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#### Static and Microwave Transport Properties of Aluminum Nanobridge Josephson Junctions

by

Eli Markus Levenson-Falk

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy

 $\mathrm{in}$ 

#### Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Irfan Siddiqi, Chair Professor John Clarke Professor Sayeef Salahuddin

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#### Abstract

Static and Microwave Transport Properties of Aluminum Nanobridge Josephson Junctions

by

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Josephson junctions are the basis of superconducting qubits, amplifiers, and magnetometers. Historically, tunnel-style junctions have been most common. However, for some applications—those requiring a small, highly transparent, all-superconducting junction—nanobridge weak link junctions may be preferable. This thesis presents extensive characterization of aluminum nanobridge Josephson junctions. The junction behavior is simulated by numerically solving the Usadel equations. These simulations are then tested and confirmed via low- and microwave-frequency transport measurements. The data confirm that nanobridge junctions approach the ideal weak-link Josephson limit.

Such a near-ideal junction can be used in a superconducting quantum interference device (SQUID) to form an ultra sensitive magnetometer. This thesis presents the first nanobridgebased dispersive SQUID magnetometer. The devices show near-zero dissipation, with bandwidth and sensitivity on par with the best reported results for any SQUID-based magnetometer. These magnetometers have several flexible modes of operation, allowing for optimization of sensitivity, bandwidth, or backaction. They also provide insight into the internal dynamics of dispersive measurement with nonlinear cavities, as the magnetometer backaction depends on the bias point and can readily be measured.

Nanobridge junctions also provide a useful tool for diagnosing sources of decoherence in superconducting qubits. By replacing the usual tunnel junction with a nanobridge, the contribution of the junction to decoherence processes may be probed. In particular, the interaction of nanobridge junctions with quasiparticles provides information both about the junctions themselves and about quasiparticle generation and relaxation mechanisms. This thesis reports measurements of nonequilibrium quasiparticles trapping in phase-biased nanobridge junctions. By probing the quasiparticle-induced changes in resonant frequency of a high-Q nanoSQUID oscillator, one may measure the mean distribution of trapped quasiparticles and study its temperature dependence. These measurements also provide spectroscopy of the junctions' internal Andreev states and the dynamics of quasiparticle excitation and retrapping. The work presented here demonstrates the utility of nanobridge junctions as a tool for quantifying the population and distribution of quasiparticles in a superconducting circuit.

#### Static and Microwave Transport Properties of Aluminum Nanobridge Josephson Junctions

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# List of Selected Symbols and Abbreviations

| 1D, 2D, 3D          | one-, two-, three-dimensional |
|---------------------|-------------------------------|
| А                   | anisole resist solvent        |
| А                   | amperes                       |
| AFM                 | atomic force microscope       |
| Å                   | angstroms                     |
| В                   | integration bandwidth         |
| $C, C_S$            | capacitance                   |
| CPR                 | current-phase relation        |
| CPW                 | co-planar waveguide           |
| dB                  | decibels                      |
| dc                  | direct current                |
| $R_S$               | resonator real impedance      |
| D                   | diffusion constant            |
| e                   | charge of an electron         |
| eV                  | electron volts                |
| $E_A$               | Andreev energy                |
| $\operatorname{EL}$ | ethyl lactate resist solvent  |
| f                   | frequency                     |
| $f_{\rm res}$       | resonant frequency            |
| $f_{ m exc}$        | excitation tone frequency     |
| F                   | anomalous Green's function    |
| F                   | farads                        |
| G                   | normal Green's function       |
| G                   | power gain                    |
| $G_{\rm sys}$       | system power gain             |
| h                   | Planck's constant             |
| Н                   | henries                       |

| HEMT                      | high electron mobility transistor                        |
|---------------------------|--|
| Hz                        | hertz  |
| $\hbar$                   | reduced Planck's constant                                |
| Ι                         | current  |
| $I_0$                     | junction critical current                                |
| $I_B$                     | bias current   |
| $I_C$                     | junction/SQUID critical current                          |
| $I_{RF}$                  | microwave drive current                                  |
| $I_S$                     | current through a SQUID                                  |
| $I_{\rm switch}$          | junction/SQUID switching current                         |
| IDC                       | interdigitated capacitor                                 |
| IPA                       | isopropyl alcohol  |
| $k_B$                     | Boltzmann constant                                       |
| Κ                         | kelvin   |
| KO-1                      | Kulik-Omel'yanchuk theory 1                              |
| l                         | mean free path   |
| L                         | inductance   |
| $L_J$                     | Josephson inductance                                     |
| $L_S$                     | SQUID inductance   |
| LJPA                      | lumped-element Josephson parametric amplifier            |
| m                         | meters   |
| MAA                       | methacrylic acid   |
| MIBK                      | methyl isobutyl ketone                                   |
| MMA                       | methyl methacrylate                                      |
| n                         | an integer   |
| $\bar{n}$                 | mean cavity photon occupation                            |
| $n_i$                     | number of quasiparticles trapped in a conduction channel |
| $\bar{n}_{\mathrm{trap}}$ | mean trapped quasiparticle number                        |
| $\{n_i\}$                 | trapped quasiparticle configuration                      |
| N                         | number of conduction channels                            |
| $N_E$                     | effective number of conduction channels                  |
| $N_{\rm on}$              | noise with paramp pump on                                |
| $N_{\rm off}$             | noise with paramp pump off                               |
| $p(\{n_i\})$              | probability of a trapped quasiparticle configuration     |
| $p(E_A)$                  | probablity of occupation of a state with energy $E_A$    |
| $P_C$                     | critical drive power at resonance bifurcation            |
| $P_k$                     | probability of trapping $k$ quasiparticles               |

| PCB                  | printed circuit board                                 |
|----------------------|---|
| PMMA                 | poly(methyl methacrylate)                             |
| $q(\phi)$            | inductive participation ratio                         |
| $q_0$                | zero-flux inductive participation ratio               |
| Q                    | resonator quality factor                              |
| $Q_{\mathrm{ext}}$   | external (coupled) resonator quality factor           |
| $Q_{ m int}$         | internal resonator quality factor                     |
| $\operatorname{rpm}$ | revolutions per minute                                |
| $\mathbf{RF}$        | radio-frequency                                       |
| $R_N$                | junction/SQUID normal state resistance                |
| S                    | seconds   |
| $S(\omega)$          | resonant response function                            |
| $\bar{S}(\omega)$    | averaged resonant response function                   |
| $S_{11}(\omega)$     | reflection response function                          |
| SEM                  | scanning electron microscope                          |
| $\operatorname{SiN}$ | silicon nitride                                       |
| $\operatorname{SNR}$ | signal-to-noise ratio                                 |
| SQUID                | superconducting quantum interference device           |
| T                    | temperature   |
| Т                    | teslas  |
| $T_1$                | relaxation time                                       |
| $T_2$                | coherence time  |
| $T_C$                | superconducting critical temperature                  |
| $T_N$                | amplifier noise temperature                           |
| $T_{\rm sys}$        | system noise temperature                              |
| U                    | junction/SQUID metapotential                          |
| V                    | voltage   |
| V                    | volts   |
| $V_{in}$             | drive voltage amplitude                               |
| $V_{\rm SNR}$        | voltage signal-to-noise ratio                         |
| $\frac{dV}{d\Phi}$   | transduction factor                                   |
| $x_{eq}$             | thermal quasiparticle density                         |
| $x_{neq}$            | nonequlibrium quasiparticle density                   |
| $x_{qp}$             | total quasiparticle density                           |
| y                    | noise rise ratio                                      |
| $\beta_L$            | ratio of SQUID loop inductance to junction inductance |
| $\gamma$             | numerical factor indicating sideband correlations     |

| Γ                  | Lorentzian resonance width                              |
|--------------------|---|
| $\delta$           | gauge-invariant superconducting phase drop across a     |
|                    | junction  |
| $\delta(x)$        | Dirac delta function                                    |
| $\Delta, \Delta_0$ | superconducting energy gap                              |
| $\Delta$           | superconducting pair potential                          |
| $\Delta_A$         | Andreev gap   |
| $\Delta \Phi$      | flux signal amplitude                                   |
| $\epsilon_F$       | Fermi energy  |
| $\theta$           | deposition angle  |
| $	heta_t$          | transduction angle                                      |
| ξ                  | superconducting total coherence length                  |
| $\xi_0$            | BCS coherence length                                    |
| $\rho(	au)$        | distribution of conduction channel transmittivities     |
| au                 | conduction channel transmittivity                       |
| $	au_R$            | quasiparticle recombination time                        |
| $	au_T$            | quasiparticle retrapping time                           |
| $\phi$             | superconducting phase drop across a circuit element     |
| $\phi$             | normalized flux   |
| $\Phi$             | flux  |
| $\Phi_0$           | flux quantum  |
| $\varphi$          | reduced flux  |
| $arphi_0$          | reduced flux quantum                                    |
| $\omega$           | angular frequency                                       |
| $\omega_0$         | angular resonant frequency                              |
| $\omega_0(	au)$    | angular resonant frequency with a quasiparticle trapped |
|                    | in a channel with transmittivity $\tau$                 |
| $\delta\omega_0$   | Gaussian resonance width                                |
| $\omega_d$         | angular drive frequency                                 |
| $\omega_n$         | nth Matsubara frequency                                 |
| $\omega_s$         | angular signal frequency                                |
| Ω                  | ohms  |

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# Chapter 1 Introduction

A superconductor can carry current with vanishingly small dissipation. This near-lossless transport allows for the realization of superconducting resonant circuits with extremely high internal quality factors  $(Q_{int})$ . Such linear resonators have been used for applications ranging from the storage of quantum information [1, 2] to the detection of dark matter [3]. However, for many practical circuits, a nonlinear resonator is required. For instance, a superconducting quantum bit (qubit) can be formed by a strongly anharmonic oscillator, where the anharmonicity makes it possible to isolate a two-level qubit transition [4]. Superconducting amplifiers require nonlinearity to mix power from the pump into the amplified signal [5, 6]. Superconducting magnetometers depend on a parametric nonlinearity modulating the circuit parameters to detect magnetic flux [7]. All of these applications depend on low loss in the resonant circuit, in order to maintain coherence (for qubits) and reduce noise (for amplifiers and magnetometers). Therefore, in order to fully take advantage of the low-loss transport of the rest of the superconducting circuit, the nonlinearity must come from a non-dissipative circuit element. This non-dissipative nonlinearity is typically provided by a circuit element known as the superconducting Josephson junction: two superconducting electrodes, with some form of interruption (e.g. a tunnel barrier, a normal metal, or a constriction) between them. While tunnel-barrier junctions are most commonly used, weak-link constriction junctions have shown promise for certain applications. The small size of these junctions—typical dimensions are 1-100 nm—makes them ideal for coupling to nanoscale magnets [8], while their engineerable internal state structure makes them suitable for experiments in basic solid-state physics [9].

Nanobridge junctions, which consist of a short, thin, narrow superconducting bridge connecting two large banks, can give nearly ideal junction behavior in a readily fabricated, allsuperconducting geometry. These junctions typically come in two flavors: the 2D or "Dayem" bridge, in which a narrow bridge connects wide banks with a constant thickness throughout the junction; and the 3D or "variable-thickness" bridge, in which a thin, narrow bridge connects thick, wide banks. The behavior of a junction depends sensitively on its geometry, the materials used in fabrication, and the circuit environment in which it is embedded. In this thesis, I describe the characterization and application of aluminum nanobridge Josephson junctions.

## 1.1 The Josephson Relations

First described by Josephson in his seminal paper [10], the canonical Josephson tunnel junction consists of two superconducting electrodes separated by a thin tunnel barrier. In the ideal case, a steady current may be passed through the junction with no voltage drop, provided the current is less than the junction's critical current  $I_0$ . The dc Josephson effect describes this current as a function of  $\delta$ , the gauge-invariant phase difference of the superconducting order parameter across the junction. For the tunnel junction described, Josephson derived the relation

$$I(\delta) = I_0 \sin(\delta) \tag{1.1}$$

In general, a junction may have any arbitrary current-phase relation (CPR), with the condition that the CPR is  $2\pi$ -periodic. When a current greater than  $I_0$  is applied, the junction switches out of the *superconducting state* and into the *voltage state*, so named because the junction develops a voltage drop and so becomes dissipative. As long as  $I < I_0$ , current may be passed through the junction with no dissipation.

The ac Josephson relation relates the voltage across the junction to the time derivative of this phase:

$$V(t) = \frac{\hbar}{2e} \frac{\partial \delta}{\partial t} \tag{1.2}$$

Combining these two equations, one may view the Josephson junction as a purely reactive element, with a nonlinear inductance —that is, an inductance which depends on the phase (and thus the current) passing through the junction:

$$V(t) = \frac{\hbar}{2e} \frac{\partial \delta}{\partial t}$$
  
=  $\left[\frac{\hbar}{2e} (\frac{\partial I}{\partial \delta})^{-1}\right] \frac{\partial I}{\partial t}$   
=  $L_J \frac{\partial I}{\partial t}$   
 $L_J \equiv \frac{\hbar/2e}{\partial I/\partial \delta} = \frac{\varphi_0}{\partial I/\partial \delta}$  (1.3)

For the tunnel junction described by Eq. (1.1), the inductance is thus

$$L_J = \frac{\varphi_0}{I_0 \cos \delta}$$

Note that the inductance depends on  $\delta$ , and thus on the current passing through the junction. Thus, a Jospehson junction is a non-dissipative, nonlinear inductor.



Figure 1.1: Josephson junction and SQUID circuit symbols

A Josephson junction, represented by the circuit symbol shown in (a) and parametrized by its critical current  $I_0$ , carries a supercurrent which is a periodic function of the phase  $\delta$  across the junction. A SQUID, consisting of a loop of superconductor interrupted by two Josephson junctions, is represented by the circuit symbol shown in (b). It carries a supercurrent which depends on the phase across the SQUID  $\phi$ , the flux threading the loop  $\Phi$ , and the critical currents of the two junctions.

### **1.2** SQUIDs and Magnetometry

When two Josephson junctions are arranged in a superconducting loop, they form a twojunction superconducting quantum interference device, or dc SQUID; See Fig. 1.1. Because the total flux through any loop of superconductor is quantized in units of  $\Phi_0 \equiv \frac{h}{2e}$ , an imposed external flux  $\Phi_{app}$  will induce a circulating current in the loop, which will ensure that the total flux is an integer multiple of  $\Phi_0$ . This can alternately be viewed as imposing a phase bias on the two junctions which ensures that the total phase around the loop is an integer multiple of  $2\pi$ :

$$2\pi \frac{\Phi}{\Phi_0} + \delta_1 - \delta_2 = 2\pi n$$

where  $\delta_1$  and  $\delta_2$  are the phase drops across the two junctions<sup>1</sup>. Like a single junction, a SQUID has a critical current  $I_C$ . Due to this flux quantization condition, the critical current becomes flux dependent:  $I_C = I_C(\Phi)$ . One may view this dependence as the circulating current "using up" some of the critical current of the junctions; an equivalent view is that the phase bias causes the junction currents to interfere destructively (hence the "interference" in the name of the device). Similarly, since the junction inductance is a function of the phase, the inductance of the SQUID depends on the flux:  $L_S = L_S(\Phi)$ . Often it is practical to treat the SQUID as an effective single junction which has a critical current and inductance which are flux-dependent. Viewed in this way it is easy to see applications of SQUIDs in superconducting circuits. Simply replacing a single junction with a SQUID allows for in-situ

<sup>&</sup>lt;sup>1</sup>This assumes that the loop has a negligible inductance, and so there is no phase drop in the loop itself.



Figure 1.2: Flux coupling into a superconducting loop

The Meissner effect will cause flux to lens away from any superconductor (grey). For a small spin (blue arrow), much of the flux will be contained very close to the spin. A wide superconducting trace, as in (a), will cause this flux to lens away, preventing it from being efficiently coupled into a SQUID loop. A narrow constriction, as shown in (b), will allow most of the flux to be coupled through the loop without being deflected.

tuning of the Josephson inductance. This capability has been used to modulate the energy structure of superconducting qubits [11], to tune the frequency of superconducting resonators [12], and to observe parametric downconversion of microwave photons (a process equivalent to the dynamical Casimir effect) [13], to name just a few examples. SQUIDs also provide a convenient way to controllably phase-bias a junction, allowing for the internal junction properties to be probed as phase is varied.

Perhaps the most obvious application for a SQUID is as a sensitive detector of magnetic flux. One can measure the flux-dependent SQUID critical current by performing switching measurements or by measuring the voltage across a shunt resistor with the SQUID pre-biased into the voltage state [14]. Another approach is to measure the flux-dependent SQUID inductance by reading out the resonant frequency of a tank circuit incorporating the SQUID [7]. Both of these approaches provide a sensitive measurement of the flux through the SQUID and thus its magnetic field environment. SQUID sensors have been used to perform ultra-low-field MRI [15], as scanning probes of transport through condensed-matter systems [16], in bolometer circuits [17], and as detectors of nanoscale magnets [8]. In these nanomagnetometers it is crucial to efficiently couple flux into the SQUID loop. As illustrated in Fig. 1.2, this is best accomplished through use of a narrow constriction in the loop, and so nanobridge-style junctions are preferred for these circuits.

### **1.3** Superconducting Quasiparticles

The BCS theory of superconductivity describes a condensate formed of paired electrons (a.k.a. Cooper pairs) [18]. There is a gap of  $2\Delta$ , above which exist single-particle states

which are unoccupied at zero temperature. The Cooper pairs carry supercurrent without any dissipation. A quasiparticle excitation of the condensate is a dressed electron, which may carry a normal current with dissipation. Quasiparticles may also tunnel across a Josephson junction [19, 20], absorbing energy as they do, or become trapped inside [21], altering the junction properties. Any circuits which depend on junctions being low-loss or lownoise, such as superconducting qubits, amplifiers, or magnetometers, will thus be adversely affected by the presence of quasiparticles. While thermal quasiparticles may be easily eliminated in some common superconductors (such as aluminum) simply by cooling below  $\sim 100$ mK—temperatures easily obtained with a commercial helium dilution refrigerator—recent experiments have shown that nonequilibrium quasiparticles are ubiquitous in many superconducting circuits [22, 23]. These quasiparticles can be created by the interaction of a high-energy  $(hf > 2\Delta)$  photon with the condensate, and so excellent radiation shielding is required to minimize quasiparticle generation. In order to best optimize the design and enclosure of superconducting circuits, a full understanding of the quasiparticle generation and relaxation mechanisms, and the exact nature of the quasiparticles' effect on the circuit. is required.

## 1.4 Structure of Thesis

This thesis begins with a general overview of the theory of weak-link Josephson junctions. It then describes how this theory may be applied to calculate the properties of physical junctions, with numerical simulations showing results for common nanobridge geometries. Next, these simulated results are extended to calculate the behavior of nanobridge junctions in superconducting circuits.

Chapters 3 and 4 describe initial characterization measurements of nanobridge junctions and SQUIDs, providing experimental confirmation of the calculated junction properties. In Ch. 3, I detail switching measurements of the critical current modulation of nanobridge SQUIDs (nanoSQUIDs) as a function of applied flux. In Ch. 4 I describe the behavior of nanoSQUIDs in low-Q and high-Q resonant circuits. I demonstrate flux tuning of the oscillators' resonant frequencies, showing good agreement with theoretical predictions. I also show resonant bifurcation of the nonlinear oscillators, and demonstrate how they can be used as quantum-noise-limited parametric amplifiers.

Chapter 5 describes the design, operation, and characterization of a dispersive nanoSQUID magnetometer. I begin with a discussion of the principles of dispersive magnetometry, and the advantages of using a nanoSQUID (rather than an ordinary tunnel junction SQUID). The chapter continues with a description of the measurement apparatus, and benchmarking results for the device operated as a linear magnetometer. I then show how noise from the rest of the measurement amplification chain limits device performance, and describe two approaches to overcoming this noise, with experimental results for each. Next, I show measurements of how internal parametric amplification processes in the device enhance and limit its performance as a magnetometer. Finally, I discuss ideas for possible extensions and

optimizations of the design.

Chapter 6 describes measurements of quasiparticle trapping in phase-biased nanobridge junctions. I begin by establishing a basic theoretical framework for interpreting the measurements. I then show evidence of quasiparticle trapping in phase-biased nanobridges, through dispersive measurements of a high-Q nanoSQUID oscillator. I describe how using the theory to fit these measurements provides insight into the distribution of trapped quasiparticles. Finally, I use these quasiparticles to perform spectroscopy on internal junction states, and measure the dynamics of clearing and retrapping quasiparticles.

The thesis concludes with a brief summary of my results. Finally, I present some ideas for future experiments utilizing nanobridge junctions.

# 1.5 Summary of Key Results

The work presented in this thesis provides full theoretical descriptions of 2D and 3D aluminum nanobridge Josephson junctions [24], as well as experimental tests of these predictions at low [25] and microwave frequencies [26]. I show that 3D nanobridges act as robust and useful Josephson junctions, with CPRs which approach the ideal limit of a weak link. These 3D nanobridges, when integrated into a low-Q nanoSQUID resonator, form a sensitive dispersive magnetometer; flux through the SQUID may be read out by measuring the flux-dependent resonant frequency of the device. I demonstrate ultra-high sensitivity and bandwidth with a non-dissipative, near-quantum-noise-limited magnetometer [27].

I also present the first measurements of quasiparticle trapping in many-channel weak link junctions [28]. When a quasiparticle traps inside a nanobridge, it alters the junction inductance, which is read out by measuring the shift in resonant frequency of a high-QnanoSQUID oscillator. These measurements agree well with a simple thermal trapping theory at temperatures above 75 mK. Furthermore, by exciting the trapped quasiparticles with an intense microwave tone, it is possible to measure their excitation and retrapping dynamics and to perform spectroscopy on the internal junction states they were trapped in. These measurements provide valuable insight into the behavior of quasiparticles in superconducting circuits, and demonstrate that dispersive measurement is a promising technique for quasiparticle characterization.

# Chapter 2

# **Fundamental Nanobridge Theory**

In order to fully understand the experimental behavior of nanobridge Josephson junctions, a detailed theoretical framework is necessary. This chapter works from the ground up to derive the Josephson behavior of nanobridge junctions, using two approaches: an effective semiconductor transmission-channel model, and a full superconducting state equation model originally developed by Sau and Cohen [24]. I then discuss how these approaches may be used to model physical junctions, and give some examples for geometries used later in the thesis. The chapter concludes with some examples of simulated nanobridges in superconducting circuits. The first is a low-Q resonant circuit similar to those used in superconducting parametric amplifiers. The next is a nanobridge SQUID (nanoSQUID). The chapter ends with a detailed discussion of how the nanoSQUID properties change with flux and with design parameters.

## 2.1 The Semiconductor Picture

A common way to model any Josephson junction is as a parallel combination of onedimensional normal conduction channels. These are equivalent to the 1D channels that can arise in a semiconductor constriction. Each channel has a transmittivity  $\tau$  which defines the probability that a Cooper pair is transmitted (or, equivalently, that it is not reflected) to the other side of the junction. In a process first described by Andreev in 1968 [29], a Cooper pair incident on one side of the junction (for simplicity, we will assume current flows left-to-right, so the Cooper pair starts in the left junction electrode) is converted to a normal electron in a conduction channel, while annihilating a counter-propagating hole. The electron is transported to the other side of the channel, where it is converted into a hole and reflected; this causes a net change in charge of +2e, which is canceled out by the creation of a Cooper pair (charge -2e) in the right electrode. Via this process, known as Andreev reflection, a Cooper pair is transmitted from one side of the junction to the other, even though the junction interior is modeled as a normal conductor.

#### Andreev States

The existence of the superconducting gap  $\Delta$  means that there are no allowed states at energies within  $\Delta$  of the Fermi energy  $\epsilon_F = 0$  in the junction electrodes. Thus, the electrodes form potential barriers on each side of the junction. This results in the formation of a pair of "particle in a box" *Andreev states* for each conduction channel, with energies which depend on the phase difference  $\delta$  across the junction:

$$E_{A\pm}(\delta) = \pm \Delta \sqrt{1 - \tau \sin^2 \frac{\delta}{2}}$$
(2.1)

Each state will carry a supercurrent, given by

$$I_{\pm}(\delta) = \frac{1}{\varphi_0} \frac{\partial E_{A\pm}}{\partial \delta} = \mp \frac{\Delta}{4\varphi_0} \frac{\tau \sin \delta}{\sqrt{1 - \tau \sin^2(\frac{\delta}{2})}}$$
(2.2)

Now, at zero temperature, all states with energies below  $\epsilon_F$  are occupied and all those above are unoccupied. This means that only the lower Andreev state is occupied, and so it is the only one which carries a supercurrent. The channel's critical current is

$$I_0 = \frac{\Delta}{2\varphi_0} (1 - \sqrt{1 - \tau})$$

which, for perfect transmission in typical aluminum thin films, gives a value of 41.4 nA.

#### Tunnel Junction and KO-1 CPRs

Let us now derive the current-phase relation (CPR) of a junction in two important limits. The first is the limit of a parallel combination of N nearly opaque channels. Expanding Eq. (2.2) to first order in the limit where  $\tau \to 0$  gives

$$I(\delta) = \frac{\Delta}{4\varphi_0} \tau \sin \delta$$

If we simply define  $I_0 \equiv N \frac{\Delta}{4\varphi_0} \tau$ , we see that these parallel channels give exactly the result (1.1) for the CPR of a tunnel junction. Thus, even though the tunnel barrier in such a junction is typically an insulator, the semiconductor picture of Andreev states gives the correct result for the junction CPR.

The other important limit is the so-called dirty limit, where the mean free path l of a superconducting quasiparticle is much less than the superconducting BCS coherence length  $\xi_0$ . This coherence length may be thought of as the mean effective radius of a Cooper pair. Typical aluminum superconducting thin films fall well into the dirty limit. In this limit, a zero-length contact between two electrodes will have a parallel combination of channels with transmittivities given by the Dorokhov distribution [30]

$$\rho(\tau) = \frac{N_e}{\tau\sqrt{1-\tau}} \tag{2.3}$$

where  $N_e$  is the effective number of channels in the junction. Integrating the single-channel CPRs given in Eq. (2.2) over this distribution gives

$$I(\delta) = \frac{\pi\Delta}{eR_N} \cos\frac{\delta}{2} \tanh^{-1} \sin\frac{\delta}{2}$$
(2.4)

where  $R_N$ , the normal state resistance of the junction, is given by  $R_N = \frac{2\pi\hbar}{e^2 N_e} = \frac{2\varphi_0}{eN_e}$ .

## 2.2 Full Quasiparticle Theory: the Usadel Equations

A full description of an arbitrary junction geometry may be achieved by using a Green's function formalism. The full historical record of the development of Green's function theory for superconductors is beyond the scope of this thesis. I will instead present a brief summary of the results and how they are applied. For those interested in the full theory, I refer you to papers by Gor'kov, Eilenberger, and Usadel [31, 32, 33].

We first define the anomalous Green's function  $F(\omega, \boldsymbol{r}, \boldsymbol{v})$ . This is a complex quantity that may be thought of as representing the density of Cooper pairs, normalized to the total charge carrier density. The related normal Green's function  $G(\omega, \boldsymbol{r}, \boldsymbol{v})$ , which is real, represents the normalized density of quasiparticles (i.e. normal electrons). A full description of a junction using these functions would typically require accounting for all possible quasiparticle momenta. However, in the dirty limit, quasiparticle transport is diffusive. This means that their momenta can be assumed to be spherically symmetric and only the average momentum is of importance, so the  $\boldsymbol{v}$  dependence of the functions is eliminated. These approximations lead to the Usadel equations for a superconductor, written in terms of the diffusion constant D:

$$\omega F_{\omega}(\mathbf{r}) + \frac{D}{2} \left( F_{\omega} \nabla^2 G_{\omega} - G_{\omega} \nabla^2 F_{\omega} \right) = \Delta^* G_{\omega}(\mathbf{r})$$
(2.5)

The pair potential  $\Delta$  is a spatially-varying, complex quantity which is analogous to the superconducting order parameter<sup>1</sup>; in the case of a spatially homogenous superconductor with no phase gradients, it reduces to the ordinary gap  $\Delta_0$ . The equation is written in terms of the *Matsubara frequencies* 

$$\omega_n = (2n+1) \, \pi \frac{T}{\Delta_0}$$

which account for the  $\omega$  dependence of F and G. Thus, we have reduced the problem to a simple differential equation in  $\mathbf{r}$ . The normalization condition on F and G gives  $G^2 + |F|^2 = 1$ . A self-consistency condition gives

$$\Delta \ln \frac{T_c}{T} = 2\pi k_B T \sum_n \left(\frac{\Delta}{\hbar\omega_n} - F_\omega\right) \tag{2.6}$$

<sup>&</sup>lt;sup>1</sup>I apologize for reuse of symbols. It is an unavoidable consequence of the use of complicated theories developed over several decades, integrating concepts from different fields, and often written in different languages.

#### The KO-1 Limit

Consider a junction formed by a short, narrow (i.e. dimensions much smaller than the total coherence length  $\xi$ ) weak link between two electrodes. Assume that the electrodes are completely rigid phase reservoirs—that is, that the phase drop from the left electrode to the right occurs entirely in the weak link. In this limit, the gradient terms of Eq. (2.5) are dominant, and other terms may be neglected. In this case the equations can be solved analytically to give the CPR in Eq. (2.4). This relation, derived by Kulik and Omel'Yanchuk in 1975, is known as the KO-1 CPR [34].

#### Arbitrary Geometries

It is helpful to parametrize the Usadel equations with a new variable  $\Phi$  (which is not related to magnetic flux):

$$F_{\omega} = \frac{\Phi}{\sqrt{\omega_n^2 + |\Phi|^2}}, G_{\omega} = \frac{\omega_n}{\sqrt{\omega_n^2 + |\Phi|^2}}$$

In a bulk superconductor,  $\Phi = \Delta$ . Let us also define a new variable  $g(\omega_n) = G_{\omega}/\omega_n$ . Using this substitution, we can write the Usadel equation (2.5) as

$$g(\omega_n) \left[ \Phi(\omega_n) - \Delta \right] = \frac{D}{2} \nabla \left[ g(\omega_n) \nabla \Phi(\omega_n) \right]$$

and the self-consistency equation as

$$\Delta = \frac{\sum_{n} g(\omega_n) \Phi(\omega_n)}{\sum_{n} (\omega_n^2 + 1)^{-1/2}}$$

The problem of solving for the CPR of a junction then becomes a relatively straightforward boundary-value problem. For the nanobridge structures we wish to simulate, we assume that the left and right boundaries have equal gaps, and so define  $\Delta_R = \Delta_L * e^{-i\delta}$ . We can then solve for  $\Delta$ , g, and  $\Phi$  across the entire geometry, and so compute the total current I through the junction. Repeating this procedure while varying  $\delta$  from 0 to  $2\pi$  gives a complete  $I(\delta)$ .

#### Calculated CPRs

Previous theoretical work on weak links has shown that the junction characteristics are strongly influenced by geometry [35]. In order to study some practical junction geometries, Vijay et al. solved the Usadel equations via the procedure described above in the 2D and 3D nanobridge geometries shown in Fig. 2.1 [24]. Later, we reproduced the results for slightly modified geometries more applicable to the devices used in experiment. Calculated CPRs for 2D and 3D nanobridges are shown in Fig. 2.2. We can see that 3D nanobridge junctions have CPRs which are more nonlinear (i.e. more sinusoidal) than those of 2D nanobridges. It



Figure 2.1: Simulated nanobridge geometries

Nanobridge geometries used in our simulations. A rectangular 2D bridge (a) or a semicylindrical 3D bridge (b) of width W and length L connect banks of larger dimensions. Phase boundary conditions are placed at the edges of the banks, providing conditions over which the Usadel equations may be solved. The semicylindrical geometry is chosen for 3D bridges (rather than rectangular) in order to maximize the symmetry of the system and thus reduce computational difficulty.

is also apparent that shorter bridges have more nonlinear CPRs. The KO-1 CPR is plotted for reference, demonstrating that short 3D nanobridges approach the KO-1 limit. The CPRs shown are for bridges which are 0.75 coherence lengths wide. For a typical aluminum thin film,  $\xi \approx 30-40$  nm, so these bridges are 25-30 nm wide, dimensions readily achievable with standard electron-beam lithography.

#### Phase Evolution

We also calculated the phase evolution across 2D and 3D junctions, including the banks. Results are plotted in Fig. 2.3. The 3D junction has fairly constant phase reservoirs in the banks and a steep phase drop in the bridge region, similar to the idealized KO-1 geometry. In contrast, the 2D junction phase evolves quite strongly in the banks; indeed, it is difficult to define the edges of the junction, since the phase gradient is large right up to the edges of the geometry simulated. This is another indication of the 3D bridges showing a more ideal weak link behavior.

This phase evolution may be understood on an intuitive level, albeit a mathematically imprecise one, by examining the current density through a junction. The current density in any superconductor is directly proportional to the gradient of the superconducting order parameter. For a junction with a constant carrier density, only the phase of the order parameter changes; a faster change indicates a greater current density. The total current through the junction must be conserved at all points along its axis. Since the nanobridge has a smaller cross-section than the banks, it must have a greater current density and thus a larger phase gradient. This difference in cross-sectional area is more pronounced for 3D



Figure 2.2: Simulated nanobridge current-phase relations

We calculate CPRs for 2D (left) and 3D (right) nanobridges of various lengths, all 20 nm wide. All the 2D nanobridge junctions show nearly-linear CPRs, with hysteretic branch jumps due to the current being nonzero at  $\delta = \pi$ . In contrast, the shorter 3D nanobridges show strongly nonlinear behavior, similar to the distorted sinusoid of the KO-1 CPR. The long 3D nanobridges give a much more linear CPR which shows hysteresis, similar to the 2D nanobridges.

junctions, and so they better confine the phase drop to the nanobridge.

# 2.3 Nanobridges in Circuits

An important application of nanobridge junctions is their use in low-Q resonant circuits for magnetometry and amplification. The nonlinear Josephson inductance of the junction causes the resonator to have a nonlinear response. These circuits are generally driven into nonlinear resonance and even into the *bifurcation* regime, where there exist two stable states of oscillation with different resonant frequencies [36]. These Josephson bifurcation amplifiers (JBAs) have been used successfully in superconducting qubit readout [37], while devices driven in the regime of continuous nonlinearity form effective parametric amplifiers (paramps) [5] and dispersive magnetometers [7]. In order for these devices to function properly, the resonator must be able to stably enter the bifurcation regime before the onset of chaos. For a detailed discussion of nonlinear Josephson oscillators, I direct the reader to R. Vijay's excellent thesis [38]. I will present a brief summary below, and then discuss the results of our nanobridge simulations.



Figure 2.3: Phase evolution across 2D and 3D nanobridges

We set a phase of  $\pm 1$  radians at the left and right boundaries of 2D (a) and 3D (b) nanobridge junctions, of the geometry shown in Fig. 2.1, with 50 nm long, 30 nm wide bridges. We then solve for the phase evolution across the junction and plot the result here. The phase evolution in the 2D junction happens both in the banks and in the bridge, with only a slightly larger phase gradient in the bridge. Phase evolution would continue out farther into the banks, were it not for the artificial constraint imposed by our boundary conditions. In contrast, most of the phase drop in the 3D junction happens in the nanobridge region, with the banks acting as good phase reservoirs.

#### Nonlinear Resonators

A resonators is classified as *nonlinear* if it has a potential energy U which is not quadratic in the displacement; equivalently, the restoring force is nonlinear in the displacement. An example is a pendulum consisting of a point mass m on the end of a massless string of length l. The potential as a function of the angular displacement  $\theta$  is a cosine function, and the restoring force is a sine. In fact, this is also the potential that is obtained for a tunnel junction, so the analogy is exact. If the pendulum is driven by a force  $F_0 \sin \omega_d t$ , the resulting equation of motion is given by

$$mgl\sin\theta + ml^2\frac{d^2\theta}{dt^2} = F_0\sin\omega_d t$$

where g is the gravitational acceleration. At low drive amplitudes, the displacement is small, and the potential is essentially parabolic and the restoring force is linear, since  $\sin \theta \approx \theta$ . This leads to a linear resonance, of the type shown in Fig. 2.4. As drive power increases, the displacement increases and the non-parabolic potential (i.e. nonlinear restoring force) becomes evident. For a softening potential like the cosine discussed—that is, a potential which grows slower than a parabola—this leads to a drop in the resonant frequency and a sharper nonlinear resonance, as shown in Fig. 2.4.



Figure 2.4: Phase response of a nonlinear resonator

Above, I plot the phase shift on a drive signal reflected off a nonlinear resonator. At low drive powers, the response is linear and the phase shift in reflection is symmetric about the resonant frequency, as shown in blue. At higher drive powers, the response sharpens and becomes nonlinear, with an asymmetric resonance centered about a lower frequency, as shown in red.

As drive power increases, the resonance response narrows until it becomes infinitely sharp, i.e. a sudden phase jump. This is the characteristic signature of resonator bifurcation, where the resonator supports two stable states of oscillation with different resonant frequencies. A sample simulation of a bifurcating resonator with a cosine potential is shown in Fig. 2.5(a). I have plotted the phase shift on a reflected drive as a function of drive frequency and amplitude. The frequency is stepped, while the amplitude is swept continuously up or down, with the two drive directions interleaved vertically. A phase shift of  $0^{\circ}$ , indicated in yellow, occurs at the resonant frequency. At low amplitude, the resonance is linear and has a frequency which is amplitude-independent. As drive amplitude rises, the resonance bends down to lower frequency and becomes nonlinear, with an amplitude-dependent resonant frequency. Above a critical power  $P_C$ , the resonator bifurcates, remaining hysteretically in one of two states of oscillation depending on the direction of drive amplitude sweep. This hysteresis is indicated by the vertical striping in the figure.

#### Simulated Nanobridge Resonator Response

We model a nanobridge resonant circuit as a parallel RLC oscillator, with a resistance  $R_S$  (which may be a physical resistor, an internal loss, or just the impedance of the microwave environment), a capacitance  $C_S$ , and an inductance given by the junction. If the oscillator is subject to a driving current  $I_{RF} \cos(\omega_d t)$ , the resulting equation of motion is given by



Figure 2.5: Simulated resonant response of Josephson oscillators

We simulate the reflected phase shift as a function of drive current  $(I_{RF})$  and drive frequency for Josephson oscillators incorporating (a) a tunnel junction, (b) an 80 × 40 nm 3D nanobridge, and (c) a 80 × 40 nm 2D nanobridge. A 0° phase shift, indicated in yellow, occurs at the resonant frequency. All three oscillators show a linear resonance at low frequency, which becomes nonlinear and bends back to lower frequency as drive amplitude is increased. Both the tunnel junction and the 3D nanobridge oscillator deterministically enter the bifurcation regime at the critical drive power  $P_c$ , switching hysteretically between two stable states of oscillation as the drive amplitude is swept up and down. In contrast, the 2D nanobridge oscillator exhibits a well-defined bifurcation region, instead entering into chaotic oscillations (indicated by the random striping at high amplitudes).

equating this drive to the sum of currents through the three elements:

$$C_S \varphi_0 \frac{d^2 \delta(t)}{dt^2} + \frac{\varphi_0}{R_S} \frac{d\delta(t)}{dt} + I(\delta(t)) = I_{RF} \cos(\omega_d t)$$
(2.7)

For a given junction CPR, we numerically solve the equation to find the resonator response. We simulated resonators incorporating a tunnel junction, an  $80 \times 40$  nm 2D nanobridge, and an  $80 \times 40$  nm 3D nanobridge, scaled to have the same inductance. The values of  $C_S$  and  $R_S$  were chosen to give a resonant frequency of 1.5 GHz and a Q of 50, parameters typical for such circuits. Results as a function of drive frequency and amplitude, plotted as the phase shift of a microwave signal reflecting off the device, are shown in Fig. 2.5. A yellow color indicates zero phase shift, i.e. the resonant frequency. For all junction types, the resonance is linear at low drive amplitudes, with a frequency which is amplitude-independent. At higher drive amplitudes the resonance becomes nonlinear and the resonant frequency drops. At even higher drive amplitudes, the difference between junction geometries becomes apparent. Both the tunnel junction and the 3D nanobridge resonators enter the bifurcation regime, switching hysteretically between two oscillation states as the drive amplitude is ramped up and down (indicated by the striping in the phase response plots). In contrast, the 2D nanobridge resonator never stably enters the bifurcation regime, instead showing evidence of

chaotic behavior at higher drive amplitudes. These results indicate that only 3D junctions will be suitable for use in such low-Q nonlinear resonators.

# 2.4 Nanobridge SQUIDs

#### The SQUID Equations

For the purposes of this thesis, when I write "SQUID" I mean a two-junction device, also known commonly as a dc SQUID. Such a device behaves as an effective single junction, with a CPR which depends periodically on the flux  $\Phi$  threading the loop, with a period of a flux quantum  $\Phi_0 \equiv \frac{h}{2e}$ . Thus, for a total phase  $\phi$  across the SQUID,  $I_S = I_S(\phi, \Phi)$ . This flux dependence comes from satisfying the condition that the phase shift acquired by traveling around any superconducting loop must sum to a multiple of  $2\pi$ ; defining the reduced flux  $\varphi \equiv 2\pi \Phi/\Phi_0 = \Phi/\varphi_0$ :

$$\delta_1 - \delta_2 + \beta_{L1} I_1(\delta_1) - \beta_{L2} I_2(\delta_2) + \varphi = 2\pi n$$
(2.8)

Here the quantity  $\beta_L \equiv I_0 L/\varphi_0$  can be thought of as the ratio between the junction inductance and the linear inductance of the SQUID arm containing that junction; terms involving  $\beta_L$  represent the phase drop due to the linear inductance of the SQUID loop. The total phase across the SQUID is just given by the average phase across the two arms of the loop:

$$\phi = \left[\delta_1 + \beta_{L1}I_1(\delta_1) + \delta_2 + \beta_{L2}I_2(\delta_2)\right]/2 \tag{2.9}$$

Together, Eqs. (2.8) and (2.9) form a system which may be solved for  $\delta_1$  and  $\delta_2$ . Then summing the currents through the two junctions gives

$$I_S(\phi, \Phi) = I_1(\delta_1(\phi, \Phi)) + I_2(\delta_2(\phi, \Phi))$$

#### Flux Modulation

We will concern ourselves with two quantities of interest. The first is the SQUID critical current  $I_C$ . This is simply the maximum of  $I_S$  as a function of  $\phi$ . The other quantity is the SQUID inductance  $L_S$ , which can be found by taking the derivative of  $I_S$  with respect to  $\phi$  and plugging in to Eq. (1.3). For a tunnel junction SQUID with identical junctions and negligible loop inductance (i.e.  $\beta_L \approx 0$ ), these are simply given by

$$I_C(\Phi) = 2I_0 \left| \cos(\pi \frac{\Phi}{\varphi_0}) \right|$$
$$L_S(\Phi) = \frac{\varphi_0}{I_C(\Phi)}$$

Note that the critical current goes to zero (and the inductance to  $\infty$ ) at half integer flux quanta. This is due to perfect interference between junction currents; at half integer flux,



Figure 2.6: Critical current modulation of an asymmetric SQUID

For a SQUID with junctions of unequal critical currents, and with nonzero  $\beta_L$  or non-sinusoidal CPRs, the critical current modulation may be asymmetric about zero flux. Plotted here is the critical current of a SQUID containing two tunnel junctions, one with 60% the critical current of the other, and  $\beta_L = 0.25$ . The flux modulation asymmetry is readily apparent to the eye. If the junction CPRs are not known ahead of time, such asymmetry will make it very difficult to draw any quantitative conclusions about the junction behavior from the flux modulation.

 $\delta_1 - \delta_2 = \pi$ , so  $I = I_0 \sin \delta_1 + I_0 \sin(\delta_1 - \pi) = 0$ . The depth of this modulation will be suppressed if  $\beta_L$  is not zero, because the linear inductance of the SQUID arm (which has a linear effective CPR, rather than the sinusoidal CPR of a junction) makes the interference imperfect [14]. One can think of the junction and SQUID arm as forming an extended effective junction, with a CPR which is a distorted sinusoid formed by the combination of a sine and a linear function; this non-sinusoidal CPR need not obey the relation  $I(\delta) =$  $-I(\delta + \pi)$ . Thus, we see that the depth of the critical current modulation is a proxy for the nonlinearity of the junction CPR. For instance, a SQUID with two linear "junctions" will have a critical current modulation with a triangle-wave shape, with minima which are only 25% lower than the maxima.

Asymmetry between the critical current of the two junctions will also suppress the depth of the critical current modulation. When the asymmetric junctions do not have sinusoidal CPRs (or  $\beta_L$  is large enough that the effective junction CPR differs appreciably from a sine wave), the critical current modulation will be asymmetric about zero flux; see Fig. 2.6. While this skewing of the modulation can be fit to correct for the asymmetry if the junctions' CPR is known, it is not possible to do this for unknown CPRs. Therefore it is necessary to minimize any asymmetry if we want to use the SQUID modulation to gain information about the CPR. Furthermore, if the junctions do not confine the phase drop, as in the case of the 2D nanobridges discussed above, then the two junction phases are no longer well-defined independent quantities. That is, it does not make sense to say where one junction ends and the other begins. This type of junction-junction interaction will also suppress the critical current modulation [39]. We can then study the length scale over which two junctions will interact by measuring the SQUID modulation as a function of SQUID loop size.

# Chapter 3

# Static Transport Measurements of Nanobridges and NanoSQUIDs

As explained in Chapter 2, the depth and shape of SQUID critical current modulation is a proxy for the nonlinearity of the SQUID's junctions. Thus, switching measurements of the SQUID critical current provide a test of the theoretical predictions made using the Usadel calculations. In particular, we would like to confirm that short 3D nanobridges show strongly nonlinear CPRs. We would also like to confirm that the theory correctly predicts the dependence of the CPR on the nanobridge length and pad thickness. To this end, I present experimental measurements of SQUID critical current modulation for 2D and 3D nanoSQUIDs with varying junction lengths. This chapter closely follows the experimental results reported in [25].

### **3.1** Switching Measurements

When a current bias greater than the critical current  $I_C$  is applied to a junction, it switches into the voltage state, and develops a voltage across its leads  $V = I_C R_N$  [40]. The resistance  $R_N$  is the normal state resistance of the junction; if the junction were driven normal (for example, by a strong magnetic field), it would show this resistance across its leads. For a tunnel junction,  $V = 2\Delta/e$ , as a single normal electron will have to overcome the gap between the condensate band (below  $-\Delta$ ) and the quasiparticle band (above  $+\Delta$ ). In general, the  $I_C R_N$  product for a given junction geometry is a constant determined only by its CPR; for instance, the KO-1 CPR given in Eq. (2.4) gives  $I_C R_N = 1.32\pi\Delta/2e$ . For junctions made of aluminum, this voltage is easily measurable, reaching ~ 400  $\mu$ V. Thus, the method for measuring the critical current of a junction is simple: perform a 4-wire voltage measurement of the junction while ramping the applied current, and record the current at which a voltage jump occurs [41]. The same method applies for measuring the critical current of a SQUID.

In practice, as always, there are some experimental complications. It turns out that a junction does not, in general, switch to the voltage state exactly at its critical current.



Figure 3.1: Tilted washboard potential of a current-biased junction

When biased with a current flow, the junction metapotential acquires an overall slope, leading to the tilted washboard potential shown above. When the junction switches out of the superconducting state, the phase escapes from one of the wells and continues to evolve down the slope.

Thermal, electrical, and even quantum noise sources will induce current noise on the junction, causing it to switch at a value lower than the critical current. The mean value of this switch is called, appropriately enough, the *switching current*. Perhaps the clearest way to see the difference between the critical current and the switching current is to examine the potential energy of the junction. Recall that the voltage across a junction is directly related to the time derivative of the junction phase. This means that in the voltage state the phase is rapidly increasing. The junction energy as a function of phase, referred to as the *metapotential*, provides an intuitive view of this process. For an unbiased tunnel junction the metapotential is just a cosine function, with a height of  $2\varphi_0 I_0$ . When the junction is biased with a current  $I_B$  the potential acquires a term  $-\varphi_0 I_B \delta$ . This linear dependence adds to the junction energy to create what is known as a *tilted washboard potential*; see Fig. 3.1. The junction phase will sit at a local minimum of this potential, but if the right-hand barrier is small enough, the phase can be excited over into the next well by thermal or electrical noise. The phase may also tunnel through the barrier, a process known as macroscopic quantum tunneling [41]. In any case, if one wishes to accurately measure the critical current of a junction, it is necessary to carefully thermalize and filter the current bias lines, and to shield the junction from radiation. We are only interested in the modulation of the SQUIDs critical current (not its absolute magnitude), and so measurements of the switching current (not the actual critical current) will be sufficient.


Figure 3.2: Optical interferometer image of nanoSQUIDs

Our devices consist of 6 nanoSQUIDs, each with different bridge lengths, arranged in a 4-wire transport measurement geometry. An optical interferometer image is shown above. Current bias is applied via two lines on opposite sides of the SQUID, while the voltage across the SQUID is sensed via the other two lines.

## 3.2 Experimental Details

#### Devices

We fabricated chips consisting of 6 nanoSQUIDs, each arranged in a 4-wire transport measurement geometry. See Fig. 3.2. The devices were made by spinning a bilayer electron-beam resist, consisting of PMMA on top of its copolymer, on a silicon substrate. The device design was patterned using an SEM and developed. Aluminum was then evaporated onto the device at normal incidence, depositing metal at a constant thickness everywhere. For 3D devices, the sample was then tilted in-situ and aluminum was evaporated at a steep angle, thus depositing metal in the banks region but not in the nanobridge region; see Fig. A.2. More fabrication details are contained in Appendix A. Each sample contained either all 2D or all 3D bridges, with lengths of 75, 100, 125, 150, 250, and 400 nm. These bridges were 8 nm thick and 30 nm wide, connecting banks which were 750 nm wide and 8 nm (for 2D bridges) or 80 nm (for 3D) thick; see Fig. 3.3. The bridges were integrated into a SQUID loop with a washer size<sup>1</sup> of  $1 \times 1.5 \ \mu m$ ; this loop had a total linear inductance calculated to be  $L_{\text{loop}} = 5$ pH. Using the diffusion constant D calculated from the measured film resistivity, we estimate the coherence length in these thin-film devices to be  $\xi = \sqrt{\hbar D/2\pi k_b T_C} \approx 40$  nm. Thus, the shortest devices are only a few coherence lengths long, and all devices are less than a coherence length in width; the theoretical predictions of [24] imply that the short 3D bridges

<sup>&</sup>lt;sup>1</sup>The SQUID *washer* is the central opening in the SQUID loop.



Figure 3.3: AFM image of a 2D nanobridge and a 3D nanoSQUID

In order to demonstrate the difference in geometries between the two junctions, I show AFM images of a 2D nanobridge (a) and a 3D nanoSQUID (b). The 3D junctions are much thinner than the banks they connect, as shown in the image.

will closely approximate a short metallic weak link connected to ideal phase reservoirs, and thus exhibit a CPR well-described by the KO-1 result.

#### Measurement Apparatus

The samples were each placed in a shielded copper box and wirebonded to a printed circuit board (PCB) with 24 traces, shown in Fig. 3.4. The box was anchored to the base stage of a <sup>3</sup>He refrigerator with a copper mounting bracket and electrically connected via 12 twisted pairs of manganin wire. The electrical lines ran through a distributed copper-powder filter and a lumped-element 2-pole RC Pi filter at base temperature and  $\sim 1$  K, respectively, before running up to room temperature through a stainless steel shield. An electromagnet, made of superconducting NbTi wire wound around a copper bobbin, fit under the box, allowing us to provide a controllable flux bias to the SQUIDs. The sample was cooled down to the fridge base temperature of 265 mK. Custom electronics provided a ramping current bias and sensitive voltage detection, while a commercial Keithley 2400 sourcemeter was used to apply a current to the flux bias coil.

## **3.3** *IV* Measurements

In order to detect the switching current of a nanoSQUID, we ramp the applied current in a triangle wave pattern while monitoring the voltage across the SQUID. This gives a hysteretic IV curve, like the one shown in Fig. 3.5. As the current is ramped away from zero, the voltage remains at zero up to the switching current. At this point, the voltage suddenly jumps to  $V = I_C R_N$  and the IV curve becomes linear, as is characteristic of a normal resistor with resistance  $R_N$ . As current is ramped back down towards zero, the SQUID remains



Figure 3.4: Shielded sample box used for DC transport measurements

We enclose the sample in a copper box, pictured above, in order to shield it from electrical and thermal radiation and to thermalize it to the fridge base plate. The chip, visible at the center, is wire-bonded to the 24-trace PCB, which serves to transition between the chip and the micro-D connector (visible at the right). A superconducting coil magnet, not visible in this picture, presses up against the bottom of the PCB through a hole in the bottom of the box.



Figure 3.5: Measured IV curve for a typical nanobridge junction

Here, I plot a typical IV curve for a nanoSQUID consisting of two 75 nm long 3D nanobridge junctions. The voltage across the SQUID is zero up to the switching current  $I_{\text{switch}}$ , at which point the voltage jumps to  $V = I_C R_N$ . The voltage then increases linearly with current. As current is ramped back down, a finite voltage exists down to current much smaller than  $I_{\text{switch}}$  before switching hysteretically back to the zero-voltage state, as indicated by the black arrows. resistive down to a current which is much lower than the switching current. We attribute this hysteresis in the *retrapping current* to thermal heating [42]. When a junction is in its normal state, it dissipates energy, thus raising its temperature, and therefore suppressing its critical (i.e. switching) current. We note that the retrapping current depends on the ramp rate, as would be expected for this heating mechanism. As current is ramped down to a negative value (that is, current flowing in the opposite direction), the SQUID switches again in the same way; note, however, that this switching current need not be the same as the positive switching current if the SQUID is asymmetric, as shown in Fig. 2.6.

The switching current is easily measured in software with a simple edge-detection routine. Several IV curves are averaged together and then run through this edge-detection procedure in order to sensitively detect  $\bar{I}_{\text{switch}}$ , the mean switching current. If the switching current varied significantly each time the junction switched, then the IV curve would show a broad switching region, rather than the near-horizontal jump in V we observe in all measurements. From this we determine that the distribution of switching currents is narrow, and so  $\bar{I}_{\text{switch}} \approx I_{\text{switch}}$ . We ramp the current in a triangle wave symmetric about zero with a frequency of 1 kHz. This ramp rate, i.e. the switching repetition rate, was chosen to be sufficiently slow so that  $I_{\text{switch}}$  did not depend on the rate. A few thousand traces were required in order to achieve an excellent signal-to-noise ratio (SNR), and so a typical  $\bar{I}_{\text{switch}}$  measurement took a few seconds.

We plot the  $I_C R_N$  product for 2D and 3D bridges as a function of bridge length in Fig. 3.6. We first measure  $R_N$  by ordinary 4-wire resistance measurements just above the superconducting critical temperature for aluminum. This resistance increases linearly with bridge length, with an offset resistance of  $\approx 1.5 \ \Omega$  for 3D SQUIDs and  $\approx 20 \ \Omega$  for 2D SQUIDs. These offset values are consistent with the normal-state resistance of the SQUID arms, and so we subtract them to find the junction  $R_N$ . For short nanobridges (below 150 nm) the  $I_C R_N$  product is a constant at  $\approx 380 \ \mu$ V for 3D bridges and  $\approx 250 \ \mu$ V for 2D bridges. For longer bridges the  $I_C R_N$  product increases linearly, as one would expect from a long superconducting wire: the critical current of a wire does not depend on its length, only its cross-sectional area, but the normal-state resistance scales linearly with the length [40]. We also plot, for reference, the KO-1 theory value  $I_C R_N = 1.32\pi\Delta/2e$  and note that it is very close to the values measured for short 3D bridges.

#### 3.4 Flux Modulation

We next measured the switching current as a function of flux for all SQUIDs. The flux bias was varied by changing the steady current through the superconducting coil magnet. While is it difficult to precisely calculate the flux coupled into a SQUID loop by a coiled electromagnet, the flux may be exactly calibrated by measuring the modulation of the switching current. This modulation is periodic with a period of exactly  $\Phi_0$ . Recall that the modulation will be asymmetric for a nanoSQUID whose junctions have different critical currents. This asymmetry also reduces the depth of the modulation, complicating quantitative comparisons



Figure 3.6: Measured  $I_C R_N$  product for 2D and 3D nanobridge junctions

We measure the voltage across the SQUIDs as they switch to the voltage state (i.e. the  $I_C R_N$  product) at zero flux and plot this for all SQUID geometries measured. For short ( $\leq 150$  nm) bridges of both geometries, the  $I_C R_N$  product is a constant, with a value of 380  $\mu$ V for 3D bridges (red squares) and 250  $\mu$ V for 2D bridges (blue circles). Longer bridges show an  $I_C R_N$  product that increases linearly with bridge length, as expected for a superconducting wire. The black dashed line indicates the value of  $I_C R_N$  given by the KO-1 theory.

of the flux modulations of different junction geometries. While the asymmetry can be corrected for if the junction CPRs are known, the purpose of these experiments is to probe the CPRs themselves. Therefore, we compared the measured modulation with theoretical modulation curves calculated using our simulated CPRs with 10% asymmetry in the SQUID. Data from any devices that showed modulations which were more asymmetric than these theory curves were not used in this experiment.

Flux modulation of the critical current for 2D and 3D nanoSQUIDs with 75, 150, and 250 nm bridges are shown in Fig. 3.7. There are a few key features to note. The first is that all the modulation curves are nearly symmetric about zero flux, indicating good junction symmetry. While the 3D SQUID modulation is deep and distinctly curved, the 2D SQUID modulation is much more shallow and triangular in shape. A triangular flux modulation curve is characteristic of junctions with nearly linear CPRs, as stated in Ch. 2. We also note that for both geometries, the SQUIDs with shorter junctions show deeper and more curved  $I_C$  modulation.

We can make these statements more quantitative by defining the modulation depth  $(I_{C,max} - I_{C,min})/I_{C,max}$  and plotting this quantity as a function of junction design. See Fig. 3.8. It is immediately apparent that SQUIDs made with short (< 150 nm) 3D bridges have a strong modulation of almost 70%. This is approaching the 81% modulation of two



Figure 3.7: Measured  $I_C$  modulation for 2D and 3D nanoSQUIDs

The critical current of a SQUID modulates periodically with flux; the depth and curvature of this modulation is a proxy for the nonlinearity of the CPRs of the junctions. Here we plot measured  $I_C$  modulation curves for 2D (a) and 3D (b) nanoSQUIDs with 75 nm (solid red lines), 150 nm (dotted blue line), and 250 nm (dashed green line) long bridges. All the 2D nanoSQUIDs show shallow modulation with a nearly linear triangle-wave shape. In contrast, the 3D nanoSQUIDs with shorter bridges show deep, curved modulation, indicating increased CPR nonlinearity.

KO-1 junctions. In contrast, 2D SQUIDs never reach 50% modulation. For both geometries, the modulation depth decreases at longer bridge lengths, indicating the decrease in junction nonlinearity as the bridges grow longer than several coherence lengths. The dotted lines indicate the theoretical predictions made by using the CPRs calculated using the procedure outlined in Ch. 2, including the effect of measured  $\beta_L$ . These predictions fit the experimental data well, providing evidence that the calculated CPRs are representative of the junctions' behavior. Finally, we note the slight suppression of the 2D SQUID modulation for very short bridges. We attribute this to the fact that the phase drop in a 2D junction is not confined to the nanobridge, but rather leaks into the banks [24]. Thus, for two weak links in close proximity it is difficult to impose a phase difference between the two arms of the SQUID and the modulation is suppressed. Our theoretical predictions do not reproduce this effect, as they are made for isolated junctions; a numerical calculation of the full two-junction SQUID, of the kind done in [39], would be necessary to accurately predict the effect.

### 3.5 Conclusions

These results confirm that weak-link junctions made with 3D nanobridges exhibit greater CPR nonlinearity than those made with 2D nanobridges. Transport in 3D bridges up to a few coherence lengths long is may be well approximated by the KO-1 formula for an ideal,



Figure 3.8: NanoSQUID critical current modulation depth

The SQUID critical current modulation depth provides a direct proxy for the nonlinearity of the junctions' CPRs. Here, we have measured the modulation depth for 2D (blue squares) and 3D (red triangles) nanoSQUIDs with various different bridge lengths. The nanoSQUIDs with short ( $\leq 150$  nm) 3D bridges show strong modulation nearing 70%, which approaches the limit of the KO-1 theory (shown as a black dashed line). In contrast, 2D nanoSQUIDs are limited to modulation depths below 50%. The modulation depth drops for longer bridges in both geometries, indicating reduced nonlinearity. The red and blue dashed lines are predictions made using our simulated CPRs, showing good agreement with the data. The 2D SQUIDs with the 75 nm long bridges show a suppressed modulation compared to those with slightly longer bridges, perhaps indicating junction-junction interactions due to the reduced phase confinement in 2D nanobridge junctions.

short, metallic weak link. Even better agreement with experiment may be attained by using CPRs calculated by solving the Usadel equations for the exact geometry used. As such, 3D nanobridge devices should have sufficient nonlinearity for use in sensitive magnetometers and ultra-low-noise amplifiers, as suggested in [24] and Ch. 2.

## Chapter 4

## Nanobridge Microwave Resonators

While the dc measurements reported in the previous chapter are an important confirmation of some of the CPR characteristics predicted by theory based on the Usadel equations, they do not fully characterize the nanobridge junctions. In particular, we would like to probe the *inductive* properties of the junctions, which our switching measurements were not sensitive to. Our approach is to integrate the nanobridges (sometimes in nanoSQUIDs) into resonant circuits. This allows us to measure the nonlinearity of the junction inductance, as well as probing whether there are any strong microwave loss mechanisms. In addition, many devices such as amplifiers and magnetometers are made of resonant circuits [7], so measuring the junctions in these devices provides an important test of their behavior when embedded in a practical circuit. In this chapter, I describe microwave-frequency measurements of nanoSQUID-based resonant circuits with both low and high quality factors (Q). This chapter closely follows the experimental results reported in [26].

#### 4.1 Device Geometries

The first device type is designed for low total Q. A device image is shown in Fig. 4.1. The resonator consists of a nanoSQUID with 100 nm long 3D nanobridges in a 2 × 2  $\mu$ m loop, shunted by a large lumped-element capacitance; this capacitance comes from two aluminum pads over a large niobium ground plane with a 275 nm thick silicon nitride (SiN) dielectric, forming two parallel-plate capacitors in series with a total C = 7 pF. The total inductance of the device with zero flux through the SQUID is 80 pH, leading to a resonant frequency of 6.6 GHz. The resonator is directly coupled to the microwave environment via a 180° hybrid to obtain Q = 30.

The second device type is a quasi-lumped-element resonator designed for a higher total Q. A device image is shown in Fig. 4.2. The resonator is formed by an interdigitated capacitor (IDC) shunting a series combination of a nanoSQUID (of the same design as the one in the low-Q resonator) and a meander inductor. Device geometry simulations made using Microwave Office show a total capacitance of 0.53 pF and an inductance of 1.25 nH.



Figure 4.1: SEM image of the low-Q resonator device

The low-Q resonator is formed by a nanoSQUID shunted by two aluminum parallel-plate capacitors over a niobium ground plane, with a silicon nitride dielectric. A false-colored SEM image is shown above. An AFM scan of the nanoSQUID is shown in the inset. A terminated co-planar waveguide, at the top right, may be used to couple flux signals from dc up to GHz frequencies into the SQUID; it was not used in the experiments discussed in this chapter.



Figure 4.2: SEM image of the high-Q resonator device

The high-Q resonator is formed by a nanoSQUID in series with a meander inductor, shunted by an interdigitated capacitor. A false-colored SEM image is shown above. An AFM scan of the nanoSQUID is shown in the inset.

Coupling capacitors, made from small IDCs, controllably isolate the device from the 50  $\Omega$  microwave environment, leading to a total Q = 3500.

Both devices were fabricated using electron-beam lithography on a bilayer resist and

double-angle evaporation with a liftoff process. The fabrication procedure is similar to that used in the dc devices discussed in Ch. 3. Fabrication details are discussed in Appendix A.

#### **Optimizing Circuit Parameters**

In order to sensitively probe the behavior of nanobridge junctions in resonant circuits we must carefully design the circuits to optimize several factors. We would like to minimize the internal loss in the resonator from all other circuit elements (i.e. capacitors and linear inductors) so that any loss due to the nanobridges is easily detected. The capacitive element is usually the dominant source of loss in such resonators [43]. The SiN dielectrics used in our devices have a low-power microwave loss tangent  $\tan \delta \sim 10^{-3} - 10^{-4}$ , thus limiting internal Q to  $Q_{\rm int} < 10^3 - 10^4$ . This loss is negligible compared to the  $Q_{\rm ext} = 30$  of the low-Q device, and so the condition is satisfied. However, this internal loss would limit the total Q of any device designed with a high  $Q_{\rm ext}$ , so a lower-loss capacitor is required. IDCs often reach  $\tan \delta < 10^{-5}$  at low powers, allowing  $Q_{\rm int} > 10^5$ , and so we use an IDC for the high-Q device.

Of course, the resonant frequency of the device must fall in an easily measurable band. Our measurement electronics typically function from 4 - 8 GHz. This condition creates an added difficulty, as IDCs are typically limited to total capacitances below 1 pF. If the IDC is enlarged in an effort to increase this capacitance, stray inductance in the structure becomes significant, and it is no longer correct to treat it as a lumped-element capacitor. Typical nanoSQUIDs have inductances on the order of 10-100 pH; increasing this inductance requires making the nanobridge cross section smaller, which is a significant fabrication challenge. Thus, the capacitance is limited to 1 pF, and the Josephson inductance to 100 pH, which would give a resonant frequency of 16 GHz. In order to lower the resonant frequency of the high-Q device to the 4-8 GHz band, we introduce extra linear inductance into the circuit with a meander inductor. The inductance of this circuit is now a series combination of the meander inductor and the nanoSQUID. This causes the *participation ratio* of the Josephson inductance in the circuit to be greatly reduced. This reduces the sensitivity of the resonance to any nanoSQUID properties; for instance, any loss due to the nanoSQUID will be diluted by roughly a factor of the participation ratio. Thus, designing nanoSQUID resonators requires optimizing the trade-offs between  $Q_{int}$ , resonant frequency, and participation ratio.

#### 4.2 Measurement Apparatus

Devices were diced to size and glued to the surface of microwave circuit boards using GE varnish. The low-Q device was wire-bonded to a microwave 180° microstrip "rat-race" hybrid launch made of copper traces on a low-loss dielectric substrate<sup>1</sup> [44]. The high-Q device was wire-bonded to a similar microwave launch, designed in a single-ended co-planar waveguide (CPW) geometry. Pictures of both launch styles are shown in Fig. 4.3. The launches were

<sup>&</sup>lt;sup>1</sup>We use TMM boards from Rogers Corp.



Figure 4.3: Picture of CPW and rat-race hybrid microwave launches

The high-Q device is placed in a CPW launch, pictured at left. The launch shown is designed to be used either for a transmission resonator, or for two reflection resonators back-to-back. The low-Q device is placed on a rat-race hybrid launch, pictured at right. The device is placed on the copper backplate and wire-bonded to the pair of traces that extend to the right of the circuit board.

enclosed in copper radiation shields and surrounded with superconducting and cryoperm magnetic shielding. We anchored the devices to the base stage of a cryogen-free dilution refrigerator with a base temperature of 30 mK. In this limit of  $T \ll T_c$ , thermal quasiparticles in the superconductor are minimized and loss is reduced. We performed reflectometry measurements by injecting a microwave tone (between 4-8 GHz) into the devices through a circulator; the reflected tone was then passed through two more isolators before reaching a low-noise high electron mobility transistor (HEMT) amplifier at T ~ 3 K. The injection lines were made of stainless steel (which is lossy at microwave frequencies) and contained successive packaged attenuators totaling 60 dB of attenuation, thus insulating the device from thermal photons. Superconducting coil magnets anchored underneath the samples provide controllable static flux through the SQUIDs.

### 4.3 Linear and Nonlinear Resonance

We performed microwave reflectometry on the devices, measuring the real and imaginary parts of a reflected tone on a vector network analyzer (VNA) as a function of drive frequency and power. All resonators measured exhibited ordinary linear resonances at low input power  $\approx -105$  dBm and zero net flux. The real part of the reflected signal is an ordinary Lorentzian, with a shape given by

$$\operatorname{Re}(S_{11}) = \frac{A}{(\omega - \omega_r)^2 + \frac{\Gamma^2}{2}}$$

We can then find  $Q_{\text{int}} = \Gamma \omega_r / (\Gamma^2 - A)$ . By fitting the resonance curve for the high-Q device, we extracted  $Q_{\text{int}}$  as a function of power. When the average photon number in the resonator



Figure 4.4: Measured phase response for low-Q and high-Q nanoSQUID resonators

We sweep the microwave drive power and measure the phase shift of the reflected signal for the low-Q (a) and high-Q (b) resonators as a function of frequency. Power sweeps in opposite directions are vertically interleaved. Both devices show a linear resonance at low power, indicated in yellow (0° phase shift). As the drive power rises, the resonance bends to lower frequency and becomes nonlinear. Above the critical power  $P_C$ , both devices show stable hysteretic bifurcation, indicated by the vertical striping.

 $\bar{n} \approx 1$  we find  $Q_{\text{int}} = 5 \times 10^4$ , significantly larger than the total Q = 3500. As  $\bar{n}$  rises,  $Q_{\text{int}}$  also rises, which is consistent with the low- $\bar{n}$  loss being mainly due to a bath of two-level systems (TLSs) [43, 45]. Resonators made during the same fabrication process with the same IDC and inductor but without nanoSQUIDs showed similar  $Q_{\text{int}}$ , including the power dependence, suggesting that the nanoSQUID does not contribute significantly to loss in the device. We did not observe any decrease in  $Q_{\text{int}}$  in the nanoSQUID resonators even as drive power is increased, which might occur if *phase slips*—jumps of the phase across a junction by  $2\pi$ —were a significant source of loss. Due to impedance variations across the bandwidth of the device, it was not possible to accurately fit the internal loss of the low-Q resonator. We are only able to say that internal loss is negligible, so  $Q_{\text{int}} \gg Q = 30$ .

As drive power is increased even more, the resonances begin to become nonlinear. Phase response diagrams are shown in Fig. 4.4, similar to those calculated in Ch. 2. We plot the phase of the reflected signal, ranging from  $+180^{\circ}$  below resonance to  $-180^{\circ}$  above resonance. At the resonant frequency  $f_{\rm res}$  the phase shift is 0° (indicated in yellow). The data were taken at a fixed frequency with drive power swept up or down; once several sweeps in each direction were averaged, the frequency was stepped to the next value. The data shown have vertically interleaved sweeps of increasing and decreasing amplitude. Both devices exhibit linear resonances at low power. As drive power increases, the resonance becomes nonlinear—that is, the phase shift becomes sharper and asymmetric about 0°, as shown in Fig. 4.5—and the resonant frequency decreases. At drive powers of  $P_C \approx -90$  dBm for the



Figure 4.5: Phase response of linear and nonlinear resonances

At low drive powers, the response of a Josephson oscillator is linear and the phase shift in reflection is symmetric about the resonant frequency, as shown in blue. At higher drive powers, the response sharpens and becomes nonlinear, with an asymmetric resonance centered about a lower frequency, as shown in red.

high-Q resonator and  $P_C \approx -85$  dBm for the low-Q resonator, the resonance bifurcates into two stable states with different oscillation amplitudes. The resonators switch hysteretically between these states depending on the direction of power sweep, as indicated by the striping in the phase response. This behavior is characteristic of an anharmonic oscillator with a softening potential [40, 38]. Using the Duffing model, we approximate the SQUID as an inductor with a cubic nonlinearity. From this we can extract the Q of the resonator, giving Q = 29 and Q = 3400 for the two devices, in excellent agreement with our measurements of the linear resonance linewidth. The bifurcation regime can be stably accessed even in the low-Q device, as predicted in [24] and Ch. 2, thus indicating strong nonlinearity in the nanobridge junctions.

#### 4.4 Flux Tuning

By changing the current through the flux bias coil we can control the magnetic flux threading the nanoSQUID loop. Because the inductance of a SQUID modulates periodically with flux, we thus tune the devices' resonant frequencies with a period of  $\Phi_0$ . This modulation allows us to tune  $f_{res}$  in the 4-8 GHz band, an important functionality for practical circuits. This modulation, plotted in Fig. 4.6, can be theoretically modeled using our numerically-computed CPR for a 100 nm long 3D nanobridge, thus providing another test of our theoretical pre-



Figure 4.6: Flux tuning of resonant frequencies

Since the inductance of a nanoSQUID increases at finite flux bias, a nanoSQUID oscillator's resonant frequency will decrease as it is tuned away from zero flux. Above, I plot flux tuning for the low-Q (a) and high-Q (b) nanoSQUID resonators. The flux tuning is hysteretic, as shown by the differing curves taken sweeping flux up (empty red circles) and down (filled blue circles). The black lines are theoretical fits using our calculated CPRs, showing excellent agreement with the data.

dictions. We fit the observed  $f_{res}$  to a formula

$$f_{res} = \frac{1}{2\pi\sqrt{C(L+L_S(\Phi))}}$$

where the SQUID inductance  $L_S$  is given by

$$L_S(\Phi) = L_S(0)F(\Phi)$$

 $F(\Phi)$  is numerically calculated from the CPR. We fit the data using  $L_S(0)$  as the only free parameter. The results are plotted as solid lines; they describe the entire flux response very well. The values of  $L_S(0) \sim 50$  pH used to fit these curves agree with the previous DC switching measurements of similar nanoSQUIDs' critical currents  $(I_C)$ . These two measurements complement each other; while  $I_C$  measures the maximum supercurrent that can be passed through the nanoSQUID, i.e. the peak of the SQUID's effective CPR,  $L_S$  measures the inverse slope of the SQUID CPR near zero phase. These two quantities are not simply related in nanobridge SQUIDs, in contrast to an ideal tunnel-junction SQUID where  $L_S(\Phi) = \varphi_0/I_C(\Phi)$ . In particular, since the CPR of nanobridge junctions is not a perfect sinusoid,  $I_C$  does not modulate to zero, even for a symmetric SQUID with zero loop inductance; the minimum of  $I_C$  occurs at  $\Phi_0/2$ . In contrast,  $L_S$  continues to increase (i.e.  $f_{res}$ continues to decrease) past  $\Phi_0/2$ , as shown by the hysteretic modulation of  $f_{res}$  in Fig. 4.6. This hysteresis is due to the formation of two wells in the SQUID metapotential, as shown in Fig. 4.7, which give different inductances; as flux bias increases the central well shrinks, until finally the SQUID escapes into the next well, detectable as a sudden jump of the resonator frequency.



Figure 4.7: Metapotential of a hysteretic flux-biased nanoSQUID

Around half a flux quantum the metapotential of a nanoSQUID develops multiple wells. Here I plot a simulated metapotential for a nanoSQUID at 0.55  $\Phi_0$ . The SQUID may remain in the central well past half a flux quantum, and then suddenly switch to either of the side wells, changing its inductance. The metapotential is determined by integrating the CPR to find the Josephson energy as a function of phase, then plugging in junction phases given by Eqs. (2.8) and (2.9).

## 4.5 Parametric Amplification

When biased into the nonlinear regime with a strong pump tone, these resonators function as parametric amplifiers (paramps). The paramp amplifies signals within the bandwidth (centered at the pump frequency) with a power gain G, while adding noise which can be quantified by the noise temperature  $T_N$ . As pump power is increased and pump frequency is decreased (following the nonlinear resonance as it drops in frequency), gain increases until the critical bifurcation power  $P_C$  is reached, at which point the resonator becomes bistable. All devices measured could be stably biased into the paramp regime. Standard amplifier characterization data for the low-Q device is shown in Fig. 4.8, showing the gain and amplifier noise temperature as a function of frequency. We probe the device with a small signal tone with and without the strong pump, and measure the increase in the reflected signal when the pump is turned on; this gives us a measurement of G. The amplifier performance is excellent, with > 20 dB of gain over  $\approx 40$  MHz of bandwidth. The amplifier has a large dynamic range, with a 1 dB compression point (the signal power at which the amplifier begins to saturate and gain drops by 1 dB) of -115 dBm.

We can characterize the noise temperature of the device by measuring the ratio y between the total measured noise with the paramp pump on and the noise with the pump off. With the pump off, the noise is simply a half-photon of quantum fluctuations plus the noise temperature of the system  $T_{sys}$  (dominated by the HEMT amplifier noise temperature),



Figure 4.8: Amplifier characterization measurements of the low-Q resonator

When biased into the nonlinear regime, a nanoSQUID resonator acts as a paramp. Here, I plot gain and noise as a function of frequency for the low-Q device. The amplifier pump (not shown) is at 6.49 GHz. The device shows greater than 20 dB of gain (blue line) over roughly a bandwidth of roughly 40 MHz. The noise (red circles) is within the measurement error of being quantum-limited at  $\hbar\omega/2$  (black dashed line).

multiplied by the overall system gain. When the pump is on, the quantum fluctuations are added to the paramp noise temperature and amplified by the paramp gain G before passing to the rest of the system. In the ratio y the system gain divides out, leaving:

$$y = \frac{N_{on}}{N_{off}} = \frac{\left(\frac{\hbar\omega}{2} + k_B T_N\right)G + k_B T_{sys}}{\frac{\hbar\omega}{2} + k_B T_{sys}}$$
$$k_B T_N = \frac{(y-1)k_B T_{sys} + y\frac{\hbar\omega}{2}}{G} - \frac{\hbar\omega}{2}$$

As plotted, the noise temperature is near  $\hbar\omega/2k_B$  across the entire amplification band, which is the minimum added noise allowed by quantum mechanics for phase-preserving amplification [46]. There is some uncertainty in  $T_N$ , as indicated by the error bars in the figure, which is mainly due to uncertainty in our measurement of  $T_{sys}$ . For more details on this measurement, see [47]. We verified that there are no apparent spurious frequency components in the amplified output signal, thus confirming that these resonators can be used as effective low-noise paramps. The high-Q resonator shows similar paramp behavior, but with a bandwidth which is roughly 100 times smaller due to its higher Q, limiting its usefulness as a practical amplifier.

## 4.6 Conclusion

These measurements present the microwave response of anharmonic oscillators based on 3D nanobridge SQUIDs at  $T \ll T_C$ . We have observed strong nonlinearity in the nanobridge CPRs, enabling the resonators to stably bifurcate even with low total Q. We do not observe any increased loss due to the presence of the nanoSQUIDs, although the low external Q of one resonator style and the low participation ratio in the other reduce the sensitivity to such loss. We measured hysteretic flux tuning of the nanoSQUID inductance which is well described by our simulated nanobridge CPR. Finally, we have demonstrated the use of a low-Q nanoSQUID oscillator as a parametric amplifier, with near-quantum-limited noise, above 20 dB gain, and 40 MHz of bandwidth. These results confirm our theoretical calculations of the nanobridge CPR, and demonstrate the utility of nanobridges in resonant circuits.

## Chapter 5

# Dispersive NanoSQUID Magnetometry

The results of Chapters 3 and 4 confirm the theoretical prediction that short 3D nanobridges behave as nonlinear Josephson junctions. However, merely demonstrating this junction behavior is not the main goal of this project; we want to use these junctions to make practical devices. In this chapter, I will discuss a promising application of nanobridge junctions as the basis for sensitive magnetometers. In particular, I will present measurements of dispersive nanoSQUID magnetometers, where a flux signal is read out by measuring the change in inductance of the nanoSQUID. This chapter closely follows the experimental results reported in [27].

## 5.1 NanoSQUIDs in Magnetometers

SQUID sensors [48] have been used in many applications including ultra-low-field MRI [15], as scanning sensors of local current [49], and for magnetization studies of single magnetic molecules and ferromagnetic clusters [8, 50, 51, 52, 53]. In general the intrinsic sensitivity of a SQUID magnetometer depends on two things: the efficiency with which a flux signal is coupled into the SQUID; and the magnitude of the electrical signal (usually a voltage) transduced from the flux signal. For studies of small magnetic objects, such as a nanoscale ferromagnet or a single electron spin, most of the magnetic flux is concentrated very near the spin(s). In order to efficiently thread this flux through a SQUID loop, the loop must contain a narrow constriction. See Fig. 5.1 for an illustration. The Meissner effect prevents any magnetic field from penetrating into a superconductor superconductor [40]; thus, any magnetic field lines that would have passed through the plane of the SQUID will be lensed away if there is superconducting material in their path. For a wide SQUID trace, there is a large area where this lensing occurs, preventing much of the flux from threading the SQUID loop; a narrow constriction enables the lensing to be minimized [54].

If this constriction is narrower or thinner than a few coherence lengths ( $\xi \approx 40$  nm in



Figure 5.1: Flux coupling via a constriction

The Meissner effect will cause flux to lens away from any superconductor (gray). For a small spin (blue arrow), much of the flux will be contained very close to the spin. A wide superconducting trace, as in (a), will cause this flux to lens away, preventing it from being efficiently coupled into a SQUID loop. A narrow constriction, as shown in (b), will allow most of the flux to be coupled through the loop without being deflected.

typical aluminum thin films) it will naturally act as a Josephson junction, and so a 3D nanobridge geometry is the optimal choice for making these junctions strongly nonlinear. A tunnel junction could be made with similar length scales, but it would typically have orders of magnitude lower critical current than a nanobridge. The critical current of the SQUID (which is proportional to the critical currents of its junctions) determines the optimal driving current (which may be greater or less than  $I_C$ , depending on the type of device); a larger driving current leads to a larger transduced signal, thus increasing device sensitivity. For these reasons, nanoSQUIDs are typically preferred to tunnel junction SQUIDs for nanoscale magnetometry.

### 5.2 Dispersive Magnetometry: an Overview

Typical nanoSQUID flux measurements are performed by measuring the switching current of the SQUID using a method similar to that described in Ch. 3 [8, 50, 51, 52, 53]. This approach has some drawbacks. The bandwidth of these devices is typically limited to a few kHz, as the nanoSQUID must cool down and reset from the voltage state. In addition, the dissipation and junction dynamics associated with switching to the voltage state can produce significant measurement backaction [49], potentially making this mode of operation incompatible with sensing the quantum state of a nanoscale spin.

I will present a different approach, called *dispersive magnetometry*. A detailed explanation of dispersive magnetometry is presented in [55], with a prototype experimental demonstration using tunnel junctions in [7]. A SQUID is integrated into a resonant circuit in such a way that the SQUID inductance strongly influences the resonant frequency. The SQUID



Figure 5.2: SEM image of magnetometer device

The magnetometer is a lumped-element nonlinear resonator, shown here in a false-colored SEM image. Two aluminum capacitor pads (pale orange) sit above a niobium ground plane (purple), separated by a silicon nitride dielectric (not visible). These two series capacitors shunt the nanoSQUID (blue, also shown in detail in the inset), forming a parallel LC resonator. Flux signals at frequencies from DC up to several GHz are coupled into the nanoSQUID via the fast flux line (blue-green), which is just a terminated section of CPW. The nanoSQUID, shown in the inset viewed from a 45° angle, contains two 100 nm long 3D nanobridge junctions in a  $2 \times 2\mu$ m loop.

is biased with a static flux to a regime where its inductance changes strongly with flux. The resonator is then driven with a microwave frequency carrier tone at  $\omega_d$ , and read out by comparing the phase of the output tone with the input. A low frequency flux signal (that is, one within the bandwidth of the sensor) at  $\omega_s$  will modulate the inductance of the SQUID. This in turn modulates the resonant frequency of the device, and thereby modulates the phase shift on the output signal, again at a frequency of  $\omega_s$ . This phase modulation is equivalent to two coherent microwave sidebands at  $\omega_d \pm \omega_s$ . Thus the transduction of a flux signal to a voltage involves up-conversion of the signal from low frequency to the microwave domain.

#### 5.3 Devices

The device discussed here consists of a 3D nanoSQUID shunted by a lumped-element capacitor. A false-colored scanning electron micrograph of the device is shown in Fig. 5.2. The nanoSQUID is comprised of 100 nm long, 30 nm wide, 20 nm thick nanobridges connecting banks which are 750 nm wide and 80 nm thick in a  $2 \times 2 \mu$ m loop, and has a critical current  $I_C \approx 20 \mu$ A at zero flux. The capacitor is formed by two plates of aluminum over a niobium ground plane, separated by a SiN dielectric, and has a capacitance C = 7 pF. A terminated section of coplanar waveguide transmission line near the nanoSQUID is used



Figure 5.3: Magnetometer experimental schematic

Shown here is a simplified schematic of the measurement apparatus for the magnetometer. The magnetometer circuit is a nanoSQUID shunted by a lumped-element capacitor. The device is driven with a microwave tone at  $\omega_d$ ; the drive reflects off the device, carrying sidebands created by the influence of a flux signal at  $\omega_s$ . The reflected drive tone and sidebands pass through a circulator, where they are directed through amplification stages and up to room temperature. A cryogenic microwave transfer switch allows us to switch an LJPA in or out of the circuit to provide a low-noise preamplifier. The drive and sidebands are measured using a spectrum analyzer; the sideband SNR allows us to calibrate the effective flux noise.

to couple in flux signals ranging in frequency from dc up to several GHz. The resonator is directly coupled to the 50 $\Omega$  microwave environment via a 180° hybrid to achieve a low Q = 30. A superconducting coil magnet is placed under this structure to provide static flux bias. The design and launching of the magnetometer is nearly identical to the low-Q device discussed in Ch. 4; indeed, that device was used as an early prototype magnetometer. The only difference between the devices is the thickness of the junctions; the newer device was made with thicker nanobridges in an effort to increase their critical current.

## 5.4 Measurement Apparatus

The magnetometer was placed inside superconducting and cryoperm magnetic shields to minimize external magnetic noise, and cooled to 30 mK in a cryogen-free dilution refrigerator. A simplified measurement setup (omitting some details such as filters and attenuators) is shown in Fig. 5.3. The magnetometer pump is injected via a heavily attenuated input line and passes through a circulator before reflecting off the device. The reflected signal passes through to the third port of the circulator and continues to the amplification chain. Static flux bias is applied by running a current through the coil magnet via twisted-pair dc lines, which pass through lumped-element and distributed lossy filters to eliminate all higher-frequency noise. Low-frequency flux signals are applied via the CPW fast flux line, creating microwave sidebands on the pump. These signals are amplified by a low-noise HEMT amplifier at 4 K before being passed up to room temperature. A lumped-element Josephson parametric amplifier (LJPA), similar to the amplifier device discussed in Ch. 4, may be inserted in-situ between the magnetometer and HEMT via a microwave transfer switch [7, 5]. The magnetometer drive is split at room temperature into two signals with independent phases, and the LJPA is pumped by a one of these drives injected via a directional coupler (not shown in the figure). Once the microwave signals reach room temperature, they are further amplified before being passed into a spectrum analyzer; a sample spectrum in shown in the figure. The flux signal information is contained in the sidebands at  $\omega_d \pm \omega_s$ , and their height above the noise floor sets the SNR.

## 5.5 Magnetometer Characterization

#### Flux Tone Calibration

We first calibrate a test flux signal by passing a small voltage signal at very low frequency (typically at less than 100 Hz) through the fast flux line and observing the modulation of the phase on a reflected microwave probe tone. We compare this modulation to the change in phase when a small change in the static flux (originating from the coil magnet) occurs; the static flux may be easily calibrated by sweeping it over several flux quanta, using the fact that the frequency modulation of the resonator will be periodic in  $\Phi_0^{-1}$ . Thus, a given voltage at room temperature may be converted to a calibrated flux signal at the device.

#### Flux Noise

Next, we inject a known flux signal  $\Delta \Phi$  at a frequency between  $10^4 - 10^8$  Hz. We observe the height above the noise floor of the sidebands on the drive tone, and from this determine the voltage signal to noise ratio  $V_{\rm SNR}$  for a given integration bandwidth *B* of the spectrum analyzer. This allows us to calculate the *flux noise*  $S_{\phi}^{1/2}$  of the device. The flux noise of a flux sensor is just the noise level in the sensor at a particular frequency, given in flux units. Equivalently, it may be thought of as the size of a flux signal which the sensor can resolve with SNR of 1; a lower flux noise indicates a more sensitive device. We calculate the flux

<sup>&</sup>lt;sup>1</sup>It would be conceptually easier to just run a static current through the fast flux line and observe the period of resonant frequency modulation from it. However, typically it is not possible to pass enough current so that the device modulates through multiple periods, as the fast flux line has a limited critical current.



Figure 5.4: Linear regime flux noise

We plot the effective flux noise as a function of flux signal frequency from 10 kHz to 80 MHz. In the linear regime, the flux noise is constant at 210  $n\Phi_0/Hz^{1/2}$ , with a bandwidth of about 100 MHz. Below 1 kHz the noise rises with a 1/f character due to intrinsic SQUID flux noise.

noise as

$$S_{\phi}^{1/2} = \frac{\Delta \Phi}{V_{\rm SNR}\sqrt{\gamma B}} \tag{5.1}$$

where  $\gamma = 1$  or 2 (depending on the mode of operation) is a numerical factor that accounts for the correlation between sidebands, as will be discussed later. This quantity has units of flux per root bandwidth, and is typically reported in  $\Phi_0/\text{Hz}^{1/2}$ . We further define the bandwidth of the magnetometer as the frequency at which the flux noise has risen by a factor of  $\sqrt{2}$ .

#### Linear Regime Results

When the drive tone is sufficiently weak (< -75 dBm) the device is a linear resonator, with a resonant frequency which may be modulated by flux through the nanoSQUID. We term this mode of operation the *linear regime*. In this mode, the magnetometer can be viewed as an upconverting transducer of low frequency flux signals to microwave voltages, with no power gain (symbolized in the legend of Fig. 5.4 as a circular transducer). Here we measure the flux noise using Eq. (5.1) with  $\gamma = 2$ ; this factor of 2 occurs because each sideband contains identical information, while the noise is uncorrelated. Fig. 5.4 shows the effective flux noise as a function of flux excitation frequency with the device operated in the linear regime. Here, the LJPA has been switched out of the measurement chain. We obtain a flux noise of 210 n $\Phi_0/\text{Hz}^{1/2}$  with about 100 MHz of instantaneous bandwidth. In the linear regime, this bandwidth is limited by the resonator linewidth, as the resonator acts as a low-pass filter for any changes that occur faster than its intrinsic bandwidth. The device discussed had a frequency of 6 GHz and Q = 30, consistent with the measured flux bandwidth. At low



Figure 5.5: Flux noise with following LJPA

With an LJPA following the magnetometer, the system noise is greatly reduced to near the quantum limit. This allows a much greater sensitivity, as shown by flux noise measurement (light blue crosses). This gives a flux noise of 30  $n\Phi_0/Hz^{1/2}$ , with a bandwidth of 60 MHz set by the LJPA bandwidth.

frequencies (below 1 kHz) we observe a flux noise with a 1/f character and a value at 1 Hz of ~ 1  $\mu \Phi_0/\text{Hz}^{1/2}$ . This 1/f flux noise is intrinsic to all SQUID sensors, although its origin is not fully understood [56].

#### 5.6 System Noise

At the output of the magnetometer the flux sidebands have an intrinsic SNR, where the noise is just the quantum vacuum fluctuations of half a photon. These correspond to a noise temperature of 144 mK at 6 GHz. However, the amplification chain has a system noise temperature of 8 K (which is mainly dominated by the noise of the low-temperature HEMT amplifier). This introduces excess voltage noise onto the microwave sidebands, degrading SNR and thus raising the effective flux noise. While one may account for this added noise and calibrate it out to calculate the intrinsic sensitivity of the magnetometer (i.e. its flux noise in a quantum-limited noise environment), we would like to minimize the actual measured flux noise, as this indicates the device's true sensitivity in applications as a flux sensor.

In order to minimize the system noise temperature, we switch the LJPA into the measurement chain, immediately following the magnetometer. Because the two sidebands are phase coherent, they can be amplified *noiselessly* by using the LJPA to perform *phase sensitive amplification*. In this process, only one quadrature of the microwave field (i.e. signals with a certain phase, such as a sine wave) is amplified, while the other (i.e. signals 90° rotated from this phase, such as a cosine wave) is deamplified. The LJPA is pumped with a phase-shifted copy of the magnetometer drive; the phase is chosen so that the sidebands are aligned with this pump when they reach the amplifier. This style of amplifier has been measured to give a noise temperature that is near the quantum limit [7], thus greatly reducing the system noise. With the magnetometer in the linear regime and the LJPA operating at 20 dB of gain (symbolized in the legend of Fig. 5.5 as a circular transducer followed by a triangular amplifier), we measure a greatly reduced effective flux noise of  $30 n\Phi_0/\text{Hz}^{1/2}$ . The bandwidth is slightly reduced to 60 MHz, with the reduction due to the finite bandwidth of the LJPA. In this mode of operation we must use  $\gamma = 1$  in Eq. (5.1), as the parametric amplification process creates correlations between the noise surrounding the two sidebands [46].

### 5.7 Amplification

As shown in Ch. 4, the magnetometer device itself may act as an amplifier, due to the nonlinearity of the nanoSQUID inductance. By driving the resonator harder with a slightly lowered frequency, we bias it into the *paramp regime*. In this regime the device will amplify any signals within its amplification bandwidth, including the upconverted flux sidebands [7]. We can model the magnetometer as an effective two-stage device: a transducer followed by a near-noiseless parametric amplifier (symbolized in Fig. 5.6 as a combined circular transducer and triangular amplifier). The low-noise phase sensitive amplification process performs the same role as the LJPA in the previous section, lowering the system noise and thus improving SNR. Running the magnetometer in the paramp regime with 20 dB gain (with the LJPA switched out of the circuit) gives a flux noise of 30  $n\Phi_0/Hz^{1/2}$  with a bandwidth of 20 MHz, again set by the amplification bandwidth of the device. This reduced bandwidth is characteristic of parametric amplifiers involving a resonant circuit, where the product of the voltage gain and the instantaneous bandwidth is conserved [7]. At this level of power gain the system noise temperature is nearly quantum-limited. As such, additional improvements in SNR can only be obtained by increasing the magnitude of the transduced microwave sidebands, since the noise floor cannot be lowered any further.

## 5.8 Flux Transduction

#### **Bias Point Selection**

Once the system noise nears the quantum limit, improving transduction is the only way to decrease flux noise. We define the transduction factor  $dV/d\Phi$  as the sideband voltage created for a unit flux signal. The transduction factor increases with the drive signal amplitude  $V_{in}$  and the flux dependence of the device's resonant frequency  $df_{res}/d\Phi$ . Thus, flux noise may be lowered by increasing the microwave drive amplitude and by tuning the static flux bias farther away from zero (where the frequency modulation is steeper). However, the onset of resonator bifurcation at high drive powers limits the maximum carrier amplitude, as the bifurcated resonator no longer acts as a linear flux transducer. This threshold is further suppressed at finite flux bias, as the critical current of the nanoSQUID drops [25] and its



Figure 5.6: Flux noise in paramp regime

When the magnetometer is biased with a strong pump into nonlinear resonance, it performs phase-sensitive amplification of the up-converted flux signal. This lowers the system noise to near the quantum limit and provides greater sensitivity, as shown by flux noise measurement (red triangles). This gives a flux noise of  $30 \text{ n}\Phi_0/\text{Hz}^{1/2}$ , with a bandwidth of 20 MHz set by the nonlinear resonance linewidth.

effective nonlinearity increases. In practice, we find the operating points with the lowest flux noise near  $\Phi = \Phi_0/4$ .

#### **Device Design Optimization**

The critical drive amplitude for a Josephson oscillator (that is, the drive amplitude at which bifurcation occurs) depends linearly on the junction critical current [38]. Thus, the maximum magnetometer drive amplitude also increases linearly with the critical current. For reference, the tunnel junction prototype dispersive magnetometer reported in [7] had a flux noise  $S_{\Phi}^{1/2} = 140 \text{ n}\Phi_0/\text{Hz}^{1/2}$ . We attribute the factor of 5 improvement in flux noise present in our nanoSQUID device mainly to the increased critical current of the nanobridges relative to the tunnel junctions. In addition, the nanobridge junctions' reduced nonlinearity (compared to tunnel junctions) means that they give a slightly higher critical drive amplitude, although at the expense of a reduced  $df_{res}/d\Phi$ . In principle, simply raising the critical current of the junctions will allow flux noise to be lowered without limit. However, as critical current is raised Josephson inductance is lowered, making the stray linear inductance in the circuit more and more significant. This linear inductance dilutes the participation of the junctions, reducing the flux tuning. Also, when the participation ratio is low enough, there is no longer enough nonlinearity in the total inductance to stably reach the paramp regime, as seen for simulations of 2D junctions in Ch. 2. Thus, careful circuit design is necessary to minimize this stray inductance. One promising avenue is to minimize the size of the capacitor pads by raising the specific capacitance. This has been accomplished in newer devices by replacing the  $\sim 300$  nm thick SiN dielectric with a 15 nm thick AlO<sub>x</sub> dieletric.



Figure 5.7: Illustration of transduction angle

(a) As the magnetometer drive power increases, the transduction angle  $\theta_t$  between the up-converted signal (ghosted green arrow) and the amplified quadrature (black line) grows. At the same time, the gain rises, further deamplifying the component of the up-converted signal orthogonal to the amplified quadrature (ghosted blue arrow). The measured transduction (green arrow) is a combination of the amplified and deamplified components, and for high gain (b) lies completely along the amplified quadrature. As drive power rises, the magnitude of the up-converted signal grows, but the measured transduction shrinks, due to increasing  $\theta_t$  and G. At high gain  $\theta_t$  saturates at 60° and the measured transduction is exactly half the up-converted signal.

#### Transduction Angle

When the device is operated in the parametric regime there is another complication that reduces flux sensitivity. Recall that the resonator acts as a phase sensitive amplifier; one quadrature of an input signal is amplified noiselessly, while the other quadrature is deamplified [5]. When a flux signal is up-converted to microwave sidebands, these sidebands are phase-coherent and so represent a single-quadrature signal. However, this quadrature is not aligned perfectly with the magnetometer drive, and so does not lie fully align the amplified axis [7]. As a result, only the component parallel to the amplified quadrature is amplified, resulting in reduced sensitivity. This process is illustrated in Fig. 5.7. The relative angle  $\theta_t$  between the up-converted signal and the amplified quadrature is an intrinsic property of the nonlinear oscillator (which is well-described by the Duffing model) and approaches a value of  $60^{\circ}$  for large parametric gain [7, 57]. In the large-gain limit, only the amplified component of the signal is measurable. This causes the measured signal to be a factor of two smaller—i.e. the effective flux noise to be a factor of two higher—than the theoretical limit which could be achieved if one could control the relative angle  $\theta_t$ . In effect, the flux transduction has been reduced. In general, the total transduced signal T leaving the magnetometer, normalized by the power gain G, for a unit flux excitation is given by

$$T \sim \frac{dV}{d\Phi} \frac{\sqrt{\frac{\sin^2 \theta_t}{G} + G \cos^2 \theta_t}}{\sqrt{G}}$$
(5.2)

Note that the term  $\frac{dV}{d\Phi}$  grows linearly with the drive voltage, while G and  $\theta_t$  are also functions of the drive amplitude.

This degradation in transduction does not occur when the device is operated with unity gain (i.e. in the linear regime). As gain rises,  $\theta_t$  increases and its effects become more pronounced, as shown in Eq. (5.2). However, working in the linear regime limits performance,



Figure 5.8: Transduction and gain as a function of drive power

To quantify the misalignment between the up-converted signal and the amplified quadrature, we measure the t-factor (green, left axis) and the gain (blue, right axis) as a function of drive power. At low powers, gain is close to unity, and increasing the power increases the up-converted signal and thus the t-factor. As drive power rises, the gain begins to increase and  $\theta_t$  grows, leading to a decrease in t-factor. The t-factor drops by roughly a factor of 2 from its peak value, indicating that  $\theta_t$  has saturated at 60°. The dashed gray line indicates the bias point at which we found the minimum flux noise (without a following LJPA), optimizing the trade-off between maximizing transduction and minimizing system noise.

as even using a fully quantum-limited LJPA to follow the magnetometer does not give a system noise temperature at the quantum limit, due to losses in the microwave components between the magnetometer and amplifier [58]. Furthermore, the magnetometer drive power is higher in the paramp regime than in the linear regime, leading to a larger up-converted signal  $V_{up}$ . Thus there is a trade-off between minimizing  $\theta_t$  and its effects, maximizing  $V_{up}$ , and providing the lowest possible system noise temperature.

To quantify this effect, it is possible to observe the effective transduction. We implemented the following experiment: first, we pumped the magnetometer at a frequency where the maximum power gain G (at the optimal drive amplitude) was about 20 dB (i.e. a voltage gain of 10). We then stepped the drive amplitude, and measured the associated paramp gain by reflecting an additional weak microwave tone (near the drive frequency) off the biased nonlinear resonator. The voltage gain  $\sqrt{G}$  is plotted in Fig. 5.8 as the solid blue line (right axis), and reaches a maximum value of about 13. At each drive amplitude we also inject a 1 MHz flux tone via the fast flux line and measure the amplitude of the up-converted sidebands. This amplitude is proportional to the magnitude of the flux tone  $\Delta \Phi$ , the effective transduction coefficient T, and the total gain of the entire measurement chain. We divide the sideband amplitude by the magnitude of the flux tone and the paramp gain to infer a value we term the t-factor, which is equal to the transduction coefficient times the net voltage gain of the measurement chain (~ 10<sup>5</sup> in our setup):  $t = T\sqrt{G_{\rm sys}}^2$ . We plot the t-factor in Fig. 5.8 as the solid green line (left axis). At low drive powers, the transduction increases with power, since  $dV/d\Phi$  is directly proportional to drive amplitude. However, as the drive power increases the transduction quickly drops by about a factor of two as the paramp gain reaches its maximum, as expected due to the increase in  $\theta_t$ . The dashed gray line in Fig. 5.8 indicates the power at which the minimum flux noise was achieved, optimizing the trade-off between large transduction and low noise temperature (i.e. high gain).

#### **Two-stage Operation**

In order to optimize transduction and amplification independently, we switched the LJPA back into the circuit. In this configuration we can reduce the magnetometer drive amplitude to the optimal transduction point without sacrificing system noise, as the LJPA will provide sufficient gain to replace the reduced magnetometer gain. We operated the magnetometer at the point of maximum transduction and adjusted the LJPA such that the combined gain of both stages was 20 dB. We then adjusted the LJPA drive phase to align the flux tones fully along the amplified quadrature. This procedure resulted in an effective flux noise of  $23 n\Phi_0/Hz^{1/2}$ , a noticeable improvement over single-stage operation. The bandwidth was 20 MHz, again limited by the combined amplification bandwidth of the two devices. We note that we did not obtain the full factor of two improvement in flux noise predicted by the tfactor measurements. We believe this is due to transmission losses between the magnetometer and the LJPA, as the transduced signals propagate from the magnetometer to the LJPA via several passive microwave components (circulators, directional couplers, and interconnects). Transmission losses in this chain of components can be as high as 2-3 dB, thus nullifying some of the SNR improvement. By minimizing this loss, it should be possible to approach optimal amplification and flux noise performance.

#### 5.9 Benchmarking and Spin Sensitivity

In order to appreciate the meaning of the flux noise and bandwidth figures quoted, it is necessary to have some context. In Fig. 5.9 I have plotted flux noise and bandwidth for some commonly reported SQUID sensors. Sensors with a dissipative readout—a switching readout for nanoSQUIDs, or an  $I_C R_N$  voltage measurement for other geometries—are indicated in red, while those with a non-dissipative (i.e. dispersive) readout are indicated in blue. The two triangles at the top right are the device discussed in this chapter, in linear and paramp modes of operation. As you can see, this dispersive nanoSQUID magnetometer reaches near

<sup>&</sup>lt;sup>2</sup>Note that this gain is only due to the semiconductor following amplifiers; the paramp gain has been divided out. No precise calibration of  $G_{\text{sys}}$  was made, so it cannot be divided out, but it remains constant over the span of our measurement. The *t*-factor is thus a proxy for *T*.



Figure 5.9: Overview of SQUID magnetometer performance

In order to give some context for the flux noise and bandwidth reported, I plot an overview of some widely reported SQUID magnetometry results. Various types of SQUID sensors are shown, including metallic and carbon nanotube (CNT) nanoSQUIDs, ordinary shunted tunnel-junction SQUIDs, and single-junction (a.k.a. RF) SQUIDs where the inductance is measured dissipatively. Please note that flux noise *decreases* towards the right, indicating increased sensitivity. Our device, indicated by the two blue triangles in the upper right (one for linear regime operation, the other for paramp regime operation), shows near-record flux noise and bandwidth simultaneously, while implementing a non-dissipative readout. The results reported are published in: (1) Cleuziou et al. [59]; (2) Rogalla and Heiden [60]; (3) Troeman et al. [53]; (4) Drung et al. [61]; (5) Hao et al. [62]; (6) Mates et al. [63]; (7) Hatridge et al. [7]; (8) Van Harlingen et al. [64]; (9) Awschalom et al. [65]; (10) Levenson-Falk et al. [27].

the best reported values for flux noise and bandwidth, while operating in a non-dissipative mode.

The device will only be of practical use if it can actually couple strongly to a nanoscale magnet. To model this interaction, we approximate the SQUID as an infinitesimally thin  $1 \times 1 \mu m$  square loop. We then calculate the flux through this loop from a single Bohr magneton  $\mu_B$  (i.e. a single electron spin) as a function of the spin's distance from the loop edge. The result is plotted in Fig. 5.10. The horizontal line indicates the sensitivity of our magnetometer assuming its lowest flux noise and a 1 Hz integration bandwidth. As you can see, the device is capable of detecting a single spin in a 1 Hz bandwidth even at a distance of 67 nm. Since the nanobridge is typically less than 30 nm wide, reaching this limit is a challenge only of spin placement. Typical single-spin relaxation  $T_1$  times are on the order of 1 s [66], and so this device implements a single Bohr magneton detector.

N. Antler et al. tested the operation of similar nanoSQUID magnetometers in high parallel magnetic fields [67]. Such fields, typically ranging from 1 - 100 mT, are often necessary to obtain a desired energy level splitting in a nanoscale magnet. This work found that devices made with both 2D and 3D nanobridge junctions could each tolerate in-plane fields up to 60 mT, limited by flux trapping in the capacitor ground plane.



Figure 5.10: Flux coupled into a SQUID loop from a single electron spin

In order to estimate the flux coupled into the magnetometer from a spin, we simply calculated the flux through a  $1 \times 1\mu$ m square loop from a spin located in the plane of the loop and oriented normal to that plane as a function of the distance betweent the spin and the loop edge. We use a total spin 1, and so calculate the flux change that results from a single electron flipping spin. The result is plotted above as the red line. The dashed black line indicates the flux sensitivity of our device at its optimal bias point, assuming a 1 s integration time. We reach single-spin resolution at a distance of 67 nm, which is achievable with our nanobridge geometry.

## 5.10 Conclusion

We have demonstrated an ultra-low-noise dispersive nanoSQUID magnetometer. The device may be operated both as a linear flux sensor and as a parametric amplifier, allowing for optimization of bandwidth or signal amplification simply by changing bias conditions. In the linear regime we report flux noise of  $210 \text{ n}\Phi_0/\text{Hz}^{1/2}$  with a bandwidth of 100 MHz, while in the paramp regime we report flux noise of  $30 \text{ n}\Phi_0/\text{Hz}^{1/2}$  with a bandwidth of 20 MHz. The flux noise in the linear regime is limited by the system noise temperature; following the device with an ultra-low-noise LJPA allows a flux noise of  $30 \text{ n}\Phi_0/\text{Hz}^{1/2}$  with a bandwidth of 60 MHz. We have also demonstrated the internal dynamics of up-conversion and amplification in the device, showing the reduction in flux transduction as the flux signal rotates out of the amplified quadrature with increasing gain. Optimizing for transduction and system noise simultaneously by using the LJPA, we achieve flux noise of  $23 \text{ n}\Phi_0/\text{Hz}^{1/2}$  and a bandwidth of 20 MHz. Finally, the flux coupling into the device seems sufficient to allow for measurement of a single Bohr magneton in less than a typical spin  $T_1$ . Combined with other work showing that this device geometry can tolerate up to 60 mT of in-plane magnetic field [67], these results demonstrate the usefulness of the device as a practical nanoscale magnetometer.

## Chapter 6

# Quasiparticle Trapping in Nanobridges

One of the most exciting applications of Josephson junctions is in superconducting quantum bits (qubits) [4]. Typically based around superconducting resonant circuits, the Josephson element provides the anharmonicity essential to isolating two quantum states for use in computation. Currently, a major limitation of superconducting qubits is their coherence times, with relaxation time  $T_1$  and phase coherence time  $T_2$  both typically on the order of 100  $\mu$ s. Recent experiments have suggested that dielectric losses in the tunnel junctions typically used in such qubits do not contribute strongly to relaxation [22, 68], and that critical current fluctuations do not contribute strongly to dephasing [69]. Still, we cannot say that the junctions do not limit the qubit coherence; their interactions with quasiparticles may cause both loss and noise in the qubit circuit. In order to optimize qubit performance, it is necessary to fully understand the interaction between quasiparticles and Josephson junctions.

In weak-link junctions such as nanobridges it is possible for quasiparticles to trap inside the junction itself, altering the junction properties. By probing the behavior of these trapped quasiparticles it is possible to learn both their steady-state distribution, and to gain information about their creation and thermalization mechanisms. Combined with experiments which study the tunneling behavior of bulk quasiparticles across a junction [20], these measurements provide a detailed picture of the loss- and noise-inducing interactions between quasiparticles and Josephson junctions, and can guide efforts towards mitigating their effects. In this chapter, I discuss dispersive measurements of quasiparticle trapping in nanobridge Josephson junctions. The chapter closely follows the experimental results reported in [28].

## 6.1 Superconducting Quasiparticles

In an ideal superconductor at zero temperature, all of the electrons have condensed into Cooper pairs capable of carrying supercurrent. The Cooper pairs occupy the *condensate* 



Figure 6.1: Andreev state energies

Each conduction channel in a junction supports a pair of Andreev bound states, with energies depending on the phase bias and the channel transmittivity. The excited states are shown in red, ground states in blue. Andreev energies for  $\tau = 1$  and 0.5 are plotted in the solid and dashed lines, respectively. More transmissive channels have bound states whose energies approach closer to zero (i.e. the Fermi level) as phase bias approaches  $\pi$ . The Andreev gap  $\Delta_A$ , defined as the energy difference between the excited state and the bottom of the quasiparticle band, is illustrated by the black arrows.

band, which has a maximum at an energy  $-\Delta$ . There are then no available states in the superconducting gap (energies between  $\pm \Delta$ ), and then another band symmetric to the condensate band above  $+\Delta$ : the quasiparticle band. At nonzero temperatures a finite population of Cooper pairs will split into normal electrons (i.e. quasiparticles). The fractional density of thermally-generated quasiparticles  $x_{eq}$  is given by

$$x_{eq} = \sqrt{\frac{2\pi k_B T}{\Delta}} e^{-\Delta/k_B T} \tag{6.1}$$

Recent experiments have shown that a non-negligible population of quasiparticles exist in typical aluminum superconducting circuits even at very low temperatures [22, 23]. This non-equilibrium quasiparticle density  $x_{neq}$  adds to the thermal quasiparticles to produce a total density

$$x_{qp} = x_{neq} + x_{eq}$$

which, again, is non-zero even at very low temperature.

#### 6.2 Andreev States: a Brief Review

As explained in Ch. 2, a nanobridge junction may be treated as a set of 1D normal conduction channels, which transmit supercurrent via Andreev reflection. For each conduction channel

there is a pair of Andreev states, with energies  $E_A$  given by

$$E_{A\pm} = \pm \Delta \sqrt{1 - \tau \sin^2 \frac{\delta}{2}} \tag{6.2}$$

where  $\tau$  is the channel transmittivity and  $\delta$  is the phase across the junction. See Fig. 6.1. When occupied, each state carries a current given by

$$I_{\pm}(\delta) = \frac{1}{\varphi_0} \frac{\partial E_{A\pm}}{\partial \delta} = \mp \frac{\Delta}{4\varphi_0} \frac{\tau \sin \delta}{\sqrt{1 - \tau \sin^2(\frac{\delta}{2})}}$$
(6.3)

Note that the upper and lower Andreev states carry equal and opposite currents. At zero temperature, only the lower state is occupied, and the channel transmits positive current. However, if the upper state is also occupied, the two currents interfere to zero and the channel carries no net supercurrent. We say that the channel has been *poisoned* by a quasiparticle.

For a conduction channel with  $\tau > 0$ , the upper Andreev state will drop below the gap energy  $\Delta$  at finite phase bias. Any quasiparticle which exists in the bulk superconductor near the junction must have an energy above  $\Delta$ , since there are no available intragap states. Thus, the junction provides lower-energy quasiparticle states, which may exist at a much higher density than any locally available states in the condensate<sup>1</sup>. Because it is energetically favorable, the upper Andreev state acts as a quasiparticle trap. A channel poisoned by this trapping no longer contributes to transport of supercurrent, and so the poisoning modifies both the critical current and the inductance of the junction. This modification is measurable, thus providing a probe of the quasiparticle trapping process. Previous experiments have probed the trapping of quasiparticles in quantum point contact junctions with only a few conduction channels by performing switching current measurements [21], and have used the trapped quasiparticles as probes of the Andreev energies [70].

## 6.3 Dispersive Measurements of Quasiparticle Trapping: Theory

The theory discussed here was developed by Filip Kos and Prof. Leonid Glazman at Yale University; I have summarized their results in this section. F. Kos performed all the theoretical fits shown in this chapter.

Consider a resonator consisting of a capacitor in parallel with a series combination of a linear inductor L and a symmetric nanoSQUID. Each junction in the nanoSQUID has an

<sup>&</sup>lt;sup>1</sup>Assuming conservation of charge, any quasiparticles made must come from a Cooper pair, so there will be an available condensate state for each quasiparticle. However, these states are somewhat localized, and the quasiparticle may travel some distance before it reaches the junction, so there may be no *locally* available states in the condensate. Furthermore, a single junction may have thousands of available Andreev states, providing a large density of states for the quasiparticle to relax to.

inductance  $L_J$ , which sum in parallel to a SQUID inductance  $L_S = L_J/2$ . Thus, the resonant frequency of the device is

$$\omega_0 = [C(L + L_J/2)]^{-1/2}$$

The static phase across each junction will be equal to half the flux phase,  $\delta = \frac{1}{2}\varphi = \pi\phi$ , where  $\phi$  is the normalized flux  $\phi \equiv \frac{\Phi}{\Phi_0}$ . At nonzero flux, the Andreev states in the junctions will begin to act as quasiparticle traps. If a quasiparticle traps in one of the junctions, then that junction will have a higher inductance corresponding to the poisoning of a single conduction channel. We will treat this as a junction wherein that channel has been completely eliminated. Thus, the new resonant frequency is

$$\omega' = [C(L + L'_S)]^{-1/2}$$

If the resonator is probed with a microwave tone near  $\omega_0$ , the reflected signal will have a phase shift which depends on the magnitude of the frequency shift and the resonator Q.

If we average the reflected signal over a time scale which is long compared to the quasiparticle trapping and escape times, then we effectively integrate the resonator response function  $S(\omega)$  over all quasiparticle configurations, weighted by their probabilities. Let us consider weak links with effective channel number  $N_e$ , i.e.  $N_e$  pairs of Andreev states. Assume that  $N_e$  is large, so that any single trapped quasiparticle only slightly changes the junction inductance. Let  $n_i = 0, 1$  denote the number of quasiparticles trapped in the *i*-th channel. We then define the quasiparticle configuration  $\{n_i\}$ ; the resonant frequency is a function of this configuration,  $\omega_0 = \omega_0(\{n_i\})$ . Since the response function  $S(\omega)$  depends on the resonant frequency, the average response function we measure is given by

$$\bar{S}(\omega) = \sum_{\{n_i\}} p(\{n_i\}) S(\omega, \omega_0(\{n_i\}))$$
(6.4)

where  $p(\{n_i\})$  is the probability to find the system in the configuration  $\{n_i\}$ . We assume that trapping events are independent, so we can write this as the product of the probabilities of trapping in each channel:

$$p(\{n_i\}) = \prod_i p_i(n_i)$$

We can then define the average number of quasiparticles trapped in the junction  $\bar{n}_{\text{trap}} = \sum_{i} p_i$ , where  $p_i \equiv p_i(n_i = 1)$ . We can then write the average response function as a convolution:

$$\bar{S}(\omega) = \int d\Omega F(\Omega) S(\omega, \Omega)$$

where

$$F(\Omega) = \sum_{\{n_i\}} p(\{n_i\})\delta(\Omega - \omega_0(\{n_i\}))$$
(6.5)

where  $\delta(x)$  is the Dirac delta function. For a linear resonator<sup>2</sup>, the response function is Lorentzian:

$$S(\omega, \Omega) = \frac{\Gamma/\pi}{(\omega - \Omega)^2 + \Gamma^2}$$
(6.6)

The resonance linewidth  $\Gamma$  can easily be measured if all trapped quasiparticles can be eliminated (i.e. at zero flux). Thus, the problem of fitting the response function reduces to solving for  $F(\Omega)$ .

#### The Gaussian Approximation

In general,  $F(\Omega)$  may be difficult to solve for. However, there are two limits in which we can simplify the sum: the limits of large  $(\bar{n}_{trap} \gg 1)$  and small  $(\bar{n}_{trap} \lesssim 1)$  average number of trapped quasiparticles. In the case of large average number of trapped quasiparticles, we can use the saddle point approximation to evaluate  $F(\Omega)$ :

$$F(\Omega) = \int \frac{d\alpha}{2\pi} \sum_{\{n_i\}} p(\{n_i\}) e^{i\alpha(\Omega - \omega_0(\{n_i\}))} = \int \frac{d\alpha}{2\pi} \sum_{\{n_i\}} p(\{n_i\}) \exp[i\alpha(\Omega - \omega_0^{(0)} - \sum_i \frac{\partial\omega_0}{\partial n_i} n_i)]$$

Here we expanded  $\omega_0(\{n_i\})$  around  $\omega_0^{(0)}$  (the resonant frequency with no trapped quasiparticles) because each quasiparticle only slightly shifts the resonant frequency. Writing the probability  $p(\{n_i\})$  as a product of  $p_i$ 's we get:

$$F(\Omega) = \int \frac{d\alpha}{2\pi} \exp[i\alpha(\Omega - \omega_0^{(0)}) + \sum_i \ln(1 - p_i + p_i e^{-i\alpha\partial\omega_0/\partial n_i})]$$
(6.7)

The saddle point approximation of this integral then gives a Gaussian for  $F(\Omega)$ :

$$F(\Omega) \propto \exp\left[-\frac{(\Omega - \omega_0^{(0)} - \sum_i p_i \partial \omega_0 / \partial n_i)^2}{2\sum_i p_i (\partial \omega_0 / \partial n_i)^2}\right]$$
(6.8)

This Gaussian has a center frequency

$$\omega_0 = \omega_0^{(0)} + \sum_i p_i \frac{\partial \omega_0}{\partial n_i}$$

and a width

$$(\delta\omega_0)^2 = \sum_i p_i (\frac{\partial\omega_0}{\partial n_i})^2$$

Thus, in the case of a large average number of trapped quasiparticles, the resonance is a convoluted Gaussian-Lorentzian with its maximum shifted away from the 0-quasiparticle value.

 $<sup>^2 \</sup>rm While$  the nanoSQUID adds nonlinearity to the circuit, at low drive powers the resonance is essentially linear.
#### **Few Quasiparticles**

In the case of a small average number of trapped quasiparticles, we may consider only a few terms of the sum in Eq. (6.5), corresponding to 0, 1, or 2 trapped quasiparticles:

$$F(\Omega) = A[\delta(\Omega - \omega_0^{(0)}) + \sum_i p_i \delta(\Omega - \omega_0^{(i)}) + \sum_{i,j} p_i p_j \delta(\Omega - \omega_0^{(i,j)})]$$
(6.9)

where A is a normalization constant and  $\omega^{(i)}$  is the resonant frequency with one quasiparticle trapped in the *i*-th channel. In the case where the probability  $P_k$  of trapping k quasiparticles is Poisson-distributed (i.e. trapping events are completely uncorrelated), then  $A = e^{-\bar{n}_{\text{trap}}}$ and

$$P_k = \frac{\bar{n}_{\rm trap}^k}{k!} e^{-\bar{n}_{\rm trap}}$$

#### **Resonant Frequency Shift**

We define the participation ratio q as a function of flux:

$$q(\phi) \equiv \frac{L_J(\phi)/2}{L + L_J(\phi)/2}$$
$$q_0 \equiv q(\phi = 0)$$

We can then write the resonant frequency with a quasiparticle trapped in a channel with transmittivity  $\tau^{(i)}$  and energy  $E_A^{(i)}$  as

$$\omega_0^{(i)} \equiv \omega_0(\tau^{(i)}) = \omega_0^{(0)} - \frac{q}{2}\omega_0^{(0)}L_J^{(0)}\Delta\frac{1}{L_J^{(i)}}, \quad \Delta\frac{1}{L_J^{(i)}} = \frac{2e^2\Delta\tau^{(i)}\left(\cos\delta + \tau^{(i)}\sin^4\frac{\delta}{2}\right)}{\hbar^2(E_A^{(i)}/\Delta)^3} \tag{6.10}$$

We can use Eq. (6.10) in the expression given in (6.9) and transform the sums into integrals by assuming the trapping probability for a channel depends only on the Andreev energy,  $p^{(i)} = p(E_A(\tau^{(i)}))$ . For a distribution of channel transmittivities  $\rho(\tau)$  this gives

$$F(\Omega) = e^{-\bar{n}_{\text{trap}}} [\delta(\Omega - \omega_0^{(0)}) + \int_0^1 d\tau \rho(\tau) p(E_A(\tau)) \delta(\Omega - \omega_0(\tau)) + \int_0^1 d\tau_1 \int_0^1 d\tau_2 \rho(\tau_1) \rho(\tau_2) p(E_A(\tau_1)) p(E_A(\tau_2)) \delta(\Omega - \omega_0(\tau_1, \tau_2))]$$

Finally, convolving this expression with the Lorentzian resonance lineshape gives the average response function:

$$\bar{S}(\omega) = P_0 S(\omega, \omega_0^{(0)}) + P_1 \int_0^1 d\tau \rho(\tau) p(E_A(\tau)) S(\omega, \omega_0(\tau))$$
$$+ P_2 \int_0^1 d\tau_1 \int_0^1 d\tau_2 \rho(\tau_1) \rho(\tau_2) p(E_A(\tau_1)) p(E_A(\tau_2)) S(\omega, \omega_0(\tau_1, \tau_2))$$
(6.11)

For this discussion we will assume that the channel transmittivities are given by the Dorokhov distribution [30]:

$$\frac{N_e}{3\tau\sqrt{1-\tau}}\tag{6.12}$$

This assumption is equivalent to assuming that the junction has a KO-1 CPR, as explained in Ch. 2. While typical nanobridges do not have exactly this CPR, it is a reasonable approximation, as shown by [25] and [26]. If the quasiparticles are thermally distributed, then the trapping probability will be given by the Gibbs distribution:

$$p(E_A(\tau)) = \bar{n}_{\text{trap}} f(E_A(\tau)) = \bar{n}_{\text{trap}} \mathcal{N} e^{-E_A(\tau)/k_B T}$$
(6.13)

where the normalization factor  $\mathcal{N}$  is determined from the condition  $\int d\tau \rho(\tau) p(E_A(\tau)) = \bar{n}_{\text{trap}}$ .

We thus have a theoretical framework to fit the results of measurements in both the lowand high- $\bar{n}_{trap}$  limits. Now we must see what the experiments tell us!

## 6.4 Device Design and Apparatus

The device studied consists of a nanoSQUID formed by two 100 nm long, 25 nm wide, 8 nm thick 3D nanobridges arranged in a  $2 \times 2 \ \mu$ m loop. The SQUID is placed in series with a linear inductance of 1.2 nH and shunted by an interdigitated capcitor (IDC) with C = 0.93 pF. This arrangement forms a parallel LC oscillator with resonant frequency  $\omega_0 = 2\pi \times 4.72$  GHz at zero flux. False-colored SEM images of the device are shown in Fig. 6.2. Coupling capacitors isolate the device from the microwave environment, giving  $Q_{ext} = 5.3 \times 10^4$ . The device was optimized for a few considerations. The first is a high total Q, necessitating the use of an IDC rather than a capacitor with a lossy dielectric; the measured  $Q_{int} \approx Q_{ext}$  and was likely limited by the IDC. The second consideration was participation ratio; we wished to have it be as high as possible so that the resonance would be more sensitive to quasiparticle trapping, and so the nanobridges were made very thin so as to have a large inductance. The goal behind these considerations was to make the frequency shift from trapping a single quasiparticle move the resonance more than a linewidth, making it easier to interpret the



Figure 6.2: SEM images of the quasiparticle trapping detection resonator

The device discussed is a quasi-lumped-element resonator. A nanoSQUID (circled, not visible) is placed in series with a linear inductance (orange). This is shunted by an interdigitated capacitor (light blue). A fast flux line (light grey), not used in this experiment, is visible to the left. The right panel shows a zoom-in of a single 3D nanobridge, showing the bridge (orange) connecting the thick banks (dark blue). The thinner first layer of evaporated material, shifted from the thick banks, is visible at the top (green).

data by eye. Finally, we needed the device's resonant frequency to be as low as possible, in order to avoid resonant interactions between the oscillator and the Andreev states.

The device was wire-bonded to a microwave circuit board and enclosed in a copper box. Flux bias was applied via a superconducting coil magnet underneath the box. The box was anchored to the base stage of a cryogen-free dilution refrigerator with a base temperature of 10 mK and enclosed in blackened radiation shields, as well as superconducting and cryoperm magnetic shielding. Microwave reflectometry was performed via a directional coupler, which also allowed signals to be coupled into the device up to 20 GHz for use in spectroscopy.

## 6.5 Resonance Measurements

We performed microwave reflectometry on the device to determine the resonant response function  $S(\omega)$ ; in this case the response is just the complex reflection coefficient  $S_{11}$ . We began by measuring the response as a function of flux, averaged over many seconds. Traces showing the real part of the reflected signal at several flux values are shown in Fig. 6.3. At low flux bias ( $\phi \leq 0.2$ ) the resonance has an ordinary Lorentzian lineshape. However, at larger flux biases the resonance peak shrinks and begins to develop a second "hump" at lower frequency, indicating the development of another, broader resonance. At the highest flux biases an even broader third resonance is discernible. These multiple resonance peaks are indicative of quasiparticles trapping in the junctions, raising their inductance and thus lowering the resonant frequency of the device; the three peaks correspond to 0, 1, and 2 quasiparticles trapped in the device. As the 0-quasiparticle peak is only somewhat suppressed



Figure 6.3: Flux dependence of quasiparticle trapping

We plot resonance lineshapes as a function of flux at 10 mK. At low flux values, the resonance is a single Lorentzian peak whose height is constant in flux. At higher flux, the peak begins to shrink and another "hump" forms at lower frequency. At the highest flux biases, a third hump is discernible at even lower frequency. These humps are the 1-quasiaprticle and 2-quasiparticle resonance peaks; they grow, as the 0-quasiparticle peak shrinks, with increasing flux bias, due to the increased trapping probability in the deepening Andreev trap states.

from its 0-flux height, it appears that we are in the low- $\bar{n}_{trap}$  limit discussed above. Note that as flux bias grows  $E_A$  drops, and so the likelihood of a quasiparticle trapping in the Andreev state grows. This is shown in the data, as  $\bar{n}_{trap}$  grows with flux, and so the 0-quasiparticle peak shrinks. In a junction with many conduction channels with different transmittivities there are a range of values of  $E_A(\tau)$ , leading to differing trapping probabilities. Since the different channels have different inductances when they are not poisoned, there are a broad range of resonant frequencies with one trapped quasiparticles, and thus the 1-quasiparticle peak is broadened. A similar argument applies to the 2-quasiparticle peak; the range of 2-quasiparticle trapping configurations is even broader, and so the 2-quasiparticle peak is as well.

#### **Temperature Dependence**

We next measured the resonance at finite flux bias as a function of temperature by controllably heating the sample stage. Data at  $\phi = 0.464$  is shown in Fig. 6.4. At low temperature, both the 1-quasiparticle and 2-quasiparticle peaks are visible. As temperature rises, first the 2-quasiparticle and then the 1-quasiparticle peak are suppressed, while the 0-quasiparticle peak grows, indicating a drop in  $\bar{n}_{trap}$ . This is to be expected, as quasiparticles at higher temperature are less likely to occupy lower-energy states (i.e. trap states). At much higher temperature (T = 200 mK)  $\bar{n}_{trap}$  raises again due to rising quasiparticle density  $x_{qp}$ , as



Figure 6.4: Temperature dependence of quasiparticle trapping

We plot resonance lineshapes as a function of temperature at  $\phi = 0.464$ . At low temperature, the number of trapped quasiaprticles is high, and the 0-quasiparticle resonance is strongly suppressed, with 1- and 2-quasiparticle peaks visible. As temperature rises, first the 2-quasiparticle and then the 1-quasiparticle peak disappear as the trapping probability decreases. Above 250 mK, the resonance broadens and shrinks, indicating increased loss at all fluxes.

in Eq. (6.1). When the sample is heated to 250 mK the resonance broadens and shrinks, even at 0 flux. We attribute this effect to increased loss (i.e. reduced  $Q_{int}$ ) in the resonator due to bulk quasiparticle transport, as  $x_{qp}$  grows quite significant for such a high-Q device  $(x_{eq} \sim 4 \times 10^{-4})$ .

## 6.6 Fits to Theory

We next attempted to fit the data using the theory developed in Section 6.3. First, we needed to determine  $q(\phi)$ . The resonant frequency is given by

$$\omega_0(\phi) = \omega_0(0) [1 + q_0 \frac{L_J(\phi) - L_J(0)}{L_J(0)}]^{-1/2}$$

Again assuming a KO-1 current-phase relation for the nanobridges, we can write this as

$$\omega_0(\phi) = \omega_0(0) \left[1 + q_0 \frac{\sin\frac{\delta}{2} \tanh^{-1}\sin\frac{\delta}{2}}{1 - \sin\frac{\delta}{2} \tanh^{-1}\sin\frac{\delta}{2}}\right]^{-1/2}$$
(6.14)

assuming that  $\delta = \pi \phi$ . We then fit the flux tuning of the resonator, as shown in Fig. 6.5. This gives  $q_0 = 0.015$ . The linear inductance of the resonator was simulated using Microwave Office, giving L = 1.2 nH. Thus, we find  $L_J(0) = 2Lq_0/(1-q_0) = 36$  pH. From this inductance we find  $N_e$ :

$$N_e = \frac{3\hbar^2}{2\Delta e^2 L_J} = 680$$



Figure 6.5: Theoretical fit to nanobridge inductance participation ratio

By fitting the flux tuning of the resonant frequency, it is possible to determine the participation ratio of the Josephson inductance in the total inductance. Here, I have plotted resonant frequency (red circles) as a function of flux. The best fit line using Eq. (6.14) is shown in blue, giving a value  $q_0 = 0.015$ . (Fit performed by F. Kos)

This should be a large enough  $N_e$  to treat a single trapping event as a small change in the junction. We assume thermal trapping (i.e. Gibbs-distributed quasiparticles) using the fridge temperature and fit the response using Eq. (6.11) with the  $P_k$  as the only free parameters.

Sample fits at T = 75 mK for  $\phi = 0$  and 0.464 are shown in Fig. 6.6. The theory produces excellent agreement with the data, suggesting that the trapped quasiparticles are thermally distributed with a temperature equal to the fridge temperature. We note that in regimes where  $\bar{n}_{trap} \sim 1$ , the 3-quasiparticle contribution to the resonance is non-negligible; this causes the fit  $P_k$  to sum to less than 1. Fitting the 3-quasiparticle peak requires a triple integral and is thus quite computationally intensive, so we restrict our analysis to the first two quasiparticle peaks. We note that in general the  $P_k$  are not Poisson-distributed (even after accounting for  $P_3$ );  $P_2$  is larger and  $P_1$  smaller than would be expected from Poisson statistics. For instance, at T = 100 mK and  $\phi = 0.464$ ,  $P_0 = 0.66$ ,  $P_1 = 0.16$ , and  $P_2 = 0.12$ , while Poisson statistics would predict  $P_0 = 0.66$ ,  $P_1 = 0.26$ ,  $P_2 = 0.06$ . This indicates that quasiparticle trapping may be correlated, although I hesitate to make any strong statements regarding this.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup>What follows is merely speculation: the local quasiparticle density may be time-varying, with very few quasiparticles usually and then a few large groups occasionally present. This would lead to a higher probability of multiple-quasiparticle trapping. Such a process is easy to imagine physically, as a stray infrared photon impinging on the resonator near the junction could create many quasiparticles, which would then recombine over some time scale which may be short compared to the time between photon impacts. In addition, a junction with a trapped quasiparticle has a lower critical current, introducing asymmetry in the nanoSQUID. This asymmetry causes the poisoned junction to have a higher phase bias, lowering its  $E_A^{(i)}$ and thus making additional trapping events more likely.



Figure 6.6: Theoretical fits to resonance lineshapes with trapped quasiparticles

Here, I plot the resonance lineshape at zero flux (brown) and  $\phi = 0.464$  (green) at 75 mK. At finite flux, the average number of trapped quasiparticles is nonzero, and the 1-quasiparticle and 2-quasiparticle resonances are visible. We fit the data using our small- $\bar{n}_{trap}$  theory (black lines), showing excellent agreement. (Fits performed by F. Kos)

#### **Temperature Dependence**

We repeated our fitting procedure at all temperatures below 250 mK. We plot  $\bar{n}_{trap}$  as well as extracted values of  $x_{ap}$  as a function of temperature between 75-200 mK in Fig. 6.7. While  $x_{qp}$  does increase with temperature, the increase does not seem to follow the thermal function of Eq. (6.1). The density at 75 mK,  $x_{qp} = 1 \times 10^{-6}$ , is consistent with other measurements of aluminum superconducting circuits [22, 71]. While our theory fits the data well in the range 75-200 mK, we find below 75 mK we cannot fit the resonance lineshapes using a Gibbs distribution at the fridge temperature. There are likely two causes for this discrepancy. First, at low temperatures  $\bar{n}_{trap}$  grows, and so the 3-quasiparticle peak becomes more significant. In addition, below 75mK the quasiparticles may not be well thermalized to the fridge temperature, and indeed may not be thermally distributed at all, as thermalization mechanisms such as inelastic electron-phonon scattering will be suppressed at low temperatures due to the falling phonon density [72]. The overall population of quasiparticles at low temperatures is likely due to thermal radiation from warmer stages of the fridge [23]. If such high-energy photons leak through the radiation shields and impact the resonator, they can be absorbed by Cooper pairs, breaking the pairs and creating high-energy nonequilibrium quasiparticles. Indeed, previous iterations of this experiment with less radiation shielding had much higher values of  $x_{neq}$ .



Figure 6.7: Trapped quasiparticle number and quasiparticle density as a function of temperature

We extract values of  $\overline{n}_{\text{trap}}$  from our fits as a function of temperature, plotted on the left axis as red circles. The average number of trapped quasiparticles drops as temperature increases, as hotter quasiparticles are less likely to occupy the low-energy trap states. At 200 mK, the trapped quasiparticle number rises again, as the quasiparticle density  $x_{qp}$  (blue squares, right axis) begins to rise faster than the occupation probability drops. (Values calculated by F. Kos)

#### Large Numbers of Quasiparticles

In order to test our theory in the high- $\bar{n}_{trap}$  limit, we must artificially raise the population of quasiparticles. To reach this regime we heated a radiator near the copper sample box, inside the radiation shields. The radiator consisted of a 50  $\Omega$  SMA termination on the end of a thermally-insulating stainless steel SMA cable; running DC through the cable heated the radiator. The extra thermal load on the fridge base stage prevented stable operation of the radiator below 100 mK. A sample resonance trace at 100 mK and  $\phi = 0$  and 0.432 are shown in Fig. 6.8. A theory fit is shown as the dashed line, giving  $\bar{n}_{trap} = 7$ . The best fit, shown as the solid line, is obtained by allowing the Gaussian width to vary independently from the peak's center point (in contrast to the theory presented above, which gives them both as a function of  $\bar{n}_{trap}$ ); these give  $\bar{n}_{trap} = 19$  and 7, respectively. This discrepancy may be due to the fact that the majority of the quasiparticles are not being thermally generated. In addition, it may be due to the non-Poissonian statistics observed in the few-quasiparticle limit.

## 6.7 Quasiparticle Excitation

For aluminum thin films, typical values of the superconducting gap  $\Delta$  are around 170  $\mu$ eV, or 41 GHz in frequency units. This means that the gap between the upper Andreev state and the quasiparticle band—which we will term the Andreev gap  $\Delta_A \equiv \Delta - E_A$ —will be below about 15 GHz in the range of flux bias studied ( $\phi = 0$ -0.55). These frequencies



Figure 6.8: Theoretical fits to resonance lineshapes with trapped quasiparticles in the Gaussian limit

In the limit of many trapped quasiparticles, the resonance becomes a Lorentzian convolved with a Gaussian. Here, I plot the resonance lineshape at zero flux (brown) and  $\phi = 0.432$  (green) at 100 mK, with the quasiparticle-generating radiator on. At finite flux, the resonance is broadened, with a non-Lorentzian shape. The dashed black line is a fit using our high- $\bar{n}_{trap}$  theory. The solid black line is a fit using the theory but allowing the width and centerpoint of the Gaussian to vary independently, showing improved agreement with the data. (Fits performed by F. Kos)

are easily generated and injected into the device. Thus, it should be possible to promote trapped quasiparticles out of the junction and back into the continuum by illuminating the resonator with an excitation tone with a frequency  $f_{exc} > \Delta_A$ . Resonance lineshapes as a function of flux with an excitation tone at 17.5 GHz are shown in Fig. 6.9. As you can see, the resonance maintains its Lorentzian character throughout the range of flux values studied. At the highest flux biases, the resonance peak shrinks slightly; we attribute this to incomplete clearing of quasiparticles from the junction. The excitation efficiency increases with increasing excitation tone power, as expected; we see this as a rise in the 0-quasiparticle peak as excitation tone power rises. At all but the highest flux biases, the peak height saturates as power is raised, reaching its zero-flux height. However, at all fluxes, a very strong excitation tone begins to suppress the critical current of the nanosSQUID, moving the resonance down in frequency. At the highest flux biases, we do not observe saturation of the peak height before this movement occurs, thus indicating that we could not fully clear the quasiparticles at a low enough power.

We can use this excitation of quasiparticles to directly compare the resonance with and without quasiparticle trapping over a short time scale. In fact, we used the quasiparticlefree resonance to calibrate the 0-quasiparticle resonant frequency for use in our fits. Things become even more interesting when we stop using a constant tone at a single frequency and begin to change the tone.



Figure 6.9: Resonance lineshaps in the presence of a quasiparticle excitation tone

By illuminating a junction with a high-frequency microwave tone, it is possible to excite trapped quasiparticles and thus clear them out of the junction. Here, I plot resonance lineshapes as a function of flux at 10 mK, with a strong 17.5 GHz bias tone clearing the quasiparticles from the junction. The resonance now retains its single-peak Lorentzian shape at all fluxes. The peak shrinks slightly at the highest flux values, which we attribute to imperfect excitation efficiency.

#### Andreev Gap Spectroscopy

We can probe the energy spectrum of trapped quasiparticles using our excitation tone. A quasiparticle trapped in a channel with energy  $E_A^{(i)}$  will only be excited by a tone with  $f_{exc} > \Delta_A^{(i)}$ . Thus, by sweeping the frequency of the excitation tone, we can probe the distribution of  $E_A^{(i)}$ . Spectroscopy data as a function of flux at 100 mK is shown in Fig. 6.10. We plot  $\Delta \operatorname{Re}(S_{11})$ , that is, the change in the real part of the reflected signal (probed on the 0-quasiparticle resonance) referred to its value with no excitation tone. A positive shift indicates a greater probability of zero quasiparticles, i.e. that quasiparticles have been cleared from the junction. Several features are apparent. First, at low flux bias there is no effect of the excitation tone at any frequency. This is expected, as there will only be a change in the response if quasiparticles are being cleared; at low flux bias, there are none trapped in the first place. As flux bias grows, so does the magnitude of the response at high excitation frequencies, indicating that more and more quasiparticles are being trapped and then cleared. At the same time, a minimum frequency gap becomes apparent, with no response for excitation frequencies below this gap. The frequency of this gap grows with increasing flux and has a value which is consistent with  $\Delta_A(\phi, \tau)$  for  $\tau \approx 0.8 - 1$ . At high excitation frequencies  $(f_{exc} > \Delta_A(\phi, 1))$  the response saturates, as the frequency is now high enough to excite even the quasiparticles in the deepest traps. The width of the transition from no response to saturation is consistent with the Dorokhov distribution for channel transmittivities.

In order to accurately perform spectroscopy, we must ensure that we are coupling a constant amount of power into the junction at all frequencies. This is non-trivial, since the quasi-lumped-element resonator will have higher order modes, and various parasitic couplings



Figure 6.10: Andreev gap spectroscopy

A quasiparticle will only be excited out of the *i*-th trap state if it absorbs a microwave photon of energy  $hf_{exc} > \Delta_A^{(i)}$ . Thus, by sweeping the frequency of the excitation tone, we can measure the distribution of Andreev gaps, and thus the distribution of Andreev energies, of the junction trap states. Here, I plot spectroscopy data, taken by measuring the change in the real part of the response at the 0-quasiparticle resonance frequency, as a function of excitation frequency and flux bias. At low flux bias, there is no change in the response, as there are no trapped quasiparticles to excite. As flux bias grows, the response grows with it, indicating more and more trapped quasiparticles. There is no response below a certain excitation frequency and a saturation above a certain frequency (both of which grow with flux), indicating the distribution of  $\Delta_A^{(i)}$ .

and spurious reflections from microwave components can change the power greatly over such a wide frequency band. In order to correct for these effects, we measured the resonance response near zero flux as a function of excitation power and excitation frequency. At a high enough power, the critical current of the junctions is suppressed and the resonant frequency drops. By measuring the power at which this drop occurs at all frequencies, we can calibrate the power coupled into the junction. The spectroscopy data referred to above used this calibration method; uncalibrated data shows "stripes" as a function of frequency, due to the varying coupling strengths (and thus varying spectroscopic response).

## **Trapping Dynamics**

By pulsing the excitation tone and monitoring the response at the 0-quasiparticle resonant frequency, we can probe the dynamics of quasiparticle trapping and excitation. We mix the reflected measurement tone down to dc with an IQ demodulation setup and measure the two quadratures of the signal with a fast digitizer card. We test the pulse's efficacy by increasing



Figure 6.11: Sample measurement of quasiparticle excitation and retrapping

By pulsing the quasiparticle excitation tone we can measure the dynamics of the quasiparticle excitation and retrapping. Here, I plot the real part of the response, measured at the 0-quasiparticle resonance, during such a measurement. The data is averaged over many pulse cycles. The pulse is turned on, indicated by the grey shaded region, and the resonance evolves to a new value with an exponential envelope. The time constant of this rise decreases with increasing pulse amplitude, thus indicating faster excitation. Once the pulse is turned off, the resonance decays back to its original value, indicating quasiparticles retrapping in the junction. The time constant for this retrapping is independent of the pulse amplitude.

its power, and choose the power at which the response has saturated. See Fig. 6.11 for a sample measurement at 10 mK, averaged over many iterations of the pulse (at 17.5 GHz, far above  $\Delta_A$ ). The data shown is the I quadrature (i.e. real part), referred to its value before pulsing; the grey section indicates the time during the pulse. The resonance shifts to a new equilibrium with the excitation on and decays back to its old value once the pulse is turned off. Both processes occur with an exponential envelope. The first exponential corresponds to the clearing of quasiparticles from the junction. Its time constant depends linearly on the excitation pulse amplitude and is roughly constant as a function of  $\phi$  as shown in Fig. 6.12(a). The second exponential corresponds to quasiparticles retrapping in the junctions as the system returns to steady state. It has a time constant which is independent of both the pulse amplitude and its duration, even if the pulse amplitude is below saturation. The time constant decreases as a function of flux from  $60 - 20 \ \mu s$  as flux increases from  $\phi = 0.35$ , as shown in Fig. 6.12(b). At lower flux values the pulse response signal was too small to accurately measure. All the excitation and retrapping times measured were on the order of  $1-100 \ \mu s$ , significantly slower than the many-second averaging times used in the lineshape measurements. This confirms the validity of our assumption that we had been averaging over all configurations on a time scale which is slow compared to the trapping dynamics.

We attempt to fit the retrapping time by assuming electron-phonon relaxation is the



Figure 6.12: Quasiparticle excitation and retrapping times as a function of flux

Here, I plot the excitation time (a) and retrapping time (b) extracted from data like that in Fig. 6.11 as a function of flux. All data was taken with a 17.5 GHz excitation tone. The excitation time shows no clear trend as a function of flux, although there is a large spread in its value. The retrapping time drops sharply with flux. The solid line in (b) is a fit using our electron-phonon relaxation theory, showing reasonable agreement with the data. (Fit performed by F. Kos)

dominant mechanism for a quasiparticle in the continuum dropping into an Andreev state [73]. The electron-phonon interaction takes the form

$$H_{e-ph} = \frac{1}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q},\sigma} \alpha q^{1/2} (a^{\dagger}_{\mathbf{k}+\mathbf{q},\sigma} a_{\mathbf{k},\sigma} b_{\mathbf{q}} + a^{\dagger}_{\mathbf{k},\sigma} a_{\mathbf{k}+\mathbf{q},\sigma} b^{\dagger}_{\mathbf{q}})$$

where a and b are the annihilation operators and **k** and **q** are the momenta for electrons and phonons, respectively. F. Kos and L. Glazman have used the bulk superconductor Green's functions to calculate the relevant matrix elements and find the retrapping time  $\tau_T$  in terms of the bulk quasiparticle recombination time  $\tau_R$ :

$$\tau_T = \left(\frac{2\Delta}{\Delta - E_A}\right)^2 \tau_R \tag{6.15}$$

We fit the relaxation time using this function (using  $E_A(\tau = 1)$ ), as shown in Fig. 6.12(b). This gives  $\tau_R = 0.3 \ \mu$ s. Previous experiments have measured  $\tau_R = 100 \ \mu$ s at 250 mK [74], a clear discrepancy with our data. We do not yet have an explanation for this discrepancy, although it be due to the fact that we have used bulk Green's functions in a constriction geometry.

## 6.8 Conclusion

We have performed dispersive measurements of thermal and non-equilibrium quasiparticle trapping in phase-biased nanobridge junctions integrated in a narrow-linewidth resonator. We measure resolved resonance peaks from single quasiparticles trapped in junctions with  $\sim 1000$  channels. The trapped quasiparticles obey a thermal Gibbs distribution for temperatures above 75 mK. Both low and high average number of trapped quasiparticles seem to obey slightly non-Poissonian trapping statistics, perhaps indicating correlations between trapping events. Applying a microwave tone to the resonator enables us to clear trapped quasiparticles from the junctions. By sweeping the frequency tone we are able to spectroscopically probe the energies of the trapped quasiparticles and thus measure the Andreev gap as a function of flux. Pulsing the bias tone allows us to measure the quasiparticle retrapping times, which range from 15 – 60  $\mu$ s in the flux range studied.

Future work can further probe the mechanisms of quasiparticle trapping and thermalization and investigate any correlations between trapping events. I suggest an experiment in which the resonance response is monitored and the higher moments  $\overline{S^2}$  and  $\overline{S^3}$  measured, in order to further probe the trapping statistics. Measurement with a near-quantum-limited amplifier may provide enough sensitivity to resolve quasiparticle trapping/untrapping events continuously with single-shot resolution, allowing measurements of quantum jumps in Andreev levels [58]. Finally, we note that the number of trapped quasiparticles is a sensitive probe of the bulk quasiparticle density. This allows our device to be used to evaluate the quality of the radiation shielding used in a cryogenic setup.

## Chapter 7

## **Conclusions and Future Directions**

At last, we have come to the end of the thesis. It is time to look back on what has been learned, and look forward to new experiments that can be done in the future.

## 7.1 Conclusions

We have demonstrated the utility of aluminum 3D nanobridges as nonlinear Josephson elements. We have theoretically calculated current-phase relations for both 2D and 3D nanobridge junctions of various different lengths. These predictions are confirmed by DC switching experiments and microwave inductance measurements of 2D and 3D nanoSQUIDs as a function of flux. Both theory and experiment show that short ( $\leq 150$  nm) 3D bridges approach the ideal KO-1 limit for a diffusive weak link. In contrast, 2D nanobridges behave much more like linear wires, with weakly nonlinear CPRs and poor phase confinement.

We have used 3D nanobridge junctions in dispersive magnetometer devices. These devices show very low flux noise  $(23 - 140 \text{ n}\Phi_0 \text{ Hz}^{-1/2})$  and broad bandwidth (20 - 100 MHz), with near-negligible dissipation. The magnetometers may be operated as linear detectors optimized for bandwidth, or may be biased into parametric amplification, thus providing near-quantum-limited noise performance. The sensitivity measured should be high enough to detect a single electron spin in less than a typical spin relaxation time. Furthermore, we have measured the internal dynamics of the magnetometry process, observing the mismatch between the up-converted signal and the amplified quadrature.

Finally, we have used phase-biased 3D nanobridge junctions as traps for superconducting quasiparticles. Dispersively measuring the nanobridge inductance allows us to probe the mean distribution of trapped quasiparticles in the junction. We find that quasiparticles are thermally distributed above 75 mK, with non-Poissonian trapping statistics. Our apparatus allows us to perform spectroscopy on the internal Andreev states in the nanobridges, showing good agreement with qualitative theoretical predictions. We have also measured the dynamics of quasiparticle trapping, finding trapping times orders of magnitude faster than those predicted by a simple electron-phonon relaxation theory.

## 7.2 Future Experiments

## Magnetometry

The dispersive nanoSQUID magnetometer has many readily-apparent applications. Its constriction geometry makes it ideal for coupling to small spin ensembles, nanoscale ferromagnets, and single spins. Its low flux noise gives it the sensitivity to detect these spins, while broad bandwidth will allow us to measure fast spin dynamics. For instance, this device may have enough sensitivity and bandwidth to detect multiple domain switches as a ferromagnet switches magnetization. We may also be able to perform continuous monitoring of the Rabi oscillations of a single electron spin or an ensemble of spins, and even to perform feedback on this oscillation as in [58]. Finally, further optimization of the devices may be made by reducing stray inductance, increasing critical current, and moving to materials more tolerant of large magnetic fields.

## Quasiparticle Trapping

The first experimental step will be to optimize the sensitivity of the quasiparticle trapping measurement. This means that the participation ratio of the nanobridges in the resonator inductance should be increased, while the system noise temperature should be lowered (probably through the use of an LJPA). This may allow for experiments such as observation of single trapping events in real time. It will also provide increased SNR for measurements of the noise and "noise of noise" of the nanobridge inductance. These noise measurements will provide more information about the correlations between quasiparticle trapping events. It is important to note that, for optimal sensitivity and ease of interpretation, these measurements should be made with a lower-Q resonator, so that a single trapping event does not move the resonance more than a fraction of a linewidth. This will allow the measurement to detect multiple trapping numbers while probing at a single frequency, which is essential for good noise measurements. Finally, the nanoSQUID resonator provides an easy test device for any cryogenic setup's radiation shielding, as the quasiparticle population (and thus the trapping probability) will be greatly increased by any stray infrared radiation.

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# Appendix A

## Nanobridge Fabrication

Just a few decades ago, the prospect of controllably defining a 100 nm long 3D nanobridge would have been nearly impossible. Luckily for my graduate career and for you, the interested reader, modern nanofabrication techniques have made this task merely extremely difficult. It will come as no surprise that nanobridge fabrication was *the* major challenge in the early stages of this project. The following appendix details the fabrication best practices that I have developed. I will also do my best to explain the options available at each step of fabrication, and to show whether my choices were empirically motivated, based on a few suggestive data points, or just pure voodoo. Finally, I will make suggestions for improving the fabrication process, including experimental tests that could be done to determine which effects are most important.

## A.1 Basic Principles

The fabrication process begins by spinning two layers of electron-beam resist on a substrate. An e-beam writer exposes the resist, lithographically defining the features of the device. Dipping the chip in a chemical developer removes the exposed resist. The bottom resist layer is more sensitive to the e-beam than the top layer, and so the resist profile develops an undercut, as shown in Fig. A.1. The chip is then placed in an aluminum evaporation system and pumped down to high vacuum. The device metal is deposited in two steps; see Fig. A.2. In the first step, metal is deposited at normal incidence. The sample is then tilted to a steep angle, and metal is deposited again. Because of the steep aspect ratio of the window in the top-layer resist in the nanobridge region, there is no line of sight between the evaporation source and the substrate, and so no metal reaches the substrate surface. In the rest of the device the resist window is wide, and so metal is deposited on top of the first layer, with a small shift due to the angled deposition. In this way a 3D nanobridge structure may be defined without breaking vacuum, i.e. without oxidizing the metal interface between the thick pads and thin bridge. Finally, the chip is soaked in acetone, which dissolves the resist and removes all the metal except that which reached the substrate surface.



Figure A.1: Schematic of the standard nanobridge resist stack

Here, I show a cross-section of our standard resist stack. We define the nanobridge lithographically in a bilayer of positive electron-beam resist, typically consisting of PMMA (dark blue) and copolymer (light blue). (a) The e-beam (red arrows) exposes an area, with some stray dosing towards the edges. The copolymer is more sensitive to e-beam than the PMMA, so a larger area is exposed. (b) When the resist is developed, the exposed resist is dissolved, leading to an undercut in the resist profile.



Figure A.2: Schematic of double-angle shadow mask evaporation

Viewed from the top, the nanobridge and banks are defined as a "dogbone" structure, as shown by the dashed black lines in the center of the figure. (a) In the nanobridge region, metal (shown in black) deposited at normal incidence can reach the substrate, but metal deposited at a steep angle will be occluded by the narrow opening in the PMMA. (b) Metal may reach the substrate both at normal incidence and via angled deposition in the wide banks region. The metal deposited at an angle will be slightly shifted from the first layer, due to the finite thickness of the resist stack. This shift makes a large undercut necessary.

This simple description hides the complexity inherent in any nanofabrication. I will now discuss in detail each step of the process, explaining the choices made and the reasoning behind them.

## A.2 Methods

#### **Resist Stack**

Several factors go into choosing the optimal resist stack. The first is resolution. The top layer of resist must be a very high-resolution resist so that a narrow nanobridge may be defined. The standard high-resolution e-beam resists are poly(methyl methacrylate) with a molecular weight of 950,000 (PMMA 950K), and the proprietary ZEP 520. The bottom layer resist need not be very high-resolution, but it must either be more sensitive to e-beam than the top layer or it must have an orthogonal developer chemistry—that is, its developer does not affect the other resist and vice versa—in order to provide sufficient undercut. We typically use the PMMA copolymer of methyl methacrylate mixed with 8.5% methacrylic acid (MMA (8.5) MAA), as it has good resolution, is more sensitive than PMMA, and has a nominally orthogonal developer chemistry to ZEP. In practice, it turns out that the standard copolymer developer (a 1:3 solution of methyl isobutyl ketone and isopropyl alcohol, or MIBK:IPA) does have some effect on ZEP. Thus, we choose a PMMA/copolymer bilayer.

The choice of resist solvent is also important. If a resist is spun on top of another which has the same solvent, the two layers will mix at the interface, degrading performance. For this reason we use copolymer dissolved in ethyl lactate (EL) and PMMA dissolved in anisole (A).

Finally, the resist thickness must be optimized. The bottom copolymer layer must be thicker than the total device thickness, otherwise the resist will not lift off correctly. The top PMMA layer must be thick enough that the aspect ratio of the bridge window is steep enough to prevent deposition without requiring too extreme of a deposition angle. However, a narrower line may be made in a thinner resist, and so the thinnest top layer possible is desirable. A higher concentration of resist in solution will spin to a thicker layer, while a faster spin will result in a thinner layer. We choose a bottom layer of MMA (8.5) MAA EL6 and a top layer of PMMA 950K A2 (6% and 2% solutions, respectively), and a spin speed of 4000 rpm. This leads to a bottom layer which is  $\approx 100$  nm thick and a top layer which is  $\approx 50$  nm thick.

The spin is ramped at 4000 rpm/s, near the maximum for the Headway spinner system we use. This fast ramp is chosen to minimize the buildup of resist at the edges or corners of a chip, a phenomenon known as *edge bead*. The spin lasts 60 s, which is a sufficient time to fully planarize the resist.



Figure A.3: SEM image of a thin film with insufficient undercut

If there is insufficient undercut, an angled deposition will cause metal to ride up the sides of the underlayer resist. In the SEM image shown above, the bright structures visible at the bottom edges of the aluminum traces are large "flags" of aluminum caused by insufficient undercut. In this case, the issue was caused by overdosing the undercut boxes, resulting in exposure of the top-layer PMMA. All features were overdosed, leading to the shorts visible between the junction banks.

## Pattern Design and Lithography

The main challenge of nanobridge lithography is defining as thin a bridge as possible between two large pads, which are separated by only  $\sim 100$  nm. Writing with an electron beam is not a perfectly local process, and exposing a feature which is several microns away may impart some extra dose to the nanobridge. For this reason, it is important to write the junction pads with as low a dose as possible, in order to minimize this proximity dose. It is also important to leave several microns between the nanobridges and any large features such as capacitor pads or ground planes.

Because the second metal layer is deposited at an angle, it will be laterally shifted relative to the PMMA window. If there is insufficient undercut in the copolymer layer, metal will ride up this sides of this resist, producing "flag" structures like those shown in Fig. A.3. In order to ensure sufficient undercut, we place low-dose "undercut boxes" in the design. These are exposed only lightly, imparting a dose which does not significantly affect the PMMA but which is sufficient to clear away the copolymer. It is also important to design the nanobridge slightly off center of the banks, so that it is centered in the banks once this shift is taken into account. See Fig. A.4 for a typical design pattern. The size of the shift will depend both on the angle of evaporation and on the total thickness of the resist stack; for our standard process it is 110 nm. Note that the undercut box dose is fairly sensitive; a dose that is too low will not expose the copolymer, while a dose that is too high will also expose the PMMA top layer.

In order to make the narrowest, most rectangular nanobridge possible, the electron beam must be a tightly focused spot. In general one should use the smallest aperture, the lowest current, and the highest accelerating voltage possible. Our process regularly achieves 25



Figure A.4: A typical nanobridge SQUID lithography pattern

Most nanoSQUID lithography patterns used in this thesis are similar to the one shown above. The bridges are written as single-pass lines, shown in red. An extra line is written outside each junction as a test of the dose parameters. The undercut boxes (written at a lower dose), shown in purple, allow for angled deposition.

nm wide bridges with a 20  $\mu$ m aperture, a ~ 30 pA current, and a 30 keV accelerating voltage. It is also crucial to focus the beam as tightly as possible. This involves not only ordinary focusing, but also ensuring that the lens alignment and stigmation are completely optimized. I find that the best focus is achieved by burning a contamination spot in the resist by aiming the beam at one pixel for 10-60 s (with a low current). This spot both provides a small feature to focus on and gives information about the beam characteristics: a small, round spot or ring is indicative of good focus, while a large spot indicates poor focus and an ellipsoidal shape indicates some lens astigmatism. Finally, if several devices will be written across more than a few hundred microns, a focal plane must be defined in order to maintain good focus across a tilted sample. In practice, no sample is perfectly level, and height variations of just a few microns can bring the chip out of focus.

## Development

In order to attain the highest resolution lithography, it is necessary to cool the resist developer. For PMMA resists, we use a 1:3 mixture of methyl isobutyl ketone (MIBK) and isopropyl alcohol (IPA) at  $-15^{\circ}$  C. This temperature has been shown to be optimal for resolution and contrast [75]. We achieve accurate temperature control by placing a small beaker of developer in a large bath of IPA in an insulated dewar. The IPA bath is stirred continually and temperature-controlled by an immersion chiller with a resistive thermometer

probe. In practice the chiller is only stable to  $\pm 1.5^{\circ}$  C. The IPA bath temperature oscillates around  $-15^{\circ}$  C; the warming stage of this oscillation is quite slow, and so the developer temperature closely tracks the measured bath temperature. The standard practice is to begin development at  $-15.1^{\circ}$  C and develop for 60 s; typically the bath temperature will only rise to  $-14.9^{\circ}$  during the development. Once the chip is removed it is important to blow it dry immediately; if the developer is allowed to warm up on the surface of the chip it will eat away much more resist, causing all the features to be overexposed. This has been demonstrated empirically<sup>1</sup>. Continuing to blow dry nitrogen on the chip for 30 s ensures that the chip will warm up before it is exposed to room air, thus preventing water from condensing on the surface.

Because the optimal e-beam dose is sensitive to developer temperature, it is important to set up the chiller bath consistently each time. That means the position of the chiller coil, the thermometry probe, and the developer beaker should be the same; the bath level should be kept constant; and the bath stir rate should be the same each time. I typically arrange the beaker directly opposite the chiller coil, with the thermometer tucked behind the beaker holder, and the stir rate set to roughly 250. I do not claim that this is the optimal arrangement, but it has shown dose consistency.

## Ashing

Most standard nanofabrication recipes will tell you to do an oxygen plasma ash on your chip before putting it in the evaporator. This is designed to remove any resist residue that was not developed away. **DO NOT DO THIS!** For unknown reasons, doing this pre-evaporation ash is associated with greatly reduced 3D nanobridge yield. This will be discussed in greater detail in Section A.4. A post-liftoff ash is perfectly fine, although it should be kept under 60 s in length in order to avoid etching the nanobridge too much (i.e. until it breaks).

#### Evaporation

The evaporator used must have a the ability to tilt the sample stage so that metal can be deposited at varying angles. The chip should be aligned on this stage so that the nanobridges are parallel to the axis of this tilt. The sample chamber must be pumped down to a low enough pressure for a clean evaporation; I have had good results with any pressure under  $10^{-6}$  Torr. Finally, the evaporation rate will affect the graininess of the aluminum film. A slower evaporation will result in a more finely-grained film; see Fig. A.5. Typically a rate between 2 and 6 Å/s will give good results.

The first deposition is performed at normal incidence. The angle of the second deposition must be chosen so that no metal is deposited on the nanobridge during this step. This means that, for deposition angle  $\theta$ , top-layer resist thickness t, and resist window width w,

<sup>&</sup>lt;sup>1</sup>By accident, of course—it is always a good idea to test your blower nozzle before beginning development!



Figure A.5: SEM images showing aluminum grain size variation

When aluminum is evaporated at a slow (< 1 Å/s) rate, the film forms small grains, as shown in the SEM image in (a). A film deposited at a faster rate (2 - 5 Å/s), as in (b), will have a much larger grain size, leading to a smoother appearance.

 $t \tan \theta > w$ . Typically we define a < 30 nm wide line in a 50 nm thick resist layer, so we choose an angle of 35°.

#### Liftoff

Acetone dissolves most e-beam resists, including those used in this process. When the resist is dissolved, any material (i.e. aluminum) sitting on top will lift off. It is important to make sure that the metal on the substrate is not strongly connected to the metal on top of the resist, hence the need for a thick bottom layer resist. Immersing the chip in acetone will begin to dissolve the resist, although this process can be very slow. Heating the acetone greatly speeds up the dissolution, although one must be careful not to overheat it due to acetone's low boiling point (and high flammability!). Soaking in a beaker at 65° C for at least one hour usually fully dissolves the resist. Once the resist is dissolved, a short (between 1 and 20 s) burst of ultrasound will usually lift off all the undesired material, leaving only the correct pattern. A longer sonication time will give a higher probability of full liftoff, but may also cause some of the material on the substrate to peel up; luckily, aluminum is a fairly "sticky" metal on most substrates, so sonication is generally safe.

Once the excess deposited material has been removed from the chip, it will be floating in the acetone bath. It is important to remove the chip from the bath without any of this material landing on the surface of the chip, as it will then stick to the surface and be very difficult to remove. For this reason, the standard procedure is to squirt a strong jet of acetone at the surface of the bath, and raise the chip up through this stream of debris-free acetone. In the case that sonication is not possible (i.e. if there is a delicate substrate or other structure on the chip that would be damaged by ultrasound), this stream of acetone may be used to remove the material that was on top of the resist. Such jet-based liftoff may require a few seconds, so it is important to make sure that the acetone squirt bottle is full enough!

Once the chip is removed from the acetone bath, a quick rinse with a jet of IPA will rinse off the acetone. We then blow the chip dry and examine it optically. If there is any extra material that did not lift off, the chip may be placed back in the acetone and sonicated again. However, my experience is that it is very difficult to lift off material after the chip has been removed once.

## A.3 Basic Fabrication Recipe

What follows is the nanobridge fabrication recipe as it is currently used in the autumn of 2013:

## Spinning Resist Stack

- 1. Take a clean, fresh-out-of-the-cassette silicon wafer. For any application requiring low loss at microwave frequencies, an undoped (a.k.a. intrinsic) wafer with high resistivity (> 10 k $\Omega$ -cm) is best. We typically use single-side polished wafers with a [100] orientation and a thickness of 250-350  $\mu$ m, as they are easy to handle and to dice into smaller chips.
- 2. Remove the wafer from its cassette in a clean environment, ideally a class-100 or better cleanroom or laminar flow hood. Place onto a resist spinner, programmed to run at 4000 rpm for 60 s, with a 4000 rpm/s ramp rate. Center the wafer as precisely as possible; if necessary, run a very slow spin program to determine if it is centered.
- 3. Using a clean syringe or pipette, slowly drop MMA (8.5) MAA copolymer EL6 onto the wafer surface until it is completely covered. Be careful not to bubble the resist out of the dropper. Once the wafer is covered, move your hand away from the spinner and immediately start the spin program. You will notice that almost all of the resist is flung off the wafer; this is fine, although you will probably want to arrange some aluminum foil in your spinner to catch this waste so that it can be safely disposed of.
- 4. Once the spin program is complete, carefully lift the wafer off the spinner chuck, contacting it with tweezers only at the edge to avoid scratching the resist. Inspect it to make sure there was no debris caught under the resist; if there was, you will need to clean the wafer off and start again. Assuming the spin was clean, put the wafer on an ordinary hot plate (*not* one with a vacuum chuck) at 170° C. Cover with a petri dish or similar glass lid. Bake for 5 minutes, then remove.
- 5. Allow the wafer to cool for at least 3 minutes, then place back on the spinner and center it as before. During this time, change the hot plate to  $180^{\circ}$  C

- 6. Repeat the resist deposition and spinning, this time using PMMA 950K A2. Again inspect for debris.
- 7. If the spin was clean, place the wafer on the hot plate (at 180° C) and cover. Bake for 5 minutes, then remove and put in a clean wafer carrier.

You should now have a complete, clean wafer with a bilayer resist stack. This wafer can be diced into smaller chips for easier fabrication. If you see a large amount of edge bead, it is likely due to the wafer not being centered on the spinner chuck. If you see "comet" features in the resist, it means that there was dust on the surface of the wafer when the resist was spun on. In either case, you will need to clean the wafer off and start again. For a complete wafer cleaning procedure, see Section A.3.

#### Electron-beam Lithography

Pattern the chip with an accelerating voltage of 30 keV, using an area dose between 600 and 700  $\mu$ C/cm<sup>2</sup> for main features, an area dose of 280  $\mu$ C/cm<sup>2</sup> for undercut boxes, and a line dose between 2.5 and 3.5 nC/cm. Exact doses will vary slightly depending on the design, and will also vary between different resist spins (even if the spins are nominally identical). Ensure that the focus, lens alignent, and lens stigmation are all optimized. This step is deliberately vague, as the details of the lithography process will vary greatly depending on the exact e-beam system used.

#### Development

- 1. Cool a small beaker of developer—a 1:3 mixture of methyl isolbutyl ketone (MIBK) and isopropyl alcohol (IPA)—in a chiller bath to  $-15^{\circ}$  C.
- 2. Dip the chip into the developer and gently agitate for 60 s.
- 3. Remove the chip and immediately blow dry with a strong jet of dry nitrogen for 30 s.

#### Deposition

- 1. Anchor the chip to the tiltable sample stage of the evaporator. Ensure that the chip is aligned correctly so that the axis of rotation for the stage tilt is parallel to the nanobridges.
- 2. Pump the sample chamber down to a pressure below  $10^{-6}$  Torr.
- 3. Evaporate a luminum at between 2-5 Å/s. Deposit whatever thickness you would like the bridge to be.
- 4. If you are making a 2D nanobridge, you may skip to venting the evaporator. Otherwise, proceed to the next step.

- 5. Tilt the sample to  $35^{\circ}$ .
- 6. Evaporate again in order to define the thick banks. If the first layer was  $t_{bridge}$  thick, and you want a total bank thickness of  $t_{bank}$ , then evaporate  $(t_{bank} t_{bridge})/\cos 35^\circ$ .
- 7. Tilt the sample back to level.
- 8. Vent the sample chamber, remove the chip, and pump back down.

#### Liftoff

- 1. Heat a beaker of acetone on a hot plate at  $65^\circ$  C.
- 2. Gently place the chip in the beaker, cover tightly with foil, and leave on the hotplate for at least 1 hour.
- 3. Remove the beaker from the hot plate and put in a sonication bath. Sonicate the beaker for 1-20 s, using longer sonication only if metal liftoff has not been achieved. If sonication would damage the substrate or other structures on the chip, skip this step.
- 4. Grip the chip edge tightly with tweezers. Squirt a strong jet of acetone into the beaker. Slowly remove the chip from the beaker, drawing it up through this stream of acetone. If sonication was not performed, keep the chip in the acetone stream until liftoff occurs.
- 5. Rinse the chip with a gentle stream of IPA. Blow dry with dry nitrogen.

## Ashing

For applications where low microwave loss is desired, an oxygen plasma ash may be performed after liftoff. Ashing in an oxygen atmosphere at between 240 and 260 mTorr with a plasma current of 85 mA for 60 s is safe and effective for most nanobridge geometries.

## **Insulating Substrates**

You may wish to fabricate nanobridges on insulating substrates such as sapphire or diamond. When such an insulating substrate is put in the path of an electron beam, it will build up charge, bending the beam away from its initial target. In order to perform high-resolution lithography it is therefore necessary to deposit a thin conducting layer on top of the resist. This conducting layer will eliminate charge buildup, but will have a minimal effect on the beam path (assuming it is a thin enough film). The conductor must be removed before resist development, as it will block the developer from reaching the exposed resist. In the past, I have had reasonable success by evaporating 10 nm of aluminum on top of the resist after spinning. This aluminum may be removed by using any of several standard aluminum etches; I use MF-319 developer, which is a solution of tetramethyl ammonium hydroxide



Figure A.6: SEM image of a well-shaped nanobridge

An ideal nanobridge will have a smooth rectangular shape, as defined by the design file. Here, I show an SEM image of a 250 nm long 3D nanobridge with this ideal shape.

in water. Soaking for at least 1 minute seems to effectively remove the aluminum. Once the aluminum has been removed, the rest of the fabrication (development, deposition, etc.) proceeds as usual. Another type of conductive layer, which goes by the name AquaSAVE, may be spun onto the top of the resist stack, and will dissolve away in water. It has not been tested for nanobridge fabrication, so use with caution.

## Wafer Cleaning

If a resist spin goes wrong, or if the nanobridges are not the first layer of lithography on your substrate, it will be necessary to thoroughly clean the chip. A simple cleaning recipe follows:

- 1. Put the chip in a beaker of acetone and soak for at least 5 minutes. If possible, heat this beaker to at least  $45^{\circ}$  C and sonicate it during this soak.
- 2. Remove the chip and rinse in a jet of IPA. Put the chip in a beaker of room-temperature IPA. Soak for at least 5 minutes; again, sonicate if possible.
- 3. Blow the chip dry thoroughly.
- 4. Place the chip in an oxygen plasma asher and ash it for at least 5 minutes.

## A.4 Yield and Failure Mechanisms

The fabrication recipe outlined above gives a 3D nanobridge yield—the fraction of bridges which are unbroken and rectangular, as in Fig. A.6—of roughly 75%. This has not always

been the case! Improving the bridge yield is a continuing process. I will now discuss some of the discoveries made during these improvements.

## Film Strain

The most obvious mode of nanobridge failure is for the bridge to be broken, thus failing to connect the junction banks, as in Fig. A.7(b). A related problem is the distortion of the bridge profile, often visible as a narrow constriction, as in Fig. A.7(a). At first glance, these problems would appear to be due to underexposure of the resist in the nanobridge region. However, other evidence would point to lithography not being the issue. Some empirical findings about bridge yield:

- 2D nanobridges almost never show this breakage or distortion, and generally have yields above 90%. Note that 2D and 3D nanobridges have identical lithography steps.
- Isolated nanobridges or nanoSQUIDs show significantly higher yield (typically 80-90%) than those that have been integrated into a circuit with large leads.
- Chips which are ashed between the development and evaporation steps show significantly reduced yield (typically less than 25%).
- Allowing the aluminum to oxidize between the two layers of deposition increases the yield<sup>2</sup>.
- $\bullet$  Certain substrates, such as silicon-on-insulator wafers, are very difficult to fabricate nanobridges on, with yields down below 10%.

These results suggest that the bridges break due to residual strain in the aluminum thin film. This strain is apparently much more severe in 3D nanobridges than 2D nanobridges; this may be due to the thicker banks, or due to the angled deposition of the second aluminum layer. The role of ashing in this strain is unclear; perhaps a small residue of resist helps pin the aluminum film to the substrate and prevent breakage, and ashing removes this residue. In any case, this remains an unsolved problem. Here are some proposed solutions, which I invite future researchers to test:

- Use a thicker top-layer resist and a smaller angle of deposition for the second aluminum layer. If the angled deposition causes the excess strain, this should alleviate the problem.
- Cool the sample stage during aluminum deposition. This should reduce atomic migration and may prevent breakage.

 $<sup>^2\</sup>mathrm{Of}$  course, such a structure is not a good 3D nanobridge junction due to the tunnel barrier the oxide creates



Figure A.7: SEM images of failed nanobridges

Film strain can cause a nanobridge to deviate from its ideal rectangular shape. (a) Strain in the film leads to bulges and narrowings of the 150 nm long bridge shown. (b) Film strain has caused the 115 nm bridge shown to break completely; this device will not conduct at DC.

- Deposit a thin (~ 1 − 3 nm) layer of titanium either on the substrate or in between the two aluminum layers. This may or may not be deposited in the nanobridge region. Titanium is a very hard material that sticks well to silicon and to aluminum, and can act to pin the aluminum in place and relieve strain.
- Cleverly design the device so that strain is minimized<sup>3</sup>.

## A.5 Two-step Fabrication

Another technique to make 3D nanobridges is to abandon double-angle evaporation and move to a two-step process. This involves first writing a long (~ 1 $\mu$ m) nanobridge without banks, aligned to a set of alignment marks, and evaporating an aluminum layer to define the thin nanobridge. After liftoff and a cleaning step, a new resist stack is spun onto the chip and the rest of the circuit—including the banks, but with no bridge between them—is lithographically defined using the same alignment marks. The sample is then put in an evaporator, and the oxide is removed from the aluminum surface with an ion-mill cleaning step. Without breaking vacuum, a thick aluminum layer is deposited, contacting the thin bridge and thus forming a 3D nanobridge structure.

In practice, this procedure has some problems. Because the contact between aluminum layers is made in the very small area on top of the nanobridge, any oxide remaining on the nanobridge will cause a large contact resistance. In practice, it is extremely difficult to deposit the 2nd layer of aluminum without a delay of at least a few minutes after the ion milling. This delay allows any stray oxygen in the evaporator (such as oxygen contamination in the ion mill working gas) to cause enough oxidation to make the contact resistance unacceptably

 $<sup>^{3}\</sup>mathrm{I}$  do not have any suggestions for how to do this. Good luck!
high. This problem can be alleviated by designing the bottom layer with wide contact pads, but then these pads must then be spaced by the final bridge length. This means that the alignment between the first and second layers is extremely sensitive, with misalignments as small as a few tens of nanometers being unacceptable. Two-step fabrication may be possible, but it appears to be far more difficult than double-angle evaporation.