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# Integrating Demand Flexibility in Power System Markets 

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in the

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of the
University of California, Berkeley

Committee in charge:
Professor Kameshwar Poolla, Chair
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# Integrating Demand Flexibility in Power System Markets 

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Deepan Raj Prabakar Muthirayan

Abstract<br>Integrating Demand Flexibility in Power System Markets<br>by<br>Deepan Raj Prabakar Muthirayan<br>Doctor of Philosophy in Engineering - Mechanical Engineering<br>University of California, Berkeley<br>Professor Kameshwar Poolla, Chair

Retail Demand Response (DR) serves to reduce the demand for electricity especially during times when supply is scarce or expensive. The program entails calling the recruited consumers to reduce their energy consumption from an established baseline. Baseline is an estimate of the counterfacutual against which load reductions are measured to determine payments to consumers. This creates an incentive for consumers to inflate their baseline so that their payments are inflated. This is not fictitious. There have been reported cases where consumers have inflated their baseline to increase their payments. To address this, we propose a novel selfreported baseline mechanism. That is the consumers report their baseline and their marginal utility to an aggregator who manages the DR program. Based on the reports, the aggregator selects a set of consumers for each DR event to meet the load reduction requirement. The consumers are then paid based on the measured reductions from their baseline report. The mechanism is specified by the design of rewards and penalties for consumption deviation from the reported baseline. For a particular design choice, we show that reporting true baseline consumption and marginal utility is incentive compatible and individually rational. Also, the proposed mechanism meets the load reduction requirement and the allocation/selection is nearly efficient. We then compare the self-reported baseline mechanism with the current mechanisms used to estimate baseline like the CAISO's (California Independent System Operator) $\mathrm{m} / \mathrm{m}$ method and show that the self-reported mechanism is either reliable or more efficient.

We also show how to integrate retail DR in to electricity markets. Electricity scheduling in US typically operates as a two settlement system, a day-ahead market (DAM) for bulk power scheduling and a real-time market (RTM) for supply-demand balancing. Under the two market system, demand response (DR) can be used as intermediate recourse, which has several benefits from efficiency and operational perspective. We show that an intermediate market for DR an be created. This enables the Load Serving Entity (LSE) to exploit the
improved forecast available at the intermediate time (as compared with the day-ahead) thus improving overall efficiency and gives the aggregator enough lead time for organizing and delivering the load curtailment. We analyze this intermediate market, characterize the equilibrium and study the efficiency properties.

In the thrid chapter, we provide an algorithm for Wholesale DR programs that maximizes the benefit that the SO derives from deploying the DR resources. To emphasize the efficiency of the proposed mechanism we compare it with current methods used for baseline calculation like CAISO's $m / m$ method. We show that the proposed mechanism achieves better efficiency when compared to $m / m$ method especially when the variability in consumption is high. Here we do not concern ourselves with the transient aspect of the wholesale DR algorithm.

In the final chapter, we provide a baseline algorithm for transient performance. For this, we consider a repeated setting where the DR events repeat. In such a setting it is not sufficient that the optimal price is attained at steady state, becuase the transient losses have to be taken in to account. In the full information setting it is trivial to define the optimal pricing policy. In the incomplete information setting, one has to consider the trade-off between learning the consumer behavior and maximizing the savings based on the learnt information. Here we propose a pricing policy that achieves sub-linear regret.

To my Mother Sathyabama Maruthanayagam
§

The collective is greater 'by many orders' than the sum of its parts

## Contents

Contents ..... ii
List of Figures ..... iv
List of Tables ..... v
1 Introduction ..... 1
2 Baseline Mechanism for Retail Market ..... 4
2.1 Model and Problem Formulation ..... 6
2.2 Self-Reported Baseline Mechanism ..... 8
2.3 Uniform Payment Mechanisms ..... 10
2.3.1 Simulations ..... 12
2.3.2 Illustrative Example ..... 13
2.4 Comparison with SRBM for the Retail Market ..... 14
2.5 Conclusion ..... 15
3 Market Mechanisms for Selling Retail Demand Response ..... 16
3.0.1 Related Work ..... 16
3.1 Preliminaries ..... 17
3.2 Optimal Scheduling for the Entity ..... 19
3.2.1 Optimal Scheduling without DR ..... 20
3.2.2 Optimal Scheduling with DR ..... 20
3.2.3 Examples ..... 21
3.3 Spot Markets with Contingent Prices ..... 22
3.3.1 Examples ..... 24
3.4 Monopsony Contracts ..... 24
3.4.1 Principal-Agent Problem: Formulation ..... 24
3.4.2 Complete Information: First Best Contract ..... 26
3.4.3 Incomplete Information: Continuum of Types ..... 27
3.5 Concluding Remarks and Options Market ..... 28
4 Baseline Mechanism for Wholesale Markets ..... 29
4.1 Consumer Model ..... 30
4.1.1 Mechanism ..... 30
4.1.2 Second Stage - Optimal Consumption ..... 32
4.2 Optimal Forecast ..... 35
4.2.1 Optimal Forecast Without Penalty ..... 36
4.2.2 Optimal Forecast With Penalty ..... 36
4.3 Mechanism Efficiency ..... 38
4.4 Comparison with SRBM for Wholesale Market ..... 38
4.5 Threshold Market Clearing Price ..... 43
4.6 General Non-Linear Utility ..... 44
4.7 Further Remarks ..... 44
5 Learning and Pricing Demand Response ..... 46
5.1 Formulation ..... 47
5.1.1 Consumer Model ..... 48
5.1.2 Demand Response Mechanism ..... 48
5.1.3 Baseline Assignment $\hat{b}_{t}^{i}$ ..... 49
5.1.4 Measured Reduction and Payment ..... 49
5.1.5 Utility's Objective ..... 50
5.2 DEMAND MODEL LEARNING ..... 53
5.3 Pricing Policy ..... 55
5.4 Bound on Regret ..... 56
5.5 Conclusion ..... 57
Bibliography ..... 58
A Appendix ..... 63
A. 1 Proof of Theorem 1 ..... 63
A. 2 Proof of Theorem 2 ..... 69
A.2.1 Proof of Theorem 3 ..... 74
A.2.2 Proof of proposition 1 ..... 75
A.2.3 Proof of proposition 2 ..... 76
A.2.4 Proof of Theorem 4 ..... 77
A. 3 Proof of Proposition 5 ..... 77
A. 4 Proof of Proposition 6 ..... 80
A. 5 Consumption and Cost Sensitivities ..... 81
A. 6 Proof of Lemma 3 ..... 82
A. 7 Proof of Lemma 4 ..... 83
A. 8 Proof of Theorem 5 ..... 85

## List of Figures

2.1 CDF of deviations from the truthful reporting ( $\delta$ ) ..... 12
3.1 Players, interactions, and decision time-line. ..... 17
3.2 Purchase Decisions - Uniform Forecast Error ..... 21
3.3 Load Curtailment Decisions (equation (3.10)) ..... 21
3.4 Purchase Decisions - Gaussian Forecast Error ..... 22
3.5 Purchase Decisions - Uniform Forecast Error ..... 22
5.1 Learning and Pricing:Time-line ..... 48

## List of Tables

2.1 Notations - Retail Market ..... 7
4.1 Notations - Wholesale Market ..... 30

## List of Algorithms

1 (Self-Reported Baseline Mechanism (SRBM)) - Retail Market ..... 9
2 (Self-Reported Baseline Mechanism (SRBM)) - Wholesale Market ..... 33

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## Chapter 1

## Introduction

The core problem in power systems operations is to maintain a fine balance of electricity supply and demand. Achieving balance in electricity systems given its operational constraint is a demanding proposition. Also this has to be done economically. For example, consider a hot summer afternoon where the supply of additional electric power is scarce and expensive. At these times, it is more cost-effective alternative to reduce demand than to increase supply to maintain power balance. Because increasing supply would entail deploying very expensive generators.

Demand Response (DR) programs [1] are widely used tools to reduce the demand for electricity in such scenarios. It is widely believed that DR can improve system efficiency and reliability. The 2005 Energy Policy Act provides the Congressional mandate to promote demand response in organized wholesale electricity markets. With increased renewable integration, DR promises to be a better alternative than other expensive and polluting reserves to address the variability in renewable generation. The FERC order 745 [12] met this mandate by prescribing that demand response resource owners should be allowed to offer their demand reduction as if it were a supply resource rather than a bid to reduce demand so that the market operates justly. Dynamic pricing [39] can ideally achieve the desired market efficient outcomes. But state regulatory hurdles hinders its implementation [4]. Constant retail price is still widely used and is believed to protect consumers from price volatility. Also, dynamic pricing has limitations when it comes to implementation requiring Advanced Metering Infrastructure (AMI) [33]. Alternatively the operator could use Incentive-based DR programs or Demand Reduction programs.

Incentive-based DR schemes require an established baseline against which consumer's load reduction will be measured. The baseline is an estimate of the consumption of the consumer had the consumer not been participating in the DR program. Hence it is a counter-factual and requires estimation. The current methods for baseline estimation can be broadly classified as: i) averaging methods, which use some linear combination of hourly load values from previous days to predict the load on the event day, and ii) explicit weather models [14]. In its current scheme, the California Independent System Operator (CAISO) uses the mean of the
consumption on the ten most recent non-event days as the baseline estimate. The CAISO method also uses a morning adjustment factor to account for any variability in consumption pattern during the day of the DR event from the past. Authors in [14, 6, 9, 27] provide a comparative study of different methods for estimating baseline. Baseline estimation schemes have several concerns. One is that consumers have an incentive to artificially inflate their baseline to increase their profits [5, 49, 7]. There are celebrated cases where the participants artificially inflated their baseline for increasing payments [40]. Also, there is a problem of under payment and over payment because of inaccurate baseline. This can lead to inadequate demand response from the consumers.

Wolak et al. [49] provide a case study of critical peak pricing (CPP) experiment involving residential customers of the City of Anaheim Public Utilities (APU) over the period June 1, 2005 to October 14, 2005. They conclude that vast majority of reductions that were paid for, would have occurred anyway without the payments. They suggest that the incentives to increase the consumption at similar times during non-CPP days could have been a possible cause for the observed effect.

In chapter 2, we propose a baseline mechanism for the retail market. We approach baseline reporting as a mechanism design problem from the perspective of the demand response aggregator. Consumers self-report their estimated baseline consumption and marginal utility to the aggregator. The objective of the aggregator is to design an incentive mechanism such that (a) each consumer reports their true baseline and true marginal utility, (b) meet any (random) load reduction requirement by effective scheduling, and (c) scheduling is efficient.

We propose a mechanism where reporting the true baseline consumption and marginal utility is a dominant strategy for each consumer. We also show that each consumer sticks to their baseline consumption when she is not called for DR and reduces maximum possible load when she is called for DR. Also, the aggregator can ensure adequate response to meet the load reduction requirement. The proposed mechanism is also nearly efficient since it selects consumers with the smallest marginal utilities where each selected consumer contributes the maximum possible load reduction. We also propose a second mechanism with a uniform payment scheme. In this second mechanism, under some assumptions, we show that truthful reporting of baseline consumption and marginal utility is a Nash equilibrium. Using simulations, we argue that such a Nash equilibrium is an expected outcome when the number of consumers is very large.

In the next chapter we show how retail DR programs can be integrated in to electricity markets. Electricity scheduling in US typically operates as a two settlement system, a day-ahead market (DAM) for bulk power scheduling and a real-time market (RTM) for supply-demand balancing. Under the two market system, demand response (DR) can be used as intermediate recourse, which has several benefits from both efficiency and operation point of view. To elaborate on this, the Load Serving Entity (LSE) can signal the aggregator for load reduction at one of the three possible time instants - (i) the Day Ahead Market (DAM), (ii) the Real Time Market or (iii) at an intermediate time which is between the DAM or

RTM. The LSE will have benefit less from requesting load curtailment one day in advance because it will require more accurate forecasts about the random renewables and the prices to determine the optimal amount of required load curtailment. So, from the LSE point of view it is best to signal for load curtailment closer to the RTM. However, the aggregator typically needs some lead time to organize the demand flexibility and to achieve a given load reduction reliably. This is because the cost for achieving a given reduction reliably can be considerably high especially when the consumers are informed in short notice. From reliability and cost point of view, it is better to request load reduction at an intermediate time. So we consider a setting where the LSE requests load curtailment from the aggregator at an intermediate time, between the DAM and the RTM. Here, we show that an intermediate market for DR can be created. As mentioned before, this has the twin advantages of a better forecast and gives the aggregator enough lead time for delivering the load curtailment reliably. This DR market is analyzed, its equilibrium and the efficiency properties are characterized.

In chapter 4, we propose a self-reported DR scheme for residential consumers that integrates DR resources into the wholesale markets efficiently. In this scheme the consumers are required to self-report their baseline and are paid at a pre-determined price level for every unit of reduction they provide. The price is set such that the SO's benefit is maximized when the recruited DR resources are deployed. We then compare the self-reported DR mechanisms with the CAISO's averaging method for estimating baseline. We show that the self-reported baseline DR mechanism establishes a better estimate of the mean baseline in scenarios where there is high variability in consumption. And in scenarios where there is low variability, both methods establish a similar baseline estimate. Because the payments are proportional to the baseline estimae, overall the self-reported DR mechanism is cost-effective.

The wholesale market mechanism proposed in chapter 4 achieves optimality in steady state. But in a repeated setting the transient losses also needs to be taken in to account. In the final chapter 5, we provide a baseline algorithm for transient performance. For this, we consider a repeated setting where the DR events repeat. In such a setting it is not sufficient that the optimal price is attained at steady state, becuase the transient losses have to be taken in to account. In the full information setting it is trivial to define the optimal pricing policy. In the incomplete information setting, one has to consider the trade-off between learning the consumer behavior and maximizing the savings based on the learnt information. Here we propose a pricing policy that achieves sub-linear regret.

## Chapter 2

## Baseline Mechanism for Retail Market

We motivated the design of retail DR programs in the previous chapter. In Retail Demand Response (DR) programs, there is an aggregator who recruits and manages residential or industrial customers who are willing to reduce their electricity consumption at times when they are called to do so. These could be times of peak load where the load curtailment is a better alternative to balance the grid. The aggregator role is to serve as an intermediary by representing these flexible consumers to the system operator (SO). The SO pays the aggregator a capacity payment for the resources it acquires to provide the necessary load reduction at short notice. And when the consumer responds with the required load reduction the aggregator pays the customers it represents for their flexibility.

Though most DR programs cater to peak shaving applications, it is recognized that demand flexibility can offer more lucrative ancillary services such as frequency regulation or loadfollowing. Hence demand flexibility can be used to compensate the variability of renewable generation thereby balancing supply and demand without polluting or consuming more resources. This is recognized and encouraged by the Federal Energy Regulatory Commission through its Order 745, which mandates that demand response be compensated on par with the conventional generation that supplies grid power. In this work, however, we are concerned only with peak shaving DR applications.

There are three key components of any DR program that need to be designed: (a) a baseline against which load reduction is measured, (b) the payment scheme to recruited customers for the load reduction they provide, and (c) various contractual clauses which specifies limits on the frequency of DR events or penalties for nonconforming customers. Typically, historical averages of consumption on similar days (by the consumer, or by a peer group of similar consumers) are used as baseline estimate. However, there are several reported cases where consumers artificially inflated their baseline for generating more payment [40]. Incorrect baselines result in under or over-payment. Under payments affect consumer participation and over payments affect the efficiency of these programs. Neither are desirable. Apart from this adverse effect one needs to worry about the fairness aspect of the payments. A customer who happens to be on vacation during a DR event receives a payment for load reduction without
suffering any hardship faced by other customers who actually curtail their consumption.
Traditionally DR programs reward customers for load reduction during peak consumption periods. It is widely agreed upon that consumers have an incentive to artificially inflate their baseline to increase their profits [5], [7], [4], [49]. These incentives persist even when the probability of occurrence of the DR event is low [7]. Alternative payment mechanisms that avoid resulting inefficiencies are offered in [7], but these do not explicitly address baseline inflation concerns. Adverse selection and double payment effects are two other issues that arise as a result of rewarding consumers based on load reduction from estimated baselines [5]. Addressing these gaming issues while ensuring fairness to participating consumers is essential to encourage and sustain wider use of DR programs.
Related Work: Chen et al. [10], consider penalties when the consumer deviates from the baseline. The authors propose a two-stages game for DR . It is shown that the penalty (linear in deviations) induces users to report their true baseline assuming knowledge of consumer's utility function. The aggregator realizes its DR objective by tuning the retail price. In [42], assuming fixed costs for DR participation and linear costs for deviating from a known baseline, a centralized DR scheduling algorithm is proposed guaranteeing incentive compatibility in the case of two participants. A DR market assuming known baselines is proposed in [37, 36] where the objective is to maximize a social benefit function in the DR market. The approaches in [10], [42], [37] and [36] either assume knowledge of utility function or baselines.

The authors in [10] propose a two-stage game for DR. It is shown that the penalty (linear in deviations) induces users to bid their true demand. If the SO knows the utility function of customers it can choose a retail price to tune demand. In [42] assuming fixed costs for DR participation and linear costs for deviating from baseline (known), a central dispatch algorithm is proposed guaranteeing incentive compatibility in the case of two participants. A DR market assuming known baselines is proposed in [37,36] where the objective is to maximize a social benefit function of the DR market. In all the above works either the baseline or the utility of the consumer is assumed to be known. In our current work we do not assume that the aggregator has knowledge of either of the variables

We approach retail DR problem as a mechanism design problem from the perspective of the demand response aggregator. Consumers self-report their estimated baseline consumption and marginal utility to the aggregator. The objective of the aggregator is to design an incentive mechanism such that (a) each consumer reports their true baseline and true marginal utility, (b) meet any (random) load reduction requirement by effective scheduling, and (c) scheduling is efficient.

We propose a mechanism where reporting the true baseline consumption and marginal utility is a dominant strategy for each consumer. We also show that each consumer sticks to their baseline consumption when she is not called for DR and reduces maximum possible load when she is called for DR. Also, the aggregator can ensure adequate response to meet the load reduction requirement. The proposed mechanism is also nearly efficient since it selects consumers with the smallest marginal utilities where each selected consumer
contributes the maximum possible load reduction. We also propose a second mechanism with a uniform payment scheme. In this second mechanism, under some assumptions, we show that truthful reporting of baseline consumption and marginal utility is a Nash equilibrium. Using simulations, we argue that such a Nash equilibrium is an expected outcome when the number of consumers is very large.

### 2.1 Model and Problem Formulation

We consider a setting where the baseline-based DR program is managed by a single aggregator with $N$ participating consumers. The aggregator's role is to manage the DR program for these consumers - enrolling them for the DR program, sending signals whenever a load reduction is required, and giving rewards for participating in the program.

Consumer Model: Let $u_{k}\left(q_{k}\right)$ be the utility of consumer $k$ derived by consuming $q_{k}$ units of energy. We assume that $u(\cdot)$ has the following form.

$$
u_{k}\left(q_{k}\right)= \begin{cases}\pi_{k} q_{k} & \text { if } q_{k}<b_{k}  \tag{2.1}\\ \pi_{k} b_{k} & \text { if } q_{k} \geq b_{k}\end{cases}
$$

Here $b_{k}$ is the maximum energy requirement of consumer $k$. Any additional consumption will not increase her utility. We call $\pi_{k}$ the true marginal utility of consumer $k$. We assume that $\pi_{k} \leq \pi_{\max }, \forall k$. Let $\pi^{e}$ be the retail price for unit energy. Then the net utility $U_{k}(\cdot)$ of consumer $k$ is given by,

$$
U_{k}\left(q_{k}\right)= \begin{cases}\pi_{k} q_{k}-\pi^{e} q_{k} & \text { if } q_{k}<b_{k}  \tag{2.2}\\ \pi_{k} b_{k}-\pi^{e} q_{k} & \text { if } q_{k} \geq b_{k}\end{cases}
$$

We assume that $\pi_{k}>\pi^{e}, \forall k$. The optimal consumption for consumer $k$ which maximizes her net utility is clearly $b_{k}$. We call this the true baseline consumption of consumer $k$. We formulate baseline reporting and subsequent load reduction as a two stage mechanism.

Stage 1 (Reporting): At the beginning of stage 1, aggregator announces a selection procedure and a reward function $R(\cdot, \cdot, \cdot)$ for consumers who are selected to reduce load. Aggregator also announces a penalty function $\Phi(\cdot, \cdot, \cdot)$ for consumers who deviate from their reported baseline. Depending on the selection procedure, the reward $R(\cdot, \cdot, \cdot)$ and the penalty $\Phi(\cdot, \cdot, \cdot)$, each consumer $k$ reports two parameters, the baseline consumption $\hat{b}_{k}$ and marginal utility $\hat{\pi}_{k}$. Consumers can potentially give incorrect reports, i.e., $\hat{b}_{k}$ and $\hat{\pi}_{k}$ need not be equal to $b_{k}$ and $\pi_{k}$ respectively.

Stage 2 (Load reduction and Payment): A DR event occurs where the aggregator has to collate a total load reduction of $D$ units. We assume that $D$ is random, and only the aggregator knows the value of $D$. This is reasonable considering the fact that a DR event and the extent of supply deficit is an exogenous event and is not observed by the consumer. Aggregator selects a set of consumers, depending on their reports $\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$, to meet the total reduction

Table 2.1: Notations - Retail Market

| $b_{k}$ | Baseline consumption of consumer $k$ |
| :---: | :--- |
| $q_{k}$ | Energy consumption of consumer $k$ |
| $\pi_{k}$ | Marginal utility of consumer $k$ |
| $u_{k}$ | Utility of consumer $k$ |
| $\pi^{e}$ | Retail price of energy |
| $U_{k}$ | Net utility of consumer $k$ |
| $R(\cdot, \cdot, \cdot)$ | Reward function for load reduction |
| $\Phi(\cdot, \cdot, \cdot)$ | Penalty function for deviation from baseline |
| $\pi_{k}^{r}$ | Reward/kWh awarded to consumer $k$ |
| $\pi_{k}^{p}$ | Penalty/kWh imposed on consumer $k$ |
| $\hat{b}_{k}$ | Baseline report of consumer $k$ |
| $\hat{\pi}_{k}$ | Marginal utility report of consumer $k$ |
| $D$ | Load reduction requirement |

$D$ and sends a load reduction signal to each of these selected consumers. If consumer $k$ is selected, it gets a reward $R\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)$ when consuming $q_{k}$. Every consumer $k$ gets a penalty $\Phi\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)$ if the consumption $q_{k}$ is different from the reported baseline $\hat{b}_{k}$.

We assume that the net load reduction requirement $D$ is independent of the reports of the consumers. Even though, only the aggregator observes the actual realization of $D$, we assume that the distribution $D$ is a common information, i.e., it is known to the aggregator and all consumers. In particular, each consumer knows the probability of a DR event, i.e., $\alpha=\mathbb{P}(D>0)$.

Consumer's problem: We assume that the consumers are non-cooperative. In particular, each consumer faces a two stage decision problem. In the first stage, it has to decide the value of the reports, i.e. $\hat{b}_{k}$ and $\hat{\pi}_{k}$. In the second stage it has to decide on the energy consumption $q_{k}$. The value of $q_{k}$ will depend on the consumer $k$ being called for DR or not. The objective of each consumer is to maximize her expected benefit, $\bar{J}_{k}$. Consumer $k$ 's two-stage optimization problem can be formulated as

$$
\begin{align*}
& (C P) \max _{\hat{b}_{k}, \hat{\pi}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right), \text { where, } \\
& \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)=\underset{D, \hat{b}_{-k}, \hat{\pi}_{-k}}{\mathbb{E}}\left[\max _{q_{k}}\left(U_{k}\left(q_{k}\right)+R(\cdot, \cdot, \cdot)-\Phi(\cdot, \cdot, \cdot)\right)\right] \tag{2.3}
\end{align*}
$$

Here $\hat{b}_{-k}=\left(\hat{b}_{1}, \ldots, \hat{b}_{k-1}, \hat{b}_{k+1}, \hat{b}_{N}\right)$ and $\hat{\pi}_{-k}=\left(\hat{\pi}_{1}, \ldots, \hat{\pi}_{k-1}, \hat{\pi}_{k+1}, \hat{\pi}_{N}\right)$.

### 2.2 Self-Reported Baseline Mechanism

Here we introduce the Self-Reported Baseline Mechanism (SRBM). In the first stage, (each) consumer $k$ reports her baseline consumption $\hat{b}_{k}$ and marginal utility $\hat{\pi}_{k}$ to the aggregator. Aggregator reorders the consumers in increasing order of their marginal utility. Formally, the aggregator forms the vectors $\hat{B}$ and $\hat{\Pi}$ such that

$$
\begin{equation*}
\hat{B}=\left(\hat{b}_{1}, \ldots, \hat{b}_{N}\right), \hat{\Pi}=\left(\hat{\pi}_{1}, \ldots, \hat{\pi}_{N}\right), \quad \hat{\pi}_{j} \leq \hat{\pi}_{j+1}, \forall j . \tag{2.4}
\end{equation*}
$$

In the second stage, aggregator observes the net load reduction requirement $D$. Then, the aggregator selects the first $k^{*}(D)$ consumers (in the increasing order of their marginal utility) such that

$$
\begin{equation*}
\sum_{j=1}^{k^{*}(D)-1} \hat{b}_{j}<D \leq \sum_{j=1}^{k^{*}(D)} \hat{b}_{j} \tag{2.5}
\end{equation*}
$$

If $\hat{b}_{j}$ is the true baseline $\forall j$ and consumers reduce their load by the maximum possible amount, this selection process would meet the load reduction requirement.

From now on we shorten the notation to $k^{*}$ instead of $k^{*}(D)$. Also, let $S(D)=\left\{1, \ldots, k^{*}\right\}$ be the set of selected consumers. Aggregator then sends DR signal to every consumer in the set $S(D)$. Note that not all consumers are selected at each time. The number of selected consumers depends on $D$. Aggregator then observes the consumption $q_{k}$ of all consumers.

Let $S_{-k}(D)$ be the set of consumers who would be selected by the aggregator according to (2.5), if consumer $k$ is not participating in the DR program. Let

$$
\begin{equation*}
\pi_{k}^{r}(D)=\max \left\{\hat{\pi}_{j}\right\}-\pi^{e}, j \in S_{-k}(D) \tag{2.6}
\end{equation*}
$$

The reward function for consumer $k$ is then defined as,

$$
R\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)=\left\{\begin{array}{cc}
\pi_{k}^{r}(D)\left(\hat{b}_{k}-q_{k}\right)_{+} & \text {if } k \text { is selected }  \tag{2.7}\\
0 & \text { otherwise }
\end{array}\right.
$$

Clearly, consumer $k$ gets a reward for load reduction only if it is selected. The consumer reward is proportional to the measured load reduction $\left(\hat{b}_{k}-q_{k}\right)_{+}$. In the following, we denote $\pi_{k}^{r}(D)$ simply as $\pi_{k}^{r}$ making the dependence on $D$ implicit.
The aggregator also imposes a penalty if the consumption $q_{k}$ of consumer $k$ is different from her reported baseline consumption $\hat{b}_{k}$. We define the penalty function as

$$
\Phi\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)=\left\{\begin{array}{lc}
\pi_{k}^{r}\left(q_{k}-\hat{b}_{k}\right)_{+} & \text {if } k \text { is selected }  \tag{2.8}\\
\hat{\pi}_{k}\left|\left(q_{k}-\hat{b}_{k}\right)\right| & \text { otherwise }
\end{array}\right.
$$

Clearly, no penalty for decreasing the load when the consumer is selected for DR. However, we impose a penalty for positive and negative deviation from the reported baseline when consumers are not selected for DR.

Let $J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k}\right)$ be the ex-post benefit of consumer $k$ given the first stage reports $\hat{b}_{k}$ and $\hat{\pi}_{k}$ and the second stage consumption $q_{k}$. Then using (2.7) and (2.8) the ex-post benefit, a consumer receives when selected for DR can be written as,

$$
\begin{align*}
& J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\pi_{k} \min \left\{q_{k}, b_{k}\right\}-\pi^{e} q_{k} \\
& +\pi_{k}^{r}\left(\hat{b}_{k}-q_{k}\right) \tag{2.9}
\end{align*}
$$

The ex-post benefit when the consumer is not selected for DR can be written as,

$$
\begin{gather*}
J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right)=\pi_{k} \min \left\{q_{k}, b_{k}\right\} \\
-\pi^{e} q_{k}-\hat{\pi}_{k}\left|q_{k}-\hat{b}_{k}\right| . \tag{2.10}
\end{gather*}
$$

We now specify the SRB mechanism formally, from the perspective of the aggregator.

1. Receive reports $\hat{b}_{j} \mathrm{~S}$ and $\hat{\pi}_{j} \mathrm{~s}$ from all consumers
2. Observe the net load reduction requirement $D$
3. Select $S(D)$ consumers as specified by (2.5)
4. Observe consumption $q_{j}$ of every consumer $j$
5. Award the payment and impose the penalty as specified by (2.7)-(2.8)

Algorithm 1: (Self-Reported Baseline Mechanism (SRBM)) - Retail Market
Before characterizing the equilibrium outcome under the SRB mechanism, we first formally define the following notions.

Definition 1 (Dominant strategy). Let $\bar{J}_{k}\left(\left(\hat{b}_{k}, \hat{\pi}_{k}\right) ;\left(\hat{b}_{-k}, \hat{\pi}_{-k}\right)\right)$ be the expected net benefit of consumer $k$ when she reports $\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$ and other consumers report $\left(\hat{b}_{-k}, \hat{\pi}_{-k}\right)$. The pair $\left(\hat{b}_{k}^{*}, \hat{\pi}_{k}^{*}\right)$ is a dominant strategy report for consumer $k$ if $\left(\hat{b}_{k}^{*}, \hat{\pi}_{k}^{*}\right)=\arg \max _{\hat{b}_{k}, \hat{\pi}_{k}} \bar{J}_{k}\left(\left(\hat{b}_{k}, \hat{\pi}_{k}\right) ;\left(\hat{b}_{-k}, \hat{\pi}_{-k}\right)\right)$, for any report $\left(\hat{b}_{-k}, \hat{\pi}_{-k}\right)$ from other consumers.

We make the following assumption on the probability a DR event.
Assumption 1. Let $\alpha=\mathbb{P}(D>0)$ be the probability of a $D R$ event. Then, $\alpha \pi_{\max }<(1-\alpha) \pi^{e}$.
This is a mild assumption when the frequency of DR events is small, which is indeed the case in the current DR programs. We now give the main result which establishes that the proposed mechanism achieves the desired properties.

Theorem 1. In the $S R B$ mechanism, under Assumption 1,
(i) Reporting baseline and marginal utility truthfully is a dominant strategy i.e. $\left(\hat{b}_{k}^{*}, \hat{\pi}_{k}^{*}\right)=$ $\left(b_{k}, \pi_{k}\right)$
(ii) When consumer $k$ is not selected for $D R$, her optimal consumption is the same as the reported baseline consumption, i.e. $q_{k}=\hat{b}_{k}$
(iii) When consumer $k$ is selected for $D R$, her optimal consumption is zero, i.e. $q_{k}=0$

Proof is given in the appendix.
Remark 1. Note that under SRBM, each consumer reports its true baseline and sticks to its baseline consumption when it is not selected for DR. Hence the reduction that is measured from the reported baseline is indeed the true load reduction. Also, the selection process (2.5) meets the load reduction requirement because the consumers reduce their consumption to zero when selected. Note that, consumers reducing their consumption to zero is an artifact of the piece-wise linear utility function model.
Remark 2. From a social welfare point, neglecting the payment/penalty transactions, the selection process is also nearly efficient because it selects consumers with the smallest marginal utilities, where each selected consumer contributes the maximum possible load reduction. Hence the total disutility $\left(-\sum_{k} u_{k}(\cdot)\right)$ is minimized. However it is not clear if it is the best mechanism from the perceptive of the aggregator: i.e., minimize the total payment to the consumers while (i) eliciting truthful reports from the consumers and (ii) achieving required load reduction.

Remark 3. Here we assumed that the true baseline consumption $b_{k}$ is deterministic. However, $b_{k}$ depends on (exogenous) random parameters like temperature. For example, a possible model can be $b_{k}=\bar{b}_{k}+a_{k}\left|\theta-\theta_{0}\right|$ where $\theta$ is the temperature and $\theta_{0}$ is the nominal temperature. Here the consumers can report two parameters, $\bar{b}_{k}$ and the temperature sensitivity $a_{k}$. Historical consumption data can be used to estimate these parameters. SRB mechanism can also be extended to such cases.

Remark 4. Clearly, the results depends on the form of the utility function; in particular, the deterministic and the piece-wise linear assumption on utility function $u_{k}(\cdot)$. This can be considered as the first step approximation, where such a self-reported baseline mechanism achieves desirable properties.

### 2.3 Uniform Payment Mechanisms

In the SRB mechanism the reward rate $\pi_{k}^{r}$ is different for different consumers. In this section, we introduce a uniform payment mechanism for the baselining problem. We show that under certain conditions, this mechanism achieves the desired properties.

Mechanism: In the beginning of the first stage, consumer $k$ submits her baseline consumption $\hat{b}_{k}$ and marginal utility report $\hat{\pi}_{k}$ to the aggregator. As in the previous mechanism, the
aggregator reorders the consumers in increasing order of their reported marginal utility. Formally the aggregator forms the vectors $\hat{B}$ and $\hat{\Pi}$ where

$$
\begin{equation*}
\hat{B}=\left\{\hat{b}_{1}, \ldots, \hat{b}_{N}\right\}, \hat{\Pi}=\left(\hat{\pi}_{1}, \ldots, \hat{\pi}_{N}\right) \text { such that } \hat{\pi}_{j} \leq \hat{\pi}_{j+1}, \forall j \tag{2.11}
\end{equation*}
$$

In the second stage the aggregator receives the load reduction requirement $D$. The aggregator then selects the first $k^{*}=k^{*}(D)$ consumers such that

$$
\begin{equation*}
\sum_{k=1}^{k^{*}(D)-1} \hat{b}_{k}<D \leq \sum_{k=1}^{k^{*}(D)} \hat{b}_{k} \tag{2.12}
\end{equation*}
$$

The consumers who are selected are paid according to,

$$
\begin{equation*}
\pi^{r}(D)=\hat{\pi}_{\left(k^{*}(D)+1\right)}-\pi^{e}, \tag{2.13}
\end{equation*}
$$

while others are paid zero. Then the reward function is given by,

$$
R\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)=\left\{\begin{array}{cc}
\pi^{r}(D)\left(\hat{b}_{k}-q_{k}\right)_{+} & \text {if } k \text { is selected }  \tag{2.14}\\
0 & \text { otherwise }
\end{array}\right.
$$

The penalties are set as,

$$
\Phi\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)=\left\{\begin{array}{lc}
\pi_{\max }\left(q_{k}-\hat{b}_{k}\right)_{+} & \text {if } k \text { is selected }  \tag{2.15}\\
\pi_{\max }\left|\left(q_{k}-\hat{b}_{k}\right)\right| & \text { otherwise }
\end{array}\right.
$$

In the following theorem, we show that reporting marginal utility and baseline truthfully is a Nash equilibrium strategy. Also we show that the consumer provides maximum possible load reduction when signaled to reduce. We define the Nash equilibrium concept below.
Definition 2 (Nash equilibrium). Let $\bar{J}_{k}\left(\left(\hat{\pi}_{k}, \hat{b}_{k}\right),\left(\hat{\pi}_{-k}, \hat{b}_{-k}\right)\right)$ be the expected benefit of consumer $k$ when she reports $\left(\hat{\pi}_{k}, \hat{b}_{k}\right)$ and other consumers report $\left(\hat{\pi}_{-k}, \hat{b}_{-k}\right)$. Then $\left(\hat{\pi}^{*}, \hat{b}^{*}\right)=$ $\left(\left(\hat{\pi}_{1}^{*}, \hat{b}_{1}^{*}\right), \ldots,\left(\hat{\pi}_{N}^{*}, \hat{b}_{N}^{*}\right)\right)$ is a Nash equilibrium if $\bar{J}_{k}\left(\left(\hat{\pi}_{k}, \hat{b}_{k}\right),\left(\hat{\pi}_{-k}^{*}, \hat{b}_{-k}^{*}\right)\right) \leq \bar{J}_{k}\left(\left(\hat{\pi}_{k}^{*}, \hat{b}_{k}^{*}\right),\left(\hat{\pi}_{-k}^{*}, \hat{b}_{-k}^{*}\right)\right)$

We need the following assumption. In the next subsection, we also argue that this assumption is not really necessary when the number of consumers is very large.

Assumption 2. Let $\left\{\pi_{k}, b_{k}\right\}$ be the marginal utility and baseline consumption of consumers. If $\pi_{j} \leq \pi_{k}$, then $b_{j} \leq b_{k}, \forall j, k$.
Theorem 2. Under Assumption 2, with the condition that $\pi_{\max }<(1-\alpha) \min \left\{\pi_{k}-\right.$ $\left.\pi^{e}, \pi^{e}\right\} / \alpha \forall k$, reporting the baseline and marginal utility truthfully is a Nash equilibrium of mechanism SRBM-UP. Under this Nash-Equilibrium strategy,
(i) When consumer $k$ is not selected for $D R$, consumer $k$ consumes the reported baseline i.e. $q_{k}=\hat{b}_{k}$
(ii) When consumer $k$ is selected for $D R$, consumer reduces load consumption to zero i.e. $q_{k}=0$

Proof and a detailed explanation is given in appendix.

### 2.3.1 Simulations

In Section 2.3 we proved that reporting truthfully is a Nash equilibrium under Assumption 2. However this assumption may not be necessary when the number of consumers are very large, which we argue by simulation. More specifically, simulation experiments suggest that the fraction of consumers who give incorrect reports approaches zero as the total number of consumers increases.

Given the number of consumers $N$, we generate each consumer's baseline consumption $b_{k}$ and marginal utility $\pi_{k}$ uniformly at random. We assume $b_{k} \sim U[0,1]$ and $\pi_{k} \sim U[0,1]$. Let $Q=\left(b_{1}, \ldots, b_{N}\right)$ and $\Pi=\left(\pi_{1}, \ldots, \pi_{N}\right)$. Also, we assume that the net load reduction requirement $\mathbb{P}(D \mid D>0) \sim U[0,50]$. Simulation procedure is as follows:
(i) For a given $N$ generate $Q$ and $\Pi$ as specified above.
(ii) In [35] we show that reporting baseline truthfully is a dominant strategy. Hence, we set $\hat{b}_{k}=b_{k}$ as the optimal baseline report of consumer $k$. Then we compute the optimal report $\hat{\pi}_{k}$ by numerically solving the consumer problem (4.4) assuming that other consumers report truthfully. While solving, we assume that consumer $k$ knows both $Q$ and $\Pi$ perfectly. So, the deviation $\delta_{k}=\hat{\pi}_{k}-\pi_{k}$ is in a way the worst case deviation because $\hat{\pi}_{k}$ is computed with perfect information, which is not available to consumer $k$ in the original setting.
(iii) Compute the cdf of the vector of deviations $\Delta=\left(\delta_{1}, \ldots, \delta_{N}\right)$.
(iv) Repeat this for different realizations of $Q$ and $\Pi$. Plot the averaged cdf.

We repeat the above procedure for different values of $N$. Figure 2.1 shows that as $N$ increases, the cdf approaches a step function. So, the fraction of consumers who deviate from truthful reporting decreases to zero as $N$ increases. This extends the scope of uniform payment mechanisms to more general conditions provided the number of consumers is large.


Figure 2.1: CDF of deviations from the truthful reporting $(\delta)$

### 2.3.2 Illustrative Example

We give an example for SRBM and SRBM-UP based demand response.

Example 1: Consider the setting: $N=3, \pi_{1}=1, \pi_{2}=2, \pi_{3}=3, b_{1}=3, b_{2}=1$ and $b_{3}=2$. To keep this simple, assume that $\pi^{e}=0$.
(i) SRBM: Reporting truthfully is a dominant strategy for all consumers. Let the realized load reduction be $D=3$ (and assume $\mathbb{P}(D \leq 3) \leq \alpha$ ). By the selection process (2.5) consumer 1 will be selected, that is $k^{*}=1$ and $\hat{S}_{-1}=\{2,3\}$. Then the reward for unit reduction offered to consumer 1 is $\pi_{1}^{r}=\max \left\{\pi_{2}, \pi_{3}\right\}-\pi^{e}=3$. Because $\pi_{1}^{r}>\pi_{1}$ consumer 1 will reduce consumption to zero i.e. $q_{1}=0$ and is paid $P=\pi_{1}^{r} b_{1}=9$.
(ii) SRBM-UP: We now solve for the payments under SRBM-UP if the consumers report truthfully. Here too $k^{*}=1$ and hence $\pi^{r}=\pi_{k^{*}+1}=\pi_{2}=2$. Hence consumer 1 reduces on being selected and is paid $P=\pi^{r} b_{1}=6$. Note that this is less than what the aggregator pays using SRBM. However, we can construct other example where the total payment by the aggregator is more in SRBM-UP as compared to SRBM. In this section we provide an illustrative example. Using SRBM-UP, the aggregator cannot always expect the payment to equal the payment that would result from consumers reporting truthfully. Such intractability could be avoided by choosing a very large price for unit reduction, but that would result in a large payment and the aggregator could instead use SRBM. On the other hand in scenarios where the consumers report truthfully SRBM-UP is more effective in terms of the total payment. We explain this using examples. Next we provide an example where SRBM-UP performs better in terms of the total payment. In this example, we assume consumers report truthfully under SRBM-UP.

In the following example we show that SRBM achieves the same level of load reduction with a lesser payment in comparison to SRBM-UP.
Example 2: For the same setting consider the scenario where the true marginal utility of consumer 1 and consumer 3 is common knowledge. First we show that it is profitable for consumer 2 to under report it's marginal utility under SRBM-UP and then compare the total payment with that of SRBM. Let the realized load reduction be $D=3$ (and assume $\mathbb{P}(D \leq 3) \leq \alpha)$. If consumer 2 reports truthfully then by Mechanism SRBM-UP, $k^{*}=1$ and $\pi^{r}(D)=\pi_{k^{*}+1}=\pi_{2}=2$. Since $k^{*}=1$, consumer 2 will not be selected and will consume it's baseline report which is the true baseline i.e. $q_{2}=b_{2} \Rightarrow \Phi_{2}=0$. Hence in this case, $J_{2}\left(\hat{\pi}_{2}=\pi_{2}\right)=U_{2}+R-\Phi=\pi_{2} b_{2}=2$ and the total payment $P\left(\hat{\pi}_{2}=\pi_{2}\right)=\pi^{r}(D) b_{1}=6$. On the other hand if consumer 2 deviates by reporting a $\hat{\pi}_{2}$ that is less than $\pi_{1}$, then both consumer 1 and consumer 2 will be selected i.e. $k^{*}=2$ and will be paid according to $\pi^{r}(D)=\pi_{k^{*}+1}=\pi_{3}$ per unit of reduction. As a result both consumer 1 and consumer 2 will reduce their consumption to zero. Hence consumer 2 will be made the payment $R_{2}=$ $\pi^{r}(D) b_{2}=\pi_{3}=3$ for the observed reduction $b_{2}$ and $J_{2}\left(\hat{\pi}_{2}<\pi_{1}\right)=R_{2}=3$. Total payment $P\left(\hat{\pi}_{2}<\pi_{1}\right)=R_{1}+R_{2}=\pi^{r}(D) b_{1}+\pi^{r}(D) b_{2}=12$. It is clear that $J_{2}\left(\hat{\pi}_{2}<\pi_{1}\right)>J_{2}\left(\hat{\pi}_{2}=\pi_{2}\right)$,
and so consumer 2 will under report it's marginal utility. As a result the aggregator makes a total payment $P\left(\hat{\pi}_{2}<\pi_{1}\right)=12$ which is less than the payment made in mechanism SRMB-UP (refer above example).

### 2.4 Comparison with SRBM for the Retail Market

Here we compare the proposed mechanism with that of CAISO's method. In the CAISO method of payment for the consumers, the SO or the Utility computes the mean of the consumption of the most recent $m$ similar but non-event days and then multplies this mean estimate by an adjustment factor to estimate the baseline. The adjusment factor accounts for any variation in the consumption pattern from the past. The adjustment factor is calculated based on the consumption in the hours prior to the DR event on the DR event day [14]. In this chapter we discuss CAISO's mechanism and compare it's mechanism with the self-reported mechanism (SRBM) proposed in 2

Baseline Calculation: After the DR event the baseline is calculated in the following way. Denote the consumption of the most recent $m$ similar but non-event days collectively by $\mathbf{b}_{m}$ and the calculated adjustment factor by $C_{b}$. Then the estimated baseline is

$$
\bar{b}_{k}=b_{k}^{c} C_{b}
$$

Where $b_{k}^{c}=\left(\operatorname{sum}\left\{\mathbf{b}_{m}\right\} / m\right)$. Denote the consumption in the hours prior to the DR event by $q_{k}^{-}$and the consumption during the hours prior to the hour that corresponds to the DR event hour on the DR event day, of the most recent $m$ similar but non event days collectively as, $\mathbf{b}_{m}^{-}$. Let $b_{k}^{-}=\left(\operatorname{sum}\left\{\mathbf{b}_{m}^{-}\right\} / m\right)$. Then the adjustment factor is given by $C_{b}=q_{k}^{-} / b_{k}^{-}$. This completes baseline calculation. Let the price for unit reduction be $\pi$. So the payment for reduction in the CAISO method is given by $R\left(q_{k}, b_{k}^{c}, \pi\right)=\pi\left(\bar{b}_{k}-q_{k}\right)$. Below we provide the theorem that characterizes CAISO mechanism

Theorem 3. The following holds for the CAISO mechanism, (i) When $\pi<\pi_{\max }-\pi^{e}$ CAISO mechanism cannot guarantee the required load reduction
(ii) When $\pi \geq \pi_{\max }-\pi^{e}$, CAISO mechanism payment is larger than the payment in SRBM (iii) When consumers are informed of the $D R$ event several hours prior to the $D R$ event then $\bar{b}_{k} \rightarrow \infty$ when $\pi>\pi^{e}$, i.e. the baseline estimate is inflated

Proof is given in appendix
Remark 5. Condition (i): In case $\pi<\pi_{\max }-\pi^{e}$, then there could be players who may not have adequate incentive to reduce. In such a case, there is a possibility that the marginal utility of consumer $k, \pi_{k}^{u}>\pi$ and the consumer will not have the incentive to reduce. Note that in SRBM the selected consumers will necessarily provide the required load reduction (Theorem 1)

Remark 6. Condition (ii): Note that the reward/kWh in SRBM is always $\pi_{k}^{r} \leq \pi_{\max }-\pi^{e}$. And in this case, CAISO's reward $/ \mathrm{kWh}$ is $\pi>\pi_{\max }-\pi^{e}$. So even if CAISO's allocation is as efficient as that of SRBM, the relation between $\pi$ and $\pi_{k}^{r}$ i.e. $\pi>\pi_{k}^{r}$ would imply that CAISO's payment would exceed that of the payment made in SRBM.
Remark 7. Condition (iii): The adjustment factor in CAISO's baseline estimate depends on the consumption few hours before the DR event. So if the DR event is announced several hours before the DR event then the incentive for the consumer to inflate is very high. So it is intuitive that the consumer will inflate by a large amount.

### 2.5 Conclusion

The unanswered question is whether we can design a mechanism that performs better than the mechanism proposed here. Characterizing this is difficult for the following reason. First, it is difficult to estimate the payment of the optimal mechanism in this setting and secondly it is difficult to estimate the upper bound on the factor by which the payment in SRBM will differ from the optimal mechanism.

To summarize, we proposed a mechanism where the consumers self-report their baseline and their marginal utility. In the proposed mechanism the consumers reveal their true baseline and also provide the maximum load reduction when signaled for DR. Also they stick to their baseline consumption when they are not signaled. So, the aggregator is able to meet any random load reduction requirement reliably by selecting consumers whose measured reductions adds up to the total reduction. Finally we show that in the current CAISO method either (i) one cannot guarantee reliable load reduction (ii) Or the payments are larger (iii) And in some scenarios the baseline estimate is inflated.

## Chapter 3

## Market Mechanisms for Selling Retail Demand Response

Electricity scheduling in US typically operates as a two settlement system, a day-ahead market (DAM) for bulk power scheduling and a real-time market (RTM) for supply-demand balancing. Under the two market system, demand response (DR) can be used as intermediate recourse, which has several benefits from efficiency and operational perspective. We show that an intermediate market for DR an be created. This enables the Load Serving Entity (LSE) to exploit the improved forecast available at the intermediate time (as compared with the day-ahead) thus improving overall efficiency and gives the aggregator enough lead time for organizing and delivering the load curtailment. We analyze this intermediate market, characterize the equilibrium and study the efficiency properties.
We formulate the energy scheduling problem from the perspective of LSE as a three stage stochastic control problem. We characterize the optimal decisions in each of the three stages - the optimal energy purchase from the DAM, the optimal load curtailment decision at the intermediate market and finally the energy purchase from the RTM. Here we refer to the LSE and the aggregator as a single entity.

First we consider the entity problem. Characterize the opimal purchase decisions for the entity. We then consider the interaction of LSE and the aggregator as two different entitties. The intermediate market is modelled as a a spot-market with contingent prices. We show the existense of a competitive equilibrium in this market. Also all equilibria are socially optimal. We characterize the equilibrium prices and purchase decisions in such a market. We also comment on the efficiency of this mechanism. In the final section we consider the contract setting assuming that the LSE has full market power.

### 3.0.1 Related Work

There are several benefits of coordinating demand side resources to balance the variability of intermittent renewable generation [13]. Many market or pricing mechanisms and scheduling
algorithms have been proposed for DR. Roscoe and Ault [43], Samadi et al. [44] and Chen et al. [11] also studied various real-time pricing based algorithms for DR. Li et al. [31] showed that time-varying prices can be used as signal to elicit demand response from consumers in such a way that it will achieve social optimality. Alternatively game theoretic models were proposed by Wu et al. [50] to understand the interactions among EVs and aggregators in a V2G market. In this setup, EVs role were in providing frequency regulation service to the grid. Different game theoretic fomulations for DR were also studided by Mohsenian-Rad et al. [34] and Yang et al. [52].

Gabriel et al., [17] and Haring et al., [20] studied contract mechanisms from the perspective of a retailer whose objective is to maximize its own profits and minimize settlement risks. Thus, a retailer acts as a mediator between the supplier and the consumer. Authors in [16] also considered a contract design method for load curtailment. Authors in [30] and [51] considered a setting where each consumer submits a parameter which characterizes their supply function and the utility company acts as the market maker who computes the market clearing price.
Varaiya et al. [47] formulated a multi-stage stochastic control problem where at each stage a utility company can decide the amount of power purchase depending on the available information. This idea was extended by Rajagopal et al. [41] to characterize threshold based decision policies for power procurement. Huang et.al. [21] also considered a similar problem of maximizing social welfare, where they jointly optimized demand response and power procurement. They proposed time varying price algorithm which the Utility company can announce to control the users' consumption while guaranteeing quality-of-usage to the consumer. There are also works that have extended ideas from financial engineering to interruptible power markets [19] [38]. Our approach is different from these works.

### 3.1 Preliminaries



Figure 3.1: Players, interactions, and decision time-line.

The setting that we consider is shown in Figure 3.1. The LSE serves the loads who consumes an energy of $l$ units. The LSE has access to zero-marginal cost random renewables $w$ which
are revealed at $T$. The LSE purchases $q^{d a}$ from the DAM at price $\pi^{d a}$ and $q^{r t}$ from the RTM at price $\pi^{r t}$. The LSE can also extract a curtailment of $y$ units from the loads at the intermediate time which is between the real time market and the day ahead market. The load suffers a disutility of $\phi(y)$ for the $y$ units of load reduction they provide. The total energy purchase and curtailment should be such that

$$
\begin{equation*}
l \leq q^{d a}+q^{r t}+w+y \tag{3.1}
\end{equation*}
$$

We assume that an aggregator manages a collection of flexible consumers or loads for demand response. It recruits the loads for DR participation, designs incentives for them and sends load reduction signals to a set of selected consumers when a DR event occurs. In the previous chapter 2 (refer [35] and [24]), we addressed the problem of mechanism design for reliably accruing a given net load reduction. Given that such mechanisms are possible to design, we assume that the aggregator can reliably deliver a net load reduction of $y$, if it commits. Here we assume that the LSE interacts only with one aggregator for achieving the load reduction, as shown in Figure 3.1.

The LSE decides curtailment decision $y$ based on information state $f_{1}$ at $t_{1}$ which comprises information about $w$ and $\pi_{r t}$ (as denoted in the figure 3.1). We have already discussed the benefits of deciding the curtailment decision $y$ at the intermediate time. The purchase decision $q_{d a}$ is made based on the information state $f_{0}$ which comprises information about $w$ and $\pi_{r t}$ at time $t_{0}$.

Consider the state at $t_{0}, f_{0}$, to be single valued. This is a reasonable assumption. Let $\alpha(s)$ be probability density function of the information state, i.e.,

$$
\alpha(s)=\mathbb{P}\left(f_{1}=s \mid f_{0}\right)
$$

We assume that the day-ahead price $\pi^{d a}$ is known at time $t_{0}$. We denote the expected RTM price conditioned on the realized state $f_{1}$ at $t_{1}$ by,

$$
\bar{\pi}_{s}^{r t}=\mathbb{E}\left[\pi^{r t} \mid f_{1}=s\right] .
$$

Let $p\left(w \mid f_{1}\right)$ be the conditional probability of the wind given the intermediate state $f_{1}$ at time $t_{1}$. We assume that $f_{1} \in S=[0,1]$. We call this an information state. Let

$$
p_{s}(w)=p\left(w \mid f_{1}=s\right), \quad P_{s}(z)=\int_{w=0}^{z} p_{s}(w) d w
$$

So, $P_{s}(z)$ is the probability that the wind at time $T$ is less than $z$ given the information state $s$. We use $\mathbb{E}_{s}[\cdot]$ and $\mathbb{E}_{w}[\cdot]$ to denote the expectation w.r.t. to the information state $s$ and the wind respectively and $\mathbb{E}[\cdot]$ denotes the joint expectation. The following assumptions are made regarding the random variables

Assumption 3. (i) $\mathbb{P}\left(w \geq z \mid f_{1}=s_{1}\right)<\mathbb{P}\left(w \geq z \mid f_{1}=s_{2}\right)$, $\forall z$, if $s_{2}>s_{1}$
(ii) $\bar{\pi}_{s_{2}}^{r t}<\bar{\pi}_{s_{1}}^{r t}$, if $s_{2}>s_{1}$
(iii) $\pi^{r t}$ and $w$ are conditionally independent given the information state $s$.

Then intuitive interpretation of assumption (i) is that, higher $s$ indicates (stochastically) more wind. Also, this assumption guarantees that $P_{s_{2}}(z)<P_{s_{1}}(z), \forall z$, if $s_{2}>s_{1}$ so that the cdfs $P_{s_{1}}(\cdot)$ and $P_{s_{2}}(\cdot)$ don't cross each other. Assumption (ii) similarly imposes an order on the expected value of the real-time price conditioned on the information state. We assume that higher $s$ indicates a lower expected real-time price (because of the higher wind). We note that the above assumptions are not necessary for most of the results we show. However, they considerably simplify the notations and give better insight to the problems.

LSE: As mentioned above it is the LSE that manages or serves the load. Its main objective is to make sure there is load balance. Based on the description above, it is clear that the LSE can buy $q^{d a}$ units of energy at a price $\pi^{d a}$ from the day ahead market (DAM). It can also request a load reduction of $y_{s}$ units from the aggregator when the information state $s$ is revealed at time $t_{1}$ by making a payment $R_{s}\left(y_{s}\right)$ to the aggregator. Based on the previous decisions $q_{d a}$ and $y_{s}$, the LSE can make the right amount of purchase, $q^{r t}$, such that it balances the load requirement, i.e., $q^{r t}=\left(l-q-y_{s}-w\right)_{+}$. The LSE will make these decisions such that it is optimal from the view of the cost it incrus. The ex-post cost for the LSE given the information state $s$ is,

$$
\begin{equation*}
J_{s}^{l s e}=\pi^{d a} q^{d a}+R_{s}\left(y_{s}\right)+\pi^{r t}\left(l-q^{d a}-y_{s}-w\right)_{+} \tag{3.2}
\end{equation*}
$$

Aggregator: As mentioned above the aggregator can provide load reduction of $y_{s}$ reliably. On doing so, it gets a payment $R_{s}\left(y_{s}\right)$ and suffers a disutility $\phi\left(y_{s}\right)$. The aggregator's objective is to minimize it's cost. The ex-post cost for the aggregator, given the information state $s$ is,

$$
\begin{equation*}
J_{s}^{a g g}=\phi\left(y_{s}\right)-R_{s}\left(y_{s}\right) \tag{3.3}
\end{equation*}
$$

We make the assumption that the disutility function is strictly convex i.e.
Assumption 4. $\phi^{\prime \prime}(y)>0$ for all $y$.

### 3.2 Optimal Scheduling for the Entity

In this section we consider the optimal scheduling of energy from the perspective of the entity. The entity is the combination of the LSE and the aggregator. This entity can buy $q^{d a}$ units of energy from the DA market, get a load curtailment of $y_{s}$ units at an intermediate time $t_{1}$ when the information state is $s$, get power $w$ from wind at time $T$ and purchase the remaining energy $\left(l-q^{d a}-y_{s}-w\right)_{+}$from the RTM for load balance (c.f. equation (3.1)). The ex-post cost for the entity is then,

$$
\begin{equation*}
J_{s}^{e}=\pi^{d a} q^{d a}+\phi\left(y_{s}\right)+\pi^{r t}\left(l-q^{d a}-y_{s}-w\right)_{+} \tag{3.4}
\end{equation*}
$$

So, the function of the entity is equivalent to that of a social planner. It considers only the system cost, not the payment transfer between the agents. First, we consider optimal scheduling without DR.

### 3.2.1 Optimal Scheduling without DR

Without DR, the entity can buy energy only from the DA market and the RT market. Let

$$
\begin{align*}
& J_{n d r}^{e}(q)=\pi^{d a} q+\mathbb{E}\left[\pi^{r t}(l-q-w)_{+}\right]  \tag{3.5}\\
& \quad q_{n d r}^{e} \in \arg \min _{q} J_{n d r}^{e}(q) \tag{3.6}
\end{align*}
$$

$J_{n d r}^{e}(q)$ is the net expected cost for the entity as a function of the day-ahead purchase $q$ when there is no DR . $q_{n d r}^{e}$ is the optimal day-ahead purchase.

Proposition 1. $J_{n d r}^{e}(\cdot)$ is convex. The minimizer $q_{n d r}^{e}$ is given by the solution of

$$
\pi^{d a}-\mathbb{E}_{s}\left[\bar{\pi}_{s}^{r t} P_{s}\left(l-q_{n d r}^{e}\right)\right]=0
$$

Remark 8. In order to avoid trivial results, we can assume that $\pi^{d a}<\mathbb{E}_{s}\left[\bar{\pi}_{s}^{r t} P_{s}(l)\right]$. This will ensure that $q_{n d r}^{e}>0$.

### 3.2.2 Optimal Scheduling with DR

Here we consider the energy scheduling with DR from the perspective of the entity. The net expected cost for the entity as a function of the first-stage purchase $q, J^{e}(q)$, is

$$
\begin{gather*}
J^{e}(q)=\pi^{d a} q+\mathbb{E}_{s}\left[\min _{y_{s}} J_{s}^{e}\left(y_{s}\right)\right], \text { where, }  \tag{3.7}\\
J_{s}^{e}\left(y_{s}\right)=\phi\left(y_{s}\right)+\mathbb{E}_{w}\left[\bar{\pi}_{s}^{r t}\left(l-q-y_{s}-w\right)_{+} \mid f_{1}=s\right] \tag{3.8}
\end{gather*}
$$

Here $J_{s}^{e}(\cdot)$ is the expected second-stage cost conditioned on information state $s$ and the first-stage decision $q$. Let

$$
\begin{equation*}
q^{e} \in \arg \min _{q} J^{e}(q), J^{* e}=J^{e}\left(q^{e}\right), y_{s}^{e} \in \arg \min _{y_{s}} J_{s}^{e}\left(y_{s}\right) \tag{3.9}
\end{equation*}
$$

Proposition 2. $J^{e}(\cdot)$ and $J_{s}^{e}(\cdot)$ are convex. For any given first stage decision $q$, the secondstage decision $y_{s}^{e}$ is given by solution of

$$
\begin{equation*}
\phi^{\prime}\left(y_{s}^{e}\right)-\bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{e}\right)=0 \tag{3.10}
\end{equation*}
$$

if $\phi^{\prime}(0)<\bar{\pi}_{s}^{r t} P_{s}(l-q)$ and zero otherwise. The first-stage decision $q^{e}$ is given by solution of

$$
\begin{equation*}
\pi^{d a}-\mathbb{E}_{s}\left[\bar{\pi}_{s}^{r t} P_{s}\left(l-q^{e}-y_{s}^{e}\right)\right]=0 . \tag{3.11}
\end{equation*}
$$

Remark 9. It is trivial to check that $\min J_{n d r}^{e} \geq \min J^{e}$
Remark 10. The scheduling decisions $q^{e}$ and $y_{s}^{e}$ of the entity corresponds to the efficient solution and we will use the corresponding cost $J^{* e}$ as the benchmark

### 3.2.3 Examples

A simple example is worked to illustrate the scheduling problem from the point of view of the entity. We will later use the same example for comparing with the market based allocations in Section 3.3.

1. Uniform forecast error: Let $l=3$. Let's assume that information set $S$ has three elements, $s_{L}, s_{M}$ and $s_{H}$ indicating low, medium and high wind forecast states respectively. Let $p\left(w / s_{L}\right)=U[0,3], p\left(w / s_{M}\right)=U[0.25,3.25]$ and $p\left(w / s_{H}\right)=U[0.5,3.5]$ where $U[a, b]$ is the uniform distribution of $[a, b]$. Also, let $\alpha\left(s_{L}\right)=\alpha\left(s_{M}\right)=\alpha\left(s_{H}\right)=1 / 3$. Take $\pi^{d a}=50, \pi^{r t}=1000$ and $\phi(y)=50 y+50 y^{2}$. Figure 3.5 shows the purchasing decision at each stage. Figure 3.3 illustrate the way we compute the intermediate load curtailment decisions $y_{s}^{e}$ as specified by equation (3.10). Note that, as intuition suggests, load curtailment is small when the wind forecast is high.
2. Gaussian forecast error: Let $l=5$. Here also, we assume that that information set $S$ has three elements, $s_{L}, s_{M}$ and $s_{H}$ with equal $(=1 / 3)$ probability. Let $p\left(w / s_{L}\right)=\tilde{\mathcal{N}}(0,0.5)$, $p\left(w / s_{M}\right)=\tilde{\mathcal{N}}(0.25,0.5)$ and $p\left(w / s_{H}\right)=\tilde{\mathcal{N}}(0.5,0.5)$ where $\tilde{\mathcal{N}}(\mu, \sigma)$ is a truncated normal distribution with mean $\mu$ and standard deviation $\sigma$. We truncated the distribution for negative values and re-normalized the integral to 1 . Take $\pi^{d a}=50, \pi^{r t}=1000$ and $\phi(y)=50 y+25 y^{2}$. We can compute $q^{e}=4.49,\left(y_{s_{L}}^{e}, y_{s_{M}}^{e}, y_{s_{H}}^{e}\right)=(0.464,0.222,0)$.


Figure 3.2: Purchase Decisions - Uniform Forecast Error


Figure 3.3: Load Curtailment Decisions (equation (3.10))

### 3.3 Spot Markets with Contingent Prices

Note that, entity is a fictitious system planner and the actual agents in the systems are the LSE and the aggregator. Hence it is realistic to consider them as two different entities. The best case scenario would be to design a market mechanism that achieves similar decision outcomes as the entity $\left(q^{e}, y_{s}^{e}\right)$ ? Because that would ensure that the social optimum. We show here that such a spot market for the intermediate DR market can achieve this market efficiency provided the spot market prices are chosen appropriately.

Let $\pi_{s}^{i n}$ be the price for unit load reduction in the intermediate DR market when the information state is $s$. This corresponds to a spot market. And the price $\pi_{s}^{i n}$ depends on the realized information state $s$. We assume that the LSE and the aggregator are price takers. Their obective would be to choose the decision variables such that it maximizes their net benefit.

First we highlight the time line and the decision variables from the point of view of LSE. The LSE buys $q$ from the DA market at the day ahead market price $\pi^{d a}$. Then, at the intermediate state, the LSE pays for a curtailment of $y_{s}$ units of energy from the aggregator at the intermediate market price $\pi_{s}^{i n}$. At the final time $T$, the LSE gets $w$ energy from wind and purchases the remaining energy $\left(l-q-y_{s}-w\right)_{+}$from the RTM at the RTM price $\pi^{r t}$. The net expected cost for the LSE as a function of the first-stage purchase $q$, $J^{l s e}(q)$, is given by


Figure 3.4: Purchase Decisions - Gaussian Forecast Error


Figure 3.5: Purchase Decisions - Uniform Forecast Error

$$
\begin{align*}
J^{l s e}(q) & =\pi^{d a} q+\mathbb{E}_{s}\left[\min _{y_{s}} J_{s}^{l s e}\left(y_{s}\right)\right], \quad \text { where }  \tag{3.12}\\
J_{s}^{l s e}\left(y_{s}\right) & =\pi_{s}^{i n} y_{s}+\mathbb{E}_{w}\left[\bar{\pi}_{s}^{r t}\left(l-q-y_{s}-w\right)_{+}\right] \tag{3.13}
\end{align*}
$$

Here, $J_{s}^{l s e}(\cdot)$ is the expected second stage cost for the LSE given the first stage purchase $q$. Also, let

$$
\begin{equation*}
q^{l s e} \in \arg \min J^{l s e}(q), \quad y_{s}^{l s e} \in \arg \min _{y_{s}} J_{s}^{l s e}\left(y_{s}\right) \tag{3.14}
\end{equation*}
$$

Note that $q^{l s e}$ and $y_{s}^{l s e}$ are the optimal first and second stage purchase decisions for the LSE. Similarly, the net expected cost and the optimal selling decision for the aggregator when the information state is $s$ are given by,

$$
\begin{equation*}
J_{s}^{a g g}\left(y_{s}\right)=\phi\left(y_{s}\right)-\pi_{s}^{i n} y_{s} \text { and } y_{s}^{a g g} \in \min _{y_{s}} J^{a g g}\left(y_{s}\right) \tag{3.15}
\end{equation*}
$$

The intermediate market decision of LSE and aggregator, $y_{s}^{a g g}$ and $y_{s}^{l s e}$, are functions of the information state $s$ and the spot market price $\left\{\pi_{s}^{i n}\right\}$ in the information state $s$. Note that the LSE procures load curtailment and the aggregator provides the load curtailment. Hence market equilibrium is acheived if the market prices are such that the optimal buying and selling decisions of the agents in the market balance each other. We formally define the market equilibrium below.

Definition 3 (Competitive Equilibrium with Contingent Prices). The spot market prices $\left\{\pi_{s}^{* i n}\right\}$, the optimal buying decisions of the LSE $q^{* l s e},\left\{y_{s}^{* l s e}\right\}$, optimal selling decisions of the aggregator $\left\{y_{s}^{* a g g}\right\}$ constitute a competitive equilibrium with contingent prices, if ( $y_{s}^{* l s e}-$ $\left.y_{s}^{* a g g}\right)=0, \forall s \in S$.

Let $J^{* l s e}$ be the expected cost for the LSE and let $J^{* a g g}$ be the expected cost for the aggregator at equilibrium. And so the total system cost is given by $J^{* c p}$

$$
\begin{equation*}
J^{* c p}=J^{* l s e}+J^{* a g g} \tag{3.16}
\end{equation*}
$$

Note that the competitive equilibrium is socially optimal if the total system cost $J^{* c p}$ is equal to $J^{* e}$ which is the optimal system cost from the perspective of the entity (c.f. (3.9)). We define this formally below.

Definition 4 (Socially Optimal Equilibrium with Contingent Prices). An equilibrium with contingent prices is said to be socially optimal, if $J^{* c p}=J^{* e}$

We now give the main result of this section.

Theorem 4. There exists a competitive equilibrium with contingent prices. All competitive equilibria are socially optimal. At equilibrium, denoting $q^{*}=q^{* l s e}, y_{s}^{*}=y_{s}^{* l s e}=y_{s}^{* a g g}$,

$$
\begin{array}{r}
\phi^{\prime}\left(y_{s}^{*}\right)=\pi_{s}^{* i n}=\bar{\pi}_{s}^{r t} P_{s}\left(l-q^{*}-y_{s}^{*}\right) \\
\text { if } \phi^{\prime}(0) \leq \bar{\pi}_{s}^{r t} P_{s}\left(l-q^{*}\right) \text { and } y_{s}^{*}=0 \text { if } \phi^{\prime}(0)>\bar{\pi}_{s}^{r t} P_{s}\left(l-q^{*}\right) . \text { Also, } \\
\pi^{d a}-\mathbb{E}_{s}\left[\bar{\pi}_{s}^{r t} P_{s}\left(l-q^{*}-y_{s}^{*}\right)\right]=0 \tag{3.18}
\end{array}
$$

### 3.3.1 Examples

The equilibrium allocations are the same as the solution of the entity's problem given in Section 3.2. Equilibrium prices $\left\{\pi_{s}^{* i n}\right\}$ can be found easily using the equation (3.17).
Remark 11. We have shown that it is possible to design an intermediate spot market that will achieve the socially optimum cost

### 3.4 Monopsony Contracts

In Section 3.3, we showed that a spot market can be created such that the buying and selling decision achieve the socially optimum outcome. However, this may not be a realistic scenario. Note that our setting in the spot market considered a two agent system. This suggests that in a real setting one agent might exercise market power and will be able to influence the prices over the other agent. For example, the agent can be a monopoly where he is only seller or a monopsony where he is the only buyer. In either cases, this agent will have more market power

Here we consider a situation where the LSE has a monopsony position over the aggregator(s). This can arise in a scenario where the LSE has the option of choosing from multiple aggregators. So the LSE can use this market power over the aggregator and extract the maximum benefit from trade. We analyze this scenario using a Principal-Agent Model. Below we analyse this scenario.

### 3.4.1 Principal-Agent Problem: Formulation

Here we describe the principal agent formulation. Since the principal has market power it is appropriate to choose LSE as the principal and the aggregator as the agent. In the principal agent formulation the principal offers a menu of contracts to the agent. From the point of view of the LSE it would want to design a menu of contracts such that its net utility is maximized. The agent can either accept one of these contracts or not choose any of the contracts at all. Clearly, the agent chooses a contract if the contract maximizes its net utility. Also it only makes sense for the agent to buy the contract if it provides a benefit (i.e., accepting the best contract) greater than the benefit derived from not participating (i.e., rejecting all contracts).

Definition 5. reservation utility: The net utility of the agent when not participating in the contract mechanism

Now we define the contract between LSE and Aggregator. A contract is specfied by the conditional payment $r_{s}$ made by the LSE to the aggregator for the load curtailment it provides and the load curtailment $y_{s}$ the aggregator should make if it accepts the contract from the LSE. So we denote a contract by $\left\{r_{s}, y_{s}, s \in S\right\}$, which specifies two quantities for each information state $s$ : (i) the amount of load reduction $y_{s}$ that the LSE demands from the aggregator when the information state is $s$, (ii) the payment $r_{s}$ that the aggregator will receive from the LSE for reducing the load by $y_{s}$ units.

The variable $\theta \in \Theta:=\left[\theta_{\min }, \theta_{\max }\right]$ parametrizes the type of the aggregators. Lets denote the true type of agent by $\bar{\theta}$. This is private information to the aggregator. Similar to the general formulation outlined in the previous section, we denote the disutility function of the aggregator by $\phi(\cdot, \theta)$. In this case, we incorporate the type of the agent by type $\theta$ in the disutility function and so the disutility of an aggregator of type $\theta$ for $y$ units of load curtailment is $\phi(y, \theta)$. Denote the partial derivatives of $\phi(\cdot, \cdot)$ w.r.t. the first $(y)$ and the second $(\theta)$ arguments as $\phi_{y}$ and $\phi_{\theta}$. We make the following assumptions.

Assumption 5. (i) $\phi(y, \theta)$ is strictly increasing in $y$ and $\theta$, i.e., $\phi_{y}(\cdot, \cdot)>0, \phi_{\theta}(\cdot, \cdot)>0$
(ii) (Spence-Mirrlees condition) $\phi_{\theta y}(\cdot, \cdot)>0$
(iii) $\phi_{y y \theta}(\cdot, \cdot)>0, \phi_{y \theta \theta}(\cdot, \cdot)>0$ and $\frac{d \frac{F(\theta)}{f(\theta)}}{d \theta}>0$

From the LSE point of view the true type of the aggregator i.e. $\bar{\theta}$ is randomly distributed over the set $\Theta$ according to the probability density (distribution) function $f(\cdot)(F(\cdot))$. The expected cost that the LSE incrurs in the RTM after receiving $y$ units of load reduction in the state $s$ and buying $q$ in the day-ahead market is denoted by $V_{s}(y ; q)$. From Section 3.2.2, we define

$$
V_{s}(y ; q)=\mathbb{E}_{w}\left[\bar{\pi}_{s}^{r t}(l-q-y-w)_{+}\right] .
$$

Clearly, $V_{s}(y ; q)$ is a decreasing function in $y$. Also, from Assumption 3, $V_{s}(y ; q) \leq V_{s^{\prime}}(y ; q)$ for $s>s^{\prime}$.

As specifie above, the LSE procures $q$ from the day-ahead market and offers a set of such contracts to the aggregator in the first stage. The aggregator facing these choices will select the contract that will maximize its net utility. So the optimization problem for the LSE given the day-ahead procurement $q$ is given by,

$$
\begin{array}{ll}
\min _{r_{s}(\cdot)} & \mathbb{E}_{\theta}\left[r_{s}\left(y_{s}(\theta)\right)+V_{s}\left(y_{s}(\theta) ; q\right)\right] \text {, where } \\
\text { s.t. } & y_{s}(\theta)=\arg \max _{y_{s}}\left(r_{s}\left(y_{s}\right)-\phi\left(y_{s}, \theta\right)\right), \forall \theta \in \Theta \\
& r_{s}\left(y_{s}(\theta)\right)-\phi\left(y_{s}(\theta), \theta\right) \geq 0, \forall \theta \in \Theta \tag{3.20}
\end{array}
$$

The first constraint is called incentive compatibility (IC) constraint and the second constraint is called individual rationality (IR) constraint. The incentive compatibiliy constraint ensures that the aggregator chooses the contract that corresponds to its true type. The IR constraint ensures that the aggregator benefits by participating. This is a non-trivial optimization problem to solve because the constraint itself is an optimization problem. However, thanks to revelation principle. We can simplify this problem as below.

$$
\begin{array}{rc}
\min _{r_{s}(\theta), y_{s}(\theta)} & \mathbb{E}_{\theta}\left[r_{s}(\theta)+V_{s}\left(y_{s}(\theta) ; q\right)\right](\text { Cont-Cont }) \\
\text { s.t. I.R. } & r_{s}(\theta)-\phi\left(y_{s}(\theta), \theta\right) \geq 0, \quad \forall \theta \in \Theta, \forall s \in \mathcal{S} \\
& \text { I.C. } \\
& r_{s}(\theta)-\phi\left(y_{s}(\theta), \theta\right) \geq r_{s}\left(\theta^{\prime}\right)-\phi\left(y_{s}\left(\theta^{\prime}\right), \theta\right),  \tag{3.24}\\
& \forall \theta, \theta^{\prime} \in \Theta, \forall s \in \mathcal{S}
\end{array}
$$

Where the contract payment and load curtailment is a function of the type of the aggregator. That is $r_{s}(\theta)$ is the payment to the aggregator of type $\theta$ for a load curtailment of $y_{s}(\theta)$ at stage $s$. Also, since the I.C. constraints guarantee that aggregator of type $\theta$ will only select the contract of type $\theta$, its net utility will be $U_{s}(\theta)=r_{s}(\theta)-\phi\left(y_{s}(\theta), \theta\right)$. This is called information rent. In the first stage, the LSE procures $q$ from the day-ahead market and offers the set of optimal contingent contracts $\left\{r_{s}^{*}(\theta ; q), y_{s}^{*}(\theta ; q)\right\}$ (contracts that solve the LSE's optimization problem for the given day-ahead procurement $q$ ) to the aggregator. The net expected cost of the LSE as a function of the first stage decision $q$ is then given by,

$$
\begin{aligned}
& \tilde{J}^{l s e}(q)=\pi^{d a} q+\mathbb{E}_{s}\left[\min _{r_{s}(\cdot), y_{s}(\cdot)} \tilde{J}_{s}^{l s e}\left(r_{s}(.), y_{s}(.)\right)\right] \text { Where } \\
& \tilde{J}_{s}^{l s e}\left(r_{s}(.), y_{s}(.)\right)=\mathbb{E}_{\theta}\left[r_{s}(\theta)+V_{s}\left(y_{s}(\theta) ; q\right)\right]
\end{aligned}
$$

Where $\tilde{J}_{s}^{l s e}\left(r_{s}().\right)$ is the expected second stage cost conditioned on information state $s$. Also, let

$$
\tilde{q}^{l s e} \in \arg \min _{q} \tilde{J}^{l s e}(q)
$$

Which is the optimal first stage procurement from the day-ahead market.

### 3.4.2 Complete Information: First Best Contract

In the complete information case, the LSE knows the true type of the agent. Then the LSE doesn't have to worry about the I.C. constraint. The I.R. constraint will be binding at the optimum. Then we have the following result.

Proposition 3. Under complete information, the optimal monopsony contracts, $\left\{r_{s}^{*}(\theta ; q), y_{s}^{*}(\theta ; q)\right.$ $, \theta \in \Theta\}$, are given by,

$$
\begin{equation*}
\phi_{y}\left(y_{s}^{*}(\theta ; q), \theta\right)+V_{s}^{\prime}\left(y_{s}^{*}(\theta ; q) ; q\right)=0 \tag{3.25}
\end{equation*}
$$

if $\phi_{y}(0, \theta)<-V_{s}^{\prime}(0 ; q)=\bar{\pi}_{s}^{r t} P_{s}(l-q)$ and $y_{s}^{*}(\theta ; q)=0$ otherwise.

$$
r_{s}^{*}(\theta ; q)=\phi\left(y_{s}^{*}(\theta ; q), \theta\right)
$$

Remark 12. The above optimization problem is the same as the optimal scheduling problem that we considered in Section (3.2.2). Thus, the first best contract achieves social optimality. Also, the information rent is zero, $U=0$, and all the surplus is taken by the LSE.

We now characterize the optimal first stage decision $\tilde{q}^{l s e}$.
Proposition 4. $\tilde{J}^{l s e}($.$) is strictly convex. The unique first-stage decision \tilde{q}^{l s e}$ is given by the solution of

$$
\pi^{d a}-\mathbb{E}_{s} \mathbb{E}_{\theta} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{*}(\theta ; q)\right)=0
$$

### 3.4.3 Incomplete Information: Continuum of Types

In the incomplete information case, the LSE does not know the true type of the agent. The naive way to solve the above problem is via applying Lagrangian techniques directly. However, due to the underlying structure, there is an easier procedure to solve this problem. There are many works [32]. We follow the procedure in [28].

One important difference is that, we address a contingency dependent contract design problem. We make the assumption that the aggregator is completely risk-averse, i.e., it demands $U_{s} \geq 0$ for all contingency $s \in \mathcal{S}$. Below we provide the optimal contract in this setting as a theorem.

Proposition 5. Under incomplete information, the optimal monopsony contracts, $\left\{r_{s}^{*}(\theta ; q), y_{s}^{*}(\theta ; q)\right.$ $, \theta \in \Theta\}$, are given by,

$$
V_{s}^{\prime}\left(y_{s}^{*}(\theta ; q) ; q\right)+\phi_{y}\left(y_{s}^{*}(\theta ; q), \theta\right)=-\frac{F(\theta)}{f(\theta)} \phi_{y \theta}\left(y_{s}^{*}(\theta), \theta\right)
$$

if $-V_{s}^{\prime}(0 ; q)>\phi_{y}(0, \theta)+\frac{F(\theta)}{f(\theta)} \phi_{y \theta}(0, \theta)$ and $y_{s}^{*}(\theta)=0$ otherwise.

$$
r_{s}^{*}(\theta)=\phi\left(y_{s}^{*}(\theta), \theta\right)+\int_{\theta}^{\theta_{\max }} \phi_{z}\left(y_{s}(z), z\right) d z
$$

We now characterize the optimal first stage decision $q$.
Proposition 6. $\tilde{J}^{l s e}($.$) is strictly convex. The unique first-stage decision \tilde{q}^{l s e}$ is given by the solution of

$$
\pi^{d a}-\mathbb{E}_{s} \mathbb{E}_{\theta} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{*}(\theta ; q)\right)=0
$$

Proof is given in the appendix
Remark 13. Note that the relation between equilibrium social cost under incomplete information and complete information is not clear. In propositions 5 and 6 we only give the conditions satisfied by the contract payment and scheduling decisions.

### 3.5 Concluding Remarks and Options Market

Till now we showed that the optimal scheduling of energy from the DR market can be implemented using a market. More precisely, the competitive equilibrium with contingent prices achieves a socially optimal outcome. However, there are many difficulties in implementing such an intermediate spot market with contingent prices. Firstly, spot market price can vary greatly and rapidly. Moreover this price fluctuation is very unpredictable. This is highly unattractive to a risk-averse market player which can possibly go bankrupt during such an extended extreme real-time price period. Forward contract enables hedging in such scenarios. But forward contracts do not allow the flexibility of scheduling the load reduction based on the improved forecast available in the intermediate market. This motivates selling demand flexibility thorough options which can potentially combine the positive aspects of both the forward market and the real time market.

In the options setting, the LSE will buy $q$ units of energy from the DA market. Also, at the same time, the LSE will buy $x$ units of options from the aggregator at a price $\pi^{o}$. So, the LSE has the right, but not the obligation to get $y$ units of load reduction, $y \leq x$, from the aggregator. The LSE can give the notification for a load reduction at any time $t \leq t_{1}$ before the intermediate market closes, by paying a strike price $\pi^{s p}$ for unit reduction. However, to exploit the better wind prediction available at a later time, the LSE will give load curtailment notification only at time $t_{1}$. Clearly, the amount of load reduction the LSE asks for, $y_{s}$, will depend on the information state $s$ realized at time $t_{1}$. However, note that the strike price $\pi^{s p}$ doesn't depend on the information state. Given the notification at time $t_{1}$, the aggregator will deliver the load reduction at time $T$. The LSE will observe the wind energy $w$ at time $T$ and purchase the remaining amount of energy $\left(l-q-y_{s}-w\right)_{+}$from the RT market.
Unlike the spot market with continent prices, here we have only two prices $\pi^{o}$ and $\pi^{s p}$. Also, in the spot market with contingent prices the trading takes place in the intermediate time. Here, in options market, the trading of options takes place in the day ahead market. LSE buys $x$ units of options for a specified option price $\pi^{o}$ and strike price $\pi^{s p}$. In the intermediate market, only the exercise of the options takes place. In a spot market with contingent prices, the equilibrium notion is that, the optimal trading decisions of the LSE and the aggregator should balance each other for every possible contingency of the intermediate market. So, at equilibrium $y_{s}^{l s e}=y_{s}^{a g g}$ for all $s \in S$. In an options market, the equilibrium notion is that, the number of options that the LSE is willing to buy and the number of options that the aggregator is willing to sell should balance each other. So, $x^{l s e}=x^{a g g}$. Not that the contingent information state $s$ doesn't appear explicitly here.

Replacing the intermediate spot market with an options market is an attractive proposition from the point of view of implementation, hedging risk and the flexibility of scheduling load reduction in the intermediate market. But showing existence of equilibrium (though sufficient conditions could be derived) and deriving upper bound on the ratio between optimal social cost for the options market and the social optimum is a hard problem and an ongoing work.

## Chapter 4

## Baseline Mechanism for Wholesale Markets

In this section we propose a self-reported DR scheme for residential consumers that integrates DR resources into the wholesale markets efficiently. Similar to the retail market, the consumers are required to self-report their baseline and are paid at a pre-determined price level for every unit of reduction they provide. The consumers are recruited such that the SO's benefit is maximized when the recruited DR resources are deployed. We then compare the self-reported DR mechanisms with the CAISO's averaging method for estimating baseline. We show that the self-reported baseline DR mechanism establishes a better estimate of the mean baseline in scenarios where there is high variability in consumption. And in scenarios where there is low variability, both methods establish a similar baseline estimate. Because the payments are proportional to the baseline estimae, overall the self-reported DR mechanism is cost-effective in either of the markets.

Here we do not restate the importance of baseline estimation and the issues that accompanies it. Also note that we restrict ourselves to peak shaving DR applications. In this setting, the consumer or a group of consumers (through an aggregator) interact directly with the operator. We refer the reader to sections 1 and 2 for a discussion on this aspect. A natural question to ask is 'why do we need to redesign the mechanism for the wholesale market '? Can't we use the mechanism described in 2. Recall the setting in section 2. The aggregator is sent an external load reduction requirement $D$ that the aggregator has to provide. The aggregator schedules sufficient number of consumers so that the load reduction provided meets the load reduction requirement $D$. The System Operator on the other hand has direct access to the market outcomes and can determine the extent of load reduction, that the consumers have provided, by measuring the market price. The required load reduction is met when the market price falls to the threshold market clearing price after deploying the DR resources (Refer section 4.5 for a detailed discussion on how this price is estimated). Hence the setting for the wholesale market is simpler. And we do not need to consider the load reduction requirement explicitly.

Table 4.1: Notations - Wholesale Market

| $q$ | Energy consumption of consumer |
| :---: | :--- |
| $u$ | Utility of consumer |
| $\pi^{e}$ | Retail price of energy |
| $R(\cdot, \cdot, \cdot)$ | Reward function for load reduction |
| $\Phi(\cdot, \cdot)$ | Penalty function for deviation from baseline |
| $\pi^{r}$ | Reward/kWh awarded to consumer $k$ |
| $\hat{b}$ | Baseline report of consumer |
| $\pi^{*}$ | Threshold Market Clearing Price (TMC) |
| $\theta$ | Exogenous random variable |

### 4.1 Consumer Model

Consider a residential consumer. Denote her consumption as $q$. Let $\theta$ be the random variable on which her consumption depends. We assume that the distribution of $\theta$ includes every possible source of uncertainty. For example, $\theta$ could represent the consumer's state where the consumer could either be at home or outside home. It could also model the randomness induced due to external weather conditions like temperature. Let the private utility function be $u(q, \theta)$ which is concave monotone increasing in $q$ and is dependent on $\theta$. We assume that the random variable $\theta$ is realized at the time when consumption is decided. Define the marginal utility,

$$
\begin{equation*}
\mu(q, \theta)=\frac{\partial u(q, \theta)}{\partial q} \tag{4.1}
\end{equation*}
$$

Note that since $u(q, \theta)$ is monotone increasing in $q$, we have $\forall q: \mu(q, \theta)>0$. Also since $u(q, \theta)$ is concave in $q$, we have

$$
\forall q: \frac{\partial \mu(q, \theta)}{\partial q}<0 .
$$

### 4.1.1 Mechanism

We prescribe a self-reported baseline mechanism which utility and/or SO can use to acquire DR resources for wholesale energy market operation. DR events are called when the market price exceeds a threshold market clearing price (TMC price). The 'threshold market clearing price' (TMC Price) is defined as the market price (after deploying the DR resources) that maximizes SO's benefit. In a later section we show how this price is estimated. The TMC Price is announced every month based on past supply function data that excludes DR resources, the peak load estimate $L$ and the retail price of electricity $\left(\pi^{e}\right)$.

Assumption 6. Probability of $D R$ event is small

From the consumer point of view the mechanism has two stages. In the first stage the consumers report their baseline. In the second stage, the participating consumers are signaled to reduce when there is a DR event. The SO can appoint a DR program manager (DRM) to manage the program. The two stages of the mechanism are the following,

Stage 1 (Reporting) In this stage, the DRM announces the 'threshold market clearing price' $\pi^{*}$ as the reward per unit reduction $\left(\pi^{r}\right)$, the probability $\alpha$ of a DR event $(\alpha \ll 1)$ and the reward function $R\left(\pi^{r}, \hat{b}, q\right)$ where $\hat{b}$ is the report of baseline consumption. The DRM also announces a penalty function $\Phi(\hat{b}, q)$ for consumers who deviate from their reported baseline when they are not signaled to reduce. This penalty is critical to ensure that the consumers do not inflate their baseline report. At the same time the penalty should not over penalize in a way that will be not profitable for the consumers to participate. Depending on price or reward per unit reduction $\pi^{r}$, and the penalty $\Phi(\hat{b}, q)$, each consumer submits the baseline report $\hat{b}$. As noted above the consumers can inflate their baseline report.

Stage 2 (DR Event) In the second stage a DR event occurs. DR events are triggered when the SO expects the market price to shoot above the threshold market clearing price. The DRM signals the participating consumers to reduce. The DRM then observes the final consumption $q$ of these consumers. If the resulting market price is greater than the TMC price, then the SO recruits more consumers till the market operates at the TMC price in the future DR events. At steady state we expect the market to operate at the TMC price and so the conventional suppliers will be paid the TMC price during DR events. And by the mechanism, the recruited consumers are paid the TMC price for the load reduction they provide. By the definition of TMC price, this should maximize the benefit of SO.
Remark 14. After deploying the DR capacity of the recruited consumers, even though the market operates at the TMC price, there is an incurred loss because of inflation in the baseline report of the consumers.
Remark 15. The operator does require some time to learn or decide which consumers to call. As specified in the mechanism above, it decides to recruit more or remove consumers depending on the realized market price during the current DR event. If the market price (after calling the DR resource) is more than the TMC price then the SO will call more consumers in the next round. This repeats till the market price falls to the TMC price. Hence there is a learning curve for the Operator till the market operates at the TMC price during DR events.
Remark 16. Provided that the number of participating consumers are large enough, the SO's strategy to deploy more consumers every round till the market price drops to TMC price should, ensure that the market price drops to the TMC price in finite time (no of rounds/iterations). In the next chapter we shall address the transient aspects of scheduling DR resources.

Given the 'threshold market clearing price' $\pi^{r}$ the reward function in the mechanism is set as,

$$
R\left(\pi^{r}, \hat{b}, q\right)= \begin{cases}\pi^{r}(\hat{b}-q), & \text { if consumer is signaled. }  \tag{4.2}\\ 0, & \text { otherwise }\end{cases}
$$

Note that the SO pays the consumers according to the measured reduction $\hat{b}-q$, where $\hat{b}$ is the consumer's report of baseline and $q$ is the consumption. The following penalty is applied to the consumer,

$$
\Phi(\hat{b}, q)= \begin{cases}0, & \text { if consumer is signaled }  \tag{4.3}\\ \phi(\hat{b}-q), & \text { otherwise }\end{cases}
$$

The penalty function $\phi$ in (4.3) is chosen such that it has the following properties,
i) $\phi(0)=\phi^{\prime}(0)=0$.
ii) $\forall r: \phi^{\prime \prime}(r) \geq 0$ (Convexity).

From the mechanism it follows that the consumer's optimization problem is given by,

$$
\begin{equation*}
\mathbf{C P}: \min _{\hat{b}} H(\hat{b}) \tag{4.4}
\end{equation*}
$$

where

$$
\bar{J}(\hat{b})=\mathbb{E}\left[\min _{q}\left\{\pi^{e} q-u(q, \theta)+\Phi(\hat{b}, q)-R\left(\pi^{r}, \hat{b}, q\right)\right\}\right]
$$

is the cost that is incurred by the consumer. Hence the consumer problem (4.4) is a two stage decision problem. In the first stage the consumer decides the report $\hat{b}$ and in the second stage decides the optimal consumption $q$. First we solve for the second stage decision and then use the second stage decision to characterize the optimal forecast.

### 4.1.2 Second Stage - Optimal Consumption

The optimal consumption in the second stage depends on the following cases: i) Consumer is not participating in the DR program. ii) Consumer is participating but is not signaled to reduce. iii) Consumer is participating and is signaled to reduce. First, we solve the optimal consumption for the three cases. The consumption when the consumer is not participating corresponds to the true baseline. Hence we can use this to derive the true mean baseline. The cases (ii) and (iii) will determine the optimal first stage report $\hat{b}^{*}$. From here we can characterize the inflation in baseline as the difference between the optimal forecast and the true mean baseline. Since $\theta$ is realized by the second stage, we assume $\theta$ is fixed in the calculations to follow.

1. Set $\pi^{r}=\pi^{*}$ (TMC Prize)
2. Announce the Reward function $R$ and the Penalty function $\Phi($.
3. Receive forecast report $\hat{b}$ from the consumer
4. DR Event: Signal consumer to reduce
5. Observe consumption $q$
6. Reward the consumer according to (4.2)
7. No DR Event: Impose penalty as given by (4.3)

Algorithm 2: (Self-Reported Baseline Mechanism (SRBM)) - Wholesale Market

Consumer is not participating In this case, $R=0$ and $\Phi=0$. The realized cost function is then given by,

$$
J^{a}(q, \theta)=\pi^{e} q-u(q, \theta)
$$

and the optimal consumption is given by

$$
q^{a}(\theta)=\arg \min _{q} J^{a}(q, \theta) .
$$

The optimal consumption is a function of $\theta$ because the value of $\theta$ is realized when the consumption decision is made. Note that $q^{a}(\theta)$ solves the optimality condition,

$$
\begin{equation*}
\pi^{e}-\frac{\partial u(q, \theta)}{\partial q}=0 \tag{4.5}
\end{equation*}
$$

Hence $q^{a}(\theta)$ is given by $q^{a}(\theta)=\mu^{-1}\left(\pi^{e}\right)$ where $\mu^{-1}$ is the inverse function of the marginal utility (4.1) that always exists for each $\theta$. Having characterized the consumption when the consumer is not participating we can define baseline inflation $\delta \hat{b}$ as,

Definition 6. Baseline inflation: $\delta \hat{b}=\hat{b}^{*}-\mathbb{E}_{\theta} q^{a}(\theta)$
Where $\mathbb{E}_{\theta} q^{a}(\theta)$ is the true mean baseline because it corresponds to the mean consumption when the consumer is not participating.

Participating but not signaled to reduce The reward and penalty functions are given by (4.2) and (4.3). Then the realized function cost function is given by

$$
J^{b}(\hat{b}, q, \theta)=\pi^{e} q-u(q, \theta)+\phi(\hat{b}-q) .
$$

As before, the value of $\theta$ is realized when the consumption decision is made. Note that in this scenario the realized cost $J^{b}$ is also a function of $\hat{b}$ apart from the consumption decision and the value of $\theta$. The optimal consumption is given by

$$
q^{b}(\hat{b}, \theta)=\arg \min _{q} J^{b}(\hat{b}, q, \theta)
$$

So $q^{b}(\hat{b}, \theta)$ solves

$$
\begin{equation*}
\pi^{e}-\frac{\partial u(q, \theta)}{\partial q}-\phi^{\prime}(\hat{b}-q)=0 . \tag{4.6}
\end{equation*}
$$

Hence the optimal consumption $q^{b}(\hat{b}, \theta)$ satisfies the implicit equation $q^{b}(\hat{b}, \theta)=\mu^{-1}\left(\pi^{e}-\right.$ $\phi^{\prime}\left(f-q^{b}(\hat{b}, \theta)\right)$. And $q^{b}(\hat{b}, \theta)$ is also a function of $\hat{b}$ because the deviation from $\hat{b}$ incurs a penalty.

Participating and signaled to reduce Again the reward and penalty functions are given by (4.2) and (4.3). Then the realized cost function is given by

$$
J^{c}(\hat{b}, q, \theta)=\pi^{e} q-u(q, \theta)-\pi^{r}(\hat{b}-q)
$$

and the optimal consumption is given by

$$
q^{c}(\hat{b}, \theta)=\arg \min _{q} J^{c}(\hat{b}, q, \theta)
$$

So $q^{c}(\hat{b}, \theta)$ solves

$$
\begin{equation*}
\pi^{e}-\frac{\partial u(q, \theta)}{\partial q}+\pi^{r}=0 \tag{4.7}
\end{equation*}
$$

Hence the optimal consumption $q^{c}$ is independent of $\hat{b}$ and is given by,

$$
\begin{equation*}
q^{c}(\theta)=\mu^{-1}\left(\pi^{e}+\pi^{r}\right) . \tag{4.8}
\end{equation*}
$$

Below we state the general relation between $q^{a}(\theta), q^{b}(\theta, \hat{b})$ and $\hat{b}$.
Lemma 1. The optimal consumption $q^{b}(\hat{b}, \theta)$ is convex combination of $q^{a}(\theta)$ and $\hat{b}$.
Proof $J^{b}(\hat{b}, q, \theta)$ is composed of two convex functions $U_{1}(q)=\pi^{e} q-u(q, \theta)$ and $U_{2}(\hat{b}, q)=$ $\phi(\hat{b}-q)$. The minimizer of $U_{1}$ is $q^{a}(\theta)$ and the minimizer of $U_{2}$ is $q=\hat{b}$. Then the minimizer of $J^{b}=U_{1}+U_{2}$ has to necessarily lie between the minimizers of $U_{1}$ and $U_{2}$ which implies that $q^{b}(\hat{b}, \theta)$ is a convex combination of $q^{a}(\theta)$ and $\hat{b}$.

Also from (4.8) it follows that

$$
q^{c}(\theta)<q^{a}(\theta)
$$

Hence a rational consumer necessarily provides load reduction.

### 4.2 Optimal Forecast

Let $\alpha$ denote the probability of a DR event. Then $\alpha$ also equals the probability of being signaled to reduce. The optimal forecast $\hat{b}^{*}$ minimizes the expected cost, which is given by

$$
\begin{equation*}
\bar{J}(\hat{b})=\alpha \mathbb{E}_{\theta} J^{c}\left(\hat{b}, q^{c}, \theta\right)+(1-\alpha) \mathbb{E}_{\theta} J^{b}\left(\hat{b}, q^{b}, \theta\right) \tag{4.9}
\end{equation*}
$$

Define $M(\hat{b})$ as the expected marginal utility under the proposed DR mechanism. Then, $M(\hat{b})$ is given by

$$
\begin{equation*}
M(\hat{b})=\alpha \mathbb{E}_{\theta} \frac{\partial u\left(q^{c}, \theta\right)}{\partial q}+(1-\alpha) \mathbb{E}_{\theta} \frac{\partial u\left(q^{b}, \theta\right)}{\partial q} \tag{4.10}
\end{equation*}
$$

We now present the main lemma which specifies the condition that the optimal forecast $\hat{b}^{*}$ satisfies.

Lemma 2. The optimal forecast $\hat{b}^{*}$ satisfies $\pi^{e}=M\left(\hat{b}^{*}\right)$.
Proof The optimal forecast $\hat{b}^{*}$ satisfies the first order condition:

$$
\bar{J}^{\prime}(\hat{b})=(1-\hat{b}) \mathbb{E}_{\theta} \frac{d J^{b}\left(\hat{b}, q^{b}, \theta\right)}{d \hat{b}}+\alpha \mathbb{E}_{\theta} \frac{d J^{c}\left(\hat{b}, q^{c}, \theta\right)}{d \hat{b}}=0
$$

Using (A.70) and (A.71) we get,

$$
\begin{equation*}
\mathbb{E}_{\theta} \phi^{\prime}\left(\hat{b}^{*}-q^{b}\left(\hat{b}^{*}, \theta\right)\right)=\frac{\alpha \pi^{r}}{1-\alpha} . \tag{4.11}
\end{equation*}
$$

Then using (4.6) and (4.7)

$$
\begin{equation*}
\pi^{e}=(1-\alpha) \mathbb{E}_{\theta} \frac{\partial u\left(q^{b}, \theta\right)}{\partial q}+p \mathbb{E}_{\theta} \frac{\partial u\left(q^{c}, \theta\right)}{\partial q} \tag{4.12}
\end{equation*}
$$

The right hand side by definition is the expected marginal utility which implies that $\pi^{e}=$ $M\left(\hat{b}^{*}\right)$.
Next we show that the function $\bar{J}(\hat{b})$ is convex in its argument $\hat{b}$.
Lemma 3. $\bar{J}(\hat{b})$ is (strictly) convex if and only if $\phi$ is (strictly) convex.
From Lemma 2 and Lemma 3, we derive the following corollary.
Corollary 1. The global minimizer of the cost function is given by the condition $\pi^{e}=M\left(\hat{b}^{*}\right)$. The minimizer is unique when $\phi$ is strictly convex.

### 4.2.1 Optimal Forecast Without Penalty

We now solve for the optimal forecast $\hat{b}^{*}$ when there is no penalty i.e. when $\phi=0$. This implies,

$$
\frac{d \bar{J}}{d \hat{b}}=\alpha \mathbb{E}_{\theta} \frac{d J^{c}\left(\hat{b}, q^{c}, \theta\right)}{d \hat{b}}=-p \pi^{r}
$$

This means that the consumer will forecast a very high baseline, which is clearly undesirable. This clearly suggests the importance of imposing a penalty on the consumers. Next, we derive the optimal forecast when a penalty is imposed on the consumer.

### 4.2.2 Optimal Forecast With Penalty

Above we emphasized the importance of imposing a penalty. Analysis becomes difficult with a general utility function. So we consider only quadratic utility function for illustration purposes. First we illustrate the most simple case that is when there is no randomness in $\theta$.

### 4.2.2.1 No Variability in Consumption

Here we illustrate the case where there is no variability in consumption. So this case corresponds to the scenario where there is no dependency on the random variable $\theta$. Then from (4.5) and (4.8)

$$
\begin{align*}
q^{a} & =\mu^{-1}\left(\pi^{e}\right)  \tag{4.13}\\
q^{c} & =\mu^{-1}\left(\pi^{e}+\pi^{r}\right) . \tag{4.14}
\end{align*}
$$

Also from (4.12) we get

$$
\begin{align*}
\pi^{e} & =(1-\alpha) \frac{\partial u\left(q^{b}\right)}{\partial q}+\alpha \frac{\partial u(q)}{\partial q^{c}} \\
& =(1-\alpha) \frac{\partial u\left(q^{b}\right)}{\partial q}+\alpha\left(\pi^{e}+\pi^{r}\right) \tag{4.15}
\end{align*}
$$

This implies,

$$
\begin{equation*}
q^{b}=\mu^{-1}\left(\pi^{e}-\frac{\alpha}{1-\alpha} \pi^{r}\right) \tag{4.16}
\end{equation*}
$$

Using (4.11) we get

$$
\begin{equation*}
\hat{b}^{*}=q^{b}+\phi^{\prime-1}\left(\frac{\alpha}{1-\alpha} \pi^{r}\right) . \tag{4.17}
\end{equation*}
$$

Substituting for $q^{b}$ we get

$$
\begin{equation*}
\hat{b}^{*}=\mu^{-1}\left(\pi^{e}-\frac{\alpha}{1-\alpha} \pi^{r}\right)+\phi^{\prime-1}\left(\frac{\alpha}{1-\alpha} \pi^{r}\right) . \tag{4.18}
\end{equation*}
$$

Note that these expressions also imply that, $q^{c}<q^{a}<q^{b}<f$. For this example, actual load reduction is given by

$$
\begin{equation*}
q^{a}-q^{c}=\mu^{-1}\left(\pi^{e}\right)-\mu^{-1}\left(\pi^{e}+\pi^{r}\right), \tag{4.19}
\end{equation*}
$$

and the corresponding measured reduction is

$$
\begin{align*}
\hat{b}^{*}-q^{c}= & \mu^{-1}\left(\pi^{e}-\frac{\alpha}{1-\alpha} \pi^{r}\right)+\phi^{\prime-1}\left(\frac{\alpha}{1-\alpha} \pi^{r}\right) \\
& -\mu^{-1}\left(\pi^{e}+\pi^{r}\right) . \tag{4.20}
\end{align*}
$$

So Baseline Inflation is given by,

$$
\begin{align*}
f^{*}-q^{a}= & \mu^{-1}\left(\pi^{e}-\frac{\alpha}{1-\alpha} \pi^{r}\right)+\phi^{\prime-1}\left(\frac{\alpha}{1-\alpha} \pi^{r}\right) \\
& -\mu^{-1}\left(\pi^{e}\right) \tag{4.21}
\end{align*}
$$

Let $q^{a}-q^{c}$ be the load reduction required for the market to operate at the TMC price $\pi^{*}$. Then by the recruitment process it would suffice to recruit this consumer only. The price per unit of kWh paid for the DR services provided by the consumer is,

$$
\begin{align*}
\text { Price } / \mathrm{kWh} & =\frac{\pi^{*}\left(\hat{b}-q^{c}\right)}{q^{a}-q^{c}} \\
& =\pi^{*}\left(1+\frac{\hat{b}-q^{a}}{q^{a}-q^{c}}\right) \\
& =\pi^{*}+\pi^{*} \frac{\delta \hat{b}}{q^{a}-q^{c}} \tag{4.22}
\end{align*}
$$

where the first term is the threshold market clearing price and the second one characterizes the loss or the excess payment made to the consumer for a unit of kWh reduction provided by the consumer. It follows that the incurred loss is directly proportional to baseline inflation. Later we argue that this is in general the case. We also observe that for very small $\alpha, \hat{b}$ approaches $q^{a}$ and Price/kWh approaches $\pi^{*}$.

### 4.2.2.2 Variable Consumption

Here we consider the case where the consumption is dependent on the exogenous random variable. Also we assume that consumer's utility is quadratic. The following theorem gives the expression for baseline inflation $\delta \hat{b}$,

Lemma 4. $\delta \hat{b}=\frac{\alpha \pi^{*}(1+d / \lambda)}{(1-\alpha) d}$ where
i) $\alpha>0$.
ii) $\forall q:-\mu^{\prime}(q)=d$
iii) $\forall r: \phi^{\prime \prime}(r)=\lambda \geq d$.

Proof See Appendix.
From Lemma 4, it is clear that the excess payment per unit of kWh reduction is proportional to the inflation in baseline $\delta \hat{b}$. Also it follows that as $d \rightarrow \infty, \delta \hat{b} \rightarrow 0$. This is expected, since the consumer looses by a large margin for any positive inflation in baseline when $d \rightarrow \infty$.

The above results on baseline inflation gives us a way to characterize mechanism's efficiency in terms of baseline inflation. Below we comment on how we characterize a baseline mechanism's efficiency.

### 4.3 Mechanism Efficiency

Because of inflation in baseline the DR program manager pays in effect a price that is more than the price that is set for unit reduction that the consumer provides. This is evident from the discussion in section 4.2. As a result the SO incurs losses. Let the number of recruited consumers be $N$ and the net load reduction offered by the consumers be $\Delta L$, which is different from the measured reduction. From previous section it follows that the excess payment is directly proportional to the inflation in baseline. Given that $\delta \hat{b}$ is the amount by which consumers inflate their baseline the loss to the SO can be characterized by $\frac{\pi N \delta \hat{b}}{\Delta L}$ where $\pi$ is the price $/ \mathrm{kWh}$. Later we show that the proposed mechanism achieves better control over baseline inflation when compared to the $m / m$ baseline method used by system operators like CAISO.

### 4.4 Comparison with SRBM for Wholesale Market

Here we compare the mechanism with the CAISO scheme as in chapter 2. Restating the CAISO mechanism, the CAISO computes the mean of the consumption on the most recent $m$ similar but non-event days and then multplies this mean estimate by an adjustment factor to estimate the baseline. The adjusment factor accounts for any variation in the consumption pattern from the past. The adjustment factor is calculated based on the consumption in the hours prior to the DR event on the DR event day. In the next section we discuss how this scheme influences the behavior of consumers and compare the baseline estimate with the self-reported scheme.

During the DR event day the optimal consumption decision depends on whether the consumer is signaled for DR or whether the consumer is not signaled for DR . As discussed above
the baseline estimate used for the payments depend on the consumption in the days prior to the DR event day. So the payments made during the DR event day can influence the consumer to inflate their consumption before the DR event day. We consider a simple model to characterize the baseline inflation for the CAISO scheme. Then using this we show that the self-reported scheme achieves better control over baseline inflation thereby improving the efficiency of the DR program.

### 4.4.0.1 DR Event Day

During the DR event day the consumer can be either signaled to reduce or not signaled to reduce. We consider these two cases below.

Consumer is signaled for DR Let $\mathbf{q}_{m}$ denote the $m$-dimensional vector consisting of the consumption on the most recent $m$ similar but non-event days. Define $\hat{b}^{c}$ as,

$$
\hat{b}^{c}=\operatorname{sum}\left\{\mathbf{q}_{m}\right\} / m
$$

Denote the consumption in the hours prior to the DR event by $q^{-}$. Also denote the consumption during the hour, prior to the hour that corresponds to the DR event hour on the DR event day, on the days corresponding to the consumption vector $\mathbf{q}_{m}$, as $\mathbf{q}_{m}^{-}$and let $b^{-}=\left(\operatorname{sum}\left\{\mathbf{q}_{m}^{-}\right\} / m\right)$. Then the adjustment factor is $C_{b}=q^{-} / b^{-}$. So the CAISO baseline with adjustment factor is given by,

$$
\begin{equation*}
\bar{b}^{c}=\hat{b}^{c} C_{b} \tag{4.23}
\end{equation*}
$$

The payment for reduction is $R\left(\pi^{r}, q, \bar{b}^{c}\right)=\pi^{r}\left(\bar{b}^{c}-q\right)$ and the total cost is given by $J^{c}=$ $\pi^{e} q-u(q, \theta)-R\left(\pi^{r}, \bar{b}^{c}, q\right)$.

Consumer is not signaled for DR No penalty is imposed on the consumers in the current CAISO DR scheme and so the total cost is given by $J^{b}(q)=\pi^{e} q-u(q, \theta)$.

### 4.4.0.2 Baseline Estimate

The timing of DR signal alters the behavior of the consumer and so modifies the baseline estimate. Hence we consider the following two scenarios: (i) Consumer is informed just before the DR event (ii) Consumer is informed day ahead. For each scenario we calculate the baseline estimate for the CAISO scheme and compare with the baseline estimate of the self-reported scheme.

Consumer is informed just before the DR event Let $J^{-}$represent the cost for consuming $q^{-}$prior to the DR event. Then the average cost on the DR event day is the average of $J^{-}$and the cost during the hour when the DR event is supposed to occur. At the
beginning of the day the consumer only knows the probability of occurence of a DR event, which is $\alpha$. So the average cost during the DR event hour is $\alpha J^{c}+(1-\alpha) J^{b}$. The payments made during the DR event day can influence the consumer to inflate the consumption $q^{-}$. To model this effect we include the cost $J^{-}$in the total cost for the DR event day. So the total average cost for the DR event day is given by,

$$
\begin{aligned}
& J\left(\bar{b}^{c}\right)=\frac{1}{2}\left(J^{-}+\alpha J^{c}+(1-\alpha) J^{b}\right) \\
& J^{-}=\pi^{e}\left(q^{-}\right)-u\left(q^{-}, \theta\right)
\end{aligned}
$$

Note that the average cost depends on the baseline estimate $\bar{b}^{c}$. From (4.23) it follows that the baseline estimate depends on $\hat{b}^{c}$ which is the average of consumption on the similar non-event days in the past. Denote the consumption on the $k$ th similar non-event day of the past by $q_{-k}$. Then the consumer's decision $q_{-k}$ will be influenced by the payments to be made in the future DR event. To model this effect we consider the average total cost over the $m$ recent non-event days and the event day,

$$
\begin{equation*}
\bar{J}=\frac{1}{m+1}\left(\sum_{k=1}^{m} \mathbb{E}_{\theta} J^{b}\left(q_{-k}\right)+\mathbb{E}_{\theta} J\left(\bar{b}^{c}\right)\right) \tag{4.24}
\end{equation*}
$$

Remark 17. Let $\theta_{-k}$ be the realized value of $\theta$ on the $k$ th similar non-event day. For illustration purposes, we shall assume that $\theta_{-k}=\theta^{\prime}$ for all $k$ where $\theta^{\prime}$ is such that $q^{a}\left(\theta^{\prime}\right)=\mathbb{E}_{\theta} q^{a}(\theta)$.

Optimal consumption decision $q^{-}$: Optimal $q^{-}$satisfies $\frac{\partial \bar{J}}{\partial q^{-}}=0$. W.r.t $q^{-}, \hat{b}^{c}$ and $b^{-}$are constants, so differentiating $\bar{J}$ w.r.t $q^{-}$and equating to zero we get,

$$
\begin{equation*}
\frac{\partial \bar{J}}{\partial q^{-}}=\pi^{e}-\mu\left(q^{-}\right)-\alpha \pi^{r} \frac{\hat{b}^{c}}{b^{-}}=0 \tag{4.25}
\end{equation*}
$$

The lemma below specifies the relation between $\hat{b}^{c}$ and $b^{-}$
Lemma 5. $\hat{b}^{c}>b^{-}$

Considering space limitations we do not provide the details of the proof. From lemma 5 and (4.25) it follows that,

$$
\begin{equation*}
\pi^{e}-\mu\left(q^{-}\right)-\alpha \pi^{r}>0 \tag{4.26}
\end{equation*}
$$

From the concavity of $u$ it follows that,

$$
\begin{equation*}
q^{-}(\theta)>\mu^{-1}\left(\pi^{e}-\alpha \pi^{r}\right) \tag{4.27}
\end{equation*}
$$

When $\mu^{\prime}=-d$, this implies that,

$$
\begin{equation*}
q^{-}(\theta)>q^{a}(\theta)+\frac{\alpha \pi^{r}}{d} \tag{4.28}
\end{equation*}
$$

Optimal consumption decision $q_{-1}: q_{-1}$ is the consumption on the 1st similar non-event day before the DR event day. Let $\theta_{-1}$ be the realized value of the random variable $\theta$ on this day. Optimal $q_{-1}$ satisfies $\frac{\partial \bar{J}}{\partial q_{-1}}=0$. With respect to $q_{-1}, b^{-}$is a constant because $b^{-}$depends on consumption prior to the hour that corresponds to the consumption $q_{-1}$. So differentiating $\bar{J}$ w.r.t $q_{-1}$ and equating to zero we get that optimal $q_{-1}$ satisfies,

$$
\begin{equation*}
\frac{\partial u\left(q_{-1}\right)}{\partial q_{-1}}+\alpha \pi^{r} \frac{\partial \hat{b}^{c}}{\partial q_{-1}} \mathbb{E}_{\theta} C_{b}-\pi^{e}=0 \tag{4.29}
\end{equation*}
$$

Note that $\frac{\partial \hat{b}^{c}}{\partial q_{-1}}=1 / m$. So we get,

$$
\begin{aligned}
& \frac{\partial u\left(q_{-1}\right)}{\partial q_{-1}}+\alpha \pi^{r} \mathbb{E}_{\theta} C_{b} / m-\pi^{e}=0 \\
& q_{-1}=\mu^{-1}\left(\pi^{e}-\alpha \pi^{r} \mathbb{E}_{\theta} C_{b} / m\right)
\end{aligned}
$$

Now $\mathbb{E}_{\theta} C_{b}=\left(\mathbb{E}_{\theta} q^{-}\right) / b^{-}$because $b^{-}$is a constant here. In this analysis we will assume that $\mu^{\prime}=-d$. This is just for illustration purposes. Then $\mathbb{E}_{\theta} C_{b}=\left(\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{d} \frac{\hat{b}^{c}}{b^{-}}\right) / b^{-}$.

$$
\begin{equation*}
q_{-1}=\mu^{-1}\left(\pi^{e}-\frac{\alpha \pi^{r}\left(\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{d} \frac{\hat{b}^{c}}{b^{-}}\right)}{\left(m b^{-}\right)}\right) \tag{4.30}
\end{equation*}
$$

That is to say,

$$
\begin{align*}
& q_{-1}=q^{a}\left(\theta^{\prime}\right)+\frac{\alpha \pi^{r}}{m d}\left(\mathbb{E}_{\theta} q^{a}(\theta) / b^{-}\right)+\frac{\alpha^{2} \pi^{r 2}}{m d^{2} b^{-}} \frac{\hat{b}^{c}}{b^{-}} \\
& q_{-1}=\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{m d}\left(\mathbb{E}_{\theta} q^{a}(\theta) / b^{-}\right)+\frac{\alpha^{2} \pi^{r 2}}{m d^{2} b^{-}} \frac{\hat{b}^{c}}{b^{-}} \tag{4.31}
\end{align*}
$$

In the lemma below we establish the relation between $b^{-}$and $\mathbb{E}_{\theta} q^{a}(\theta)$
Lemma 6. $b^{-}<\mathbb{E}_{\theta} q^{a}(\theta)$

The proof of this lemma is part of the proof of lemma 5. From lemma 6 and lemma 5 it follows that,

$$
\begin{equation*}
q_{-1}>\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{m d}+O\left(\alpha^{2}\right) \tag{4.32}
\end{equation*}
$$

Similarly, we can show that,

$$
\begin{equation*}
q_{-k}>\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{m d}+O\left(\alpha^{2}\right) \tag{4.33}
\end{equation*}
$$

Baseline Inflation: If $\delta \hat{b}$ denotes the inflation in baseline estimate for the CAISO scheme. Then $\delta \hat{b}$ is given by,

$$
\begin{equation*}
\delta \hat{b}=\mathbb{E}_{\theta} \bar{b}^{c}-\mathbb{E}^{a}(\theta)=\mathbb{E}_{\theta}\left(q_{-}\right) \frac{\hat{b}^{c}}{b^{-}}-\mathbb{E} q^{a}(\theta) \tag{4.34}
\end{equation*}
$$

From (4.28) it follows that $\mathbb{E}_{\theta}\left(q^{-}\right)>\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{d}$. So,

$$
\begin{equation*}
\delta \hat{b}>\mathbb{E}_{\theta} q^{a}(\theta) \frac{\hat{b}^{c}}{b^{-}}+\frac{\alpha \pi^{r}}{d} \frac{\hat{b}^{c}}{b^{-}}-\mathbb{E} q^{a}(\theta) \tag{4.35}
\end{equation*}
$$

From (4.33) it follows that $\hat{b}^{c}=1 / m \times \sum_{k=1}^{m} q_{-k}>\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{m d}+O\left(\alpha^{2}\right)$. This implies,

$$
\begin{equation*}
\delta \hat{b}>\left(\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{d}\right)\left(\frac{\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}}{m d}+O\left(\alpha^{2}\right)}{b^{-}}\right)-\mathbb{E} q^{a}(\theta) \tag{4.36}
\end{equation*}
$$

Then from lemma 6 it follows that,

$$
\begin{equation*}
\delta \hat{b}>\frac{\alpha \pi^{r}}{d}+\frac{\alpha \pi^{r}}{m d}+O\left(\alpha^{2}\right) \tag{4.37}
\end{equation*}
$$

Since $\alpha$ is small ( i.e. $\alpha \ll 1$ ), $\delta b<\delta \hat{b}$, that is the self-reported scheme achieves better control of inflation in baseline.

Consumer is informed day ahead Here we assume that consumers are signaled a day ahead of the DR event. The consumer cost for the DR event day is the average of the cost during the DR event and the cost during the hours prior to the DR event. Let $J^{-}$represent the cost for consuming $q^{-}$prior to the DR event. Here the consumer knows that there is going to be a DR event. So the average cost for the DR event day is,

$$
\begin{align*}
J & =\frac{1}{2}\left(J^{-}+J^{c}\right)  \tag{4.38}\\
& =\frac{1}{2}\left(\pi^{e}\left(q^{-}\right)-u\left(q^{-}, \theta\right)-u(q, \theta)-\pi^{r}\left(\bar{b}^{c}-q\right)\right) .
\end{align*}
$$

Now, we use the optimal consumption decision $q^{-}$to derive a lower bound on the customer baseline. Note that on the DR event day $\hat{b}^{c}$ and $b^{-}$are constants. So, differentiating $J$ in (4.38) w.r.t $q^{-}$gives,

$$
\begin{equation*}
\frac{\partial J}{\partial q^{-}}=\frac{1}{2}\left(\pi^{e}-\mu\left(q^{-}, \theta\right)-\pi^{r} \frac{\hat{b}^{c}}{b^{-}}\right) \tag{4.39}
\end{equation*}
$$

Optimal $q^{-}$satisfies $\frac{\partial J\left(q^{-}\right)}{\partial q^{-}}=0$ and is given by

$$
\begin{equation*}
q^{-}(\theta)=\mu^{-1}\left(\pi^{e}-\pi^{r} \frac{\hat{b}^{c}}{b^{-}}\right) \tag{4.40}
\end{equation*}
$$

From lemma 5 we know that $\hat{b}^{c}>b^{-}$. This implies that

$$
\begin{equation*}
q^{-}(\theta) \geq \mu^{-1}\left(\pi^{e}-\pi^{r}\right) \tag{4.41}
\end{equation*}
$$

Then, the baseline estimate is given by,

$$
\begin{equation*}
\bar{b}^{c}=\hat{b}^{c} C_{b}=q^{-} \frac{\hat{b}^{c}}{b^{-}} \geq \mu^{-1}\left(\pi^{e}-\pi^{r}\right) \tag{4.42}
\end{equation*}
$$

Then the inflation in baseline in this scenario is atleast,

$$
\begin{equation*}
\delta \hat{b} \geq \mathbb{E}_{\theta} \mu^{-1}\left(\pi^{e}-\pi^{r}, \theta\right)-\mathbb{E}_{\theta} \mu\left(\pi^{e}, \theta\right)=\frac{\pi^{r}}{d} \tag{4.43}
\end{equation*}
$$

From previous section we know that the baseline inflation in the self-reported scheme is

$$
\begin{equation*}
\delta b=\hat{b}^{*}-\mathbb{E}_{\theta} q^{a}(\theta)=\frac{\alpha \pi^{r}}{d} . \tag{4.44}
\end{equation*}
$$

Clearly $\delta b \ll \delta \hat{b}$.
Remark 18. From the above analysis, it is clear that the CAISO scheme is prone to greater baseline inflation than the self reported mechanism, especially in scenarios where there is high variability in consumption. So the self-reported scheme improves the efficiency of DR programs

### 4.5 Threshold Market Clearing Price

The threshold market clearing price is integral to the DR mechanism. As mentioned in section 4.1.1, DR events are called by the SO when the load forecast corresponds to the peak load scenario to a high degree of confidence. In our formulation peak load scenario is realized when the market price corresponding to the load forecast crosses the threshold market clearing price. In this section we demonstrate how the SO calculates this threshold market clearing price.

TMC Price Let $\Pi(l)$ denote the inverse supply function, i.e. the price at which conventional suppliers are willing to provide $l$ units of electricity, that is assumed to be monotone increasing and continuously differentiable. Denote $L$ as the peak load. In our notation retail price of electricity is $\pi^{e}$. Total cost to the SO/DRM when the net DR resource $\Delta L$ is deployed and all resources are paid at the market price is given by,

$$
\begin{equation*}
C(\Delta L)=\Pi(L-\Delta L) L^{p}-\pi^{e}(L-\Delta L) \tag{4.45}
\end{equation*}
$$

Hence the optimal DR capacity $\Delta L^{*}$ that maximizes the SO/DRM's savings satisfies

$$
\begin{equation*}
0=\frac{d C\left(\Delta L^{*}\right)}{d \Delta L}=-\Pi^{\prime}\left(L-\Delta L^{*}\right) L+\pi^{e} \tag{4.46}
\end{equation*}
$$

It follows that,

$$
\begin{equation*}
\Delta L^{*}=L-\Pi^{\prime-1}\left(\frac{\pi^{e}}{L}\right) \tag{4.47}
\end{equation*}
$$

Let $\pi^{*}$ be the market price corresponding to the optimal DR deployment, i.e. $\Pi\left(L-\Delta L^{*}\right)=\pi^{*}$. Then the SO sets $\pi^{*}$ as the threshold market clearing price. In the proposed mechanism the SO announces the reward as $\pi^{r}=\pi^{*}$ and then it recruits consumers such that when their DR resources are deployed the market operates at the same price $\pi^{*}$. This means that the combined load reduction of the recruited consumers adds up to $\Delta L^{*}$ and that all resources are paid at the market price which is $\pi^{*}$. By the above derivation this maximizes the savings for $\mathrm{SO} / \mathrm{DRM}$.

### 4.6 General Non-Linear Utility

Here we highlight the extent of baseline inflation for a general non-linear utility funciton. The following theorem gives the expression for baseline inflation $\delta \hat{b}$ for a general utility function. Note that we only provide an upper-bound here.
Lemma 7. $\delta \hat{b} \leq \frac{\alpha \pi^{*}(1+d / \lambda)}{(1-\alpha) d}$ where
(i) $\alpha>0$.
(ii) $\forall q:-\mu^{\prime}(q) \geq d$
(iii) $\forall r: \phi^{\prime \prime}(r)=\lambda \geq d$.

The proof steps follow from the proof of Lemma 4. Note that what the lemma provides is only an upperbound and not the exact inflation in baseline. The exact expression might be difficult to derive.

### 4.7 Further Remarks

The discussion in this chapter signifies two important aspects of self reported baseline mechanism. First, self-reporting offers a better baseline estimate than averaging methods especially when variability in consumption is high. Secondly the mechanism required to integrate DR resources in to wholesale markets is a simpler when compared to the retail baseline mechanism described above 4.1.1. It is counter-intuitive that integration in wholesale markets is easier compared to retail market. In the case of wholesale market the Operator can measure the extent of aggregate load reduction offered by the DR providers using realized market price (after deploying DR ) and the load forecast (assuming load forecast is accurate). The aggregator has no such external reference to rely up on. It is also this simplifying aspect of the wholesale market that allows us to characterize the baseline report of the mechanism for general non-linear utility functions. Also, this enables the operator to decide independently
whether to call more consumers during the next DR event. So the operator does require some time to learn or decide which consumers to call so that its savings are maximized. If this external reference such as the market price was not available then the setting inevitably reduces to the retail DR setting considered in section 2.

## Chapter 5

## Learning and Pricing Demand Response

The wholesale market mechanism proposed in section 4 achieves optimality in steady state. But in a repeated setting the transient losses also needs to be taken in to account. Here, we consider a repeated setting where the DR events repeat. In such a setting its not sufficient that the optimal price is attained at steady state, becuase the transient losses have to be taken in to account. In the full information setting it is trivial to define the optimal pricing policy. In the incomplete information setting, one has to consider the trade-off between learning the consumer behavior and maximizing the savings based on the learnt information. Here we propose a sub-linear regret pricing policy mechanism.
Framework Outline: We consider the following setting where the electric power Utility maximizes its cumulative risk-sensitive payoff over a sequence of $T$ days. Here the Utility's payoff on a given day is the largest return the utility is guaranteed to receive with a probability of atleast $1-\alpha$. We assume that the consumer is myopic in its behavior and the consumer's response to price incentive is affected by additive random shocks. In order to maximize the payoff, the Utility is best served if it tries to learn the consumer behavior by dynamically adjusting the prices offered for load reduction. So we consider a strategy where the Utility strategically adjusts the price such that it learns the consumer behavior and at the same time maximizes its savings.
Related Work Jia et al. [23] consider the problem of pricing demand response when the underlying demand function is unknown, affine, and subject to 'normally distributed random shocks'. They propose a stochastic approximation-based pricing policy, and establish an upper bound on the T-period regret that is of the order $O(\log T)$. Authors in [46, 25, 22, 48] use the mulit-armed bandit setting to study the problem of eliciting demand response under uncertainty. Kalathil and Rajagopal [25] consider a similar multi-armed bandit setting in which a customer's load curtailment is subject to an exogenous shock, and there is attenuation due to fatigue resulting from repeated requests for reduction in demand over time. They propose a policy that guarantees that the T-period regret is bounded by $O(\sqrt{T \log T})$.

In this work we do not make any assumption on the distribution nor do we assume explicit knowledge of the baseline. Also we consider a risk-sensitive setting. Baseline is very critical to estimate the payments for the load reduction that the consumer provides. Hence incorporating baseline in to the setting that we consider here is very critical.

### 5.1 Formulation

We consider the setting where Utility serves a set of $N$ consumers, indexed by $i$. On any given day (ex: the $t$ th day), DR event occurs with probability $\beta$ i.e.

$$
\text { Probability of DR Event }=\beta
$$

When the DR event occurs, the Utility elicits demand response from this group of $N$ consumers indexed by $i=1,2, \ldots N$. And this repeats. The Utility has to set the reward or price $/ \mathrm{kWh}$ of reduction so that its savings are maximized for the entire sequence of days. But in order to set the optimal price the Utility should have knowledge of the optimal behavior of the consumers. This leads to a trade-off between learning and the opportunity to maximize the savings based on the information learnt till the current day. The time-line of the DR mechanism is given below (refer figure 5.1)

1. After the $t$ th day Reward $/ \mathrm{kWh} p_{t}$ is announced (based on prior information)
2. Consumer $i$ reports $\hat{f}_{t}^{i}$
3. DR event occurs with probability $\beta$. If there is a DR event then the consumers are called.
4. Measure reduction of the called consumer w.r.t to the 'assigned baseline' $\hat{b}_{t}^{i}$.
5. Pay $p_{t}$ for every unit of measured reduction

We explain the time line in more detail. Before day $t$, all consumers are informed the reward $/ \mathrm{kWh}$ for load reduction $p_{t}$. Following which consumer $i$ reports value of it's baseline $\hat{f}_{t}^{i}$, corresponding to day $t$. The consumer is assigned a baseline $\hat{b}_{t}^{i}$ based on the reported value $f_{t}^{i}$. The baseline is the counterfactual against which consumer's load reduction is measured. When the consumer is called, Utility measures the load reduction of all the consumers w.r.t the assigned baseline. Pays them $p_{t}$ for every unit of measured load reduction. The objective of the Utility is to maximize it's risk-sensitive savings for its life time. The time line is shown in Figure 5.1.


Figure 5.1: Learning and Pricing:Time-line

### 5.1.1 Consumer Model

We assume that consumer $i$ 's demand function $q_{t}^{i}(p)$, where $p_{t}$ is the reward per kWh of load reduction, is linear i.e.,

$$
\begin{equation*}
q_{t}^{i}\left(p_{t}\right)=a^{i}-d^{i} p_{t}-\epsilon_{t}^{i} \tag{5.1}
\end{equation*}
$$

Where $a^{i}$ and $d^{i}$ are model parameters of consumer $i$ and are unknown to the Utility. Note that a linear demand funciton is implied and implied by quadratic utility. So the linear deand function model assumed here directly corresponds to the quadratic utility model of theorem 4. $\epsilon^{i}$ models the uncertainty in consumer $i$ 's demand function. The distribution of $\epsilon^{i}$ is also unknown to the Utility. The true baseline is given by the consumption of the consumer when $p_{t}=0$. If we denote the true baseline on day $t$ by $b_{t}^{i}$ then from (5.1)

$$
\begin{equation*}
b_{t}^{i}=q^{i}(0)=a^{i}-\epsilon_{t}^{i} \tag{5.2}
\end{equation*}
$$

We make the following assumption on the consumers
Assumption 7. Consumers are myopic
This implies that consumer $i$ will report $\hat{f}_{t}^{i}$ only based on $p_{t}$ and its utility for the $t$ th day.

### 5.1.2 Demand Response Mechanism

Similar to sections 2 and 4, the consumer is called to reduce when the DR event occurs and is paid for the load reduction it provides. If there is no DR event, the consumer is not called and a penalty is imposed for any deviation from its report.

### 5.1.2.1 When the Consumer is called for DR

When consumer $i$ is called for DR it is paid by $p_{t}$ for every unit of measured reduction. Let the consumer's assigned baseline be $\hat{b}_{t}^{i}$. Then the consumer is paid $p_{t}\left(\hat{b}_{t}^{i}-q_{t}^{i}\right)$.

### 5.1.2.2 When the Consumer is not called for DR

When consumer $i$ is not called for DR the following penalty $\phi$ is imposed on the consumer,

$$
\phi(.)=\left\{\begin{array}{cc}
\lambda\left(\hat{f}_{t}^{i}-q_{t}^{i}\right)^{2} & \Gamma=1 \\
0 & \Gamma=0
\end{array}\right.
$$

$$
\begin{equation*}
\text { Where } \lambda \gg 1 \text { and } \mathbb{P}\{\Gamma=1\}=\gamma_{t}=g_{\gamma}(t)=\gamma \tag{5.3}
\end{equation*}
$$

So when the consumers is not called, a large penalty is imposed for any deviation from the consumer report $\hat{f}_{t}^{i}$. And so it consumes the assigned baseline $\hat{b}_{t}^{i}$.

### 5.1.3 Baseline Assignment $\hat{b}_{t}^{i}$

Here we discuss how the Utility assigns the baseline to consumer $i$. A baseline $\hat{b}_{t}^{i}$ is assigned to consumer $i$ based on past information, in particular $\left\{\hat{f}_{1}^{i}, \hat{f}_{2}^{i}, \hat{f}_{3}^{i}, \ldots, \hat{f}_{t-1}^{i}\right\}$, and the current report $\hat{f}_{t}^{i}$. Define $\bar{f}_{t}^{i}$,

$$
\begin{equation*}
\bar{f}_{t}^{i}=\frac{\sum_{j=1}^{t} \hat{f}_{j}^{i}}{t} \tag{5.4}
\end{equation*}
$$

Then the baseline of consumer $i$ is equated to $\bar{f}_{t}^{i}$,

$$
\begin{equation*}
\hat{b}_{t}^{i}=\bar{f}_{t}^{i} \tag{5.5}
\end{equation*}
$$

### 5.1.4 Measured Reduction and Payment

When the consumers are called, the total consumption $Q_{t}\left(p_{t}\right)$ of all the $N$ consumers is given by,

$$
\begin{align*}
& Q_{t}\left(p_{t}\right)=\sum_{i=1}^{N} q_{t}^{i}\left(p_{t}\right)=a-d p_{t}-\epsilon_{t} \\
& \text { Where } a=\sum_{i=1}^{N} a^{i}, d=\sum_{i=1}^{N} d^{i}, \epsilon_{t}=\sum_{i=1}^{N} \epsilon_{t}^{i} \tag{5.6}
\end{align*}
$$

The true aggregate baseline is given by $B_{t}=\sum_{i}^{N} a^{i}-\epsilon_{t}^{i}=a-\epsilon_{t}$. This implies the true aggregate reduction $\Delta Q_{t}$ is given by,

$$
\begin{equation*}
\Delta Q_{t}=B_{t}-Q_{t}\left(p_{t}\right)=d p_{t} \tag{5.7}
\end{equation*}
$$

The corresponding measured aggregate reduction $\Delta \hat{Q}_{t}$ is given by,

$$
\begin{align*}
& \Delta \hat{Q}_{t}=\hat{B}_{t}-Q_{t}\left(p_{t}\right) \\
& \text { Where } \hat{B}_{t}=\sum_{i=1}^{N} \hat{b}_{t}^{i} \tag{5.8}
\end{align*}
$$

Define the inflation in baseline as $\delta B_{t}=\hat{B}_{t}-B_{t}$. Then it follows that $\Delta \hat{Q}_{t}-\Delta Q_{t}=\delta B_{t}$. From 5.1.3 and lemma 4 from section 4 we can write $\delta B_{t}$ explicitly as,

$$
\begin{equation*}
\delta B_{t}=\frac{\beta \sum_{j=1}^{t} \frac{p_{j}}{j}}{(1-\beta) \gamma t} d+\epsilon_{t}<\frac{\beta(\log t+1)}{(1-\beta) \gamma t} d \bar{p}+\epsilon_{t} \tag{5.9}
\end{equation*}
$$

Where $\bar{p}=\max p_{i}$
Remark 19. Property 1: As $T \rightarrow \infty, \mathbb{E}\left\{\delta B_{t}\right\}=0$.
Remark 20. Property 1 is necessary for a consistency. Because any amount of inflation in the baseline only results in excess payments.
Remark 21. Property 1 also avoids the under-payment concerns that we highlighted in 2. Because asymptotically the baseline estimate is not biased on either side of the true baseline it avoids the possibility of under-payment and over-payment.

The consumer is paid $p_{t}$ for every unit of load reduction it provides. Then the total payment made to the consumer for the load reduction they provide is given by $p_{t}$ times the aggregate measured reduction i.e. $p_{t}\left(\Delta Q_{t}+\delta B_{t}\right)$. Note that the payments made to the consumer is inflated by $p_{t} \delta B_{t}$.

### 5.1.5 Utility's Objective

We number the days as $t=1,2,3, \ldots, T$. On any given day let $c_{t}(\$ / k W h)$ denote the wholesale price of electricity. If a DR event occurs on day $t$ the consumers are called up on to provide load reduction. The consumers respond based on the price $p_{t}$ that was announced before day $t$. Then the aggregate demand reduction $\Delta Q_{t}$ is realized. The Utility saves the market price times the load reduction that was provided i.e. $c_{t} \Delta Q_{t}$. But there is a payment that the Utility makes to the consumers for the load reduction they provide. From 5.1.1 the payment made to the consumer is given by $p_{t}\left(\Delta Q_{t}+\delta B_{t}\right)$.
So the net savings the Utility makes is given by $\left(c_{t}-p_{t}\right) \Delta Q_{t}-p_{t} \Delta B_{t}$. Henceforth, we will refer to the net savings $\left(c_{t}-p_{t}\right) \Delta Q_{t}-p_{t} \Delta B_{t}$ in period $t$ as revenue. Below we introduce the risk-sensitive measure that the Utility maximizes,

$$
\begin{equation*}
r_{\alpha}\left(p_{t}\right)=\sup \left\{x \in \mathbb{R}: \mathbb{P}\left\{\left(c_{t}-p_{t}\right) \Delta Q_{t}-p_{t} \Delta B_{t} \geq x\right\} \geq 1-\alpha\right\} \tag{5.10}
\end{equation*}
$$

So the optimal price in this risk-sensitive model maximizes the revenue that the Utility is guaranteed to receive with probability no less than $1-\alpha$ where the parameter $\alpha \in(0,1)$ encodes the degree to which the Utility is sensitive to risk. We can simplify this further as,

$$
r_{\alpha}\left(p_{t}\right)=\left(c_{t}-p_{t}\right) d p_{t}-p_{t} \frac{\beta \sum_{j=1}^{t} \frac{p_{j}}{j}}{(1-\beta) \gamma t} d-p_{t} F^{-1}(\alpha)
$$

Finally,

$$
\begin{equation*}
r_{\alpha}\left(p_{t}\right)=\left(c_{t} d-\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j} d}{(1-\beta) \gamma t}-F^{-1}(\alpha)\right) p_{t}-p_{t}^{2}\left(d+\frac{\beta d}{(1-\beta) \gamma t^{2}}\right) \tag{5.11}
\end{equation*}
$$

Let $p_{t}^{*}$ denote the optimal price, which maximizes the risk-sensitive revenue during a DR event in period $t$. Namely,

$$
p_{t}^{*}=\arg \max \left\{r_{\alpha}\left(p_{t}\right): p_{t} \in\left[0, c_{t}\right]\right\}
$$

Then it follows by explicit maximization that $p_{t}^{*}$ satisfies,

$$
\begin{equation*}
\left(c_{t}-2 p_{t}^{*}\right) d-\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j}}{(1-\beta) \gamma t} d-2 p_{t}^{*} \frac{\beta d}{(1-\beta) \gamma t^{2}}-F^{-1}(\alpha)=0 \tag{5.12}
\end{equation*}
$$

This implies $p_{t}^{*}$ is given by,

$$
\begin{align*}
& p_{t}^{*}=\frac{c_{t}}{2\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)}-\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j} d+(1-\beta) \gamma t F^{-1}(\alpha)}{2 d\left((1-\beta) \gamma t+\frac{\beta}{t}\right)} \\
& p_{t}^{*}=\frac{c_{t}}{2\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)}-\frac{\frac{\beta d}{(1-\beta) \gamma t} \sum_{j=1}^{t-1} \frac{p_{j}}{j}+F^{-1}(\alpha)}{2 d\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)} \tag{5.13}
\end{align*}
$$

Substituting for $p_{t}^{*}$ in (5.11) we get,

$$
\begin{equation*}
r_{\alpha}\left(p_{t}^{*}\right)=d\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)\left(p_{t}^{*}\right)^{2} \tag{5.14}
\end{equation*}
$$

Risk-sensitive optimal revenue: Define,

$$
\begin{equation*}
R^{*}(T)=\sum_{t=1}^{T} r_{\alpha}\left(p_{t}^{*}\right) \tag{5.15}
\end{equation*}
$$

We call this the oracle risk sensitive optimal revenue because the $p_{t}^{*}$ which by definition maximizes the (5.11) requires explicit knowledge of consumer behavior parameters. Note that when there is no DR event the Utility does not elicit demand response and its savings are zero. So we can simplify the cumulating oracle risk sensitive revenue as,

$$
\begin{equation*}
R^{*}(T)=\sum_{t=1}^{T} r_{\alpha}\left(p_{t}^{*}\right) I\{\text { DR Event }\} \tag{5.16}
\end{equation*}
$$

Risk-sensitive optimal revenue - Baseline: Also denote the optimal risk-sensitive revenue when the baseline is explicitly known by $r_{\alpha}^{b}$. It follows that the risk-sensitive savings is given by,

$$
\begin{equation*}
r_{\alpha}^{b}=\left(c_{t}-p_{t}\right) \Delta Q_{t}-p_{t} F^{-1}(\alpha) \tag{5.17}
\end{equation*}
$$

Note that the term that depends on $\Delta B_{t}$ is absent. If we denote the corresponding optimal price by $p^{* b}$ then it follows that,

$$
\begin{equation*}
p^{* b}=\frac{c_{t}}{2}-\frac{F^{-1}(\alpha)}{2 d} \tag{5.18}
\end{equation*}
$$

And from here it follows that,

$$
\begin{equation*}
r_{\alpha}\left(p^{* b}\right)=d\left(p^{* b}\right)^{2} \tag{5.19}
\end{equation*}
$$

Define the corresponding cumulative savings as,

$$
\begin{equation*}
R^{* b}(T)=\sum_{t=1}^{T} r_{\alpha}^{b}\left(p^{* b}\right) \mathrm{I}\{\text { DR Event }\} \tag{5.20}
\end{equation*}
$$

A general feasible pricing policy is an infinite sequence of pricing functions $\Pi=\left(p_{1}, p_{2}, \ldots\right)$, where each function in the sequence depends on the past history. More precisely, the function $p_{t}$ is a measure of the $\sigma$ - algebra generated by the history of past decisions, measured demand reductions during DR events and measured consumption during normal event for all $t \geq 2$, and that $p_{1}$ be a constant function. Then the expected risk-sensitive revenue generated by a feasible pricing policy $\Pi$ over the DR events during the $T$ time periods is defined as

$$
\begin{equation*}
R_{\Pi}(T)=\mathbb{E}_{\Pi} \sum_{t=1}^{T} r_{\alpha}\left(p_{t}\right) I\{\text { DR Event }\} \tag{5.21}
\end{equation*}
$$

$R_{\Pi}(T)$ is the risk-sensitive revenue corresponding to policy $\Pi$. So the difference between $R_{\Pi}(T)$ and $R^{* b}(T)$ will give us the extent of loss incurred with respect to the optimal pricing policy. As highlighted before, in the absence of oracle, the estimation of optimal pricing policy is nearly impossible without learning the demand function parameters. Hence to measure the performance of any pricing policy with respect to the optimal pricing policy, we use regret as the metric.

### 5.1.5.1 Performance Metric

The performance metric for a feasible pricing policy is the difference between the cumulative savings for the feasible pricing policy and the optimal savings defined by $R^{* b}(T)$

$$
\begin{equation*}
\Delta_{\Pi}(T)=R^{* b}(T)-R_{\Pi}(T) \tag{5.22}
\end{equation*}
$$

Note that the oracle risk-sensitive revenue $R^{* b}(T)$ which requires explicit knowledge of the baseline is the maximum savings one can achieve. It might be impossible to build a zero regret pricing policy, so we rather seek a pricing policy whose $T$-period regret is sub-linear in the horizon $T$.

Definition 7. A feasible pricing policy $\Pi$ is said to exhibit no-regret if

$$
\lim _{T \rightarrow \infty} \frac{\Delta_{\Pi}(T)}{\left(\sum_{i=1}^{T} \mathrm{I}\{D R \text { Event }\}\right)}=0
$$

### 5.2 DEMAND MODEL LEARNING

It is very clear that the $T$ - period regret is going to grow linearly if the prices are chosen randomly. Hence it is important that the parameters that determine the behavior of the consumers is learnt. Clearly, the ability to price with no-regret will rely centrally on the rate at which the unknown parameters, $\theta=(a, d)$, and quantile function, $F^{-1}(\alpha)$, are leart from past data. In what follows, we describe a learning approach based on the method of least squares estimation. Let $\bar{t}$ correspond to the count of number of DR events till time $t$. Then,

Definition 8. $\bar{t}=n$ when $\sum_{k=1}^{t} \mathrm{I}\{D R$ Event $\}=n$
Remark 22. Let $x_{T}$ denotes the value of variable $x$ when $t=T$. Then the value of variable $x$ in the $\bar{t}$ time scale when $\bar{t}=\bar{T}$ is $x_{\bar{T}}=x_{T}$ where $T=\min \left\{t \mid \sum_{k=1}^{t} \mathrm{I}\{\mathrm{DR}\right.$ Event $\left.\}=\bar{T}\right\}$.
Remark 23. We use $r$ as the index in summations over $\bar{t}$ time scale.
Parameter Estimation: Given the history of past decisions and demand observations in the $\bar{t}$ time scale $\left(p_{1}, \ldots, p_{\bar{t}}, Q_{1}, \ldots, Q_{\tilde{t}}\right)$, define the least squares estimator (LSE) of $\theta$ as

$$
\begin{align*}
& \theta_{\bar{t}}:=\arg \min \left(q_{r}-\lambda\left(p_{r}, \theta\right)\right)^{2}: \theta \in \mathbb{R}_{2} \\
& \text { where } \lambda\left(p_{r}, \theta\right)=a-d p_{r} \tag{5.23}
\end{align*}
$$

Note that $r$ is an index that runs over the $\bar{t}$ time scale. If $k$ denotes the index that runs over $t$ time scale then $q_{r}$ and $p_{r}$ are given by,

$$
\begin{align*}
& q_{r}=q_{k} \text { where } k=\min \left\{t ; \sum_{j=1}^{t} \mathrm{I}\{\mathrm{DR} \text { Event }\}=r\right\} \\
& p_{r}=p_{k} \text { where } k=\min \left\{t ; \sum_{j=1}^{t} \mathrm{I}\{\mathrm{DR} \text { Event }\}=r\right\} \tag{5.24}
\end{align*}
$$

For time periods $\bar{t}=1,2, \ldots$. The LSE at period $\bar{t}$ admits an explicit expression of the form

$$
\theta_{\bar{t}}=\left(\sum_{r=1}^{\bar{t}}\left[\begin{array}{c}
p_{r}  \tag{5.25}\\
1
\end{array}\right]\left[\begin{array}{c}
p_{r} \\
1
\end{array}\right]^{T}\right)^{-1}\left(\sum_{r=1}^{\bar{t}}\left[\begin{array}{c}
p_{r} \\
1
\end{array}\right] Q_{\bar{t}}\right)
$$

provided the indicated inverse exists. It will be convenient to define the $2 \times 2$ matrix

$$
\Psi_{\bar{t}}:=\sum_{r=1}^{\bar{t}}\left[\begin{array}{c}
p_{r}  \tag{5.26}\\
1
\end{array}\right]\left[\begin{array}{c}
p_{r} \\
1
\end{array}\right]^{T}
$$

Utilizing the definition of the aggregate demand model (5.6), in combination with the expression in (5.25), one can obtain the following expression for the parameter estimation error:

$$
\theta_{\bar{t}}-\theta=\Psi_{\bar{t}}^{-1}\left(\sum_{r=1}^{\bar{t}}\left[\begin{array}{c}
p_{r}  \tag{5.27}\\
1
\end{array}\right] \epsilon_{\bar{t}}\right)
$$

The Role of Price Dispersion: From the expression for the parameter estimation error in (5.27) it is clear that the asymptotic spectrum of the matrix $\Psi_{\bar{t}}$ determines the consistencey of LSE estimation. The parameter estimation error will converge to zero provided the asymptotic eigenvalues grow unbounded. It is sufficient that the minimum eigenvalue of $\Psi_{\bar{t}}$ is bounded from below (up to a multiplicative constant) by the sum of squared price deviations defined as

$$
\begin{equation*}
J_{\bar{t}}:=\sum_{r=1}^{\bar{t}}\left(p_{r}-\bar{p}_{\bar{t}}\right)^{2}, \tag{5.28}
\end{equation*}
$$

where $\bar{p}_{\bar{t}}:=(1 / \bar{t}) \sum_{r=1}^{\bar{t}} p_{r}$. The result is reliant on the assumption that the underlying pricing policy $\Pi$ yields a bounded sequence of prices $\left\{p_{\bar{t}}\right\}$.

Finally, given the underlying assumption that the unknown model parameters $\theta$ belong to a compact set defined $\Theta:=[\underline{d}, \bar{d}] \times[0, \bar{a}]$, one can improve upon the LSE at time $\bar{t}$ by projecting it onto the set $\Theta$. Accordingly, we define the truncated least squares estimator as

$$
\begin{equation*}
\hat{\theta}_{\bar{t}}:=\arg \min \left\{\|v-\theta\|_{2} \mid v \in \Theta\right\} \tag{5.29}
\end{equation*}
$$

Quantile Estimation: Building on the parameter estimator specified in (5.29), we construct an estimator of the unknown quantile function $F^{-1}(\alpha)$ according to the empirical quantile function associated with the demand estimation residuals. Namely, in each period $t$, define the sequence of residuals up to the current time by

$$
\begin{equation*}
\hat{\epsilon}_{r, \bar{t}}=Q_{r}-\lambda\left(\hat{\theta}_{\bar{t}}, p_{r}\right) \tag{5.30}
\end{equation*}
$$

for all $r=1,2,3, \ldots \bar{t}$. Define their empirical distribution as

$$
\begin{equation*}
\hat{F}_{\bar{t}}(x):=\frac{1}{\bar{t}} \sum_{r=1}^{\bar{t}} \mathrm{I}\left\{\hat{\epsilon}_{r, \bar{t}} \leq x\right\} \tag{5.31}
\end{equation*}
$$

The order statistics $\hat{\epsilon}_{(1), \tilde{t}}, \hat{\epsilon}_{(2), \bar{t}}, \hat{\epsilon}_{(3), \bar{t}}, \ldots, \hat{\epsilon}_{(\hat{t}, \bar{t}}$ of $\hat{\epsilon}_{1, \bar{t}}, \hat{\epsilon}_{2, \bar{t}}, \hat{\epsilon}_{3, \bar{t}}, \ldots, \hat{\epsilon}_{\bar{t}, \bar{t}}$ is a permutation such that $\hat{\epsilon}_{(1), \bar{t}} \leq \hat{\epsilon}_{(2), \bar{t}} \leq \hat{\epsilon}_{(3), \bar{t}} \leq \ldots \leq \hat{\epsilon}_{(t), \bar{t}}$. Define,

$$
\begin{equation*}
\hat{F}_{\bar{t}}^{-1}(\alpha)=\hat{\epsilon}_{(i), \bar{t}} \tag{5.32}
\end{equation*}
$$

where the index $i$ is chosen such that $\frac{i-1}{\bar{t}}<\alpha \leq \frac{i}{t}$. Then one can relate the quantile estimation error to the parameter estimation error according to the following inequality,

$$
\begin{equation*}
\left|\hat{F}_{\bar{t}}^{-1}(\alpha)-F^{-1}(\alpha)\right| \leq\left|F_{\bar{t}}^{-1}(\alpha)-F^{-1}(\alpha)\right|+\left(1+p_{(i)}\right)| | \hat{\theta}_{\bar{t}}-\theta \|_{1} \tag{5.33}
\end{equation*}
$$

where $F^{-1}($.$) is defined as the empirical quantile function associated with the sequence of$ demand shocks $\epsilon_{1}, \ldots, \epsilon_{t}$. Their empirical distribution is defined as

$$
\begin{equation*}
F_{\bar{t}}(x)=\frac{1}{\bar{t}} \sum_{r=1}^{\bar{t}} \mathrm{I}\left\{\epsilon_{t} \leq x\right\} \tag{5.34}
\end{equation*}
$$

We state the following proposition which bounds the probability of deviation of the empirical estimate of the distribution from the true CDF $F$. This theorem will be used later in the proof for bouding the regret.

Proposition 7. There exists a finite positive constant $\mu_{1}$ such that $\mathbb{P}\left\{\left|F_{\bar{t}}^{-1}(\alpha)-F^{-1}(\alpha)\right|>\right.$ $\gamma\} \leq 2 \exp \left(-\mu_{1} \gamma^{2} \bar{t}\right)$

This completes parameter estimation. In the next section we use the determined parameters to define our pricing policy.

### 5.3 Pricing Policy

At each stage $t+1$, the utility estimates the demand model parameters and quantile function according to 5.27 and 5.31. Based on the learnt parameters and distribution we define a myopic price according to,

$$
\begin{equation*}
\hat{p}_{t+1}=\frac{\tilde{c}_{t+1}}{2}-\frac{\hat{h}_{\bar{t}}+\hat{F}_{\bar{t}}^{-1}(\alpha)}{2 \hat{e}_{\bar{t}}} \tag{5.35}
\end{equation*}
$$

Where,

$$
\begin{align*}
\tilde{c}_{t+1} & =\frac{c_{t+1}}{\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)} \leq c_{t+1} \quad \hat{h}_{\bar{t}}=\frac{\beta \hat{d}_{\bar{t}}}{(1-\beta) \gamma t} \sum_{j=1}^{t} \frac{p_{j}}{j} \\
\hat{e}_{\bar{t}} & =\hat{d}_{\bar{t}}\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right) \tag{5.36}
\end{align*}
$$

Here we remark on the rationale behind the definition of the myopic price $\hat{p}_{t+1}$. One could say, the Utility is treating its model parameter estimate in each period as correct, and disregarding the subsequent need to learn the model parameters. Hence the danger inherent to a myopic approach is that the resulting price sequence may fail to elicit information from demand at a rate that is fast enough to enable consistent model estimation. As a result, the model
estimates may converge to incorrect values and so will have undesirable consequences on the regret. Such behavior is well documented in the literature [15], [26] and [29]. We propose a pricing policy that is different from the myopic approach. In the new pricing policy, we add a perturbation to the myopic pricing policy but not on all the days. In particular we alternate between turning on the pertubration and turning it off as given below,

$$
\begin{align*}
& p_{t+1}=\left\{\begin{array}{cc}
\hat{p}_{t+1} & \bar{t} \text { even And I\{DR Event }\}=1 \\
\hat{p}_{\bar{t}}+\frac{1}{2}\left(\tilde{c}_{t+1}-\frac{\hat{h}_{\bar{t}}}{\hat{e}_{\bar{t}}}-\left(\tilde{c}_{\bar{t}}-\frac{\hat{h}_{\bar{t}-1}}{\hat{e}_{\bar{t}-1}}\right)\right)+\delta_{t+1} & \bar{t} \text { odd And I\{DR Event }\}=1 \\
0 & \text { Otherwise }
\end{array}\right. \\
& \text { Where } \delta_{t+1}=\operatorname{sgn}\left(\tilde{c}_{t+1}-\frac{\hat{h}_{\bar{t}}}{\hat{e}_{\bar{t}}}-\left(\tilde{c}_{\bar{t}}-\frac{\hat{h}_{\bar{t}-1}}{\hat{e}_{\bar{t}-1}}\right)\right) \bar{t}^{-1 / 4} \tag{5.37}
\end{align*}
$$

Here we describe couple of aspects of the proposed pricing policy 5.37. First, the model parameter estimate, $\theta_{t}$, and quantile estimate, $F^{-1}(\alpha)$, are updated only at every other DR event. Second, to enforce sufficient price exploration, an offset is added to the myopic price at every other DR event. In the next section 5.4, we show that the combination of these two features is enough to ensure consistent parameter estimation and a sublinear growth rate for the $T$-period regret, which is bounded from above by $\sqrt{T}$.

### 5.4 Bound on Regret

We can show that for any pricing policy $\Pi$,

$$
\begin{equation*}
\Delta_{\Pi}(T) \propto \sum_{t=1}^{T} \mathbb{E}_{\Pi}\left(p_{t}-p_{t}^{*}\right)^{2} I\{\text { DR Event }\}+o(\sqrt{\bar{T}}) \tag{5.38}
\end{equation*}
$$

Under any pricing policy $\Pi$, It becomes apparent that (5.38), that is the rate at which regret grows is directly proportional to the rate at which pricing errors accumulate. We, therefore, proceed in deriving a bound on the rate at which the absolute pricing error $\left|p-p^{*}\right|$ converges to zero in probability, under the pricing policy (5.37).
We can show that the absolute pricing error incurred in period $t+1$ is upper bounded by,

$$
\begin{equation*}
\left|p_{t+1}-p_{t+1}^{*}\right| \leq k_{1}\left|\theta-\hat{\theta}_{\bar{t}}\right|+k_{2}\left|F_{\bar{t}}^{-1}(\alpha)-F^{-1}(\alpha)\right|+\left|\delta_{t+1}\right| \tag{5.39}
\end{equation*}
$$

The following Lemma establishes a bound on the rate at which the parameter estimates converges to the true model parameters in probability

Lemma 8. There exists finite positive constants $\mu_{2}$ and $\mu_{3}$ such that, under the pricing policy (5.37)

$$
\begin{equation*}
\mathbb{P}\left\{\left|\hat{\theta}_{\bar{t}}-\theta\right|_{1}>\gamma\right\} \leq 2 \exp \left(-\mu_{2} \gamma^{2}(\sqrt{\bar{t}}-1)\right)+2 \exp \left(-\mu_{3} \gamma^{2} \bar{t}\right) \tag{5.40}
\end{equation*}
$$

Proof of lemma is ignored. The above lemma is needed in the proof of the theorem we state below, which follows from the fact that the regret is direclty proportional to error in parameter estimation. Next we present the theorem that shows that the proposed policy is a no-regret policy,

Theorem 5. There exists finite constants $C_{0}, C_{1}, C_{2}$ and $C_{3}$ such that,

$$
\begin{equation*}
\Delta_{\Pi}(T) \leq C_{0}+C_{1} \sqrt{\bar{T}}+C_{2} \bar{T}^{1 / 4}+C_{3} \log (\bar{T}) \tag{5.41}
\end{equation*}
$$

Proof refer in appendix.
Remark 24. Note that $\bar{T} \rightarrow \infty$ as $T \rightarrow \infty$ a.s. So from Theorem 5 it follows that $\Delta_{\Pi}(T) \propto \sqrt{\bar{T}}$ as $T \rightarrow \infty$.
Remark 25. Then $\lim _{T \rightarrow \infty} \frac{\Delta_{\Pi}(T)}{\left(\sum_{i=1}^{T} \mathrm{I}\{\mathrm{DR} \text { Event }\}\right)}=\lim _{T \rightarrow \infty} \frac{\Delta_{\Pi}(T)}{T}=\lim _{T \rightarrow \infty} \frac{\sqrt{\bar{T}}}{T}=0$ a.s.

### 5.5 Conclusion

Here we propose a sub-linear regret policy and in particular show that the regret grows as $O(\sqrt{T})$. Also we showed that the proposed method of baseline assignment avoids any under-payment or over payment concerns that we highlighted in 2. Asymptotically the baseline estimate is not biased on either side of the true baseline and hence it avoids the possibility of under-payment and over-payment. However it is not clear if the proposed pricing policy is the optimal policy. For the risk-neutral cost setting it is established that $O(\sqrt{T})$ is the optimal regret. But an equivalent result for the risk-sensitive setting is not known. The similarity of the regret of the proposed policy to the optimal regret in the risk-neutral setting is encouraging but further research is required to establish the optimality in the risk-sensitive setting.

## Bibliography

[1] Mohamed H Albadi and EF El-Saadany. A summary of demand response in electricity markets. Electric power systems research, 78(11):1989-1996, 2008. $\uparrow 1$
[2] Omar Besbes and Assaf Zeevi. On the (surprising) sufficiency of linear models for dynamic pricing with demand learning. Management Science, 61(4):723-739, 2015.
[3] Severin Borenstein, Michael Jaske, and Arthurenfeld Ros. Dynamic pricing, advanced metering, and demand response in electricity markets. Journal of the American Chemical Society, 128(12):4136-45, 2002.
[4] James Bushnell, Benjamin F Hobbs, and Frank A Wolak. When it comes to demand response, is ferc its own worst enemy? The Electricity Journal, 22(8):9-18, 2009. $\uparrow 1, \uparrow 5$
[5] Hung Chao. Price-responsive demand management for a smart grid world. The Electricity Journal, 23(1):7-20, 2010. $\uparrow 2, \uparrow 5$
[6] Hung-po Chao. Demand response in wholesale electricity markets: the choice of customer baseline. Journal of Regulatory Economics, 39(1):68-88, 2011. $\uparrow 2$
[7] Hung Po Chao and Mario DePillis. Incentive effects of paying demand response in wholesale electricity markets. Journal of Regulatory Economics, 43(3):265-283, 2013. $\uparrow 2, \uparrow 5$
[8] Phani Chavali, Peng Yang, and Arye Nehorai. A distributed algorithm of appliance scheduling for home energy management system. Smart Grid, IEEE Transactions on, 5(1):282-290, 2014.
[9] Charalampos Chelmis, Muhammad Rizwan Saeed, Marc Frincu, and Viktor Prasanna. Curtailment estimation methods for demand response: Lessons learned by comparing apples to oranges. In Proceedings of the 2015 ACM Sixth International Conference on Future Energy Systems, pages 217-218. ACM, 2015. $\uparrow 2$
[10] Yan Chen, W Sabrina Lin, Feng Han, Yu-Han Yang, Zoltan Safar, and KJ Liu. A cheat-proof game theoretic demand response scheme for smart grids. In Communications (ICC), 2012 IEEE International Conference on, pages 3362-3366. IEEE, 2012. $\uparrow 5$
[11] Zhi Chen, Lei Wu, and Yong Fu. Real-time price-based demand response management for residential appliances via stochastic optimization and robust optimization. Smart grid, IEEE transactions on, 3(4):1822-1831, 2012. $\uparrow 17$
[12] Federal Energy Regulatory Commission. Demand response compensation in organized wholesale energy markets. Final Rule Report, 2011. $\uparrow 1$
[13] Federal Energy Regulatory Commission et al. Assessment of demand response and advanced metering. 2008. $\uparrow 16$
[14] Katie Coughlin, Mary Ann Piette, Charles Goldman, and Sila Kiliccote. Statistical analysis of baseline load models for non-residential buildings. Energy and Buildings, 41(4):374-381, 2009. $\uparrow 1, \uparrow 2, \uparrow 14$
[15] Arnoud V den Boer and Bert Zwart. Simultaneously learning and optimizing using controlled variance pricing. Management Science, 60(3):770-783, 2013. $\uparrow 56$
[16] Murat Fahrioĝlu and Fernando L Alvarado. Designing incentive compatible contracts for effective demand management. Power Systems, IEEE Transactions on, 15(4):1255-1260, 2000. $\uparrow 17$
[17] S Gabriel, Antonio J Conejo, Miguel A Plazas, and S Balakrishnan. Optimal price and quantity determination for retail electric power contracts. IEEE Transactions on Power Systems, 21(1):180-187, 2006. $\uparrow 17$
[18] Nikolaos Gatsis and Georgios B Giannakis. Residential load control: Distributed scheduling and convergence with lost ami messages. Smart Grid, IEEE Transactions on, $3(2): 770-786,2012$.
[19] Thomas W Gedra and Pravin P Varaiya. Markets and pricing for interruptible electric power. Power Systems, IEEE Transactions on, 8(1):122-128, 1993. $\uparrow 17$
[20] Tobias Haring, Johanna L Mathieu, and Goran Andersson. Decentralized contract design for demand response. In European Energy Market (EEM), 2013 10th International Conference on the, pages 1-8. IEEE, 2013. $\uparrow 17$
[21] Longbo Huang, Jean Walrand, and Kannan Ramchandran. Optimal power procurement and demand response with quality-of-usage guarantees. In Power and Energy Society General Meeting, 2012 IEEE, pages 1-8. IEEE, 2012. $\uparrow 17$
[22] Shweta Jain, Balakrishnan Narayanaswamy, and Y Narahari. A multiarmed bandit incentive mechanism for crowdsourcing demand response in smart grids. In AAAI, pages 721-727, 2014. $\uparrow 46$
[23] Liyan Jia, Lang Tong, and Qing Zhao. An online learning approach to dynamic pricing for demand response. arXiv preprint arXiv:1404.1325, 2014. $\uparrow 46$
[24] Dileep Kalathil and Ram Rajagopal. Online learning for demand response. 53rd Annual Allerton Conference on Communication, Control, and Computing, Sept 29-Oct 22015. $\uparrow 18$
[25] Dileep Kalathil and Ram Rajagopal. Online learning for demand response. In 2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton), pages 218-222. IEEE, 2015. $\uparrow 46$
[26] N Bora Keskin and Assaf Zeevi. Dynamic pricing with an unknown demand model: Asymptotically optimal semi-myopic policies. Operations Research, 62(5):1142-1167, 2014. $\uparrow 56$
[27] Taehoon Kim, Dongeun Lee, Jaesik Choi, Anna Spurlock, Alexander Sim, Annika Todd, and Kesheng Wu. Predicting baseline for analysis of electricity pricing. Available at SSRN 2773991, 2016. $\uparrow 2$
[28] Jean Jacques Laffont and David Martimort. The theory of incentives: the principal-agent model. Princeton university press, 2009. $\uparrow 27$
[29] TL Lai and Herbert Robbins. Iterated least squares in multiperiod control. Advances in Applied Mathematics, 3(1):50-73, 1982. $\uparrow 56$
[30] Na Li, Lijun Chen, Munther Dahleh, et al. Demand response using linear supply function bidding. $\uparrow 17$
[31] Na Li , Lijun Chen, and Steven H Low. Optimal demand response based on utility maximization in power networks. In Power and Energy Society General Meeting, 2011 IEEE, pages 1-8. IEEE, 2011. $\uparrow 17$
[32] Eric Maskin and John Riley. Monopoly with incomplete information. The RAND Journal of Economics, 15(2):171-196, 1984. $\uparrow 27$
[33] Johanna L Mathieu, Tobias Haring, John O Ledyard, and Goran Andersson. Residential demand response program design: Engineering and economic perspectives. In European Energy Market (EEM), 2013 10th International Conference on the, pages 1-8. IEEE, 2013. $\uparrow 1$
[34] Amir Hamed Mohsenian Rad, Vincent WS Wong, Juri Jatskevich, Robert Schober, and Alberto Leon-Garcia. Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid. Smart Grid, IEEE Transactions on, 1(3):320-331, 2010. $\uparrow 17$
[35] Deepan Muthirayan, Dileep Kalathil, Kameshwar Poolla, and Pravin P Varaiya. Mechanism design for self-reporting baselines in demand response. American Control Conference, 2016. $\uparrow 12, \uparrow 18$
[36] Duy Thanh Nguyen, Michael Negnevitsky, and Martin De Groot. Pool-based demand response exchange - concept and modeling. Power Systems, IEEE Transactions on, 26(3):1677-1685, 2011. $\uparrow 5$
[37] Duy Thanh Nguyen, Michael Negnevitsky, and Martin de Groot. Market-based demand response scheduling in a deregulated environment. Smart Grid, IEEE Transactions on, $4(4): 1948-1956,2013$. $\uparrow 5$
[38] Shmuel S Oren. Integrating real and financial options in demand-side electricity contracts. Decision Support Systems, 30(3):279-288, 2001. $\uparrow 17$
[39] Catherine D. Wolfram Paul L. Joskow. Dynamic pricing of electricity. The American Economic Review, 102(3):381-385, 2012. $\uparrow 1$
[40] Jim Pierobon. Two FERC settlements illustrate attempts to 'game' demand response programs, 2013. $\uparrow 2, \uparrow 4$
[41] Ram Rajagopal, Eilyan Bitar, Pravin Varaiya, and Felix Wu. Risk-limiting dispatch for integrating renewable power. International Journal of Electrical Power $\mathfrak{G}$ Energy Systems, 44(1):615-628, 2013. $\uparrow 17$
[42] Angel Ramos, Cedric De Jonghe, Daan Six, and Ronnie Belmans. Asymmetry of information and demand response incentives in energy markets. In European Energy Market (EEM), 2013 10th International Conference on the, pages 1-8. IEEE, 2013. $\uparrow 5$
[43] Andrew J Roscoe and G Ault. Supporting high penetrations of renewable generation via implementation of real-time electricity pricing and demand response. IET Renewable Power Generation, 4(4):369-382, 2010. $\uparrow 17$
[44] Pedram Samadi, Hamed Mohsenian-Rad, Vincent WS Wong, and Robert Schober. Realtime pricing for demand response based on stochastic approximation. Smart Grid, IEEE Transactions on, 5(2):789-798, 2014. $\uparrow 17$
[45] Wenbo Shi, Na Li, Xiaorong Xie, Chi Cheng Chu, and Rajit Gadh. Optimal residential demand response in distribution networks. Selected Areas in Communications, IEEE Journal on, 32(7):1441-1450, 2014.
[46] Joshua A Taylor and Johanna L Mathieu. Index policies for demand response. IEEE Transactions on Power Systems, 29(3):1287-1295, 2014. $\uparrow 46$
[47] Pravin P Varaiya, Felix F Wu, and Janusz W Bialek. Smart operation of smart grid: Risk-limiting dispatch. Proceedings of the IEEE, 99(1):40-57, 2011. $\uparrow 17$
[48] Qingsi Wang, Mingyan Liu, and Johanna L Mathieu. Adaptive demand response: Online learning of restless and controlled bandits. In Smart Grid Communications (SmartGridComm), 2014 IEEE International Conference on, pages 752-757. IEEE, 2014. $\uparrow 46$
[49] Frank A Wolak. Residential customer response to real-time pricing: The anaheim critical peak pricing experiment. Center for the Study of Energy Markets, 2007. $\uparrow 2, \uparrow 5$
[50] Chenye Wu, Hamed Mohsenian-Rad, and Jianwei Huang. Vehicle-to-aggregator interaction game. Smart Grid, IEEE Transactions on, 3(1):434-442, 2012. $\uparrow 17$
[51] Yunjian Xu , Na Li, and Steven H Low. Demand response with capacity constrained supply function bidding. $\uparrow 17$
[52] Peng Yang, Gongguo Tang, and Arye Nehorai. A game-theoretic approach for optimal time-of-use electricity pricing. Power Systems, IEEE Transactions on, 28(2):884-892, 2013. $\uparrow 17$

## Appendix A

## Appendix

## A. 1 Proof of Theorem 1

We first prove the following two propositions. Let

$$
\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=\arg \max _{\hat{b}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right) .
$$

Proposition 8. In SRB mechanism, under Assumption $1, \hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=b_{k}$ and is unique.
Proof. Each consumer's decision can be formulated as a two stage optimization problem. In the first stage, consumer $k$ reports $\hat{b}_{k}$ and $\hat{\pi}_{k}$. In the second stage, the consumption $q_{k}$ depends on whether consumer $k$ is selected or not. We first consider the second stage optimization problem.
Second Stage optimization: Let,

$$
\begin{equation*}
J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k}\right)=U_{k}\left(q_{k}\right)+R\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)-\Phi\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right) \tag{A.1}
\end{equation*}
$$

which is the net benefit of consumer $k$ in the second stage as a function of $q_{k}$, given the first stage reports $\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$. Note that the exact form of $J_{k}$ depends on whether consumer $k$ is selected for DR or not. So, we separately consider these two cases. Case: Consumer $k$ is selected for DR.
Let $q_{k}^{d r}$ be the optimal consumption of consumer $k$ when she is selected for DR. Formally,

$$
\begin{align*}
& q_{k}^{d r}=\arg \max _{q_{k}} J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right), \text { where }, \\
& \begin{array}{l}
J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\pi_{k} \min \left\{q_{k}, b_{k}\right\}-\pi^{e} q_{k} \\
\quad+\pi_{k}^{r}\left(\hat{b}_{k}-q_{k}\right) .
\end{array}
\end{align*}
$$

Of course, $q_{k}^{d r}$ will depend on the first stage reports $\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$. In particular, there are two possible values for $q_{k}^{d r}$.

Sub-Case $\pi_{k}^{r} \geq \pi_{k}-\pi^{e}$ : It is straightforward to show that $q_{k}^{d r}=0$. Then, substituting for $q_{k}^{d r}$ back in (A.2) we get,

$$
\begin{equation*}
J_{k}\left(q_{k}^{d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\pi_{k}^{r} \hat{b}_{k} \tag{A.3}
\end{equation*}
$$

Sub-Case: $\pi_{k}^{r}<\pi_{k}-\pi^{e}$
It is again straightforward to show that $q_{k}^{d r}=b_{k}$. Then we get,

$$
\begin{equation*}
J_{k}\left(q_{k}^{d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\left(\pi_{k}-\pi^{e}\right) b_{k}+\pi_{k}^{r}\left(\hat{b}_{k}-b_{k}\right) \tag{A.4}
\end{equation*}
$$

Case Consumer $k$ is not selected for DR: Let $q_{k}^{n d r}$ be the optimal consumption of consumer $k$ when she is not selected for DR. Formally,

$$
\begin{align*}
& q_{k}^{n d r}=\arg \max _{q_{k}} J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right), \text { where, } \\
& \begin{array}{c}
J_{k}\left(q_{k} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right)=\pi_{k} \min \left\{q_{k}, b_{k}\right\} \\
\quad-\pi^{e} q_{k}-\hat{\pi}_{k}\left|q_{k}-\hat{b}_{k}\right| .
\end{array}
\end{align*}
$$

We make the following observation: From the assumption $\pi_{k}-\pi^{e}>0$ it follows that $\hat{\pi}_{k}>\pi^{e}$. Similar to Case 1, we consider different sub-cases on the first stage reports ( $\hat{b}_{k}, \hat{\pi}_{k}$ ).
Sub-case $\hat{\pi}_{k} \geq \pi_{k}-\pi^{e}$ : Using $\hat{\pi}_{k}>\pi^{e}$, we get $q_{k}^{n d r}=\hat{b}_{k}$ and

$$
\begin{equation*}
J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\pi_{k} \min \left\{\hat{b}_{k}, b_{k}\right\}-\pi^{e} \hat{b}_{k} \tag{A.6}
\end{equation*}
$$

Sub-case $\hat{\pi}_{k}<\pi_{k}-\pi^{e}$ and $\hat{b}_{k} \leq b_{k}$ : we get $q_{k}^{n d r}=b_{k}$ and

$$
\begin{equation*}
J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\left(\pi_{k}-\pi^{e}\right) b_{k}+\hat{\pi}_{k}\left(\hat{b}_{k}-b_{k}\right) \tag{A.7}
\end{equation*}
$$

Sub-case $\hat{\pi}_{k}<\pi_{k}-\pi^{e}$ and $\hat{b}_{k}>b_{k}$ : Using $\hat{\pi}_{k}>\pi^{e}$, we get $q_{k}^{n d r}=\hat{b}_{k}$ and

$$
\begin{equation*}
J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right)=\pi_{k} b_{k}-\pi^{e} \hat{b}_{k} \tag{A.8}
\end{equation*}
$$

This completes the characterization of the second-stage decisions $q_{k}^{d r}$ and $q_{k}^{n d r}$ of consumer $k$. First stage optimization: Let $\rrbracket_{k}(D)$ denote the indicator function of selection of consumer $k$. Then,

$$
\mathbb{\square}_{k}(D)=\left\{\begin{array}{cc}
0 & \text { if } k \text { is not selected }  \tag{A.9}\\
1 & \text { if } k \text { is selected }
\end{array}\right.
$$

By the selection process (2.5), consumer $k$ will not be selected up to a $D_{k}$ where $D_{k}=\sum_{m=1}^{k-1} \hat{b}_{m}$ and will always be selected for $D>D_{k}$ i.e.,

$$
\mathbb{\square}_{k}(D)= \begin{cases}0 & \text { if } D \leq D_{k}  \tag{A.10}\\ 1 & \text { if } D>D_{k}\end{cases}
$$

And it follows that,

$$
\begin{equation*}
\frac{\partial D_{k}}{\partial \hat{b}_{k}}=0 \tag{A.11}
\end{equation*}
$$

The expected benefit consumer $k$ receives in the first stage is given by,

$$
\begin{align*}
& \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)=\mathbb{E}\left[\square\{k \text { is selected }\} J_{k}(. \mid k \text { is selected })\right] \\
& \left.+\mathbb{E}[\square\{k \text { is not selected }\}) J_{k}(. \mid k \text { is not selected })\right] \tag{A.12}
\end{align*}
$$

Recall that $\mathbb{P}(D>0)$ is the probability that a $D R$ event occurs. From (A.10) we get

$$
\begin{align*}
& \left.\mathbb{E}[0\{k \text { is selected }\}) J_{k}(. \mid k \text { is not selected })\right] \\
& =\mathbb{E}\left[\square_{k}(D) J_{k}(. \mid k \text { is selected })\right] \\
& =\mathbb{P}(D>0) \mathbb{E}\left[\square_{k}(D) J_{k}(. \mid k \text { is selected }) \mid D>0\right] \tag{A.13}
\end{align*}
$$

And similarly,

$$
\begin{align*}
& \left.\mathbb{E}[0\{k \text { is not selected }\}) J_{k}(. \mid k \text { is not selected })\right] \\
& =\mathbb{P}(D>0) \mathbb{E}\left[\left(1-\rrbracket_{k}(D)\right) J_{k}(. \mid k \text { is not selected }) \mid D>0\right] \\
& +(1-\mathbb{P}(D>0)) J_{k}(. \mid k \text { is not selected }) \tag{A.14}
\end{align*}
$$

Using (A.14) and (A.13) we can express $\bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$ as,

$$
\begin{align*}
& \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)=\mathbb{P}(D>0) \mathbb{E}\left[\square_{k}(D) J_{k}(. \mid k \text { is selected }) \mid D>0\right] \\
& +\mathbb{P}(D>0) \mathbb{E}\left[\left(1-\square_{k}(D)\right) J_{k}(. \mid k \text { is not selected }) \mid D>0\right] \\
& +(1-\mathbb{P}(D>0)) J_{k}(. \mid k \text { is not selected }) \tag{A.15}
\end{align*}
$$

Also let, $\mathbb{P}\left(D \leq D^{\prime} \mid D>0\right)=\int_{0+}^{D^{\prime}} f(D)$. Then using (A.10) and $\alpha=\mathbb{P}(D>0)$ we get,

$$
\begin{align*}
& \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)=\alpha \int_{D_{k}}^{\bar{D}} J_{k}\left(q_{k}^{d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right) f(D) \\
& +\alpha \int_{0+}^{D_{k}} J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right) f(D) \\
& +(1-\alpha) J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right) \tag{A.16}
\end{align*}
$$

Define,

$$
\begin{equation*}
\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=\arg \max _{\hat{b}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right) \tag{A.17}
\end{equation*}
$$

Where, $\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)$ is the optimal report of baseline consumption that maximizes the expected net utility of consumer $k$. Next we solve this optimization problem to derive consumer's optimal
report $\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)$ corresponding to the marginal utility report $\hat{\pi}_{k}$. Differentiating $\bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)$ w.r.t $\hat{b}_{k}$ and using (A.11) we get,

$$
\begin{align*}
& \frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \frac{\partial J_{k}(. \mid k \text { is selected })}{\partial \hat{b}_{k}} f(D) \\
& +\alpha \int_{0+}^{D_{k}} \frac{d J_{k}(. \mid k \text { is not selected })}{\partial \hat{b}_{k}} f(D) \\
& +(1-\alpha) \frac{\partial J_{k}(. \mid k \text { is not selected })}{\partial \hat{b}_{k}} \tag{A.18}
\end{align*}
$$

Using the fact that $\pi_{k}^{r}$ is independent of consumer $k$ 's reports, it follows from (A.3) and (A.4) that $\frac{\partial J_{k}(\cdot \mid k \text { is selected })}{\partial \hat{b}_{k}}=\pi_{k}^{r}$. Substituting for $\frac{\partial J_{k}(\cdot \mid k \text { is selected })}{\partial \hat{b}_{k}}$ in (A.18) we get,

$$
\begin{align*}
& \frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi_{k}^{r} f(D) \\
& +\alpha \int_{0+}^{D_{k}} \frac{\partial J_{k}(. \mid k \text { is not selected })}{\partial \hat{b}_{k}} f(D) \\
& +(1-\alpha) \frac{\partial J_{k}(. \mid k \text { is not selected })}{\partial \hat{b}_{k}} \tag{A.19}
\end{align*}
$$

To complete the analysis we consider the following two cases, (i) when $\hat{\pi}_{k} \geq \pi_{k}-\pi^{e}$ (ii) when $\hat{\pi}_{k}<\pi_{k}-\pi^{e}$. In each case we show that $\frac{\partial \bar{J}_{k}}{\partial \hat{b}_{k}}>0$ when $\hat{b}_{k} \leq b_{k}$ and $\frac{\partial \bar{J}_{k}}{\partial \hat{b}_{k}}<0$ when $\hat{b}_{k}>b_{k}$ which establishes that $\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=b_{k}$ is the unique maximizer.
Case $\hat{\pi}_{k} \geq \pi_{k}-\pi^{e}$ : We consider the following two sub-cases (i) $\hat{b}_{k} \leq b_{k}$ and (ii) $\hat{b}_{k}>b_{k}$. Sub-case Consumer reports a baseline that is less than her true baseline i.e. $\hat{b}_{k} \leq b_{k}$ : Using (A.6) in (A.19) we get,

$$
\begin{align*}
& \frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi_{k}^{r} f(D)+\alpha \int_{0+}^{D_{k}}\left(\pi_{k}-\pi^{e}\right) f(D) \\
& +(1-\alpha)\left(\pi_{k}-\pi^{e}\right) \tag{A.20}
\end{align*}
$$

From (2.6) we get $\pi_{k}^{r} \geq \hat{\pi}_{k}-\pi^{e}$. Since $\hat{\pi}_{k}>\pi^{e}$ it follows that $\pi_{k}^{r} \geq \hat{\pi}_{k}-\pi^{e}>0$. This implies $\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}>0$ when $\hat{b}_{k} \leq b_{k}$. Sub-case The consumer reports a baseline that is greater than her true baseline i.e. $\hat{b}_{k}>b_{k}$ : Using (A.6) in (A.19) we get,

$$
\begin{align*}
& \frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi_{k}^{r} f(D)+\alpha \int_{0+}^{D_{k}}-\pi^{e} f(D)-(1-\alpha) \pi^{e} \\
& =\int_{D_{k}}^{\bar{D}}\left(\alpha \pi_{k}^{r}-(1-\alpha) \pi^{e}\right) f(D)-\int_{0+}^{D_{k}} \pi^{e} f(D) \tag{A.21}
\end{align*}
$$

Note that consumers cannot report a value more than $\pi_{\max }$ which implies $\pi_{k}^{r} \leq \pi_{\max }-\pi^{e}$. Then from assumption 1 it follows that $\alpha \pi_{k}^{r}-(1-\alpha) \pi^{e}<0$. And from (A.21) it follows that $\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}<0$ when $\hat{b}_{k}>b_{k}$.
Case $\hat{\pi}_{k}<\pi_{k}-\pi^{e}$ : As before we have two sub-cases.
Sub-case Consumer reports a baseline that is less than her true baseline i.e. $\hat{b}_{k} \leq b_{k}$ : Using (A.7) in (A.19) we get,

$$
\begin{equation*}
\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi_{k}^{r} f(D)+\alpha \int_{0+}^{D_{k}} \hat{\pi}_{k} f(D)+(1-\alpha) \hat{\pi}_{k} \tag{A.22}
\end{equation*}
$$

Since $\hat{\pi}_{k}>\pi^{e}>0$ and $\pi_{k}^{r} \geq \hat{\pi}_{k}-\pi^{e}>0$ it follows that $\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}>0$ when $\hat{b}_{k} \leq b_{k}$.
Sub-case The consumer reports a baseline that is greater than her true baseline i.e. $\hat{b}_{k}>b_{k}$ : Using (A.8) in (A.19) we get,

$$
\begin{align*}
& \frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi_{k}^{r} f(D)+\alpha \int_{0+}^{D_{k}}-\pi^{e} f(D) \\
& -(1-\alpha) \pi^{e} \\
& =\int_{D_{k}}^{D}\left(\alpha \pi_{k}^{r}-(1-\alpha) \pi^{e}\right) f(D)-\int_{0+}^{D_{k}} \pi^{e} f(D) \tag{A.23}
\end{align*}
$$

This expression is exactly the same as (A.21) and it follows that $\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)}{\partial \hat{b}_{k}}<0$.
The two cases together imply $\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=b_{k}$ is the unique maximizer for any given $\hat{\pi}_{k}$.
Let

$$
\hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)=\arg \max _{\hat{\pi}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)
$$

Proposition 9. In the SRB mechanism, under Assumption 1, $\hat{\pi}_{k} \in\left\{\pi \mid \bar{J}_{k}\left(\hat{b}_{k}=b_{k}, \pi\right)=\right.$ $\bar{J}_{k}\left(\hat{b}_{k}=b_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}=b_{k}\right)\right\}$.

Proof. Here we show that reporting marginal utility truthfully maximizes consumer's utility when $\hat{b}_{k}=b_{k}$. Let $J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)$ be the ex-post benefit of consumer $k$ when it reports $\left(\hat{\pi}_{k}, \hat{b}_{k}\right)$ and when the realized value of load reduction requirement is $D$. Define, $\hat{S}\left(D ; \hat{\pi}_{k}\right)$ to be the set of consumers who would be selected (according to (2.5)) when consumer $k$ reports $\hat{\pi}_{k}$ and $\hat{S}_{-k}(D)$ to be the set of consumers who would be selected when consumer $k$ is excluded. From (2.6) the payment for unit reduction when consumer $k$ gets selected is then given by,

$$
\begin{equation*}
\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right)=\max \left\{\hat{\pi}_{j}\right\}-\pi^{e}, j \in \hat{S}_{-k}(D) \tag{A.24}
\end{equation*}
$$

To show that reporting marginal utility truthfully maximizes consumer's utility when $\hat{b}_{k}=b_{k}$, we consider the following two cases; (i) consumer $k$ gets selected on reporting truthfully (ii) consumer $k$ does not get selected on reporting truthfully. And show that in either of the scenarios consumer $k$ does not gain in terms of the net benefit by deviating from reporting her marginal utility truthfully.
Case 1: consumer $k \notin \hat{S}\left(D ; \pi_{k}\right)$ :
Here, on reporting truthfully consumer $k$ will not be selected. Hence by (A.6) consumer $k$ consumes $q_{k}=b_{k}$ and her ex-post benefit is given by $J_{k}\left(q_{k}=b_{k} ; \pi_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\left(\pi_{k}-\pi^{e}\right) b_{k}$. We then compare the net benefit of consumer $k$ on over-reporting and under-reporting her marginal utility from that of reporting truthfully.
Over Reporting, $\hat{\pi}_{k}>\pi_{k}$ :
By the selection process (2.5) consumer $k$ will not be selected on over reporting her marginal utility. Hence by (A.6) $J_{k}\left(q_{k}=b_{k} ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\left(\pi_{k}-\pi^{e}\right) b_{k}$ and consumer $k$ is indifferent to over reporting in this case.
Under Reporting, $\hat{\pi}_{k}<\pi_{k}$ :
Let consumer $k$ under report such that consumer $k$ gets selected. Because $k \notin \hat{S}\left(D ; \pi_{k}\right)$, $\hat{S}\left(D ; \pi_{k}\right)=\hat{S}_{-k}\left(D ; \pi_{k}\right)$. Note that $\hat{S}_{-k}$ is not dependent on the report $\hat{\pi}_{k}$. Hence $\hat{S}\left(D ; \pi_{k}\right)=$ $\hat{S}_{-k}\left(D ; \hat{\pi}_{k}\right)$. And because $k \notin \hat{S}\left(D ; \pi_{k}\right), \pi_{k}>\max \left\{\hat{\pi}_{j} \mid j \in \hat{S}\left(D ; \pi_{k}\right)\right\}$ i.e. $\pi_{k}>\max \left\{\hat{\pi}_{j} \mid j \in\right.$ $\left.\hat{S}_{-k}\left(D ; \hat{\pi}_{k}\right)\right\}$. Then from (A.24) it follows that, $\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right)<\pi_{k}-\pi^{e}$. By (A.4) consumer $k$ consumes $q_{k}=b_{k}$ and her ex-post benefit is given by $J_{k}\left(q_{k}=b_{k} ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\left(\pi_{k}-\pi^{e}\right) b_{k}$. Hence consumer $k$ is indifferent to under reporting in this case.
For this case we have established that consumer $k$ is indifferent to deviating from reporting her marginal utility truthfully.
Case 2: consumer $k \in \hat{S}\left(D ; \pi_{k}\right)$ :
Similarly for this case we show that consumer $k$ does not gain by over-reporting or underreporting her marginal utility. Since $k \in \hat{S}\left(D ; \pi_{k}\right), \pi_{k}^{r}\left(D ; \pi_{k}\right) \geq \pi_{k}-\pi^{e}$. By (A.3), consumer $k$ consumes $q_{k}=0$. Her ex-post benefit is given by, $J_{k}\left(q_{k}=0 ; \pi_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\pi_{k}^{r}\left(D ; \pi_{k}\right) b_{k}$ Over Reporting, $\hat{\pi}_{k}>\pi_{k}$ :
On over reporting if the consumer gets selected then $\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right)=\pi_{k}^{r}\left(D ; \pi_{k}\right) \geq \pi_{k}-\pi^{e}$. By (A.3) consumption $q_{k}=0$ and $J_{k}\left(q_{k}=0 ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right) b_{k}=\pi_{k}^{r}\left(D ; \pi_{k}\right) b_{k}=$ $J_{k}\left(q_{k}=0 ; \pi_{k}, \hat{b}_{k}=b_{k} \mid D\right)$. Hence consumer $k$ 's ex-post benefit remains same if it gets selected on over reporting. On the other hand if the consumer does not get selected, then by (A.6) $J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\left(\pi_{k}-\pi^{e}\right) b_{k} \leq \pi_{k}^{r}\left(D ; \pi_{k}\right) b_{k}=J_{k}\left(q_{k}=0 ; \pi_{k}, \hat{b}_{k}=b_{k} \mid D\right)$. And the consumer can strictly loose but never gain in terms of ex-post benefit. Hence over-reporting is a loss to the consumer in this case.
Under Reporting, $\hat{\pi}_{k}<\pi_{k}$ :
Here, on under reporting consumer $k$ will always be selected and so $\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right)=\pi_{k}^{r}\left(D ; \pi_{k}\right) \geq$ $\pi_{k}-\pi^{e}$. By (A.3) consumption $q_{k}=0$ and $J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k}=b_{k} \mid D\right)=\pi_{k}^{r}\left(D ; \hat{\pi}_{k}\right) b_{k}=\pi_{k}^{r}\left(D ; \pi_{k}\right) b_{k}=$ $J_{k}\left(q_{k} ; \pi_{k}, \hat{b}_{k}=b_{k} \mid D\right)$ and so it does not gain by under-reporting.
Both cases together imply that reporting marginal utility maximizes consumer's utility when $\hat{b}_{k}=b_{k}$. And it follows from the proof that this maximizer may not be unique. Hence
$\hat{\pi}_{k} \in\left\{\pi \mid \bar{J}_{k}\left(\hat{b}_{k}=b_{k}, \pi\right)=\bar{J}_{k}\left(\hat{b}_{k}=b_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)\right\}\right.$
We now complete the proof of Theorem 1
Proof. For any given $\hat{b}_{k}$ and $\hat{\pi}_{k}, \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right) \leq \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)\right)$. From proposition 8 it follows that, $\bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)\right) \leq \bar{J}_{k}\left(b_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)\right)$. And from proposition 9 it follows that $\bar{J}_{k}\left(b_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}\right)\right) \leq$ $\bar{J}_{k}\left(b_{k}, \pi_{k}\right)$. Hence $\overline{\bar{J}}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right) \leq \bar{J}_{k}\left(b_{k}, \pi_{k}\right)$. This implies that reporting the baseline and marginal utility truthfully is a dominant strategy, i.e. $\left(\hat{b}_{k}^{*}, \hat{\pi}_{k}^{*}\right)=\left(b_{k}, \pi_{k}\right)$. When consumer is not selected, it follows from $\hat{\pi}_{k}=\pi_{k}, \hat{b}_{k}=b_{k}$ and (A.5) that $q_{k}^{n d r}=b_{k}$, which gives assertion (ii). When consumer $k$ is selected, $\hat{\pi}_{k}=\pi_{k}$ implies that $\pi_{k}^{r}>\pi_{k}-\pi^{e}$. Then from (A.2) it follows that $q_{k}^{d r}=0$, which gives assertion (iii)

## A. 2 Proof of Theorem 2

We first prove the following two propositions. Let

$$
\hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=\arg \max _{\hat{b}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right) .
$$

Definition 9. $\bar{D}=$ Maximum possible load reduction requirement
Definition 10. $F(D)=$ Cumulative distribution function of load reduction requirement $D$
Assumption 8. $b_{m}=\max _{k} b_{k} \ll 1$
Assumption 9. $F^{\prime \prime} \geq-\frac{1}{\bar{D} b_{m}}$
Proposition 10. Under Assumption 2, with the condition that $\pi_{\max }<(1-\alpha) \min \left\{\pi_{k}-\right.$ $\left.\pi^{e}, \pi^{e}\right\} / \alpha \forall k, \hat{b}_{k}^{*}\left(\hat{\pi}_{k}\right)=b_{k}$.

Proof: The consumer's problem is a two-stage optimization problem as specified by (4.4). Here we compute the optimal solution for an arbitrary consumer $k$ using dynamic programming approach. First we solve the second stage decision variable $q_{k}$ followed by the first stage decision variable $\hat{b}_{k}$ assuming $\hat{\pi}_{k}$ is fixed.

Second stage optimization: Let

$$
\begin{equation*}
J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k}\right)=U_{k}\left(q_{k}\right)+R\left(q_{k}, \hat{b}_{k}, \hat{\pi}_{k}\right)-\Phi\left(q_{k}, f_{k}, \hat{\pi}_{k}\right) \tag{A.25}
\end{equation*}
$$

which is the net benefit of consumer $k$ in the second stage as a function of $q_{k}$. The second stage decision $q_{k}$ depends on whether consumer $k$ is selected or not. Hence we consider the following cases.

Case 1: Consumer $k$ is selected for DR.
Let $q_{k}^{d r}$ be the optimal consumption of consumer $k$ when she is selected for DR. Formally,

$$
q_{k}^{d r}=\arg \max _{q_{k}} J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is selected }\right), \text { where }
$$

$J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k\right.$ is selected $)=\pi_{k}^{u} \min \left\{q_{k}, q_{m, k}\right\}-\pi^{e} q_{k}+\pi^{r}\left(D ; \hat{b}_{k}\right)\left(\hat{b}_{k}-q_{k}\right)_{+}-\pi_{\max }\left(q_{k}-\hat{b}_{k}\right)_{+}$
Note the dependency of $\pi^{r}\left(D ; \hat{b}_{k}\right)$ on $\hat{b}_{k}$. This is because the reward for unit reduction, which is $\pi^{r}($.$) decreases with \hat{b}_{k}$ for a particular realization of $D$. Hence the reward for unit reduction depends on $\hat{b}_{k}$ as well
When, $\pi^{r}\left(D ; \hat{b}_{k}\right) \geq \pi_{k}^{u}-\pi^{e}$, it is easy to show that $q_{k}^{d r}=0$. Also we get,

$$
\begin{equation*}
J_{k}\left(q_{k}^{d r} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is selected }\right)=\pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k} \tag{A.26}
\end{equation*}
$$

When $\pi^{r}\left(D ; \hat{b}_{k}\right)<\pi_{k}^{u}-\pi^{e}, q_{k}^{d r}=\hat{b}_{k}$ and we get,

$$
\begin{equation*}
J_{k}\left(q_{k}^{d r} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is selected }\right)=\left(\pi_{k}^{u}-\pi^{e}\right) \min \left\{\hat{b}_{k}, q_{m, k}\right\}+\pi^{r}\left(D ; \hat{b}_{k}\right)\left(\max \left\{\hat{b}_{k}, q_{m, k}\right\}-q_{m, k}\right) \tag{A.27}
\end{equation*}
$$

Case 2: When consumer $k$ is not selected for DR:
Let $q_{k}^{n d r}$ be the optimal consumption of consumer $k$ when she is not selected for DR. Formally,

$$
\begin{gathered}
q_{k}^{n d r}=\arg \max _{q_{k}} J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is not selected }\right), \text { where, } \\
J_{k}\left(q_{k} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is not selected }\right)=\pi_{k}^{u} \min \left\{q_{k}, q_{m, k}\right\}-\pi^{e} q_{k}-\pi_{\max }\left|q_{k}-\hat{b}_{k}\right| .
\end{gathered}
$$

It then follows that,

$$
\begin{equation*}
J_{k}\left(q_{k}^{n d r} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is not selected }\right)=\pi_{k}^{u} \min \left\{\hat{b}_{k}, q_{m, k}\right\}-\pi^{e} \hat{b}_{k} \tag{A.28}
\end{equation*}
$$

First stage optimization: The baseline report of consumer $k$ is the solution of the first stage optimization problem,

$$
\hat{b}_{k}^{*} \in \arg \max _{\hat{b}_{k}} \bar{J}_{k}\left(\hat{b}_{k}\right)
$$

Where $\bar{J}_{k}\left(\hat{b}_{k}\right)$ is the expected net benefit of consumer $k$ and is given by,

$$
\begin{align*}
& \bar{J}_{k}\left(\hat{b}_{k}\right)=\mathbb{E}\left[\square\{k \text { is selected }\} J_{k}\left(q_{k}^{d r} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is selected }\right)\right] \\
& +\mathbb{E}\left[\square\{k \text { is not selected }\} J_{k}\left(q_{k}^{n d r} ; \hat{\pi}_{k}, \hat{b}_{k} \mid k \text { is not selected }\right)\right] \tag{A.29}
\end{align*}
$$

Also, let $D_{k}$ be the smallest realization of $D$ for which consumer $k$ gets selected. Then from the selection rule (2.5) it is clear that consumer $k$ will get selected for all realizations of $D \geq D_{k}$ up to the maximum, $\bar{D}$. We consider the following cases to complete the analysis.
$\underline{\text { Case } \hat{\pi}_{k} \geq \pi_{k}^{u}-\pi^{e}:}$

In this case when consumer $k$ is selected $\pi^{r}\left(D ; \hat{b}_{k}\right) \geq \pi_{k}^{u}-\pi^{e}$. Hence from (A.26) and (A.28) we get,

$$
\begin{align*}
& \bar{J}_{k}\left(\hat{b}_{k}\right)=\alpha \int_{0}^{\bar{D}} \mathbb{1}\{k \text { is selected }\} \pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k} f(D)+ \\
& \alpha \int_{0}^{\bar{D}} \mathbb{1}\{k \text { is not selected }\}\left(\pi_{k}^{u} \min \left\{q_{m, k}, \hat{b}_{k}\right\}-\pi^{e} \hat{b}_{k}\right) f(D)+(1-\alpha)\left(\pi_{k}^{u} \min \left\{q_{m, k}, \hat{b}_{k}\right\}-\pi^{e} \hat{b}_{k}\right) \tag{A.30}
\end{align*}
$$

Using the observation that consumer $k$ will get selected for all realizations of $D \geq D_{k}$ and will not be selected for any realization of $D<D_{k}$.

$$
\begin{align*}
\bar{J}_{k}\left(\hat{b}_{k}\right) & =\alpha \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k} f(D)+\alpha \int_{0}^{D_{k}}\left(\pi_{k}^{u} \min \left\{q_{m, k}, \hat{b}_{k}\right\}-\pi^{e} \hat{b}_{k}\right) f(D) \\
& +(1-\alpha)\left(\pi_{k}^{u} \min \left\{q_{m, k}, \hat{b}_{k}\right\}-\pi^{e} \hat{b}_{k}\right) \tag{A.31}
\end{align*}
$$

When $\hat{b}_{k}<q_{m, k}$ : Differentiating $\bar{J}_{k}\left(\hat{b}_{k}\right)$ w.r.t $\hat{b}_{k}$ we get,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}}=\alpha \frac{d}{d \hat{b}_{k}} \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k} f(D)+\alpha \int_{0}^{D_{k}}\left(\pi_{k}^{u}-\pi^{e}\right) f(D)+(1-\alpha)\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.32}
\end{equation*}
$$

Ignoring the second term in A. 32 we get,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}}=\alpha \frac{d}{d \hat{b}_{k}} \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k} f(D)+(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.33}
\end{equation*}
$$

Using product rule for differentiation,

$$
\begin{align*}
& \frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) f(D)+\alpha \hat{b}_{k} \frac{\partial}{\partial \hat{b}_{k}} \sum_{j=k}^{m} \hat{\pi}_{j+1}\left(F\left(D_{k}+. .+\hat{b}_{j}\right)-F\left(D_{k}+\ldots+\hat{b}_{j-1}\right)\right) \\
& +(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.34}
\end{align*}
$$

This implies

$$
\begin{align*}
& \frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}}=\alpha \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) f(D)+\alpha \hat{b}_{k} \sum_{j=k}^{m} \hat{\pi}_{j+1}\left(f\left(D_{k}+. .+\hat{b}_{j}\right)-f\left(D_{k}+. .+\hat{b}_{j-1}\right)\right) \\
& +(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.35}
\end{align*}
$$

Lets assume for the time being, $f^{\prime \prime} \leq 0 \forall D \geq D_{k}$. Then the following holds,

$$
f\left(D_{k}+. .+\hat{b}_{j}\right) \leq f\left(D_{k}+\ldots . .+\hat{b}_{j-1}\right)
$$

From $\hat{\pi}_{k} \leq \pi_{\text {max }}$ it follows that,

$$
\begin{align*}
& \sum_{j=k}^{m} \hat{\pi}_{j+1}\left(f\left(D_{k}+. .+\hat{b}_{j}\right)-f\left(D_{k}+\ldots+\hat{b}_{j-1}\right)\right) \\
& \geq \pi_{\max } \sum_{j=k}^{m}\left(f\left(D_{k}+. .+\hat{b}_{j}\right)-f\left(D_{k}+\ldots+\hat{b}_{j-1}\right)\right) \tag{A.36}
\end{align*}
$$

Substituting the above inequality in (A.35) gives,

$$
\begin{align*}
& \frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}} \geq \alpha \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) f(D)+\alpha \pi_{m a x} \hat{b}_{k} \sum_{j=k}^{m}\left(f\left(D_{k}+. .+\hat{b}_{j}\right)-f\left(D_{k}+\ldots+\hat{b}_{j-1}\right)\right) \\
& +(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.37}
\end{align*}
$$

Hence,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}} \geq \alpha \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) f(D)+\alpha \pi_{\max } \hat{b}_{k} f(\bar{D})+(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right) \tag{A.38}
\end{equation*}
$$

From assumption 9 it follows that $f(\bar{D}) \geq-\frac{\bar{D}}{\bar{D} b_{m}}$. This implies,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}} \geq(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right)+\alpha \pi_{\max } \hat{b}_{k} f(\bar{D}) \tag{A.39}
\end{equation*}
$$

And in this case $\hat{b}_{k} \leq b_{k} \leq b_{m}$. So it follows that,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}} \geq(1-\bar{\alpha})\left(\pi_{k}^{u}-\pi^{e}\right)-\alpha \pi_{\max } \geq(1-\alpha)\left(\pi_{k}^{u}-\pi^{e}\right)-\alpha \pi_{\max } \tag{A.40}
\end{equation*}
$$

Then from the condition $\pi_{\max }<(1-\alpha) \min \left\{\pi_{k}^{u}-\pi^{e}, \pi^{e}\right\} / \alpha$ it follows that,

$$
\begin{equation*}
\frac{d \bar{J}_{k}\left(\hat{b}_{k}\right)}{d \hat{b}_{k}} \geq(1-\alpha)\left(\pi_{k}^{u}-\pi^{e}\right)-\alpha \pi_{\max }>0 \tag{A.41}
\end{equation*}
$$

This implies $\hat{b}_{k}^{*} \geq b_{k}=q_{m, k}$.
When $\hat{b}_{k} \geq q_{m, k}$ :

$$
\begin{equation*}
\bar{J}_{k}\left(\hat{b}_{k}\right)=\frac{\alpha}{\bar{D}} \int_{D_{k}}^{\bar{D}} \pi^{r}\left(D ; \hat{b}_{k}\right) \hat{b}_{k}+(1-\bar{\alpha})\left(\pi_{k}^{u} q_{m, k}-\pi^{e} \hat{b}_{k}\right) \tag{A.42}
\end{equation*}
$$

Differentiating $\bar{J}_{k}\left(\hat{b}_{k}\right)$ w.r.t $\hat{b}_{k}$ and using the observation that $\pi^{r}\left(D ; \hat{b}_{k}\right)$ is decreasing in $\hat{b}_{k}$ for a given $D$ gives us

$$
\begin{equation*}
\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}\right)}{\partial \hat{b}_{k}} \leq \alpha \bar{\pi}\left(\hat{b}_{k}\right)-(1-\bar{\alpha}) \pi^{e} \quad \text { Where } \bar{\pi}\left(\hat{b}_{k}\right)=\frac{1}{\bar{D}} \int_{D_{k}}^{\bar{D}} \pi\left(D ; \hat{b}_{k}\right)<\pi_{\max } \tag{A.43}
\end{equation*}
$$

Then using the condition $\pi_{\max }<(1-\alpha) \min \left\{\pi_{k}^{u}-\pi^{e}, \pi^{e}\right\} / \alpha$ in (A.43) we get,

$$
\begin{equation*}
\frac{\partial \bar{J}_{k}\left(\hat{b}_{k}\right)}{\partial \hat{b}_{k}}<0 \Rightarrow \hat{b}_{k}^{*} \leq q_{m, k} \tag{A.44}
\end{equation*}
$$

Before we showed that $\hat{b}_{k}^{*} \geq q_{m, k}$ and so $\hat{b}_{k}^{*}=q_{m, k}$ when $\tilde{\pi}_{k} \geq \pi_{k}^{u}-\pi^{e}$.

Case $\hat{\pi}_{k} \geq \pi_{k}^{u}-\pi^{e}$ is similar.

Both cases together imply $\hat{b}_{k}^{*}=q_{m, k}$.
Denote the set of other consumers' marginal utility report by $\hat{\pi}_{-k}$. Let

$$
\hat{\pi}_{k}^{*}\left(\hat{b}_{k} ; \hat{\pi}_{-k}\right)=\arg \max _{\hat{\pi}_{k}} \bar{J}_{k}\left(\hat{b}_{k}, \hat{\pi}_{k}\right)
$$

Proposition 11. In SRBM-UP, under Assumption 2, $\hat{\pi}_{k} \in\left\{\pi \mid \bar{J}_{k}\left(\hat{b}_{k}=b_{k}, \pi\right)=\bar{J}_{k}\left(\hat{b}_{k}=\right.\right.$ $\left.b_{k}, \hat{\pi}_{k}^{*}\left(\hat{b}_{k}=b_{k} ; \hat{\pi}_{-k}=\pi_{-k}\right)\right\}$.

Let $J_{k}\left(D ; \hat{\pi}_{k}, \hat{b}_{k}\right)$ be the net benefit of consumer $k$ when it reports $\left(\hat{\pi}_{k}, \hat{b}_{k}\right)$ as the marginal utility and baseline consumption respectively and when the realized value of load reduction requirement is $D$. So,

$$
\begin{aligned}
& J_{k}\left(D ; \hat{\pi}_{k}, \hat{b}_{k}\right)=I\{k \text { is selected } \mid D\} J_{k}\left(q_{k}^{d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is selected }\right) \\
& +I\{k \text { is not selected } \mid D\} J_{k}\left(q_{k}^{n d r} ; \hat{b}_{k}, \hat{\pi}_{k} \mid k \text { is not selected }\right)
\end{aligned}
$$

Now, using the results of Proposition 10 i.e. $\hat{b}_{k}=q_{m, k}$, we get

$$
\begin{aligned}
& J_{k}\left(D ; \hat{\pi}_{k}, \hat{b}_{k}=q_{m, k}\right)=I\{k \text { is selected } \mid D\}\left(\pi_{k}-\pi^{e}\right) q_{k}^{d r}+I\{k \text { is selected } \mid D\} \pi^{r}(D)\left(\hat{b}_{k}-q_{k}^{d r}\right) \\
& \quad+I\{k \text { is not selected } \mid D\}\left(\pi_{k}-\pi^{e}\right) \hat{b}_{k}
\end{aligned}
$$

Consider consumer $k$ who reports $\hat{\pi}_{k}$. Suppose that all other consumers report their true marginal utility, i.e., $\hat{\pi}_{j}=\pi_{j}, \forall j, j \neq k$. Let $\pi^{r}\left(D ; \hat{\pi}_{k}\right)$ be value of $\pi^{r}(D)$ as a function of $\hat{\pi}_{k}$. From our mechanism and under Assumption 2, it is clear that $\pi\left(D ; \hat{\pi}_{k}\right)$ is a non-decreasing function of $\hat{\pi}_{k}$ for a given $D$. Below we consider the only two possible cases for $\pi\left(D ; \hat{\pi}_{k}\right)$ and show that for either of the cases consumer $k$ does not gain by deviating from reporting
truthfully.
Case 1: $\left(\pi_{k}-\pi^{e}\right) \geq \pi^{r}\left(D ; \hat{\pi}_{k}=\pi_{k}\right)$ : In this case, consumer $k$ will not be selected if it reports truthfully and its net benefit on reporting truthfully is $\left(\pi_{k}-\pi^{e}\right) \hat{b}_{k}$. If consumer $k$ over-reports, i.e., $\hat{\pi}_{k}>\pi_{k}$, it will not be selected. And it's net benefit will remain the same as the truthful reporting case.
The only way to change the net benefit is to get selected, which is possible only by underreporting. If consumer $k$ under-reports and gets selected, it's net benefit will be either $\pi^{r}\left(D ; \hat{\pi}_{k}\right) \hat{b}_{k}$ or $\left(\pi_{k}-\pi^{e}\right) \hat{b}_{k}$. However,

$$
\pi^{r}\left(D ; \hat{\pi}_{k}\right) \leq \pi^{r}\left(D ; \pi_{k}\right) \leq\left(\pi_{k}-\pi^{e}\right)
$$

The first inequality is by the non-decreasing property of $\pi^{r}(D ; \cdot)$. The second inequality follows from this case. Hence the consumer is at most indifferent to any deviation from reporting truthfully.
Case 2: $\left(\pi_{k}-\pi^{e}\right)<\pi^{r}\left(D ; \hat{\pi}_{k}=\pi_{k}\right)$ :
By the assumption of this case consumer $k$ will be selected on reporting truthfully and its net benefit on reporting truthfully is $\pi\left(D ; \hat{\pi}_{k}=\pi_{k}\right) \hat{b}_{k}$. If consumer $k$ deviates by under-reporting i.e. $\hat{\pi}_{k} \leq \pi_{k}$, she will still be selected. Also, $k^{*}(D)$ will be the same. So, $\pi^{r}\left(D ; \pi_{k}\right)=\pi^{r}\left(D ; \hat{\pi}_{k}\right)$ and the consumer's net benefit will not change.
The only way to increase the net benefit is to over-report such that $\left.\pi^{r}\left(D ; \hat{\pi}_{k}\right)>\pi^{( } D ; \pi_{k}\right)$. However, consumer $k^{*}(D)+1$ will be selected before consumer $k$. Excluding consumer $k$ and including consumer $k^{*}+1$ will still satisfy $D$ since $q_{m, k^{*}+1}-q_{m, k}>0$ by Assumption 2. So, in effect, consumer $k$ will not be selected if it over-reports to make $\pi^{r}\left(D ; \hat{\pi}_{k}\right)>\pi^{r}\left(D ; \pi_{k}\right)$. This will lead to a decrease in its net benefit.
Combining Case 1 and Case 2, we can conclude that truthful reporting of marginal utility is a best response for consumer $k$, if all others are reporting their marginal utility truthfully. Since $k$ is arbitrary, truthful reporting is a Nash equilibrium for our mechanism. Rest of the theorem follows from here.

We now complete the proof of Theorem 2. This step is similar to the proof of Theorem 1

## A.2.1 Proof of Theorem 3

We give a rough outline of the proof here.
(i) For all $\pi<\pi_{\max }-\pi^{e}$, we can construct a set of consumers such that when a particular consumer $j$ is selected $\pi<\pi_{j}-\pi^{e}$. So when the reward per unit reduction $\pi$ is such that $\pi<\pi_{\max }-\pi^{e}$ one cannot guarantee that all selected consumers would reduce
(ii) For a particular load reduction requirement $D$, even if CAISO's selection is as efficient as in SRBM, each selected consumer is paid $\pi$ per unit of reduction where $\pi \geq \pi_{\max }-\pi^{e} \geq \pi_{k}-\pi^{e}$ for all $k$. Hence the total payment made by CAISO would be greater than that made in SRBM.
(iii) When the consumers are informed of the $D R$ event several hours prior to the $D R$ event:

This is done in scenarios where the participating consumers need sufficient time to prepare for the DR event. We model the average benefit of consumer $k$ on the DR event day as the mean of the utility during the DR event $J_{k}^{d r}$ and the utility during the hours prior to the DR event $J_{k}^{-1}$. Then,

$$
\begin{align*}
& J_{k}(. \mid k \text { is selected })=\left(J_{k}^{-1}+J_{k}^{d r}\right) / 2 \\
& =\left(u_{k}\left(q_{k}^{-}\right)-\pi^{e} q_{k}^{-}+u_{k}\left(q_{k}\right)-\pi^{e} q_{k}+R\left(q_{k}, b_{k}^{c}, \pi\right)\right) / 2 \\
& =\left(u_{k}\left(q_{k}^{-}\right)-\pi^{e} q_{k}^{-}+u_{k}\left(q_{k}\right)-\pi^{e} q_{k}+\pi\left(b_{k}^{c}-q_{k}\right)\right) / 2 \tag{A.45}
\end{align*}
$$

Here we compute the consumption decisions of the consumer $q^{-}$prior to the DR event hour on the DR event day and use it to derive a lower bound on the customer baseline. On the DR event day, $\mathbf{b}_{m}$ and $\mathbf{b}_{m}^{-}$are constants so differentiating the average utility $J_{k}($.$) in (A.45)$ w.r.t $q_{k}^{-}$gives,

$$
\begin{align*}
& \frac{\partial J_{k}(.)}{\partial q_{k}^{-}}=\left(\frac{\partial u_{k}\left(q_{k}^{-}\right)}{\partial q_{k}^{-}}-\pi^{e}+\pi \frac{\partial b_{k}^{c}}{\partial q_{k}^{-}}\right) / 2 \\
& =\left\{\begin{array}{cc}
\pi_{k}-\pi^{e}+\pi \frac{\bar{b}_{k}}{\bar{b}_{k}^{-}} & \text {If } q_{k}^{-} \leq b_{k} \\
-\pi^{e}+\pi \overline{\bar{b}}_{k}^{-} & q_{k}^{-}>b_{k}
\end{array}\right. \\
& \geq \frac{\partial J_{k}(.)}{\partial q_{k}^{-}} \geq\left\{\begin{array}{cc}
\pi_{k}-\pi^{e}+\pi & \text { If } q_{k}^{-} \leq b_{k} \\
-\pi^{e}+\pi & q_{k}^{-}>b_{k}
\end{array} \quad \text { since } \bar{b}_{k}>\bar{b}_{k}^{-}\right. \tag{A.46}
\end{align*}
$$

Note that $\frac{\partial J_{k}(.)}{\partial q_{k}^{-}}>0$ for all values of $q_{k}^{-}$when $\pi>\pi^{e}$. This implies $q_{k}^{-} \rightarrow \infty$ when $\pi>\pi^{e}$. This implies that

$$
\begin{equation*}
b_{k}^{c}=\bar{b}_{k} C_{b}=\bar{b}_{k} q_{k}^{-} / \bar{b}_{k}^{-}>q_{k}^{-} \rightarrow \infty \text { when } \pi>\pi^{e} \tag{A.47}
\end{equation*}
$$

## A.2.2 Proof of proposition 1

Proof. We have

$$
\begin{equation*}
J_{n d r}^{e}(q)=\pi^{d a} q+\int_{s=0}^{1} \int_{w=0}^{(l-q)} \bar{\pi}_{s}^{r t}(l-q-w) p_{s}(w) d w \alpha(s) d s \tag{A.48}
\end{equation*}
$$

By taking partial derivative we get

$$
\begin{aligned}
\frac{d J_{n d r}^{e}}{d q} & =\pi^{d a}-\int_{s=0}^{1} \bar{\pi}_{s}^{r t} P_{s}(l-q) \alpha(s) d s \\
\frac{d^{2} J_{n d r}^{e}}{d q^{2}} & =\int_{s=0}^{1} \bar{\pi}_{s}^{r t} p_{s}(l-q) \alpha(s) d s
\end{aligned}
$$

$J_{n d r}^{e}$ is convex since the second derivative is positive. Hence $J_{n d r}^{e}$ is strictly convex. Solution is obtained by equating the first derivative to zero and uniqueness follows from strict convexity.

## A.2.3 Proof of proposition 2

Proof. We have

$$
J_{s}^{e}\left(y_{s}\right)=\phi\left(y_{s}\right)+\int_{w=0}^{\left(l-q-y_{s}\right)} \bar{\pi}_{s}^{r t}\left(l-q-w-y_{s}\right) p_{s}(w) d w
$$

Taking partial derivatives

$$
\begin{aligned}
\frac{\partial J_{s}^{e}}{\partial y_{s}} & =\phi^{\prime}\left(y_{s}\right)-\bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}\right) \\
\frac{\partial^{2} J_{s}^{e}}{\partial y_{s}^{2}} & =\phi^{\prime \prime}(y)+\bar{\pi}_{s}^{r t} p_{s}\left(l-q-y_{s}\right)
\end{aligned}
$$

Second derivative is positive. This implies $J_{s}^{e}\left(y_{s}\right)$ is strictly convex. Using first derivative the optimal load curtailment $y_{s}^{e}$ is given by,

$$
y_{s}^{e}=0 \text { if } \phi^{\prime}(0)>\bar{\pi}_{s}^{r t} P_{s}(l-q)
$$

else the optimal load curtailment $y_{s}^{e}$ is given by equating the first order condition to zero i.e.,

$$
\frac{\partial J_{s}^{e}}{\partial y_{s}}=\phi^{\prime}\left(y_{s}\right)-\left.\bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}\right)\right|_{y_{s}=y_{s}^{e}}=0
$$

By strict convexity it follows that the optimal load curtailment decision is unique. Also, when $\phi^{\prime}(0)>\bar{\pi}_{s}^{r t} P_{s}(l-q), \frac{\partial y_{s}^{e}}{\partial q}=0$. Otherwise from (3.10), by taking partial derivative w.r.t. $q$ we get,

$$
\phi^{\prime \prime}\left(y_{s}^{e}\right) \frac{\partial y_{s}^{e}}{\partial q}=-\bar{\pi}_{s}^{r t} p_{s}\left(l-q-y_{s}^{e}\right)\left(1+\frac{\partial y_{s}^{e}}{\partial q}\right)
$$

From this it can be deduced that $-1<\frac{\partial y_{s}}{\partial q} \leq 0$. Taking partial derivatives of $J^{e}$ w.r.t $q$ and using the optimality condition for $y_{s}^{e}$ gives

$$
\begin{aligned}
\frac{d J^{e}}{d q} & =\pi^{d a}-\int_{s=0}^{1} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{e}\right) \alpha(s) d s \\
\frac{d^{2} J^{e}}{d q^{2}} & =\int_{s=0}^{1} \bar{\pi}_{s}^{r t} p_{s}\left(l-q-y_{s}^{e}\right)\left(1+\frac{\partial y_{s}^{e}}{\partial q}\right)
\end{aligned}
$$

Second derivative is positive since $\frac{\partial y_{s}}{\partial q}<-1$. Hence we get the optimality condition (3.11), by equating the first derivative to zero.

## A.2.4 Proof of Theorem 4

Proof. The unique solution of (3.15) is given by,

$$
y_{s}^{a g g}\left(\pi_{s}^{i n}\right)=\left\{\begin{array}{cc}
0 & \text { if } \phi^{\prime}(0)>\pi_{s}^{i n} \\
\left(\phi^{\prime}\right)^{-1}\left(\pi_{s}^{i n}\right) & \text { otherwise }
\end{array}\right.
$$

Now, for a given $q, y_{s}^{l s e}$ is unique and is given by,

$$
y_{s}^{l s e}\left(\pi_{s}^{i n}\right)=\left\{\begin{array}{cc}
0 & \text { if } P_{s}(l-q)<\left(\pi_{s}^{i n} / \bar{\pi}_{s}^{r t}\right) \\
l-q-P_{s}^{-1}\left(\pi_{s}^{i n} / \bar{\pi}_{s}^{r t}\right) & \text { otherwise }
\end{array}\right.
$$

Proof for the above is similar to that of Proposition 2 and hence we skip the details. $y_{s}^{a g g}\left(\pi_{s}^{i n}\right)$ is a continuous and increasing function of $\pi_{s}^{i n} . y_{s}^{l s e}\left(\pi_{s}^{i n} ; q\right)$ is a continuous and decreasing function of $\pi_{s}^{i n}$. So, for any given $q$, there exists a $\pi_{s}^{i n}(q)$ such that $y_{s}^{a g g}\left(\pi_{s}^{i n}(q)\right)=y_{s}^{l s e}\left(\pi_{s}^{i n}(q) ; q\right)$. Similar to proposition 2 the minimizer $q^{l s e}$ satisfies the first order condition,

$$
\pi^{d a}-\int_{s=0}^{1} \bar{\pi}_{s}^{r t} P_{s}\left(l-q^{l s e}-y_{s}^{l s e}\right) \alpha(s) d s=0
$$

Note that $J^{l s e}$ is not strictly convex (which was not the case in 2 ). Choose $\pi_{s}^{* i n}=\pi_{s}^{\text {in }}\left(q^{e}\right)$ as contingent prices. Then uniqueness of second-stage purchase implies $y_{s}^{* l s e}=y_{s}^{* a g g}=y_{s}^{e}$. As a result, $q=q^{e}$ satisfies the optimality condition and is one of the minimizers. So we can set, $q^{* l s e}=q^{e}$. From Proposition 2 we know that this decision is socially optimal. Hence by convexity equilibria are socially optimal.

## A. 3 Proof of Proposition 5

Step 1: Replace the set of I.R. constraints (3.22) by a single constraint (A.49) : For any arbitrary $\theta$,

$$
\begin{aligned}
r_{s}(\theta)-\phi\left(y_{s}(\theta), \theta\right) & \geq r_{s}\left(\theta_{\max }\right)-\phi\left(y_{s}\left(\theta_{\max }\right), \theta\right) \\
& \geq r_{s}\left(\theta_{\max }\right)-\phi\left(y_{s}\left(\theta_{\max }\right), \theta_{\max }\right) \geq 0
\end{aligned}
$$

The first inequality is from the I.C. constraint (3.23). The second inequality is from Assumption 5 and the fact that $\theta_{\max } \geq \theta$. Third inequality is from the I.R. constraint (3.22). So, we can replace the I.R. constraints (3.22) by

$$
\begin{equation*}
r_{s}\left(\theta_{\max }\right)-\phi\left(y_{s}\left(\theta_{\max }\right), \theta_{\max }\right) \geq 0, \quad \forall s \in \mathcal{S} \tag{A.49}
\end{equation*}
$$

Step 2: Replace the set of I.C. constraints by a monotonicity constraint and a first order constraint:

First note that the net utility of the aggregator of type $\bar{\theta}$ by accepting a contract of type $\theta$ is

$$
R(\theta ; \bar{\theta})=r_{s}(\theta)-\phi\left(y_{s}(\theta), \bar{\theta}\right)
$$

Considering $R(\theta ; \bar{\theta})$ as a function of $\theta$, I.C. constraints (3.23) imply that $\theta=\bar{\theta}$ is a global optimum for $R(\theta ; \bar{\theta})$. Below we show that it is sufficient to consider two simple first order conditions for verifying these global optimality.

The first and second order conditions on $R(\theta ; \bar{\theta})$ are,

$$
\begin{aligned}
& r_{s}^{\prime}(\theta)-\phi_{y}\left(y_{s}(\theta), \bar{\theta}\right) y_{s}^{\prime}(\theta)=0 \\
& r_{s}^{\prime \prime}(\theta)-\phi_{y y}\left(y_{s}(\theta), \bar{\theta}\right)\left(y_{s}^{\prime}(\theta)\right)^{2}-\phi_{y}\left(y_{s}(\theta), \bar{\theta}\right) y_{s}^{\prime \prime}(\theta) \leq 0
\end{aligned}
$$

Due to the global maximum at $\theta=\bar{\theta}$, from the above conditions we get,

$$
\begin{aligned}
& r_{s}^{\prime}(\bar{\theta})-\phi_{y}\left(y_{s}(\bar{\theta}), \bar{\theta}\right) y_{s}^{\prime}(\bar{\theta})=0 \\
& r_{s}^{\prime \prime}(\bar{\theta})-\phi_{y y}\left(y_{s}(\bar{\theta}), \bar{\theta}\right)\left(y_{s}^{\prime}(\bar{\theta})\right)^{2}-\phi_{y}\left(y_{s}(\bar{\theta}), \bar{\theta}\right) y_{s}^{\prime \prime}(\bar{\theta}) \leq 0
\end{aligned}
$$

But, since the LSE doesn't know the true parameter $\bar{\theta}$, the above conditions should be true for all $\theta \in \Theta$, i.e.,

$$
\begin{array}{ll}
r_{s}^{\prime}(\theta)-\phi_{y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime}(\theta)=0 \\
r_{s}^{\prime \prime}(\theta)-\phi_{y y}\left(y_{s}(\theta), \theta\right)\left(y_{s}^{\prime}(\theta)\right)^{2}-\phi_{y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime \prime}(\theta) \quad \leq 0 \tag{A.51}
\end{array}
$$

Differentiating the first order condition above (A.50), we get

$$
\begin{align*}
r_{s}^{\prime \prime}(\theta) & -\phi_{y y}\left(y_{s}(\theta), \theta\right)\left(y_{s}^{\prime}(\theta)\right)^{2}-\phi_{y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime \prime}(\theta) \\
& -\phi_{\theta y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime}(\theta)=0 \tag{A.52}
\end{align*}
$$

Now, from (A.51) and (A.52) and using the assumption that $\phi_{\theta y}>0$, we get the following monotonicity condition.

$$
\begin{equation*}
y_{s}^{\prime}(\theta) \leq 0 \tag{A.53}
\end{equation*}
$$

Note that the above monotonicity condition is quite intuitive that is less load curtailment from costly type. Now, we show that the (local) conditions (A.50) and (A.53) are sufficient to guarantee the global optimality of $R(\theta: \bar{\theta})$ at $\theta=\bar{\theta}$.
Let $\theta>\bar{\theta}$. Then, from Assumption $5, \phi_{y}\left(y_{s}(\theta), \bar{\theta}\right)<\phi_{y}\left(y_{s}(\theta), \theta\right)$. Using the monotonicity condition (A.53) and then using the first-order condition (A.50),

$$
\phi_{y}\left(y_{s}(\theta), \bar{\theta}\right) y_{s}^{\prime}(\theta) \geq \phi_{y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime}(\theta)=r_{s}^{\prime}(\theta)
$$

Integrating on both sides from $\bar{\theta}$ to $\tilde{\theta}$

$$
\phi\left(y_{s}(\tilde{\theta}), \bar{\theta}\right)-\phi\left(y_{s}(\bar{\theta}), \bar{\theta}\right) \geq r_{s}(\tilde{\theta})-r_{s}(\bar{\theta})
$$

which is the same as the I.C. constraints.
Step 3: Modified Optimization Problem:
Using Step 1 and Step 2, we can rewrite the original optimization problem as, for every $s \in S$,

$$
\begin{align*}
\min _{r_{s}(\theta), y_{s}(\theta)} & \mathbb{E}_{\theta}\left[r(\theta)+V_{s}\left(y_{s}(\theta) ; q\right)\right]  \tag{A.54}\\
\text { s.t. I.R. } & r_{s}\left(\theta_{\max }\right)-\phi\left(y_{s}\left(\theta_{\max }\right), \theta_{\max }\right) \geq 0  \tag{A.55}\\
\text { I.C.1 } & y_{s}^{\prime}(\theta) \leq 0  \tag{A.56}\\
\text { I.C.2 } & r_{s}^{\prime}(\theta)-\phi^{\prime}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime}(\theta)=0 \tag{A.57}
\end{align*}
$$

Step 4: Simplify the Optimization Problem:
It is easy to see that the I.R. constraint will be binding. If not, the LSE can decrease $r_{s}\left(\theta_{\max }\right)$ until it is binding and thus minimize its cost. Recall that we defined $U_{s}(\theta)=r_{s}(\theta)-\phi\left(y_{s}(\theta), \theta\right)$. Also, since I.R. constraint is binding, $U_{s}\left(\theta_{\max }\right)=0$. By differentiating $U_{s}(\theta)$, we get,

$$
\begin{aligned}
U_{s}^{\prime}(\theta) & =r_{s}^{\prime}(\theta)-\phi_{y}\left(y_{s}(\theta), \theta\right) y_{s}^{\prime}(\theta)-\phi_{\theta}\left(y_{s}(\theta), \theta\right) \\
& =-\phi_{\theta}\left(y_{s}(\theta), \theta\right)
\end{aligned}
$$

So, we can replace I.C. 2 by the above condition. Then the optimization problem will be

$$
\begin{array}{cl}
\min _{y_{s}(\theta)} & \mathbb{E}_{\theta}\left[U_{s}(\theta)+\phi\left(y_{s}(\theta), \theta\right)+V_{s}\left(y_{s}(\theta) ; q\right)\right] \\
\text { I.C. } 1 & y_{s}^{\prime}(\theta) \leq 0 \\
\text { I.C. } 2 & U_{s}^{\prime}(\theta)=-\phi_{\theta}\left(y_{s}(\theta), \theta\right) \tag{A.60}
\end{array}
$$

From I.C. 2, by integrating,

$$
U_{s}(\theta)=\int_{\theta}^{\theta_{\max }} \phi_{z}\left(y_{s}(z), z\right) d z
$$

where we used the fact that $U_{s}\left(\theta_{\max }\right)=0$. Now,

$$
\begin{aligned}
\mathbb{E}_{\theta}\left[U_{s}(\theta)\right]= & \int_{\theta \in \Theta} U_{s}(\theta) f(\theta) d \theta \\
= & \left(U_{s}\left(\theta_{\max }\right) F\left(\theta_{\max }\right)-U_{s}\left(\theta_{\min }\right) F\left(\theta_{\min }\right)\right) \\
& -\int_{\theta \in \Theta} U_{s}^{\prime}(\theta) F(\theta) d \theta
\end{aligned}
$$

Since $U_{s}\left(\theta_{\max }\right)=0, F\left(\theta_{\min }\right)=0$, we get,

$$
\begin{aligned}
\mathbb{E}_{\theta}\left[U_{s}(\theta)\right] & =-\int_{\theta \in \Theta} U_{s}^{\prime}(\theta) F(\theta) d \theta \\
& =\int_{\theta \in \Theta} \phi_{\theta}\left(y_{s}(\theta), \theta\right) F(\theta) d \theta
\end{aligned}
$$

So, optimization problem is

$$
\begin{aligned}
\min _{y_{s}(\theta)} & \int_{\theta \in \Theta} \phi_{\theta}\left(y_{s}(\theta), \theta\right) F(\theta) d \theta \\
& +\int_{\theta \in \Theta}\left(\phi\left(y_{s}(\theta), \theta\right)+V_{s}\left(y_{s}(\theta) ; q\right)\right) f(\theta) d \theta \\
& \text { s.t. } y_{s}^{\prime}(\theta) \leq 0
\end{aligned}
$$

Taking derivative of the objective w.r.t., $y_{s}(\theta)$ and equating to zero we get the condition that $y_{s}^{*}(\theta)$ should satisfy,

$$
\begin{equation*}
V_{s}^{\prime}\left(y_{s}^{*}(\theta) ; q\right)=-\phi_{y}\left(y_{s}^{*}(\theta), \theta\right)-\frac{F(\theta)}{f(\theta)} \phi_{y \theta}\left(y_{s}^{*}(\theta), \theta\right) \tag{A.61}
\end{equation*}
$$

Next we check whether $\frac{y_{s}^{*}(\theta)}{d \theta}<0$. Consider the expression

$$
\psi(y, \theta)=-\phi_{y}(y, \theta)-\frac{F(\theta)}{f(\theta)} \phi_{y \theta}(y, \theta)
$$

From assumption (5) $\frac{\partial \psi(y, \theta)}{\partial \theta}>0$. Which implies that for a fixed $y, \psi(y, \theta)$ increases with $\theta$. On the other hand, $V_{s}^{\prime}(y)$ does not vary with $\theta$. Hence from (A.61) $\frac{y_{s}^{*}(\theta)}{d \theta} \leq 0$

## A. 4 Proof of Proposition 6

The expected cost $\tilde{J}^{l s e}()$ is given by,

$$
\begin{align*}
& \tilde{J}^{l s e}(q)=\pi^{d a} q+\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} r_{s}^{*}(\theta) \\
& +\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} \bar{\pi}_{s}^{r t} \int_{w=0}^{l-q-y_{s}^{*}(\theta)}\left(l-q-y_{s}^{*}(\theta)-w\right) p_{s}(w) d w \tag{A.62}
\end{align*}
$$

From proposition (5), we can rewrite $\tilde{J}^{l s e}()$ by,

$$
\begin{align*}
& \tilde{J}^{l s e}(q)=\pi^{d a} q+\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} U_{s}(\theta) \\
& +\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} \phi\left(y_{s}^{*}(\theta), \theta\right)+V_{s}\left(y_{s}(\theta) ; q\right) \tag{A.63}
\end{align*}
$$

Differentiating $\tilde{J}^{l s e}(q)$ w.r.t $q$ we get,

$$
\begin{align*}
\frac{d \tilde{J}^{l s e}}{d q}(q) & =\pi^{d a}-\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} \frac{\partial V_{s}\left(y_{s}(\theta) ; q\right)}{\partial q} \\
& =\pi^{d a}-\int_{s=0}^{1} \int_{\theta=\theta_{\min }}^{\theta_{\max }} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{*}(\theta)\right) \tag{A.64}
\end{align*}
$$

Which can be rewritten as,

$$
\begin{equation*}
\frac{d \tilde{J}^{l s e}(q)}{d q}=\pi^{d a}-\mathbb{E}_{s} \mathbb{E}_{\theta} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{*}(\theta)\right) \tag{A.65}
\end{equation*}
$$

From proposition (5), we know that $y_{s}^{*}(\theta)$ satisfies,

$$
V_{s}^{\prime}\left(y_{s}^{*}(\theta) ; q\right)+\phi_{y}\left(y_{s}^{*}(\theta), \theta\right)=-\frac{F(\theta)}{f(\theta)} \phi_{y \theta}\left(y_{s}^{*}(\theta), \theta\right)
$$

Differentiating w.r.t $q$ we get,

$$
\begin{align*}
& \left(\phi_{y y}\left(y_{s}^{*}(\theta), \theta\right)+\frac{F(\theta)}{f(\theta)} \phi_{y y \theta}\left(y_{s}^{*}(\theta), \theta\right)\right) \frac{\partial y_{s}^{*}(\theta)}{\partial q} \\
& =-\bar{\pi}_{s}^{r t} p_{s}\left(l-q-y_{s}^{*}(\theta)\right)\left(1+\frac{\partial y_{s}^{*}(\theta)}{\partial q}\right) \tag{A.66}
\end{align*}
$$

Then the condition $\phi_{y y \theta}\left(y_{s}^{*}(\theta), \theta\right)>0$ implies that $-1<\frac{\partial y_{s}^{*}(\theta)}{\partial q}<0$. Now differentiating $\frac{d \tilde{\mathcal{I}}^{l s e}(q)}{d q}$ w.r.t $q$ we get,

$$
\begin{equation*}
\frac{d^{2} \tilde{J}^{l s e}(q)}{d q^{2}}=\mathbb{E}_{s} \mathbb{E}_{\theta} \bar{\pi}_{s}^{r t} p_{s}\left(l-q-y_{s}^{*}(\theta)\right)\left(1+\frac{\partial y_{s}^{*}(\theta)}{\partial q}\right)>0 \tag{A.67}
\end{equation*}
$$

This implies that $\tilde{J}^{l s e}(q)$ is strictly convex. And the unique minimizer is given by equating the first derivative (A.65) to zero i.e. $\tilde{q}^{l s e}$ satisfies,

$$
\pi^{d a}-\mathbb{E}_{s} \mathbb{E}_{\theta} \bar{\pi}_{s}^{r t} P_{s}\left(l-q-y_{s}^{*}(\theta)\right)=0
$$

## A. 5 Consumption and Cost Sensitivities

Note that the optimal consumption decision $q^{b}(\hat{b}, \theta)$ depends on $\hat{b}$. Of course $q^{a}(\theta)$ and $q^{c}(\theta)$ do not depend on $\hat{b}$. Here we calculate the sensitivity of the realized cost $J^{b}$ and $J^{c}$ with respect to $\hat{b}$. Again, we hold $\theta$ fixed. Define

$$
\begin{equation*}
\zeta(\hat{b}, \theta)=\frac{d q^{b}(\hat{b}, \theta)}{d \hat{b}} \tag{A.68}
\end{equation*}
$$

The sensitivity of optimal cost $J^{b}\left(\hat{b}, q^{b}, \theta\right)$ with respect to $\hat{b}$ is given by,

$$
\begin{align*}
\frac{d J^{b}\left(\hat{b}, q^{b}, \theta\right)}{d \hat{b}}= & \pi^{e} \zeta(\hat{b}, \theta)-\frac{\partial u\left(q^{b}, \theta\right)}{\partial q} \zeta(\hat{b}, \theta) \\
& -\phi^{\prime}\left(\hat{b}-q^{b}\right)(\zeta(\hat{b}, \theta)-1) \tag{A.69}
\end{align*}
$$

Then, taking into account that $q^{b}(\hat{b}, \theta)$ satisfies (4.6), we get

$$
\begin{equation*}
\frac{d J^{b}\left(\hat{b}, q^{b}, \theta\right)}{d \hat{b}}=\phi^{\prime}\left(\hat{b}-q^{b}\right) \tag{A.70}
\end{equation*}
$$

Define $\beta(\theta)=\frac{d q^{c}(\theta)}{d \hat{b}}$. The optimal cost $J^{c}\left(\hat{b}, q^{c}, \theta\right)$ depends on $f \hat{b}$ and its sensitivity with respect to $\hat{b}$ is given by

$$
\frac{d J^{c}\left(\hat{b}, q^{c}, \theta\right)}{d \hat{b}}=\pi_{0} \beta(\theta)-\frac{\partial u\left(q^{c}, \theta\right)}{\partial q} \beta(\theta)-\pi^{r}(1-\beta(\theta))
$$

As before, $q^{c}(\theta)$ satisfies (4.7) and we get

$$
\begin{equation*}
\frac{d J^{c}\left(\hat{b}, q^{c}, \theta\right)}{d \hat{b}}=-\pi_{2}=\pi_{0}-\frac{\partial u\left(q^{c}, \theta\right)}{\partial q} \tag{A.71}
\end{equation*}
$$

In the next section we derive the condition that the optimal forecast $\hat{b}^{*}$ has to satisfy and use it to derive an expression for $\hat{b}^{*}$. We then use the inflation in baseline to characterize efficiency of the proposed mechanism.

## A. 6 Proof of Lemma 3

First we show that $0 \leq \alpha(\hat{b}, \theta)<1$, where $\alpha$ is the cost sensitivity defined in (A.68). The optimal consumption $q^{b}(\hat{b}, \theta)$ satisfies (4.6). Holding $\theta$ fixed and differentiating (4.6) further we get,

$$
\begin{equation*}
\left(\phi^{\prime \prime}(\hat{b}-q)-\frac{\partial^{2} u(q, \theta)}{\partial q^{2}}\right) \frac{d q^{b}(\hat{b}, \theta)}{d \hat{b}}-\phi^{\prime \prime}(\hat{b}-q)=0 \tag{A.72}
\end{equation*}
$$

Convexity of $\phi$ and strict convexity of $-u$ implies the existence of $\frac{d q b}{d \hat{b}}$ and is given by:

$$
\alpha(f, \theta)=\left(\phi^{\prime \prime}\left(\hat{b}-q^{b}\right)-\frac{\partial^{2} u\left(q^{b}, \theta\right)}{\partial q^{2}}\right)^{-1} \phi^{\prime \prime}\left(\hat{b}-q^{b}\right) .
$$

and satisfies $0 \leq \alpha<1$.
Next we differentiate $H(f)$ twice to show that $H^{\prime \prime}(f)>0$. Differentiating $H(f)$ we get

$$
\begin{align*}
\bar{J}^{\prime}(\hat{b}) & =(1-\alpha) \mathbb{E}_{\theta} \frac{d J^{b}\left(\hat{b}, q^{b}, \theta\right)}{d \hat{b}}+\alpha \mathbb{E}_{\theta} \frac{d J^{c}\left(\hat{b}, q^{c}, \theta\right)}{d \hat{b}} \\
& =(1-\alpha) \mathbb{E}_{\theta} \phi^{\prime}\left(\hat{b}-q^{b}\right)-\alpha \pi^{r} \tag{А.73}
\end{align*}
$$

Differentiating once again we get

$$
\begin{equation*}
\bar{J}^{\prime \prime}(\hat{b})=(1-\alpha) \mathbb{E}_{\theta}(1-\alpha(\hat{b}, \theta)) \phi^{\prime \prime}\left(\hat{b}-q^{b}\right) \tag{A.74}
\end{equation*}
$$

Before we showed that $(1-\alpha(\hat{b}, \theta))>0$. Then, it follows that $\bar{J}(\hat{b})$ is (strictly) convex if and only if $\phi$ is (strictly) convex.

## A. 7 Proof of Lemma 4

Due to space limitations, we only provide a rough outline of the proof here. The optimal forecast $f^{*}$ satisfies the first order condition $\pi^{e}=M\left(\hat{b}^{*}\right)$ and is unique. Then from (4.6) and (4.7) it follows that,

$$
\begin{equation*}
\mathbb{E}_{\theta}\left\{\phi^{\prime}\left(\hat{b}^{*}-q^{b}\left(\hat{b}^{*}, \theta\right)\right\}=\frac{\alpha \pi^{r}}{(1-\alpha)}\right. \tag{A.75}
\end{equation*}
$$

Also the second stage decision variable $q^{b}(\hat{b}, \theta)$ for a given $\hat{b}$ has to satisfy the second stage first-order optimality condition (4.6) which is

$$
\begin{align*}
\phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right) & =\pi_{0}-\mu\left(q^{b}(\hat{b}, \theta), \theta\right) \\
& =\mu\left(q^{a}(\theta), \theta\right)-\mu\left(q^{b}(\hat{b}, \theta), \theta\right) \tag{A.76}
\end{align*}
$$

We consider the two possible scenarios, $\hat{b}<q^{a}(\theta), \hat{b} \geq q^{a}(\theta)$. For each of the scenarios we obtain a lower bound on $\phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right.$. Then using the first order optimality condition (A.75), we derive the upper bound on $f^{*}$. We consider each scenario separately.
Case $f \geq q^{a}(\theta)$ : It follows that $\hat{b} \geq q^{b}(\hat{b}, \theta)$ and $q^{b}(\hat{b}, \theta) \geq q^{a}(\theta)$. From (A.76), and taking into account that $-\mu=d$,

$$
\begin{align*}
\phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right) & =\mu\left(q^{a}(\theta), \theta\right)-\mu\left(q^{b}(\hat{b}, \theta), \theta\right) \\
& =d\left(q^{b}(\hat{b}, \theta)-q^{a}(\theta)\right) \tag{А.77}
\end{align*}
$$

Since $\phi^{\prime}(||)=.\lambda||,. \phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)=\lambda\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)$, and from (A.77) it follows that,

$$
\begin{align*}
& \lambda\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)=d\left(q^{b}(\hat{b}, \theta)-q^{a}(\theta)\right) \\
& \Rightarrow q^{b}(\hat{b}, \theta)=\frac{f+d / \lambda q^{a}(\theta)}{1+d / \lambda} \\
& \Rightarrow \hat{b}-q^{b}(f, \theta)=\frac{d\left(\hat{b}-q^{a}(\theta)\right)}{\lambda(1+d / \lambda)} \tag{A.78}
\end{align*}
$$

From here it follows that,

$$
\begin{equation*}
\Rightarrow \phi^{\prime}\left(\hat{b}-q^{b}(f, \theta)\right)=\frac{\lambda d\left(\hat{b}-q^{a}(\theta)\right)}{\lambda(1+d / \lambda)} \tag{А.79}
\end{equation*}
$$

Case $\hat{b}<q^{a}(\theta)$ : Here too it follows that $\hat{b}<q^{b}(f, \theta)<q^{a}(\theta)$. Then (A.76) implies,

$$
\begin{align*}
& \phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)=\mu\left(q^{a}(\theta), \theta\right)-\mu\left(q^{b}(f, \theta), \theta\right) \\
& =d\left(q^{b}(f, \theta)-q^{a}(\theta)\right) \tag{A.80}
\end{align*}
$$

Since $\phi^{\prime}(||)=.\lambda|$.$| it follows that,$

$$
\begin{align*}
& \lambda_{l}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)=d_{u}\left(q^{b}(\hat{b}, \theta)-q^{a}(\theta)\right) \\
& \Rightarrow q^{b}(\hat{b}, \theta)=\frac{\hat{b}+d / \lambda q^{a}(\theta)}{1+d / \lambda} \\
& \Rightarrow \hat{b}-q^{b}(\hat{b}, \theta)=\frac{d\left(\hat{b}-q^{a}(\theta)\right)}{\lambda_{l}(1+d / \lambda)} \tag{A.81}
\end{align*}
$$

From here it follows that,

$$
\begin{equation*}
\phi^{\prime}\left(\hat{b}-q^{b}(\hat{b}, \theta)\right)=\frac{\lambda d\left(\hat{b}-q^{a}(\theta)\right)}{\lambda(1+d / \lambda)} \tag{A.82}
\end{equation*}
$$

We now solve for the upper bound of the optimal forecast $f^{*}$. The first order condition for the optimal forecast $f^{*}$ (A.75) can be written as,

$$
\begin{align*}
& \mathbb{E}_{\theta}\left[\phi^{\prime}\left(\hat{b}^{*}-q^{b}(\hat{b}, \theta)\right) \mid \hat{b}^{*} \geq q^{a}(\theta)\right] \\
& +\mathbb{E}_{\theta}\left[\phi^{\prime}\left(\hat{b}^{*}-q^{b}(\hat{b}, \theta)\right) \mid \hat{b}^{*}<q^{a}(\theta)\right]=\frac{\alpha \pi^{r}}{(1-\alpha)} \tag{A.83}
\end{align*}
$$

Using (A.79), (A.82) gives,

$$
\begin{align*}
& \mathbb{E}_{\theta}\left[\left.\frac{\lambda d\left(\hat{b}^{*}-q^{a}(\theta)\right)}{\lambda(1+d / \lambda)} \right\rvert\, \hat{b}^{*} \geq q^{a}(\theta)\right] \\
& +\mathbb{E}_{\theta}\left[\left.\frac{\lambda d\left(\hat{b}^{*}-q^{a}(\theta)\right)}{\lambda(1+d / \lambda)} \right\rvert\, \hat{b}^{*}<q^{a}(\theta)\right] \\
& =\frac{\alpha \pi^{r}}{(1-\alpha)} \tag{A.84}
\end{align*}
$$

Multiplying on either side by $(1+d / \lambda)$ and rearranging terms we get,

$$
\frac{\lambda d \hat{b}^{*}}{\lambda}-\frac{\lambda d \mathbb{E}_{\theta} q^{a}(\theta)}{\lambda}=\frac{\alpha \pi^{r}}{(1-\alpha)}(1+d / \lambda)
$$

From here it follows that,

$$
\hat{b}^{*}=\mathbb{E}_{\theta} q^{a}(\theta)+\frac{\alpha \pi^{r}(1+d / \lambda)}{(1-\alpha) d}
$$

## A. 8 Proof of Theorem 5

First we prove the following lemma
Lemma 9. $\Delta_{\Pi}(T) \propto \sum_{t=1}^{T} \mathbb{E}_{\Pi}\left(p_{t}-p_{t}^{*}\right)^{2} I\{D R$ Event $\}+o(\sqrt{\bar{T}})$
Proof. The saving at time $t$ is given by $r_{\alpha}(p)=\hat{s}_{1} p-\hat{s}_{2} p^{2}$ where

$$
\begin{align*}
& \hat{s}_{1}=\left(c_{t} d-\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j} d}{(1-\beta) \gamma t}-F^{-1}(\alpha)\right) \\
& \hat{s}_{2}=\left(d+\frac{\beta d}{(1-\beta) \gamma t^{2}}\right) \tag{A.85}
\end{align*}
$$

Define,

$$
\begin{align*}
& s_{1}=\left(c_{t} d-F^{-1}(\alpha)\right) \\
& s_{2}=d \tag{A.86}
\end{align*}
$$

Then $r_{\alpha}^{b}=s_{1} p-s_{2} p^{2}$. Note that $r_{\alpha}^{b}$ is the risk-sensitive savings if the baseline is known before hand. Note that the cumulative regret for the pricing policy $\Pi=\left\{p_{t}\right\}$ is given by,

$$
\begin{align*}
& \Delta_{\Pi}(T)=\sum_{t=1}^{T} \delta_{\Pi}(t) \\
& \text { Where } \delta_{\Pi}(t)=r_{\alpha}^{b}\left(p^{* b}\right)-r_{\alpha}\left(p_{t}\right) \tag{A.87}
\end{align*}
$$

Where $\delta_{\Pi}(t)$ is the loss that is incurred at a particular time step $t$. Expanding on $\delta_{\Pi}(t)$ we get,

$$
\begin{equation*}
\delta_{\Pi}(t)=r_{\alpha}^{b}\left(p^{* b}\right)-r_{\alpha}\left(p_{t}\right)=r_{\alpha}^{b}\left(p^{* b}\right)-r_{\alpha}^{b}\left(p_{t}\right)+r_{\alpha}^{b}\left(p_{t}\right)-r_{\alpha}\left(p_{t}\right) \tag{A.88}
\end{equation*}
$$

Call $\delta_{\Pi}^{1}(t)=r_{\alpha}^{b}\left(p^{* b}\right)-r_{\alpha}^{b}\left(p_{t}\right)$ and $\delta_{\Pi}^{2}(t)=r_{\alpha}^{b}\left(p_{t}\right)-r_{\alpha}\left(p_{t}\right)$. Then it follows that $\delta_{\Pi}^{1}(t)=$ $2 s_{2}\left(p_{t}-p^{* b}\right)^{2}$. And,

$$
\begin{equation*}
\delta_{\Pi}^{2}(t)=\left(s_{1}-\hat{s}_{1}\right) p_{t}-\left(s_{2}-\hat{s}_{2}\right)\left(p_{t}\right)^{2} \tag{A.89}
\end{equation*}
$$

From the fact that $p_{t}=0$ when $\mathrm{I}\{\mathrm{DR}$ Event $\}=0$, it follows that,

$$
\begin{align*}
\delta_{\Pi}^{2}(t) & \approx\left(O(\log (t) / t) p_{t}+O\left(1 / t^{2}\right)\left(p_{t}\right)^{2}\right) \mathrm{I}\{\mathrm{DR} \text { Event }\} \\
& \leq\left(O\left(1 / t^{2}\right)+O(\log \bar{t} / \bar{t})\right) \bar{p}, \text { When } t \text { is large } \tag{A.90}
\end{align*}
$$

This implies

$$
\begin{equation*}
\sum \delta_{\Pi}^{2}(t)=o(\sqrt{\bar{T}}) \tag{A.91}
\end{equation*}
$$

Note that $\delta_{\Pi}^{1}(t)=2 s_{2}\left(p_{t}-p^{* b}\right)^{2}$. This can be rewritten as,

$$
\begin{equation*}
\delta_{\Pi}^{1}(t)=2 s_{2}\left(p_{t}-p^{* b}\right)^{2}=2 s_{2}\left(p_{t}-p_{t}^{*}+p_{t}^{*}-p^{* b}\right)^{2} \tag{A.92}
\end{equation*}
$$

This can be rewritten as,

$$
\begin{equation*}
\delta_{\Pi}^{1}(t) \leq 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2}+2 s_{2}\left(p_{t}^{*}-p^{* b}\right)^{2} \tag{A.93}
\end{equation*}
$$

Recall that,

$$
\begin{equation*}
p_{t}^{*}=\frac{c_{t}}{2\left(1+\frac{\beta}{(1-\beta) \gamma t^{2}}\right)}-\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j} d+(1-\beta) \gamma t F^{-1}(\alpha)}{2 d\left((1-\beta) \gamma t+\frac{\beta}{t}\right)} \tag{A.94}
\end{equation*}
$$

And $p^{* b}$ is given by,

$$
\begin{equation*}
p^{* b}=\frac{c_{t}}{2}-\frac{F^{-1}(\alpha)}{2 d} \tag{A.95}
\end{equation*}
$$

Then $p_{b}^{*}-p^{* t}$ is given by,

$$
\begin{equation*}
p^{* b}-p_{t}^{*}=\frac{c_{t}}{2} \frac{\beta}{(1-\beta) \gamma t^{2}}+\frac{\beta \sum_{j=1}^{t-1} \frac{p_{j}}{j} d}{2 d\left((1-\beta) \gamma t+\frac{\beta}{t}\right)}-\frac{F^{-1}(\alpha)}{2 d} \frac{\beta}{(1-\beta) \gamma t^{2}} \tag{A.96}
\end{equation*}
$$

From the fact that $p_{t}^{*}=p_{t}^{* b}=0$ when $I\{D R$ Event $\}=0$, it follows that,

$$
\begin{equation*}
p^{* b}-p_{t}^{*} \leq\left(O\left(1 / t^{2}\right)+O(\log t / t)\right) \mathrm{I}\{\mathrm{DR} \text { Event }\} \leq O\left(1 / \bar{t}^{2}\right)+O(\log \bar{t} / \bar{t}), \text { When } t \text { is large } \tag{A.97}
\end{equation*}
$$

Then

$$
\begin{align*}
& \sum \delta_{\Pi}^{1}(t) \leq \sum 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2}+\sum 2 s_{2}\left(p_{t}^{*}-p^{* b}\right)^{2} \\
& \sum \delta_{\Pi}^{1}(t) \leq \sum 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2}+o(\sqrt{\bar{T}}) \tag{A.98}
\end{align*}
$$

Now (A.91) and (A.98) implies that,

$$
\begin{equation*}
\Delta_{\Pi}(T) \propto \sum 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2}+o(\sqrt{\bar{T}}) \tag{A.99}
\end{equation*}
$$

In particular, when there is no DR event $p_{t}^{*}=p^{* b}=p_{t}=0$. This implies we can further simplify the above regret as,

$$
\begin{equation*}
\Delta_{\Pi}(T) \propto \sum 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2} \mathrm{I}\{\mathrm{DR} \text { Event }\}+o(\sqrt{\bar{T}}) \tag{A.100}
\end{equation*}
$$

This completes the proof

Next we prove the Theorem 5. Lemma 9 suggests that it is enough to focus on the first term i.e. $\sum 2 s_{2}\left(p_{t}-p_{t}^{*}\right)^{2} \mathrm{I}\{\mathrm{DR}$ Event $\}$. Call $\bar{\Delta}(T)=\mathrm{E} \sum\left(p_{t}-p_{t}^{*}\right)^{2} \mathrm{I}\{\mathrm{DR}$ Event $\}$. Then,

$$
\begin{equation*}
\bar{\Delta}(T)=\mathrm{E} \sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\left(\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}\right)^{2}+\left(\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}+\delta_{2 \bar{t}}\right)^{2}\right)+o(\sqrt{\bar{T}}) \tag{A.101}
\end{equation*}
$$

Remark 26. We ignore some of the details. Only main steps of the proof re given.
Now this implies that,

$$
\begin{equation*}
\bar{\Delta}(T) \leq \frac{1}{2} \sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\delta_{2 \bar{t}}\right)^{2}+2 \mathrm{E} \sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}+\frac{\delta_{2 \bar{t}}}{2}\right)^{2}+o(\sqrt{\bar{T}}) \tag{A.102}
\end{equation*}
$$

For a continuous random variable $X$ which is positive, $\mathbb{E}\{X\}=\int_{0}^{\infty} \mathbb{P}\{X>x\} d x$. So the following holds

$$
\begin{align*}
& \mathrm{E} \sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}+\frac{\delta_{2 \bar{t}}}{2}\right)^{2} \leq \sum_{\bar{t}=1}^{(\bar{T}+1) / 2} \int_{0}^{\infty} \mathbb{P}\left\{\left|\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}\right|>\sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right\} d \gamma \\
& =\sum_{\bar{t}=1}^{(\bar{T}+1) / 2} \int_{\frac{\delta_{2 \bar{t}}^{2}}{4}}^{\infty} \mathbb{P}\left\{\left|\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}\right|>\sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right\} d \gamma+\sum_{\bar{t}=1}^{(\bar{T}+1) / 2} \int_{0}^{\frac{\delta_{2 \bar{t}}^{2}}{4}} d \gamma \tag{A.103}
\end{align*}
$$

When $\bar{t}$ is even, it is straight forward to show that

$$
\begin{equation*}
\left|\hat{p}_{\bar{t}+1}-p_{\bar{t}+1}^{*}\right| \leq k_{3}\left|\theta-\hat{\theta}_{\bar{t}}\right|+k_{4}\left|F_{\bar{t}}^{-1}(\alpha)-F^{-1}(\alpha)\right| \tag{A.104}
\end{equation*}
$$

This implies that,

$$
\begin{equation*}
\left|\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}\right| \leq k_{3}\left|\theta-\hat{\theta}_{2 \bar{t}-2}\right|+k_{4}\left|F_{2 \bar{t}-2}^{-1}(\alpha)-F^{-1}(\alpha)\right| \tag{A.105}
\end{equation*}
$$

Then using Lemma 8 and Theorem 7 it follows that,

$$
\begin{align*}
& \mathbb{P}\left\{\left|\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}\right| \geq \sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right\} \leq 2 \exp \left(-\frac{\mu_{2}}{4 k_{3}^{2}}\left(\sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right)^{2}(\sqrt{2} \sqrt{\bar{t}-1}-1)\right) \\
& +2 \exp \left(-\frac{\mu_{3}}{2 k_{3}^{2}}\left(\sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right)^{2}(\bar{t}-1)\right)+2 \exp \left(-\frac{\mu_{1}}{2 k_{4}^{2}}\left(\sqrt{\gamma}-\frac{\delta_{2 \bar{t}}}{2}\right)^{2}(\bar{t}-1)\right) \tag{A.106}
\end{align*}
$$

Using the following integrals, $\int_{0}^{\infty} x \exp -c x^{2}=-\frac{1}{2 c} \exp -c x^{2}$ and $\int_{0}^{\infty} \exp -a x^{2}=\frac{1}{2} \sqrt{\frac{\pi}{a}}$

$$
\begin{align*}
& \mathrm{E} \sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\hat{p}_{2 \bar{t}-1}-p_{2 \bar{t}-1}^{*}+\frac{\delta_{2 \bar{t}}}{2}\right)^{2} \leq \sum_{\bar{t}=1}^{(\bar{T}+1) / 2} \frac{\delta_{2 \bar{t}}^{2}}{4}+\sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(k_{2} \sqrt{\frac{2 \pi}{\mu_{1}}}+\sum_{\bar{t}=1}^{(\bar{T}+1) / 2} k_{3} \sqrt{\frac{2 \pi}{\mu_{3}}}\right) \frac{\delta_{2 \bar{t}}}{\sqrt{\bar{t}-1}} \\
& +\sum_{\bar{t}=1}^{(\bar{T}+1) / 2}\left(\frac{2 k_{2}^{2}}{\mu_{1}}+\frac{2 k_{3}^{2}}{\mu_{3}}\right) \frac{1}{\bar{t}-1}+\sum_{\bar{t}=1}^{(\bar{T}+1) / 2} \frac{4 k_{3}^{2}}{\mu_{2}} \frac{1}{\sqrt{\bar{t}-1}-1}+\sum_{\bar{t}=1}^{(\bar{T}+1) / 2} k_{3} \frac{\pi}{\mu_{2}} \frac{\delta_{2 \bar{t}}}{\sqrt{\bar{t}-1}-1} \tag{A.107}
\end{align*}
$$

Using this expression and (A.102) the statement of the theorem follows

