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UNIVERSITY OF CALIFORNIA, IRVINE

Essays on Monetary Policy and Financial Markets

DISSERTATION

submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

David Conway

Dissertation Committee: Associate Professor Fabio Milani, Chair Associate Professor Ivan Jeliazkov Professor David Brownstone

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DEDICATION

For my mother, father and brother

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Communication from the Federal Reserve about the future 'path' of the federal funds target rate can have just as great an impact on financial markets as an actual target rate change. On March 18, 2009 the target rate remained at the zero lower bound (ZLB) but 10-year Treasury yields fell by more than 50bp. The market reaction may have been to the official Federal Open Market Committee statement that the duration for which the target rate was expected to remain at zero had changed from "some time" to "an extended period". To model the effect of path shocks on yields this paper implements the first Dynamic Nelson-Siegel (DNS) model for daily data. Time-varying coefficients account for the changing degree to which the ZLB constrains medium-term yields. My results indicate that yields were most sensitive to path shocks in 2006-2007 with a rapid decline in sensitivity in 2008-2011. By the third quarter of 2011 the effect of a one standard deviation path shock on maturities under two years had fallen by more than 50% from early 2007. In 2013 that trend began to reverse, as the market now seems to expect the target rate to lift off from the ZLB sometime in 2015.

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ABSTRACT OF THE DISSERTATION

Essays on Monetary Policy and Financial Markets

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My dissertation is composed of three chapters that contribute to the fields of Applied Econometrics and Macroeconomics.

The first chapter, "A Copula Model for Discrete Duration Data with Sample Selection," presents a copula model to account for sample selection in a model of unemployment duration data. I apply two Markov Chain Monte Carlo (MCMC) methods to determine posterior distributions for the model parameters. In particular, a version of the Gibbs sampler is applied to evaluate the integrals that result from the copula representation of the likelihood, and the Random Walk Metropolis Hasting (RW-MH) algorithm for sampling from the posterior distribution. The model is applied to discrete data on unemployment duration from the 2011 Current Population Survey. Joint estimation of the selection and duration equations indicates that selection bias is present, and the data are informative about the model parameters.

The second chapter, "Monetary Policy and Equity Prices in a Multivariate GARCH Model," develops a model for the high-frequency analysis of a fundamental relationship in macroeconomics, between monetary policy and equity prices. A market-based approach to estimating daily changes in unexpected monetary policy is incorporated into a multivariate copula-GARCH model. This allows for efficient estimation of parameters in the mean equations for each of the variables, as well as the conditional heteroskedasticity and spillovers in volatility. I find little evidence for a relationship between monetary surprises and equity prices on this scale, with a mean correlation of -0.0503 between the two time series, which does not vary systematically over time.

The third chapter, "Federal Reserve Communication and its Time-Varying Impact on the Yield Curve: A Dynamic Nelson-Siegel Model for Daily Data," implements the first Dynamic Nelson-Siegel (DNS) model for daily data. This allows for estimation of the effect of daily shocks to monetary policy and macroeconomic factors. Time-varying coefficients account for the changing degree to which the zero lower bound (ZLB) constrains medium-term yields. My results indicate that yields were most sensitive to shocks to the future path of monetary policy in 2006-2007 with a rapid decline in sensitivity from 2008 to 2013. By the second quarter of 2013 the effect of a one standard deviation path shock on maturities under two years had fallen by more than 50% from the fourth quarter of 2007. As of July 2014 the sensitivity of 1-year yields began to show a modest increase, suggesting the market expects lift-off from the ZLB by mid-2015.

Chapter 1

A Copula Model for Discrete Duration Data with Sample Selection

1.1 Introduction

Sample selection is a common problem in applied econometrics. Beginning with the original treatment of sample selection in [15], empirical economic literature was forced to address this source of estimation bias. Much of the early work in this field assumed a multivariate normal distribution for the latent variables of the outcome and selection equations. However, as pointed out in [23] and others, in cases where the outcome variable is discrete this assumption of joint normality leads to misspecification of the model. A better approach is to utilize a copula function which creates a tractable joint distribution yet maintains the intended combination of discrete and continuous marginal distributions.

In the paper I use the Gaussian copula function to construct the likelihood of a two-equation model for the analysis of individuals durations of unemployment during the year 2010. The duration of unemployment takes on a modified geometric marginal distribution and the probability of incidence of unemployment is given by a probit representation. More specifically, the duration variable considered is the number of weeks in which individuals were considered members of the labor force but not in a state of paid employment, as measured by the March 2011 Current Population Survey Supplement. Incidence of unemployment is a binary variable, which indicates whether an individual experienced unemployment at any point in the sample period. The copula function couples the distributions of these two variables to determine a bivariate joint distribution.

To estimate the model I use two Markov Chain Monte Carlo approaches. First, the density formed by the copula lacks a closed-form solution, so the Gibbs sampler allows for evaluation of the likelihood contributions via the CRT method from [18]. The Random-Walk Metropolis Hastings algorithm (RW-MH) facilitates sampling from the posterior distributions of the model parameters. The likelihood is highly non-convex so Bayesian estimation is indispensable in this situation and far more computationally efficient than existing algorithms for obtaining maximum likelihood estimates.

It should be noted that this application is intended as an illustration for the basic model and techniques I propose in the paper, rather than an addition to the unemployment duration literature *per se*. In Section 6 I consider potential solutions for some of the shortcomings of this particular application, as well as ways of strengthening the interpretation and meaningfulness of the empirical results.

The rest of the paper is organized in the following way. Section 2 outlines the basic selection model and how the Gaussian copula function improves model estimation. In Section 3 an explanation of the data is given. Section 4 presents the estimation strategy. I offer results and discussion in Section 5, and offer concluding thoughts in Section 6.

1.2 The Selection Model

1.2.1 Gaussian Copula Function

Over the past twenty years the prevailing approaches to correcting for selection bias have come under scrutiny, especially in situations where the outcome variable is discrete. The issue of how to properly specify the joint distribution of the outcome and selection equations was first addressed in [23], and later in [34] and [26]. Rather than a top-down approach in which a functional form is first assumed for the joint distribution, these authors suggested that a better approach would be to first specify marginal distributions and then form a joint distribution in a way that maintains the original margins. One such method is to couple univariate distributions with copula functions. As described in [36], and [24] a copula is any function C that meets the following criteria:

1. $C(1, ..., 1, a_p, 1, ..., 1) = a_p$ for all $a_p \in [0, 1]$

2.
$$C(a_1, ..., a_q) = 0$$
 if $a_p = 0$ for any $p \in \{1, ..., q\}$

3. C is q-increasing, i.e. any hyperrectangle in $[0,1]^q$ has non-negative C-volume

Since we know that the inverse of a cumulative distribution function is distributed uniformly, inserting inverse distribution functions $F_1^{-1}, ..., F_q^{-1}$ as arguments of $C(\cdot)$ leads to a joint distribution with margins $F_1, ..., F_q$. This can be illustrated by a flexible and easily interpretable parametric form of the copula function, the Gaussian copula, given by (1.1).

$$C(z|\Omega) = \Phi_q(\Phi^{-1}(z_1), ..., \Phi^{-1}(z_1)|\Omega)$$
(1.1)

where $z = (z_1, ..., z_q)$, Φ is the standard normal cumulative distribution, and Ω is the correlation matrix. The elements $z_1, ..., z_q$ are distributed according to $F_1, ..., F_q$ (which can be

discrete or continuous), with a resulting joint distribution H. The key point here is that H is a joint distribution with any desired marginal distributions. Few multivariate distributions exist with both discrete and continuous margins, so this property of copulas makes them extremely valuable in cases where joint modeling is required. The sacrifice for using such a model is that the joint distribution does not have a closed-form solution, so that evaluation of a copula model requires MCMC sampling.

The data generating process associated with the Gaussian copula is given by (1.2). The latent variables z_{ij} come from a q-variate standard normal distribution with correlation matrix Ω . The y_{ij} 's we observe take on separate marginal distributions F_{ij} , while maintaining the correlation created by Ω .

$$y_{ij} = F_{ij}^{-1}(\Phi(z_{ij})) \text{ for } z_i \sim N(0, \Omega)$$
 (1.2)

Though an array of different copula functions abound in the literature for usage in sample selection problems (as in [24]), the Gaussian copula has desirable properties for our uses. Most significantly, the Gaussian copula can accommodate any correlation on the [-1, 1] interval, a property that does not hold for many other classes of copulas.

1.2.2 Likelihood Function

For application to the two-equation selection model, the Gaussian copula combines the distribution of the error term in the selection equation with the distribution of the discrete duration variable. For the present model I have binary data on an individuals incidence of unemployment (y_{i1}) to indicate selection into the state of unemployment, and the duration of an unemployment spell in number of weeks (y_{i2}) . The real quantity of interest is the covariate effects on an individual's duration of unemployment but I incorporate y_{i1} to account for the individuals selection into a state of unemployment. Since y_{i2} is measured in weeks it should be represented as having a discrete distribution, with no recorded y_{i2} for those agents who were employed for the entire period for which they were observed.

I choose a modified geometric distribution for duration variable y_{i2} as in [29]. Therefore y_{i2} has probability distribution and hazard rate given by:

$$f(y_{i2}|p) \sim p((y_{i2}+1)!)^{-1}(1-p)(2-p)...(y_{i2}-p)$$
(1.3)

$$H(y_{i2}|p) \sim \frac{p}{(y_{i2}+1)}$$
(1.4)

Here p is the probability of leaving the state of unemployment in any period. The arguments of the copula function are cumulative distribution functions, provided in (1.5).

$$F(y_{i2}|p) = \sum_{k=1}^{y_{i2}} p((k+1)!)^{-1} (1-p)(2-p)...(k-p)$$
(1.5)

The hazard rate makes it clear why this distribution is chosen for the present application. Unlike a geometric distribution which has a hazard rate of p, or the Poisson and negative binomial distributions that have constant hazard rates, the hazard for this distribution decreases as y_{i2} increases. This has intuitive appeal in the case of unemployment duration by capturing the hysteresis effect. Being unemployed for an extra week may lower the probability of finding a job in the next period, in the sense that an individual's skills diminish relative to the progress of processes and technology used by firms. Firms will be less likely to hire a worker the longer the workers unemployment spell has persisted. This implies that the rate at which individuals leave the state of unemployment in period t+1 given that they have been in unemployment for t periods, should decline.

Of course hysteresis is not the only effect present in the determination of the rate at which individuals find employment. One could postulate a functional form for the duration variable likelihood with an increasing hazard rate. This would be consistent with the decrease in reservation wages experienced by workers as the unemployment duration increases. As the duration of unemployment grows for a particular individual, the lowest wage for which they will work should drop since they become more desperate to find employment. While hysteresis captures the decrease in a firms willingness to hire a worker as the workers duration of unemployment increases, the decrease in a workers reservation wage makes it easier for a firm to fill an open position. This implies an increase in the probability of leaving the state of unemployment as the duration of the unemployment spell increases, suggesting an increasing hazard rate. It is unclear whether hysteresis or falling reservation wages is the dominant effect but my functional form for the duration variable assumes the dominance of hysteresis.

Covariates enter the distribution through p:

$$p = \frac{1}{(1 + exp(x_{i2}\beta_2))} \tag{1.6}$$

Taking the exponential function of $x_{i2}\beta_2$ in the denominator implies that p is bounded by [0, 1]. The selection equation is modeled as a univariate probit equation, such that the error term ν_i is assumed to follow the standard normal distribution. y_{i1} takes on a value of 1 if the individual experiences at least one week of unemployment in the sample period, or 0 in the case of full employment.

$$y_{i1} = \begin{cases} 1 \text{ if } x_{i1}\beta_1 + \nu_i > 0 \\ 0 \text{ if } x_{i1}\beta_1 + \nu_i \le 0 \text{ where } \nu_i \sim N(0, 1) \end{cases}$$
(1.7)

From the marginal distributions it is possible to specify the joint likelihood. For individuals with no unemployment $(y_{i1} = 0)$, data on unemployment duration is missing, so the likelihood contribution of these individuals is based solely on the marginal distribution of ν_i . In the case where an individual does experience some unemployment $(y_{i1} = 1)$, we must evaluate the joint likelihood of y_{i1} and y_{i2} .

$$f(y_1, y_2|X) = \prod_{t:y_{i1}=0} f(y_{i1}|x_{i1}) \prod_{t:y_{i1}=1} f(y_{i1}, y_{i2}|x_{i1}, x_{i2})$$
(1.8)

As discussed above, the first term on the right-hand side represents the selection variable modeled as univariate probit. In the second term the copula constructs the joint distribution as a function of the marginal distributions. Implementing the Gaussian copula and following [40], and [34], the full likelihood can be represented as (1.9).

$$f(y_1, y_2|X) = \prod_{t:y_{i_1}=0} \Phi(-x_{i_1}\beta_1) \prod_{t:y_{i_1}=1} \Phi_2(\Phi^{-1}(F(y_{i_1})), \Phi^{-1}(F(y_{i_2})), \rho)$$
(1.9)

The next step is to evaluate the likelihood. The probit term is straightforward, since a closed-form expression exists for the standard normal cdf. However, evaluating the bivariate cdf in the second term is significantly more complex, since it requires evaluating a double integral. From [19] the expression for the joint distribution can be given in terms of the latent variables (z_{i1}, z_{i2}) from (1.2):

$$Pr(y_{i1}, y_{i2}|X_i, \beta, \rho) = \int_{\beta_{i2}} \int_{\beta_{i1}} f_N(z_i|0, \rho) dz_i \text{ where } B_{ij} = (\gamma_{ij,L}, \gamma_{ij,U})$$
(1.10)

By using the latent variable representation the marginal distributions F_j for (j = 1, 2)are present in the cutpoints $\gamma_{ij,U}$ as $\gamma_{ij,U} = \Phi^{-1}(F_j(y_{ij}|\beta_j))$ and $\gamma_{ij,U} = \Phi^{-1}(F_j(y_{ij}|\beta_j) - Pr(y_{ij}|\beta_j))$. The problem has therefore become one of evaluating a bivariate truncated normal distribution. The way in which the bivariate normal is truncated is represented graphically in Figure 1.1 borrowed from [19]. Furthermore, since y_{i1} is modeled as a univariate probit the cutpoints in B_{i1} can be simplified to:

$$\gamma_{ij,U} = \Phi^{-1}(1) = \infty \text{ and } \gamma_{ij,U} = \Phi^{-1}(\Phi(-x_{i1}\beta_1)) = -x_{i1}\beta_1$$
 (1.11)



Figure 1.1: Truncated area for bivariate normal distribution.

In order to estimate β_1 , β_2 and the off-diagonal element of Ω (correlation parameter ρ), I estimate the value of the bivariate truncated normal with the CRT method from [18], and draw from the posterior distributions of the model parameters with the RW-MH algorithm.

1.3 Estimation

1.3.1 CRT Method

Several methods exist for evaluating problems with multiple integrals and no closed-form solution, as discussed in [18]. One possible approach is known as the CRT method, which relies on MCMC sampling to create an efficient algorithm for directly drawing from the double integral in (1.12). As a first step, the probability of observing the realization $y_i =$ (y_{i1}, y_{i2}) can be rewritten as:

$$Pr(y_{i}|\beta,\rho) = \int_{\beta_{i2}} \int_{\beta_{i1}} f_{N}(z_{i}|0,\rho) dz_{i} = \int z_{i} \in B_{i} f_{N}(z_{i}|0,\Omega)$$

$$= \frac{1(z_{i} \in B_{i}) f_{N}(z_{i}|0,\Omega)}{f_{TN_{B_{i}}}(z_{i}|0,\Omega)}$$
(1.12)

where $B_i = B_{i1} * B_{i2}$. From Bayes rule $1[z_i \in B_i]$ can be seen as the likelihood, $f_N(z_i|0,\Omega)$ the prior distribution, and the posterior distribution. The left-hand side is therefore the integrating constant. Taking logs of both sides this problem can be written as:

$$\ln \hat{P}r(y_i|\beta,\rho) = \ln f_N(z_i^*|0,\Omega) - \ln \hat{f}_{TN_{B_i}}(z_i^*|0,\Omega)$$
(1.13)

The log of $f_N(z_i^*|0,\Omega)$ can be found directly, where any z_i^* can be chosen but is commonly taken to be the mean of the values from the Gibbs sample. The second term on the righthand side is estimated by the Gibbs sampling algorithm for a multivariate standard normal distribution, presented in (1.14) and (1.15).

$$z_{i1}^{(g)}|z_{i2}^{(g-1)} \sim (z_{i1}^{(g)}|y_{i1}, z_{i2}^{(g-1)}, \Omega) = TN_{B_i}(\mu_{i1}^{(g)}, \sigma_{i1}^{2(g)})$$
(1.14)

$$z_{i2}^{(g)}|z_{i1}^{(g)} \sim (z_{i2}^{(g)}|y_{i1}, z_{i1}^{(g)}, \Omega) = TN_{B_i}(\mu_{i2}^{(g)}, \sigma_{i2}^{2(g)})$$
(1.15)

where $\mu_{ij}{}^{(g)} = \rho \mu_{ij}{}^{(g-1)}$ and $\sigma_{ij}{}^{2(g)}$. This process is iterated G times, creating a sample of draws from the joint distribution for (z_{i1}, z_{i2}) . These values are then used to estimate the Gibbs transition kernel in (1.16).

$$K(z_i, z_i^*) = f(z_{i1}|y_i, z_{i2}, \beta, \Omega) \times f(z_{i2}^*|y_i, z_{i1}^*, \beta, \Omega)$$
(1.16)

Finally, (1.17) shows that the truncated normal distribution evaluated at z_i^* is given by the average of the Gibbs kernel evaluated at z_i^* .

$$\hat{f_{TN}}_{B_i}(z_i * | 0, \Omega) = \frac{1}{G} \sum_{g=1}^G K(z_i, z_i^* | y_i, \beta, \Omega)$$
(1.17)

The value of the likelihood contribution for a particular observation in (1.17) can then be calculated, and is repeated across all observations to determine the value of the likelihood for a particular set of parameter values from (1.9).

1.3.2 Metropolis Hastings Algorithm

The estimation of the model continues by applying an MCMC approach known as the Random Walk Metropolis Hastings (RW-MH) algorithm, in order to sample from the posterior distributions of the model parameters. I first define my priors in Table 1.1. Since I am Table 1.1: Prior parameter distributions

Variable	Prior distribution
ρ	N(0,0.10)
β_1, β_2	N(0,0.50)

initially agnostic as to the signs of the parameters they each receive a standard normal prior. The algorithm for sampling directly from the posterior distribution proceeds in the following steps:

- 1. Begin with parameter values, $\theta^{(i)}$ (when $i=1, \theta^{(i)}=\bar{\theta}$)
- 2. Draw $\theta^{(\star)}$ from $N(\theta^{(i)}, \sigma^2)$ where σ^2 is calibrated to achieve an optimal acceptance rate
- 3. If $\frac{\pi(\theta^*)}{\pi(\theta^{(i)})} = r > 1$ the draw is accepted and $\theta^{(i+1)} = \theta^*$
 - If $\frac{\pi(\theta^*)}{\pi(\theta^{(i)})} = r < 1$ the draw is accepted and $\theta^{(i+1)} = \theta^*$ with probability r

4. Repeat steps 2-3 for 12,500 iterations.

This creates a sample from the posterior distribution, which has no closed-form solution. In the application to follow I use 12,500 draws, discarding the first 2,500 as a burn-in that is dropped from the sample. This is to only retain samples that are more likely to have been drawn after the convergence of the algorithm. For the entire sample the acceptance rate was 38.1%, within the efficient range of 23.4% to 44.4% suggested in [13].

1.4 Application

Following the Great Recession of 2008/2009, the high-water mark for the unemployment rate in the United States came in October of 2009, eclipsing 10% for the first time since 1983. Throughout the course of 2010 the rate of unemployment remained above 9.4% .An equally important point to consider is the duration for which these individuals were out of work. As of November 2010 the BLS found the average duration of unemployment to be 34.5 weeks, well above values that had obtained in all other recessions in the post-war era. It is appropriate then to think about who these people are that experience long durations of unemployment, and specifically what factors contribute to the length of time for which individuals are out of work. Descriptive statistics for the data set are presented in Table 1.2.

For the application of the copula model I have constructed, I use individual level crosssectional data from the Current Population Survey March 2011 Supplement. The Supplement is done once a year and is different from the typical CPS survey in that it includes data on the number of weeks of employment in the previous year. The survey includes more than 204,000 people randomly chosen from throughout the U.S. Exogenous variables in the outcome are given as a dummy for gender, race, and highest level of education obtained. I

			Weeks o	of Uner	nployment	,			
		_		Mear	n	3.40	_		
		_		Std. D	ev.	18.31			
(% of Po	pula	tion tha	t expe	rienced un	employn	nent:	27.62	_
			Sta	te of F	Residence ((%)			_
CT	DE	MF	MD	MA	NH	NJ	NY	PA	RI
2.34	1.62	1.63	3 2.60	1.56	2.06 2.23	4.19	2.76	1.61	
				Gen	der (%)				
				Male	Female				
				51.95	48.05				
				Ra	ce (%)				
			White	Black	k Asian	Other			
			79.89	10.08	8 5.69	3.54			
				Educa	ation $(\%)$				
Hig	h Schoo	ol F	ligh Sch	ool A	Associates	Bachel	ors	¿Bachel	ors
	9.88		28.14		28.87	20.64	4	8.05	

Table 1.2: Descriptive Statistics

also include each respondents age and marital status. In the selection equation I use all of these variables and additionally include an instrument, state of residence. For the selection equation the dependent variable takes on a value of 1 if the individual did experience unemployment in 2010, and 0 if they did not. The outcome variable is measured as the number of weeks in 2010 for which an individual was in the labor force but not employed for pay.

At this point several additional clarifications concerning the data are needed. First, serious measurement error seems to be at work in the responses on the duration variable, with many values at 10, 20, 26, 30 etc. This is likely due to the fact that the survey was given in March 2011, asking individuals to recollect their number of weeks of unemployment during 2010. Since they may not be able to recall an exact figure, many seem to respond by rounding their answer to the closest multiple of 10, or 26 as it marks half of the year. To handle this I group responses into tens of weeks, such that 1-10 weeks of unemployment takes on the value 1, 11-20 takes on the value 2 etc. As the histograms in Figure 1.2 and Figure 1.3 show, this transformation of the data leads to a distribution of the data that better reflects

actual behavior. Another approach to this measurement error would be to use dummies for roundability as in [31] but grouping the data into tens of weeks also facilitates easier evaluations of the likelihood so I continue in this way.



Figure 1.3: Histogram of tens of weeks of unemployment (frequency).

Second, since the survey does not include reliable information on the number of different spells of unemployment experienced by each respondent I assume that all unemployment was experienced consecutively. It may be the case that individuals had multiple spells for which they were unemployed during the year, but from the given data I cannot distinguish between respondents on this basis.

Third, selection enters the model due to an individual's geographic location. The assumption here is that an individuals state of residence is a significant variable in determining an individual's probability of being unemployed but does not affect his or her duration of unemployment. In this sense, becoming unemployed in different states should not impact for how long one is unemployed. At the time when a workers job is eliminated the worker enters a labor market national in scope, and is willing to take a job in any other state in the U.S. To make a stronger case for the geographic flexibility of workers in the labor market, I focus on the 10 states mentioned in the descriptive statistics and individuals who are currently unmarried and without dependents. The areas under study include all states roughly considered part of the Northeast region of the U.S., where the flow of workers across state lines is common. The lack of spouse or children makes it less likely to have it a family attachment to any state. The subsample acquired in this way is then weighted by the CPS sample weights provided by the Census Bureau.

Lastly, there are many observations subject to censoring at a duration of 52 weeks. In order to account for this I set the value of the distribution function at this point equal to one, such that F(6, p) = 1. Here I use the value 6 because that is the group into which 52 weeks falls, once I have grouped data into tens of weeks as explained above.

1.5 Results

The estimation algorithm returns some strong results concerning the distributions of the model parameters. The relevant posterior distributions are presented in Table 1.3 and are shown graphically alongside the prior distributions in Figure 1.4. In several cases the data are quite informative about the model parameters. To interpret the sign of the results we can refer to (1.11) and (1.12), which indicate that a negative value for a parameter in β_2 parameter implies a decrease in the duration of unemployment, while a positive value implies an increase.

Variable	Posterior Mean	Standard Deviation
ρ	0.312	0.215
β_2		
Intercept	1.121	0.344
Female	4.093	0.901
Age	-0.893	0.462
White, Non - Hispanic	-1.102	0.453
Black, Non-Hispanic	1.943	0.561
White, Hispanic	-0.521	1.049
Black, Hispanic	0.065	0.599
Non-White * Female	-1.524	0.804
Income > 50,000	-2.044	0.913
College Educated	-2.510	0.823

Table 1.3: Posterior means and standard deviations

In Table 3 we see that being male, being white, having a college education, having a job with annual income above \$50,000, and increasing age are all factors that decrease the length of time for which one is unemployed. It should be pointed out that here White includes those who responded as having race either White or Asian. I have also included an interaction term for Black females, since the data indicate that this is the only group of women that tends to be unemployed less than their male counterparts of the same race. Though the effect does not seem to be particularly strong, the sign of the posterior mean for this interaction term in the table suggests that being Black and female does lower duration of unemployment.

The results also provide an estimate of the correlation coefficient ρ from the Gaussian copula. This parameter measures the correlation between the distributions of selection and duration variables. In the estimation of the model I find that the posterior mean for ρ is 0.283, with a standard deviation of 0.191. This implies a positive correlation between the distributions that were coupled by the copula function, such that the higher an individuals probability of becoming unemployed, the greater the duration of the individuals unemployment spell. Though the correlation is not particularly strong and the standard deviation is relatively large, the results suggest that the probability of unemployment and duration of unemployment are likely linked, making the case for the joint estimation of the model via the



Figure 1.4: Prior (black) and posterior (blue) distributions

copula representation. The fact that a correlation exists between the selection and duration equations implies that selection bias does indeed exist in the unemployment duration data.

While the plots and signs of the posterior means are telling of the relationship among the variables, the magnitude of the effect of each covariate on the probability of experiencing a duration of unemployment of a certain length is not straightforward. These covariate effects can be estimated by averaging over the entire sample population and other parameter values; a process I consider in the next subsection.

1.5.1 Covariate Effects

In interpreting the coefficients it should be pointed out that the variables have been rescaled, such that all of the dummy variables take on values in the set [0, 0.1], rather than the typical [0, 1]. As well, the age variable was rescaled such that each year counts as 0.1. For example, a thirty year old person would have age variable equal to 3.00.

In the nonlinear model I have used thus far, the interpretation of the *beta* coefficients is not that of a marginal effect. The sign of the coefficient is informative, in that (1.6) tells us a negative value implies lower duration of unemployment, whereas a positive value implies a longer duration of employment. However, the magnitude of a change in unemployment duration due to a change in the value of a covariate is dependent upon the other parameter values as well as other covariate values.

A better approach to determine the effect on the dependent variable of changing the value of a single covariate is discussed in [16] and [17]. When looking at the effect of covariate β_i on the probability of a particular realization of the dependent variable, the value in question is:

$$Pr(y_{i2} = k | x_{i2j}^{\dagger}, x_{i2-j}, \beta_2) - Pr(y_{i2} = k | x_{i2j}^{\dagger}, x_{i2-j}, \beta_2)$$
(1.18)

where x_{i2j}^{\dagger} and x_{i2j}^{\dagger} are the desired different values of the covariate. This value differs across the population from the data, so I average these values over the distribution of the data and the posterior parameters. It can be shown that the difference in (1.18) can be represented as:

$$\int (Pr(y_{i2} = k | x_{i2j}^{\dagger}, x_{i2-j}, \beta_2) - Pr(y_{i2} = k | x_{i2j}^{\dagger}, x_{i2-j}, \beta_2)) \pi(x_{i2-j}) \pi(\beta_2 | y_{i2}) dx_{i2-j} d\beta_2 \quad (1.19)$$

Since we have already sampled from the posterior distribution of the parameters in the RW-MH process, this only requires taking a random draw from the β_2 parameters and matching them with a random individual from the empirical distribution of the covariates. By varying covariate x_{i2j} from x_{i2j}^{\dagger} to x_{i2j}^{\ddagger} in the bracketed term in (1.19), the difference in the probability of realization y_{i2} is determined. Table 1.4 provides estimates of the covariate effects.

Table 1.4: Covariate effects for gender, race and education

	$\Delta Pr(y_2 = 1)$	$\Delta Pr(y_2 = 3)$	$\Delta Pr(y_2 = 5)$	$\Delta Pr(y_2 = 6)$
Male - > Female	0.091	0.045	0.037	0.062
White - > Non	0.110	0.048	0.007	0.033
NoCollege - > College	-0.144	-0.071	-0.062	-0.094

According to Table 1.4, gender, race and education all have similar effects on the probability that an individual experiences a particular duration of unemployment, especially for small values of y_2 . For example, a female is 9.1% more likely to be unemployed for 1-10 weeks than a male with all other covariates the same. Over time the difference in probabilities diminish, which we would expect due to the fact that the probability of experiencing long duration of unemployment is lower than the probability of a short duration. However, since the data is censored at 52 weeks, the covariate effects for $y_2 = 6$ reflect the point mass at this value, and the fact that nearly 20% of the population falls into this category. The greatest difference among the covariate effects is that having a college education is much more important at longer durations of unemployment than the other variables. Having a college education creates almost a 6% gap in the probabilities of experiencing between 40 and 50 weeks of unemployment. This is about four times the gap for the gender and race variables. In this sense, college education is the most persistent of the covariate effects over time. Not having a college education is the most important factor for increasing the probability one experiences a particular duration of unemployment, particularly when looking at longer durations.

1.6 Conclusion

In this paper I have estimated the posterior distribution of the parameters of a two-equation model for a discrete duration variable in the presence of sample selection. Using a copula function to form the likelihood of the model, I use two MCMC approaches to both evaluate the likelihood function, and then sample from the posterior distributions of the parameters.

I apply this model to discrete data on duration of unemployment, and estimate covariate effects and the correlation parameter of the Gaussian copula. I find that in many cases the data are informative on parameter values. A positive correlation parameter is found, suggesting that the distributions of incidence and duration of unemployment are positively related. Due to the Gaussian copula function the marginal distributions are properly specified, and joint estimation is possible.

Another area for improvement is to make the empirical results more comprehensive. This can be done by using different functional forms for the distribution of the duration variable, as well as estimating the copula model with alternatives like the quadrature method or simple maximum likelihood. Additionally, the introduction of other dependent variables can take advantage of the relative ease with which the Gaussian copula can estimate higher dimensional problems. This line of inquiry seems promising and applications of copula functions to problems of sample selection are numerous.

Chapter 2

Monetary Policy and Equity Prices in a Multivariate GARCH Model

2.1 Introduction

The relationship between monetary policy and equity markets has been a topic of interest for financial institutions and economists for some time. For investors, knowing the correlation between equity prices and monetary policy expectations can be a valuable aspect of any investment strategy. Determining the impact of possible changes to current stated policy or expected future policy for an equity portfolio relies on an understanding of how stocks co-move with monetary policy. For central bankers, the interest in the equity-monetary policy relationship stems from the immediate effect that equity price changes have on the real wealth of consumers. While the transmission of monetary policy to the real economy via interest rates and capital investment may take a considerable amount of time to play out, policymakers may be able to create an immediate impact on current levels of consumption by altering the value of consumers' equity portfolios. An estimate of the correlation between policy and equity prices is therefore essential in calibrating the optimal policy action.

Most of the focus on the correlation between equity prices and monetary policy is concentrated at meetings of the Federal Open Market Committee (FOMC), usually held eight times per year. At these meetings the FOMC can make adjustments to the Federal Funds Rate (FFR) target for the current period, or (particularly since the 2007/2008 financial crisis) announce unconventional monetary policy such as quantitative easing (QE) or forward guidance on the future path of the short rate. However, there are two considerations that suggest an event-study model in this framework is misspecified. First, interest in the FOMC is not limited to days on which the committee has a regularly scheduled meeting. Whether through unconventional monetary policy, speeches by FOMC members, or research published by the Federal Reserve, a variety of means exist for dissemination of monetary policy information. This suggests that a full understanding of the correlation between stock markets and monetary policy might be gained by incorporating data from all trading days, not simply those on which an official FOMC meeting is due to take place. Second, if a model is to incorporate observations from non-meeting days, the causation can not be assumed to strictly flow from monetary policy expectations to equity markets. An equally viable hypothesis is that changes in the value of equity reflects the overall health of the economy and therefore will influence the policy choices of the monetary authority. While the assumption is that a policy shock should 'cause' a change in equity prices in some small window of time around the policy event, on a daily basis there is likely to be a causal link in both directions.

The question that I address in this paper is how to properly measure the relationship between daily fluctuations in monetary policy and equity prices. While overall changes to equity prices is most easily captured by a stock index such as the Dow Jones Industrial Average or the S&P 500, the measurement of daily changes to monetary policy is non-trivial. The effective FFR could present an option, but this is flawed for two reasons. First, since December 2008 the FFR target as has been fixed in a range of 0-25bp, and daily fluctuations in the effective

FFR are a reflection of the supply and demand for bank reserves. Information content in the effective FFR therefore seems to be limited and the channel by which this is related to changes in equity prices is not obvious. More importantly, this is not the focus of this paper. Second, ideally this model should capture the impact of changes to expectations regarding future monetary policy. If equity markets are assumed to be forward looking the target FFR expected in future periods will be essential to equity valuations, more than just the current target rate.

In order to evaluate the equity/monetary policy relationship on a high-frequency basis this paper implements a multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) model. More specifically, I apply a variant of MGARCH known as copula-MGARCH, which allows for more efficient specification for the distributions of the financial assets in the model. I find that the monetary policy shock is conditionally correlated with equity price changes at about -0.05 throughout the sample, while conditional variances of each of the time-series do vary greatly over time. As well, the low conditional correlation suggests that there is little evidence for spillovers in volatilities between the two series.

I begin in Section 1 with a literature review of previous work in this field. Section 2 presents the copula-MGARCH model, while Section 3 addresses the data and measurement of the monetary policy shock variable. In Section 4 I discuss estimation methodology. Section 5 provides results and discussion.

2.2 Literature Review

This paper synthesizes two strains in the financial economics literature concerning the interaction of monetary policy and stock returns, and models for high-frequency data that incorporate time-varying conditional heteroskedasticity. I discuss the former here and the
latter in Section 3 as I work through the development of the model.

Much of the literature on the relationship between monetary policy and equity prices takes the form of event studies, where returns and rate changes are measured only on days where the FOMC meets. This makes intuitive sense, since only on these days is there a possibility of a target rate change. Moreover, anecdotal evidence from the financial press might suggest that these are the only days that really matter for creating shocks to monetary policy. In this sense, [35] consider a simple linear regression of the change in major index returns on the change in the target FFR, where the events in the study are only meeting days. For data from 1974-1979 the authors find that a higher FFR target significantly depresses equity prices. This can be explained as a consequence of higher interest rates slowing the pace of growth in the economy and thereby lowering the market values of firms in the economy, or as the result of a lower present value of returns. Using a similar methodology [20] find that tighter monetary policy is correlated with decreases in returns for stock markets in a handful of (mostly) developed economies.

Notably, the preceding papers have emphasized the role of observed changes to the policy rate, when the focus should have been on the unexpected component of the policy action. For this reason [11] consider the role of monetary policy surprises in relation to equity markets, calculating the surprise monetary policy as the difference between projections from a survey of market analysts and the actual FOMC decision. The difference in the mean expected value from the survey and the actual target rate change should reflect the component of the FOMC action that the market did not expect. Assuming that the information regarding the expected rate is already included in asset prices at the time of the meeting, the unexpected component of the rate change is what may possibly move equity prices. Looking at a crosssection of firms listed on the S&P 500, the authors find that this measure of monetary surprise is a good indicator of equity returns for firms with certain balance sheet characteristics, but not for all equities in general. In the same paper [11] suggest that the correlation between monetary policy shocks and asset price changes is best observed on a high-frequency basis, though this would greatly increase the difficulty of estimating the model. Their observation, that identifying monetary policy surprises at times other than policy event days could be quite difficult, has apparently led the issue to be dismissed in the literature thus far. For the reasons mentioned in the introduction a better understanding of how policy and equity markets interact could be very valuable to policymakers and market participants alike.

While maintaining an event study approach, [4] and [22] improve upon the survey methodology of [11] by incorporating a financial product called the Federal Funds Futures (FFF) contract to measure the unexpected element of monetary policy. These futures contracts are sold by CME Group at monthly maturities up to twelve months into the future. Each FFF contract pays out \$100 minus the value of the average target FFR for that month. This implies that the changes in the value of these contracts are an immediate indicator of updates to the expectations for the target rate at particular horizons into the future. [14] finds the FFF rate is an approximately unbiased predictor of the Fed Funds rate target, so this seems like a reasonable measure for the purposes of this paper.

On FOMC meeting days [4] calculate the monetary policy shock for the next month as $\frac{D}{d-D}(ff_t - ff_{t-1})$, where D is the number of days in the month, d is the current day and ff_t is the implied value of the average FFR target on day t. Because the FFF rate is a predictor of the average target rate for a particular month, changes in the rate must be weighted so that an equal change at the end of the month implies a much stronger expectation change than at the beginning of the month. For example, if an FOMC meeting is held on January 1^{st} a 1.00 increase in ff_t for the spot month suggests that the average target FFR over the course of the rest of the month will be 100bp lower than was expected on the day prior to the meeting. However, if the same change to ff_t occurs on January 20^{th} this implies that the average target FFR for the final 10 days of the month will be 300bp lower than was

expected. Using this methodology in an event study setting the authors find that a 25bp surprise decrease in the target rate leads to a 1% increase in equity prices. [37] follows up on this study with a similar methodology where equity and futures prices are calculated in the 10-20 minutes directly prior to and following FOMC announcements. The author finds that in international markets a 25bp decrease in the unexpected target rate leads to 1.5%-2.0% increase in equity indexes.

A different approach to the same topic is that of [28]. The authors address the issue of determining correlation between monetary policy and equity shocks with the assumption that the variance of monetary policy shocks is greatest on FOMC meeting days, in what they call "identification through heteroskedasticity". Using data from these event days and 1 day prior to the events the authors find that increases in short-term interest rates have the predictable effect of depressing stock markets, of a similar magnitude to previous studies. This paper is related to [28] in that it incorporates volatility in monetary policy and equities in order to better understand their correlation, but the measurement of the monetary surprise variable originates in [22].

2.3 Model

2.3.1 GARCH

Beginning with [12] economists began to take seriously the idea that in order to study high frequency time-series data one must account for the clustering of volatility over time. This implies that a given time-series variable (say, daily change in return for a stock market index) experiences periods of low and high variance, such that the variance at any point in time depends on past squared errors and lagged variances. [2] formalized this approach in the generalized autoregressive conditional heteroskedasticity (GARCH) model, later adapted to a multivariate setting. The foundational univiariate GARCH (1,1) model can be represented as:

$$Y_t = \mu_t(\theta) + \sqrt{h_t(\theta)}\epsilon_t \quad \epsilon_t \sim N(0, 1)$$
(2.1)

$$h_t = \omega + \beta h_{t-1} + \alpha \epsilon_{t-1}^2 \tag{2.2}$$

Here the unconditional variance of Y_t is the variance of the normally distributed unconditional error term ϵ_t , but by conditioning on information from previous periods Y_t has a conditional variance of h_t . This h_t value varies over time in a manner similar to an ARMA(1,1) process, so that h_t is observed to have many consecutive periods of large values followed by periods of small values.

Applying this model to the multivariate case is not straightforward. A naive approach would be for the covariance matrix H_t to progress over time in the same way as (2.2), simply in matrix form. However, this ignores the problems of imposing positive definiteness and stationarity on H_t , as well as how to reduce the (potentially quite large) number of parameters and achieve identification. While there are many possible parameterizations in the literature [3], I implement the Dynamic Conditional Correlation (DCC) model of [12] because it achieves the requirements just mentioned and maintains easy interpretability of the parameters. The model is parameterized according to 2.3-2.6.

$$Y_t = \mu_t(\Theta) + H_t(\Theta)^{\frac{1}{2}} \epsilon_t \quad \epsilon_t \sim N(0_n, 1_n)$$
(2.3)

$$H_t = D_t R_t D_t \tag{2.4}$$

$$D_t = diag(h_{11t}^{\frac{1}{2}}, \dots, h_{NNt}^{\frac{1}{2}})$$
(2.5)

$$R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \Psi_{t-1} + \theta_2 R_{t-1} \quad \theta_1 + \theta_2 < 1$$
(2.6)

Here the elements of D_t represent the conditional variances while R_t is the conditional correlation matrix. The constant R matrix is fixed at the unconditional correlation matrix

of the data, and Ψ_t is in correlation form with off-diagonal elements determined by the standardized residuals, $u_{i,t}$:

$$\Psi_{ij,t-1} = \frac{\left(\sum_{m=1}^{M} u_{i,t-m} u_{j,t-m}\right)}{\sqrt{\left(\sum_{m=1}^{M} u_{i,t-m}^{2}\right)\left(\sum_{m=1}^{M} u_{j,t-m}^{2}\right)}}$$
(2.7)

The value of m is chosen to be any value greater than n (the number of dependent variables) in order to assure positive definiteness of Ψ_{t-1} and therefore R_t . Application of this model to financial data has one major shortcoming: the assumption of joint normality. Many types of high frequency time series data exhibit much fatter tails than are allowed by the normal distribution, so it is possible that more efficient inference can be achieved by a more appropriate specification of the marginal distributions of the data.

2.3.2 Copula Functions

Since [30], copula functions have become a popular and tractable way to specify a joint distribution for a set of known margins. What Sklar found in this seminal work was that while a given multivariate distribution defines the entire set of univariate margins by simply integrating out the other parameters, a copula function can do the opposite: begin with *any* univariate margins and determine the joint distribution. For a function to qualify as a valid copula it must meet the following criteria:

- 1. $C(1, ..., 1, a_p, 1, ..., 1) = a_p$ for all $a_p \in [0, 1]$
- 2. $C(a_1, ..., a_q) = 0$ if $a_p = 0$ for any $p \in \{1, ..., q\}$
- 3. C is q-increasing, i.e. any hyperrectangle in $[0,1]^q$ has non-negative C-volume

This general definition is met by a wide array of functions, many of which are mentioned in [24]. In estimation of the model I use the Gaussian copula function because it is based on a common and tractable distribution, and most importantly because it maintains symmetric dependence (a property not shared by most common copulas) which implies that the correlations of the dependent variables are also symmetric. The Gaussian copula takes the following form:

$$C(u|\Omega_t) = \Phi_q(\Phi^{-1}(X_{1t}), ..., \Phi^{-1}(X_{qt})|\Omega_t)$$
(2.8)

Here Φ_q is the q-variate normal cumulative distribution function with correlation matrix Ω , Φ^{-1} is the normal inverse cdf, and $u_1, ..., u_q$ are the q dependent variables following (possibly different) univariate distributions. It is easily seen that $C(u|\Omega)$ is a copula function according to the properties listed above, and that the output will be a properly specified multivariate distribution function according to the probability integral transform. According to [32] we can express the joint density function as:

$$C(Y_1, Y_2|\Omega) = \Omega|^{-\frac{1}{2}} exp(-\frac{1}{2}Z'\Omega Z) \ln(f_1(y_1)) \times \dots \times \ln(f_N(y_N))$$

where $Z = (\Phi^{-1}(F_1^{-1}(Y_1)), \dots, \Phi^{-1}(F_N^{-1}(Y_N)))$ (2.9)

2.3.3 Copula-GARCH

Following on the work of [27] and [21], the copula-MGARCH model improves on the assumption of joint normality found in the standard multivariate GARCH model by specifying marginal distributions for each dependent variable separately and then determining a joint distribution using a copula function. In the case of just two variables the marginal distributions can be given as:

$$Y_1 \sim f_1(\theta), \quad Y_2 \sim f_2(\theta) \tag{2.10}$$

The Gaussian copula then forms a joint distribution of these two variables with the assumption of a data generating process for an underlying variable Z, such that $Z \sim N(0, \Omega)$, with some correlation matrix Ω . This underlying vector of variables gives us the observed variables according to:

$$Y_i = F_i^{-1}(\Phi(z_i))$$
(2.11)

Following [32], the Gaussian copula density and log-likelihood contributions are given by:

$$c(y_1, y_2|\Omega) = |\Omega|^{-\frac{1}{2}} exp(-\frac{1}{2}Z'\Omega Z)(f_1(y_1))(f_2(y_2))$$
(2.12)

$$l_t(y_{1t}, y_{2t}|\Omega_t) = \sum_{t=1}^T -\frac{1}{2}\ln|\Omega_t| - \frac{1}{2}Z'\Omega_t Z) + \ln(f_1(y_1)) + \ln(f_2(y_2))$$
(2.13)

The process that determines H_t remains the same as in (2.13) but this enters the Gaussian copula in correlation form, such that:

$$\Omega_t = corr(H_t)$$

In the application to monetary policy and equities I also specify parameters for the marginal distributions. Changes in both SP and MP follow a random-walk with Student-t distributed error terms, as in 2.14

$$\Delta M P_t \sim t_{\nu_1}$$

$$\Delta S P_t \sim t_{\nu_2}$$
(2.14)

2.4 Data

The equity price variable is calculated from daily percent changes in the S&P 500 index, which incorporates many stocks from various sectors and is considered to be representative of the equity market as a whole. Figure 2.2 presents data on this variable from January 1, 1998 until March 1, 2013. It appears that several periods of high volatility exist, followed by sustained periods of low volatility. This suggests that a GARCH representation for the variance of this data series is reasonable and should improve inference in the model. As well, a fat-tailed distribution would seem to be a wise choice to represent these returns, as there are a high number of large deviations relative to the mean, which is approximately 0. This agrees with the finance literature on this subject, which tends to find asset prices to display greater kurtosis than that of a normal distribution.

Measurement of the monetary shock variable is similar to that in [4]. The policy shock for a given day is calculated as the difference in the implied rate on the FFF contract on that day, weighted according to the timing of the next scheduled FOMC meeting. For each day I use FFF contract data for the month in which the next scheduled FOMC meeting is to be held. The FFF values are then weighted according to (2.15). Here d is the day of the next meeting, D is the number of days in the month of the next meeting, and $ffr_{i,t}$ is the implied rate of the average target FFR from the FFF contract for i months ahead. Due to the spacing of FOMC meetings approximately every 6 weeks, this means that a scheduled meeting could be 0, 1 or 2 months ahead. Data is collected on FFF contracts of these maturities. This is convenient because FFF contracts less than 3 months forward tend to have the highest liquidity. An additional point regarding (2.15) is that for observations that reference a meeting to occur within the last 3 days of a month, the weighting becomes so large that these points outweigh the rest of the sample, so I follow [4] by including these observations in unweighted form.

$$MP_t = \frac{d}{D-d} (ffr_{0,t} - ffr_{0,t-1})$$
(2.15)

While MP_t does reflect the change in price of an FFF contract a given number of months

ahead and therefore the unexpected change in the average target FFR for that month, it should be noted that this does not capture changes to future monetary policy as a whole. It could be the case that the real monetary policy shock on a given day comes in the form of unexpected forward guidance for 2 or 3 years into the future, which the measure I have constructed here will not be able to measure. A specification that does consider incorporating unexpected changes to monetary policy expectations at all horizons is considered in the third chapter of this dissertation. The approach there is to decompose shocks to monetary policy in terms of the 'target', or current rate, and the future 'path' of the policy rate, as in [14]. However, for this paper I restrict the analysis to the next meeting.



Figure 2.1: Plots of daily changes in FFF contracts for 0, 1 and 2 months ahead (bp)



Figure 2.2: Plots of daily % changes in S&P500 index and the monetary policy shock (100bp).

A time-series plot of MP_t is given in Figure 2.2. This plot indicates the possibility of sustained periods of high and low volatility, as well as many more observations in the tails of the distribution than would be likely under a Gaussian distribution. Descriptive statistics for both MP_t and S&P500 variables can be found in Table 2.1.

	S&P% daily change	Monetary policy surprise (100 bps)
Mean	0.017	0.0
Variance	2.015	0.48
Minimum	-9.000	-2.90
Maximum	10.800	2.27

Table 2.1: Descriptive Statistics

2.5 Estimation

I use Bayesian methods to estimate the model. Prior distributions are set on each of the parameters and then the mean and variance of the joint posterior distribution are calculated to describe the distribution of each parameter individually. Since the likelihood function for the copula-MGARCH model is completely intractable, no closed form solution for the posterior parameter distributions exist. The solution is to instead draw samples from the posterior distributions by way of the Metropolis-Hastings (MH) algorithm, so that the posterior moments are estimated from the sample of draws.

The prior distributions on the model parameters are chosen to satisfy stationarity of the volatility evolution equations, by amending the basic model given by (2.3)-(2.6). The model should work best if I can parameterize it in a way such that all the parameters are unbounded, but still meet the positive-definiteness requirements. All of the parameters in (2.4)-(2.6) are restricted to be positive, so I set a normal prior on the square root of each of these. The specific means and variances for the priors on each parameter are decided according the MLE estimates of the likelihood, as explained later. The variances are set at what I expect

is a large enough value to encompass any reasonable parameter values within two standard deviations. All priors are specified in Table 2.2.

Because the posterior distribution is intractable, a numerical sampling method is needed. I use a common form of the MH algorithm known as the Random Walk Metropolis-Hastings (RW-MH) algorithm from [6]. This proceeds in the following way, where θ is taken to be the set of all parameters:

- 1. Begin with parameter values, $\theta^{(i)}$ (when $i = 1, \theta^{(i)} = \bar{\theta}$ from the prior mean
- 2. Draw $\theta^{(\star)}$ from $N(\theta^{(i)}, \sigma^2)$ where σ^2 is calibrated to achieve an optimal acceptance rate

4. Repeat steps 2-3 for 25,000 iterations.

According to [13] an optimal acceptance rate is between 25-70%, which the acceptance rates for each parameter approximately fall into.

The copula-MGARCH model relaxes the joint normality assumption found in the standard DCC model. I assume that the degrees of freedom parameter on each marginal distribution is fixed, according to the apparent kurtosis of the observations found in Figure 2.2. I set $\nu_1 = 5$ which reflects very heavy tail observations in monetary policy shocks and $\nu_2 = 10$ for S&P returns. This reflects the fact that the monetary policy shock variable is observed to have a very fat-tailed distribution, while equity returns are fat-tailed but not to as great an extent.

2.6 Results

Beginning from the maximum likelihood estimates given by the dcc.estimation function in R (shown in columns 1 and 2 of Table 2.2), I run 25,000 iterations of the algorithm enumerated above. After the initial 5,000 iteration burn-in, I keep the remaining sample that is found to have an acceptance rate of 31.2%. The means and variances of the sampled marginal posterior distributions are given in columns 3 and 4 of Table 2.2). The posterior moments indicate similar results for point estimates relative to the maximum likelihood estimates. However, in some cases the variances of the sample of MCMC draws from the posterior distributions are quite different than the variances found from the MLE solution, but with an even split in terms of which estimation method finds smaller variances. To be clear, the MLE method was used to estimate the MGARCH model without a copula representation for the joint distribution so the estimates can not indicate which estimation method was superior.

	PriorMean	PriorVariance	PosteriorMean	PosteriorVariance
Parameters in D_t				
α_1	0.023	0.010	0.013	0.002
$lpha_2$	0.016	0.020	0.014	0.024
β_{11}	0.112	0.079	0.131	3.0e-5
β_{12}	1.2e-07	0.020	-0.001	0.024
eta_{21}	0.015	0.005	0.053	0.002
eta_{22}	5.031	0.013	4.906	0.032
γ_{11}	0.873	0.001	0.799	2.4e-4
γ_{12}	0.002	0.747	0.019	0.164
γ_{21}	3.5e-06	0.007	0.003	1.3e-4
γ_{22}	1.0e-07	0.001	3.2e-05	0.002
Parameters in R_t				
θ_1	2.43e-07	0.021	0.009	4.0e-5
$ heta_2$	0.250	1.244	0.201	$3.0\mathrm{e}{-5}$

Table 2.2: Copula-MGARCH

Previous studies on this topic were done in an event study setting, so it seems reasonable

that less of an effect would be found in my model, where less monetary policy information is conveyed on days when the FOMC does not meet. Most of the coefficients in (2.13) is indistinguishable from zero considering that the posterior variance is greater than the mean. This implies that on a daily basis changes in equity prices do not impact unanticipated monetary policy.

Regarding the volatility parameters, interpretability is much easier when considering how the parameters affect the time-path of the variances and correlations over time. The plots of the parameters and their associated 95% HPD intervals present a similar picture to what we observe in the original time series. The plot of the conditional correlation is tightly distributed around the mean of -0.0503. This is barely different from zero, and seems to vary little over time. From this it seems that the conditional correlation between the two variables is mildly negative but also does not vary in a noticeable trend over time.



Figure 2.3: Conditional correlation over time (with 95% HPD interval)

In terms of volatility parameters, the results seem to be very similar to those derived as maximum likelihood estimates from the standard DCC model. It does seem that the conditional correlations are significantly negative at all times but the magnitude is quite low. This is evidence against a relationship between the monetary policy variable and equity markets in terms of a spillover in volatilities. Conditional volatilities tend to vary over time as seen in Figure 2.4 and Figure 2.5, and conditional correlations are quite low and relatively constant



Figure 2.4: Conditional variance of S&P returns over time (with 95% HPD interval).



Figure 2.5: Conditional variance of MP returns over time (with 95% HPD interval).

through time.

These results are not necessarily an indication that the copula-MGARCH model does not offer improvements on the standard multivariate GARCH model. From the estimation results it seems that parameter estimates are very close, even though the normality assumption is likely to be restrictive for these two variables. It may be that the presence of many zeros in the monetary policy variable is adversely affecting my ability to estimate the parameters. Allowing the degrees of freedom parameters in the specification of the marginal distributions to vary might improve the fit of the copula-MGARCH model overall, but it seems unlikely this would make a large difference in the current application. My findings here suggest that although there is merit to representing each of the univariate time-series by either a GARCH or stochastic volatility model, there is little gain from modeling the two series jointly. Correlations between the monetary policy shock I have calculated and changes in equity prices do not exist in the sample I have studied.

Chapter 3

Federal Reserve Communication and its Time-Varying Impact on the Yield Curve: A Dynamic Nelson-Siegel Model for Daily Data

3.1 Introduction

On December 16, 2008 the U.S. economy entered unfamiliar territory when the Federal Funds Rate (FFR) target was lowered to an historic low of 0-25bp. In the months prior there had been a series of often large rate cuts but this particular policy action was unique: the target was now constrained from below, according to the zero lower bound (ZLB) on nominal interest rates. At the ZLB the Federal Reserve had given up the power to exert downward pressure on short term interest rates. In a time when expansionary policy is especially necessary this limit on the Fed is problematic. Policymakers have responded by

shifting the emphasis of monetary policy from the short end of the yield curve to longer maturities.

With its shift in focus to medium and long maturity yields, the Fed has been forced to rely more heavily on unconventional monetary policy. This has primarily come in two forms: 1. quantitative easing (QE), and 2. 'communication' regarding future plans for the target rate (termed the 'path' of policy). Unlike quantitative easing, communication (explicit or implicit) concerning the path has been disseminated to the markets via official Federal Open Market Committee (FOMC) announcements for over twenty years. Communication therefore represents the only aspect of monetary policy that spans the pre- and post-crisis period, and offers the best glimpse into how market sensitivity to policy has changed over time.

Communication can be a difficult policy tool to manage for two reasons. First, the important aspect of communication, like more traditional policy, is the surprise component. The impact of a rate change on financial markets will derive from how unexpected the rate change is; the expected component of a rate change should already be priced into the market. The same principle applies to path changes: the expected component of the Fed's published statements should be anticipated by the market, so that changes to asset prices on an event day are only correlated with whatever is unexpected. Market perceptions must be understood if Fed communication is to be effective.

The second complication for communication is that the Fed's actual intended path change is up to market interpretation. Whereas the actual target change on an FOMC meeting day is a matter of fact, the precise meaning of what the Fed says in its press release is often up for debate. When the minutes from the meeting are released two weeks later, market agents may have to revise their interpretation once again. Likewise with communication, the surprise element of a path change is the difference between what the market expects prior to the change and how the market interprets the Fed statement. This can make the impact of the Fed's words highly uncertain. To understand how the ZLB alters the impact of communication, it is first important to have the correct interpretation for how the bound affects the yield curve in general. The ZLB is traditionally viewed as a binary state: either the lower end of the FFR target range is zero, or it is not. At any point in time whether the ZLB binds on the target rate is solely determined by the Fed and is directly observable. However, the ZLB takes on a continuous, unobservable form in reference to the yield curve as a whole. The value at which the yield of a particular maturity is bound from below depends on: 1. the rate at which the market discounts future returns 2. the length of time which the ZLB is expected to bind on the short rate and 3. the time-varying risk premium. Suppose the market expects the Fed to maintain a target rate at zero for the next 12 months. Any event that would typically lower the yield on 9 or 12 month bonds, whether macroeconomic or monetary shocks, will have much less of an effect than if the short rate was unbounded. The actual value at which the yield is bound is then a function only of the market discount rate and the risk premium, according to the term structure of interest rates. As both of these factors are unobservable, the value at which the yield of a particular maturity is bound can not be known a priori and may change with time. The largest maturity for which the ZLB binds can be thought of as its "tightness".

The continuous nature of the ZLB creates a dilemma for monetary policy makers: as communication becomes more successful in altering market expectations the sensitivity of interest rates to communication will likely decrease. If the market expects the ZLB to persist for the next 24 months, say, maturities under 24 months should be much less sensitive to path shocks. No information can cause the market to lower its expectation for the short rate any further up to the 24 month horizon. Likewise, as the longer maturity yields are a function of expected future medium-term yields as well, sensitivity of long-term rates to a path shock should also fall. This implies two hypotheses: 1. path shocks should have less of an impact on the short end of the yield curve beginning in December 2008 and 2. the effectiveness of monetary policy should tend to decrease as the tightness of the ZLB increases. In order to study this aspect of monetary policy I implement a Dynamic Nelson-Siegel (DNS) model for daily changes in yields. This requires the first-differenced DNS model in order to circumvent the issue of non-stationarity that arises with high-frequency data. Bayesian estimation via the Gibbs sampler is then used for estimation, generating draws from the conditional posterior distributions of the parameters and the latent states. The mean and variance of the conditional posteriors for the parameters and latent states are derived from the Bayesian update for Gaussian conjugate priors and the Kalman filter, respectively.

This paper makes a significant contribution to the literature on unconventional monetary policy. By expanding the first-differenced DNS model to daily data and including macroeconomic and monetary policy factors, I obtain a model that reflects yield curve changes at a shorter horizon than in any previous work. Whereas DNS models are typically estimated with quarterly data to facilitate the inclusion of stock variables like capacity utilization and inflation, my approach can measure the sensitivity of the yield curve to macroeconomic or financial variables on a daily basis. This allows the monetary authority to have an accurate picture of the effects its communication to the market is having, and to adjust policy accordingly. I find that the sensitivity of yields to path shocks was at its height in 2006-2007 with a rapid decline in 2008-2011, so that by the third quarter of 2011 the effect of a one standard deviation path shock on yield maturities under two years was less than 50% of what it was in early 2007. By 2013 that trend began to reverse, as the market seems to expect the target rate to move away from the ZLB sometime in 2015.

The rest of the paper is structured as follows: Section 2 provides a literature review and Section 3 offers a description of the data used in my analysis. In Section 4 the model is presented. In Section 5 I lay out the estimation scheme and Section 6 includes results on parameter estimates and impulse response functions. Section 7 concludes with discussion of the results and plans for future work.

3.2 Literature Review

This paper draws primarily from the literature on central bank communication, which has seen renewed interest since the recent financial crisis. This line of research began with the creation of a method to measure monetary policy shocks from the principal components of asset prices. Originally, this was to calculate the surprise component of a target rate change. [4] find the unexpected target rate shock from the difference in the value of the current month's Federal Funds Futures (FFF) contracts. These contracts are priced based on the *average* federal funds rate expected to obtain for a given month. On an event day (FOMC meeting or minutes release), the daily change in the price of an FFF contract can be used to extract an expectations surprise. The authors then use this measure in regressions to explain variation in equity market markets.

A similar approach can be used in regards to communication. In the period from 1990-2004 the FFR target was free to vary but FOMC announcements often contained content on future policy. To understand the impact of communication concerning future periods [14] collect data on assets that reflect daily changes to monetary policy expectations. Daily changes in FFF contracts for up to three months ahead reflect how the market revises expectations for the next scheduled meeting, in the same way spot month contracts reflect the change in expectations for the current meeting. As well, daily changes to Eurodollar futures contracts indicate the change to expected interest rates at longer horizons. For a set of FFF and Eurodollar futures contracts up to two years in maturity the authors find that the first two principal components of the data mimic surprise changes to monetary policy very closely. The first principal component can be seen to correlate with unexpected changes to the current target rate, as it has a 95% correlation with the spot month FFF changes. The second principal component is then observed to follow shocks to the path of rates. By comparing to the actual content of FOMC press releases, the second principal component seems to track changes to the expected future target. [14] estimate that a 100bp positive innovation in the path rate leads to a statistically significant 40bp, 36bp and 27bp increase in 2-, 5- and 10-year yields, respectively.

Adapting this methodology to different settings, a variety of papers suggest that path shocks have less of an effect on yields for periods in which the ZLB binds on the short rate. For data only in the ZLB period, [38] finds the first principle component of FFF and Eurodollar futures contracts to reflect surprise path changes. This is to be expected, as the short rate is bounded and the first principal component explains most of the variation in the underlying asset prices should only indicate changes to the path. The author finds that a 1 standard deviation negative path shock leads to a 6bp drop in the 2-year Treasury yield and a 12bp drop in the yield on 10-year Treasuries. This is a similar direction of effect to what was found in [14]. [10] include further data points in their sample from both inter-meeting announcements and minutes release days. The authors find that a negative 100bp path surprise leads to a 32bp, 35bp and 29bp decrease on 2-, 5- and 10-year yields in the period from 1990-2007. From 2007-2013 the effect is slightly less, leading to a decrease of 24bp, 32bp and 29bp for the same maturities. This agrees with |5| which finds that the effect of a path shock has decreased somewhat from 2008, particularly for maturities up to two years. The consensus from the literature is that a negative (positive) surprise to the expected path of future rates is correlated with a decrease (increase) in yields on Treasury securities, and this effect is diminished in magnitude during the ZLB period.

As the short rate is expected to stay at zero for a longer duration yields should become less sensitive to shocks. [33] find that yields of maturities up to two years are less responsive to macroeconomic shocks the longer the ZLB is expected to persist into the future. By April 2012, 3-, 6- and 12- month yields were found to be completely insensitive to news about the labor market, prices and production. The sensitivity of 2-year yields was not quite zero but had declined by more than 50% and no significant change in the responsiveness of 5 or 10 year yields was found. The authors match this to survey data from professional forecasters. From December 2008 to October 2011 forecasters expected the federal funds target rate to stay below 25bp for roughly four quarters, at which point the expected duration increased to seven quarters or more. From 12-month rolling regressions [10] find that the effect of a path surprise on Treasuries up to 10-years in maturity has tended to decrease within the zero lower-bound period. The authors suggest that this is due to the increasing tightness with which the ZLB binds during this period, which is validated by [33]. However, a different approach would be to explicitly describe changes to the shape of the yield curve as a function of path shocks. In Section 4 I provide a model that does just that.

3.3 Data

Data for this paper covers the period from September 2004 to April 2014, including 2390 trading days. Yield data is collected from the St. Louis Federal Reserve's FRED database. One reason that a Dynamic Nelson-Siegel model is needed for this data can be found in Figure 3.1. The figure contains plots of daily changes in empirical estimates of the level, slope and curvature of the yield curve for each year from 2004-2014. The empirical level factor is given by the daily change in the 10-year yield, the slope by the daily change in the difference between the 10-year and 6-month yield, and the curvature by daily change in the 'butterfly' spread, calculated as (10year + 6month - 2 * 5year). Days on which the FOMC had a meeting or published a press release are plotted in red. This figure shows that there can be quite a bit of variation in the variability of the shape of the yield curve over time. The greatest volatility in the shape factors is found roughly between 2007 and 2011, consistent with a period of overall market turmoil. As we will see in Section 4, these changes in the amount of variation in yields can be accounted for by the parametrization of the DNS model.

In the spirit of a traditional macro-finance DNS model like [8] I must include in my model



Figure 3.1: Daily changes in level, slope and curvature factor (bp), 2004-2013 (FOMC meeting days in red).

variables for inflation and growth that may affect the evolution of the yield curve shape factors. Daily changes in inflation are calculated from Treasury Inflation Protected Securities (TIPS). The par value of these securities is linked to the value of inflation so that daily changes should reflect the update to the market expectations for inflation.

To measure daily changes in production I use the Aruoba-Diebold-Scotti Business Index (BI) found on the Philadelphia Federal Reserve website. This measure is a daily, seasonally



adjusted index that aggregates data on: weekly initial jobless claims, monthly payroll employment, industrial production, personal income less transfer payments, manufacturing and trade sales, and quarterly real GPD. The index is scaled to have mean zero, with positive values reflecting a growing economy. Though this measure is far from a perfect analog to capacity utilization (as is typically found in a DNS model), it should mimic changes to the health of the economy fairly well. Plots of TIPS and BI are provided in Figure 3.2.

The measurement of daily fluctuations in the expected path of the FFR target is the most involved aspect of the data collection for this paper. I follow [14] and [10] in using Federal Funds Futures (FFF) rates and Eurodollar futures to calculate monetary policy shocks as the principal components of all of these asset prices. FFF contracts change in value based on the expected average federal funds rate for a particular month. Specifically, at the end of each month the value of a 100 dollar contract for the spot month is worth 100-(the average federal funds rate for all trading days in that month). This can then be translated into a value for how much the market expects the federal funds rate to change at the next FOMC



meeting. An increase (decrease) in the price of a spot month contract indicates an expected decrease (increase) in the federal funds rate of:

$$mp_{1t} = \frac{d}{D-d}(ffr_{0,t} - ffr_{0,t-1})$$
(3.1)

where d is the day of the next meeting, D is the number of days in the month, and $ffr_{0,t}$ is the implied rate of the federal funds rate from the FFF contract for the spot month.

The above measure in (3.1) gives the change in expectations for the federal funds rate at the time of the next scheduled FOMC policy meeting. A similar calculation for the expectational change in the target at the next policy meeting determines the shock to the path of monetary policy. The change to the FFF contract at the policy meeting j months ahead should be the change to $ffr_{j,t}$ that is not due to the change in $ffr_{0,t}$. For this reason the expected future change of the FFF rate is given by:

$$mp_{2t} = \left((ffr_{j,t} - ffr_{j,t-1}) - mp_{1t} \frac{d_2}{D_2} \right) \frac{D_2}{D_2 - d_2}$$
(3.2)

where d_2 and D_2 are the number of days in the month at the next event day. In general, FOMC meetings are scheduled approximately six weeks apart, so for the calculation of mp_1 and mp_2 I acquire data on FFF contracts for the spot month and from 1-3 months ahead.



Figure 3.4: 1-year rolling regression of yields on the path shock, in bp.

While FFF contracts are available up to 12 months into the future, volume for these securities is quite low more than 3 months ahead. If the market is using Fed communication to update its monetary policy expectations far beyond the next three months, an accurate formulation of the path variable must include changes in expectations for at a significantly longer horizon. For this I use daily changes in Eurodollar futures. Like FFF contracts the value of a Eurodollar futures contract increases (decreases) as the expected interest rate on Eurodollars decreases (increases). The value of Eurodollar futures contracts are tied to international interest rates beyond just the federal funds rate, so I must make the assumption that the only factor moving the value of these contracts on FOMC event days is the FOMC announcement. Daily changes in Eurodollars futures from 1 to 10 quarters ahead are included for the present application.

Collectively, data from FFF and Eurodollar futures contracts make up 12 different variables on 159 event days, an unwieldy amount of information to include as distinct measures of monetary policy shocks. To condense information and have a more straightforward measure of path shocks I use principal components analysis on these variables, as in earlier work on this subject. The first two principal components seem to follow the pattern we would expect of shocks to both the short-term target rate and the medium-term path. Of note is the order of the principal components: in the period covered by [14] shocks to the short rate were much more important than shocks to communication, so the first principal component is related to shocks to the target rate. In my application the interpretation of the principal components is flipped. This agrees with the observation that the target rate has been bounded by the ZLB for all of the 2009-2014 period, so that most policy shocks should be related to expected future rate. The target and path shocks are displayed in Figure 3.3.

Since the data on target and path shocks are not readily available in [10], to compare results of the principal component analysis I replicate the rolling regressions from that paper in Figure 3.6. This also acts as a prelude to later results on the time-varying effect of a path shock. Though I would not expect identical results because the time frame of the data in each paper is slightly different, I find results similar to those in [10]. I plot the effect of a 1 standard deviation increase in the path shock on yields, resulting in a 14bp increase in 2-year yields in 2007, while by 2013 the same shock led to only a 2bp increase in the same yield. Yields on 5- and 10-year bonds also decrease in sensitivity to a path shock during the same period, but only slightly. From 2007-2013 the effect of a 100bp path shock on 5-year yields falls from 12bp to 5bp, while the effect on 10-year yields falls from 10bp to 6bp. These results are consistent with the findings of [10]. Importantly, these results are also consistent with increasing tightness of the ZLB on medium-term yields during this period. Two-year yields became significantly less sensitive to path shocks, while the fall in sensitivity of 5- and 10-year yields is comparatively small.

It is not as easy to verify the validity of the target shock variable, as there is no point of reference for this in previous work. However, the fact that the path shock is similar to previous papers suggests the target shock is also estimated appropriately. In Table 3.1 I provide correlations of the target and path shocks with each of the underlying asset prices. The path variable is somewhat correlated with 3 to 12 month changes to interest rate expectations, and much more highly correlated with 12 to 30 month rate expectations. The converse is true of the target shock.

Table 3.1: Correlations between estimated monetary policy shocks and underlying asset price changes.

	mp_1	mp_2	edf_3	edf_6	edf_{12}	edf_{18}	edf_{30}
$target_t$	0.965	0.551	0.454	0.291	0.248	0.225	0.156
$path_t$	0.185	0.291	0.670	0.839	0.944	0.950	0.901

3.4 Model

To model the shape of the yield curve one must account both for its cross-sectional properties and variation over time. As well, it is desirable to impose a smooth curve connecting yields of all maturities. A popular approach to this problem was developed by [25]. Looking only at the cross-sectional properties of the yield curve, the authors give Treasury yields the functional form:

$$y(\tau) = \beta_1 + \beta_2 \frac{1 - e^{-\lambda\tau}}{\lambda\tau} + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$
(3.3)

Here β_1 , β_2 , β_3 and λ are parameters, while τ is the maturity (in months) of the yield (y). This function has many nice properties that fit the yield curve. First, as τ goes to ∞ , the yield approaches the constant β_1 which signifies the long rate. This coincides with the observation that long-term yields appear to approach a constant limit as maturity increases. Second, under this parameterization the curve can take on any shape that is either globally concave of globally convex, which goes along with historical data on the yield curve, like the plots presented in 3.1. This implies that either upward or downward sloping portions of the yield curve are possible, but with only one hump to its shape.

The third advantage of the functional form of the DNS model is that it can easily be transformed to account for the time-varying nature of the yield curve. Coefficients from the cross-sectional model take on the interpretation of the level, slope and curvature of the dynamic yield curve. This is seen by rewriting the yield curve with time-varying β 's as:

$$y_t(\tau) = \beta_{1t} + \beta_{2t} \frac{1 - e^{-\lambda\tau}}{\lambda\tau} + \beta_{3t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$
(3.4)

The time-constant factor loadings are therefore given by $(1, \frac{1-e^{-\lambda\tau}}{\lambda\tau}, \frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau})$. Unlike the cross-sectional model, we can now interpret the time-varying β s as latent factors, while λ is the free parameter that governs the time-constant factor loadings. If we vary the maturities for a fixed λ , we see that plots of the factor loadings look very much like what we expect to be the three principle shape factors of the yield curve. The long run level (β_1) of the yield curve stays constant, the slope (β_2) tends to decrease as maturity increases and curvature (β_3) tends to peak in the 24-36 month range. This can be seen in the plots of factor loadings when $\lambda = 0.0609$, found in FIgure 3.5. Note that the 'slope' factor as estimated in the model



Figure 3.5: Factor loadings by maturity

is the inverse of the actual slope of the yield curve. By stacking the yields and factor loadings over maturities, (3.2) can be rewritten in matrix form as

$$y_t = \Lambda f_t + \epsilon_t \quad t = 1, \dots T \tag{3.5}$$

Here y_t is an $(n \times 1)$ vector, where *n* is the number of distinct maturities being considered. Λ is an $(n \times 3)$ matrix with row *j* given by $(1, \frac{1-e^{-\lambda \tau_j}}{\lambda \tau_j}, (\frac{1-e^{-\lambda \tau_j}}{\lambda \tau_j} - e^{-\lambda \tau_j})$. f_t is a 3×1 vector of the shape factors:

$$f_t = \begin{bmatrix} l_t \\ s_t \\ c_t \end{bmatrix} \quad t = 1, \dots T \tag{3.6}$$

where l_t is the level, s_t the slope and c_t the curvature of the yield curve.

For the latent factors the transition equation is given as a VAR(1):

$$f_t = A f_{t-1} + \eta_t \quad t = 1, \dots T \tag{3.7}$$

A is a 3×3 matrix of parameters. The error terms in the observation and state equations

are assumed to be orthogonal, with $\eta_t \sim N(0, \Omega_{\eta})$ and $\epsilon_t \sim N(0, \Omega_{\epsilon})$.

The application I consider in this paper requires daily data. This creates a problem for estimation of (3.7). Even in the typical DNS model for monthly data the coefficient matrix A is nearly non-stationary. With daily data the latent factors are in fact all approximately non-stationary as indicated by my preliminary analysis. In this case, variance estimates for parameters and confidence intervals do not have the properties that they would under stationarity, so that inference in this model becomes very difficult.

A solution is to use the first-differenced DNS model of [39]. One can see from timedifferencing (3.5) that the factor loadings will stay the same, while f_t is replaced by $\tilde{f}_t = (f_t - f_{t-1})$. Similarly y_t has been replaced by $\tilde{y}_t = (y_t - y_{t-1})$. The structure of the observation equation is therefore unchanged except that the data is now the daily difference in the yields and the daily difference in the underlying latent factors, which I write as \tilde{y}_t and \tilde{f}_t , respectively. Without loss of generality the error terms $\tilde{\epsilon}_t = (\epsilon_t - \epsilon_{t-1})$ and $\tilde{\eta}_t = (\eta_t - \eta_{t-1})$ are kept in the observation and state equation, though their actual distribution is likely not the same as in the original model. The yields-only first-differenced DNS model can then be written as

$$\begin{split} \tilde{y}_t &= \Lambda \tilde{f}_t + \tilde{\epsilon}_t t = 1, \dots T \\ \tilde{f}_t &= A \tilde{f}_{t-1} + \tilde{\eta}_t \quad t = 1, \dots T \\ \begin{bmatrix} \tilde{\epsilon}_t \\ \tilde{\eta}_t \end{bmatrix} \sim N_{n+3} \begin{bmatrix} \Omega_{\tilde{\epsilon}} & 0 \\ 0 & \Omega_{\tilde{\eta}} \end{bmatrix} \quad t = 1, \dots T \end{split}$$

Macroeconomic variables have also been incorporated in the DNS model, beginning with [9] and [1]. Beyond the three latent factors, additional observed factors are typically included for real activity, inflation and monetary policy. Only the latent factors are present in the observation equation, while the macro and latent factors are included in the VAR(1) state equation. Monthly data on macro variables typically take the form of the level of capacity utilization, the level of inflation and the monthly average federal funds rate. To briefly summarize this line of literature, he federal funds rate is closely correlated with the slope factor and capacity utilization is correlated with the level factor. The curvature factor has been found to be unrelated to any of the macro factors.

Macroeconomic measures TIPS and BI are combined into $T \times 2$ matrix m, and shocks to the monetary policy target and path are given by $target_t$ and $path_t$. As is common in the literature, the state (3.7) is reformulated as:

$$\begin{bmatrix} \tilde{f}_t \\ m_t \end{bmatrix} = A \qquad \qquad \begin{bmatrix} \tilde{f}_{t-1} \\ m_{t-1} \end{bmatrix} + b(target_t) + c(path_t) + \eta_t \quad t = 1, \dots T$$

Here the A matrix is redefined to have dimensions (5×5) .

As discussed earlier, an additional element to include in the model is that the effect of the path shock on yields likely varies considerably around the time that the zero lower bound began to restrict yields. One possible solution to this problem is to allow the coefficients to be governed by a regime switch at the point when the target rate came up against the ZLB, as in [7]. However, this approach misses the potentially continuous coefficient change that has been observed in [10]. If agents believe that short-term nominal rates will be fixed at zero for longer periods of time, medium-term bonds will also begin to have less sensitivity to monetary policy shocks. This has been found to be true of shocks to macroeconomic factors in [33]. If this is also true in regards to path rate surprises short and medium yields should not react as much to path surprises the longer the market expects the zero lower-bound to restrict the target rate. The state equation is further modified to account for time-varying

parameters on the path shock with the addition of c_t :

$$\begin{bmatrix} \tilde{f}_t \\ m_t \end{bmatrix} = A \qquad \qquad \begin{bmatrix} \tilde{f}_{t-1} \\ m_{t-1} \end{bmatrix} + b(target_t) + c_t(path_t) + \eta_t \quad t = 1, \dots T$$

I assume that c_t follows a random walk with Gaussian error term:

$$c_t = c_{t-1} + \zeta_t \quad t = 1, ... T$$

where $\zeta_{tj} \sim N(0, \sigma_{\zeta j})$ for j = 1, ...5. The full macro-yields model with time-varying parameters can now be written in state space form. In what follows I suppress the *tilde* on the variables and error terms for simplicity, though the variables maintain a first-differenced interpretation as above.

$$y_t = \Lambda f_t + \epsilon_t \quad t = 1, \dots T \tag{3.8}$$

$$\begin{bmatrix} f_t \\ m_t \end{bmatrix} = A \begin{bmatrix} f_{t-1} \\ m_{t-1} \end{bmatrix} + b(target_t) + c_t(path_t) + \eta_t \quad t = 1, \dots T$$
(3.9)

$$c_t = c_{t-1} + \zeta_t \quad t = 1, \dots T \tag{3.10}$$

$$\begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix} \sim N_{n+3} \begin{bmatrix} \Omega_{\epsilon} & 0 \\ 0 & \Omega_{\eta} \end{bmatrix} \quad t = 1, \dots T$$

$$\zeta_{tj} \sim N(0, \sigma_{\zeta j}) \quad j = 1, \dots 5$$
(3.11)

In the next section I estimate the model with Bayesian methods.

3.5 Estimation

(3.8)-(3.11) represent a standard latent factor model with time-varying parameters and a nonlinear parameter λ in the factor loadings. In line with the DNS literature I calibrate λ to be fixed over time, which greatly simplifies estimation of the model. Previous work has found little reason to believe that different values of λ (within a reasonable range) have a significant impact on the results. I therefore calibrate $\lambda = 0.0609$ as in [8].

The model allows for straightforward Bayesian estimation, wherein the parameters and latent factors are estimated with the Gibbs sampler. I place multivariate Gaussian priors on A and b so that the full-conditional posterior distributions are readily available. The conditional posteriors for the latent states also have a multivariate Gaussian distribution with mean and variance determined by the Kalman filter and smoother. Additionally, to achieve identification in the model I calibrate the $\sigma_{\zeta j}$ variance parameters to be 0.01. As I will show in the next section, the value of each c_{jt} is in the range of -0.2 to 0.2, so this variance is large enough to accommodate any variation in the coefficients over time.

3.5.1 **Prior Distributions**

For the sets of parameters A and b I choose multivariate normal priors with mean and covariance given from the two-step estimation procedure of [8]. Two-step estimation works by regressing y_t on Λ for each day separately, so that the coefficients in these daily regressions estimate the f_t latent factors. In the second step of the process these estimated latent factors are regressed on both their own lags and the monetary policy shocks by simple OLS. This provides an estimate of A and b, and residuals from the observation and state equations can be used to calculate moment estimators of Ω_{η} and Ω_{ϵ} . The coefficient estimates are inconsistent approximations to the likelihood solutions but nonetheless serve as a decent starting point for more robust estimation. These prior mean values are given by \bar{A} , \bar{b} , $\bar{\Omega}_{\epsilon}$ and $\bar{\Omega}_{\eta}$, reported in Table 2.3.

In the top part of Table 2.3 the first five columns give the prior mean for A, while the last column has the prior mean for b. The lower part of the table contains $\bar{\Omega}_{\eta}$ and the prior covariances for the mean parameters can be found as $\Sigma_{\bar{A}} = (\bar{\Omega}_{\eta}^{-1} \otimes X_{A}^{\ '} X_{A})$ and $\Sigma_{\bar{b}} = (\bar{\Omega}_{\eta}^{-1} \otimes X_{b}^{\ '} X_{b})$ where

$$X_{A} = \begin{bmatrix} \tilde{f}_{1} & m_{1} \\ \tilde{f}_{2} & m_{2} \\ \vdots & \vdots \\ \tilde{f}_{T-1} & m_{T-1} \end{bmatrix} \quad X_{b} = \begin{bmatrix} target_{1} \\ target_{2} \\ \vdots \\ target_{T-1} \end{bmatrix} \quad Y = \begin{bmatrix} \tilde{f}_{2} \\ \tilde{f}_{2} \\ \vdots \\ \vdots \\ \tilde{f}_{T} \end{bmatrix}$$

The prior mean $\overline{\Omega}_{\epsilon}$ is assumed to be diagonal, with entries taken from the squared residuals of the first stage of the two-step estimation procedure:

$$\bar{\Omega}_{\epsilon} = \text{diag}(2.5e - 04, 2.6e - 04, 1.9e - 04, 1.1e - 04, 1.04e - 04, 9.7e - 05, 5.9e - 05, 8.5e - 05)$$

For the purpose of identification I assume that the covariance matrices Ω_{ϵ} and Ω_{η} are fixed at their prior values. With the assumption that both the observation and state equation have normally distributed errors, conjugate priors for the state equation parameters are given by the multivariate normal distributions:

$$A \sim N(\bar{A}, \Sigma_{\bar{A}})$$

$$b \sim N(\bar{b}, \Sigma_{\bar{b}})$$

(3.12)

Table 3.2: Prior means of A, Ω_{η} and Ω_{ϵ}

			\bar{A}			\overline{b}
	l_{t-1}	s_{t-1}	c_{t-1}	$TIPS_{t-1}$	BI_{t-1}	$target_t$
l_t	-0.019	-0.001	-0.001	0.036	0.099	0.128
s_t	0.314	0.190	0.033	-0.177	-0.088	0.098
c_t	-0.643	-0.328	-0.080	0.273	-0.139	0.052
$TIPS_t$	-0.204	-0.062	-0.030	0.178	0.067	-0.103
BI_t	0.012	0.002	0.000	-0.012	0.661	-0.019

			$ar{\Omega}_\eta$		
	l_t	s_t	c_t	$TIPS_t$	BI_t
l_t	5.70 e-03	-4.91 e-03	-4.59 e-03	2.85 e-03	-4.56 e-06
s_t	-4.91 e-03	7.89 e-03	-1.77 e-03	-2.91 e-03	6.25 e-05
c_t	-4.59 e-03	-1.77 e-03	3.31 e-02	3.80 e-04	-1.25 e-04
$TIPS_t$	2.85 e-03	-2.91 e-03	3.80 e-4	2.80 e-03	-2.77 e-06
BI_t	-4.56 e-06	6.25 e-05	-1.24 e-04	-2.77 e-06	4.04 e-04

			$\bar{\Omega}_{\epsilon}$		
	l_t	s_t	c_t	$TIPS_t$	BI_t
l_t	5.96e-03	-5.38e-03	-4.24e-03	4.70e-05	1.028e-06
s_t	-5.38e-03	8.18e-03	-1.71e-03	-1.00e-04	-6.61e-06
c_t	-4.24e-03	-1.71e-03	3.21e-02	2.14e-04	1.76e-05
$TIPS_t$	4.70e-03	-1.00e-03	2.14e-04	3.50e-03	-8.05e-07
BI_t	2.85e-03	-2.91e-03	3.80e-4	2.80e-03	-2.77e-06

3.5.2 Posterior Distributions

The full likelihood can be written as

$$p(c_t, f_t, y_t; \theta) = p(y|f_t, c_t, \theta)p(f_t, c_t|\theta)$$
(3.13)

where $\theta = (A, b, \Omega_{\epsilon}, \Omega_{\eta}, \sigma_j)$ for j = 1, ..., 5. Prior distributions were chosen as conjugates of the likelihood so the conditional posterior distributions of the parameters are easily found. According to Bayes' Rule, multiplying the likelihood by the joint prior gives the joint posterior distribution:

$$p(c_t, f_t, \theta | y_t) \propto p(y | f_t, c_t, \theta) p(f_t, c_t | \theta) p(\theta)$$
(3.14)
Because I have assumed that the covariance matrices of the error terms are known (setting a dogmatic prior on these parameters), if f_t and c_t are held constant the posterior reduces to a multivariate Gaussian distribution. In turn, when all parameters are assumed fixed, f_t and c_t are distributed as multivariate Gaussian. The mean and variance of the posterior for the parameters comes from the standard Bayesian update, and for the latent states the mean and variance are estimated via the Kalman filter and smoother.

Sampling Scheme

Estimation of the model follows in three steps within each run of the MCMC chain. First, the latent factors f_t are sampled from a multivariate Gaussian distribution with mean and variance found by the Kalman filter and smoother on (3.8) and (3.9). The same is true of the time-varying coefficients c_t where instead equations (3.9) and (3.10) are used for filtering. Finally, the coefficient matrices A and b are sampled from their multivariate normal posterior conditional on the latent factors and the time-varying covariates. The sampling algorithm can be summarized by:

- 1. Initialize parameters $A^{(0)}$, $b^{(0)}$
- 2. In the i^{th} iteration, posterior mean and variance of the latent factors are given by $m_{ft}^{(i)}$ and $v_{ft}^{(i)}$, the smoothed estimates from the Kalman filter on equations (7) and (8). The draw from the posterior is given by $f_t^{(i)} \sim N(m_{ft}^{(i)}, v_{ft}^{(i)})$ for t = 1, ...T.
- 3. In the i^{th} iteration, posterior mean and variance of the time-varying coefficients are given by $m_{ct}{}^{(i)}$ and $v_{ct}{}^{(i)}$, the smoothed estimates from the Kalman filter on equations (8) and (9). The draw from the posterior is given by $c_t{}^{(i)} \sim N(m_{ct}{}^{(i)}, v_{ct}{}^{(i)})$ fort = 1,...T.

4. Draw $A^{(i)} \sim N(\hat{A}, \hat{\Sigma}_A)$, where $\hat{\Sigma}_A = (\Sigma_{\bar{A}}^{-1} + \Omega_\eta \otimes (X'_A X_A)^{-1}$ and $\hat{A} = \hat{\Sigma}_A (\Sigma_{\bar{A}}^{-1} \bar{A} + \Omega_\eta \otimes (X'_A Y))$

5. Draw
$$B^{(i)} \sim N(\hat{B}, \hat{\Sigma}_b)$$
, where $\hat{\Sigma}_b = (\Sigma_{\bar{b}}^{-1} + \Omega_\eta \otimes (X_b'^T X_b))^{-1}$ and $\hat{b} = \hat{\Sigma}_b (\Sigma_{\bar{b}}^{-1} \bar{b} + \Omega_\eta \otimes (X_b' Y))^{-1}$

6. Repeat steps 2-5

This is consistent with Bayesian updating for a multivariate Gaussian likelihood and prior. The sampling process is repeated for 10,000 iterations with a burn-in of 500 draws. The sample moments of these draws are used to categorize the posterior distributions.

3.6 Results

3.6.1 Latent Factors

The first check on the validity of the model is the behavior of the estimated factors f_t . The median values along with a 90% Bayesian credible interval at each point in time are plotted in Figure 3.6. All three factors are estimated with relatively little variance between simulations, so that the credible intervals are not distinguishable from the median estimates. To be assured that these factors are reasonable approximations to the actual shape components of the yield curve, I calculate the linear correlation between the estimated factors and the empirical factors. The correlations are approximately 0.82 for curvature, 0.90 for slope and 0.95 for level. Thus, the factors estimated in the first-differenced DNS model are a reasonable approximation to the actual shape factors of the yield curve.



Figure 3.6: Estimated latent factors

3.6.2 Parameters

Results for parameters from the MCMC sampling scheme are presented in Table 3.3. Median estimates are quite similar to the prior values for A and b but the standard deviations of the draws from the posterior are noticeably smaller than those of the prior distributions. This suggests that the sample is quite informative about the values of coefficients. It should first be noted that inflation, business activity and the target shock were rescaled, so that the coefficients in the table are the effects of a one standard deviation change for each of these three variables.

There are several significant results from Table 3.5. The first is that the target rate shock only has a slightly positive effect on the slope factor. According to the interpretation of the slope given in section 3, the positive coefficient implies that a positive one standard deviation target shock leads to a mild 2bp decrease in the slope of the yield curve. This makes intuitive sense, as a target shock should push up short-term rates while long rates remain unaffected.

I also find reason to believe that the yield curve has more of a significant effect on my macro factors than vice versa. While the coefficients on BI and TIPS is very small, a 1 unit increase

	l_{t-1}	s_{t-1}	c_{t-1}	$TIPS_t$	BI_{t-1}	$target_t$
l_t	-0.004	-0.013	0.001	0.005	0.128	0.034
	(0.048)	(0.032)	(0.012)	(0.047)	(0.063)	(0.023)
s_t	0.301	0.216	0.031	-0.124	-0.083	0.082
	(0.058)	(0.039)	(0.015)	(0.056)	(0.074)	(0.026)
c_t	-0.687	-0.325	-0.073	0.284	-0.114	0.085
	(0.074)	(0.115)	(0.081)	(0.110)	(0.155)	(0.052)
$TIPS_t$	-0.201	-0.067	-0.026	0.161	0.098	0.063
	(0.034)	(0.022)	(0.008)	(0.034)	(0.042)	(0.016)
BI_t	0.011	0.002	0.000	-0.011	0.655	0.006
	(0.012)	(0.008)	(0.003)	(0.012)	(0.015)	(0.006)

Table 3.3: Posterior median and standard deviation

in the level factor or slope factor leads to a -0.208 and -0.071 decrease in inflation on the following day. This implies that higher long-term rates or a less steep yield curve today leads to a decrease in inflation tomorrow. The first part of this is easily interpretable: higher expected interest rates in the future are related to slower growth and lower inflation in the future. The interpretation of the coefficient on the slope factor is not as obvious. However, a more complete picture comes from plotting the impulse response functions in Section 6.3.

Table 3.4: Initial impulse response of $TIPS_t$, BI_t and $target_t$ on yields, in bp. 90% credible intervals in parentheses.

	TIPS	BI	target
3-month	-0.476	0.121	3.402
	(-0.681, -0.200)	(-0.601, 0.234)	(2.771, 4.172)
6-month	-0.420	0.096	3.224
	(-0.504, -0.078)	(-0.054, 0.266)	(2.413, 3.572)
1-year	-0.073	0.114	3.189
	(-0.312, 0.133)	(-0.121, 0.314)	(2.395, 3.826)
2-year	0.134	0.165	2.895
	(-0.109, 0.309)	(0.232, 0.501)	(2.355, 3.504)
5-year	0.221	0.198	2.358
	(-0.178, 0.478)	(-0.002, 0.443)	(1.500, 3.050)
10-year	0.345	0.187	1.611
	(-0.193, 0.421)	(0.032, 0.406)	(0.141, 2.248)

3.6.3 Macro Impulse Responses

Table 3.6 presents medians and 90% credible intervals for the initial impulse response to a 1 standard deviation increase in the two macro variables and the target shock. Because the impulses in all cases revert to almost zero by the second period, only the initial (and therefore peak) response is shown. The results in Table 3.6 suggest that daily changes in INF and BI have little impact on yields of any maturity. In no case is the median value from the MCMC samples greater than 1bp. The target shock does have some impact on yields, though it is very mild. A 1 standard deviation target shock leads to between a 2.7bp to 4.2bp increase in the 3-month yield on the event day, which falls to a 1.4bp to 2.3bp increase for 10-year yields. This is consistent with the interpretation that the target shock mainly affects the short end of the yield curve.

3.6.4 Sensitivity to Path Shocks

The most interesting aspect of this study is the role of the time-varying sensitivity of yields to monetary policy path shocks. From the results I find that the coefficients c_t are tightly estimated, with little variance between simulations. Figure 3.7 and Figure 3.8 present plots of the five coefficients over time.

A noticeable result is that the sensitivity of $TIPS_t$ and BI_t to path shocks increases significantly during the time around the most recent recession. Except for the period around the financial crisis, path shocks have had approximately the same effect on macro factors. In terms of the latent yield factors the pattern is different. From 2007 onward both the level and slope have increased their sensitivity to path shocks. These are consistent with one another, as an increase in the level is associated with a steeper slope (and therefore a decrease in the slope factor). As well, the sensitivity of curvature increases from 2004-2008, only to drop quickly and change sign by 2010. For interpretation of these results I calculate



Figure 3.7: Time-varying sensitivity of latent factors to a path shock



Figure 3.8: Time-varying sensitivity of macro factors to a path shock

the time-varying peak impulse response to a path change, according to the estimates of c_t .

3.6.5 Impulse Responses to Path Shocks

The time-varying peak impulse responses for path shocks is given in Figure 3.9. Because the sensitivity of yields to path shocks has been seen to vary over time, the impulse responses will also change depending upon when the initial impulse occurs. For this reason, in Figure 3.9 I plot the initial (and therefore peak) reaction of 6 different yield maturities to a 1 standard deviation positive path shock.



Figure 3.9: Peak impulse response of yields to a path shock, in basis points.

A second way of looking at the response of yields to a path shock is to observe the estimated response on particular event days. In Table 3.6 I show the value of the IRF on five different days, each roughly two years apart.

Both Figure 3.9 and Table 3.6 present a similar picture: yields of up to two years in maturity see a significant drop in sensitivity to path shocks from 2007 to 2011, while long yields of 5- and 10-year maturities see a modest decrease in sensitivity. The difference is illustrated most clearly by the difference in the effect of a path shock on 12/11/07 and 8/9/2011, during which time the impact of a path shock fell by more than 50% for maturities under two years.

	4-12-2005	12-11-2007	03-18-2009	08-09-2011	06-19-2013	07-30-2014
3-month	0.73	1.90	-0.38	0.00	-0.88	0.02
	(0.42, 0.99)	(0.74, 3.10)	(-0.61, -0.19)	(-0.87, 0.81)	(-1.02, -0.73)	(-0.24, 0.91)
6-month	1.72	4.11	0.74	0.54	-0.25	0.21
	(1.47, 1.99)	(2.90, 5.31)	(0.50, 0.91)	(-0.35, 1.41)	(-0.37, -0.16)	(-0.51, 1.24)
1-year	4.73	10.22	5.36	3.52	2.74	3.14
Ŭ	(2.82, 4.99)	(6.15, 11.52)	(2.29, 5.29)	(0.70, 4.19)	(0.77, 2.81)	(1.02, 3.11)
2-year	5.38	11.50	7.20	5.12	4.13	5.20
Ŭ	(5.10, 5.61)	(10.41, 11.12)	(6.91, 7.23)	(4.32, 5.74)	(4.05, 4.22)	(4.61, 5.97)
5-year	5.29	11.04	9.24	7.42	5.92	7.11
-	(5.20, 5.49)	(10.11, 12.01)	(9.10, 9.43)	(6.29, 8.10)	(5.73, 6.07)	(6.43, 8.29)
10-year	5.01	9.20	11.49	9.82	7.79	9.12
	(4.88, 5.14)	(8.47, 9.91)	(11.27, 11.59)	(9.49, 10.36)	(7.64, 7.86)	(8.50, 10.54)

Table 3.5: Initial response of yields to a positive 1 standard deviation path shock, at 6 points in time. 90% credible intervals in parentheses.

3.7 Discussion

Figure 3.9 presents a similar picture to the rolling regressions of Figure 3.4. There are slight differences in the values however, as the yield changes from the DNS model tend to be about 1 to 2bp lower than with the rolling regression method. This suggests that the earlier work on this subject by [10] was accurate, without a more fully specified model. However, in my results the effect of path shocks are estimated with much tighter credible intervals than the confidence intervals found in [10].

The results from section 6 are clear: shocks to the path of monetary policy have had decreasing influence on the latent factors in the DNS model between 2008 and 2013. The question is to what extent this changing effect of path shocks impacts the yield curve. According to Figure 3.9 and Table 3.6 the impact of a path shock becomes particularly weak in 2011 and 2012, in which the response of even 2-year yields was significantly muted. Interestingly, by 2013 2-year yields began to increase in responsiveness to path shocks, while the sensitivity of maturities up to 1-year was unchanged. These findings coincide with the results of [33] and [10] discussed above. As the ZLB is expected to bind on the target rate at longer time horizons, yields of longer maturities lose their sensitivity to shocks. The responsiveness of a medium-term yield falls to nearly zero if the short rate is expected to stay at zero beyond the yield maturity. This suggests that from 2009 to early 2013 the market generally increased its expectation of the duration of the ZLB period up to at least two years.

An interesting result of this paper is that it appears the market assumes the ZLB period is going to end sometime in 2015. In Figure 3.8 the coefficients change little from about 2010/2011 onward, but a slight up-tick in the path shock sensitivity of 2-year yields begins in early 2013. As well, by 2014 the sensitivity of 1-year yields began to modestly increase. This is consistent with a market that expects the target rate to finally lift-off from the ZLB in 2015. From what I have shown here, this belief in a target rate bump in 2015 has been held since mid-2013.

3.7.1 Future Work

There are a few areas in which this paper can be expanded to include realistic refinements to the model. One of these is to use a different distribution for the model likelihood, specifically to account for the fact that target and path shocks do not appear to be Gaussian. Looking back at Figure 3.2, both of these variables display several large events suggesting that the kurtosis of the underlying distributions is greater than 1. A starting point for this type of analysis would be for the factors and yields to be distributed with multivariate Student-t distribution.

A second area of potential future work is more broad. Now that I have shown the value of daily estimation in a DNS model this methodology can be adapted to show the impact of all kinds of macroeconomic shocks on the yield curve. Any macroeconomic variables that have survey or asset price data that reflect expectations can be used in the same way as the target or path shock. This could be useful for announcements on labor market, consumption or capacity utilization data. There are innumerable possible variables to be used for a better understanding of yield curve dynamics for both investors and policymakers.

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