraint), and (b) $\theta \geqslant \underline{\theta}$ (i.e., supplier's participation constraint), where $\underline{\theta}$ is derived from the condition $\Pi_{s,1}^g(\theta) \geqslant (r - e)E(D)$ as follows:

$$(\theta r - c)E(D) - (e - c)E[D - x_g^*]^+ - (c - s)E[x_g^* - D]^+ \geqslant (r - e)E(D) \tag{2.24}$$
$$\Rightarrow \theta \geqslant \frac{1}{rE(D)}\left[(r - e + c)E(D) + (e - c)E[D - x_g^*]^+ + (c - s)E[x_g^* - D]^+\right] \equiv \underline{\theta}.$$

Hence, any delegation contract θ that has $\theta \in [\underline{\theta}, 1)$ will ensure the supplier is not worse off and make the manufacturer strictly better off under the discount contract with delegation. Therefore, a delegation contract that coordinates the decentralized supply chain and is Pareto-improving is possible if, and only if, $\underline{\theta} < 1$. Proposition 8 provides the necessary and sufficient for such a delegation contract to exist when the demand is normally distributed.

Proposition 8. *Let $D \sim N(\mu, \sigma^2)$. Then $\underline{\theta} < 1$ if and only if $\frac{\sigma}{\mu} < \frac{\Phi(k)}{\phi(k)}$, where $k = \Phi^{-1}\left(\frac{e-c}{e-s}\right)$.*

Proposition 8 provides the necessary and sufficient condition for a Pareto-improving delegation contract that can coordinate the supply chain to exist. This condition depends on the magnitude of the demand uncertainty (measured in terms of the coefficient of variation, $\frac{\sigma}{\mu}$). A delegation contract exists only when the demand uncertainty is below a certain threshold that depends on the underlying cost parameters e, c, and s. Thus, we demonstrate that when the demand uncertainty is sufficiently low, the discount contract with delegation is favorable to both the supplier and the manufacturer, who delegates the ordering and salvage decisions to the supplier and benefits from a lower price θr.

Proposition 8 has other implications as well. Specifically, when the stated condition holds, such a Pareto-improving delegation contract is not unique; any delegation contract with $\theta \in [\underline{\theta}, 1)$ can ensure the supplier is not worse off (i.e., $\Pi^g_{s,1}(\theta) \geqslant (r-e)\mu$) and the manufacturer is strictly better off. In the light of this observation and our discussion hitherto, we conclude the following. First, if the delegation contract is proposed by the supplier, then the supplier will propose the highest discount factor θ that is as close to 1 as possible so that the manufacturer is strictly better off and the supplier is substantially better off (because $\Pi^g_{s,1}(\theta)$ given in (2.23) is linearly increasing in θ). Second, if the delegation contract is proposed by the manufacturer, then the manufacturer will propose the lowest discount factor $\theta = \underline{\theta}$ so that the supplier is not worse off and yet the manufacturer is considerably better off (because the manufacturer's profit $\Pi^g_m(\theta) = (p - \theta r)E(D)$ is linearly decreasing in θ). In practice, the implementation of this form of delegation contract involves negotiations between the supplier and the manufacturer, and the actual value of $\theta \in [\underline{\theta}, 1)$ that is agreed upon would depend on many factors including the bargaining power of each party.

2.7 Optimal Advance-Order Discount Contract with Two Manufacturers

In this section we extend our model to analyze the advance-order discount contract when there is more than one manufacturer. We consider a model with two manufacturers who simultaneously source from a single supplier. It is well known from the past literature that manufacturers compete for supplier's capacity by strategically inflating their orders to game the rationing policy that the supplier adopts when capacity is finite (see Lee et al. (1997), Cachon and Lariviere (1999), Cho and Tang (2014), and the references therein). Even in a single-period setting, the analysis for finding an efficient allocation rule $A_i(x_1, x_2)$, $i = \{1, 2\}$, that allocates the supplier's capacity between the manufacturers, for a given order quantities x_i, $i = \{1, 2\}$, they place, is complicated. For the optimal advance-order discount contract in a two-period problem, the supplier's decisions include the discounts δ_i and the capacity allocations $A_i^t(x_1, x_2)$, for $i = \{1, 2\}$ and $t = \{1, 2\}$. On the other hand, the decisions of manufacturer i include the advance-order quantity x_i and the second-period order-up-to quantity y_i, $i = \{1, 2\}$. Thus, the two-stage game with three players consists of 10 decisions (6 for supplier and 2 for each manufacturer), rendering it intractable. Moreover, because our focus in this paper is on the advance-order discount contracts, minimum-order quantity, and delegation, and on designing an efficient combined contract to coordinate the supply chain in a Pareto-improving manner, we defer the inclusion of capacity constraints and the analysis of the potential allocation rules to future research.

Though we do not analyze the generic case of the two-period game with finite supplier capacity in each of the periods, we examine a specific setting in which the supplier has a finite capacity K in the first period, but unlimited capacity in the second period. Also, the manufacturer pays a discounted price for all the units that are ordered in advance. Note that we do not restrict the sum of the advance orders from the two manufacturers to be less than K. We show that:

- The advance-order discount contract cannot coordinate the supply chain.
- When advance-order discount δ_i is combined with minimum order quantity $\tilde{q}_i$ for each manufacturer $i \in \{1, 2\}$, the combined contract can coordinate the supply chain.
- The coordinating delegation contract exists if and only if the coefficients of variation of the demands of the manufacturers are small (specifically, we require $\frac{\sigma_1+\sigma_2}{\mu_1+\mu_2}$ to be below a threshold, where σ_i and μ_i are the standard deviation and mean of the demands of manufacturer $i \in \{1, 2\}$, respectively).

For a detailed analysis of the two-manufacturer case, we refer the reader to Chintapalli et al. (2017).

2.8 Conclusions and Future Directions

In this paper, we have examined three supply chain contracts that are applicable in the context when the supplier can afford to offer advance-order discounts to its manufacturer, who places its order before the uncertain demand is realized. We showed that the optimal advance-order discount contract is Pareto-improving, but it can never coordinate the supply chain because of some loss in efficiency from decentralization.

This finding led us to examine whether the supplier can leverage the advance-order discount contract to design a mechanism that can coordinate the supply chain. We found that if the supplier offers a *combined contract* that is based on the advance-order discount and a minimum advance-order quantity, then such a contract can coordinate the supply chain. More importantly, the supplier can achieve the first-best solution by extracting the entire surplus from the manufacturer.

Finally, we considered another contract that could be proposed by the manufacturer or the supplier where, in exchange to a discount θ on all the items procured from the supplier, the manufacturer delegates its ordering decisions and the salvaging activities to the supplier. We found that, under some mild conditions on the demand distribution, the delegation contract can coordinate the supply chain and that the total profit could be arbitrarily (within a range) apportioned between the manufacturer and the supplier. We showed that the combined advance-order discount and minimum order quantity contract that coordinates a supply chain always exists and we derived a necessary and sufficient condition for a combined advance-order discount and delegation contract to exist. We found that our results continue to hold in the case when there are two manufacturers, and when the supplier has limited capacity K in the first period and unlimited capacity in the second period.

The model presented in this paper has several limitations that can serve as potential directions of future research. First, in our model we assume that the demand is fully realized in the second period (i.e., the manufacturer receives firm orders from its retailers). However, there may be practical cases when the demand uncertainty is not fully resolved. Even though the same solution procedure applies, it is of interest to extend our analysis to the case of demand updating over multiple time periods and explore the nature of contracts over the sale horizon. Second, our model did not capture supplier's capacity constraints in both periods. When the supplier has limited capacity in both periods, the strategic interaction between the supplier and the manufacturer becomes quite intricate. Specifically, it is of interest to explore the impact of supplier's capacity on the supplier's decisions regarding the advance-order discount factor, the minimum order quantity, capacity rationing policy and the like. For instance, when the supplier's capacity is limited, manufacturers may anticipate capacity rationing and hence inflate their orders (see Cho and Tang (2014) and the references therein) making the analysis substantially more complex. We shall defer these issues to future research.

Chapter 3 The Impact of Crop Minimum Support Prices on Crop Selection and Farmer Welfare in the presence of Strategic Farmers and Complementary Production Costs

Abstract

In many developing countries, governments often use *Minimum support prices* (MSPs) as interventions to (i) safeguard farmers' income against crop price falls, and (ii) ensure sufficient and balanced production of different crops. In this paper, we examine two questions: (1) What is the impact of MSPs on the farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits? (2) What is the impact of strategic farmers on crop selection and production decisions, future crop availabilities, and farmers' expected profits? To explore these questions, we present a model in which the market consists of two types of farmers (with heterogeneous production costs): myopic farmers (who make their crop selection and production decisions based on recent market prices) and strategic farmers (who make their decisions by taking all other farmers' decisions into consideration). By examining the dynamic interactions among these farmers for the case when there are two (complementary or substitutable) crops for each farmer to select to grow, we obtain the following results. First, we show that, regardless of the values of the MSPs offered to the crops, the price disparity between the crops worsens as the complementarity between the crops increases. Second, we find that MSP is not always beneficial. In fact, offering MSP for a crop can hurt the profit of those farmers who grow that crop especially when the proportion of strategic farmers is sufficiently small. Third, a bad choice of MSPs can cause the expected quantity disparity between crops to worsen. By taking these two drawbacks of MSPs into consideration, we discuss ways to select effective MSPs that can improve farmers' expected profit and reduce quantity disparity between crops.

Keywords

Minimum support prices, subsidies, agricultural supply chains, government and public policy

3.1 Introduction

In many developing countries, the agricultural sector is important because: (1) it offers a source of income to a large number of small rural households, and (2) it provides a stable food supply for the country. As such, developing efficient and effective agro-policies to improve farmers' earnings and to stabilize crop availabilities and prices are critical (Thorbecke, 1982). While governments in developing countries design and develop a wide variety of agro-policies ranging from input subsidies (for seeds and fertilizers, power, etc.) to output subsidies (for storage and transportation), in this paper, we shall focus on a particular type of output subsidies that is called the *Minimum Support Price* (MSPs). MSPs for different crops are offered by governments in many developing countries like Bangladesh, Brazil, China, India, Pakistan, and Thailand. For example, in 2017, the Indian government offers MSPs for 23 crops, which comprise 7 cereals, 5 pulses (grain seeds of legumes), 7 oil seeds, and 4 commercial crops. Essentially, MSP of a crop serves as a form of "contingent subsidy" to farmers who grow that crop: when the market price of a crop falls below its MSP, government purchases the crop from the farmers at the pre-announced MSP of the crop by absorbing the price shortfall (i.e., the difference between the market price and the MSP). By guaranteeing minimum prices for certain crops, a government intends to provide incentives for farmers to grow a more balanced mixture of crops.

This paper examines the implications of MSPs on: (1) farmers' earnings, and (2) quantity disparity between two crops. Our model is based on the setting of a developing country. To motivate our research questions involving MSPs, let us consider the role played by MSPs in the Indian agricultural sector. MSPs have been introduced as a part of the Green Revolution in 1965 when India's cereal imports reached an alarming stage. This event has triggered the Indian government to establish the Commission for Agricultural Costs & Prices (CACP) with the mandate to develop crop-price policies. As a part of these reforms, MSPs were introduced as incentives to benefit Indian farmers and consumers by increasing food supply at affordable prices (Chand, 2003; Malamasuri et al., 2013).[1] With an efficient MSP scheme developed by CACP over the years, India evolved from a grain "deficient" country in mid-1960's to a grain "surplus" country by early 1980's. However, due to the fact that MSPs in India were geared towards rice and wheat production, there was a severe shortage of coarse cereals and oil seeds (Chand, 2003; Parikh and Chandrashekhar, 2007) and an over-production of rice and wheat. Such an imbalance in the availability of agricultural commodities can lead to micro-nutrient malnutrition (or hidden hunger) (Byerlee et al., 2007). This observation suggests that the efficacy of MSPs should be measured in terms of the availability of different crops and the farmers' expected earnings.

In this paper we develop a parsimonious model to analyze the impact of MSPs on farmers'

[1] When determining MSPs, CACP takes into account six factors, namely (i) demand and supply, (ii) cost of production, (iii) market price trends, (iv) inter-crop price parity, (v) terms of trade between agriculture and non-agriculture, and (vi) likely implications of MSP on consumers of *that* product (Commission for Agricultural Costs & Prices, 2017). In our analysis we take into account the factors (i), (ii), (iii), (iv) and (vi). We account for the supply through considering the response of the farmers' sowing decisions towards the MSPs announced. We account for the demand through the inverse demand functions of the crops. By assuming that the farmers are heterogeneous in their production costs for the two crops, we account for (ii). Based on his type – strategic or myopic – each farmer considers the past price of the crops in a specific way. Thus, we account for (iii) through farmers' perceptions of past prices. We account for (iv) through farmers' individual rationality and their choice of the crops in the light of the past prices and the MSPs of the crops. Finally, we account for (vi) by analyzing the impact of MSP, in confluence with the past market prices, on the future market prices. Thus, we make an attempt to develop a unified framework using a parsimonious model with two crops to comprehensively capture the main features of an MSP scheme.

earnings, crop availabilities, and crop prices by considering a setting in which there are two (complementary or substitutable) crops from which each farmer can choose one crop to cultivate. In addition to heterogeneous production costs for each crop, we also consider the case when the market is comprised of myopic farmers (who make their crop selection and production decisions based on recent market pries) and strategic farmers (who make their decisions by taking all other farmers' decisions into consideration). By examining the dynamic interactions among myopic and strategic farmers, our model enables us to examine two research questions:

1. What is the impact of MSPs on the farmers' crop selection and production decisions, future crop availabilities, and farmers' expected revenues?

2. What is the impact of strategic farmers on crop selection and production decisions, future crop availabilities, and farmers' expected revenues?

Our equilibrium analysis enables us to obtain the following results. First, we find in Corollaries 2 and 7 that, regardless of the values of MSPs, the price disparity between the crops worsens as the complementarity between the crops increases. Second, we show in Proposition 13 that MSP is not always beneficial. In Proposition 13, specifically, we show that moderately low MSP for a crop will degrade the expected profits of the farmers growing the crop if the number of strategic farmers is very small. Thus, choosing an inappropriate MSP for a crop, especially when there are very few strategic farmers, can actually defeat the intended goal of offering MSP for the crop, which is to benefit the farmers growing the crop. Also, we show in Proposition 12 that when the proportion of strategic farmers is small, offering moderately low MSP for a crop can actually cause fewer strategic farmers to grow that crop. Third, in Proposition 11, we find that the total production of a crop is increasing in the MSP offered for the crop. Therefore, a bad choice of MSPs can cause the production quantity disparity between crops to worsen. Hence, to reduce quantity disparity between crops, a carefully designed MSP policy is critical. Finally, through formulating an optimization problem for a policy-maker to choose crop MSPs in order to maximize social welfare, we illustrate that offering MSPs to complementary (i.e., dissimilar) crops has the potential to achieve higher social welfare at a lower expected expenditure to the policy-maker.

The paper is organized as follows. Section 2 reviews literature related to MSPs. In Section 3 we introduce the model and discuss various assumptions. To explicate our analysis about myopic and strategic farmers' crop selection and production decisions, we examine the case when MSPs are not present in Section 4. Section 5 extends our analysis to the case when MSPs are present. In Section 6 we formulate and discuss the optimization problem of the government whose objective is to set MSPs in order to improve farmers' welfare and crop balance. We conclude in Section 7.

3.2 Literature Review

Our research pertains to agro-policies that affect both crop selection and crop production by myopic and strategic farmers. The literature on MSPs is vast in the agricultural economics discipline and the reader is referred to Tripathi et al. (2013) and the references therein for a good synopsis on MSPs in developing countries. Without accounting for the price interactions between crops with MSP support and those crops without MSP support, Fox (1956) develops macro-economics analysis to evaluate the impact of MSPs and finds that MSPs can mitigate the fall in GNP during a recession. Dantwala (1967) finds that in spite of the increasing MSPs, the crop market prices continue to rise. More recently, Subbarao et al. (2011) shows evidence that the increase in market price is caused by the increase in MSPs. In the same vein, Chand (2003) presents qualitative assessment of the ill-effects of the wheat- and-rice-centric MSPs on the Indian economy. Chhatre et al. (2016) point out that many farmers in India moved to cultivating high-yield varieties of rice and wheat due to the wheat- and-rice-centric MSPs offered by the Indian government. The authors also identify the various socio-economic and environmental problems associated with an improper choice of MSPs. Besides the Indian context, Spitze (1978) analyzes the impact of federal policy (The Food and Agriculture Act of 1977) on agriculture in the United States. The author states that continuous improvement in gathering and analyzing information is a prerequisite for the design of effective MSPs.

Recent papers on agricultural operations in OM literature include: (i) Tang et al. (2015); Chen and Tang (2015); Parker et al. (2016); Liao et al. (2017) focus on the economic value of disseminating agricultural information to the farmers, (ii) Kazaz and Webster (2011); Dawande et al. (2013); Huh and Lall (2013) examine the issue of resource and inventory management, (iii) Huh et al. (2012); Federgruen et al. (2015); An et al. (2015) focus on contract farming and farmer aggregation, and (iv) Hu et al. (2016); Alizamir et al. (2015); Guda et al. (2016) examine social responsibility and public policy issues arising from the agricultural sector.

While our paper is related to group (iv), it differs from the these papers in the following manner. First, Hu et al. (2016) focus on the value of strategic farmers in the context of a single crop with a deterministic demand function. They show that a tiny fraction of strategic farmers can stabilize the steady state prices. They also extend their analysis to two crops with independent market prices. In contrast, our goal is to evaluate the impact of MSPs on farmers' crop selection and production decisions, and on the market prices of two crops with dependent and yet stochastic market price.

Second, Alizamir et al. (2015) focus on the impact of federal policy on agriculture industry in the United States. They compare two schemes (Price Loss Coverage (PLC) and the Agriculture Risk Coverage (ARC) programs) with respect to (i) farmers' welfare, (ii) federal expenditure, and (iii) consumer welfare. While PLC is akin to MSP, our paper differs from Alizamir et al. (2015) in three aspects. First, they assume there are finite number of farmers, and the production of each farmer can affect the market price (i.e., farmers are price setters). In contrast, our context is that of developing countries, and we consider infinitesimally small farmers whose individual decisions do not affect the market price (i.e., farmers are price-takers). Second, they analyze the case of only one crop, while we consider two crops that can be substitutes or complements. Hence, by capturing the interaction between two crops in our model, we analyze the simultaneous impact of the MSP of each crop on the production of both the crops. Third, they do not consider the existence of myopic and strategic farmers, while we consider a mixture of both myopic and strategic farmers in our model. Our model fits well in the context of developing countries where a large portion of the farming communities are smallholders who are myopic: their crop selection and production decisions are purely based on the most recently oberved market price.

Finally, Guda et al. (2016) examine the role of MSPs in emerging economies, but there are two fundamental differences between our paper and theirs. The first difference is that we assume heterogeneity in farmers' production costs, while they assume homogeneous production costs. In general, the cost of cultivating a crop can vary across farmers depending on the local soil, the climatic conditions, and the farming practices they employ. Second, they consider a single crop and relegate the case of multiple crops as future research due to the inherent complexity. As such, our paper attempts to examine the impact of the MSPs of two crops on the availabilities of one another.

3.3 Model Preliminaries

We consider two crops (A and B) to be produced by heterogeneous farmers whose production costs are uniformly distributed over the interval $[-0.5, 0.5]$ as in the Hotelling's model. These two crops can be substitutes (e.g., rice and wheat) or complements (e.g., rice and pulses/lentils). For a farmer located at $x \in [-0.5, 0.5]$, his costs of producing crops A and B are given by $c_A(x) = x + 0.5$ and $c_B(x) = 0.5 - x$, respectively. We assume that the farmers are infinitesimally small so that each farmer can produce 1 unit of a crop and each farmer is a price taker.

In our model, the market price of a crop depends on the available quantity of the crop. Let q_t^{kT} denote the "total" availability of crop $k \in \{A, B\}$ in period t and let p_t^k denote the market price of crop $k \in \{A, B\}$ in period t. For ease of exposition, we normalize the size of markets to 1 so that $q_t^{kT} \leqslant 1$ for $k \in \{A, B\}$. Throughout this paper, we assume that the market price p_k^t for crop $k \in \{A, B\}$ in period t satisfies:

$$p_t^A = a - \rho q_t^{AT} - \alpha q_t^{BT} + \epsilon_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A, \quad \text{and}$$
$$p_t^B = a - \alpha q_t^{AT} - \rho q_t^{BT} + \epsilon_t^B = \mathbb{E}[p_t^B] + \epsilon_t^B, \tag{3.1}$$

where ρ (> 0) is the price sensitivity, and α is a measure of substitutability (if $\alpha > 0$) or complementarity (if $\alpha < 0$) between the two crops. As commonly assumed in the literature for substitutable/complementary products, we shall assume that $\alpha < \rho$. The random variables ϵ_t^k ($k \in \{A, B\}$) denote the market uncertainty in period t. We assume ϵ_t^k are iid (across t and k) with mean 0, variance σ^2 and with distribution and density functions $F(\cdot)$ and $f(\cdot)$, respectively.[2] We also assume that the distribution $F(\cdot)$ has support over a range of value so that the market price p_t^k in non-negative. Let $\overline{F}(\cdot) = (1 - F(\cdot))$ denote the complementary cumulative distribution of ϵ_t^k. The expected profit of a farmer at location x who grows crop $k \in \{A, B\}$ is given by:

$$\Pi_t^k(x) = \mathbb{E}[p_t^k] - c_k(x) = a - \rho q_t^{kT} - \alpha q_t^{jT} - c_k(x), \; j \neq k. \tag{3.2}$$

For ease of exposition, we define $r \equiv \rho - \alpha$ (> 0), so that r measures the "dissimilarity" between the two crops, and $\phi \equiv a - \frac{\rho+\alpha}{2}$, which corresponds to the expected market price when half of the market grows A (grows B) (i.e., when $q_t^{AT} = q_t^{BT} = 0.5$). Finally, wherever applicable, we denote

[2] To keep the notation simple, we assume that ϵ_t^A and ϵ_t^B follow the same distribution. However, our analysis can be extended to the case of different distributions.

the price vector in period t by $\mathbf{P}_t = [p_t^A, p_t^B]^T$. To simplify our exposition and our analysis (e.g., by ruling out the boundary equilibrium solution), we shall make the following assumptions:

Assumption 4. *In each period, each farmer will not be idle and will select exactly one crop to grow.*

First, the non-idling assumption is reasonable especially when the farmer's production cost is lower than the market price p_t^k in general. Second, due to economies of scale, small land-holders in emerging markets cannot afford to grow multiple crops.

Next, let Δp_t be the price disparity between crops A and B in period t. By applying (3.1) and the fact that $r = \rho - \alpha$, we obtain:

$$\Delta p_t = p_t^A - p_t^B = -r(q_t^{AT} - q_t^{BT}) + \xi_t, \ \forall t,$$

where $\xi_t = \epsilon_t^A - \epsilon_t^B$. To ensure that the price disparity Δp_t is stable over time so that we can rule out boundary equilibrium solution, we shall make the following assumption.

Assumption 5. *The dissimilarity between two crops r satisfies: $0 < r \equiv (\rho - \alpha) < 1$. Also, the variance of the market uncertainty is sufficiently less than 1 (i.e., $\sigma^2 << 1$).*

Since, r measures the "dissimilarity" between two crops, we can treat the crops to be substitutes if r is small and to be complements if r is large. Furthermore, because $0 < r < 1$, $|q_t^A - q_t^B| \leqslant 1$, and $\mathbb{E}[\xi_t] = 0$, it is easy to check that $|\mathbb{E}[\Delta p_t]| \leqslant r < 1$, for all t. Furthermore, when $|\mathbb{E}[\Delta p_t]| \leqslant 1$ and $\sigma^2 << 1$, we can ascertain that $|\Delta p_t| < 1$ nearly always holds so that we can effectively assume $\mathbb{P}(|\Delta p_t| < 1) \approxeq 1$.[3]

[3] We formalize this finding in the lemma below.

Lemma 4. *Let the random variable $X \sim U[-\beta, \beta]$ denote the type of the farmer so that the production costs of crops*

Assumption 6. *There are two types of farmers in the market: myopic and strategic. Also, the proportion of strategic farmers is* $\theta \in [0, 1]$.

In our model, we assume that myopic farmers are those who make their crop selection and production decisions purely based on recent market prices. However, strategic farmers are forward looking, and they make their decisions by taking all other farmers' decisions into consideration. For the convenience of notation, we define $z^+ = \max\{z, 0\}$ and let $\overline{\theta} \equiv (1 - \theta)$ throughout this paper.

3.4 Model Analysis: In the Absence of MSPs

To explicate the analysis that involves crop selection and crop production by myopic and strategic farmers with heterogeneous production costs, we first examine the case when MSPs are absent. (We shall extend our analysis to the case when MSPs are present in Section 3.5.) By considering different decision making mechanisms adopted by different types of farmers, we now determine their crop selection and production decisions in period t for any realized market prices in period $t - 1$ (i.e., p^k_{t-1} for $k \in \{A, B\}$).

Myopic farmers' crop selection and production decisions in period t

Let q^{km}_t denote the quantity of crop $k \in \{A, B\}$ to be produced by the myopic farmers in period t, and let p^{km}_t denote the price of crop k in period t as "anticipated" by the myopic farmers. In our model, each myopic farmer anticipates that $p^{km}_t = p^k_{t-1}$, $k \in \{A, B\}$. Hence, a myopic farmer at $x \in [-0.5, 0.5]$ will grow crop A if $p^{Am}_t - c_A(x) \geqslant p^{Bm}_t - c_B(x)$, and will grow crop B,

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A and B for farmer who is located at $X = x$ *are given by* $x + \beta$ *and* $\beta - x$ *respectively. Then,*

$$P\left(|p^A_t - p^B_t| > 2\beta\right) \leqslant P\left(|\xi_t| > 2\beta(1 - r)\right) \leqslant \frac{\sigma^2}{2\beta^2(1 - r)^2}, \text{ where } \xi_t = \epsilon^A_t - \epsilon^B_t.$$

Hence, for a given $r \in (0, 1)$, *we have* $P\left(|\Delta p_t| \geqslant 2\beta\right) \to 0$ *if* $\beta >> \sigma$.

Without loss of generality, we scale β to 1 in our model and assume that σ is sufficiently small (i.e., $\sigma << 1$). Hence, by virtue of Lemma 4, there will be a positive production of each crop in every period.

otherwise. Observe that the myopic farmer located in τ^m is indifferent between the two crops, where $\tau^m = \{x : p_t^{Am} - c_A(x) = p_t^{Bm} - c_B(x)\}$. Because $p_t^{km} = p_{t-1}^k$ for $k \in \{A, B\}$, $\tau^m = \frac{p_{t-1}^A - p_{t-1}^B}{2}$. By applying Assumption 5, we can conclude that $\tau^m \in (-0.5, 0.5)$. Given the threshold τ^m, the segment $\{x : -0.5 \leqslant x < \tau^m\}$ of myopic farmers will grow only crop A, while the segment $\{x : \tau^m < x \leqslant 0.5\}$ of myopic farmers will grow only crop B.

Strategic farmers' crop selection and production decisions in period t

Let q_t^{ks} denote the quantity of crop $k \in \{A, B\}$ to be produced by the strategic farmers in period t, and let p_t^{ks} denote the price of crop k in period t as "anticipated" by the strategic farmers. By taking all other farmers' decisions into consideration, we shall show that strategic farmers can actually anticipate the expected market price in equilibrium so that $p_t^{ks} = \mathbb{E}[p_t^k]$. Also, we shall show later that, among the strategic farmers, the segment $\{x : -0.5 \leqslant x < \tau^s\}$ will grow only A and the segment $\{x : \tau^s < x \leqslant 0.5\}$ will grow only B, where $\tau^s \equiv \tau^s(p_t^{As}, p_t^{Bs}) = \{x : p_t^{As} - c_A(x) = p_t^{Bs} - c_B(x)\}$. (We shall determine the threshold τ^s value in Proposition 9).

Let us illustrate the decisions of different types of framers graphically. Without loss of generality, let us consider the case when $p_{t-1}^A > p_{t-1}^B$. Figure 3.1 depicts the crop selection and production decisions of myopic and strategic farmers. Also, by noting that the market consists of θ strategic and $\overline{\theta} \equiv (1-\theta)$ myopic farmers, the figure depicts the overall crop selection and production. Recall that τ^s and τ^m are the threshold values associated with the myopic and the strategic farmers, respectively. Therefore, the total quantities of crop A produced by the myopic and the strategic farmers are $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$ and $q_t^{As} = \theta(\tau^s + 0.5)$, respectively. Thus, the total availability of crop A in period t is given by $q_t^{AT} = q_t^{Am} + q_t^{As} = \theta(\tau^s + 0.5) + \overline{\theta}(\tau^m + 0.5) = \tau + 0.5$, where $\tau = \theta\tau^s + \overline{\theta}\tau^m$. (Regarding the availability of crop B, it is easy to see that the fraction of myopic farmers producing crop B is given by $0.5 - \tau^m$ and that of strategic farmers producing crop B is

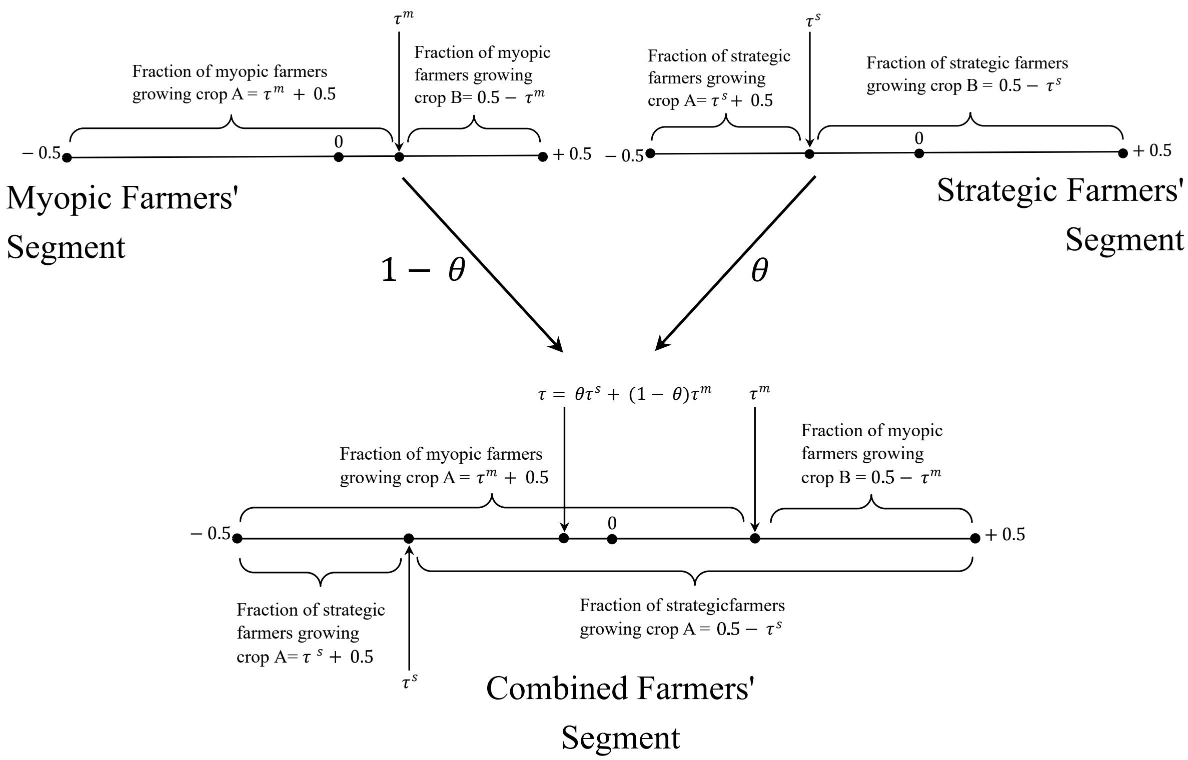


Figure 3.1: Total product availability when $\theta \in [0, 1]$ farmers are strategic and $p^A_{t-1} > p^B_{t-1}$.

$0.5 - \tau^s$. Hence, the total availability of crop B is $q^{BT}_t = \theta(0.5 - \tau^s) + \overline{\theta}(0.5 - \tau^m) = 0.5 - \tau$.)

3.4.1 Farmers' crop selection and production decisions in period t in equilibrium

While the threshold τ^m has been established earlier, the determination of the threshold τ^s is more involved because each strategic farmer takes the crop selection and production decisions of all other farmers into consideration. We now present the following proposition that states the farmers' crop selection and production decisions in period t as depicted in Figure 3.1. In preparation, let us define a term that will prove useful in our analysis. Let:

$$\hat{r} \quad = \quad \frac{\overline{\theta} r}{1 + r\theta}, \tag{3.3}$$

where $r \equiv (\rho - \alpha) > 0$ measures the "dissimilarity" between the two crops. Notice that $\hat{r}$ is increasing in r.

Proposition 9. (Crop selection and production decisions in period t for any realized $\mathbf{P}_{t-1}$) *For any realized prices $\boldsymbol{P}_{t-1}$, the equilibrium crop selection and production decisions of the farmers in period t can be described as follows:*

1. ***Myopic farmers' decisions:*** *The amount of crop A produced by myopic farmers is given by $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$, where $\tau^m = \frac{p_{t-1}^A - p_{t-1}^B}{2} = \frac{\Delta p_{t-1}}{2} \in [-0.5, 0.5]$.*

2. ***Strategic farmers' decisions:*** *The amount of crop A produced by strategic farmers is given by $q_t^{As} = \theta(\tau^s + 0.5)$, where $\tau^s = -\hat{r}\tau^m \in [-0.5, 0.5]$.*

3. ***Total production:*** *The total production of crop A is given by $q_t^{AT} = \tau + 0.5$, where $\tau = \theta\tau^s + \overline{\theta}\tau^m = \left(\frac{\hat{r}}{r}\right)\tau^m \in [-0.5, 0.5]$.*

Even though we focus on crop A in the above proposition, the quantity of crop B produced by myopic and strategic farmers can be obtained through symmetry as $q_t^{Bm} = 0.5 - \tau^m$ and $q_t^{Bs} = 0.5 - \tau^s$, respectively. Also, the total production of crop B is $q_t^{BT} = 0.5 - \tau$.

For any given proportion of strategic farmers θ in the market, the first and the second statements of Proposition 9 describe the equilibrium production decisions of the myopic and strategic farmers through the threshold values τ^m and τ^s, respectively. By combining the corresponding production decisions of these two types of farmers, the third statement gives the total availability of each crop in equilibrium. It is interesting to note that, when $\theta = 1$ (i.e., all the farmers are strategic), $\tau = \tau^s = 0$ so that $q_t^{As} = q_t^{Bs} = 0.5$. Hence, when the market consists of strategic farmers only, half of the strategic farmers will grow A and the remaining half will grow B. Also, the realized market price in period $(t-1)$ has no bearing on the strategic farmers' crop selection and production decisions in period t.

Before we proceed, let us calculate the equilibrium expected crop prices as follows. Recall that $\phi = a - \frac{\rho+\alpha}{2}$ and $\Delta p_{t-1} = p_{t-1}^A - p_{t-1}^B$. Also, by recalling from the third statement of Proposition

9 that $q_t^{AT} = \tau + 0.5$ and $q_t^{BT} = 0.5 - \tau$, we can apply (3.1) to show that:

$$\mathbb{E}[p_t^A] = \phi - r\tau = \phi - \hat{r}\tau^m = \phi - \frac{\hat{r}}{2}\left(p_{t-1}^A - p_{t-1}^B\right) = \phi - \frac{\hat{r}}{2}\Delta p_{t-1}, \text{ and}$$
$$\mathbb{E}[p_t^B] = \phi + r\tau = \phi + \hat{r}\tau^m = \phi + \frac{\hat{r}}{2}\left(p_{t-1}^A - p_{t-1}^B\right) = \phi + \frac{\hat{r}}{2}\Delta p_{t-1}. \tag{3.4}$$

Also, for any location $x \in [-0.5, 0.5]$, let $\pi_t^m(x)$ and $\pi_t^s(x)$ denote the equilibrium profits of a myopic and a strategic farmers who is located at x, respectively. By using (3.2) and Proposition 9 that a farmer of type $v \in \{m, s\}$ will grow crop A if $x \leqslant \tau^v$, and will grow crop B, otherwise, we can apply (3.4) and the production costs $c_A(x) = 0.5 + x$ and $c_B(x) = 0.5 - x$ to show that the profit of a farmer of type $v \in \{m, s\}$ located at x is given as:

$$\pi_t^v(x) = \begin{cases} \Pi_t^A(x) = \mathbb{E}[p_t^A] - c_A(x) = \phi - \frac{\hat{r}}{2}\Delta p_{t-1} - (x + 0.5) & \text{if } x \leqslant \tau^v, \\ \Pi_t^B(x) = \mathbb{E}[p_t^B] - c_B(x) = \phi + \frac{\hat{r}}{2}\Delta p_{t-1} - (0.5 - x) & \text{if } x > \tau^v. \end{cases} \tag{3.5}$$

3.4.2 Impact of crop dissimilarity r

Now, let us use the results stated in Proposition 9 to examine the effect of dissimilarity between the crops (i.e., r) on the crop availability disparity (i.e., $\Delta q_t \equiv q_t^{AT} - q_t^{BT}$) and crop price disparity (i.e., $\Delta p_t \equiv p_t^A - p_t^B$) in period t. First, from the third statement of Proposition 9, it is easy to check that $\Delta q_t = q_t^{AT} - q_t^{BT} = 2\tau$, where $\tau = \left(\frac{\hat{r}}{r}\right)\tau^m$. In this case, by considering (3.3), we can conclude that the crop availability disparity $|\Delta q_t|$ is decreasing in the crop dissimilarity r when $\theta > 0$ and it is independent of r when $\theta = 0$. This result implies that the presence of strategic farmers can improve the balance of crop availability.

Next, let us examine the crop price disparity (i.e., $\Delta p_t \equiv p_t^A - p_t^B$) in period t. From (3.4) we obtain $|\mathbb{E}[\Delta p_t]| = |\mathbb{E}[p_t^A - p_t^B]| = \hat{r} \cdot |-\Delta p_{t-1}|$. Because the term $\hat{r}$ given in (3.3) is increasing in r, we can conclude that the expected crop price disparity is increasing in crop dissimilarity r. Moreover, because $\hat{r} < r < 1$, we can conclude that the expected crop price disparity will be dampened over time. The key results can be summarized in the following corollary:

Corollary 2 (Impact of crop dissimilarity r)**.**

1. ***Crop availability disparity:*** *The disparity between the total production quantities of the crops decreases with r if there are strategic farmers. That is $\frac{\partial|\Delta q_t|}{\partial r} < 0$ if $\theta > 0$, where $\Delta q_t = q_t^{AT} - q_t^{BT}$. However, if $\theta = 0$, then $\frac{\partial|\Delta q_t|}{\partial r} = 0$.*

2. ***Crop price disparity:*** *The expected disparity between the two crop prices increases with the crop dissimilarity r. That is $\frac{\partial|\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$.*

3.4.3 Impact of recent market prices $\mathbf{P}_{t-1}$

We now use the results stated in Proposition 9 to examine the impact of the realized market prices $\mathbf{P}_{t-1}$ in period $t-1$ on the production decisions of different types of farmers in period t. To avoid repetition, we shall focus on the case when $\Delta p_{t-1} = p_{t-1}^A - p_{t-1}^B > 0$. (The case when $\Delta p_{t-1} = p_{t-1}^A - p_{t-1}^B < 0$ can be analyzed in the exact manner.) By applying the results in Proposition 9 (i.e., $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$, $q_t^{As} = \theta(\tau^s + 0.5)$, and $q_t^{AT} = \tau + 0.5$), we obtain the following results:

Corollary 3 (Impact of realized market prices $\mathbf{P}_{t-1}$)**.** *Suppose $\Delta p_{t-1} > 0$. Then:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \tau^m}{\partial \Delta p_{t-1}} = \frac{1}{2} > 0$, *and* $\frac{\partial q_t^{Am}}{\partial \Delta p_{t-1}} = \frac{\overline{\theta}}{2} \geqslant 0$.

2. ***Stratetic farmers' decisions:*** $\frac{\partial \tau^s}{\partial \Delta p_{t-1}} = -\frac{\hat{r}}{2} < 0$, *and* $\frac{\partial q_t^{As}}{\partial \Delta p_{t-1}} = -\frac{\theta \hat{r}}{2} \leqslant 0$.

3. ***Total production:*** $\frac{\partial \tau}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} > 0$, *and* $\frac{\partial q_t^{AT}}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} > 0$.

4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \pi_t^v(x)}{\partial \Delta p_{t-1}} = -\frac{\hat{r}}{2} \leqslant 0$ *if* $x < \tau^v$, *and* $\frac{\partial \pi_t^v(x)}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2} \geqslant 0$ *if* $x > \tau^v$.

Because myopic farmers make their crop selection and production decisions in period t based on the realized market prices $\mathbf{P}_{t-1}$ observed in period $t-1$, more myopic farmers will select to grow the crop that has the higher price in the previous period. This observation explains the first statement of Corollary 3, which stipulates that the larger the price disparity $|\Delta p_{t-1}|$ in period $t-1$, the larger is the disparity in the production quantities of the myopic farmers in period t.

Next, let us consider the second statement. Because each strategic farmer knows the behavior of the myopic farmers and anticipates the behavior of all the other strategic farmers, he anticipates an increase in the production quantity of crop A can cause the price of the crop to go down further. For this reason, fewer strategic farmers will choose to grow A in period t as stated in the second statement.

While the realized market prices $\mathbf{P}_{t-1}$ have opposite effects on the myopic and strategic farmers as shown in the first two statements, the third statement shows that the strategic farmers can never nullify the impact of the decisions of the myopic farmers (and hence the impact of $\mathbf{P}_{t-1}$) on the aggregate product availability in period t. Specifically, the product with higher price in period $t-1$ is always produced more in period t than the product with lower price in period $t-1$. Furthermore, according to the fourth statement of the corollary, a higher value of Δp_{t-1} causes a higher availability of crop A in period t and hurts the expected profits of the farmers (both myopic and strategic) who grow crop A in equilibrium in period t due to the increased production of crop A. Figure 3.2 pictorially illustrates these three effects that are stated in Corollary 3.

3.4.4 Impact of the proportion of strategic farmers θ

Let us examine the impact of the proportion of strategic farmers θ on the farmers' decisions. By considering the equilibrium outcomes as stated in Proposition 9 along with the fact that $\hat{r} = \frac{\bar{\theta}r}{1+r\theta}$ as given in (3.3), it is easy to show that:

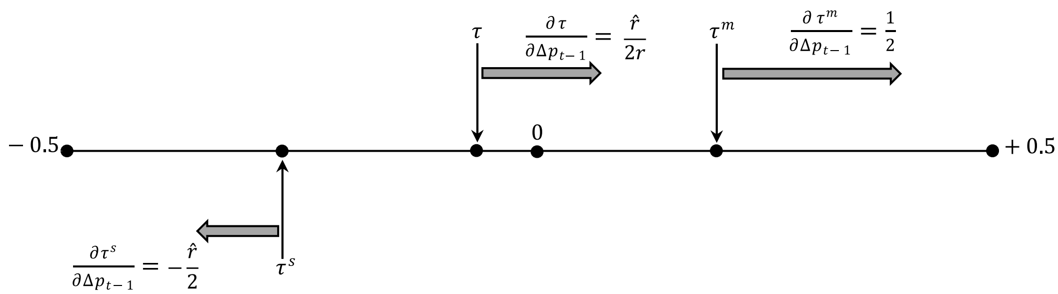


Figure 3.2: Sensitivities of τ^m, τ^s and τ in equilibrium to Δp_{t-1}. Note that $|\frac{\partial \tau^s}{\partial \Delta p_{t-1}}| = |-\frac{\hat{r}}{2}| < \frac{\partial \tau}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} < \frac{\partial \tau^m}{\partial \Delta p_{t-1}} = \frac{1}{2}$.

Corollary 4 (Impact of the proportion of strategic farmers θ)**.** *Suppose* $\Delta p_{t-1} > 0$ *so that* $\tau^m = \frac{\Delta p_{t-1}}{2} > 0$. *Then,*

1. ***Myopic farmers' decisions:*** $\frac{\partial \tau^m}{\partial \theta} = 0$ *and* $\frac{\partial q_t^{Am}}{\partial \theta} = -(\tau^m + 0.5) < 0$.

2. ***Strategic farmers' decisions:*** $\frac{\partial \tau^s}{\partial \theta} = \frac{r(1+r)}{(1+r\theta)^2}\tau^m > 0$ *and* $\frac{\partial q_t^{As}}{\partial \theta} > 0$

3. ***Total production:*** $\frac{\partial q_t^{AT}}{\partial \theta} = \frac{\partial \tau}{\partial \theta} = -\frac{(1+r)}{(1+r\theta)^2}\tau^m < 0$ *and* $\frac{\partial^2 q_t^{AT}}{\partial \theta \partial \Delta p_{t-1}} < 0$.

4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \pi_t^v(x)}{\partial \theta} > 0$ *and* $\frac{\partial^2 \pi_t^v(x)}{\partial \theta \partial \Delta p_{t-1}} > 0$ *if* $x < \tau^v$. *Similarly,* $\frac{\partial \pi_t^v(x)}{\partial \theta} < 0$ *and* $\frac{\partial^2 \pi_t^v(x)}{\partial \theta \partial \Delta p_{t-1}} < 0$ *if* $x > \tau^v$.

The first two statements show that the production quantity of crop A produced by the myopic (strategic) farmers is decreasing (increasing) in θ. As stated in statement 3, the submodularity of q_t^{AT}(or τ) in $(\theta, \Delta p_{t-1})$ asserts that the strategic farmers "counteract" the impact of past market prices on the total production quantity q_t^{AT} in period t, and this "counteracting" effect is more pronounced as the proportion of strategic farmers θ increases. The fourth statement shows that the profit of a farmer (either myopic or strategic) growing crop A (B) in equilibrium is increasing (decreasing) in θ. Moreover, the supermodularity of π_t^v in $(\theta, \Delta p_{t-1})$ for $x < \tau^v$ indicates that the negative impact of past price difference on the farmers growing crop A is mitigated. In summary, the destabilizing effect of past prices on the current expected equilibrium profits of the farmers is mitigated as the proportion of strategic farmers increases.

To summarize, we have shown that past prices will have an impact on the farmers' crop selection and production decisions, product availability, and crops' market prices in the future periods. If a large portion of the farmers are myopic (i.e., θ is small) then the crop with higher price in period $t-1$ (say, crop A) will be grown in abundance in period t. Due to the high availability of crop A, its price in period t is very likely to be low, which hurts the earnings of the those farmers who grow crop A. Consequently, high fluctuations in the past crop prices will destabilize farmers' profits in the current period. To safeguard the earnings of the farmers, many governments in developing countries offer MSPs. However, will MSPs create economic value to farmers? We examine this question in the next section.

3.5 Minimum Support Prices

We now extend our analysis presented in the last section to incorporate crop MSPs. To begin, let m_t^k denote the MSP associated with crop $k \in \{A, B\}$ in period t.[4] Also, let $\hat{p}_t^{km}$ and $\hat{p}_t^{ks}$ denote the *effective* market prices of crop $k \in \{A, B\}$ in period t as "anticipated" by myopic and strategic farmers, respectively.[5] Because each myopic farmer anticipates that the future selling price is equal to the most recently observed market price p_{t-1}^k, myopic farmers will anticipate that $\hat{p}_t^{km} = \max\{p_{t-1}^k, m_t^k\}$ for crop k. However, because each strategic farmer accounts for the actions of all the other farmers, each strategic farmer can anticipate the *effective* market price in equilibrium based on its expected value so that $\hat{p}_t^{ks} = \mathbb{E}_{\epsilon_t^k} \max\{p_t^k, m_t^k\}$ for crop k, where p_t^k is the actual market price as given in (3.1).

The decision making process employed by the farmers remains the same as explained in Section

[4] In general, the MSPs are announced before the crop sowing season; the farmers make their sowing decisions with the complete knowledge of the MSPs and the price history of the crops.

[5] To differentiate between the base case and the case when positive MSPs are offered, we use ˆ over the variables of interest in the latter case.

3.4, except that the anticipated prices p_t^{km} and p_t^{ks} are now replaced by $\hat{p}_t^{km}$ and $\hat{p}_t^{ks}$ for $k \in \{A, B\}$. To ease our exposition and to identify the conditions under which offering higher MSPs is detrimental to the farmers, we shall assume throughout this section that the difference between the MSPs of two crops is bounded by 1 (i.e., $|m_t^A - m_t^B| < 1$). (However, except Propositions 12 and 13 that discuss possible disadvantages of MSPs, all other results described in this section can be extended to MSPs such that $|m_t^A - m_t^B| > 1$, with additional notation.) We first characterize the unique equilibrium in the presence of MSPs in Proposition 10, which is analogous to Proposition 9.

Proposition 10 (Equilibrium under MSPs)**.** *For any realized prices $\boldsymbol{P}_{t-1}$ and for any given MSPs (m_t^A, m_t^B), the equilibrium crop selection and production decisions of the farmers in period t can be described as follows:*

1. ***Myopic farmers' decisions:*** *The amount of crop A produced by myopic farmers is given by $\hat{q}_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$, where*

$$\hat{\tau}^m = \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2} \in [-0.5, 0.5], \tag{3.6}$$

$\hat{p}_t^{km} = \max\{p_{t-1}^k, m_t^k\}$, $k \in \{A, B\}$.

2. ***Strategic farmers' decisions:*** *The amount of crop A produced by strategic farmers is given by $\hat{q}_t^{As} = \theta(\hat{\tau}^s + 0.5)$, where*

$$\hat{\tau}^s = -\hat{r}\hat{\tau}^m - \left[\frac{1}{2(1+r\theta)} \int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon\right] \in [-0.5, 0.5], \tag{3.7}$$

$\hat{r} = \frac{\overline{\theta} r}{1 + r\theta}$ *and* $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$.[6]

[6]Note that (3.7) can be alternatively written as $\hat{\tau}^s = -r\hat{\tau} - \frac{1}{2}\int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon \in [-0.5, 0.5]$. We will use either of these two definitions of $\hat{\tau}^s$ in our analysis, based on convenience.

3. ***Total production:*** *The total production of crop A is given by* $\hat{q}_t^{AT} = \hat{\tau} + 0.5$ *where* $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m \in [-0.5, 0.5]$.

By using $\hat{\tau}^m$ from (3.6) and the fact that $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$, we can obtain $\hat{\tau}^s$ by solving (3.7) as an equation that involves $\hat{\tau}^s$ as the only variable. Once we determine $\hat{\tau}^s$, we can retrieve $\hat{\tau}$ accordingly. Also, it can be shown that Proposition 10 reduces to Proposition 9 when $m_t^A = m_t^B = 0$.[7]

Next, consider a special case when all farmers are strategic so that $\theta = 1$. In this case, statement 2 reveals that, when $\theta = 1$, $\hat{r} = 0$, $\hat{\tau} = \hat{\tau}^s$, $\hat{q}_t^{AT} = (0.5 + \hat{\tau}^s)$, $\hat{q}_t^{BT} = (0.5 - \hat{\tau}^s)$, and (3.7) can be simplified as:

$$\hat{\tau}^s = -\frac{1}{2(1+r)} \int_{m_t^A - \phi + r\hat{\tau}^s}^{m_t^B - \phi - r\hat{\tau}^s} F(\epsilon)\, d\epsilon. \tag{3.8}$$

By noting that $\hat{\tau}^s$ is independent of $\mathbf{P}_{t-1}$, we can conclude that, when all farmers are strategic, the production quantity of each crop k is increasing in its own MSP m_t^k. Hence, a policy-maker can always select appropriate MSPs to attain a balanced mixture of both crops when all farmers are strategic. However, when the market consists of both myopic and strategic farmers, the selection of proper MSPs is much more complex, and we shall discuss this in Section 6.

Finally, the results stated in Proposition 10 possess the same characteristics as the results stated in Proposition 9. First, observe that the threshold associated with the strategic farmers given in (3.7) involves two components: (i) the response to the actions of myopic farmers (which is the first term in the RHS of (3.7), i.e., $-\hat{r}\hat{\tau}^m$, which is analogous to the expression of τ^s given in the second statement of Proposition 9), and (ii) the response to the crop MSPs announced (which is the second

[7] First, to ensure that the crop prices are non-negative we require, $p_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A = \phi - r\hat{\tau} + \epsilon_t^A \geqslant 0$, which implies that $\epsilon_t^A \geqslant -\phi + r\hat{\tau}$ for all values of ϵ_t^A. Second, by using the same arguement for crop B, we can conclude that $\epsilon_t^B \geqslant -\phi - r\hat{\tau}$ for all values of ϵ_t^B. Using these two observations and the fact that ϵ_t^A and ϵ_t^B follow the same distribution $F(\cdot)$, we can conclude that $F(\epsilon) = 0$ for all values of $\epsilon \leqslant \max\{-\phi + r\hat{\tau}, -\phi - r\hat{\tau}\}$. Hence, $\int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon = 0$ so that $\hat{\tau}^s$ is reduced to τ^s when $m_t^A = m_t^B = 0$. Similarly, $\hat{\tau}^m$ is reduced to τ^m and $\hat{\tau}$ is reduced to τ when $m_t^A = m_t^B = 0$. Hence, we can conclude that Proposition 10 reduces to Proposition 9 when $m_t^A = m_t^B = 0$.

term in the RHS of (3.7)). Thus, MSPs influence the decisions of the strategic farmers in two ways. First, they influence the decisions of strategic farmers via the decisions of the myopic farmers as explained in (i), and we term this effect as the *indirect* effect. Second, the MSPs influence the decisions of strategic farmers directly as explained in (ii), and we term this effect as the *direct* effect. There two effects play an important role in our analysis of the impact of MSPs.

It can be shown that the threshold $\hat{\tau}^s$ for strategic farmers is decreasing and the total product availability threshold $\hat{\tau}$ is increasing in the threshold $\hat{\tau}^m$ for myopic farmers. Specifically, it is easy to observe from Proposition 10 that $\frac{\partial \hat{\tau}^s}{\partial \hat{\tau}^m} < 0$ and $\frac{\partial \hat{\tau}}{\partial \hat{\tau}^m} > 0$. The same characteristics of the thresholds can be observed from Proposition 9 as well. Essentially, these two characteristics of $\hat{\tau}^s$ and $\hat{\tau}$ imply that strategic farmers "counteract" the actions of myopic farmers; however, strategic farmers' counter-actions cannot fully nullify the impact of myopic farmers even when MSPs are offered. Also, it can be shown that the findings made in Corollary 2 regarding the impact of crop disimilarity r continued to hold for any given MSPs of the crops (we refer the reader to Corollary 7 in Appendix C.1).

In addition to the production quantities as stated in Proposition 10, we can compute the farmers' profits in equilibrium in the presence of MSPs (m_t^A, m_t^B). Analogous to (3.2), we can express the expected profit of a farmer who is located at x and growing crop $k \in \{A, B\}$ as:

$$\begin{aligned}
\hat{\Pi}_t^k(x) &= \mathbb{E}_{\epsilon_t^k}\left[\max\{p_t^k, m_t^k\}\right] - c_k(x) \\
&= \mathbb{E}[p_t^k] + \mathbb{E}_{\epsilon_t^k}\left[\max\{\epsilon_t, m_t^k - \mathbb{E}[p_t^k]\}\right] - c_k(x) \\
&= \mathbb{E}[p_t^k] + \left(m_t^k - \mathbb{E}[p_t^k]\right) F\left(m_t^k - \mathbb{E}[p_t^k]\right) + \int_{m_t^k - \mathbb{E}[p_t^k]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - c_k(x). \qquad (3.9)
\end{aligned}$$

By considering (3.9), we can use the thresholds $\hat{\tau}^m, \hat{\tau}^s$ and $\hat{\tau}$ stated in Proposition 10 along with the production cost $c_A(x) = 0.5 + x$ and $c_B(x) = 0.5 - x$ to determine the expected profit in equilibrium for a farmer of type $v \in \{m, s\}$ and who is located at x in period t as:

$$\hat{\pi}_t^v(x) = \begin{cases} \hat{\Pi}_t^A(x) = \mathbb{E}[p_t^A] + \left(m_t^A - \mathbb{E}[p_t^A]\right) F\left(m_t^A - \mathbb{E}[p_t^A]\right) + \int_{m_t^A - \mathbb{E}[p_t^A]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - (0.5 + x) \text{ if } x \leqslant \hat{\tau}^v, \\ \hat{\Pi}_t^B(x) = \mathbb{E}[p_t^B] + \left(m_t^B - \mathbb{E}[p_t^B]\right) F\left(m_t^B - \mathbb{E}[p_t^B]\right) + \int_{m_t^B - \mathbb{E}[p_t^B]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - (0.5 - x) \text{ if } x > \hat{\tau}^v. \end{cases} \tag{3.10}$$

Also, by using statement 3 of Proposition 10 stating that $\hat{q}_t^{AT} = 0.5 + \hat{\tau}$ and $\hat{q}_t^{BT} = 0.5 - \hat{\tau}$, we can apply (3.1) to determine the expected market price $\mathbb{E}[p_t^A]$ and $\mathbb{E}[p_t^B]$ in equilibrium as a function of $\hat{\tau}$, which in turn depends on the MSPs via (3.7).

3.5.1 Impact of $\mathbf{P}_{t-1}$ and θ

We now examine the impact of the most recently realized prices $\mathbf{P}_{t-1}$ and the fraction of strategic farmers θ on the equilibrium outcomes, which are as stated in Proposition 10, in the presence of MSPs. Corollary 5, which is an analogue to Corollary 3, explains the impact of $\mathbf{P}_{t-1}$ on the equilibrium. For ease of exposition, we shall focus on crop A only.

Corollary 5 (Impact of the most recently realized prices $\mathbf{P}_{t-1}$ under MSPs)**.** *For any given MSPs (m_t^A, m_t^B), the impact of $\boldsymbol{P}_{t-1}$ can be described as follows:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} \geqslant 0$ *and* $\frac{\partial \hat{q}_t^{Am}}{\partial p_{t-1}^A} \geqslant 0$.

2. ***Strategic farmers' decisions:*** $\frac{\partial \hat{\tau}^s}{\partial p_{t-1}^A} \leqslant 0$ *and* $\frac{\partial \hat{q}^{As}}{\partial p_{t-1}^A} \leqslant 0$.

3. ***Total production:*** $\frac{\partial \hat{\tau}}{\partial p_{t-1}^A} \geqslant 0$ *and* $\frac{\partial \hat{q}^{AT}}{\partial p_{t-1}^A} \geqslant 0$.

4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \hat{\pi}_t^v(x)}{\partial p_{t-1}^A} \leqslant 0$ *if* $x < \hat{\tau}^v$ *and* $\frac{\partial \hat{\pi}_t^v(x)}{\partial p_{t-1}^A} \geqslant 0$ *if* $x > \hat{\tau}^v$.

It is easy to check that Corollary 4 resembles Corollary 2 (for any given p_{t-1}^B) even when MSPs are present; hence, it can be interpreted in the same manner.

Next, we examine the impact of the proportion of strategic farmers θ on the equilibirium outcomes. Corollary 6 is analogous to Corollary 4. However, because of the MSPs, the analysis is more involved in the sense that the result hinges on the comparison between the threshold $\hat{\tau}^m$, as defined in (3.6), and the threshold $\hat{\tau}_0^s$, where $\hat{\tau}_0^s$ is the value of $\hat{\tau}^s$ (as defined in (3.7)) evaluated at $\theta = 0$. In other words, $\hat{\tau}_0^s \equiv \hat{\tau}^s|_{\theta=0} = -2r\hat{\tau}^m - \frac{\int_{m_t^A-\phi+2r\hat{\tau}^m}^{m_t^B-\phi-2r\hat{\tau}^m} F(\epsilon)d\epsilon}{2}$. It can shown that depending on the parameters and the distribution $F(\cdot)$, the difference between $\hat{\tau}^m$ and $\hat{\tau}_0^s$ can be positive or negative, but explicit conditions are not available.

Corollary 6 (Impact of strategic farmers under MSPs)**.** *For any given MSPs (m_t^A, m_t^B), the impact of θ can be described as follows:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \hat{\tau}^m}{\partial \theta} = 0$ *and* $\frac{\partial \hat{q}^{Am}}{\partial \theta} = -(\hat{\tau}^m + 0.5) \leqslant 0$.

2. ***Strategic farmers' decisions:*** $\frac{\partial \hat{\tau}^s}{\partial \theta} \geqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}_0^s$, *and* $\frac{\partial \hat{q}_t^{As}}{\partial \theta} \geqslant 0$.

3. ***Total production:*** $\frac{\partial \hat{\tau}}{\partial \theta} \leqslant 0$, *and* $\frac{\partial \hat{q}_t^{AT}}{\partial \theta} \leqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}_0^s$.

4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** *If* $x \leqslant \hat{\tau}^v$, *then* $\frac{\partial \hat{\pi}_t^v(x)}{\partial \theta} \geqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}_0^s$. *Else, if* $x > \hat{\tau}^v$, *then* $\frac{\partial \hat{\pi}_t^v(x)}{\partial \theta} \leqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}_0^s$.

When $\hat{\tau}^m \geqslant \hat{\tau}_0^s$, the above corollary exhibits the same characteristics as Corollary 3 (for the case when $\tau^m \geqslant \tau^s$, which holds when the supposition $p_{t-1}^A \geqslant p_{t-1}^B$ holds). Hence, it can be interpreted in the same manner.

However, the above corollary exhibits opposite results when $\hat{\tau}^m < \hat{\tau}_0^s$, where this condition depends on the value of MSPs. This condition is not present in Corollary 4 because, in the absence of MSPs, strategic farmers respond only to myopic farmers' decisions that are determined by the realized prices $\mathbf{P}_{t-1}$. However, in the presence of MSPs, MSPs have a *direct* impact (along with $\mathbf{P}_{t-1}$) on the decisions of myopic farmers as described in (3.6). Also, MSPs have both *direct* and *indirect* (via the actions of myopic farmers) impacts on strategic farmers as described in (3.7), which makes the decisions of strategic farmers more intricate. This observation calls for more attention to the analysis of the impact of MSPs on farmers' decisions. We explore this in the following section.

3.5.2 Impact of MSPs

We now examine the impact of MSPs on the farmer's crop selection and production decisions (again, we focus on crop A alone). In preparation, let us define the following two bounds on the MSP of crop A.

$$\underline{m}_t^A \equiv \underline{m}_t^A(\mathbf{P}_{t-1}, m_t^B) = \max\{p_{t-1}^A, \max\{m_t^B, p_{t-1}^B\} - 1\} \text{ and}$$
$$\overline{m}_t^A \equiv \overline{m}_t^A(\mathbf{P}_{t-1}, m_t^B) = \max\{p_{t-1}^A, \max\{m_t^B, p_{t-1}^B\} + 1\}.$$

With these two bounds, MSP m_t^A is considered to be *low* when $m_t^A < \underline{m}_t^A$, *moderate* when $\underline{m}_t^A \leqslant m_t^A \leqslant \overline{m}_t^A$, and *high* when $m_t^A > \overline{m}_t^A$. The two bounds $\underline{m}_t^A$ and $\overline{m}_t^A$ are intended to establish the necessary and sufficient conditions under which $\hat{\tau}^m$, which represents myopic farmers' crop selection decisions and that is defined in (3.6) in Proposition 10, is independent of m_t^A, the MSP of crop A. It can be shown that $\hat{\tau}^m$ is independent of m_t^A if and only if either m_t^A is *low* (i.e., $m_t^A \leqslant \underline{m}_t^A$) or m_t^A is *high* (i.e., $m_t^A \geqslant \overline{m}_t^A$). [8] By doing this, we can observe the impact of MSPs when (i) they have only the *direct* effect, and (ii) they have both the *direct* and the *indirect* effects, on the

[8] Clearly, when $m_t^A \geqslant \overline{m}_t^A$ then $\hat{p}_t^A = \max\{m_t^A, p_{t-1}^A\} = m_t^A \geqslant \hat{p}_t^B + 1$ so that all the myopic farmers grow crop A and hence $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5) = \overline{\theta}(0.5 + 0.5) = \overline{\theta}$, which is independent of m_t^A. On the other hand, if $m_t^A \leqslant \underline{m}_t^A$ then, we consider two cases: (i) $p_{t-1}^A \geqslant \max\{m_t^B, p_{t-1}^B\} - 1$ and (ii) $p_{t-1}^A < \max\{m_t^B, p_{t-1}^B\} - 1$. Under case (i), we have $m_t^A \leqslant \underline{m}_t^A = p_{t-1}^A$ and $|p_{t-1}^A - \hat{p}_t^{Bm}| < 1$ because $m_t^A - m_t^B| < 1$ and $|p_{t-1}^A - p_{t-1}^B| < 1$. Hence, $\hat{\tau}^m = \frac{p_{t-1}^A - \hat{p}_t^{Bm}}{2} > -0.5$ so that $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$. Under case (ii), we have $m_t^A \leqslant \underline{m}_t^A = \max\{m_t^B, p_{t-1}^B\} - 1$, hence we have $\hat{\tau}^m = -0.5$ so that $q_t^{Am} = 0$. Therefore, if $m_t^A \leqslant \underline{m}_t^A$, the total production quantity by myopic farmers can we written as $q_t^{Am} = \overline{\theta}\left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + \frac{1}{2}\right]^+$.

decisions of strategic farmers given by $\hat{\tau}^s$ in (3.7). By using the two bounds $\underline{m}_t^A$ and $\overline{m}_t^A$, along with the results as stated in Proposition 10, we obtain the following results:

Proposition 11 (Impact of MSPs on Equilibrium). *For any given MSP m_t^B of crop B, the MSP of crop A, m_t^A, affects the production decisions of myopic and strategic farmers as follows:*

1. ***Total production:*** *The total availability of crop A is always increasing in the MSP of A so that $\frac{\partial \hat{q}_t^{AT}}{\partial m_t^A} = \frac{\partial \hat{\tau}}{\partial m_t^A} \geqslant 0$.*
2. ***Low MSP:*** *When $m_t^A \leqslant \underline{m}_t^A$, then: (a) $\hat{q}_t^{Am} = \overline{\theta}\left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + \frac{1}{2}\right]^+$ so that $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = 0$, and (b) $\frac{\partial \hat{q}_t^{As}}{\partial m_t^A} \geqslant 0$.*
3. ***High MSP:*** *When $m_t^A \geqslant \overline{m}_t^A$, then: (a) $\hat{q}_t^{Am} = \overline{\theta}$ so that $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = 0$, and (b) $\frac{\partial \hat{q}_t^{As}}{\partial m_t^A} \geqslant 0$.*
4. ***Moderate MSP:*** *When $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then: (a) $\hat{q}_t^{Am} \in (0, \overline{\theta})$, and (b) $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = \frac{\overline{\theta}}{2} \geqslant 0$.*

The first statement of Proposition 11 shows that the availability of a crop is always increasing in the MSP offered for the crop. Due to this increase in the availability of the crop, its market price drops as its MSP increases. Hence, the equilibrium expected market price of crop A is decreasing in m_t^A (and increasing in m_t^B with details omitted). Therefore, to achieve a better balance of different crops, a policy-maker has to account for the effect of MSP of one crop on the production of the other crop. Further, it is always possible to obtain a desired production-mix of the crops using MSPs.[9]

Now, when MSP m_t^A is *low* (i.e., $m_t^A \leqslant \underline{m}_t^A$), the decisions of myopic farmers are independent of m_t^A (as explained in footnote 8). Hence, when MSP m_t^A is *low*, a slight increase in the MSP

[9] To see why, suppose $\hat{\tau}_{\text{target}}$ is the targeted production of crop A (so that $1 - \hat{\tau}_{\text{target}}$ is the targeted production of crop B). Without loss of generality, assume that initially we set $m_t^A = m_t^B = 0$ so that $\hat{\tau} = \tau$, which is as defined in Proposition 9. If $\hat{\tau} = \tau > \hat{\tau}_{\text{target}}$ then we can set m_t^B sufficiently high so that $\hat{\tau} = \hat{\tau}_{\text{target}}$ is attained. This is possible because from (3.7) we see that $\lim_{m_t^B \to \infty} \hat{\tau}^s = \max\{-\infty, -0.5\} = -0.5$ and from (3.6) we see that $\lim_{m_t^B \to p_{t-1}^A + 1} \hat{\tau}^m = -0.5$ so that $\lim_{m_t^B \to \infty} \hat{\tau} = \lim_{m_t^B \to \infty}\{\theta \hat{\tau}^s + \overline{\theta}\hat{\tau}^m\} = -0.5$. Likewise, on the other hand, if $\hat{\tau} = \tau < \hat{\tau}_{\text{target}}$ then we can set m_t^A sufficiently high so that $\hat{\tau} = \hat{\tau}_{\text{target}}$ is attained because it can be shown that $\lim_{m_t^A \to \infty} \hat{\tau} = 0.5$.

m_t^A will not affect the sowing decisions of myopic farmers as stated in part (a) of statement 2. Anticipating the myopic farmers' sowing decisions, more strategic farmers will grow crop A as MSP m_t^A increases. This explains part (b) of statement 2.

Next, when MSP m_t^A is *high* (i.e., $m_t^A \geqslant \overline{m}_t^A$), all myopic farmers will grow crop A (as explained in footnote 8). As such, increasing m_t^A will not increase myopic farmers' production of crop A any further. Anticipating the myopic farmers' sowing decisions, more strategic farmers will grow crop A as MSP m_t^A increases. This explains statement 3. Essentially, the second and the third statements imply that, as long as myopic farmers are "unaffected" by the increase in m_t^A, strategic farmers will increase their production of crop A in order to benefit from the increase in m_t^A.

Finally, let us examine the fourth statement of Proposition 11 in which the MSP m_t^A is *moderate* (i.e., $\underline{m}_t^A < m_t^A < \overline{m}_t^A$). In this case, it can be shown that the production of crop A by the myopic farmers is strictly increasing in m_t^A (and decreasing in m_t^B with details omitted). As shown in the fourth statement, when the MSP is moderate so that $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, more myopic farmers will grow crop A as the MSP m_t^A increases (i.e., $\hat{\tau}^m$ is increasing so that $\hat{q}_t^{Am}$ is increasing in the MSP m_t^A). Anticipating myopic farmers' behavior, strategic farmers make decisions in a more intricate manner, when the MSP m_t^A is *moderate*. However, as it turns out, the amount of crop A produced by strategic farmers q_t^{As} (or equivalently $\hat{\tau}^s$) is not necessarily monotonic in the MSP m_t^A: offering a higher MSP for a crop can cause strategic farmers to produce less of the crop. We shall explore this seemingly counter-intuitive result in more detail.

Due to the complexity of the analysis, we shall consider a special case when the market uncertainty $\epsilon_t^k \sim U[-\delta, \delta],\ k \in \{A, B\}$, instead of a general probability distribution $F(\cdot)$. In preparation, we let:

$$\widetilde{m} = \phi - \delta \left(\frac{1-r}{1+r} \right) < \phi = a - \frac{\rho + \alpha}{2}.$$

Notice that $\widetilde{m} > 0$ when a is sufficiently large and δ is sufficiently small. By considering $\widetilde{m}$, We obtain the following result:

Proposition 12 (Impact of MSPs on strategic farmers)**.** *Suppose the given MSP m_t^B of crop B is such that $m_t^B \geqslant p_{t-1}^B$. Then, for any moderately low MSP of A such that $m_t^A \in \left(\underline{m}_t^A, \min\{m_t^B + 1, \widetilde{m}\}\right)$, there exists a threshold $\theta_0 \equiv \theta_0(m_t^A, m_t^B) > 0$ such that $\frac{\partial \hat{\tau}^s}{\partial m_t^A} < 0$ if and only if $0 \leqslant \theta < \theta_0$.*[10]

While Proposition 12 is based on the assumption that the market uncertainty $\epsilon_t^k \sim U[-\delta, \delta]$, $k \in \{A, B\}$, the results stated in the proposition continue to hold for general distribution. (Please see Proposition 15 in Appendix C.1 for details.)

Figure 3.3 provides a numerical example to verify the results that are stated in Propositions 11 and 12. The parameter values used are $a = 1$, $\rho = 0.7$, $\alpha = -0.25$, $p_{t-1}^A = 0.1$, $p_{t-1}^B = 0.5$, $m_t^B = 0.55$, $r = 0.95$, $\theta = 0.1$, and $\epsilon_t^k \sim U[-0.1, 0.1]$ (i.e., $\delta = 0.1$). As illustrated in the figure, the thresholds $\hat{\tau}^m$ and $\hat{\tau}$ are always increasing in m_t^A. This conforms with the findings as shown in Proposition 11. The threshold $\hat{\tau}^s$ is however not monotonic in m_t^A (it is decreasing in m_t^A until $m_t^A \approx 0.74$), which verifies Proposition 12.

Proposition 12 shows that when m_t^A, the MSP for crop A, is *moderately low* (i.e., $\underline{m}_t^A < m_t^A < \widetilde{m}$), the proportion of strategic farmers producing crop A (i.e., $\hat{\tau}^s$) can be decreasing in m_t^A. Intuitively, one expects that more farmers grow crop A as the MSP of the crop increases. While this is always true in the case of myopic farmers, as shown in Proposition 11 when the MSP is moderately low, it is not true for strategic farmers when $\theta < \theta_0$, as shown in Proposition 12.

The rationale for this counter-intuitive result as stated in Proposition 12 is as follows. Strategic

[10] It can be shown that for any given m_t^B, there exist values of m_t^A that satisfy the conditions listed in Proposition 12 when δ is sufficiently small and crop A is produced more in the previous period. To elaborate, suppose $0 < \delta \leqslant \frac{r(1+r)}{2}(q_{t-1}^{AT} - \frac{1}{2})$ where q_{t-1}^{AT} is the total production of crop A in the previous period. Then $\delta \leqslant \frac{r(1+r)}{2}(q_{t-1}^{AT} - \frac{1}{2}) \Leftrightarrow \delta + \delta\left(\frac{1-r}{1+r}\right) \leqslant r\left(q_{t-1}^{AT} - \frac{1}{2}\right) \Rightarrow \epsilon_{t-1}^A + \delta\left(\frac{1-r}{1+r}\right) \leqslant r\left(q_{t-1}^{AT} - \frac{1}{2}\right)$ (because $\epsilon_{t-1}^A \in [-\delta, \delta]$) $\Leftrightarrow a - \alpha - rq_{t-1}^{AT} + \epsilon_{t-1} < a - \frac{\rho+\alpha}{2} - \delta\left(\frac{1-r}{1+r}\right) \Leftrightarrow p_{t-1}^A < \phi - \delta\left(\frac{1-r}{1+r}\right) = \tilde{m}$. Hence, there exists m_t^A such that $p_{t-1}^A < m_t^A < \tilde{m}$. Next, m_t^B can be chosen sufficiently close to $\tilde{m}$ such that $m_t^B + 1 > \tilde{m}$ so that $\underline{m}_t^A < m_t^A < \min\{m_t^B + 1, \widetilde{m}\}$. Note that this provides only a sufficient condition, but not a necessary condition, for the conditions listed in the proposition to hold simultaneously.

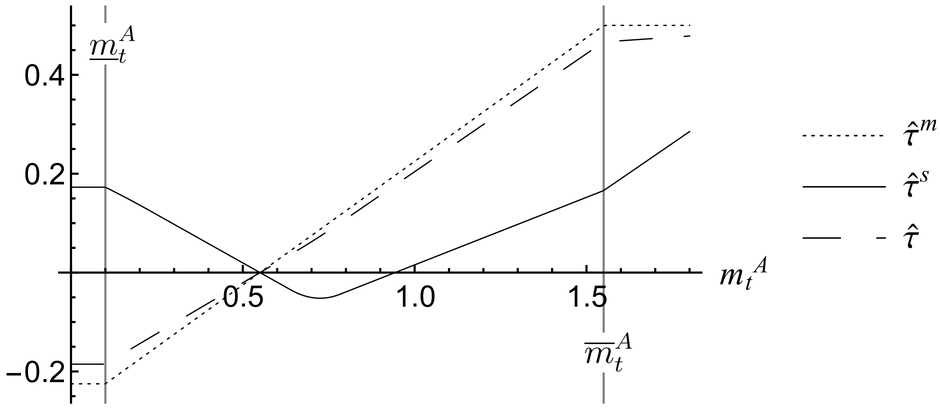


Figure 3.3: $\hat{\tau}^m$, $\hat{\tau}^s$ and $\hat{\tau}$ as a function of MSP m_t^A.

farmers know that, when the MSP of crop A is moderate, more myopic farmers will grow crop A as m_t^A increases. The resulting increase in production of crop A is substantial when θ is small because, by using statement 1 of Proposition 10 (i.e., $\hat{q}^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$), it is easy to see that: $\frac{\partial \hat{q}^{Am}}{\partial m_t^A} = \overline{\theta} \frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{\overline{\theta}}{2}$ when $m_t^A > p_{t-1}^A$. This substantial increase in the total production quantity q_t^{AT} causes a significant drop in the price of crop A (and causes a steep increase in the price of crop B). By anticipating myopic farmers' behavior, strategic farmers are better off by producing less of crop A and more of crop B. This explains the seemingly counter-intuitive result that is stated in Proposition 12.

To summarize, we find that, when the MSP of crop A is moderately low, increasing the MSP m_t^A can cause fewer strategic farmers to grow crop A (and more strategic farmers to grow crop B). This seemingly counter-intuitive finding offers a hint regarding the condition(s) under which offering MSP for a crop can hurt the earnings of farmers who grow that crop. We shall explore this next.

3.5.3 Impact of MSPs on farmers' profits

We now examine the impact of the MSP of crop A on the ex-ante expected profits of farmers of each type $v \in \{m, s\}$ as given by (3.10). By differentiating (3.10) with respect to m_t^A and by using the fact that the expected market price $\mathbb{E}[p_t^A]$ and $\mathbb{E}[p_t^B]$ in equilibrium depend on the MSPs via $\hat{\tau}$, we obtain:

$$\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = \begin{cases} F\left(m_t^A - \mathbb{E}[p_t^A]\right) + \overline{F}\left(m_t^A - \mathbb{E}[p_t^A]\right) \cdot \frac{\partial \mathbb{E}[p_t^A]}{\partial m_t^A} & \text{if } x \leqslant \hat{\tau}^v \\ \overline{F}\left(m_t^B - \mathbb{E}[p_t^B]\right) \cdot \frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} & \text{if } x > \hat{\tau}^v \end{cases} \tag{3.11}$$

where $\frac{\partial \mathbb{E}[p_t^A]}{\partial m_t^A} = -r \frac{\partial \hat{\tau}}{\partial m_t^A} \leqslant 0$ and $\frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} = r \frac{\partial \hat{\tau}}{\partial m_t^A} \geqslant 0$. As before, we focus on the impact of the MSP of crop A on the expected profits of the farmers. We introduce the following lemma.

Lemma 5 (Impact of MSPs on farmers' profits)**.** *Consider a farmer of type $v \in \{m, s\}$ who is located at $x \in [-0.5, 0.5]$.*

1. ***Farmers growing crop B:** If $x > \hat{\tau}^v$, then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*

2. ***Low or high m_t^A on farmers growing crop A:** If $x \leqslant \hat{\tau}^v$ and m_t^A is either low or high (i.e., $m_t^A < \underline{m}_t^A$ or $m_t^A > \overline{m}_t^A$), then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*

The lemma explains the *indirect* benefit that m_t^A offers to the farmers growing crop B (i.e., farmers who are located at $x > \hat{\tau}^v$) in equilibrium. When m_t^A is increased, the total availability of crop A (B) increases (decreases) according to statement 1 of Proposition 11. Hence, the expected market price of crop B increases, which will increase the expected profit of those farmers who grow crop B in equilibrium. Furthermore, the lemma proves that, as long as the decisions of the myopic farmers are not "affected" by m_t^A (i.e., m_t^A is low so that $m_t^A \leqslant \underline{m}_t^A$ or m_t^A is high so that $m_t^A \geqslant \overline{m}_t^A$), an increase in m_t^A will always increase the equilibrium expected profit of the farmers who grow crop A. This indicates that, when the myopic farmers are not influenced by the changes in m_t^A, the strategic farmers will make decisions in such a way that the expected profit of all the farmers growing crop A will increase if m_t^A is increased.

It remains to analyze the impact of MSP of crop A on the farmers' expected profits when it is moderate (i.e., $\underline{m}_t^A < m_t^A < \overline{m}_t^A$). To simplify our analysis as before, let us consider a special case when ϵ_t^A and ϵ_t^B are independent random variables that follow $U[-\delta, \delta]$. Further, assume $m_t^k \geqslant p_{t-1}^k$, $k \in \{A, B\}$, so that both the MSPs are effective. Also, we define another threshold that will prove useful in our analysis. Let:

$$\widetilde{m}^A(m_t^B) = \left(\frac{r}{r+2}\right) m_t^B + \frac{2}{r+2}\left[\phi - \delta\left(\frac{2-r}{2+r}\right)\right].$$

Akin to $\tilde{m}$ as defined earlier, $\widetilde{m}^A \equiv \widetilde{m}^A(m_t^B) > 0$ when ϕ is sufficiently large and δ is sufficiently small. The following proposition shows that increasing the MSP of crop A can hurt the expected profits of those farmers who grow crop A in equilibrium.

Proposition 13 (Impact of moderate m_t^A on farmers' profits)**.** *For any given MSP for crop B m_t^B and suppose that the MSP for crop A is moderately low so that $m_t^A \in (\underline{m}_t^A, \widetilde{m}^A)$. Then there exists a threshold $\theta_1 \equiv \theta_1(m_t^A, m_t^B)$ such that $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} < 0$ for all $\theta < \theta_1$, for each farmer of type $v \in \{m, s\}$ located at $x \leqslant \hat{\tau}^v$. Furthermore, if θ is sufficiently high, then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*

While Proposition 13 is based on the assumption that the market uncertainty $\epsilon_t^k \sim U[-\delta, \delta]$, $k \in \{A, B\}$, the results stated in the proposition continued to hold for general distribution. (Please see Proposition 16 in Appendix C.1 for details.)

In Proposition 13, we identify a scenario in which increasing the MSP of a crop can decrease the expected profits of the farmers who grow that crop. According to the proposition, when the MSP of crop A is moderately low so that $m_t^A \in (\underline{m}_t^A, \widetilde{m}^A)$ and when there are very few strategic farmers (i.e., θ is sufficiently small so that $\theta < \theta_1$), then increasing m_t^A will hurt the expected profits of the farmers who grow crop A (i.e., for farmers who are of type $v \in \{m, s\}$ and located at x with $x \leqslant \hat{\tau}^v$). This is because, even with a small increase in m_t^A, there is a substantial increase in the production of crop A by the myopic farmers (because the proportion of myopic farmers $(1 - \theta)$ is large when θ is small). Consequently, there is a drop in the price of crop A. This drop in the market price of crop A, coupled with the moderately low value of m_t^A, will reduce the expected profits of those farmers who grow crop A.

We can conclude that, when θ is sufficiently small, there exists a threshold, say, m_t^{A*} (as shown in Figure 3.4) such that offering MSP of A in $(\underline{m}_t^A, m_t^{A*})$ is disadvantageous to the farmers who grow crop A. In other words, by choosing m_t^A in the interval $(\underline{m}_t^A, m_t^{A*})$, the policy-maker creates an undesirable frenzy among the myopic farmers who switch to crop A thereby substantially increasing the production of crop A that causes a significant drop in the price of the crop, which overrides the benefit accrued by the increase in m_t^A at moderately low values, thereby hurting the expected profits of the farmers growing crop A in equilibrium.

Figure 3.4 provides a numerical example when the equilibrium revenue of farmers growing crop A decreases with an increase in m_t^A. The parameter values used for the example are $a = 1$, $\rho = 0.7$, $\alpha = -0.25$, $p_{t-1}^A = 0.1$, $p_{t-1}^B = 0.5$, $m_t^B = 0.55$, $r = 0.95$, $\theta = 0.1$, and $\epsilon_t^k \sim U[-0.1, 0.1]$. Note that it suffices to observe the sensitivities of expected revenues from the crops with respect to the MSP m_t^A, because the expected profits of a farmer from growing the crops are the expected revenue less the production cost of the corresponding crop, where the latter are independent of the MSPs for all $x \in [-0.5, 0.5]$.

While the expected profit of the farmers who grow crop B is always non-decreasing in m_t^A (as shown in the first statement of Lemma 5), the profit of a farmer who grows crop A is non-monotonic in m_t^A. From Figure 3.4 we can draw the following conclusions about the value of MSPs. First, relative to the case when MSP is absent, offering a higher MSP that has $m_t^A > m_t^{A*}$ can benefit farmers who grow A as well as those who grow B. Second, relative to the case when MSP is absent, offering a moderately low MSP for a crop, say, crop A, can make those farmers who grow A to

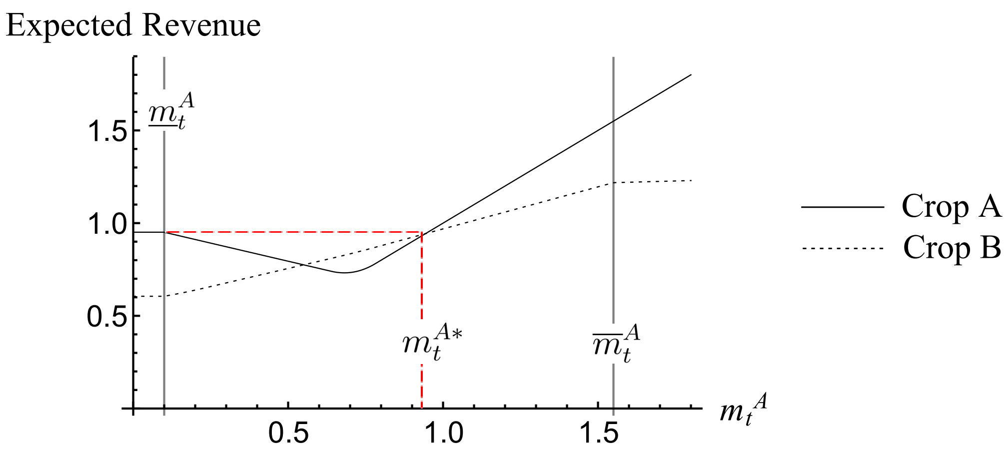


Figure 3.4: Expected revenue from crop A and crop B as a function of MSP m_t^A.

become worse off and make those farmers who grow B to become better off. When this happens, the actual impact of MSP for crop A violates the intended goal for offering MSP for crop A (which is intended to benefit farmers who grow crop A). Therefore, selecting an appropriate level of MSP is crucial and a policy-maker has to exercise sufficient care in choosing the right MSPs m_t^k, $k \in \{A, B\}$ to ensure that they: (i) benefit the farmers, especially those who grow crop k, and (ii) balance the crop availabilities for the consumer. We explore this topic further in Section 6.

3.6 Selection of efficient MSPs

Lastly, in this section, we formulate the optimization problem of a social planner (i.e., the policy-maker or the government) whose objective is to choose crop MSPs such that the farmers and the consumers can be benefited to the largest extent at the lowest possible total expenditure. First, we define *farmer surplus* in period t as follows:

$$
\begin{aligned}
\mathscr{F}_t(m_t^A, m_t^B) =& \theta \int_{-0.5}^{0.5} \pi_t^s(x)\, dx + \overline{\theta} \int_{-0.5}^{0.5} \pi_t^m(x)\, dx \\
=& \theta \left[\int_{-0.5}^{\tau^s} \left(\mathbb{E}\left[\max\{p_t^A, m_t^A\}\right] - (x + 0.5) \right)\, dx + \int_{\tau^s}^{0.5} \left(\mathbb{E}\left[\max\{p_t^B, m_t^B\}\right] - (0.5 - x) \right)\, dx \right] \\
&+ \overline{\theta} \left[\int_{-0.5}^{\tau^m} \left(\mathbb{E}\left[\max\{p_t^A, m_t^A\}\right] - (x + 0.5) \right)\, dx + \int_{\tau^m}^{0.5} \left(\mathbb{E}\left[\max\{p_t^B, m_t^B\}\right] - (0.5 - x) \right)\, dx \right],
\end{aligned}
$$

where $p_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A = \phi - r\hat{\tau} + \epsilon_t^A$, $p_t^B = \mathbb{E}[p_t^B] + \epsilon_t^B = \phi + r\hat{\tau} + \epsilon_t^B$ and $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$, as in Proposition 10. Second, We capture the *disutility of the consumers* through the imbalance of crop availability as follows:

$$\mathscr{C}_t(m_t^A, m_t^B) = -\left(q_t^{AT} - q_t^{BT}\right)^2 = -4\hat{\tau}^2.$$

Third, the *total expected expenditure* incurred by the policy-maker by setting MSPs m_t^A and m_t^B is given by:

$$\begin{aligned}\mathscr{K}_t(m_t^A, m_t^B) =& \hat{q}_t^{AT} \cdot \mathbb{E}[m_t^A - p_t^A]^+ + \hat{q}_t^{BT} \cdot \mathbb{E}[m_t^B - p_t^B]^+ \\ =& (\hat{\tau} + 0.5)\mathbb{E}[m_t^A - \phi + r\hat{\tau} - \epsilon_t^A]^+ + (0.5 - \hat{\tau})\mathbb{E}[m_t^B - \phi - r\hat{\tau} - \epsilon_t^B]^+,\end{aligned}$$

because government has to bear an expected expenditure of $\mathbb{E}[m_t^k - p_t^k]^+$ for all the quantity of $\hat{q}_t^{kT}$ of crop $k \in \{A, B\}$ produced. The quantity $\hat{q}_t^{kT}$ is as given in Proposition 10.

Using $\mathscr{F}_t$, $\mathscr{C}_t$ and $\mathscr{K}_t$, we can define the social welfare (maximization) problem ($\mathbf{SWP}_t$) in period t as below:

$$\begin{aligned}\mathbf{SWP}_t : \quad & \max_{m_t^A, m_t^B} \mathscr{W}_t(m_t^A, m_t^B) = \{\lambda\mathscr{F}_t(m_t^A, m_t^B) + (1-\lambda)\mathscr{C}_t(m_t^A, m_t^B)\} - \eta\mathscr{K}_t(m_t^A, m_t^B) \\ & \text{such that } 0 \leqslant m_t^k \leqslant M, k \in \{A, B\}, \\ & \qquad \mathscr{K}_t(m_t^A, m_t^B) \leqslant B,\end{aligned}$$

where $\lambda \in (0, 1)$ and $(1 - \lambda) \in (0, 1)$ are the exogenous weights associated by the policy-maker to farmers' welfare and consumers' welfare, respectively, η is the sensitivity of the policy-maker (or the government) to its expenditure, M is the maximum limit of the MSP to be awarded to a crop, and B is a bound on the expected expenditure to be incurred (we can consider the constraint

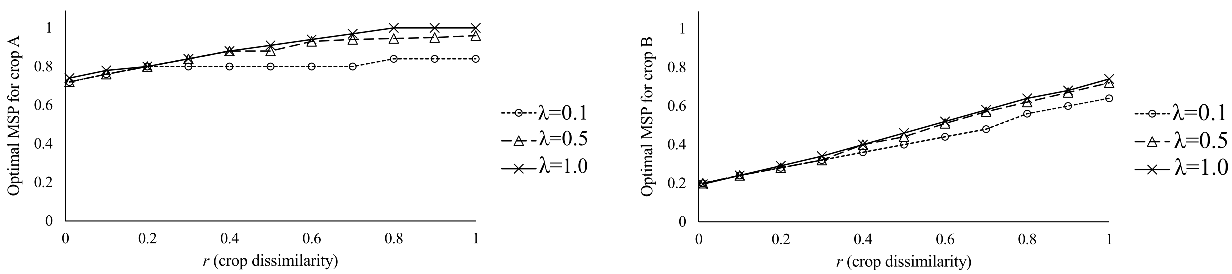
$\mathscr{K}_t(m_t^A, m_t^B) \leqslant B$ as a budget constraint).

Having analyzed the impact of MSPs chosen by a policy-maker on farmers' crop selection and production decisions in the earlier section, we now focus on the effect of crop dissimilarity r (i.e., substitutability or complementarity) on the optimal choice of crop MSPs and crop balance. Offering crop MSPs without understanding the degree of complementarity (or substitutability) between the crops being supported by the MSPs can destabilize the availability of those crops to the consumers. For instance, MSPs focused on wheat and rice (which are substitutes) caused a severe shortage of coarse cereals and oil seeds and an over-production of rice and wheat in the Indian economy (Chand, 2003; Parikh and Chandrashekhar, 2007). Hence, we note that it is important to explore the impact of r, which measures the "dissimilarity" between the two crops, on the choice of MSPs and the consequent production decisions of farmers.

Given the complexity of the above problem, we solve it numerically and draw some insights. The parameter values used in our numerical example are $a = 1$, $\rho = 0.7$, $p_{t-1}^A = 0.1$, $p_{t-1}^B = 0.9$, $\theta = 0.1$, $\eta = 0.3$, $B = 0.2$, $M = 1$ and $\epsilon_t^k \sim U[-0.1, 0.1]$. We take the "weight" $\lambda = 0.1, 0.5, 1$, which correspond to low, medium and high values, respectively. In our discussion we focus on the impact of crop dissimilarity (r) on the optimal choice of MSPs. We change r by varying α while retaining ρ constant (i.e., $\rho = 0.7$).

As shown in Figure 3.5, the optimal value of MSP for crop A is higher than that for crop B, for each value of λ, because our example is based on the case when the previous period price of crop A is lower than that of crop B (i.e., $0.1 = p_{t-1}^A < p_{t-1}^B = 0.9$). Because of this past price differential, more myopic farmers choose to grow crop B and so a larger MSP should be offered for crop A in order to entice a few of these farmers to switch to growing crop A from growing crop B. Furthermore, we notice that the optimal MSPs of the crops are increasing in r, which can be explained as follows. When r increases (i.e., α decreases while ρ is left unchanged), the expected prices of the crops increase, for any given production quantities of the crops.[11] Hence it is less likely that the realized market prices are lower than the crop MSPs. As such, the government can afford to increase the MSPs in order to benefit the farmers. Thus, for any given budget, government will be able to offer higher MSPs for complementary crops (like rice/wheat and pulses/vegetables) than for substitutable crops (like rice and wheat).

Furthermore, when a policy-maker gives higher importance to the welfare of the farmers (i.e., as λ increases), the crop MSPs also increase, because, when appropriately chosen, higher MSPs improve farmers' revenues. The case when $\lambda = 1$ corresponds to the extreme case when a policy-maker is concerned only about the welfare of the farmers but not at all about the welfare of the consumers.



(a) Crop A

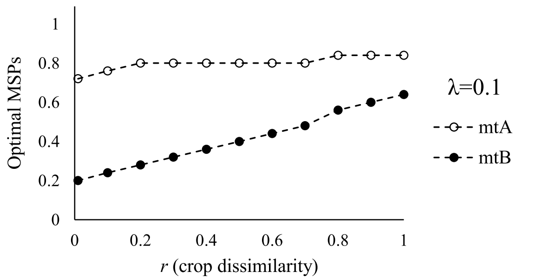
(b) Crop B

Figure 3.5: Optimal MSPs for crops A and B for low, medium, and high λ values.

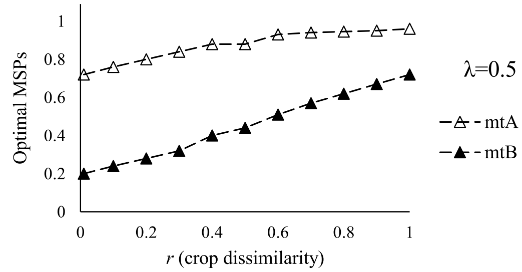
Next, the plots in Figure 3.6 indicate that the difference between the MSPs of crops A and B is decreasing in r, for any given value of λ. That is, as the complementarity between the crops (i.e., r) increases the crop MSPs have to be set in such a way that the difference between them decreases, in order to maintain a balance in crop production quantities. In other words, to maintain a balance of complementary crops (eg., rice and vegetables), a policy-maker should offer comparable MSPs for both the crops.

The total crop production quantities for our example are given in Figures 3.7a and 3.7b. (Note

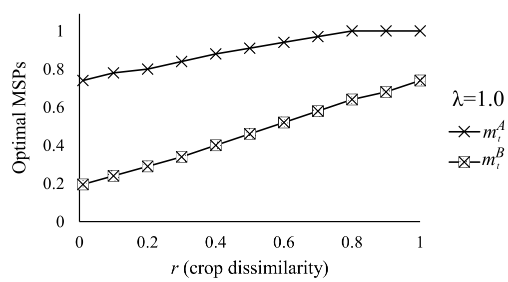
[11] Note that by differentiating (3.1) with respect to α, we obtain for $k \in \{A, B\}$ that $\frac{\partial \mathbb{E}[p_t^k]}{\partial \alpha} = -q_t^{jT} \leqslant 0$, $j \neq k$.



(a) $\lambda = 0.1$



(b) $\lambda = 0.5$

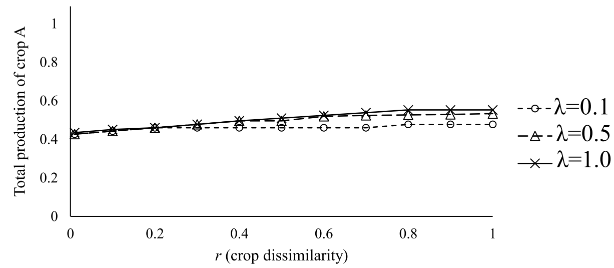


(c) $\lambda = 1$

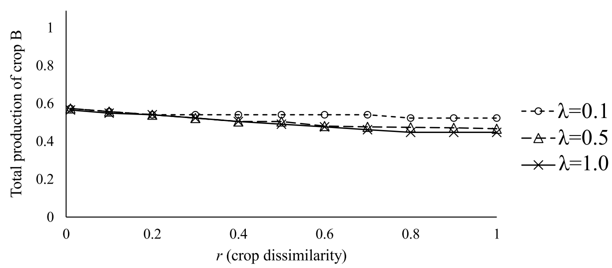
Figure 3.6: MSPs of crops A and B for low, medium, and high λ values.

that the production of each crop is approximately 0.5 so that the production of crops is balanced.) Furthermore, we can observe from Figures 3.7a and 3.7b that the crop production quantity disparity decreases as λ decreases because lower values of λ give more importance to consumer welfare, which increases as the production quantity disparity between the crops decreases (we omit separate plots for individual values of λ due to space constraints).

Finally, Figure 3.8 gives the plots of farmer surplus ($\mathscr{F}$), total expected expenditure incurred by policy-maker ($\mathscr{K}$), and social welfare ($\mathscr{W}$). (We omit the consumer disutility ($\mathscr{C}$) graph due to space constraints. The consumer disutility values can be easily obtained from Figures 3.7a and 3.7b by using the fact that $\mathscr{C} = -(q_t^{AT} - q_t^{BT})^2$). It is interesting to observe from Figure 3.8a that farmer surplus is increasing in crop disparity (r). This is due to the fact that, for any given production quantities of the crops, the expected prices of the crops increase as the complementarity



(a) Crop A



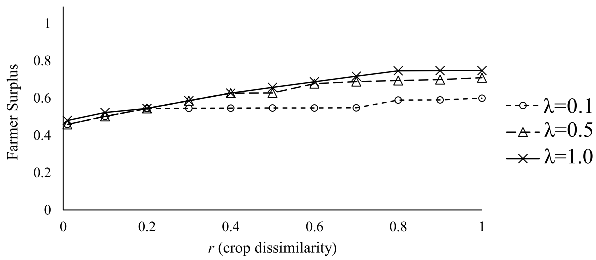
(b) Crop B

Figure 3.7: Total production of crops A and B for low, medium, and high λ values

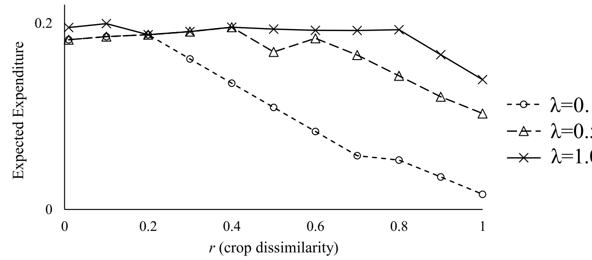
between the crops increases. From Figure 3.8b we observe that the total expenditure incurred by a policy-maker in administering the MSP program is decreasing in r, when r is sufficiently high. Because the expected market prices of the crops are high when r is high, in many instances the crop market prices tend to be higher than the crop MSPs, which obviates the need for the policy-maker to purchase the crop at MSP, thereby reducing the expected expenditure incurred from the MSP program. Hence, by combining the farmer surplus (Figure 3.8a) and expected expenditure (Figure 3.8b) plots, we can infer that a policy-maker will achieve a higher farmer surplus at a lower expense by offering MSPs to diverse crops. Finally, from Figure 3.8c, we observe that the total social surplus increases as r increases.

3.7 Conclusions

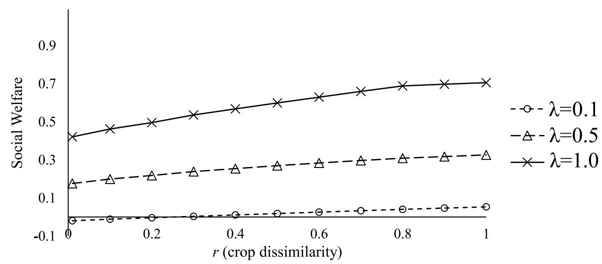
In this paper, we analyzed the role of *minimum support prices* (MSPs), which is a government intervention to safeguard farmers' incomes against crop price falls and, at the same time, to ensure



(a) Farmer surplus



(b) Expected expenditure



(c) Social welfare

Figure 3.8: Farmer surplus, expected expenditure and social welfare for low, medium, and high λ values

sufficient and balanced production of different crops. First, by considering a mixture of myopic and strategic farmers, we analyzed the behavior of myopic and strategic farmers, and their crop selection and production decisions, in the absence of MSPs. Later, we extended our analysis to incorporate MSPs and to analyze their impact, along with past prices, on farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits. Second, we discussed the impact of strategic farmers on farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits. By examining the interactions among a mixed population of myopic and strategic farmers for the case when there are two (complementary or substitutable) crops, we made the following findings.

First, we showed that, regardless of the MSPs offered to the crops, the price disparity between the crops always worsens as the complementarity between the crops increases. Second, we found that MSPs may not always be beneficial to farmers. We proved that when there are very few strategic farmers, an improper choice of MSP of a crop can negatively impact the profits of the farmers, both myopic and strategic, who grow that crop. This defeats the actual goal of MSP for a crop, which is to benefit the farmers who grow that crop. Third, we showed that the total production of a crop is increasing in the MSP offered for the crop and decreasing in the MSP offered for the other crop. Therefore, a carefully chosen MSPs can always be used to balance crop productions. Hence, to reduce quantity disparity between crops, a carefully designed MSP policy is critical.

Finally, we formulated the optimization problem of a policy-maker (i.e., government) with an objective to maximize social welfare (which is the sum of farmers' surplus and consumers' welfare less the policy-maker's expenditure) subject to a budgetary constraint on the expected expenditure incurred by the policy-maker in administering the MSP program. Given the complexity of the problem, we solved it numerically to draw a few practical insights, especially those pertaining to the impact of nature of crops (i.e., crop complementarity or dissimilarity) on the optimal choice of crop MSPs. First, we noted that, even though crop MSPs are increasing in the complementarity (or dissimilarity) between the crops, the difference between the crop MSPs decreases. Second, we observed that offering MSPs to dissimilar crops is efficient in achieving higher farmer surplus and higher social welfare at a lower expected expenditure. Hence, we inferred that it is more advantageous to offer MSPs to complementary crops like rice and pulses (or vegetables) than to offer MSPs to crops that are close substitutes like rice and wheat.

Our paper represents an initial attempt to examine the efficacy of MSPs of two (complementary or substitutable) crops in the presence of market price uncertainty and strategic farmers. However, there are plenty directions for future research. A natural and a challenging extension of our model is to incorporate multiple periods in the presence of hoarding; i.e., each farmer can sell his perishable crop over the next few period periods). In doing so, one can explore the impact of MSPs on the farmers crop planning and selling decisions over time. Another direction of future research is to examine the economic value of agricultural advisory services to farmers. Specifically, one can analyze the impact of long-term farming assistance programs that can enable farmers to take more strategic production decisions. Such a study will provide insights on the design and choice of such long-term programs vis-à-vis short-term (contingent) subsidy programs such as MSPs.

Appendices

Appendix A Appendix to Improving Supplier Compliance Through Joint and Shared Audits with Collective Penalty

A.1 Supplemental Results.

Lemma 6. *Under the joint mechanism J, the buyer's joint audit level z^J and the supplier's compliance level x^J given in* (1.10) *possess the following properties:*

(i) The supplier's compliance level is always higher than the buyer's audit level (i.e., $x^J = 2rz^J > z^J$).

(ii) Both the supplier's compliance level x^J and the buyer's audit level z^J are increasing in the buyer's damage cost d and are decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^J is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^J is increasing in γ.

(iv) The supplier's compliance level x^J is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^J is decreasing in g.

(v) The supplier's compliance level x^J is increasing in the wholesale price w. However, the buyer's audit level z^J is increasing in w if, and only if, $w < \sqrt{\alpha\gamma} - (d - p)$.

Lemma 7. *Under the shared mechanism S, the buyer's joint audit level z^S and the supplier's compliance level x^S given in Lemma 2 possess the following properties:*

(i) The supplier's compliance level is higher than the buyer's audit level (i.e., $x^S > z^S$).

(ii) Both the supplier's compliance level x^S and the buyer's audit level z^S are increasing in the buyer's damage cost d and are decreasing in the buyer's audit cost α.

(iii) The supplier's compliance level x^S is decreasing in the supplier's compliance cost γ. However, the buyer's audit level z^S is increasing in γ.

(iv) The supplier's compliance level x^S is increasing in the supplier's goodwill cost g. However, the buyer's audit level z^S is decreasing in g.

(v) The supplier's compliance level x^S is increasing in the wholesale price w. The buyer's audit level z^S is decreasing in w when w is sufficiently large.

Proposition 14. *The total supply chain profit under the shared mechanism is higher than that under the independent mechanism if the total cost of non-compliance for both buyers is larger than the cost of non-compliance for the supplier (i.e., $2(d-m) > g+w$) and $\alpha \geq \widetilde{\alpha}$, where $\widetilde{\alpha}$ is defined as in Proposition 1*

A.2 Proportional Sharing of Joint Audit Cost under the Joint Mechanism.

A.2.1 Exogenous Wholesale Prices

Here we provide the details of the non-cooperative game under the joint mechanism. To ensure that there is an implementable joint audit, we assume that the consortium will agree to adopt the "minimum-audit-level rule" that we describe shortly. This rule embodies the notion of the weakest link or minimum effort that underpins many coordination problems that are modeled as non-cooperative games, see Camerer (2003). Though this is one particular rule to reach consensus, it should be noted that the same results shown here below can be obtained by formulating the joint mechanism as a unanimous game, see Caro et al. (2015).

The buyers have to agree on the joint audit level and the audit cost sharing. In a non-cooperative setting, buyer i would have to propose an audit level z_i and a share θ_i of the audit cost. Hence, each buyer has a two-dimensional strategy space. Analyzing such kind of game is complex. Moreover, without additional structure the profit of buyer i might not be jointly concave in z_i and θ_i. To avoid these problems, recall that Equation (1.9) provides a one-to-one mapping between the share θ_i and buyer i's "ideal" joint audit level. We use this relation to reduce buyer i's strategy space to $\theta_i \in [0, 1]$ as shown next.

We now introduce the audit level selection process that is agreed upon by both buyers a priori. Specifically, the buyers play a game in which they simultaneously propose the share of the auditing cost each one of them would like to pay. In other words, buyer i proposes θ_i and buyer j proposes θ_j.[1] The outcome of the game is determined according to the following rules:

1. If $\theta_i \neq \theta_j$, then the audit level adopted by the consortium is $z = \min\{z_i(\theta_i), z_j(\theta_j)\}$, where $z_i(\theta_i)$ is given in Equation (1.9), and the total audit cost will be shared according to the proportion that is proposed by the buyer whose audit level is adopted.

2. If $\theta_i = \theta_j = \theta \geq \frac{1}{2}$, then the joint audit level is $z = z_i(\theta) = z_j(\theta)$ and each buyer pays a proportion θ of the auditing cost.

3. If $\theta_i = \theta_j < \frac{1}{2}$, then the consortium is not formed and the independent mechanism takes place.

Since $z_i(\theta_i) < z_j(\theta_j)$ if and only if $\theta_i > \theta_j$, the minimum-audit-level rule reduces to verifying which buyer is willing to pay a higher share of the auditing cost. With this audit selection process, buyer

[1]The supplier also participates in the game by simultaneously choosing the compliance level x.

i's profit can be written as:

$$\Pi_i^J(\theta_i;\theta_j,x) = \begin{cases} m(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i\,\alpha\,z_i(\theta_i)^2 & \text{if } \theta_i > \theta_j \\ m(1-z_i(\theta)(1-x)) - d(1-z_i(\theta))(1-x) - \theta\,\alpha\,z_i(\theta)^2 & \text{if } \theta_i = \theta_j = \theta \geqslant \frac{1}{2} \\ m(1-z_j(\theta_j)(1-x)) - d(1-z_j(\theta_j))(1-x) - (1-\theta_j)\,\alpha\,z_j(\theta_j)^2 & \text{if } \theta_i < \theta_j \\ \Pi^I(z^I) & \text{if } \theta_i = \theta_j < \frac{1}{2}. \end{cases} \tag{A.1}$$

The buyers' simultaneous actions θ_i and θ_j are essentially a coordination game and as such there are multiple equilibria (Fudenberg and Tirole 1991). In fact, any $\theta \in [0,1]$ such that $\theta_i = \theta_j = \theta$ corresponds to an equilibrium. To select one equilibrium point, we adopt the payoff dominance refinement proposed by Harsanyi and Selten (1988). Specifically, we show that the equilibrium $\theta_1 = \theta_2 = \frac{1}{2}$ in which the buyers equally share the joint audit cost is payoff dominant. This is formalized in Lemmas 8 and 9.

Lemma 8. *Under the minimum-audit-level rule, each buyer will agree to share the joint audit cost equally, i.e.,* $\theta_1 = \theta_2$ *in equilibrium.*

Lemma 9. *The payoff dominant equilibrium of the joint mechanism game is given by* $\theta_1 = \theta_2 = \theta = \frac{1}{2}$.

A.2.2 Endogenous Wholesale Prices

Note that since $z_i(\theta_i) = \frac{(d-m_i)(1-x)}{2\alpha\theta_i}$, we have $z_i(\theta_i) < z_j(\theta_j)$ if, and only if, $\frac{\theta_i}{d-m_i} > \frac{\theta_j}{d-m_j}$. Thus, the profit of buyer i under J with unequal wholesale prices and "minimum-audit-level rule" is given by

$$
\Pi_i^J(\theta_i;\theta_j,x) = \begin{cases} m_i(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i\alpha z_i(\theta_i)^2 & \text{if } \frac{\theta_i}{d-m_i} > \frac{\theta_j}{d-m_j} \\ m_i(1-z_i(\theta)(1-x)) - d(1-z_i(\theta))(1-x) - \theta\alpha z_i(\theta)^2 & \\ \qquad \text{if } \frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} = \frac{1}{2d-m_1-m_2} & \\ m(1-z_j(\theta_j)(1-x)) - d(1-z_j(\theta_j))(1-x) - (1-\theta_j)\alpha z_j(\theta_j)^2 & \text{if } \frac{\theta_i}{d-m_i} < \frac{\theta_j}{d-m_j} \\ \Pi^I(z^I) \ \text{ if } \frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} < \frac{1}{2d-m_1-m_2} & \end{cases} \tag{A.2}
$$

where $m_i = p - w_i$. In the last case, when $\frac{\theta_i}{d-m_i} = \frac{\theta_j}{d-m_j} < \frac{1}{2d-m_1-m_2}$, the consortium is not formed and each buyer resorts to an independent audit. The following lemmas are equivalent to Lemmas 8 and 9.

Lemma 10. *For a given wholesale prices w_1 and w_2, the buyers' equilibrium choice of θ_1 and θ_2 satisfy the condition $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$. Hence, the buyers choose the same audit level in equilibrium.*

Lemma 11. *The equilibrium given by $\theta_i = \frac{d-m_i}{2d-m_1-m_2}$, $i = 1, 2$, is payoff dominant.*

A.3 Numerical Study with $d >> g$

Here we present numerical results when the collateral penalty of the buyers d is much larger than the goodwill cost g experienced by the supplier. This scenario is arguably more realistic because in cases of non-compliance the market tends to punish more the buyers and put less blame on the supplier (due to the fact it is located in developing country). The following figures assume $d = 2g = 2000$. All the other parameters remain the same as in Sections 1.3 and 1.4.

Figures A.2 and A.1 show that the audit and compliance levels are higher compared to the

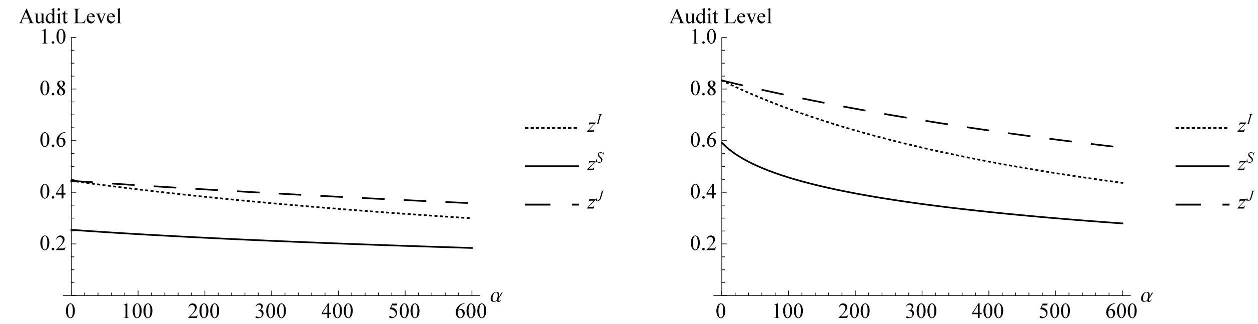


Figure A.1: Audit levels when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.

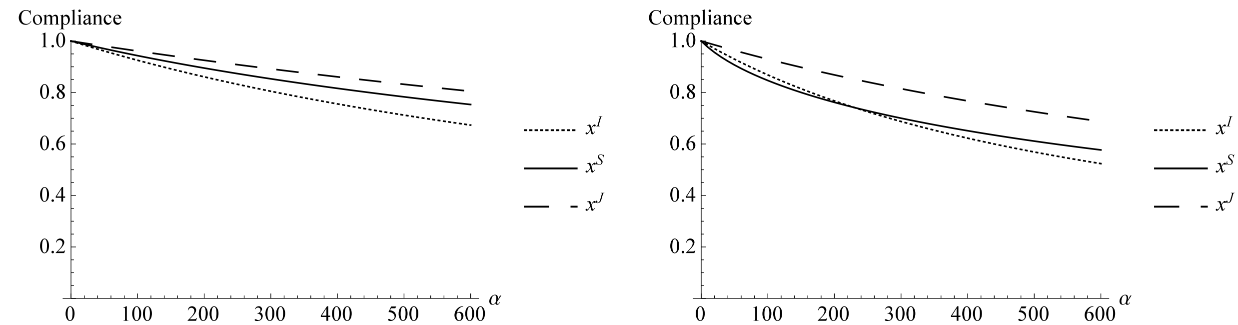


Figure A.2: Compliance levels when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.

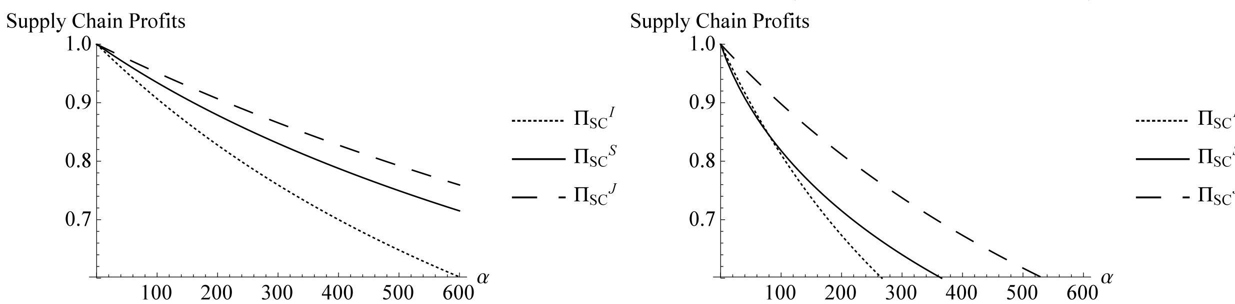


Figure A.3: Supply chain profits (normalized) when $d = 2g = 2000$. Left plot has $\gamma = 800$ and right plot has $\gamma = 1500$.

scenarios with $d = g = 1000$, especially for high values of the audit cost α. This follows from Lemmas 1, 6, and 7. Figure A.3 shows the supply chain profits. Note that $d > \gamma$ so from Proposition 4(ii) it follows that the joint mechanism achieves higher supply chain profits for all values of α. We omit the figures when the wholesale price is endogenous because they look very similar to the exogenous case. In contrast to Figure 1.9, when $d = 2000$ the wholesale price in equilibrium is constant for all relevant values of α. The reason is that a high penalty d pushes the buyers to audit more, which in turn increases the compliance level, so they do not have to use the wholesale price to incentivize the supplier. Consequently, the buyers lower the wholesale price as much as possible and the constraint $w_1 + w_2 - 2c \geq \gamma$ becomes active.

A.4 Proofs

Proof of Lemma 1: The first statement follows immediately from the fact that $2r > 1$. All other statements can be obtained from differentiating x^I and z^I given in (1.3) with respect to the corresponding parameter. However, to prove the last statement, one needs to account for the fact that $r = \frac{g+w}{2\gamma}$ and $m = (p - w)$. We omit the details. ■

Proof of Lemma 2: Observe from (1.15) that $z_1 = z_2 = z$ by symmetry and apply (1.15) to show that $x^S = 2rz^S(2 - z^S)$. This proves the second statement. Next, by substituting $x = 2r(2z - z^2)$ into (1.15) and by rearranging the terms, the buyer's audit level z is the solution to $V(z) = 0$. By showing that $V(0) < 0, V(1 - \sqrt{\frac{2r-1}{2r}}) > 0$ and $V(z)$ is concave over $[0, 1 - \sqrt{\frac{2r-1}{2r}}]$, we prove the first statement and that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$. Next, observe that $x^S(z) = 2rz(2 - z)$ is increasing in z when $z \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$, that $x^S(0) = 0$ and that $x^S(1 - \sqrt{\frac{2r-1}{2r}}) = 1$, we can use the fact that $z^S \in (0, 1 - \sqrt{\frac{2r-1}{2r}})$ to show that $x^S \in (0, 1)$. ■

Proof of Lemma 3: We have that $S^J - S^I = x^I(1 - x^I) - x^J(1 - x^J) = \frac{\alpha r(d-m)\left(2r^2(d-m)^2 - \alpha^2\right)}{(\alpha+2r(d-m))^2(\alpha+r(d-m))^2}$. Hence, $S^J > S^I$ if and only if $\sqrt{2}\, r(d - m) > \alpha$. Similarly, $S^S - S^J = x^J(1 - x^J) - x^S(1 - x^S) = (x^J - x^S)(1 - x^J - x^S)$, so that $S^S - S^J \to 0^-$ as $\alpha \to 0^+$. Further, we know: (i) from Lemmas 6 and 7 that $\frac{dx^J}{d\alpha} < 0$ and $\frac{dx^S}{d\alpha} < 0$, (ii) from Equation (1.10) that $\lim_{\alpha\to\infty} x^J = 0$, and (iii) from Proposition 1 that $0 \leqslant x^S < x^J$, which indicates that $\lim_{\alpha\to\infty} x^S = 0$. Hence, we conclude that there exists a threshold α_J such that $S^S - S^J < 0$ if and only if $\alpha < \alpha_J$.

When comparing S^I and S^S, we have $S^S - S^I = x^I(1-x^I) - x^S(1-x^S) = (x^I - x^S)(1-x^I - x^S)$.

By noting that $\dfrac{dx^I}{d\alpha} < 0$, $\dfrac{dx^S}{d\alpha} < 0$, $\lim_{\alpha\to\infty} x^I = 0$, and $\lim_{\alpha\to\infty} x^S = 0$, we conclude that there exists a threshold α_I such that $(1 - x^I - x^S) < 0$ if and only if $\alpha < \alpha_I$. Further, by Proposition 1, $x^S > x^I$ if and only if $\alpha > \widetilde{\alpha} = \max\{(d-m)(\widetilde{r} - r), 0\}$. Therefore, if $\widetilde{r} \leqslant r$, then $x^S > x^I$ for all $\alpha \geqslant 0$, and hence $S^S - S^I > 0$ if and only if $\alpha < \alpha_I$.

When $\widetilde{r} > r$, as $\alpha \to 0^+$ we have $(x^I - x^S) \to 0^+$ and $(1 - x^I - x^S) \to -1$. Thus, $S^S - S^I < 0$ when α is sufficiently small (i.e., when $\alpha < \min\{\alpha_I, \widetilde{\alpha}\}$) and sufficiently large (i.e., when $\alpha > \max\{\alpha_I, \widetilde{\alpha}\}$). On the other hand, when α is moderate (i.e., when $\min\{\alpha_I, \widetilde{\alpha}\} < \alpha < \max\{\alpha_I, \widetilde{\alpha}\}$) then $S^S - S^I > 0$. ■

Proof of Lemma 6: The proof follows the same approach as the proof for Lemma 1. We omit the details. ■

Proof of Lemma 7: To prove the first statement, we use the fact that $2r > 1$ and the fact that $z^S \in (0,1)$ to show that $x^S = 2rz^S(2 - z^S) > z^s + z^S(1 - z^S) > z^S$. To prove the second statement, we differentiate (1.16) with respect to $k \equiv \frac{2\alpha}{d-m}$ and apply the implicit function theorem, getting: $U(z^S) \cdot \frac{dz^S}{dk} + z^S = 0$, where $U(z) = [6rz^2 - 12rz + (1 + 4r + k)]$. By noting that $U(z)$ is increasing in z and that $U(0) > 0$ and $U(1 - \sqrt{\frac{2r-1}{2r}}) > 0$, we can conclude that $U(z^S) > 0$. Hence, we can conclude that $\frac{dz^S}{dk} < 0$. Also, observe that $\frac{dx^S}{dk} = 4r(1 - z^S) \cdot \frac{dz^S}{dk} < 0$. Combine these results with the fact that k in increasing in α and decreasing in d, we obtain the desirable properties about z^S and x^S as stated in the second statement.

To prove the third statement, differentiate (1.16) with respect to r and apply the implicit function theorem, getting: $U(z^S) \cdot \frac{dz^S}{dr} + W(z^S) = 0$, where $U(z)$ is defined above and $W(z) = 2z(z^2 - 3z + 2)$. By using the fact that both $U(z) > 0$ and $W(z) > 0$ for any $z \in (0,1)$, we can conclude that $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)} < 0$. Next, observe that $\frac{dx^S}{dr} = 2z^S(2 - z^S) + 4r(1 - z^S) \cdot \frac{dz^S}{dr}$. By substituting $\frac{dz^S}{dr} = -\frac{W(z^S)}{U(z^S)}$ and by rearranging the terms and by using the fact that $V(z^S) = 0$, it can be shown that: $\frac{dx^S}{dr} = \frac{2z^S \cdot (4r(z^S)^2 - 4rz^S + 3 + 2k)}{U(z^S)} = \frac{2z^S \cdot (-2x^S + 4rz^S + 3 + 2k)}{U(z^S)} > 0$, where the last inequality is due to the fact that $x^S < 1$. Finally, by combining the result that $\frac{dz^S}{dr} < 0$ and $\frac{dz^S}{dr} > 0$ and by using the fact that $r = \frac{g+w}{2\gamma}$, we obtain the third statement.

To prove the fourth statement, implicitly differentiating the equation $V(z^S) = 0$ and the expression for x^S with respect to r, we get:

$$\begin{aligned}
\frac{\partial z^S}{\partial r} &= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha - 6r(d-m)(2-z^S)z^S + (4r+1)(d-m)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)} \\
&= -\frac{2(d-m)(2-z^S)(1-z^S)z^S}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} < 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial x^S}{\partial r} &= 2(2-z^S)z^S + 4r(1-z^S)\frac{\partial z^S}{\partial r} \\
&= -\frac{2(z^S-2)z^S(2\alpha + 2r(d-m)(z^S-2)z^S + d - m)}{2\alpha + (d-m)(1-2rz^S(2-z^S)) + 4r(d-m)(1-(2-z^S)z^S)} \\
&= \frac{2(2-z^S)z^S(2\alpha + (d-m)(1-2r(2-z^S)z^S))}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} \\
&= \frac{2(2-z^S)z^S(2\alpha + (d-m)(1-x^S))}{2\alpha + (d-m)(1-x^S) + 4r(d-m)(1-\frac{x^S}{2r})} > 0.
\end{aligned}$$

It remains to prove the last statement. By noting that $r = \frac{g+w}{2\gamma}$ and $m = (p-w)$, we differentiate

(1.16) with respect to w and apply the implicit function theorem to get:

$$\frac{dz^S}{dw} = \frac{z^S\left(2\gamma\alpha - (2-z^S)(1-z^S)(d-p+w)^2\right)}{(d-p+w)\left[2\gamma\alpha + (d-p+w)(\gamma + 2(g+w) - 3(g+w)(2-z^S)z^S)\right]}$$
$$\Rightarrow \frac{dz^S}{dw} \geqslant 0 \Leftrightarrow 2\gamma\alpha - (2-z^S)(1-z^S)(d-p+w)^2 \geqslant 0$$

because, by using the fact that $x^S = 2rz^S(2-z^S) \leqslant 1$ it can be easily verified that the denominator of the above expression is positive. Now,

$$2\gamma\alpha - (2-z^S)(1-z^S)(d-p+w)^2 \geqslant 0 \Leftrightarrow (2-z^S)(1-z^S) \leqslant \frac{2\gamma\alpha}{(d-p+w)^2}$$
$$\Leftrightarrow 2rz^S(2-z^S)(1-z^S) \leqslant \frac{4r\gamma\alpha z^S}{(d-p+w)^2} = \frac{2\alpha z^S(g+w)}{(d-p+w)^2}$$
$$\Leftrightarrow 2r{z^S}^3 - 6r{z^S}^2 + 4rz^S + z^S\left[1 + \frac{2\alpha}{d-p+w}\right] \leqslant \frac{2\alpha z^S(g+w)}{(d-p+w)^2} + z^S\left[1 + \frac{2\alpha}{d-p+w}\right]$$
$$\Leftrightarrow 1 \leqslant z^S\left[1 + \frac{2\alpha}{d-p+w} + \frac{2\alpha(g+w)}{(d-p+w)^2}\right] \quad \text{by using (1.16)}$$
$$z^S \geqslant \left[1 + \frac{2\alpha}{d-p+w} + \frac{2\alpha(g+w)}{(d-p+w)^2}\right]^{-1} = [f(w)]^{-1} \text{ (say)}$$

By noting that $\alpha > 0$, $f'(w) < 0$, $\lim_{w\to\infty} f(w) = 1$, and $\lim_{w\to\infty} z^S = 0$ (from (1.16)) we infer that there exists a threshold value of w above which $z^S < f(w)^{-1}$, that is, $\frac{dz^S}{dw} < 0$.

Next, by noting that $x^S = 2rz^S(2-z^S)$ and by using the expression for $\frac{dz^S}{dw}$, we get:

$$\begin{aligned}\frac{dx^S}{dw} &= \frac{2(g+w)(1-z^S)\left(\frac{dz^S}{dw}\right) + (2-z^S)z^S}{\gamma}, \\ &= \frac{z^S}{\gamma}\left\{(2-z^S) + \frac{2(g+w)(1-z^S)\left[2\alpha\gamma - (2-z^S)(1-z^S)(d-p+w)^2\right]}{(d-p+w)\left[2\alpha\gamma + (d-p+w)\left(\gamma - 3(g+w)(2-z^S)z^S + 2(g+w)\right)\right]}\right\}\end{aligned}$$

It follows from Assumptions 2 and the fact that $r = \frac{g+w}{2\gamma} > \frac{1}{2}$ and $(d-m) = (d-p+w) > 0$, the denominator of the second term is also positive. Hence, the sign of the above term depends on the sign of the numerator alone. By expanding and rearranging the terms, the numerator can be simplified as: $\gamma\left(2\alpha\left[(d-m)(2-z^S)+4r\gamma(1-z^S)\right]+(d-m)^2(2-z^S)(1-x^S)\right)>0$, where the last inequality is due to the fact that both x^S and z^S are bounded above by 1. This completes our proof. ■

Proof of Lemma 8: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1 \leqslant \theta_2$. Suppose buyer 1's best response has $\theta_1 > \theta_2$. Then for every fixed compliance level x,

$$\begin{aligned}&\Pi_1^J(\theta_1;\theta_2,x) = m(1-z_1(\theta_1)(1-x)) - d(1-z_1(\theta_1))(1-x) - \theta_1\,\alpha\,z_1(\theta_1)^2\\ &\Rightarrow \frac{\partial \Pi_1^J(\theta_1;\theta_2,x)}{\partial\theta_1} = -\alpha\,z_1(\theta_1)^2 < 0.\end{aligned}$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1$. Hence, $\theta_1 = \theta_2$ in equilibrium. ■

Proof of Lemma 9: Given the symmetry of the buyers, we drop the indexes in this proof. By Lemma 8 it is true that the equilibrium under J comprises of symmetric cost sharing. Let $\Pi^J(\theta;\theta,x)$ be the profit under J when $\theta_i=\theta_j=\theta$ as obtained from from (A.1) and let x^J be the equilibrium compliance level when $\theta=\frac{1}{2}$. We prove that $\theta=\frac{1}{2}$ is the payoff dominant equilibrium.

Suppose $\theta<\frac{1}{2}$, then by (A.1) the profit of each buyer under J is given by (1.4). Let z^I and $x^I=2rz^I$ be the equilibrium audit and compliance levels in the independent mechanism. Then,

$$\Pi^I(z^I) < \Pi^I(z^I) + \frac{1}{2}\alpha {z^I}^2 = \Pi^J(\theta=\frac{1}{2};\theta=\frac{1}{2},x^I) \leqslant \Pi^J(\theta=\frac{1}{2};\theta=\frac{1}{2},x^J)$$

where x^J is the equilibrium compliance level in the joint mechanism with $\theta=\frac{1}{2}$, and the last inequality follows by noting that $\frac{\partial\Pi^J}{\partial x}=mz+d(1-z)>0$ (obtained from using Envelope theorem on (1.8)) and $x^J \geqslant x^I$. Hence, the equilibrium with $\theta < \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$.

Now, suppose $\theta > \frac{1}{2}$. Let $z^J(\theta)$ and $x^J(\theta) = 2rz^J(\theta)$ be the equilibrium audit and compliance levels. Then,

$$z^J(\theta) = \frac{(d-m)}{2\alpha\theta + 2r(d-m)} \Rightarrow \frac{dz^J(\theta)}{d\theta} < 0 \text{ and}$$

$$\Pi^J(\theta;\theta,x) = m(1 - z(\theta)(1-x)) - d(1-z(\theta))(1-x) - \theta\alpha z(\theta)^2$$

$$\Rightarrow \frac{d\Pi^J(\theta;\theta,x^J(\theta))}{d\theta} = -\alpha z^J(\theta)^2 + \frac{\partial \Pi^J}{\partial x} \cdot 2r\frac{dz^J(\theta)}{d\theta} < 0 \text{ since } \frac{\partial \Pi^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta)}{d\theta} < 0.$$

Hence, the equilibrium with $\theta > \frac{1}{2}$ is dominated by $\theta = \frac{1}{2}$. ■

Proof of Lemma 10: For a given θ_2 of buyer 2, we show by contradiction that buyer 1's best response must satisfy $\theta_1 \leqslant \theta_2\left(\frac{d-m_1}{d-m_2}\right)$. Suppose buyer 1's best response has $\theta_1 > \theta_2\left(\frac{d-m_1}{d-m_2}\right)$. Then,

$$\Pi_1^J(\theta_1;\theta_2,x) = m_1(1 - z_1(\theta_1)(1-x)) - d(1-z_1(\theta_1))(1-x) - \theta_1\alpha z_1(\theta_1)^2$$

$$\Rightarrow \frac{\partial \Pi_1^J(\theta_1;\theta_2,x)}{\partial \theta_1} = -\alpha z_1(\theta_1)^2 < 0.$$

Hence, buyer 1 sets θ_1 such that $\theta_1 \leqslant \theta_2\left(\frac{d-m_1}{d-m_2}\right)$. Similarly, buyer 2 sets θ_2 such that $\theta_2 \leqslant \theta_1\left(\frac{d-m_2}{d-m_1}\right)$. Hence, $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$ in equilibrium.

Clearly, for every given compliance level x of the supplier,

$$\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} \Rightarrow z_1(\theta_1) = \frac{(d-m_1)(1-x)}{2\alpha\theta_1} = \frac{(d-m_2)(1-x)}{2\alpha\theta_2} = z_2(\theta_2). \quad ■$$

Proof of Lemma 11: For ease of notation, let $\hat{\theta}_i = \frac{d-m_i}{2d-m_1-m_2}$. By Lemma 10 that in equilibrium $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2}$. Let $\Pi_i^J(\theta_i;\theta_j,x)$ be the profit of buyer i and x^J be the compliance level when $\theta_i = \hat{\theta}_i$ and $\theta_j = \hat{\theta}_j$. We prove that $\theta_i = \hat{\theta}_i,\ i = 1,2$ is the payoff dominant equilibrium. We argue for buyer i and the argument for buyer j is similar.

Suppose $\frac{\theta_1}{d-m_1} = \frac{\theta_2}{d-m_2} < \frac{1}{2d-m_1-m_2}$, then by (A.2) the profit of buyer i under J is given by

$$\Pi_i^I(z_i^I; z_j^I, x^I) = m_i(1 - z_i^I(1 - 2rz_i^I)) - d(1 - z_i^I)(1 - 2rz_i^I) - \alpha {z_i^I}^2. \tag{A.3}$$

Let z_i^I and $x^I = r_1 z_1^I + r_2 z_2^I$ be the equilibrium audit and compliance levels under independent audits. Then for buyer i we have

$$\begin{aligned}
\Pi_i^J(z^J) =& m_i(1 - z^J(1 - x^J)) - d(1 - z^J)(1 - x^J) - \hat{\theta}_i \alpha {z^J}^2 \\
\geqslant& m_i(1 - z_i^I(1 - x^J)) - d(1 - z_i^I)(1 - x^J) - \hat{\theta}_i \alpha {z_i^I}^2 \text{ since } z^J \text{ maximizes (1.25) for every} \\
& \text{fixed value of } x \\
\geqslant& m_i(1 - z_i^I(1 - x^I)) - d(1 - z_i^I)(1 - x^I) - \hat{\theta}_i \alpha {z_i^I}^2 \text{ because } x^I < x^J \text{ and} \\
& \frac{\partial \Pi_i^J}{\partial x} = mz + d(1 - z) > 0 \\
\geqslant& m_i(1 - z_i^I(1 - x^I)) - d(1 - z_i^I)(1 - x^I) - \alpha {z_i^I}^2 \text{ because } \hat{\theta}_i \in [0, 1] \\
=& \Pi_i^I(z_i^I),
\end{aligned}$$

and $x^J \geqslant x^I$ because

$$\begin{aligned}
& \theta_i = \hat{\theta}_i \Rightarrow x^J = \frac{(r_1 + r_2)(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)} \\
& \Rightarrow x^J - x^I = \frac{(r_1 + r_2)(2d - m_1 - m_2)}{2\alpha + (r_1 + r_2)(2d - m_1 - m_2)} - \frac{r_1(d - m_1) + r_2(d - m_2)}{2\alpha + r_1(d - m_1) + r_2(d - m_2)} \\
& = \frac{2\alpha((d - m_1)r_2 + (d - m_2)r_1)}{(2\alpha + (r_1 + r_2)(2d - m_1 - m_2))(2\alpha + r_1(d - m_1) + r_2(d - m_2))} \geqslant 0.
\end{aligned}$$

Hence, the equilibrium with $\theta_i < \hat{\theta}_i (\Leftrightarrow \theta_j < \hat{\theta}_j)$ is dominated by $\theta_i = \hat{\theta}_i (\Leftrightarrow \theta_j = \hat{\theta}_j)$. Now, suppose

$\theta_i > \hat{\theta}_i (\Leftrightarrow \theta_j > \hat{\theta}_j)$. Then

$$z^J(\theta_i) = \frac{(d-m_i)}{2\alpha\theta_i + (d-m_i)(r_1+r_2)} \Rightarrow \frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ and}$$

$$\Pi_i^J(\theta_i;\theta_j,x) = m_i(1-z_i(\theta_i)(1-x)) - d(1-z_i(\theta_i))(1-x) - \theta_i \alpha z_i(\theta_i)^2$$

$$\Rightarrow \frac{d\Pi_i^J(\theta_i;\theta_j,x^J(\theta_i))}{d\theta_i} = -\alpha z^J(\theta_i)^2 + \frac{\partial \Pi_i^J}{\partial x}\cdot(r_1+r_2)\frac{dz^J(\theta_i)}{d\theta_i} < 0 \text{ since } \frac{\partial \Pi_i^J}{\partial x} > 0 \text{ and } \frac{dz^J(\theta_i)}{d\theta_i} < 0.$$

Hence, the equilibrium with $\theta_i > \hat{\theta}_i$ is dominated by $\theta_i = \hat{\theta}_i$. ■

Proof of Proposition 1: Observe from (1.3) and (1.10) that $z^I = \frac{d-m}{2(\alpha+r(d-m))} < \frac{d-m}{\alpha+2r(d-m)} = z^J$. Next, by sustituting $z^I = \frac{d-m}{2(\alpha+r(d-m))}$ into (1.16) and by rearranging the terms, one can show that $V(z^I) = 2(2r+1)\alpha^2 + 2r(d-m)(4r-1)\alpha + r(1-2r)^2(d-m)^2 > 0 = V(z^S)$. By using the fact that $V(z)$ is increasing in z, we can conclude that $z^I > z^S$. Therefore, we prove the first statement: $z^S < z^I < z^J$.

Noting that $x^J = 2rz^J$ and $x^I = 2rz^I$, it follows that $x^J > x^I$.

Before we proceed further, we define the function $L(z) = z(2-z)$, which is an inverted parabola with roots 0 and 2, and mode at 1, for better exposition and shorthand notation.

In the region $[0,1]$, we note that $L(z) > z^J \Leftrightarrow z > \overline{z}$ where $\overline{z}$ is the solution of $L(z) = z^J$. The solution is given by $\overline{z} = 1 - \frac{\sqrt{\alpha + (2r-1)(d-m)}}{\sqrt{\alpha + 2r(d-m)}}$ and, hence $\frac{(d-m)V(\overline{z})}{\alpha} = 2 - 2\sqrt{1-z^J} - z^J\sqrt{1-z^J} = 2 - (2-z^J)\sqrt{1-z^J} > 0$. Thus, $\overline{z} > z^S \Leftrightarrow z^J = L(\overline{z}) > L(z^S) \Leftrightarrow x^J > x^S$.

Similarly, to compare x^I and x^S, we need to compare z^I and $z^S(2-z^S)$. To compare z^I and z^S we consider the solution of the equation $L(z) = z^I$ in the region $[0,1]$. On solving, we get $\underline{z} = 1 - \sqrt{\frac{2\alpha + (2r-1)(d-m)}{2(\alpha + r(d-m))}}$. Now, in order to compare $\underline{z}$ and z^S, we consider $V(\underline{z})$. On substituting the value of $\underline{z}$ in $V(z)$ we get $\dfrac{(d-m)V(\underline{z})}{\alpha} = 1-(1+z^I)\sqrt{1-z^I}$. Hence,

$$V(\underline{z}) > 0 \Leftrightarrow z^I \geqslant \frac{\sqrt{5}-1}{2} \Leftrightarrow (d-m)\left[1-r(\sqrt{5}-1)\right] \geqslant (\sqrt{5}-1)\alpha.$$

When $r \geqslant \frac{1}{\sqrt{5}-1}$, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2-z^S) \Leftrightarrow x^I < x^S$. On the other hand, if r is small (i.e., $r < \frac{1}{\sqrt{5}-1}$) and when α is sufficiently small then $V(\underline{z}) > 0 \Leftrightarrow \underline{z} > z^S \Leftrightarrow z^I = L(\underline{z}) > L(z^S) = z^S(2-z^S) \Leftrightarrow x^I > x^S$. And, when r is small but α is sufficiently large, then $V(\underline{z}) < 0 \Leftrightarrow \underline{z} < z^S \Leftrightarrow z^I = L(\underline{z}) < L(z^S) = z^S(2-z^S) \Leftrightarrow x^I < x^S$. This concludes the proof.

■

Proof of Proposition 2: First, it follows from (1.5) and (1.12) that $\pi_s^I(z^I) - \pi_s^J(z^J) = \gamma[(1-2r\cdot z^I)+(1-2r\cdot z^J)]\cdot[2r(z^J-z^I)]$. By applying the first statement of Proposition 1 (i.e., $z^J > z^I$), we prove the first statement. By using the same approach, we obtain the second statement. Finally, observe from (1.5) and (1.12) that $\pi_s^I(z^I)-\pi_s^S(z^S) = \gamma[(1-x^I)+(1-x^S)]\cdot(x^S-x^I)$. We prove the third statement by applying (2) and (3) of Proposition 1 (i.e., $x^S > x^I$ when α is sufficiently large). This completes our proof. ■

Proof of Proposition 3: First, we note that from (1.2), we note that for every fixed audit level z_i of buyer i, the buyer's profit is increasing in the supplier's compliance level x. That is:

$$\frac{\partial \Pi_i(z_i; z_j, x)}{\partial x} = mz_i + d(1-z_i) > 0. \tag{A.4}$$

Now, the joint mechanism profits at the payoff-maximizing equilibrium $\theta_1 = \theta_2 = \frac{1}{2}$ is

$$\begin{aligned}
\Pi_i^J(z^J) &= m(1 - z^J(1 - x^J)) - d(1 - z^J)(1 - x^J) - \frac{1}{2}\,\alpha\, {z^J}^2 \\
&\geqslant m(1 - z^I(1 - x^J)) - d(1 - z^I)(1 - x^J) - \frac{1}{2}\,\alpha\, {z^I}^2 \\
&\qquad\qquad\qquad \text{since } z^J \text{ maximizes } \Pi_i^J(z; x^J) \\
&\geqslant m(1 - z^I(1 - x^I)) - d(1 - z^I)(1 - x^I) - \frac{1}{2}\,\alpha\, {z^I}^2 \quad \text{using (A.4) and } x^J \geq x^I \\
&= \Pi_i^I(z^I) + -\frac{1}{2}\,\alpha\, {z^I}^2 \; > \Pi_i^I(z^I)
\end{aligned}$$

Next, it follows from (1.4) and (1.17), we get: $\Pi^I(z^I) - \Pi^S(z^S) = \alpha\left({z^S}^2 - {z^I}^2\right) + \left(z^I - (2 - z^S)z^S\right) T_I(z^S)$, where $T_I(z^S) = 2r(d-m)(z^S)^2 - 4r(d-m)z^S + (d-m)(1-2rz^I) + 2dr > 0$. By noting that the term $T_I(z^S) > 0$ for $z^S \in (0, 1)$, we can prove our second statement by applying Proposition 1 to show that the terms $\left((z^S)^2 - (z^I)^2\right)$ and $\left(z^I - (2 - z^S)z^S\right)$ are both negative. This proves second statement. ■

Proof of Proposition 4:

$$[2\,\Pi^J + \pi_s^J] - [2\,\Pi^I + \pi_s^I] = \frac{\alpha(d - m)}{2(\alpha + 2r(d - m))^2(\alpha + r(d - m))^2}\, f(\alpha)$$

where $f(\alpha) = (d - m + 4r(d - \gamma))\,\alpha^2 + 6r^2(2d - \gamma)(d - m)\alpha + 2r^2(d - m)^2\,(4dr - (d - m))$, which is a quadratic in α. Note that $f(0) > 0$ always and $f(\alpha)$ is continuous in α. It follows that $f(\alpha) > 0$ for α sufficiently low. This proves the first statement. For the second statement: when $d > \gamma$, we have $f(\alpha) > 0$. Finally, for the third statement: if $2d > \gamma$, then $f'(0) > 0$. Further, if $2d > \gamma$ and $d - m > g + w$, then we get $f''(\alpha) = 2\,[d - m + 4r(d - \gamma)] \geqslant 2\,[d - m - 2r\gamma] = 2\,[(d - m) - (g + w)] > 0$, which indicates that

f is convex. Thus, $f > 0$ for all positive values of α. This completes the proof. ■

Proof of Proposition 14:

$$\begin{aligned}[2\,\Pi^S + \pi_s^S] - [2\,\Pi^I + \pi_s^I] \quad &= \left(\frac{d-m}{r} - \gamma\right)(x^S - x^I)(2 - x^I - x^S) + 2\left(d - \frac{d-m}{2r}\right)(x^S - x^I) \\ &\quad + 2\alpha(z^I - z^S)(z^I + z^S)\end{aligned}$$

From Proposition 1, the last term in the above expression is positive. If $\frac{d-m}{r} > \gamma$ ($\Leftrightarrow d - m > \frac{g+w}{2}$) then the first term in brackets is always positive. Hence, if the compliance of supplier under S is higher than that under I, then $2\,\Pi^S + \pi_s^S > 2\,\Pi^I + \pi_s^I$. From Proposition 1, $x^S > x^I$ if and only if $\alpha \geq \widetilde{\alpha}$. This concludes the proof. ■

Appendix B Coordinating Supply Chains via Advance-Order Discounts, Minimum Order Quantities, and Delegations

B.1 Proofs

Lemma 12. *The manufacturer's profit $\Pi_1^d(\delta)$ given in (2.7) is decreasing in δ. Also, the supplier's profit $\Pi_s^d(\delta)$ given in (2.8) is decreasing in δ when δ approaches 1 from the right and increasing in δ when δ approaches δ^* from the left.*

Proof of Proposition 5: By applying Lemma 12 (that the manufacturer's profit $\Pi_1^d(\delta)$ is decreasing in δ) we conclude the manufacturer will always participate in the discount contract for any $\delta \in (0,1)$. Next, we see from (2.8) that

$$\begin{aligned}\frac{d\Pi_s^d}{d\delta} &= rx_d^* + [(\delta r - c) - (r-e)(1-F(x_d^*))]\frac{dx_d^*}{d\delta} \\ &= rx_d^* + [(\delta r - c) - (r-e)(1-F(x_d^*))]\left(\frac{-r}{(r-s)f(x_d^*)}\right) \\ &= rx_d^* + \left[(\delta r - c) - (r-e)\left(\frac{\delta r - s}{r-s}\right)\right]\left(\frac{-r}{(r-s)f(x_d^*)}\right). \qquad \text{(B.1)}\end{aligned}$$

By using the fact that $\frac{d\Pi_s^d}{d\delta}|_{\delta=1} < 0$ and $\frac{d\Pi_s^d}{d\delta}|_{\delta=\delta^*} > 0$, and that the supplier's profit function $\Pi_s^d(\delta)$ is unimodal because the probability distribution of D satisfies the IGFR properties (c.f., Lariviere (2006)), we can conclude that (a) $\hat{\delta} \in (\delta^*, 1)$; and (b) $\hat{\delta}$ satisfies the first order condition so that $\frac{d\Pi_s^d}{d\delta} = 0$. This proves the first statement.

To prove the second statement, let us first show the uniqueness of the solution of $\frac{d\Pi_s^d}{d\delta} = 0$ for an IGFR demand distribution. Let $g(\cdot)$ denote the generalized failure rate of the demand distribution, where $g(x) = \frac{xf(x)}{1-F(x)}$. We rewrite (B.1) as follows:

$$\begin{aligned}
\frac{d\Pi_s^d}{d\delta} &= rx_d^* + [(\delta r - c) - (r-e)(1 - F(x_d^*))]\left(\frac{-r}{(r-s)f(x_d^*)}\right) \\
&= \frac{r(1-F(x_d^*))}{f(x_d^*)}\left[\frac{x_d^* f(x_d^*)}{(1-F(x_d^*))} - \frac{\delta r - c}{(r-s)(1-F(x_d^*))} + \frac{r-e}{r-s}\right] \\
&= \frac{r(1-F(x_d^*))}{f(x_d^*)}[U(\delta) - V(\delta)]
\end{aligned}$$

where $U(\delta) = g(x_d^*) + \frac{r-e}{r-s}$ and $V(\delta) = \frac{\delta r - c}{\delta r - s}$. Hence, $\frac{d\Pi_s^d}{d\delta} = 0$ if, and only if, $U(\delta) = V(\delta)$.

By differentiating U and V with respect to δ, by noting that $g(x)$ is increasing in x, and x_d^* is decreasing in δ, we can conclude that $\frac{dU}{d\delta} = g'(x_d^*)\frac{dx_d^*}{d\delta} < 0$ and $\frac{dV}{d\delta} = \frac{r(c-s)}{(\delta r - s)^2} > 0$. Combining this with knowledge that $\frac{d\Pi_s^d}{d\delta}\big|_{\delta=\delta^*} = U(\delta^*) - V(\delta^*) > 0$ and $\frac{d\Pi_s^d}{d\delta}\big|_{\delta=1} = U(1) - V(1) < 0$, we conclude that there exists a unique $\hat{\delta}$ such that $\frac{d\Pi_s^d}{d\delta}\big|_{\delta=\hat{\delta}} = U(\hat{\delta}) - V(\hat{\delta}) = 0$.

Before we prove the remainder of the second statement for the case when D is $N(\mu, \sigma^s)$, let us prepare some prerequisites. First, because $F(\cdot)$ follows the normal distribution with mean μ and standard deviation σ, the manufacturer's initial order quantity $x_d^*(\delta)$ given in (2.6) can be simplified as $x_d^*(\delta) = \mu + k\sigma$, where $k = \Phi^{-1}\left(\frac{(1-\delta)r}{r-s}\right)$. For notational convenience, we suppress the argument that k is a function of δ. Second, we use the fact that $\int_k^\infty z\phi(z)dz = \phi(k)$ to simplify the supplier's profit function $\Pi_s^d(\delta)$ given in (2.10) as:

$$\Pi_s^d(\delta) = (\delta r - c)(\mu + k\sigma) + \sigma(r-e)[\phi(k) - k(1 - \Phi(k))]. \tag{B.2}$$

Third, by using the result established in Lemma 1 of Brown and Tang (2006) that $\frac{d\phi(z)}{dz} = -z\phi(z)$ and $\frac{d\Phi^{-1}(z)}{dz} = \frac{1}{\phi(\Phi^{-1}(z))}$ and by considering (B.2), it can be shown that the optimal discount $\hat{\delta}$

satisfies the first order condition $\frac{d\Pi_s^d(\delta)}{d\delta} = 0$, which can be expressed as:

$$r(\mu + k\sigma) + (\delta r - c)\sigma k' - (r - e)\sigma(1 - \Phi(k))k' = 0, \tag{B.3}$$

where $k' = \frac{dk}{d\delta}$. By letting $k^{''} = \frac{d^2 k}{d\delta^2}$, we can use the fact that $\Pi_s^d(\delta)$ is concave in δ and $\frac{d^2\Pi_s^d(\delta)}{d\delta^2} \leq 0$ to show that:

$$2r\sigma k' + (\delta r - c)\sigma k^{''} - (r - e)\sigma(1 - \Phi(k))k^{''} + (r - e)\sigma\phi(k)[k']^2 \leq 0. \tag{B.4}$$

We now establish our proof. Because $\hat{\delta}$ satisfies (B.3), we can apply the implicit function theorem to differentiate (B.3) with respect to σ, getting (after some algebra):

$$\begin{aligned} &\big(rk + (\delta r - c)k' - (r - e)(1 - \Phi(k))k'\big) \\ &+ \Big(2r\sigma k' + (\delta r - c)\sigma k^{''} - (r - e)\sigma(1 - \Phi(k))k^{''} + (r - e)\sigma\phi(k)[k']^2\Big) \cdot \frac{d\hat{\delta}}{d\sigma} = 0. \end{aligned} \tag{B.5}$$

By applying (B.3), it can be easily checked that the first term is negative. Also, by applying (B.4), the second term is also negative. Consequently, after re-arranging terms, we can conclude that $\frac{d\hat{\delta}}{d\sigma} < 0$. This completes our proof for the second statement.

It remains to prove the third statement by showing that the optimal discount contract $\hat{\delta}$ is Pareto-improving. First, consider the case when $\delta = 1$. In this case, $x_d^* = F^{-1}\left(\frac{(1-\delta)r}{r-s}\right) = 0$ because, without discount, the manufacturer has no incentive to place the early order. Consequently, when $\delta = 1$, the supplier's problem reduces to the base case so that the manufacturer's profit $\Pi_1^d(1) = \Pi_m^o$ and the supplier's profit $\Pi_s^d(1) = \Pi_s^o$. Second, by noting that $\hat{\delta} < 1$ and that the manufacturer's profit $\Pi_1^d(\delta)$ is decreasing in δ, we can conclude that $\Pi_1^d(\hat{\delta}) > \Pi_1^d(1) = \Pi_m^o$. Hence, the manufacturer is better off under the supplier optimal discount contract. Finally, by noting that both $\delta = \hat{\delta}$ and $\delta = 1$ are feasible solutions to the supplier's problem given in (2.10) and that $\delta = \hat{\delta}$ is the optimal solution, we can conclude that the supplier is also better off under the optimal discount contract $\hat{\delta}$. ■

Proof of Proposition 6: Recall from Lemma 12 and Proposition 5 that $\frac{d\Pi_s^d}{d\delta}$ is positive when $\delta = \delta^*$, negative when $\delta = 1$, and zero when $\delta = \hat{\delta}$. Hence, we conclude that $\hat{\delta} > \delta^*$. ■

Proof of Proposition 7: We first solve the supplier problem to determine the optimal discount δ for any given q. Let us consider the supplier's problem (2.15) for the case when $x_q^*(\delta, q) = q$ (which will occur if $q \geqslant x_d^*(\delta) = F^{-1}\left(\frac{(1-\delta)r}{r-s}\right)$). For any given q, it is easy to check the supplier's profit function is increasing in δ. Also, the manufacturer's profit function, $pE(D) - rE[D-q]^+ + sE[q-D]^+ - \delta rq$, is decreasing in δ. These two observations imply that the manufacturer's participation constraint, $\Pi_1^q \geq \Pi_m^o$, is binding for any optimal $\tilde{\delta}$ so that:

$$\tilde{\delta} = \frac{r[E(D) - E[D-q]^+] + sE[q-D]^+}{rq}.$$

By using $r > c > s$, it is easy to check that $\tilde{\delta} \in (\frac{c}{r}, 1)$.

By substituting $\tilde{\delta} = \frac{r[E(D)-E[D-q]^+]+sE[q-D]^+}{rq}$ into the supplier's profit function, we obtain:

$$\Pi_s^q(\tilde{\delta}, q) = q(e-c) + (r-e)E(D) + (s-e)E[q-D]^+. \tag{B.6}$$

By considering the first order condition with respect to q, we can show that the optimal $\tilde{q}$ satisfies (2.17). Through direct substitution, we can show that the optimal $\tilde{\delta}$ is as given in (2.16).

Using $(D-x)+[x-D]^+ = [D-x]^+$, the supply chain profit under the combined contract $(\tilde{\delta}, \tilde{q})$

can be simplified as:

$$\begin{aligned}\Pi(\tilde{\delta},\tilde{q}) &= \Pi_1(\tilde{\delta},\tilde{q}) + \Pi_s(\tilde{\delta},\tilde{q}) \\ &= [pE(D) - rE[D-\tilde{q}]^+ + sE[\tilde{q}-D]^+ - \tilde{\delta}r\tilde{q}] \\ &\quad + \tilde{q}(\tilde{\delta}r - c) + (r-e)(E(D) - \tilde{q} + E[\tilde{q}-D]^+) \\ &= -c\tilde{q} + pE(D) - eE[D-\tilde{q}]^+ + sE[\tilde{q}-D]^+ = \Pi^c.\end{aligned} \tag{B.7}$$

The last equality is due to the fact that the optimal $\tilde{q}$ given in (2.17) is equal to the initial order quantity in the centralized system x_c^* as given in (2.3). This implies that, if $\tilde{q} \geqslant x_d^*(\tilde{\delta}) = F^{-1}\left(\frac{(1-\tilde{\delta})r}{r-s}\right)$ then a minimum-order quantity of $\tilde{q}$ is imposed by the supplier on the manufacturer. We may then conclude that the combined contract $(\tilde{\delta},\tilde{q})$ is an optimal coordinating contract for two reasons: (a) it coordinates the supply chain so that $\Pi(\tilde{\delta},\tilde{q}) = \Pi^c$ and (b) it enables the supplier to extract the entire surplus from the manufacturer so that $\Pi_1^d(\tilde{\delta},\tilde{q}) = \Pi_m^o$. Also, because the combined contract $(\tilde{\delta},\tilde{q})$ enables the supplier to achieve the highest possible profit, we do not need to consider the case when $x_q^*(\delta,q) = F^{-1}\left(\frac{(1-\delta)r}{r-s}\right) \geqslant q$.

We now complete our proof by showing that $\tilde{q} \geqslant x_d^*(\tilde{\delta}) = F^{-1}\left(\frac{(1-\tilde{\delta})r}{r-s}\right)$. By considering the fact that $\tilde{q} = x_c^* \equiv F^{-1}\left(\frac{e-c}{e-s}\right)$ and from (2.13) that $x_q^*(\tilde{\delta},\tilde{q}) = \max\{F^{-1}\left(\frac{(1-\tilde{\delta})r}{r-s}\right),\ \tilde{q}\}$, we aim to show that $\tilde{\delta} \geqslant \delta^* \equiv 1 - \frac{r-s}{r}\left(\frac{e-c}{e-s}\right)$, where δ^* corresponds to the supplier optimal discount contract as examined in Section 2.4.1. We show this by contradiction. Suppose not so that $\tilde{\delta} < \delta^*$. Hence, by using the fact that the manufacturer's profit $pE(D) - rE[D-q]^+ + sE[q-D]^+ - \delta rq$ is decreasing in δ for any q, $\tilde{\delta} < \delta^*$ and the fact that $\tilde{q} = x_c^*$ imply that $pE(D) - rE[D-x_c^*]^+ + sE[x_c^*-D]^+ - \tilde{\delta}rx_c^* > pE(D) - rE[D-x_c^*]^+ + sE[x_c^*-D]^+ - \delta^* rx_c^* \geqslant \Pi_m^o$, where the last weak inequality comes from the fact that $x_d^* = x_c^*$ when $\delta = \delta^*$ and $x = 0$ is a feasible point in problem (2.5). This contradicts the fact that the manufacturer's participation constraint is binding under the optimal combined contract $(\tilde{\delta},\tilde{q})$. ■

Proof of Corollary 1: When $D \sim N(\mu, \sigma^2)$, it is easy to check from (2.17) that the minimum order quantity $\tilde{q} = \mu + k\sigma$, where $k = \Phi^{-1}\left(\frac{e-c}{e-s}\right)$. By substituting $\tilde{q} = \mu + k\sigma$ into (2.16) and by using $\int_{-\infty}^{k} z\phi(z)dz = -\phi(k)$ and $\int_{k}^{\infty} z\phi(z)dz = \phi(k)$, it can be shown that $\tilde{\delta} = 1 - \frac{(r-s)(\phi(k)+k\Phi(k))\sigma}{r(\mu+k\sigma)}$ after some algebra. All other results can be obtained immediately by differentiating $\tilde{\delta}$ with respect to μ and σ. We omit the details. ■

Proof of Proposition 8: Before we prove our result, when $D \sim N(\mu, \sigma^2)$, x^* given in (2.22) can be simplified as $x^* = \mu + k\sigma$, where $k = \Phi^{-1}\left(\frac{e-c}{e-s}\right)$. Also, by transforming D into a standard normal random variable and by using $\int_{-\infty}^{k} z\phi(z)dz = -\phi(k)$ and $\int_{k}^{\infty} z\phi(z)dz = \phi(k)$ it can be shown that the supplier's profit given in (2.23) can be simplified to

$$\Pi_{s,1}^{g}(\theta) = (\theta r - c)\mu - \sigma(e-s)\phi(k). \tag{B.8}$$

To ensure the existence of a delegation contract $\theta < 1$ that can ensure $\Pi_{s,1}^{g}(\theta) \geqslant (r-e)E(D)$, we can use (B.8) to show that such a delegation contract exists if, and only if:

$$(\theta r - c)\mu - (e-s)\sigma\phi(k) \geqslant (r-e)\mu, \text{ and } \theta < 1. \tag{B.9}$$

By rearranging the terms, the above conditions can be simplified as:

$$\theta \geqslant 1 - \frac{e-c}{r} + \frac{\sigma}{\mu} \cdot \frac{e-s}{r} \cdot \phi(k), \text{and } \theta < 1.$$

Hence, we can conclude that a delegation contract with $\theta < 1$ can exist if, and only if,

$$1 > 1 - \frac{e-c}{r} + \frac{\sigma}{\mu} \cdot \frac{e-s}{r} \cdot \phi(k).$$

By rearranging the terms and by using the fact that $\frac{e-c}{e-s} = \Phi(k)$, we obtain the desired result. ■

Proof of Lemma 12: Observe from (2.6) that $\frac{dx_d^*}{d\delta} = -\frac{r}{(r-s)f(x_d^*)}$ so that x_d^* is decreasing in δ. By using the chain rule, we can differentiate the manufacturer's profit under the discount contract ($\Pi_1^d(\delta)$, given in (2.7)) with respect to δ,

$$\begin{aligned}\frac{d\Pi_1^d}{d\delta} =& \frac{dx_d^*}{d\delta} \cdot (r(1-\delta)(1-F(x_d^*)) \\ & - (\delta r - s)F(x_d^*)) - r\left(E(D) - E[D - x_d^*]^+ + E[x_d^* - D]^+\right).\end{aligned}$$

By using the fact that $F(x_d^*) = \frac{(1-\delta)r}{r-s}$, it is easy to check that the first term equals zero. Then by noting that $E(D - [D - x_d^*]^+ + [x_d^* - D]^+) > 0$ for any realized value of D, we can conclude that the manufacturer's profit Π_1^d under the discount contract is strictly decreasing in δ as $\frac{d\Pi_1^d}{d\delta} < 0$. This proves the first statement.

To prove the second statement, we only need to show $\frac{d\Pi_s^d(\delta)}{d\delta} < 0$ when $\delta < 1$ and $\frac{d\Pi_s^d(\delta)}{d\delta} > 0$ when $\delta = \delta^*$. From the supplier's profit function, we obtain

$$\begin{aligned}\frac{d\Pi_s^d}{d\delta} &= rx_d^* + ((\delta r - c) - (r - e)(1 - F(x_d^*)))\,\frac{dx^*}{d\delta} \\ &= rx_d^* + ((\delta r - c) - (r - e)(1 - F(x_d^*)))\left(\frac{r}{(s-r)f(x_d^*)}\right)\end{aligned}$$

Further, $\lim_{\delta\to 1} F(x_d^*) = 0$, which implies that $x_d^* \to 0$ as $\delta \to 1$. This is intuitive because if there is zero discount offered for the advance order then the manufacturer prefers to place the order with the supplier after its demand is realized. Therefore,

$$\lim_{\delta\to 1}\frac{d\Pi_s}{d\delta} = r\cdot\lim_{\delta\to 1}\{x_d^* - \frac{((\delta r - c) - (r - e)(1 - F(x_d^*)))}{(r-s)f(x_d^*)}\} = \frac{r(e-c)}{(s-r)f(0)} < 0.$$

On the other hand, the supplier's profit when $\delta = \delta^* + \epsilon$, for some ϵ that is positive but sufficiently close to 0, we have

$$\begin{aligned}\frac{d\Pi_s}{d\delta} &= rx_d^* + (\epsilon - (r-e)(1-F(x_d^*)))\frac{dx_d^*}{d\delta} \\ &= rx_d^* + \left(\epsilon - (r-e)\left(1 - \frac{r-c+\epsilon}{r-s}\right)\right)\frac{dx_d^*}{d\delta} \\ &= rx_c^* > 0\end{aligned} \tag{B.10}$$

as $\epsilon \to 0$. The last equality is obtained by using the fact that $\lim_{\delta\to\delta^*} x_d^*(\delta) = x_c^*$. ■

Appendix B: Supplier's Over-production Strategy During the First-Period

In this section, we analyze the case when the supplier can produce, during the first period, more than the quantity x_d^* that the manufacturer orders. The supplier's profit is given by (2.9) and the supplier's optimization problem is to maximize its profit with respect to its decision variables: the first-period production quantity (z), and the advance-order wholesale price discount (δ). By noting that $z \geqslant x_d^*$, (2.9) can be written as

$$\begin{aligned}\Pi_s^d(z,\delta) &= \begin{cases}\delta r x_d^* - cz + rE[D-x_d^*]^+ \text{ if } D \leqslant x_d^* \\ \delta r x_d^* - cz + rE[D-x_d^*]^+ - eE[D-z]^+ \text{ if } D \geqslant x_d^*\end{cases} \\ &= \delta r x_d^* - cz + rE[D-x_d^*]^+ - eE[D-z]^+.\end{aligned} \tag{B.11}$$

Hence, for a given discount δ, the optimal production quantity is obtained as $z = \max\{x_d^*, F^{-1}\left(\frac{e-c}{e}\right)\}$. With algebraic manipulation it can be shown that the supplier produces the exact order-quantity x_d^* if, and only if, $\delta \leqslant 1 - \frac{(r-s)(e-c)}{re}$. For ease of notation, let $\delta_0 = 1 - \frac{(r-s)(e-c)}{re}$. It is easy to check that $\delta^* < \delta_0 < 1$. Though the objective function lacks a nice structure, in the following lemma we show that the optimal discount contract will not coordinate the supply chain even when the supplier overproduces during the first period.

Lemma 13. *The optimal discount contract does not coordinate the supply chain even when the supplier produces more than the advance-order quantity* x_d^* *during the first period.*

Proof of Lemma 13: We optimize (B.11) over δ by considering the following two cases:

Case 1 ($\frac{c}{r} < \delta \leqslant \delta_0$): By using Proposition 5 we argue that the optimal discount in the region $(\frac{c}{r}, 1 - \frac{(r-s)(e-c)}{re}]$ is given by $\overline{\delta}_1 = \min\{\hat{\delta}, \delta_0\}$ where $\hat{\delta}$ is the solution to the first-order condition stated in the second statement of Proposition 5. Since, $\hat{\delta} > \delta^*$ and $\delta_0 > \delta^*$, we have $\overline{\delta}_1 > \delta^*$.

Case 2 ($\delta_0 \leqslant \delta \leqslant 1$): Let $\overline{\delta}_2$ be the optimizer in the region $[\delta_0, 1]$. Hence, $\overline{\delta}_2 \geqslant \delta_0 > \delta^*$.

Since, the global optimizer $\overline{\delta}$ is either $\overline{\delta}_1$ or $\overline{\delta}_2$, we have $\overline{\delta} > \delta^*$ and hence $\overline{\delta}$ cannot coordinate the supply chain. ■.

Appendix C The Impact of Crop Minimum Support Prices on Crop Selection and Farmer Welfare in the presence of Strategic Farmers and Complementary Production Costs

C.1 Supplementary and Additional Results

Corollary 7 (Impact of crop dissimilarity)**.** *For a given pair of MSPs (m_t^A, m_t^B) the following statements hold:*

1. ***Crop availability disparity:*** *The disparity between the total production quantities of the crops decreases with r if there are strategic farmers. That is $\frac{\partial|\Delta q_t|}{\partial r} < 0$ if $\theta > 0$ where $\Delta q_t = q_t^{AT} - q_t^{BT}$. If $\theta = 0$, then $\frac{\partial|\Delta q_t|}{\partial r} = 0$.*

2. ***Crop price disparity:*** *However, the expected disparity between the two crop prices increases with the crop dissimilarity r. That is $\frac{\partial|\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$.*

Proposition 15 (Some strategic farmers may forgo low MSPs)**.** *Let $\tilde{\tilde{m}}$ be the unique value of m_t^A satisfying the equation $F\left(m_t^A - \phi + r\hat{\tau}^m\right) = \frac{r}{2+r}$. Then for each m_t^A such that $\underline{m}_t^A < m_t^A < \min\left\{\overline{m}_t^A, \tilde{\tilde{m}}\right\}$*[1]*, there exists a θ_0 such that $\frac{\partial\hat{\tau}^s}{\partial m_t^A} \leqslant 0$ for all $\theta < \theta_0$. Further, if θ is sufficiently high then $\frac{\partial\hat{\tau}^s}{\partial m_t^A} \geqslant 0$ always (i.e., $\lim_{\theta\to 1}\frac{\partial\hat{\tau}^s}{\partial m_t^A} \geqslant 0$ always).*

[1] Note that if $\tilde{\tilde{m}} < \underline{m}_t^A$ then the range of interest is empty. Hence, this condition is likely to be encountered when p_{t-1}^A is sufficiently low.

Proposition 16 (Effect of moderate MSPs). *Let* $\tilde{\tilde{m}}$ *be the unique value of* m_t^A *satisfying the equation* $F\left(m_t^A - \phi + r\hat{\tau}^m\right) = \frac{r}{2+r}$. *Then for each* m_t^A *such that* $\underline{m}_t^A < m_t^A < \min\left\{\overline{m}_t^A, \tilde{\tilde{m}}\right\}$[2], *there exists a threshold* θ_1 *such that* $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \leqslant 0$ *for all* $\theta < \theta_1$, *for each farmer of type* $v \in \{m, s\}$ *located at* $x \leqslant \hat{\tau}^v$. *Further, if* θ *is sufficiently high then* $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ *always (i.e.,* $\lim_{\theta \to 1} \frac{\partial \hat{\pi}_t^A}{\partial m_t^A} \geqslant 0$ *always).*

C.2 Proofs

Proof of Lemma 4: First, we note that

$$|\xi_t| \leqslant 2\beta(1-r) \Rightarrow -2\beta(1-r) \leqslant \xi_t \leqslant 2\beta(1-r) \Rightarrow \{-2\beta \leqslant \xi_t - 2r\beta\} \wedge \{\xi_t + 2r\beta \leqslant 2\beta\}$$

$$\Rightarrow -2\beta \leqslant -2r\beta + \xi_t \leqslant p_t^A - p_t^B \leqslant 2r\beta + \xi_t \leqslant 2\beta \Rightarrow |p_t^A - p_t^B| \leqslant 2\beta.$$

Hence,

$$|p_t^A - p_t^B| > 2\beta \Rightarrow |\xi_t| > 2\beta(1-r)$$

$$\Leftrightarrow \mathbb{P}\left(|p_t^A - p_t^B| > 2\beta\right) \leqslant \mathbb{P}\left(|\xi_t| > 2\beta(1-r)\right) \leqslant \left(\frac{\sigma}{\sqrt{2}\beta(1-r)}\right)^2,$$

where the last inequality is obtained by using Chebyshev's inequality. ■

Proof of Lemma 5: The first statement is proved by using (3.11) and the fact that $\frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} = r\frac{\partial \hat{\tau}}{\partial m_t^A} > 0$.

For the second statement, by substituting (C.13) in (3.11) and simplifying, we obtain that for

[2]Note that if $\tilde{\tilde{m}} < \underline{m}_t^A$ then the range of interest is empty. Hence, this condition is likely to be encountered when p_{t-1}^A is sufficiently low.

every $x \leqslant \hat{\tau}^v$, $v \in \{m, s\}$,

$$\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = \frac{2F\left(m_t^A - \phi + r\hat{\tau}\right) + r\theta F\left(m_t^A - \phi + r\hat{\tau}\right)\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right) - 2r\overline{\theta}\,\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right)\frac{\partial \hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left(\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right)} \tag{C.1}$$

which is non-negative if $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = 0$. Hence, by using Proposition 11 we obtain the desired result. ■

Proof of Proposition 9:

1. The myopic farmers anticipate the price in period t to be the same as the price in period $t-1$. A farmer produces crop A as long as the anticipated benefit from crop A is more than that from crop B, otherwise the farmer produces crop B (by Assumption 4). Therefore, the fraction of myopic farmers growing crop A is then given by:

$$\begin{aligned}\mathbb{P}\left(p_{t-1}^{Am} - c_A(x) \geqslant p_{t-1}^{Bm} - c_B(x)\right) &= \mathbb{P}\left(p_{t-1}^{Am} - \left(x + \frac{1}{2}\right) \geqslant p_{t-1}^{Bm} - \left(\frac{1}{2} - x\right)\right) \\ &= \mathbb{P}\left(x \leqslant \frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}\right) = \left(\frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}\right) + 0.5 \end{aligned} \tag{C.2}$$

   since $\frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2} \in [-0.5, 0.5]$ (by Assumption 5). Thus, we obtain the threshold value as $\tau^m = \frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}$.

2. The strategic farmers on the other hand are forward-looking and hence anticipate the market price in period t by taking into account the total availability of the crops, which takes into account the behaviors of the myopic farmers and the other strategic farmers. Hence, by using the principle of rational expectations, the fraction of the strategic farmers growing crop A is given by:

$$\mathbb{P}\left(\mathbb{E}[p_t^A] - c_A(x) \geqslant \mathbb{E}[p_t^B] - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]}{2}\right). \tag{C.3}$$

From (3.1), and the fact that $q_t^{BT} = 1 - q_t^{AT}$ we obtain

$$\mathbb{E}[p_t^A] = a - \alpha - rq_t^{AT} \text{ and } \mathbb{E}[p_t^B] = a - \rho + rq_t^{AT} \Rightarrow \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]$$
$$= r(1 - 2q_t^{AT}) \in (-r, r) \subset (-1, 1),$$

where $q_t^{AT} \in [0, 1]$ is the total production quantity of crop A. Therefore, from (C.3) we obtain the threshold τ^s as:

$$\tau^s = \frac{\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]}{2} = \frac{r(1 - 2q_t^{AT})}{2}. \tag{C.4}$$

and the total production quantity of crop A by strategic farmers is $q_t^{As} = \theta(\tau^s + 0.5)$. Further, using the fact that $q_t^{AT} = q_t^{As} + q_t^{Am} = \theta\tau^s + \overline{\theta}\tau^m + 0.5$ and substituting it in (C.4) we obtain:

$$\tau^s = \left(\frac{-r\overline{\theta}}{1 + r\theta}\right)\tau^m = -\hat{r}\tau^m.$$

Note that $|\tau^s| = |\hat{r}||\tau^m| < |\tau^m| \Rightarrow \tau^s \in [-0.5, 0.5]$ since $r < 1$ by Assumption 5.

3. The total availability of crop A is given by

$$q_t^{AT} = q_t^{As} + q_t^{Am} = \theta\tau^s + \overline{\theta}\tau^m + 0.5 = \tau + 0.5, \tag{C.5}$$

and by using (C.4), we obtain $\tau = \frac{\overline{\theta}\tau^m}{1+r\theta}$. Note that $|\tau| = |\frac{\hat{r}}{r}||\tau^m| < |\tau^m| \Rightarrow \tau \in [-0.5, 0.5]$ since $\hat{r} < r$.

Proof of Proposition 10:

1. The fraction of myopic farmers sowing crop A is given by:

$$\mathbb{P}\left(\hat{p}_t^{Am} - c_A(x) \geqslant \hat{p}_t^{Bm} - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2}\right).$$

Since $|p_t^A - p_t^B| < 1$ and $|m_t^A - m_t^B| < 1$ by assumptions, we obtain $|\hat{p}_t^{Am} - \hat{p}_t^{Bm}| < 1$ so that the threshold value $\hat{\tau}^m$ is given by $\hat{\tau}^m = \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2} \in [-0.5, 0.5]$. The total quantity of crop A produced by myopic farmers is given by $\hat{q}_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$.

2. On the other hand, the price anticipated by the strategic farmers for crop $k \in \{A, B\}$ is given by $\hat{p}_t^{ks} = \mathbb{E}_{\epsilon_t} \max\{p_t^k, m_t^k\}$ where $p_t^k = \mathbb{E}[p_t^k] + \epsilon_t$. Hence, the fraction of strategic farmers growing crop A is given by:

$$\mathbb{P}\left(\hat{p}_t^{As} - c_A(x) \geqslant \hat{p}_t^{Bs} - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\hat{p}_t^{As} - \hat{p}_t^{Bs}}{2}\right). \tag{C.6}$$

Using the fact that $\hat{p}_t^{ks} = \mathbb{E}[p_t^k] + \mathbb{E}_{\epsilon_t} \max\{\epsilon_t, m_t^k - \mathbb{E}[p_t^k]\}$, we can write

$$\begin{aligned} \hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{\infty} F(\epsilon)\, d\epsilon + \int_{m_t^B - \mathbb{E}[p_t^B]}^{\infty} F(\epsilon)\, d\epsilon \\ &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon. \end{aligned} \tag{C.7}$$

We know that $\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] = r(1 - 2\hat{q}_t^{AT}) \in (-1, 1)$, where $\hat{q}_t^{AT}$ is the total availability of crop A. Hence, (C.7) can we written as:

$$\hat{p}_t^{As} - \hat{p}_t^{Bs} = \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon. \tag{C.8}$$

For any given set of MSPs (m_t^A, m_t^B), we have exactly one of these three cases to hold: (i) $\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] > m_t^A - m_t^B$, or (ii) $\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] < m_t^A - m_t^B$, or (iii) $\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] = m_t^A - m_t^B$.

When (iii)holds, then trivially $\hat{p}_t^{As} - \hat{p}_t^{Bs} = \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] \in (-1, 1)$. Hence, it remains to check if $\hat{p}_t^{As} - \hat{p}_t^{Bs} \in (-1, 1)$ for cases (i) and (ii).

In case (i), we note that

$$
\begin{aligned}
\hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon \leqslant \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] < 1, \text{ and} \\
\hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon \geqslant \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] \\
&\quad - \{m_t^B - \mathbb{E}[p_t^B] - (m_t^A - \mathbb{E}[p_t^B] + 2r\hat{\tau})\} \\
&\quad = m_t^A - m_t^B > -1.
\end{aligned}
$$

In case (ii), we get

$$
\begin{aligned}
\hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] + \int_{m_t^B - \mathbb{E}[p_t^B]}^{m_t^A - \mathbb{E}[p_t^A]} F(\epsilon)\, d\epsilon \geqslant \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] > -1, \text{ and} \\
\hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] + \int_{m_t^B - \mathbb{E}[p_t^B]}^{m_t^A - \mathbb{E}[p_t^A]} F(\epsilon)\, d\epsilon \\
&\quad \leqslant -2r\hat{\tau} + \{m_t^A - \mathbb{E}[p_t^B] + 2r\hat{\tau} - (m_t^B - \mathbb{E}[p_t^B])\} \\
&\quad = m_t^A - m_t^B < 1.
\end{aligned}
$$

Hence, if $|m_t^A - m_t^B| < 1$ then $\hat{p}_t^{As} - \hat{p}_t^{Bs} \in (-1, 1)$ always and so we obtain the threshold τ^s as

$$
\hat{\tau}^s = \frac{\hat{p}_t^{As} - \hat{p}_t^{Bs}}{2} = \frac{\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]}{2} - \frac{1}{2} \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon \in (-0.5, 0.5) \qquad \text{(C.9)}
$$

and the total production quantity of crop A by strategic farmers as $\hat{q}_t^{As} = \theta(\hat{\tau}^s + 0.5) \in (0, 1)$. The total production of crops A and B are then given by $\hat{q}_t^{AT} = \hat{q}_t^{Am} + \hat{q}_t^{As} = \hat{\tau} + 0.5$ and

$\hat{q}_t^{BT} = 0.5 - \hat{\tau}$. Hence, we obtain $\mathbb{E}[p_t^A] = a - \alpha - r\hat{q}_t^{AT} = \phi - r\hat{\tau}$ and $\mathbb{E}[p_t^B] = a - \rho + r\hat{q}_t^{AT} = \phi + r\hat{\tau}$. Substituting these values in (C.9), we obtain

$$\hat{\tau}^s = -r\hat{\tau} - \frac{1}{2}\int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon \in [-0.5, 0.5]. \tag{C.10}$$

By substituting $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$ in (C.10) we obtain (3.7). Note that the above equation is an implicit definition of $\hat{\tau}^s$ so that $\hat{\tau}^m$, $\hat{\tau}^s$ and $\hat{\tau}$ are all functions of of p_{t-1}^A, p_{t-1}^B, m_t^A and m_t^B. Hence, it is important to check the existence of equilibrium and, if possible, show that (3.7) is satisfied by a unique value of $\hat{\tau}$ in order to prove uniqueness of the equilibrium.

Proof of Uniqueness of $\hat{\tau}^s$: Let $RHS_{(C.10)}$ and $LHS_{(C.10)}$ denote the right-hand side and the left-hand side of (C.10), respectively. We note that $\frac{\partial LHS_{(C.10)}}{\partial \hat{\tau}^s} = 1$, $LHS_{(C.10)}|_{\hat{\tau}^s=-0.5} = -0.5$ and $LHS_{(C.10)}|_{\hat{\tau}^s=0.5} = 0.5$. Next, we proved that $RHS_{(C.10)} \in [-0.5, 0.5]$. Hence, $RHS_{(C.10)}|_{\hat{\tau}^s=-0.5} \geqslant -0.5$ and $RHS_{(C.10)}|_{\hat{\tau}^s=0.5} \leqslant 0.5$. Further,

$$\begin{aligned}\frac{\partial RHS_{(C.10)}}{\partial \hat{\tau}^s} &= -r\theta - \frac{1}{2}\left[-r\theta F\left(m_t^B - \phi - r\hat{\tau}\right) - r\theta F\left(m_t^A - \phi + r\hat{\tau}\right)\right] \\ &= -\frac{r\theta}{2}\left[\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right) + \overline{F}\left(m_t^A - \phi + r\hat{\tau}\right)\right] < 0\end{aligned}$$

Hence, by intermediate value theorem, there exists a unique solution to (C.10).

3. By definition, $q_t^{As} = \theta(\hat{\tau}^s + 0.5)$, $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$ and $q_t^{AT} = q_t^{Am} + q_t^{As} = \hat{\tau} + 0.5$. ■

Proof of Proposition 11:

1. First, we note that $\frac{\partial \tau^m}{\partial m_t^A} \geqslant 0$ and $\frac{\partial \tau^m}{\partial m_t^B} \leqslant 0$ by its definition given in (3.6). Next, after differentiating (3.7) implicitly with respect to m_t^A and simplifying by using $\hat{r} = \frac{r\overline{\theta}}{1+r\theta}$, we

obtain

$$2(1+r\theta)\frac{\partial\hat{\tau}^s}{\partial m_t^A} = -2r\overline{\theta}\frac{\partial\hat{\tau}^m}{\partial m_t^A} + F\left(m_t^A - \phi + r\hat{\tau}\right) \tag{C.11}$$

$$+ r\left[F\left(m_t^A - \phi + r\hat{\tau}\right) + F\left(m_t^B - \phi - r\hat{\tau}\right)\right]\ \frac{\partial\hat{\tau}}{\partial m_t^A}. \tag{C.12}$$

However, by definition of $\hat{\tau}$, we obtain $\frac{\partial\hat{\tau}^s}{\partial m_t^A} = \frac{1}{\theta}\left[\frac{\partial\hat{\tau}}{\partial m_t^A} - \overline{\theta}\frac{\partial\hat{\tau}^m}{\partial m_t^A}\right]$. Hence, we obtain:

$$\frac{\partial\hat{\tau}}{\partial m_t^A} = \frac{\theta F\left(m_t^A - \phi + r\hat{\tau}\right) + 2\overline{\theta}\frac{\partial\hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]} \geqslant 0. \tag{C.13}$$

Similarly, we obtain

$$\frac{\partial\hat{\tau}}{\partial m_t^B} = -\frac{\theta F\left(m_t^B - \phi - r\hat{\tau}\right) - 2\overline{\theta}\frac{\partial\hat{\tau}^m}{\partial m_t^B}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]} \leqslant 0. \tag{C.14}$$

This proves the first statement. Further, by using the equation (C.11) for $\frac{\partial\hat{\tau}^s}{\partial m_t^A}$, we obtain

$$\frac{\partial\hat{\tau}^s}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}\right) - r\overline{\theta}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]\frac{\partial\hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}. \tag{C.15}$$

Similarly, we obtain

$$\frac{\partial\hat{\tau}^s}{\partial m_t^B} = -\frac{F\left(m_t^B - \phi - r\hat{\tau}\right) + r\overline{\theta}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]\frac{\partial\hat{\tau}^m}{\partial m_t^B}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}. \tag{C.16}$$

Note that the signs of $\frac{\partial\hat{\tau}^s}{\partial m_t^A}$ and $\frac{\partial\hat{\tau}^s}{\partial m_t^B}$ cannot be ascertained easily.

2. By the definition of $\underline{m}_t^A$, there are two values of $\underline{m}_t^A$ that are possible: (i) $\underline{m}_t^A = p_{t-1}^A > (\max\{m_t^B, p_{t-1}^B\} - 1)$ and (ii) $\underline{m}_t^A = (\max\{m_t^B, p_{t-1}^B\} - 1) \geqslant p_{t-1}^A$. In case (i) we have $m_t^A \leqslant \underline{m}_t^A = p_{t-1}^A$ then $\hat{\tau}^m = \frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2}$ because $|m_t^A - m_t^B| < 1$. In case (ii)

we have $m_t^A < (\max\{m_t^B, p_{t-1}^B\} - 1)$ which implies that $\hat{\tau}^m = -0.5$. Hence, $q_t^{Am} = \left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + 0.5\right]^+$, which is independent of m_t^A. Hence, $\frac{\partial q_t^{Am}}{m_t^A} = 0 = \frac{\partial \hat{\tau}^m}{m_t^A}$. Using $q_t^{As} = \theta(\hat{\tau}^s + 0.5)$ we obtain $\frac{\partial q_t^{As}}{\partial m_t^A} = \theta \frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ by noting from (C.15) that $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ if $\frac{\partial \hat{\tau}^m}{m_t^A} = 0$.

3. The fact that $\hat{\tau}^m = +\frac{1}{2}$ when $m_t^A \geqslant \overline{m}_t^A$ follows directly from the fact that all the farmers produce crop A when $m_t^A \geqslant \overline{m}_t^A$. The proof of the remaining results follows as in part 2.

4. If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then $\hat{\tau}^m = \frac{m_t^A - \max\{p_{t-1}^B, m_t^B\}}{2} \in (-0.5, 0.5) \Rightarrow q_t^{Am} \in (0, \overline{\theta})$. Hence, $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{1}{2} \Rightarrow \frac{\partial q_t^{Am}}{\partial m_t^A} = \frac{\overline{\theta}}{2}$. Further, from (C.15) we obtain, $\frac{\partial \hat{\tau}^s}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}\right) - \frac{r\overline{\theta}}{2}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}$, whose sign cannot be ascertained. ■

Proof of Proposition 12: From (3.7), we have

$$2\hat{\tau}^s = -2r\hat{\tau} - \int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon = -2r\hat{\tau} - \int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} \frac{\epsilon + \delta}{2\delta}\, d\epsilon.$$

By substituting $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$ and $\hat{\tau}^m \equiv \hat{\tau}^m(m_t^A, m_t^B) = \frac{m_t^A - m_t^B}{2}$, we obtain

$$\hat{\tau}^s(m_t^A, m_t^B) = \left(\frac{m_t^A - m_t^B}{2}\right)\left[\frac{2\delta - 2\phi + m_t^A + m_t^B - r\overline{\theta}\left(2\delta + 2\phi - m_t^A - m_t^B\right)}{4\delta + r\theta\left(2\delta + 2\phi - m_t^A - m_t^B\right)}\right] \tag{C.17}$$

$$\Rightarrow \frac{\partial \hat{\tau}^s}{\partial m_t^A} = \frac{1}{2} + \frac{2\delta(1+r)(m_t^A - m_t^B)}{\left(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B)\right)^2} - \frac{(1+r)(2\delta + 2\phi - m_t^A - m_t^B)}{2\left(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B)\right)}$$

$$= \frac{V(\theta)}{2\left(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B)\right)^2} \tag{C.18}$$

where

$$V(\theta) = r^2(2\delta + 2\phi - m_t^A - m_t^B)^2\theta^2$$
$$+ r(2\delta + 2\phi - m_t^A - m_t^B)\left(8\delta - (1+r)(2\delta + 2\phi - m_t^A - m_t^B)\right)\theta$$
$$- 8\delta\left((1+r)(\phi - m_t^A) - (1-r)\delta\right),$$

which is a convex quadratic in θ and $V(0) < 0$ if $\underline{m}_t^A < m_t^A \leqslant \phi - \delta\left(\frac{1-r}{1+r}\right) (< \phi)$, that is if the MSP of A is moderately small. Hence, there exists a $\theta_0 > 0$ such that $V(\theta) < 0$ if and only if $\theta < \theta_0$, that is $\frac{\partial \hat{\tau}^s}{\partial m_t^A} < 0$ if and only if $\theta < \theta_0$. ■

Proof of Proposition 13: First, we obtain $\hat{\tau}^s(m_t^A, m_t^B)$ as given in (C.17). Second, since $|m_t^A - m_t^B| < 1$, we have $\hat{\tau}^m(m_t^A, m_t^B) = \frac{m_t^A - m_t^B}{2}$. Using these values of $\hat{\tau}^s(m_t^A, m_t^B)$ and $\hat{\tau}^m(m_t^A, m_t^B)$, we obtain

$$\hat{\tau}(m_t^A, m_t^B) = \theta\hat{\tau}^s(m_t^A, m_t^B) + \overline{\theta}\hat{\tau}^m(m_t^A, m_t^B) = \left(\frac{m_t^A - m_t^B}{2}\right)\left[\frac{4\delta - \theta\left(2\delta + 2\phi - m_t^A - m_t^B\right)}{4\delta + r\theta\left(2\delta + 2\phi - m_t^A - m_t^B\right)}\right].$$

On substituting the value of $\hat{\tau}(m_t^A, m_t^B)$ in (3.11) we obtain

$$\lim_{\theta \to 0} \frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = \frac{(2+r)^2}{8\delta}\left[m_t^A - \widetilde{m}^A\right] < 0$$

for $v \in \{m, s\}$ and $x \leqslant \hat{\tau}^v$. Hence the result.

Further, from (C.13), we have $\lim_{\theta \to 1} \frac{\partial \hat{\tau}}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]}$ so that, from (3.11), we obtain

$$\lim_{\theta \to 1} \frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = F\left(m_t^A - \phi + r\hat{\tau}^s\right) - r\frac{F\left(m_t^A - \phi + r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]}$$
$$= \frac{2F\left(m_t^A - \phi + r\hat{\tau}^s\right) + rF\left(m_t^A - \phi + r\hat{\tau}^s\right)\overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]} \geqslant 0 \;\forall x \leqslant \hat{\tau}^v, v \in \{A, B\}.$$

$\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ for all $x > \tau^v$ by Lemma 5. Hence, $\lim_{\theta \to 1} \frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ for all $x \in (-0.5, 0.5)$ because of continuity of $\hat{\pi}_t^v(x)$ at $x = \hat{\tau}^v$, for $v \in \{A, B\}$. ■

Proof of Proposition 15: If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then from Proposition 11, we have $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{1}{2}$. Hence, from (C.15) we find that if

$$F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\overline{\theta}}{2 + r\overline{\overline{\theta}}}\left[1 + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right] \tag{C.19}$$

then $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \leqslant 0$. Note that $m_t^A - \phi + r\hat{\tau}^m$ is increasing in m_t^A and hence the equation $F\left(m_t^A - \phi + r\hat{\tau}^m\right) = \frac{r}{2+r}$ has a unique solution (which we denote by $\tilde{\tilde{m}}$). Hence, for m_t^A such that $\underline{m}_t^A < m_t^A < \min\left\{\overline{m}_t^A, \tilde{\tilde{m}}\right\}$ we have

$$\begin{aligned}\lim_{\theta \to 0} F\left(m_t^A - \phi + r\hat{\tau}\right) = F\left(m_t^A - \phi + r\hat{\tau}^m\right) < \frac{r}{2+r} = \lim_{\theta \to 0} \frac{r\overline{\theta}}{2 + r\overline{\overline{\theta}}} \\ < \lim_{\theta \to 0} \frac{r\overline{\theta}}{2 + r\overline{\overline{\theta}}}\left[1 + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]\end{aligned}$$

Hence, there exists θ_0 (sufficiently close to 0) such that $F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\overline{\theta}}{2 + r\overline{\overline{\theta}}}\left[1 + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]$ for all $\theta \in [0, \theta_0)$. The proof is completed by using (C.19). ■

Proof of Proposition 16: If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then from Proposition 11, we have $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{1}{2}$. Hence, from (C.1), we observe that if

$$F\left(m_t^A - \phi + r\hat{\tau}\right) \leqslant \frac{r\overline{\theta}}{2 + r\overline{\theta} + r\theta\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)} \tag{C.20}$$

then $\frac{\partial \hat{\pi}_t^A}{\partial m_t^A} \leqslant 0$. Hence, we have

$$\lim_{\theta \to 0} F\left(m_t^A - \phi + r\hat{\tau}\right) = F\left(m_t^A - \phi + r\hat{\tau}^m\right) < \frac{r}{2+r} = \lim_{\theta \to 0} \left\{\frac{r\overline{\theta}}{2 + r\overline{\theta} + r\theta\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)}\right\}$$

Hence, there exists θ_1 (sufficiently close to 0) such that $F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\bar{\theta}}{2+r\bar{\theta}+r\theta\bar{F}\left(m_t^B-\phi-r\hat{\tau}\right)}$ for all $\theta \in [0, \theta_1)$. The proof is completed by using (C.20). Further, $\lim_{\theta\to 1} \frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ is shown the same way as in Proposition 13. ■

Proof of Corollary 2:

1. $|\Delta q_t| = |q_t^{AT} - q_t^{BT}| = 2|\tau| = \frac{2\bar{\theta}}{1+r\theta}|\tau^m| \Rightarrow \frac{\partial |\Delta q_t|}{\partial r} < 0$ if $\theta > 0$. Clearly, if $\theta = 0$ then $|\Delta q_t|$ is independent of r.

2. $|\mathbb{E}\Delta p_t| = |\Delta p_{t-1}|\hat{r} \Rightarrow \frac{\partial |\mathbb{E}\Delta p_t|}{\partial r} = |\Delta p_{t-1}|\frac{\partial \hat{r}}{\partial r} > 0$. ■

Proof of Corollary 3: The proof follows directly from the expressions derived in Proposition 9 and (3.5). ■

Proof of Corollary 4:

1. The proof follows from the definition of τ^m and q_t^{Am} given in the first statement of Proposition 9.

2. From the second statement of Proposition 9 we obtain:

$$\tau^s = -\hat{r}\tau^m \Rightarrow \frac{\partial \tau^s}{\partial \theta} = -\tau^m \frac{\partial \hat{r}}{\partial \theta} = \frac{r(1+r)}{(1+r\theta)^2}\tau^m \text{ and} \tag{C.21}$$

$$q_t^{As} = \theta(\tau^s + \frac{1}{2}) \Rightarrow \frac{\partial q_t^{As}}{\partial \theta} = \tau^s + \frac{1}{2} + \theta\frac{\partial \tau^s}{\partial \theta} = -\hat{r}\tau^m + \frac{1}{2} + \theta\frac{r(1+r)}{(1+r\theta)^2}\tau^m, \tag{C.22}$$

which gives the desired result on simplification.

3. The expression for $\frac{\partial \tau}{\partial \theta}$ (or equivalently $\frac{\partial q_t^{AT}}{\partial \theta}$) is obtained by differentiating the expression for τ (or equivalently q_t^{AT}) given in the third statement of Proposition 9. Next, from the third statement of Corollary 3 we obtain $\frac{\partial^2 \tau}{\partial \theta \partial \Delta p_{t-1}} = \frac{\partial^2 q_t^{AT}}{\partial \theta \partial \Delta p_{t-1}} = -\frac{r+1}{2(1+r\theta)^2} < 0$.

4. The result is obtained by successively differentiating (3.5) with respect to θ followed by Δp_{t-1}.

■

Proof of Corollary 5:

1. By differentiating (3.6) with respect to p_{t-1}^A, p_{t-1}^B and Δp_{t-1}, we get

$$\frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} = \frac{1}{2} \cdot \mathbb{I}_{\{m_t^A < p_{t-1}^A\}} \geqslant 0, \text{ and } \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^B} = -\frac{1}{2} \cdot \mathbb{I}_{\{m_t^B < p_{t-1}^B\}} \leqslant 0,$$

   where $\hat{p}^{km} = \max\{m_t^k, p_{t-1}^k\}$, $k \in \{A, B\}$. The expressions for $\frac{\partial q_t^{Am}}{\partial p_{t-1}^k}$ can be obtained by using $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$.

2. By differentiating (3.7) with respect to p_{t-1}^A, we get

$$\frac{\partial \hat{\tau}^s}{\partial p_{t-1}^A} = -\frac{r}{2} \cdot \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right] \frac{\partial \hat{\tau}}{\partial p_{t-1}^A}$$
$$\Rightarrow \frac{\partial \hat{\tau}^s}{\partial p_{t-1}^A} = -\left[\frac{\overline{\theta} r \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}{2 + \theta r \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}\right] \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} \leqslant 0$$

   where the second equation is obtained by using $\hat{\tau} = \theta \hat{\tau}^s + \overline{\theta} \hat{\tau}^m$. Similarly, we obtain

$$\frac{\partial \hat{\tau}^s}{\partial p_{t-1}^B} = -\left[\frac{\overline{\theta} r \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}{2 + \theta r \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}\right] \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^B} \geqslant 0.$$

   The expressions for $\frac{\partial q_t^{As}}{\partial p_{t-1}^k}$ can be obtained by using $q_t^{As} = \theta(\tau^s + 0.5)$.

3. By using the fact that $\hat{\tau} = \theta \hat{\tau}^s + \overline{\theta} \hat{\tau}^m$ and $q_t^{AT} = \tau + 0.5$, we obtain

$$\frac{\partial \hat{\tau}}{\partial p_{t-1}^A} = \frac{\partial q_t^{AT}}{\partial p_{t-1}^A} = \left[\frac{2\overline{\theta}}{2 + r\theta \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}\right] \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} \geqslant 0$$
$$\frac{\partial \hat{\tau}}{\partial p_{t-1}^B} = \frac{\partial q_t^{AT}}{\partial p_{t-1}^B} = \left[\frac{2\overline{\theta}}{2 + r\theta \left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}\right] \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^B} \leqslant 0.$$

4. If $x \leqslant \hat{\tau}^v$, $v \in \{m, s\}$, then the farmer at x produces crop A. Hence, from (3.10) we obtain

$$\begin{aligned}
\frac{\partial \hat{\pi}_t^v(x)}{p_{t-1}^A} &= \frac{\partial \hat{\Pi}_t^A(x)}{p_{t-1}^A} = -r\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right)\frac{\partial \hat{\tau}}{\partial p_{t-1}^A} \leqslant 0 \text{ and} \\
\frac{\partial \hat{\pi}_t^v(x)}{p_{t-1}^B} &= \frac{\partial \hat{\Pi}_t^A(x)}{p_{t-1}^B} = -r\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right)\frac{\partial \hat{\tau}}{\partial p_{t-1}^B} \geqslant 0.
\end{aligned}$$

Similarly, when $x > \hat{\tau}^v$, $v \in \{m, s\}$, then the farmer at x produces crop B. Hence, from (3.10) we obtain

$$\begin{aligned}
\frac{\partial \hat{\pi}_t^v(x)}{p_{t-1}^A} &= \frac{\partial \hat{\Pi}_t^B(x)}{p_{t-1}^A} = r\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\frac{\partial \hat{\tau}}{\partial p_{t-1}^A} \geqslant 0 \text{ and} \\
\frac{\partial \hat{\pi}_t^v(x)}{p_{t-1}^B} &= \frac{\partial \hat{\Pi}_t^A(x)}{p_{t-1}^B} = r\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\frac{\partial \hat{\tau}}{\partial p_{t-1}^B} \leqslant 0. \quad \blacksquare
\end{aligned}$$

Proof of Corollary 6 : For exposition, we define the sign function as: $sgn[x] = -1$ if $x < 0$, $sgn[x] = +1$ if $x > 0$ and $sgn[0] = 0$.

1. From (3.6) we obtain $\frac{\partial \hat{\tau}^m}{\partial \theta} = 0$. Since $\hat{q}_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$, we obtain $\frac{\partial \hat{q}_t^{Am}}{\partial \theta} = -(\hat{\tau}^m + 0.5) \leqslant 0$.

2. From (3.7) we obtain

$$\begin{aligned}
\frac{\partial \hat{\tau}^s}{\partial \theta} &= -2r\frac{\partial \hat{\tau}}{\partial \theta} + r\frac{\partial \hat{\tau}}{\partial \theta}\left(F(m_t^A - \phi - r\hat{\tau}) + F(m_t^B - \phi + r\hat{\tau})\right) \\
&= -r\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\frac{\partial \hat{\tau}}{\partial \theta}, \\
\text{and } \hat{\tau} &= \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m \Rightarrow \frac{\partial \hat{\tau}}{\partial \theta} = \hat{\tau}^s + \theta\frac{\partial \hat{\tau}^s}{\partial \theta} - \hat{\tau}^m \text{ so that} \\
\frac{\partial \hat{\tau}^s}{\partial \theta} &= \frac{r\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)} \cdot (\hat{\tau}^m - \hat{\tau}^s) \Rightarrow \operatorname{sgn}\left[\frac{\partial \hat{\tau}^s}{\partial \theta}\right] \\
&= \operatorname{sgn}\left[\hat{\tau}^m - \hat{\tau}^s\right] \geqslant 0 \Leftrightarrow \hat{\tau}^m \geqslant \hat{\tau}_0^s.
\end{aligned}$$

Since $q_t^{As} = \theta(\hat{\tau}^s + 0.5)$ we obtain

$$\begin{aligned}\frac{\partial q_t^{As}}{\partial \theta} &= \hat{\tau}^s + 0.5 + \theta\frac{\partial \hat{\tau}^s}{\partial \theta} \\ &= \frac{(\hat{\tau}^s + 0.5) + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)(\hat{\tau}^m + 0.5)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)} \geqslant 0\end{aligned}$$

because $\hat{\tau}^m, \hat{\tau}^s \in [-0.5, 0.5]$.

3. As shown in part 2, $\frac{\partial \hat{\tau}}{\partial \theta} = \hat{\tau}^s + \theta\frac{\partial \hat{\tau}^s}{\partial \theta} - \hat{\tau}^m$. Hence,

$$\begin{aligned}\frac{\partial \hat{\tau}}{\partial \theta} &= \frac{(\hat{\tau}^s - \hat{\tau}^m)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)} \\ \Rightarrow \text{sgn}\left[\frac{\partial \hat{\tau}}{\partial \theta}\right] &= \text{sgn}[\hat{\tau}^s - \hat{\tau}^m] \leqslant 0 \Leftrightarrow \hat{\tau}^m \geqslant \hat{\tau}_0^s.\end{aligned}$$

4. The result is obtained by differentiating (3.10) with respect to θ. ■

Proof of Corollary 7: By differentiating (3.7) implicitly by r we obtain

$$\begin{aligned}\frac{\partial \hat{\tau}^s}{\partial r} &= -2\hat{\tau} - 2r\frac{\partial \hat{\tau}}{\partial r} + \left(F(m_t^A - \phi + r\hat{\tau}) + F(m_t^B - \phi - r\hat{\tau})\right)\frac{\partial}{\partial r}(r\hat{\tau}) \\ &= -\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\left(\hat{\tau} + r\frac{\partial \hat{\tau}}{\partial r}\right).\end{aligned}$$

Further, by definition of $\hat{\tau}$ we obtain $\frac{\partial \hat{\tau}}{\partial r} = \theta\frac{\partial \hat{\tau}^s}{\partial r}$. Hence, we obtain

$$\begin{aligned}\frac{1}{\theta}\frac{\partial \hat{\tau}}{\partial r} &= -\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\left(\hat{\tau} + r\frac{\partial \hat{\tau}}{\partial r}\right) \\ \Rightarrow \frac{\partial \hat{\tau}}{\partial r} &= -\frac{\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\hat{\tau}}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}.\end{aligned}$$

Therefore, we have the following:

1.

$$\Delta q_t = q_t^{AT} - q_t^{BT} = 2q_t^{AT} - 1 = 2\hat{\tau} \Rightarrow \frac{\partial \Delta q_t}{\partial r} = 2\frac{\partial \hat{\tau}}{\partial r}$$
$$= -\frac{2\theta\hat{\tau}\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}.$$

Now, consider the value of $\hat{\tau}$ at $r = 0$. We consider two cases: (i) $\hat{\tau}|_{r=0} > 0$ and (ii) $\hat{\tau}|_{r=0} < 0$. It is easy to see that when $\hat{\tau}|_{r=0} = 0$ then $\frac{\partial \Delta q_t}{\partial r} = 0$. When (i) $\hat{\tau}|_{r=0} > 0$, then crop A is produced more than crop B at $r = 0$ (i.e.,$\Delta q_t|_{r=0} > 0$) and hence, since strategic farmers are present (i.e., $\theta > 0$), $\frac{\partial \Delta q_t}{\partial r} < 0$. That is the disparity between the quantities of crops A and B decreases because of the strategic farmers. When (ii) $\hat{\tau}|_{r=0} < 0$, then crop A is produced less than crop B at $r = 0$ (i.e.,$\Delta q_t|_{r=0} < 0$) and hence, since strategic farmers are present (i.e., $\theta > 0$), $\frac{\partial \Delta q_t}{\partial r} > 0$. That is the disparity between the quantities of crops A and B decreases because of the strategic farmers. Hence,

2.

$$\mathbb{E}[\Delta p_t] = \mathbb{E}[p_t^A - p_t^B] = \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] = -2r\hat{\tau} \Rightarrow \frac{\partial \mathbb{E}[\Delta p_t]}{\partial r} = -2\left(\hat{\tau} + r\frac{\partial \hat{\tau}}{\partial r}\right)$$
$$\Rightarrow \frac{\partial \mathbb{E}[\Delta p_t]}{\partial r} = -2\left(\hat{\tau} + r\frac{\partial \hat{\tau}}{\partial r}\right) = \frac{-2\hat{\tau}}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}$$

Now, consider the value of $\hat{\tau}$ at $r = 0$. We consider two cases: (i) $\hat{\tau}|_{r=0} > 0$ and (ii) $\hat{\tau}|_{r=0} < 0$. It is easy to see that when $\hat{\tau}|_{r=0} = 0$ then $\frac{\partial \mathbb{E}\Delta p_t}{\partial r} = 0$. When (i) $\hat{\tau}|_{r=0} > 0$, then crop A is produced more than crop B at $r = 0$ and hence $\mathbb{E}[\Delta p_t]|_{r=0} < 0$. Hence, $\frac{\partial \mathbb{E}\Delta p_t}{\partial r} < 0$ (i.e., as r increases the price of crop A goes further down while that of crop B goes up, thus widening the gap between the two prices). When (ii) $\hat{\tau}|_{r=0} < 0$, then crop A is produced less than crop B at $r = 0$ and hence $\mathbb{E}[\Delta p_t]|_{r=0} > 0$. Hence, $\frac{\partial \mathbb{E}\Delta p_t}{\partial r} > 0$ (i.e., as r increases the price of crop

A goes further up while that of crop B goes down, thus widening the gap between the two prices). Hence, $\frac{\partial |\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$. ■

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