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Wolfgang von *Schweinitz's Plainsound Brass Trio*  
in Theory and Practice: A Guide for Performers

A dissertation submitted in partial satisfaction of the  
requirements for the degree Doctor of Musical Arts  
in Music

by

Lukas Timothy Storm

2017

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# ABSTRACT OF THE DISSERTATION

Wolfgang von *Schweinitz's Plainsound Brass Trio*

in Theory and Practice: A Guide for Performers

by

Lukas Timothy Storm

Doctor of Musical Arts in Music

University of California, Los Angeles, 2017

Professor Ian Krouse, Chair

This dissertation essay explores the *Plainsound Brass Trio* by Wolfgang von Schweinitz, a work for horn, trombone, and tuba written in 2008. This work uses a system of tuning and harmony known as microtonal just intonation and is notated with the Helmholtz-Ellis Just Intonation Pitch Notation, a collection of precisely defined accidentals developed by Schweinitz and fellow composer Marc Sabat. A discussion of intonation theory is presented along with a rudimentary overview of historical tuning practices in an effort to integrate the concepts of just intonation with a preexisting understanding of traditional music theory. Twentieth-century uses of just intonation as a compositional device are examined, with a focus on the works and theories of the composers who most directly influenced Schweinitz: Harry Partch, Ben Johnston, La Monte Young, and James Tenney. A description of Schweinitz's oeuvre is given, with a case study of the *Plainsound-Litany* to illustrate the composer's commitment to using the inherent untempered tuning capabilities of musical instruments. After a description of the Helmholtz-Ellis notation



and discussion of developing aural skills for just intonation, the *Plainsound Brass Trio* is analyzed in terms of its notation, interpretation, and performance challenges. The aim of this dissertation is to provide the reader with the knowledge to understand the *Plainsound Brass Trio* and to encourage brass players to perform this work, which occupies a unique place in the repertoire.

The dissertation of Lukas Timothy Storm is approved.

Münir Beken

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2017

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## VITA

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## CHAPTER 1

### INTRODUCTION

Wolfgang von Schweinitz wrote the *Plainsound Brass Trio* in 2008, shortly after assuming James Tenney's former position as professor of composition at the California Institute of the Arts. The work was written for and premiered by Trio Kobayashi—at that time composed of CalArts graduate students—a group of which I am a founding member. This piece is among the most difficult in the brass repertoire due to its theoretical complexity, unusual notation, and technical challenges. It is written in what the composer calls “microtonal just intonation,” an exploration of tuning in untempered intervals that makes use of the complex harmonies found between the higher partials of the harmonic series. This system is used to create a richly detailed spectrum of sonorities ranging from sweet and pure consonance to pungent dissonance.

A theoretical and historical background of tuning theory is given at length in the first section of this paper. A basis of knowledge in tonal theory—a theory that is ordinarily taught without much consideration of intonation—is assumed. It is my belief that an ingrained knowledge of traditional tonal theory actually makes the learning of intonation theory more difficult; the two have conflicting nomenclature that is full of misleading “false cognates.” The intention of section I is to impart an understanding of intonation theory in a way that seeks to be fully compatible with and mutually beneficial to traditional music theory. It is my opinion that intonation theory should supplement and deepen one's understanding of tonal harmony, not seek to displace or diminish it.

A brief history of tuning practice is interwoven with the theoretical concepts so that the reader can more fully understand the musical implications of tuning choices and gain an appreciation of the relationship between intonation and music theory, which were historically



treated as closely interrelated, if not one and the same. While the extensive theoretical discussion presented here may seem impractical and prohibitively arcane in the context of learning to play a single piece of music, knowledge of intonation theory can open the door to a deeper understanding of traditional harmony, as well as the growing repertoire of contemporary music in just intonation. In that spirit, the *Plainsound Brass Trio* can be seen as a fruitful opportunity to take another look at the questions of temperament and tuning that were once central to musical discourse.

Putting the theory of just intonation into practice is a difficult task, as few tools are readily available to demonstrate the sounds of untempered intervals and chords. Deep listening and slow exercises with drones can train the ears to seek out the purity of untempered intonation and gain comfort with the abundant variety of intervals produced thereby. The questions of how to use these intervals, how to notate them, and how to produce them in performance gives rise to a fascinating history of experimentation and creativity by some of modern music's most captivating personalities.

Wolfgang von Schweinitz's answer has been to develop his own system of microtonal accidentals and to write each piece based upon a meticulously conceived use of instrumental acoustics and technique. This is not to say that Schweinitz's music is easy; on the contrary, the performer can find it more difficult knowing that everything in such complex music has been so carefully thought through. To perform Schweinitz's music, the performer must become fully engaged with its guiding principles of harmony, notation, and instrumental technique. In the *Plainsound Brass Trio*, Schweinitz provides precise tunings for each valve and more than double the number of standard trombone slide positions. While these might seem cumbersome and unnecessary at first, a deeper look reveals that they are the only way to fully realize Schweinitz's

microtonal conception. The *Plainsound Brass Trio* represents an opportunity for performers to become active participants in the beautiful and groundbreaking experiment that Schweinitz has set forth.

## CHAPTER 2

### FOUNDATIONS OF INTONATION THEORY

To discuss the just intonation music of Wolfgang von Schweinitz, it is necessary to be conversant in two separate musical “languages:” the theory of tonal harmony and that of intonation theory. The first is cultural, evolved from centuries of use and adaptation by composers and theorists to describe observed musical practices current and past. The second seeks a scientific level of specificity and clarity in pitch relationships and attempts to be more universally applicable across cultural traditions, although it has also been used by composers and theorists to justify claims of the “superiority” or “correctness” of musical systems.

The language of ratios,<sup>1</sup> which describes musical intervals as ratios between frequencies, is a return to the origins of music theory. It is a language that feels unfamiliar only because of the predominance of the equal-tempered, twelve-note keyboard as a teaching tool. As Harry Partch writes: “If ratios seem a new language, let it be said that it is in actual fact a language so old that its beginnings as an expression of the essential nature of musical sound can only be conjectured.”<sup>2</sup>

These two languages will, from time to time, come into conflict with one another, as in the case of what is known in tonal theory as a major triad. Traditionally, its components are labelled as “root,” “third,” and “fifth.” Described as ratios of frequencies, however, the components of this chord are in a relationship of 6/5/4, the number 5 representing the third of the chord and 6 the fifth. In second inversion, spelled 5/4/3, these labels become yet more conflicting, with numbers 5, 4, and 3 denoting the third, root, and fifth, respectively. For the sake

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1. So-called by composer Harry Partch in *Genesis of a Music*, 2nd ed. (New York: Da Capo Press, 1974).

2. Ibid., 77.

of clarity, Arabic numerals are used throughout this paper to denote rational frequency relationships and should carry this association in the mind of the reader, while the interval classifications of traditional theory (e.g., perfect fourth) are spelled out.

Relationships written as fractions generally denote harmonic (vertical) sonorities with the upper number indicating a higher position in the sonority. For example, the fraction  $6/5$  can be read as “six over five.” Fractions are also used in this paper to describe the position of a pitch in a particular tuning system relative to a given “fundamental” or “tonic.” Chords of more than two voices are represented by stacked fractions, again in descending order. Ratios, with numbers separated by a colon, denote melodic intervals or a more general relationship between pitches. The number to the left of the colon represents the first pitch and the number to the right, the following pitch. Thus,  $3/2$  (read “three over two”) represents a dyad, while  $3:2$  (“three to two”) represents a descending melodic interval.

Although this language of “music as numbers” is essential to discussion of just intonation theory, elements of traditional theory will frequently be used to provide a point of reference for the reader. This is especially useful considering that the work to be examined is notated in a system derived in large part from the traditional notation of European classical music. Because the intervals of just intonation differ from those of equal temperament, it might seem imprecise, for example, to use the term “major third” interchangeably with a frequency ratio. However, the nomenclature of music theory and intonation practice have historically been treated with some degree of separation. While a variety of differing tuning systems were employed in Europe before the twentieth century, nomenclature was generally determined by the notation of the intervals, not the precise tuning thereof, which varied by geography, time period, and the preferences of composers and performers. This paper adheres to such practice.

To further aid the interface between these two musical languages, cents provide a simple expression of interval size that is applicable to both systems. Cents are defined as 1200 equal divisions of an octave (the interval at which the higher pitch is double the frequency of the lower) and there are 100 cents per equal-tempered semitone. This measurement allows for easy comparison between intervals in different tuning systems. For example, pitches in a relationship of 10:9 are 182.4 cents apart, roughly eighteen cents narrower than an equal-tempered whole step of 200 cents. Throughout this paper, cents values are given to the nearest tenth of one cent, a degree of specificity that will suffice for players of non-fixed-pitch instruments. If the reader desires greater precision, cents can be calculated by the following formula, where  $c$  = cents, and  $f$  = frequency:  $c = 1200 \times \log_2(f_1 / f_2)$ . This higher level of precision is useful for electronic realization of intervals, avoidance of rounding errors in compounded intervals, and other calculations.

Any tuning system is no more than a Platonic ideal when put into practice. Although the frequencies of notes in tuning systems are mathematically definable with complete precision, absolute adherence to those frequencies is impossible except in sounds generated by electronics. Thus, I propose that errors of tuning do not affect the validity of the performer's choice of tuning system. For example, a piano is still considered to be "in equal temperament" if some notes have gone out of tune. For the purposes of this paper, the choice of tuning system is defined by the intent of the performer, not by the degree of precision with which that system is executed. As the pioneering just intonation composer La Monte Young is fond of saying, "tuning is a function of time,"<sup>3</sup> which is to say that our ability to determine pitch relationships by ear is affected by the

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3. Richard Kostelanetz, "Conversations with La Monte Young," in *Selected Writings*, ed. Michael H. Tencer (Ubuclassics, 2004), 7, [http://www.ubu.com/historical/young/young\\_selected.pdf](http://www.ubu.com/historical/young/young_selected.pdf).

duration for which we hear them. A mistuned octave, for example, could sound acceptable when heard briefly but intolerable when sustained for several seconds. In the context of the quickly changing harmonies in Schweinitz's *Plainsound Brass Trio*, this distinction is important.

What, then, makes an interval “pure” or “impure,” “in-tune,” or “out-of-tune?” For the purposes of this paper, pure intervals are those that can be expressed as small-number frequency ratios.<sup>4</sup> They are in-tune when executed precisely enough to eliminate or minimize the perception of beating, a phenomenon that occurs at near-unisons between pitches. Beating is perceived as wavering loudness of a combined sound and is caused by alternating cancellation and reinforcement of sound waves. The rate of this beating is equal to the difference in frequency between two pitches.<sup>5</sup> Upper partial tones play an important but subtle role in tuning. They are the sounds, along with the fundamental frequency, that make up the characteristic timbre of musical instruments or voices and are often referred to as “overtones.” Most pitched instruments have upper partial tones that are harmonic, meaning that they are at or near integer multiples of the fundamental, while many percussion instruments have mostly inharmonic components, making their fundamental pitch more difficult, if not impossible, to discern. A colloquial definition might be to term harmonic sounds as being “musical” and inharmonic sounds as “noise.” However, this prevents the inclusion of percussive instruments into the realm of the “musical” and draws a distinction that is too binary. In the words of Hermann von Helmholtz,

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4 An exact definition of what qualifies as a “small number” is deliberately unspecified. “Purity” and the related concept of “consonance” are subjective qualities that have been debated for centuries.

5. Arthur H. Benade, “The Brass Wind Instruments,” in *Fundamentals of Musical Acoustics*, 2nd ed. (New York: Dover, 1990), 239–259.

“noises and musical tones may certainly intermingle in very various degrees, and pass insensibly into one another, but their extremes are widely separated.”<sup>6</sup>

Sustaining instruments, such as those in the string and brass families, have upper partial tones at exact integer multiples of the fundamental. These completely harmonic timbres are particularly well-suited to illustrating pure intervals because of the presence of unisons between upper partial tones when the fundamental pitches are in simple-ratio relationships. The phenomenon of beating mentioned earlier can occur at near-unisons between the upper partial tones of two separate sounds and are used by musicians to determine whether an interval is in tune or not.<sup>7</sup>

The perceptibility of very precise tuning relationships is a complex subject and one that is open to debate. In the words of Ll. S. Lloyd:

The only instrument which the musician has for measuring musical intervals is his ear. ... No theoretical conjectures about acoustics and music are of any significance unless they take these limitations of the ear into account. ... There is entertainment in calculating the arithmetical niceties of the diesis, the diaschisma, the great limma, and so forth. The occupation is harmless so long as we remember it has very little to do with music.<sup>8</sup>

While the modern composer or performer of contemporary music might balk at such a restrictive limitation, this pejorative statement can also be taken as a challenge to compose and perform music in such a way that emphasizes perceptibility of pitch relationships, including those outside the traditional limits of the tonal system. Composer Marc Sabat and tubist Robin Hayward explore the concept of intonation perception in their co-authored paper “Towards an Expanded

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6. Hermann L. F. Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music*, trans. Alexander J. Ellis (New York: Dover, 1954), 7.

7. Ibid., 152–181.

8. Ll. S. Lloyd, “Intonation and the Ear,” *Music & Letters* 19 (1938) 443–444, <http://www.jstor.org/stable/727727>.

Definition of Consonance.” They draw a distinction between intervals that can be approximated by memory, as a musician might employ when playing with a piano in equal temperament, and those that can be objectively recognized as in tune solely by their sound. Their unusual definitions are worth quoting at length:

Consider two pitches, melodically close to each other, sounding simultaneously. As they approach unison, we perceive amplitude modulation or *beating* which gradually slows down. At a certain point, this beating is replaced by a phenomenon of spectral fusion (*perceptible periodicity*), which may be likened to a focussing of the sound at the interval 1/1. Any interval that may be determined by ear in a similar manner is called a *tuneable interval*. Note that one of the characteristics of tuneability is a *region of tolerance* in which the interval is perceived as being nearly in tune. The existence of such regions, together with the fact that the frequency range of human hearing is limited, implies that there can only be a *finite number* of tuneable intervals.

Tuneability is not proposed as an absolute property. It varies depending on the register, relative volume and timbre of the sounds, as well on the experience of the listener. Nevertheless, it suggests a precise perceptual definition of consonance: namely, *a consonant interval is one which may be precisely tuned by ear*. Relative consonance may be described as the degree of difficulty in achieving a precise intonation.<sup>9</sup>

Sabat and Hayward define a second set of intervals as being “indirectly tuneable by construction.” This class of intervals can be tuned with two or more intervening intervals that *are* directly tuneable. For example, the 9/8 major whole tone was found in the experiments to not be directly tuneable, but can be found with the intervening directly tuneable intervals of 3/2 and 3/4 (up a perfect fifth, then down a perfect fourth) or 9/4 and 1/2 (up a major ninth and down an octave), all of which were found to be easily tuneable by the authors.<sup>10</sup>

The concept of resultant tones is closely related to the phenomenon of beating. First described in the eighteenth century, resultant tones are often referred to as “Tartini tones,” although the famous violinist himself called them *terzi suoni* (third sounds). Tartini described a

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9. Marc Sabat and Robin Hayward, “Towards an Expanded Definition of Consonance: Tuneable Intervals on Horn, Tuba, and Trombone,” accessed March 20, 2017, <http://www.marcsabat.com/pdfs/tuneable-brass.pdf>.

10. Ibid., 15.



subset of resultant tones called difference tones, which are heard at the difference of the frequencies between two pitches.<sup>11</sup> For example, sounds at 440 Hz and 660 Hz could produce a difference tone of 220 Hz,<sup>12</sup> thereby creating a sonority that is greater than the sum of its parts. Difference tones are most easily heard between two harmonic sounds that are sustained at loud volume without vibrato. In my experience, a pair of trumpets playing a close interval such as a major second often produces a noticeable difference tone.

While the difference tone of the fundamentals is usually the strongest, additional resultant tones can be heard at the *sum* of the fundamentals and, in theory, at the sum or difference of upper partial tones or upper partial tones and fundamentals, although these last examples are practically impossible to discern.<sup>13</sup> The musical consequences and applications of these resultant tones will be explored further in the analysis of Wolfgang von Schweinitz's music.

### **Music as Numbers: Dealing with Frequency Ratios**

While it is customary in tonal theory to describe the addition or subtraction of intervals, these same operations applied to frequency ratios require multiplication or division. A reference pitch of A<sub>4</sub>=440 Hz sounded in different octaves requires a doubling of the frequency for each ascending octave and a halving of the frequency for each descending octave. Higher octaves are found at A<sub>5</sub>=880 Hz and A<sub>6</sub>=1,760 Hz, while lower octaves are found at A<sub>3</sub>=220 Hz and A<sub>2</sub>=110 Hz. Note that the quotient (found by division) is the same between the frequencies of adjacent As, while the difference (found by subtraction) varies. Similarly, the expression *perfect fourth*

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11. Giuseppe Tartini, "Tartini's 'Trattato di musica secondo la vera scienza dell' armonia: ' an Annotated Translation with Commentary," trans. Fredric Bolan Johnson (PhD diss., Indiana University, 1985), 38–52.

12. Benade, *Fundamentals of Musical Acoustics*, 256. Resultant tones are perceived as a result of the physiology of human hearing and are not produced by instruments or in the air as a matter of acoustics.

13. Helmholtz, *On the Sensations of Tone*, 152–155.

*plus major third equals major sixth* can be expressed as  $4/3 \times 5/4 = 5/3$ . To invert an interval, multiply the smaller number by 2 or divide the larger number by 2 and invert the fraction. For example, an inverted  $5/4$  (major third) is  $8/5$  (minor sixth).

Discovery of the ratios underlying musical harmony is commonly attributed to Pythagoras, the Greek philosopher and mathematician.<sup>14</sup> As described by Nicomachus of Gerasa, Pythagoras's experiments in acoustics and musical theory were inspired by the harmonious clanging of blacksmiths' hammers, the weights of which were measured and compared. The combinations of hammers that gave consonant intervals were found to have weights related to one another by simple ratios, such as 2:1, now called an octave, and 3:2, a perfect fifth. Although this legend is, at best, acoustically improbable, the theory of musical intervals as ratios endured.<sup>15</sup> These experiments led to the classification of intervals as either consonant (*symphonos*) or dissonant (*diaphonos*). Although ancient Greek musical systems included ratios approximating quarter tones, sixth tones, and other intervals now considered microtonal,<sup>16</sup> only the perfect fourth (4:3), perfect fifth (3:2), octave (2:1), and octave compounds of these intervals (e.g., perfect fifth plus an octave, or 3:1) were considered to be consonant.<sup>17</sup>

This basis of consonance in fifths, fourths, and octaves held sway in Europe for centuries and provided the framework for the Pythagorean tuning system, in which all intervals are derived

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14. *Grove Music Online*, s.v. "Pythagoras," by André Barbera, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/22603>. Harmony is used throughout this paper in its broadest sense — a fitting together of multiple sounds — unless given other context.

15. Gene H. Anderson, "Pythagoras and the Origin of Music Theory," *Indiana Theory Review* 6 (1983), 35–61. The pitch of the clanging would more likely have been a factor of the size and composition of the anvil, not the hammers striking it.

16. J. Murray Barbour, *Tuning and Temperament* (Mineola, NY: Dover, 2004), 16–21.

17. James Tenney, *A History of Consonance and Dissonance* (New York: Excelsior Music Publishing Co., 1988), 9–16.

from a cycle of pure fifths, a 3:2 frequency relationship that is 2.0 cents wider than an equal-tempered fifth. Tuning a keyboard using 3:2 intervals reveals the first complication of tuning by simple ratios. A cycle of pure fifths must include one wolf interval, placed at the discretion of the tuner, as a result of the fact that twelve pure fifths exceed seven octaves by a small amount: 23.5 cents. This interval is known as the “Pythagorean comma” and is mathematically defined as 531,441:524,288, or  $(3/2)^{12} \times (1/2)^7$ . This lack of enharmonic equivalence is a defining characteristic of many tuning systems, in which, for example, a note must be tuned as either A $\flat$  or G $\sharp$ . In the case of the Pythagorean system, the familiar circle of fifths more closely resembles a spiral.

Pythagorean intonation is characterized by wide major thirds and sixths and narrow minor thirds, sixths, and semitones. These wide major intervals should be familiar to players of string instruments, the strings of which are ordinarily tuned in pure fourths or fifths. Theoretically, the major sixth plus one octave between the A string and C string of a viola or cello is a ratio of  $3^3/2^3$  or 27/8, 5.9 cents wider than in equal temperament. The interval between the violin E string and the viola C string is 81/16, a major third plus two octaves that is 7.8 cents wider than in equal temperament. Recalling Pythagoras’s preference for intervals of simple ratios, these intervals are noticeably discordant and it is unsurprising that they were treated as such by medieval theorists.<sup>18</sup>

A major scale in Pythagorean tuning is comprised of 1/1, 9/8, 81/64, 4/3, 3/2, 27/16, and 243/128. The intervals between successive notes are 8:9, 8:9, 243:256, 8:9, 8:9, 8:9 and 243:256. The 8:9 whole steps are slightly wider than in equal temperament at 203.9 cents, while the

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18. *Grove Music Online*, s.v. “Pythagorean Intonation,” by Mark Lindley, accessed February 4, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/22604>.

243:256 semitones are narrower at 90.2 cents. This tuning is well-suited to the medieval and early-Renaissance music in which it was predominantly used. Mark Lindley writes:

The major seconds ... are handsomely large; and ... all the minor seconds are rather small, lending melodic incisiveness especially to polyphonic cadences with double leading notes. The impurity of these Pythagorean diatonic sixths and thirds was apparently not a blemish in most medieval music. ... Medieval theorists did not regard the major third as primarily a harmonic interval. They commonly referred to it by a name suggesting, rather, melodic implications: “ditone” (that is, a pair of whole steps). As a harmonic interval they never gave it unqualified status as a concord, and they generally restricted its use to those moments when a harmonic interval of perfect consonance (a fifth, fourth, or unison) was being approached.<sup>19</sup>

As a curiosity of the Pythagorean system, major and minor triads formed with notes from opposite ends of the chain of fifths are very close to mathematically pure major ( $6/5/4$ ) and minor ( $15/12/10$ ) triads.<sup>20</sup> In a commonly used form of the Pythagorean tuning system, the wolf fifth (narrower than pure by a Pythagorean comma) is placed between B and F $\sharp$ . The latter is actually tuned as a G $\flat$ , the last in a series of descending fifths or ascending fourths. An A-major triad, for example, in this case A–D $\flat$ –E,<sup>21</sup> contains a diminished fourth of 384.4 cents—a mere 1.9 cents narrower than the pure major third of  $5/4$ —and a pure perfect fifth of  $3/2$ . While this is a close approximation of a much simpler ratio, this Pythagorean diminished fourth has a complex pitch ratio of  $8,192/6,561$ .

If Pythagoras is correct that simpler ratios yield greater consonance, how can this interval sound more sonorous than the ordinary Pythagorean major third of  $81/64$ ? Composer and theorist

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19. Mark Lindley, “Pythagorean Intonation and the Rise of the Triad,” *Royal Musical Association Research Chronicle* 16 (1980), 6, <http://www.jstor.org/stable/25099353>.

20. Note that the just minor triad shares the intervals of the just major triad, but in reverse order.  $15/12$  simplifies to  $5/4$ ,  $12/10$  to  $6/5$ , and  $15/10$  to  $3/2$ .

21. Mark Lindley, “Pythagorean Intonation and the Rise of the Triad,” 4–61. This is not to say that chords were thus notated in the Renaissance, when Pythagorean keyboard tuning was widespread. This system allows the following triads to be virtually pure: D major, A major, E major, D–sharp minor, G $\sharp$  minor, C $\sharp$  minor, and F $\sharp$  minor.

James Tenney states that the “‘language of ratios’ is ... assumed to be the appropriate language for the analysis and description of harmonic relations—but only if it is understood to be qualified and limited by the concept of interval tolerance.”<sup>22</sup> In other words, the ear and brain categorize musical intervals within general regions anchored by the simplest ratio within that region. A poorly tuned octave is still recognized as an octave within limits determined by the extent of the mistuning, the pitch perception of the listener, the timbre of the sounds, and the duration of the interval.

It is precisely this allowance for approximation that facilitates temperament, the process of compromising the tuning of some intervals to improve others or to mitigate wolf intervals. A great variety of tempered tuning systems were employed prior to the widespread adoption of equal temperament, each of which has a unique character and necessitates choices between a variety of compromises and possibilities. As with the Pythagorean systems, the placement of the wolf fifth,<sup>23</sup> the sizes of melodic intervals, and the relative consonance or dissonance of harmonic intervals in each type of temperament inform the compositional choices of the composers who chose these systems. The converse is surely more correct: that composers chose tuning systems to suit their music and the circumstances of its performance, sometimes adopting a flexible and pragmatic approach not rigidly tied to a mathematical scheme. That the theory and practice of temperament were important considerations to composers and performers in the seventeenth and eighteenth centuries is evident in the frequency with which they are discussed in sources from that time. Jean-Philippe Rameau, Georg Philipp Telemann, Francesco Geminiani,

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22. James Tenney, “John Cage and the Theory of Harmony,” in *Writings about John Cage*, ed. Richard Kostelanetz (Ann Arbor: University of Michigan Press, 1996), 151.

23. And, in the case of non-Pythagorean tunings, the severity of these wolf intervals, which varies in size.

Giuseppe Tartini, Leopold and Wolfgang Amadeus Mozart, and many others all found it necessary to weigh in on matters of tuning in their treatises, practical methods, and lessons.<sup>24</sup>

Before the ubiquity of equal temperament, a family of tuning systems known as “meantone” provided the first widely adopted alternative to the Pythagorean system. The aim of meantone temperament is to improve the Pythagorean thirds and sixths by compromising the purity of fifths and fourths in varying degrees. The difference between the  $5/4$  pure major third and the  $81/64$  Pythagorean ditone is 21.5 cents, an interval known as the syntonic comma.<sup>25</sup> Meantone temperaments are identified by the degree to which fifths are narrowed, expressed as a fraction of the syntonic comma. In  $1/4$ -comma meantone, the prototypical form of the meantone principle, fifths are narrowed so that four of them (minus two octaves) reach a pure  $5/4$  major third, which is narrower than the equal-tempered major third by 13.7 cents. In other versions of meantone, such as  $1/6$ -comma or  $2/7$ -comma, thirds are slightly less pure to favor other intervals or to avoid the intense sourness of  $1/4$ -comma meantone’s 736-cent wolf fifth.<sup>26</sup> Within a limited set of keys, the meantone tunings allow some chords to be far more pure than in equal temperament; the mistuning of perfect fifths in every version of meantone being less severe than that of thirds in equal temperament.

Common to all meantone systems is the equal size of the whole steps that form a major third (hence the term “mean tone”) and that notes written as flats are higher than enharmonic

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24. Bruce Haynes, “Beyond Temperament: Non-Keyboard Intonation in the 17th and 18th Centuries,” *Early Music* 19 (1991): 356–81.

25. ‘Syntonic’ refers to the ‘Diatonic Syntonon’ ratio-scale described by Claudius Ptolemy, which contains pure thirds of  $5/4$ . Barbour, 20.

26. *Grove Music Online*, s.v. “Mean-tone,” by Mark Lindley, accessed February 22, 2017, <http://www.oxfordmusiconline.com:80/subscriber/article/grove/music/18222>.

notes written as sharps.<sup>27</sup> Applying a meantone tuning to a keyboard instrument therefore necessitates a choice between tuning each black key as a sharp or a flat. This family of tuning systems also shares the trait of unequal semitones: the diatonic semitones—such as from F♯ to G—are larger than the chromatic semitones—such as from G♭ to G. The mastery of these unequal semitones was evidently an important skill in the Baroque and Classical periods, as is shown in a variety of woodwind fingering charts and in descriptions by composers and performers.<sup>28</sup>

Scales with both pure fifths ( $3/2$ ) and pure thirds ( $5/4$  major and  $6/5$  minor) satisfy the historical definition of just intonation. These tunings rely upon unequal sizes of both whole steps and semitones. In just intonation, all intervals are untempered and can therefore be expressed as ratios, a characteristic that is shared with the Pythagorean system, although the latter employs more complex ratios for many intervals. Applying a just tuning system to a keyboard instrument presents several challenges due to the fact that the precise tuning of these pure intervals precludes any notes from functioning in multiple ways. For example, an A♭ tuned 4:5 below C cannot also be used as a G♯ 5:4 above E (which itself is 5:4 above C).

A simple keyboard tuning in just intonation relates F, C, G, and D by the Pythagorean series of pure fifths, with the remaining notes tuned in pure thirds above or below these notes, as shown in table 1.<sup>29</sup> In this table, pure major triads— $6/5/4$ —are found where the third and fifth are located, respectively, one cell above and one cell to the right of the chord's root (as in F–A–

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27. This is the opposite of Pythagorean tuning, in which sharps are higher than flats.

28. Haynes, “Beyond Temperament,” 358–61.

29. J. Murray Barbour, “The Persistence of the Pythagorean Tuning System,” *Scripta Mathematica* 1 (1933): 289.

C) and pure minor triads—15/12/10—one cell below and one to the left of the fifth (as in C–E $\flat$ –G). Note that many commonly used chords are not pure in this tuning. The D minor triad, for example, contains a fifth that is too narrow by a syntonic comma and a Pythagorean minor third.

Table 1  
Keyboard Tuning in Just Intonation

← Perfect fifths (3:2) →				
↑	C $\sharp$	G $\sharp$	—	—
Major thirds	A	E	B	F $\sharp$
(5:4)	F	C	G	D
↓	—	—	E $\flat$	B $\flat$

Table 2 shows the ratio for each note in this tuning as related to a reference pitch of C and the distance in cents from this reference pitch. Some of these cents-deviations should look familiar to musicians who have learned the most basic rules of thumb regarding intonation, usually taught in relation to the tuning of triads: that major thirds should be 14 cents narrower than equal-tempered, minor thirds 16 cents wider, and fifths 2 cents wider. Because the entirety of this tuning system is formed by pure fifths and pure thirds, a simple analysis of its ratios makes it possible to quickly figure out the deviation of each note from equal temperament. Recalling the operations of multiplying and dividing ratios to add and subtract intervals, the seemingly complex ratios of this tuning can be better understood by factoring. For example, the 25/16 G $\sharp$  can be thought of as  $(5 \times 5) / (2 \times 2 \times 2 \times 2)$ —in other words, this note is formed by two pure major thirds above the tonic. Multiplying any frequency by 5 yields a pitch that is two octaves and a pure major third higher. Multiplying by 3 gives a pitch one octave and a perfect fifth higher. Octaves—any number of 2s as factors in the numerator or denominator—can be ignored because they are tuned the same in all Western tuning systems. After factoring the ratio



for an interval, subtract 13.7 cents for every 5 in the numerator and add 2.0 cents for every 3 in the numerator. In the denominator, cents values are negated: for example, each 5 adds 13.7 cents and each 3 subtracts 2.0 cents. For example, the F $\sharp$  of 45/32 is factored  $(5 \times 3 \times 3) / (2 \times 2 \times 2 \times 2 \times 2)$ . The 600-cent augmented fourth of equal temperament is altered by  $-13.7 + 2.0 + 2.0 = -9.7$ .

Table 2

Ratios and Cents Values of Chromatic Scale in Just Intonation

C	C $\sharp$	D	E $\flat$	E	F	F $\sharp$	G	G $\sharp$	A	B $\flat$	B
1/1	25/24	9/8	6/5	5/4	4/3	45/32	3/2	25/16	5/3	9/5	15/8
0	70.7	203.9	315.6	386.3	498.0	590.2	702.0	772.6	884.4	1,017.6	1,088.3

A chromatic scale in this tuning primarily uses two sizes of semitones: the 15:16 diatonic semitone of 111.7 cents and the 24:25 chromatic semitone of 70.7 cents; and two sizes of whole steps: the Pythagorean scale's 8:9 major whole step of 203.9 cents and the 9:10 minor whole step of 182.4 cents. These stepwise intervals can be practiced easily by playing or singing a sequence of the following carefully tuned intervals over a drone: minor third, major third, perfect fourth, perfect fifth, major sixth. The intervals between these pitches are: chromatic semitone, diatonic semitone, major whole step, minor whole step. The semitones differ in size by 41.1 cents, while the whole steps differ by 21.5 cents (the syntonic comma). The practicality and aesthetics of the melodic use of these unequal steps in the performance of classical music might now seem debatable given the pervasiveness of equal temperament, but many harmonic and contrapuntal situations make them indispensable to producing pure sonorities.

Figure 1 shows an excerpt from the first movement of Gustav Mahler's Sixth Symphony in which these two sizes of semitone could be used. At measures 59 and 60, the harmony moves

from A major to A minor as the middle oboes move from C $\sharp$  to C $\flat$ . If each of these chords is tuned with pure intervals and the common tones are unchanged, this semitone will be noticeably small. Similarly, the descending chromatic line found two bars later in the first flute and first oboe could be tuned with alternating chromatic (e.g., from G $\sharp$  to G) and diatonic (e.g., G to F $\sharp$ ) semitones. The difference in size between these steps could be strikingly expressive, although it would certainly sound unusual to modern audiences.

**Figure 1:** Mahler, *Symphony No. 6*, mm. 59–68 (woodwinds only).  
Source: Gustav Mahler, *Symphonies Nos. 5 & 6* (New York: Dover, 1991).

Thus far, intonation theory has been discussed primarily in relation to closed systems that are limited to the twelve notes of the standard keyboard. Instruments with some degree of pitch flexibility have the capability of applying the concepts behind just intonation far more broadly. In traditional tonal music, this is particularly useful for the alteration of notes by a syntonic comma to avoid the problems of the twelve-note just intonation tuning.

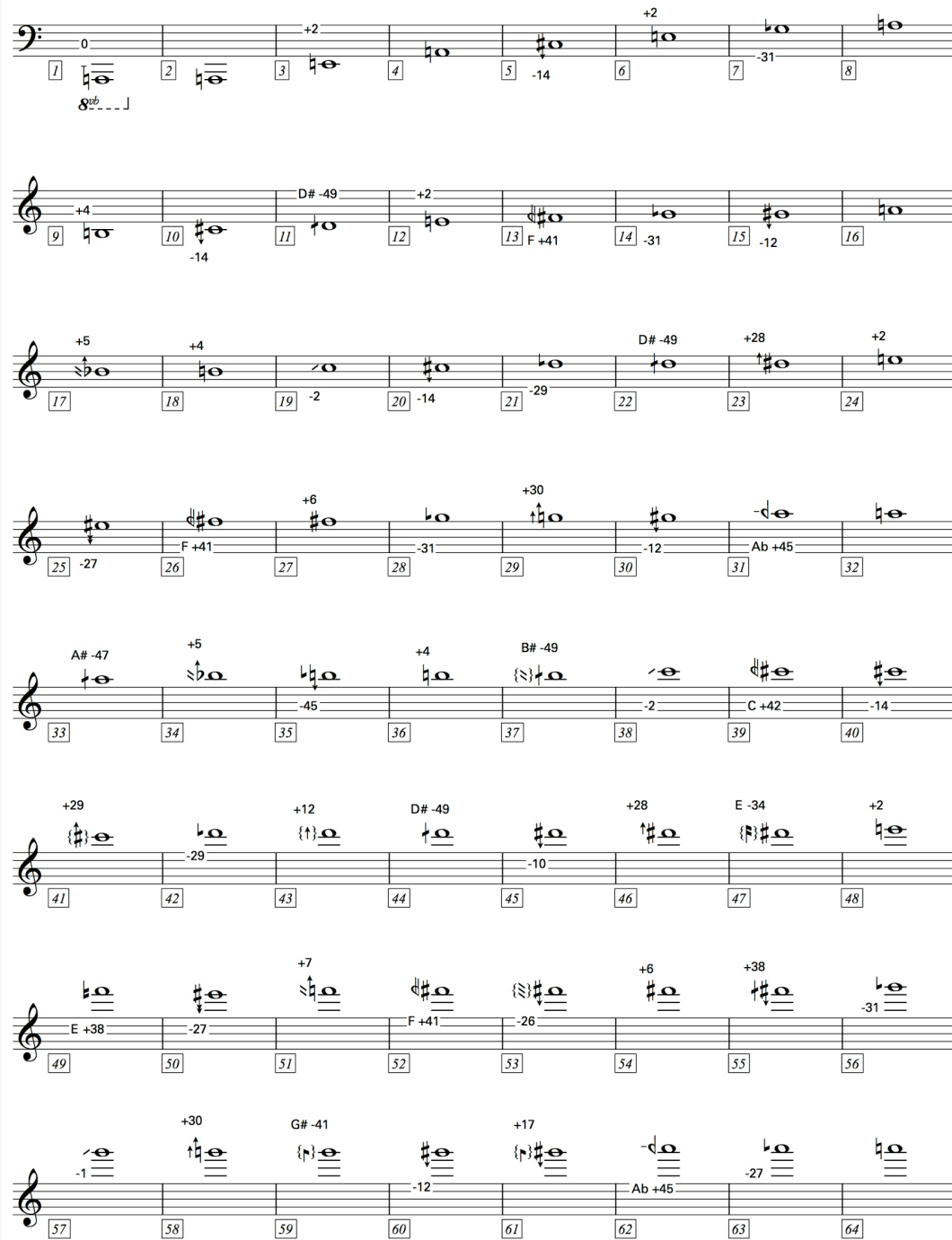
## Intonation and the Harmonic Series

The harmonic series is composed of a fundamental pitch and whole-number multiples of that pitch. It is a theoretical construct that contains every interval of extended just intonation as well as the frequencies of the upper partial tones that comprise the timbre of harmonic musical sounds. Analysis of the harmonic series shows simple and clear patterns to make the seemingly complex set of numbers into a manageable and easily comprehensible resource that can be of use to any musician. An understanding of the harmonic series and its implications is invaluable for the avoidance or identification of tuning problems, the ability to analyze the harmonic meaning of various chords, and for the appreciation and analysis of historical performance practice and alternative tuning systems.

Figure 2 shows the first sixty-four partials of the harmonic series of the lowest note of the piano, A<sub>0</sub>. The positive and negative numbers above each note show the deviation, in cents, of the partial from its counterpart in twelve-tone equal temperament, which is found by looking at the accidental closest to the note head, but ignoring any upward or downward arrows attached to it. First, notice that octaves of the fundamental, or first partial tone, are identical to those in equal temperament. The absence of a positive or negative number above each A shows that there is no pitch deviation from equal temperament. In the harmonic series, octaves of the fundamental are always powers of two:  $2=2^1$ ,  $4=2^2$ ,  $8=2^3$ ,  $16=2^4$ ,  $32=2^5$ , and  $64=2^6$ . Next, observe that the third partial and its octaves (6, 12, 24, 48, ...  $3 \times 2^n$ ) all have the same pitch deviation from equal temperament, +2.0 cents. The same is true for octave repetitions of any partial. Any even-numbered partial, therefore, is an octave repetition of a lower partial. Each odd-numbered partial is a “new” note, if the harmonic series is considered as an upward progression from the fundamental.

# THE HARMONIC SERIES 1 - 64 above "A0" (overtone row)

*notated using the Extended Helmholtz-Ellis JI Pitch Notation  
microtonal accidentals designed by Marc Sabat and Wolfgang von Schweinitz, 2004*



**Figure 2:** First 64 partials of the harmonic series of A.

Source: Marc Sabat, "The Harmonic Series 1–64" (2005), <http://www.marcsabat.com/pdfs/HS64.pdf>.

Reducing the number of any partial in the harmonic series to its prime factors allows derivation of its tuning. For example, the fifteenth partial can be factored as  $3 \times 5$ . To find its pitch deviation from equal temperament, add the amounts by which the third and fifth partials deviate: +2.0 cents and -13.7 cents, respectively. Therefore, the fifteenth partial is 11.7 cents lower than its equal-tempered counterpart. In this way, one can think of any partial with multiple factors as a sort of partial of a partial; in the example above, the 15th partial can be thought of as the fifth partial of the third partial or as the third partial of the fifth partial. Each successive prime-numbered partial has a unique cents deviation, listed in table 3.

Table 3

Prime Partial and Cents Deviations, Separated by Octave of the Harmonic Series

2, octave ±0 cents	5, major third -13.7 cents	11, perfect fourth +51.3 cents	17, minor second +5.0 cents	37, major second +51.3 cents
3, perfect fifth +2.0 cents	7, minor seventh -31.2 cents	13, major sixth -59.5 cents	19, minor third -2.5 cents	41, major third +29.1 cents
			23, augmented fourth +28.3 cents	43, perfect fourth +11.5 cents
			29, minor seventh +29.6 cents	47, augmented fourth +65.5 cents
			31, octave -55.0 cents	53, major sixth -26.5 cents
				59, minor seventh +59.2
				61, major seventh +16.9

Each successive prime number introduces new categories of intervals and new harmonic possibilities. Thus, any tuning system can be defined by the highest prime number used as a factor in its frequency ratios. Pythagorean tuning, limited to the simplicity of primes 2 and 3 (3-limit), enhances the melodic focus of the Renaissance, with evenly sized whole steps and narrow semitones giving strident dissonance to passing intervals and strong consonance to nearly all perfect fifths. 5-limit just intonation—and the meantone temperaments that approximate it—is the model for triadic harmony, with rich sonorities facilitated by unequal semitones and whole steps, strictly limited to certain tonal centers by comma errors or wolf intervals.

Historically, the meantone temperaments gave way to more complex systems that enabled the freer use of modulation seen in the Romantic era. The eventual widespread adoption of equal temperament took this trend to its logical conclusion, being a system in which each tonal center sounds the same as any other.<sup>30</sup> Although the equal-tempered system consists entirely of irrational intervals, it is similar to the Pythagorean system, with fifths a mere 2.0 cents narrower than  $3/2$ , major thirds closer to the wide Pythagorean ‘ditone’ of  $81/64$  than the pure  $5/4$ , and narrower semitones than the  $16/15$  diatonic semitones of just intonation (though not as narrow as Pythagorean semitones). In addition to facilitating modulations and the enharmonic “pivots” used in the late Romantic, equal temperament also helped to smooth the use of chord extensions and altered chords. In a closed system of 5-limit just intonation, seventh chords frequently contain comma errors, such as perfect fifths that are too narrow. In meantone systems,

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30. Mark Lindley, “Some Thoughts Concerning the Effects of Tuning on Selected Musical Works (From Landini to Bach),” *Performance Practice Review* 9 (1996): 114–118. The irregular temperaments of the late Baroque lent different intonational “flavors” to different keys in varying degrees of subtlety. Such differences were not as present in regular meantone temperaments (with the exception of wolf intervals), such as 1/6-comma meantone, widely used in the 18th century.

the problem is less severe, but equal temperament generally provides more satisfactory seventh and ninth chords.<sup>31</sup>

7-limit harmony, which can be termed “septimal,”<sup>32</sup> is more complex still. A closed, twelve-note tuning of septimal harmony for use in triadic tonal music is practically impossible, which should be unsurprising in light of the complexities already presented by 5-limit tuning. Singers and players of instruments with flexible pitch, however, can use the harmonic seventh to create consonant dominant seventh chords, the most prominent feature of 7-limit tuning. Barbershop quartet harmony is largely defined by the use of such chords, in which the minor seventh is 31.2 cents—nearly a sixth-tone<sup>33</sup>—lower than in equal temperament. If the 13.7- and 15.6-cent corrections for pure major and minor thirds can, perhaps, be done subconsciously, the intervals of 7-limit harmony require pronounced adjustments of the pitch. A few notable intervals using the 7th partial are listed in table 4.

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31. R.H.M. Bosanquet, “On Some Points in the Harmony of Perfect Consequences,” *Proceedings of the Musical Association* 3 (1876), 145–153. In 5-limit just intonation tunings allowing more than twelve notes per octave, sometimes called “enharmonic” or “multiple division,” minor sevenths narrowed by two commas are possible and can sound very consonant.

32. “Septimal,” “undecimal” (relating to prime 11), and “tridecimal” (relating to prime 13), are used in the present paper, but “tertial” and “quintal” are replaced with “Pythagorean,” and “syntonic” to avoid confusion with “thirds” and “fifths” in traditional interval nomenclature. That partials 7 through 14 align perfectly with their interval classifications is purely a coincidence — one that is bound to cause problems to readers expecting the same pattern of other intervals and partials.

33. Recalling that an equal-tempered whole step, or “tone,” measures 200 cents.

Table 4

## Septimal Intervals in Just Intonation

Interval name	Typical use in tonal music	Frequency ratio	Cents deviation from equal temperament
“Harmonic” or “natural” seventh	Root to seventh in dominant seventh chord	4:7	-31.2
Septimal tritone	Third to seventh in dominant seventh chord or root to fifth in diminished chord	5:7	-17.5
Septimal minor third	Fifth to seventh in dominant seventh chord or third to fifth in diminished chord	6:7	-33.1
Septimal whole step	Seventh to upper-octave root in dominant seventh chord or melodically as 8:7 when moving downward to the seventh of a chord	7:8	+31.2
Septimal major third	Seventh to ninth in a dominant seventh chord with added ninth or fifth to seventh in a half-diminished seventh chord	7:9	+35.1
Septimal semitone	Melodically as 21:20, a downward resolution from the seventh of one chord to the major third of the next	20:21	-15.5



## Just Intonation as Compositional Practice

In European classical music, practical application of intonation based on higher primes than 7 was never widely implemented between the time of the ancient Greeks and the twentieth century.<sup>34</sup> Use of these higher primes, therefore, belongs squarely in the realm of modern music. The 11th partial—used in what can be called “undecimal” harmony—is 48.7 cents lower than an equal-tempered tritone, akin to the quarter-tone music of Charles Ives, Pierre Boulez, and others. The 13th partial of “tridecimal” harmony, 41.5 cents higher than an equal-tempered major sixth, forms something of a “neutral sixth” with the fundamental, sounding neither major nor minor. The 17th and 19th partials, on the other hand, are so close to notes on the equal-tempered scale that they might not be recognized as rational intervals when heard in isolation. Each successive prime number introduces a new collection of intervals and new possibilities for composition.

Contemporary composers and theorists have proposed the use of these higher primes and greater degrees of tuning refinement—along with quartertones and other irrational microtonality—as paths beyond the limits of both tonality and serialism.<sup>35</sup> In the American experimental tradition, ratio tuning came to take on rebellious, philosophical, moral, and even quasi-spiritual dimensions. Through its rejection of equal temperament, it played an important role in the movement’s opposition to the European classical tradition and the serial system that

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34. J. Murray Barbour, *Tuning and Temperament*, 15–24. Greek sources describe several scales with higher primes including 11, 13, 17, 19, and 23.

35. Ben Johnston, “Proportionality and Musical Pitch Relations” in *Maximum Clarity*, edited by Bob Gilmore (Urbana and Chicago: University of Illinois Press, 2006), 35. Ben Johnston writes that using quartertones and other equal divisions as an expansion of the equal-tempered idea “complicates the problem without helping to solve it.”

came to supersede it.<sup>36</sup> At the same time, just intonation was also seen as a return to music theory's first principles: the experiments and ratio-scale tuning of Pythagoras and the ancient Greeks.

Henry Cowell (1897–1965), whom John Cage (1912–1992) described as “the open sesame for new music in America,”<sup>37</sup> described the relationship between the harmonic series and music theory in his book *New Musical Resources*, first published in 1930. Cowell writes that its purpose,

... is not to attempt to explain the materials of contemporary music, ... but to point out the influence that the overtone [harmonic] series has exerted on music throughout history, how many musical materials of all ages are related to it, and how, by various means of applying its principles in many different manners, a large palette of musical materials can be assembled.<sup>38</sup>

Cowell goes on to show the basis of traditional tonal language in the lower portions of the harmonic series and potential for new possibilities in its higher reaches. Because the high partials are more prominent in the timbre of modern instruments than in that of older instruments, the intervals between the higher partials, Cowell argues, are already “within the experience of the listener” and will be received more favorably than sounds that are not related to the harmonic series.<sup>39</sup> Cowell's theories of the dissonant possibilities of the harmonic series align closely with Arnold Schoenberg's “emancipation of the dissonance” and Schoenberg's subsequent calls that

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36. Colin Holter, “The Spiritual Construction of Tuning in American Experimental Music,” in *The Second Century of New Music*, ed. Franklin Cox, et al. (Lewiston, New York: Edwin Mellen Press, 2011).

37. John Cage, *Silence* (Hanover, NH: Wesleyan University Press, 1961), 71.

38. Henry Cowell, *New Musical Resources* (Cambridge: Cambridge University Press, 1996), x–xi.

39. *Ibid.*, 5

“comprehensibility of the dissonance is considered as important as the comprehensibility of consonance.”<sup>40</sup>

Around the time of the publication of Cowell’s *New Musical Resources*, composer Harry Partch (1901–1974), then aged twenty-one, discovered Hermann von Helmholtz’s *On the Sensations of Tone*. Partch describes this nineteenth-century investigation into acoustics, physiology of hearing, and music “the key for which [he] had been searching.”<sup>41</sup> A few years later, in a symbolic display of the iconoclasm that would come to define his presence in American music, Partch burned his early attempts at composition and turned his focus solely to the creation of music in just intonation. Partch’s manifesto, *Genesis of a Music*, also saw its first drafts during this time, eventually growing to an extensive work of the composer’s philosophies and theories.<sup>42</sup>

Partch begins *Genesis of a Music* with a recasting of the history of European music as a struggle between the competing tendencies of composers toward the “Abstract” and the “Corporeal,” a passage that Alex Ross describes as “the most startling forty-five page history of music ever written.”<sup>43</sup> “Corporeal” encompasses Partch’s ideal for his own music:

For the essentially vocal and verbal music of the individual—a Monophonic concept—the word Corporeal may be used, since it is a music that is vital to a time and place, a here and now. The epic chant is an example, but the term could be applied with equal propriety to almost any of the important ancient and near-ancient cultures—the Chinese, Greek, Arabian, Indian, in all of which music was physically allied with poetry or the

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40. Arnold Schoenberg, “My Evolution,” *The Musical Quarterly* 75 (1991): 527, accessed March 20, 2017, <http://www.jstor.org/stable/741842>.

41. Harry Partch, *Genesis of a Music*, 2nd ed. (New York: Da Capo Press, 1974), vii.

42. *Ibid.*, x.

43. Alex Ross, “The Critics: Musical Events: Off the Rails: A Rare Performance of Harry Partch’s ‘Oedipus,’” *The New Yorker* 81, no. 9 (Apr 18, 2005): 199–201. <https://search.proquest.com/docview/1745448>.

dance. Corporeal music is emotionally “tactile.” It does not grow from the root of “pure form.” It cannot be characterized as either mental or spiritual.<sup>44</sup>

The “Abstract,” on the other hand, “grows from the root of non-verbal ‘form’” and is “always ‘instrumental,’ even when it involves the singing of words, because the emotion of an individual conveyed through vitally rendered words would instantly end the characteristic domination of non-verbal ‘form.’”<sup>45</sup> With this dichotomy, Partch traces a Corporeal thread from Plato through a handful of select works written by an unlike lineage of heirs. Those composers who “preserved the continuity of music-drama,”<sup>46</sup> includes Monteverdi, Gluck, Berlioz, Wagner, Mahler, and Debussy. Partch’s rejection of the European classical tradition extends to contemporary developments as well:

I too was encompassed by the popular assumption that the *present*, in relation to the past, means *progress*. It involved no small inner struggle to emerge from that spell, to discover that present “progress” clothed a skeleton of bondage to a specific and limited past, and to emerge from mental turmoil to a realization that what was called progress was not necessarily progress to me, however sincerely it might be accepted as such by many, perhaps even a great majority.<sup>47</sup>

To Partch, Corporeal music necessitates simple-ratio tuning, calling temperament in all forms “a system which deliberately robs its intervals of their purity in order to implement the idea of every-tone-in-several-senses.”<sup>48</sup> Partch invokes the names of philosophers, monks, and scientists to lend credibility to his ideas, but rarely other musicians, their use of temperament and abstraction evidently being incompatible with Monophony and Corporeality.

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44. Harry Partch, *Genesis of a Music*, 8.

45. *Ibid.*, 8.

46. *Ibid.*, 28.

47. *Ibid.*, 5–6.

48. *Ibid.*, 74.

Partch outlines a theory of Monophony, beginning with a definition of a consonance-dissonance spectrum that “begins with absolute consonance (1 to 1), and gradually progresses into an infinitude of dissonance, the consonance of the intervals decreasing as the odd numbers of their ratios increase.”<sup>49</sup> Each ratio on this scale, which in this case marks the tuning of an individual note rather than a melodic or harmonic interval, has a two-part identity: the “over” number, or “odentity,” and the “under” number or “udentity,” both of which are divided by 2 until an odd number is reached. Thus, the ratio 9/8 has an oidentity of 9 and a uidentity of 1. These identities generate tonalities: otonalities (overtone tonalities) are those in which the oidentities increase while the uidentity remains fixed, while the reverse is true of utonalities. Partch terms the fixed number the “numenary nexus.”<sup>50</sup> Otonality is associated with the historical concept of major, while Utonality is associated with minor.<sup>51</sup>

Self-described as a “philosophic music-man seduced into carpentry,”<sup>52</sup> Partch had little use for traditional instruments. He constructed instruments of his own design or modified existing ones to fit the needs of his tuning systems. “The plucked strings in general represent the soul of my work and percussion the bodily structure,” writes Partch in *Genesis of a Music*.<sup>53</sup> This preference is curious, as the upper partial tones of plucked and struck sounds always contain

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49. Ibid., 87. By this unique definition — which conflicts markedly with that of Marc Sabat and Robin Hayward’s “tuneability” spectrum — major thirds (5/4), minor thirds (6/5), major sixths (5/3), and minor sixths (8/5) are equally consonant, as are the septimal intervals 7/4, 7/5, 7/6, and 7/8. Partch considers only intervals within an octave, extending the concept of octave equivalence to include “identities” in a ratio.

50. Ibid., 72.

51. Ibid., 110–111. With all ratios reduced to one octave and placed above a fundamental, the undertone series of 1/1, 1/2, 1/3, 1/4, 1/5, etc. becomes 1/1, 2/1, 4/3, 2/1, 8/5, etc. These intervals are identical to the inversions of the harmonic series.

52. Quoted in Lou Harrison, “I do not quite Understand you, Socrates,” in *Harry Partch: An Anthology of Critical Perspectives*, ed. David Dunn (Amsterdam: Harwood Academic Publishers: 2000), 136.

53. Harry Partch, *Genesis of a Music*, 196.

some degree of inharmonicity—causing beating between combinations of them even when the fundamental pitches are in tune—and because such instruments are non-sustaining, limiting the perceptibility of Partch’s careful tuning. Perhaps it is for this reason that Partch repeatedly dismisses the importance of the upper partial tones in his system, stating that, “the ear is not impressed by partials as such. The faculty—the prime faculty—of the ear is the perception of small-number intervals ... and the ear cares not a whit whether these intervals are in or out of the overtone series.”<sup>54</sup>

By abandoning any ties to traditional pitch nomenclature—even in text—Partch sees “inestimable [advantages] in opening new tonal vistas, in getting to the analyzable root of music and the core of the universe of tone. ... If time is taken out to translate each ratio into what is assumed to be a synonymous word value, these vistas are dimmed or lost altogether, and the values, which are not synonyms, are nevertheless convicted of fraud by alleged synonyms.”<sup>55</sup> Partch developed a separate tablature notation for each of his many instruments, a solution that he saw as a necessary evil and one that he admits falls far short of being a comprehensive system.<sup>56</sup> His pitch notation for voice and for the adapted viola simply uses ratios, while others employ traditional staff notation that indicates only fingered pitch, which often does not correspond to a scalar configuration of sounding pitches. In some works for larger ensemble—where an expectation of proficiency in intonation theory would be impractical—a color-coding

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54. Ibid., 87.

55. Ibid., 76. This purist approach, which rejects the notion that the term *pure third* could be synonymous with the ratio 5/4, leads to plenty of awkward prose in *Genesis of a Music*, such as descriptions of instruments tuned in “2/1’s” or “4/3’s” and tessitura described as higher or lower “2/1’s.”

56. Ibid., 197–198.

system signifies approximate alterations to the equal-tempered pitches presumed to be more familiar to the musicians.<sup>57</sup> As musicologist Bob Gilmore writes:

This system works very well for the performing musician but not so well for the would-be analyst. One has first to translate a note on a staff to its pitch value: but that pitch value is not a familiar entity like B-flat or E-natural but a frequency ratio, one of the forty-three possible tones in the octave of Partch's microtonal scale—and in fact, he often uses pitches outside those of the forty-three-tone scale he analyzes in his book *Genesis of a Music*.<sup>58</sup>

The work of Harry Partch has been described here at length due to his outsize influence as a pioneer of just intonation music. His utterly steadfast commitment to non-tempered tuning and consequent invention of new tonal systems provided a model to a generation of composers, including Lou Harrison (1917–2003), Ben Johnston (b. 1926), and James Tenney (1934–2006). He is worth quoting one last time, as his words eloquently sum up the challenges faced by this next generation:

In grappling with notation, the composer-pioneer is constantly on the horns of dilemma, a situation that becomes so thoroughly normal to him that when an integrated and rational solution seems to present itself, he is more than likely to remain incredulously perched—and with good reason. In a sense, a notation is the least of a music's ingredients, one that might be supplied on little more than a moment's notice. To provide a notation is a matter of paper and pencil and a good night's sleep; but to evolve a theory, to develop instruments based upon it, and to write and present music conceived there-from is easily the matter of a lifetime.<sup>59</sup>

Ben Johnston was strongly influenced by Partch, his first exposure to just intonation concepts coming directly from *Genesis of a Music* and through a six-month residency at the composer's California workshop. Johnston was tasked with tuning and maintaining Partch's

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57. Ibid., 202–203.

58. Bob Gilmore, "Harry Partch," *Notes — Quarterly Journal of the Music Library Association* 59 (2003): 740.

59. Harry Partch, *Genesis of a Music*, 197.

handmade instruments, which quickly developed his ability to hear simple-ratio intervals.<sup>60</sup>

Nearly ten years later, Johnston turned his efforts to the composition of music using ratio-scales and undertook the task that Partch never completed: the development of a comprehensive notation system for intervals in what Johnston calls “extended just intonation.” Due to his lack of carpentry skills, Johnston did not wish to follow Partch’s path in the invention of new instruments and he found early electronic instruments unsatisfactory. This left him with traditional instruments as the primary medium for the performance of his music and necessitated a notation system that could be approached by players of those instruments.<sup>61</sup>

Johnston sought to create a notation that hewed as closely as possible to the Western classical tradition, with a five-line staff and flat, sharp, and natural accidentals. Johnston adds to this plus and minus symbols to indicate raising or lowering of a pitch by the syntonic comma (the 21.5-cent difference the Pythagorean  $81/64$ —a major third derived from a series of fifths—and a pure  $5/4$  third) along with a numerals and arrow symbols to accommodate ratios using prime numbers higher than 5.<sup>62</sup> Johnston explains: “From the very beginning, my whole aim was to keep my notation as close to ordinary usage as I could; in fact, for a long time I was only dealing with extended triadic usage, that is, no prime overtones higher than five.”<sup>63</sup>

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60. Ben Johnston, interview with Bruce Duffie, 1987, <http://www.bruceduffie.com/johnston.html>.

61. Ben Johnston, interview with Derek Bermel, *Paris Transatlantic Magazine*, 1995, <http://www.paristransatlantic.com/magazine/interviews/johnston.html>.

62. The plus symbol, ironically, is one used in nineteenth-century string-instrument methods to show raised leading tones in the tradition known as “expressive intonation,” a practice that is in direct opposition to equal temperament. See Patrizio Barbieri, *Enharmonic Instruments and Music 1470–1900* (Latina, Italy: Il Levante, 2008), 165–169.

63. Ben Johnston, interview with Derek Bermel.



Johnston’s notation begins with just intonation in the historical sense: a 5-limit system with pure thirds and pure fifths. Unaltered naturals (the white keys of the piano) are found by tuning pure 6/5/4 triads on C, F, and G, which are related to each other by pure 3/2 fifths, yielding a C major scale of  $1/1 - 9/8 - 5/4 - 4/3 - 3/2 - 5/3 - 15/8 - 2/1$ .<sup>64</sup> Unlike untempered keyboard tunings, Johnston’s notation system is not limited to twelve notes and allows for distinction between flats and sharps. Sharps are tuned by building 6/5/4 triads on the remaining scale degrees: 9/8, 5/4, 5/3, and 15/8. Flats are tuned as pure major thirds below the naturals.

Table 5  
Ben Johnston’s Notation of 5-Limit Pitches

← Perfect fifths (3/2) →									
↑ Major thirds (5/4) ↓	E–	B–	F♯	C♯	G♯	D♯	A♯+	E♯+	B♯+
	C–	G–	D–	A	E	B	F♯+	C♯+	G♯+
	A♭–	E♭–	B♭–	F	C	G	D	A+	E+
	F♭–	C♭–	G♭–	D♭–	A♭	E♭	B♭	F+	C+
	D <sub>bb</sub> –	A <sub>bb</sub> –	E <sub>bb</sub> –	B <sub>bb</sub> –	F♭	C♭	G♭	D♭	A♭+

While this system may seem simple at first glance, it is complicated by the fact that notes with traditional accidentals are tuned with differing syntonic comma alterations. For example, a pure D major chord must be spelled D, F♯+, A+ or D–, F♯, A depending upon whether the chord uses the 9/8 D or 10/9 D–; the 5/3 A or 27/16 A+. In either circumstance, the spelling of this chord does not correspond to the spelling of several other 6/5/4 chords, such as G major, which is simply spelled G, B, D. Similarly, the standard tuning of the violin strings is spelled G, D, A+,

64. C is considered the “unity” or 1/1 in most of Johnston’s tuning schemes, although this should not be taken to mean that Johnston writes in the key of C.

E+ or G-, D-, A, E, both of which seem to imply a wider interval between the second and third strings.<sup>65</sup> Finally, the plus symbol, especially in Johnston's manuscripts, looks quite similar to a quarter-sharp symbol with one vertical and one horizontal bar. While it is unlikely that a performer familiar with Johnston's music would confuse the two, the similarity might give pause to those uninitiated into this system.

Venturing beyond the 5-limit confines of triadic music complicates this system further. The symbol introduced by Johnston to notate 7-limit harmonies, the Arabic numeral 7, which can be attached to other accidentals, alters a note by  $36/35$ , or the amount by which the  $9/5$  minor seventh exceeds the  $7/4$  minor seventh, an interval of 48.8 cents. That the  $9/5$  seventh is the basis for the altered interval makes sense in that it is the simplest minor seventh found in 5-limit harmony, although many might find it counterintuitive that the minor seventh is the inversion of the  $10/9$  minor whole step rather than the  $9/8$  major whole step. For the 11th partial, Johnston saw the use of a numeral as impractical, opting instead for upward and downward arrows to raise or lower a pitch by  $33/32$ , the amount by which the 11th partial exceeds a perfect fourth of  $4/3$ : 53.3 cents. Similar processes govern the higher primes of 13, 17, 19, 23, 29, and 31, the last being the highest prime used by Johnston.<sup>66</sup>

Some performers of Johnston's works are certain to be tempted to use electronic tuners to verify the accuracy of their pitch. However, this is made rather difficult in Johnston's system due to the fact that accidentals do not refer to equal-tempered notes but to those of Johnston's 5-limit tuning scheme. To check a note with a tuner, it is therefore necessary to figure out how that note

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65. Ben Johnston, *Maximum Clarity*, 77. Johnston's system is not tied to a particular reference pitch, so that 440 Hz, 442 Hz, 438 Hz, etc. could each be used to define A, A-, or even A+.

66. Ibid., 82-88.

is related to a given reference pitch and calculate a ratio from it. For example, E+↓ can be calculated as follows: E is a 5/4 above C, the plus symbol raises this note by a syntonic comma of 81/80, and the downward arrow lowers this note by the 11th-partial chroma of 32/33. Multiply these numbers:  $5/4 \times 81/80 \times 32/33 = 12,960/10,560$  and simplify the product to 27/22. Because C is 3/5 under a reference pitch of A, another ratio is involved before the frequency of E+↓ can be calculated. Suffice it to say that the process is laborious and still only results in the calculation of a frequency and not its cents-deviation from a note in equal temperament.

In choosing the just major scale as the framework of his notation system, Johnston shows an interest in the performance practice of classically trained musicians and familiarity and comfort with the tonal tradition so reviled by Partch.<sup>67</sup> On the subject of coaching performers on his music, Johnston says:

This is why the notation is as close to ordinary notation as possible; I would tell them, “Your triads ought to be as free of beats as you can make it. That’s all, just aim for the same goals as when you’re playing Mozart or Haydn. If you have a natural seventh, it’s about a quarter-tone lower than a just minor seventh, not than temperament.” But, come on, these string players don’t have temperament in their minds anyway! It’s actually much harder to play in equal temperament than it is to play in just intonation. As you know, it takes a professional to tune a tempered keyboard instrument. You can’t do that as an amateur. You have to count beats, and use all sorts of devices; it’s too difficult.<sup>68</sup>

However, the capacity of skilled musicians to seek out consonance and gravitate toward simple intervals is about as far as Johnston takes his concern for instrumental technique: “I decide what I want, then I find out whether it can be done by checking with a good player. Then when I know that it works, I simply say to the player, ‘find this.’”<sup>69</sup> The tuning guide found at

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67. Ibid., 62–69. Although his views were not always so accepting. In his 1976 essay, “Rational Structure in Music,” Johnston calls equal temperament an “acoustical lie ... [that] has led to the impoverishment of pitch usage in our music,” and states that calling combinations of equal-tempered pitches “harmony” is a further lie.

68. Ben Johnston, interview with Derek Bermel.

69. Ibid.

the beginning of most of Johnston's scores simply says: "play in extended just intonation," followed by a brief description of the pitch notation. A short essay titled *On the Performance Practice of Extended Just Intonation* is sometimes also included, which states in part:

Just intonation is simply the easiest way to tune intervals by ear. It results in greatly heightened purity and clarity of sound for two reasons: first, it eliminates acoustic beats to the maximum possible, and second, it exploits resonance by utilizing harmonically simple combinations of pitches. ... The notational symbols I have devised are explained elsewhere. They provide a precisely accurate description of what the extended just intonation requires. The actual realization is achieved by ear. ... The aim of all this is to provide a harmonic logic to the ear which is even more compelling than traditional tonal logic.<sup>70</sup>

Johnston's own examples for learning just intervals primarily mention 5-limit harmonies: the major and minor thirds and sixths, differing sizes of whole and half steps, and extensions of those principles to augmented and diminished triads, chord extensions, and comma modulations. Observations of musicians' abilities to make minute adjustments of intonation are used as evidence for the performability of Johnston's complex microtonality.<sup>71</sup> Learning to hear and perform septimal and other higher-prime harmonies is given only cursory acknowledgment under an assumption that improvements in a performer's 5-limit pitch perception will naturally stimulate skills in the execution of more complex intervals.<sup>72</sup>

The perceptibility of just intervals is paramount in the works of La Monte Young, which often stretch to several hours in length, or, in the case of his *Dream House* installation, days, weeks, and months. Unlike Ben Johnston, Young received little direct influence from Harry Partch, instead finding inspiration in a diverse range of musical traditions and collaborative

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70. Ben Johnston, *Pursuit* (Baltimore: Smith Publications: 1996).

71. Ben Johnston, *Maximum Clarity*, 13–15.

72. *Ibid.*, 36–38.

relationships. Among these: performing as a jazz saxophonist alongside the likes of Eric Dolphy and Ornette Coleman; studying ethnomusicology and composition at the University of California, Los Angeles (UCLA); studying at Darmstadt under Karlheinz Stockhausen; collaborations with Yoko Ono, Anna Halprin, and Andy Warhol; North Indian Raga singing; and many more.<sup>73</sup> He was in the vanguard of American minimalism but stood out by eschewing the pulsing rhythm of Terry Riley, Philip Glass, and Steve Reich. Instead, Young's minimalism favored long durations of sustained sounds and an exploration of timbre and just intonation.<sup>74</sup>

Young's predilection for Japanese and Indian musical traditions, drone-based textures, and slow harmonic movement intuitively led him to just intonation before he even knew to give it such a title, a marked contrast to Partch and Johnston, who both started their efforts on firm theoretical ground. Some of Young's earliest pieces were primarily constructed of very long durations while still heavily under the twelve-tone influence of his early teachers, themselves students of Arnold Schoenberg. By focusing on sustained tones as a primary element of musical composition, Young says, "the longer notes make harmonic analysis by ear a reality and these integral relationships soon sound much more beautiful and harmonious and correct than their irrational equal-tempered counterparts."<sup>75</sup> In other words, performers invested in making such music sound "beautiful and harmonious" will favor pure intervals by intuition alone.

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73. *Grove Music Online*, s.v. "Young, La Monte," by Jeremy N. Grimshaw, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/A2225888>.

74. *Grove Music Online*, s.v. "Minimalism," by Keith Potter, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/40603>.

75. Richard Kostelanetz, "Conversations with La Monte Young," 25.

Young attributes the birth of his intellectual grasp of just intonation to violinist Tony Conrad, with whom Young collaborated in the Theater of Eternal Music group. Conrad recounts describing aspects of intonation theory in rehearsal and being met with skepticism:

At first this invocation of seemingly arcane arithmetical terms went down with difficulty. However, La Monte quickly realized the richness and utility of this alternative characterization of musical pitch relationships, and before too very long we began an endless train of constructive conversations almost completely centered on arithmetical whole number ratios.<sup>76</sup>

Young found new possibilities through intonation theory and used them to expand the sonic horizons of his drone-based compositions. Unlike Partch, with his forty-three-tone just scale, and Johnston, with his focus on notation and the traditional concert format, Young often limits his pitch complexity to no more than a handful of notes, exploring them over the course of his compositions, improvisations, and installations.

Young began the development of his iconic piece, *The Well-Tuned Piano*, in 1964. In keeping with Young's elusive and transient artistic presence, no score exists and the "work" is improvised on a harmonic progression to illustrate the piano's tuning and explore its possibilities and details. In its first performance, *The Well-Tuned Piano* lasted forty-five minutes but has since expanded to several hours in length. A five-hour recording of 1987 has come to represent a definitive—or, at least analyzable—version of the piece and its tuning system, which has changed over time from 31-limit to a 2, 3, 7 tuning: 7-limit without factors of 5.<sup>77</sup>

From the first chord of *The Well-Tuned Piano*, the listener is struck by how unusual the piano sounds when tuned in pure intervals, having a calmness and resonance that cannot be

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76. Quoted in Colin Holter, "The Spiritual Construction of Tuning in American Experimental Music," 63–64.

77. Kyle Gann, "La Monte Young's *The Well-Tuned Piano*," *Perspectives of New Music* 31 (Winter 1993), 141–143.

present in the equal-tempered system. In the words of Kyle Gann, who analyzes the work in *Perspectives of New Music*: “My experience of hearing the *W.T.P.* live has been that you spend the first four hours becoming familiar with the cozy septimal minor third, the expansive septimal major third, and by the fifth hour you can hardly remember that intervals had ever been any other size.”<sup>78</sup>

Young ascribed spiritual dimensions to his pure harmonies and even hinted that they might have some healing effect:

To my knowledge there have been no previous studies of the long term effects [of] continuous periodic composite sound waveforms on people. ... [Theories of pitch perception] suggest that when a specific set of harmonically related frequencies is continuous, as is often the case in my music, it could more definitively produce (or stimulate) a psychological state that may be reported by the listener since the set of harmonically related frequencies will continuously trigger a specific set of the auditory neurons which in turn will continuously perform the same operation of transmitting a periodic pattern of impulses to the corresponding set of fixed points in the cerebral cortex. When these states are sustained over longer periods of time they may provide greater opportunity to define the psychological characteristics of the ratios of the frequencies to each other.<sup>79</sup>

Young does not consider the reality that the tiniest deviation from mathematical purity in his tuning negates his concept of perfect periodicity or that the presence of any outside sounds might do the same. The *Dream House* installation comes closest to his ideal, a multi-sensory bath of electronically generated drones and purple light (designed by Young’s wife and long-time collaborator, Marian Zazeela), with pillows to lie on. Ken Johnson of the *New York Times* reports that “if you give in to it ... you may find yourself in an altered state of consciousness, on the

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78. Ibid., 144.

79. La Monte Young and Marian Zazeela, *Selected Writings*, ed. Michael H. Tencer (Ubuclassics, 2004) 6–9. [http://www.ubu.com/historical/young/young\\_selected.pdf](http://www.ubu.com/historical/young/young_selected.pdf).

verge of some ineffable, transcendental revelation.”<sup>80</sup> Such mysticism has become decidedly less fashionable since the 60s and 70s. Composer Larry Polansky cautioned the readers of *1/1: The Journal of the Just Intonation Network* against their enthusiasm devolving into “preclusionary fanaticism, or cultism” and stated unequivocally that just intonation is simply one of many interesting and useful choices that can be made by a composer.<sup>81</sup>

James Tenney is similarly skeptical of any metaphysical connection in just intonation, although he does see Partch as “an indispensable technical point of departure.”<sup>82</sup> Tenney’s oeuvre inhabits a wide variety of stylistic and formal frameworks, yet always place the listener’s experience at the focal point, a concept inspired by John Cage, whose obsession with “sounds heard” is echoed in Tenney’s works in just intonation, which can be thought of as “*harmonic sounds heard*:”

When it comes to microtonality, well, any kind of music, I don’t see how we can avoid considering how people hear. With respect to tuning, I think one of my contributions to the field is the concept of tolerance. ... It’s unrealistic to think that we do not have to take that into account. It’s unrealistic to believe that we can be absolutely in tune. You can be more and more in tune but it’s an asymptote. You’re never there; you’re just getting closer and closer to it. But it’s more than just a recognition of human frailty; it’s a recognition that within a certain tolerance range, we are going to interpret an interval. My hypothesis is that our auditory systems (and this is not a conscious thing) interpret an interval as meaning harmonically the simplest ratio interval within the tolerance range of what’s actually being sounded. So if you’re within 5 cents of a 5:4, you’re going to hear 5:4. That’s what the meaning is going to be.<sup>83</sup>

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80. Ken Johnson, “Review: ‘Dream House’ at Dia:Chelsea, a Hallucinatory Show,” *The New York Times*, August 27, 2015, <https://www.nytimes.com/2015/08/28/arts/design/review-dream-house-at-diachelsea-a-hallucinatory-show.html>.

81. Larry Polansky, guest editorial, *1/1: The Journal of the Just Intonation Network* 1 (1985): 2, 13.

82. James Tenney, “Reflections after ‘Bridge,’” *Bridge* (Baltimore: Smith Publications, 1984). Quoted in Colin Holter, “The Spiritual Construction of Tuning in American Experimental Music,” 89. Tenney worked with Partch for six months at his workshop in Urbana, where Ben Johnston taught.

83. James Tenney, interview with Frank J. Oteri, <http://www.newmusicbox.org/articles/james-tenney-postcards-from-the-edge/2/>.



This practical limit on the philosophies of Partch, the theories of Johnston, and the spirituality of Young represents a unifying hypothesis of just intonation and intonation theory in general.

In “John Cage and the Theory of Harmony,” Tenney provides a model for “harmonic space,” a refinement of the generalized notions of consonance and dissonance described by Partch and Young. To represent this space visually, Tenney uses a lattice, first proposed by Leonhard Euler in 1774.<sup>84</sup> A “multidimensional space of pitch-perception,” harmonic space is a theory of how intervals are understood by the listener. Even without any theoretical or musical background, it is clear that some intervals have special qualities not shared by others. Partch is content to say that it is the intervals of small-number ratios have these special qualities, which excludes all equal-tempered intervals other than the octave.

Tenney defines harmonic space as having a number of dimensions, one for each of the prime factors used in a given tuning system, scale, or set of pitches. Each available note in the harmonic space is an isolated coordinate point within a “tolerance range.” All pitches within the tolerance range are understood (by the ear) as being of the same pitch or interval class, even if they are slightly mistuned. Pythagorean tuning is two-dimensional, using only primes 2 and 3, while 5-limit triadic music is three-dimensional. Extended just intonation adds more dimensions to this harmonic space: La Monte Young’s *Well-Tempered Piano* is three-dimensional;<sup>85</sup> Partch’s 11-limit, 43-tone scale is five-dimensional; and the 31-limit extreme of Ben Johnston’s system inhabits eleven dimensions.<sup>86</sup> These coordinates in harmonic space can be mapped

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84. Patrizio Barbieri, *Enharmonic Instruments and Music 1470–1900*, 11–12.

85. The *W.T.P.* does use a 7-limit tuning, but does not use intervals with any factors of 5. Therefore, it could be said to inhabit a 2, 3, 7 harmonic space.

86. James Tenney, “John Cage and the Theory of Harmony,” 136–161.

against an axis of pitch-height using a logarithmic measure such as cents or equal-tempered semitones.<sup>87</sup>

Harmonic distance is derived from the number of steps between two pitches in harmonic space. These steps do not refer to the interval class of traditional theory, but a number of discrete moves on the axis of each prime factor involved in the harmonic space. Table 6 shows a 2, 3 harmonic space representing the old Pythagorean tuning. We can see that a perfect fifth (3/2) is two steps from 1/1 and a perfect fourth (4/3) is three steps away. Extending this table further reveals intervals that begin to come very close to one another in pitch-height even though they are widely separated in harmonic space. Tenney writes that, in this case “an interval is represented by *the simplest ratio within the tolerance range*.”<sup>88</sup>

Table 6  
2, 3 Harmonic Space Representing the Pythagorean Tuning

← 3 →					
↑ 2 ↓	4/9	4/3	4/1	12/1	36/1
	2/9	2/3	2/1	6/1	18/1
	1/9	1/3	1/1	3/1	9/1
	1/18	1/6	1/2	3/2	9/2
	1/36	1/12	1/4	3/4	9/4

At the risk of drawing an overly simplistic chronology—one which deliberately excludes a number of important composers of just intonation music—themes and trends of Wolfgang von

87. Ibid., 152.

88. Ibid., 153.

Schweinitz's forerunners must be considered. Drawing from a historical and theoretical basis, Harry Partch saw "truth" and a return to theoretical and aesthetic purity in the use of just intonation as part of his widespread condemnation of many classical and popular musical traditions. Finding no other options suitable, he was forced to invent and construct his own collection of instruments and trained whomever he could find to play them. Ben Johnston praises Partch as a musical pioneer in the strongest sense of the term:

Partch was determined to get this kind of music out of the limbo of theorizing, and he did it. ... The training and coaching of performers, the maintenance and carrying about of his instruments, the persuading people that so unconventional a project is worth so much effort and expense, not to speak of the theoretical research, the design and building of instruments, and the composing itself – all these constitute a monumental life-work, mostly carried out, moreover, in conditions of poverty, public indifference, and rejection by his colleagues. Anyone interested in this field owes him not merely a debt but an apology.<sup>89</sup>

Partch proved to the world that a fully ratio-based theory of music was not only possible, but aesthetically compelling, as well. Through his eloquence and the force of his convictions, Partch inspired a small but devoted following, which bordered on the fanatical at its extreme.<sup>90</sup>

Ben Johnston sees three options for anyone wishing to follow in Partch's footsteps: "(1) rely mostly on fixed-pitch instruments or (2) realize the music directly, without the intervention of performers, or (3) reeducate the listening habits of performers."<sup>91</sup> Choosing the last option, Johnston's life work has been dedicated to the development and teaching of a comprehensive notation system for just intonation. Johnston writes, "my own music deliberately attempts to relate the practice of extended just intonation to mainstream currents in traditional Western

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89. Ben Johnston, *Maximum Clarity*, 113.

90. Bob Gilmore, "The Climate Since Harry Partch," *Contemporary Music Review* 22 (2003), 16–17, <http://dx.doi.org/10.1080/0749446032000134724>.

91. Ben Johnston, *Maximum Clarity*, 112.

music, to reassert continuity.”<sup>92</sup> Johnston shares many of Partch’s views concerning the illegitimacy of the tempered system, often stating them in equally polemical terms, but hoped that his own music could coexist fruitfully alongside other musical traditions. Perhaps because of his lacking carpentry skills or his tenure in musical academia, Johnston’s efforts took a long stride in the direction of pragmatism and inclusion.

La Monte Young, by contrast, demands close control over his artistic output and has taken steps to limit the access of performers and even audiences to his music. Scores remain unpublished and recordings scarce to the point of becoming collector’s items. Young says:

I’ve had enough mediocre performances to realise that they are not helpful ... When people hear a great performance, they can be transformed to a high level of enlightenment. And that’s what you want to do – you want to accomplish something on that level. And you don’t just need to have your name on every concert. ... Short-term relationships are dangerous.<sup>93</sup>

Like Partch, Young rejects the norms of European classical performance—not from philosophical and rhetorical reasons, but from the fact that he never truly took part in them in the first place, finding more compelling traditions in jazz, blues, and world music. By delving deeply and persistently into a limited collection of pitches—in contrast to the complex systems of Partch and Johnston—Young develops meditative explorations of his chords and motives, which take on vivid characterisations, such as the “Romantic Chord,” “189/98 Ancestral Lake Region,” “Tamiar Dream Chord” and “Ancestral Boogie.”<sup>94</sup> With the exception of his statement that “tuning is a function of time,” Young’s influence is not primarily one of theoretical development. Rather, it is his aesthetic choices that have created an impact to rival that of most contemporary

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92. Ibid., 258.

93. Quoted in Seth Colter Walls, “La Monte Young: ‘I’m only interested in putting out masterpieces,’” *The Guardian*, July 30, 2015, <https://www.theguardian.com/music/2015/jul/30/la-monte-young-dia-chelsea-exhibit>.

94. Kyle Gann, “La Monte Young’s *The Well-Tuned Piano*,” 142–148.

composers: despite of the fact that (legal) access to his recordings and scores is virtually nonexistent, Young has attained status as an icon of American music, known to audiences both inside and outside of the classical music establishment.

Through his brilliantly reasoned yet approachable theoretical writings and compositions, James Tenney serves as a bridge between the uncompromising orthodoxy of the just intonation purists and the skeptics and intonation-curious of contemporary music. Without disrespect to the views of Partch et al., Tenney sets aside any hyperbolic claims to truth, purity, or transcendence, describing acoustical phenomena and their relations to musical practices in terms that strive to be aesthetically and culturally neutral. Tenney's concept of tolerance range serves to both define a limit to the comprehensibility of interval complexity and soften the ideological divide between "pure" tunings and tempered tunings. Separate measures of melodic distance—heard linearly as a logarithm of frequency ratio—and harmonic distance—the number of steps in harmonic space between two coordinates—provide equitable theoretical basis for both ratio-based and equal-tempered systems.

Thus far, theories of just intonation have been presented largely in the words of the composers who use them. Debunking of, opposition to, and outright rejection of these theories have been constant since they were first described. Perhaps the strongest rebuke of all is the fact that just intonation has not gained the widespread acceptance that its claims of acoustic/aesthetic superiority, conceptual simplicity, and ease-of-use would suggest. In "Adding Pitches,"<sup>95</sup> a reflection on the tenth anniversary of *Perspectives of New Music's* special issue on microtonality, Julia Werntz takes these claims head-on, calling them "grave and misleading," and engendering

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95. Julia Werntz, "Adding Pitches: Some New Thoughts, Ten Years after *Perspectives of New Music's* 'Forum: Microtonality Today,'" *Perspectives of New Music* 39 (2001), 159–210.

a “narrow, one-dimensional musical conception” that is a purely “vertical view of music.”<sup>96</sup> The most unequivocal statements of Partch, Harrison, Johnston, and Young are taken apart by Werntz and made to face the consequences of their implied and insinuated rejection of all non-tempered music, as is their picking-and-choosing of elements acoustical science, ethnomusicology, and tonal theory to suit their arguments.

The preceding is included here to encourage a healthy sense of skepticism in the mind of the reader when confronted with the clearly biased claims made by composers to justify their compositional choices. I have deliberately limited my own statements of criticism or enthusiasm because I believe that composers can—and should—present and defend their methods in the strongest terms possible. Musical movements are not born from half-measures and self-skeptical statements make poor rallying cries.

The work of these pioneering just intonation composers is the primary inspiration for the recent works of Schweinitz. He credits Partch with establishing “a completely new field with his music,” Young with demonstrating the “utterly new and fascinating sound and musical presence” of untempered tuning, Johnston with his “solitary and monumental achievement of inventing a whole new harmonic and contrapuntal language,” and Tenney with the creation of a “universal conceptual model of our brain’s harmonic perception.”<sup>97</sup> Schweinitz sees his compositional efforts as closest to the goals of Johnston, but sees an opportunities to improve upon Johnston’s notation system and to deal more directly with the experiences and challenges faced by performers. With the utmost respect for his predecessors, Schweinitz attempts to continue the development of just intonation music and to actively involve performers in this development.

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96. Ibid., 163, 167–168.

97 Schweinitz, e-mail message to author, May 27, 2017.

## CHAPTER 3

### WOLFGANG VON SCHWEINITZ AND IDIOMATIC JUST INTONATION

Born in 1953 in Hamburg, Germany, Wolfgang von Schweinitz studied composition with Esther Ballou (1915–1973), Ernst Gernot Klusmann (1901–1975), György Ligeti (1923–2006), and John Chowning (b. 1934).<sup>98</sup> Even before his commitment to music in just intonation, Schweinitz explored artistic paths beyond serialism, which dominated late-twentieth-century European composition. In the 1970s, he introduced elements of and references to tonality in some works for traditional ensembles, such as string orchestra, voice and piano, and string quartet.<sup>99</sup> In his 1980 “Points of View” essay in the journal *Tempo*, Schweinitz writes: “although the idea of tonality plays a central role in my harmonic thinking, it is only as an intuitive allegory of a paradise still desirable but no longer attainable—a paradise in which a common language was psychologically and sociologically integrated.”<sup>100</sup> Schweinitz wrote prolifically in the late 1970s and early 1980s, gaining recognition with pieces such as the *Mozart Variations* (1976) and a Mass (1981–3). During this time, Schweinitz came to be considered a member, along with Wolfgang Rihm, of a so-called *Neue Einfachheit* (New Simplicity) movement, a label that Schweinitz rejects as “inappropriate and misleading.”<sup>101</sup>

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98. “Wolfgang von Schweinitz,” Californita Institute of the Arts, accessed May 28, 2017, <https://directory.calarts.edu/directory-search/wolfgang-von-schweinitz>

99. *Grove Music Online*, s.v. “Schweinitz, Wolfgang von,” by David Murray, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/O006174>.

100. Wolfgang von Schweinitz and Harriet Watts, “Points of View,” *Tempo* 132 (1980) 12–14, accessed May 19, 2017, <http://www.jstor.org/stable/946198>.

101. *Grove Music Online*, s.v. “Neue Einfachheit,” by Christopher Fox, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/51675>; Schweinitz, e-mail message to author, May 27, 2017. See appendix B for full interview.

In 1981, Schweinitz left Berlin for the German countryside, where he spent the next twelve years. Emerging from this “quiet seclusion,”<sup>102</sup> Schweinitz was beginning to formulate his untempered tuning theories when he met James Tenney. Schweinitz writes: “In 1994 I had the great opportunity to meet James Tenney during his residency in Berlin, and his beautiful music, thought and enthusiasm have been very inspiring and encouraging for my ongoing aesthetic reorientation at that time.”<sup>103</sup> The following years saw the completion of Schweinitz’s first published works in just intonation: *Passacaglia: Zwölf Versuche zu einer reineren Stimmung* (1995) and *Helmholtz-Funk* (1997). In a recent interview, Schweinitz elaborates on the influences and convictions that led him to concentrate solely on just intonation music:

My interest in the sound and performance practice of microtonal just intonation is rooted in a feeling that has haunted me since I first encountered twentieth-century atonal music as a teenager about fifty years ago; it was the notion that our ears and our music may perhaps have lost something very substantial when giving up for good the fundamental principle of tonality, which appears to be closely related to our brain’s astounding capability of harmonic perception.<sup>104</sup>

Like Ben Johnston, Schweinitz came to the conclusion that a microtonal notational system was necessary for the continuation of his work in just intonation. Schweinitz developed a such a system for his solo trombone piece, *JUZ*, in 1999, but revisited his efforts after meeting Marc Sabat (b. 1965) in 2000. “No one has exerted such an important influence upon my work as my friend, the Canadian composer Marc Sabat,” writes Schweinitz. The two developed the Helmholtz-Ellis Just Intonation Pitch Notation together, as well as the valve-tuning ideas used in their works for brass instruments. Their friendship and collaboration continues to this day with a

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102. Schweinitz, “Biography,” accessed May 23, 2017. <http://www.plainsound.org/bio.html>.

103. Schweinitz, e-mail message to author, May 27, 2017.

104. Ibid.



symbiotic interchange of ideas and experimentation. Sabat and Wolfgang publish many of their compositions, transcriptions, writings, and research on the Plainsound Music Edition website, a loose affiliation of composers who use just intonation and the Helmholtz-Ellis notation.

Wolfgang von Schweinitz has set himself apart from his predecessors by treating the inherent just intonation capabilities of musical instruments as the organizing principles of his works. Whereas Harry Partch invented new instruments for his tunings and Ben Johnston entrusted his performers to develop their own microtonal techniques, Schweinitz has gone to great lengths to create an idiomatic language for each instrument that he uses and to think about how that language is learned by performers. His dedication to understanding technique is typified by the fact that he purchased his own cello to compose a piece for the instrument.<sup>105</sup>

In Schweinitz's writing for strings, natural harmonics are used as a harmonic series of partials. These serve as waypoints for stopped notes and complex chords as the performer is guided from note to note. Figure 3 shows the opening measures of *Plainsound-Litany*, a solo cello work from 2004.<sup>106</sup> In the first nine measures, the performer simply verifies the tuning of the strings. This is both a practical necessity to achieve precise intonation and an invitation to what cellist Ashley Walters calls, "a concentrated, shared ritual of listening for the performer and audience."<sup>107</sup>

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105. Ashley Walters, "Six New and Recent Works for Solo Cello in Recording and Discussion" (DMA diss., UC San Diego, 2013), 4–5, <https://search.proquest.com/docview/1420352211?accountid=14512>.

106. Schweinitz, *Plainsound-Litany*, Plainsound Music Edition, 2004.

107. Ashley Walters, "Six New and Recent Works for Solo Cello in Recording and Discussion," 3.

**Plainsound-Litany**  
for Louise and Liane

*Lento e libero* (♩ circa 24 - 30)  
Every note may be prolonged or shortened ad libitum.

Wolfgang von Schweinitz, op. 46 (2004)

Violoncello

3/2      ad lib.      3/2      ad lib.      4/3      3/2      ad lib.

retune G string if necessary      retune C string if necessary      retune A string if necessary

sempre tutto d'arco

10      3/2      5/3      5/3      3/2      2/1      5/2      sempre non vibrato      pp      sonore

20      5/4      5/2      5/4      5/4      6/5      5/4      7/3      9/2      cresc.

**Figure 3:** Wolfgang von Schweinitz, *Plainsound-Litany*, mm. 1–29.

In the score’s prefatory notes, Schweinitz explains that the opening harmonics are tuned in “pure, non-beating fifths that produce a clear and steady octave bass.”<sup>108</sup> This “bass” is a resultant tone, heard at the difference between the frequencies of the two sounding pitches. In measures one and two, the 3/2 G–D produces a difference tone at 1, the common fundamental of the dyad an octave below (1:2) the lower played note. The 4/3 A–D at measure 7 again produces a difference tone of 1, which now refers to a note a perfect twelfth below (1:3) the lower note. At measure 10, the piece begins in earnest, charting a course that leads the performer and audience from harmonies that are simple and closely related to the open strings into ones that are increasingly distant and complex. Throughout the piece, natural harmonics function as a “trail of breadcrumbs—showing the way for both the hand and the ear,” in the words of Walters. Stopped

108. Schweinitz, *Plainsound-Litany*.

notes are tuned to these harmonics, which then function as common tone pivots to the following chord.

### **The Helmholtz-Ellis Pitch Notation and Its Predecessors**

The Helmholtz-Ellis Just Intonation Pitch Notation (HE notation), developed by Marc Sabat and Wolfgang von Schweinitz, is indebted to its namesakes in two major ways. First, the impetus to explicitly notate differences of intonation comes from Hermann von Helmholtz, who writes, “in accurate theoretical researches both kinds [Pythagorean and syntonic] of tones must be kept distinct, although in modern music they are practically confused,” while the definition of cents, an easily understood measure of melodic distance, comes from Alexander J. Ellis.<sup>109</sup> The core of the HE notation is its ability to communicate the harmonic meaning of intervals and chords, with the possibility of modifying intervals with the notation of cents deviations. However, both Helmholtz and Ellis limit their efforts to text in theoretical writings and neither sought to develop a complete musical notation.

Comma differences in 5-limit just intonation or meantone are specified in many sixteenth- and seventeenth-century instrumental methods, but again only in text. In such sources, standard notation suffices to specify these differences: flats are played a comma higher than their equivalent sharps.<sup>110</sup> Similarly, Leopold Mozart specifies violin fingerings that imply that chromatic semitones are smaller than diatonic semitones.<sup>111</sup> Such a performance-practice-based just intonation is today intermingled with the tradition of “expressive intonation”—in which

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109. Helmholtz, *On the Sensations of Tone*, 276, 446.

110. Bruce Haynes, “Beyond Temperament,” 356–365.

111. Leopold Mozart, *Versuch einer gründlichen Violinschule* (Augsburg: Potter, 1756), 66–67. Quoted in Patrizio Barbieri, *Enharmonic Instruments and Music 1470–1900* (Latina, Italy: Il Levante, 2008), 134–135.

tuning is influenced by its voice-leading and sharps are generally higher than flats—and with equal temperament, the only practical option for fixed-pitch instruments.

Expanding upon a practice used by theorist Moritz Hauptmann,<sup>112</sup> the first edition of *On the Sensations of Tone* uses uppercase letters to show notes related by fifths and lowercase letters to show relationships of pure major and minor thirds—that is, lowered or raised by a syntonic comma—with a stroke above or below a letter to show alteration by two syntonic commas. This system was abandoned due to its conflict with Helmholtz’s own pitch nomenclature, which used upper- and lowercase letters and prime symbols to delineate octaves.<sup>113</sup> In later editions, Helmholtz and Ellis adopt a system of superscript and subscript numbers to show the comma level of notes. Thus, a C major triad consists of C, E<sub>1</sub>, and G; or C<sup>1</sup>, E, and G<sup>1</sup>. To show the intervals between these notes, pluses and minuses show major and minor thirds, respectively, and  $\pm$  shows a perfect fifth ( $5/4 \times 6/5 = 3/2$ ). Thus, the C major triad is written C + E<sub>1</sub> – G and a C minor triad is written C – E<sub>b</sub><sup>1</sup> + G. This specificity was evidently important enough to Helmholtz to justify a three-month delay in the printing of the first English edition.<sup>114</sup> Septimal intervals are given in text in two different ways: with plus and minus symbols (e.g., B<sub>b</sub>–), and with superscript 7 numerals (e.g., <sup>7</sup>B<sub>b</sub>).<sup>115</sup> These correspond to a septimal comma alteration of 64:63.

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112. Moritz Hauptmann, *The Nature of harmony and Metre*, trans. W.E. Heathcote (New York: Da Capo Press, 1991), xlv. Hauptmann uses uppercase letters for what he calls “Root-notes” and “Fifth-notes,” and lowercase letters for “Third-notes.” He calls staff notation “useless for [his] purpose.”

113. *Grove Music Online*, s.v. “Pitch Nomenclature,” by L.S. Lloyd and Richard Rastall, accessed May 9, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/21857>.

114. Helmholtz, *On the Sensations of Tone*, 276–277.

115. *Ibid.*, 187, 195.

This system, however, only hints at the possibility of a staff notation. Notated examples in *On the Sensations of Tone* do not use any system of differentiation of the syntonic comma, but do use a downward-sloping line to show septimal and undecimal intervals.<sup>116</sup> These symbols are used ambiguously, however, since the intervals they are used to represent are not tuned in the same manner. Furthermore, these notated examples demonstrate only acoustical concepts, such as 7th and 11th partials and resultant tones, not musical usage.

Mathematician and musicologist Alexander J. Ellis proposes elements of a staff notation “for theoretical and experimental purposes.”<sup>117</sup> Ellis’s article, a version of which appears in his additions to *On the Sensations of Tone*, outlines a system of related pitches within a tuning system, similar to the tone lattices first introduced by Leonhard Euler and used by Harry Partch and James Tenney. Ellis calls these “duodenes,” when limited to twelve notes to fit the manuals of keyboard instruments, which were often undergoing “enharmonic” experimentation in the nineteenth century. Ellis first introduces yet another set of symbols to denote rational tuning in text only. In it, naturals follow the just intonation major scale in the key of C, that is  $C=1/1$ ,  $D=9/8$ ,  $E=5/4$ ,  $F=4/3$ ,  $G=3/2$ ,  $A=5/3$ , and  $B=15/8$ . Added to this are sharp and flat symbols to indicate a raising or lowering of the pitch by the 92.2-cent chromatic semitone of 135:128; dagger (†) and double dagger (‡) to indicate raising or lowering by the 21.5-cent syntonic comma of 81:80; and pilcrow (¶) and upside-down pilcrow to indicate raising or lowering by the 2.0-cent schisma of 32805:32768, the difference between the Pythagorean comma (the amount by which an augmented seventh of a cycle of pure fifths exceeds an octave) and the syntonic

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116. Ibid., 215. Helmholtz writes that this line “represents a tone slightly deeper than that of the note in the scale which it precedes.”

117. Alexander J. Ellis, “On Musical Duodenes, or the Theory of Constructing Instruments with Fixed Tones in Just or Practically Just Intonation,” *Proceedings of the Royal Society of London* 32 (1874–5), 3–31. <http://www.jstor.org/stable/113266>.

comma.<sup>118</sup> These alterations, if not the symbols used to notate them, bear some resemblance to Ben Johnston's notation, which similarly begins with naturals of the just C major scale and includes alterations by syntonic comma (Johnston's plus and minus symbols).

The only staff notation shown by Ellis in "On Musical Duodenes" is an adaptation of his in-text syntonic comma symbols † and ‡, which are replaced by upward- and downward-facing flags. A more complete staff notation is suggested by theorist R.H.M. Bosanquet in a paper read to the Royal Musical Association in 1875,<sup>119</sup> in which upward and downward sloping lines—such as those used by Helmholtz to imprecisely show septimal and undecimal relationships—represent syntonic comma alterations, with naturals, sharps, and flats derived from Pythagorean tuning. An "H" placed by a note shows that it is in a harmonic seventh (septimal) relation to the tonic, although such intervals are approximated in Bosanquet's system by being lowered by two syntonic commas. In discussion following Bosanquet's presentation, the author is asked if music could be performed in just intonation by singers and instrumentalists, to which he replies that "much of it could not. No one could execute those small intervals with accuracy by the voice; they could only be examined by an instrument."<sup>120</sup> Thus Bosanquet, despite putting forth a working model of 7-limit notation, concludes that its use in performance is unrealistic outside of the confines of the fifty-three-note-per-octave harmonium specially built for him to demonstrate this system.

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118. Ellis, "On Musical Duodenes," 3.

119. R. H. M. Bosanquet, "Temperament: Or, the Division of the Octave (part II)," *Proceedings of the Royal Musical Association* 1 (1874–5), 112–58.

120. *Ibid.*, 155.

Around the same time, another novel system of notation was proposed to the Royal Musical Association by theorist Sedley Taylor.<sup>121</sup> Taylor's system explicitly shows the tonal center of a given passage of music, thereby allowing singers and non-fixed-pitch instrumentalists to use their knowledge of intonation to make adjustments to produce pure intervals. Citing Helmholtz, Taylor insists that tuning "natural" intervals against the tonic is not only the preference of great musicians, but is "clearly and directly exhibited by the established [i.e., traditional] notation," but only in the key of C.<sup>122</sup> This last point is an interesting one: Taylor's view is that it is a matter of common performance practice to perform in just intonation on the C major scale and that a clearer representation of tonal center would make this same practice more easily performed in other keys.

Taylor's simplified system uses a wavy line to mark the position of the tonic note on the staff, with looped lines if the tonic is a flat or sharp note. Accidentals are only used in relation to the major key, with naturals rendered superfluous, sharps and flats only indicating modifications to the diatonic notes of the major key. To distinguish minor key relations, L-shaped brackets function in place of chromatic accidentals. This notation functions like a combination of the "tonic sol-fa" (i.e., "movable *do*") solfege system and ordinary staff notation.

Taylor's notation also circumvents the excessive specificity and compounded accidentals that often arise when marking syntonic comma differences. However, it fails to address some ambiguities in the sizes of intervals. The ratios of the just chromatic scale and the sizes of chromatic alterations must be decided upon ahead of time, explained in writing, and studied by

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121. Sedley Taylor, "On a Suggested Simplification of the Established Pitch-Notation, *Proceedings of the Royal Musical Association* 1 (1874–5), 18–40.

122. *Ibid.*, 21.

the performer. Otherwise, adding a sharp to the tonic note, for example, could yield either the 25/24 relationship if tuned as 5/4 above the 5/3 submediant, or 135/128 if tuned 5/4 above the Pythagorean 27/16 submediant. Such finer points are ignored by Taylor along with other consequences of just intonation, as is evidenced, for example, by his assertion that there is no change of pitch in an enharmonic modulation.<sup>123</sup> This lack of detail, combined with the deliberately tonal-diatonic nature of the notation, means that Taylor's system fails to satisfy the requirements of a comprehensive system of tuning-specific notation.

Given the familiarity that most performers have with twelve-tone equal temperament, it is worth examining microtonal notation systems based upon it. The simplest such system defines each pitch in cents-deviation from the nearest equal-tempered note. Many performers will find this to be the most user-friendly possibility of all, as an electronic tuner can be used to verify pitch-accuracy and no arcane theoretical knowledge is needed. Ever the pragmatist, James Tenney uses this method in *Saxony*, a piece for sustaining instrument or instruments and tape-delay. He describes this work as a “‘stochastic canon’ on the harmonic partials of a given fundamental,”<sup>124</sup> a free improvisation on a harmonic series, beginning with a long drone on the fundamental and gradually adding higher partials up to 32.

A number of factors support the use of this notation as being the best choice for Tenney's *Saxony*: (1) because the fundamental of the harmonic series is never changed in this work, the number of discrete pitches used is relatively small; (2) when learning to tune the pitches of the harmonic series, the performer can easily use an electronic tuner to check that he or she is tuning the partials correctly; (3) the fundamental of the harmonic series is a pitch from the equal-

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123. Ibid., 27–28. This is, of course, true in equal temperament, but not in just intonation.

124. James Tenney, *Saxony* (Baltimore: Smith Publications, 1978).



tempered system, making it possible to practice with a drone without the need to perform any calculations of frequency; and (4) there is no loss of clarity in the representation of harmony, since the work's harmonic content is explicitly stated.

Expansions of the equal-tempered system beyond twelve notes per octave to include quarter-, sixth-, third-, and eighth-tones have been explored at length throughout the twentieth century. Using such intervals to approximate the sonorities of just intonation can be successful in certain circumstances, although many nuances of a fully ratio-based system are lost. The lack of notational consensus found in just intonation music is similarly problematic in equal-tempered microtonality, with identical or similar symbols used in conflicting ways by different composers. That said, the eighth-tone can be used to approximate the 5th partial, the sixth-tone the 7th partial, and the quarter-tone the 11th.<sup>125</sup>

Composer Ezra Sims (b. 1928) developed a highly detailed system of seventy-two-tone equal temperament to approximate the ratios of extended just intonation. Entirely new symbols for quarter-tones, sixth-tones, and twelfth-tones supplement the traditional naturals, sharps, and flats.<sup>126</sup> Sims finds this system satisfactory for his music and states that performers of his music “seem to have had no particular difficulty with all this.”<sup>127</sup> The performance practice that Sims envisions is “the older practice of tuning the current key in something like Just [intonation], but adjusting the relations between keys to something like equal temperament in order to avoid going off the instruments. Modulation on these terms is something done every day by good singers and

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125. David Lefkowitz, “Analysis of Post-Tonal Music: A Parametric Approach” (unpublished manuscript, September, 2015), PDF file, 71–85.

126. Ezra Sims, “Yet Another 72-Noter,” *Computer Music Journal* 12 (1988), 28–45, <http://www.jstor.org/stable/3680151>. Sims rejects the pre-existing twelfth-tone accidentals of Czech composer Alois Hába as too numerous for practical use. Hába's accidentals are shown in the article by Sims.

127. *Ibid.*, 34.

orchestral instrumentalists.”<sup>128</sup> This notation system, like the cents-deviations in Tenney’s *Saxony*, requires no advanced knowledge of intonation theory. It also allows relatively easy use of an electronic tuner, though the microtonal accidentals must first be converted into cent-deviations and added together. What this system lacks, however, is a clear representation of harmonic meaning. It would take a great deal of intonation theory, for example, to deduce that a Db raised by a sixth-tone is in a 13/12 relation to C.

“The Extended Helmholtz-Ellis JI Pitch Notation” (HE notation), was developed by composers Marc Sabat and Wolfgang von Schweinitz “for the composition and performance of new music using the sonorities of Just Intonation.”<sup>129</sup> It seeks to be a complete resource to demonstrate harmonic relationships and precise tuning in a traditional staff notation. Only Ben Johnston thoroughly confronted this challenge before Sabat and Schweinitz, though his system is marred by an easily confused method of notating syntonic commas. In the HE notation, families of intervals are kept distinct through the use of unique accidentals, an idea inspired by James Tenney’s concept of dimensions in harmonic space.<sup>130</sup>

Ordinary natural, flat, sharp, double-flat, and double-sharp accidentals refer to the 3-limit system of untempered fifths, tuned outward from a reference pitch of A. Marc Sabat explains that “this Pythagorean diatonic division is compatible with many currently used musical instruments,”<sup>131</sup> which likely refers to the tuning of string instruments in pure fifths and fourths.

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128. Ibid., 31.

129. Marc Sabat and Wolfgang von Schweinitz, “The Extended Helmholtz-Ellis JI Pitch Notation” (Plainsound Music Edition, 2004), <http://www.marcabat.com/pdfs/notation.pdf>.

130. Ibid.

131. Marc Sabat, “Three Tables for Bob,” *Tempo* 70 (2016), 49. doi:10.1017/S0040298216000334.

In Johnston's 5-limit diatonic, on the other hand, the standard tuning of violin strings must be notated G, D, A+, E+.

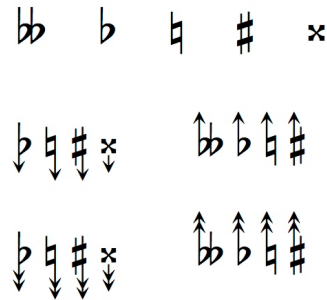
In Pythagorean tuning, flats are lower than equivalent sharps<sup>132</sup> and the spelling of intervals is important: an augmented unison, such as D–D#, is in the ratio 2048:2187 ( $3^7:2^{11}$  or 113.7 cents), showing the seven-degree separation on the gamut of pure fifths. It should be noted that only the reference pitch of A will match equal temperament. Therefore, not even a note with ordinary accidentals can be checked by tuner without first figuring out how many fifths separate it from A. For example, Bb is five fifths below, while C# is four fifths above. Because an ascending fifth is 2.0 cents wider than in equal temperament, these Pythagorean notes are respectively 10 cents lower and 8 cents higher than in equal temperament.

In the HE notation, the syntonic comma is notated by attaching a downward or upward arrow to a standard accidental. Multiple comma alterations are shown with multiple arrows. This notation is likely taken from similar use as an eighth-tone (25 cents) symbol by composers such as Tristan Murail (b. 1947).<sup>133</sup> This comma of 81:80 (21.5 cents) is a modification of the Pythagorean notes, not a deviation from equal temperament or the just diatonic of Johnston. The 3-limit and 5-limit accidentals are shown in Figure 4.

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132. The opposite is true in Johnston's 5-limit tuning.

133. Lefkowitz, "Analysis of Post-Tonal Music," 75.



**Figure 4:** 5-Limit accidentals in HE notation.

The practical use of these Pythagorean and syntonic accidentals is obvious: the lowering of majors thirds and raising of minor thirds can be explicitly notated with the unobtrusive and intuitive arrows, while the raised leading tones and exaggerated tendency tones of “expressive intonation”<sup>134</sup> are represented in the Pythagorean symbols. Figure 5 shows the first phrase of J.S. Bach’s chorale harmonization “Aus meines Herzens Grunde” in standard notation with the addition of 5-limit HE accidentals to show its tuning in just intonation. This notation is not meant to be prescriptive or to imply any relation to historical performance practice; it is solely intended to show how the HE accidentals can graphically demonstrate just intonation.



**Figure 5:** Bach chorale example with HE accidentals added by the author.  
Source: Johann Sebastian Bach, *371 Four-Part Chorales, Vol. 1* (n.p.: Kalmus, n.d.), 5.

134. Patrizio Barbieri and Sandra Mangsen, “Violin Intonation: A Historical Survey,” *Early Music* 19 (1991): 82–85.

Septimal intervals are shown with a small flag resembling a reversed or reversed-and-inverted numeral seven. This symbol is similar to that used by Giuseppe Tartini to show the natural seventh, though his more closely resembles a *w* with an elongated leftmost vertical.<sup>135</sup> The flags used in the HE notation are identical to those used by Ellis to represent *syntonic* comma alterations. Such symbols, Ellis explains, are expedient for engraving purposes, being the stems and uncurved tails of eighth notes and sixteenth notes.<sup>136</sup> The HE notation's septimal comma is a ratio of 64:63 (27.3 cents) and is the amount by which a Pythagorean minor seventh of 16/9 exceeds the harmonic seventh of 7/4. It differs from that defined by Johnston, who compares the harmonic seventh to 9/5, a solution that uses fewer accidentals in his system.<sup>137</sup> Johnston's comma is very close to an equal-tempered quarter-tone at 48.8 cents, significantly larger than that in the HE notation.

Figure 6 shows “Auld Lang Syne” in an arrangement for barbershop quartet with the addition of HE accidentals to demonstrate the tuning of pure chords in 7-limit intonation. Just intonation is more than a theoretical matter to barbershop singers; the ideal of “ringing chords” free of beating is favored by such ensembles<sup>138</sup> and they perform in this tuning with a high

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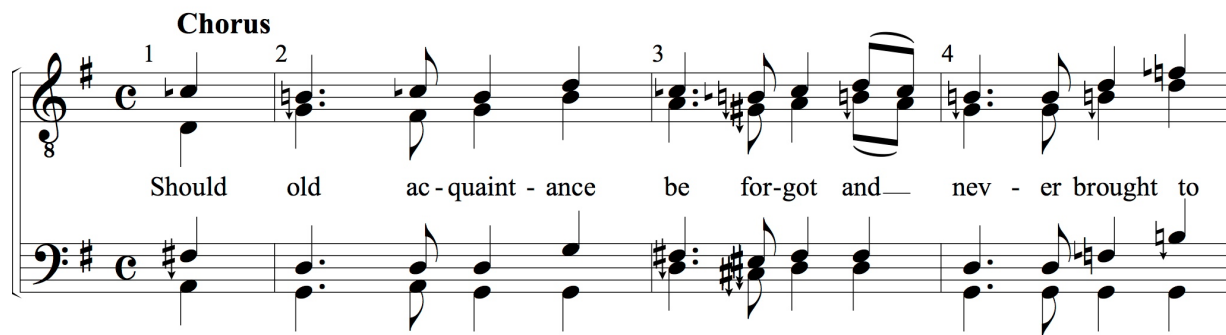
135. Giuseppe Tartini, “*Tartini's 'Trattato di musica secondo la vera scienza dell' armonia: ' an Annotated Translation with Commentary,*” 322. Tartini was a strong advocate for the inclusion of the natural seventh in musical practice, calling it “very easy to intone on the violin [and] desired by harmonic nature.”

136. Alexander J. Ellis, “On Musical Duodenes,” 20.

137. Johnston, *Maximum Clarity*, 82–88. In the HE notation, showing septimal intervals with Johnston's comma of 36:35 requires *more* accidentals as the minor seventh would first need to be widened by a syntonic comma, before being narrowed by the septimal comma.

138. Gage Averill, “Bell Tones and Ringing Chords: Sense and Sensation in Barbershop Harmony,” *The World of Music* 41 (1999), 37–51.

degree of accuracy.<sup>139</sup> This example also shows a note lowered by two syntonic commas in the baritone part in measure 3, which comes as a result of the entire first chord of the measure being shifted down by a half-step of 16:15. Notable intervals in this example are the 21:20 septimal half steps (84.5 cents) and 7:8 septimal whole steps (231.2 cents) that predominate in the tenor (top) voice, as well as the 6:7:10 arpeggiation in the baritone voice in measure 4.



**Figure 6:** Septimal harmony in barbershop arrangement with HE accidentals added by author.  
Source: The Barbershop Harmony Society, [www.harmonymarketplace.org](http://www.harmonymarketplace.org).

For most primes above 5, Ben Johnston chooses numerals over symbols for the sake of practicality and readability, while Sabat and Schweinitz continue to add new symbols for accidentals representing primes up to 61. Undecimal (11-limit) harmonies are shown in the HE notation with symbols traditionally associated with equal-tempered quarter-tones: a sharp with only one vertical and one horizontal stroke and a backwards flat. The ratio represented is 32:33 and is the amount by which the 11th partial exceeds a perfect fourth, 53.3 cents. Accidentals for primes 11 to 61 and the alterations they represent are shown in Figure 7.

139. B. Hagerman and J. Sundberg, “Fundamental Frequency Adjustment in Barbershop Singing,” *Department for Speech, Music and Hearing Quarterly Status and Progress Report* 21 (1980), 28–42. Through scientific measurements and experimentation with two barbershop quartets, Hagerman and Sundberg do find deviation from the expected ratios of just intonation due, perhaps, to a melodic preferences for diatonic half steps narrower than 15:16. Taking this finding into account, m. 3 in figure 6, might be more accurately notated with a Pythagorean diatonic half step from A to G# in the lead voice (second from top). This change would shift the entire second chord up by a syntonic comma, removing all of the downward arrows from the chord’s notation.

$\sharp$	$\flat$	<i>Raises / lowers the pitch by an 11-limit undecimal quarter-tone</i> $(33/32) = \text{circa } 53.3 \text{ cents}$
$\sharp$	$\sharp$	<i>Lowers / raises the pitch by a 13-limit tridecimal third-tone</i> $(27/26) = \text{circa } 65.3 \text{ cents}$
$\sharp$	$\sharp$	<i>Lowers / raises the pitch of the subsequent accidental by a 17-limit schisma</i> $(16/17) * (16/15) = (256/255) = \text{circa } 6.8 \text{ cents}$
$\sharp$	$\sharp$	<i>Raises / lowers the pitch of the subsequent accidental by a 19-limit schisma</i> $(19/16) * (27/32) = (513/512) = \text{circa } 3.4 \text{ cents}$
$\sharp$	$\flat$	<i>Raises / lowers the pitch of the subsequent accidental by the 23-limit comma</i> $(23/16) * (8/9) * (8/9) * (8/9) = (736/729) = \text{circa } 16.5 \text{ cents}$
$\sharp$	$\sharp$	<i>Lowers / raises the pitch of the subsequent accidental by a 29-limit comma</i> $(29/16) * (5/9) = (145/144) = \text{circa } 12.0 \text{ cent}$
$\flat$	$\sharp$	<i>Lowers / raises the pitch of the subsequent 11-limit accidental by a 31-limit schisma</i> $(32/31) * (32/33) = (1024/1023) = \text{circa } 1.7 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Lowers / raises the pitch of the subsequent 11-limit accidental by a 37-limit schisma</i> $(36/37) * (33/32) = (297/296) = \text{circa } 5.8 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Lowers / raises the pitch of the 5-limit accidental by a 41-limit schisma</i> $(32/41) * (81/64) * (81/80) = (6561/6560) = \text{circa } 0.3 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Raises / lowers the pitch of the subsequent accidental by a 43-limit comma</i> $(43/32) * (3/4) = (129/128) = \text{circa } 13.5 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Lowers / raises the pitch of the 7-limit accidental by the 47-limit schisma</i> $(32/47) * (48/49) * (3/2) = (2304/2303) = \text{circa } 0.8 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Lowers / raises the pitch of the subsequent 5-limit accidental by a 53-limit comma</i> $(32/53) * (5/3) = (160/159) = \text{circa } 10.9 \text{ cent}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Lowers / raises the pitch of the 13-limit accidental by a 59-limit schisma</i> $(32/59) * (24/13) = (768/767) = \text{circa } 2.3 \text{ cents}$
$\{ \} \sharp$	$\{ \} \flat$	<i>Raises / lowers the pitch of the 7-limit accidental by a 61-limit schisma</i> $(61/32) * (21/40) = (1281/1280) = \text{circa } 1.4 \text{ cents}$

**Figure 7:** Accidentals for primes 11–61.

Figure 8 shows the application of the Helmholtz-Ellis accidentals to the opening horn solo of Benjamin Britten's *Serenade for tenor, horn and strings* in HE notation. Britten requests that this passage "be played on natural harmonics," which necessitates the use of partials usually avoided due to their discrepancy with equal temperament. Partial 7, 11, and 13 are all used in this striking passage.<sup>140</sup>



**Figure 8:** Britten *Serenade*, Prologue with accidentals added by author.  
Source: Benjamin Britten, *Serenade for Tenor Solo, Horn and Strings* (London: Boosey & Hawkes, 1944).

In my personal experience of playing the music of Schweinitz and Sabat, the accidentals for primes up to 13 have proven to be recognizable and memorable. Primes 17 and higher, however, are more confusing, with symbols that are increasingly similar to those of lower primes. For example, it is difficult to remember whether it is the 17th or the 19th partial that uses one or two sloping lines, or if is the 23- or 29-limit arrow that is raised above the other accidental. In a recent article, Marc Sabat acknowledges these difficulties and describes his adoption of an alternative solution:

140. Robert Evans, "The Post-Classical Horn" in *The Cambridge Companion to Brass Instruments*, ed. Trevor Herbert and John Wallace (Cambridge: Cambridge University Press, 1997), 214. Britten later made clear that he preferred the use of the 14th partial to the 13th.



In my recent music, I generally have not exceeded the 23rd partial, and for the most part am exploring intervals and aggregates up to the 13th harmonic. Higher primes above 23 are perhaps more usefully notated by means of cents and a text indication rather than by using special accidentals, which I now prefer to reserve for the more easily perceived lower primes.<sup>141</sup>

In his 2016 piece *Asking Ocean*, Sabat uses a new approach to address the forms of the HE accidentals that Partch calls “utonal,” most often used to tune a just interval underneath another, more conventionally notated pitch. These include raising pitches by the syntonic or septimal comma and lowering pitches by the undecimal quarter-tone. These forms are opposite how they appear in the harmonic series. In an acknowledgment that such accidentals can be counterintuitive, Sabat supplies an additional indication to clarify how these notes should be tuned. For just intervals tuned above another pitch, Sabat uses a numeral followed by a degree symbol as in “7°.” For intervals tuned below, the numeral is preceded by a *u*, as in “u11.” Figure 9 shows a 13/8 interval with the lower voice marked “u13.” To further aid the performer, cents-deviations from equal temperament are shown in roman type and melodic intervals are shown in italics as both a frequency ratio and an interval size in cents.<sup>142</sup>

**Figure 9:** Marc Sabat *Asking Ocean*: Fl. and bass fl., m.20.

141. Marc Sabat, “Three Tables for Bob,” 50, 53.

142. Marc Sabat, *Asking Ocean: for Solo String Quartet and 18 Musicians* (n.p.: Plainsound Music Editions, 2016).

Becoming familiar with the Helmholtz-Ellis notation takes time and is perhaps most easily accomplished by constructing exercises similar to the notated examples in this chapter. The accidentals can be drawn by hand into printed music or simply visualized. Scale and tuning exercises are a good starting point, progressing into duets, trios, and quartets. Performing these exercises while observing the new notation is the next step.

Passages from the Baroque and Classical repertoire make for a good starting point with 5-limit accidentals. Begin by thinking about the primary chords, scale degrees, and tonal centers in the passage. Observing the general rule of major-key just intonation, lower the third, sixth, and seventh scale degrees by a comma with the appropriate syntonic accidentals. Beware of false fifths, which commonly occur in chords built on the supertonic. To avoid these, raise the sixth scale degree to its Pythagorean tuning of  $27/16$  above the tonic or lower the second scale degree to  $10/9$ . Let context be your guide: common tones should be maintained unless doing so causes the tonic to sink by a comma.

Music with an abundance of diminished triads and seventh chords provides an opportunity to explore the use of septimal accidentals. The 7th partial fits best in a dominant seventh chord—a relationship of  $7/6/5/4$  that forms a surprising consonance—but does not fit when the chord contains a minor third. The minor triad has a ratio of  $15/12/10$ , which becomes  $35/30/24/20$  with a harmonic seventh added. Replacing the harmonic seventh with a pitch a perfect fifth above the third creates a simpler and sweeter  $18/15/12/10$ . Diminished triads can be conceived as a dominant seventh chord with a missing root. In 7-limit tuning, this triad is  $7/6/5$ , which highlights the relatively consonant tritone  $7/5$ . Adding a ninth to the  $7/6/5/4$  dominant seventh chord creates a sonority with four different sizes of thirds: the  $5/4$  just major third (386.3 cents), the  $6/5$  just minor third (315.6 cents), the  $7/6$  septimal minor third (266.9 cents), and the

9/7 septimal major third (435.1 cents). Removing the root of this ninth chord gives a half-diminished seventh chord of 9/7/6/5.

Harmony using primes 11 and higher presents challenges of application because there is less established tradition with which to experiment. Certainly, the chord extensions of late Romantic music and jazz could be reinterpreted with the 11th partial, the latter especially so given the tradition of experimentation with altered and extended chords. The dominant eleventh chord becomes 11/9/7/6/5/4, the top interval of 11/9 sounding halfway between a major and a minor third. This interval could be called a “neutral third” and has a size of 347.4 cents. It is important to remember that such experiments are not presented as improvements upon equal-temperament or other tuning practices but as an open-ended invitation to explore new sonorities and musical materials. In the words of Ben Johnston: “if the use of pitch as a basis of musical organization is not to fall into eclipse in serious contemporary music, a considerably broadened horizon of possibilities must be explored.”<sup>143</sup>

### **Just Intonation and “Aural Skills”**

While the study of microtonal notation and tuning theory is necessary to develop a vocabulary of composition and interpretation, performance of just intonation is impossible without ear training. While the basic 5-limit intervals will be quite familiar to most musicians, those involving primes 7 and higher will sound increasingly foreign. The goal of these exercises is not only to improve intonation accuracy, but to discover the unique character of the interval families associated with each prime partial.

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143. Ben Johnston, *Maximum Clarity*, 39.

The majority of these exercises simply involve singing against a drone pitch, with an electronic tuner used to check if the interval is approximately in the correct tuning. Although the human voice is not nearly as precise as a fixed-pitch instrument or electronically generated sound, there is, perhaps, greater benefit in the constant searching, refinement, and adjustment that singing requires. However, interruptions of breathing and inconsistencies of vocal technique will yield only approximate results. In these exercises, it is more important to obtain a *sense of ideal tuning* than to hear the interval in its purest form.

The piano serves as an invaluable tool in traditional ear training, for which there is no widely used equivalent for just intonation. Attempts were made to solve this deficiency in the nineteenth century by theorists such as R.H.M. Bosanquet<sup>144</sup> and more recently by tubist and composer Robin Hayward, whose “Tuning Vine” software turns the harmonic lattice of Euler and Tenney into an interactive electronic instrument.<sup>145</sup> The efforts of Bosanquet and Hayward are beyond the scope of the present work but merit further study and use. The simple exercises presented below are intended to be convenient to use and are imagined as an extension and, in some ways, a refinement of the traditional ear-training. Intonation exercises should generally be slow-moving to focus attention on tuning precision and will therefore appear to be rather basic in comparison to traditional sight-singing exercises. Singing in a slow glissando in the region of an interval’s correct tuning will help to hear the speeding-up and slowing-down of beating. Beating becomes progressively slower as the pure interval is approached.

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144. R.H.M. Bosanquet, “On Some Points in the Harmony of Perfect Consequences,” *Proceedings of the Musical Association* 3 (1876), 145–153.

145. “Hayward Tuning Vine,” Robin Hayward, <http://tuningvine.com/>.

For generating drone pitches, it is recommended to use a smartphone or tablet tuner application. Many tuner applications allow the user to change the timbre and octave of the sound output. Using earphones will also allow the sung pitch to be distinctly heard and analyzed by the tuner application, although it is encouraged to “tune with the eyes” as little as possible. At the time of this writing, TonalEnergy Chromatic Tuner and Metronome, available for iOS and Android devices, fulfills all of the above requirements and can produce the sound of some 5-limit just intervals, a helpful feature.<sup>146</sup> In lieu of this application, any software that is capable of generating tones at custom frequencies can be used to demonstrate intervals and produce drones.

Figure 10 presents strong 5-limit consonances within a one-octave range. The top staff shows the sung pitches, which should be transposed to a comfortable singing range, while the bottom shows the drone. Harmonic intervals are reduced to a one-octave range and indicated between staves as a fraction, e.g., “6/5.” Melodic intervals are notated above the top staff both as a ratio and in cents, e.g., “8:9, +203.9¢.” A modified natural symbol with a T is used to show that C is equal-tempered and is the reference pitch for this exercise. Each note is sung for as long as it takes to minimize beating and establish confidence with the tuning of the interval. While these notes are presented in an ascending scale, they can be sung in any order.

Melodic Interval	Ratio	Cents	Harmonic Interval
24:25	+70.7¢	6/5	
15:16	+111.7¢	5/4	
8:9	+203.9¢	4/3	
9:10	+182.4¢	3/2	
24:25	+70.7¢	8/5	
		5/3	

**Figure 10:** 5-limit tuning exercise.

146. Sonosaurus LLC, *Tonal Energy*, computer software, v.1.5.3, <http://tonalenergy.com/>.

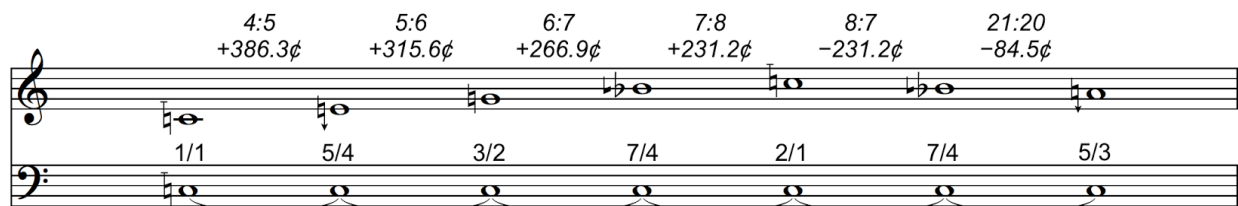
It is suggested to begin with those pitches related to the drone by the closest harmonic distance, which can be roughly determined by the size of the numbers in the ratio. Begin with  $3/2$ , a perfect fifth, then try  $4/3$ , a perfect fourth. Once the sonority of each of these dyads is established, concentrate on the 8:9 Pythagorean whole step between the two by alternating between the fourth and fifth. Although these first intervals are sure to sound familiar, it is important to establish precision of listening, both in terms of intonation and the acoustical result of beat-free resonance.

Once these Pythagorean intervals are mastered, add the syntonic major third ( $5/4$ ) and major sixth ( $5/3$ ). It may be useful to check these with the tuner; while  $3/2$  and  $4/3$  require only a 2.0 cent adjustment from equal temperament,  $5/4$  is 13.7 cents lower and  $5/3$  is 15.6 cents lower. Again, establish the intonation of the dyads separately first, then the intervals between them. Between  $5/4$  and  $4/3$  is a melodic interval of 15:16, while that between  $3/2$  and  $5/3$  is 9:10. First, repeat the alternation between  $4/3$  and  $3/2$  several times, then alternate between  $3/2$  and  $5/3$ . The difference in size between these two whole steps is the syntonic comma of 21.5 cents.

Finally, add the  $6/5$  minor third and the  $8/5$  minor sixth, which are 15.6 and 13.7 cents wider than their equal-tempered counterparts, respectively. The narrow half step between the minor and major thirds or major and minor sixths is the lesser chromatic half step of 24:25 of 70.7 cents. 41.1 cents narrower than the 15:16 diatonic half step, this interval is the first to feel microtonal, being closer in size to equal-temperament's 50-cent quarter-tone than its half step. Once the minor sonorities are established individually, alternate between major and minor thirds or sixths to practice the 24:25 melodic interval, then contrast it with the larger 15:16 by singing  $6/5$ – $5/4$ – $4/3$  in sequence.

Mastery of these simple and familiar 5-limit intervals establishes a foundation of accurate and concentrated listening that is essential to hearing more complex harmonies. Septimal intervals add considerable complexity to the challenge of tuning by ear because they are often in close proximity to other simple intervals. The harmonic seventh of  $7/4$ , for example, is 31.2 cents lower than an equal-tempered minor seventh and closely neighbors the Pythagorean seventh of  $16/9$  (3.9 cents lower than ET) and the syntonic seventh of  $9/5$  (17.6 cents higher). Adding a major third to the drone helps the ear to hear the  $7/4$  more clearly and can be done on the TonalEnergy app if the temperament is set to “just.” This setting gives pure intervals in a 5-limit tuning centered on an equal-tempered C. With this drone of  $5/4$ , only the  $7/4$  minor seventh will sound consonant. Adding the  $16/9$  seventh to this major third drone would result in an unlikely chord of  $64/45/36$ ; the syntonic seventh,  $36/25/20$ .

Figure 11 shows an exercise to sing the pitches of the third octave of the harmonic series, a melodic sequence of  $4:5:6:7:8$ , with a short descending sequence to illustrate the  $21:20$  septimal half step. The first three intervals are the familiar sonorities of the major triad, while the  $7/4$  has a notably different character, perhaps heard as sweet or bluesy. Once the tuning of the  $7/4$  has been established, contrast the  $5:6$  syntonic minor third with the  $6:7$  septimal minor third. Next, practice the  $7:8$  septimal whole step,  $21:20$  septimal half step and the  $5:7$  septimal tritone between the second and fourth notes. This exercise can be extended by raising the drone to E-comma-down (E in the just temperament setting in TonalEnergy), which creates a sequence of harmonic intervals  $4/5-1/1-6/5-7/5-8/5-7/5-4/3$ ; or lowering the drone to  $2/3$ , which creates the sequence  $4/3-5/3-2/1-7/3-8/3-7/3$  (the last dyad,  $20/9$ , is very complex and should be omitted).



**Figure 11:** 7-limit tuning exercise.

The preceding examples are merely an introduction to creating further exercises in just intonation. Additional exercises can be devised to suit the needs of the repertoire and the interests of the musician, with drones, composed études, or electronically realized backing tracks to aid the ear. The personality that each new prime factor and its characteristic intervals add to the sonic palette should be explored. To me, the Pythagorean intervals sound austere and solemn, syntonic tuning brings a warm fullness, and septimal harmony sounds melancholy but sweet. Schweinitz refers to “the somewhat drunken beauty of 13-limit harmonies, the touching intensity of the various septimal intervals, and the relaxed sound of the familiar pure major and minor triads.”<sup>147</sup> These attributes can be creative and imaginative, unique to the individual who studies them. Such metaphors guide the ear to familiar sounds and enrich the experience of hearing music in just intonation.

147. Schweinitz, e-mail message to author, March 29, 2009.



## CHAPTER 4

### THE PLAINSOUND BRASS TRIO

#### **Valve Tunings in the *Plainsound Brass Trio***

Schweinitz has carefully considered the inherent just intonation capabilities of brass instruments in a manner that is similar to his writing for strings. He utilizes the harmonic series that forms the basis of brass technique as a collection of pitches related by simple ratios. Much of traditional brass technique, however, is a process of imposing the pitches of equal temperament on the sometimes-unwilling instruments through fingerings, slide positions, and lip-adjustments. Schweinitz, by contrast, asks brass players to use unadjusted partials in combination with precise valve-tunings or trombone slide positions.

The partials of brass instruments, however, do not correspond precisely to the harmonic series. More correctly called modes of vibration, partials can only be exact integer multiples of the fundamental on a perfectly conical instrument.<sup>148</sup> With the addition of valves, a handslide, or even a single tuning slide, perfect conicity is impossible and the partials must deviate from the harmonic series to some degree. For example, the sixth partial is rather sharp on many brass instruments, far in excess of the 2.0-cent deviation expected from the harmonic partial 6.<sup>149</sup> Furthermore, each partial is not a precisely fixed pitch but a narrow range within which the player can adjust the frequency by changes in air and lip pressure. This tolerance is smaller on trumpets and trombone, which have bores that are primarily cylindrical, and somewhat larger on

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148. Arnold Myers, "How Brass Instruments Work," in *The Cambridge Companion to Brass Instruments*, ed. Trevor Herbert and John Wallace (Cambridge: Cambridge University Press, 1997), 20–21.

149. This is not to say that the timbre of a brass instrument contains inharmonic upper partial tones: brass instruments, like bowed string instruments, have upper partial tones that are precisely harmonic. The inharmonicity of "modes of vibration" does affect the instrument's timbre, however, when one of these modes of vibration cannot support harmonic upper partial tones produced by the played note.

horns and tubas, which have bores that are primarily conical. Brass instrument manufacturers go to great lengths to manage these acoustical characteristics through minute refinements in bore size, bell flare, valve design, and many other factors. These factors affect not only the intonation of the instrument, but timbre, response, and loudness, as well. For a detailed discussion of brass instrument acoustics, see “The Brass Wind Instruments” in Arthur H. Benade’s *Fundamentals of Musical Acoustics*.<sup>150</sup>

The valves of brass instruments are ordinarily tuned in an attempt to provide a descending, equal-tempered chromatic scale of available fundamentals. The open horn of each instrument—without valves depressed or with the trombone’s handslide in first position—is the shortest tube length and therefore has the highest fundamental. The second valve lowers the open horn by a half step; the first valve by a whole step. In combination, the first and second valves lower the open horn by a minor third. The third valve ostensibly lowers the open horn by a minor third as well, but is usually used only in combination with other valves; 2–3 for a major third, 1–3 for a perfect fourth, and 1–2–3 for a tritone. Tubas commonly come with four, five, or even six valves, which serve to provide a full octave of chromatic fundamentals.

Because the valves add lengths of tubing, they can be defined in terms of ratios to the open horn. Like string length, tubing length is inversely proportional to frequency: longer tubes give lower fundamentals. For example, increasing the tubing length by  $1/4$  lowers the frequency of the fundamental by  $1/4$ , a ratio of 4:5. It may seem logical that the intervals produced by the valves can be added together, as in “whole step valve plus half step valve equals minor third.” However, the length of each valve’s tubing is defined in relation to the open horn. Therefore, if the first valve lowers the open horn by a whole step and the second valve lowers the open horn

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150. Arthur H. Benade, “The Brass Wind Instruments,” 391–429.

by a half step, the combination of the two lowers the horn by *slightly less* than a minor third. For example, take a trumpet in C with its first and second valves tuned as described. If the first valve is depressed, the trumpet is “in B $\flat$ .” If the second valve is of the right length to lower the C trumpet to B, it is not quite long enough to lower the B $\flat$  trumpet to A.

To define this situation more accurately, it is helpful to conceive of the valve lengths as simple ratios. Let us decide that the first valve is tuned to lower the open horn by a Pythagorean whole step of 9:8 and the second valve by a syntonic half step of 16:15. In this tuning scheme, the first valve tubing must be  $\frac{1}{8}$  of the length of the open horn, while the second valve must be  $\frac{1}{15}$ . Recalling that addition of intervals requires multiplication of frequency ratios, but tubing lengths are added, not multiplied—it is clear that  $\frac{1}{8}$  plus  $\frac{1}{15}$  will not equal the  $\frac{1}{5}$  length that would yield a 6:5 minor third. The tubing added is, in fact,  $\frac{23}{120}$  of the open horn, yielding a complex 23-limit interval.

The trombone has the capability to precisely define any tubing length within the tritone range of the handslide, although only seven basic positions are used in standard trombone technique. It is common knowledge among advanced trombonists that the physical distance between slide positions producing equal intervals must increase the farther the slide is out. Many modern trombones are equipped with an F-attachment, colloquially known as a “trigger.” This valve, operated by the left thumb, lowers the pitch of the trombone in first position—with the slide all the way in—by a perfect fourth, adding  $\frac{1}{3}$  to the length of the instrument. In a similar fashion to the problem of valve combinations, adding the trigger to slide positions other than the first lowers the pitch by less than a perfect fourth. If the slide is in third position and tuned to a 8:9 Pythagorean whole step, adding the trigger yields a tube  $\frac{11}{24}$  longer than the trombone in

first position. The interval between the fundamentals of first position and this third-position-plus-trigger is 24:35.

In equal-tempered playing, such unwanted microtonality is easily mitigated by the experienced player through “lipping” the notes and making tuning slide or handslide adjustments. Schweinitz, however, uses this microtonality as the basis for his brass writing in the *Plainsound Brass Trio*. By precisely defining the tubing-length ratio produced by each valve and each valve combination, Schweinitz creates a microtonal collection of fundamentals. The tuning schematic for the *Plainsound Brass Trio* is shown in Figure 12. Note that all of the open horns are tuned up by a syntonic comma, so that the F and D partials can be easily tuned as  $4/5$  and  $2/3$  below the reference pitch of A. The horn and tuba valves are tuned in proportions with common denominators, which avoids the complex intervals created by the valve combinations described earlier. The first three valves of the F tuba and the F side of the double horn are tuned identically. The second and third valves are tuned to lower the open horn by a  $16:15$  half step ( $1/15$  length) and  $6:5$  minor third ( $1/5$  length), respectively. The first valve is in an unusual tuning of  $15:17$  ( $2/15$  length), which produces a complex interval to the open horn but a simple  $6:5$  in combination with the second valve ( $1/15 + 2/15 = 1/5$ ) and  $4:3$  in combination with the third valve ( $2/15 + 3/15 = 1/3$ ). The 2–3 combination lowers the open horn by  $19:15$  and 1–2–3 lowers the open horn by  $7:5$ . The B $\flat$  side of the horn is identical to the F side but transposed up by  $3:4$ .

The fourth and fifth valves of the F tuba add considerable complexity to the horn’s three-valve system. They are tuned here in the standard way: the fourth valve lowers the open horn by a perfect fourth like the trombone’s trigger valve, while the fifth valve is used in combination

with the fourth to lower the open horn by a perfect fifth.<sup>151</sup> The length of the fifth valve by itself must be  $1/6$  of the open horn, so that the fourth valve— $1/3$  of the open horn length—plus the fifth valve equals  $1/2$  of the length. The fifth valve is ordinarily not used by itself as it gives a septimal interval of 7:6. Because of the fifth valve's length of  $1/6$ , the common denominator for the valve lengths of the tuba is 30 in Schweinitz's tuning scheme.

The tuning of the trombone is treated somewhat differently: the handslide provides eighteen distinct positions, with some positions separated by as little as an eighth-tone. Each position has a second fundamental available with the addition of the trigger valve, which is tuned to lower the open horn by 4:3. In a few slide positions, these pairs of fundamentals have a close harmonic relation; in others, the interval between them is highly complex. For example, the second slide position lowers the horn by an interval of 49:48 (35.7 cents), meaning that the slide adds  $1/48$  to the length of the tubing. The addition of the trigger creates a fundamental 65:48 below the first position ( $1/48 + 1/3 = 17/48$ ). The interval between the second slide position and second position plus trigger is therefore 65:49.

Finding a reliable way to tune the valve slides is the first intonation challenge presented by the *Plainsound Brass Trio*. The instructions included by Schweinitz, shown in Figure 13, explain the objective of the valve tuning in idealistic terms. Because these instructions do not take the inharmonicity of brass partials into account, they can serve only as something of a Platonic ideal. The previous sentence is not meant as a criticism of Schweinitz; the discrepancies of intonation are unique to each brass instrument and even each fingering, making it simply not possible to devise a more accurate generalized tuning scheme.

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151. This fifth-valve tuning is often referred to as the “flat whole step” fifth valve and is a common arrangement in the United States. Other fifth valve tunings include one in which the fifth valve lowers the open horn by 3:2 in combination with the second and third valves.

# Jl Tuning for Double Horn, F-Tuba with 5 valves, and Tenor-Bass Trombone

HORN : The valve slides 1, 2, and 3 are tuned to the rational proportions  $2/15$ ,  $1/15$ , and  $3/15$  of the open horn's length (both in B-flat and F), producing, in various combinations, a Utonal chromatic Series of fundamental pitches with wavelengths in the proportions  $15 : 16 : 17 : 18 : 19 : 20 : 21$ .  
F-TUBA : The valve slides 1, 2, 3, 4, and 5 are tuned to the rational proportions  $2/15$ ,  $1/15$ ,  $3/15 = 1/5$ ,  $5/15 = 1/3$ , and  $5/30 = 1/6$  of the open horn's length, producing, in various combinations, a Utonal micro-chromatic Series of fundamental pitches with wavelengths in the proportions  $30:32:34:35:36:37:38:39:40:41:42:43:44:45:46:47:48:49:50:51:52:53:55:57$ .

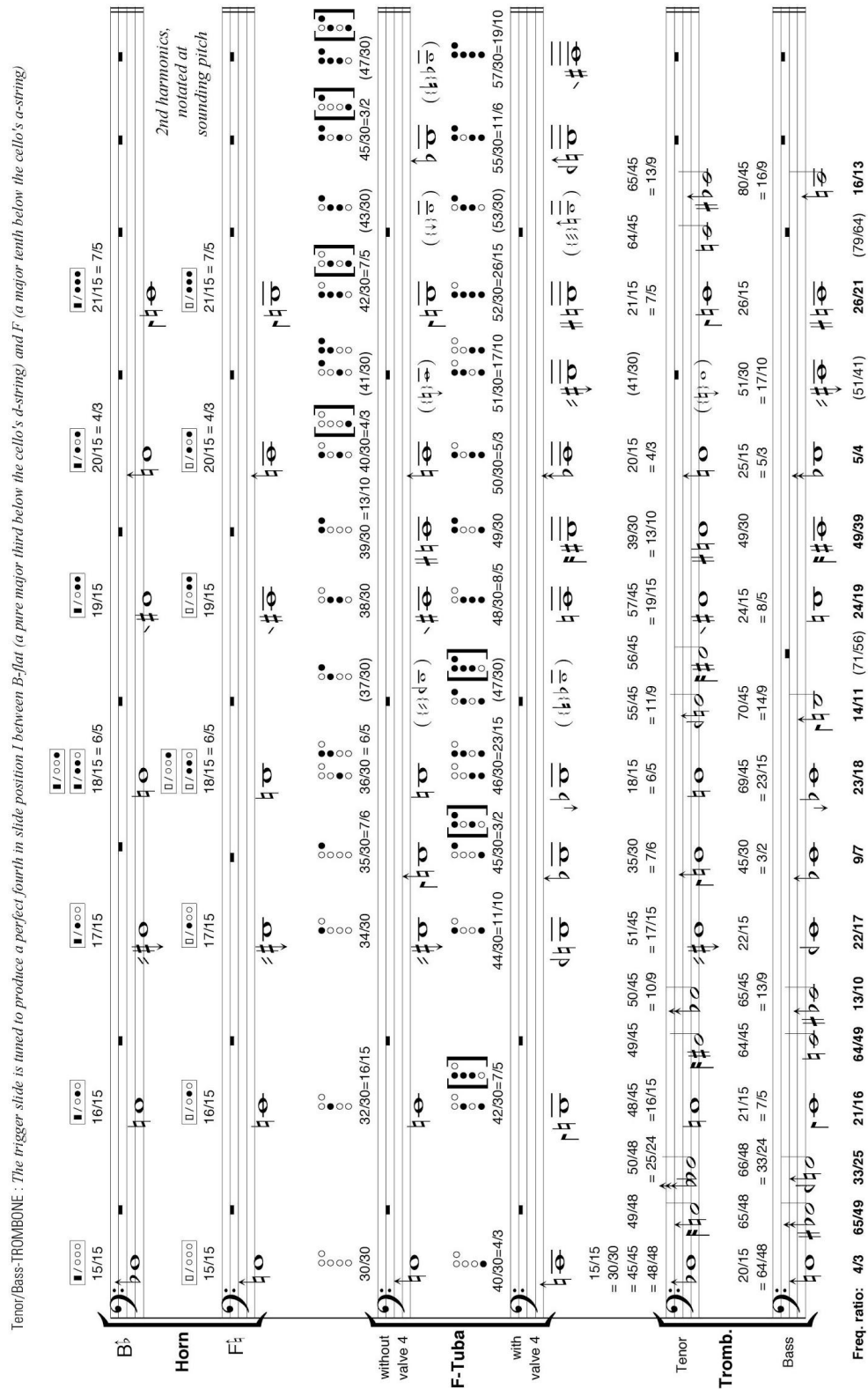


Figure 12: Valve tuning for the Plainsound Brass Trio.

## TUNING INSTRUCTIONS

### DOUBLE HORN

*The open F-Horn is tuned a pure major third below A, so that its 5<sup>th</sup> harmonic produces 220 Hertz. The open B-flat-Horn is tuned a perfect fourth above the F-Horn, so that its 6<sup>th</sup> harmonic produces the same pitch as the 8<sup>th</sup> harmonic of the open F-Horn. The 2<sup>nd</sup> valve slide of the B-flat-Horn is tuned such that the 4<sup>th</sup> harmonic of the A-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz), and the 2<sup>nd</sup> valve slide of the F-Horn is tuned such that the 8<sup>th</sup> harmonic of the E-Horn produces the same pitch as the 6<sup>th</sup> harmonic of the A-Horn (330 Hz). The 3<sup>rd</sup> valve slide of the F-Horn is tuned such that the 6<sup>th</sup> harmonic of the D-Horn produces the same pitch as the 4<sup>th</sup> harmonic of the A-Horn and the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz). The 3<sup>rd</sup> valve slide of the B-flat-Horn is tuned such that the 6<sup>th</sup> harmonic of the G-Horn produces the same pitch as the 8<sup>th</sup> harmonic of the D-Horn and the 5<sup>th</sup> harmonic of the open B-flat-Horn. Then the 1<sup>st</sup> valve slide of the B-flat-Horn is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (G-Horn), and likewise the 1<sup>st</sup> valve slide of the F-Horn is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (D-Horn).*

### 5-VALVE F-TUBA

*The open horn is tuned a pure major third below A, so that its 5<sup>th</sup> harmonic produces 220 Hertz. The 4<sup>th</sup> valve slide is tuned such that the 4<sup>th</sup> harmonic of the C-Horn produces the same pitch as the 3<sup>rd</sup> harmonic of the open F-Horn. The 3<sup>rd</sup> valve slide is tuned such that the 6<sup>th</sup> harmonic of the D-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz). The 2<sup>nd</sup> valve slide is tuned such that the 4<sup>th</sup> harmonic of the E-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the C-Horn (115 Hz). Then the 1<sup>st</sup> valve slide is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (D-Horn). The 5<sup>th</sup> valve slide is tuned in combination with the 4<sup>th</sup> valve such that the 6<sup>th</sup> harmonic of the valve combination 4+5 (B-flat-Horn) produces the same pitch as the 4<sup>th</sup> harmonic of the open F-Horn.*

### TENOR-BASS-TROMBONE

*Once the tuning slide is adjusted such that the harmonics produced in the 1<sup>st</sup> position of the tenor trombone will match those of the open B-flat-Horn, the trigger valve slide is tuned such that the trigger valve will lower the pitch by a perfect fourth in the first position, so that the harmonics produced in the 1<sup>st</sup> position of the bass trombone will match those of the open F-Horn. Thus the trigger valve acts exactly like the 4<sup>th</sup> valve of the tuba. This is demonstrated in the table showing the 18 microtonal slide positions used for the performance of this piece (see below).*

**Figure 13:** Plainsound Brass Trio Instructions page.

The actual process of tuning the valve slides must accept some compromises to provide the greatest number of in-tune notes for the *Plainsound Brass Trio*. Slide positions that provide some in-tune partials may be out-of-tune for others. The tuning process should be done carefully and methodically, with a number of double-checks and a thoughtful approach to analyzing potential compromises. Try each partial at loud and soft dynamics to determine the best possible average. Because small errors in tuning individual valve slides can be compounded in valve combinations and lead to difficulties in rehearsal and performance, time dedicated to tuning can save time in rehearsal. My suggestions for a tuning routine follow.

Begin by setting a drone to a reference pitch of A<sub>3</sub>, a frequency of 220 Hz. This A can be also be calibrated to 442 Hz, 438 Hz, or any other reference at the preference of the performer, but the lower octave is helpful for tuning lower partials. The open horn of the tuba and the F-side of the horn are tuned so that their fifth partials match the A drone.<sup>152</sup> The tenth partial of the F horn should be checked as well. The effect of this tuning should be to raise the pitch of the open horn by 13.7 cents above its standard tuning in equal temperament.

Verify the tuning of the horn and tuba with the F-comma-up and C-comma-up partials (partials two and three, plus higher octaves), which should form pure thirds and sixths with the A. Make note of which partials are the most difficult to play in tune and experiment with changing the position of the main tuning slide. Find a compromise that allows the greatest number of open-horn partials to be in tune with the A without causing any individual partial to be unusably out of tune.

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152. The F side of the horn and the F tuba are the same tube length. The narrow-bore horn generally plays on much higher partials than the wide-bore tuba. The B $\flat$  side of the horn is the same length as the trombone or euphonium.



The trombone (in first position) and the B $\flat$  horn are tuned so that the third, sixth, and twelfth partials match the F-comma-up partials of the F horn and tuba. Tune these notes to produce pure thirds and sixths against the A. The D of the fifth and tenth partials should produce pure perfect fifths and fourths, and the ninth partial C-comma-up a pure minor third. Next, turn off the drone and tune the second and fourth partial B $\flat$ -comma-up against a sustained F from the horn or tuba. For extra precision, the drone can be left on but played on earphones worn by the sustaining player. Repeat with the B $\flat$  horn, adding higher partials. Once the trombone's first position is satisfactorily in tune, repeat the open F horn and tuba process for the trigger valve.

Next, turn the drone back on to tune the second valves of the tuba and horn. The B $\flat$  horn's second valve yields a fundamental of A and provides a plethora of partials easily tuned to the drone. On the F horn and tuba, the second valve gives an E, best tuned to the A at the third and sixth partials. With the drone off or played through earphones, alternate having one player sustain an E while the other explores the tuning of partials on the "E horn."

Tune the first valve in combination with the second. On the F horn and tuba, this combination a D series, all the partials of which can be tuned to the A drone. Use the fifth partial of the trombone to sustain a D to tune 1–2 on the B $\flat$  horn to a G fundamental. Explore the sound of all of the available partials on this "G horn" to begin to go beyond the 5-limit intervals. Next, tune the third valves of the tuba and horn to match the 1–2 combinations. Note that the problematic partials may be different these fingerings and think about switching between the two to create more ideal G and D series of partials. Having these two options for the same fundamental can also help to address compromises for the remaining valve combinations. On the horn, all the valves have now been tuned and it will hopefully be found that there are no major problems with 2–3, 1–3, or 1–2–3.

Check the 1–3 combination first. On the B $\flat$  horn, this fingering produces an F-comma-up fundamental that should match the open F horn, albeit with a greater likelihood of out-of-tune partials. Test the partials that were found to be problematic on the open F horn to see if they might be better as 1–3 on the B $\flat$  side. On the F horn and tuba, the 1–3 combination is a C-comma-up fundamental. The fifth and tenth partial E should match that of the second valve and can be tuned against the drone A. Again, alternate holding a fourth or eighth partial on the tuba or horn while testing the partials of the other instrument. Tune the fourth valve of the tuba to match 1–3, noting which partials are better or worse on each fingering.

The 2–3 combination lowers the open horn by 19:15, a fundamental that is difficult to check against a drone. It will suffice to check that the tuning matches between the F horn and the tuba and that the partials on the B $\flat$  horn are a perfect fourth above those on the tuba. Check partials against sustained notes and observe significant discrepancies.

The 1–2–3 combination is a septimal relation of 7:5. On the F horn and tuba, this B-septimal-comma-up series has a seventh partial of A, which can be tuned to the drone as a unison. The other partials can be tuned if it is imagined that the drone is holding the seventh partial. As with the 2–3 fingering, concentrate on finding discrepancies between the horn and tuba and check that the B $\flat$  horn's partials are in parallel fourths to those on the tuba. Check to see how well the tuba's 2–4 combination matches 1–2–3, again noting problems and advantages for each.

Finally, tune the tuba's fifth valve. Check its tuning first in combination with the fourth valve. This B $\flat$ -comma-up is one octave lower than the open B $\flat$  horn and first-position trombone. With such a long tube length, it should be expected that the partials of 4–5 will differ significantly from the harmonic series. Luckily, many partials of this fundamental are duplicated

in partials of shorter tube lengths, such as the F-comma-up of the open horn and the D of 1–2 or 3. Confirm the tuning of the fifth valve by itself by checking it against the 1–2–3 series of the F horn, the partials of which should be a pure minor third below the tuba. Finally, check the 2–3–4 fingering, which supplies an A series that matches the drone, and 1–3–4, an A $\flat$ -two-commas-up that has fifth and tenth partials of C-comma-up, a major sixth below or minor third above the drone. The remaining fingerings provide few useful opportunities for “double-checks” and the tuning of the simpler fingerings should not be compromised to accommodate them.

The tubist should be especially careful of his or her tuning. Because of the complexity added by having five valves, errors of slide tuning can be compounded more harmfully. As the lowest voice in most of the *Plainsound Brass Trio*, the tuba’s intonation is paramount to the success of the piece. Many of the trombone’s tuning modulations are performed against sustained notes from the tuba and tuning errors at these points will cause significant problems. The reader has probably noticed that the trombone tuning has not been addressed since discussion of its first position and trigger valve. Although the trombonist has the most complex slide-tuning task in the *Plainsound Brass Trio*, each position other than the first is found during the course of the piece. In these moments, which I call “tuning modulations,” the trombonist has only a few moments to find the next slide position.

The preceding tuning sequence is meant to be useful in a rehearsal setting and require only a basic tuner for the drone. Although it is time-consuming, this full process only needs to be done once and can be partially revisited as needed. For individual practice and to further refine the valve tunings, the frequency of each fingering’s second partial is provided in appendix A. The frequency of any other partial of each fingering can be calculated by multiplying the second-partial frequency by the desired partial number and dividing by two. For example, in learning to

hear the undecimal intervals, a drone set to the eleventh partial can be helpful. Individual work with a drone set to these frequencies will save time and prevent frustration in rehearsal and will help especially with the fingerings that are harmonically distant from the A reference pitch. A number of audio software programs can generate drone pitches at user-defined frequencies. At the time of this writing, [onlinetonegenerator.com](http://onlinetonegenerator.com) provides a simple and easy-to-use solution.

Throughout the tuning process, take note of the intonation tendencies of problematic partials and compile these notes into a list for reference. Use a small ruler to measure the position of each slide as the tuning process is refined. Once these measurements are made, they can be quickly found at the start of rehearsals. As a group, compare the tuning of the common partials on each instrument and discuss the merits and drawbacks of the compromises chosen. As the piece is rehearsed, such discussions allow the group to solve tuning problems quickly.

### **Interpreting the *Plainsound Brass Trio***

It is easy to be fooled into thinking that the complex tuning system of the *Plainsound Brass Trio* is the only important element of the piece. On the contrary, Schweinitz uses this intonation to realize a musical language that is full of expressivity. The *Plainsound Brass Trio* is subtitled “18 microtonal variations exploring the trombone's trigger valve action at various tuned slide positions.” This organizing principle divides the piece into harmonic regions, with each slide position giving a collection of available pitches from the trombone that are harmonized by the horn and tuba. In variations where the slide position is harmonically close to available fingerings of the horn and tuba, the chords tend to be simpler, represented by ratios of smaller numbers. Where the slide positions are not closely related to fingerings in the tuba and/or horn, Schweinitz uses more complex harmonies, sometimes using the same chord in parallel motion, as is seen in Figure 14.

Figure 14: MM. 93–96. Parallel chords.

This structure of dividing the piece into regions of harmonic possibilities can be emphasized by adding a fermata in the cadences of the tuning modulations and elongating the following rests before the next section begins. Each region should feel unique, with a character that is mutually agreed upon in the rehearsal process. This character is determined by the type of harmonies that predominate the region, the dynamics and articulations, and the melodic shapes and rhythms used. In determining these expressive decisions, it is also helpful to recall the personalities of each prime limit as perceived by each player. These subjective and emotional qualities can help to break through the barriers presented by the work’s theoretical complexity.

In performance, the performers must find a basic tempo that strikes a balance between a freely expressive interpretation and a deliberate presentation of the work’s harmonic language. Recalling the words of La Monte Young, “tuning is a function of time,”<sup>153</sup> and many of the chords in the *Plainsound Brass Trio* are difficult to tune. As a result, there can be a nearly constant impulse for the piece to slow down. Giving great care to the tuning in rehearsal can actually exacerbate the problem, as the higher standards of pitch refinement can lead to

153. Richard Kostelanetz, “Conversations with La Monte Young,” 7.

dissatisfaction with even minor intonation issues. Balance the need for tuning precision with mindfulness of the shape of each phrase and the larger structure of each movement.

The texture of this thirty-minute work is nearly all homophonic, with only isolated moments of variety. This presents a challenge to the performers to maintain interest and excitement as the piece progresses. Wide extremes of soft and loud playing will help this issue, with these changes in volume, intensity, and timbre substituting for textural variety. The passages that do employ different textures should be emphasized through a prolongation of preceding cadences or rests. There should also be a marked difference in such sections between soloistic and accompanimental playing. Figure 14 shows an opportunity for soloistic playing. Although marked *molto piano*, the horn should dominate the texture, with the *pianissimo* indication in the trombone and tuba strictly observed. The first measure can be played with liberal rubato to linger on the horn melody's beautiful, expressive intervals, especially the 5:7 septimal tritone and the 21:20 resolution. Although the trombone and tuba have the more melodically active parts after the first measure, it is perhaps more musically satisfying to maintain the soloist/ensemble texture for the remainder of the line by emphasizing the horn's microtonal steps and treating the descending fifth in measure 215 as a mock cadence.

**REGION III**  
*lo stesso tempo* (♩ ca. 52 - 54)

212 **51**

20/3 13/2 65/64 16/5 13/4 39/38 13/4 13/6 13/10

40:39 (-44 c) 65:64 (-27 c) 39:38 (-45 c)

*molto p* *espr.* *mp marc.* *espr.*

T:B = 24:19 Same slide position as before B 19:18 (-94 c) T 12:19 (+789 c) B 13:8 (-841 c) T:B = 49:39 Tune the new slide position T (-45 c)

*pp* *mp marc.* *espr.*

19:18 (-94 c) 48:95 (+1182 c) 39:38 (-45 c)

*pp* *mp marc.* *espr.*

Figure 15: MM. 212–215.

In all three movements of the piece, the time signature remains  $\frac{4}{1}$ , a nod to early styles of rhythmic notation in long note-values.<sup>154</sup> With a tempo indication of 52–54 to the whole note, the music does proceed steadily, yet it *looks* spacious and slow. This notation provides a visual cue to treat each sonority as if it were a long chord, with only the shortest notes resembling passing tones. One effect of this meter is to engage the interpreter in vertical listening, which undoubtedly has a positive effect on the intonation and blend of chords. However, it also has the potential to hinder melodic listening, as well as expressivity, phrasing, and the momentum of the performance. The slowness implied by a predominance of long note values can too easily translate into slowness in performance. The  $\frac{4}{1}$  meter also causes difficulties of reading rhythm. Double whole, whole, and half rests are easily confused, as are dotted whole notes and dotted rests. Some of these durations appear rarely enough in modern music that most musicians are unaccustomed to reading them quickly.

Schweinitz makes frequent use of short rests immediately preceding accented chords or arrival points. These can be interpreted as a notated version of the type of accentuation employed especially by harpsichordists and organists (who cannot generate accents by means of dynamic contrast). In his manual of Baroque style, Robert Donington describes this as “normal accentuation” and writes that it “results from a momentary silence of articulation.”<sup>155</sup> Figure 16 shows several instances of these short rests.

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154. *Grove Music Online* s.v. “Note Values,” by John Morehen and Richard Rastall, accessed May 28, 2017, <http://www.oxfordmusiconline.com/subscriber/article/grove/music/20127>.

155. Robert Donington, *Baroque Music: Style and Performance* (New York: Norton, 1982), 38.

II

same tempo (♩ ca. 56 - 63) 51 : 50 (-34 c)

106

17 10 17 17 17 11 11 11  
7 7 5 3 3 4 5 5  
(-105 c) (+105 c) (+105 c)

pp p p espr. mf

T: B = 22:17

Tune the new slide position T (-34 c)

51 : 50 (-34 c) 5/2 5/2 pp p espr. mf

33 : 34 (+52 c) B

pp p pp p espr. mf

25 11 11 11 11 11 11 11 11 11 11 12  
6 7 3 6 5 5 6 6 6 6 6 8  
5 4 2 4 3 3 16 6 6 6 6 8  
110

f più f ff (+165 c) 17 : 16 (-105 c)

15 : 16 (+112 c) 10 : 11 (+165 c) 12 : 11 (-151 c) 17 : 16 (-105 c)

f più f ff

Figure 16: Schweinitz, *Plainsound Brass Trio*, mm. 106–113.

An imaginative performer can successfully form an interpretation of these rests as a highly specified matter of performance practice, with the goal of making the accents sound natural or even spontaneous. The eighth rests should be nearly unrecognizable as a specific duration of silence—being a mere eighth of one beat—and should be felt as an imperceptible lift in the sound before the accented chord. Quarter rests should be more noticeable, while half rests give an obvious separation. The longer the rest, the more pronounced the accent of the following chord, even if it is played quietly.

The sheer amount of information on the page presents a challenge to concentration and expression. With unfamiliar accidentals, specified fingerings, copious expression markings, and supplemental tuning information, the *Plainsound Brass Trio* score is visually cluttered. Figure 16



shows the extent of this profusion of notation. The Helmholtz-Ellis accidentals, explained earlier, are compounded here in a complex manner, with some notes having several accidentals of different prime-factor families. Stacked numbers above the top staff show the frequency ratios of each chord. When only two voices are playing, these ratios are given as fractions *between* staves. Some melodic intervals—usually very small or harmonically distant ones—are shown as numbers separated by a colon and as a cents value in parentheses and italic type, e.g., “51:50 (*−34 ¢*).”

Fingerings are given above the horn and tuba staves as open or filled-in circles. Filled-in circles indicate valves that are depressed. The rectangle to the left of each horn fingering indicates the position of the F/Bb change valve: a filled-in rectangle denotes that the horn is in Bb. The first four valves of the tuba fingerings are indicated vertically, with the fifth valve offset to the right, corresponding intuitively to the standard valve layout on the tuba.

Trombone slide positions are not indicated for each note since each position is held for a section of the piece before being re-tuned for the next. The trigger valve is shown with T for tenor (trigger disengaged) and B for bass (trigger engaged). This is somewhat counterintuitive; T is often used in many slide position charts to indicate that the trigger *is* engaged. Measures 106 and 107 also show the notation at one of the tuning modulations. The new slide position is indicated by the interval between the position and its counterpart with the trigger. A cents value is given to show the melodic distance between the previous slide position and the new one.

The notation of the *Plainsound Brass Trio* is dense and complex, but every marking serves a clear purpose. Although it is not the simplest notation, it is, perhaps, the best option. Removing the fingering and trigger indications would require the performers to memorize a large number of fingerings and all of the partials to which they correspond. Removing the ratio and

cents indications for the small or complex melodic intervals is possible, but the loss of these helpful indications only provides a minimal reduction in visual clutter. The melodic intervals could be indicated only in cents, not ratios, but this would be less helpful for intervals on the same fingering, such as the 10:11:12 sequence in the horn part at measure 113 in Figure 16, above.

The inclusion of chord ratios in addition to the HE accidentals could be seen as redundant. Omitting the ratios takes away a useful shortcut to understanding the harmonic makeup of each chord and assumes a stronger knowledge of the HE notation than can reasonably be expected of most performers. Including only the ratios and no special accidentals creates a host of notational problems. Would the nearest equal-tempered pitches be shown instead? If so, the inclusion of cents deviations would be necessary, a solution more untidy and far less elegant than the HE accidentals. Cents deviations would also tempt performers to use electronic tuners, which would harm the quality of the music-making.

Finally, the *Plainsound Brass Trio* presents a number of technical challenges to its interpreters. Chief among these is the preponderance of playing on high partials. On all brass instruments, the higher partials are more difficult to produce than lower ones.<sup>156</sup> Long and thin tubes like that of the horn provide greater acoustical support to the production of high partials, but these too demand increasing effort for high notes. Because of the decreasing melodic distance between the high partials and the complex intervals between them, the high reaches of the horn, trombone, and tuba contain the greatest number of pitches and variety of intervals. To write convincing music with each of the trombone's eighteen slide positions, Schweinitz makes

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156. Arthur H. Benade, "The Brass Wind Instruments," in *Fundamentals of Musical Acoustics*, 2nd ed. (New York: Dover, 1990), 391–429.

use of the high partials often. In every performance of this piece by Trio Kobayashi, endurance has proved to be a major challenge.

One strategy to deal with this challenge is to ensure that the tempo does not become too slow, especially in the high and loud sections of the piece. Mentally adding a “*ma non troppo*” qualifier to the tempo indication of “*lento rubato*” will address issues of both interpretation and endurance. While the use of rubato is both requested by the composer and an effective expressive tool, it is important to find places for the tempo to push steadily ahead. Otherwise, the continuous high playing can become unmanageable and the piece can become tedious.

Another strategy is the use of alternate fingerings. When a note is a common partial of two different fingerings, Schweinitz often chooses the fingering that is closest in length to the surrounding fingerings. Doing so has the advantage of maintaining a more consistent timbre and reducing the number of fingering changes. Figure 17 shows such instances in the tuba part. Three notes in this passage can be played on the open horn but are notated on longer fingerings: C-comma-up, F-comma-up, and A. Choosing to play these with the shortest possible tube length—thus on the lowest possible partial—saves energy. Although its effect might seem negligible, consistent use of these alternate fingerings can make a significant difference by the end of the piece. Furthermore, these lower-partial alternatives are also more stable and reliable. Using them as checkpoints in an extended passage of complex intervals and long fingerings can help to prevent mistakes.



Figure 17: MM. 25–29 (tuba only).

## CHAPTER 5

### CONCLUSION

Wolfgang von Schweinitz's *Plainsound Brass Trio* is undoubtedly a difficult piece to learn, presenting formidable barriers to entry that may seem unreasonable to many musicians. Before the premiere performance by Trio Kobayashi, the work was rehearsed for most of a year, including regular meetings with the composer and significant outside study of just intonation theory. I hope that this dissertation will serve to ease the difficulties of understanding, rehearsing, and performing this work so that other ensembles may feel encouraged to learn and present it. I believe that the *Plainsound Brass Trio* opens the door to a vast range of musical possibilities that cannot be fully realized without the inquiry and discussion of a larger group of performers and a wider audience.

The *Plainsound Brass Trio* is unique in the literal sense of the term. It is difficult to find an equivalent example in the entirety of the brass repertoire that is as uncompromisingly conceived and executed. Even in the rarified company of the masterworks of just intonation—pieces such as La Monte Young's *The Well-Tuned Piano*, Ben Johnston's *Amazing Grace*, and Harry Partch's *And on the Seventh Day Petals Fell on Petaluma*—the *Plainsound Brass Trio* stands out as radically ambitious in the combination of its romantic expressiveness and harmonic complexity. Within Schweinitz's oeuvre, it is the most complete exploration of brass tuning, building upon that of the *Kantate* (2002–3) and the *Plainsound-Sinfonie* (2003–5). It also employs the most frequent and radical shifts of harmony of any of his instrumental works, a stylistic departure from the meditative and deliberate character of the *Plainsound-Litany* and *Plainsound Glissando Modulation* (2006–7).

Schweinitz intends many of his pieces to serve as something of a “proof of concept,” which explains why the titles of several of his works contain the words “étude” or “study.” “One

of the objectives of all my JI compositions has been the desire to provide the interested performers with some appropriate musical material for exploring the tuning and ensemble playing techniques needed to establish the sound of just intonation,” writes Schweinitz.<sup>157</sup> To composers, the *Plainsound Brass Trio* demonstrates how just intonation can move from abstract theory into a musical practice that can be communicated to interpreters. For performers, it shows how a selectively applied performance practice can evolve into a logical and uniform principle of untempered tuning that can be applied to modern and traditional repertoire alike.

Through this piece, Schweinitz has made an extraordinary contribution to the repertoire by creating a just intonation music based upon thorough investigation of instrumental technique, with consideration of the perceptibility of its pitch relationships and the direct experience of its performers. Such a compositional style, which I propose be called “idiomatic just intonation,” is perhaps still in its infancy, but the *Plainsound Brass Trio* represents a dramatic exposition of the intonation possibilities of brass instruments. Although string instruments offer a finer degree of pitch control, brass instruments have the unparalleled ability to quickly modulate to distant harmonic centers through the use of the tuned valves. This ability can be used to create music that would be unreasonably difficult for singers or string players to execute because their tuning relies to a greater extent on the performer’s ability to first hear the interval. Brass instruments also have a richness of timbre, power, and dynamic range that makes them especially compelling for just intonation harmony, with the potential for strong resultant tones and clear presentation of interval character.

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157. Schweinitz, e-mail message to author, March 29, 2009. Although these works are certainly intended for public performance, Schweinitz acknowledges the role of his works in the larger context of a shared experiment between composer and performer.

The broader implications of just intonation—such as its use in performance of the Baroque, Classical, or Romantic repertoire—have been avoided throughout this paper but warrant further discussion and investigation. It has not been my intention to convince the reader of the superiority of just intonation or to “indoctrinate” the reader into any ideology. In the words of composer Larry Polansky, “the use of rational frequency relationships is simply one part of the complex and beautiful web of translations between the acoustic and psychoacoustic. ... There can be no best intonation, as there can be no best form.”<sup>158</sup> That said, learning the theory of just intonation as a complement to that of equal temperament provides a far more complete understanding of the music of many traditions and styles and can establish a bridge between the fields of music theory, performance practice, acoustics, and the phenomenology of music.

Why a performer should choose to take on this challenge can only become clear through a committed effort to realize just intonation’s potential. Schweinitz has gone to extraordinary lengths to reimagine and elevate the capabilities of brass instruments and it would be a shame to leave this call unanswered. In my personal experience, the effort has been worth the trouble. I strongly feel that my involvement in the creation and performance of this piece is among the most important projects of my performing career. Through a substantial—yet reasonable—amount of research, experimentation, and perseverance, I have discovered that the *Plainsound Brass Trio* is a surmountable challenge. Moreover, I have found this work to be stunningly beautiful, containing a sonic and expressive richness that has the ability to transport performer and listener alike. It has opened my ears and mind to a more intricate and abundant world of harmonic possibilities that simultaneously speaks to the most ancient of musical practices and the most modern.

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<sup>158</sup> Larry Polansky, guest editorial, 1/1: The Journal of the Just Intonation Network 1 (1985): 2.

## APPENDIX A

### FREQUENCIES OF SECOND PARTIALS FOR FINGERINGS AND SLIDE POSITIONS IN THE *PLAINSOUND BRASS TRIO*

Fingering	Ratio to open horn	Frequency of 2nd partial
F Horn		
0	1:1	88
2	16:15	82.5
1	17:15	77.65
1-2 or 3	6:5	73.33
2-3	19:15	69.47
1-3	4:3	66
1-2-3	7:5	62.86
B $\flat$ Horn		
0	1:1	117.33
2	16:15	110
1	17:15	103.53
1-2 or 3	6:5	97.78
2-3	19:15	92.63
1-3	4:3	88
1-2-3	7:5	83.81
Tuba		
0	1:1	88
2	16:15	82.5
1	17:15	77.65
5	7:6	75.43
1-2 or 3	6:5	73.33
2-5	37:30	71.35
2-3	19:15	69.47
1-5	13:10	67.69
1-3 or 4	4:3	66
1-2-5 or 3-5	41:30	64.39
1-2-3 or 2-4	7:5	62.86
2-3-5	43:30	61.4
1-4	22:15	60
1-3-4 or 4-5	3:2	58.67
1-2-4 or 3-4	23:15	57.39
1-2-3-5 or 2-4-5	47:30	56.17
2-3-4	8:5	55
1-4-5	49:30	53.88
1-3-4	5:3	52.8
1-2-4-5 or 3-4-5	17:10	51.76
1-2-3-4	26:15	50.77
2-3-4-5	53:30	49.81
1-3-4-5	11:6	48
1-2-3-4-5	19:10	46.32

Measure of tuning modulation	2nd partial frequency without valve ("T")	2nd partial frequency with valve ("B")
Trombone		
1	117.33	88
14	114.94	86.65
31	112.64	85.33
47	110	83.81
65	107.76	82.5
82	105.6	81.23
102	103.53	80
126	100.57	78.22
150	97.78	76.52
161	96	75.43
186	94.29	not used
195	92.63	73.33
214	90.26	71.84
223	88	70.4
246	85.85	69.02
255	83.81	67.69
276	82.5	not used
300	81.23	66



## APPENDIX B

### TRANSCRIPT OF E-MAIL INTERVIEW WITH WOLFGANG VON SCHWEINITZ (MAY, 2017)

#### **Compositional Style and Influences**

- LS: What influences led you to pursue just intonation as the basis of pitch in your compositions?
- WvS: My interest in the sound and performance practice of microtonal just intonation is rooted in a feeling that has haunted me since I first encountered 20th century atonal music as a teenager about fifty years ago; it was the notion that our ears and our music may perhaps have lost something very substantial when giving up for good the fundamental principle of tonality, which appears to be closely related to our brain's astounding capability of harmonic perception.
- LS: You were once identified as a member of a so-called 'Neue Einfachkeit' movement in German contemporary music. Is this an apt description of your compositional style?
- WvS: At first I could not do much more but deplore this historical loss (like 1976 in my Mozart-Variations, for example), emphatically looking back at what tonality once had been and trying to inject some quasi-tonal sense into my microtonal cluster sounds. This could certainly be understood as a neo-romantic, post-modern enterprise; but I think that the habitually reiterated classification of my early music as representing part of a so-called German "New Simplicity" movement (which did not even ever exist as such) is inappropriate and quite misleading.
- The term "Neue Einfachheit" was coined in the mid 1970s by the composer Walter Zimmermann, as an enthusiastic brand name for his radical artistic project of starting as it were from scratch in the wake of John Cage. Some of Walter's decomposed folklore turned out sounding like tonal music, and it didn't take long until some impertinent journalist in the German contemporary music scene stole the word from Walter and his original music and maliciously applied it to the music of a bunch of other young German composers instead. (And I think it is a pity that the German musicologists and music historians have later picked up and perpetuated this meaningless and disparaging label in such a rather uncritical way.)
- LS: What connection do you see in your music to the 'Spectral' school of composition (however you choose to define it)?
- WvS: I met Gérard Grisey in 1980 at the International Summer Courses in Darmstadt; we immediately liked each other a lot, and we remained friends even though we saw each other only two more times thereafter. Gérard always handled the tuning issues with greatest care, and I love all of his magnificent music very much. But I was never enthralled by the severely tempered sound of Tristan Murail's spectral music (with its distorted "natural sevenths" that are almost an entire comma flat).

LS: To the extent that such a thing exists, the 'JI school' is often associated with the United States. To what extent were/are you inspired by the American JI composers and their ideas?

WvS: With their particular focus on the musical relevance of tuning, the American JI pioneers have been far more important to me, and they have encouraged me to move forward in my endeavor to better understand the phenomenon of tonality and its future potential as a compositional concept. Harry Partch opened up a completely new field with his music, and he instilled in me his passionate love for the great physicist Hermann von Helmholtz, whose groundbreaking book “On the Sensations of Tone as a Physiological Basis for the Theory of Music” inspired me to follow Harry Partch and adopt the principle of non-tempered just intonation as the basis of pitch organization in all my compositions from the past twenty years.

La Monte Young’s radical sonic explorations have also been a major inspiration for my work, since his music has demonstrated early on that it is in fact possible to establish an utterly new and fascinating sound and musical presence simply by applying the old performance practice of just intonation without compromise. I’d like to mention in this context that the composer and violinist Christopher Otto has greatly enriched the repertoire of impressive hard-core JI drone music in recent years.

I am very impressed by Ben Johnston’s solitary and monumental achievement of inventing a whole new harmonic and contrapuntal language based on extended just intonation. I am pursuing such a project myself, trying to develop an effective microtonal counterpoint that is based on the physics of the musical instruments, derived from the involved tuning tasks, and serving to optimize the chances for all the musicians to tune their notes with such a degree of precision that the specific sonority of just intonation may indeed be produced and perceived.

In 1994 I had the great opportunity to meet James Tenney during his residency in Berlin, and his beautiful music, thought and enthusiasm have been very inspiring and encouraging for my ongoing aesthetic reorientation at that time. As a brilliant scholar of psychoacoustics, James Tenney has also supplied us with a universal conceptual model of our brain’s harmonic perception, which he called “harmonic space”. This infinite multi-dimensional matrix arranging all non-tempered pitches in the order suggested by the unisons of their partials, which also contains a so-called “pitch-height projection axis” representing the basilar membrane of the inner ear and the glissando continuum, may also be considered as the classic definition of the JI tuning system – if one wants to understand just intonation as a tone system. I prefer to regard JI as a performance practice based on the simple principle of never tempering the intervals between simultaneous and successive pitches, but I am always thinking in terms of this pitch lattice when composing my melodies and harmonic modulations.

No one has exerted such an important influence upon my work as my friend, the Canadian composer Marc Sabat, whom I met in 2000 shortly after he had settled in Berlin. We conceived and developed a method of staff notation for JI pitch ratios called “The Extended Helmholtz-Ellis JI Pitch Notation” together in 2001, as well as a system for tuning the valve slides of the French horn and the tuba, and we have continued to share our ideas, experiences and the results of our research ever since we got to know each other (like his comprehensive and extremely useful “List of Intervals Tunable by Ear” from 2005, for example).

- LS: Most JI composers organize their pitch content by defining a scale and then choosing combinations from it. How is pitch organized in your works, especially those for brass and strings?
- WvS: Lou Harrison's scales are very beautiful, especially when performed on a diatonic harp, and they still sound as amazingly fresh and new as fifty years ago. – But when I got involved with just intonation twenty years ago, I was very interested in exploring the microtonal aspects of modulation, and so I was thinking that using the concept of a pre-structured scale would be somewhat old-fashioned and too limiting. I prefer to allow my music to freely pick its pitches right out of harmonic space, thus creating various tetrachords, scales or other pitch sets on the fly as needed in each moment.
- LS: Since your op. 39 string trio, many of your pieces have had the words 'etude' or 'study' in the title or subtitle. Is there a pedagogical or investigative aspect to these pieces? Do you see them as primarily intended for public performance? Do they differ from pieces without these designations?
- One of the objectives of all my JI compositions has been the desire to provide the interested performers with some appropriate musical material for exploring the tuning and ensemble playing techniques needed to establish the sound of just intonation. This is why I have called some of my pieces “intonation studies” in the subtitle, but I consider all of them as concert etudes, primarily intended for public performance.
- LS: What general advice do you have for performers interested in your JI works? How did you develop your own skills with hearing and producing these intervals?
- WvS: Refining our capacity of “spectral listening” (focused listening as it were into the sounds, perceiving partial unisons, beats, difference and summation tones, and what Marc Sabat has called the characteristic “periodic signatures” of the interval timbres) can apparently be trained most easily by playing non-tempered double stops on string instruments. – Refining our ear's melodic feeling, e.g. for such subtle melodic differences as that between a major whole tone (9:8, or 204c) and a minor whole tone (10:9, or 182 c), can best be trained by singing. Our voice seems to be equipped with some innate intuitive “body knowledge” regarding the acoustically defined “srutis”, and it feels more secure and immediately involved in the intonation procedure than the instrumentalist's ear, which seems to be as it were a bit further away from the instrument and from the tuning job while trying to react on the spot – perceiving, evaluating and instantly correcting the melodic step created by the pitch that has just been produced, adjusting the embouchure or finger position.

### **The Extended Helmholtz-Ellis JI Pitch Notation**

- LS: When did you first meet Marc Sabat and how did the two of you begin to collaborate on the Helmholtz-Ellis system?

When I met Marc Sabat in 2000, I had already worked out a full-fledged notation system for the harmonics (up to prime number 63), which I used in the score of the original version of my trombone solo piece “JUZ 1999” to notate the pitches of the sound aggregates produced in the instrument by simultaneous singing and playing. – This precursor of the “Extended Helmholtz-Ellis II Pitch Notation” was based on the notation system which Hermann von Helmholtz first introduced in Chapter 16 of his book (with standard accidentals for the 3-limit pitches, and with attached arrows for the 5-limit pitches), and it used quarter sharps and flats for the 11-limit pitches and a number of different additional arrows (specifying pitch deviations from the nearest 5-limit or 11-limit pitch) for the 7-limit pitches and for the pitches of all the other primes. – As Marc Sabat was also very interested in having an optimized alternative to Ben Johnston’s notation system, we sat down together to figure out how we could improve my first attempt.

WvS: With no agreed-upon standard of microtonal notation, was there a great deal of discussion to arrive at the design of the HE accidentals?

It seemed to be desirable to devise easily recognizable logo signs for each prime and to notate as many of them as possible as deviations from the basic set of Pythagorean or 3-limit pitches (to keep the notation of the higher primes legible). Thus we decided to employ the sign used by Giuseppe Tartini for notating the septimal comma  $64/63$  to write up the 7-limit pitches and a third-tone accidental for the alteration  $27/26$  to notate the 13-limit pitches.

LS: Helmholtz uses several approaches to in-text notation the syntonic comma in various editions of *On the Sensations of Tone*: lowercase and capital letters, pluses and minuses, and superscript and subscript numbers. Why did you decide to instead use arrows attached to standard accidentals?

WvS: As plus signs and minus signs in front of the notes are somewhat difficult read in staff notation, the most elegant notation for the syntonic comma seems to be the use of upward and downward arrows attached to the standard, i.e. Pythagorean accidentals. – This straight-forward transcription of the in-text notation used by Helmholtz (and of the modified version used by Ellis in his translation) has the additional advantage that not only the 3-limit pitches, but also all 5-limit pitches are notated with a single accidental in front of the note.

LS: Similarly, the downward or upward sloping lines used by Helmholtz to show septimal and undecimal intervals (and by Bosanquet and others to show the syntonic comma) appear in your system as 19- (single line) and 17-based (double line) alterations. How did you choose this symbol?

WvS: We used the tiny single downward and upward sloping line to notate the very small pitch alteration  $513:512$  (ca. 3.4 cents), by which the 19-limit minor third ( $19/16$ ) deviates from the Pythagorean minor third ( $32/27$ ); and we used the tiny double line to notate the 17-limit schisma  $256/255$  (ca. 6.8 cents), which is the difference between the diatonic semitone  $16/15$  and the 17-limit semitone  $17/16$ . Thus the tiniest signs notate the tiniest pitch alterations; and both the 17-limit and the 19-limit pitches are notated with an

accidental or logo that makes sense in an intuitive way, as these 17-limit and the 19-limit pitches are actually not very far away from the familiar pitches of Equal Temperament.

LS: Ben Johnston was the only other composer to develop a comprehensive system of pitch notation based on the traditional five-line staff. What, in his system, did you see as needing improvement or clarification?

WvS: Harry Partch deplored (somewhere in his book “Genesis of a Music”) the fact that he never managed to invent a method of staff notation for the Extended JI pitch ratios he used in his music, and his pupil Ben Johnston deserves great credit for his historical achievement of developing a well devised, comprehensive system of microtonal JI pitch notation based on the traditional five-line staff. But by being based on the C major scale (rather than on the series of non-tempered fifths, i.e. on the basic 3-limit pitches of the Pythagorean tuning), Ben Johnston’s notation system unfortunately does not fulfill two highly desirable conditions, namely that the same interval between two pitches should always be written in the same way, and that two intervals spelled in the same way should always have the same size. For example: The perfect fifth between D and A is not spelled like most other fifths; and the minor whole tone (10:9) between 3<sup>rd</sup> and the 2<sup>nd</sup> degree of the C major scale is notated with two standard accidentals just like the major whole tone (9:8) between D and C.

LS: When learning microtonal music, many performers may wish to use an electronic tuner to verify their pitch accuracy, but the HE system makes this rather difficult. Did you and Sabat at any time consider a system in which the accidentals alter the equal-tempered pitches? Would such a system even be possible?

WvS: The “Extended Helmholtz-Ellis JI Pitch Notation” generally includes the option to notate any pitch with an additional pitch bend cents-number specifying the pitch in terms of its deviation from the respective pitch of Equal Temperament. (That is why we have included the name of Alexander J. Ellis, who invented the cents, in the name of our notation system.) I’m using these additional pitch bend numbers

In my scores whenever I really want the instrumentalists to employ an electronic tuner at some stage of the rehearsal procedure, for example in order to choose and practice an appropriate microtonal fingering on a woodwind instrument. But if this is not necessary, I consider these pitch bend numbers as useless information in nearly all cases. – Generally, I do not like these electronic tuning devices at all, because they are not precise enough and usually force the musicians to tune the notes by using their eyes instead of their ears.

### **The Plainsound Brass Trio No. 1**

LS: Before your conversations with Matt Barbier, what was your level of interest in writing for brass instruments and why did you write for the unusual combination of horn, trombone, and tuba?

WvS: I had already successfully tried out the u-tonal tuning system for the valve slides of the French horn and the tuba, which I had devised together with Marc Sabat, in two major

compositions (“The Cantata” 2002, and “Plainsound-Symphony” 2003-2005), when the newly founded Trio Kobayashi invited me to write a piece for them. This gave me the most welcome opportunity to continue my microtonal explorations of the brass instruments together with the wonderful musicians of this ensemble.

LS: This piece defies traditional methods of musical analysis. How did you conceive of this work's structure?

WvS: Even though the “Plainsound Brass Trio 1” may often sound quite romantic and free, the piece is actually structured in a very strict and conceptual way. Its form is derived from its harmonic material, which is predetermined by the tuning of the valve slides of the horn and tuba, as well as by the fact that the trigger valve slide of the trombone is tuned in such a way that it can precisely mimic the action of the tuba’s 4<sup>th</sup> valve slide when applied to various tube lengths (lowering the pitch by a perfect fourth in the trombone’s 1<sup>st</sup> position, and by less than a major third in the lowest position). In each of the 18 harmonic regions or episodes of the piece, the trombone’s slide position is carefully tuned to a specific valve combination of the tuba and then remains unaltered for the entire duration of that section. This allows the trombonist to explore all the microtonal melodies that can be played on the instrument simply by employing the thumb valve, without ever changing the slide position, and without having to tune any of these crazy microtonal pitches at all. Both the French horn and the tuba accompany and harmonize the trombone’s voice with their melodic counterpoints; and this is something they can easily do, because the trombonist has taken care to get the slide position precisely tuned at the beginning of each episode.

LS: What specific advice do you have for performers interested in this piece?

WvS: Well, in theory just about every note in this piece should automatically be in tune if it is centered well enough. But in the real world reality, things are much more complicated than that; and it is an enormous challenge for the three players to determine how to best deal with these problems. Which instrument is providing the reference pitch in each instance, and which two instruments must therefore adjust their pitches if a particular chord is not in tune? – Since there is a lot more leeway in the low register, it is the high notes (from harmonics # 6 or 7 upwards) that are most trustworthy, I think. This seems to suggest that the best results may be expected if most of the chords are in fact tuned below the treble voice, instead of being tuned above their bass note.

I think it should also be mentioned that many of the unfamiliar complex harmonic sounds involving the higher partial relationships are of course very hard to tune in any case. It is probably best to simply bang them out, like all the other dissonant chords, articulating the contrasts and the musical flow of the sound progressions, as well as the microtonal contrapuntal melodies that are bringing them about.

APPENDIX C

PLAINSOUND BRASS TRIO SCORE

Wolfgang von Schweinitz

Plainsound Brass Trio 1

18 microtonal variations exploring the trombone's  
trigger valve action at various tuned slide positions

op. 50

2008

*composed for the*  
***Trio Kobayashi***

*Anna Robinson, Matthew Barbier, Lukas Storm*

PLAINSOUND MUSIC EDITON

## Performance Notes

### INTONATION

*This piece is a study exploring new playing techniques for microtonal just intonation. All the valve slides must be carefully tuned according to the instructions given below; and as all notes should be played with the valve combinations specified in the score, they “only” need to be centered in order to produce the intended microtonal pitches. The trombone part is composed in such a way that all notes within each of the 18 sections of the piece are to be played exactly in the same slide position, no matter whether the trigger valve is employed or not; and whenever the slide position is changed (at the beginning of every new section) it must be carefully tuned as specified in the score.*

### TUNING INSTRUCTIONS

#### DOUBLE HORN

*The open F Horn is tuned a pure major third below A, so that its 5<sup>th</sup> harmonic produces 220 Hertz. The open B-flat-Horn is tuned a perfect fourth above the F-Horn, so that its 6<sup>th</sup> harmonic produces the same pitch as the 8<sup>th</sup> harmonic of the open F-Horn. The 2<sup>nd</sup> valve slide of the B-flat-Horn is tuned such that the 4<sup>th</sup> harmonic of the A-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz), and the 2<sup>nd</sup> valve slide of the F-Horn is tuned such that the 8<sup>th</sup> harmonic of the E-Horn produces the same pitch as the 6<sup>th</sup> harmonic of the A-Horn (330 Hz). The 3<sup>rd</sup> valve slide of the F-Horn is tuned such that the 6<sup>th</sup> harmonic of the D-Horn produces the same pitch as the 4<sup>th</sup> harmonic of the A-Horn and the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz). The 3<sup>rd</sup> valve slide of the B-flat-Horn is tuned such that the 6<sup>th</sup> harmonic of the G-Horn produces the same pitch as the 8<sup>th</sup> harmonic of the D-Horn and the 5<sup>th</sup> harmonic of the open B-flat-Horn. Then the 1<sup>st</sup> valve slide of the B-flat-Horn is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (G-Horn), and likewise the 1<sup>st</sup> valve slide of the F-Horn is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (D-Horn).*

#### 5-VALVE F-TUBA

*The open horn is tuned a pure major third below A, so that its 5<sup>th</sup> harmonic produces 220 Hertz. The 4<sup>th</sup> valve slide is tuned such that the 4<sup>th</sup> harmonic of the C-Horn produces the same pitch as the 3<sup>rd</sup> harmonic of the open F-Horn. The 3<sup>rd</sup> valve slide is tuned such that the 6<sup>th</sup> harmonic of the D-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the open F-Horn (220 Hz). The 2<sup>nd</sup> valve slide is tuned such that the 4<sup>th</sup> harmonic of the E-Horn produces the same pitch as the 5<sup>th</sup> harmonic of the C-Horn (115 Hz). Then the 1<sup>st</sup> valve slide is tuned in combination with the 2<sup>nd</sup> valve such that all harmonics of the valve combination 1+2 have the same pitches as those produced with the 3<sup>rd</sup> valve (D-Horn). The 3<sup>rd</sup> valve slide is tuned in combination with the 4<sup>th</sup> valve such that the 6<sup>th</sup> harmonic of the valve combination 4+3 (B-flat-Horn) produces the same pitch as the 4<sup>th</sup> harmonic of the open F-Horn.*

#### TENOR-BASS-TROMBONE

*Once the tuning slide is adjusted such that the harmonics produced in the 1<sup>st</sup> position of the tenor trombone will match those of the open B-flat-Horn, the trigger valve slide is tuned such that the trigger valve will lower the pitch by a perfect fourth in the first position, so that the harmonics produced in the 1<sup>st</sup> position of the bass trombone will match those of the open F-Horn. Thus the trigger valve acts exactly like the 4<sup>th</sup> valve of the tuba. This is demonstrated in the table showing the 18 microtonal slide positions used for the performance of this piece (see below).*

### PERFORMANCE DURATION *circa 25 minutes (without the pauses between the movements)*

*The first performance was given by Trio Kobayashi on June 13, 2009 in Los Angeles at The Wulff.*



# ACCIDENTALS EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION for *Just Intonation*

The exact intonation of each pitch is written out by means of the following harmonically defined accidentals:

$\flat \flat \quad \flat \quad \natural \quad \sharp \quad \times$	Pythagorean series of pure fifths (based on the open strings: – c g a e –)
$\downarrow \flat \sharp \times \quad \uparrow \flat \flat \sharp \sharp$	Lowers / raises the pitch by a syntonic comma $(81/80) = \text{circa } 21.5 \text{ cents}$
$\downarrow \flat \sharp \times \quad \uparrow \flat \flat \sharp \sharp$	Lowers / raises the pitch by two syntonic commas $(81/80)^2 = \text{circa } 43.0 \text{ cents}$
$\flat \quad \natural$	Lowers / raises the pitch by a septimal comma $(64/63) = \text{circa } 27.3 \text{ cent}$
$\flat \text{ or } \flat \flat \quad \natural \text{ or } \natural \natural$	Raises / lowers the pitch by two septimal commas $(64/63)^2 = \text{circa } 54.5 \text{ cents}$
$\dagger \quad \dagger$	Raises / lowers the pitch by an 11-limit undecimal quarter-tone $(33/32) = \text{circa } 33.3 \text{ cents}$
$\dagger \quad \sharp$	Lowers / raises the pitch by a 13-limit tridecimal third-tone $(27/26) = \text{circa } 61.3 \text{ cents}$
$\approx \flat \quad \nearrow \sharp$	Lowers / raises the pitch of the subsequent accidental by a 17-limit schisma $(16/17)^*(16/15) = (256/231) = \text{circa } 6.8 \text{ cents}$
$\nearrow \flat \quad \searrow \flat$	Raises / lowers the pitch of the subsequent accidental by a 19-limit schisma $(19/16)^*(27/32) = (513/512) = \text{circa } 3.4 \text{ cents}$
$\uparrow \sharp \quad \downarrow \flat$	Raises / lowers the pitch of the subsequent accidental by the 23-limit comma $(23/16)^*(8/9)^*(8/9)^*(8/9) = (736/729) = \text{circa } 16.5 \text{ cents}$
$\uparrow \flat \quad \downarrow \flat$	Lowers / raises the pitch of the subsequent accidental by a 29-limit comma $(29/16)^*(3/9) = (345/144) = \text{circa } 12.0 \text{ cent}$
$-\dagger \quad +\dagger$	Lowers / raises the pitch of the subsequent 11-limit accidental by a 31-limit schisma $(32/31)^*(32/33) = (1024/1023) = \text{circa } 1.7 \text{ cents}$
$\{ \approx \dagger \quad \{ \nearrow \dagger$	Lowers / raises the pitch of the subsequent 11-limit accidental by a 37-limit schisma $(36/37)^*(33/32) = (297/296) = \text{circa } 5.8 \text{ cents}$
$\{ \uparrow \sharp \quad \{ \downarrow \flat$	Lowers / raises the pitch of the 5-limit accidental by a 41-limit schisma $(32/41)^*(81/64)^*(81/80) = (6561/6560) = \text{circa } 0.3 \text{ cents}$
$\{ \uparrow \flat \quad \{ \downarrow \flat$	Raises / lowers the pitch of the subsequent accidental by a 43-limit comma $(43/32)^*(3/4) = (129/128) = \text{circa } 13.5 \text{ cents}$
$\{ \flat \sharp \quad \{ \flat \flat$	Lowers / raises the pitch of the 7-limit accidental by the 47-limit schisma $(32/47)^*(48/49)^*(8/2) = (2304/2303) = \text{circa } 0.8 \text{ cents}$
$\{ \approx \sharp \quad \{ \nearrow \flat$	Lowers / raises the pitch of the subsequent 5-limit accidental by a 53-limit comma $(52/53)^*(3/3) = (160/159) = \text{circa } 10.9 \text{ cent}$
$\{ \sharp \quad \{ \dagger$	Lowers / raises the pitch of the 13-limit accidental by a 59-limit schisma $(32/59)^*(24/13) = (768/767) = \text{circa } 2.3 \text{ cents}$
$\{ \dagger \quad \{ \uparrow \flat$	Raises / lowers the pitch of the 7-limit accidental by a 61-limit schisma $(61/32)^*(21/40) = (1281/1280) = \text{circa } 1.4 \text{ cents}$

In addition to the harmonic definition of a pitch by means of its accidentals, it is also possible to indicate its absolute pitch-height as a cents-deviation from the respectively indicated chromatic pitch in the 12-tone system of Equal Temperament.

The attached arrows denoting the pitch alteration by a syntonic comma are transcriptions of the notation that Hermann von Helmholtz used in his book "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863). The annotated English translation "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) is by Alexander J. Ellis, who refined the definition of pitch within the 12-tone system of Equal Temperament by introducing a division of the octave into 1200 cents. – The accidental for the pitch alteration by a septimal comma was devised by Giuseppe Tartini (1692-1770), the composer-violinist and researcher who first studied the production of difference tones by means of tuned double-stops.

# OVERVIEW

## of the 18 microtonal Trombone slide positions used for the performance of this piece

*HORN* : The valve slides 1, 2, and 3 are tuned to the rational proportions  $2/15$ ,  $1/15$ , and  $3/15$  of the open horn's length (both in B-flat and F), producing, in various combinations, two related sets of Utoral chromatic Series of fundamental pitches with wavelengths in the proportions  $15 : 16 : 17 : 18 : 19 : 20 : 21$ .

*F-TUBA* : The valve slides 1, 2, 3, 4, and 5 are tuned to the rational proportions  $2/15$ ,  $1/15$ ,  $3/15$ ,  $1/5$ ,  $5/15$ ,  $1/3$ , and  $5/30$  –  $1/6$  of the open horn's length, producing, in various combinations, a Utoral micro-chromatic Series of fundamental pitches with wavelengths in the proportions  $30 : 32 : 34 : 35 : 36 : 37 : 38 : 39 : 40 : 41 : 42 : 43 : 44 : 45 : 46 : 47 : 48 : 49 : 50 : 51 : 52 : 53 : 55 : 57$ .

*Tenor/Bass-TROMBONE* : As the trigger valve slide is tuned to produce a perfect fourth in slide position 1 between B-flat and F, it acts exactly like the 4th valve of the Tuba.

The musical score is divided into four staves, each representing a different instrument. The Horn staff (B-flat and F) shows 18 microtonal slide positions with frequency ratios and valve settings. The F-Tuba staff (B-flat and F) shows 18 microtonal slide positions with frequency ratios and valve settings. The Tenor Trombone staff (B-flat and F) shows 18 microtonal slide positions with frequency ratios and valve settings. The Bass Trombone staff (B-flat and F) shows 18 microtonal slide positions with frequency ratios and valve settings. The frequency ratios are listed below the staves, and the valve settings are indicated by numbers 1 through 5.

**Horn**

B $\flat$ : 15/15, 16/15, 17/15, 18/15 = 8/5, 19/15, 20/15 = 4/3, 21/15 = 7/5

F: 15/15, 16/15, 17/15, 18/15 = 8/5, 19/15, 20/15 = 4/3, 21/15 = 7/5

**F-Tuba**

without valve 4: 30/30, 32/30=16/15, 34/30, 35/30=7/6, 36/30=6/5, 37/30, 38/30, 39/30=13/10, 40/30=4/3, 41/30, 42/30=7/5, 43/30, 45/30=3/2, 47/30

with valve 4: 40/30=4/3, 42/30=7/5, 44/30=11/10, 45/30=3/2, 46/30=23/15, 47/30, 48/30=8/5, 49/30, 50/30=5/3, 51/30=17/10, 52/30=28/15, 53/30, 55/30=11/6, 57/30=19/10

**Tenor Tromb.**

15/15 = 30/30, 45/45 = 49/48, 50/48, 48/45 = 46/45, 50/45 = 51/45, 35/30 = 16/15, 55/45 = 58/45, 57/45 = 39/30, 20/15 = 41/30, 65/45 = 64/45, 65/45 = 13/6 (TUBE LENGTH=5)

**Bass Tromb.**

20/15 = 64/48, 65/48 = 33/24 = 7/5, 21/15 = 64/45, 65/45 = 22/15, 45/30 = 80/45, 70/45 = 71/45, 24/15 = 49/30, 25/15 = 51/30, 78/45 = 79/45, 80/45 = 15/9

**T&B trigger valve frequency ratio:** 4/3, 65/49, 33/25, 21/16, 64/49, 13/10, 22/17, 9/7, 23/18, 14/11, 71/53, 24/19, 49/39, 5/4, 51/41, 26/21, 75/54, 16/13

# Microtonal Pitch Repertoire of the Double Horn

## HORN TUNING

The valve slides 1, 2, and 3 are tuned to the rational proportions  $2/15$ ,  $1/15$ , and  $3/15$  of the open horn's length (both in B-flat and F), producing, in various combinations, two related sets of Uttoni chromatic Series of fundamental pitches with wavelengths in the proportions  $15 : 16 : 17 : 18 : 19 : 20 : 21$

All pitches sound a fifth lower than written.

The musical score is organized into two systems of staves, each containing seven staves numbered 15 through 21. The top system uses treble clefs, and the bottom system uses bass clefs. Each staff contains musical notation with various accidentals and numerical values indicating pitch adjustments. The notation is complex, featuring many accidentals and numerical values above and below the notes. The values are often in the form of fractions or integers, such as  $1/15$ ,  $2/15$ ,  $3/15$ ,  $15$ ,  $16$ ,  $17$ ,  $18$ ,  $19$ ,  $20$ , and  $21$ . The score is a detailed representation of the microtonal pitch repertoire of the double horn, showing the relationship between the fundamental pitches and the resulting microtonal series.

# Microtonal Pitch Repertoire of the 5-Valve F-Tuba

with valve slides 1, 2, 3, 4, and 5 tuned to the rational proportions  $2/15$ ,  $1/15$ ,  $3/15 = 1/5$ ,  $5/15 = 1/3$ , and  $5/30 = 1/6$  of the open horn's length

The score displays microtonal pitch repertoire for a 5-valve F-tuba. The staves are organized into groups of five, representing different valve combinations. Each staff shows notes with microtonal adjustments in cents, indicated by numbers above the notes. The adjustments are as follows:

- Staff 1 (Valve 1): -13.7, 2, 13.7, 3, -15.6, 4, -13.7, 5, -9.0, 6, -15.6, 7, 17.5, 8, +13.7, 9, +17.6, 10, 11, 12, 13, 14, 15, 16
- Staff 2 (Valve 2): -7.0, +2.0, +4.0, +2.0, -11.7, -3.9, -29.2, +2.0, +5.9, -16.7
- Staff 3 (Valve 3): 3.0, 3.0, 1.0, 3.0, -16.7, -1.0, -34.2, -3.0, +0.9, -16.7
- Staff 4 (Valve 4): -46.9, -46.9, +48.8, +48.8, +33.1, -48.8, +15.6, +46.3, +50.7, +33.1
- Staff 5 (Valve 5): 2.0, 2.0, +6.0, 2.0, -15.6, +6.0, -33.1, -2.0, +2.0, -15.6
- Staff 6 (Valve 1+2): 49.4, 49.4, -51.4, -49.4, -63.1, 51.4, -51.4, -49.4, -45.5, -63.1, -1.9
- Staff 7 (Valve 1+3): -4.4, +4.4, +6.4, +4.4, -9.2, -6.4, -26.7, +4.4, +8.4, -9.2, -55.7
- Staff 8 (Valve 1+4): +59.4, +59.4, +61.4, +59.4, +45.7, -41.4, +28.3, +59.4, +63.4, +45.7, +110.7, +61.4
- Staff 9 (Valve 1+5): +15.6, +15.6, +17.6, +15.6, +2.0, -17.6, -15.6, +15.6, +19.6, +2.0, +66.9, +17.6
- Staff 10 (Valve 2+3): -27.2, -28.2, -27.2, -40.8, -25.2, -58.3, -27.2, -23.2, +9.8, +24.7, -25.2
- Staff 11 (Valve 2+4): +33.1, +31.2, +17.5, -33.1, +0.0, +31.2, +35.1, +17.5, +32.5, +33.1
- Staff 12 (Valve 2+5): -7.6, -9.6, -23.3, -7.6, -48.8, -9.6, -5.7, -23.3, -41.8, -7.6, -69.1
- Staff 13 (Valve 3+4): -9.6, -51.4, -49.4, -63.0, -51.4, -38.5, -49.4, -45.5, -63.0, +2.0, -51.4, -100.3
- Staff 14 (Valve 3+5): +13.7, +11.7, -2.0, -13.7, -19.5, +11.7, +15.6, -2.0, -63.0, +13.7, -47.7
- Staff 15 (Valve 4+5): +11.7, -24.4, -26.3, -40.0, -24.4, -57.5, -26.3, -22.4, -40.0, +25.0, -24.4, -85.8
- Staff 16 (Valve 1+2+3): -26.3, -61.6, -63.6, -77.8, -61.6, -94.8, -63.6, -59.7, -77.8, -12.3, -61.6, -123.0, -94.8
- Staff 17 (Valve 1+2+4): -63.6, +2.0, +0.0, -13.7, -2.0, -31.2, +0.0, -13.7, +51.4, -2.0, -58.5, 31.2
- Staff 18 (Valve 1+2+5): +6.0, -66.3, +64.3, +50.6, -66.3, +33.1, +64.3, +68.2, +50.6, -115.6, -66.3, +4.0, +33.1
- Staff 19 (Valve 1+3+4): -64.3, +31.3, +29.3, +15.6, -31.3, -1.5, -29.3, +33.2, +15.6, +50.6, +31.3, +30.1, +17.6
- Staff 20 (Valve 1+3+5): -29.3, -3.0, -5.0, -18.7, -3.0, -36.2, -5.0, -1.0, -18.7, +46.3, -3.0, +35.6, -36.2, -16.7
- Staff 21 (Valve 1+4+5): -5.0, +63.4, +61.4, +47.7, -63.4, +30.2, +61.4, +65.4, -47.7, -112.7, +63.4, +2.0, +30.2, +49.7
- Staff 22 (Valve 2+3+4): -61.4, +30.4, +28.4, +14.7, -30.4, -2.8, +28.4, +32.4, +14.7, +79.7, -30.4, -31.0, -2.8, +18.7, -28.4
- Staff 23 (Valve 2+3+5): +28.4, -33.7, -35.7, -49.4, -33.7, +66.9, -35.7, -31.7, -49.4, +15.6, 33.7, -95.2, +66.9, -51.4, -35.7
- Staff 24 (Valve 2+4+5): -35.7, +4.4, +2.4, -11.2, +4.4, -28.7, +2.4, +6.4, -11.2, +53.8, -4.4, -57.0, -28.7, -9.2, -2.4
- Staff 25 (Valve 3+4+5): +2.4, -2.4, -11.2, +4.4, -28.7, +2.4, +6.4, -11.2, +53.8, -4.4, -57.0, -28.7, -9.2, -2.4

# Plainsound Brass Trio 1

18 microtonal variations exploring the trombone's trigger valve action at various tuned slide positions

## REGION I

*lento rubato* (ca. 52 - 54) \*

Wolfgang von Schweinitz  
op. 50 (2009)

notation in C

T: B = 4:3

Tune the first slide position

Horn

Trombone

Tuba

pp

p

28 : 27 (- 63 c)

21 : 20 (- 84 c)

\* The performers may add a short fermata to any note or rest at their discretion.

5

1

20 : 21 (+ 84 c)

15 : 16 (+ 112 c)

28 : 21 12

pp

p

21 : 20 (- 84 c)

28 : 27 (- 63 c)

pp

p espr.

9

2

64 : 63 (- 27 c)

11 : 10 2 1

11 : 10 2 1

espr.

sf

cresc.

15 : 14 (- 119 c)

33 : 32 (- 53 c)

espr.

sf

cresc.

13

3

13 : 8 5

65 : 64 (- 27 c)

Tune the new slide position

T: B = 65:49

51 : 52 (+ 34 c)

5 : 4 7 : 4

5 : 4 7 : 4

f

ff

pp

mf

ff

p

f

8 : 13

39 : 40 (+ 44 c)

ff

pp

mf

ff

p

f

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111

34 8 11/4 11/4 11/4 11/4 11/2

*pp* *mf* *p* *espr.* *f*

*espr.* *pp* *mf* *p* *f* *p*

132-125 (-94 c)

SLIDE PAGE

*p* *espr.*

38 11/5 5/3 5/4 9 11/4 11/4 11/4 11/4 11/7 5/3

*p* *espr.* *pp* *p* *mf* *p* *mf* *p*

*espr.* *pp* *p* *mf* *p* *mf* *p*

5/4 2/1 99:100 (+17 c) B 6:7 35:33 (-192 c)

*p* *pp* *p* 10:9:10

42 10 11/4 7/4 8/5 25/16 25/16 25/16

*ff* *ff* *mf* *f* *ff*

*pp* *molto crescendo* *ff* *ff* *p* *mf* *f* *ff*

3/2 (+17 c) 75:77 (+46 c) T B 132:125 (-94 c)

*p* *pp* *ff* *ff* *p* *mf* *f* *ff*

46 11/8 5/4 11 4/3 12 16:15

*f* *p* *dolce* *pp* *p*

*f* *p* *dolce* *pp* *p*

10:11 (+165 c) 16:15 (-112 c) 63:64 (+27 c)

*pp* *pp* *p* *pp* *espr.*

SLIDE PAGE

SLIDE 2 PAGES

same slide position

63 : 64 (+ 27 c) 20 : 21 (+ 84 c) 48 : 49 (+ 36 c) 63 : 64 (+ 27 c) **13**

51

*p marc.* *mp* *pp* *p* *espr.*

*p marc.* *mp* *pp* *p* *espr.*

*p marc.* *mp* *pp* *p* *espr.*

21 : 20 (- 84 c)

35 : 14 7 : 4 32 : 33 (+ 53 c) 24 : 25 (+ 71 c) **14** 20 : 21 (+ 84 c)

55

*f* *p* *cresc.* *f* *p* *dolce* *pp*

*f* *p* *cresc.* *f* *p* *dolce* *pp*

*f* *p* *cresc.* *f* *p* *dolce* *pp*

5 : 6 5 : 7 21 : 25 (+ 392 c)

35 : 36 (+ 49 c) 39 : 40 (+ 44 c) 49 : 48 (- 36 c) 12 : 23 8 : 28 6 : 106 7 : 21 24 : 81 5 : 21 6 : 56 **15** 8 : 5

59

*espr.* *f* *sub. p* *mf* *p* *ff* *ff* *p cresc.*

*espr.* *f* *sub. p* *cresc.* *mf* *p* *ff* *ff* *p cresc.*

*espr.* *f* *sub. p* *cresc.* *mf* *p* *ff* *ff* *p cresc.*

20 : 21 (+ 84 c) 48 : 49 (+ 36 c) 49 : 48 (- 36 c)

11 : 5 3 9 : 5 4 7 : 4 3 63 : 64 (+ 27 c) **SLIDE 2 PAGES**

63

*f* *p* *mp* *p* *pp* *p*

*f* *p* *mp* *p* *pp* *p*

*f* *p* *mp* *p* *pp* *p*

22 : 21 (- 81 c) 16 : 15 (- 112 c) 16 : 15 T:B = 21:16 Use the new slide position (- 76 c) 49 : 48 (- 36 c) **SLIDE 2 PAGES**



16

67

27 : 28 (+63 c) 49 : 48 (-36 c) 7 48 : 49 7 9 7 6 4 (+36 c) 4 6

(+76 c) 48 : 49 (+36 c) *espr.* *p* *espr.* *mf* *mf*

17

71

13 7 7 48 : 49 7 9 7 6 91 : 96 3 (+93 c) (+36 c) 3 3 63 : 64 3 (+27 c) 3 63 : 64 3 (+27 c) 48 : 49 (+36 c)

*f* *ff* *pp* *p marc.* *espr.*

(+195 c) (+36 c) 63 : 64 (+27 c) T B T SLIDE PAGE 12/15 7/13 7/14 7/14 7/14

18

75

7 5 1 8 : 7 (-119 c) 16 14 11 10 7 8 63 : 64 7 6 (-27 c) (+27 c) 33 28 12 (+36 c) 9 7 3 (-36 c) (+36 c)

*pp* *p cresc.* *mf* *f* *più f*

same slide position T (-76 c) B (+76 c) T (+89 c) B (+113 c) T (+36 c)

*pp* *p cresc.* *mf* *f* *più f*

(-45 c) (+195 c) (+36 c) (-36 c) (+36 c)

19

79

9 7 3 48 : 49 21 : 20 (+36 c) (-84 c) 49 : 48 (-36 c) 7 : 10

*ff* *p* *pp* *p* *pp* *p*

SLIDE PAGE 7/12 7/12 5/11 7/11 7/11 7/12 5/11 same slide position 50 : 49 (-35 c) T B = 13 : 10 Tune the new slide position

*ff* *p* *pp* *p* *pp* *p*

48 : 49 (+36 c) 49 : 48 (-36 c) 7 : 5 (-35 c) 50 : 49 (-35 c)

*ff* *p* *pp* *p* *pp* *p*

6

40 13 20 13 16 20  
15 5 11 5 5 15  
12 4

83

*espr.* *poco f* *p* *cresc.* *f* *mf*

39 : 40  
(+44 c) B

13 : 4 13 : 2 13 : 8 13 : 7 13 : 8

*espr.* *poco f* *p* *cresc.* *molto f* *mf*

39 : 40  
(+44 c)

*espr.* *poco f* *mf* *f* *mf*

21

88

8 : 5 8 : 5 6 : 5 12 : 5 7 : 3

*p* *pp* *p* *espr.* *mf* *f* *p* *espr.*

39 : 40  
(+44 c) B

8 : 5 25 : 28  
(+68 c) T

90 : 91  
(+19 c)

13 : 8 B

*p* *pp* *p* *espr.* *mf* *f* *p* *espr.*

39 : 40  
(+44 c)

(-114 c)

26 : 25  
(-68 c)

*p* *pp* *p* *espr.* *mf* *f* *p* *espr.*

(+44 c) (+68 c)

23

93

13 13 13 13 13 13 15 40 13 13 15  
8 8 8 8 8 8 8 24 8 8 14  
5 5 5 5 5 5 5 15 5 5 6

*f* *intenso* *mf* *p* *espr.* *mf*

14 : 13 : 12

(+68 c) (+44 c) (+19 c) T

*f* *intenso* *mf* *p* *espr.* *mf*

(-114 c)

14 : 13 : 12

(+44 c) (+68 c)

*f* *intenso* *mf* *p* *espr.* *mf*

24

97

13 16 12 16 24 16 22 9 16 6  
12 13 10 13 20 13 13 6 13 6  
10 9 10 10 15 10 10 4 10 4

*f* *pia f* *ff* *fff* *fff*

(+53 c)

B (+68 c) T (+97 c) B (+107 c) T B T (+44 c) B T

*f* *pia f* *ff* *fff* *fff*

(+68 c) 12 : 13 : 10 (+97 c) (+68 c) (-68 c)

*f* *pia f* *ff* *fff* *fff*

SIDE 2 PAGES

SIDE 2 PAGES

SIDE 2 PAGES

# REGION II

lo stesso tempo (ca. 52-54) 51:50 (-34 c)

101

pp p pp p espr. mf

T.B = 22:17

Tune the new slide position

51:50 (-34 c) 5/2 5/2 pp p espr. mf

33:34 (+52 c) B

pp p pp p espr. mf

25

105

f più f ff (+165 c) 17:16 (-165 c)

f più f ff (+165 c) 10:11 (+165 c) 12:11 (-151 c) 17:16 (-165 c)

f più f ff

26

109

f mf dim p pp espr.

f mf dim p pp espr.

f mf dim p pp espr.

(-60 c) B

(-60 c)

27

113

SLIDE next PAGE

p pp (-60 c) 33:34 (+52 c) 5/3 8/5

p pp p espr. mf p marc.

p pp p espr. mf p marc.

(-19 c) p marc.

8

28

117

4/3 4/3

(+11 c)

*f* *ff* *f* *mf cresc.* *f* *più f*

16:17 (+105 c)

12 11 28 7 11 13  
9 24 21 6 5 51:52 6  
(+34 c)

29

121

11 11 11 12 24:25 45:44 11:10 28:27  
6 6 4 7 (+71 c) (-39 c) (-165 c) (-63 c)

*ff* *f* *mf* *p* *cresc. molto espr.* *ff* *mf* *p*

SLIDE PAGE

45:44 (-39 c)

11/4 11/4 11/4 11/4 11/3 11/2 11/1 11/1 7/3 27/14

15:16 24:25 45:44  
(+112 c) (+71 c) (-39 c)

30

126

21:20 (-84 c)

*pp* *p* *pp* *espr.* *p* *pp* *ppp* *p dolce*

Tune the new slide position  
T:B=9:7 B (-39 c)

7/6 *pp* *p* *pp* *espr.* *p* *pp* *ppp* *p dolce*

27:28 (+63 c) 35:36 (+49 c)

31

131

14:15 5:6:7 20:21 24:25 35:36 8 11 12 24 8  
(+119 c) (+84 c) (+71 c) (+49 c) 5 9 10 10 27:28 5  
(+119 c) (+49 c) (+116 c) (+88 c) (+63 c)

*p* *pp* *mp* *espr.* *mf* *f* *più f* *p*

14:15 14:15 35:36 14:15 12 24 8  
(+119 c) (+119 c) (+49 c) (-119 c) 5 9 10 10 27:28 5  
(+119 c) (+49 c) (+116 c) (+88 c) (+63 c)

*p* *pp* *mp* *espr.* *mf* *f* *più f* *p*

14:15 35:36 14:15 12 24 8  
(+119 c) (+49 c) (-119 c) 5 9 10 10 27:28 5  
(+119 c) (+49 c) (+116 c) (+88 c) (+63 c)

*p* *pp* *mp* *espr.* *mf* *f* *più f* *p*

32 21:20 15:14 21:20 21:20 7/4 7/4 7/4 9

136 *f* *mf* *mp* *p* *più p* *pp*

SLIDE 2 PAGES

33 26 24 21

142 SLIDE PAGE

SLIDE PAGE

146 8/7 6

*mf* *p* *pp* *p* *mf* *pp*

*mf* *p* *pp* *p* *mf* *pp*

*mf* *p* *pp* *p* *mf* *pp*

34 15 23 23 23 23 18 18  
10 10 12 6 8 8  
6 6 6 3 3 3

150 *p* *espr.* *p* *pp*

Tune the new slide position 46:45 (-39c)

1:1 *p* *espr.* *p* *pp*

46:45 (-38c) 24:23 (-74c)

*p* *espr.* *p* *pp*





[illegible]



200

9 6  
7 5  
4 4

16:19 (+298 c) 19:18 (-94 c) 9 9 24 9 24  
7 7 19 7 16 6 7 16

49

*mf* *p* *mf* *p* *f* *ff* *mf*

19:20 B (+89 c) T 19:18 (-94 c) T 56:57 (+31 c) B (-31 c) T (+31 c) B

*mf* *p* *mf* *p* *f* *ff* *mf*

16:19 (+298 c) 19:18 (-94 c)

*mf* *p* *mf* *p* *f* *ff* *mf*

204

7 19 28 56:57 28 46:49 8 21:20 15:14 19:18 11 12 19  
4 16 21 (+31 c) 24 21 (+36 c) 6 (-84 c) (-119 c) (-94 c) 7 19:18 7 21:20 10  
3 10 21 (+31 c) 24 21 (+36 c) 6 (-84 c) (-119 c) (-94 c) 4 (-94 c) 4 (-84 c) 9

50

*p* *pp* *p* *espr.* *mf*

(+31 c) B T (+31 c) B (+18 c) T (+34 c) B (+94 c) B

*p* *pp* *p* *espr.* *mf*

56:57 (+31 c) 19:18 (-94 c) 96:95 (-18 c) 15:16 (+112 c) (-94 c)

*p* *pp* *p* *espr.* *mf*

208

40 18:19 13 26 22 96:95 14  
15 (+94 c) 5 19:18 (-94 c) 24 3 (-18 c) 5 2

SLIDE 2 PAGES

*f* *più f* *ff*

38:39 B (-94 c) T 132:133 B T B

*f* *più f* *ff*

18:19 (+94 c) (-94 c) 18:19 (+94 c)

*f* *più f* *ff*

SLIDE 2 PAGES

SLIDE 2 PAGES

13 8 5 3 2 7 6 4 54 6 4 11 7 4 11 7 5 11 9 2 10 10 3

223

*p* *pp* *p espr.* *mf*

*Tune the new slide position*

40 : 39 (-44 c) T: B = 5:4 T (-44 c) B 14 15 (+119 c) T B T B 4:5 T 7:11 B (-97 c)

*p* *pp* *p espr.* *mf*

25 : 26 (+68 c) 35 : 36 (+49 c)

*p* *pp* *p espr.* *mf*

226

13 10 3

11 6 5

7 5 3

12 10 7

7 6 4

55

26:25 (-68 c)

25:24 (-71 c)

45:44 (-39 c)

36:35 (-49 c)

21:20 (-84 c)

*f*

*più f*

230

56

7:5

*ff*

*p dolce*

*espr.*

*molto espr.*

15:14 (-119 c)

28:25 (-196 c)

25:24 (-71 c)

20:27

28:25 (-196 c)

*ff*

*p dolce*

*espr.*

*molto espr.*

234

57

SLIDE 2 PAGES

*mf*

*p*

*molto p*

SLIDE 2 PAGES

*mf*

*p*

238

*pp*

*più p*

*pp*

*pp*

58  
242

51:50 (-34 c)

28:27 (-63 c)

*p* *pp* *p*

SLIDE 2 PAGES

51:50 (-34 c)

5/2 7/2 7/2 5/1 5/2 5/1 5/1 5/2 9/2 7/2 5/2 7/3 9/5

49:50 (+35 c) 50:49 (-35 c) 51:50 (-34 c)

59  
246

9 21 16 10 3 8 5 4 3

(-93 c) (+6c) (+6c) (+6c)

T.B = 51:41

Tune the new slide position

B (-34 c)

3 3

(-205 c)

(-822 c)

*pp* *p* *pp* *p* *pp* *p* *pp* *p*

60  
250

12 9 7 17 12 9 21 17

8 8 6 12 6 5 14 8

5 6 5 5 7 3 12 6

(-111 c) (-105c) (-120c) (+6c) (-111 c) (-216 c)

(+57c) B (-120c) (+205c) B

204:205 (+8 c)

(+822c)

*ff* *mf* *p* *espr.* *ff* *mf* *p* *espr.*

17 3 or 4 18 12 5 40 40 5

11 (-21 c or -1221 c) 2 3 12 8 4 24 24 4

6 1 2 5 5 3 15 15 3

42:41 (-42 c)

253

ossia: *f* *molto p* *dolce* *p* *espr.*

(+30 c)

T 42:41 T.B = 28:21

Tune the new slide position

T (-42 c)

*f* *molto p* *dolce* *p* *pp* *p* *espr.*

(-120 c)

42:41 (-42 c)

*f* *molto p* *dolce* *p* *espr.*

61 7 --> 49:48 6:7 26:27 15 9:8 "10:9" 5 9:8 8 "16:15" 9 6:7 17  
 4 --> 3 --> (-36 c) (-63 c) 2 (-11c) -11c -4.5c 4.5c 4.5c  
 257 U... U... U... U... U...  
 pp cresc. ff espr. p  
 T pp cresc. mf p  
 5:3 (-103c) B  
 pp cresc. mf p

62 13 13 13 13  
 5 4 4 2  
 261 mf p p pp ossia: espr. dolce pp  
 SLIDE PAGE  
 mf p 13/4 --> start sliding page here  
 mf espr. p pp espr. dolce pp

63 13 13 --> or 21 23 28 21  
 9 5 8 --> 14 21 21 14  
 6 --> 6 12 12 12  
 265 p pp p cresc. ossia: mf dolce  
 13/4 B  
 p pp p cresc. mf dolce  
 SLIDE PAGE  
 p cresc. mf dolce

64 5 7 12 7 7 10 13 10 7 13 14 5 13  
 4 4 3 11 6 3 9 12 7 11 12 4 12 7  
 3 3 7 4 3 6 7 6 4 7 6 5 3 7  
 268 p cresc. f p mf p pp espr.  
 20:21 B T B T (-54 c) B T (-22 c) B T (+17 c) B  
 p (+84 c) 6:7 cresc. f p mf p pp espr. p  
 p cresc. (-103 c) f p mf p pp espr. p  
 5/4

18

**65**

272

13 13 13 8 7 13  
8 7 4 5 3 8 6  
63:64 (+27 c) 15:14 (-119 c) 16:15 (-112 c) 64:63 (-27 c)

27:25 13:12 14:13 52:49 147:130 65:63  
(-65 c) (-138c) (-128c) (-103c) (-213 c) (-54 c)

*mf* *f* *mf* *p* *marc.* *più p*

9/17 4/13 → *espr.* 7/15 *marc.* 4/13 4/13 *mf* *f* *mf* *p* *marc.* *più p*

104:105 (+17 c) 64:63 (-27 c)

105:104 (-17 c) 64:63 (-27 c)

(-128c) (-138c) (-128c) (-103c) (-84c) (-128c) (-54 c)

*espr.* *marc.* *mf* *f* *mf* *p* *marc.* *più p*

**66**

276

*pp* *pp* *p*

T (-27 c) Tune the new slide position (-49 c)

SLIDE PAGE

**67**

280

mf *f marc.* *più f* *ff* *f*

3/13 13/14 8/11 80:79 (-22 c) 13/14 16/15 16/7 13/8 2/1 10/5 6/3

*mf* *f marc.* *più f* *ff* *f*

13/4 13/4 5/6

**68**

284

*f* *ff* *sfz* *mf* *p* *pp*

8/7 80:79 (-22 c) 21:20 (-84 c) 80:79 (-22 c)

*ff* *f* *ff* *sfz* *mf* *p* *pp*

**69**

SLIDE 2 PAGES

128

304

*f* *p* *mf*

308

75

13/4 13/4

21:20 (-84c) 20 12 13 13 13 13

10 10 7 2

*f* *mf* *p* *espr.* *f* *mf* *p* *mf*

13:8 T 13:14 (+128c) B T B 65:32 (-1173c) T

SLIDE PAGE

4:7 39:40 (+44c)

*f* *mf* *p* *mf*

312

76

13 13 12 40 13 15:14 28:27 (-119c) (-63c)

4 64:65 (+27c) 5 4 4 24 15 5 4

*f* *ff* *f* *espr.* *p dolce* *espr.*

64:65 (+27c) B 39:40 (+44c) T

SLIDE PAGE

13/8 → 15:14 28:27 (-119c) (-63c) 13/4 or 13/8 →

64:65 (+27c) 39:40 (+44c)

*f* *ff* *f* *espr.* *p dolce*

316

77

13 13 13 40 11 12 13 13 13 13 13 13 →

4 6 12 8 5 24 7 9 7 8 8 7 8 →

*espr.* *molto espr.* *f intenso* *più f* *mf*

(-44c) B (+93c) T 14:13 (-128c) 13:12 (-139c)

*p* *espr.* *molto espr.* *f intenso* *più f* *mf*

(-44c) (+44c) 14:13 (-128c) 13:12 (-139c)

*espr.* *molto espr.* *f intenso* *più f* *mf*



319

13 13 8 9 20 13 13 13 15 21  
6 6 5 5 3 7 3 15 6 8 5 5 3 10 6

78

64 : 65 (+27 c) B 13 : 6 T 8 : 13 B

40 : 39 (-44 c) (+112 c) (-71 c) (-68 c) 8 : 13

*espr.* *p* *espr.* *mf* *f* *ff* *mf*

323

28 13 40 7 : 6 (-84 c) *largo* (ca. 48)

18 8 24 15

*ritenuto* (+119 c) *ritardando* *espr.* *p* *pp*

(-223 c) (-44 c) *espr.* *p* *pp*

8 : 5 8 : 5 17 : 12 7 : 6 7 : 4 4 : 7

(-68 c) (-44 c) (-105 c) (-50 c) (-119 c) (-112 c) (-84 c)

*espr.* *p* *pp*



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