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by

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TABLE OF CONTENTS

SIGNATURE PAGE iii

TABLE OF CONTENTS iv

VITA…… vi

ABSTRACT OF THE DISSERTATION vii

CHAPTER ONE INTRODUCTION: THEORETICAL VIRTUES 1

1 The Ontological Question Reformulated 1

2 Ontological Commitments in Mathematics and Modal Theories 6

CHAPTER TWO COUNTERFACTUALS AND POSSIBLE WORLDS 11

1 Kinds of Conditional Statements 12

2 Early Analysis of Counterfactuals 20

3 Propositional Modal Logic 36

4 Semantics for Modal Systems 42

CHAPTER THREE REALISM VS. CONSTRUCTIVISM 52

1 Platonist Interpretation of Mathematics 54

2 An Alternative to Platonism 61

3 Problems with Platonism 66

CHAPTER FOUR A REALIST VIEW OF POSSIBLE WORLDS 72

1 Counterpart Theory and an Indexical Notion of Actuality 74

2 Truth Conditions for Counterfactuals (Lewis) 79

3 Advantages of Lewis' Theory 87

4 Lewis' Quantified Modal Logic 89

5 The Lack of Empirical Data 95

CHAPTER FIVE NICHOLAS RESCHER: A CONSTRUCTIVIST VIEW 99

1 Mind-Dependent Existence of Possible Objects 99

2 Construction of Preferred Maximal Mutually Consistent Sets 103

3 Kinds of Properties and Essentialism 108

4 Rescher's Quantified Modal Logic 115

5 Truth Condition for Counterfactuals (Rescher) 124

CHAPTER SIX AN ONTOLOGY OF IDEAL CONSTRUCTIONS 132

1 Theoretical Virtues of Realism 132

2 Objections to Realism 135

3 Objections to an Ontology of Idealized Operations? 142

4 The Choice between Rival Theories 151

BIBLIOGRAPHY 160

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ABSTRACT OF THE DISSERTATION

An Actualist Ontology for Counterfactuals

by

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Doctor of Philosophy in Philosophy

University of California, San Diego, 2008

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What theory along with its ontological commitments should we adopt? David Lewis and Nicholas Rescher offer competing theories for the interpretation of counterfactuals and modality involving statements. Using Quine's criterion for theory selection, this paper will argue that, when one weighs the various merits and difficulties of these two theories, it appears that a constructivist theory like Nicholas Rescher's offers a greater balance of theoretical advantages than does David Lewis' realist view.

I begin by surveying the early attempts made by logicians to understand counterfactuals and modal statements and then proceed with a similar inquiry into a mathematician's efforts at interpreting mathematical statements. I try to show how similar ontological problems arise in the histories of both disciplines and how both disciplines first attempt to meet these difficulties by espousing a realist perspective, that of mathematical Platonism and David Lewis' possibilism. I then argue that a host of problems may be avoided and a number of metaphysical questions may be answered if we adopt a constructivist approach to modal and mathematical statements. With respect to mathematical statements, Philip Kitcher offers a constructivist theory which interprets mathematical statements as referring to idealized constructions of an ideal mind rather than to abstract Platonic entities.

This paper concludes that we should adopt Rescher's actualist theory over David Lewis' possibilism and shift our ontological commitments from possible worlds to ideal constructions. Accordingly, modal statements are true in virtue of the constructions which the mind is ideally able to perform upon the actual physical world. Among its theoretical virtues, Rescher's theory explains the truth of counterfactuals in connection with ordinary experience, allows for the continued use of a uniform semantics for modal discourse, and avoids certain epistemological problems which rival theories face.

CHAPTER ONE INTRODUCTION: THEORETICAL VIRTUES

1 The Ontological Question Reformulated

What kinds of objects are there? It's less problematic to be a realist about physical objects such as tables and chairs. Through our sense perception of the world, we have evidence that such macroscopic, material things exists. It is a more challenging task, however, to be a realist with respect to the kinds of objects our scientific, mathematical, and modal theories suppose. Is science right to assume the existence of unobserved entities such as quarks and electron-spins? Do numbers or sets actually exist as some mathematicians may presume? Along similar lines, we may also question our various modal theories and their ontological commitments to the existence of possible worlds. Ontological claims about the existence of abstract objects like numbers or possible worlds are neither directly verifiable nor falsifiable. We cannot simply check to see whether such objects really do exist. Nor can we gather physical evidence to prove the contrary.

Notwithstanding, our theories, even our best scientific ones, may sometimes allow for the existence of unobservable objects. Scientists have proposed the existence of entities that cannot be directly seen nor detected and about which they know very little. For example, physicists consider whether their theories should include quarks. Our best theories of the proton and neutron, they argue, indicate that there are three particles inside every proton and neutron. Although these particles cannot be observed, their existence accords with the evidence we do have and reference to 'quarks' would help to explain various other phenomena. It has been argued that an object's explanatory power is reason enough to suppose that such an object exists.

Scientific explanation also makes use of ideal models that offer a more simplified and orderly system of the world than may actually exist. Newton recognized the operations of three laws of motion and the law of force which pull two bodies together according to their mass and the distance between them. Although Newton devised a system of the world subject to these four laws he did not claim that the real world was exactly like the model he constructed. The laws of Newtonian mechanics are to be understood as describing the behavior of idealized particles in idealized inertial systems. Today, scientists tend to view Newtonian world-models as mathematical structures which correspond to the phenomena of physical motion at a certain level. Newton imagined that there are likely other unknown forces operative in the world in accord with his laws.

One school of thought considers such ideal models acceptable even if all the elements of the model do not have perfect correspondence with the real world. Models may be empirically adequate as long as observable phenomena can be isomorphically embedded in these models. If this is the case, then our best scientific theories may construct world-models that only save the phenomena but do not correspond to any real objects at all. Indeed, many of the most fundamental physical laws do not apply directly to actual, physical systems; nonetheless these laws are widely accepted by the scientific community not because physical systems really obey these laws but because their behavior can be successfully explained by them. That is, science assumes the existence of unobserved objects when doing so would help to develop a more coherent and comprehensive scientific world-picture. We cannot rely on science to provide us with direct answers to our ontological question, "what kind of objects exist?" In effect we allow our scientific theories to make claims about existence without supplying proof. If this is allowed in our scientific theories, then the same should be permissible for our mathematical and modal theories as well. Questions about the existence of unobservable objects such as quarks or electron spins are on par with ontological concerns about the existence of abstract entities such as numbers, sets, and possible worlds. Answers to these ontological questions will not directly proceed from investigating the empirical data.

W.V. Quine agrees that investigations into the noumenal existence of entities seek answers to problems which are "the shining candidates of questions which man in principle could never answer."[1] If we want to engage in answerable questions over the existence of objects we must move away from the realm of existence to imputations of existence. If we are to advance over any ontological ground, we must direct ourselves not to what actually exists but to what our best theories say exist. The objects assumed by any given theory do exist in an internal sense within the confines of that theory. Quine states that "it is meaningless, while working within a theory to question the reality of its objects or the truth of its laws, unless in so doing we are thinking of abandoning the theory and adopting another."[2] Theory acceptance is prior to ontological commitments. Our inquiry into the existence of objects must be reformulated as a question about our best theories. Instead of asking what kinds of entities really do exist, our focus is turned to the kinds of objects which our best theories are committed to. Our task is to uncover

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[1] W.V. Quine, *The Ways of Paradox and Other Essays* (Cambridge: Harvard University Press, 1976), 64.

[2] Ibid., 65.

the ontological commitments that are implicit in our scientific theories. And the kinds of objects which our best theories will probably be committed to are exactly those entities indispensable to the development of a coherent and comprehensive scientific world-picture.

In other words, matters of ontology are settled for us after we decide amongst competing theories. We choose which theories to accept and which to reject based on a number of criteria. In choosing a particular theory, we automatically commit ourselves to the kinds of objects that our accepted theory assumes. So, if matters of ontology are what we are interested in, we must redirect our focus away from what really does exist to what our theories say exist. Thus, we must reformulate our initial ontological question "what kinds of objects exist?" into the question "what theory along with its ontological commitments should we adopt?"

Quine offers a criterion to help in the choice among different competing theories and their range of ontological commitments. According to Quine, we can best decide what theories to keep "by considerations of simplicity plus a pragmatic guess as to how the overall system will continue to work in connection with experience."[3] Thus, the criteria for determining ontology may be reduced down to two considerations:

(1) the simplicity of the overall system, and

(2) its utility in connection with experience

When we set out to form theories, what we are attempting is to integrate into the simplest system the fragments of sensory experience. In our efforts at systematization,

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[3] Ibid., 223.

we may surprisingly find that, for the sake of simplicity and the overall integration of our system, a theory must adopt the existence of additional, unobserved objects.

Quine does not see much difference between one theory's commitment of macroscopic physical objects and another theory's commitment to abstract entities. The difference between these ontologies is only a matter of degree. For Quine, questions about existence in the more metaphysical cases such as numbers and sets are just more broadly systematic. Thus, physical objects and abstract objects are on much "the same footing" when it comes to their being assumed by a theory; their right to be posited as objects by a theory depends on their contribution to the overall simplicity of our linguistic or conceptual organization of experience. Quine takes a methodologically holistic approach when it comes to judging theories. Theories should not only fit the data, but pragmatic considerations are also allowed to count as theoretical virtues in the determination of the best theory.

Nicholas Rescher also adopts a similar stance on ontology. Rescher recognized the need to promote comprehensiveness or fecunditas (adequacy to the full variety of the phenomena) and economy or simplicitas (under the aegis of the usual standards of effective systematization – simplicity, uniformity, etc.). Although he sees both empirical adequacy and simplicity as being important theoretical virtues, Rescher is more reluctant to sacrifice the comprehensiveness of a theory for the sake of economy or simplicity:

> As I see it, we must never let our regulative concern for economy, simplicity, elegance, or the like stand in the way of substantive adequacy of the systems we create under their aegis – the capacity of those system

> adequately to accommodate the phenomena with which they are designed to deal.[4]

2 Ontological Commitments in Mathematics and Modal Theories

Platonism and possibilism have assumed the existence of abstract objects in order to explain the truth values of certain kinds of statements: modal and mathematical statements. Both Platonists and possibilists have argued that there are definite advantages in postulating the existence of abstract objects. For one, admitting abstract entities into the universe of discourse allows us to interpret mathematical and modal statements as having an obvious logical form. Both the Platonist approach and the possibilist approach take the surface form of arithmetical statements and modal statements, respectively, to be their logical form, thus enabling us to read at face value the sentence we assert in mathematics and in modal theory.

In mathematics, the postulation of abstract mathematical objects furnishes us with referents for the singular terms (e.g. 3, π, etc.) and the variables which abound in mathematical statements. According to the Platonist view, mathematical statements say something about a particular abstract mathematical object and the structural properties that this object possesses. Interpreting mathematical statements as talk about abstract entities enables us to stick with a standard Tarskian semantics for first order languages. All the formal machinery developed by Tarski and widely employed by logicians can then be made use of in the discipline of mathematics. Furthermore, for purely heuristic reasons, it just seems to help our mathematical thinking to talk about abstract

[4] Brian Ellis, "Hypothetical Reasoning and Conditionals," *The Philosophy of Nicholas Rescher*, E. Sosa, ed., (Boston: D. Reidel Publishing Company, 1979), 17.

mathematical objects such as numbers or sets. Similarly it also seems to help our modal reasoning to imagine possible worlds.

Furthermore, postulating the existence of possible worlds enables us to replace the intensional notions of necessity and possibility by quantifiers over possible worlds. The language in which such quantifiers appear can then be given an ordinary Tarski semantics. David K. Lewis asserts that we can choose to restrict existential quantification to range over just actual things or we can broaden its range to include everything without exception. Lewis, like other possibilists, chooses to treat modal idioms as quantifiers whose scope is broad enough to include even possible worlds. Any sentence which expresses modality, states Lewis, can be taken at face value. So the sentence *"There are many ways things could have been besides the way they actually are"*[5] can be given a straightforward treatment. At face value, this sentence appears to be an existential quantification which asserts the existence of entities that might be called "ways things could have been" or "possible worlds."

Lewis adds that he does not make it an inviolable principle to take seeming existential quantifications in ordinary language at their face value. However, he does recognize a presumption in favor of taking sentences at their face value, "unless (1) taking them at face value is known to lead to trouble, and (2) taking them some other way is known not to."[6] Although Philip Kitcher does not support a realist view of these kinds of abstract entities, Kitcher concedes that "*when other things are equal*, an account of a

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[5] David Lewis, *Counterfactual* (Cambridge: Harvard University Press, 1973), 84.

[6] Ibid.

particular body of discourse which reads that discourse at face value is to be preferred to one which suggests a complicated reformulation of it."[7]

On the other hand, the theoretical virtue of simplicity may conflict with the drive for economy. Bertrand Russell presented us with the principle of Occam's razor which demands that entities not be multiplied beyond necessity. Wherever possible, he advised, we should substitute logical constructions for inferred entities. The risk of error in philosophical theories, he argued, would be minimized if ontological economy is preserved. W.V. Quine holds an even stronger support of ontological economy. Quine claims that with everything else being equal, a theory which assumes the existence of fewer kinds of objects is to be preferred to a theory which postulates more.

Keeping the theoretical virtues of empirical adequacy, comprehensiveness, simplicity and economy in mind, we must weigh each theory according to its different merits. In the area of mathematics, Philip Kitcher argues for an interpretation of mathematics that seems to offer a greater balance of economy and comprehensiveness without too great a sacrifice in the area of simplicity. Kitcher claims that, with his non-realist approach to mathematical objects one can still take advantage of a Tarskian semantics in which quantifiers range over abstract objects. Where one would depart from the Platonist approach is in a reinterpretation of the language in which Tarskian semantics is given. References to abstract mathematical objects such as sets, on Kitcher's reinterpretation, would have to be replaced by references to collectings. For convenience, we can still choose to talk in terms of abstract objects such as sets or collections but we must recognize that our ontological commitments are to the mental

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[7] Philip Kitcher, *The Nature of Mathematical Knowledge* (New York: Oxford University Press, 1984), 141.

activities (e.g. collectings) which seem to manufacture these abstract objects (e.g. collections). Kitcher's reinterpretation of abstract objects as mental constructions would allow for the use of a uniform semantics for modal discourse (Tarskian semantics) without requiring a commitment to abstract objects.

I propose a similar reinterpretation of possible worlds. Analogous to the mathematical constructions introduced by Kitcher, constructions by an ideal mind may also prove helpful in the analysis of counterfactual statements. Under this alternative interpretation, modal statements may be viewed true in virtue of the modal activities of ideal minds acquainted with the actual world. This nonrealist approach to possible worlds will not force us to abandon the standard language of contemporary modal theory. We may even in some sense maintain that modal statements concern possible worlds – so long as we regard this claim to be ultimately interpreted in terms of ideal operations. What I will show is that the adoption of this alternative ontology will help us to understand counterfactual statements in a way that avoids epistemological difficulties, preserves ontology economy and maintains overall simplicity. Instead of postulating the existence of possible worlds, we need only to posit the existence of the actual world and our mental abilities to reconstrue this world. Moreover, when we interpret modal statements such as counterfactuals or when we analyze mathematical statements, we do not have to resort to talk about abstract objects.

In principle we are free to assume abstract entities for our mathematical and modal theories, as the realist proposes, but this assumption is warranted only if it leads us towards the development of better theories for interpreting the kind of statement we are concerned with. I believe, however, that realism does not offer us the best theory for interpreting mathematical or modal statements. In the following chapters, I will argue for the adoption of an alternative ontology with which to interpret counterfactuals and other modality-involving statements, an ontology that does not postulate the existence of abstract objects. In the first two chapters, I begin by chronicling the early attempts made by logicians to understand counterfactuals and other modal propositions and then proceed with a survey of the mathematician's efforts to interpret mathematical statements. I try to show how similar ontological problems arise in the histories of both disciplines and how both disciplines attempt to meet these difficulties by espousing a realist perspective, that of mathematical Platonism and David Lewis' possibilism. In chapters three and four, I discuss the merits and the difficulties of a realist approach to mathematics and modality. I then argue that a host of problems may be avoided and a number of metaphysical questions may be answered if we adopted a constructivist approach to modal and mathematical statements. The main objective of this work is to investigate how best to interpret counterfactuals and other modality-involving statements and to identify the kind of ontology requisite for an adequate understanding of these types of sentences. My interest in the debate over mathematical ontology is limited to the extent that this (ongoing) debate sheds some light on related question in modal theory. In the final chapters, I leave behind ontological concerns about mathematics to focus on counterfactual statements in more depth. I try to show that, when one weighs the various merits and difficulties of realist vs. constructivist approaches, it appears that a constructivist theory like Nicholas Rescher's offers a greater balance of theoretical advantages than does David Lewis' realist view

CHAPTER TWO COUNTERFACTUALS AND POSSIBLE WORLDS

Use of counterfactuals abound in ordinary discourse and scientific studies. Counterfactual conditionals play a key role in our everyday thinking about the world and in our decision-making process. When we reassess the course of action we've taken, voice regrets, consider foregone options or hypothesize about unrealized events, we are apt to communicate our thoughts by way of contrary-to-fact conditionals. Speakers of our language readily grasp the meaning of this special linguistic construction without having to formally analyze the modal concepts involved. Rational-minded people unacquainted with the techniques of formal semantics but informed of the nature of the physical world believe in the truth of certain counterfactuals despite the fact that the hypothetical event described in the antecedent is contrary to fact. Consider, for example, the following counterfactuals:

If Socrates were beheaded, then he would die.

If this cube of salt were immersed in water, then it would dissolve.

If a moving body were free of all external forces, then it would stay in motion.

On the other hand we are convinced of the falsity of other counterfactuals:

If Socrates were beheaded, then he would remain alive.

If my golden ring were immersed in water, then it would dissolve.

If a moving body were free of all external forces, then it would come to a halt.

But, what rational grounds can we have to support our convictions about such counterfactual conditionals? Counterfactual conditionals make claims regarding unrealized and sometimes unrealizable situations or states of affairs. They deal with events that have not taken place and are contrary to the actual facts and may include nonexistent individuals with whom we have no direct acquaintance. Naturally we cannot rely on an investigation of the actual world to uncover truths about such hypothetical states of affairs and unreal individuals. As native speakers of the language, we know how to use contrary to fact conditionals in ordinary speech but we are left with the task of making precise and explicit this knowledge which we all share.

Still other counterfactuals conditionals present us with added problems. Even our prephilosophical intuitions offer us little help with the following cases:

If Zeus were human, then Zeus would be mortal.

If kangaroos had no tails, then they would topple over. (David Lewis)

If Bizet and Verdi had been compatriots, Bizet would have been Italian. (Quine)

These counterfactuals appear less clear-cut than the earlier examples and without further qualifications their truth-values seem undeterminable.

In this chapter, we shall begin with a general discussion of conditionals, treating counterfactuals as a special case. Next we shall explore the early attempts made by logicians such as Chisholm and Goodman to analyze counterfactuals. Third, this chapter will examine standard propositional and quantified modal systems developed by logicians in the early half of the 1900's but will presume that the reader is familiar with elementary symbolic logic.

1 Kinds of Conditional Statements

Language to a large extent is conventional and we learn how to employ certain linguistic constructions by following the way speakers of our language normally use them. Thus, in our analysis of counterfactual statements, we shall start by distinguishing the different kinds of conditionals and examining the ways we use them.

Conditional statements ordinarily consist of a main clause and an if-clause. The clause in the scope of "if" is called the antecedent (or protasis) and the main clause is called the consequent (or apodosis). A counterfactual conditional is a special type of "if . . ., then . . ." statement in which the antecedent is known or believed to be false but the falsity is not explicitly stated; it is implied by the use of the subjective mood in English. Moreover, the terms "counterfactuals" and "subjunctive conditionals" are often used interchangeably; however, not all counterfactuals are expressed in the subjunctive mood nor are all subjunctive conditionals counterfactual in nature. The counterfactual "If Lincoln had not been born, then the Gettysburg Address would not have been written" could be shortened to the non-subjunctive statement: "No Lincoln, No Gettysburg Address." Likewise, all conditionals in the subjunctive mood are not necessarily counterfactuals for they can contain antecedents that are not contrary to facts, for example "If Abraham Lincoln were president, then Mary Todd Lincoln would be first lady." Typically a counterfactual is expressed by a sentence of the form "If . . . had. . . , then . . . would . . ." or "If . . . had . . . , . . . would have . . ."

Asserting a conditional statement is not the same thing as advancing an argument. An argument contains one conclusion and one or more premises. One who advances an argument asserts the truth of all the premises and is committed to the truth of the conclusion on the basis of the premises. On the other hand, one who asserts a conditional statement is neither claiming the truth of the antecedent nor the truth of the consequent but rather is commenting on the relation between the two. Likewise, asserting a counterfactual statement does not require that the speaker believe in the truth of the antecedent or the consequent. Often the speaker of a counterfactual conditional implies the very falsity of the antecedent. The hypothesis described in the antecedent is believed to be false and the intent behind the assertion of a counterfactual is to point out some connection between the nonfactual event hypothesized in the antecedent and another event described in the consequent. Moreover, the antecedent of the counterfactual is not required to be actually false as long as it is simply believed to be false. As Nicholas Rescher explains, "a supposition is rendered belief-contravening (equally well applicable to the counterfactuals) not through failure to square with actual facts but simply and solely through discord with what is believed.[8]

Roderick Chisholm distinguishes two kinds of subjunctive conditionals with respect to their antecedents: (1) conditionals whose antecedents we know or believe to be false and (2) conditionals whose antecedents have truth-values that are either undetermined or irrelevant for one's purposes. Counterfactuals fall under this first group of conditionals since they are conditionals whose antecedents describe a state of affairs known to be false: e.g. If Lincoln had not been elected president, then the Civil War would not have occurred." The antecedent is inconsistent with the historical events that have already occurred in the actual world. The first group also includes conditionals whose antecedents assume ideal situations that can never be realized in the actual world given the physical laws that obtain. The sciences often make use of ideal states of affairs as an explanatory device for defining nonobservable concepts such as momentum: "If a body in motion were free of all external forces, then that body would remain in motion."

[8] Nicholas Rescher, "Belief Contravening Suppositions," *Philosophical Review*, 70, (April 1961), 179.

However, in the natural state of the actual world one would never find a moving body uninfluenced by external forces. These ideal possibilities, like contrary-to-fact possibilities, describe non-actual happenings to actual things. In contrast, counterfactual possibilities are events that generally conform with the physical laws of the actual world, whereas ideal situations violate this natural order. Thus, ideal possibilities broaden the range of events that could happen to actual things beyond what is naturally permitted by physical laws.

We might add a special kind of conditional to Chisholm's first group which Nicholas Rescher identifies as "the merely hypothetical possibilities." These conditionals deal with the possibility of non-actual events occurring to non-actual individuals. This conditional diverges even more from the actual state of affairs for it may assume not only "contrary-to-fact" events and "contrary-to-physical law" situations, but additionally it assumes the existence of non-actual objects. These merely hypothetical possibilities then would alter the actual world on an ontological level by introducing nonexistent objects. The merely hypothetical conditional "If you caught a leprechaun, then you would win his pot of gold" entertains the nonfactual event of your catching a leprechaun and in doing so introduces the existence of non-actual entities such as leprechauns and their pots of gold. In such cases, the antecedent of this counterfactual is said to be vacuous, i.e. the class designated in the antecedent may be a null class.

Chisholm identifies three principle uses for a second type of conditional which contains an antecedent whose truth-value may remain unknown or irrelevant to the purpose for which the conditional is employed. First, it may be employed for precautionary purposes. One might warn prospective trespassers, that "if anyone were to trespass, then he or she would be attacked by a dog." A second use of this type of conditional is for purposes of deliberation. This can occur when one takes the truth-value of the antecedent to be irrelevant and so deliberately ignores it, considering it only "for the sake of argument." This deliberative use is employed in methods of reasoning such as reductio arguments and conditional proofs. This kind of subjunctive conditional may also be employed in a third way, in order to point out the dispositional quality of objects. One characteristic feature of dispositional predicates is that they may be analyzable in terms of subjunctive conditionals. The conditional statement, "if this cube of salt were to be immersed in water, it would dissolve," is an example of a subjunctive conditional which characterizes the dispositional quality of being "soluble." Dispositions constitute the realizable potential or abilities of actual things.

A subjunctive conditional could be proven false if there were some way to observe a case in which the antecedent actually obtained and yet the consequent did not. It is important to note that the opportunity of falsification clearly cannot happen with the first group of conditionals, counterfactuals whose antecedents are contrary to facts or whose antecedents are unrealizable in the actual world. However, with subjunctive conditionals of the second type, the opportunity of falsification is open since the antecedent is not known to be false. One might be able to investigate or even facilitate cases in which the antecedent turns out true and then test whether the consequent does in fact result. In the case of the warning to trespassers, we can falsify the claim that "if anyone were to trespass, he would be attacked by a dog." We'd simply check for future cases in which someone trespasses and observe whether in every case the trespasser is attacked by a dog. Just one unattacked trespassor would serve as a counterexample to falsify the subjunctive conditional. Likewise, the attributed dispositional qualities of a class may be tested by finding a member of that class that fails to manifest the potential characteristic in question. The subjunctive conditional, "if a member of class X is immersed in water, then it would dissolve" may be falsified by finding at least one member of X that satisfies the antecedent condition of "being immersed in water" and yet fails to satisfy the consequent, of "dissolving."

Initially, what this seems to suggest is that the defensibility of the first kind of conditional is significantly more difficult than that of the second because the opportunity of falsification is not available. Without recourse to the possibility of future evidence, the truth-value of the first type seems more suspect than the truth-value of the second. Nevertheless, both kinds of subjunctive conditionals are equally problematic in the sense that neither, at the moment of utterance, can be confirmed by one's current knowledge of the world. That is, for both types of conditionals, there seems to be a lack of available empirical data at hand to enable one to draw definite conclusions about their truth-values. For the latter case, future events may have the power to falsify the conditional, but from the vantage point of the present state of affairs the truth-values of both appear equally indeterminable.

Goodman's classification of conditional statements also focuses on the truth-value of their component parts. Goodman distinguishes counterfactuals from semifactual and factual conditionals. He restricts the use of the term "counterfactual" to a conditional in which both the if-clause and the main clause are known or believed to be false. However, Goodman distinguishes conditionals whose if-clause is false but whose main clause is true apart from other counterfactuals and calls such conditionals "semifactuals" ("Even if

. . . , still . . ."). Conditionals which contain both a true antecedent and a true consequent are classified as "factual" conditionals.

Both Goodman and Chisholm take for granted that the component clauses of subjunctive conditionals have truth-values and at present we shall go along with their assumption. They attempt to overcome the difficulties of assigning truth values to the subjunctive components of counterfactuals by considering the corresponding indicative instead. A subjunctive conditional such as, "If Abraham Lincoln were not born, then the Gettysburg Address would not have been written" presents us with subjunctive clauses which have no truth-values. The antecedent clause, "Abraham Lincoln were not born" does not constitute a complete sentence at all. Likewise, the consequent "the Gettysburg Address would not have been written" cannot be intelligibly said to be either true or false. According to Chisholm's and Goodman's analysis, we must first give an indicative rendering of the subjunctive clause "Abraham Lincoln were not born" in order to assign truth value to the if-clause. "Abraham Lincoln was not born" is the indicative corresponding to the antecedent and may be assigned a false truth-value. Similarly, the indicative version of the main clause reads "the Gettysburg Address was not written" is taken to be the proper bearer of truth-value and given the actual course of history turns out to be false. We shall proceed with the assumption that the components of subjunctive conditionals do have the same truth-values as their indicative counterparts but later question whether this strategy helps to resolve the problem of counterfactuals.

By eliminating the subjunctive mood from counterfactuals, Chisholm and Goodman entertain the hope that something like a truth-functional treatment of indicative conditionals can serve as a model for analyzing counterfactuals. Our standard logic equips us with a fairly mechanical way of analyzing indicative material conditions which are composed of subsentences connected together by truth-functional logical operators.

The operator of an ordinary material conditional is truth functional in the sense that the truth-value of the whole statement may be computed just by knowing the values of its simpler component sentences. The non-modal operator found in a material conditional obeys logical principles which are fixed by truth tables. The truth of the material conditional of the form $p \supset q$ requires only that it not be the case that both p is true and q is false. A material conditional is interpreted as true just in case its antecedent is false or its consequent is true. A question which early logicians like Chisholm and Goodman faced is whether subjunctive conditionals could be suitably represent by "$\supset$" of a material conditionals if certain other conditions were met.

Several problems result if subjunctive conditionals are treated merely as an ordinary material conditional whose truth-value is a function of the truth-values of its antecedent and consequent. If interpreted like a material conditional, a counterfactual would automatically be true by the mere falsity of its contrary-to-fact antecedent. Yet, we want to claim that some counterfactuals with false antecedents are false, e.g. "If George Washington were beheaded, George Washington would remain alive." Also, we would want to reject some subjunctive conditionals in cases where the antecedent is false but the consequent is true, conditionals which Goodman refers to as "semifactuals." Consider the following assertion: "If John F. Kennedy were not elected U.S. president, then Jacqueline Kennedy would have been first lady." Naturally we would reject the truth of this conditional statement and no one familiar with these public figures and aware of the political office would make such a claim. However, as a material implication, this conditional would be true since the indicative statement corresponding to the antecedent is false (Kennedy was elected) and the indicative statement corresponding to the consequent is also true (Jacqueline Kennedy was first lady). According to the truth-value definition of the logical operator "⊃," a conditional is true so long as the antecedent is not true while the consequent is false.

Furthermore, if treated like a material conditional, a counterfactual would be true independently of whatever consequent followed so that there would be no difference in the truth-values of the following two statements:

If Socrates had remained quiet, then he would not have been executed.

If Socrates had remained quiet, then he would have been executed.

But we cannot logically accept the idea that the same counterfactual assumption can at the same time lead to a consequent C and also to its negation ~C. Clearly counterfactuals cannot without additional qualifications be interpreted simply as material conditionals.

2 Early Analysis of Counterfactuals

Given that counterfactuals involve more than just material implication, we must determine what truth conditions apply to subjunctive conditionals. Just what is required for a subjunctive conditional to be true or false? In their interpretation of counterfactuals, Roderick Chisholm and Nelson Goodman were in agreement as to the nature of the problem and how it is to be solved. They sought to render counterfactual statements more intelligible by eliminating the subjunctive mood of a counterfactual statement.

They had hoped to reduce counterfactual statements into something we can better understand – i.e. into indicative conditionals which would assert the same thing as their subjunctive counterparts. Both Goodman and Chisholm proceeded toward a characterization of the form of counterfactual statements and attempted to find the relevant conditions and constraints which applied to this form.

Chisholm proposed that a counterfactual having the subjunctive form

$(x)(y)$ [if x were ϕ and y were ψ, then y would be χ][9]

may be replaced with an equivalent expression in the indicative mood

There is a true statement p such that: p and 'x is f and y is g' entail 'y is c.'

Chisholm's proposal would reduce a subjunctive conditional into an indicative statement which asserts that some true sentence p conjoined with the antecedent strictly implies the consequent. Along similar lines, Goodman suggests that a counterfactual conditional may be translated by the general formula *"If (A • S), then C"* where A and C stand for a counterfactual's antecedent and consequent reworked as statements in the indicative mood. Like the true statement 'p' (possibly consisting of a series of conjuncts) in Chisholm's formula, S represents the set of true sentences which in conjunction with the antecedent serves as a basis for inferring the consequent. A counterfactual does not explicitly specify what true sentences make up the members of S (hereafter, we shall use Goodman's symbolization), but it does imply that some set of relevant conditions must obtain along with the counterfactual assumption. For example, the subjunctive conditional "If the vase fell, it would break" implies that certain relevant background conditions attend this event. Among the attendant circumstances listed in S, we would

[9] Chisholm, "The Contrary to Fact Conditional," *Mind*, 55 (1949), 293.

have to include that "the laws of gravity apply," "the vase is not lighter than air," "there is nothing to intercept the vase," etc. The set of statements, S, assumed by a counterfactual must include any such relevant facts, attendant circumstances, or laws that are implied by the context.

Here the connection between the conjunction A • S with the consequent C is taken to be an entailment relation or strict implication and is not to be confused with material implication. Strict conditionals require not just the mere falsity of 'p & ~q' as the material conditional does but claim the impossibility of 'p & ~q'. The strict implication 'p implies q' is symbolized 'p –϶ q' where the modal operator is the fishhook –϶ rather than the truth-functional operator, the horseshoe ⊃. Although a strict conditional is a much stronger claim than its corresponding material conditional, the truth-values of both conditionals are relative to the truth-values of their antecedents. The material conditional and the strict conditional are automatically true when its antecedent is false or impossible, respectively, regardless of the consequent. Also, both types of conditionals are true just in case the consequent is true for the material conditional or necessarily true for the strict conditional. Thus the horseshoe and the fishhook represent different ways by which to formalize conditional sentences. Material implication is a truth function defined on a finite four row truth table while a strict implication is a modal concept whose semantics often involves an infinite set of possible worlds.

Since ancient times philosophers have disputed which way best captures the semantics of indicative conditional statements. The debate originated back in the third century B.C. when Megarian philosophers, a school of philosophers founded by Euclides, a student of Socrates, disagreed about the right way to represent conditionals. Two of the main characters in this philosophical dispute were Diodorus Cronus and his student Philo.[10] Diodorus interpreted conditional statements in terms of the fishhook while Philo defended a truth-functional analysis of conditionals using the horseshoe. This debate later resurfaced as Frege adopted the Philonian conditional in his logical system and made explicit the truth-functional nature of material implication.

Strict implication is different from the notion of entailment. A statement entails another statement in virtue of some genuine relation between them. Thus an entailment relation is an even stronger claim than a strict implication since it demands the extra requirement of genuine relatedness or relevance between statements. Goodman required that, for a true counterfactual, the consequent C be logically entailed by the antecedent along with certain other relevant conditions. Chisholm also demanded a genuine relation between the antecedent and the consequent of a counterfactual but allowed nonlogical as well as logical laws to be the connecting principles.

Instead of treating a counterfactual like a material conditional, Chisholm and Goodman attempt to interpret a counterfactual statement as the entailment of a consequent C from the conjunction of some contrary-to-fact antecedent A and some set S of relevant conditions. For them, a given counterfactual would be true if there were a suitable S, which conjoined with A, logically entails C and if there were no such suitable S, then that counterfactual would be false. The challenge both faced consisted in identifying those specific conditions which would determine the suitability of a given set,

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[10] Paul Herrick, *The Many Worlds of Logic* (Fort Worth, TX: Harcourt Brace College Publishers, 1998, 302-303.

S. What true statements, when conjoined with the antecedent, entail the consequent if and only if the counterfactual is true? To answer this question, they had to consider the impact of altering the actual course of events in which ~A is true to an alternative course of events in which the contrary to fact antecedent A is assumed to be true.

Goodman and Chisholm both agreed that the set S cannot include all true statements and each identifies a number of different restrictions to be applied to the set of true statements, S. Clearly, one true statement that must be excluded from S is the negation of the antecedent, ~A, because the conjunction of the antecedent with the set S would then logically imply any statement given that any statement may be derived from a contradiction of the form (A • ~A). Secondly, Goodman asserts that any statement logically incompatible with the antecedent cannot be included as a member of S. Hence we could not include in S statements such as (~A ∨ ~A) or (A ≡ ~A). Not only must sentences of S be logically compatible with A but they must also be non-logically compatible. That is, any statement that is logically compatible with A but is in violation of some non-logical law must also be excluded from S. With the match example, the statement, "The match was beyond reach" is physically incompatible with the claim that "the match was struck." It would be physically impossible for the match to be struck when it is out of the reach of anyone who could do the striking. Goodman concludes that the set (A • S) must be self-compatible according to natural laws as well as logical ones.

Goodman adds that the evaluation of a counterfactual involves more than just finding an adequate S which in conjunction to A leads by law to C. Goodman claims that "we are testing whether our criterion not only admits the true counterfactual we are concerned with but also excludes the opposing conditional."[11] A suitable account of counterfactuals should not support each of a pair of opposite conditionals, of the forms "if A, then C" and "if A, then ~C." Even if a suitable S were found and the conjunction (A • S) is shown to lead by law to the consequent, it is crucial, asserts Goodman, that no other set S' exists such that (A • S') is self-compatible and leads by law to the negation of the consequent, ~C. An example of such an S is a set which consists of ~C. In most cases ~C is compatible with A except for the situations where A alone independent of other conditions leads by law to C. However, as in most cases, ~C is compatible with A and if we take ~C to be our S, then (A • S) constitutes a self-compatible set which leads by law to ~C. Hence, this shows that the requirement that S be compatible with A is not enough because that would still allow for the possibility of taking ~C for S and then our criterion would not do the job of excluding the opposing conditional.

Goodman also points out that his criterion must not only recognize that the antecedent is in most cases compatible with C and ~C, but also requires that the S be compatible with both C and ~C. For, our criterion must show that A in conjunction with S leads by law to C and not that S alone decides between C and ~C. Combining all these considerations, Goodman concludes that:

> . . . a counterfactual is true if and only if there is some set S of true sentences such that S is compatible with C and with ~C, and such that A•S is self-compatible and leads by law to C; while there is no set S' compatible with C and with ~C, and such that A•S' is self-compatible and leads by law to ~C.[12]

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[11] Goodman, *Fact, Fiction and Forecast* (Indianapolis: Bobbsmerrill, 1965), 12.

[12] Ibid.

Goodman also argues that the problem of counterfactuals involves not only the problem of avoiding incompatibility with the antecedent but the problem of cotenability. A must not only be compatible but must also be cotenable with S. In order to restore consistency, Goodman recognized that other minimal omissions or changes were needed, including the exclusion of all the statements not cotenable with the antecedent. This cotenability condition requires that it not be the case that if A were true, then S would be false. That is, any statement that would fail to be true if A were true is said to be not cotenable with the antecedent and thereby cannot be a member of S. It is clear that the elimination of statements that are not cotenable with the antecedent succeeds in eliminating all the statements that are incompatible with the antecedent. Thus the problem of eliminating incompatibilities with the antecedent is subsumed by the broader problem of resolving cotenability with the antecedent.

After reducing the analysis of counterfactuals as the problem of cotenability – how to exclude true statements not cotenable with the antecedent – Goodman admitted that his search for an adequate criterion led to an infinite regress:

> In order to determine the truth of a given counterfactual it seems that we have to determine, among other things, whether there is a suitable S that is cotenable with A and meets certain further requirements. But in order to determine whether or not a given S is cotenable with A, we have to determine whether or not the counterfactual "If A were true, then S would not be true" is itself true. Thus we find ourselves involved in an infinite regressus or a circle; for cotenability is defined in terms of counterfactuals, yet the meaning of counterfactuals is defined in terms of cotenability.[13]

In other words, Goodman involves himself in a circle for in order to determine if this counterfactual is true, he would again have to identify yet another set of true statements

[13] Ibid, 16.

which in conjunction with A leads by law to ~S. Thus, Goodman's attempt to evaluate a counterfactual led to the attempt to define cotenability, which in turn brought him back to the evaluation of another counterfactual.

Among the restrictions, Chisholm placed on S is the condition that S should not contain vacuous truths. That means, that S must not contain any material conditionals that are trivially true (that have false antecedents or true consequents) nor can S contain any universal conditionals whose antecedents determine an empty class. Chisholm also excludes from S any universal conditional whose consequent includes any two functions logically equivalent to '*x is f and y is g and y is c.'*

Another important restriction on S is the exclusion of accidentally true general statements. At face value, accidentally true general statements appear no different from natural laws. Both law and non-law statements can be reformulated into a statement of the form, "For every x, if x is S, x is P." Chisholm viewed the enterprise of differentiating accidental universal conditionals from natural laws as "the basic problem in the logic of science" and not one he expected to be able to answer fully. In a later article "Law Statements and Counterfactual Inference," Chisholm emphasized that counterfactuals must depend on law rather than non-law statements. According to Chisholm, law statements can warrant counterfactual inference whereas non-law statements cannot. A statement which we recognize to be law-like such as All gold is malleable" can be reexpressed "For all x, if x is gold, then x is malleable." Likewise, the accidental general statement "Everyone in this room is a philosopher" can be translated "For all x, if x is in this room, then x is a philosopher." Chisholm asserts that if a universal statement of the form "For all x, if x is S, then x is P" is law-like, then one could logically infer a counterfactual statement of the form “For every x, if x were S, x would be P” where “x were S” means “x had the property S.” With respect to our examples, the law statement about gold would warrant the counterfactual inference that “If a, which is not gold, were gold, a would be malleable.” In contrast, the non-law statement about our room filled with philosophers would not warrant the counterfactual inference “If a, who is not in this room, were in this room, a would be a philosopher.” A necessary condition then for the truth of a counterfactual statement is that S contain true law-like statements and exclude true accidental generalizations. Chisholm adds that “the point of asserting the counterfactual may be that of calling attention to, emphasizing, or conveying, one or more of the premises which, taken with the antecedent, logically imply the consequent.” Thus, the whole point of uttering a counterfactual may just be to call attention to some law-like statement upon which the counterfactual inference depends.

The truth of a counterfactual statement evidently requires that the general statements in S be law-like, but the difficulty remains in providing a criterion which would distinguish laws from non-laws. Chisholm doubted that the difference between these two kinds of statements can be explained in familiar terminology without the employment of such modal notions as “causal necessity,” “necessary condition,” and “physical possibility” or without reference to metaphysical terms such as “real connections between matters of fact.”

Goodman also explored the problem of what makes a general statement lawlike. Unlike Chisholm, Goodman did not choose to treat laws as general statements contained in the set S and to allow the inference of C from (A • S) to be simply one of logical relation. Instead, Goodman preferred to exclude law statements from among the true statements included in S. Goodman treats laws as the connecting principles which allow for the inference of C from (A • S). These law statements taken as connecting principles may be expressed in the form of a generalization "Every case for which A and S hold, C holds."

Although Goodman singles out lawlike statements as having special status from other true statements in S, he maintains that the same restrictions placed on statements in S apply to the lawlike statements that inferentially connect (A • S) to C. Thus, the counterfactual assumption A must be compatible and cotenable with the laws that warrant the counterfactual inference. The general statement that acts as the connecting principle must be able to sustain the counterfactual assumption. That is, the contrary-to-fact event assumed in the antecedent should not pose a counterexample that threatens the universal status of connecting general principles. If not further qualified, the counterfactual statement "If Zeus were human, then Zeus would be mortal" seems to contain an antecedent which is not cotenable with the law that would otherwise underlie this inference. The law that warrants this counterfactual inference is the universal generalization "For every case for which x is human, x is mortal." However, if the counterfactual assumption "Zeus (who is an immortal being) is human" obtained, then at least one human, Zeus, would be immortal. This alteration would then contradict the connecting principle that "all humans are mortal." Goodman emphasizes a Humean approach to the concept of laws. What makes a statement lawlike according to Goodman is simply our use of it. By serving as the basis of our predictions, general statements take on the status of being lawlike. Laws are no more than the general sentences we use for making predictions. The emphasis here is not that we make predictions based on statements we take to be laws but that some statements are taken to be laws because we use them as the basis for making predictions. According to Goodman, we make predictions based on general statements we accept as true even though many cases still remain undetermined; those unexamined cases are only predicted to conform to these general statements. That is, we base our predictions on generalizations that are acceptable prior to the determination of all instances and that are acceptable independently of the determination of any give instance.

Goodman equates the kinds of general statements we employ in making predictions with the kinds of lawlike statements that underlie our counterfactual inferences. Both counterfactuals and predictions he claims make inductive inferences from known to unknown cases. Predictions project hypotheses confirmed for the past cases to future cases that are yet undetermined. In a similar fashion, counterfactuals can be viewed as projecting predicates that are manifest in actual things to other objects similar in kind but which due to attendant circumstances do not exhibit the manifest property. Goodman claims that counterfactual statements may be treated as a special case that falls under the more general problem of induction. The strength of a counterfactual inference, like all other inductive inferences, depends on the laws that warrant the inference.

Goodman also points out that there is an acceptable circularity that occurs, for while counterfactual inferences are justified by laws, these laws in turn are justified to the extent that they warrant only such inferences as we are willing to accept. Thus, inductive inferences which include counterfactual inferences are warranted by laws and the laws are justified by warranting the right kind of inferences. According to Goodman, "a rule is amended if it yields an inference we are unwilling to accept; an inference is rejected if it violates a rule we are unwilling to accept." Goodman views this circularity as a virtuous one and the only justification to be found for the counterfactuals lies in the mutual agreement of its underlying laws with particular inferences.

Leaving aside the issue of law vs. non-law statements for now, the central question that must concern us is whether the general approach taken by Chisholm and Goodman adequately captures the complexity of counterfactual statements. Namely, can we eliminate the subjunctive element from counterfactual statements and reduce them to indicative conditionals whose truth-value depends on the existence of some set S which meets particular restrictions?

Let us apply such an approach to our earlier example "If Zeus were human, then Zeus would be mortal." When one utters a counterfactual statement, one proposes to do two things: (1) to entertain a false belief, namely the counterfactual assumption A, and (2) to exclude other true beliefs that are incompatible with A. In this case, the antecedent "Zeus is human" contradicts the belief held by ancient Greek mythology that "Zeus is not human." Thus, if we are to assume the antecedent, we must exclude from our set of beliefs the negation of the antecedent, i.e. we must exclude ~A. The counterfactual assumption will likely lead to the exclusion of other background beliefs. The person asserting the statement "If Zeus were human, then Zeus would be mortal" might hold to the following beliefs (beliefs shared by the ancient Greeks):

(1) Zeus is not human.

(2) Zeus is immortal.

(3) All humans are mortal.

(4) Nothing can be both mortal and immortal.

In asserting our counterfactual, we have already noted the statement (1) because it is the negation of the antecedent must be excluded from S. Depending on the emphasis of the statement, that is depending on the point that the speaker wants to convey, other statements must also be excluded to allow for the inference of the consequent. In order to make the counterfactual inference work one must also exclude one of the following statements: the statement (2) Zeus is immortal, or the logically true statement (4) Nothing can be both mortal and immortal. As a result, there are a number of different sets of beliefs that comprise S. The only fixed requirements are that we exclude from set S statement (1) since it is the negation of the contrary-to-fact assumption and that we forgo either (2) or (4). However, we must keep statement (3) “All humans are mortal” in S to serve as the law-like statement justifying the counterfactual inference.

Suppose the speaker of the counterfactual “If Zeus were human, then Zeus would be mortal” really meant to exclude statements (1) and (2) but to maintain statements (3) and (4). Then a less ambiguous rendering of his counterfactual statement would be Chisholm’s version “If Zeus were different from what we have believed him to be and had instead the attributes which all men possess, then he would be mortal.” As qualified, this counterfactual inference seems warranted, since the conjunction of the statements “Zeus is human (antecedent)” and “All humans are mortal (3)” logically entails the consequent “Zeus is mortal (consequent).” Thus, it appears that we have found a suitable set S which in conjunction with the antecedent entails the consequent. However, the suitability of this set S assumes that we consistently exclude the statements “Zeus is not human" and "Zeus is an Olympian god" while including in S statements "Zeus is human" and "All humans are mortal."

What we must keep in mind, however, is the fact that besides the five statements we have identified, there are many others beliefs we might hold about Greek mythology, humans, and mortality, and these two must be considered with respect to our counterfactual situation. These other beliefs along with the above five statements can comprise a very complex system of statements and a seemingly slight alteration of a single fact can possibly lead to a number of unexpected adjustments in the system. By simply focusing only on a manageable number of relevant statements we may run the risk of oversimplification, oversimplifying what really is a more complex system of beliefs than appears at face value.

To summarize, if we made the counterfactual assumption that "Zeus is human" and adopted a set of relevant condition, S*, which excluded the beliefs that

(1) Zeus is not human

(2) Zeus is immortal.

And included the beliefs that

(3) All humans are mortal

(4) Nothing can be both mortal and immortal.

it would appear that the consequent "Zeus is mortal" logically follows. Yet, altering the state of affairs by supposing Zeus to be human and not an Olympian god could render unexpected changes in one's system of beliefs. In accord with the supposition that Zeus is human, more than one state of affairs becomes possible. For example, it is possible that this counterfactual assumption could change the lawlike status of a general statement in S. According to Goodman, a general statement belonging to S must be lawlike and not merely accidental. What makes a generalization lawlike is the fact that it is used for making predictions and we can use a general statement as a basis for predictions only if it has not been violated by any past nor present occurrences. We cannot be certain about the outcome of future cases but they are projected to conform to our general laws.

To illustrate such a possibility, let us also include in S* the following statements which we will grant to be true according to Greek Mythology:

(1) Zeus is married to Hera.

(2) Hera is immortal.

(3) Zeus and Hera can have children.

What might the child of a mortal human, Zeus, and an immortal goddess, Hera, be like? A possible consequence is that some child is born to Zeus and Hera who inherits all of Zeus' human-like qualities except for his mortal nature; the one thing the child inherits from Hera is her property of immortality. Then, one might argue that we now obtain a counterexample of someone who is both human and yet not mortal. The possibility of such a counterexample would render the general statement "All humans are mortal" either false or accidentally true until the birth of Zeus' child. In either case what we took for a lawlike statement could no longer be sustained as a member of the set of true statements S*, since any suitable S is restricted from including false statements or merely accidental generalizations.

This may seem a far-fetched possible consequence to our mythological story but the main point is to illustrate that a set of background assumptions that frame a counterfactual conditional can be more complex than may initially appear. The approach which Chisholm and Goodman take with respect to counterfactuals does not provide us with a comprehensive enough framework to meet the complexity of the task. For an alteration of even a miniscule detail within the actual state of affairs may entail radical and unexpected revisions elsewhere in one's system of beliefs. When we change certain circumstances about the present state of the world, we can no longer simply focus on the actual world and accept the law-like status of certain generalizations. With the alteration of a single fact, other possibilities open up that may not yet or ever obtain in the actual world.

David Lewis proposes an alternate approach for analyzing counterfactuals. He suggests that one can view the various possibilities as whole other worlds uniquely distinct from the actual world we presently inhabit. These possible worlds are like the actual one except for the changes specified by and resulting from the counterfactual assumption. The notion of possible worlds helps us to consider a variety of other worlds which differ from the actual world with respect to events, processes, physical laws, etc. Lewis claims that in analyzing counterfactual statements, we must consider other worlds besides the original, unaltered world we began with. Non-modal propositional systems outlined by Chisholm and Goodman overlook multiple possible worlds when evaluating the validity of a formula. For them, a proposition is true if it corresponds to the way things are in just one possible world, the actual one with only minimal changes introduced. The validity of a formula in non-modal propositional systems is a structural property of the formula itself. Determining the validity of a counterfactual, however, cannot be a just a matter of simply assigning a value to each variable in the formula and then calculating the value of the whole formula from there. The value assigned to each variable in the formula would depend on facts which are presumed to obtain in the actual world. The semantical principles of truth-functional logic may be specified by means of truth-tables which can contain only a finite numbers of rows. The truth-functional analysis of a counterfactual would be inadequate since such a statement cannot be defined in terms of a finite manifold. Analyses of counterfactuals as pursued by Chisholm and Goodman were succeeded by a number of other theories which employed a "possible worlds" approach to conditionals. In the 1960's, philosophical attention increasingly moved toward a possible worlds analysis of counterfactuals like that offered by Robert Stalnaker and David Lewis.

3 Propositional Modal Logic

Later figures of the twentieth century imported the Leibnizian notion of possible worlds in order to provide a semantics for the various modal systems that had been developed in the earlier part of the century. By the early fifties, logicians like C.I. Lewis and others had succeeded in giving axiomatic presentations of the different modal systems and specified the syntax for those systems. The formalization of modal inference resulted in a large number of nonequivalent systems which lacked a clear cut semantics. This meant that logicians couldn't identify models for the various systems, couldn't define the notion of a valid formula, and were not able to provide completeness proofs. Without a model for these systems, logicians could not know what set of objects the formulas of the system referred to. Modal logicians were left in the precarious position of having formalized a number of nonequivalent systems but could not provide us with a model so that we could interpret what the formulas different systems were about and thereby have some understanding of how to discriminate between the different modal systems.

Although logicians were able to provide a semantical theory for truth-functional logic in the early part of the century, a semantical theory for modal logic had eluded logicians until the late 1950's when Kripke and a number of other philosophers developed some promising theories. Several logicians were able to develop a semantic theory by utilizing the Leibnizian idea of possible worlds. Leibniz had believed in a principle of theodicy which claimed that among the different worlds God could have created God chose this one because it is the best of all possible worlds. Although not interested in this theological point, logicians did find the notion of possible worlds useful as they began to define necessary truth as truth across all possible worlds. The concept of "possible worlds" would allow for a uniform reading of the modal operators throughout the diverse modal systems.

There are more than ten different modal systems that have been developed which reflect the different interpretations that may be given to modalities. Systems are counted distinct just in case at least one theorem contained in one system is not contained in the other. We will focus centrally on three distinct systems. The variety of different modal systems (e.g. M, S4, S5) reflects the different senses of "necessity" that may be employed. The concept of a necessity in system S5 is much stronger than the one operating in system S4 and it is weakest in system M. System S5 employs the strongest version of necessity because when it claims that a proposition p is necessarily true in w_1

then it is true in all possible worlds accessible to w_1 but since all possible worlds are accessible to w_1 in the S5-model, p must be true in every possible world. S5 reflects the Leibnizian idea of necessity. The modal operator in systems S5 more correctly represents "broadly logical necessity" and so may be interpreted to mean "it is analytically the case that." On the other hand, the modal operator in system S4 is often translated as meaning "it is informally provable in mathematics that." Hence, the different systems of modal logic were motivated by different conceptions of "necessity" and divergent treatments of statements with iterated modalities.

Before investigating the differences among the distinct modal systems, we shall begin by pointing out the minimum qualifications that would make a logical system "modal." What conditions must a system satisfy to qualify as a modal system? The way to approach this question is first to consider the concepts that one wants to represent, concepts like necessity, possibility, contingency. Additionally, one must decide upon formal rules which would lead to acceptable consequences. With respect to the concepts of necessity and possibility, modal system should hold the following definitional equivalences as valid: **Lp ≡ ~M~p** and **Mp ≡ ~L~p**

Another condition for a modal system that seems intuitively reasonable is the qualification that a logically necessary proposition p is true. Likewise, we would want to qualify that a true proposition is also possibly true. Hence the following propositions must be valid in a modal system, the axiom of necessity and the axiom of possibility, respectively: **Lp ⊃ p**

p ⊃ Mp

There has been philosophical controversy on how to interpret the operator $-\!\ni$ "entail," "necessary implies," but what remains undisputed is the following valid proposition:

$$(p \;-\!\ni q) \supset \sim\!M\,(p \bullet \sim\! q)$$

If a modal system were to interpret $-\!\ni$ as a "strict implication" then that system could also claim that the implication holds in the other direction as well and following would also be valid:

$$(p \;-\!\ni\; q) \equiv \sim\!M\,(p \bullet \sim\! q) \text{ or}$$

$$(a \;-\!\ni\; b) \equiv L(a \supset b)$$

According to a modal system, when two propositions strictly imply each other that means each is said to be strictly equivalent to the other. This strict equivalence may be symbolized by =. Thus by definition, the following are valid for modal systems:

$$a{=}b \;=df\; ((a \;-\!\ni\; b) \bullet (b \;-\!\ni a))$$

$$(a{=}b) =df\; L(a \equiv b)$$

Since modal operators are not truth-functional, Lp must not be equivalent to any truth-function of p. Therefore, we can deduce that the following are not valid for modal systems:

$$Lp \equiv \sim\! p$$

$$Lp \equiv p$$

$$Lp \equiv (p \vee \sim\! p)$$

$$Lp \equiv (p \bullet \sim\! p)$$

It also seems intuitively acceptable that any proposition which has the form of a valid formula is not merely true but is necessarily true. Hence, a modal system should contain the transformation rule that if a is a thesis, so is La:

$$\vdash a \;\rightarrow\; \vdash La$$

A modal system must also reflect the fact that the conclusion of valid inferences runs no greater risk of falsification than its premises do. So, necessary truths must logically imply a conclusion which is itself a necessary truth. Therefore, the following is valid:

$$\mathbf{(Lp \bullet (p -\!\!\ni q)) \supset Lq}$$

A modal system can also express this formula by the variant form:

$$\mathbf{L(p \supset q) \supset (Lp \supset Lq)}$$

One modal system, System M, is a relatively weaker system than other developed alternatives. System M is weaker than systems S4 and S5 since every thesis of M is a thesis of both S4 and S5, while S4 and S5 contain theses which do not belong to M. Thus, both S4 and S5 are modal systems that can identify more valid inferences than M. A logical system can be defined by its axiomatic basis which consists of the following:

(a) primitive symbols & definitions

(b) formation rules for wffs

(c) set of wffs known as axioms

(d) transformation rules

The axiomatization of System M build upon the axiomatization of **the nonmodal system PM** derived from Principia Mathematica. Nonmodal system PN contains the following axiomatic basis:

(a) Primitive symbols of PC (Propositional Calculus):

propositional variables: set of letters p, q, r, …

monadic operator: ~ dyadic operator: v brackets: (,)

(b) Formation rules of PC:

FR1 A letter standing alone is a wff

FR2 If a is a wff, so is ~a

FR3 If a and b are wffs, so is (a v b)

(c) System PM axioms (derived from Principia Mathematica)

A1 $(p \vee p) \supset p$

A2 $q \supset (p \vee q)$

A3 $(p \vee q) \supset (q \vee p)$

A4 $(q \supset r) \supset ((p \vee q) \supset (p \vee r))$

(d) Transformation rules

TR1 The Rule of Substitution: The result of uniformly replacing any variable in a thesis by any wff is itself a thesis.

TR2 Modus Ponens: If a and $(a \supset b)$ are theses, so is b.

The axiomatic basis of **System M** may then be presented as follows (Feys 1937):

(a) Primitive symbols: same as PC but add
modal operator L (necessity operator, "it is necessary that…")

Definitions: Def •, Def ⊃, Def ≡, as in PC plus

[Def M] $Ma =_{df} \sim L \sim a$

[Def –∋] $(a \multimap b) \equiv L(a \supset b)$

[Def =] $a{=}b =_{df} ((a \multimap b) \bullet (b \multimap a))$

(b) Formation rules: same as PC but change FR2
FR2 If a is a wff, so is ~a and La

(c) Axioms: same as PM above A1-A4 but add

A5 $Lp \supset p$ [The axiom of Necessity]

A6 $L(p \supset q) \supset (Lp \supset Lq)$

(d) Transformation rules: same as PM but add

TR3 The Rule of Necessitation (N): if a is a thesis, La is a thesis

$\vdash a \rightarrow \vdash La$

System S4 is a stronger modal system which contain System M and its axiomatic basis includes that of M but with the addition of the following axiom:

A7 $Lp \supset LLp$

System S5 contains both of the previous modal systems and thus its axiomatic basis builds on S4. S5 contains the additional axiom:

A8 $Mp \supset LMp$

Another quite similar modal system worth mentioning is the **Brouwerian system**, B. System B may be formed by adding one of the following as an extra axiom to M:

$p \supset LMp$ or $MLp \supset p$

The resulting system is one that is relatively weaker than S5 and stronger than M but is neither contained by nor contains S4.

4 Semantics for Modal Systems

In the early half of the twentieth century, C.I. Lewis and other logicians accomplished important work in the formalization of modal inference and showed that there are a number of these kinds of nonequivalent modal theories. Despite these advances in the syntax for modal systems, a more thoroughgoing semantics still had to be developed. By the 1960's, modal logic still needed to identify models by which to interpret the formulas of these systems, to provide a definition for a valid formula of a modal system and to construct completeness proofs for these modal systems.

Kripke's semantics for modal systems works as follows: First he defined a model structure as an ordered triple (G, K, R) where K is a nonempty set of objects, G is a member of K, and R is an accessibility relation defined over the members of K. The relation R varies according to the particular modal system in question. For modal system M, R has to be reflexive. For the other modal systems R is further restricted. The Brouwer system requires that R be symmetrical as well as reflexive, while an S4-model structure demands that R be transitive and reflexive. An S5-model structure takes R to be reflexive, symmetrical, and transitive; hence, for S5 the accessibility relation is an equivalence relation. According to Kripke's informal semantics, we may think of K as being the set of all possible worlds and G to be one specific possible world, namely the actual world. Then, intuitively we may think of R as the accessibility of one world to others.

Given this definition of a model structure, we obtain a model for a formula p by introducing a function $\phi(p, w)$ where the first argument p ranges over the atomic formulas in p and the second argument w ranges over members of K. The values of this function are members of the set {T, F}. Thus, the function ϕ is a binary function from atomic sentences of a modal system and the various possible worlds to the truth values.

Intuitively, we may view a model as assigning a truth value to each atomic formula in each world. The truth-values of propositions are said to be distributed across the infinite set of possible worlds so that a proposition may be true in some worlds but false in others. The truth value for compound sentences of the modal system are inductively defined in the natural way using the standard truth-value definitions given to the truth-functional connectives ~ and v and by using the following definitions for the modal operators:

(1) Mp is true in w if and only if there is at least one possible worlds, w', such that w' is accessible to w and p is true in w'.

(2) Lp is true in w if and only if for every world, w', such that w' is accessible to w, p is true in w'.

Kripke defines a valid formula as a formula that comes out true in all accessible possible worlds under all assignments of truth value to its atomic parts in those worlds. Given these definitions, Kripke succeeded in producing completeness proofs for the modal systems and thereby showed that every formula that is valid in a given modal system may be derived by the syntactic rules of that system.

In modal systems, the validity of a formula depends not only on the truth-value it holds in the actual world but also on the truth-value that it may hold in other possible worlds. Propositions are recognized as having modal properties which reflect the way a proposition's truth values are distributed across an infinite number of possible worlds. In the modal system called S5, unless a formula is true in every possible world in that model, the formula would fail to be valid. In a modal system, a given proposition is valid or necessarily true in a given world w_1 if and only if it is true in every world accessible to

w_1. For the modal systems M and S4, the validity of a proposition p requires only that p be true in a specified set of worlds and not in all worlds in every model of the system. However, this specified set of accessible worlds may be infinite in number without constituting the totality of all possible worlds. In the S5-model, every world is accessible to every other world, so that the validity of p in a given world w_1 requires that p is true in every possible world in every S5-model. Whereas, in M and S4, p is necessarily true in w_1 if it is true in every world accessible to w_1 and all worlds may not be accessible to w_1.

So far we have focused on propositional modal systems, but perhaps the most interesting insights into counterfactual analysis are to be found in quantified modal logic. To each of the four propositional modal systems there corresponds a quantified modal system incorporating n-placed predicate letters, individual variables and quantifiers. Each quantified modal system contains the theses of its corresponding propositional modal system as well as those of first order predicate logic. A quantificational model structure adds another function $\psi(w)$ that assigns to each w in K a domain of individuals, i.e. ψ assigns to each world in K a set of objects. Intuitively these individuals represent the objects that exist in each possible world. Kripke calls the union of all sets of objects U which we are to think of as the set of all possible objects.

A quantificational model must build on the propositional model and provide additional directions for determining the truth value of predicate expressions and quantified formulas. Kripke proceeds to inductively define the truth values of formulas incorporating predicates and quantifiers. If p is an unquantified atomic formula of the form $P^n(x_1, \ldots, x_n)$, V(p, w) = T, relative to the assignments of $a_1, \ldots, a_n$ to x_i if and only if the n-tuple $(a_1 \ldots, a_n)$ is a member of ϕ (P^n, w). Thus, a quantificational model ϕ (P^n, w)

on a quantificational model structure is a binary function from predicate expressions and possible worlds to sets of n-tuples of members of U. Intuitively, the sets may be seen as the extensions of the predicates in given worlds. If p is a universally quantified formula of the form $(\forall x)F(x, y_1,\ldots, y_n)$ where $F(x, y_1,\ldots, y_n)$ is a formula with $x, y_1, \ldots, y_n$ as free variables, then $V(p, w) = T$, relative to the assignment of $b_1, \ldots, b_n$ to y_i if and only if $V(p(x, y_1,\ldots,y_n), W) = T$ for every assignment of a member of $\psi(w)$ to x. In other words, $(\forall x)F(x, y_1,\ldots,y_n)$ is true in w just in case $F(x, y_1,\ldots,y_n)$ comes out true in w regardless of how we replace the value of x with objects from w. Existentially quantified formulas may be defined similarly. $(\exists x)F(x, y_1,\ldots,y_n)$ is true in w just in case there is at least one object in w taken as the value of x such that $F(x, y_1,\ldots,y_n)$ is true in w. The notion of validity for quantificational model structure may be defined just as Kripke had for propositional logic: a formula is valid in a quantified modal system just in case it comes out true in all quantificational models on a quantificational model structure.

The important advances made in the semantics of modal logic provide us with models for interpreting the formulas of various modal systems, determining validity and establishing completeness proofs. Logicians like Kripke were able to give us a uniform reading to modal operators in diverse nonequivalent systems. To achieve this, Kripke had to import new kinds of entities called “possible worlds” into his ontology. Along with “possible worlds” is admittance of the set of all possible objects contained in these worlds and a variety of new problems and unanswered questions that had to be dealt with. It is argued that the possible worlds framework leads to certain unacceptable logical consequences which seem either absurd or highly implausible. Critics of the possible worlds approach argue that the possible worlds approach to analyzing modal statements and counterfactuals should be rejected for various reasons.

Among them is a problem concerning the identity of possible individuals in these possible worlds. Questions arise such as: Who occupies these possible worlds? Does each possible world have the same number of occupants or do some worlds have more and other fewer residents? Can the same individual inhabit different worlds or is there a whole different set of unique individuals per world?

Some people welcome the idea of transworld individuals over a theory which assumes worldbound individuals. Certain problems seem to arise with worldbound individuals which may be avoided when a theory allows for transworld identity. A theory committed to worldbound individuals restricts the existence of individuals to only one world. If a worldbound individual exists in the actual world, then it does not exist in any other possible worlds and similarly if a worldbound individual inhabits some other possible world, then it does not exist in any other possible world including the actual world. The belief that no individual exists in more than one possible worlds leads to the consequence that it is impossible for things to have gone differently for individuals. One would have to grant that every property of an individual turns out to be essential.

Nonetheless there have been proponents of worldbound individuals such as Leibniz and David Lewis who have claimed that an object exists in only one world. Idealists committed to the doctrine of internal relations also seem compelled to believe in worldbound individuals since their belief that all relations between individuals are internal means that all relational properties possessed by an individual are essential to them. This latter idea seems to entail that all properties are essential to their possessor and hence an individual could not lack any of its properties and is bound to one world.

People who think that things could have been different from the way they actually are or who think that some properties of individuals are merely accidental are more likely to favor the idea of "transworld" individuals over the idea of "worldbound" individuals. The Theory of Transworld Identity claims that one and the same individual may exist in different possible worlds. To say that Socrates could have been a politician rather than a philosopher amounts to saying that in some other possible world the individual Socrates exists and in this other possible world Socrates is a politician. A transworld individual exists in more than one possible world.

Two main objections raised against the notion of transworld individuals deal with the formal properties of identity: The Indiscernibility of Identicals and Transitivity. The Indiscernibility of Identicals is the principle that for any object x and any object y, x is identical with y if and only if every property of x is a property of y. Transitivity is another formal property of identity which stipulates that for any three objects x, y and z, if x is identical to y and y is identical to z, then x is identical to z. It appears that commitment to transworld individuals would violate both of these principles.

The argument from the Indiscernibility of Identicals begins with the supposition that the same individual x exist in different worlds. But the Indiscernibility of Identicals demands that if two person, x and W^1 and x in W^2, are identical than they ought to share the same properties. One obvious difference in properties between x in W^1 and x in W^2 is that x in W^1 exemplifies the property of "existing in W^1" and x in W^2 exemplifies the property of "existing in W^2." Even differences in the circumstances and events which occur in W^1 and W^2 present differences between x in W^1 and x in W^2. For example, perhaps the only difference between W^1 and W^2 is that Socrates is snub-nosed in W^1 but not in W^2. Hence, x in W^1 exemplifies the property a thing has just in case it is a person and Socrates is snub-nosed whereas x in W^2 exemplifies the property a thing has just in case it is a person and Socrates is not snub-nosed.

Another argument claims that, even granted the Indiscernibility of Identicals and the view that individuals may exist in different worlds, the theory of transworld individuals would be incompatible with the principle of transitivity. Roderick Chisholm illustrates this problem of transitivity as he entertains the idea of a possible world inhabited by Adam and Noah who are much like the real biblical figures except for having swapped certain properties with each other:

> Suppose Adam had lived for 931 years instead of 930 and suppose Noah had lived for 949 years instead of 950… Both Noah and Adam, then, may be found in W^2 as well as W^1 (i.e. the actual world). Now let us move from W^2 to still another possible world W^3… In W^3 Adam lives for 932 years and Noah for 948. Then moving from one possible world to another, but keeping our fingers, so to speak, on the same two entities, we arrive at the world in which Noah lives for 930 years and Adam for 950. In that world, therefore, Noah has the age that Adam has in this one, and Adam has the age that Noah has in this one; the Adam and Noah that we started with might thus be said to have exchanged their ages. Now let us continue on to still other possible worlds and allow them to exchange still other properties. We will imagine a world in which they have exchanged the second, then one in which they have exchanged the fourth, with the result that Adam in this new possible world will be called "Noah" and Noah "Adam."[14]

Let us suppose that we continue with slight interchanges of properties between Adam and Noah. Inevitably we will come to some world W^n where the accumulating interchanges

[14] Roderick Chisholm, "Identity Through Possible Worlds: Some Questions," *The Possible and the Actual*, Michael Loux, ed., (Ithaca: Cornell University Press, 1979), 150.

result in Adam's having in W^n all the properties which our original Noah had in W^1. On the other hand, Noah in W^n would have assumed all the properties of our original Adam in W^1. Apparently Adam in W^n is identical to Noah in W^1 and Noah in W^n is identical to Adam in W^1. This is in conflict with the principle of transitivity since we began with two distinct individuals Adam and Noah in W^1. At every stage we can picture the slight alterations as preserving identity such that Adam in W^1 is identical with Adam in W^2, Adam in W^2 is identical with Adam in W^3, … Adam in W^{n-2} is identical with Adam in W^{n-1}, and Adam in W^{n-1} is identical with Adam in W^n, but the accumulated alterations prevent us from applying transitivity so as to conclude that Adam in W^1 is then identical with an Adam in W^n who in this world possesses all the properties and hence the identity of the original Noah in W^1.

Another conflict philosophers have with the use of possible worlds is the apparent lack of a criteria for identifying and individuating possible objects, or as Roderick Chisholm calls the problem of "knowing who." Quine's famous reproach of possible objects consists of a string of apparently unanswerable questions:

> Wyman's slum of possibles is a breeding ground for disorderly elements. Take for instance the possible fat man in that doorway; and again the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no two possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles? But what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct to one another?[15]

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[15] Quine, "On What There Is," *From a Logical Point of View* (Cambridge: Harvard University Press, 1980, 4.

Until such questions can be answered Quine views possible worlds as “slums of possibles,” “a breeding ground for disorderly elements.”

CHAPTER THREE REALISM VS. CONSTRUCTIVISM

Ontological commitments to abstract objects are also defended in mathematics. The main goal of this chapter is to examine and to evaluate the Platonist mathematical theory in order to draw parallels between this realist approach to possible worlds. Both possibilism and Platonism assume the existence of abstract objects in order to explain the truth-values of certain kinds of statements: modal statements and mathematical statement. Like the possible worlds for Lewis, mathematical objects such as sets or numbers are posited by Platonist as having a mind-independent existence.

Traditionally, working mathematicians have adopted a Platonistic attitude toward their subject matter. The general assumption is that classical mathematics can only be accommodated by a Platonist ontology which presupposes the existence of abstract mathematical objects. Often the mathematician may accept the existence of certain entities without further inquiry into the ontological nature of their being. Such a mathematician might view the symbol “3” as denoting an entity in the real world and likewise the expression “2+1” or “the successor of 2” as denoting the same entity. Mathematical objects such as sets, numbers, functions, points, etc. are posited by Platonist as having a mind-independent existence. These abstract mathematical objects are claimed to exist in some non-spatial, non-temporal realm. They existed prior to any human’s first engaging in mathematical activities and they would continue to exist even if human minds ceased to perform mathematical operations. Thus mathematical objects exist in a way that is both independent of cognitive operations and unamenable to the possibilities of verification.

Mathematicians who assume that mathematical objects exist independently of us and that mathematical truths are discovered rather than created have been categorized either as “Platonists” or “realists.” Realism holds the philosophical position that universals or abstract entities exist in the external world independently of our consciousness or perception of them. The title “Platonism” originates from Plato’s ideal theory of forms. According to Plato’s theory, ideal objects he called “forms” exist in the real world, but the forms did not exist as physical objects in the material universe and neither as mental objects in the human mind. These ideal forms may be represented in the physical world by concrete realizations which are only imperfect copies of the true ideal forms. Like Plato’s forms, “Platonists” insist that mathematical objects exist as ideal objects that may have concrete realizations, in the way that a dozen eggs is a concrete instance of the number 12.

However, the Platonist would deny that mathematical objects such as numbers or sets can be identified with any group of physical objects. First, our material universe is not infinite and so numbers cannot be material objects. Secondly, it makes no sense to ask where numbers or sets are located, when they came into existence or how long they will last. Thus, the Platonist claims that mathematical objects are not ordinary physical objects but are abstract, lying outside of space and time.

Third, the truths of mathematical statements are believed to be more necessary than statements or laws about the physical world. Yet in order to conceive of the idea of a set, we might need to depend on a model composed of physical objects such as marbles or children’s building blocks, but such a model would serve only as heuristic device.

Similarly simple mathematical statements such as "2+3=5" can be checked by observation. Collect two marbles together, then form another group of three marbles, and finally combine both groups together. Each time you perform this physical activity regardless of the physical objects you use, you will be able to verify if indeed the resulting group of physical objects amounts to 5. So, in one sense, the concept of number does apply to empirically observable objects since they can be counted and the corresponding numbers can be said to obey the law "2+3=5." However, despite the agreement of actual manipulations of physical objects with mathematical statements (that is, as far as we know), no set of physical objects can be deemed the proper object of mathematics since the certainty with which we hold our mathematical statements exceeds the confidence we place on laws based on observation.

Instead the Platonist accounts for the certainty of mathematical truths as flowing from the existence of abstract objects. A powerful tradition in philosophy has regarded mathematical truths as a case of a priori knowledge. Platonists attribute the abstract character, generality, exactitude and certainty of mathematical truths to the nature of abstract mathematical objects. Moreover, Platonists see mathematicians as discoverers of truths that are independent of us, as opposed to constructivists who claim that the activity of mathematicians is essentially one of creating rather that finding?

1 Platonist Interpretation of Mathematics

Hence, the debate over the existence of mathematical objects can be reformulated as a question about the role of the mathematical subject, that is the mathematician. Is the mathematician's role primarily that of a discoverer or a creator? Charles Chihara compares two mathematicians' characterization of their mathematical activity:

> The discovery of the proof that pi is transcendental did not create any logical relations but showed us what the relation always has been. [A proof] makes new connections, and it creates the concept of these connections.
>
> [A proof] does not establish that they are there; they do not exist until [a proof] makes them.[16]

Traditionally, it has been the idea of the mathematicians as a discoverer of independent mathematical truths which has dominated the history of mathematics. This Platonist position was adopted by early mathematicians and logicians such as Frege, Russell, and Godel. Throughout the nineteenth century, mathematicians endeavored to make arithmetic and analysis more rigorous. This required the development of an axiomatized system and the use of a theory of natural numbers to define the concepts of arithmetic and analysis. The axiomatization and definition of mathematics was developed under Platonist assumptions in the sense that both sets and numbers were treated as existing themselves. Georg Cantor's development of set theory further provided a general framework for this enterprise which led to even a greater abstraction and stronger Platonist assumptions.

Upon the discovery of the paradoxes of set theory in the early twentieth century, the concept of class or set began to appear problematic and in need of clarification. In response to the problems posed by these paradoxes, other general theories (formalism, intuitionism, logicism) were developed which questioned the early

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[16] Charles Chihara, "Mathematical Discovery and Concept Formation," *The Philosophical Review*, 72 (January 1963), 17.

assumptions made by Platonists. Because our primary goal is the analysis of counterfactual statements and our investigation into mathematical statements and Platonism are conducted toward that end, we will treat these other schools of thought as one body of various criticisms launched against Platonism. Also, we will limit our attention to the more developed and perhaps convincing versions of Platonism such as Penelope Maddy's theory about the sets. Of course other forms of Platonism have assumed different kinds of abstracts entities as its mathematical objects such as the totality of natural numbers or the totality of the points of the continuum; however, I think one strong version of Platonism should suffice to reveal the general advantages and drawbacks with the Platonist enterprise as a whole.

Despite counter-theories, Platonism is still a dominant position in the modern philosophy of mathematics. The working mathematician is content to be given some "entities" like sets, numbers, spaces or points to work with and, unlike the philosophical logician, for the mathematician's main concern is not with the inner character of such entities but with the mathematical structure that they exhibit. Our special interest in ontological matters leads us to questions which mathematicians do not standardly ask: What is the nature of these abstract entities and how do they relate to other kind of entities (e.g. minds)? Are these mathematical entities to be taken as primitive or are they reducible to more fundamental entities?

Debates over these questions have lead to arguments in support of the Platonist ontology as well as to objections against it. An understanding of the theoretical merits and drawbacks of adopting a Platonist view of mathematics will hopefully bring us to a better position of evaluating realist theories about possible worlds.

One version of Platonism, once defended by Penelope Maddy, helps to reveal the general theoretical virtues of Platonism and the apparent success of Platonism in classical mathematics. Maddy refers to her version of "Platonism" by the title "set theoretic realism." She prefers the term "realism" to "Platonism" since her theory postulates sets which should be regarded as particulars rather than universals. Her view is ontologically committed to sets as existing independently of the mind. Like Godel, Maddy claims that mathematicians can causally interact with mathematical objects through a special faculty she calls "mathematical intuition." Maddy draws an analogy between sense perception and mathematical intuition, insisting that mathematical intuition gives us knowledge of sets of numbers (i.e. properties of sets) analogous to the way sense perception gives us knowledge about physical objects.

In defense of her set-theoretic realism, Penelope Maddy identifies four advantages in adopting a Platonistic view:

(1) It allows a straightforward Tarskian semantics for set theoretic discourse.

(2) It makes no mystery of how mathematical premises can combine with physical ones to yield testable consequences in physical science.

(3) It squares with the prephilosophical views of most working mathematicians.

(4) It allows set theoretic practice to remain as it is; it does not demand reform[17]

These consequences add to the attractiveness of the Platonist ontology for it allows us to preserve past practices and assumptions made in mathematics, language and science.

[17] Penelope Maddy, "Perception and Mathematical Intuition," *The Philosophical Review*, 89, no. 2 (April 1980), 163-164.

With respect to (3), we have already noted that most working mathematicians do postulate independent abstract entities such as numbers or sets as the objects of their mathematical activity. The well-know mathematician, G.H. Hardy, suggests that "mathematical reality lies outside us, that our function is to discover or to observe it, and that the theorems which we prove and which we describe grandiloquently as our creation, are simply our notes of our observations." Likewise, Godel claimed mathematical objects as necessary for the development of a satisfactory system of mathematics as physical objects are necessary for the satisfactory theory of sense perceptions. Godel characterized mathematicians as having certain experiences of "axioms forcing themselves upon us as being true."[18] This kind of mathematical experience for Godel was a kind of perception of mathematical objects which he claimed humans are capable of.

Another advantage in accepting a realist perspective of mathematics is the opportunity of benefitting from the accomplishments made in set-theory and also in semantics. It adds the comprehensiveness of our theories in different fields, as well as to overall convenience, to possess a homogenous semantical theory in which semantics for the propositions of mathematics is similar to the semantics for the rest of the language. Adopting a Platonist ontology would furnish us with an account of truth that treats mathematical discourse in a way that is uniform to our treatment of nonmathematical discourse. The Platonist takes the surface syntax of mathematical statements to be their logical form. Thus it seem appropriate for the mathematician to include abstract objects such as sets or numbers within their domain of objects since mathematical statements

[18] K. Godel, "Russell's Mathematical Logic," *Philosophy of Mathematics* (Prentice-Hall, 1964), P. Benacerraf and H. Putnam, eds., 230.

appear to make reference to such mathematical entities. Mathematicians can then rely on Tarskian semantics with quantification ranging over abstract objects such as particular kind of entities like 'natural numbers' or 'sets' or even specific entities like 'the null set' or 'pi'. According to the Platonist view, mathematical statements say something about abstract mathematical objects and the structural properties that these objects possess. Mathematical statements are true in virtue of the nature of abstract mathematical objects. In other words, true mathematical statements are descriptive of this realm of mind-independent abstract objects that is causally distinct from the ordinary realm of physical objects. Both Frege and Russell belonged to the Platonist camp, seeing abstract mathematical objects as the referents of the singular terms and values of the variables contained in true mathematical statements. An acceptable general theory of truth that is uniform across our discourse, i.e. that treats superficially similar sentences in similar ways, is assumed to be more favorable than a less comprehensive theory of truth.

In addition the objectivity of mathematics seems to support a Platonist ontology. For, if mathematical objects are external to the mind, then statements about these objects would not be subjective to any one mind. Mathematical statements would be true in an objective sense in contrast to the subjectivity of aesthetic judgments. Rarely is there a disagreement over the acceptability of a mathematical proof or calculation. Thus, the independent existence of abstract mathematical objects would account for the agreement of results found in mathematical operations.

Although critics of Platonism are willing to concede the above advantages to a realist view of mathematics, they insist that weightier drawbacks attend these gains. In the article "Mathematical Truth" which stirred renewed interest over these matters, Benacerraf point out two opposing camps: those concerned with ontological matters vs. those concerned with epistemological ones.[19] Benacerraf distinguishes realism as belonging to the first group whose main goal is to address questions about what mathematical objects exist and what mathematical statements mean at the expense of questions of how we know mathematical facts. Usually what is lacking in a realist theory of mathematics is a coherent and believable account of the causal interaction between knower and abstract objects. This means we lack an adequate explanation for how mathematical objects are casually involved in the production of our mathematical beliefs.

Hence, philosophers who think Platonism is suspect usually take its shortcomings to be of a certain broad epistemological sort. According to the Platonist interpretation of mathematics, mathematical statements are true in virtue of the fact that they describe the abstract mathematical objects which manifest the structural features of the world. Our knowledge of mathematical truths would then depend not only on the existence of such abstract mathematical objects but also on our ability to investigate that existence. On a realist view of mathematics, our mathematical beliefs are reliable, but this high degree of reliability appears to remain inexplicable given realist assumptions about ontology. For many critics, the Platonist conception of mathematics seems to preclude any credible nonmystifying account of our mathematical knowledge. Even if we accepted the idea of "mathematical intuition," many questions would still remain: How many people have had these experiences? What sort of people? And under what conditions? Appealing to mathematical experiences is much like appealing to so called "mystical experiences;" justification on these grounds are often received with much skepticism.

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[19] Paul Banacerraf, "Mathematical Truth," *Journal of Philosophy*, 70, no. 19 (November 1973), 661-679.

2 An Alternative to Platonism

But must our mathematical language assume abstract mathematical objects such as number or sets to be its objects of discourse? Are we forced to accept a Platonistic ontology in order to explain mathematics? In his book *The Nature of Mathematical Knowledge*, Philip Kitcher advances a different interpretation for mathematical statements and proposes an alternative ontology. He takes certain "mathematical activities" to be the proper objects of mathematics. His project is to replace the notion that mathematical statements asserts the existence of abstract entities with the notion that they assert the existence of constructive operations performed by an ideal subject, i.e. an idealized agent free of biological limitations who can perform ideal operations.

It will not be my aim in this section to defend Kitcher's mathematical theory against opposing views. However, to the extent that it may be relevant to the problem of counterfactuals, I will discuss both the theoretical drawbacks and merits of Kitcher's theory. Despite a number of objections, Kitcher's mathematical ontology is an innovative idea that provides an illuminating picture about mathematical knowledge and deals with important epistemological issues unaddressed by Platonists. My eventual goal is to show that even if Kitcher's recommended ontology of "mental constructions" may not fare well in the field of mathematics, a similar ontology may prove useful when applied in the area of modal discourse.

Kitcher's theory diverges from traditional ideas about mathematical knowledge by rejecting the assumption that mathematical knowledge is a priori. This view has been widely accepted by mathematicians and philosophers including such notable apriorists as

Descartes, Locke, Kant, Frege, Hilbert, Brouwer, and Carnap. Although their respective theories contain internal differences, these theorists believed that humans do not acquire mathematical knowledge through sense perceptions. So unlike the discipline of science, the study of mathematics does not depend on one's observations of the physical world or experimentation on material objects. Kitcher breaks from the view of mathematical apriorism in favor of his own version of mathematical empiricism. Kitcher claims that the rejection of mathematical apriorism is not entirely without precedent citing J.S. Mill, W.V. Quine, Hilary Putnam, Lakatos as early proponents of the empirical nature of mathematical knowledge. His goal is to develop a more complete formulation of this thesis.

Kitcher rejects not only mathematical apriorism but also an apsychological conception of knowledge. The apsychologistic account of knowledge claims that knowledge consists of true beliefs that are independent of the causal events which produced the belief. Some proposition can be an item of knowledge so long as it is logically connected to other propositions which are believed. Logical relations among propositions not psychological processes of the believer determine whether a true belief counts as an item of knowledge. The method of mathematical proofs is one way in which a given proposition can be determined to be mathematical truth in a particular system. Proofs show the logical relations between the assumed axioms of a system and the theorem in question. If a proposition is a theorem, then a proof may be constructed which consists of a sequence of sentences in the language of the system such that every member of that sequence is either an axiom or a sentence derived by rules of inference from previous lines.

Instead of an apsychological view of knowledge, Kitcher favors a more psychologistic approach which takes into consideration the psychological events that produced the particular mental state in the believer. Kitcher sees the progress of mathematical knowledge as a historical process consisting of psychological events experienced by individuals. Each generation inherits a store of mathematical knowledge from their predecessors who in turn also acquired their knowledge from prior knowers. Most individuals learn mathematics from competent mathematicians and so can be said to obtain knowledge of mathematics based on the testimony of authorities. This chain of knowers of course must have started at some initial stage when one member of the community was first able to establish the truth of a particular mathematical proposition. Kitcher's account identifies one's sense experience of the physical world as the initial source of mathematical knowledge. Kitcher states:

> If [my] explanation is to be ultimately satisfactory, we must understand How the chain of knowers is itself initiated. Here I appeal to ordinary perception. Mathematical knowledge arises from rudimentary knowledge acquired by perception…learning through practical experience some elementary truths of arithmetic and geometry. I shall try to explain how perpetual origins for mathematical knowledge are possible.[20]

Mathematical knowledge may be linked back to our causal interaction with the physical world. Through our senses we are able to observe and manipulate physical objects allowing us to delineate the structure these objects exemplify. From these observations we can infer the structure that all physical objects exemplify and view our own actual operations with physical objects as instances of operations we can perform with respect to any objects. This link between mathematical knowledge and its perceptual origins

[20] Kitcher, *The Nature of Mathematics*, 105.

reveal the constructivist nature of Kitcher's theory. Yet, it is important to clarify that Kitcher does not ground mathematical knowledge on actual operations but on an idealized description of such operations that an ideal subject can perform as opposed to the actual activities which human agents do or can engage in.

One such operation that a human agent (and hence and ideal agent) performs in mathematic is the activity of "collection." We can acquire mathematical knowledge through mathematical operations of collecting and combining. At a very rudimentary level collecting occurs when we manipulate physical objects, combining and separating them into groups or collections.

> A young child is shuffling blocks on the floor. A group of his blocks is segregated and inspected, and then merged with a previously scrutinized group of three blocks. The event displays a small part of the mathematical structure of reality…Children come to learn the meanings of 'set,' 'number,' 'addition' and to accept basic truths of arithmetic by engaging in activities of collecting and segregating. Rather than interpreting these activities as an avenue to knowledge of abstract objects, we can think of the rudimentary arithmetical truths as true in virtue of the operations themselves. By having experiences…we learn that particular types of collective operations have particular properties: we recognize, for example, that if one performs the collective operation called 'making two,' then performs on different objects the collective operation called 'making three,' then performs the collective operation of combining, the total operation is an operation of 'making five.'[21]

Analogous to the child's "shuffling" of blocks, when we do mathematics we engage in a more abstract and or idealized form of collecting, but it is a collecting which disregards the identity of the things which are being collected. It makes no difference whether our activity of collecting proceeds as the manipulation of building blocks or symbols or

[21] Ibid., 107-108.

names. Our mathematical statements say something directly about the nature of such "collectings." Kitcher claims that if we must picture something as being the objects over which our mathematical variables range then that something could just as well be these kinds of constructive operations and not abstract mathematical objects.

The constructive operations we perform when we do mathematics, even for the idealized subject, is ultimately linked to the way the world will allow itself to be structured. Our constructive activities however we may undertake them are framed by the structural possibilities which the physical world allows. Human mathematical activities are part of the world and so are constrained by it. According to Kitcher, "we might consider arithmetic to be true in virtue not of what we can do to the world but rather of what the world will let us do to it."[22] At another level, mathematical statements owe their truth to the structure of the world or the "permanent possibilities of manipulation" which inhere in the world. Nonetheless, we must not view these two levels as separate contributors to the truth of mathematical statements. Both the structure of the world and our abilities at construction are to be understood as being integrally linked to one another. Thus, to posit that mathematical truths owe their truth to idealized human activities and that they also owe their truth to the structural features of the world are compatible statements. Hence, Kitcher successfully reconciles the claim that mathematics is true in virtue of human abilities with the competing claim that mathematics is true in virtue of the structure of the world.

Kitcher emphasizes that even though mathematics is the activity of "collecting," it does not have to be thought of as the activity of forming "collections." He admits that

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[22] Ibid.

when we think of the operations which we perform in mathematics it is “convenient” to think of them as having a product. It is easy to conceive of our constructive activities as bringing about some new entity for which we have descriptive predicates. But, we are assured that we need not resort to talk of “collections” when we have the option of talking about “the collecting” themselves.

3 Problems with Platonism

Let us now reconsider the arguments for Platonism in light of Kitcher’s arguments for an alternative mathematical theory. I hold the view that Kitcher’s theory of mathematics is to be preferred over the Platonist account. I will justify this conclusion in two ways (1) by showing that an ontology of “ideal constructions” avoids certain problems which an ontology of “abstract objects” faces and (2) by arguing that this preferred ontology preserves the theoretical advantages which the Platonist ontology offers. I will save discussion of (2) along with other objections unique to Kitcher’s theory until Chapter Six. This will allow me to draw parallels between arguments in support of Kitcher’s mathematical theory with my own defense of a modal theory that is built on a similar kind of ontology.

One major problem that Platonists confront lies with the notion of intuitions. Despite attempts by some philosophers such as Maddy, we are left with an incomplete account of the nature of these mathematical intuitions. Godel and Maddy attempt to explain intuitions by means of analogy to sense perception. Our knowledge of the world originates from our ability to causally interact with physical objects in the world.

Knowledge of physical objects is gained through our sense-perception. According to Platonist, mathematical intuition allows us access to other kinds of objects – abstract mathematical objects. Much like a scientist gathers data about the material world through sense-perception, the mathematician obtains facts about a non-material world through mathematical intuition. Mathematical intuition is the means by which we may causally interact with mathematical objects and learn about mathematical truths.

Yet, this analogy not only reveals very little about the nature mathematical intuition but it provides no proof that mathematical intuition in fact exists. At most, the Platonist has shown that mathematical intuition might exist since sense perception exists. But, this line of reasoning is as weak as claiming that humans can communicate with spirits since they can speak to other humans. The way to prove that people can interact with abstract mathematical entities (as with the case of communicating with spirits) is to point to certain people who have had such experiences. The obvious candidates the Platonists would point to are mathematicians. To prove that mathematical intuition exists and is the source of mathematical knowledge, the Platonist need not claim that every human has this ability nor has ever exercised it.

The problem with mathematical intuition for the Platonist can be viewed as an offshoot of an underlying problem – the notion of abstract mathematical objects. If there are such things as abstract mathematical objects, what kinds of things are they? Mathematicians have a number of options to choose from: numbers, spaces, functions, groups, etc. However, given the fact that all these entities may be reduced to sets, the likely answer for the Platonist is that mathematical objects are sets. With this, the issue is still unresolved for the Platonist since there are a number of different ways to identify sets. For example, '2' might be the set {{O}} or {O, {O}}. Platonists confront the difficulty of deciding in a non-arbitrary way the reference of the sets.

An additional problem with abstract objects concerns their connection with the physical world. Clearly we find that by studying mathematics we are able to better explain and predict the behavior of physical objects. We know that there is survival value in having true beliefs about the physical world but how can there be survival value in having true beliefs about abstract mathematical objects. After all, the objects of mathematics according to Platonists lies outside of the spatio-temporal realm of the physical world and so cannot causally interact with physical objects. Yet, we depend on mathematical and scientific premises to tell us something about the physical world. Hence an explanation is in order to account for the apparent usefulness of mathematics despite the causal independence of mathematical and physical objects. Platonists must explain how the physical world exemplifies mathematical propositions that are supposed to be true of abstract and not physical objects.

On the other hand, Kitcher's psychologistic account of mathematical knowledge does not require the postulation of a special intuitive faculty. The initial source of mathematical knowledge is perception. Perception is the process in which we causally interact with physical objects. Thus, the faculty which allows us to build our mathematical knowledge is the same faculty we employ when we causally interact with ordinary material objects. Of course a mathematical proposition is not true of only a single act of perception upon one particular group of physical objects. If we recognize various groups of physical objects as multiple instances of a common structure, then we can view our actual perceptual operations as acts which delineate the structure exemplified by all physical objects. Thus, perception can be viewed as the basis of our mathematical knowledge since it is by means of this faculty that we are able to recognize the common structure exemplified by the physical objects. Our perceptual powers enable us to recognize a common structure which any given group of physical objects instantiates. The activity of mathematics may be viewed as consisting of mental operations which we can ideally perform on any given set of physical objects. These ideal operations uncover mathematical truths that reflect the general structure which all physical objects exemplify.

By identifying perception as the fundamental source of mathematical knowledge, Kitcher's theory need not posit mathematical intuition and consequently is not forced to assume the objects of intuition, i.e. abstract mathematical entities. We would be freed of the Platonist complicated research problem of investigating abstract mathematical objects if we substitute mathematical operations as the real objects of mathematics. Kitcher advocates that

> Instead of supposing initially that mathematics is about abstract objects and then, when we find multiple instances of a common structure, reinterpreting statements as descriptive of the structure exemplified in those objects, why do we not begin from the thesis that mathematics is descriptive of structure without making the initial move to Platonistic objects?[23]

There are several other advantages to be gained by not assuming abstract objects. For one, by taking idealized operations as objects of mathematics, we are freed from having to identify what mathematical abstract objects are. As we have noted earlier, Platonists have traditionally run into difficulties when defining what kind of

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[23] Ibid, 110.

mathematical object numbers can be (e.g. are numbers sets? If so what sets are they?) Reference to independently existing sets or collections would be superfluous when we can explain mathematical truths as being true in virtue of the collecting and combining operations themselves. Instead of having to investigate the properties of sets and natural numbers, our focus would then turn to the mathematical operations themselves rather than to the seeming products of such operations.

Another desirable consequence of Kitcher's theory is the apparent utility of mathematics. According to Kitcher's theory, mathematics is descriptive of a common structure which all physical objects share. Therefore, a mathematical proposition may be viewed as a general statement about the abstract structure manifest in the world. Consequently, from mathematical truths, we may deduce true beliefs about the physical world. Any arrangement of physical objects exemplifies this general structure and so can be taken as a concrete instance of some mathematical truth. What is true of our mathematical operations on the general structure of the world is thus reflective of the actual configuration of physical objects. That is why mathematical and scientific knowledge can be combined to give us practical knowledge of our surroundings, enabling us to better describe, explain and predict facts about our physical world.

Thus, the reforms advocated by Kitcher's alternative theory does not commit us to problematic abstract entities such as sets. As a consequence, we are not troubled with having to explain the nature of mathematical intuition and to account for the utility of mathematics. In chapter Six I will argue that in adopting an ontology of "constructions," we can still preserve much of the theoretical merits which makes Platonism so attractive in the first place.

In the next chapter I will explore the ontological commitments of the modal realism developed by David Lewis. Parallels will be drawn between the enterprise undertaken by the Platonist in the field of mathematics with that undertaken by Lewis in modal discourse. Similar epistemological questions will have to be raised concerning a realist view of possible worlds. Does the modal realist run into the same epistemological stumbling blocks as the Platonist? How are we to determine the truth-value of modal statements if they refer to inaccessible abstract objects? How can we, inhabitants of the actual world, investigate the nature of the possible worlds which we don't inhabit? Why do truths about possible worlds prove so useful to us in making predictions and explaining events which occur in the actual world? I will argue that answers to these questions remain far from satisfactory and recommend that we rid ourselves of these problems by making amends to the theory which gives them rise. This will lead us to a reinterpretation of possible worlds that corresponds to the Kitcher's reformulation of abstract mathematical objects.

CHAPTER FOUR A REALIST VIEW OF POSSIBLE WORLDS

The idea that reality has a modal structure was first accepted in traditional metaphysics. Leibniz' metaphysical myth about the creation of the world had introduced the notion of possible worlds. Leibniz believed in an omniscient and all-powerful God who could have chosen to create one of an infinite number of possible worlds, each world competing for the privileged position of being actualized. In God's perfect wisdom, God chose to create the best of all possible worlds and hence the universe as we know it came to be. Since Leibniz' day, logicians have borrowed the notion of possible worlds not so much for want of a creation story but in the hopes of giving content to the formal semantics for modal logic. Recent work on the semantics of modal logic and forms of modal discourse have renewed interest in the Leibnizian idea that our world is not the only possible world.

The concept of possible worlds offers a powerful heuristic device for interconnecting the formulae of modal logic and giving meaning to out modal discourse. Nothing about the formal semantics proposed by Saul Kripke restricts us from adopting abstract possible worlds for our domain of objects. Kripke offers a formal or pure semantics which commits itself to a purely set theoretical construction that does not illuminate modal notions. This pure semantics defines the meaning of a valid formula but does not assign a meaning to modal operators or explain what it means for an object to have a property essentially. Kripke's semantics will generate the same S-valid sentences regardless of the objects we may assign to the elements of K.

Nonetheless, if we wish our modal concepts ('necessity', 'possibility') to convey a particular meaning, we must be discriminating in our choice of objects. For example, we would not want to choose the set of apples to be K and the relation "x is on the same tree as y" to be R. For then this would mean that a sentence is necessarily true just in case it is true on every apple that is on the same tree as some given apple. If we are to preserve an account of the meaning of modal statements that accords with our intuitions, it appears that we should not posit any arbitrary set of objects but be selective in our choice. Proponents of a possibilist interpretation of modal statements believe that one is forced to accept the existence of possible worlds if one is to render modal discourse meaningful. Kripke's informal semantics even suggests that we adopt the set of possible worlds for K and interpret G to be the actual world:

> Intuitively we look at matters thus: K is the set of all 'possible worlds'; G is the 'real world'. If H_1 and H_2 are two worlds, H_1RH_2 means intuitively that H_2 is 'possible relative to' H_1; i.e., that every proposition true in H_2 is possible in H_1….Then, we would understand a sentence to be necessarily true just in case it is true in every possible world that is accessible to some given possible world.[24]

It is convenient for modal systems to utilize the concept of "possible worlds" in its informal semantics. But if we admit possible worlds into our ontology, we then face the problem of explaining what exactly is the nature of possible worlds. The realist view of possible worlds, otherwise knows as "possibilism," provides us with one account that tries to explain the nature of possible worlds. This position is one defended by philosophers such as David Lewis and Robert Stalnaker who insist that modal logician's

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[24] Saul Kripke, *Naming and Necessity* (Cambridge: Harvard University Press, 1980), 24.

talk of possible worlds should be taken literally. A possibilist believes that necessity and possibility are analyzable in terms of quantification over some kind of entities. Lewis' realism about possible worlds can be categorized as an extreme form of possibilism. Like other possibilists, Lewis believes in the existence of nonfactual possible worlds and maintains that the notion of a possible world is not to be analyzed in terms of actual things.

Thus, a possibilist sees possible worlds as concrete entities much like the actual world. This approach begins with the whole system of possible worlds and sees the actual world first as a possible world, a member of that system. Since possibilism confers an independent existence to possible worlds, it must also provide a world-relative concept of truth. In contrast to possibilism, actualism takes the opposite approach. Actualism begins with the actual world and treats talk about the system of possible worlds as a way of talking about a proper part of the actual world. Actualists, unlike Lewis, insist that the use of the term "actual" is not world-relative and can apply properly only to one world.

1 Counterpart Theory and an Indexical Notion of Actuality

Lewis develops a version of possibilism which he calls his "Counterpart Theory." According to Counterpart Theory, transworld identity is false. Lewis denies that the same individual can exist in several possible worlds in which it may have somewhat different properties and may experience different events. On the contrary, Lewis insists that all individuals are worldbound, i.e. every individual can exist only in one world. He attempts to reconcile the notion of worldbound individuals with the claim that things might have been different does not entail that there are worlds in which different things do happen to the same individuals that inhabit the actual world. Lewis claims that he is not forced into the existence of transworld individuals.

Lewis replaces the need for transworld individuals by introducing the notion of “counterparts.” While each individual is worldbound, Lewis allows that an individual existing in one possible world may have a counterpart in another possible world. Your counterpart is an individual who occupies a different world yet closely resembles you in significant ways. If we reinterpret Goodman’s counterfactual about the match according to Lewis’ counterpart theory, we would then need to posit the existence of counterparts for the match and the striker of the same match in some other world. Hence, the counterfactual “If I struck the match, then it would have lighted” would mean that in some other possible world there is an individual who closely resembles me, i.e. my counterpart, and in this other world there is also a counterpart to the match. However, unlike the actual world in which I exist and in which the match remains unlit, this other world contains my counterpart who successfully manages to strike and light a similar but nonidentical match. In Lewis’ scheme, essential properties may also be explained with respect to counterpart relations. To say that Socrates is essentially human amounts to saying that all of Socrates’ counterparts who inhabit different worlds are human though they may differ with respect to their other properties.

Lewis argued that his Counterpart Theory avoids the problems of transworld identity and solves the difficulties traditionally associated with worldbound individuals. As noted earlier, Quine had problems with accepting unactualized possibles because he could not comprehend how the principle of individuation would work for such objects.

For Quine, there appeared no way to decide whether things in different world were identical. Lewis skirts this problem by claiming that no individual was ever identical to another individual in some other world. According to Lewis, the relation between an individual and his counterpart is a relation of similarity and not of identity. Lewis denied that individuals inhabiting different worlds are ever identical. Hence, the question of whether two individuals were identical or not does not pose a problem for Lewis' counterpart theory. The answer simply was that they were not. The counterpart relation was Lewis' substitute for identity between individuals in different worlds. Your counterparts are similar to you in many important respects but are never identical to you. At the very least, counterparts exist in their own world while the actual object exist in this world. Lewis emphasizes that the similarity between oneself and one's counterparts may be so close as to approach identity, but never quite:

> Indeed we might say, speaking casually, that your counterparts are you in other worlds, that they and you are the same; but this sameness is no more a literal identity than the sameness between you today and you tomorrow. It would be better to say that your counterparts are men you would have been, had the world been otherwise. [25]

Possibilists must define the actual world relative to other possible worlds. The notion of "actuality" may be given different interpretations by proponents of a possibilist ontology. Even though possibilists presuppose the existence of this world as well as other possible worlds, they may differ in their use of the term "actual." One version of possibilism takes the notion of actuality to be absolute. The most plausible theory of this kind defines "actuality" to be a simple and irreducible, perhaps even an unexplainable

[25] David Lewis, "Counterpart Theory and Quantified Modal Logic," *The Journal of Philosophy*, 65 (Mar 1968), 120.

property. According to this version, all possible worlds have being, but only this world has existence or actuality.

For Lewis, actuality is a relative matter. Lewis' possibilism claims that this world is actual with respect to itself, but so is every other world actual with respect to itself. So, all worlds are on an ontological par; all may be considered actual with respect to itself. Other possible worlds are distinct from the one we call the actual world but are not different in kind. According to Lewis, his theory of possible worlds can be seen as qualitatively parsimonious but no quantitatively parsimonious, for possible worlds abound in number not in kind. No two possible worlds are the same and all of them lack spatiotemporal relations but Lewis insists that worlds are not distinct by any categorical difference. Lewis does not even give this world the privileged status of being the only actual world. It is the actual world from our perspective since we inhabit this world, but all possible worlds may be called "actual" by their own inhabitants.

To explain the relation between possible worlds and the world we inhabit, Lewis presents us with an "indexical theory of actuality." Lewis insists that our world is just one world among an infinite set of other possible worlds. What makes our world different from other possible worlds is that it happens to be the particular world that we inhabit. As residents of this world, we can use the indexical adjective "actual" to describe our world. Yet, Lewis adds that our ability to call this world "the actual world" does not set our world apart as having a special ontological status. For any world w, the term "the actual world" denotes w and the predicate "is actual" is true of w and the things that exist in w. Hence any resident of a possible world may properly name his world the actual one. "Actual" is a world-relative attribute which our world has relative to itself, but which all other worlds also have relative to themselves.

This does not imply that the term "actual" has different meanings in the languages spoken in different worlds. Lewis does not intend "the actual world" to serve as the proper name of w in the native language of w. Instead, "actual" has fixed meaning such that, at any world w, "actual" refers in our language to w. Indexical terms depend for their reference on the place, the speaker, the intended audience, and the speaker's acts of pointing. If we ignore our own location among the worlds we cannot use indexicals like "actual." Lewis explains that such terms as 'the actual world', 'actual', and 'actually' are indexical expressions just like the notions 'I', 'here', or 'now'. Indexical expressions are context-sensitive and so must depend for their reference on the circumstances of utterance, on the relevant features of the context of use. The actuality of a world consists in its being the world in which the linguistic utterance occurs. Just as the present time differs from other times not in kind but because it is the time we're in, so the actual world stands in relation to other possible worlds; that is, the actual world is the possible world that we inhabit.

Consequently, no one can say that all possible worlds are actual. Although the sentence "This is the actual world" is true in whatever world it is uttered, the sentence "All worlds are actual" is false whenever it is uttered by anyone no matter the location. Thus although everyone may truly call his own world actual, no one may truly call all the worlds actual. This follows from the fact that no one may inhabit more than one world at a time and the adjective "actual" must refer back to the world in which it is uttered. Hence, the term "actual" uttered by someone picks out all the objects that bear a certain relation to this speaker, namely the speaker's world and all the things that are in the same world as the speaker.

Among the things that bear this kind of relation include the speaker's "worldmates." According to Lewis, "being a worldmate of x" is a relational property that is purely extrinsic. It is a property that does not reflect the natures of the things which share in the relation. Some relations are a consequence of the nature of things, such as the relations "being taller than x," "being older than x" or "being faster than x." These relations are consequences of the height, age and speed of individuals and hence stem from their nature. The relation of "being a worldmate of x" is not an intrinsic property possessed by x's worldmates in virtue of the natures of x or x's worldmates. Rather it is more like the relation of "being in the same room as x" which is a property possessed by some individual only extrinsically and not due to his particular intrinsic nature. One may occupy the same room as another independently of any particular attributes either occupant may possess. Likewise, "being a worldmate of x" does not necessarily reflect any intrinsic characteristics of x or x's worldmate. Hence, to call some possible world "the actual world" is similar to calling someone "a worldmate" for both in effect identify a relational link between the speaker and his world or the coinhabitants of his world.

2 Truth Conditions for Counterfactuals (Lewis)

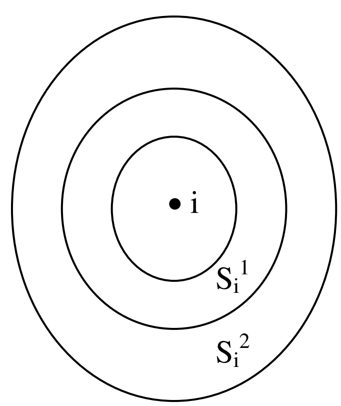
Lewis considers counterfactuals "notoriously vague" but nonetheless endeavors to give a clear account of their truth conditions. According to Lewis' theory, counterfactuals may be compared to ordinary strict conditionals of the form $\mathbf{L}(\phi \supset \psi)$, i.e.

a material conditional within the scope of the necessity operator. This necessity operator **L** acts like a restricted universal quantifier ranging over possible worlds. Given that there are different sorts of necessity, a strict conditional or any modal statement preceded by the necessity operator may be true at some given world as long as the statement within the scope of the necessity operator is true in all the relevant worlds. Depending on the type of necessity we're concerned with, we might place restrictions specifying which world are relevant to our discussion and hence exclude some possible worlds from consideration.

These restrictions are specified by stipulating accessibility relations. We say that a possible world is accessible to another if it satisfies the restriction associated with the necessity under question. By definition **L**p is true at i if and only if p is true at every world j accessible from i. Thus, the strict conditional **L**($\phi \supset \psi$) is true at i if and only if the material conditional $\phi \supset \psi$ is true at every world j that is accessible from i. The material conditional $\phi \supset \psi$ is true just in case ϕ is false or ψ is true. Hence the strict conditional **L**($\phi \supset \psi$) is true at some world i iff at every world j which is accessible to i, either ϕ is false or ψ is true. Moreover, the strict conditional **L**($\phi \supset \psi$) is true at some world i if and only if in every accessible ϕ-world (a possible world in which ϕ holds true) ψ is true. Interdefinable with the necessity operator is the possibility operator which may also be seen as a restricted existential quantifier over possible worlds.

Lewis sees counterfactuals as a very special kind of strict conditional that differs from other kinds of strict conditionals with respect to its accessibility assignment. For counterfactuals, the accessibility assignment is to be determined by the similarity among possible worlds. Lewis proposes that a counterfactual is true at a world i if and only if it holds true in all accessible worlds. Those possible worlds that are said to be accessible from some given world will be just those worlds which share "an overall similarity, with respects of difference balanced off somehow against respects of similarity."[26] A counterfactual would be true if its corresponding strict conditional is true at i and this strict conditional is true at i just in case the material conditional is true throughout a set of worlds, S_i, whose members are relevantly similar to i. In other words, a counterfactual is true at some world i if and only if the consequent holds at all antecedent-worlds that are equally (or roughly so) similar to i.

Lewis makes further qualifications on the accessibility relations connected to counterfactuals. Instead of employing the standard accessibility relation, Lewis devises a different but equivalent formulation which he calls "spheres of accessibility." Lewis assigns to each possible world i a set of worlds S_i which are similar to the world i in varying degrees and in different respects. The worlds in S_i are arranged within concentric circles around the given world i according to the extent to which these possible worlds resemble i. We can envision the possible world i forming a nucleus encircled by various spheres of worlds, S_i^1, S_i^2, S_i^3, etc.



[26] Ibid.

Lewis calls this assignment of spheres to worlds the accessibility assignment corresponding to the modal operator and defines truth conditions for modal sentences with respect to these spheres. The following are truth conditions Lewis gives to the counterfactual statement $\phi \rightarrow \psi$, "If it were the case that ϕ, then it would be the case that ψ."

> $\phi \rightarrow \psi$ is true at a world i (according to a system of spheres \$) if and only if either
>
> (1) no ϕ-world belongs to any sphere S in $\$_i$, or
>
> (2) some sphere S in $\$_i$ does contain at least one ϕ-world, and $\phi \supset \psi$ holds at every world in S.

For some counterfactuals, it may be the case that there is a smallest sphere containing ϕ–worlds closest to i. Lewis calls this the Limit Assumption since it presupposes that there is a limit to the ϕ-worlds that approach closer and closer to i. Of course, the Limit Assumption may not be justified in all counterfactual situations where there is no smallest sphere containing ϕ-worlds closest to i. Consider the counterfactual, "If Smith were taller than 6 feet, then Smith could reach the top of the shelf." In this case, there is no closest world to the possible world in which Smith is exactly 6 feet. To satisfy the contrary to fact antecedent of being taller than, we might consider the world in which Smith were 7 feet tall, or 6'5," or 6'3," or 6'1," etc. But there is no "closest world" in which Smith is taller than 6 feet, because no matter how close Smith's adjusted height is from 6 feet, we can always find a height that is even closer.

In those cases where the limit assumption does hold, that is, for those counterfactuals for which there is a smallest sphere which contains ϕ-worlds closest to i, truth condition (2) above is equivalent to the following truth condition:

(2) the smallest sphere S in $\$_i$ around i does contain at least one ϕ-world, and $\phi \supset \psi$ holds at every world in S.

Thus, for Lewis, the sentence **L**p is true at world i if and only if p is true in some sphere of accessibility S in $\$_i$ around i. Interdefinable with the necessity operator is the possibility operator **M**p. A sentence **M**p is true at a world i if and only if p is true somewhere in the sphere S_i. The strict conditional **L**$(\phi \supset \psi)$ is true at i if and only if $\phi \supset \psi$ is true throughout the sphere S_i in $\$_i$, i.e. if and only if ψ is true at every ϕ-world in S_i.

As mentioned earlier, the accessibility relations which Lewis expresses by means of his concentric spheres correspond to different kinds of necessity. Lewis recognizes various sorts of necessity. The broadest type of necessity and hence the largest and most encompassing sphere according to Lewis is logical necessity. Since the laws of logic hold of all possible worlds in Lewis' theory, all possible worlds are accessible to one another with respect to logical necessity. The logical strict conditional **L**$(\phi \supset \psi)$ is true at i if and only if is true at all worlds in which ϕ holds true and no possible ϕ-world is excluded as being inaccessible.

In another case there is the necessity of the physical laws which imposes restrictions that may exclude accessibility to other possible worlds. This physical necessity is truth at *all* worlds where the same laws of nature operate while physical possibility is truth at *some* worlds where the same laws of nature obtain. Worlds are accessible with respect to physical necessity if they run according to the same physical laws of nature. So the only possible worlds accessible to the actual world are those worlds that operate according to the actual laws of nature which happen to govern our world. Accessibility between possible worlds in this case is a relative matter. The physical strict conditional **L**($\phi \supset \psi$) is true at i if and only if ψ is true at all ϕ-worlds governed by the physical laws of i.

Three other types of necessities which Lewis distinguishes include a time-dependent necessity, a fact-dependent necessity and deontic necessity. Possible worlds are defined as being accessible to one another in this time-dependent sense if they are exactly the same up to a certain time t. With respect to time-dependent necessity, the strict conditional **L**($\phi \supset \psi$) is true at i if and only if ψ is true at all worlds in which ϕ holds true and which are exactly like i up to time t. Fact-dependent necessity stipulates that accessible worlds be the same with respect to all facts of a specified kind. With this version of necessity, the strict conditional **L**($\phi \supset \psi$) is true at i if and only if ψ is true at all ϕ-worlds which resembles i with respect to all facts of a certain kind.

Deontic or moral necessity restricts accessibility to the set of all morally perfect worlds so that the proposition is true at i if and only if it holds true in every morally perfect world. In this case each world i does not have to belong in the set of possible worlds to which it is accessible. For example, the set of morally perfect worlds is accessible from our actual world even though the actual world does not belong to this set since the actual world is itself morally imperfect.

Lewis is not the only realist about possible objects to claim that there are more things that actually exist. Alexius Meinong is another philosopher who is also committed to a large class of possible objects which extends beyond the narrower class of actual objects. Both Meinong and Lewis have proposed that extra objects are needed for an adequate foundation of metaphysics. Meinong's theory of objects follows Aristotelian tradition in distinguishing different categories of existence. These three senses of 'is' convey different ways of having properties: the 'is' of existence (there is an F), the 'is' of predication (a is F), and the 'is' of identity (a is b). Meinong's theory of objects distinguishes between objects which have being or "subsistence" yet fail to actually exist (e.g. unicorns, round squares) from those objects which have actual existence. Meinong presupposes two senses of 'is': "to subsist" vs. "to exist." Meinong's predicate 'exists' picks out only the present, material objects found in this spatiotemporal world.

David Lewis' ontological commitments do not extend as far as Meinong's which admits not only concrete particulars, fictional objects (Tom Sawyer), nonexistents (golden mountain) but also allow for impossible objects (non-square squares) and incomplete objects ("the fat, bald man in the doorway"). The ontology proposed by Lewis rejects the existence of incomplete and inconsistent Meinongian objects. Lewis does not allow for worlds that do not obey the laws of logic and arithmetic. He calls a world in which p and ~p are both true "the impossible world" and insists that such a world cannot exist. The laws of physics may differ from world to world but the laws of logic and math must stay fixed. In *Counterfactuals*, Lewis states the following:

> I believe that there are worlds where physics is different from the logic and arithmetic of our world. This is nothing but the systematic expression of my naïve, pre-philosophical opinion that physics could be different, but not logic or arithmetic.[27]

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[27] Lewis, *Counterfactuals*, 88.

Thus, Lewis rejects the Meinongian category of impossible objects such as non-square squares. Lewis also stipulates that all possible worlds and the objects contained therein must be completely determinate, so that for any property p, and object in Lewis' ontology either has p or not. Of course, we may not be aware of all the properties which particular objects possess, but Lewis is not reluctant to admit that there is much about entities, especially possible worlds, which we cannot know.

Even though Meinong may admit more kinds of entities that Lewis is willing to, they are both noted for their ontological extravagance. Lewis' exists/actual distinction may be seen as restricted version of the Meinongian being/exists distinction. Lewis' 'actual' has the same extension as Meinong's 'exists' with respect to domain. Meinong's being/exists distinction and Lewis exists/actual distinction may be expressed with the use of two separate quantifiers. For Meinong, we can designate one quantifier to express "there is" and another restricted quantifier to express "there exists." The broader quantifier would range over those objects that have being but may not necessarily exist in the spatiotemporal world, whereas the restricted quantifier would range over those objects which exist and not merely subsist.

Similarly, Lewis asserts that we can choose to restrict existential quantification to range over just actual things or we can broaden its range to include everything without exception. Lewis chooses to treat modal idioms as quantifiers whose scope is broad enough to include even possible worlds. Any sentence which expresses modality, states Lewis, can be taken at face value. To be committed to possible objects requires an all-inclusive quantifier ranging over the entire domain of 'being'. This type of quantifier is distinguished from the more restricted kind whose range is limited only to actual objects in our world. For Lewis this all-inclusive quantifier ranges over exactly what exist with possible objects existing in the same sense that actual spatiotemporal objects do.

3 Advantages of Lewis' Theory

David Lewis argues that his possibilist theory is the best among a number of alternative approaches. He claims that he does not know of any successful argument that "shows his realism leads to trouble…[while] all the alternatives [he] knows, on the other hand, do lead to trouble."[28] Lewis defends his theory by pointing out what he sees to be attractive features of his view:

(1) it allows a straightforward Tarskian semantics for modal discourse, i.e. it treats modal operators as quantifiers over possible words.

(2) it reflects our prephilosophical intuitions about "the way things might have been"

(3) it provides an ontology that is qualitatively parsimonious in line with Ockham's razor.

(1) does present us with the more convenient option of taking modal statements at their face value. To say that "that things could have been different" could be interpreted simply as an existential statement that asserts the existence of "those ways things could have been" or more concisely of "possible worlds." Lewis' theory agrees with Kripke's suggestion that we assume possible worlds into our domain of objects, but rejects the need to use modal operators to express the notions of "possibility" and "necessity." Instead, Lewis explains that these modal notions may be formalized with the use of

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[28] Lewis, *Plurality of Worlds*, 27.

existential and universal quantifiers ranging over possible worlds. The statement "possibly p" would ontologically imply that there exists at least one possible world in which p is true whereas "necessarily p" would mean that p is true in all possible worlds. Lewis claims that we don't need special modal operators when we can depend on our standard quantification theory enriched with special predicates and a suitable domain of quantification. In "Counterpart Theory and Quantified Modal Logic," Lewis begins to formalize modal discourse into an extensional logic according to standard Tarskian semantics. He does not see why we must depart from our usual practice when it comes to modal discourse:

> We can conduct formalized discourse about most topics perfectly well by means of our all-purpose extensional logic, provided with predicates and a domain of quantification suited to the subject matter at hand. That is what we do when our topic is numbers, or sets, or wholes and parts, or strings of symbols. That is what we do when our topic is modality: what we might be and what we must be, essence and accident. Then we introduce modal operators to create a special-purpose, nonextensional logic. Why this departure from our custom? Is it a historical accident, or was it forced on us somehow by the very nature of the topic of modality? …It was not forced on us. We have an alternative.[29]

Lewis identifies a second reason to support his realist approach. Lewis claims that his theory is one that best respects our prephilosophical intuitions about modal statements. According to Lewis, we intuitively think that things might have gone differently from the way they did. In Lewis' words, "things could have been different in countless ways." Lewis claims that we might express the same proposition by saying that "there are many ways things could have been besides the way they actually are." Taken at face value, this appears to be an existential quantification, revealing our belief in the

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[29] Lewis, *Counterpart Theory and Quantified Modal Logic, 125*.

existence of a certain set of abstract entities which we may describe as "ways things could have been" or refer to as "possible worlds." Admitting possible worlds into our ontology does not conflict, but rather reflects our preexisting intuitions on the matter.

A third feature Lewis wishes to attribute to his theory is the theoretical virtue of ontological economy. Lewis anticipates the likely objection that his ontology is too extravagant allowing for unnecessary abstract objects. Lewis' reply is that at most his theory can be accused of allowing many more entities of the same kind that we already admit but none that is categorically different in nature. According to Lewis, because possible worlds are on an ontological par with the actual world, his ontology has not committed him to new kinds of objects but more of the same kind, namely worlds and their contents. In this light, his theory of possible worlds can be seen as qualitatively parsimonious but not quantitatively parsimonious. In other words, possible worlds abound in number not in kind and, therefore, his ontology does not commit him to more kinds of entities. Hence, Lewis argues that his theory is not ontologically extravagant but submits to Ockham's razor in not multiplying kinds of entities beyond necessity.

4 Lewis' Quantified Modal Logic

Davis Lewis develops a quantified modal logic that formalizes modal discourse along the lines of standard extensional logic without the use of special modal operators. He claims that his Counterpart Theory may be adapted to create an extensional logic much like standard quantification theory with identity but enriched with added predicates and a suitable domain of quantification to handle modal statements. Lewis argues that not only are modal operators eliminable but their addition to quantification theory does not add to the expressive power of our language any more than the addition of counterpart theory. In fact, Lewis claims that adding counterpart theory to our standard extensional first order quantified system would far enrich our modal discourse and enable us to express thoughts that we could not in our present quantified modal language. According to Lewis, every sentence of quantified modal logic can be translated into a sentence of counterpart theory, but it is not the case that every sentence of counterpart theory has an equivalent translation in a language that uses only standard quantified logic with modal operators. Thus, Lewis claims that adding counterpart theory would allow us to say more than if we merely stuck to the use of nonextensional modal operators.

In order to eliminate the need for the modal operators, **L** (necessity) and **M** (possibility), Lewis introduces the following predicates to his system which he calls Counterpart Theory:

W_x x is a possible world

A_x x is actual

I_{xy} x is possible world in y

C_{xy} x is a counterpart of y

For Lewis' Counterpart Theory the universe of discourse is not restricted to the objects contained in the actual worlds but includes all possible worlds and all the things contained in these worlds. In place of an identity relation, Lewis employs the notion of a counterpart relation existing between two individuals. Lewis rejects the idea of transworld identity between things that exist in different possible worlds and denies that individuals, no matter how closely they may resemble one another, share the same identity. Instead Lewis confers a counterpart relation between two objects that exist in different worlds. This counterpart relation is a relation of similarity which may be weighted in a variety of respects and according to degrees of resemblance. Although the counterpart relation takes the place of identity in Lewis' theory, it is important to note that this counterpart relation is not to be understood as an equivalence relation as the identity relation is. Lewis' realist theory of possible does not render the counterpart relation to be a transitive one.

Lewis' Counterpart Theory is committed to at least eight postulates:

P1 $\forall x \forall y(Ixy \supset Wy)$

Nothing is in anything except in a world

P2 $\forall x \forall y \forall z((Ixy \,\&\, Ixz \;.\!\supset\; y=z)$

Nothing is in two worlds

P3 $\forall x \forall y(Cxy \supset \exists z Ixz)$

Whatever is a counterpart is in a world

P4 $\forall x \forall y(Cxy \supset \exists z Iyz)$

Whatever has a counterpart is in a world.

P5 $\forall x \forall y \forall z(Ixy \,\&\, Izy \,\&\, Cxz \;.\!\supset x = z)$

Nothing is a counterpart of anything else in its world

P6 $\forall x \forall y\, (Ixy \supset Cxx)$

Anything in a world is a counterpart of itself

P7 $\exists x(Wx \,\&\, \forall y(Iyx \equiv Ay)$

Some World contains all and only actual things

P8 $\exists x Ax$

Something actual

The first postulate establishes that the only type of membership denoted by the predicate Ixy is of something being in a world. That is, nothing is in anything except in a world. The second postulate follows from Lewis' rejection of transworld identity and denies that anything can exist in two worlds. (P2) says that if an individual exists in some world y and exists in some world z, then world y and z are the same world. The third and fourth postulates ensure that given any two things that share counterpart relation, both things (i.e. the individual and its counterpart) must exist in some world. The fifth postulate adds the requirement that the two different individuals which share the counterpart relation must exist in different worlds. That is, a thing and its counterpart cannot belong to the same world, unless by the sixth postulate the two things we are considering are identical. According to P6 an individual in any world is always counted as a counterpart of itself. The seventh postulate establishes one world as containing all and only actual things. The eight postulate asserts the existence of some actual things. By (P2) and (P8), the world containing all and only actual things is a unique one. Lewis symbolizes the actual world as follows: $@ =_{df} \iota x \forall y\, (Iyx \equiv Ay)$

Lewis proceeds to provide a translation scheme which converts sentences of quantified modal logic containing modal operators into the sentences of Counterpart theory having the same meaning. A closed (0-place) sentence with a single, initial modal operator may be translated with the use of universal or existential quantifiers that range over wider domain, i.e. the domain of all possible things and worlds:

Lϕ may be replace by $\forall\beta(W\beta \supset \phi^{\beta})$

Mϕ may be replace by $\exists\beta(W\beta \;\&\; \phi^{\beta})$

ϕ^{β}: "ϕ holds in world β"

However, quantifiers contained ϕ must have a restricted range to the domain of things in world β. This may be arranged by replacing occurrences of $\forall\alpha$ with $\forall\alpha(I\alpha\beta \supset \ldots)$ and occurrences of $\exists\alpha$ with $\exists\alpha(I\alpha\beta \;\&\; \ldots)$

A 1-place open sentence with a single, initial modal operator may also be translated by replacing expressions on the right-hand side with the expressions on the left hand side:

L$\phi\alpha$ $\forall\beta\forall\gamma(W\beta \;\&\; I\gamma\beta \;\&\; C\gamma\alpha \;.\!\supset \phi^{\beta}\gamma)$

M$\phi\alpha$ $\exists\beta\exists\gamma(W\beta \;\&\; I\gamma\beta \;\&\; C\gamma\alpha \;\&\; \phi^{\beta}\gamma)$

$\phi^{\beta}\gamma$: "ϕ holds of every counterpart γ of α in any world β"

Only parts of a sentence including modal operators and the subsentence within its scope need translation. If the modal operator is not initial, only the subsentence that proceeds the modal operator and that lies within its scope must be replaced as noted above. In the case of a nonmodal sentence, one that does not contain any modal operators, or in the case of a subsentence which lies outside the scope of a modal operator, the only revision needed involves the restriction of quantifiers. In such cases the quantifier should be understood as in usual contexts to range over the domain of actual things in the actual world. The unrestricted quantifier in counterpart theory on the other hand ranges over all possible things and worlds.

Sentences with modal operators in the scope of other modal operators are translated by working inwards, restricting quantifiers in ϕ and translating subsentences of ϕ with initial modal operators. This translation procedure may be defined recursively and is patterned after the translation of a standard sentence ϕ of quantified modal logic. The translation of ϕ is $\phi^{@}$ which reads "ϕ holds in the actual worlds @," is expressed in primitive notation as follows:

T1: ϕ is $\exists\beta(\forall\alpha(I\alpha\beta \equiv A\alpha) \mathbin{\&} \phi^{\beta})$

We follow this by a recursive definition of ϕ^{β}, which reads "ϕ holds in world β":

T2a: ϕ^{β} is ϕ , if is ϕ atomic

T2b: $(\sim\phi)^{\beta}$ is $\sim\phi^{\beta}$

T2c: $(\phi \mathbin{\&} \psi)^{\beta}$ is $\phi^{\beta} \mathbin{\&} \psi^{\beta}$

T2d: $(\phi \vee \psi)^{\beta}$ is $\phi^{\beta} \vee \psi^{\beta}$

T2e: $(\phi \supset \psi)^{\beta}$ is $\phi^{\beta} \supset \psi^{\beta}$

T2f: $(\phi \equiv \psi)^{\beta}$ is $\phi^{\beta} \equiv \psi^{\beta}$

T2g: $(\forall\alpha\phi)^{\beta}$ is $\forall\alpha(I\alpha\beta \supset \phi^{\beta})$

T2h: $(\exists\alpha\phi)^{\beta}$ is $\exists\alpha(I\alpha^{\beta} \mathbin{\&} \phi^{\beta})$

T2i: $(\mathbf{L}\phi\alpha_1 \ldots \alpha_n)^{\beta}$ is $\forall\beta_1\forall\gamma_1 \ldots \forall\gamma_n$
$(W\beta_1 \mathbin{\&} I\gamma_1\beta_1 \mathbin{\&} C\gamma_1\alpha_1 \mathbin{\&} \ldots \mathbin{\&} I\gamma_n\beta_1 \mathbin{\&} C\gamma_n\alpha_n .\supset \phi^{\beta}{}_1\gamma_1\ldots\gamma_n)$

T2j: $(\mathbf{M}\phi\alpha_1 \ldots\alpha_n)^{\beta}$ is $\exists\beta_1\exists\gamma_1\ldots\exists\gamma_n$
$(W\beta_1 \mathbin{\&} I\gamma_1\beta_1 \mathbin{\&} C\gamma_1\alpha_1 \mathbin{\&} \ldots \mathbin{\&} I\gamma_n\beta_1 \mathbin{\&} C\gamma_n\alpha_n .\mathbin{\&} \phi^{\beta}{}_1\gamma_1\ldots\gamma_n)$

5 The Lack of Empirical Data

One criticism that Lewis encounters targets the lack of empirical evidence for the existence of possible worlds. Just like the Platonists, Lewis believes in the existence of certain abstract entities with which we have no spatiotemporal relations and no ordinary means of communication. With abstract mathematical objects, Platonists like Maddy posited the special faculty of mathematical intuition that allows mathematicians to acquaint themselves with the nature of mathematical objects. Lewis flatly denies that humans have any epistemological access to possible worlds. He admits that there is much we don't know and cannot know about possible worlds because they exist in a realm apart from our own actual world and hence beyond physical reach.

Brian Skyrms finds this lack of evidence an unacceptable aspect of Lewis' theory. Although I will ultimately reject Lewis' approach to possible worlds for other reasons, it appears that Lewis' theory can withstand this objection raised by Skyrms in his article, *Possible Worlds, Physics and Metaphysics*. Skyrms sees Lewis as presenting a physical theory and thinks that such a theory ought to be held to the same standards by which other physical theories are judged. Indeed, Lewis claims that other possible worlds exist in a real sense, that they have a physical reality as concrete and robust as the actual world. To justify his claim, Lewis must show that the best physical theory available supports a reality composed of many worlds. According to Skyrms, the kind of arguments Lewis offers is inappropriate for deciding what is essentially a physical question. Skyrms argues:

> If I were convinced that the smoothest semantical theory that could make sense of ordinary talk about the Easter rabbit, goblins, angels, and Pegasus was one which assumed that such things really existed, I would not thereby be convinced of their existence. I would hold such ordinary talk suspect. What I require here are physical reasons (rabbit tracks, etc.) If possible worlds are supposed to be the same sorts of things as our actual world; if they are supposed to exist in as concrete and robust a sense as our own; then they require the same sort of evidence for their existence as other constituents of physical reality.[30]

Skyrms concedes that there is a sense in which an ontology of possible worlds is consistent with our semantics for modal logic. However, he disagrees that a possible-world semantics requires Lewis' realist theory of possible worlds. It is not metaphysical and modal-semantical arguments that will justify a reality of many worlds, but a scientific demonstration that supports such a reality. Skyrms grants that such a physical theory is possible, but denies that anyone has yet come up with one. The best candidate so far is Everett's "relative state' formulation of quantum mechanics. Everett's interpretation of quantum mechanics implies that the universe consists of many worlds:

> Our universe must be viewed as constantly splitting into a stupendous number of branches, all resulting from the measurementlike interaction between its myriads of components. Because there exists neither a mechanism within the framework of the formalism nor, by definition, an entity outside of the universe that can designate which branch of the grand superposition is the 'real' world, all branches must be regarded as equally real… From the viewpoint of the theory all elements of a superposition (all 'branches') are 'actual', none any more 'real than the rest. It is unnecessary to suppose that all but one are somehow destroyed, since all the separate elements of a superposition individually obey the wave equation with complete indifference to the presence or absence ('actuality' or not) of any other elements this total lack of effect of one branch on another also implies that no observer will ever be aware of any 'splitting' process.[31]

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[30] Brian Skyrms, "Possible Worlds, Physics and Metaphysics," *Philosophical Studies*, 30 (1976), 327-328.

[31] Ibid.

Although a fascinating analysis of wave mechanics, this theory is only one interpretation of quantum mechanics while there are many other competing interpretations which require only one world. Skyrms claims that justification of Lewis' many worlds supposition requires a physical theory such as Everett's; however, he denies that Everett's interpretation of quantum mechanics is convincing enough to qualify. Therefore, as far as science can tell, the claim that there are more possible worlds than this one is false.

Evaluating theories from a Quinean perspective, I am not inclined to object to Lewis' line of reasoning in the way that Skyrms does. An assumption that underlies Skyrms' thinking is the position held by scientific realists who judge a model to be successful if its elements purporting to represent unobservable entities can be proven to have corresponding elements in reality. I think Lewis has the right way of settling existence questions in philosophy. When it comes to ontological matters the availability of empirical evidence is not the deciding factor for determining a theory's worth. Although Lewis presents a physical theory in the sense that possible worlds are concrete entities, I believe that it is legitimate for him to claim that possible worlds are inaccessible to human minds in the actual world. This does not leave the existence of possible worlds more suspect. The reason we cannot know much about the nature of these real entities, Lewis explains, has to do with epistemological difficulties, not ontological ones. Our knowledge of possible worlds is limited not because possible worlds do not exist, but because they exist independently of our thinking of them. It is entirely consistent for Lewis to claim that possible worlds exist and that their existence lies outside the scope of our observations.

We hold our science to certain standards of evidence, but we also permit it some liberties. In science, we sometimes assume the existence of physical entities that are either too small or too far for us to directly observe. We permit science to postulate unobserved entities, so why not in philosophy? What justifies the assumption of unobserved entities in science is their auxiliary role in the development of a consistent and unified system of knowledge. According to Quine's methodological holism, we may determine the "best" theory to be the one which claims the most theoretical virtues (ontological economy, empirical adequacy, etc.) including purely pragmatic advantages (simplicity) without demands for direct verifiability. Hence, the hypothesis that there is more than one world should not be dismissed solely for its lack of confirming empirical data. Instead, we must evaluate Lewis' theory according to other theoretical virtues it may possess or lack.

CHAPTER FIVE NICHOLAS RESCHER: A CONSTRUCTIVIST VIEW

Unlike Lewis' theory, Rescher's semantics for modal logic does not require "possible worlds" nor any ontological commitments to objects that exist beyond the actual world. He develops a constructive approach to a theory of possible worlds. He begins with the actual world and the actual individuals it contains to the fabrication of possible individuals with which to stock other possible worlds. The possible worlds are defined relative to the actual world which serves as a basis. The actual world furnishes one with an inventory of individuals and their properties from which to construct other possible worlds. Rescher proceeds with the specification of individuals to the specification of worlds as suitable sets of "prefabricated individuals." Rescher's approach may be seen as an attempt to reduce the concept of "possible worlds" to the notion of "ideal constructions" reflecting the nature of the actual world. This reinterpretation of worlds in terms of ideal constructions is analogous to Kitcher's representation of "abstract mathematical objects" as "mathematical constructions of an ideal agent." This chapter will discuss Rescher's actualistic interpretation of possible worlds and a formulation of a quantified modal logic to suit his constructivist approach. Under this alternative interpretation, modal statements are regarded true in virtue of the conceptual operations of ideal minds acquainted with the actual world.

1 Mind-Dependent Existence of Possible Objects

Unlike possibilists such as David Lewis, Nicholas Rescher rejects a realist view of possible worlds. Instead, Rescher takes an actualist approach to counterfactuals grounded on conceptualist ideas. Rescher denies that any possible worlds other than the actual world unqualifiedly exist, or if possible worlds do exist in some sense then they exist only as the objects of certain intellectual processes. According to Rescher, "only actual things or states of affairs can unqualifiedly be said to exist…Of course, unactualized possibilities can be conceived, entertained, hypothesized, assumed, and so on."[32] Possible worlds would then be mind-dependent since their existence would be the product of certain mental operations. Rescher insists that the actual world is the only mind-independent objective reality that exists and possible worlds should not be viewed as somehow existing within this natural world order. Like Kitcher, Rescher recognizes no mind-accessible Platonic realm in which abstract entities such as hypothetical possibles can exist. Unactualized hypothetical possibilities would be unfeasible if there were no rational minds to conceive of them. For possibilities to be (esse), therefore, is to be conceived (concipi). "If the conceptual resources that come into being with rational minds and their capabilities were abolished, the real of supposition and counterfact would be abolished too, and with it the domain of unrealized, albeit possible, things would also have to vanish."[33]

Rescher claims that the mind-independent existence possessed by actual objects manifests a dualism which possible objects lack. Rescher explains that it makes sense to assert "This stone I am looking at would exist even if nobody ever saw it." In connection to actual objects, two things may be said to obtain:

(1) There is the actual existing thing or state of affair (e.g. the actual stone)

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[32] Nicholas Rescher, *Topics in Philosophical Logic* (Dorderecht: D. Reidel Publishing Co., 1968) 169.

[33] Ibid, 172.

(2) There is the thought or entertainment of this thing or state of affair (e.g. the thought of the actual stone)

However, a similar sentence about a possible but nonactual stone would not make sense: *"This nonexistent but possible stone I am thinking of would be there even if nobody ever saw it."* The reason one cannot presume the independent existence of a possible object is due to the fact that only the second of the two entities listed above can be said to obtain. Whereas, actual objects manifest a dualism, possible objects manifest a monism:

(1) There is not the possible thing or state of affair (e.g. the nonexistent stone)

(2) There is the thought or entertainment of this thing or state of affair (e.g. the thought of the nonexistent stone)

According to Rescher, actual objects involve "a condition of factuality": the possession of properties by objects, as a matter of fact. In contrast, possible unactualized objects involve "a condition of possibility" which merely allows the attribution of properties to objects by someone's mind. Thus, with respect to the ontology of possibles what concerns us is "modality de re," i.e. the ascription of properties to objects.

Although the secondary and mind-dependent existence of possible objects is generated by minds, Rescher emphasizes that this dependence does not rest on any specific mind nor does it depend on any actual mental operation. Just as mathematics is rooted in an idealized agent's ability to perform ideal mathematical operations, the "reality" of possibles resides in the capability of minds to perform certain possibility-involving processes. The mind-dependence of possible objects then is not particularistic but generic, not dependent on a specific mind but on a "mind-in-general." As Kitcher had stipulated for mathematical operations, Rescher also states that unactualized possibles need not actually be conceived but they obtain their ontological footing "only insofar as it lies within the generic province of minds to conceive (or to entertain, hypothesize, and so on) them."[34]

The ability to reason about hypotheticals like the ability to engage in mathematical thinking is learned during childhood. Even to consider future possibilities is to engage in one kind of hypothetical reasoning. A proponent of Rescher's conceptualist view, Brian Ellis depicts the learning process as follows:

> A child learns, early in life, how to make both absolute and conditional predictions when he says that X will occur. He makes a conditional prediction when he says that if X occurs, then Y will occur. The making of such conditional predictions is the beginning of hypothetical reasoning. He learn that his conditional prediction has been successful if both X and Y occur, and unsuccessful if X occurs, but Y does not occur. He also learns that if X does not occur, then this, in itself, has no bearing on the propriety of his having made the conditional prediction. If he stills believes that he was justified in making the conditional prediction, then he has no reason to retract his statements. On the contrary, he learns now how to express his belief that if X occurs then Y will occur in the past tense, and in light of knowledge that X did not occur. He learns to say that if X had occurred, then Y would have occurred.[35]

This ontology of constructive activities raises the question of circularity? Our analysis of counterfactuals may avoid other modal notions but one kind of modality still remains, namely the notion of "conceivability." We have attempted to explain the concept of "possible states of affairs" or more specifically "counterfactual situation," i.e. a state of affairs possible given certain contrary-to-fact conditions, by referring to "operations conceivable to an ideal agent." Toward this goal, it was important for us to establish that idealized conceptual activities are possible for an idealized agent although

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[34] Nicholas Rescher, *On Conceptualism* (Oxford: Basil Blackwell, 1973), 169.

[35] Ellis, 38.

accidentally impossible for actual individual minds. In effect it appears that to say that possibility resides in conceivability amounts to defining possibility in terms of the possible – to what can be conceived. Does explaining "possible states of affairs" in terms of such "possible ideal constructions" then involve a circularity? Are we simply trading one set of modal concepts for another? It appears that we are defining the notions of possible states in terms of a primitive understanding of possibility. This criticism, however, assumes that definition in terms of nonmodal concepts is the only way to illuminate the notion of possibilities.

Our proposal avoids circularity because it explains one class of possibilities, i.e. possible states of affairs and things, by referring to possibility-involving processes. Nicholas Rescher explains that this way of illuminating modal notions does not force us into circular reasoning:

> Here actuality is indeed prior to possibility – the actuality of one category of things, namely, minds with their characteristic modes of functioning, underwrites the construction of the totality of nonexistent possible that can be contemplated. Nonexistent possibilities thus have an amphibious ontological basis: They root in the capability of minds to perform certain operations – to describe and to hypothesize (assume, conjecture, suppose) – operations to which the use of language is essential, so that both thought processes and language enter the picture.[36]

2 Construction of Preferred Maximal Mutually Consistent Sets

Another similarity between Kitcher's theory about mathematical operations and Rescher's conceptual idealism is the relationship between the mind and the world. According to Kitcher, "the slogan that arithmetic is true in virtue of human operations

[36] Nicholas Rescher, "The Ontology of the Possible," *Logic and Ontology* (1973), 174.

should not be treated as an account to rival the thesis that arithmetic is true in virtue of structural features of reality."[37] The way our minds can mathematically manipulate the world reflects the structure of the world which allows for such manipulation. Similarly, Rescher rejects the accusation that his theory only teaches us things about the human mind but nothing about the way the world is. Rescher claims that "there is no clean separation between the world and the domain of thought." The way we think about the world is necessarily patterned by the kinds of cognitive operations we can perform. "Our only possible route to cognitive contact with the world" according to Rescher, "is through mediation of our conceptions about it, so that for us, 'the world' is inevitably 'the world as we can manage to conceive of it'…, how we conceive of the world has to be seen as a fact not just about us but about 'the world' as well…"[38]

By linking possible objects with mental processes, we may construe "possible worlds" as being grounded on the actual. What is actual is the real world and the human operations performed in it. In this sense, my proposal, takes on an actualistic approach. It is actualistic to the extent that it is able to reinterpret "possible worlds" as being about actual objects. This reduction of "possible worlds" requires that we substitute in its place some set of actual entities that are structurally analogous or isomorphic to the system of possible worlds. The main task before me is to describe the kinds of objects that may serve as world-surrogates to "possible worlds."

The nature of the actual world grounds the plausibility of a counterfactual statement although the counterfactual itself does not describe any actual event. Since a

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[37] Kitcher, *The Nature of Mathematics*, 109.

[38] Nicholas Rescher, *A System of Pragmatic Idealism, Vol I, (Princeton University Press,1992)*, 319.

counterfactual statement does not correspond to any actual event or object, its truth-value cannot be determined through direct verification. Rescher also emphasizes that traditional deductive or inductive logic will not afford much help either. Counterfactual statements entertain assumptions which conflict with one or more beliefs we accept as true. According to Rescher, purely formal modes of logic cannot resolve problems with hypothetical reasoning over belief-contravening suppositions. The aim of hypothetical reasoning is to restore consistency to a belief-set which has been disrupted by the addition of a proposition incompatible with those already accepted. Deductive logic and probabilistic logic are applicable only to situations in which we are given a consistent set of premises from which to draw necessary or probable inferences. Deductive logic can reveal to us the self-contradictory nature of a set of beliefs, but does not help us determine which proposition to abandon.

Rescher suggests that we must rely on plausibility theory, a "relatively primitive but for that no less important – mode of reasoning,"[39] as the tool for resolving an inconsistent set of givens. Plausibilistic reasoning provides us with a procedure for rationally dealing with cognitive conflict. In such cases, plausibilistic reasoning appeals to the material factors involved and offers a mechanism for restoring consistency to discordant belief-sets along an index that reasonable people would agree to. Plausibility theory, Rescher claims, provides a way out of conflicts that still lie within the limits of rationality.

A counterfactual introduces claims that are inconsistent to a set of mutually compatible beliefs we have about the world. To restore consistency to our belief-set we

[39] Nicholas Rescher, *Plausible Reasoning* (Amsterdam: Van Gorcum, Assen, 1976), 12.

must give up a subset of these beliefs which conflict with the counterfactual assumption. There may be various alternative ways of reconciling hypothetical suppositions with our original belief-set. But which alternative should be taken? That is, given the choice, which beliefs should we give up and which ones should we maintain? According to Rescher, we must make our selections so as to produce minimal disruptions to our belief-set. We should be more willing to give up "lower-order" beliefs to which we have less fundamental commitment than our "higher-order" beliefs. Goodman also recognized a hierarchy among beliefs and claimed that we should strive to preserve our "well-entrenched" beliefs over those that are more dispensable to our theoretical commitments.[40]

Rescher offers a plausibility-indexing of beliefs which order beliefs into two main categories:

(1) Lawful generalizations

(2) Statements of particular facts: invariable states or fixed circumstances receive a higher index value than activities or manipulable responses

Thus, when presented with the choice of giving up a lawlike statement over a matter of fact, it is more natural for one to reject an empirical fact before a general law, assuming, of course, that the negation of fact would not prove a counterexample to the general law.

Given Rescher's plausibility indexing, let us reconsider the counterfactual statement "If Zeus were human, then Zeus would be mortal." This counterfactual introduces an assumption, that is clearly in conflict with a maximally consistent set of beliefs that we may hold about the actual world. Specifically, the assumption, "Zeus is

[40] Goodman, 84.

human," is inconsistent with one or more statements in the following subset of commonly held beliefs:

(1) Zeus in not human.

(2) Zeus is immortal.

(3) All humans are mortal.

(4) Nothing can be both mortal and immortal.

If we introduce the hypothetical supposition that "Zeus is human," what would be the most plausible and natural way of restoring consistency to our original belief-set? Clearly, we would have to reject (1) since it contradicts our hypothesis. According to Rescher's plausibility indexing, before sacrificing a general statement such as (3) or a logical law such as (4), we should first reject the particular fact expressed in statement (2) Thus, the most plausible counterfactual statement that would result from preserving statements (3) and (4) and hypothesizing the claim "Zeus is human" is the conditional "If Zeus were human, then he would be mortal." So, Rescher's analysis would insist that such a counterfactual is plausible in virtue of the beliefs that we maintain about the actual world.

Rescher calls a set of beliefs that reconciles a contrary to fact assumption with the greatest number of our higher-order beliefs "preferred maximal mutually consistent subsets" (PMMC-subsets). A counterfactual statement would be true in virtue of our ability to organize a set of beliefs that includes both the contrary-to-fact assumption and our "higher-order" beliefs. Nicholas Rescher outlines a procedure for generating this set of beliefs such that the set is

(1) consistent: all the statements of the set may be true at the same time

(2) maximal: no additional distinct statement may be added to the set without making the set inconsistent

(3) preferred: higher-order beliefs are prioritized for membership into the set before lower-order beliefs

This decision procedure is able to output sets which are to be preferred over others because it precategorizes true sentences or our accepted beliefs about the actual world into modal families of statements. These modal families are indexed according to the varying degrees of plausibility we attach to its members. The hierarchy of modal families starts with the set of beliefs we are least willing to give up such as logical truths and laws to other kinds statements we are more inclined to abandon such as contingent facts.

3 Kinds of Properties and Essentialism

Rescher's suggestions could prove more useful to the analysis of counterfactuals, if we could extend his approach down to the level of individuals and their properties. We hold varying degrees of commitment not only to different kinds of statements, but also to the kinds of properties we attribute to individuals. Just as general laws take precedence over empirical facts, we may be able to distinguish some properties of an individual as having priority over their other properties (e.g. essential vs. accidental properties). Instead of constructing preferred maximally consistent sets corresponding to a hierarchy of beliefs, we might engage in counterfactual reasoning by constructing sets whose members are not statements but individuals and their properties. Breaking up the components of the world into individuals and their properties would allow for the analysis of the subsentential relationships that would otherwise go undetected.

Let us reconsider the counterfactual statement "If Zeus were human, then Zeus would be mortal" from the perspective of competing properties. This counterfactual attributes to the individual designated by the name Zeus the property of "being human." Immediately, we see that the addition of this property precludes the otherwise accepted idea that Zeus lacks the property of "being human." Adding the property of "humanness" forces us to make other changes to Zeus' character. For example, we must exclude from Zeus' usual set of properties the property of "being a god." It is important to recognize that the properties of "being human" or "being a god" are complex ones which entail the presence or absence of other properties as well. In particular, the property of "being mortal" or "being immortal" is contingent upon the individual's possessing the property of "being human" or "being a god." Because our counterfactual example introduces the hypothesis that "Zeus is human" and if we are committed to the idea that the property of "being human" entails the property of "being mortal," then we would have to include this latter property among the properties of Zeus and exclude the property of "being immortal Thus, deciding which properties to retain and which to abandon given the introduction of the counterfactual hypothesis will ultimately depend on our knowledge of the actual world and the individuals and properties that it contains. Much counterfactual reasoning requires the reorganization of the properties of individuals that belong to the actual world. Although there may be no single best way to reconstruct the contents of the actual world, the way in which properties are actually grouped and ordered in the world can serve as a guideline for our reconstructions.

Entertaining a counterfactual situation will very seldom involve the introduction of a single property to an individual's actual set of properties. What is added and removed often comprise whole subsets of properties, not only the property specified but other properties as well which we recognize to be either the precondition and/or consequences of the property in question. When we evaluate counterfactual statements about individuals we engage in the construction of alternative sets of properties each of which contains some counterfactual property X and the other properties entailed by X. To this set, we would attempt to preserve as many of the individual's actual properties as can be consistently maintained giving priority to those properties that we deem most characteristic and essential to the identity of the individual. A counterfactual statement would then be plausible if it corresponds to the most preferred set of properties we can (ideally) construct for that individual.

The construction of preferred maximally consistent sets of properties will require a means of individuating properties and of arranging these properties from lower ranking ones which we are more willing to abandon to higher ranking properties to which we are more strongly committed. That is, some properties will have to be stipulated as being essential to an individual while others are seen as being merely accidental features. Thus Rescher must account for a way of dividing an individual's properties into those which belong to the individual essentially and those merely accidentally. Rescher proceeds by defining four sets of properties:

(1) the actual-property set of an individual: all the properties that an individual actually has

(2) the essential-property set of an individual: all of the properties that an individual necessarily has, i.e. that are essential to the individual

(3) the possible-property set of an individual: all of the properties that x possibly has, i.e. properties the complements of which are not essential to the individual

(4) the dispositional-property set of an individual: the set of those properties that an individual must have just in case the individual has some other properties.

Around these four property-sets, Rescher builds his theory of essentiality. Rescher believes that his version of essentialism may be give an extensional, set-theoretic treatment where the extension is in terms of properties rather than whole individuals.

The essentialism that Rescher adopts does not identify one single way to determine essential properties. He allows for the possibility of designating various different bases upon which the essential-accidental distinction can be placed, denying that there is only one unique and universally correct way of differentiating essential from accidental properties. Rescher maintains that this distinction may vary depending on the particular problem context. The distinction between essential and accidental properties may be made along various distinct lines according to the particular contexts of application rather than one specially privileged standard based on general principles.

We are free to choose from several distinct approaches to essentialism, this choice to me made according to our specific pragmatic concerns. One way to categorize properties is from canonical description. This categorization assumes that an individual may be identified by certain basic or canonical descriptions. A property is taken to be essential for an individual just in case it follows from at least one of the individuating descriptions from among the individual's family of canonical descriptions. The essential properties of an individual are relative to the particular way we choose to describe the individual. Hence, this approach is context-relative, i.e. relative to the how the individual is specified. This way of interpreting essentialism is unlike Quine's view which sets out the essential-accidental distinction as an absolute, context-free one that does not depend on how we may choose to describe this individual. The "canonicity" of a description according to this approach is left an open question to be determined relative to the perspective which one is prepared to employ. The same "wooden kitchen chair" from the carpenter's perspective may be designated as having the property of being made of wood as its essential property and being a piece of furniture, specifically a chair, as merely an accidental feature. Yet from the homemaker's viewpoint, its being made of wood may be the incidental property while its being a kitchen chair is the essential one.

Another way to distinguish essential properties is according to what Rescher calls "Difference-Maintaining Properties." On this approach an essential property is one needed to maintain the uniqueness of an individual, the descriptive differentiation of an individual from all others. A property is essential just in case the loss of property for an individual would make it impossible for us to distinguish that individual from another. Here the principle of "identity of indiscernibles" motivates the essential-accidental distinction among properties.

A different way of making this distinction is according to "Uniformity-Maintaining Properties," which Rescher calls Typological or Taxonomic Essentialism. Typological Essentialism assumes that individuals may be distinguished into disjoint types and an individual's essential properties are those which belong to all individuals of its type. Hence, a property is essential if its presence is necessary to maintain uniformity across members of the same type.

Essentialism may also be defined according to the temporally invariant properties of an individual. The essential properties are identified to be those properties that an individual keeps throughout its entire life. That is, essential properties are those that remain invariable and persist for as long as the individual exists. This sort of essentialism reflects the very conception of essential properties adopted by Aristotle, Arabic logicians and medieval scholars.

A fifth way of distinguishing essential properties is from invariance under transformations. This version of essentialism picks out those properties which remain constant throughout temporal change as being essential. For example those properties that water maintains despite its exposure to different temperatures would be regarded as essential. Also essential would be those properties which a human being possesses throughout its different stages of life and despite the various events experienced.

The five criteria offered by Rescher constitute some possible ways to account for essentiality. Other criteria may be devised that may better suit a different perspective of characterizing individuals and properties. The distinction Rescher draws between essential and accidental properties is thus a flexible one which varies according to the context of discussion and the perspective from which individuals and their properties are being considered. Rescher emphasizes that matters of essentiality reflect the descriptive way we characterize the actual world. The actual world serves as the foundation upon which we draw the distinction between essential vs. accidental features. Should the descriptive constitution of the actual world change so would our adoption of a particular view of essentialism. Thus, Rescher sees essentialism as actuality-derivative since the distinction between essential and accidental properties rests on a fundamentally factual and contingent description of the actual world. Rescher claims that "the essentiality of some property of a thing is not to be detached by its inspection in isolation; it is imputed to the thing on the basis of synoptically holistic or global considerations regarding the makeup of the wider world to which it belongs.[41] Rescher's treatment of essentialism overcomes the objections which Quine had against possible individuals and their properties.

Allowing for different versions of essentialism permits greater flexibility in the interpretation of counterfactual statements. The construction of a preferred maximally consistent set of properties will inevitably reflect our perspective of the actual world and our way of characterizing individuals and their properties. We would distinguish "higher-order" from "lower-order" properties based on the criteria provided by the particular version of essentialism that best suits the problem-context of philosophical discussion. Modal families of properties are indexed in such a way that the essential properties of an individual are to be ranked as "higher-order" properties and thus considered the properties which we are more reluctant to give up than the accidental properties which are of a "lower-order." We can then proceed to construct a maximally consistent set of properties that would include both the contrary-to-fact property assigned to an individual and as many of the original properties possessed by the individual, giving the "higher-order" properties priority.

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[41] Rescher, *A Theory of Possibility*, 36-37.

4 Rescher's Quantified Modal Logic

We are in need of a workable quantified modal logic to serve as the guiding mechanism for the construction of these preferred maximally consistent sets. The present section will consider how a system of quantified modal logic can be formulated according to Rescher's constructivist account of possible worlds. The quantified modal logic which Rescher develops is articulated in the usual way with the standard logical operators ~, & v, ⊃, ≡ along with the universal and existential quantifiers ∀, ∃ ranging over the individuals of the actual worlds. As with standard first order logic, the language will contain propositional variables p, q, r, … and individual variables x, y, z, …which range over possible and actual individuals. In addition, Rescher includes the symbols w, w', w," … as variables which range over possible worlds. Individual constants and possible world constants are formed with the use of subscripts: x_1, x_2, x_3, … and w_1, w_2, w_3, …, respectively. The actual world or the set of all real individuals is denoted by the symbol w*. Rescher uses Greek lower case alphabet ϕ, ψ, χ, for predicate variables and English capital letters F, G, H, … for predicate constants. Relational constants are distinguished from atomic predicate constants and are symbolized by R, R', R," …

Rescher's quantified modal system employs three kinds of universal and existential quantifiers which range over different sets of objects. Like standard first order predicate logic, Rescher's Rw-calculus uses the symbols ∀, ∃ to act as quantifiers but they are restricted to range over actual individuals only. Since modal systems must also consider nonfactual objects, Rescher introduces the symbols Π, Σ to serve as quantifiers over both actual and possible individuals. In this way, the standard quantifiers of quantified modal logic ∀, ∃ may be seen as restricted versions of the quantifiers Π, Σ with a more extensive range. In addition, Rescher employs quantifiers A, E to range over possible worlds. Rescher distinguishes an existence predicate E from existential quantifiers such that Ex iff $x \in w^*$. Identity of individuals within a given world is symbolized by =, while identity of individuals in general is expressed by ≅. Rescher also makes use of the following expressions:

$x \in w$	the individual x is a member of the possible world w
Rw(p)	the proposition p is realized in the world w
S^{w}_{yx}	in the world w, the individual y is the surrogate for the actual individual x; equivalent to $(x \in w^* \ \& \ y \in w \ \& \ x \cong y)$
P(x)	the set of all actual properties x, i.e., $\{\phi: \phi x\}$
$\phi!x$	ϕ is an essential property of x
$\phi?x$	ϕ is a possible property of x

One interesting aspect of Rescher's symbolization is his use of a special existence predicate E to designate actual existence, i.e. existence in the actual world. This along with the introduction of the more comprehensive pair of quantifiers Π, Σ that range over all possible individuals, actual or not, allows him to eliminate the standard quantifiers ∀, ∃ generally interpreted as ranging over only actual objects. Unlike Leibniz, Rescher considers an individual's description to be incomplete until the question of its existence is settled. Rescher feels justified to count (real) existence as a property which ought to be treated like any other predicate characteristic of objects.

> Existence Precedes Essence. The existence or nonexistence of individuals is a pivotal and crucial aspect of them – and so why not a descriptive aspect? Existence represents a fundamentally descriptive fact, something

> that must be dealt with from the very first. One must assume an emphatically "man's eye view" of the matter: a purported "description" of something that omits to indicate whether or not this thing exists leaves out of account something crucial to an adequate characterization of the sort of thing at issue. The (descriptive) essence of an individual must be recognized as itself hinging on a prior determination of its existence.[42]

Of course, the E predicate would have the special status of being universally true only of the actual individuals, i.e. the individuals in the real world. By including E as a predicate we can reduce $(\forall x)\phi x$ to $(\Pi x)(Ex \supset \phi x)$ and $(\exists x)\phi x$ to $(\Sigma x)(Ex \,\&\, \phi x)$. The E predicate takes on the role of the standard quantifier since it picks out only those individuals that actually exist. Hence the E predicate characterizes only those individuals which belong to the domains of $\forall$, $\exists$.

We must then set aside a different formulation for existence in other possible worlds. The claim "x exists" as expressed in the formula "Ex" is restricted to mean "x belongs to w*," that is

$$Ex \text{ iff } x \in w^*$$

It is also important to note that we cannot make use of the expression Rw(Ex) for existence in other worlds. Rw(Ex) is not equivalent to $x \in w^*$ in cases where $w \neq w^*$. In order to express existence in other worlds, "x exists in w," Rescher introduces a generalized existence operator:

$$E^w x \quad \text{iff} \quad x \in w$$

Ex is to be understood then as a special case of $E^w x$ when $w = w^*$. Although Rescher treats existence as a predicate, he confers on this predicate a special status of belonging only to real world individuals. Although unqualified existence may be a property of all

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[42]Ibid., 129.

actual individuals, this property cannot however be among the essential properties of actual individuals since it is not transposable from world to world. The surrogates of actual world individuals cannot take with them the property of non-relativized (real) existence, which is inherently bound to the actual world.

Rescher continues by introducing five axioms governing the Rw-operator along with a rule of inference:

(R_1) $Rw(\sim P) \equiv \sim Rw(P)$

(R_2) $Rw(P \,\&\, Q) \equiv [Rw(P) \,\&\, Rw(Q)]$

(R_3) $(Aw)\ [Rw'(Rw(P))]] \equiv Rw'(Aw)Rw(P)]$

(R_4) $P \equiv Rw^*(P)$

(R_5) $Rw'(Rw(P)) \equiv R\langle w' * w\rangle(P)$ for some suitable *function that is subject only to the stipulation that $\langle w * w^* \rangle = w$

(R) If $\vdash P$, then $\vdash Rw\,(P)$

The ∗ function conveys the notion of accessibility in Rescher's Rw-calculus. For any two possible worlds w and w', the ∗ function yields the image of w from the perspective of w'. Rescher stipulates that $\langle w * w^* \rangle = w$ means that the actual world is a mirror in which all the worlds can see themselves. The accessibility relation (w'∗ w) is to be read "w' is accessible from w" and is defined by the following equivalence:

$\alpha\,(w',w)$ iff $(Ew'')\ [\langle w'' * w\rangle = w']$

The first two axioms are necessary to establish that the Rw-operator distributes over all truth-functional connectives. The third axiom (R_3) may be seen as a consequence of the first two axioms generalized over an unrestricted domain of worlds, rather than a finite domain of worlds. The intuition behind (R_4) is that an absolute proposition P is proposition that is realized with respect to the actual world.

In order to establish that logical truths obtain in all possible worlds, Rescher offers the following rule of inference

(R) If $\vdash P$, then $\vdash Rw(P)$

From the above rule, we can derive the following results

If $\vdash P$, then $\vdash Rw(P)$, for arbitrary w

and

If $\vdash P \equiv Q$, then $\vdash Rw(P) \equiv Rw(Q)$

Rescher's Rw-calculus may also be extended to open proposition by adding another rule of inference:

(R†) $Rw(\phi x) \equiv (\Sigma y)\ [y \in w\ \&\ y \cong x\ \&\ \phi \in P(y)]$

This rule of inference yields the following equivalences:

$\sim Rw(\phi x) \equiv (\Pi y)\ [y \in w\ \&\ y \cong x \supset \phi \notin P(y)]$

$Rw(\sim\phi x) \equiv (\Sigma y)\ [y \in w\ \&\ y \cong x\ \&\ \phi \notin P(y)]$

Given that there is only one version of an individual per world, $(\Sigma y)\ [y \in w\ \&\ y \cong x\ \&\ \phi \notin P(y)]$ entails $(\Pi y)\ [y \in w\ \&\ y \cong x \supset \phi \notin P(y)]$. This leads to the important consequence that $Rw(\sim\phi x)$ entails $\sim Rw(\phi x)$. Under the condition that $(\Sigma y)\ [y \in w\ \&\ y \cong x]$, then $\sim Rw(\phi x)$ entails $Rw(\sim\phi x)$. This means that rule (R_1) does not apply to open formulas unless certain conditions hold, namely that some version of x belongs to w.

The relationship between the actuality restricted quantifiers and the quantifiers of Rescher's extended system is defined by an additional axiom:

(R_6) $Rw[(\forall x)\ \phi x] \equiv (\Pi x)\ [x \in w \supset Rw(\phi x)]$

Which consequently leads to the equivalence

$$Rw[(\exists x)\ \phi x] \equiv (\Sigma x)\ [x \in w \supset Rw(\phi x)]$$

If we set w in the above equivalences to w* and then apply (R_4) we obtain the following consequences:

$$(\forall x)\ \phi x \equiv (\Pi x)\ [x \in w^* \supset\ \phi \in P(x)]$$

$$(\exists x)\ \phi x \equiv (\Sigma x)\ [x \in w^*\ \&\ \phi \in P(x)]$$

These consequences reflect the exact relationship Rescher had in mind between the restricted quantifiers $\forall$, $\exists$ and the more comprehensive quantifiers Π, Σ. Rescher notes that it would be useful to specify the worlds to which the variables bound by Π and Σ quantifier belong. Hence he adopts the convention of placing a world-placement indicator, either $\in$ or S, inside the scope of these quantifiers. The following equivalence may be applied towards this end:

$$(\Pi x)\phi \in P(x) \text{ iff for every w: } (\Pi x)\ [x \in w \supset\ Rw(\phi x)]$$

$$(\Sigma x)\phi \in P(x) \text{ iff for every w: } (\Sigma x)\ [x \in w\ \&\ Rw(\phi x)]$$

Rescher proceeds to define modal operators with respect to another set of quantifiers, A and E, which range over possible worlds. Necessity is defined according to the following equivalences:

(R_7) $Rw(\mathbf{L}\phi x) \equiv (\Pi y)\ [y \cong x\ \supset\ \phi \in P(y)]$,or

(R_7) $Rw[\mathbf{L}\phi x] \equiv (\mathsf{A}w')\ (\Pi y)[(y \in w'\ \&\ y \cong x\ ) \supset Rw'(\phi y)]$

From (R_7) and the relation between necessity and possibility ($\mathbf{M} \equiv \sim\mathbf{L}\sim$), we also obtain:

$$Rw[\mathbf{M}\phi x] \equiv (\mathsf{E}w')(\Sigma y)[(y \in w'\ \&\ y \cong x\ \&\ Rw'(\phi y)]$$

This means that a property is possible for an individual if some versions of this individual have this property in some possible world.

Another important consequence follows from a special case of R_7 and the application of (R_4). We are able to derive Leibniz's principle that:

$$\mathbf{L}P \equiv (\mathsf{A}w)Rw(P)$$

In the special case of (R_7) when ϕx is the closed formula P, we obtain $Rw(\mathsf{L}P) \equiv (\mathsf{A}w')Rw'(P)$. Then if we set w to w*, we may apply R4, $P \equiv Rw^*(P)$, and derive Leibniz's principle $\mathbf{L}P \equiv (Aw)Rw(P)$ which states that a necessary proposition is one which is realized in all possible worlds. Given the relation between necessity and possibility, we may define possibility: $\mathbf{M}(P) \equiv (\mathsf{E}w)Rw(P)$.

Rescher insists that when the w at issue is w*, (R_7) is transformed to

$$\mathbf{L}\phi x \equiv (\mathsf{A}w)\ (\Pi y)[(y \in w\ \ \&\ \ y \cong x\ ) \supset Rw(\phi y)]$$

According to this formula, a property necessarily characterizes an individual x iff it characterizes all of x's versions that may exist in other possible worlds. This means that the essential properties of an actual individual x will be all the properties that characterize all of x's surrogates in all the possible worlds that contain surrogates for x. Hence, it holds that for any actual x:

$$\phi!x\ \text{ iff }\ \phi x\ \&\ (\mathsf{A}w)(\Pi y)[Sw_{yx} \supset Rw(\phi y)]$$

So when the x we are dealing with is an actual individual in the real world w*, we are left with the desired consequence.

$$\mathbf{L}\phi x\ \text{ iff }\ \phi!x$$

This would then set de re necessity as equivalent to de dicto necessity. We establish that an individual possesses an essential property (de re necessity) iff it is a necessary truth that the individual have this property (de dicto necessity). With respect to possibility, (R_7) and (R_4) yields $\mathbf{M}\phi x$ iff $\phi?x$ allowing for a similar correspondence between de re and de dicto necessity.

An interesting consequence of Rescher's formal system is that the "Barcan Formula does not hold. That is, the formula $\mathbf{M}(\exists x)\phi x$ does not imply $(\exists x)\mathbf{M}\phi x$. According to Rescher's theory the following equivalents hold:

$$\mathbf{M}(\exists x)\phi x \equiv (Ew)Rw[(\exists x)\phi x] \equiv (Ew)(\Sigma y)[y \in w \ \& \ Rw(\phi y)] \equiv (\Sigma y)\phi y$$

$$(\exists x)\mathbf{M}\phi x \equiv (\exists x)(Ew)(\Sigma y)[Sw_{yx} \ \& \ Rw(\phi y)]$$

$(\exists x)\mathbf{M}\phi x$ is equivalent to saying that "in some possible world w there exists a surrogate of some actual individual which has the property ϕ." On the other hand, $\mathbf{M}(\exists x)\phi x$ means something very different. $\mathbf{M}(\exists x)\phi x$ reads that "some possible individual, not restricted to being a surrogate of an actual individual, has ϕ." Given these equivalences, the Barcan Formula would fail to hold in Rescher's theory.

Rescher shows that his Rw-calculus is consistent with the system S5. The modal logic of the purely propositional part of his system which consists of modal operators followed only by closed formulas is exactly the modal system S5. The Rule of Necessitation and the three axioms of S5 may be derived from Rescher's modal system. Furthermore the accessibility relations of Rescher's system are reflexive, symmetric and transitive like the modal system S5. This is established by (R_5) and the added stipulation that from the possible world w' is accessible from w iff there is a w" such that for all P:

$$Rw'(P) \equiv Rw''(Rw(P))$$

Given R_5, we obtain $Rw''(Rw(P)) \equiv R\langle w''*w\rangle(P)$. It follows that $Rw'(P) \equiv R\langle w''*w\rangle(P)$. Hence, there is a world w” such that

$$\langle w''* w\rangle \equiv w'$$

Except for the special case when $w \neq w^*$, then $\langle w'' *w\rangle$ amounts to w itself, i.e. the image one sees when looking at w from w” is simply w. Thus the only accessibility relation that exists is from a given world to itself. This arrangement insures that the relationship of accessibility is reflexive, symmetric and transitive.

Thus along constructivistic lines Rescher is able to build an appropriate semantical foundation for quantified logic. Rescher succeeds in developing a workable form of modal logic based on the assumption that possible worlds are to be actually constructed from the realm of the real. The developments of such a quantified modal system significantly add to the credibility of Rescher’s constructivist views as a viable alternative over Lewis’ Platonistic assumptions. We are not forced to settle for Lewis’ theory of possible worlds and logical system simply because there is no other existing theory that is formally developed and rigorously systematized enough to compete.

If we can show that the plausibility of certain counterfactual statements depends on our ability to construct maximal mutually consistent sets, then it would appear unnecessary for our ontology to assume the existence of abstract objects such as possible worlds. Instead of postulating the existence of possible worlds, we can then posit the existence of the actual world and our mental abilities to reconstrue this world. Nor do we have to view PPMC-sets as being abstract entities existing independently of the mind; we may simply regard these as being the byproducts of certain mental activities which a human mind can ideally engage in. Moreover, when we interpret modal statements such as counterfactuals or when we analyze mathematical statements, we do not have to resort to talk about abstract objects. It would be ontologically superfluous for us to maintain as well as epistemologically difficult for us to investigate such abstract objects purported to lie in some Platonic realm outside of the spatio-temporal boundaries of the actual world. It would be a welcomed idea indeed to be able to restrict our ontological commitments to the existence of this actual world and our mental abilities to understand, order, and hypothesize about it.

5 Truth Condition for Counterfactuals (Rescher)

How are counterfactuals conditionals to be formalized in the Rw-calculus? Rescher interprets counterfactuals as assertions about the dispositional properties of actual individuals. Rescher defines dispositional properties with respect to an individual and a pair of properties: Given an individual x and any pair of properties ϕ and ψ, x has the dispositional property ϕ/ψ just in case ψ constrains or requires ϕ. An individual who possesses both property ϕ and ψ has the dispositional property ϕ/ψ iff it has ϕ because it possesses ψ. Rescher distinguishes that a thing may possess properties in either a categorical or conditionalized way. Actual or essential properties possessed by an individual are categorical properties which we can definitely say belongs to a particular individual whereas dispositional properties belong to an individual in a conditionalized way, i.e. the possession of one property by an individual depends on its possession of another property.

Rescher focuses on counterfactual conditional of the "property-modificatory" sort having the following form:

> If an actual individual x ($x \in w^*$) in fact has the property ϕ, then x would have ψ.

The counterfactual thesis "if x had ϕ, then it would have ψ" implies that x lacks ϕ; however, it does not also guarantee that x does not have ϕ. The hypothesis contained in the antecedent of a counterfactual statement is contrary-to-fact and may take the form:

> -If x had ϕ (which x does not), then x would have ψ.
>
> -If x lacked ψ (which x does not), then x would lack ϕ.

Rescher provides his own set of truth conditions for counterfactual conditionals. Rescher claims that a counterfactual of the form "If x had ϕ (which it doesn't), then x would have ψ." is true just in case three conditions are met.

(i) the condition at issue is indeed contrary to the fact: x in fact lacks ϕ – i.e. $\sim\phi x$.

(ii) the antecedent envisages a feasible supposition: x might have ϕ –i.e. $\phi?x$,

(iii) the linkage from ϕ to ψ is at least of the strength of an ordinary disposition for x – i.e. $(\psi/\phi)x$.

The first requirement simply states that the antecedent hypothesis is contrary-to-fact. A counterfactual statement implies that the supposition under consideration does not actually obtain, that x actually does not possess the property in question. The second condition establishes the possibility of x's having the property ϕ which it actually lacks.

This requirement prevents us from entertaining counterfactual conditionals containing infeasible or inadmissible fact-contravening hypothesis. According to Rescher, if a counterfactual conditional contains an impossible supposition, then it would be rendered nonsensical or meaningless. The viability of some conceivable but contrary-to-fact hypothesis presupposes that the supposition is possible under the circumstances. Hence the hypothesis ϕx may serve as the antecedent of a counterfactual conditional only if **M**ϕx or ϕ?x obtains.

The third condition requires that the relation between ϕ and ψ is at least of the strength of an ordinary disposition for x, i.e. x must have the dispositional property ϕ/ψ This last condition employs the notion of dispositional properties. In addition to actual properties and possible properties, Rescher recognizes dispositional properties of objects such as solubility, malleability, or fragility. According to Rescher dispositions do describe real objects but they point to characteristics about the actual things which are never manifest to our inspection. Nonetheless Rescher identifies dispositions to be unobservable properties that may be imputed to things based on theoretical considerations and the possession of these properties accounts for the way a thing behaves in certain hypothetical situations. A vase is said to be "fragile" because it possesses the disposition to break when it is accidentally dropped or roughly handled. An object x is said to have some dispositional property D if and only if *Given x's having* ϕ, *it must have* ψ. In the case of the vase, we say that the vase has the dispositional property of "fragility" because given the vase's being dropped, it must be the case that it is broken. Rescher sees counterfactuals and dispositions to be interdefinable. For a counterfactual asserts the same idea that *"if x were to have ϕ (which it doesn't) then it would have to have ψ.* Thus, a dispositional property relates two possible properties of an object; it requires the possession of a certain property ψ when the object possesses the property of ϕ. In order to accommodate the possession of the property ψ, other requirements might have to be met as well. Any logical consequence of ψ must also obtain if ψ is to be a property of an object.

The third condition may also be expressed with reference to a set of D-possible worlds. This is useful if we wish to interpret the modal operators as restricted universal or existential quantifiers which range over some set of worlds. Th third condition demands that the linkage from ϕ to ψ is at least of the strength of an ordinary disposition. So, to assert the counterfactual "If x had ϕ, then x would have ψ," one would be claiming that all ϕ-endowed x-surrogates have ψ in all suitably possible worlds. Let us call the set of all suitably possible worlds Γ. Hence, we can translate the above counterfactual in terms of Rescher's special universal quantifiers: (Aw) which ranges over possible worlds and (Πy) which ranges over all possible individuals.

$$(\mathsf{A}w)(\Pi y)\ [(w \in \Gamma\ \&\ S^{w}_{yx}\ \&\ \phi y) \supset \psi y]$$

The important question is what possible worlds are to be counted as suitable? What worlds belong to Γ? In other words, how are we to stipulate the accessibility assignment? Must we consider all logically possible worlds in deciding whether the counterfactual is true? Or would the set of physically possible worlds suffice? Rescher identifies the suitably possible worlds to comprise of every world w that is D-possible with respect to

x. That is, Γ is the set of all D-possible worlds. In order to understand Rescher's notion of D-possible worlds we must consider four other modes of possibility.

Like Lewis, Rescher distinguishes two of the same modes of possibility, but he refers to them by different terms. Rescher calls possible worlds that obey the laws of logic "L-possible," i.e. logically-possible. He uses the term "nomically possible" or "N-possible" to describe worlds that operate according to the same laws of nature which the actual worlds follows. Recall Lewis identifies this same kind of possibility simply as "physical possibility (or necessity)." Like Lewis, Rescher sees N-possible worlds to be a subset of L-possible worlds.

Lewis felt that the analysis of counterfactuals may be aided by introducing three other modes of possibility: time-dependent possibility (i.e. possible in worlds that are exactly similar up to some time t), fact-dependent possibility (i.e. possible in worlds that are the same in respect to certain facts), and deontic possibility (i.e. possible in morally perfect worlds). Rescher consider three other sorts of possibility which he thinks is useful in the analysis of counterfactual statements. The truth conditions he sets out for counterfactuals can be expressed in terms of one of these modes of possibility, namely D-possibility or "dispositional possibility." The definition of D-possibility depends on two other modes of possibility: "E-possibility" and "M-possibility." E-possibility applies to possible individuals if they possess all the essential properties of its prototype. If the possible individual happens to be a non-actual surrogate of an actual object x, then x's surrogate is said to be E-possible if x's surrogate does not lack any of x's essential properties. M-possibility represents metaphysical possibility and this mode of possibility obtains only if E-possibility also does. A possible world is M-possible if the individuals which inhabit that world are each separately E-possible, and also conjointly compossible in order to mutually coexist in a single possible world.

Finally, we are able to explain D-possible worlds in terms of the concept of M-possible worlds. A D-possible (dispositionally possible) world is an M-possible world which contains individuals who possess all of the dispositional properties (both the essential and the inessential dispositional properties) of its real world prototype. Hence, to claim the counterfactual "If x had ϕ, then x would have ψ" is true is to make the assertion that in all D-possible worlds every x-surrogate that has the property ϕ also has the property ψ.

Although the case of subjunctive conditionals with contrary-to-fact antecedents is problem enough, there is the added question of how to treat counterfactuals whose antecedent is impossible as well. Both Lewis' and Rescher's analysis of counterfactuals begin with a comment on impossible antecedents. Lewis takes a noncommittal stance about counterfactuals with impossible antecedents, claiming that he is "fairly content to let counterfactuals with impossible antecedents be vacuously true."[43] A correct account of truth conditions ought to account for why the things we want to assert are true, but not why the things we do not want to assert are false. Counterfactuals with impossible antecedents fall into this second category, being a kind of statement which is pointless for anyone to assert. An adequate account of truth conditions need not discriminate the truth value among different counterfactuals with impossible antecedents and so Lewis adopts the convention of making all of them alike come out true in his theory. He adds, however, that his reasons for this are "less than decisive" and entertain other alternatives.

[43] Lewis, Counterfactuals, 26.

Counterfactuals with impossible antecedents might be interpreted as subjunctive conditionals of a special sort or might be taken to be non-vacuously true by allowing impossible possible worlds along with possible worlds. Possible worlds differ from the actual world in matters of contingent or empirical fact but not with respect to logical laws, whereas impossible possible worlds do differ along philosophical, logical, mathematical lines. Although Lewis entertains these alternative hypotheses, he opts not to admit impossible possible worlds into his ontology but rather makes all counterfactuals with impossible antecedents vacuously true.

Rescher takes a stronger stance against counterfactuals with impossible antecedents, i.e. counterfactuals whose antecedent may be regarded as implausible, inadmissible, improper, etc. When an antecedent of a counterfactual fails to be possible under the circumstances, then the counterfactual, according to Rescher's theory, must be regarded as meaningless or nonsensical. Rescher's account of truth conditions applies only to counterfactuals whose antecedent is possible. Those with impossible hypotheses are to be looked upon as "blocked," untenable or improper, and hence uninterpretable. Different sorts of possibility correspond to the different kinds of blockage of antecedent suppositions.

The attribution or denial of a property to an individual is blocked whenever the lack or presence of that property is supposed to be essential to it. That is, ϕx is blocked whenever it is not the case that ϕ?x. Thus, some of the more "far-fetched" contrary-to-fact hypotheses may be dismissed as being too unrealistic to consider. For example in a given context of discussion, we may be unwilling to give up certain properties which we attribute to Socrates, e.g. being a man, human, and a philosopher. We may view the hypothesis that Socrates is a centipede or a female politician as being far too ludicrous to meaningfully entertain. If we assume that a certain property ϕ is essential to a particular individual x we cannot entertain the hypothesis that x lacks ϕ. Certain hypotheses would then be blocked due to our presumptions about the essential nature of individuals or perhaps due to certain laws we believe should obtain.

Counterfactuals which postulate a change in one individual often entail revision to be made elsewhere as a result of this initial alteration. As Rescher points out

> If one postulates a change in one thing (e.g., Caesar's crossing of the Rubicon) one must make due hypothetical revisions in the history of others (Cassius, Brutus, etc), and effect these changes so that essentialistic requirements are honored throughout. Item-revisions entail world-revisions modificatory changes in things come not as single spies but in battalions.[44]

The admissibility of an antecedent hypothesis of a counterfactual implies the existence of a suitable possible world. If contrary-to-fact hypothesis ϕx is to be admissible and not blocked, then not only must ϕ be a possible property for x but also there must be a suitable possible world where x (or more precisely x's surrogate of that possible world) does have a property ϕ.

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[44]Rescher, *A Theory of Possibility*, 165.

CHAPTER SIX AN ONTOLOGY OF IDEAL CONSTRUCTIONS

1 Theoretical Virtues of Realism

It is hard to deny the general virtues of realism. One of the chief reasons that realism in mathematics and modality appear initially plausible is that they account for the objectivity of mathematical or modal statements. Moreover, it was thought to be the only coherently formulated theory that explains the objectivity of these statements. Realism establishes this objectivity by postulating abstract objects in virtue of which certain statements are either true or false, independent of our knowledge or ability to prove them. A mathematical or modal assertion is said to be objectively true if its truth is independent of what people may think. We take our assertions about the physical world to be true or false as a result of the nature of the real objects to which they refer. In the philosophy of mathematics, Platonists identify a domain of mathematical objects such as numbers or sets in virtue of which mathematical statements derive their truth values. Similarly, modal realists postulate a set of modal objects such as possible worlds and, as in Lewis' theory, counterparts inhabiting these worlds which provide the objective basis for the truth or falsity of modal statements.

A second reason for the appeal of realism is its consistency with our preexisting beliefs, so proponents argue. Penelope Maddy, an advocate of set-theoretic mathematical realism points out her version of realism "squares with the prephilosophical views of most working mathematicians." Many modern mathematicians take the surface syntax of mathematical statements as reflective of their logical form, seeing terms such as numerals or sets as singular terms which refer to objects. Working mathematicians along with most philosophers attest to the truth of some mathematical statements. But, how are mathematical statements containing singular terms to be interpreted? What set of objects did. “Among my common opinions that philosophy must respect (if it is to deserve credence) are not only my naïve belief in tables and chairs,” Lewis insists, “but also my naïve belief that these tables and chairs might have been otherwise arranged.” In another place, Lewis also points out that “I believe, and so do you , that things could have been different in countless ways.” What the philosopher does is to call “alternative ways or arrangements” by a more technical term “possible worlds,” but no matter what linguistic convention or no matter what our preferred choice of words Lewis emphasizes that we are conveying the same belief about modality. We are simply referring to these different alternative ways things might have been when we talk about possible worlds and counterparts. And, as we discussed in Chapter 4, Lewis maintains the more specific thesis that his realist theory, which denies the existence of individuals in more than one world, can accommodate all of the prephilosophical intuitions motivating the doctrine of transworld individuals. Thus, Lewis concludes that his modal realism satisfies the prime requirement on a semantic theory – to assign truth conditions to sentences of our natural language that are in accord with our intuitive judgments.

This brings us to a third point that renders realism so appealing – the convenience of being able to utilize already developed well-accepted theories and standard semantics which logicians and mathematicians uniformly rely upon. Realist formal semantics has its formal elegance to commend it. One advantage that Platonist theories of mathematics and possibilist interpretations of modal statements share is their use of standard Tarskian semantics for first order languages. The big concern is that use of such a powerful mechanism of discourse would no longer be available for alternative theories which rejected abstract objects such as numbers or sets for mathematics and possible worlds for modal statements. Admitting abstract entities into the universe of discourse allows classical mathematicians and logicians to interpret mathematical and modal statements as having an obvious logical form. According to the Platonist view, mathematical statements say something about a particular abstract mathematical object and the structural properties that this object possesses. Interpreting mathematical statements as talk about abstract entities enables us to stick with a standard Tarskian semantics for first order languages. All the formal machinery developed by Tarski and widely employed by logicians can then be made use of in the discipline of mathematics.

With respect to modal statements, postulating the existence of possible worlds enables us to replace the intensional notions of necessity and possibility by quantifiers over possible worlds. The language in which such quantifiers appear can then be given an ordinary Tarski semantics. David K. Lewis asserts that we can choose to restrict existential quantification to range over just actual things or we can broaden its range to include everything without exception. Lewis, like other possibilists, chooses to treat modal idioms as quantifiers whose scope is broad enough to include even possible worlds. Any sentence which expresses modality can be taken at face value. So the sentence “There are many ways things could have been besides the way they actually are"[45] can be given a straightforward treatment. At face value, this sentence appears to be an existential quantification which asserts the existence of entities that might be called "ways things could have been "or "possible worlds."

2 Objections to Realism

Modal realism shares in the theoretical gains which mathematical realism does, but these advantages come at a cost. The advantage of realist interpretation of modal statements may be outweighed by the added epistemological and ontological difficulties incurred when we admit abstract objects into our domain of discourse. In his article, Modal Realism: The Poisoned Pawn," Frabrizio Mondadori points out that "The general problems of objectivity and realism have been most explicitly discussed not in the philosophy of modality but in the philosophy of mathematics. By referring to it we can explain our intentions with respect to modality most easily." The problems that have traditionally worried Platonists in the area of mathematics help to illuminate similar complications that confront the modal realist.

An area of concern for both realists is how to explain the nature of these mysterious abstract objects to which they are ontologically committed. Both mathematical objects and possible worlds are shrouded in mystery and we are left with at best an incomplete and sketchy picture of these strange, foreign entities.

[45] David Lewis, Counterfactuals, 84.

According to the Platonist account, mathematical objects are supposed to be mind-independent and their existence bears no spatio-temporal relation to the physical world. They are assumed to exist unlike ordinary objects but as timeless unchanging entities, independent of all human consciousness. Hence abstract mathematical objects such as numbers or sets cannot causally interact with mathematicians, or anyone else for that matter. This renders mathematical realism susceptible to ontological and epistemological questions.

One question involves the ontological status of numbers, namely what kind of objects are we referring to when we talk about numbers or sets? These ontological challenges apply to certain versions of mathematical realism such as those with set-theoryist tendencies like Penelope Maddy's. Her Platonist picture of mathematics identifies numbers as sets. Set-theoretic Platonism about numbers appears attractive because we are able to reduce the assortment of entities (natural numbers, rational numbers, functions, groups, spaces, matrices, etc.) which mathematicians talk about into one kind of thing-sets. Developments in set theory have shown that all sorts of mathematical entities can be identified as sets. This is a reduction Platonists are eager to take since one comprehensive theory which contains fewer kinds of entities better meets the theoretical demands of parsimony and explanatory unification. A single, all encompassing theory containing fewer entities is preferable. However, the identification of numbers as sets poses the added difficulty of identifying what particular sets numbers are – a problem given that there are so many ways to reduce numbers to sets. Without any other overriding directive to guide us it would seem a purely arbitrary choice to identify a number, for example "2," with a particular set. For any of a number of sets would suffice {{∅}} or {∅,{∅}} or {∅,{{∅}}}, etc. The Platonist finds that he must arbitrarily pick from among too many abstract instantiations. Thus, although philosophers are willing to grant that realism provides a formally acceptable account of the objectivity of mathematical truths, there are still broad epistemological and ontological problems that have not been fully resolved.

The Platonist's inability to identify what numbers or sets is related to a much broader epistemological problem about our knowledge of mathematics given an ontology of mind-independent objects. Platonists have difficulty singling out the entity or entities in question because humans have no ordinary means of contact with abstract mathematical objects. This brings up the epistemological question of how human beings can interact with ethereal mathematical objects in such a way that they can ever know about them. Since mathematical entities are not sensible, it appears that Platonists cannot turn to the method of ostension via our ordinary sense perception. To identify a mathematical object ostensively would seem to require that we possess some sort of faculty of non-sensible intuition, some especial mental faculty that allows us to investigate the properties of numbers. In our consideration of Penelope Maddy's conception of mathematical intuition, such a picture of a special faculty appeared indefinite and unsatisfactory. But if we are provided with no satisfactory explanation to account for our acquaintance with numbers, we are left to puzzle about how we acquire arithmetical knowledge. The realist about mathematics is caught in the epistemological perplexity of claiming to know things about abstract mathematical objects which are not causally involved in the production of that knowledge. Thus, the Platonist theory of mathematics lacks explanatory power since it cannot explain how mathematicians come to believe in certain mathematical statements that are purported to be about existing abstract objects.

Similar epistemological questions may be asked of the modal realist. What do we know of possible worlds? How does knowledge about such abstract objects aid us in our hypothetical reasoning and everyday deliberation? Just as with abstract mathematical objects, we may ask how is it possible for us to know anything about other worlds given that they are supposed to exist independently of our minds and are causally and spatiotemporally inaccessible to the actual world in which we live. Quine complained that unactualized possible individuals are troublesome disorderly elements that present us with problems of individuation:

> Wyman's slum of possible is breeding ground for disorderly elements. Take, for instance, the possible fat man in that doorway; and again the possible bald man in that doorway. Are they the same possible man, or two possible men? How do we decide? How many possible men are there in that doorway? Are there more possible thin ones than fat ones? How many of them are alike? Or would their being alike make them one? Are no two possible things alike? Is this the same as saying that it is impossible for two things to be alike? Or, finally, is the concept of identity simply inapplicable to unactualized possibles? But what sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another? These elements are well-nigh incorrigible.

As we had mentioned earlier, the issue of transworld identity poses deep conceptual difficulties to those who hold a realist view of possible worlds. What is needed is adequate criteria for identifying the same objects from world to world.

Alvin Plantinga also suspects a difficulty with identifying the same individual in a different world. Plantinga's illustration asks us to imagine a different Socrates who occupies another world, W. Among other changes, this Socrates is characterized as having lived in Corinth, was six feet tall, and remained a bachelor throughout his life. Along with the other inhabitants of W, Socrates resides in W. The problem, Plantinga points out, lies with the task of locating which one of the occupants in W is Socrates. How are we to know who he is? It appears that we would need some kind of criteria that would enable us to identify Socrates in W, some principle that demarcates him from the rest such as an empirically manifest property which Socrates in W possesses and no other inhabitant in W does. We can no longer depend on the actual properties Socrates possessed in the real world for all we know someone else in W possesses the same properties which Socrates possessed in the real world. However in order to talk about Socrates in W, it seems we should at least be able to identify him, know who it is in W we're referring to. If we cannot even identify him, we would not know whom we were talking about, in saying that Socrates exists in that world or has this or that property therein. In order to make sense of such talk, we must have a criterion or principle that enables us to identify Socrates from world to world. If we were to pick him out by certain of his properties they would have to be empirically manifest. How otherwise could we use it to pick out or identify him?

David Lewis admits that there is difficulty in identifying an individual's counterparts in other worlds. His realist theory claims that possible worlds lie outside of the ordinary spatio-temporal bounds of the actual world and that we have no causal connection to other worlds and no epistemic link to our counterparts. Many questions about possible worlds must remain unanswered for they exist in some mind-independent realm forever out of our observational reach. Lewis concedes that we are at an epistemic loss with respect to knowing possible worlds.

> How many are there? In what respects do they vary, and what is common to them all? Do they obey a nontrivial law of identity of indiscernibles? ... If worlds were creatures of my imagination, I could imagine them to be any way I liked, and I could tell you all you wish to hear simply by carrying on my imaginative creation. But a I believe that there really are other worlds, I am entitled to confess that there is much about them that I do not know, and that I do not know how to find out.[46]

This raises the question of whether it is worthwhile to posit abstract objects for the sake of ontological and semantical purposes when there seems no epistemic advantage in doing so.

Another shortcoming of realist theories of mathematics lies in its inability to explain the usefulness of mathematical statements. If we have no causal connection to abstract objects, it is not only difficult to understand how we gain our knowledge of these objects but it is also unclear how their existence has bearing on the actual, physical world and why their properties prove so useful to our lives. How are we to explain the usefulness of arithmetic? How is it that by studying mathematical objects we improve our ability to describe and manipulate the physical environment about us? According to Platonists, mathematics distinguishes abstract objects as existing apart from the ordinary objects of the physical world. Yet, they still maintain that investigations into this ethereal domain of abstract entities provide us with a greater understanding of our own world.

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[46] Lewis, *Counterfactuals*, 128.

Somehow the nature of abstract mathematical objects reflects the structural properties of the physical objects. The burden lies with the Platonist theory to explain why mathematical statements which are true in virtue of the nature abstract objects are also representative of our physical world. After all, a strong motivation for regarding mathematical statements as having truth values is the hope of being able to account for the usefulness of arithmetic. Thus, Platonism offers us a theory which holds that there are true statements in mathematics without delivering a satisfactory explanation of how their truth has such a useful impact on our physical surroundings.

In addition to their lack of explanatory power, realist theories are also criticized for their extravagance in ontology. A theory which posits abstract objects appears to violate the principle of parsimony, the aesthetic demand on theories to limit the number of objects to which they are ontologically committed. This principle often referred to as Occam's razor is an aesthetic principle recognized both by Russell and Quine. Russell had argued that ontological economy would minimize the risk of error and had recommended that we ought to opt for substituting logical constructions before adding more objects into our ontology. Quine was even more stern and puritanical on this point. Quine equated the ontological commitments of a theory with the values of the variables of a theory's quantified statements and often criticized theories which indulged in ontological excess.

Few philosophers would admit to positing entities they think to be totally gratuitous for purposes of philosophical examination. Platonist argue that the existence of abstract objects is the most coherent means for understanding the objectivity of mathematics and so their ontological commitment to extra abstract entities are deemed necessary. Lewis takes a different approach in defending his ontology of possible worlds. In admitting such things as possible worlds and counterparts, he claims that he has not admitted any new kind of entities and so has remained consistent with the principle of ontological economy. Lewis argues that possible worlds are the same kind of thing as the actual world and that counterparts are just more of the same kind of thing as actual individuals. His ontology consists of a greater number of objects but does not admit entities of a different sort. Hence his theory is to be regarded as "qualitatively parsimonious." Lewis explains that "a doctrine is qualitatively parsimonious if it keeps down the number of fundamentally different kinds of entities … You believe in our actual world already. I ask you to believe in more things of that kind, not in things of some new kind."

3 Objections to an Ontology of Idealized Operations?

An ontology of ideal constructions meets with different sorts of objections. One such objection that may arise is that "there are not enough actual conceivings for an uncountable number of possible worlds." The notion of possibilities that have never been conceived by anyone is a viable one. It makes sense to talk about a possibility which no one has yet or may never imagine. Such unconceived possibilities should be no less real than ones that have been thought of. The fact that we do not perform certain mental acts, or perhaps cannot, given the accidental limitations of our existence, is no reflection on the range of possibilities. If the notion of possible worlds is to be successfully reduced to the notion of "constructive activities," then we'll need an ontology committed to more than just a finite number of actual conceivings. If our ontology limited itself to actual conceivings, i.e. to possibilities which one particular human mind has actually imagined, then this objection would be a legitimate one. This objection is misdirected since our view does not stipulate the existence of possibles to be dependent on their being conceived but rather on their conceivability. That is the "existence" of possibles is not grounded on the actual mental activities of any one specific mind.

This independence from any specific mind means that possibilities are not subjective to different minds. Possibilities establish an objectivity despite their mind dependence, since they result not from the abilities of one particular mind but from the generic capability of minds in general. Thus, possibilities are not conceptions that would vary according to the conceptual powers of any one person but are determined by the ideal operations of an idealized agent. The range of possibilities is not affected by one person's lack of imagination or another's misguided expectations. Nonetheless, the capabilities attributed to our idealized agent are abstracted from our own human constructive activities. We imagine the constructive activity of our ideal subject as the best idealization of our own actual constructive practice.

Another criticism directed against Rescher and Kitcher's theories is the claim that idealized agents and idealized constructions are incoherent notions. Can we justify this idealization? Can we specify a coherent idealization to account for modal concepts? In response to the first question, other disciplines such as ethics and mathematics have incorporated the notion of idealized agents into their theories. An idealized agent, then, is not a new concept. Appeal to the notion of an ideal agent may gain legitimacy to the extent that it helps elucidate other concepts at issue.

Ethical and social-political theories have relied on this notion of an idealized agent to explicate moral concepts. Rawl's egalitarian theory identifies principles of justice as those selected by ideal moral agents through a fair procedure. For the selection of principles of justice, Rawl's rational decision-makers would be ideal in their ignorance of one another's, as well as their own, social position and personal characteristics. Ignorance of one's personal interests would ensure that rational agents could fairly decide a just distribution of benefits and burdens of society. Any actual group of people would, of course, be aware of their own social circumstances and personal abilities, values and aspirations. Thus, the existence of such unbiased rational agents "veiled by ignorance" in the original position is completely hypothetical and must be abstracted from actual self-interested persons belonging to a particular social setting.

The notion of an idealized agent has also been utilized by constructivist theories for mathematics. Brouwer's intuitionism held a constructivist view that mathematical constructions are mental. Such possibilities of construction according Brouwer must refer to idealized possibilities of construction. Mathematical constructions are possibilities that originate from our perception of external objects, but involve an abstraction from concrete qualities and human limitations. For example, the construction of an infinite sequence of symbols is an incompletable task by an actual person restricted by their biological time constraints, but not for an idealized agent capable of ideal operations.

As we have mentioned earlier, Phillip Kitcher's theory of mathematics also refers to the idealized operations of an ideal agent. According to Kitcher's theory, the extensions of predicates in mathematical statements are not actually satisfied by anything at all but in some sense are approximately satisfied by operations we perform which include physical operations. If we posit idealizations of these operations, we can say that mathematics is true in virtue of the constructive activities which an ideal agent is capable of. Such idealized constructions are abstractions of the actual manipulation of reality performed by human agents. Unlike actual human agents, however, this ideal subject is not restricted by accidental limitations such as the constraint of time or the possibility of error. Thus the mathematical constructions of the ideal agent can be regarded as complete although our own merely finite performances fail to survey infinite domains. Kitcher claims that this abstraction of ideal mathematical operations from our actual limited practices is analogous to the description of ideal gases based on the actual accidental properties of actual gases. The successor operation is one mathematical practice which must eventually come to an end due to our limited life spans. However, this end is nothing more than accidental. For given our knowledge of past practices we know that whenever we have attempted to perform a successor operation, we have always succeeded and we expect that were we to follow the last operation with yet another we would again succeed. So in cases where we did not try, there is no reason to think that we would fail to perform a successor operation. By principle of induction, we expect that the iteration indefinitely continues and that there will never come a point in the sequence when the successor operation is no longer possible.

Can we describe an idealized agent to account for modal concepts? What powers would our idealized modal agents have to possess? Like an idealized mathematical agent, our modal agent must also be freed of temporal constraints. This ideal agent should not be limited by a finite life span, but be able to participate in acts of conceiving that may consist of an infinite number of steps. In addition, unlike the Rawlsian moral agent, the idealized modal agent is not to be shrouded with a veil of ignorance but equipped with an infinite knowledge about the actual world. In order to evaluate the plausibility of counterfactual statements, our modal agent must be acquainted with the all the facts that obtain in the actual world as well as any laws that may operate in the actual world. To entertain a contrary-to-fact assumption, this idealized agent must possess the cognitive ability to reconcile contrary statements into consistent sets with minimum disruption to the status quo. Thus our modal agent must have superhuman, error-free abilities of organizing, rearranging, and assessing infinite bits of data.

Have we gone too far in the abstraction of human conceptual abilities? Has our idealization of a modal agent granted abilities that are qualitatively superior to our actual human capabilities? The idealized mathematical agents were freed of merely quantitative limitations allowing them to complete simple mathematical operations that required an infinite number of steps. However, one might object that the kinds of powers we endow our idealized modal agent are not merely quantitatively unrestricted. For our modal agents are freed not only from the accidental limits that our life span imposes, but one might regard these agents as possessing capacities that are qualitatively more complex than any which a human mind could possess. It would appear that such powers are more in line with God's than man's.

If this is the case, why should our view bother to develop a distinct notion of an idealized subject? It would be simpler to adopt an ontological view like Leibniz' or certain Scholastic philosophers who made reference to the notion of "God" as a way of explaining possibilities. Instead of an ideal human agent, Leibniz relied on a divine agent to explain the notion of possibilities or possible worlds. By doing so, he did not have to deal with the problem of "unconceived possibilities" as we do with human agents. All possibilities or possible worlds are known to an omniscient, omnipotent God. In creating the world, God chooses freely and could have chosen to create any possible world. What distinguishes the actual world from other possible worlds is that it is the object of God's actual choice.

The notion of an idealized agent is different from Leibniz' conception of God. Our view may make reference to the ideal operations of an idealized agent but it in no way sees this ideal agent as having any kind of independent existence of its own, whereas Leibniz' view is committed to the existence of God. Secondly, we want to idealize the powers of our modal agent only to the extent that our agent is free of limits imposed by a finite existence. It is important for our ideal agent to preserve the distinctly human-like quality of his conceptual abilities. The reason we strive to maintain the human character of constructive activities stems from the very motivation behind this proposal. This after all is an attempt to analyze modal statements in terms of things we can understand and one kind of thing we have difficulty comprehending is possible worlds. Our view would err in the same way if it attempted to reduce possible worlds into just another set of entities that are equally mysterious. At least we can better understand human conceptual activities over Lewis' ethereal "possible worlds." We do not want to be charged with introducing god-like beings which are as elusive to the intellect as the ageless mystery of God. The challenge we face then is to provide a coherent picture of "idealized human operations."

Thus, we wish to maintain that our idealized agents are only capable of the kind of constructive activities that actual human minds are. The powers of our idealized agent should reflect the generic conceptual abilities possessed by specific minds. The difference lies not in the nature of the constructive moves themselves but with the ideal conditions that attend our ideal agent's activities. Unlike actual minds, the ideal agent may engage in constructive activities in an idealized setting free of temporal constraints. In this arena, the ideal agent can be seen as completing infinite task without the risk of human error or biological strain. So far, we have conceded nothing to this modal agent which the idealized mathematical subject did not also possess.

In addition, we will require that our modal agents have knowledge of an infinite number of the facts and laws that obtain in the actual world. Is this any different from the kind of knowledge that an actual human mind can possess? Again, the difference would be a quantitative one. Every person actually knows a large, though finite, number of facts about the world. Particular persons know mathematical facts ('two is even"), historical facts ('Kennedy was assassinated'), geographical facts ('Sacramento is the capital of California'), scientific facts ('water is composed of hydrogen and oxygen'), etc. No one person knows all the facts, because that would be an infinite amount of information which no human being would have the time nor the opportunity to learn during one lifetime. We merely allow our idealized agent to know quantitatively more data concerning the actual world. Along with particular facts, we expect the ideal agent to understand the various laws that govern the actual world. This type of knowledge is possessed by actual persons as well. Although we may not know all the conditions and the scope of certain laws, we are acquainted with some laws that obtain in the actual world, for example the laws of gravitation and Newtonian physics. We attribute to our ideal agent a perfect knowledge of all laws which actual persons may not fully understand, have not yet realized, or may accidentally never discover.

Along with the impeccable knowledge of data about the actual world, we attribute to our modal agent the same kinds of cognitive abilities as actual minds possess. The ideal agent is able to remember the past, to analyze concepts, to synthesize ideas and to compare thoughts. The power to engage in hypothetical reasoning is also a necessary ability. Thus the ideal agent can imagine and hypothesize about unactual events or situations. The ideal agent is able to engage in the broad range of hypothetical reasoning which actual minds do from reductio ad absurdum proofs and contingency planning to make-believe games and story-telling. Our ideal agent must be especially capable of the mental operations needed to evaluate counterfactual statements. So, the idealized mind must be able to entertain contrary-to-fact assumptions despite having true beliefs about the actual world and must be able to adjust a given set of beliefs so as to reconcile conflicting data. These mental operations according to this analysis of hypothetical reasoning will entail the ability to construct preferred maximally consistent sets. It is important to stipulate that despite the idealized mental powers of our ideal agent these conceptual abilities do not differ in kind from those possessed by particular minds.

Even if we restrict our ideal agent to characteristically human mental activities, another objection may be raised. Namely, could there be possibilities that no human mind could ever ideally conceive of? Even if we grant the idealized agent's abilities to operate over infinite domains, there may still be possibilities our the ideal agent cannot imagine? The interesting question is whether there are such "unconceivable possibilities," possibilities which are beyond the conceptual powers of even our ideal modal agent to imagine.

Prior to the twentieth century, technological innovations such as televisions, computers and microwaves appear to be possibilities which no one could have dreamed of. Given even full knowledge of the actual world at a particular time, the human mind may not have the power to anticipate all future possibilities, let alone predict the probabilities of possible outcomes. However admitting that there are possibilities which no one has actually imagined is different from believing the stronger claim that there are possibilities that no one could ever imagine. Furthermore, both of these statements must be distinguished from an even stronger third claim that there are possibilities that no one could "ideally" or "in principle" ever imagine. One may concede to the first and second of these claims without committing to the third. Unimagined and unimaginable possibilities are viable notions with respect to actual minds. There may be cases of underdeterminability when the given set of facts are insufficient for picking out one outcome or possibility as the most plausible. In such cases no one prediction or counterfactual may be preferable even from the perspective of the ideal mind. For example with complete knowledge of physical laws and the given position of matter at a particular time, the ideal mind may still not be able to anticipate the arbitrary movements of certain subatomic particles. Also the counterfactual assumption "Bizet and Verdi are compatriots" favors neither the inference that "Bizet is Italian" nor that "Verdi is French." The underdeterminability of such counterfactual cases does not show a weakness in our ideal agent's conceptual powers. Instead it reflects the existence of a special class of modal statements which must properly be labeled as underdeterminable. What this constructivist view of possible worlds must defend is the thesis that all possibilities exist as products of the idealized operations of our ideal agent. This claim requires only that every possibility is reducible to the constructive activities of an ideal agent. It is enough that the ideal agent can conceive of every possibility, but in some special cases the ideal agent may not have a decided preference for one possibility over others.

4 The Choice between Rival Theories

How are we to decide between the modal theories developed by Lewis versus the theory offered by Rescher? Just as in the case of mathematical theories, the choice between these rival positions stems from the more general debate between scientific realism and constructive empiricism. The main difference between these two philosophical views is their requirements for a good theory. Scientific realism places a high value in the truth of a theory and requires that all the elements of a model even those which purport to represent unobservable structures refer to real features of the world. On the other hand constructivist theories regard empirical adequacy as the sufficient condition of a good theory and a model is acceptable as long as observable phenomena can be isomorphically embedded.

If we take the Quinean approach, we do not have to chose one philosophical view over the other, i.e. scientific realism over constructive empiricism, in order to decide between Lewis' modal realism or Rescher's constructivism. Instead we can judge all theories by weighing all its virtues and limitations against those of its rivals. According to Quine, our best theories should not only fit the data but yield the greatest overall balance of theoretical as well as pragmatic advantages. A theory is to be valued to the extent that it explains the data, answers philosophical questions, resolves paradoxes and leads to true beliefs. Ontological questions about what objects really do exist must give way to the question of what our best theories say exist. A theory is not chosen because it offers us the "correct" ontology, but ontological commitments result from our determination of the best theory. Nonetheless since our best theories would offer a good balance of power and empirical adequacy, they would consequently require an ontology comprehensive enough to solve philosophical puzzles and yet parsimonious enough to risk giving us false beliefs about what there is. Hence, to determine whether our ontology should include possible worlds or ideal constructions, we must first decide which philosopher has offered us the better theory about modal statements. Brian Skyrms views theory selection as the determinant of ontological commitment:

> If possible worlds are supposed to be same sorts of things as our actual world; if they are supposed to exist in as concrete and robust a sense as our own; if they are supposed to be as real as Afghanistan, or the center of the sun or Cygnus A, then they require the same sort of evidence for their existence as other constituents of physical reality. What is required to show that the sort of possible worlds Lewis wants exist, is their presumption in the best physical theory.[47]

The reason realist theories have appeared so plausible is due to the absence of an attractive alternative. With the emergence of constructivist theories, we are given another viable option for interpreting mathematical and modal statements, an option which I have argued we have good reason to take. Philosophers like Philip Kitcher have argued that one need not assume the existence of abstract objects in order to account for the truths in mathematics. Certain ideal mental activities may serve as the proper objects of mathematics. We are to replace the notion that mathematical statements assume the existence of abstract entities with the notion that they assume the existence of constructive operations performed by an ideal subject. These idealized operations reflect the structure or general makeup of the physical world. For although these constructions are idealized, they are still subject to the constraint of conforming to the structure of the physical world. That is, these operations reflect not only what the mind can do to the physical world but at the same time reflect what the structure of the physical world will allow the mind to do. As Philip Kitcher suggests this approach takes the more direct route of describing the structure exemplified by the physical world.

Similarly, we are presented with the alternative theory of interpreting modal statements as descriptive of mental operations of ideal agents. Rescher argued that

[47] Skyrms, 327.

possible worlds are products of intellectual construction. This construction begins with a view of the real world given to us by our empirical and scientific study of nature. According to Rescher, a descriptive account of the real world must contain the following information:

(A) an inventory of actual individuals

(B) an essentialistically laden description of actual individuals including

  (i) Absolute properties

  (ii) Dispositional properties

(C) a specification of the laws of nature or universally essential dispositions[48]

From the basic components of the real world, we have the building blocks to form alternative ways the real world could have been, i.e. other possible worlds. We can then give "free rein" to the unfettered imagination in constructing countless combinatorial variations in this description. Alternative possible worlds are viewed as emerging from such an initial picture of reality through a process of hypothetical alterations.

Several advantages make an ontology of ideal constructions an attractive option over a realist ontology. First, the realist ontology has traditionally been criticized for ignoring the principle of parsimony in admitting new kinds of entities. Furthermore, the realist's postulation of additional entities was further complicated by the fact that these abstract entities did not even belong to the ordinary physical world but to an ethereal realm inaccessible to human observation. Given an ontology of ideal constructions, however, no controversial abstract entities, such as sets or possible worlds, are posited to

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[48] Nicholas Rescher, "Counterfactual Hypotheses, Laws and Dispositions, *Nous*, 5 (May 1971), 157.

exist. In positing the existence of mental operations, we would not admit any new kinds of entities to our ontology, since we ordinarily recognize the existence of such mental entities as thoughts, imagination, calculations, etc. We do however commit ourselves to a certain level of abstraction when we posit idealizations of these mental operations, but this extrapolation from actual mental entities yields a less problematic class of abstract objects ("ideal operations") than the class of abstract objects posited by Platonist and modal realists. According to this alternative ontology, the world would not have to be segmented into two existential compartments. The Platonist had divided ordinary physical objects from abstract Platonic ones, while the modal realists had segregated the actual from the unactual. This new constructivist approach opens a way for us to interpret mathematics given the assumption that only physical objects exist and to interpret modal statements from the viewpoint that only actual things exist. Mathematical and modal statements are true in virtue of the constructions which the mind is ideally able to perform upon the actual physical world. Hence, abstract objects such as Platonic entities or possible worlds do not really exist in an unqualified way as ordinary physical objects or actual entities, but they may be said to exist or subsist in a relativized way as the objects of intellectual processes.

Certain epistemological perplexities begin to dissolve when we take idealized operations and actual physical objects to be the objects of our discourse. To explain our mathematical knowledge, we no longer have to posit some mysterious intuitive powers but we can view our mathematical abilities as rooted in our causal interaction with ordinary objects which exemplify a particular structure. So, perception and not mathematical intuition can serve as the basis for our mathematical knowledge. We must revise our picture of the mathematician as primarily a discoverer of truths which are in some sense independent of us. Mathematicians, such as G.H. Hardy, have viewed mathematical reality to lie outside of us and recognized our role as that of discoverer rather than creator of mathematical truths. With an ontology of ideal operations, we are recast as constructors of mathematical reality rather than mere explorers. Of course, these idealized constructions are constrained by the structural features of the physical world, but nonetheless emphasize the role of the mind as an active participant in the formulation of mathematical truths.

There are definite advantages to construing mathematics as a product of our creative efforts. Namely, we are able to explain why mathematics proves so useful in the real world. This after all was the original motivation for regarding some arithmetical statements as true; the reason why mathematics aided us in getting around in the physical world and helped us to manipulate material objects was because mathematical statements have truth values. Yet, the problem with Platonic abstract objects was the fact that such objects are wholly removed from our spatio-temporal world and it was difficult to explain how knowledge of objects in a mysterious other-worldly realm could have practical significance to our daily lives in our ordinary physical world.

We find however that the **utility** of mathematics may also be explained by adopting an ontology of ideal operations in place of an ontology of abstract objects. Instead of viewing mathematical statements as corresponding to some Platonic realm of abstract objects, we can interpret mathematical statements to be descriptive of the actual structure exemplified by all physical objects. Mathematical statements are useful because they describe ideal operations that we may perform on objects, and these ideal operations are approximately like the actual operations which we perform on ordinary physical objects.

An ontology of ideal operations also diminishes certain epistemological worries about possible worlds. A major problem confronting realists about possible worlds was the identity of individuals among different worlds. Realists who accepted transworld identity had to decide which specific individual among the inhabitants in a world is the very same individual which they wished to talk about. Identification of individuals throughout various possible worlds implies that we have some epistemic access to these different worlds, that we can come to know facts about other worlds by observation of the other world individuals and their properties. Even Lewis' counterpart theory which substituted counterpart relation in place of transworld identity confronted the same problems with the identification of individuals and their counterparts in different possible worlds. For example, William Lycan questions whether we could determine as if with some kind of "imaginary" telescope which chimpanzee would turn out to be himself in a given other possible world:

> It is possible for me to have been a purple chimpanzee with yellow spots; therefore, we are told, there is a nonfactual world some inhabitant of which is a purple chimpanzee with yellow spots and is identical with (or is a counterpart of) me. Suppose this world contains many other chimpanzees of just the same type. Which one is, or is a counterpart of, me?[49]

[49] William Lycan, *Modality and Meaning*, (Springer, 1994), 76.

A realist picture of possible worlds seems to entail that we have this kind of epistemic link to different possible worlds, as though they are given to us as things we can somehow look into. Hence, if realists posit abstract possible worlds as the objects of our modal statements, our modal knowledge would have to be rooted somewhere in transmundane reality and our epistemic questions about how we acquire our modal knowledge would remain incomplete, if not wholly mysterious.

With the adoption of an ontology of ideal constructions, we are able to claim that our modal knowledge is the combined product of both the nature of the actual world and our idealized mental abilities. There are no possible worlds that exist unqualifiedly, but what does exist are constructions which we can mentally produce (we imagine, hypothesize, etc.) from the basic groundwork of the actual world and its objects. A possible world is merely a product of intellectual processes and does not exist independently of the mind in some Platonic realm. As a consequence, our access to possible worlds may be seen as stipulative. Questions of transworld identity are not settled by investigating different possible worlds but are settled by our own stipulation. Since possible worlds are simply products of our mental processes, i.e. of our own making, it is we who stipulate just which world we are talking about and we stipulate which individuals in that world we are referring to. Our way of characterizing the world then settles the questions of identity and does not leave the answers contingent somewhere in some transmundane reality.

This proposal to reduce possible worlds in terms of ideal constructive activities, however, does not force us to abandon a Tarskian semantics in which quantifiers appear to range over abstract objects. This was an important selling point for realist theories for it was simpler and more convenient to stick to the standard Tarskian semantics than to adopt a more complicated semantics. As Rescher has shown in his book "A Theory of Possibility," one can continue to use Tarskian semantics for counterfactual discourse. We would depart from the standard possibilist approach only in our reinterpretation of the language in which Tarskian semantics is given. Just as references to abstract mathematical objects are replaced by references to collectings in Philip Kitcher's theory of mathematics, so possible worlds are understood as referring to constructive activities. For the sake of convenience, we might talk in terms of "possible worlds" as long as we realize that our ontological commitments are really to the mental activities (e.g. collectings) that give rise to the idea of possible worlds. Like Kitcher's theory, our reinterpretation of abstract objects as mental constructions would allow for the continued use of a uniform semantics for modal discourse (Tarskian semantics) without requiring a commitment to abstract objects.

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