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Los Angeles

## **Essays in Finance**

A dissertation submitted in partial satisfaction  
of the requirements for the degree  
Doctor of Philosophy in Management

by

**Shaun William Davies**

2013

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ABSTRACT OF THE DISSERTATION

**Essays in Finance**

by

**Shaun William Davies**

Doctor of Philosophy in Management

University of California, Los Angeles, 2013

Professor Bruce I. Carlin, Chair

In this dissertation I present three theoretical papers, each as an individual chapter. The first paper is titled “The Economics of Discretion in Multi-Agent Decision Problems.” It is premised on the idea that discretion is valuable when knowledge mismatches lead principals to delegate decisions to agents with specialized knowledge. In the paper, I consider a team setting, and I characterize the optimal delegated choice set that is offered to agents when an agency conflict is present. I show that the amount of discretion increases with the value of an agent’s private information, with the degree of alignment between the principal and agents, and with the ex ante uncertainty faced by the principal and agents. The latter finding implies that discretion may be used by agents to hedge the risk that they face. Nevertheless, I demonstrate that when all participants have rational expectations, it is never optimal for agents to add strategic uncertainty to receive more discretion ex post. I conclude the paper by applying the theory to delegated portfolio management, which yields novel empirical implications in this setting.

The second paper is co-authored with Bruce I. Carlin and Andrew Iannaccone and it is titled “Competition, Comparative Performance, and Market Trans-

parency.” In the paper, we study how competition affects market transparency, taking into account that comparative performance is assessed via tournaments and contests. Extending Dye (1985) to a multi-firm setting in which top performers are rewarded, we show that increased competition usually makes disclosure *less* likely, which lowers market transparency and may decrease per capita welfare. This result appears to be robust to several model variations and as such, has implications for market regulation.

The final paper is co-authored with Bruce. I. Carlin and is titled “Political Influence and the Regulation of Consumer Financial Products.” In the paper, we explore a theoretical model of product regulation in which the social planner chooses an optimal level of market complexity, given that people have varied sophistication. We investigate how several dimensions affect the quality of regulation: the skill of the social planner, imperfect information, lobbying efforts, voting behavior in elections, and political philosophy. We find that both sophisticated and unsophisticated market participants often vote to elect the least informed and educated planners, which erodes social welfare. Further, when concerns regarding equality are sufficiently large (i.e., a socialistic agenda), the social planner limits the market to one product. In such case, adequacy suffers and all market participants are equally worse off.

The dissertation of Shaun William Davies is approved.

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2013

*To the Lord God Almighty,  
who richly blesses me  
with family, friends, colleagues,  
and interesting problems to study.*

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# CHAPTER 1

## The Economics of Discretion in Multi-Agent Decision Problems

### 1 Introduction

The use of delegation is motivated by a principal's inability to extract an agent's private information due to costly information transmission. The principal chooses to delegate her decision-making rights to the agent, instead of making an uninformed decision. Delegation, however, introduces an agency conflict as the principal and agent may have different objectives. Consequently, the principal mitigates the conflict by providing discretion that limits the decision-maker to a subset of choices. In this paper, I focus on how the principal optimally selects the decision-maker's choice set and I characterize it. My model's insights and reach extend to many economic fields, e.g., industrial organization, political economy, and corporate finance. As such, I first analyze the general model to explore the economics of discretion-limits. Subsequently, I apply the theory to delegation in the investment management industry.

The subject of discretion is particularly relevant in the investment management industry where knowledge mismatches lead agents to delegate investment decisions to those with specialized knowledge, e.g., retail investors delegate portfolio management decisions via mutual fund purchases and institutions delegate investment decisions to internal managers and sub-advisors. Delegating portfolio choices is beneficial as it leads to informed portfolio construction. The benefit, however, is limited. It is well documented that delegated investment management typically gives rise to an agency cost: compensation contracts lead managers to add extra portfolio risk (Starks 1987; Grinblatt and Titman 1989; Brown, Harlow, and Starks 1996; Chevalier and Ellison 1997), managers expend too little effort on information collection (Stoughton 1993), and behavioral biases create a wedge between manager and investor risk appetites (Eriksen and Kvaloy 2010).

The existing literature focuses primarily on compensation schemes and monitoring, but largely ignores discretion. Indeed, an understanding of compensation and incentives is important, however, it overlooks a significant dimension of delegated portfolio management: delegation is accompanied with decision constraints. For example, mutual fund managers are typically limited by their funds' prospectuses and institutional portfolio managers may be provided with risk budgets to execute on strategies. I add a new dimension to the literature by considering how decision rights should be allocated and the *quantity of discretion* that should accompany those decision rights. Furthermore, understanding discretion-limits as a substitute or complement to other mechanisms is important when compensation is constrained and monitoring is costly.

The generalized model involves two agents and a principal. The agents are bound by the outcome of a common decision and each agent has a preferred choice that maximizes his individual payoff. If complete contracting is possible, the agents find it optimal to choose the decision that maximizes the aggregate payoff, which is then split according to their Nash bargaining weights. I assume, however, that bilateral contracting between agents is costly. For example, depending on the application, it may be costly for the two to haggle, coordinate information or transfer specialized knowledge. If the contracting cost is sufficiently large, the agents do not find it efficient to bargain. The principal remedies this by providing a delegation mechanism that bypasses the cost. The mechanism, however, gives rise to an agency conflict due to misalignment between the principal and agents. Nevertheless, the principal optimally delegates all decision rights to one of the agents, but limits his choices, i.e., the principal grants a provision of discretion.

I first consider the case where it is prohibitively costly for the the agents to contract and I show that discretion is especially valuable when at least one agent has private information. The principal grants the privately informed agent a continuum of choices (Holmstrom 1977, 1984; Alonso and Matouschek 2008), which includes the principal's uninformed best guess (had she retained the decision-making rights). Naturally, I show that the discretion provided to one agent is also characterized by the needs of the other. Specifically, the continuum of choices also contains the other agent's bliss point. I augment the analysis by considering a setup where there are  $N > 2$  agents. The analysis provides new insights into the theory of delegation by considering agent-to-agent effects in addition to the principal-agent relationship.

I then consider when the contracting cost is not large enough to prevent agent-

to-agent bargaining. Indeed, it is efficient for the agents to bypass the delegation mechanism and bargain if the net gain is greater than the contracting cost. The principal takes the agents' ability to do so under consideration when she provides discretion. Consequently, the quantity of discretion is a function of the contracting cost and the agents' bargaining weights. I show that discretion increases in the contracting cost and decreases with the decision-making agent's bargaining power. This suggests that discretion is greater in competitive environments than in monopolistic ones.<sup>1</sup>

In the initial setup, I assume that both agents' payoff functions contain a quadratic loss component. I relax this assumption later in the paper and I analyze the sensitivity of my initial results by considering a generalized payoff function. I demonstrate that linear payoff functions make unlimited discretion optimal. However, this is a knife-edge case and it is optimal to limit discretion if the agents' payoff functions are anything but linear. I supplement the analysis by showing that discretion increases with the concavity of the agents' payoff functions. As the payoff function gets "steeper" around the agents' bliss points, the decision-maker's private information becomes more valuable and the principal provides greater discretion.

Next, I show that discretion increases with uncertainty in the principal's beliefs. I model this by assuming that the principal's beliefs are a probability distribution over the agents' preferences. I show that the principal allocates more discretion as the distribution's variance increases. For the analysis, I derive and use a distribution which I coin the "mean-preserving linear distribution." The distribution is attractive because the variance is characterized by a single parameter without introducing confounding factors, e.g., increasing the variance of the uniform distribution requires expanding its support. The result might suggest that the decision-maker enjoys additional ex ante uncertainty because he is granted greater discretion ex post. If the principal's beliefs are accurate, however, this turns out not to be the case; the increased discretion does not sufficiently compensate the agent against increased risk. Consequently, agents prefer an informed principal ex ante. In the supplementary appendix, I consider a general class of distributions and provide conditions for which the results hold.

I conclude the analysis by considering an application to delegated portfolio management. I consider a decentralized firm that generates profits by actively managing

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<sup>1</sup>The result also has implications for the theory of the firm, as upstream firm divisions should receive greater discretion than monopolistic downstream divisions. See Williamson (1973, 1979), Klein, Crawford, and Alchian (1978), Grossman and Hart (1986), and Hart and Moore (1990) for a thorough analysis of the hold-up problem.

investor capital. The agents in the application are the firm’s division managers and the principal is the firm’s CEO. One division, the trading division, invests capital and actively trades the firm’s portfolio. The other division, the “reputation” division, is client-facing, e.g., the division is responsible for raising and maintaining outside capital via marketing & client services and establishing services to augment investment activities. The firm’s CEO must decide how much portfolio risk the firm should take, but the division managers possess specialized information. It is costly for managers to transfer their information, so the CEO delegates the portfolio risk choice to one of them. Furthermore, the CEO limits the choice set due to an agency conflict induced by the firm’s compensation scheme.

The rest of the paper is organized as follows. In Section 2, I discuss the relevant literature. In Section 3, I pose and characterize my base model, introduce the delegation mechanism and derive optimal discretion under an imperfectly informed principal. In Section 4, I allow the agents to choose between the principal’s mechanism and costly bargaining. I demonstrate, in a rational expectations framework, that the optimal provision of discretion increases with the contracting cost and decreases with bargaining power. In Section 5, I explore the principal’s mechanism with variants of the the agents’ payoff functions. In Section 6, I consider the degree to which agents want the principal to be informed. In Section 7, I apply my model to delegated portfolio management. Section 5 concludes. All mathematical proofs are in Appendix A. In Appendix B, I consider variants to the base model. In Appendix C, I derive the mean-preserving linear distribution used in my analysis.

## 2 Literature Review

The application I explore in this paper is set in the context of an investment firm, but I should emphasize that discretion is pervasive in many contexts and my model can be easily applied to problems in industrial organization, political economy, and corporate finance. As such, it is worthwhile to discuss the related literature in both general economics and my specific application of delegated portfolio management.

## 2.1 Delegation in Contract Theory and the Theory of the Firm

The topic of delegation and the accompanying subtopic of discretion-limits were first broached by Simon (1951) who introduced the concepts within the context of an employer and employee relationship. In his setup, the employee sells his services to an employer. The employer, in turn, provides the employee various terms of service in an “acceptance set” and lets the employee choose by delegating the decision-making ability. Holmstrom (1977, 1984) also considers the relationship between a principal and a single agent. He demonstrates that a principal achieves a greater payoff by delegating decisions to better informed agents, as long as their preferences are minimally aligned. My paper continues in the spirit of Simon (1951) and Holmstrom (1977, 1984) by exploring the value of discretion in a multi-agent setting, how discretion is measured, and how much discretion is optimal.

Melumand and Shibano (1991) consider a bilateral relationship between principal and agent. Similar to my setup, they assume quadratic payoff functions and they solve for the conditions under which delegating a single continuum of actions is optimal. While Melumand and Shibano model the principal’s uncertainty as a uniform distribution, Alonso and Matouschek (2008) extend their analysis by considering general distributions. My work complements these two papers by using their results to justify discretion as the optimal, incentive compatible, truth-telling mechanism. Dessein (2002) considers a model where the principal chooses between delegation or noisy communication with the agent. He shows that delegation is value-adding because it inherently involves commitment to a certain decision-making rule. If the principal cannot commit, i.e., she maintains veto power, communication between principal and agent breaks down and results in the cheap talk framework of Crawford and Sobel (1982). My work adds another dimension to the work of Dessein; he considers the tradeoff between loss of control (delegation) and noisy communication (cheap talk), while I consider the tradeoff between delegation and costly complete contracts. Krishna and Morgan (2008) consider the delegation problem with contingent monetary transfers. I also consider contingent transfers in my paper and I show that there exists a unique pricing scheme that leads to the first-best decision by the agent. Lastly, Aghion and Tirole (1997) consider the effect of authority on the principal-agent information structure. I, however, assume that the agent is always better informed.

Discretion is often considered in the same context as control rights and contingent control. Control rights, or the designation of unlimited discretion, as explored by Klein,

Crawford, and Alchian (1978), Grossman and Hart (1986) and Hart and Moore (1990), serve as a mechanism to mitigate ex ante inefficiencies. My paper adds to this existing literature by distinguishing discretion as a form of limited control. I show that provisions of discretion enhance the aggregate payoff and substitute for costly contracting. My research also adds to the contingent control literature. Contingent control, as explored by Aghion and Bolton (1992), designates full discretion to interested parties depending on the state of the world. In their paper they consider whether a wealth constrained entrepreneur or investor should control a project. In this paper I consider whether an agent with private information or an agent with no private information should be granted decision rights.

Lastly, the agents in my model rely on rules constructed by an uninformed principal. As such, my paper adds to the existing economic literature regarding rules and uncertainty. Baron and Myerson (1982) provide, perhaps, the definitive paper on optimal regulation in the presence of private information. Specifically, they consider the optimal method for regulating a monopolist with an unknown cost function. My paper adds to their work by considering how a principal accommodates an unknown payoff function with discretion. In a similar spirit to Baron and Myerson, Milgrom and Roberts (1986) and Laffont and Tirole (1991) consider how a decision-maker efficiently balances the needs of interested parties with private information. My paper also explores this tradeoff by examining how discretion imposes both a cost and a benefit to all parties. Furthermore, my paper utilizes insights from the costly contracting literature. Anderlini and Felli (1994) and Battigalli and Maggi (2002) justify the existence of incomplete contracts by the costs associated with explicitly defining every possible contingency. In my model I assume an exogenous contracting cost but appeal to their work as a micro foundation for it.

## **2.2 Delegation in Portfolio Management**

Portfolio management requires a great deal of specialization within asset classes and market sectors. Consequently, investment firms often require decentralized decision-making. Delegation is used to match specialized knowledge to portfolio investment decisions. Chen, Hong, and Kubik (2011) explore the performance of investment funds that outsource portions of their fund portfolio. They find empirically that funds which use outsourcing underperform funds that are run internally. The authors attribute the difference to outside managers extracting rents. Cashman and Deli (2009) also examine

the outsourcing of portfolio decision rights. They find empirically that decisions are delegated to outsiders when there is likely a large degree of private information. My paper provides a theoretical basis for both of these papers: the presence of private information motivates principals to delegate decisions while an agency conflict leads to rent seeking.

Almazan, Brown, Carlson, and Chapman (2004) consider the constraints placed on managers by investors. They find empirically that constraints are more common when agency costs are likely high. My work adds a theoretical dimension to their work by solving for and characterizing the optimal quantity of discretion in delegated portfolio management. van Binsbergen, Brandt, and Kojen (2008) consider a decentralized investment firm where division managers and a centralized decision-maker have different objectives. The authors prescribe the use of a benchmark in compensation contracts to mitigate the agency cost. I, however, show that limiting manager discretion is an alternative tool for mitigating the agency cost.

Lastly, much of the theoretical research aimed at the delegated portfolio management problem focuses on compensation schemes. Admati and Pfleiderer (1997) demonstrate that benchmark-adjusted compensation is difficult to rationalize in a one-shot game. Ou-Yang (2003) and Kraft and Korn (2008), however, show that benchmark-adjusted compensation does mitigate agency conflicts when the problem is taken to a continuous-time setting. It is also well documented that some compensation contracts lead to agency conflicts, e.g., Starks (1987) shows that asymmetric incentive contracts lead managers to add extra portfolio risk and Stoughton (1993) demonstrates that a linear contract may lead to a moral hazard conflict with managerial effort. While an understanding of compensation and incentives is important, I add a new dimension to the problem by considering the effect of limiting discretion to mitigate agency conflicts.

### 3 Base Model

In this section, I establish an underlying game form which allows me to explore costly contracting and delegation. Consider two agents that are mutually bound by the outcome of a common choice  $d$  from the set of all feasible choices. The set of choices is represented by the continuum  $[0, D]$ . Each of the two non-cooperative agents has a unique choice  $x_i^* \in [0, D]$  that maximizes his individual payoff function,

$$\Pi_i = H_i(d, x_i^*) \text{ with } i \in \{1, 2\}. \quad (1.1)$$



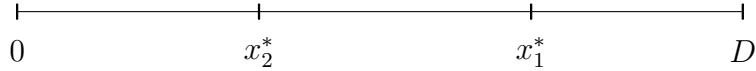


Figure 1.1: Decision space. The bliss points for agent 1 and agent 2 appear in the continuum of all possible choices.

Furthermore, for each agent assume that

1. the payoff function is everywhere continuous and differentiable in  $d$ ,
2. the payoff function is concave in  $d$ , i.e.,  $\frac{\partial^2 H_i(d, x_i^*)}{\partial d^2} \leq 0$ ,
3. and  $\frac{\partial H_i(d, x_i^*)}{\partial d} = 0$  if  $d = x_i^*$ .

The space of all possible choices and each agent's bliss point are depicted in Figure 1.1. Both agents retain the ability to walk away from the game and receive a zero payoff.

The aggregate payoff is the sum of the two agents' payoffs,

$$\mathbf{\Pi}(d, x_1^*, x_2^*) = H_1(d, x_1^*) + H_2(d, x_2^*). \quad (1.2)$$

The sum of the two concave functions yields an aggregate payoff function that is also concave in  $d$ . Consequently, there is a unique choice  $d^*$  that maximizes it. The choice is implicitly defined by

$$0 = \frac{\partial H_1(d, x_1^*)}{\partial d} + \frac{\partial H_2(d, x_2^*)}{\partial d}, \quad (1.3)$$

and it is certainly possible that  $d^* \neq x_1^*$  and  $d^* \neq x_2^*$ . For the sake of exposition, I assign an explicit form to the payoff functions, namely,

$$\Pi_i(d) = \pi_i - (d - x_i^*)^2, \quad (1.4)$$

where  $\pi_i > 0$  for  $i \in \{1, 2\}$ .<sup>2</sup> Because the agents are bound by a common choice, it is impossible to simultaneously maximize both agents' payoffs, except for the trivial case of  $x_1^* = x_2^*$ . This highlights the model's key tension and provides the focus of my paper: how is  $d$  chosen?

---

<sup>2</sup>I require that  $\pi_i \geq D^2$  for  $i \in \{1, 2\}$ . The assumption guarantees that both agents' payoffs are positive for all values of  $x_1^*, x_2^* \in [0, D]$  and for all choices  $d \in [0, D]$ . In the absence of this condition there exist regions where it is not efficient for at least one agent to participate in the game. While this is noteworthy, it does not provide additional insights.

Table 1.1: Model timing in base model.

Time	Event
$t = 1$	Agents observe bliss points and decide whether or not to pay $c/2$ . If agents participate, they choose $d$ and split aggregate payoff according to Nash bargaining.
$t = 2$	Payoffs realized.

Although the focus of this paper is delegated discretion, I begin with an alternative decision-making process to motivate the delegation mechanism. An obvious decision-making mechanism is agent-to-agent complete contracting. The agents maximize their payoffs by choosing  $d$  to maximize the aggregate payoff and then split it according to the asymmetric Nash bargaining solution. Denote agent 1's bargaining power as  $\theta_1 \in [0, 1]$  and agent 2's as  $\theta_2 = 1 - \theta_1$ . The contracting process itself is costly for the two agents. Namely, the agents each incur a fixed cost  $c/2 > 0$  when they bargain. This contracting cost is attributed to a vast list of bargaining expenses, e.g., effort and time expended on haggling, the cost of coordinating information, and the cost of explicitly contracting. As such, the aggregate payoff, net of the contracting cost, is given by

$$\mathbf{\Pi}(d) = \pi_1 + \pi_2 - (d - x_2^*)^2 - (d - x_1^*)^2 - c. \quad (1.5)$$

For now, let us start with the perfect information case. The model's timing proceeds as follows. At time  $t = 1$  the agents' bliss points are publicly observable and the agents choose whether or not to participate. When the agents bargain, they each pay the contracting cost of  $c/2$  and Nash bargaining ensues to determine  $d$ . At time  $t = 2$  the agents' combined payoffs yield the aggregate payoff and each agent receives his share. I assume that agreements made at  $t = 1$  are enforceable and cannot be renegotiated, i.e., there are no holdup issues and an agent cannot abscond with his individual payoff. The base model's timing is summarized in Table 1.1.

**Proposition 1.** *If  $c \leq \hat{c}$  the agents choose  $d^* = (x_1^* + x_2^*)/2$ . It is inefficient for the*

agents to participate if  $c > \hat{c}$  where

$$\hat{c} \equiv \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2}. \quad (1.6)$$

The economics of Proposition 1 are straightforward, but the implications are a bit more subtle. The aggregate payoff in Equation 1.5 contains the contracting cost  $c$ , which is attributed to haggling, coordinating information and drafting agreements. If  $c$  is sufficiently small, the agents are able to bargain. If, however, the cost is large, then bilateral contracting between agents is not possible and alternative decision mechanisms are valuable. In the following section, I introduce a principal that oversees both agents and provides a mechanism to overcome the cost.

### 3.1 The Delegation Mechanism

Assume that  $c > \hat{c}$  so that there is a benefit from alternative decision-making mechanisms. Furthermore, the two agents are overseen by a principal. The principal's objective is to maximize the aggregate payoff and, for consistency, I assume that the existence of the principal does not eliminate the contracting cost  $c$ . The principal is tasked with implementing a mechanism that enables decision-making without incurring  $c$ .

In the spirit of Holmstrom (1984), the principal utilizes a delegation mechanism.<sup>345</sup> Specifically, at time  $t = 0$ , the principal delegates the decision rights for  $d$  to one of the agents. If the agents comply with the mechanism, they no longer bargain at  $t = 1$ . Instead, the decision-making agent determines  $d \in [0, D]$ . Each agent realizes his payoff at  $t = 2$ . For now, without loss of generality, suppose agent 1 is delegated the decision rights.

At  $t = 1$ , both agents are committed to the mechanism, regardless of the decision-making agent's choice. Without limits on his discretion, the agent maximizes his payoff

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<sup>3</sup>Holmstrom (1984) postulates that delegation is an attractive alternative because of the "contracting costs saved by the simpler decision process." Furthermore, the claim is supported by the prevalence of the mechanism in practice.

<sup>4</sup>It is equivalent to consider a broader set of mechanisms and allow the principal to solve the contracting problem by choosing any deterministic, incentive compatible, truth-telling mechanism. This fact is redolent of the Revelation Principle. See Alonso and Matouschek (2008) for further discussion.

<sup>5</sup>There may exist a cost to implement the delegation mechanism. The important distinction, however, is that the delegation mechanism is *relatively* cheaper to implement than the alternative. As such,  $c$  captures the relative difference in implementation costs.

by choosing  $d = x_1^*$ . If  $x_1^*$  and  $x_2^*$  are equal, unlimited discretion maximizes the aggregate payoff. If, on the other hand,  $x_1^*$  and  $x_2^*$  differ, the aggregate payoff is lower than first-best. Consequently, it is possible to increase the aggregate payoff by limiting the agent's discretion.<sup>6</sup>

**Definition 1.** A *rule* is a set,  $\{\underline{\alpha}, \bar{\alpha}\} \in [0, D] \times [0, D]$  with  $\underline{\alpha} \leq \bar{\alpha}$ , determined by the principal, that restricts the decision-making agent's choice to the continuum  $[\underline{\alpha}, \bar{\alpha}]$ . A decision-making agent's discretion is increasing in  $\bar{\alpha} - \underline{\alpha}$ .

Definition 1 provides the means to limit the decision-maker's discretion.<sup>78</sup> The definition also contains a metric for comparing rules. A rule provides broad discretion if  $\bar{\alpha}$  and  $\underline{\alpha}$  differ greatly and narrow discretion when the two bounds are close. The timing of the rule mechanism is outlined in Table 1.2.

### 3.2 Optimal Rules

The principal's problem is to define a rule that maximizes the aggregate payoff,

$$\begin{aligned} & \max_{\underline{\alpha}, \bar{\alpha} \in [0, D]} \Pi_1(d) + \Pi_2(d) \\ & \text{subject to} \\ & d \in \arg \max_{d \in [\underline{\alpha}, \bar{\alpha}]} \Pi_1(d), \\ & \Pi_1(d) \geq 0, \\ & \Pi_2(d) \geq 0. \end{aligned} \tag{1.7}$$

If the agents do not possess private information, the principal dictate's the decision-maker's choice.

**Proposition 2.** A perfectly informed principal restricts the decision-making agent to a single choice,

$$\alpha^* = \underline{\alpha} = \bar{\alpha} = \frac{x_1^* + x_2^*}{2}. \tag{1.8}$$

---

<sup>6</sup>I do not allow the principal to "price" individual choices for the decision-making agent. The assumption that transfers are not contingent on  $d$  is consistent with the existing delegation literature, e.g., Holmstrom (1977, 1984), Aghion and Tirole (1997), Melumad and Shibano (1991), Dessein (2002), and Alonso and Matouschek (2008). I, however, do consider the possibility of contingent transfers in Appendix B.

<sup>7</sup>My use of the term "rule" is analogous to the term "control intervals" described by Holmstrom (1984).

<sup>8</sup>Alonso and Matouschek (2008) discuss the optimacy of a delegation mechanism that requires an agent to make his decision from a single interval, i.e., "interval delegation." They expound on the earlier work of Melumad and Shibano (1991) and provide the necessary conditions under which interval delegation is optimal. The model setup prescribed in this paper satisfies those conditions, that is, a rule consisting of a single interval is optimal.

Table 1.2: Model timing in the principals’s delegation mechanism.

<b>Time</b>	<b>Event</b>
$t = 0$	Principal anticipates the agents’ bliss points and provides discretion $[\underline{\alpha}, \bar{\alpha}]$ . Agents decide whether or not to commit to mechanism.
$t = 1$	Agents learn their respective bliss points. Agent 1 chooses $x_1^*$ if it is contained in $[\underline{\alpha}, \bar{\alpha}]$ . Otherwise he chooses his best alternative from the continuum $[\underline{\alpha}, \bar{\alpha}]$ .
$t = 2$	Payoffs realized.

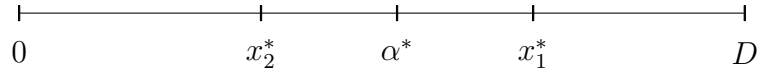


Figure 1.2: Rule with a perfectly informed principal. If both agents’ bliss points are known, the optimal rule is to restrict agent 1 to choose the midpoint of  $x_1^*$  and  $x_2^*$ .

Proposition 2 indicates that it is a misnomer to call the agent the “decision-maker” if he does not have private information. Instead, the principal maximizes the aggregate payoff by restricting the decision-maker to the first-best choice. Obviously, creating rules that provide latitude, i.e.,  $\underline{\alpha} \neq \bar{\alpha}$ , introduces the possibility of suboptimal decision-making by self-interested agents. The space of all possible choices, the agents’ bliss points and the permitted choice are depicted in Figure 1.2. I now turn my attention to optimal discretion when the agents possess private information.

In situations where agents possess better information than the principal and a decision needs to be made, the principal can either use available information to dictate the decision or delegate away the decision. Indeed, as I will show shortly, delegating

the decision is weakly dominant.<sup>9</sup> In my analysis I implicitly assume that the principal has the ability to commit to delegation, i.e., she does not retain veto-power. If the principal could not commit to the mechanism it would be appropriate to model the game in a cheap-talk framework, e.g., Crawford and Sobel (1982) and Dessein (2002). In addition to this, the existing literature often considers delegation only in the context of a vertical relationship between principal and agent. I depart from this conventional setup by considering delegation in a setting where the principal oversees multiple agents and I explore the benefits of delegation on agent-to-agent interaction. In Appendix B, I consider a setup with  $N > 2$  agents and provide additional insights.

Suppose that the principal is perfectly informed of agent 2's bliss point but she has only beliefs for agent 1's.<sup>10</sup> I denote the principal's beliefs for  $x_1^*$  as a probability density function,  $f(x)$ , on the support  $[0, D]$ . For analytic tractability I make the minimal assumptions that  $f(x)$  is continuous and differentiable. Again, assume that the principal delegates the decision rights to agent 1 and that  $c > \hat{c}$ .

Before I demonstrate the optimal quantity of delegated discretion, consider the following thought experiment. We saw in Section 3.1 that the first-best aggregate payoff occurs when agent 1 selects  $d^* = \frac{x_1^* + x_2^*}{2}$ . Here, the principal is unable to restrict the agent to this choice because his bliss point is private. The principal, however, does know agent 2's bliss point. Therefore, even without beliefs, the principal can limit discretion and increase the expected aggregate payoff. Specifically, she can establish the naive rule,

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{x_2^*}{2}, \frac{x_2^* + D}{2} \right\}. \quad (1.9)$$

The naive rule eliminates any choices that cannot be first-best, since  $x_1^*$  cannot be less than 0 or greater than  $D$ . The naive rule dominates unlimited discretion, but, as I will now show, the principal uses her beliefs to do better.

Consider a rule  $\{\underline{\alpha}, \bar{\alpha}\}$ . The principal anticipates the agent's optimal choice for each possible realization of  $x_1^*$ . The following lemma establishes the expected aggregate payoff under a given rule,

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<sup>9</sup>The existing research on delegation in economics and political economy suggests that authoritative parties should delegate decision power to better informed subordinates. See Huber and Shipan (2006) for discussion on the ‘‘Uncertainty Principle’’ in political economy.

<sup>10</sup>I consider the possibility that both agents' bliss points are private information at the conclusion of this section. I show that my results are qualitatively unchanged in that setup and that little intuition is lost by considering that only agent 1's bliss point is private information.

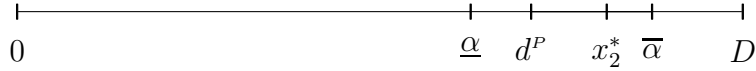


Figure 1.3: Rules with an imperfectly informed principal. If agent 1's bliss point is private information the optimal rule straddles agent 2's bliss point and the principal's best uninformed guess.

**Lemma 1.** *For a given rule  $\{\underline{\alpha}, \bar{\alpha}\}$  the expected aggregate payoff is given by,*

$$\begin{aligned}
 E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] = & \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2] f(x) dx \\
 & - \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^2 f(x) dx \\
 & - \int_{\bar{\alpha}}^D [(\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2] f(x) dx. \tag{1.10}
 \end{aligned}$$

Equation 1.10 contains three integrals because agent 1 will choose either his bliss point or one of the rule's bounds. The agent achieves his optimum when  $x_1^*$  is in the realm of discretion  $[\underline{\alpha}, \bar{\alpha}]$ . When  $x_1^*$  falls below the lower bound of the rule, the agent maximizes his payoff by choosing  $\underline{\alpha}$ . Similarly, when  $x_1^*$  is above the upper bound, the choice is  $\bar{\alpha}$ . I effectively bake-in partial alignment between the principal and agent 1 via their payoff functions. Consequently, discretion-limits provide a benefit and a cost: increased discretion lets agent 1 increase his payoff and, consequently, part of the aggregate payoff. The additional discretion, however, may adversely affect agent 2's payoff, i.e., the other component of the aggregate payoff. The principal's problem is to strike a balance between this benefit and cost by providing a rule that offers optimal discretion.

**Proposition 3.** *When the decision-maker has private information, the optimal rule does not restrict him to a single choice. Instead, the principal provides the decision-maker with discretion,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{E[x_1^* | x_1^* \leq \underline{\alpha}] + x_2^*}{2}, \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] + x_2^*}{2} \right\}. \tag{1.11}$$

Proposition 3 yields the optimal discretion provided to agent 1. Both the lower and upper bounds of the rule are defined implicitly without stating a specific distribution.

Later in this section I provide closed-form solutions for the rule’s bounds when  $f(x)$  is uniform over  $[0, D]$ . It is important to note that the optimization results in a band of permissible choices, rather than a single one. As such, providing discretion,  $\underline{\alpha} \neq \bar{\alpha}$ , at least weakly dominates “no discretion” since  $\underline{\alpha} = \bar{\alpha}$  is contained in the optimization’s choice set. The intuition of the result follows from understanding how the principal utilizes the agent’s private information. If the principal relied on her own beliefs, she would dictate her uninformed best guess,

$$d^P = \frac{E[x_1^*] + x_2^*}{2}. \quad (1.12)$$

A rule that restricts agent 1 to this lone choice is suboptimal; a provision of discretion that includes all choices between  $d^P$  and  $x_2^*$  can *only increase* the ex post aggregate payoff. This is indeed the case, and, as I demonstrate in the following corollary, the discretion provided to agent 1 always includes agent 2’s bliss point and the principal’s uninformed best guess  $d^P$ .

**Corollary 3.1.** *The discretion provided to agent 1 includes both agent 2’s bliss point  $x_2^*$  and the principal’s uninformed best guess  $d^P$ .*

Holmstrom (1984), in a setup with a principal and a single agent, demonstrates that optimal discretion always includes the principal’s uninformed best guess. Corollary 3.1, however, adds to this insight by demonstrating how the principal uses discretion with multiple agents. The principal utilizes agent 1’s private information and, simultaneously, shields agent 2 from a potentially excessive payoff loss. She does so by providing discretion-limits around agent 2’s bliss point and her own uninformed best guess. This is illustrated in Figure 1.3. The lower bound of the rule lies below agent 2’s bliss point and the principal’s uninformed best guess, while the upper bound resides above.

Until this point, my analysis has assumed that agent 1, who has private information, is the decision-maker. One might wonder if agent 2, who has no private information, would make a better decision-maker. Before I compare aggregate payoffs under different decision-making regimes, I provide the following corollary,

**Corollary 3.2.** *If agent 2 is the decision-maker, the optimal rule restricts him to the principal’s best uninformed guess,*

$$d^P = \underline{\alpha} = \bar{\alpha} = \frac{E[x_1^*] + x_2^*}{2}. \quad (1.13)$$



Corollary 3.2 implies that discretion is a function of the decision-maker's private information and not aggregate private information. I now consider the aggregate payoff differential under the two different decision-making regimes. Denote  $\Lambda_i$  with  $i \in \{1, 2\}$  to indicate which agent is the decision-maker. The difference in the aggregate payoff when agent 1 makes the decision rather than agent 2 is given by,

$$\Delta\Pi \equiv E[\Pi(d)|\Lambda_1] - E[\Pi(d)|\Lambda_2]. \quad (1.14)$$

Equation 1.14 measures the difference in payoffs under the two possible decision-making regimes. When  $\Delta\Pi$  is positive, the aggregate payoff is larger when agent 1 makes the decision. If  $\Delta\Pi$  is always positive, the aggregate payoff is maximized with agent 1 as the decision-maker. The next proposition shows that this is indeed always the case.

**Proposition 4.** *The difference in the expected aggregate payoff when agent 1 has the decision rights, rather than agent 2, is positive,*

$$\Delta\Pi \geq 0. \quad (1.15)$$

Proposition 4 supplies an important result in setting optimal rules: the principal maximizes the aggregate payoff by delegating decision rights to the agent with private information.

I conclude this section with two remarks. The analysis in this section considers that only  $x_1^*$  is private information. It is certainly possible that both agents' bliss points are private information and the following remarks demonstrate the optimal rules when that is the case.

**Remark 1.** *The optimal rule when both bliss points are private information and uncorrelated is,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{E[x_1^*|x_1^* \leq \underline{\alpha}] + E[x_2^*]}{2}, \frac{E[x_1^*|x_1^* \geq \bar{\alpha}] + E[x_2^*]}{2} \right\}. \quad (1.16)$$

**Remark 2.** *The optimal rule when both bliss points are private information and correlated is,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{E[x_1^*|x_1^* \leq \underline{\alpha}] + E[x_2^*|x_1^* \leq \underline{\alpha}]}{2}, \frac{E[x_1^*|x_1^* \geq \bar{\alpha}] + E[x_2^*|x_1^* \geq \bar{\alpha}]}{2} \right\}. \quad (1.17)$$

The analysis for Remarks 1 and 2 is found in Appendix B. Furthermore, Remarks 1 and 2 demonstrate that the results of this section are qualitatively unchanged when the principal cannot anticipate either bliss point. As such, there is little sacrificed in assuming that only one agent's bliss point is private information.

### 3.3 Uniform Distribution Example

Proposition 3 provides a solution for optimal rules under a general distribution. Here I include an analytic example to provide closed-form solutions and additional intuition. Suppose the principal believes that  $x_1^*$  is distributed uniformly over  $[0, D]$ . The principal sets a rule according to Proposition 3.

**Example 1.** *The optimal rule when the principal's beliefs are uniformly distributed is*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{2x_2^*}{3}, \frac{2x_2^* + D}{3} \right\}. \quad (1.18)$$

*Furthermore, the expected aggregate payoff differential,  $\Delta\Pi$ , is strictly positive.*

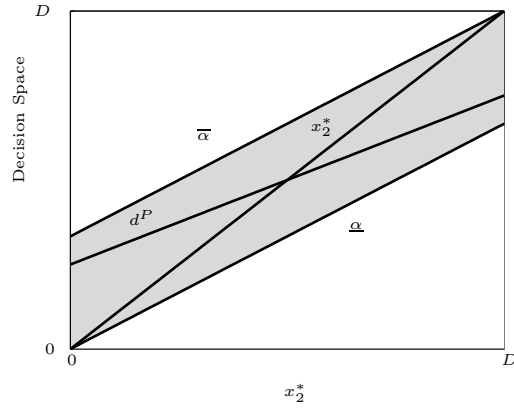
The results of Example 1 are best appreciated in a graphical representation. Figure 1.4(a) illustrates  $\{\underline{\alpha}, \bar{\alpha}\}$  as a function of  $x_2^*$ . The shaded band that follows  $x_2^*$  is the space of permissible choices. As depicted, agent 1 is provided discretion around agent 2's bliss point and the principal's best uninformed guess. The extent of discretion,  $\bar{\alpha} - \underline{\alpha}$ , is constant for all values of  $x_2^*$ . The placement of the discretion, however, depends on agent 2's bliss point. Low values of  $x_2^*$  result in providing the majority of the discretion in choices above  $x_2^*$ . The converse is true for high values of  $x_2^*$ . Only at the uniform distribution's median,  $D/2$ , is there an equal amount of discretion above and below  $x_2^*$ .

Figure 1.4(b) illustrates the expected aggregate payoff as a function of  $x_2^*$  under the two possible decision-making regimes. The curve labeled  $\Lambda_2$  is strictly less than the one labeled  $\Lambda_1$ , which emphasizes the result of Proposition 4. The expected aggregate payoff is strictly larger when agent 1 chooses  $d$ .

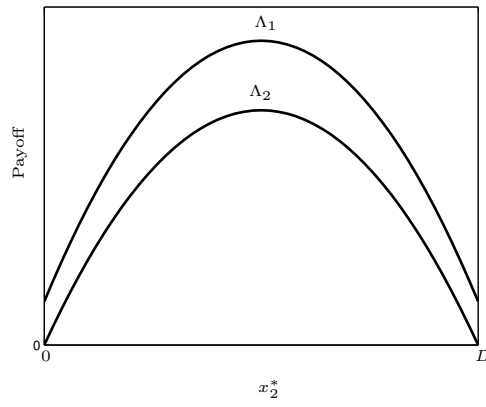
## 4 Costly Bargaining

Earlier, in Section 3, I introduced delegation as a mechanism to bypass the contracting cost. Specifically, I assumed that it was prohibitively costly for the agents to haggle or contract directly. Suppose now that the cost is not too large, i.e.,  $c \leq \hat{c}$ . In this section, I extend the base model and explore the relationship between this cost and discretion-limits.

Consider again the model setup from Section 3.1, where agent 2's bliss point is public information and agent 1's is private. The principal believes that  $x_1^*$  is distributed according to a probability density function,  $f(x)$ . For analytic tractability I assume



(a)  $\{\underline{\alpha}, \bar{\alpha}\}$  as a function of  $x_2^*$ .



(b) Expected aggregate payoff.

Figure 1.4: Optimal rules with a uniform distribution.

that the distribution is uniform over  $[0, D]$  and that agent 2 shares these beliefs. Again, in the spirit of Proposition 4, assume that agent 1 is the decision-maker.

Agent 1's private information is two fold: the agent knows his own bliss point *and* the aggregate payoff maximizing choice  $d^*$ . If  $c > \hat{c}$ , the agent is only concerned with choosing the choice  $d$  in  $[\underline{\alpha}, \bar{\alpha}]$  closest to  $x_1^*$ . This focus on his own bliss point occurs because, even with full bargaining power  $\theta_1 = 1$ , complete contracting with agent 2 is too costly. If, however,  $c \leq \hat{c}$  this is not necessarily the case. Agent 1's share of the aggregate payoff that results from directly bargaining with agent 2 may sufficiently exceed  $c$ .

Suppose agent 1 elects to pay  $c$  and engage agent 2 directly through bargaining. According to Proposition 1, the agents choose  $d^* = \frac{x_1^* + x_2^*}{2}$  to maximize the aggregate payoff. If, however, he does not pay  $c$ , his payoff depends on the discretion provided by the principal. Agent 1's payoffs under both bargaining and the principal's mechanism are given by the following lemma,

**Lemma 2.** *Agent 1's bargaining payoff is given by*

$$\Pi_1^{NB} = \theta_1 \left( \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - \delta_1 - \delta_2 \right) + \delta_1, \quad (1.19)$$

where  $\delta_1$  and  $\delta_2$  correspond to agent 1 and agent 2's disagreement points respectively. The disagreement points in this setup are each agent's payoff under the principal's mechanism.

Agent 1's payoff under the principal's mechanism is given by,

$$\Pi_1^R = \begin{cases} \pi_1 - (\underline{\alpha} - x_1^*)^2 & \text{for } x_1^* < \underline{\alpha} \\ \pi_1 & \text{for } x_1^* \in [\underline{\alpha}, \bar{\alpha}] \\ \pi_1 - (\bar{\alpha} - x_1^*)^2 & \text{for } x_1^* > \bar{\alpha}. \end{cases} \quad (1.20)$$

The cost  $c$  is a dead weight loss for agent 1 since he does not recoup any of it in his bargaining payoff. Despite this, paying the cost  $c$  ensures that the agents make the first-best choice and maximize the aggregate payoff. If agent 1's Nash bargaining weight is sufficiently large or the principal's rule restricts him to undesirable choices, it may be efficient for agent 1 to incur the cost. The agent compares his payoff under the bargaining outcome to the payoff under the principal's mechanism. If the net gain

in payoff is greater than  $c$ , the agent elects to incur the cost. Define  $\Delta G$  to be the difference between the net gain and the cost  $c$ ,

$$\Delta G \equiv (\Pi_1^{NB} - \Pi_1^R) - c. \quad (1.21)$$

The agent's optimal decision is determined by the sign on  $\Delta G$ . If it is positive, the agent pays  $c$  and contracts directly with agent 2. Conversely, if  $\Delta G$  is negative, the agent adheres to the discretion provided to him by the principal.

The principal is rational and anticipates that agent 1 may pay  $c$ . She takes this into account when establishing the optimal rule. The following proposition establishes and characterizes the optimal discretion provided to agent 1.

**Proposition 5.** *The optimal rule is given by,*

$$\underline{\alpha} = \begin{cases} x_2^* - \sqrt{\frac{2c}{\theta_1}} & \text{if } c \leq \theta_1 \left( \frac{x_2^{*2}}{18} \right) \\ \frac{2x_2^*}{3} & \text{otherwise} \end{cases} \quad (1.22)$$

$$\bar{\alpha} = \begin{cases} x_2^* + \sqrt{\frac{2c}{\theta_1}} & \text{if } c \leq \theta_1 \left( \frac{(D-x_2^*)^2}{18} \right) \\ \frac{2x_2^*+D}{3} & \text{otherwise} \end{cases}. \quad (1.23)$$

Furthermore, the decision-maker's discretion,  $\bar{\alpha} - \underline{\alpha}$ ,

1. is decreasing in  $\theta_1$
2. is increasing in  $c$

According to Proposition 5, a sufficiently small contracting cost creates regions where it is efficient for the agents to bargain. The principal accounts for this by making the rule's discretion-limits a function of the parameters  $\theta_1$  and  $c$ .

Agent 1's discretion decreases in  $\theta_1$ . Intuitively, the more bargaining power the agent has, as measured by  $\theta_1$ , the more he has to gain by paying  $c$ . Therefore, the principal does not accommodate the agent's private information with as much discretion. Instead, the agent's bargaining power substitutes for discretion and makes paying  $c$  to achieve the first-best outcome more attractive. The result provides an empirical prediction: agents with high bargaining power, for example an agent operating a downstream firm division, is provided less discretion than an agent with little bargaining power, e.g., one that operates an upstream division.

Additionally, the proposition states that discretion moves in the same direction as  $c$ . This provides another empirical prediction: bilateral relationships that involve high contracting costs should be characterized by larger provisions of discretion.

Also, as  $c$  increases to prohibitively high levels, the discretion approaches that which was prescribed in Proposition 3. More interestingly, however, is what happens to discretion as the cost goes to zero.

**Corollary 5.1.** *If  $c = 0$  the decision-maker is given no discretion,*

$$\underline{\alpha} = \bar{\alpha} = x_2^*.$$

The principal's mechanism serves as a substitute for costly bargaining between agents. As demonstrated in Section 3.1, the mechanism provides second-best outcomes if there is private information. If bargaining is costless, the principal does not settle for second-best. Corollary 5.1 confirms this intuition, that is, the principal eliminates the agency cost by limiting agent 1 to a single choice of  $x_2^*$ . If agent 1's bliss point  $x_1^* \neq x_2^*$ , the agent is compelled to directly contract with agent 2.

**Corollary 5.2.** *It is never optimal for the non-decision-maker to initiate bargaining.*

The intuition of Corollary 5.2 follows directly from the agents' outside options. If the agents engage in bargaining each agent must be at least as well off as they are under the principal's mechanism. If not, bargaining breaks down and the agents return to the principal's mechanism. Essentially, the disagreement points in Nash bargaining bake-in individual rationality constraints. This implies that the aggregate payoff must be larger under bargaining than under the principal's mechanism, otherwise there is no incentive to pay the contracting cost. Agent 2 shares the same beliefs and information as the principal and, consequently, believes that the aggregate payoff is maximized under the delegation mechanism. As such, he has no incentive to initiate bargaining with agent 1.

The results of Proposition 5 are illustrated in Figure 1.5 for a sufficiently small  $c$ . The horizontal axis corresponds to agent 1's bliss point while the vertical axis depicts the agent's payoff. The provided discretion is the region between  $\underline{\alpha}$  and  $\bar{\alpha}$ . The points  $\underline{t}$  and  $\bar{t}$  represent the points where agent 1 is indifferent between bargaining with agent 2 and adhering to the principal's rule. As such, these indifference points become thresholds. The agent adheres to the principal's mechanism for all values of  $x_1^* \in [\underline{t}, \bar{t}]$ . If  $x_1^* < \underline{t}$  or  $x_1^* > \bar{t}$ , the agent pays  $c$  and bargains with agent 2. This is depicted as the shaded "Bargaining Space."

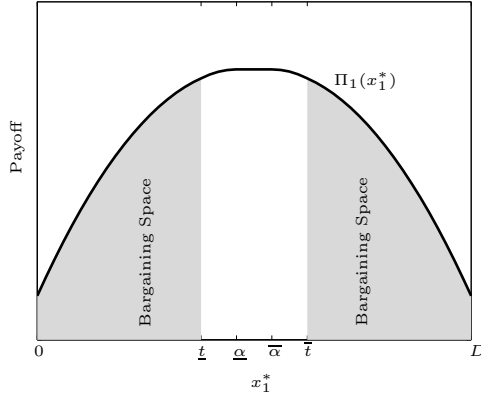


Figure 1.5: Agent 1's payoff with  $x_2^* = D/2$  and  $\theta_1 = 1/2$ .

## 5 Rules and the Payoff Functions' Concavity

The discretion-limits imposed by a rule stems from the quadratic components of each agent's payoff. If, instead, the functions were linear, should rules limit the choices? Suppose that both agents have linear payoff functions,

$$\Pi_i = \pi_i - (d - x_i^*). \quad (1.24)$$

A linear payoff alters the marginal benefit and marginal cost of limiting discretion. Unlike the payoff function in Equation 1.4, incremental deviations between  $d$  and  $x_i^*$  have a constant marginal impact. Again, assume agent 1 is the decision-maker and his preferences are private information. The following proposition addresses how much discretion the principal should grant the agent.

**Proposition 6.** *If the agents have linear payoff functions, the decision-making agent is granted unlimited discretion.*

The intuition of Proposition 6 is straightforward. Agent 1's payoff gain from choosing  $d = x_1^*$  is exactly offset by agent 2's loss. In fact, the aggregate payoff is unaffected so long as  $d$  falls anywhere between the agents' bliss points. If the principal limits agent 1's discretion, she potentially impedes efficiency. Therefore, it is weakly dominant to give the agent unconstrained freedom in choosing  $d$ .

The combination of Proposition 6 and the earlier analysis in Section 3.1 would appear to imply that discretion should *decrease* with the concavity of the payoff functions. As I will show shortly, the linear payoff result turns out to be a knife-edge case

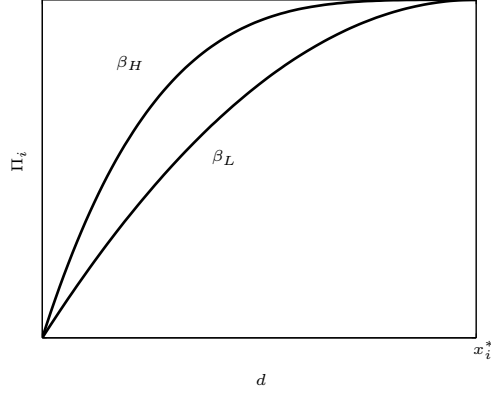


Figure 1.6: Payoff for agent  $i$  as a function of  $d$  for  $\beta = 2$  and  $\beta = 4$ .

and discretion actually *increases* with concavity. Let  $\vec{\beta}$  be the set of all positive even integers,  $\{2, 4, 6, \dots\}$  and consider the generalized payoff function,

$$\Pi_i = \pi_i - (d - x_i^*)^\beta \quad (1.25)$$

where  $\beta \in \vec{\beta}$ .<sup>11</sup> Figure 1.6 depicts two different payoff curves for an agent, where  $2 \leq \beta_L < \beta_H$ . The lower curve, labeled “ $\beta_L$ ”, represents the agent’s payoff with the less concave function, while the upper curve represents the agent’s payoff with an exponent  $\beta = \beta_H$ . Although it is difficult to compare payoff levels with the generalized function, Figure 1.6 emphasizes the effect  $\beta$  has on the concavity of the payoff function. As such,  $\beta$  reflects the sensitivity of agent payoffs to the choice  $d$ .

Similar to Equation 1.10, the expected aggregate payoff is given by,

$$\begin{aligned} E[\Pi|\{\underline{\alpha}, \bar{\alpha}\}] = & \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x)^\beta + (\underline{\alpha} - x_2^*)^\beta] f(x) dx \\ & - \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^\beta f(x) dx - \int_{\bar{\alpha}}^D [(\bar{\alpha} - x)^\beta + (\bar{\alpha} - x_2^*)^\beta] f(x) dx. \end{aligned} \quad (1.26)$$

Again, the principal maximizes the aggregate payoff by providing agent 1 limited discretion. The discretion allows the agent to use his private information to increase his own payoff and the aggregate payoff. The limits on discretion, however, ensure that the agent’s choice does not unduly decrease agent 2’s payoff. For analytic tractability I assume that the principal accurately believes that  $x_1^*$  is uniformly distributed over  $[0, D]$ . The following proposition characterizes optimal discretion as a function of  $\beta$ ,

<sup>11</sup>The choice of focusing on positive even integers is for analytic ease, but the analysis could be conducted with  $|d - x_i^*|^\beta$  for all  $\beta > 0$ .



**Proposition 7.** *The decision-maker's discretion is increasing in  $\beta$ . Furthermore, if  $\beta \geq 2$ , discretion is bounded by  $\frac{x_2^*}{2} \leq \underline{\alpha}$  and  $\bar{\alpha} \leq \frac{x_2^* + D}{2}$ .*

Proposition 7 echoes the prevailing theme in this paper and the existing delegation literature; discretion is valuable when there is private information and partial alignment of preferences. Furthermore, the proposition provides an additional insight; discretion increases with the value of private information. The principal's intent to maximize the aggregate payoff is limited by her beliefs. As  $\beta$  increases, obtaining the optimal ex post decision becomes increasingly important. The principal cannot become better informed of agent 1's private information, so instead she provides the agent additional discretion.

It is also important to note that the principal does not completely relinquish the reins as  $\beta$  increases. According to the proposition, discretion is always bounded. In fact, discretion is bounded by the naive rule outlined in Equation 1.9. Thus, as  $\beta$  goes to infinity, agent 1 is granted discretion over all possible first-best choices.

## 6 Strategic Uncertainty and Rules

In Section 3, I demonstrate that the principal increases the aggregate payoff by granting discretion if there is uncertainty. This leaves several questions to be answered: How does the quantity of discretion change with uncertainty? If it increases, do agents have the incentive to strategically add uncertainty ex ante?

In this section I consider these questions. For simplicity, I normalize the choice continuum by  $D$  to restrict the analysis to the unit continuum  $[0, 1]$ . Furthermore, I assume that agent 2's bliss point is known by the principal and, for analytic tractability, I assume it is equal to the midpoint of the continuum,  $x_2^* = \frac{1}{2}$ . I also assume that agent 1's bliss point is distributed according to the mean-preserving linear distribution I constructed. The mean-preserving linear distribution,  $f(x, \gamma)$ , outlined in Appendix C, is attractive for analytic work. The distribution's second moment is characterized by the parameter  $\gamma$  and, as its name indicates, the distribution is mean-preserving. The distribution is also appealing because its variance changes with  $\gamma$  despite the support remaining fixed, e.g., the uniform and triangular distribution are special cases of it. As such, closed-form analytic solutions can be characterized by the distribution's variance without introducing confounding factors, e.g., increasing the variance of the uniform distribution requires expanding its support. The mean and variance of the distribution

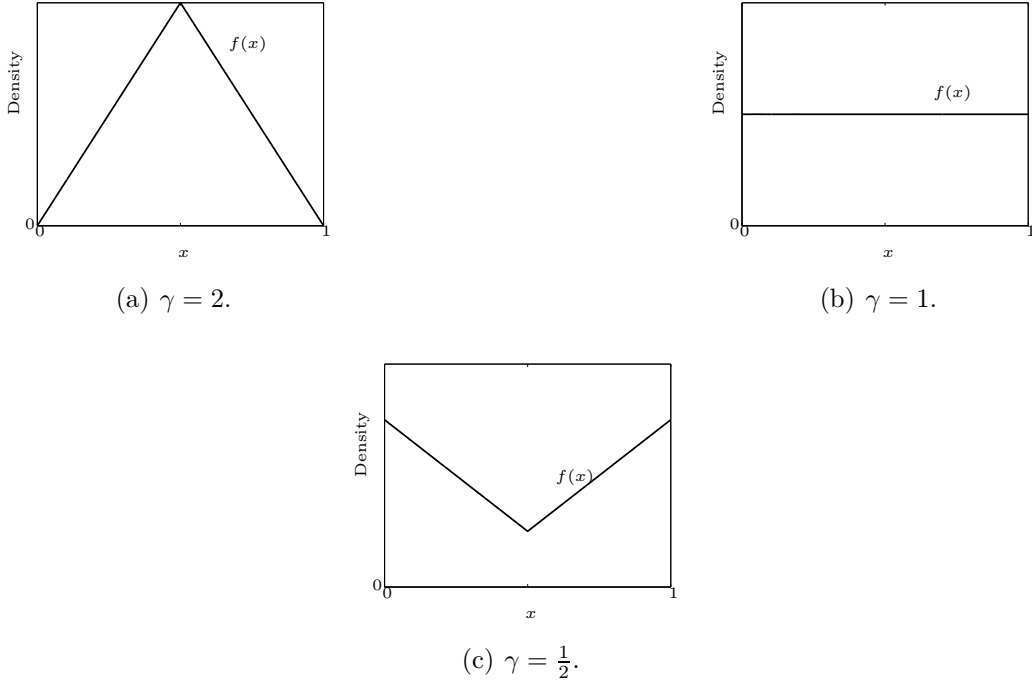


Figure 1.7: Mean-preserving linear distribution examples.

are given by,

$$E_f[x] = \frac{1}{2} \tag{1.27}$$

$$E_f[x^2] - E_f[x]^2 = \frac{3 - \gamma}{24} \tag{1.28}$$

with  $\gamma \in [0, 2]$ .

Figures 1.7(a), 1.7(b) and 1.7(c) depict the distribution with three different values of  $\gamma$ . In Appendix B, I consider a broader class of distributions and I demonstrate that the results of this section hold with mild restrictions on a distribution's hazard rate.

## 6.1 Optimal Rules Under the Mean-Preserving Linear Distribution

Consider the optimal rule outlined in Proposition 3. The generalized rule,  $\{\underline{\alpha}, \bar{\alpha}\}$ , is defined implicitly and provides a quantity of discretion equal to  $\bar{\alpha} - \underline{\alpha}$ . An explicit solution for the rule exists under the mean-preserving linear distribution and the following proposition characterizes how the rule's discretion changes with the distribution's variance.

**Proposition 8.** *The optimal rule grants a provision of discretion that is increasing in the variance of  $f(x, \gamma)$ .*

According to Proposition 8, agent 1's discretion increases with the principal's uncertainty regarding his bliss point. The intuition is straightforward; an increase in uncertainty, as measured by variance, increases the value of private information. The principal accommodates the private information by granting greater discretion when uncertainty is high.<sup>12</sup> The result of the proposition is exhibited in Figures 1.8(a) and 1.8(b). The horizontal axis in Figure 1.8(a) represents the the distribution's variance and the vertical axis represents the choice space. The optimal rule's lower and upper bounds diverge from each other as the variance increases and agent 1 is given a wider range to choose  $d$  from. Figure 1.8(b) depicts the quantity of discretion, measured by  $\bar{\alpha} - \underline{\alpha}$ , as the distribution's variance increases.

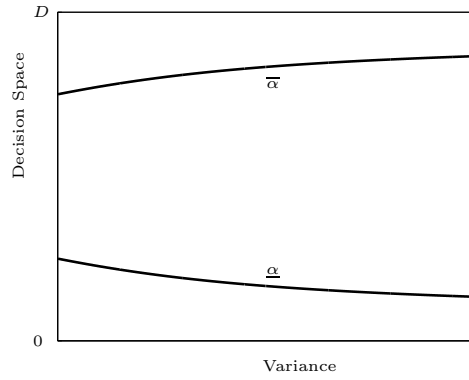
Proposition 8 suggests that agent 1 may benefit from higher uncertainty since he is granted greater discretion. Then again, because the principal establishes the optimal rule at  $t = 0$  and the agent does not realizes his bliss point until  $t = 1$ , an increase in the distribution's variance may adversely affect his ex post payoff. Higher variance increases the probability that the agent's bliss point realization falls in the distribution's tails. The following proposition demonstrates that the second effect dominates, i.e., the added discretion from increasing the distribution's variance does not compensate agent 1 sufficiently for his exposure to tail events.

**Proposition 9.** *Agent 1's ex ante payoff is decreasing in the variance of the distribution  $f(x, \gamma)$ .*

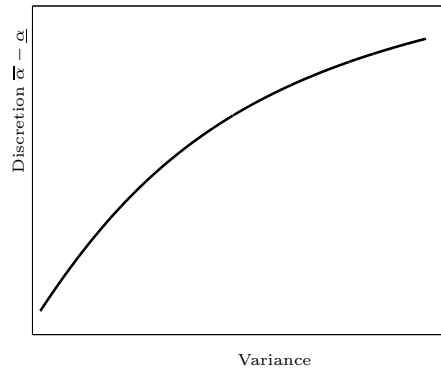
Proposition 9 yields additional insight regarding discretion. Discretion over a set of choices is valuable to agent 1, as opposed to his decision being dictated by the principal, because it lets him utilize his private information to increase his payoff. Despite this, the agent actually prefers that there is less uncertainty regarding his private information, i.e., the informational rents he extracts via discretion are dominated by the payoff he receives when the principal enjoys better information.

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<sup>12</sup>The setup of Holmstrom (1984) considers a setup with a principal and a single agent. In his setup, the principal believes that the agent's private information is normally distributed with variance  $\sigma^2$ . Holmstrom shows that  $\bar{\alpha} - \underline{\alpha} \rightarrow \infty$  as  $\sigma^2 \rightarrow \infty$ . My analysis adds a new dimension to this result by considering a distribution with a finite support.



(a) Variance versus the upper and lower bounds of the optimal rule.



(b) Variance versus the quantity of discretion provided as measured by  $\bar{\alpha} - \underline{\alpha}$ .

Figure 1.8: Optimal discretion and variance.

## 7 Application to Delegated Portfolio Management

Many important issues in industrial organization, political economy, and corporate finance relate to my results. The issue I choose to focus on involves delegated portfolio management. Specifically, I consider an application of my model to a decentralized investment firm.

Consider an investment firm that generates profits by actively managing investor capital. The firm relies on raising and maintaining outside capital via marketing & client services and establishing services to augment investment management, e.g. securing lines of credit. The firm is segmented into two profit divisions: a trading division ( $T$ ) that invests capital and a reputation division ( $R$ ) that is client-facing. Each division is run by a self-interested manager that seeks to maximize the expected profit of his own division.<sup>13</sup> The two managers represent the agents in my generalized model, i.e., agent 1 is manager  $T$  and agent 2 is manager  $R$ .

The common decision that links the two managers is what level of risk to take with the firm's current investment portfolio. Let  $\sigma$  represent this choice and assume that  $\sigma$  is restricted to the continuum  $[0, \Sigma]$ , where  $\Sigma$  is the maximum level of risk allowed by regulation or investor mandates. The trading division profit is largely influenced by the firm's contemporaneous risk choice, i.e., the returns from a particular strategy are generally realized in the short-run. Reputation profit, however, relies on outsiders' expectations regarding the firm's long-run solvency. Consequently, the two divisions respond differently to the firm's portfolio risk, *ceteris paribus*: trading profit increases with  $\sigma$  and reputation profit declines, i.e., an increase in  $\sigma$  increases the probability of insolvency.

In the absence of any spillover effects between the two divisions, it is simple to see that the trading profit is maximized when the firm's investment portfolio exhibits the maximal level of risk and, conversely, the reputation profit is highest when the firm bears the minimal level. In my setup, however, the two divisions are complementary: greater profits via trading enhance the firm's reputation and an enhanced reputation leads to more investor capital and cheaper leverage. Consequently, each manager has

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<sup>13</sup>My analysis handles the issue of compensation by assuming that managers care only about division performance rather than overall firm performance. I do not model the underlying moral hazard problem that leads to this wage contract, but instead appeal to its empirical foundation, e.g., Bushman, Indjejikian, and Smith (1995), and theoretical grounds, e.g., Bernardo, Cai, and Luo (2001).

a unique, preferred  $\sigma$  and I denote these bliss points as  $x_i^* \in [0, \Sigma]$  with  $i \in \{T, R\}$ .

I adopt the explicit form of the divisions' profit functions from the base model,

$$\Pi_i(\sigma) = \pi_i - (\sigma - x_i^*)^2, \quad (1.29)$$

where  $\pi_i > 0$  for  $i \in \{T, R\}$ . To understand the profit functions consider the trading division as an example. From the perspective of manager  $T$ , a risk choice of  $\sigma < x_T^*$  is suboptimal because it is too conservative in pursuing returns. Similarly, a choice  $\sigma > x_T^*$  is also suboptimal because it is too aggressive and decreases the complementary benefits provided by reputation activities, i.e., it decreases the amount of investor capital or increases the cost of leverage. At  $\sigma = x_T^*$ , the marginal benefit of increased portfolio risk equals the marginal cost of decreased firm reputation.<sup>14</sup> The two divisions are overseen by a chief executive officer (CEO), who is the principal in this application, and her objective is to maximize firm profit, which is the sum of the two divisions' profits,

$$\mathbf{\Pi}(\sigma) = \pi_T + \pi_R - (\sigma - x_T^*)^2 - (\sigma - x_R^*)^2. \quad (1.30)$$

Manager  $T$  has private, specialized knowledge regarding the firm's optimal choice of  $\sigma$ , but it is costly for him to transfer it to the CEO and manager  $R$ . The cost is analogous to the contracting cost  $c$  in my generalized model. The private, specialized knowledge possessed by manager  $T$  is his division's bliss point  $x_T^*$ . While the CEO does not know  $x_T^*$ , she correctly believes that it distributed according to the probability distribution  $f(x, \gamma)$ , where  $\gamma$  characterizes the variance of the distribution.

The firm must make a decision regarding its aggregate portfolio risk and the CEO determines it one of three ways:

1. via her beliefs, i.e., the CEO's uninformed best guess,
2. via costly manager-to-manager or manager-to-CEO negotiations,
3. via delegation of the decision rights to manager  $T$ .

---

<sup>14</sup>Alternatively, one could consider a setup where there are no complementary spillovers between the profit activities. Instead, the difference between  $x_T^*$  and  $x_R^*$  serves as a proxy for the extent to which stakeholders', e.g., managers, payoffs are tied to a particular activity's profit. The congruency of the stakeholders is measured by taking the difference between  $x_T^*$  and  $x_R^*$ . If the two bliss points are close, the stakeholders share similar preferences, which would follow from an incentive scheme based on overall firm performance. Conversely, values that differ greatly may reflect an incentive scheme based on division performance.

All three options are associated with a cost: the first option introduces an information cost as it neglects manager  $T$ 's private information; the second option incurs the explicit contracting cost  $c$ ; and the third option introduces an agency cost as manager  $T$ 's objective is to maximize his division's profit rather than firm profit. According to Propositions 3 and 4, delegation of the decision rights is optimal. The CEO gives manager  $T$  the continuum  $[\underline{\alpha}, \bar{\alpha}]$  to choose  $\sigma$  from. Furthermore, the CEO chooses the continuum so that it includes her uninformed best guess and  $x_R^*$ .

Manager  $T$  may find his division's bliss point in the continuum of allowed choices, and, if so, he chooses it. If, however, the bliss point is outside the continuum, the manager chooses either  $\underline{\alpha}$  or  $\bar{\alpha}$  and his division's profit incurs a quadratic loss. According to Proposition 5, the quadratic loss may be sufficiently large that paying  $c$  to transfer his specialized knowledge increases his division's profit. This occurs when his division has significant bargaining power and it extracts a large portion of the firm's profit in negotiations, i.e., the trading division is willing to relinquish private information if the division commands the firm's compensation scheme.

Proposition 5 also demonstrates that the CEO accounts for  $c$  in the discretion she provides. In fact, as  $c$  goes to zero, manager  $T$  is able to transfer his specialized knowledge freely and the CEO responds by restricting the manager to a single choice. The choice corresponds to manager  $R$ 's bliss point, which compels manager  $T$  to fully reveal his information.

Discretion is a good to manager  $T$ : additional discretion enables the manager to increase his division's profit. According to Proposition 8, the discretion provided to manager  $T$  increases with the CEO's uncertainty. The proposition would seem to suggest that the manager is incentivized to add strategic uncertainty to the CEO's beliefs. The intuition is further enforced by recent work that suggests investment managers add strategic complexity to financial products to gain market power, e.g., Carlin (2009). Proposition 9, however, shows that this is not the case, i.e., optimal discretion is immune to strategic manipulation by managers. In fact, managers prefer an informed CEO ex ante. The result suggests that optimal discretion is a robust mechanism in delegated portfolio management.

## 8 Concluding Remarks

In this paper I explored the economics of discretion-limits. I motivated the delegation mechanism as an inexpensive alternative to complete contracting, and, as such, provided a theoretical bridge between the two. My analysis, however, could be extended to a more general framework where I consider a broader class of decision-making mechanisms. This is a subject of future research.

One criticism of my setup is that I do not allow the principal to price the individual choices within the discretion-limits. Consequently, the decision-making agent's optimal choice maximizes his payoff. As I demonstrate in Appendix B, a fully-separating mechanism can be achieved if the principal prices each choice. Specifically, the price transfers the aggregate payoff to the decision-maker and he makes the first-best decision. This result is uninteresting as it simplifies the setup to a single agent optimizing over two payoff functions.

My findings are particularly timely as the debate regarding regulation-overhaul in financial markets escalates. Recent proposals to limit the pay of bankers and traders jeopardizes the effectiveness of compensation to remedy agency conflicts. As such, one should view discretion-limits as an additional tool in mitigating conflicts of interest, rather than as a panacea. Indeed, the optimal mechanism for solving a particular problem may be a scheme consisting of compensation, monitoring and discretion-limits. As such, it is important to understand both the benefits and costs associated with discretion.



## A Appendix

**Proof of Proposition 1:** The agents split the aggregate payoff according to the Nash bargaining weights  $\theta_1$  and  $\theta_2$ . Therefore, it is efficient for the agents to select the choice  $d$  that maximizes the aggregate payoff. The choice is found by taking the first-order condition of Equation 1.5 with respect to  $d$ ,

$$\begin{aligned} 0 &= \frac{\partial}{\partial d} \mathbf{\Pi}(d) \\ &= (d - x_2^*) + (d - x_1^*) \\ d^* &= \frac{x_1^* + x_2^*}{2}. \end{aligned} \tag{A1}$$

A substitution of  $d^*$  into Equation 1.5 simplifies the expression to,

$$\mathbf{\Pi}(d^*) = \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - c. \tag{A2}$$

Is is efficient for the agents to bargain if  $\mathbf{\Pi} \geq 0$ , which is true for all  $c$  less than  $\hat{c}$ ,

$$\hat{c} \equiv \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2}. \tag{A3}$$

■

**Proof of Proposition 2:** Agent 1 chooses  $d = x_1^*$  if  $x_1^* \in [\underline{\alpha}, \bar{\alpha}]$ ,  $d = \underline{\alpha}$  if  $x_1^* < \underline{\alpha}$ , or  $d = \bar{\alpha}$  if  $x_1^* > \bar{\alpha}$ . For any possible rule, the principal knows agent 1's unique best response. Therefore, she restricts the action space so that the decision is always first-best. She sets  $\underline{\alpha} = \bar{\alpha} = \alpha^*$ , since permitting any other choice is dominated.

The aggregate payoff function is continuous, differentiable and concave in  $d$ . Therefore, first-order conditions are necessary and sufficient for finding a value that maximizes it,

$$\begin{aligned} 0 &= \frac{\partial}{\partial d} \mathbf{\Pi}(d) \\ &= \frac{\partial}{\partial d} ((d - x_1^*)^2 + (d - x_2^*)^2) \\ &= (d - x_1^*) + (d - x_2^*) \\ \alpha^* &\equiv \frac{x_1^* + x_2^*}{2}. \end{aligned}$$

■

**Proof of Lemma 1:** For a given rule  $\{\underline{\alpha}, \bar{\alpha}\}$ , it is a dominant strategy for agent 1 to choose  $d = \underline{\alpha}$  if  $x_1^* < \underline{\alpha}$ ,  $d = x_1^*$  if  $x_1^* \in [\underline{\alpha}, \bar{\alpha}]$ , and  $d = \bar{\alpha}$  if  $x_1^* > \bar{\alpha}$ . Therefore, since there is no contracting cost with the principal's mechanism, the expected aggregate payoff is

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] &= E[\Pi_1(d)|\{\underline{\alpha}, \bar{\alpha}\}] + E[\Pi_2(d)|\{\underline{\alpha}, \bar{\alpha}\}] \\
&= \pi_1 + \pi_2 - \left[ \int_0^{\underline{\alpha}} (\underline{\alpha} - x)^2 f(x) dx + \int_{\underline{\alpha}}^{\bar{\alpha}} 0 f(x) dx \right. \\
&\quad \left. + \int_{\bar{\alpha}}^D (\bar{\alpha} - x)^2 f(x) dx \right] - \left[ \int_0^{\underline{\alpha}} (\underline{\alpha} - x_2^*)^2 f(x) dx \right. \\
&\quad \left. + \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^2 f(x) dx + \int_{\bar{\alpha}}^D (\bar{\alpha} - x_2^*)^2 f(x) dx \right].
\end{aligned}$$

A rearrangement yields,

$$\begin{aligned}
&= \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2] f(x) dx \\
&\quad - \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^2 f(x) dx - \int_{\bar{\alpha}}^D [(\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2] f(x) dx. \quad (\text{A4})
\end{aligned}$$

■

**Proof of Proposition 3:** I begin by taking the first-order condition of Equation 1.10 with respect to  $\underline{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \underline{\alpha}} E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\underline{\alpha} - x_2^*)^2 f(\underline{\alpha}) + \int_0^{\underline{\alpha}} [2(\underline{\alpha} - x) + 2(\underline{\alpha} - x_2^*)] f(x) dx - (\underline{\alpha} - x_2^*)^2 f(\underline{\alpha}) \\
&= (2\underline{\alpha} - x_2^*) - \frac{\int_0^{\underline{\alpha}} x f(x) dx}{F(\underline{\alpha})} \\
\underline{\alpha} &= \frac{x_2^* + E[x_1^* | x_1^* \leq \underline{\alpha}]}{2}. \quad (\text{A5})
\end{aligned}$$

Now I take first-order condition with respect to  $\bar{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \bar{\alpha}} E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\bar{\alpha} - x_2^*)^2 f(\bar{\alpha}) - (\bar{\alpha} - x_2^*)^2 f(\bar{\alpha}) + \int_{\bar{\alpha}}^D [2(\bar{\alpha} - x) + 2(\bar{\alpha} - x_2^*)] f(x) dx \\
&= (2\bar{\alpha} - x_2^*) - \frac{\int_{\bar{\alpha}}^D x f(x) dx}{(1 - F(\bar{\alpha}))}, \\
\bar{\alpha} &= \frac{x_2^* + E[x_1^* | x_1^* \geq \bar{\alpha}]}{2}. \quad (\text{A6})
\end{aligned}$$

■

**Proof of Corollary 3.1:** Consider the optimal rule outlined in Proposition 3. The lower and upper bound on the continuum of permitted choices are given in Equation 1.11. The lower bound is smaller than agent 2's bliss point,  $\underline{\alpha} \leq x_2^*$ ,

$$\begin{aligned} 0 &= \underline{\alpha} - \frac{x_2^* + E[x_1^* | x_1^* \leq \underline{\alpha}]}{2} \\ &= 2\underline{\alpha} - (x_2^* + E[x_1^* | x_1^* \leq \underline{\alpha}]) \\ &\geq 2\underline{\alpha} - (x_2^* + \underline{\alpha}) \\ &= \underline{\alpha} - x_2^* \end{aligned} \tag{A7}$$

$$\rightarrow x_2^* \geq \underline{\alpha}. \tag{A8}$$

Similarly, the upper bound is greater than agent 2's bliss point,  $\bar{\alpha} \geq x_2^*$ ,

$$\begin{aligned} 0 &= \bar{\alpha} - \frac{x_2^* + E[x_1^* | x_1^* \geq \bar{\alpha}]}{2} \\ &= 2\bar{\alpha} - (x_2^* + E[x_1^* | x_1^* \geq \bar{\alpha}]) \\ &\leq 2\bar{\alpha} - (x_2^* + \bar{\alpha}) \\ &= \bar{\alpha} - x_2^* \end{aligned} \tag{A9}$$

$$\rightarrow \bar{\alpha} \geq x_2^*. \tag{A10}$$

It is obvious that  $d^P = \frac{E[x_1^*] + x_2^*}{2}$  lies within the continuum.

■

**Proof of Corollary 3.2:** Agent 2 does not have private information. When he is delegated the decision rights, the principal knows he will choose  $d = x_2^*$  if  $x_2^* \in [\underline{\alpha}, \bar{\alpha}]$ ,  $d = \underline{\alpha}$  if  $x_2^* < \underline{\alpha}$ , or  $d = \bar{\alpha}$  if  $x_2^* > \bar{\alpha}$ . For any possible rule, the principal knows the agent's unique best response. Therefore, the principal restricts the action space so that the agent's decision maximizes the expected aggregate payoff. She sets  $\underline{\alpha} = \bar{\alpha}$ , since permitting any other choice is dominated. As a result, the expected aggregate payoff takes the form,

$$E[\mathbf{\Pi}(d) | \{d, d\}] = \pi_1 + \pi_2 - \int_0^D (d - x)^2 f(x) dx - (d - x_2^*)^2. \tag{A11}$$

The first-order condition of Equation A11 with respect to  $d$  is,

$$\begin{aligned}
0 &= \frac{\partial}{\partial d} E[\mathbf{\Pi}(d)|\{d, d\}] \\
&= \int_0^D 2(d-x)f(x) dx + 2(d-x_2^*) \\
&= 2d - x_2^* - E[x_1^*] \\
\alpha^* &\equiv \frac{E[x_1^*] + x_2^*}{2}.
\end{aligned} \tag{A12}$$

■

**Proof of Proposition 4:** For any  $\underline{\alpha} \neq \bar{\alpha}$  it must be the case that  $\Delta\mathbf{\Pi} = E[\mathbf{\Pi}(d)|\Lambda_1] - E[\mathbf{\Pi}(d)|\Lambda_2] \geq 0$ .

*Suppose Not:* According to Corollary 3.2, when agent 2 is the decision-maker, the principal limits his discretion to a single choice,  $\alpha^* \in [0, D]$ . Therefore,  $E[\mathbf{\Pi}(d)|\Lambda_1] < E[\mathbf{\Pi}(d)|\Lambda_2]$  implies that  $\{\underline{\alpha}, \bar{\alpha}\}$  is not a solution to the principal's problem in Equation 1.7, since choosing  $\underline{\alpha} = \bar{\alpha} = \alpha^*$  dominates. This directly contradicts Proposition 3.

■

**Proof of Example 1:** I begin by directly applying Equations A5 and A6 to obtain the optimal rule when agent 1 is the decision-maker. The lower bound of the rule is,

$$\begin{aligned}
\underline{\alpha} &= \frac{x_2^* + E[x_1^*|x_1^* \leq \underline{\alpha}]}{2} \\
&= \frac{x_2^* + \frac{\int_0^{\underline{\alpha}} \tilde{x}}{\underline{\alpha}}}{2} \\
&= \frac{x_2^* + \frac{\underline{\alpha}}{2}}{2} \\
\underline{\alpha} &= \frac{2x_2^*}{3}.
\end{aligned} \tag{A13}$$

Similarly, the upper bound is,

$$\begin{aligned}
\bar{\alpha} &= \frac{x_2^* + E[x_1^* | x_1^* \geq \bar{\alpha}]}{2} \\
&= \frac{x_2^* + \frac{\int_{\bar{\alpha}}^D \tilde{x}}{D - \bar{\alpha}}}{2} \\
&= \frac{x_2^* + \frac{D + \bar{\alpha}}{2}}{2} \\
\bar{\alpha} &= \frac{2x_2^* + D}{3}.
\end{aligned} \tag{A14}$$

The single choice when agent 2 is the decision-maker is given by,

$$\begin{aligned}
\alpha^* &= \frac{E[x_1^*] + x_2^*}{2} \\
&= \frac{\int_0^D x \, dx + x_2^*}{2} \\
&= \frac{\frac{D}{2} + x_2^*}{2}
\end{aligned} \tag{A15}$$

The discretion solutions under both regimes permit a comparison of aggregate payoffs,

$$\begin{aligned}
\Delta \Pi &= \int_0^D ((\alpha^* - x)^2 + (\alpha^* - x_2^*)^2) f(x) \, dx \\
&\quad - \left[ \int_0^{\underline{\alpha}} ((\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2) f(x) \, dx + \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^2 f(x) \, dx \right. \\
&\quad \left. + \int_{\bar{\alpha}}^D ((\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2) f(x) \, dx \right] \\
&> 0.
\end{aligned} \tag{A16}$$

■

**Proof of Lemma 2:** I begin with the agents' payoffs under the principal's rule. The

payoffs are derived from Proposition 3,

$$\Pi_1^R = \begin{cases} \pi_1 - (\underline{\alpha} - x_1^*)^2 & \text{for } x_1^* < \underline{\alpha} \\ \pi_1 & \text{for } x_1^* \in [\underline{\alpha}, \bar{\alpha}] \\ \pi_1 - (\bar{\alpha} - x_1^*)^2 & \text{for } x_1^* > \bar{\alpha} \end{cases} \quad (\text{A17})$$

$$\Pi_2^R = \begin{cases} \pi_2 - (\underline{\alpha} - x_2^*)^2 & \text{for } x_1^* < \underline{\alpha} \\ \pi_2 - (x_1^* - x_2^*)^2 & \text{for } x_1^* \in [\underline{\alpha}, \bar{\alpha}] \\ \pi_2 - (\bar{\alpha} - x_2^*)^2 & \text{for } x_1^* > \bar{\alpha}. \end{cases} \quad (\text{A18})$$

I now proceed to agent 1's payoff in Nash bargaining. Once agent 1 pays the cost  $c$  to bargain with agent 2, the cost is sunk and inconsequential to the bargaining process. The agents choose  $d^* = \frac{x_1^* + x_2^*}{2}$  to maximize the aggregate payoff which is,

$$\mathbf{\Pi}(d^*) = \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2}. \quad (\text{A19})$$

In addition to bargaining power, the process is governed by each agent's ability to walk away and return the principal's mechanism. As such, the outcomes under the principal's rule become the disagreement points for the agents. Denote the set of disagreement points as  $\{\delta_1, \delta_2\}$ . The asymmetric Nash bargaining solutions for the agents are given by,

$$\Pi_1^{NB} = \theta_1 \left( \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - \delta_1 - \delta_2 \right) + \delta_1 \quad (\text{A20})$$

$$\Pi_2^{NB} = (1 - \theta_1) \left( \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - \delta_1 - \delta_2 \right) + \delta_2. \quad (\text{A21})$$

■

**Proof of Proposition 5:** I begin by solving for  $\Delta G$  given the payoffs in Lemma 2,

$$\begin{aligned}
\Delta G &= (\Pi_1^{NB} - \Pi_1^R) - c \\
&= \left( \theta_1 \left( \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - \delta_1 - \delta_2 \right) + \delta_1 - \underbrace{\Pi_1^R}_{\delta_1} \right) - c \\
&= \theta_1 \left( \pi_1 + \pi_2 - \frac{(x_1^* - x_2^*)^2}{2} - \delta_1 - \delta_2 \right) - c \\
&= \begin{cases} \theta_1 \left( -\frac{(x_1^* - x_2^*)^2}{2} + (\underline{\alpha} - x_1^*)^2 + (\underline{\alpha} - x_2^*)^2 \right) - c & \text{for } x_1^* < \underline{\alpha} \\ \theta_1 \left( \frac{(x_1^* - x_2^*)^2}{2} \right) - c & \text{for } x_1^* \in [\underline{\alpha}, \bar{\alpha}] \\ \theta_1 \left( -\frac{(x_1^* - x_2^*)^2}{2} + (\bar{\alpha} - x_1^*)^2 + (\bar{\alpha} - x_2^*)^2 \right) - c & \text{for } x_1^* > \bar{\alpha}. \end{cases} \quad (\text{A22})
\end{aligned}$$

If  $\Delta G < 0$ , agent 1 follows the principal's mechanism, otherwise he will pay  $c$  and engage agent 2 in bargaining. First, consider the values of  $x_1^* \in [\underline{\alpha}, \bar{\alpha}]$ . Agent 1 is indifferent between his two options if  $\Delta G = 0$ ,

$$\begin{aligned}
0 &= \theta_1 \left( \frac{(x_1^* - x_2^*)^2}{2} \right) - c \\
c^* &= \theta_1 \left( \frac{(x_1^* - x_2^*)^2}{2} \right). \quad (\text{A23})
\end{aligned}$$

The value  $c^*$  represents a threshold cost. For values of  $x_1^* \in [\underline{\alpha}, \bar{\alpha}]$ , agent 1 follows the principal's rule if  $c > c^*$  and chooses  $d \in [\underline{\alpha}, \bar{\alpha}]$ . If  $c \leq c^*$ , agent 1 engages agent 2 in bargaining.

Now consider  $x_1^* < \underline{\alpha}$  and suppose  $c > c^*$ . The function  $\Delta G$  can be rewritten as follows,

$$\begin{aligned}
\Delta G &= \theta_1 \left( -\frac{(x_1^* - x_2^*)^2}{2} + (\underline{\alpha} - x_1^*)^2 + (\underline{\alpha} - x_2^*)^2 \right) - c + (c^* - c^*). \\
&= \theta_1 \left( -(x_1^* - x_2^*)^2 + (\underline{\alpha} - x_1^*)^2 + (\underline{\alpha} - x_2^*)^2 \right) - (c - c^*).
\end{aligned}$$

A rearrangement yields,

$$\begin{aligned}
&= 2\theta_1(x_2^* - \underline{\alpha})(x_1^* - \underline{\alpha}) - (c - c^*) \\
&< 0 \quad (\text{A24})
\end{aligned}$$

The inequality must always hold because  $x_1^* < \underline{\alpha}$  and it is known from the earlier analysis in Section 3.1 that the principal sets  $\underline{\alpha} \leq x_2^*$ . Therefore, if  $c > c^*$ , agent 1 again adheres

to the principal's rule for all  $x_1^* < \underline{\alpha}$ . A similar analysis is performed for  $x_1^* > \bar{\alpha}$ . As such, agent 1 will never engage agent 2 in bargaining if  $c > c^*$ . For the remainder of the proof, assume  $c \leq c^*$ <sup>15</sup>.

Again consider  $x_1^* < \underline{\alpha}$  and define  $\underline{t}$  to be the value of  $x_1^*$  where agent 1 is indifferent between the principal's mechanism and bargaining. Agent 1 is indifferent when  $\Delta G = 0$ , therefore  $\underline{t}$  is defined implicitly by,

$$\begin{aligned} 0 &= \theta_1 \left( -\frac{(x_2^* - \underline{t})^2}{2} + (\underline{\alpha} - \underline{t})^2 + (\underline{\alpha} - x_2^*)^2 \right) - c \\ &= \frac{\theta_1}{2} \left( 2x_2^*\underline{t} + 4\underline{\alpha}^2 + \underline{t}^2 - 4\underline{\alpha}\underline{t} + x_2^{*2} - 4\underline{\alpha}x_2^* \right) - c \\ &= \frac{\theta_1}{2} (x_2^* + \underline{t} - 2\underline{\alpha})^2 - c. \end{aligned}$$

An application of the quadratic formula yields,

$$\underline{t} = 2\underline{\alpha} - x_2^* \pm \sqrt{\frac{2c}{\theta_1}}. \quad (\text{A25})$$

Agent 1 will pay  $c$  for all values of  $x_1^* < \underline{t}$ , because his payoff is higher when he bargains with agent 2 and chooses the first-best choice. Intuitively,  $\underline{t}$  decreases with  $c$  since the benefit of bargaining shrinks relative to the cost. Therefore, Equation A25 simplifies to,

$$\underline{t} = 2\underline{\alpha} - x_2^* - \sqrt{\frac{2c}{\theta_1}}. \quad (\text{A26})$$

Now, consider  $x_1^* > \bar{\alpha}$  and define  $\bar{t}$  to be the value of  $x_1^*$  where agent 1 is indifferent between the principal's mechanism and bargaining. Similar to the analysis in solving for  $\underline{t}$ , the solution for  $\bar{t}$  is,

$$\bar{t} = 2\bar{\alpha} - x_2^* + \sqrt{\frac{2c}{\theta_1}}. \quad (\text{A27})$$

The principal knows of the agent's ability to incur  $c$  and she takes this into consideration when drafting the optimal rule. Her problem is to maximize the aggregate

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<sup>15</sup>In fact, at the end of the proof it is obvious that agent 1 never pays  $c$  if  $x_1^* \in [\underline{\alpha}, \bar{\alpha}]$ . The principal's allocation of discretion is a function of  $c$ , and as such, she never provides unneeded discretion.



payoff,

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}, c] = & \pi_1 + \pi_2 - \left[ \int_0^{\underline{t}} \left[ \left( \frac{x+x_2^*}{2} - x \right)^2 + \left( \frac{x+x_2^*}{2} - x_2^* \right)^2 + c \right] f(x) dx \right. \\
& + \int_{\underline{t}}^{\underline{\alpha}} \left[ (\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2 \right] f(x) dx \\
& + \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*)^2 f(x) dx \\
& + \int_{\bar{\alpha}}^{\bar{t}} \left[ (\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2 \right] f(x) dx \\
& \left. + \int_{\bar{t}}^D \left[ \left( \frac{x+x_2^*}{2} - x \right)^2 + \left( \frac{x+x_2^*}{2} - x_2^* \right)^2 + c \right] f(x) dx \right]. \quad (\text{A28})
\end{aligned}$$

The first-order condition of Equation A28 with respect to  $\underline{\alpha}$  yields,

$$\begin{aligned}
0 = & \frac{\partial E[\mathbf{\Pi}|\{\underline{\alpha}, \bar{\alpha}\}, c]}{\partial \underline{\alpha}} \\
= & 2 \int_{\underline{t}}^{\underline{\alpha}} [(\underline{\alpha} - x) + (\underline{\alpha} - x_2^*)] f(x) dx + (\underline{\alpha} - x_2^*)^2 f(\underline{\alpha}) \\
& - (\underline{\alpha} - x_2^*)^2 f(\underline{\alpha}).
\end{aligned}$$

A rearrangement leads to,

$$\begin{aligned}
= & \int_{\underline{t}}^{\underline{\alpha}} [(\underline{\alpha} - x) + (\underline{\alpha} - x_2^*)] f(x) dx \\
= & 2\underline{\alpha} (F(\underline{\alpha}) - F(\underline{t})) - \int_{\underline{t}}^{\underline{\alpha}} x f(x) dx - x_2^* (F(\underline{\alpha}) - F(\underline{t})) \\
= & 2\underline{\alpha} - \frac{\int_{\underline{t}}^{\underline{\alpha}} x f(x) dx}{(F(\underline{\alpha}) - F(\underline{t}))} - x_2^*.
\end{aligned}$$

The solution for  $\underline{\alpha}$  is,

$$\underline{\alpha} = \frac{E[x_1^* | \underline{t} \leq x_1^* \leq \underline{\alpha}] + x_2^*}{2}. \quad (\text{A29})$$

Recall that  $f(x)$  is uniform over  $[0, D]$  and that Equation A26 provided a solution for  $\underline{t}$ . Therefore,  $\underline{\alpha}$  is given by,

$$\begin{aligned}
&= \frac{\frac{\underline{t} + \underline{\alpha}}{2} + x_2^*}{2} \\
&= \frac{\underline{t} + 2x_2^*}{3} \\
&= \frac{2\underline{\alpha} + x_2^* - \sqrt{\frac{2c}{\theta_1}}}{3} \\
&= x_2^* - \sqrt{\frac{2c}{\theta_1}}.
\end{aligned} \tag{A30}$$

I now minimize Equation A28 with respect to  $\bar{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial E[\mathbf{\Pi}|\{\underline{\alpha}, \bar{\alpha}\}, c]}{\partial \bar{\alpha}} \\
&= (\bar{\alpha} - x_2^*)^2 f(\bar{\alpha}) + 2 \int_{\bar{\alpha}}^{\bar{t}} [(\bar{\alpha} - x) + (\bar{\alpha} - x_2^*)] f(x) dx \\
&\quad - (\bar{\alpha} - x_2^*)^2 f(\bar{\alpha}).
\end{aligned}$$

A rearrangement yields,

$$\begin{aligned}
&= \int_{\bar{\alpha}}^{\bar{t}} [(\bar{\alpha} - x) + (\bar{\alpha} - x_2^*)] f(x) dx \\
&= 2\bar{\alpha} (F(\bar{t}) - F(\bar{\alpha})) - \int_{\bar{\alpha}}^{\bar{t}} x f(x) dx - x_2^* (F(\bar{t}) - F(\bar{\alpha})) \\
&= 2\bar{\alpha} - \frac{\int_{\bar{\alpha}}^{\bar{t}} x f(x) dx}{(F(\bar{t}) - F(\bar{\alpha}))} - x_2^*.
\end{aligned}$$

Solving for  $\bar{\alpha}$  gives,

$$\bar{\alpha} = \frac{E[x_1^* | \bar{\alpha} \leq x_1^* \leq \bar{t}] + x_2^*}{2}. \tag{A31}$$

$$\begin{aligned}
&= \frac{\frac{\bar{t} + \bar{\alpha}}{2} + x_2^*}{2} \\
&= \frac{\bar{t} + 2x_2^*}{3} \\
&= \frac{2\bar{\alpha} + x_2^* + \sqrt{\frac{2c}{\theta_1}}}{3} \\
&= x_2^* + \sqrt{\frac{2c}{\theta_1}}.
\end{aligned} \tag{A32}$$

Equations A30 and A32 do not fully solve for the optimal rule. The indifference thresholds,  $\underline{t}$  and  $\bar{t}$ , used in the analysis must be contained in  $[0, D]$ . If the lower threshold falls below 0 or the upper threshold is greater than  $D$  then there are no values of  $x_1^*$  where it is efficient for agent 1 to pay  $c$ . As such, 0 serves as a lower bound for  $\underline{t}$  and  $D$  similarly serves as an upper bound for  $\bar{t}$ . Consider Equations A30 and A26 and the cost  $\underline{c}$  at which  $\underline{t} = 0$ ,

$$\begin{aligned}
0 &= 2\underline{\alpha} - x_2^* - \sqrt{\frac{2c}{\theta_1}} \\
&= 2 \left( x_2^* - \sqrt{\frac{2c}{\theta_1}} \right) - x_2^* - \sqrt{\frac{2c}{\theta_1}} \\
&= x_2^* - 3\sqrt{\frac{2c}{\theta_1}}. \\
\underline{c} &= \frac{\theta_1 x_2^{*2}}{18}.
\end{aligned} \tag{A33}$$

Similarly, define  $\bar{c}$  to be the cost at which  $\bar{t} = D$

$$\bar{c} = \frac{\theta_1 (D - x_2^*)^2}{18}. \tag{A34}$$

It is never efficient for agent 1 to bargain if  $x_1^* < \underline{\alpha}$  and  $c > \underline{c}$ . Similarly, agent 1 will never bargain if  $x_1^* > \bar{\alpha}$  and  $c > \bar{c}$ .

The optimal rule is given by,

$$\underline{\alpha} = \begin{cases} x_2^* - \sqrt{\frac{2c}{\theta_1}} & \text{if } c \leq \min \left[ \theta_1 \left( \frac{x_2^{*2}}{18} \right), \theta_1 \left( \frac{(x_1^* - x_2^*)^2}{2} \right) \right] \\ \frac{2x_2^*}{3} & \text{otherwise} \end{cases} \tag{A35}$$

$$\bar{\alpha} = \begin{cases} x_2^* + \sqrt{\frac{2c}{\theta_1}} & \text{if } c \leq \min \left[ \theta_1 \left( \frac{(D - x_2^*)^2}{18} \right), \theta_1 \left( \frac{(x_1^* - x_2^*)^2}{2} \right) \right] \\ \frac{2x_2^* + D}{3} & \text{otherwise} \end{cases}. \tag{A36}$$

The comparative statics for the discretion provided by the rule are computed directly using Equations A30 and A32. Assume that  $c$  is sufficiently small such that  $0 < \underline{t}$  and  $\bar{t} < D$ ,

$$\begin{aligned}\frac{\partial \underline{\alpha}}{\partial c} &= \frac{\partial}{\partial c} \left( x_2^* - \sqrt{\frac{2c}{\theta_1}} \right) = - (2c\theta_1)^{-1/2} \\ &< 0\end{aligned}\tag{A37}$$

$$\begin{aligned}\frac{\partial \underline{\alpha}}{\partial \theta_1} &= \frac{\partial}{\partial \theta_1} \left( x_2^* - \sqrt{\frac{2c}{\theta_1}} \right) = \frac{1}{2} \sqrt{\frac{2c}{\theta_1^3}} \\ &> 0\end{aligned}\tag{A38}$$

$$\begin{aligned}\frac{\partial \bar{\alpha}}{\partial c} &= \frac{\partial}{\partial c} \left( x_2^* + \sqrt{\frac{2c}{\theta_1}} \right) = (2c\theta_1)^{-1/2} \\ &> 0\end{aligned}\tag{A39}$$

$$\begin{aligned}\frac{\partial \bar{\alpha}}{\partial \theta_1} &= \frac{\partial}{\partial \theta_1} \left( x_2^* + \sqrt{\frac{2c}{\theta_1}} \right) = -\frac{1}{2} \sqrt{\frac{2c}{\theta_1^3}} \\ &< 0.\end{aligned}\tag{A40}$$

■

**Proof of Corollary 5.1:** This result comes directly by setting  $c = 0$  in Equations A35 and A36.

■

**Proof of Corollary 5.2:** If the agents engage in bargaining, each agent's payoff must be at least as large as the payoff under the principal's mechanism. If not, bargaining breaks down and the agents return to the principal's mechanism. Agent 2, i.e., the non-decision-making agent, has the same beliefs and information as the principal. As such, he believes that the aggregate payoff is maximized under the mechanism and has no reason to initiate bargaining with agent 1.

■

**Proof of Proposition 6:** The expected aggregate payoff with linear payoff functions

is defined by,

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] &= \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x) + (\underline{\alpha} - x_2^*)] f(x) dx \\
&\quad - \int_{\underline{\alpha}}^{\bar{\alpha}} (x - x_2^*) f(x) dx \\
&\quad - \int_{\bar{\alpha}}^D [(\bar{\alpha} - x) + (\bar{\alpha} - x_2^*)] f(x) dx.
\end{aligned} \tag{A41}$$

I begin by taking first-order conditions with respect to  $\underline{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \underline{\alpha}} E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\underline{\alpha} - x_2^*) f(\underline{\alpha}) + 2 \int_0^{\underline{\alpha}} f(x) dx - (\underline{\alpha} - x_2^*) f(\underline{\alpha}) \\
&= 2\underline{\alpha} \\
\underline{\alpha} &= 0.
\end{aligned} \tag{A42}$$

Now I take first-order conditions with respect to  $\bar{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \bar{\alpha}} E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\bar{\alpha} - x_2^*) f(\bar{\alpha}) + 2 \int_{\bar{\alpha}}^D f(x) dx - (\bar{\alpha} - x_2^*) f(\bar{\alpha}) \\
&= 2(D - \bar{\alpha}) \\
\bar{\alpha} &= D.
\end{aligned} \tag{A43}$$

■

**Proof of Proposition 7:** The expected aggregate payoff is given by Equation 1.26. I begin by taking the first-order condition with respect to  $\underline{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \underline{\alpha}} E[\mathbf{\Pi}(d) | \{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\underline{\alpha} - x_2^*)^\beta f(\underline{\alpha}) + \int_0^{\underline{\alpha}} [\beta(\underline{\alpha} - x)^{\beta-1} + \beta(\underline{\alpha} - x_2^*)^{\beta-1}] f(x) dx - (\underline{\alpha} - x_2^*)^\beta f(\underline{\alpha}) \\
&= \frac{\underline{\alpha}^{\beta-1}}{\beta} + (\underline{\alpha} - x_2^*)^{\beta-1} \\
&= \frac{\underline{\alpha}}{\beta^{1/(\beta-1)}} + (\underline{\alpha} - x_2^*) \\
\underline{\alpha} &= \frac{\beta^{1/(\beta-1)} x_2^*}{\beta^{1/(\beta-1)} + 1}. \tag{A44}
\end{aligned}$$

Now I take first-order condition with respect to  $\bar{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \bar{\alpha}} E[\mathbf{\Pi}(d) | \{\underline{\alpha}, \bar{\alpha}\}] \\
&= (\bar{\alpha} - x_2^*)^\beta f(\bar{\alpha}) + \int_{\bar{\alpha}}^D [\beta(\bar{\alpha} - x)^{\beta-1} + \beta(\bar{\alpha} - x_2^*)^{\beta-1}] f(x) dx - (\bar{\alpha} - x_2^*)^\beta f(\bar{\alpha}) \\
&= \frac{-(\bar{\alpha} - D)^{\beta-1}}{\beta} - (\bar{\alpha} - x_2^*)^{\beta-1} \\
&= \frac{(\bar{\alpha} - D)}{\beta^{1/(\beta-1)}} + (\bar{\alpha} - x_2^*) \\
\bar{\alpha} &= \frac{\beta^{1/(\beta-1)} x_2^* + D}{\beta^{1/(\beta-1)} + 1}. \tag{A45}
\end{aligned}$$

I now show that discretion is increasing in  $\beta$ . Taking partial derivatives of  $\underline{\alpha}$  and  $\bar{\alpha}$  with respect to  $\beta$  is technically incorrect because  $\beta$  comes from a set of discrete integers. However, because only the derivatives' directions matter, I show that  $\underline{\alpha}$  is strictly decreasing in  $\beta$  while  $\bar{\alpha}$  is strictly increasing.

$$\begin{aligned}
\frac{\partial \underline{\alpha}}{\partial \beta} &= \frac{\partial}{\partial \beta} \left( \frac{\beta^{1/(\beta-1)} x_2^*}{\beta^{1/(\beta-1)} + 1} \right) \\
&= \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) x_2^*}{\beta^{1/(\beta-1)} + 1} - \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) (\beta^{1/(\beta-1)} x_2^*)}{(\beta^{1/(\beta-1)} + 1)^2} \\
&= \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) ((\beta^{1/(\beta-1)} x_2^* + x_2^*) - (\beta^{1/(\beta-1)} x_2^*))}{(\beta^{1/(\beta-1)} + 1)^2}. \tag{A46}
\end{aligned}$$

Clearly the denominator is positive. The numerator is more difficult to sign. The first term,  $\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right)$ , is negative because  $(\beta-1)^2 < (\beta-1)\beta$  and  $\log(2) > 1$ . The second term,  $((\beta^{1/(\beta-1)}x_2^* + x_2^*) - (\beta^{1/(\beta-1)}x_2^*))$ , is positive because  $x_2^* \geq 0$ . This means that the product of the two terms is negative and  $\frac{\partial \bar{\alpha}}{\partial \beta} \leq 0$ .

The partial derivative of  $\bar{\alpha}$  with respect to  $\beta$  is given by,

$$\begin{aligned} \frac{\partial \bar{\alpha}}{\partial \beta} &= \frac{\partial}{\partial \beta} \left( \frac{\beta^{1/(\beta-1)}x_2^* + D}{\beta^{1/(\beta-1)} + 1} \right) \\ &= \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) x_2^*}{\beta^{1/(\beta-1)} + 1} - \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) (\beta^{1/(\beta-1)}x_2^* + D)}{(\beta^{1/(\beta-1)} + 1)^2} \\ &= \frac{\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right) ((\beta^{1/(\beta-1)}x_2^* + x_2^*) - (\beta^{1/(\beta-1)}x_2^* + D))}{(\beta^{1/(\beta-1)} + 1)^2}. \end{aligned} \quad (\text{A47})$$

Again, the denominator is positive. The first term in the numerator,

$$\beta^{1/(\beta-1)} \left( \frac{1}{(\beta-1)\beta} - \frac{\log(\beta)}{(\beta-1)^2} \right),$$

is again negative because  $(\beta-1)^2 < (\beta-1)\beta$  and  $\log(2) > 1$ . The second term in the numerator,  $((\beta^{1/(\beta-1)}x_2^* + x_2^*) - (\beta^{1/(\beta-1)}x_2^* + D))$ , is also negative because  $x_2^* \leq D$ . The product of two negative numbers is positive which indicates that  $\frac{\partial \bar{\alpha}}{\partial \beta} \geq 0$ .

It is worthwhile to note that  $\beta^{1/(\beta-1)}$  approaches 1 as  $\beta \rightarrow \infty$ . This implies that discretion is bounded by  $[\frac{x_2^*}{2}, \frac{x_2^* + D}{2}]$ .

■

**Proof of Proposition 8:** The mean-preserving linear distribution, outlined in Equation C10, is continuous and piecewise-defined. The assumption that  $x_2^* = \frac{1}{2}$  and the result from Corollary 3.1 allow the expected aggregate payoff to be written as,

$$\begin{aligned} E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] &= \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2] f(x, \gamma) dx \\ &\quad - \int_{\underline{\alpha}}^{x_2^*} (x - x_2^*)^2 f(x, \gamma) dx - \int_{x_2^*}^{\bar{\alpha}} (x - x_2^*)^2 f(x, \gamma) dx \\ &\quad - \int_{\bar{\alpha}}^1 [(\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2] f(x, \gamma) dx. \end{aligned} \quad (\text{A48})$$

A substitution of the density's explicit form, found in Equation C12, into the above expression yields,

$$\begin{aligned}
&= \pi_1 + \pi_2 - \int_0^{\underline{\alpha}} [(\underline{\alpha} - x)^2 + (\underline{\alpha} - x_2^*)^2] (-4(1 - \gamma)x + (2 - \gamma)) dx \\
&\quad - \int_{\underline{\alpha}}^{x_2^*} (x - x_2^*)^2 (-4(1 - \gamma)x + (2 - \gamma)) dx \\
&\quad - \int_{x_2^*}^{\bar{\alpha}} (x - x_2^*)^2 (4(1 - \gamma)(x - 1) + (2 - \gamma)) dx \\
&\quad - \int_{\bar{\alpha}}^1 [(\bar{\alpha} - x)^2 + (\bar{\alpha} - x_2^*)^2] (4(1 - \gamma)(x - 1) + (2 - \gamma)) dx. \quad (\text{A49})
\end{aligned}$$

The first-order condition of Equation A49 with respect to  $\underline{\alpha}$  yields the same result outlined in Proposition 3. Consequently, an explicit form of  $\underline{\alpha}$  is obtained as follows,

$$\begin{aligned}
\underline{\alpha} &= \frac{x_2^* + E[x_1^* | x_1^* \leq \underline{\alpha}]}{2} \\
&= \frac{x_2^*}{2} + \frac{\int_0^{\underline{\alpha}} x (-4(1 - \gamma)x + (2 - \gamma)) dx}{2(2(1 - \underline{\alpha}) - \gamma(1 - 2\underline{\alpha})) \underline{\alpha}} \\
&= \frac{x_2^*}{2} + \frac{\frac{1}{6}(-8\underline{\alpha}^2(1 - \gamma) + 3\underline{\alpha}(2 - \gamma))}{2(2(1 - \underline{\alpha}) - \gamma(1 - 2\underline{\alpha}))}.
\end{aligned}$$

Rearranging the expression and substituting  $x_2^* = \frac{1}{2}$  yields,

$$0 = \underline{\alpha}^2 \left( \frac{8}{3}(1 - \gamma) \right) + \underline{\alpha} \left( \frac{5}{2}\gamma - 4 \right) + \left( 1 - \frac{\gamma}{2} \right). \quad (\text{A50})$$

An application of the quadratic formula and previous insights from the proof of Proposition 3 yield,

$$\begin{aligned}
\underline{\alpha} &= \frac{-\left(\frac{5}{2}\gamma - 4\right)}{2\left(\frac{8}{3}(1 - \gamma)\right)} - \frac{\sqrt{\left(\frac{5}{2}\gamma - 4\right)^2 - 4\left(\frac{8}{3}(1 - \gamma)\right)\left(1 - \frac{\gamma}{2}\right)}}{2\left(\frac{8}{3}(1 - \gamma)\right)} \\
\underline{\alpha} &= \frac{24 - 15\gamma}{32(1 - \gamma)} - \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)}. \quad (\text{A51})
\end{aligned}$$

A similar analysis yields the explicit form for  $\bar{\alpha}$ ,

$$\bar{\alpha} = \frac{8 - 17\gamma}{32(1 - \gamma)} + \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)}. \quad (\text{A52})$$



The quantity of discretion provided by the rule is given by,

$$\begin{aligned}
\bar{\alpha} - \underline{\alpha} &= \left( \frac{8 - 17\gamma}{32(1 - \gamma)} + \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \right) \\
&\quad - \left( \frac{24 - 15\gamma}{32(1 - \gamma)} - \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \right) \\
&= \frac{-16 - 2\gamma}{32(1 - \gamma)} + \frac{2\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \\
&= \frac{-(8 + \gamma)}{16(1 - \gamma)} + \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{16(1 - \gamma)}. \tag{A53}
\end{aligned}$$

Equation A53 demonstrates that optimal discretion is a function of  $\gamma$ . According to Equation C13 in Appendix C, the variance of the distribution  $f(x)$  is also a function of  $\gamma$ ,

$$E[x^2] - E[x]^2 = \frac{3 - \gamma}{24}. \tag{A54}$$

Consequently, because both optimal discretion and the distribution's variance are tied to  $\gamma$ , a relationship between discretion and variance exists. First consider how optimal discretion changes with  $\gamma$ ,

$$\begin{aligned}
\frac{\partial(\bar{\alpha} - \underline{\alpha})}{\partial\gamma} &= \frac{\partial}{\partial\gamma} \left( \frac{-(8 + \gamma)}{16(1 - \gamma)} + \frac{\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{16(1 - \gamma)} \right) \\
&= \frac{-9}{16(1 - \gamma)^2} + \frac{120 - 39\gamma}{16(1 - \gamma)^2 \sqrt{3(11\gamma^2 - 48\gamma + 64)}} \\
&= \frac{120 - 39\gamma - 9\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{16(1 - \gamma)^2 \sqrt{3(11\gamma^2 - 48\gamma + 64)}}. \tag{A55}
\end{aligned}$$

The denominator in the above expression is clearly positive for all values of  $\gamma \in [0, 2]$ . The sign on the numerator, however, is not obvious. The numerator can be signed as

follows,

$$\begin{aligned}
120 - 39\gamma - 9\sqrt{3(11\gamma^2 - 48\gamma + 64)} &\leq 0 \\
120 - 39\gamma &\leq 9\sqrt{3(11\gamma^2 - 48\gamma + 64)} \\
\frac{40 - 13\gamma}{3} &\leq \sqrt{3(11\gamma^2 - 48\gamma + 64)} \\
\left(\frac{40 - 13\gamma}{3}\right)^2 &\leq 3(11\gamma^2 - 48\gamma + 64) \\
\frac{(40 - 13\gamma)^2}{27} - (11\gamma^2 - 48\gamma + 64) &\leq 0 \\
-1 - \gamma^2 + 2\gamma &\leq 0 \\
-(1 - \gamma)^2 &\leq 0.
\end{aligned} \tag{A56}$$

Because the numerator is negative, optimal discretion is decreasing with  $\gamma$ . Similarly, the variance of the distribution is also decreasing with  $\gamma$ . This means that optimal discretion and variance move in the same direction, i.e., optimal discretion increases as the variance of  $f(x, \gamma)$  increases.

■

**Proof of Proposition 9:** Agent 1's ex ante payoff is

$$\Pi_1 = \pi_1 - E[(d - x_1^*)^2] \tag{A57}$$

$$= \pi_1 - \left( \int_0^{\underline{\alpha}} (\underline{\alpha} - x)^2 f(x, \gamma) dx + \int_{\bar{\alpha}}^1 (\bar{\alpha} - x)^2 f(x, \gamma) dx \right). \tag{A58}$$

A substitution of the mean-preserving linear distribution Density into the expression yields,

$$\begin{aligned}
&= \pi_1 - \left( \int_0^{\underline{\alpha}} (\underline{\alpha} - x)^2 (-4(1 - \gamma)x + (2 - \gamma)) dx \right. \\
&\quad \left. + \int_{\bar{\alpha}}^1 (\bar{\alpha} - x)^2 (4(1 - \gamma)(x - 1) + (2 - \gamma)) dx \right) \\
&= \pi_1 - \left( \frac{\underline{\alpha}^3(2 - \gamma - \underline{\alpha}(1 - \gamma))}{3} + \frac{(1 - \bar{\alpha})^3(1 + \bar{\alpha}(1 - \gamma))}{3} \right).
\end{aligned} \tag{A59}$$

A utilization of the explicit forms of  $\underline{\alpha}$  and  $\bar{\alpha}$  from Equations A51 and A52 results in,

$$= \pi_1 - \left( \frac{24 - 15\gamma - \sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \right)^3 \left( \frac{40 - 17\gamma + \sqrt{3(11\gamma^2 - 48\gamma + 64)}}{48} \right). \tag{A60}$$

It can now be shown that agent 1's payoff is increasing in  $\gamma$ . Consider the partial derivative of the agent's payoff with respect to  $\gamma$ ,

$$\begin{aligned} \frac{\partial \Pi_1(\gamma)}{\partial \gamma} = & -3 \left( \frac{24 - 15\gamma - \sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \right)^2 \left( \frac{40 - 17\gamma + \sqrt{3(11\gamma^2 - 48\gamma + 64)}}{48} \right) \\ & \left( \frac{-120 + 39\gamma + 9\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)^2 \sqrt{3(11\gamma^2 - 48\gamma + 64)}} \right) \\ & - \left( \frac{24 - 15\gamma - \sqrt{3(11\gamma^2 - 48\gamma + 64)}}{32(1 - \gamma)} \right)^3 \left( \frac{33\gamma - 72 - 17\sqrt{3(11\gamma^2 - 48\gamma + 64)}}{48\sqrt{3(11\gamma^2 - 48\gamma + 64)}} \right). \end{aligned} \tag{A61}$$

It can be shown that the above expression achieves its minimum value at  $\gamma = 1$ . Evaluating the expression at this value requires multiple applications of L'Hopital's rule and results in,

$$\frac{\partial \Pi_1(1)}{\partial \gamma} = \frac{8}{729} > 0. \tag{A62}$$

Equation A62 shows that agent 1's payoff is increasing in  $\gamma$ .

■

## B Appendix

In this appendix, I explore several variants of the base model. First I explore a model where both agents' payoff functions are private information. The analysis yields results similar to those of Sections 3 and 3.1, thus I omit them from the main body of the paper. I then consider a model where there are  $N > 2$  agents and I show that discretion decreases as the decision-maker becomes less aligned with the principal's objective. Next, I consider a model where the decision-maker receives a bonus or penalty for each decision. I demonstrate that there is an optimal mechanism where the principal transfers the aggregate surplus to the decision-maker via the "price" and the first-best decision is made. I conclude the appendix by presenting a set of generalized results relating to Section 6.

### B.1 Two-Sided Asymmetric Information

A principal is often times uncertain about both agents' payoff functions. Instead of perfect information, she must rely on beliefs in setting the optimal ex ante rules. In this extension I consider such a scenario by assuming that the principal does not observe either  $x_1^*$  or  $x_2^*$ . Instead, she depends on her beliefs, a set of probability distributions  $F^1(\tilde{x})$  and  $F^2(\tilde{x})$ , when choosing  $\{\underline{\alpha}, \bar{\alpha}\}$ . I first assume that the agents' bliss points are independent and I show that the optimal rule parallels the result of Proposition 3. I then conduct the analysis when the bliss points are correlated.

#### B.1.1 Independent Bliss Points

Agent 1's and agent 2's bliss points may be based entirely on idiosyncratic factors. That is, one agent's preferred choice is completely random conditional on the other's. When this is the case, a principal can consider each agent's preferences independently.

**Lemma B1.** *The expected aggregate payoff,  $E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]$ , when both agents' bliss*

points are private information and uncorrelated is given by,

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] = & \pi_1 + \pi_2 - \left[ \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) + \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) \right. \\
& + F^1(\underline{\alpha}) \int_0^D (\underline{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2) - 2 \int_{\underline{\alpha}}^{\bar{\alpha}} \tilde{x}_1 dF^1(\tilde{x}_1) \int_0^D \tilde{x}_2 dF^2(\tilde{x}_2) \\
& + \int_{\underline{\alpha}}^{\bar{\alpha}} \tilde{x}_1^2 dF^1(\tilde{x}_1) + (F^1(\bar{\alpha}) - F^1(\underline{\alpha})) \int_0^D \tilde{x}_2^2 dF^2(\tilde{x}_2) \\
& \left. + (1 - F^1(\bar{\alpha})) \int_0^D (\bar{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2) \right]. \tag{B1}
\end{aligned}$$

**Proof of Lemma B1:** The expected aggregate payoff,  $E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]$ , when both bliss points are private information is given by,

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] = & \pi_1 + \pi_2 \\
& - \left[ \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) dF^2(\tilde{x}_2) + \int_0^D \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) dF^2(\tilde{x}_2) \right. \\
& + \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_2)^2 dF^1(\tilde{x}_1) dF^2(\tilde{x}_2) + \int_0^D \int_{\underline{\alpha}}^{\bar{\alpha}} (\tilde{x}_1 - \tilde{x}_2)^2 dF^1(\tilde{x}_1) dF^2(\tilde{x}_2) \\
& \left. + \int_0^D \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_2)^2 dF^1(\tilde{x}_1) dF^2(\tilde{x}_2) \right].
\end{aligned}$$

Because  $\tilde{x}_1$  and  $\tilde{x}_2$  are independent, it simplifies to,

$$\begin{aligned}
= & \pi_1 + \pi_2 - \left[ \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) + \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1) \right. \\
& + F^1(\underline{\alpha}) \int_0^D (\underline{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2) - 2 \int_{\underline{\alpha}}^{\bar{\alpha}} \tilde{x}_1 dF^1(\tilde{x}_1) \int_0^D \tilde{x}_2 dF^2(\tilde{x}_2) \\
& + \int_{\underline{\alpha}}^{\bar{\alpha}} \tilde{x}_1^2 dF^1(\tilde{x}_1) + (F^1(\bar{\alpha}) - F^1(\underline{\alpha})) \int_0^D \tilde{x}_2^2 dF^2(\tilde{x}_2) \\
& \left. + (1 - F^1(\bar{\alpha})) \int_0^D (\bar{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2) \right]. \tag{B2}
\end{aligned}$$

■

**Proposition B1.** *The optimal rule when both agents' bliss points are private and uncorrelated is given by,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{E[x_1^* | x_1^* \leq \underline{\alpha}] + E[x_2^*]}{2}, \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] + E[x_2^*]}{2} \right\} \tag{B3}$$

**Proof of Proposition B1:** I begin with Equation B1 from Lemma B1. The first-order conditions with respect to  $\underline{\alpha}$  and  $\bar{\alpha}$  are given by,

$$\begin{aligned}
0 &= \frac{\partial E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]}{\partial \underline{\alpha}} \\
&= 2 \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1) + F^1(\underline{\alpha}) \int_0^D (\underline{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2) \\
&\quad + 2F^1(\underline{\alpha}) \int_0^D (\underline{\alpha} - \tilde{x}_2) dF^2(\tilde{x}_2) + 2\underline{\alpha}F^1(\underline{\alpha}) \int_0^D \tilde{x}_2 dF^2(\tilde{x}_2) \\
&\quad - \underline{\alpha}^2 F^1(\underline{\alpha}) - F^1(\underline{\alpha}) \int_0^D \tilde{x}_2^2 dF^2(\tilde{x}_2) \\
&= 2\underline{\alpha} - \frac{\int_0^{\underline{\alpha}} \tilde{x}_1 dF^1(\tilde{x}_1)}{F^1(\underline{\alpha})} - E[x_2^*] \\
\underline{\alpha} &= \frac{E[x_1^* | x_1^* \leq \underline{\alpha}] + E[x_2^*]}{2}. \tag{B4}
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{\partial E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]}{\partial \bar{\alpha}} \\
&= 2 \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1) - 2\bar{\alpha}F^1(\bar{\alpha}) \int_0^D \tilde{x}_2 dF^2(\tilde{x}_2) \\
&\quad + \bar{\alpha}^2 F^1(\bar{\alpha}) + F^1(\bar{\alpha}) \int_0^D \tilde{x}_2^2 dF^2(\tilde{x}_2) - F^1(\bar{\alpha}) \int_0^D (\bar{\alpha} - \tilde{x}_2)^2 dF^2(\tilde{x}_2). \\
&\quad + 2(1 - F^1(\bar{\alpha})) \int_0^D (\bar{\alpha} - \tilde{x}_2) dF^2(\tilde{x}_2). \\
\bar{\alpha} &= \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] + E[x_2^*]}{2}. \tag{B5}
\end{aligned}$$

■

The result of Proposition B1 mirrors that of Proposition 3. The only difference is that agent 2's actual bliss point is replaced by the unconditional expectation.

## B.2 Correlated Bliss Points

**Proposition B2.** *The optimal rule when both agents' bliss points are correlated is given by,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{E[x_1^* | x_1^* \leq \underline{\alpha}] + E[x_2^* | x_1^* \leq \underline{\alpha}]}{2}, \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] + E[x_2^* | x_1^* \geq \bar{\alpha}]}{2} \right\} \tag{B6}$$

**Proof of Proposition B2:** The expected aggregate payoff,  $E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]$ , when both agents' bliss points are private information and correlated is given by,

$$\begin{aligned}
E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}] = & \pi_1 + \pi_2 - \left[ \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_1)^2 dF^1(\tilde{x}_1, \tilde{x}_2) \right. \\
& + \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_2)^2 dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D \int_{\underline{\alpha}}^{\bar{\alpha}} (\tilde{x}_1 - \tilde{x}_2)^2 dF^1(\tilde{x}_1, \tilde{x}_2) \\
& \left. + \int_0^D \int_{\bar{\alpha}}^D (\bar{\alpha} - \tilde{x}_2)^2 dF^1(\tilde{x}_1, \tilde{x}_2) \right]. \tag{B7}
\end{aligned}$$

The first-order conditions of Equation B7 with respect to  $\underline{\alpha}$  and  $\bar{\alpha}$  are given by,

$$\begin{aligned}
0 = & \frac{\partial E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]}{\partial \underline{\alpha}} \\
= & \int_0^D \int_0^{\underline{\alpha}} 2(\underline{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D \int_0^{\underline{\alpha}} 2(\underline{\alpha} - \tilde{x}_2) dF^1(\tilde{x}_1, \tilde{x}_2) \\
& + \int_0^D (\underline{\alpha} - \tilde{x}_2)^2 dF^1(\underline{\alpha}, \tilde{x}_2) - \int_0^D (\underline{\alpha} - \tilde{x}_2)^2 dF^1(\underline{\alpha}, \tilde{x}_2) \\
= & \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D \int_0^{\underline{\alpha}} (\underline{\alpha} - \tilde{x}_2) dF^1(\tilde{x}_1, \tilde{x}_2) \\
\underline{\alpha} = & \frac{E[x_1^* | x_1^* \leq \underline{\alpha}] + E[x_2^* | x_1^* \leq \underline{\alpha}]}{2}. \tag{B8}
\end{aligned}$$

$$\begin{aligned}
0 = & \frac{\partial E[\mathbf{\Pi}(d)|\{\underline{\alpha}, \bar{\alpha}\}]}{\partial \bar{\alpha}} \\
= & \int_0^D \int_{\bar{\alpha}}^D 2(\bar{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D (\bar{\alpha} - \tilde{x}_2)^2 dF^1(\bar{\alpha}, \tilde{x}_2) \\
& + \int_0^D \int_{\bar{\alpha}}^D 2(\bar{\alpha} - \tilde{x}_2) dF^1(\tilde{x}_1, \tilde{x}_2) - \int_0^D (\bar{\alpha} - \tilde{x}_2)^2 dF^1(\bar{\alpha}, \tilde{x}_2) \\
= & \int_0^D \int_{\bar{\alpha}}^D 2(\bar{\alpha} - \tilde{x}_1) dF^1(\tilde{x}_1, \tilde{x}_2) + \int_0^D \int_{\bar{\alpha}}^D 2(\bar{\alpha} - \tilde{x}_2) dF^1(\tilde{x}_1, \tilde{x}_2) \\
\bar{\alpha} = & \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] + E[x_2^* | x_1^* \geq \bar{\alpha}]}{2}. \tag{B9}
\end{aligned}$$

■

### B.3 Multiple Agent Extensions

In Section 3.2 I demonstrate that the discretion granted to agent 1 is characterized by the needs of agent 2. Specifically, in Corollary 3.1 I show that  $x_2^*$  is contained within the realm of discretion. Here, I expand the analysis and consider a setup where there are  $N > 2$  agents and without a loss of generality I assume agent 1 is the decision-maker.

The principal's problem is to define a rule that maximizes the aggregate payoff,

$$\begin{aligned}
 & \max_{\underline{\alpha}, \bar{\alpha} \in [0, D]} \sum_{i=1}^N \Pi_i(d) \\
 & \text{subject to} \\
 & d \in \arg \max_{\hat{d} \in [\underline{\alpha}, \bar{\alpha}]} \Pi_1(\hat{d}), \\
 & \Pi_1(d) \geq 0, \\
 & \dots \\
 & \Pi_N(d) \geq 0.
 \end{aligned} \tag{B10}$$

The following proposition provides the optimal rule when there are more than two agents.

**Proposition B3.** *The optimal rule with  $N > 2$  agents is given by,*

$$\{\underline{\alpha}, \bar{\alpha}\} = \left\{ \frac{\sum_{i=2}^N x_i + E[x_1^* | x_1^* \leq \underline{\alpha}]}{N}, \frac{\sum_{i=2}^N x_i + E[x_1^* | x_1^* \geq \bar{\alpha}]}{N} \right\} \tag{B11}$$

**Proof of Proposition B3:** First, the aggregate payoff with  $N$  agents is given by,

$$\begin{aligned}
 E[\mathbf{\Pi}(d) | \{\underline{\alpha}, \bar{\alpha}\}] &= \sum_{i=1}^N \pi_i + - \int_0^{\underline{\alpha}} \left[ (\underline{\alpha} - x)^2 + \sum_{i=2}^N (\underline{\alpha} - x_i)^2 \right] f(x) dx \\
 & - \int_{\underline{\alpha}}^{\bar{\alpha}} \sum_{i=2}^N (x - x_i)^2 f(x) dx \\
 & - \int_{\bar{\alpha}}^D \left[ (\bar{\alpha} - x)^2 + \sum_{i=2}^N (\bar{\alpha} - x_i)^2 \right] f(x) dx.
 \end{aligned} \tag{B12}$$



I begin by solving the principal's problem for  $\underline{\alpha}$ ,

$$\begin{aligned}
0 &= \frac{\partial}{\partial \underline{\alpha}} E[\mathbf{\Pi}(d) | \{\underline{\alpha}, \bar{\alpha}\}] \\
&= \sum_{i=2}^N (\underline{\alpha} - x_i)^2 f(\underline{\alpha}) + \int_0^{\underline{\alpha}} \left[ 2(\underline{\alpha} - x) + 2 \sum_{i=2}^N (\underline{\alpha} - x_i) \right] f(x) dx - \sum_{i=2}^N (\underline{\alpha} - x_i)^2 f(\underline{\alpha}) \\
&= N\underline{\alpha} - \sum_{i=2}^N x_i - \frac{\int_0^{\underline{\alpha}} x f(x) dx}{F(\underline{\alpha})} \\
\underline{\alpha} &= \frac{\sum_{i=2}^N x_i + E[x_1^* | x_1^* \leq \underline{\alpha}]}{N}. \tag{B13}
\end{aligned}$$

Similarly,  $\bar{\alpha}$  is given by,

$$\bar{\alpha} = \frac{\sum_{i=2}^N x_i + E[x_1^* | x_1^* \geq \bar{\alpha}]}{N}. \tag{B14}$$

■

Proposition B3 provides the analytic form for a rule when there are more than two agents. Providing the decision-maker with discretion is valuable because there is alignment between the principal's objective function and the decision-maker's payoff. The degree of alignment between the principal and decision-maker, however, is decreasing as the number of agents increases. Consequently, the provided discretion responds. The following two corollaries provide two new insights: the first corollary demonstrates that the provided discretion is characterized by the average bliss point of the other  $N - 1$  agents and the second corollary shows that the decision-maker is provided no discretion when there is no alignment between him and the principal.

**Corollary A1.** *The discretion provided to agent 1 includes the average bliss point of the other  $N - 1$  agents,*

$$\frac{\sum_{i=2}^N x_i}{N - 1}.$$

**Proof of Corollary A1:** The proof mirrors the proof of Corollary 3.1. ■

**Corollary A2.** *As  $N \rightarrow \infty$ , agent 1 is restricted to a single choice.*

**Proof of Corollary A2:** The quantity of discretion granted to agent 1 is given by the difference of the expressions in Equations B13 and B14,

$$\bar{\alpha} - \underline{\alpha} = \frac{E[x_1^* | x_1^* \geq \bar{\alpha}] - E[x_1^* | x_1^* \leq \underline{\alpha}]}{N}. \quad (\text{B15})$$

As  $N \rightarrow \infty$ , discretion goes to zero. ■

## B.4 Decision Price Discussion

The analysis in the body of this paper assumes that the principal cannot “price” the decision-maker’s choices. Suppose instead that a contingent price is charged for the agent’s choice and the price function,  $\tau(d)$ , is continuous and differentiable in  $d$ . Consequently, agent 1 chooses a  $d$  that not only considers his payoff function but also the decision’s price. As such, agent 1’s payoff is given by

$$\Pi_1(d) = \pi_1 - (d - x_1^*)^2 + \tau(d). \quad (\text{B16})$$

Agent 1 will optimize his decision according to

$$\begin{aligned} 0 &= \frac{\partial \Pi_1(d)}{\partial d} \\ &= \frac{\partial \tau(d)}{\partial d} - 2(d - x_1^*) \\ d^* &= \frac{\frac{\partial \tau(d)}{\partial d} + 2x_1^*}{2}. \end{aligned} \quad (\text{B17})$$

Clearly, if the price does not change with  $d$  then the partial derivative equals zero and agent 1’s optimal choice is his bliss point. If, however, the price changes with  $d$  he will choose something different than his bliss point. In fact, the optimal mechanism is to give agent 1 the entire aggregate payoff via the price. This is accomplished by transferring agent 2’s payoff,

$$\tau(d) = \pi_2 - (d - x_2^*)^2. \quad (\text{B18})$$

By giving agent 1 the entire aggregate payoff, the agent fully internalizes the effect of his choice, i.e., the multi-agent problem simplifies to a single agent optimizing over two

payoff functions. Consequently, agent 1 makes the first-best decision,

$$\begin{aligned}
 d^* &= \frac{\frac{\partial \tau(d)}{\partial d} + 2x_1^*}{2} \\
 &= \frac{-2(d - x_2^*) + 2x_1^*}{2} \\
 &= \frac{x_1^* + x_2^*}{2}.
 \end{aligned} \tag{B19}$$

This result has been studied extensively in the hold-up problem literature, e.g., Grossman and Hart (1986) and Hart and Moore (1990), and the result demonstrates that separating contracts are optimal. Although this is important to mention in the context of my analysis, it does not invalidate the results of this paper. Pooling contracts, for example contracts with a single price, are ubiquitous in financial markets. My analysis does not focus on the particular micro foundation by which pooling contracts arise, but research in the area of incomplete contract considers this matter, e.g., Anderlini and Felli (1994), Hart and Moore (1999) Maskin and Tirole (1999) and Segal (1999).

## B.5 Strategic Uncertainty and Rules: Generalized Results

In Section 6, I utilize the mean-preserving linear distribution and I show that discretion is increasing in the distribution's variance. Although the decision-maker enjoys the additional discretion that accompanies larger variance, I show that the decision-maker's ex ante payoff is decreasing in the distribution's variance. In this section I consider a broader class of distributions and provide the conditions under which the results of Section 6 hold.

Consider a general distribution  $f(x, \sigma)$  on the support  $[0, D]$ . Furthermore, assume that the probability density function is differentiable. The distribution  $f(x, \sigma)$  is second-order stochastically ordered such that for any  $\hat{\sigma} > \sigma$ , it can be said that  $f(x, \hat{\sigma})$  is a mean-preserving spread of  $f(x, \sigma)$ , i.e.,  $f(x, \sigma)$  second-order stochastically dominates  $f(x, \hat{\sigma})$ . The following proposition provides two conditions on the distribution's hazard rate for the decision-maker's discretion to be increasing in  $\sigma$ .

**Proposition B4.** *Discretion is increasing in the variance of  $f(x, \sigma)$  if the distribution's hazard rate satisfies the following two properties,*

1.  $\frac{f(\underline{\alpha}, \sigma)}{F(\underline{\alpha}, \sigma)} \leq \frac{2 \int_0^{\underline{\alpha}} f(x, \sigma) dx}{\int_0^{\underline{\alpha}} F(x, \sigma) dx},$

$$2. \frac{f(\bar{\alpha}, \sigma)}{(1-F(\bar{\alpha}, \sigma))} \leq \frac{2 \int_{\bar{\alpha}}^1 f(x, \sigma) dx}{\int_{\bar{\alpha}}^1 (1-F(x, \sigma)) dx}.$$

**Proof of Proposition B4:** The distribution  $f(x, \sigma)$  is second-order stochastically ordered such that for any  $\hat{\sigma} > \sigma$ , it can be said that  $f(x, \hat{\sigma})$  is a mean-preserving spread of  $f(x, \sigma)$ , i.e.,  $f(x, \sigma)$  second-order stochastically dominates  $f(x, \hat{\sigma})$ .

The implicit function theorem is used to determine the change in  $\underline{\alpha}$  and  $\bar{\alpha}$  with respect to a change in  $\sigma$ . I begin with  $\underline{\alpha}$  and define  $\underline{\Psi}(a)$

$$\underline{\Psi}(a) \equiv a - \frac{\int_0^a x f(x, \sigma) dx}{\int_0^a f(x, \sigma) dx} + x_2^*, \quad (\text{B20})$$

and note that  $\underline{\Psi}(\underline{\alpha}) = 0$ . The implicit-function theorem gives,

$$\frac{\partial \underline{\alpha}}{\partial \sigma} = - \frac{\partial \underline{\Psi} / \partial \sigma}{\partial \underline{\Psi} / \partial a} \Big|_{a=\underline{\alpha}}. \quad (\text{B21})$$

First, we begin with  $\partial \underline{\Psi} / \partial \sigma$ ,

$$\frac{\partial \underline{\Psi}}{\partial \sigma} = \frac{-F(a, \sigma) \int_0^a x \frac{\partial f(x, \sigma)}{\partial \sigma} dx + \int_0^a x f(x, \sigma) dx \int_0^a \frac{\partial f(x, \sigma)}{\partial \sigma} dx}{2F(a, \sigma)^2}, \quad (\text{B22})$$

and note that the result is positive by the definition of a mean-preserving spread. Next, we evaluate  $\partial \underline{\Psi} / \partial a$ ,

$$\begin{aligned} \frac{\partial \underline{\Psi}}{\partial a} &= 1 - \frac{aF(a, \sigma)f(a, \sigma) - f(a, \sigma) \int_0^a x f(x, \sigma) dx}{2F(a, \sigma)^2} \\ &= 1 - \frac{aF(a, \sigma)f(a, \sigma) - f(a, \sigma) (aF(a, \sigma) - \int_0^a F(x, \sigma) dx)}{2F(a, \sigma)^2} \\ &= 1 - \frac{f(a, \sigma) \int_0^a F(x, \sigma) dx}{2F(a, \sigma) \int_0^a f(x, \sigma) dx}, \end{aligned} \quad (\text{B23})$$

which is positive if the following relationship holds for the distribution's hazard rate,

$$\frac{f(a, \sigma)}{F(a, \sigma)} \leq \frac{2 \int_0^a f(x, \sigma) dx}{\int_0^a F(x, \sigma) dx}. \quad (\text{B24})$$

Therefore,

$$\begin{aligned} \frac{\partial \underline{\alpha}}{\partial \sigma} &= - \frac{\partial \underline{\Psi} / \partial \sigma}{\partial \underline{\Psi} / \partial a} \Big|_{a=\underline{\alpha}} \\ &= \frac{F(\underline{\alpha}, \sigma) \int_0^{\underline{\alpha}} x \frac{\partial f(x, \sigma)}{\partial \sigma} dx - \int_0^{\underline{\alpha}} x f(x, \sigma) dx \int_0^{\underline{\alpha}} \frac{\partial f(x, \sigma)}{\partial \sigma} dx}{2F(\underline{\alpha}, \sigma)^2 - f(\underline{\alpha}, \sigma) \int_0^{\underline{\alpha}} F(x, \sigma) dx} \\ &\leq 0, \end{aligned} \quad (\text{B25})$$

if the distribution's hazard rate satisfies Equation B24.

I now proceed to  $\bar{\alpha}$  and define  $\bar{\Psi}(a)$

$$\bar{\Psi}(a) \equiv a - \frac{\int_a^1 x f(x, \sigma) dx}{\int_a^1 f(x, \sigma) dx} + x_2^*, \quad (\text{B26})$$

and note that  $\bar{\Psi}(\bar{\alpha}) = 0$ . The implicit-function theorem gives,

$$\frac{\partial \bar{\alpha}}{\partial \sigma} = - \left. \frac{\partial \bar{\Psi} / \partial \sigma}{\partial \bar{\Psi} / \partial a} \right|_{a=\bar{\alpha}}. \quad (\text{B27})$$

We begin with  $\partial \bar{\Psi} / \partial \sigma$ ,

$$\frac{\partial \bar{\Psi}}{\partial \sigma} = \frac{-(1 - F(a, \sigma)) \int_a^1 x \frac{\partial f(x, \sigma)}{\partial \sigma} dx + \int_a^1 x f(x, \sigma) dx \int_a^1 \frac{\partial f(x, \sigma)}{\partial \sigma} dx}{2(1 - F(a, \sigma))^2}, \quad (\text{B28})$$

and note that the result is negative by the definition of a mean-preserving spread. Next, we evaluate  $\partial \bar{\Psi} / \partial a$ ,

$$\begin{aligned} \frac{\partial \bar{\Psi}}{\partial a} &= 1 - \frac{-af(a, \sigma)(1 - F(a, \sigma)) + f(a, \sigma) \int_a^1 x f(x, \sigma) dx}{2(1 - F(a, \sigma))^2} \\ &= 1 - \frac{-af(a, \sigma)(1 - F(a, \sigma)) + f(a, \sigma) \left(1 - aF(a, \sigma) - \int_a^1 F(x, \sigma) dx\right)}{2(1 - F(a, \sigma))^2} \\ &= 1 - \frac{-af(a, \sigma) + af(a, \sigma)F(a, \sigma) + f(a, \sigma) - af(a, \sigma)F(a, \sigma) - \int_a^1 F(x, \sigma) dx}{2(1 - F(a, \sigma))^2} \\ &= 1 - \frac{f(a, \sigma) \int_a^1 (1 - F(x, \sigma)) dx}{2(1 - F(a, \sigma)) \int_a^1 f(x, \sigma) dx}, \end{aligned} \quad (\text{B29})$$

which is positive if the following relationship holds for the distribution's hazard rate,

$$\frac{f(a, \sigma)}{(1 - F(a, \sigma))} \leq \frac{2 \int_a^1 f(x, \sigma) dx}{\int_a^1 (1 - F(x, \sigma)) dx}. \quad (\text{B30})$$

Therefore,

$$\begin{aligned} \frac{\partial \bar{\alpha}}{\partial \sigma} &= - \left. \frac{\partial \bar{\Psi} / \partial \sigma}{\partial \bar{\Psi} / \partial a} \right|_{a=\bar{\alpha}} \\ &= \frac{(1 - F(a, \sigma)) \int_a^1 x \frac{\partial f(x, \sigma)}{\partial \sigma} dx - \int_a^1 x f(x, \sigma) dx \int_a^1 \frac{\partial f(x, \sigma)}{\partial \sigma} dx}{2(1 - F(a, \sigma))^2 - f(a, \sigma) \int_a^1 (1 - F(x, \sigma)) dx} \\ &\geq 0. \end{aligned} \quad (\text{B31})$$

■

The following two propositions provide sufficient conditions for which the result of Proposition 9 holds in a general class of probability distributions.

**Proposition B5.** *A sufficient condition on  $f(x, \sigma)$ , such that agent 1's ex ante payoff is decreasing in the distribution's variance, is,*

$$\frac{\partial \underline{\alpha}}{\partial \sigma} \geq -\frac{\partial F / \partial \sigma}{\partial F / \partial x}. \quad (\text{B32})$$

**Proof of Proposition B6:** Recall, the distribution  $f(x, \sigma)$  is second-order stochastically ordered such that for any  $\hat{\sigma} > \sigma$ , it can be said that  $f(x, \hat{\sigma})$  is a mean-preserving spread of  $f(x, \sigma)$ , i.e.,  $f(x, \sigma)$  second-order stochastically dominates  $f(x, \hat{\sigma})$ .

Increasing  $\sigma$  yields two competing ex ante effects: it increases the decision-maker's discretion and increases the probability of an extreme realization. These effects occur for both the rule's lower bound  $\underline{\alpha}$  and the rule's upper bound  $\bar{\alpha}$ . For brevity, we consider the effect on the lower bound first,

$$\frac{\partial \Pi_1(\sigma)}{\partial \sigma} = -\int_0^{\underline{\alpha}(\sigma)} 2(\underline{\alpha}(\sigma) - x) \frac{\partial \underline{\alpha}}{\partial \sigma} f(x, \sigma) dx - \int_0^{\underline{\alpha}(\sigma)} (\underline{\alpha}(\sigma) - x)^2 \frac{\partial f(x, \sigma)}{\partial \sigma} dx \quad (\text{B33})$$

$$= -2 \int_0^{\underline{\alpha}(\sigma)} (\underline{\alpha}(\sigma) - x) \frac{\partial \underline{\alpha}}{\partial \sigma} f(x, \sigma) dx - 2 \int_0^{\underline{\alpha}(\sigma)} (\underline{\alpha}(\sigma) - x) \frac{\partial F(x, \sigma)}{\partial \sigma} dx. \quad (\text{B34})$$

$$= -2 \int_0^{\underline{\alpha}(\sigma)} (\underline{\alpha}(\sigma) - x) \left( \frac{\partial \underline{\alpha}}{\partial \sigma} f(x, \sigma) + \frac{\partial F(x, \sigma)}{\partial \sigma} \right) dx \quad (\text{B35})$$

$$= -2 \int_0^{\underline{\alpha}(\sigma)} (\underline{\alpha}(\sigma) - x) \left( \frac{\partial \underline{\alpha}}{\partial \sigma} \frac{\partial F(x, \sigma)}{\partial x} + \frac{\partial F(x, \sigma)}{\partial \sigma} \right) dx \quad (\text{B36})$$

Therefore, for all  $x \in [0, \underline{\alpha}]$ , a sufficient condition for the payoff to be decreasing in  $\sigma$  is,

$$\frac{\partial \underline{\alpha}}{\partial \sigma} \geq -\frac{\partial F / \partial \sigma}{\partial F / \partial x}, \quad (\text{B37})$$

where

$$\frac{\partial \underline{\alpha}}{\partial \sigma} = \frac{F(\underline{\alpha}, \sigma) \int_0^{\underline{\alpha}} x \frac{\partial f(x, \sigma)}{\partial \sigma} dx - \int_0^{\underline{\alpha}} x f(x, \sigma) dx \int_0^{\underline{\alpha}} \frac{\partial f(x, \sigma)}{\partial \sigma} dx}{2F(\underline{\alpha}, \sigma) \int_0^{\underline{\alpha}} f(x, \sigma) dx - f(\underline{\alpha}, \sigma) \int_0^{\underline{\alpha}} F(x, \sigma) dx}. \quad (\text{B38})$$

■

**Proposition B6.** *If  $x_1^*$  and  $x_2^*$  are both independently distributed according to  $f(x, \sigma)$ , agent 1's payoff is maximized when there is no ex ante uncertainty.*

**Proof of Proposition B6:** Consider the limiting case where the distribution's entire mass is located at the mean,  $E[x_1^*] = x_1^* = x_2^* = E[x_2^*]$ . In the limiting case, the planner restricts agent 1 to choose  $E[x_1^*]$  and both agents obtain a quadratic cost of zero.

■

## C Appendix

The uniform and symmetric-triangular distributions are attractive choices for analytic modeling because they often yield clean closed-form solutions. Additionally, if the distributions are over the same support they share the same mean but differ in their variances. In the following section I derive a generalized linear distribution for which the uniform and symmetric-triangular distributions are special cases of. As such, the generalized form provides a class of linear distributions that share the same mean but differ in second-order stochastic dominance.

### C.1 Motivation

Consider a uniform distribution  $f(x)$  over the support  $[\underline{D}, \overline{D}]$ . The probability density function is simply,

$$f(x) = \frac{1}{\overline{D} - \underline{D}}. \quad (\text{C1})$$

Now, consider a symmetric-triangular distribution  $g(x)$  over the same support. The probability density function is piecewise-defined and has the form,

$$g(x) = \begin{cases} \frac{4(\frac{\overline{D} + \underline{D}}{2} - x)}{(\overline{D} - \underline{D})^2} & \underline{D} \leq x \leq \frac{\overline{D} + \underline{D}}{2} \\ \frac{4(x - \frac{\overline{D} + \underline{D}}{2})}{(\overline{D} - \underline{D})^2} & \frac{\overline{D} + \underline{D}}{2} < x \leq \overline{D}. \end{cases} \quad (\text{C2})$$

Although both distributions have the same mean,  $E_f[x] = E_g[x] = \frac{\overline{D} + \underline{D}}{2}$ , their variances differ. The uniform distribution has a variance of  $Var_f[x] = \frac{(\overline{D} - \underline{D})^2}{12}$ , which is less than the triangular distribution's variance of  $Var_g[x] = \frac{(\overline{D} - \underline{D})^2}{8}$ . The two distributions are depicted in Figure 1.9.

It is desirable to obtain a general linear distribution for which both the uniform and symmetric-triangular distributions are special cases of. Specifically, a linear distribution which is mean preserving and characterized by a single parameter  $\gamma$  that determines the distribution's bend and, consequently, its variance.



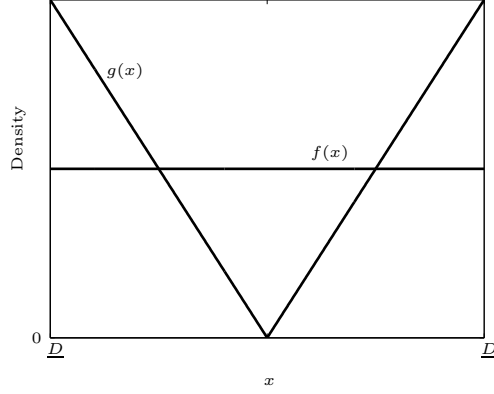


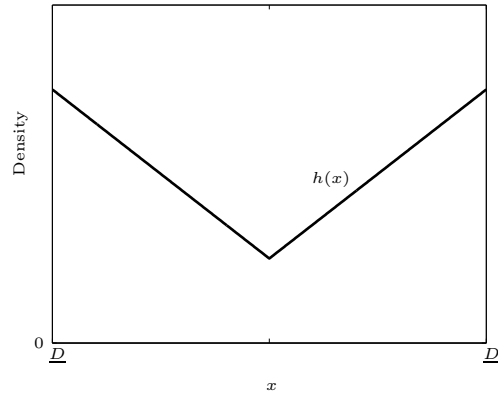
Figure 1.9: A uniform distribution  $f(x)$  and symmetric-triangular distribution  $g(x)$ . The two distributions share the same mean but differ in their respective variances.

## C.2 Mean-Preserving Linear Distribution

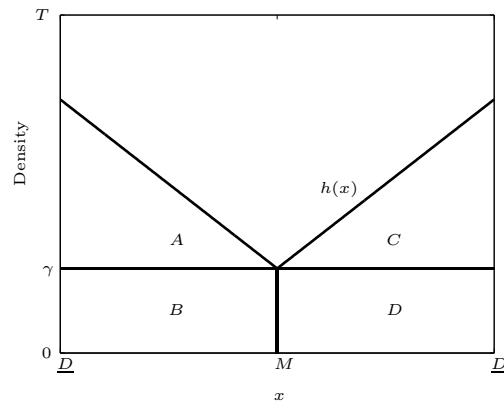
Consider the distribution in Figure 1.10(a). The distribution is neither the uniform or the symmetric-triangular distribution, rather it falls somewhere between the two. The analytic form of the distribution can be solved for with geometry. The regions  $A$ ,  $B$ ,  $C$  and  $D$  in Figure 1.10(b) represent the cumulative probability density. Denote  $M$  as the midpoint of the support  $[\underline{D}, \bar{D}]$  and  $\gamma$  to be the density evaluated at that midpoint. Additionally, let  $T$  be the density at  $g(\underline{D})$  and  $g(\bar{D})$ . The areas of  $A$  and  $C$  are given by  $(T - \gamma) \times \frac{\bar{D} - \underline{D}}{4}$  and the areas of  $B$  and  $D$  are  $\left(\frac{\bar{D} - \underline{D}}{2}\right) \times \gamma$ . The symmetric nature of the distribution ensures that  $A + B = C + D = \frac{1}{2}$ . Consequently,  $T$  is a function of  $\gamma$ ,  $\underline{D}$  and  $\bar{D}$ .

$$\begin{aligned} \frac{1}{2} &= \frac{(T - \gamma)(\bar{D} - \underline{D})}{4} + \frac{\gamma(\bar{D} - \underline{D})}{2} \\ \frac{2}{(\bar{D} - \underline{D})} &= (T - \gamma) + 2\gamma \\ T &= \frac{2 - \gamma(\bar{D} - \underline{D})}{(\bar{D} - \underline{D})}. \end{aligned} \tag{C3}$$

The distribution is piecewise-defined. The functional form of the distribution in the range of  $\underline{D} \leq x \leq \frac{\bar{D} + \underline{D}}{2}$  is the equation of a line that passes through the points



(a) Mean-preserving linear distribution.



(b) Mean-preserving linear distribution geometry.

Figure 1.10: Mean-preserving linear distribution.

$$(x_1, y_1) = (\underline{D}, T) \text{ and } (x_2, y_2) = \left(\frac{\overline{D} + \underline{D}}{2}, \gamma\right),$$

$$\begin{aligned} h(x) &= mx + b \\ &= \frac{y_2 - y_1}{x_2 - x_1}x + \left(y_1 - \frac{y_2 - y_1}{x_2 - x_1}x_1\right) \\ &= \frac{\gamma - T}{\frac{\overline{D} + \underline{D}}{2} - \underline{D}}x + \left(T - \frac{\gamma - T}{\frac{\overline{D} + \underline{D}}{2} - \underline{D}}\underline{D}\right). \end{aligned}$$

The above expression contains  $T$ , which is a function of  $\gamma$ ,  $\underline{D}$  and  $\overline{D}$  as shown in Equation C3,

$$\begin{aligned} &= \frac{-2(2 - 2\gamma(\overline{D} - \underline{D}))}{(\overline{D} - \underline{D})^2}x + \left(\frac{2 - \gamma(\overline{D} - \underline{D})}{(\overline{D} - \underline{D})} + \frac{2(2 - 2\gamma(\overline{D} - \underline{D}))}{(\overline{D} - \underline{D})^2}\underline{D}\right) \\ &= \frac{-2(2 - 2\gamma(\overline{D} - \underline{D}))(x - \underline{D}) + (2 - \gamma(\overline{D} - \underline{D}))(\overline{D} - \underline{D})}{(\overline{D} - \underline{D})^2}. \end{aligned} \quad (\text{C4})$$

Similarly, the functional form of the distribution in the range of  $\frac{\overline{D} + \underline{D}}{2} \leq x \leq \overline{D}$  is the equation of a line that passes through the points  $(x_1, y_1) = \left(\frac{\overline{D} + \underline{D}}{2}, \gamma\right)$  and  $(x_2, y_2) = (\overline{D}, T)$ ,

$$\begin{aligned} h(x) &= mx + b \\ &= \frac{y_2 - y_1}{x_2 - x_1}x + \left(y_1 - \frac{y_2 - y_1}{x_2 - x_1}x_1\right) \\ &= \frac{T - \gamma}{\overline{D} - \frac{\overline{D} + \underline{D}}{2}}x + \left(T - \frac{T - \gamma}{\overline{D} - \frac{\overline{D} + \underline{D}}{2}}\overline{D}\right). \end{aligned}$$

The above expression again contains  $T$ . An application of the result from Equation C3 yields,

$$\begin{aligned} &= \frac{2(2 - 2\gamma(\overline{D} - \underline{D}))}{(\overline{D} - \underline{D})^2}x + \left(\frac{2 - \gamma(\overline{D} - \underline{D})}{(\overline{D} - \underline{D})} - \frac{2(2 - 2\gamma(\overline{D} - \underline{D}))}{(\overline{D} - \underline{D})^2}\overline{D}\right) \\ &= \frac{2(2 - 2\gamma(\overline{D} - \underline{D}))(x - \overline{D}) + (2 - \gamma(\overline{D} - \underline{D}))(\overline{D} - \underline{D})}{(\overline{D} - \underline{D})^2}. \end{aligned} \quad (\text{C5})$$

The parameter  $\gamma$ , which represents the height of rectangles  $B$  and  $D$  in Figure 1.10(b), cannot be less than 0 or greater than  $\frac{2}{\overline{D} - \underline{D}}$ . Equations C4 and C5 yield a

probability density function of the form,

$$h(x, \gamma) = \begin{cases} \frac{-2(2-2\gamma(\bar{D}-\underline{D}))(x-\underline{D})+(2-\gamma(\bar{D}-\underline{D}))(\bar{D}-\underline{D})}{(\bar{D}-\underline{D})^2} & \underline{D} \leq x \leq \frac{\bar{D}+\underline{D}}{2} \\ \frac{2(2-2\gamma(\bar{D}-\underline{D}))(x-\bar{D})+(2-\gamma(\bar{D}-\underline{D}))(\bar{D}-\underline{D})}{(\bar{D}-\underline{D})^2} & \frac{\bar{D}+\underline{D}}{2} \leq x \leq \bar{D} \end{cases} \quad (\text{C6})$$

with  $\gamma \in \left[0, \frac{2}{\bar{D}-\underline{D}}\right]$ .

The cumulative density function is given by,

$$H(x, \gamma) = \begin{cases} \frac{(2(\bar{D}-x)-\gamma(\bar{D}-\underline{D})(\bar{D}+\underline{D}-2x))(x-\underline{D})}{(\bar{D}-\underline{D})^2} & \underline{D} \leq x \leq \frac{\bar{D}+\underline{D}}{2} \\ \frac{(\bar{D}-x)^2(1+\gamma(x-\bar{D}))+(x-\underline{D})^2(1+\gamma(\bar{D}-x))}{(\bar{D}-\underline{D})^2} & \frac{\bar{D}+\underline{D}}{2} \leq x \leq \bar{D} \end{cases} \quad (\text{C7})$$

with  $\gamma \in \left[0, \frac{2}{\bar{D}-\underline{D}}\right]$ .

The mean and variance of the distribution are given by,

$$E[x] = \frac{\bar{D} + \underline{D}}{2} \quad (\text{C8})$$

$$E[x^2] - E[x]^2 = \frac{(\bar{D} - \underline{D})^2(3 - \gamma(\bar{D} - \underline{D}))}{24}. \quad (\text{C9})$$

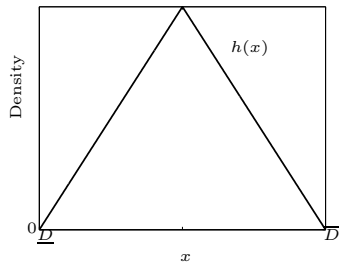
### C.2.1 Special Case: Unit Support

A special case of the mean-preserving linear distribution is when  $[0, D] = [0, 1]$ . The probability density function and corresponding cumulative density function are given by,

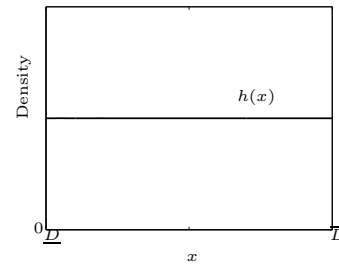
$$h(x, \gamma) = \begin{cases} -4(1-\gamma)x + (2-\gamma) & 0 \leq x \leq \frac{1}{2} \\ 4(1-\gamma)(x-1) + (2-\gamma) & \frac{1}{2} \leq x \leq 1 \end{cases} \quad (\text{C10})$$

$$H(x, \gamma) = \begin{cases} (2(1-x) - \gamma(1-2x))x & 0 \leq x \leq \frac{1}{2} \\ (1-x)^2(1+\gamma(x-1)) + x^2(1+\gamma(1-x)) & \frac{1}{2} \leq x \leq 1 \end{cases} \quad (\text{C11})$$

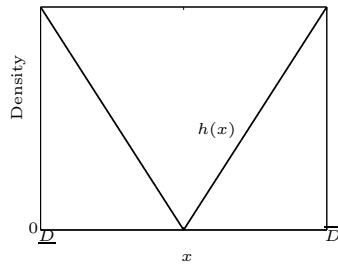
with  $\gamma \in [0, 2]$ .



(a)  $\gamma = \frac{2}{D-D}$ .



(b)  $\gamma = \frac{1}{D-D}$ .



(c)  $\gamma = 0$ .

Figure 1.11: Mean-preserving linear distribution examples.

The mean and variance of the distribution are given by,

$$E[x] = \frac{1}{2} \tag{C12}$$

$$E[x^2] - E[x]^2 = \frac{3-\gamma}{24}. \tag{C13}$$

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## CHAPTER 2

# Competition, Comparative Performance, and Market Transparency

### 1 Introduction

Early work on disclosure theory suggests that market forces are sufficient to induce full disclosure. Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) argue that in the absence of market frictions, adverse selection prompts those with good news to distinguish themselves from others by disclosing their information. This reduces the expected prospects for the remaining market and induces a cascade in which everyone discloses their information.

Subsequent work challenges this. Full disclosure may not occur because disclosure is costly (e.g. Verrecchia 1983; Fishman and Hagerty 1990), some market participants are unsophisticated (e.g., Fishman and Hagerty 2003), or there is uncertainty whether asymmetric information exists in the market (Dye 1985; Matthews and Postlewaite 1985; Jung and Kwon 1988; Shin, 2003; Acharya, DeMarzo, and Kremer 2010). In the face of such market frictions, adverse selection may prevent market transparency.

All of the prior literature, however, focuses only on absolute performance and ignores comparative performance. That is, when one entity outperforms another, there are added spoils that go to the victor, and this needs to be taken into consideration. Comparative performance is important in any tournament or contest (e.g., Rosen 1981; Lazear and Rosen 1981), especially when scarce resources are allocated or when reputation adds value. As we describe in the paper, this arises in many economic arenas: financial reporting, money-management, academic research, job applications, and marriage markets. The key insight is that when people choose whether to reveal information, they know that their comparative performance will be evaluated in addition to their absolute performance.

In this paper, we study how competition affects market transparency, taking into account that comparative performance matters. Our primary contribution is that we show that increased competition usually makes disclosure *less* likely, which lowers market transparency and may decrease per capita welfare. Especially in a tournament setting, we cannot rely on the Invisible Hand to induce informational efficiency. This has implications for market regulation.

We build on the model of Dye (1985), where incomplete disclosure results from investors' uncertainty as to whether or not a firm possesses relevant information.<sup>12</sup> In our variant, a finite number of firms compete in the market. All firms experience a random shock that changes their fundamental value. Each firm may or may not observe the precise value of their shock. Firms that make an observation simultaneously choose whether to announce it publicly, while firms with no new information have nothing to reveal. The firm with the best announcement gets a fixed prize from the market, which represents the rank-based remuneration previously described.

In the unique Nash equilibrium of the game, each firm with new information applies the same threshold in deciding whether or not to reveal its news. If the observed shock value is above this threshold, the firm announces it and competes for the prize. If the observed shock is lower, however, the firm conceals its information. The presence of uninformed firms lends plausible deniability to informed firms wishing to conceal a bad observation. As such, competing for the prize has an opportunity cost: firms who disclose give up their chance to pool with other firms. Given this, rational investors use Bayesian learning to adjust the market price of firms that do not release any news.

Because the probability of winning the prize drops when more firms compete, the benefit of making announcements decreases with competition. Increasing competition makes pooling with other firms more attractive compared to the benefit of vying for the prize, which leads to decreased information revelation and lower market transparency. In the limit, when the market is perfectly competitive, transparency is minimized because the individual probability of winning the prize goes to zero.

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<sup>1</sup>Going forward for clarity, we always refer to the solicitor who holds private information as the “firm” and the solicited agent as the “investor”, while keeping in mind the breadth of economic applications noted earlier.

<sup>2</sup>Disclosure is costly in Dye (1985) because firms give up their opportunity to pool with others. Our use of this framework does not make our analysis special, however. Competition would still have the same effects that we demonstrate in any other form of costly discretionary disclosure noted above.

Although it may not be terribly surprising that the influence of a single fixed prize decreases with the number of firms eligible to win it, we find the result to be robust to alternate prize forms: progressive reward systems (i.e., prizes are awarded to runners up), prizes that change in size as a result of competition, prizes awarded based on percentile, and sequential disclosure. Most notably, perfect competition leads to minimal market transparency in all of the model variations we analyze except one: when the prize grows exponentially with competition *ad infinitum*. Given that this is rather unlikely to occur in reality, we view this result with considerable generality.

In much of our analysis, the distribution that the firms draw their value from does not vary with competition. We relax this assumption to add a reduced-form model of product market competition, in addition to the considerations analyzed so far.<sup>3</sup> Now, the market share for each firm shrinks with competition, which makes the ratio of the prize to firm revenues grow with competition. For a small oligopoly, we show that this causes added competition to increase disclosure. However once additional firms are added, the effect of competition decreases market transparency. Intuitively, when a small number of firms are present, disclosure is a mechanism to prove oneself. But as the probability of winning the prize shrinks with competition, the incentive to hoard information grows too large.

Our work not only adds to the previous literature on disclosure and transparency, but also contributes to the work on tournaments and contests. Previous papers have focused on whether tournaments optimally solve moral hazard problems (e.g., Lazear and Rosen 1981; Green and Stokey 1983; Nalebuff and Stiglitz 1983a; Nalebuff and Stiglitz 1983b; Moldovanu and Sela 2001). These papers weigh the merits of using relative performance measures in settings in which performance is correlated. We add to this literature by considering adverse selection instead and assessing the effect of competition while taking the tournament mechanism as primitive. Indeed, as implied by Nalebuff and Stiglitz (1983a), competition disrupts the incentives to perform. In their setting, it imposes too much risk on participants. In ours, competition induces firms to hoard information and not vie for the prize.

Finally, our analysis has normative implications. We conclude that if transparency

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<sup>3</sup>This adds to the previous literature on product market competition and discretionary disclosure. See Wagenhofer (1990), Darrough and Stoughton (1990), Feltham and Xie (1992), Darrough (1993), Dye and Sridhar (1995) Clinch and Verrecchia (1997), Pae (2002), Stivers (2004), and Board (2009).

is considered a good, policy makers cannot simply depend on competition to induce transparency. They need to carefully consider the type of competition that takes place in markets before deciding whether regulation is necessary. When comparative performance matters, competition for remuneration may make disclosure less attractive, which may lower efficiency in the market. In this light, competition should *not* be viewed as a panacea to assure information disclosure and self-regulation by market participants.

The rest of the paper is organized as follows. Section 2 introduces our base model, characterizes the equilibrium, and addresses whether our results are robust to other prize structures. In Section 3, we add product market competition to the model. Section 4 concludes. Proofs of all propositions are deferred to Appendix A. In Appendix B, we explore the potential welfare implications of our results.

## 2 Discretionary Disclosure

### 2.1 Base Model

Consider that  $N$  risk-neutral firms compete in a one-period game of discretionary disclosure. Each firm  $j \in \{1, \dots, N\}$  experiences a random change  $\tilde{x}_j$ , which is distributed according to a twice continuously differentiable function  $F(x)$  on  $\mathbb{R}$ . We assume that  $f(x) > 0$  for all  $x \in \mathbb{R}$  and  $E[\tilde{x}_j] = 0$ .<sup>4</sup> Realizations are independent and identically distributed for each firm. For now,  $F(x)$  does not depend on  $N$ , but we relax this assumption in Section 3.

Each firm  $j$  observes the true realization of  $\tilde{x}_j$  with probability  $p$ , and observes nothing otherwise. As such, the parameter  $p$  measures the degree of asymmetric information and may be interpreted as a measure of strong form market (in-)efficiency. The probability  $p$  is given exogenously and we assume that firms cannot alter its value.<sup>5</sup>

Any firm  $j$  that observes  $\tilde{x}_j$  may either conceal its value ( $C_j$ ), or may credibly disclose it to investors ( $D_j$ ). For a given realization  $\tilde{x}_j = x$ , we denote the change in

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<sup>4</sup>As will become clear, the assumption that  $E[\tilde{x}_j] = 0$  is without loss of generality. For any distribution in which  $E[\tilde{x}_j] \neq 0$ , rational investors would update their valuations of the firms to take this into consideration. Therefore, by setting  $E[\tilde{x}_j] = 0$ , we are considering the news that investors cannot readily predict before any announcements are made.

<sup>5</sup>See Matthews and Postlewaite (1985) for treatment of the monopoly case in which the firm chooses  $p$  endogenously (i.e., chooses the quality of its information).

firm value following these actions as  $u_j^C(x)$  and  $u_j^D(x)$ . Following Dye (1985) and Jung and Kwon (1988), firms that do not observe  $\tilde{x}_j$ , are not permitted to fabricate one.<sup>6</sup> That is, we assume that investors can freely verify and penalize false claims of  $\tilde{x}_j$ , but cannot determine whether a non-disclosing firm is in fact concealing information.

Each informed firm  $j$  determines its action using a disclosure policy  $\sigma_j : \mathbb{R} \rightarrow [0, 1]$ , where the firm discloses with probability  $\sigma_j(x)$  and conceals its information with probability  $1 - \sigma_j(x)$ . As such, firms may choose non-deterministic strategies, although we will show shortly that deterministic strategies are optimal, almost surely. Let  $\sigma \equiv \{\sigma_1, \dots, \sigma_N\}$  and  $\sigma_{-j} \equiv \{\sigma_1, \dots, \sigma_{j-1}, \sigma_{j+1}, \dots, \sigma_N\}$ . All informed firms act simultaneously to maximize firm value and do not know which of their competitors are also informed at the time of their decision. We later extend our model and consider a sequential game of disclosure in Section 2.4.3.

Investors are competitive, risk-neutral, and have rational expectations about firm behavior. As such, investors price each firm according to  $u_j^C(x)$  and  $u_j^D(x)$ . Let  $P_j$  be the event that a firm  $j$  does not disclose new information. This may occur because the firm is legitimately uninformed or because it is concealing a bad realization (i.e, pooling with uninformed firms). Investors use Bayesian inference to calculate the firm's change in value, which is expressed as  $u_j^C(x) = E[\tilde{x}_j | P_j, \sigma, p]$ .

After all disclosures have been made, investors award a fixed prize  $\phi$  to the firm with the highest disclosed value. Given this, firm  $j$ 's expected value of revealing  $x$  is

$$u_j^D(x) \equiv x + \phi W_j(x, p, \sigma_{-j}), \quad (1)$$

where  $W_j(x, p, \sigma_{-j})$  is the probability that firm  $j$  has the highest disclosure. This value depends on the probability  $p$  of competing firms being informed, and on the strategies  $\sigma_{-j}$  they employ.

## 2.2 Applications

Before characterizing the equilibrium of the game, it is instructive to note some of the model's economic applications. The model is relevant when superior relative performance is rewarded and rankings drive remuneration. In the context of Dye (1985),

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<sup>6</sup>This assumption is very common in the disclosure literature. See page 329 of Matthews and Postlewaite (1985) for a good motivation of this assumption.

investors not only reward firms for their disclosed performance, but also take into account the future decisions by third parties who do business with each firm. When third parties allocate scarce resources to top-performing firms, this relaxes their budget constraints and increases each firm's opportunity set. Indeed, empirical evidence confirms that top firms enjoy superior access to human capital (Gatewood, Gowan, and Lautenschlager 1993), venture capital (Hsu 2004), and media attention and market coverage (Hendricks and Singhal 1996). In the context of our model, investors not only update their estimate of a firm's current operations  $\tilde{x}_j$ , but also take into account the added value  $\phi$  a firm generates from third parties when it outperforms their competition. Stock prices reflect both components of value.

Our model also applies to other economic arenas, especially when reputation is an important driver of value. For example, academic job market candidates who outperform their competitors and receive better initial job placement, often get the prize of more exposure in the future. Likewise, scientists who win grants find it easier to obtain future funding. Disclosing one's true value in these settings not only conveys information to others, but also allows the information sender to better compete for future spoils.

This consideration is particularly important when information about objective performance is difficult to obtain or interpret, such as in the money management industry. Here, the prizes are convex investor flows that funds enjoy when they outperform competitors (Brown, Harlow, and Starks 1996; Berk and Green 2004; Del Guercio and Tkac 2008). The importance of such gains is magnified in the hedge fund industry where funds are restricted by law to marketing their services to a small set of qualified investors. Because of this, there is a paucity of public performance data and disclosure is largely discretionary via for-profit databases (Ackerman, McEnally and Ravenscraft 1999; Malkiel and Saha 2005; Stulz 2007). Our model fits the strategic behavior in this industry particularly well. Funds that disclose performance compete for investor flows, whereas funds who do not disclose may do so for reasons unrelated to poor performance: they may be at their investor cap, may not be seeking new capital, or may be unable to make useful disclosure about valuations because some assets are illiquid.

Finally, our model is relevant when individual agents interact. For example, consider a marriage market in which young men court a particular woman, at the same time uncertainty about their future career prospects is being resolved. To see how our



model applies, suppose that  $N$  men court a single woman, who chooses the suitor with the best financial future  $x$ .<sup>7</sup> This  $x$  could represent the man’s salary after an important review, the companies that have made him job offers, or any other verifiable proxy of future success. With probability  $p$ , the man is already privately informed of  $x$  and may choose whether to disclose it or not. Disclosure affects both a man’s social standing among peers  $\theta(x)$ , and also makes him eligible for the “prize”  $\phi$  of marrying the woman. If the suitor does not disclose  $x$ , he is not eligible to win  $\phi$  because the woman is not able to evaluate him as a marriage prospect. Disclosing yields

$$E[u^D(x)] = \theta(x) + \phi \Pr(x \text{ highest among suitors}), \quad (2)$$

and not disclosing yields

$$E[u^C(x)] = (1 - p)E[\theta(x)] + pE[\theta(x)|x \text{ concealed}]. \quad (3)$$

These payoff functions match those in our model, except for the monotonic transformation  $\theta$ . As such, the suitor’s disclosure decision is isomorphic to the firm’s disclosure decision in our extension of the Dye (1985) model.

### 2.3 Equilibrium Disclosure

The following proposition characterizes the game’s unique Nash equilibrium.

**Proposition 1.** *There exists a unique and non-trivial Nash equilibrium, in which every firm discloses according to a common threshold  $t^*$  defined implicitly by*

$$t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx. \quad (4)$$

*Further, the threshold  $t^*$  lies below the unconditional mean of  $\tilde{x}_j$ , i.e.  $t^* < 0$ .*

The proof of the proposition, which is detailed in the appendix, proceeds in three steps. First, we show that each firm acts according to a **disclosure threshold**  $t_j < 0$ , in which each informed firm simply compares the expected utility it can obtain by revealing information to what it obtains by pooling. The threshold for each firm  $j$  is defined implicitly by

$$u_j^D(t_j) = u_j^C. \quad (5)$$

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<sup>7</sup>In reality, there may be other dimensions that are compared, but we include a single one here for purpose of example.

Second, we show that every firm uses the same disclosure threshold, which we denote as  $t^*$ . As such, there cannot be an equilibrium in which some firms are more “honest” than others. This is not too surprising, since all firms draw the observations from identical and independent distributions. Third, we show that the common disclosure threshold  $t^*$  is in fact unique.

The expression in (4) implicitly defines the unique threshold for the game. The left side is the utility that a firm enjoys if it observes  $\tilde{x}_j = t^*$  and reveals its information. In this case, the firm immediately receives its own value  $\tilde{x}_j = t^*$ , and can also win the prize  $\phi$  if its disclosure is the highest. But since competing firms never reveal values below  $t^*$ , any other disclosing firm will have a higher value almost surely. Firm  $j$  can win, therefore, only if all other firms pool and it is the only one to disclose. Each competing firm pools if either it is uninformed or it is informed with an observation below the threshold. These events occur with probabilities  $(1 - p)$  and  $pF(t^*)$ , respectively.

The right side is the utility a firm obtains by concealing its observation, which equals a pooling firm’s expected change in value. Such a firm could be uninformed and have a zero expected value for its observation, or could be hiding an observation lower than the threshold. The weighted average of these possibilities yields the right side of Equation 4.

It is important to note that the threshold  $t^*$  is lower than the distribution mean, which we’ve assumed to be zero. The average  $\tilde{x}_j$  for an uninformed firm is simply the distribution mean, and because firms disclose their best observations, rational investors expect the average concealed observation to be negative. The weighted average assigned to pooling firms must therefore be below the distribution mean. Since disclosure yields strictly greater expected utility than the value disclosed, no firm will ever conceal an above-average observation. So if firm  $j$  is indifferent between revealing and concealing a value  $x_j = t^*$ , then  $t^* < 0$ .

Given the existence of a unique  $t^*$ , we could use the expression in (4) and the Implicit Function Theorem to predict how disclosure behavior responds to exogenous parameter changes. However, a more empirically relevant characterization would describe the **disclosure frequency** with which firms opt to reveal private information. That is, what is the ex ante probability that a firm, if it observes its value change, will choose to share its observation with investors?

To address this, we define  $\omega^*$  to be the ex ante probability that an informed firm

discloses in equilibrium. By construction,  $\omega^* = 1 - F(t^*)$ , which implies that if a firm lowers its threshold, it discloses more of its realized values, and vice-versa. We also denote  $\hat{\omega}$  as the equilibrium disclosure frequency when  $\phi = 0$ , which corresponds to the equilibrium condition derived in Dye (1985) and Jung & Kwon (1988) when there is no strategic interaction. When  $\phi > 0$ ,

$$\omega^* \geq \hat{\omega} \quad \forall \phi \geq 0, \forall N < \infty. \quad (6)$$

As such,  $\hat{\omega}$  is a lower bound for  $\omega^*$  over all  $\phi$  and  $N$ . In fact, it is the largest possible lower bound. The lower bound is defined implicitly by

$$F^{-1}(1 - \hat{\omega}) = \frac{p \int_{\hat{\omega}}^1 t(\Omega) d\Omega}{1 - p\hat{\omega}}. \quad (7)$$

**Proposition 2.** *Equilibrium disclosure frequency is decreasing in  $N$  and increasing in  $\phi$ . As  $N \rightarrow \infty$ ,  $\omega^*$  converges to  $\hat{\omega}$ .*

According to Proposition 2, when firms disclose competitively in a tournament-like setting, increased competition reduces disclosure. Increased competition drives firms to hoard their informational advantage over investors. The result has immediate application in many economic settings, especially in the financial sector (e.g., money management), where disclosure is critical and where top-performing firms enjoy large rewards.

Mathematically, the cause of the competition effect is straightforward. As more firms enter the market, each firm's chance of making the highest disclosure diminishes exponentially. Since the disclosure decision is a trade-off between the desire to win the prize and the desire to conceal bad signals, additional firm entry tips the balance in favor of concealing. In the sections that follow, we will show this effect to be robust to other types of prizes and prize structures.

Proposition 2 also states simply that firms will be more inclined to disclose when the prize they can win is large. Again, a larger prize tips the balance between the opportunity to pool with other firms and to compete openly for the prize. This concept is also robust to our alternative model specifications.

The comparative statics in  $p$  turn out to be trickier. Jung and Kwon (1988) consider the special case where  $N = 1$  and  $\phi = 0$ , and find disclosure to be strictly increasing in  $p$ . We are able to confirm this result by computing our comparative statics with  $\phi = 0$ . But when there is a prize, the situation becomes more complicated.

With a prize, if  $p$  were to increase and firms do not adjust their disclosure strategies, there would be two sources of change in firm utility. First, the increase in asymmetric information would increase the Bayesian probability of a firm having inside information. Rational investors would respond by reducing their assessment  $u^C$  of pooling firms. Second, the increase in  $p$  means competing firms are more likely to be informed. Since being informed is a prerequisite to disclosing, the increase in  $p$  makes any given disclosing firm less likely to win the prize by default. Mathematically, a higher  $p$  decreases  $W_j(x)$ , which implies a lower expected utility of disclosure.

These two effects of increased  $p$  work against each other. To determine whether  $\omega^*$  will increase or decrease, we need to know which of these effect impact firm utility more. If the reduction in  $u^C(\omega^*)$  is larger than the reduction in  $u^D(\omega^*)$ , then disclosure becomes more appealing. Firms will then respond to an increase in  $p$  by disclosing more frequently. Conversely, if the reduction in  $u^D(\omega^*)$  dominates, then firms respond with less frequent disclosure.

If the prize value  $\phi$  is small or zero, then the reduction in  $W_j(x)$  is unimportant, so the reduction in  $u^C(\omega^*)$  dominates, and equilibrium disclosure increases. This echoes the Jung and Kwon (1988) result. The same result follows when  $N$  is very large, in which case the probability of winning the prize is low from the outset. In contrast, when  $\phi$  is large and  $N$  is modest, the reduction in  $W_j(x)$  is critical. The second effect dominates, so overall the incentive to disclose is reduced more than the incentive to pool. Consequently, firms pool more often, reducing the equilibrium disclosure frequency.

## 2.4 Other Prize Structures

Now, we consider other prize structures to determine whether competition's effect of reducing disclosure is robust to alternative model specifications. In what follows, we continue to assume that  $F(x)$  does not depend on  $N$ .

### 2.4.1 Increasing/Decreasing Prize Values in $N$

Consider that the prize depends on  $N$ , which we denote as  $\phi_N$ . A case might be made for either increasing or decreasing prize values. Prize value might decrease when additional firms enter because investor attention is diluted over a larger population of firms. More commonly, though, the prize might shrink because of increasing competition for

a scarce resource. It is straightforward to see, based on Proposition 2, that a prize that decreases in  $N$  only strengthens our result. If the addition of further competitors causes an exogenous reduction of prize value (i.e., lower  $\phi$ ), then equilibrium disclosure falls even faster than if the prize remained constant.

Arguing that prizes increase with competition is more challenging, but may exist in developing industries. Prizes that increase with competition may overcome the competitive effect of disclosure, and are more likely to do so when  $N$  is small. But the following proposition shows that unless the prize grows exponentially by a factor of at least  $1/(1 - p\hat{\omega})$ , disclosure will eventually decrease once  $N$  reaches some critical value.

**Proposition 3.** *If  $\phi_N$  increases with  $N$  and*

$$\lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}}, \quad (8)$$

*then there exists some  $\bar{N} \in \mathbb{R}$  such that  $N > \bar{N}$  implies that  $\omega_{N+1}^* < \omega_N^*$ .*

To gain intuition for this result, consider the case in which prize value per firm remains constant:

$$\phi_N \equiv N\phi_1. \quad (9)$$

In this case,

$$\lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} = 1 < \frac{1}{1 - p\hat{\omega}}, \quad (10)$$

so the condition in (8) is satisfied, and disclosure decreases with competition for large  $N$ .

Intuitively, the chance of winning the prize declines exponentially in  $N$ , so unless the prize grows forever at the same exponential rate, the expected winnings will eventually decline in  $N$ . Realistically, however, one must ask how a prize that continues to increase exponentially with firm entry could arise. The value of high status may well increase exponentially as the number of competing firms increases from, say  $N = 1$  to  $N' = 10$ . But it is difficult to believe the same exponential increase could continue from  $N = 10$  to  $N' = 50$ . We conjecture that exponentially increasing status prizes are uncommon at best, and may never occur in industries with a large number of firms.

## 2.4.2 Multiple Prizes

Consider that a finite number of progressive prizes  $K$  are awarded to the top firms.

**Definition 1.** A disclosure game with a **progressive prize structure** is one in which the firms that make the  $K$  highest disclosures each win a prize. The firm that makes the  $k^{\text{th}}$  highest disclosure wins  $\phi_k$ . The prizes are positive and strictly monotonic,

$$\phi_1 > \phi_2 > \dots > \phi_K > 0. \quad (11)$$

Compared to a model with a single prize of  $\phi = \phi_1$ , the addition of prizes for runners-up naturally induces greater disclosure. But although the change to a progressive prize structure may increase disclosure for any particular  $N$ , our central result remains unchanged:

**Proposition 4.** Under any particular progressive prize structure, equilibrium disclosure frequency strictly decreases as competition increases. That is,  $\omega_{N+1}^* < \omega_N^*$  for any  $N$ .

This result justifies our simplification in working with a single prize  $\phi$ . Although additional prizes may change the quantitative predictions of equilibrium disclosure, the qualitative comparative statics remain unchanged. The chance of winning a lesser prize decreases with competition just as the chance of winning a single prize does. Competition therefore reduces disclosure in this setup as well.

Now, consider that prizes are awarded based on a firm's percentile. For example, each firm in the top 20 percent of the  $N$  firms could be awarded a prize, so that the  $N/5$  highest disclosures each receive an additional  $\phi$ . This variation introduces some complications that prevent us from showing the claim from the main model, "equilibrium disclosure  $\omega_N^*$  is strictly decreasing in  $N$ ." Because the number of prizes is discrete, it cannot increase in exact proportion with  $N$ . For example, when 20 percent of the firms receive a prize, a single prize is awarded when  $N = 5, 6, 7, 8, 9$ , and we numerically find that  $\omega_5^* > \omega_6^* > \dots > \omega_9^*$ . But for  $N = 10$ , we suddenly award a second prize, which can mean that  $\omega_9^* < \omega_{10}^*$ . We must therefore content ourselves with the result that disclosure decreases to its minimum possible frequency under perfect competition.

**Proposition 5.** Suppose that for any  $N$ , a fixed fraction  $\lambda$  of the competing firms win the prize  $\phi$ . Further, suppose that  $\lambda \leq p\hat{\omega}$ . Then, disclosure converges to its lower bound in the perfectly competitive limit:

$$\omega_N^* \longrightarrow \hat{\omega} \text{ as } N \rightarrow \infty. \quad (12)$$

According to Proposition 5, as the market becomes perfectly competitive, disclosure is minimized. It should be noted that the condition that  $\lambda < p\hat{\omega}$  is weak in the sense that it allows for a large number of firms to receive prizes. If  $\lambda = p\hat{\omega}$  when  $N \rightarrow \infty$ , this would mean that all firms that observed a value above  $\hat{t}$  would receive a prize. Therefore, we limit the fraction of prizes ( $\lambda < p\hat{\omega}$ ) to keep the analysis realistic and economically interesting.

### 2.4.3 Sequential Disclosure

We complete this section with a simple sequential disclosure model to further check the robustness of our finding that disclosure is minimized under perfect competition. Suppose that firms are randomly ordered, and each in turn observes its shock value  $\tilde{x}$  with probability  $p$ , then chooses whether to disclose.

Since each firm makes a unique, history-dependent decision, we no longer have a single symmetric, deterministic disclosure threshold. Rather, each firm has a random disclosure threshold that depends upon the disclosures of the preceding firms and on the number of firms remaining to act. Let  $\nu_j$  be the ex ante probability that the  $j^{\text{th}}$  firm to act will disclose if they are informed. The average of these probabilities is the analogue of the disclosure frequency in the main model,

$$\bar{\nu}_N = \frac{1}{N} \sum_{j=1}^N \nu_j. \quad (13)$$

**Proposition 6.** (*Sequential Disclosure*) *In an equilibrium with  $N$  firms,*

1. *The ex ante probability that the  $j^{\text{th}}$  firm discloses converges to the minimum with perfect competition:*

$$\lim_{N \rightarrow \infty} \nu_j = \hat{\omega}. \quad (14)$$

2. *The ex ante probability that a randomly selected informed firm discloses also converges to the minimum with perfect competition:*

$$\lim_{N \rightarrow \infty} \bar{\nu}_N = \hat{\omega}. \quad (15)$$

According to Proposition 6, in the perfectly-competitive limit, every individual  $j^{\text{th}}$  firm discloses with frequency  $\hat{\omega}$ , the minimum possible. We can also show the slightly stronger claim that the average frequency of disclosure over all  $N$  firms converges to the minimum  $\hat{\omega}$ .

### 3 Disclosure With Concurrent Product Market Competition

In this section, we analyze firms that compete directly in the product market, as well as for prizes based on their disclosures. The distribution  $F(x)$  varies with  $N$  so that firm revenue decreases with the entry of additional firms. Accordingly, firm signals have relatively less direct importance to firm price and the prize has relatively more.

#### 3.1 Equal Shares Competition

To capture this effect simply, we assume that the distribution of value signals becomes compressed with the entry of additional firms. When  $N$  firms compete, we exchange the original distribution of signals  $x \sim F$  for a compressed distribution  $x_N \sim F_N$ , so that whenever a firm would have drawn a signal  $x$  in the original model, they instead draw a scaled-down event  $x/N$  in the new model.

The new distribution is defined as

$$F_N(x) \equiv F(Nx). \tag{16}$$

An increase in  $N$  has the effect of shifting the distribution of news events while leaving the support unchanged. For example, if  $x = \$10k$  had been a 90<sup>th</sup>-percentile result with  $N = 5$ ,  $x = \$1k$  would be the new 90<sup>th</sup>-percentile with  $N = 50$ . Increasing  $N$  scales down expectations while preserving the concavity and any other peculiarities of the value distribution.

We refer to this as “equal shares competition” for earnings, but wish to stress that this is not intended as a substitute for other models of competition. The goal here is simply to show how the value-scaling effect of competition affects disclosure. In Section 3.2, we consider more general models of competition.

In Section 2, we found that as  $N$  increases, the incentive to disclose falls as the probability of winning the prize decreases. With product market competition, though, potential revenue declines as well, which reduces the incentive to pool. These two effects oppose one another. Which effect dominates depends upon the number of competing firms.

For what values of  $N$ , then, does competition reduce disclosure? If we were to find the necessary number of firms to be in the millions, for example, then our point here



would only be academic and not of practical import. To gain a sense of how many firms is “enough,” consider the following proposition.

**Proposition 7.** *Above some threshold  $\bar{N} = 1/p\hat{\omega}$ , the equilibrium disclosure frequency  $\omega_N^*$  decreases monotonically in  $N$  and converges to  $\hat{\omega}$ .*

To appreciate Proposition 7, suppose the distribution  $\tilde{x} \sim F$  is symmetric and define  $\hat{t}$  as the threshold such that  $1 - F(\hat{t}) = \hat{\omega}$ . The fact that  $\hat{t} < E[x]$  implies that

$$\hat{\omega} = 1 - F(\hat{t}) > 1 - F(E[x]) = 0.5. \quad (17)$$

Then the sufficient condition becomes  $N > 2/p$ . If, for example, firm information arrives with probability 0.5, then  $N = 2/(0.5) = 4$  firms is enough competition that further entry will only reduce disclosure. The higher  $p$  is, the fewer firms that are required to assure that further competition decreases disclosure. We conjecture that in many industries (e.g., financial sectors), there are already enough competitors present so that disclosure responds negatively to additional competition.

Proposition 7 also shows that  $\omega_N^*$  actually converges to  $\hat{\omega}$  under perfect competition, while industry profits converge to zero. Product prices decrease to their lowest possible values, which maximizes social welfare. However, perfect competition in disclosure induces firms to retain their maximum degree of asymmetric information. Thus, while perfect competition drives product prices to their most socially efficient level, it drives firm prices to their least informationally efficient. To better appreciate this, consider the following example.

**Example 1.** *Consider the disclosure game where  $\tilde{x}$  is Gaussian with  $\mu = 0$ ,  $\sigma = 5$  and  $\phi = 1$ ,  $p = .3$  and product market competition characterized by  $F_N(x) = F(Nx)$ . Figure 2.1 shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit  $\hat{\omega} \approx 0.556$ . Note that it only takes about 5 firms for increased competition to reduce disclosure, even though  $p$  is relatively low at 0.3.*

Although the above condition of  $N > 1/(p\hat{\omega})$  is mild enough, the condition is indeed only sufficient for competition to decrease disclosure, not necessary. Typically, an even smaller number of firms will suffice. We therefore derive the constraint on  $N$  that is both necessary and sufficient for further entry to reduce disclosure.

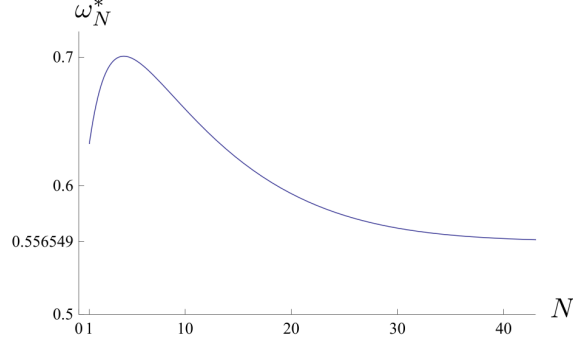


Figure 2.1: Disclosure game with product market competition. The random variable is  $\tilde{x} \sim N(0, 5)$  and  $\phi = 1$  and  $p = .3$ . The product market competition characterized by  $F_N(x) = F(Nx)$ . The curve shows how the equilibrium disclosure changes with the number of competing firms. Disclosure initially increases, then decreases asymptotically to the lower limit  $\hat{\omega} \approx 0.556$ .

Consider the position of a firm  $j$  that draws the threshold value,  $x_j = F^{-1}(1 - \omega^*)$ . With  $N$  firms competing for the prize, firm  $j$  is indifferent between disclosing and herding. If a  $(N + 1)^{th}$  competitor enters, and firm  $j$  observes the same  $x_j$ , how do the firm's prospects change? Should it disclose, the entry reduces its expected prize winnings by a factor of  $(1 - p\omega)$  because

$$\phi W(\omega; N) = \phi(1 - p\omega)^{N-1} \quad (18)$$

is exponentially decreasing in  $N$ . But the other terms,  $F_N^{-1}(1 - \omega)$  and  $u_N^C(\omega)$ , decline by a factor of  $N/(N + 1)$ , as demonstrated in Lemma A5 in the appendix. As  $N$  rises, then, this linear effect diminishes in significance compared to the exponential effect on the expected prize value. Intuitively it seems that there is a critical number of firms at which additional competition makes herding more attractive than competing for the prize.

**Proposition 8.** *Disclosure frequency decreases with firm entry if and only if the number of competing firms exceeds some threshold.:*

$$N > \frac{1 - p\omega_N^*}{p\omega_N^*} \iff \omega_{N+1}^* < \omega_N^*. \quad (19)$$

According to Proposition 8, if  $N$  exceeds the threshold specified by the relative probabilities of disclosing and pooling, then the exponential effect overwhelms the linear effect. So, the net effect of firm entry is a reduction in the incentive to disclose, which results in  $\omega_{N+1}^* < \omega_N^*$ . Note, however, that the threshold for  $N$  established by Proposition 8 is changing with  $N$ . That is, as  $N$  increases,  $\omega_N^*$  varies, and so the probability ratio in Equation 19 may also increase. Therefore, although this proposition details the necessary and sufficient condition for  $N$ , it does not provide a tighter unconditional bound than in Proposition 7.

Economically, Propositions 7 and 8 imply that when the number of firms is small, further competition increases disclosure because the benefits of the prize are large compared to the share of industry revenues that each firm receives. However, as the number of firms rises, the benefits of revealing information rapidly drop compared to sharing industry revenues with more firms, and disclosure becomes less likely.

### 3.2 Generalized Product Market Competition

Now suppose that we define the distribution as a function of  $N$  by

$$F_N(x) \equiv F\left(\frac{x}{\alpha_N}\right), \quad (20)$$

for some decreasing sequence  $\{\alpha_N\}$ . By construction, if  $\alpha_N$  decreases rapidly, then firm entry has a dramatic effect on the revenue of competing firms. If  $\alpha_N$  decreases more slowly, then the effect is less pronounced. This formulation embeds the previous set up in which  $F_N(x) = F(Nx)$ .

**Proposition 9.** *If, under generalized competition with  $F_N(x) = F(x/\alpha_N)$ ,*

$$\lim_{N \rightarrow \infty} \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega}, \quad (21)$$

*then there exists some  $\bar{N} \in \mathbb{R}$  such that  $N > \bar{N} \Rightarrow \omega_{N+1}^* < \omega_N^*$ .*

The proof follows nearly the same structure as the proof of Proposition 8. Note, however, that in this case, we need an additional restriction on the sequence  $\{\alpha_N\}$  in order to complete the proof. Roughly stated, the requirement above is that competition not reduce firm value too “quickly” as additional firms enter.

Thus, the question becomes one of whether the per-firm revenue can decrease ad-infinitum at such a rate with the entry of additional firms. Although one can posit such

a model, exponentially decreasing revenue is not a common feature of microeconomic models of competition.

**Example 2.** *Consider a Cournot competition with linear pricing. In such a model, per-firm earnings (and hence firm value) declines as  $N$  grows:*

$$\pi_N = \frac{\pi_1}{N^2}. \quad (22)$$

Therefore,  $\alpha_N = 1/N^2$ . This sequence satisfies the criterion in Proposition 9 because

$$\lim_{N \rightarrow \infty} \frac{\alpha_{N+1}}{\alpha_N} = \lim_{N \rightarrow \infty} \frac{N^2}{(N+1)^2} = 1 > 1 - p\hat{\omega}. \quad (23)$$

So under linear Cournot competition, disclosure does indeed decline with competition for large  $N$ .

## 4 Concluding Remarks

The primary result in this paper is that increased competition often reduces disclosure when tournament-like competition is present. We show this both in a parsimonious model, as well as in more sophisticated extensions. The fundamental idea, that firm entry makes attaining top status more difficult, is straightforward. But the exponential relationship between the number of competing firms and the probability of winning the prize is mathematically powerful. The result is a robustness that makes our central result widely generalizable.

If transparency is considered a good, then our analysis has straightforward welfare implications. However, competition's effect on welfare depends on the goal of screening in the market. For example, suppose that it is only necessary to identify the top performer(s) in the market. Then, even if competition causes more firms to hoard information, welfare rises because a higher number of firms makes it more likely that the best firm is identified. Indeed, this is proved analytically in Example B.1 and Proposition B1 in the Appendix.

However, if welfare depends on the behavior of the marginal firm, competition may decrease welfare. Example B.2 and Proposition B2 in the Appendix provide a situation in which welfare depends on screening all potential counterparties in the market. In such case, as competition increases, the marginal firm chooses not to disclose their

information, which worsens the efficiency of screening and causes per capita welfare to drop.

It is important to note that we do not assert that tournaments represent optimal screening mechanisms. But they do exist in many venues. In such cases, we cannot always appeal to the Invisible Hand to make markets transparent. While competition in product markets often has a favorable effect on prices, driving firms to lower and more socially efficient prices, it can have the opposite effect on disclosure. In the asymptotic limit of perfect competition, prices converge to their most efficient values, but disclosure falls to its least efficient.

In the end, our analysis implies that policy makers should consider the type of competition that takes place in markets when deciding whether to regulate them. Competition may not always cure market ailments, and may even exacerbate them.

## A Appendix

**Proof of Proposition 1:** We establish Lemmas A1, A2, and A3, to determine the game's unique subgame-perfect Nash equilibrium. Lemma A1 establishes that all firms use a threshold strategy to determine whether to disclose information when they have it. Lemma A2 shows that all firms use a common threshold. Finally, Lemma A3 shows that this common threshold is unique.

■

**Lemma A1.** *In any subgame-perfect Nash equilibrium, each firm acts according to a disclosure threshold  $t_j < 0$ ,*

$$\sigma_j(x_j) = \begin{cases} 1 & \text{for } x_j > t_j \\ 0 & \text{for } x_j < t_j. \end{cases} \quad (\text{A1})$$

*The threshold is implicitly defined by the condition that a firm observing  $x_j = t_j$  be indifferent between disclosing and concealing,*

$$u_j^D(t_j) = u_j^C. \quad (\text{A2})$$

**Proof of Lemma A1:** Suppose firm  $j$  observes the event  $x$ . In a subgame-perfect Nash equilibrium, the firm must disclose optimally given the value of  $x$ . That is, it discloses when  $u_j^D(x) > u_j^C$  and conceal  $x$  when  $u_j^C > u_j^D(x)$ .

If the firm discloses, then it is eligible to win the prize  $\phi$ . So its new market valuation is  $x$ , plus an additional  $\phi$  if no competing firm makes a higher disclosure,

$$u_j^D(x) = x + \phi W_j(x), \quad (\text{A3})$$

where  $W_j(x)$  is the probability that no competing firm discloses a higher value than  $x$ :

$$\begin{aligned} W_j(x) &= \prod_{k \neq j} (1 - P(I_k)P((x_k > x) \cap D_j)) \\ &= \prod_{k \neq j} \left( 1 - p \int_x^\infty \sigma_k(z) f(z) dz \right) \end{aligned} \quad (\text{A4})$$

Note that  $u_j^D(x)$  is differentiable, and therefore continuous. Furthermore, for any  $x$ ,

$$u_j^D(x) \leq x + \phi \quad \text{and} \quad x \leq u_j^D(x) \quad (\text{A5})$$

Evaluating at  $x = u_j^C - \phi$  and  $x = u_j^C$ , these inequalities yield

$$u_j^D(u_j^C - \phi) \leq u_j^C \quad \text{and} \quad u_j^C \leq u_j^D(u_j^C). \quad (\text{A6})$$

So if firm  $j$  observes  $x_j = u_j^C - \phi$ , then disclosure yields a lower expected utility than  $u_j^C$ ; and if it observes  $x_j = u_j^C$ , then disclosure yields a higher expected utility than  $u_j^C$ . Because  $u_j^D(x)$  is continuous, the Intermediate Value Theorem assures us there is a potential observation  $t_j \in [u_j^C - \phi, u_j^C]$  for which

$$u_j^D(t_j) = u_j^C. \quad (\text{A7})$$

This  $t_j$  is the disclosure threshold for firm  $j$ , where the firm is indifferent between disclosing and pooling. Since  $u_j^D(x)$  is strictly monotonic in  $x$ , we further obtain

$$x > t_j \Rightarrow u_j^D(x) > u_j^C \quad (\text{A8})$$

$$x < t_j \Rightarrow u_j^D(x) < u_j^C. \quad (\text{A9})$$

The subgame-optimal response of firm  $j$  is therefore to disclose any values above the threshold  $t_j$  and to conceal any values below, as desired.

Now to show that  $t_j < 0$ , we derive the value  $u_j^C$  that investors assign if the firm conceals its observation. In a rational expectations equilibrium, the beliefs of the investors with respect to the strategy must be consistent with the strategy actually used,

$$\begin{aligned} u_j^C = E[x|P_j] &= \frac{P(U_j)E[x|U_j] + P(I_j \cap C_j)E[x|I_j \cap C_j]}{P(U_j) + P(I_j)P(C_j|I_j)} \\ &= \frac{(1-p) \cdot 0 + pP(x < t_j)E[x|x < t_j]}{(1-p) + pP(x < t_j)} \\ &= \frac{pF(t_j)}{1-p + pF(t_j)}E[x|x < t_j] \\ &< \frac{pF(t_j)}{1-p + pF(t_j)}E[x] = 0, \end{aligned} \quad (\text{A10})$$

and therefore,

$$u_j^C < 0 < u_j^D(0). \quad (\text{A11})$$

Because  $u_j^D(x)$  is monotonically increasing in  $x$ , the threshold  $t_j$  must be below zero. That is, all average or better values of  $x$  will be disclosed in equilibrium.

■

**Lemma A2.** *Every firm uses the same disclosure threshold, defined as  $t^*$ .*

**Proof of Lemma A2:** Write Equation A11 as an integral, then apply integration by parts,

$$\begin{aligned}
u_j^C &= \frac{p}{1-p+pF(t_j)} \int_{-\infty}^{t_j} xf(x) \, dx \\
&= \frac{p}{1-p+pF(t_j)} \left( [xF(x)]_{-\infty}^{t_j} - \int_{-\infty}^{t_j} F(x) \, dx \right) \\
&= \frac{p}{1-p+pF(t_j)} \left( t_j F(t_j) - \int_{-\infty}^{t_j} F(x) \, dx \right). \tag{A12}
\end{aligned}$$

Using this expression, some algebraic manipulation transforms  $u_j^D(t_j) = u_j^C$  into

$$\phi W_j(t_j)(1-p+pF(t_j)) = (1-p)(-t_j) - p \int_{-\infty}^{t_j} F(x) \, dx. \tag{A13}$$

Now suppose for contradiction that a non-symmetric equilibrium exists. That is, suppose an equilibrium exists in which firms  $j$  and  $k$  use different thresholds. Without loss of generality, assume that  $t_k < t_j$ . Equation (A13) holds for firm  $k$  as well as for  $j$ . Subtracting these yields

$$\begin{aligned}
&\phi (W_j(t_j)(1-p+pF(t_j)) - W_k(t_k)(1-p+pF(t_k))) \\
&= (1-p)(-t_j + t_k) - p \int_{t_k}^{t_j} F(x) \, dx < 0, \tag{A14}
\end{aligned}$$

and therefore

$$W_j(t_j)(1-p+pF(t_j)) < W_k(t_k)(1-p+pF(t_k)). \tag{A15}$$

But we can obtain a contradiction by deriving the opposite inequality. We simplify Equation A4 with the assumption that all firms use threshold strategies, then evaluate at  $t_j$ :

$$\begin{aligned}
W_j(t_j) &= \prod_{i \neq j} \left( 1 - p \int_{t_j}^{\infty} \sigma_i(z) f(z) \, dz \right) \\
&= \prod_{i \neq j} (1 - p + pF(\max(t_i, t_j))) \\
&= (1 - p + pF(\max(t_j, t_k))) \prod_{i \neq j, k} (1 - p + pF(\max(t_i, t_j))). \tag{A16}
\end{aligned}$$



The same holds for firm  $k$ , so we obtain

$$W_k(t_k) = (1 - p + pF(\max(t_j, t_k))) \prod_{i \neq j, k} (1 - p + pF(\max(t_i, t_k))). \quad (\text{A17})$$

Since  $t_j > t_k$ , these equations show that  $W_j(t_j) > W_k(t_k)$ . Therefore,

$$W_j(t_j) (1 - p + pF(t_j)) > W_k(t_k) (1 - p + pF(t_k)). \quad (\text{A18})$$

This directly contradicts Equation (A15), so the hypothesized asymmetric equilibrium cannot exist.

■

**Lemma A3.** *The common disclosure threshold  $t^*$  is unique.*

**Proof of Lemma A3:** Suppose for contradiction there exist two distinct equilibrium thresholds  $t^*$  and  $t^{**}$ . Without loss of generality, assume  $t^* < t^{**}$ . Equation A13 holds at both thresholds. Subtracting, we obtain

$$\begin{aligned} & \phi(W(t^*) (1 - p + pF(t^*)) - W(t^{**}) (1 - p + pF(t^{**}))) \\ &= (1 - p)(t^{**} - t^*) + p \int_{t^{**}}^{t^*} F(x) dx < 0, \end{aligned} \quad (\text{A19})$$

and therefore,

$$W(t^*) (1 - p + pF(t^*)) < W(t^{**}) (1 - p + pF(t^{**})). \quad (\text{A20})$$

We now obtain a contraction by deriving the opposite inequality. Since strategies are symmetric,  $t_i = t_j$  in Equation A16, so the equation simplifies to

$$W(t^*) = (1 - p + pF(t^*))^{N-1}. \quad (\text{A21})$$

And the same holds for the other equilibrium threshold,

$$W(t^{**}) = (1 - p + pF(t^{**}))^{N-1}. \quad (\text{A22})$$

Because  $t^* > t^{**}$ , these equations show that  $W(t^*) > W(t^{**})$ . Therefore,

$$W(t^*) (1 - p + pF(t^*)) > W(t^{**}) (1 - p + pF(t^{**})), \quad (\text{A23})$$

directly contradicting Equation A20. By this contradiction, we conclude that a second distinct equilibrium threshold  $t^{**}$  cannot exist.

■

Taken together, Lemmas A1, A2 and A3 show that all firms use a common and unique disclosure threshold defined implicitly by

$$u_j^D(t^*) = u^C(t^*). \quad (\text{A24})$$

We expand this equivalence using  $u_j^D(t^*) = t^* + \phi W(t^*)$ , Equation A21, and Equation A12 to obtain the desired expression

$$t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx. \quad (\text{A25})$$

Finally, we find  $t^* < 0$  by the same argument that shows  $t_j < 0$  in Lemma A1.

**Definition A1.** For any disclosure frequency  $\omega$ , define the corresponding disclosure threshold by  $t(\omega)$ . That is,

$$t(\omega) \equiv F^{-1}(1 - \omega) \quad (\text{A26})$$

**Definition A2.** Define  $B(\omega)$  as the benefit of disclosing the threshold value relative to concealing, assuming that all firms disclose with frequency  $\omega$ ,

$$B(\omega) \equiv u^D(\omega) - u^C(\omega), \quad (\text{A27})$$

where

$$u^D(\omega) \equiv E[u_j^D | x_j = t(\omega)] = t(\omega) + \phi(1 - p\omega)^{N-1} \quad (\text{A28})$$

$$u^C(\omega) \equiv E[x_j | P_j, t_j = t(\omega)] = \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) d\Omega \quad (\text{A29})$$

Note that this definition does not require that  $\omega$  be the equilibrium frequency, which we denote distinctly by  $\omega^*$ .

**Corollary A1.** The equilibrium condition Equation 4 in Proposition 1 can be rewritten in terms of the equilibrium disclosure frequency  $\omega^*$  as

$$B(\omega^*) = 0. \quad (\text{A30})$$

**Proof of Corollary A1:** From Definition A1 we get  $F(1 - \omega^*) = t^*$ . Applying this to Equation 4 yields,

$$t^* + \phi(1 - p + pF(t^*))^{N-1} = \frac{p}{1 - p + pF(t^*)} \int_{-\infty}^{t^*} xf(x) dx, \quad (\text{A31})$$

$$t(\omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{-\infty}^{t^*} xf(x) dx, \quad (\text{A32})$$

Defining the integral substitution  $\Omega = 1 - F(x)$  yields

$$\int_{-\infty}^{t^*} x f(x) dx = \int_{1-F(-\infty)}^{1-F(t^*)} F^{-1}(1-\Omega)(-d\Omega) = \int_{\omega^*}^1 F^{-1}(1-\Omega) d\Omega. \quad (\text{A33})$$

Applying this to the right-hand side of the previous equation yields

$$t(\omega^*) + \phi(1 - p\omega^*)^{N-1} = \frac{p}{1 - p\omega^*} \int_{\omega^*}^1 F^{-1}(1-\Omega) d\Omega \quad (\text{A34})$$

$$u^D(\omega^*) = u^C(\omega^*) \quad (\text{A35})$$

$$B(\omega^*) = 0 \quad (\text{A36})$$

■

**Lemma A4.**  $B(\omega)$  is strictly decreasing for all  $\omega > \omega^*$ .

**Proof of Lemma A4:** First write  $B(\omega) \equiv u^D(\omega) - u^C(\omega)$  explicitly as

$$B(\omega, \phi, p, N) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) d\Omega. \quad (\text{A37})$$

Note that

$$\frac{\partial}{\partial \omega} t(\omega) = \frac{\partial}{\partial \omega} F^{-1}(1 - \omega) = \frac{-1}{f(F^{-1}(1 - \omega))} < 0, \quad (\text{A38})$$

so the first term is decreasing in  $\omega$ . Clearly the second term is also decreasing in  $\omega$ . In the third term,

$$\frac{\partial}{\partial \omega} \left( -\frac{p \int_{\omega}^1 t(\Omega) d\Omega}{1 - p\omega} \right) = \frac{-p^2 \int_{\omega}^1 t(\Omega) d\Omega + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2}$$

and the integrand  $t(\Omega)$  is decreasing in  $\Omega$ , so

$$\begin{aligned} \dots &< \frac{-p^2(1 - \omega)t(\omega) + pt(\omega)(1 - p\omega)}{(1 - p\omega)^2} \\ &= \frac{-p^2 + p^2\omega + p - p^2\omega}{(1 - p\omega)^2} t(\omega) \\ &= \frac{p(1 - p)}{(1 - p\omega)^2} t(\omega). \end{aligned} \quad (\text{A39})$$

By our assumption that  $\omega > \hat{\omega}$ , we know that  $t(\omega) < \hat{t} < E[\tilde{x}] = 0$ , and so the derivative of the third term is also negative. Thus,  $B(\omega)$  is strictly decreasing in  $\omega$  for all  $\omega > \hat{\omega}$ .

■

**Proof of Proposition 2:** First write  $B(\omega) \equiv u^D(\omega) - u^C(\omega)$  explicitly as

$$B(\omega, \phi, p, N) = t(\omega) + \phi(1 - p\omega)^{N-1} - \frac{p}{1 - p\omega} \int_{\omega}^1 t(\Omega) \, d\Omega. \quad (\text{A40})$$

For any set of parameter values  $(\phi, p, N)$ , the equilibrium disclosure frequency is uniquely defined by  $B(\omega^*, \phi, p, N) = 0$ . Because  $B$  is differentiable with respect to each of its parameters, the Implicit Function Theorem tells us how the equilibrium frequency changes with the parameter values. For each parameter  $\theta \in \{\phi, p, N\}$ , the IFT gives

$$\frac{\partial \omega^*}{\partial \theta} \equiv \left. \frac{\partial \omega}{\partial \theta} \right|_{B=0} = - \frac{\left. \frac{\partial B}{\partial \theta} \right|_{B=0}}{\left. \frac{\partial B}{\partial \omega} \right|_{B=0}}. \quad (\text{A41})$$

Lemma A4 tells us that  $\frac{\partial B}{\partial \omega} < 0$  for all  $\omega > \hat{\omega}$ . Differentiating with respect to the other model parameters yields

$$\frac{\partial B}{\partial \phi} = (1 - p\omega)^{N-1} > 0 \quad (\text{A42})$$

$$\frac{\partial B}{\partial N} = \phi(1 - p\omega)^{N-1} \ln(1 - p\omega) < 0 \quad (\text{A43})$$

$$\frac{\partial B}{\partial p} = -\omega\phi(N-1)(1 - p\omega)^{N-2} - \frac{1}{(1 - p\omega)^2} \int_{\omega}^1 t(\Omega) \, d\Omega. \quad (\text{A44})$$

Note that  $\int_{\omega}^1 t(\Omega) \, d\Omega < 0$  is the expected value of  $x$  for a non-disclosing firm, which is negative. So the second term of  $\frac{\partial B}{\partial p}$  is positive, while the first is negative. Which term dominates depends on the parameter values.

Applying the Implicit Function Theorem yields the desired comparative statics:

$$\frac{\partial B}{\partial \phi} > 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial \phi} = - \frac{\partial B / \partial \phi}{\partial B / \partial \omega} > 0, \quad (\text{A45})$$

$$\frac{\partial B}{\partial N} < 0 \quad \text{so} \quad \frac{\partial \omega^*}{\partial N} = - \frac{\partial B / \partial N}{\partial B / \partial \omega} > 0. \quad (\text{A46})$$

As shown already,  $\omega$  is decreasing in  $N$ . Since any monotonic bounded sequence of real numbers converges, and since we know  $\omega_N^* > \hat{\omega}$  for all  $N$ ,  $\omega_N^*$  converges as  $N \rightarrow \infty$ . Let us refer to its limit as

$$\omega_{\infty} = \lim_{N \rightarrow \infty} \omega^* \quad (\text{A47})$$

The function  $B(\cdot)$  is continuous in  $\omega_N^*$  and  $N$ , and  $B_N(\omega_N^*) = 0$  for all  $N$ . The sequence  $\{B_N(\omega_N^*)\}$  therefore converges to zero as well:

$$\begin{aligned}
0 &= \lim_{N \rightarrow \infty} B_N(\omega_N^*) \\
&= \lim_{N \rightarrow \infty} t(\omega_N^*) + \lim_{N \rightarrow \infty} \phi(1 - p\omega_N^*)^N - \lim_{N \rightarrow \infty} \frac{p \int_{\omega_N^*}^1 t(\Omega) d\Omega}{1 - p\omega_N^*} \\
&= t(\omega_\infty) + 0 - \frac{p \int_{\omega_\infty}^1 t(\Omega) d\Omega}{1 - p\omega_\infty}
\end{aligned} \tag{A48}$$

That is,

$$t(\omega_\infty) = \frac{p \int_{\omega_\infty}^1 t(\Omega) d\Omega}{1 - p\omega_\infty}, \tag{A49}$$

and therefore  $\omega_\infty = \hat{\omega}$ .

■

**Proof of Proposition 3:** We consider the base model with prizes  $\phi_N$  that increase with  $N$  according to some sequence  $\{\phi_N\}$ . Then the benefit of disclosing relative to concealing is a function of  $N$ ,

$$B_N(\omega) = t(\omega) + \phi_N(1 - p\omega)^{N-1} - u^C(\omega). \tag{A50}$$

The same holds for  $(N + 1)$  firms, so we can subtract the two equations to obtain

$$\begin{aligned}
B_{N+1}(\omega) - B_N(\omega) &= \phi_{N+1}(1 - p\omega)^N - \phi_N(1 - p\omega)^{N-1} \\
&= \phi_N(1 - p\omega)^{N-1} \left( \frac{\phi_{N+1}}{\phi_N}(1 - p\omega) - 1 \right).
\end{aligned} \tag{A51}$$

Under our assumption that  $\lim_{N \rightarrow \infty} \frac{\phi_{N+1}}{\phi_N} < 1/(1 - p\hat{\omega})$ , there exists some  $\bar{N}$  such that

$$N > \bar{N} \Rightarrow \frac{\phi_{N+1}}{\phi_N} < \frac{1}{1 - p\hat{\omega}}, \tag{A52}$$

so evaluating Equation A51 at  $\omega = \omega_N^*$  for any  $N > \bar{N}$  yields

$$B_{N+1}(\omega_N^*) - 0 < \phi_N(1 - p\omega_N^*)^{N-1} \left( \frac{\phi_{N+1}}{\phi_N}(1 - p\hat{\omega}) - 1 \right) < 0. \tag{A53}$$

By Lemma A4, we obtain the desired  $\omega_{N+1}^* < \omega_N^*$ .

■

**Proof of Proposition 4:** Define a firm’s “rank” according to the firms place among realized disclosures by competing firms. That is, if there are  $k - 1$  higher disclosures, the firm has rank  $k$  and receives  $\phi_k$ . A disclosing firm’s rank is therefore a stochastic function of its disclosed value. We define  $\tilde{r}(\omega)$  accordingly:

$$\tilde{r}(\omega) = \text{rank of a firm that discloses } x = F^{-1}(1 - \omega). \quad (\text{A54})$$

Using this notation, we would write the expected utility of disclosure in the base model as

$$u^D(\omega) = t(\omega) + \phi W(\omega) = t(\omega) + \phi P(\tilde{r}(\omega) = 1). \quad (\text{A55})$$

With prizes for the top  $K$  firms, the expected payout becomes

$$u^D(\omega) = t(\omega) + \sum_{k=1}^K \phi_k P(\tilde{r}(\omega) = k). \quad (\text{A56})$$

We wish to show that this value is decreasing in  $N$ . Unfortunately, we cannot claim that  $P(\tilde{r}(\omega) = k)$  is decreasing in  $N$  without some further restrictions. Although the chance of having *at least* the  $k^{\text{th}}$ -highest disclosure is strictly decreasing in  $N$ , the chance of having *exactly* the  $k^{\text{th}}$ -highest disclosure may be increasing in  $N$ , at least for certain parameter values. We therefore rearrange the sum in order to write it in terms we know to be unconditionally decreasing in  $N$ ,

$$\begin{aligned} u^D(\omega) &= t(\omega) + \sum_{k=1}^K \phi_k (P(\tilde{r}(\omega) \leq k) - P(\tilde{r}(\omega) \leq k - 1)) \\ &= t(\omega) + \sum_{k=1}^K (\phi_k - \phi_{k+1}) P(\tilde{r}(\omega) \leq k). \end{aligned} \quad (\text{A57})$$

Note that  $P(\tilde{r}(\omega) \leq k)$ , the probability of having at least the  $k^{\text{th}}$ -highest disclosure, is strictly decreasing in  $N$ . Since prizes are strictly decreasing in rank, we also have  $(\phi_k - \phi_{k+1}) > 0$ . Therefore,  $u^D(\omega)$  is unconditionally decreasing in  $N$ . We conclude that disclosure frequency decreases in  $N$  under a progressive prize structure.

■

**Proof of Proposition 5:** Let  $t_N^*$  be the equilibrium disclosure threshold with  $N$  firms. Suppose that a firm  $j$  observes and discloses exactly  $x_j = t_N^*$ . Then the probability  $q$  that any other given opponent observes a higher value is given by

$$q \equiv p(1 - F(t_N^*)) = p\omega_N^*. \quad (\text{A58})$$

Any such realization above the threshold will certainly be disclosed, so the number of firms who disclose values higher than  $t_N^*$  is a binomial random variable  $\tilde{S} \sim B(N, q)$ . The probability that firm  $j$  wins a prize is bounded by the probability that fewer than  $\lambda N$  other firms disclose values higher than  $\hat{t}$ . That is,

$$W_j(t_N^*) \leq P\left(\tilde{S} \leq \lambda N - 1\right). \quad (\text{A59})$$

This probability is the weight of a left tail of the binomial distribution of  $\tilde{S}$ . We may bound it using Hoeffding's inequality (Hoeffding 1963), which states that the sum  $\tilde{s}$ , of any  $N$  random variables, has the probabilistic bound

$$P(|\tilde{s} - E[\tilde{s}]| \geq c) \leq 2 \exp\left(\frac{-2c^2}{\sum_{i=1}^N (b_i - a_i)^2}\right) \quad (\text{A60})$$

where the  $i^{\text{th}}$  random variable is contained by the interval  $[a_i, b_i]$ . In our application,  $\tilde{S}$  is the sum of  $(N - 1)$  identically-distributed Bernoulli trials with success probability  $q$ , so

$$a_i = 0, \quad b_i = 1, \quad E[\tilde{S}] = q(N - 1). \quad (\text{A61})$$

We first transform our probability into the same form as Hoeffding's inequality,

$$\begin{aligned} W_j(t_N^*) &= P\left(\tilde{S} \leq \lambda N - 1\right) \\ &= P\left(\tilde{S} - E[\tilde{S}] \leq \lambda N - 1 - q(N - 1)\right) \\ &\leq P\left(|\tilde{S} - E[\tilde{S}]| \geq (q - \lambda)N - q + 1\right). \end{aligned} \quad (\text{A62})$$

We then can apply the (A60) with  $c = (q - \lambda)N - q + 1$  to obtain

$$W_j(t_N^*) \leq 2 \exp\left(-\frac{2((q - \lambda)N - q + 1)^2}{N}\right). \quad (\text{A63})$$

Note that firms will always disclose values above  $\hat{t}$ , so any equilibrium threshold  $t_N^*$  must be below  $\hat{t}$ . We therefore have

$$q = p(1 - F(t_N^*)) > p(1 - F(\hat{t})) = p\hat{\omega} > \lambda. \quad (\text{A64})$$

This ensures that as  $N \rightarrow \infty$ , the exponential in (A63) goes to  $-\infty$  and the right hand side goes to zero for any sequence of thresholds  $\{t_N^*\}$ . Since firms optimally respond to  $W = 0$  by concealing all realizations below  $\hat{t}$ , the disclosure frequency converges to  $\hat{\omega}$ , as desired.

■

**Proof of Proposition 6:** By our definition of  $\hat{t}$ , any firm with a realization  $x_j > \hat{t}$  discloses even if they have no chance of winning the prize. This establishes a lower bound for both limits:

$$\lim_{N \rightarrow \infty} \nu_j \geq \hat{\omega} \quad \text{and} \quad \lim_{N \rightarrow \infty} \bar{\nu}_N \geq \hat{\omega}. \quad (\text{A65})$$

1. Suppose firm  $j$  discloses a lower value,  $x_j < \hat{t}$ . If any of the  $N - j$  firms yet to act observes a value above  $\hat{t}$ , they will certainly disclose it. The probability that firm  $j$  wins the prize is therefore bounded above by

$$W_j(x_j) \leq W_j(\hat{t}) = (1 - p(1 - F(\hat{t})))^{N-j} = (1 - p\hat{\omega})^{N-j}. \quad (\text{A66})$$

So as  $N \rightarrow \infty$ , the probability of winning the prize converges to zero. In this limit, so firm  $j$  will optimally conceal any values below  $\hat{t}$ , disclosing no more frequently than  $\hat{\omega}$ . Together with (A65), this establishes the desired result.

2. Again, note that a firm that realizes  $x_j < \hat{t}$  will not disclose unless it has a positive probability of winning the prize. Specifically, it will not disclose if any preceding firm has already disclosed a value above  $\hat{t}$ . That is, the probability of disclosing a value below  $\hat{t}$  cannot possibly be larger than the probability that no preceding firm  $i$  has disclosed  $x_i > \hat{t}$ . This allows us to place a very loose upper bound on  $\nu_j$ :

$$\begin{aligned} \nu_j &= P(\tilde{x}_j < \hat{t}) \cdot P(D_j | x_j < \hat{t}) + P(D_j | x_j > \hat{t}) \cdot P(\tilde{x}_j > \hat{t}) \\ &\leq (1 - \hat{\omega}) \cdot \prod_{i=1}^{j-1} P(U_i \text{ or } x_i < \hat{t}) + \hat{\omega} \cdot 1 \\ &= (1 - \hat{\omega})(1 - p\hat{\omega})^{j-1} + \hat{\omega}. \end{aligned} \quad (\text{A67})$$

Averaging over all  $j$  yields

$$\begin{aligned} \bar{\nu}_N &\leq \frac{1}{N} \sum_{j=1}^N ((1 - \hat{\omega})(1 - p\hat{\omega})^{j-1} + \hat{\omega}) \\ &= (1 - \hat{\omega}) \frac{1}{N} \left( \frac{1 - (1 - p\hat{\omega})^N}{1 - (1 - p\hat{\omega})} \right) + \hat{\omega} \\ &= \frac{1 - \hat{\omega}}{p\hat{\omega}} \left( \frac{1 - (1 - p\hat{\omega})^N}{N} \right) + \hat{\omega}. \end{aligned} \quad (\text{A68})$$



As  $N \rightarrow \infty$ , the first term vanishes, so  $\lim_{N \rightarrow \infty} \bar{v}_N \leq \hat{\omega}$ . Together with (A65), this yields the desired result.

■

**Lemma A5.** *Under equal shares competition, the signal that corresponds to a given probability  $\omega$ , previously written as  $t(\omega)$  becomes*

$$t_N(\omega) = \frac{1}{N}t(\omega). \quad (\text{A69})$$

Similarly,

$$u_N^C(\omega) = \frac{1}{N}u^C(\omega) \quad (\text{A70})$$

$$u_N^D(\omega) = \frac{1}{N}t(\omega) + \phi(1 - p\omega)^{N-1}. \quad (\text{A71})$$

**Proof of Lemma A5:** Under the definition,

$$F_N(x) = F(Nx), \quad (\text{A72})$$

we find, for any  $p \in [0, 1]$ , that

$$p = F_N(F_N^{-1}(p)) \equiv F(NF_N^{-1}(p)), \quad (\text{A73})$$

which can be rearranged to

$$F_N^{-1}(p) = \frac{1}{N}F^{-1}(p), \quad (\text{A74})$$

so for  $p = 1 - \omega$ , we have

$$F_N^{-1}(1 - \omega) = \frac{1}{N}F^{-1}(1 - \omega), \quad (\text{A75})$$

and therefore

$$t_N(\omega) = \frac{1}{N}t(\omega). \quad (\text{A76})$$

Using this first result, the others follow quickly

$$\begin{aligned} u_N^D(\omega) &\equiv t_N(\omega) + \phi(1 - p\omega)^{N-1} \\ &= \frac{1}{N}t(\omega) + \phi(1 - p\omega)^{N-1} \end{aligned} \quad (\text{A77})$$

$$\begin{aligned}
u_N^C(\omega) &\equiv \frac{p}{1-p\omega} \int_{\omega}^1 t_N(\Omega) \, d\Omega \\
&= \frac{p}{1-p\omega} \int_{\omega}^1 \frac{t(\Omega)}{N} \, d\Omega \\
&= \frac{1}{N} u^C(\omega).
\end{aligned} \tag{A78}$$

■

**Proof of Proposition 7:** Suppose that  $N > 1/p\hat{\omega}$ . Since the addition of a prize can only increase equilibrium disclosure,

$$N > \frac{1}{p\hat{\omega}} \geq \frac{1}{p\omega_N^*} > \frac{1-p\omega_N^*}{p\omega_N^*} \tag{A79}$$

By Proposition 8, which is proven later in the appendix, this implies that  $\omega_N^* > \omega_{N+1}^*$ . Since the same logic holds for all larger  $N$ , the sequence  $\omega_N^*, \omega_{N+1}^*, \omega_{N+2}^*, \dots$  is monotonically decreasing. Since the sequence is also bounded below by  $\hat{\omega}$ , it must have a limit, which we will refer to as

$$\omega_{\infty} = \lim_{N \rightarrow \infty} \omega_N^* \tag{A80}$$

The function  $B(\cdot)$  is continuous in  $\omega_N^*$  and  $N$ , and  $B_N(\omega_N^*) = 0$  for all  $N$ . The sequence  $\{B_N(\omega_N^*)\}$  therefore converges to zero as well:

$$\begin{aligned}
0 &= \lim_{N \rightarrow \infty} B_N(\omega_N^*) \\
&= \lim_{N \rightarrow \infty} t(\omega_N^*) + \lim_{N \rightarrow \infty} \phi(1-p\omega_N^*)^N - \lim_{N \rightarrow \infty} \frac{p \int_{\omega_N^*}^1 t(\Omega) \, d\Omega}{1-p\omega_N^*} \\
&= t(\omega_{\infty}) + 0 - \frac{p \int_{\omega_{\infty}}^1 t(\Omega) \, d\Omega}{1-p\omega_{\infty}}
\end{aligned} \tag{A81}$$

That is,

$$t(\omega_{\infty}) = \frac{p \int_{\omega_{\infty}}^1 t(\Omega) \, d\Omega}{1-p\omega_{\infty}}, \tag{A82}$$

This the same function which implicitly defines  $\hat{\omega}$ , so  $\omega_{\infty} = \hat{\omega}$ , as desired.

■

**Proof of Proposition 8:** Applying Lemma A5 to the definition of  $B(\omega)$  under equal shares competition yields

$$\begin{aligned}
B_N(\omega) &= u_N^D(\omega) - u_N^C(\omega) \\
&= \frac{1}{N} t(\omega) + \phi(1-p\omega)^{N-1} - \frac{1}{N} u^C(\omega),
\end{aligned} \tag{A83}$$

and therefore,

$$NB_N(\omega) = t(\omega) + N\phi(1 - p\omega)^{N-1} - u^C(\omega). \quad (\text{A84})$$

The same holds for  $N + 1$ . That is,

$$(N + 1)B_{N+1}(\omega) = t(\omega) + (N + 1)\phi(1 - p\omega)^N - u^C(\omega). \quad (\text{A85})$$

Subtracting Equation A84 from Equation A85 yields

$$\begin{aligned} (N + 1)B_{N+1}(\omega) - NB_N(\omega) &= (N + 1)\phi(1 - p\omega)^N - N\phi(1 - p\omega)^{N-1} \\ &= \phi(1 - p\omega)^{N-1} ((1 - p\omega) - Np\omega). \end{aligned} \quad (\text{A86})$$

If we evaluate the expression at  $\omega = \omega_N^*$ , then  $B_N(\omega_N^*) = 0$ , so Equation A86 reduces to

$$B_{N+1}(\omega_N^*) = \frac{\phi(1 - p\omega_N^*)^{N-1}}{N + 1} ((1 - p\omega_N^*) - Np\omega_N^*). \quad (\text{A87})$$

Focusing on the sign of the term in parenthesis, we find

$$N > \frac{1 - p\omega_N^*}{p\omega_N^*} \Rightarrow B_{N+1}(\omega_N^*) < 0 \Rightarrow \omega_{N+1}^* < \omega_N^*, \quad (\text{A88})$$

where the second implication is due to Lemma A4. That is, disclosure at the frequency  $\omega_N^*$  gives  $B < 0$ , so the marginal disclosure loses value. The equilibrium frequency  $\omega_{N+1}^*$  must be lower. This shows that the entry of the  $(N + 1)^{th}$  firm reduces disclosure when  $N$  is large. When  $N$  is smaller than the threshold, the inequalities in Equation A88 are reversed, as shown by the same logic. This completes the equivalence.

■

**Proof of Proposition 9:** Under generalized competition with  $N$  firms, we have

$$\begin{aligned} B_N(\omega) &= t_N(\omega) + \phi(1 - p\omega)^{N-1} - u_N^C(\omega) \\ &= \alpha_N t(\omega) + \phi(1 - p\omega)^{N-1} - \alpha_N u^C(\omega), \end{aligned} \quad (\text{A89})$$

and therefore,

$$\frac{1}{\alpha_N} B_N(\omega) = t(\omega) + \frac{1}{\alpha_N} \phi(1 - p\omega)^{N-1} - u^C(\omega). \quad (\text{A90})$$

The same holds for  $N + 1$ , so we can subtract the two equations to obtain

$$\begin{aligned}
& \frac{1}{\alpha_{N+1}} B_{N+1}(\omega) - \frac{1}{\alpha_N} B_N(\omega) \\
&= \frac{1}{\alpha_{N+1}} \phi(1 - p\omega)^N - \frac{1}{\alpha_N} \phi(1 - p\omega)^{N-1} \\
&= \frac{1}{\alpha_{N+1}} \phi(1 - p\omega)^{N-1} \left( (1 - p\omega) - \frac{\alpha_{N+1}}{\alpha_N} \right). \tag{A91}
\end{aligned}$$

Evaluating at  $\omega_N^*$  and rearranging terms yields

$$B_{N+1}(\omega_N^*) = (1 - p\omega_N^*)^{N-1} \phi \left( (1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N} \right). \tag{A92}$$

Under our assumption that  $\lim_{N \rightarrow \infty} \alpha_{N+1}/\alpha_N > 1 - p\hat{\omega}$ , there exists some  $\bar{N}$  such that

$$N > \bar{N} \Rightarrow \frac{\alpha_{N+1}}{\alpha_N} > 1 - p\hat{\omega}. \tag{A93}$$

So for  $N > \bar{N}$ , we obtain

$$\begin{aligned}
B_{N+1}(\omega_N^*) &= (1 - p\omega_N^*)^{N-1} \phi \left( (1 - p\omega_N^*) - \frac{\alpha_{N+1}}{\alpha_N} \right) \\
&< (1 - p\omega_N^*)^{N-1} \phi((1 - p\omega_N^*) - (1 - p\hat{\omega})) \\
&= (1 - p\omega_N^*)^{N-1} \phi(\hat{\omega} - \omega_N^*) < 0. \tag{A94}
\end{aligned}$$

By Lemma A4, we conclude that  $\omega_{N+1}^* < \omega_N^*$ , as desired.

■

## B Appendix

### Monopolist Model Examples

#### B.1 Monopolist Model Example One

Consider that a risk-neutral monopolist sorts through  $N$  firms to choose a single firm to do business with. The monopolist wishes to select the firm with the best realization of  $x_j$ , but faces the problem that when  $N$  or  $p$  is small, no firm may disclose their information at all.

In this example, increasing competition is a boon to the monopolist and increases aggregate welfare, as the probability of at least one firm disclosing increases with  $N$ . We formalize this in the following proposition.

**Proposition B1.** *Define  $Z_N$  as the ex ante probability that zero of the  $N$  competing firms disclose. Then,*

1.  $Z_N$  is decreasing in  $N$ ;
2.  $\lim_{N \rightarrow \infty} Z_N = 0$ .

**Proof of Proposition B1:** For part 1, note that for all  $N$  and corresponding thresholds  $t_N^*$ , the threshold condition says

$$0 = t_N^* + \phi(1 - p + pF(t_N^*))^{N-1} + \frac{p}{1 - p + pF(t_N^*)} \int_{-\infty}^{t_N^*} xf(x) dx \quad (\text{B1})$$

Multiplying by  $(1 - p + pF(t_N^*))$  yields

$$\begin{aligned} 0 &= t_N^* (1 - p + pF(t_N^*)) + \phi(1 - p + pF(t_N^*))^N - p \int_{-\infty}^{t_N^*} xf(x) dx \\ &= t_N^* (1 - p + pF(t_N^*)) + \phi Z_N - p \int_{-\infty}^{t_N^*} xf(x) dx \end{aligned} \quad (\text{B2})$$

and therefore,

$$\phi Z_N = -t_N^* (1 - p + pF(t_N^*)) + p \int_{-\infty}^{t_N^*} xf(x) dx \quad (\text{B3})$$

Differentiating with respect to  $N$  yields

$$\begin{aligned}\phi \frac{dZ_N}{dN} &= -\frac{\partial t_N^*}{\partial N} (1 - p + pF(t_N^*)) \\ &\quad - t_N^* p f(t_N^*) \frac{\partial t_N^*}{\partial N} + p \frac{\partial}{\partial N} \left( \int_{-\infty}^{t_N^*} x f(x) dx \right) \frac{\partial t_N^*}{\partial N} \\ &= -\frac{\partial t_N^*}{\partial N} (1 - p + pF(t_N^*))\end{aligned}\tag{B4}$$

Since  $\frac{\partial t_N^*}{\partial N} > 0$ , we have  $\frac{\partial Z_N}{\partial N} < 0$ , so  $Z_N$  is decreasing in  $N$  as desired.

For part 2, note that the probability of any single firm failing to disclose converges to a number less than 1:

$$\lim_{N \rightarrow \infty} (1 - p + pF(t_N^*)) = 1 - p + pF(t_\infty) < 1\tag{B5}$$

So the probability of zero firms disclosing converges to zero as  $N \rightarrow \infty$ ,

$$\lim_{N \rightarrow \infty} Z_N = \lim_{N \rightarrow \infty} (1 - p + pF(t_\infty))^N = 0\tag{B6}$$

■

## B.2 Monoplist Model Example Two

Now suppose instead that the monopolist is unconstrained in how many firms she can choose and that it is efficient for the monopolist to choose firms with a realization of  $\tilde{x}$  above a threshold  $\underline{x}$ , and not do business with firms below  $\underline{x}$ . Denote the net gain from the partnership as  $G$ ; because the screening party is monopolistic, we assume that she extracts  $G$  from the relationship.

Consider first that the prize is equal to zero. If  $\underline{x} \geq \hat{t}$ , the monopolist can always efficiently screen counterparties because  $\hat{t} \geq t^*$ . However, when  $\underline{x} < \hat{t}$ , there may exist a region of inefficiency. In this case, for any  $\tilde{x} \in [\underline{x}, t^*]$ , an informed firm withholds their information, despite being qualified. For a given threshold  $t^*$ , the ex ante expected welfare loss from the forgone opportunity is

$$\mathcal{L} = pN [F(t^*) - F(\underline{x})] G.\tag{B7}$$

The difference  $F(t^*) - F(\underline{x})$  is the expected fraction of firms that falls in the interval  $[\underline{x}, t^*]$ . Of this group of firms, a fraction  $p$  will be informed. Therefore, of the  $N$  potential counterparties, the monopolist expects  $pN [F(t^*) - F(\underline{x})]$  of them to be qualified but not identified.

Now, consider that the monopolist can encourage additional disclosure by offering a prize  $\phi$ . We assume that  $\phi$  is a transfer between the monopolist and the firm with the highest disclosure; therefore, the size of  $\phi$  does not affect aggregate welfare in and of itself. Again by construction, it is clear that no prize is needed when  $\underline{x} \geq \hat{t}$ . However, when  $\underline{x} < \hat{t}$ , the monopolist's goal is to set the optimal prize  $\phi$  to minimize the forgone opportunity. Specifically, the monopolist desires to lower the firms' threshold  $t^*$  closer to  $\underline{x}$ . As such, we denote the threshold as a function of  $\phi$ ,  $t(\phi)$ . Obviously, there is no additional benefit to increasing the prize once  $t(\phi) = \underline{x}$ . Hence, we define  $\underline{\phi}$  to be the prize that leads to that equality. The monopolist's problem for a given  $N$  is

$$\min_{\phi \in \mathbb{R}^+} pN [F(t(\phi)) - F(\underline{x})] G + \phi(1 - Z_N). \quad (\text{B8})$$

**Proposition B2.** *The optimal prize for the monopolist to offer is*

$$\phi^* = \begin{cases} 0 & \text{if } N < \frac{1-W^N}{GpF'W^{N-1}} \\ \min \left( GW - \frac{1-W^N}{NpF'W^{N-2}}, \underline{\phi} \right) & \text{if } N \geq \frac{1-W^N}{GpF'W^{N-1}} \end{cases}, \quad (\text{B9})$$

where  $W = (1 - p + pF(t))$  and  $F' = \partial F / \partial t$ .

**Proof of Proposition B2:** We begin solving the monopolist's problem by adopting the following shorthand notation,  $W \equiv (1 - p + pF(t))$  and  $F' \equiv \partial F / \partial t$ . Using first-order conditions with respect to  $\phi$  we obtain,

$$\begin{aligned} 0 &= \frac{\partial}{\partial \phi} [\mathcal{L} + \phi(1 - Z_N)] \\ &= \frac{\partial}{\partial \phi} [pN [F(t(\phi)) - F(\underline{x})] G + \phi(1 - Z_N)]. \end{aligned} \quad (\text{B10})$$

Recalling that  $Z_N = W^N$ ,

$$0 = pNF'G \frac{\partial t}{\partial \phi} + (1 - W^N) - \phi NW^{N-1} pF' \frac{\partial t}{\partial \phi} \quad (\text{B11})$$

Recall from Proposition 2 we obtained  $\partial t / \partial \phi$  from the Implicit Function Theorem using the net benefit of disclosure formula  $B$ . That is,

$$-\frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} = \frac{\partial t}{\partial \phi}. \quad (\text{B12})$$

Therefore, we can rewrite Equation B11 as,

$$= -pNF'G \left( \frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} \right) + (1 - W^N) + \phi NW^{N-1} pF' \left( \frac{\partial B / \partial \phi}{\partial B / \partial t} \Big|_{B=0} \right). \quad (\text{B13})$$

A rearrangement yields

$$\begin{aligned}
0 &= -pNF'G \frac{\partial B}{\partial \phi} \Big|_{B=0} + \phi NW^{N-1} pF' \frac{\partial B}{\partial \phi} \Big|_{B=0} + (1 - W^N) \frac{\partial B}{\partial t} \Big|_{B=0} \\
&= -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' \\
&\quad + (1 - W^N) \left[ 1 + \phi(N-1)pF'W^{N-2} + \frac{p^2F' \int_0^t x \, dF}{W^2} - \frac{ptF'}{W} \right] \\
&= -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' \\
&\quad + (1 - W^N) \left[ 1 + \phi(N-1)pF'W^{N-2} + \frac{pF'}{W} E[\tilde{x}_j | ND] - \frac{pF'}{W} t \right] \\
&= -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' \\
&\quad + (1 - W^N) [1 + \phi(N-1)pF'W^{N-2} + pF'\phi W^{N-2}] \\
&= -pNF'W^{N-1}G + \phi NW^{2(N-1)}pF' + (1 - W^N) [1 + \phi N pF'W^{N-2}] \tag{B14}
\end{aligned}$$

Solving for the optimal prize we obtain,

$$\phi = GW - \frac{1 - W^N}{NpF'W^{N-2}}. \tag{B15}$$

However, if  $\phi > \underline{\phi}$ , then it is a dominant strategy for the monopolist to offer  $\underline{\phi}$ , because incentivizing firms to disclose below  $\underline{x}$  yields no additional benefit. This yields that the optimal prize is equal to

$$\phi^* = \min \left( GW - \frac{1 - W^N}{NpF'W^{N-2}}, \underline{\phi} \right). \tag{B16}$$

The prize that the monopolist can offer is positive so long as as

$$N \geq \frac{1 - W^N}{GpF'W^{N-1}}. \tag{B17}$$

The monopolist is restricted to  $\phi \in \mathbb{R}^+$ , so when

$$N < \frac{1 - W^N}{GpF'W^{N-1}}, \tag{B18}$$

the prize is equal to zero.

■

The condition in (B9) can be appreciated as follows. First, it always holds that  $\phi^* < G$ . That is, the monopolist never has to give up the entire surplus that she earns



from working with the most qualified counterparty. Second, when the gain is low (small  $G$ ), the probability of being informed is low (small  $p$ ), and the distribution is relatively flat locally around the threshold (low  $F'$ ), the monopolist optimally avoids paying the cost of offering a prize. In such cases, efficiency is minimized.

The effect of competition on incentives to offer the prize may be non-monotonic for some  $N$ , but is strictly decreasing once  $N$  reaches a threshold. To see this, consider the limit when  $N \rightarrow \infty$ : it is optimal for the lender to offer no prize. Mathematically, we can re-write the condition in (B9) and compute

$$\lim_{N \rightarrow \infty} \frac{NGpF'W^{N-1}}{1 - W^N} = 0 < 1. \quad (\text{B19})$$

Economically, when  $N \rightarrow \infty$ , the prize becomes ineffective: firms minimize disclosure because their chances to win the prize tends to zero. In turn, the monopolist optimally chooses not to offer a prize and efficiency is minimized.

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## CHAPTER 3

# Political Influence and the Regulation of Consumer Financial Products

### 1 Introduction

Complexity has outpaced sophistication in retail financial markets.<sup>1</sup> This mismatch appears to affect social welfare and has been cited as a contributor and catalyst of the recent financial crisis.<sup>2</sup> The traditional approach to this problem allows markets to be free and then provides assistance to people *ex post* to help them make good decisions. This might involve improved education (e.g., Mandell, 2009)<sup>3</sup>, better timely decision support (e.g., Bertrand and Morse 2009; Lynch, 2009), or a policy of libertarian paternalism (e.g., Thaler and Sunstein, 2003; Choi, Madrian, Laibson, and Metrick, 2009; Carlin, Gervais, and Manso, 2011). These types of policies aim to help market participants protect themselves, while still implicitly allowing markets to be as complete as possible (and thereby complex).

Newer proposals, however, call for limiting the types of products that can be offered or traded in markets. Such policies aim to protect people from themselves. Possibilities include, for example, limiting the types of mortgages available to home buyers or controlling the types of investments that can be accessed by consumers. The idea

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<sup>1</sup>For example, in retail settings the menu of offerings is now daunting, but financial literacy remains in short supply (Lusardi and Mitchell, 2007). Many participants in the market have limited sophistication regarding the products in the market (e.g. NASD Literacy Survey, Associated Press, 2003). See also Capon, Fitzsimons, and Prince (1996), Alexander, Jones, and Nigro (1998), Barber, Odean, and Zheng (2005), and Agnew and Szykman (2005).

<sup>2</sup>For example, many home owners did not appreciate the variable-rate clauses in their mortgages and their explicit exposure to interest rate risk. Many individuals failed to appreciate the fees and interest rate schedules used commonly in credit cards, which exacerbated the amount of household debt and number of personal defaults in the United States (Campbell, 2006).

<sup>3</sup>See also Bernheim, Garrett, and Maki (2001), Bernheim and Garrett (2003), and Carlin and Robinson (2010)

here is that, instead of improving sophistication, simplicity or standardization is enforced. Such intervention is meant to prevent less educated consumers from choosing products distant from their actual needs and avoid the problems associated with information overload (e.g., Iyengar, Huberman, and Jiang, 2004; Salgado, 2006; Iyengar and Kamenica, 2008; Heidhues and Koszegi, 2010).

In this paper, we provide a theoretical model of product differentiation to explore these new policies and investigate several dimensions that affect the quality of regulation: the skill of the social planner, imperfect information, lobbying efforts, voting behavior in elections, and political philosophy (e.g., socialism versus libertarianism). As we show, these considerations interact and impact both the success of regulation and social welfare.

In our benchmark model, a continuum of products are offered in the market. The upper bound of the continuum represents the extent of the market, that is, how complete the market is. A unit mass of agents who use these products are divided into two groups: sophisticated agents who identify the product in the market that is best suited for their needs and unsophisticated agents that cannot do this. As such, unsophisticated agents make errors because they choose randomly among the products in the market. This sets up a natural tension between the two groups. Sophisticated agents desire the market to be as complete as possible, so they have more to choose from and can identify the product that is best tailored for their needs. In contrast, unsophisticated agents are ambivalent about market complexity: while they enjoy the benefits of having more choices in aggregate, they incur the cost of making inappropriate choices. Therefore, unsophisticated agents desire less market completion than people who are sophisticated.

We solve for the optimal level of market completion when a fully informed social planner regulates the market.<sup>4</sup> As one would expect, the upper bound of the continuum is strictly increasing in both the needs of the sophisticated and unsophisticated agents, increasing in the fraction of sophisticated agents, and decreasing in the fraction of unsophisticated agents. Importantly, however, we show in equilibrium that unsophisticated consumers actually desire *less* market completion than meets their needs

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<sup>4</sup>In the Appendix, we consider two extensions: one in which the planner controls both the upper and lower extent of the product continuum and one in which the planner can simultaneously educate consumers. We considered the case in which the planner can imperfectly screen for sophistication in a prior version of this paper.

in aggregate<sup>5</sup>.

We use the benchmark model as a platform to explore political economy considerations and their corresponding effects on the quality of regulation. We start by reconsidering the model when the social planner is uninformed about the specific needs of her constituents. Before regulating the market, the planner listens to the recommendations of two advocates, each lobbying on behalf of one of the groups of constituents. Based on these reports, the planner chooses and enforces a level of market complexity that best represents the wishes of her constituents, but is regrettably second best. We consider two types of uninformed planners in this fashion. The first is naive in that she accepts and uses reports by the lobbyists at face value. The second is savvy (i.e., rational) and uses her consistent beliefs about the lobbyists' incentives to misreport and therefore unwind their messages to better appreciate their information. We use a cheap-talk framework to analyze this latter setting (e.g., Crawford and Sobel, 1982).

Not surprisingly, both advocates do in fact misreport their private information with both types of planners. Indeed, it is a weakly dominant strategy for the sophisticated agents' advocate to lobby for full market completion (i.e., a libertarian platform). Taking this into account, the advocate for the unsophisticated also shades down, but his optimal strategy depends on the parameters in the model. In many cases, he successfully lobbies the planner to set a level of complexity that exactly optimizes the utility of the unsophisticated people. In other cases, he is less successful and receives an inferior outcome.

In equilibrium, both types of uninformed planners only implement second-best regulation. However, the punchline is that the least qualified planner usually gets elected to implement regulation. We model an election in which a perfectly informed planner, a rational uninformed planner, and a naive uninformed planner run for office. We show that when a supermajority exists, the most naive planner usually gets elected, even when sophisticated agents dominate the market. This places a bound on how effective regulation can be in the market. When unsophisticated agents have the supermajority, their advocate can get their needs met perfectly. When sophisticated agents dominate, they will often support the less qualified planner so their lobbyist can maximize the

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<sup>5</sup>Kuksov and Villas-Boas (2010) arrive at a similar conclusion in a model of costly search and evaluation.



benefit of exaggerating his recommendations. This is most likely to occur when the potential extent of the market exceeds their actual needs.

Based on this, we add an important aspect to the debate on product regulation. Specifically, even though there are two groups whose wishes need to be balanced, it is the incentive problems that arise in the market that predispose the planner to be less qualified. That is, quality suffers endogenously when two groups compete for their interests, which causes welfare to deviate from first best.

Finally, we consider alternative welfare functions. First, we consider that the planner may also value the disparity between losses of the unsophisticated and sophisticated agent groups. That is, we study the tradeoff between adequacy and equality in the market. Interestingly, our analysis demonstrates that if equality is sufficiently important (e.g., represents  $\frac{2}{3}$  of aggregate utility in the welfare function), the optimal regulation involves having one, and only one, product in the market. As such, a sufficiently socialistic perspective necessarily results in making the market as incomplete as possible, even though all agents end up hurt in the effort to maintain equality. Second, if the planner is concerned with minimizing the maximum loss sustained by any one agent then she is unable to influence social welfare with her choice. The result is striking, so we consider an alternative model setup and explore the robustness of the result. Even with a more flexible model, we find that the planner has little ability to influence social welfare through her choice. Indeed, in the alternative model, the social planner offers a single product to unsophisticated and sophisticated agents. We further study the generalized model format in Appendix B.

While we study optimal regulation in this paper, one might wonder whether regulation is even necessary in retail financial markets. Previous work suggests that it is: competition in financial markets may be an unreliable driver of market transparency and education may not improve social welfare. As Carlin, Davies, and Iannaccone (2012) show, when providers compete for attention in the market, competition actually makes it less likely that they disclose private information. Likewise, Carlin (2009) shows that when competition increases, providers actually have an increased non-cooperative incentive to add complexity to their offerings. Both of these papers make a strong argument for the value of market intervention. Carlin and Manso (2010) show that educational initiatives may be welfare decreasing in some circumstances since it induces providers of financial products to decrease clarity in the market via obfuscation. In

such cases, regulation may be important to increase welfare.

Last, our paper is of general economic interest as it adds to a large literature on product differentiation and efficiency in exchange economies. Starting with, Hotelling (1929), Chamberlin (1933), and Lancaster (1966), economists have focused on oligopoly behavior when there is a demand for differentiated products and consumers are heterogeneous.<sup>6</sup> In this literature, location games using linear city and circular city models are commonplace. Our work here departs in that we model the needs of two subsets of people by superimposing two (possibly different) linear city distributions and considering that one group is less sophisticated in assessing their needs for products in the market. Kamenica (2008) also considers a product market with both unsophisticated and sophisticated consumers. In the model, unsophisticated consumers learn about their needs based on the products that are offered by strategic firms. Similarly, Kuksov and Villas-Boas (2010) consider a product market with costly sequential search. The model's takeaway is that firms strategically choose their offerings to maximize the probability of purchase. We, however, abstract away from the oligopoly behavior that might lead to the evolution of such markets and focus instead on regulating such markets.

The rest of the paper is organized as follows. In Section 2, we pose and characterize our benchmark model, and derive the socially optimal product regulation. In Section 3, we evaluate the quality of regulation by considering imperfectly informed planners and the implications of lobbying and elections. Section 5 concludes. All of the proofs are in Appendix A. In Appendix B, we consider an alternative setup where the planner chooses both the lower and upper bounds of the market's completeness. Appendix C, we consider an alternative setup where the planner considers a tradeoff between clarity and simplicity in the market. There we show that educational initiatives and product regulation are strict substitutes.

## 2 Products and Regulation

We begin by establishing an underlying game form which serves as a platform to discuss political economy considerations in Sections 3 and 4. Consider the market in Figure 3.1

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<sup>6</sup>See Lancaster (1990) or Anderson, de Palma, and Thisse (1992) for a thorough survey of this literature.

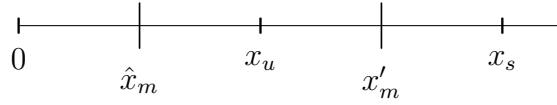


Figure 3.1: The product market. The types for unsophisticated agents are uniformly distributed along the interval  $[0, x_u]$  and the types for sophisticated agents are uniformly distributed on  $[0, x_s]$ . Products in the market are distributed on  $[0, x_m]$ . The bound  $\hat{x}_m$  is an example of an incomplete market for both sophisticated and unsophisticated agents. The bound  $x'_m$  is a market that is complete for unsophisticated agents, but is incomplete for sophisticated agents.

in which a continuum of products  $[0, x_m]$  are offered for use, where  $x_m$  measures the extent of the market. We interpret  $x_m$  as a measure of market completeness. For example, in Figure 3.1, the market with  $x'_m$  is more complete than the one with  $\hat{x}_m$ . Since people have more choices to consider with  $x'_m$  than with  $\hat{x}_m$ , we also consider  $x_m$  to be a measure of collective complexity in the market. The purpose of this paper is to study the socially optimal level of  $x_m$ . As such, we remain silent about what oligopoly behavior actually leads to any particular  $x_m$ .

There exists a unit mass of agents who participate in the market. A fraction  $\lambda_s$  of the agents are sophisticated and have a type  $\tilde{t}_s$ , which is uniformly distributed over  $[0, x_s]$ . These agents know their type exactly and maximize their payoff (to be described shortly) by choosing the product closest to their type. For example, in a product market where  $x_m = 0.55$ , a sophisticated agent with type  $\tilde{t}_s = 0.5$  would choose  $x = 0.5$ . However, if the product market was more limited in scope such that  $x_m = 0.4$ , the same sophisticated agent with type  $\tilde{t}_s = 0.5$  would choose  $x = 0.4$ , which is the best alternative available.

The remaining agents,  $\lambda_u = 1 - \lambda_s$ , are unsophisticated and have a type  $\tilde{t}_u$  distributed uniformly over  $[0, x_u]$ . We assume that unsophisticated agents do not know their own type and choose a random product  $\tilde{x}$  in  $[0, x_m]$ . By construction, unsophisticated agents make errors. This is standard in both the literature on consumer search theory (e.g., Salop and Stiglitz, 1977; Varian, 1980; and Stahl, 1989) and household finance (e.g., Carlin, 2009; Carlin and Manso, 2010).<sup>7</sup> An alternative specification

<sup>7</sup>For example, in models of “all-or-nothing” search (e.g., Salop and Stiglitz, 1977; and Varian, 1980), unsophisticated consumers are explicitly assumed to choose randomly among firms.

might allow them to see a noisy signal about their type, but the economics would be qualitatively similar to what follows here.

Sophisticated agents maximize their expected utility from participation. In this setting, this involves minimizing a loss function in which an agent is better off the closer they are to their true type. Since sophisticated agents know their type, they solve

$$\min_{x \in [0, x_m]} L(x|t_s, x_m) = |x - t_s|. \quad (1)$$

Unsophisticated agents, on the other hand, choose randomly from the menu of products offered<sup>8</sup>.

In this model, we assume that  $x_u \leq x_s$  to capture the idea that sophisticated agents may have use for more exotic products. For example, there may be a subset of home owners that demand a mortgage that amortizes in a particular way that is not appropriate for most home buyers. As we will see shortly, this induces a natural tension in the model: whereas sophisticated agents desire a more complete market, unsophisticated agents would prefer more standardization to avoid making errors and suffering losses. As we analyze in Section 2.3, the goal of the social planner is to set  $x_m$  to maximize welfare in the presence of this tension. It is important to point out, though, that the assumption that  $x_u \leq x_s$  is made for analytic convenience and is not necessary for the tension to be present: in Lemma 2, we show that the tension exists as long as  $x_s > \frac{3}{4}x_u$ .

We begin by showing some intuitive results about agent behavior that will be useful in Section 2.3 when we consider the social planner's problem.

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In sequential search models, unsophisticated consumers are randomly assigned to their first firm and then choose whether to continue searching for the best alternative. In equilibrium, unsophisticated consumers stop at the first firm, so that they in essence make purchases randomly from the firms. See either Stahl (1989) or Baye, Morgan, and Scholten (2006) for a complete review of consumer search theory.

<sup>8</sup>In a similar fashion to Kuksov and Villas-Boras (2010), we use the absolute distance  $|x - t_s|$  as the loss function for analytic tractability. Using alternative loss functions (e.g., quadratic) only allows for numerical solutions, which yield qualitatively similar results to the ones we derive in the paper.

## 2.1 Sophisticated Agents

Since each sophisticated agent knows their type, we can write the aggregate loss to these agents  $\mathcal{L}_s$  as

$$\mathcal{L}_s = \int_0^{x_m} 0 \frac{d\tilde{t}_s}{x_s} + \int_{x_m}^{x_s} |x_m - \tilde{t}_s| \frac{d\tilde{t}_s}{x_s}. \quad (2)$$

The following lemma establishes some useful results.

**Lemma 1.** *If  $x_m < x_s$ , the aggregate loss for sophisticated agents is*

$$\mathcal{L}_s = \frac{x_s^2 - 2x_mx_s + x_m^2}{2x_s}, \quad (3)$$

*which is decreasing and convex in  $x_m$ . If  $x_m \geq x_s$ , then  $\mathcal{L}_s = 0$ .*

According to Lemma 1, for  $x_m < x_s$ , some sophisticated agents are able to use a product that is identically perfect for their needs. Others, however, have to settle for a suboptimal choice. Figures 3.2(a) and 3.2(b) provide examples of this. The “hockey-stick”-shaped figure labeled  $L(\tilde{t}_s)$  plots the individual losses that sophisticated agents experience. The aggregate loss for the group is the triangular area bounded by the kink in the curve to the left and  $x_s$  to the right. This area is calculated analytically by the expression in (3).

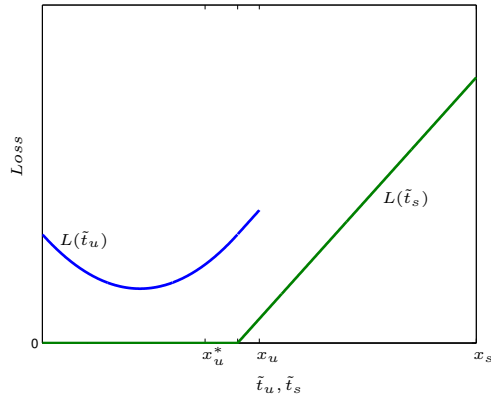
As  $x_m$  rises, this lowers the aggregate loss to sophisticated agents. This is best seen in Figure 3.3(a): the curve labeled  $\mathcal{L}_s$  is downward sloping as a function of  $x_m$ . In the limit, when  $x_m \rightarrow x_s$ ,  $\mathcal{L}_s \rightarrow 0$ . This implies that any market expansion beyond  $x_s$  adds no value to sophisticated agents. We define

$$x_s^* \equiv x_s \quad (4)$$

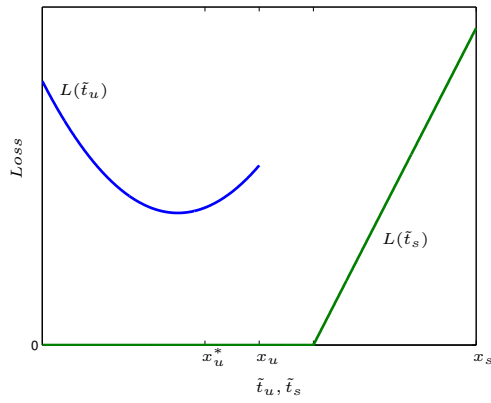
to be the point at which the aggregate loss to sophisticated agents is minimized. As we will see shortly when we consider the social planner’s problem, we will be able to allow  $x_s^*$  to serve as an upper bound for  $x_m$  without loss of generality.

## 2.2 Unsophisticated Agents

Unsophisticated agents have no information regarding their type  $\tilde{t}_u$  and choose randomly from the products in the market. The expected aggregate loss for unsophisti-



(a)  $x_m \leq x_u$ .



(b)  $x_m > x_u$ .

Figure 3.2: Individual losses given  $x_m$ .

cated agents in a product market characterized by  $x_m$  is

$$\mathcal{L}_u \equiv E[L(\tilde{t}_u, \tilde{x}|x_m)] = \int_0^{x_u} \int_0^{x_m} |\tilde{x} - \tilde{t}_u| \frac{d\tilde{x}}{x_m} \frac{d\tilde{t}_u}{x_u}. \quad (5)$$

We can now make some statements regarding the unsophisticated agents' preferences for  $x_m$ .

**Lemma 2.** *The level of  $x_m$  that maximizes the expected aggregate welfare of unsophisticated agents is given by*

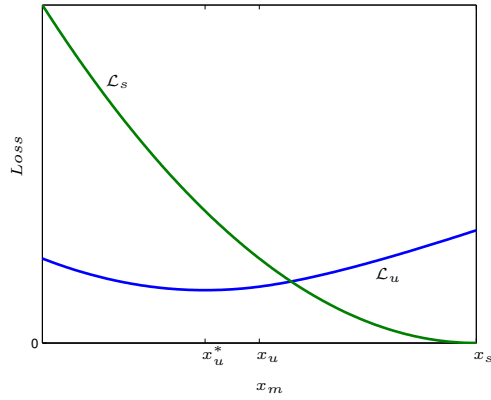
$$x_u^* \equiv \frac{3}{4}x_u. \quad (6)$$

*When  $x_m > \frac{3}{4}x_u$ ,  $\mathcal{L}_u$  is increasing and convex in  $x_m$ .*

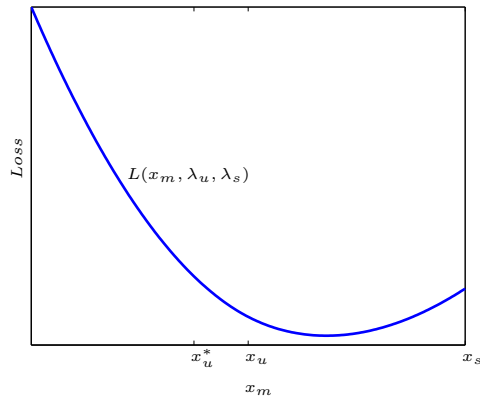
Lemma 2 tells us that unsophisticated agents are worse off when products are offered in the market that exceed their particular needs. More interestingly, they not only have an aversion to such products, but actually prefer the market to be *less* complete than meets their needs in aggregate. The intuition behind the result lies in the tradeoff between providing access to more products in the market and the cost of introducing more room for error. The highest types of unsophisticated agents benefit from more products because it improves the chances that they randomly select a product near their type. Conversely, lower types suffer as it becomes more likely that they choose wrongly. The loss for the mass of unsophisticated agents is minimized at  $\frac{3}{4}x_u$ , which results from the triangular nature of the loss function induced by the uniform distribution.

These results can be appreciated visually. The individual losses for the unsophisticated agents are plotted on the curves labeled  $L(\tilde{t}_u)$  in Figures 3.2(a) and 3.2(b). By inspection, the loss function is convex with a minimum strictly less than  $x_u$ . The aggregate loss function to the unsophisticated agents  $\mathcal{L}_u$  is plotted in Figure 3.3(a) as a function of  $x_m$ . Again, by inspection, it is clear that an  $x_m$  strictly less than  $x_u$  maximizes welfare for unsophisticated agents.

Since  $x_u^* < x_u$ , this implies a natural tension between sophisticated and unsophisticated agents, even if  $x_u = x_s$ . The social optimal level of  $x_m$  will take these forces into account, and we derive  $x_m^*$  next.



(a) Unsophisticated and sophisticated losses.



(b) Aggregate expected loss.

Figure 3.3: Ex ante expected losses as a function of  $x_m$ .



### 2.3 Optimal Product Market Complexity

The aggregate loss to all agents is given as

$$L(x_m, \lambda_u, \lambda_s) = \lambda_u \mathcal{L}_u + \lambda_s \mathcal{L}_s. \quad (7)$$

The aggregate loss has the form

$$L(x_m, \lambda_u, \lambda_s) = \begin{cases} \lambda_u \left[ \frac{x_m^2}{3x_u} + \frac{-x_m + x_u}{2} \right] + \lambda_s \left[ \frac{x_s^2 - 2x_m x_s + x_m^2}{2x_s} \right] & x_m \leq x_u \\ \lambda_u \left[ \frac{x_m^2 - x_m x_u + (2/3)x_u^2}{2x_m} \right] + \lambda_s \left[ \frac{x_s^2 - 2x_m x_s + x_m^2}{2x_s} \right] & x_m > x_u. \end{cases} \quad (8)$$

The analytic expressions for  $\mathcal{L}_u$  in (8) are derived in the proof of Lemma 2. One concern about  $L(x_m, \lambda_u, \lambda_s)$  might be whether it is discontinuous at  $x_u$ . As we show in Lemma A1 in the appendix, this function is indeed continuously differentiable at the point  $x_u$ .

The social planner solves the following problem

$$\min_{x_m \in [0, x_s]} L(x_m, \lambda_u, \lambda_s). \quad (9)$$

As noted before, the social planner can restrict her attention to  $x_m \leq x_s$  because once  $x_m \geq x_s$ , increasing it further makes the sophisticated agents no better off, but hurts the unsophisticated. The following proposition characterizes the unique socially optimal level of  $x_m^*$ .

**Proposition 1.** *There exists a unique optimal  $x_m^* \in [x_u^*, x_s^*]$  that minimizes  $L(x_m, \lambda_u, \lambda_s)$ .*

If

$$\frac{\lambda_u}{\lambda_s} < 6 \left( 1 - \frac{x_u}{x_s} \right), \quad (10)$$

then  $x_m^* > x_u$ .

The optimal  $x_m^*$  is

1. increasing in  $x_u$  and  $x_s$ ;
2. decreasing in the mass of unsophisticated agents,  $\lambda_u$ ;
3. increasing in the mass of sophisticated agents,  $\lambda_s$ .

According to Proposition 1, if the needs of either type of agent increase ceteris paribus, the optimal scope of the market is higher. However,  $x_m$  is determined based

on the proportion of types in the market. As the fraction of sophisticated agents rises,  $x_m^*$  is higher. As  $\lambda_u$  increases,  $x_m^*$  is lower. These comparative statics are a direct result of the natural tension between the two groups.

It is important to note that the market does not need to have the particular structure that we consider here to have the same comparative statics hold. For example, in Appendix B we consider an alternative specification in which the lower bound of the market is not tethered to zero. As we show, sophisticated consumers still prefer the market to be as complete as possible, whereas unsophisticated consumers desire there to be one product that is the median of their needs. We solve for the planner's optimal choice of lower and upper bounds, and show that such bounds change monotonically in the underlying parameters of the model.

Before closing this section, let us consider a special case that we will use periodically in the rest of the paper. Specifically, let

$$x_u = x_s \equiv x_p. \quad (11)$$

From Proposition 1, we know that  $\frac{\lambda_u}{\lambda_s} \geq 6 \left(1 - \frac{x_u}{x_s}\right) = 0$ , so that  $x_m^* \leq x_p$ . Taking first-order conditions with respect to the aggregate loss function, we obtain

$$0 = \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m \leq x_u) \quad (12)$$

$$= \lambda_u \left[ \frac{2x_m}{3x_p} - \frac{1}{2} \right] + \lambda_s \left[ -1 + \frac{x_m}{x_p} \right]. \quad (13)$$

Solving for  $x_m$  yields

$$x_m^* = \frac{3x_p(2 - \lambda_u)}{2(3 - \lambda_u)}. \quad (14)$$

Note that when  $\lambda_u = 0$  we obtain that  $x_m^* = x_p$ , which is intuitive since all agents are sophisticated. Likewise when all agents are unsophisticated,  $\lambda_u = 1$ , we obtain  $x_m^* = \frac{3}{4}x_p$ , which is consistent with their ideal point. If  $\lambda_u = \lambda_s = \frac{1}{2}$ , we obtain  $x_m^* = \frac{9}{10}x_p$ . Thus, the expression in (14) confirms our previous claim that even when all agents are distributed uniformly on identical supports, the presence of unsophisticated agents yields an internal optimum (i.e.,  $x_m^* < x_p$ ).

### 3 The Quality of Product Regulation

The quality of regulation depends, among other things, on the ability of elected officials to understand markets and their knowledge regarding the needs of their constituents.

In this section, we explore how these considerations affect product regulation. So far, we have assumed that the social planner perfectly observes  $x_u$  and  $x_s$ . But in reality, this is far from true. Policy makers, as benevolent as their motives might be, cannot know everything about their constituents and often resort to listening to the opinions of lobbyists and advocates before setting policy. Therefore, in this section, we consider a setting of imperfect information and study the distortions that may arise from both lobbying and voting behavior.

We consider two types of social planners. The first is *naive* in that she does not have information about  $x_u$  or  $x_s$ , and blindly accepts recommendations from advocates who represent the sophisticated and unsophisticated agents' interests. The second planner is *savvy* in that while she does not have information about market participant needs, she rationally understands the incentives of the advocates to misreport, and therefore unwinds such reports to refine her beliefs before making a final decision regarding regulation.

Compared to a social planner with perfect information, these two types of leaders do not achieve first best regulation. However, we complete the analysis by studying the voting behavior of sophisticated and unsophisticated agents, given that they participate in an election in which all three types of planners run for office: the naive uninformed, the savvy uninformed, and the perfectly informed. We determine how qualified the regulator is that gets elected and how this affects the quality of regulation.

### 3.1 Lobbying Efforts - Naive Social Planner

Consider the model setup from Section 2, except that the upper bounds  $x_u$  and  $x_s$  are unobservable to the social planner. Instead, there are two advocates that represent each group:  $A_u$  lobbies for unsophisticated agents and  $A_s$  lobbies for the sophisticated. Each advocate makes a report,  $r_u$  and  $r_s$ , about their respective bound. Since it is prohibitively costly to canvass the population to assess each person's needs, the reports are not verifiable ex post. As in Section 2,  $x_u \leq x_s$ , but we now assume that there is a finite bound on the sophisticated agents' needs (i.e.,  $x_s \leq \bar{x}$ ).

The social planner faithfully accepts the values provided by the lobbyists and uses them to choose an  $x_m(r_u, r_s)$ . However, the planner does not attempt to unwind the true observations of  $x_u$  and  $x_s$  from the reports, using the incentives that each advocate has to lobby for their constituents. We explore that consideration in the next

subsection.

The following proposition characterizes optimal reporting behavior by the advocates.

**Proposition 2.** *It is a weakly dominant strategy for  $A_s$  to always report  $r_s = \bar{x}$ . If, and only if,*

$$x_u \geq \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right), \quad (15)$$

*$A_u$  reports  $r_u > 0$  such that  $x_u^* = x_m(r_u, \bar{x})$ . Otherwise,  $A_u$  reports  $r_u = 0$  and the planner sets*

$$x_m = \bar{x} \left[ 1 - \frac{\lambda_u}{2\lambda_s} \right]. \quad (16)$$

The intuition behind the first part of Proposition 2 rests on the observation that sophisticated agents are only hurt when they are underserved. If a social planner sets  $x_m$  below  $x_s$ , the sophisticated agents with needs in  $[x_m, x_s]$  are unable to find a product with perfect fit. When  $x_m$  is set equal to or above  $x_s$ , all sophisticated agents are able to find the product they need. With this, and knowing that it is the social planner's objective to balance the needs of the two groups, the best response for  $A_s$  is to maximally exaggerate the sophisticated agents' needs. The strategy is costless and shifts  $x_m$  upward, ceteris paribus.

The advocate for unsophisticated agents takes this into account and reports  $r_u < x_u$  (i.e., shades down). According to Proposition 2, if the condition in (15) holds,  $A_u$  will make a positive report and successfully lobby the planner to restrict the market to  $x_m = x_u^*$ . If (15) does not hold, then  $A_u$  cannot push the planner to choose the unsophisticated ideal point with any report. In such case,  $A_u$  will claim that unsophisticated agents need one, and only one, product.

The condition in (15) provides some natural economic insights. First, the closer  $x_u$  is to  $\bar{x}$ , the more likely it is that unsophisticated agents are able to obtain their ideal point. The more interesting result is expressed in the following corollary.

**Corollary 1.** *When the planner depends on lobbyist reports, the unsophisticated agents obtain  $x_m = x_u^*$  if  $\lambda_u \geq 2\lambda_s$ .*

Corollary 1 tells us that if unsophisticated agents make up two thirds of the population, they will be able to persuade an imperfect social planner to choose  $x_m = x_u^*$ .

Importantly, this holds true no matter how large  $\bar{x}$  is and does not depend on any other exogenous political pressures (i.e., getting re-elected or monetary payoffs). So, as long as unsophisticated agents make up at least  $\frac{2}{3}$  of the population, they get their way by lobbying a naive social planner without full information.

### 3.2 Lobbying Efforts - Savvy Social Planner

Consider now that the social planner anticipates that the two advocates have the incentive to misreport their group's needs. Again, the social planner does not observe  $x_s$  and  $x_u$  but instead has to rely on the advocates' reports and her prior beliefs. For analytic ease, we assume that the social planner believes that  $x_u$  and  $x_s$  are distributed uniformly over  $[0, \bar{x}]$ , with  $x_u \leq x_s$ .

**Lemma 3.** *The social planner's unconditional beliefs for  $x_u$  and  $x_s$  are given by,*

$$E[x_u] = \frac{\bar{x}}{3} \tag{17}$$

$$E[x_s] = \frac{2\bar{x}}{3}. \tag{18}$$

The two agents again have the incentive to misreport their needs because the planner is imperfectly informed. Furthermore, since there is no punishment for lying and reporting is costless, a truth-telling, incentive compatible mechanism is elusive. Since this is a cheap-talk game, messages that are sent to the planner will be insubstantial if the advocates are restricted to perfectly reliable, noiseless communication channels (e.g., Farrell, 1998; Forges, 1986 and 1988). Therefore, in the spirit of Crawford and Sobel (1982), we model the described lobbying problem as a cheap-talk game in which the advocates and the planner adhere to an equilibrium message strategy and a corresponding action function.

Each of the agents independently observes their group's needs and sends the planner a report,  $r_i$  with  $i \in \{u, s\}$ . Correspondingly, the planner processes  $r_u$  and  $r_s$  and then determines an  $x_m$  conditional on the messages and her prior beliefs. Denote  $x_m^a(r_u, r_s)$  to be the decision made by the social planner. The planner's problem therefore is to minimize,

$$\min_{x_m^a \in [0, \bar{x}]} |x_m^a(r_u, r_s) - x_m^*|, \tag{19}$$

where  $x_m^*$  is the optimal level of sophistication if the planner was perfectly informed.

Similarly, denote each advocates's objective function as,

$$\min_{r_i \in [0, \bar{x}] | r_{-i}} |x_m^a(r_u, r_s) - (x_m^* - b_i)|, \quad (20)$$

where  $b_i$  represents the advocate's bias from the first-best solution. For unsophisticated agents, the bias is given by  $b_u = x_m^* - x_u^*$ , whereas for sophisticated agents it is  $b_s = x_m^* - x_s^*$ .

**Definition 1.** *An equilibrium in the cheap-talk lobbying problem consists of*

1. *a message strategy for each advocate,  $q_i(r_i|x_i)$ , such that  $\int_0^{\bar{x}} q_i(r_i|x_i) dr_i = 1$  for all  $x_i \in [0, \bar{x}]$ ,*

2. *a choice function for the principal,  $x_m^a(r_u, r_s)$  such that*

(a) *for each  $x_u, x_s \in [0, \bar{x}]$  if  $q_i(r'_i|x_i) > 0$  then it must be that*

$$r'_i \in \arg \min_{r_i} |x_m^a(r_u, r_s) - (x_m^* - b_i)|$$

,

(b) *and  $x_m^a(r_u, r_s) \in \arg \min_x \int_0^{\bar{x}} |x - x_m^*| p(x_m^*|r_u, r_s) dx_m^*$  where*

$$p(x_m^*|r_u, r_s) = \frac{q_u(r_u|x_u, r_s)q_s(r_s|x_s)}{\int_0^{\bar{x}} \int_0^{\bar{x}} q_u(r_u|\mu_u, r_s)q_s(r_s|\mu_s) d\mu_u d\mu_s}$$

Definition 1 essentially says that for any realization of  $x_i$ , advocate  $i$  will mix over a set of messages,  $\{\vec{r}_i\}$ , such that the sum of probabilities on each possible message add to one. Furthermore, given the planner's decision rule,  $x_m^a(r_u, r_s)$ , any message  $r'_i$ , sent with positive probability, must imply that  $x_m^a(r'_i|r_{-i})$  results in an outcome that is no worse than the outcome that would have resulted from sending any other message  $r''_i \in [0, \bar{x}]$ . Additionally, given the family of message rules for advocates  $A_u$  and  $A_s$ , the choice  $x_m^a(r_u, r_s)$  must be a solution to the social planner's problem.

We now appeal to the result of Crawford and Sobel (1982) that, for any  $b_i > 0$ , there exists at least one "partition" equilibrium, with specific properties to be discussed momentarily, such that each advocate reports in which partition of  $[0, \bar{x}]$  their realization lies. Correspondingly, the planner takes the midpoint of the partition as a "noisy" estimate of the true realization.<sup>9</sup>

<sup>9</sup>For any  $b_i > 0$  it is essential that each partition is "noisy", meaning that it has positive mass, for substantive communication to occur. Farrell (1988) and Forges (1986, 1988) demonstrate that messages in sender-receiver games are insubstantial if players are restricted to perfectly reliable, noiseless communication channels.

**Lemma 4.** *It is a weakly dominant strategy for  $A_s$  to always report  $r_s = \bar{x}$ . Furthermore, this implies that  $q_s(\bar{x}|x_s) = 1$  and  $q_s(r'_s|x_s) = 0$  for all  $x_s \in [0, \bar{x}]$  and  $r'_s \in [0, \bar{x}]$  and that*

$$p(x_m^*|r_u, \bar{x}) = \frac{q_u(r_u|x_u)}{\int_0^{\bar{x}} q_u(r_u|\mu_u, \bar{x}) d\mu_u}.$$

Lemma 4 greatly reduces the complexity of our lobbying problem since we now only need to concern ourselves with the message strategy of  $A_u$ . The properties of the “partition” equilibrium follow as,

1. there is a positive integer,  $N$ , such that one can define a set of  $N + 1$  real numbers, generically denoted  $\{r_u^0, r_u^1, \dots, r_u^N\}$  with  $r_u^0 < r_u^1 < \dots < r_u^N$ ,

2.

$$x_m^a(r_u^j) = \begin{cases} x_m \left( \frac{r_u^{j+1} + r_u^j}{2}, E \left[ x_s \mid x_s \geq \frac{r_u^{j+1} + r_u^j}{2} \right] \right) & N > 1 \\ x_m \left( \frac{\bar{x}}{3}, \frac{2\bar{x}}{3} \right) & N = 1, \end{cases}$$

where  $x_m(\cdot, \cdot)$  is the optimal  $x_m$  given a noisy indication of  $x_u$  and  $x_s$ .

3.  $q_u(r_u|x_u)$  is uniform over  $[r_u^i, r_u^{i+1}]$  if  $x_u \in [r_u^i, r_u^{i+1}]$ .

Now, we direct our attention to ex ante efficient message strategies and action rules, since  $x_u$  is unobservable and the advocate’s true bias is unknown to the social planner. The following proposition addresses the maximum number of partitions that can be supported in equilibrium.

**Proposition 3.** *An equilibrium exists in which the maximum number of partitions,  $\bar{N}$ , that can be supported is*

$$\left\langle -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{32 - 3\lambda_s}{5\lambda_s}} \right\rangle \quad (21)$$

where  $\langle z \rangle$  denotes the smallest integer greater than or equal to  $z$ . Additionally, only a babbling equilibrium,  $\bar{N} = 1$ , exists if  $\lambda_s > \frac{2}{3}$ , while a perfectly informative equilibrium exists if  $\lambda_s = 0$ .

According to Proposition 3, a babbling equilibrium exists as long as  $\lambda_u$  is less than  $1/3$ , in which the report is completely uninformative. Indeed, Crawford and Sobel (1982) show that a babbling equilibrium,  $N = 1$ , always exists. In such an equilibrium,

the planner would be left to choose  $x_m$  based solely on her unconditional beliefs outlined in Lemma 3.

Proposition 3 tells us that the informativeness of agent  $A_u$ 's report depends on the proportion of unsophisticated agents in the economy. Interestingly, because  $A_s$  exaggerates his report, equilibrium messages from  $A_u$  do not contain much information unless unsophisticated agents make up a substantial proportion of the population. According to (21),  $\bar{N} \geq 3$  can only be supported if  $\lambda_u > 3/4$  and  $\bar{N} \geq 4$  can only be supported if  $\lambda_u > 27/31$  (i.e., 87% of the population). This means that the savvy planner only partitions the message space into quartiles if the unsophisticated agents comprise roughly 87% of the population. This severely limits the planner's ability to conduct inference. Of course, as  $\lambda_u \rightarrow 1$ , the number of partitions goes to infinity and the planner receives a perfectly informative signal. This is not surprising because when  $\lambda_u \rightarrow 1$ , there is no longer any conflict and  $A_u$  simply tells the truth. Practically speaking, however, this is usually not the case, and we often have to settle with a savvy planner that cannot learn much from lobbying efforts.

### 3.3 Voting Behavior

We now consider what type of social planner is elected to regulate markets, which will greatly impact the quality of such regulation. We assume that all three types of planners run for office prior to the implementation of any financial policy. We assume all agents participate in the election, so outcomes will be based on their respective proportions within the population.

Let the optimal levels of market sophistication under each type of planner be denoted as,

$$\begin{cases} x_m^P & \text{Perfectly Informed} \\ x_m^V & \text{Savvy} \\ x_m^N & \text{Naive} \end{cases}$$

where  $x_m^P \neq x_m^V \neq x_m^N$ .

**Proposition 4.** *In any election, it is impossible for a unanimous decision to take place.*

Sophisticated and unsophisticated agents never agree. The reason for this is that they always have conflicting preferences as to which planner is optimal. For example,



when the condition in (15) holds, unsophisticated agents are satisfied perfectly with an imperfect planner, but sophisticated agents would rather elect a fully informed planner to drive  $x_m$  upward. However, if (15) is not satisfied, then  $r_u = 0$  and the two groups' preferences diverge based on the two differences,  $\bar{x} - x_s$  and  $\bar{x} - x_u$ . If these differences are large, sophisticated agents prefer the uninformed planner because they gain from their exaggerated report; unsophisticated agents would rather have an informed planner to minimize the cost of  $A_s$ 's exaggeration. If these differences are small, the opposite results hold.

Going forward, we analyze the election results given that there exists a supermajority of one type of agent in the population. Specifically, we consider either that  $\lambda_u \geq 2/3$  or that  $\lambda_s \geq 2/3$ . This is for mathematical convenience, but supermajority voting requirements are commonplace in U.S. legislative procedures and other political arenas.

**Proposition 5.** *When a supermajority of  $\frac{2}{3}$  exists, a savvy, uninformed social planner never gets elected. When  $\lambda_u \geq 2/3$ , a naive social planner always gets elected. When  $\lambda_s \geq 2/3$ , the planner with the higher  $x_m$  gets elected, that is  $\max(x_m^P, x_m^N)$ . If  $\lambda_s \geq 2/3$  and  $\bar{x} \geq \frac{4}{3}x_s$ , a naive social planner always gets elected.*

According to Proposition 5, the least qualified social planner often gets elected by whomever has a supermajority. Per Corollary 1, if  $\lambda_u \geq 2/3$ ,  $A_u$  always gets his way:  $x_m = x_u^*$  is always chosen. If  $\lambda_s \geq 2/3$  and  $\bar{x} \geq \frac{4}{3}x_s$ , the sophisticated agents again elect the least qualified social planner. This implies that if the potential extent of the market ( $\bar{x}$ ) is sufficiently high compared to the actual needs of the sophisticated agents ( $x_s$ ), the sophisticated advocate is able to use misreporting to his advantage to better satisfy the needs of the sophisticated agents. It is also important to note that  $\bar{x} \geq \frac{4}{3}x_s$  is a sufficient condition, but is not necessary for the sophisticated agents to elect the least qualified planner. That is, there are other parameters for which a naive planner gets elected when  $\lambda_s \geq 2/3$ . It is only when  $\bar{x}$  is close to  $x_s$  that a perfectly informed planner gets elected. This occurs because the ability of  $A_s$  to misreport is lower than that for  $A_u$ . As such, the sophisticated agents elect someone who is knowledgeable.

This analysis has several qualitative welfare consequences. Proposition 5 tells us that even if we have the ability to regulate markets, this may not be desired by market participants. Indeed, the least qualified person gets elected in many cases, and due to lobbying behavior,  $x_m$  is not set at the first best level set in Section 2. This implies

that proponents of product regulation need to take into account how qualified the leaders are who implement such policies, and incentives within the system to preclude knowledgeable people from attaining such roles.

## 4 Alternative Welfare Specifications

In the previous sections, the social planner chose the optimal level of market sophistication,  $x_m^*$ , by minimizing the total aggregate loss. The comparative statics of  $x_m^*$  in Proposition 1 reveal that the difference in expected loss for unsophisticated and sophisticated agents can be large when one group is much smaller than the other and when the upper bounds on the groups' needs are greatly dissimilar. That setup assumes that total aggregate loss is the only concern of the social planner. Here we extend the analysis to two scenarios: one where the planner is concerned about the degree of disparity between the agents and one where the planner is concerned about the maximum loss sustained by any one agent. In both scenarios we assume that  $x_u = x_s = x_p$  and  $\lambda_u \in (0, 1)$ .

### 4.1 Equality Versus Adequacy

We begin with the scenario where the social planner is interested in not only the total loss, but also the expected degree of disparity between sophisticated and unsophisticated agents<sup>10</sup>. We define the degree of disparity between agent groups as

$$D(x_m, \lambda_u, \lambda_s) \equiv |\mathcal{L}_u - \mathcal{L}_s|. \quad (22)$$

Equation 22 and Equation 8 from Section 2.3 can be combined to produce a welfare equation that incorporates both concerns,

$$W = \kappa L(x_m, \lambda_u, \lambda_s) + (1 - \kappa)D(x_m, \lambda_u, \lambda_s), \quad (23)$$

where  $\kappa \in [0, 1]$ . We assume that  $\kappa$  is exogenously given and it represents the social planner's preferences over the two matters.

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<sup>10</sup>Given that there are continua of sophisticated and unsophisticated agents, the law of large numbers makes our analysis here apply to ex post dispersion as well.

**Proposition 6.** *The optimal level of market sophistication when the social planner is concerned with both total aggregate loss and the disparity between sophisticated and unsophisticated agents is given by*

$$x_m^* = \begin{cases} 0 & \text{if } \kappa < \frac{1}{3-\lambda_u} \\ \frac{3}{2} \frac{1-\kappa(3-\lambda_u)}{1-\kappa(4-\lambda_u)} x_p & \text{if } \kappa \geq \frac{1}{3-\lambda_u}. \end{cases} \quad (24)$$

If  $\kappa \geq \frac{1}{3-\lambda_u}$ , then  $x_m^*$  is strictly increasing in  $\kappa$  and strictly decreasing in  $\lambda_u$ .

The comparative statics in  $\kappa$  and  $\lambda_u$  are straightforward. That is, as the importance of aggregate social loss increases (higher  $\kappa$ ), the optimal market completeness rises. Likewise, as the number of unsophisticated agents increases, the lower  $x_m^*$  will be.

What is interesting is that as long as  $\kappa < \frac{1}{3}$ , the social planner optimally chooses to have a one-product market. This means that if equality is most important, as it would be in a socialistic society, no differentiation is allowed in the market. Of course, as  $\lambda_u$  rises, this bound becomes larger. As  $\lambda_u \rightarrow 1$ , if  $\kappa < \frac{1}{2}$ , then a one product market is optimal.

Proposition 6 implies that there is a tradeoff between a market that provides adequate products to its constituents and the equality that people experience when they make choices. As  $\kappa$  decreases and equality is more important,  $x_m$  decreases and deviates more from the optimum derived in Section 2.3. This means that the more equality is weighted, the less adequate is the market, especially for sophisticated agents. As such, Proposition 6 captures the idea that aggregate losses may increase as equality concerns are introduced. When  $\kappa < \frac{1}{3}$ , equality is indeed achieved, but unfortunately both sophisticated and unsophisticated agents are equally worse off. This captures one of the potential drawbacks of a socialistic agenda (e.g., Stiglitz, 1994).

We complete this section with the following example.

**Example 1.** *Suppose that  $\kappa = \frac{1}{2-\lambda_u}$ . Then, plugging into (24) yields  $x_m^* = \frac{3}{4}x_u$ .*

The significance of Example 1 is that unsophisticated agents may benefit substantially at the expense of sophisticated agents. In this case,  $\kappa$  is such that  $x_m = x_u^*$ . Of course, if  $\kappa$  were to decrease further, the aggregate loss for both groups would rise, even as the two aggregate losses converged more.

## 4.2 Minimizing the Maximum Loss

The degree of disparity formulation in the previous section measures the difference between an *average* unsophisticated agent and an *average* sophisticated agent. The difference in average losses is certainly one measure of disparity, however, the planner may be concerned with the maximum loss sustained by any one agent. Let all agents, unsophisticated and sophisticated, be indexed by  $i$  and define the maximum loss sustained by any individual as,

$$\max_i |x - t_\psi^i| \text{ with } \psi \in \{u, s\}. \quad (25)$$

We assume that there is a positive measure of both agent types, and consequently, there is always a loss of  $x_p > 0$ . This is because at least one unsophisticated agent with type  $x_p$  will mistakenly choose the product located at zero. If minimizing the maximum loss is indeed a concern for the planner, our setup does not permit her choice to influence social welfare. In Appendix B, however, we consider a variant to the model: we allow the social planner to choose both the upper *and* lower bound of the market offerings. We denote the lower bound of market offerings as  $x_{m,l}$  and the upper bound as  $x_{m,u}$ . In this setup, the planner's problem is given by,

$$\min_{x_{m,l}, x_{m,u}} \max_i |x - t_\psi^i| \text{ with } \psi \in \{u, s\}. \quad (26)$$

It is straightforward to see that the preceding expression is rewritten as,

$$\min_{x_{m,l}, x_{m,u}} \max\{x_{m,u}, x_p - x_{m,l}\}. \quad (27)$$

The maximum possible losses for unsophisticated agents occur when an agent of type  $t_u = 0$  accidentally chooses the product furthest from his type, i.e., he chooses  $x_{m,u}$  and sustains a loss of  $x_{m,u}$ , or when an agent of type  $t_u = x_p$  chooses  $x_{m,l}$  and sustains a loss of  $x_p - x_{m,l}$ . The maximum possible loss for sophisticated agents is dominated by the maximum possible losses sustained by the unsophisticated. As such, the maximum possible loss across all agents is given by the expression in Equation 27. The following proposition demonstrates the optimal levels of  $x_{m,l}$  and  $x_{m,u}$  when the planner is concerned with minimizing the maximum loss sustained by any one agent.

**Proposition 7.** *The planner chooses  $x_{m,l}^* = x_{m,u}^* = \frac{x_p}{2}$  to minimize the maximum loss sustained by any one agent.*

Proposition 7 highlights a tension not obvious in our previous results: although many agents experience a loss, the losses are not felt equally agent-to-agent. Consider the unsophisticated agents; an agent with type  $t_u = 0$  or  $t_u = x_p$  is likely to sustain a greater utility loss than an unsophisticated agent of type  $t_u = \frac{x_p}{2}$ , since his mistakes tend to be larger. Sophisticated agents also sustain losses if the market is anything but complete, i.e.,  $x_{m,l} \neq 0$  or  $x_{m,u} \neq x_p$ , but these losses are dominated by the losses to unsophisticated agents. Proposition 7 addresses the tension on an agent-to-agent basis by minimizing the maximum loss sustained by any one agent. It is intuitive that the proposition prescribes the most conservative level of market completion, i.e., the market consists of a single product. The conservative nature provides agents with a robustness that no single loss is “too large.”

## 5 Concluding Remarks

The model we have explored provides an innovative framework to explore the tension that exists in offering products to agents with heterogeneous levels of sophistication. There is a natural inclination to think markets should be complete so that participants are free to make choices that best fit their needs. Skeptics, on the other hand, think such a paradigm is too idealistic. They believe that agents are prone to make mistakes and that offering too many products introduces room for error. We have characterized this friction by modeling a market where perfect and imperfect agents jointly participate. In a parsimonious model, we have analyzed the optimal level of collective complexity with respect to each group’s size and needs.

We also have explored the effects of political influence and political philosophy on the quality of regulation. Uninformed planners, whether naive or rational, will only achieve second best regulation. Sophisticated agents always support a libertarian platform, even if they will not utilize the most complex products in the market. Both sophisticated and unsophisticated agents often vote for the least qualified planner when they have a supermajority, which erodes the quality of regulation. Finally, socialism that dictates equality, necessarily decreases product differentiation and destroys the surplus that can be produced by completing markets.

In the end, our analysis provides a new dimension to consider in the debate of financial product regulation. Policy makers should carefully consider the tradeoff between

satisfying agents' needs and introducing the possibility of blunders. Furthermore, a robust policy is one that resolves the lobbying and political problems that arise in the market.

## A Appendix

**Proof of Lemma 1:** First, consider that  $x_m \geq x_s$ . The loss for a given sophisticated consumer with type  $\tilde{t}_s$  is given by

$$L(x|\tilde{t}_s, x_m) = |x - \tilde{t}_s|. \quad (\text{A1})$$

Because the consumer knows their type perfectly and their type is available in the set of financial products  $[0, x_m]$ , the agent will choose  $x = \tilde{t}_s$ . Thus,  $|\tilde{t}_s - \tilde{t}_s| = 0$  for all types of sophisticated agents. Therefore,  $\mathcal{L}_s = 0$ .

Now consider that  $x_m < x_s$ . By the same reasoning, for any sophisticated agent with type  $\tilde{t}_s \in [0, x_m]$ , their loss is zero. However, the sophisticated agents with type  $\tilde{t}_s \in [x_m, x_s]$  will choose  $x = x_m$  to minimize their losses. Since sophisticated consumers are distributed uniformly, the aggregate expected loss in this population is given by

$$\begin{aligned} \mathcal{L}_s &= \int_0^{x_m} 0 \frac{d\tilde{t}_s}{x_s} + \int_{x_m}^{x_s} |x_m - \tilde{t}_s| \frac{d\tilde{t}_s}{x_s} \\ &= \int_{x_m}^{x_s} (\tilde{t}_s - x_m) \frac{d\tilde{t}_s}{x_s} \\ &= \left. \frac{\tilde{t}_s^2}{2x_s} - \frac{x_m \tilde{t}_s}{x_s} \right|_{x_m}^{x_s} \\ &= \frac{x_s^2 - 2x_m x_s + x_m^2}{2x_s}. \end{aligned}$$

The first-order derivative of the expected loss with respect to  $x_m$  is

$$\frac{\partial}{\partial x_m} E[L(\tilde{t}_s, x|x_m)] = \frac{-x_s + x_m}{x_s},$$

The which is strictly negative because  $x_m < x_s$  by assumption.

The second-order derivative of the expected loss with respect to  $x_m$  is given by,

$$\frac{\partial^2}{\partial x_m^2} E[L(\tilde{t}_s, x|x_m)] = \frac{1}{x_s} > 0,$$

which tells us that the loss function is convex in  $x_m$ .

■

**Proof of Lemma 2:** Suppose first that  $x_m > x_u$ . Using the expected loss for a given unsophisticated consumer in Equation 5, we can compute the expected loss for the

group as a whole. The expected loss is,

$$\begin{aligned}
E[L(\tilde{t}_u, x|x_m)] &= \int_0^{x_u} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m(x_m - \tilde{t}_u)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left( \frac{(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_u} \left( \frac{x_m^2 - 2x_m\tilde{t}_u + 2\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \left( \frac{x_m^2\tilde{t}_u - x_m\tilde{t}_u^2 + (2/3)\tilde{t}_u^3}{2x_mx_u} \right) \Big|_0^{x_u} \\
&= \frac{x_m^2x_u - x_mx_u^2 + (2/3)x_u^3}{2x_mx_u} \\
&= \frac{x_m^2 - x_mx_u + (2/3)x_u^2}{2x_m}.
\end{aligned}$$

For convenience we note the following computation that was used in the last calculation:

$$\int |(ax + b)^n| dx = \frac{(ax + b)^{n+2}}{a(n+1)|ax + b|} + C,$$

where  $n$  is odd and  $n \neq -1$ .

The first-order derivative of the expected loss with respect to  $x_m$  is given by,

$$\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m)] = \frac{1}{2} - \frac{x_u^2}{3x_m^2},$$

which is strictly positive because  $x_m > x_u$  by assumption. Additionally, the loss function is convex. By second-order conditions we obtain,

$$\frac{\partial^2}{\partial x_m^2} E[L(\tilde{t}_u, x|x_m)] = \frac{2x_u^2}{3x_m^3} > 0.$$

Note that we did not solve for the optimal  $x_m$  for unsophisticated consumers using this first-order equation because our assumption that  $x_m \geq x_u$  allowed us a simplifying step that  $|x_m - \tilde{t}_u| = (x_m - \tilde{t}_u)$ . We address the optimal  $x_m$  shortly. It is useful to note here, that had we solved for the optimum using this first-order condition we would have obtained,

$$x_m^* \equiv x_u \sqrt{\frac{2}{3}}. \tag{A2}$$



In this case,  $x_m^* < x_u$ , violating our assumption that  $x_m \geq x_u$ .

Now, we consider the case when now we look at  $x_m < x_u$ . Using the expected loss for a given unsophisticated consumer in Equation 5, we can compute the expected loss for the group as a whole. The expected loss is,

$$\begin{aligned}
E[L(\tilde{t}_u, x|x_m)] &= \int_0^{x_u} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m|x_m - \tilde{t}_u|} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m(x_m - \tilde{t}_u)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left( \frac{(x_m - \tilde{t}_u)^3}{2x_m(\tilde{t}_u - x_m)} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left( \frac{(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left( \frac{-(x_m - \tilde{t}_u)^2}{2x_m} + \frac{\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \int_0^{x_m} \left( \frac{x_m^2 - 2x_m\tilde{t}_u + 2\tilde{t}_u^2}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} + \int_{x_m}^{x_u} \left( \frac{-x_m^2 + 2\tilde{t}_u x_m}{2x_m} \right) \frac{d\tilde{t}_u}{x_u} \\
&= \left( \frac{x_m^2\tilde{t}_u - x_m\tilde{t}_u^2 + (2/3)\tilde{t}_u^3}{2x_mx_u} \right) \Big|_0^{x_m} + \left( \frac{-x_m^2\tilde{t}_u + \tilde{t}_u^2 x_m}{2x_mx_u} \right) \Big|_{x_m}^{x_u} \\
&= \frac{x_m^3 - x_m^3 + (2/3)x_m^3}{2x_mx_u} - 0 + \frac{-x_m^2x_u + x_mx_u^2}{2x_mx_u} - \frac{-x_m^3 + x_m^3}{2x_mx_u} \\
&= \frac{x_m^2}{3x_u} + \frac{-x_m + x_u}{2}.
\end{aligned}$$

First-order conditions with respect to  $x_m$  yield the ideal level of market sophistication for unsophisticated consumers,

$$\begin{aligned}
0 &= \frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m)] \\
&= \frac{2x_m}{3x_u} - \frac{1}{2}.
\end{aligned}$$

Additionally, the loss function is convex. The second-order condition is given as,

$$\frac{\partial^2}{\partial x_m^2} E[L(\tilde{t}_u, x|x_m)] = \frac{2}{3x_u} > 0.$$

Therefore, the unsophisticated consumers' losses are minimized when

$$x_u^* \equiv \frac{3}{4}x_u < x_u. \tag{A3}$$

■

**Lemma A1.** *The function  $L(x_m, \lambda_u, \lambda_s)$  is continuously differentiable at  $x_u$ .*

**Proof of LemmaA1:** Consider the following:

$$\begin{aligned}
E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{x_m^2}{3x_u} + \frac{-x_m + x_u}{2} \Big|_{x_m=x_u} \\
&= \frac{x_u}{3} \\
E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{x_m^2 - x_mx_u + (2/3)x_u^2}{2x_m} \Big|_{x_m=x_u} \\
&= \frac{x_u}{3} \\
\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{2x_m}{3x_u} - \frac{1}{2} \Big|_{x_m=x_u} \\
&= \frac{1}{6} \\
\frac{\partial}{\partial x_m} E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{1}{2} - \frac{x_u^2}{3x_m^2} \Big|_{x_m=x_u} \\
&= \frac{1}{6} \\
\frac{\partial^2}{\partial x_m^2} E[L(\tilde{t}_u, x|x_m \leq x_u)] &= \frac{2}{3x_u} \Big|_{x_m=x_u} \\
&= \frac{2}{3x_u} \\
\frac{\partial^2}{\partial x_m^2} E[L(\tilde{t}_u, x|x_m > x_u)] &= \frac{2x_u^2}{3x_m^3} \Big|_{x_m=x_u} \\
&= \frac{2}{3x_u}
\end{aligned}$$

■

**Proof of Proposition 1:** Both of the loss functions of the sophisticated and unsophisticated agents are strictly convex by Lemmas 1 and 2. Since the sum of two convex functions is also convex,  $L(x_m, \lambda_u, \lambda_s)$  is strictly convex. By Lemma A1,  $L(x_m, \lambda_u, \lambda_s)$  is continuously differentiable. Finally, since  $L(x_m, \lambda_u, \lambda_s)$  evaluated on the compact set  $[0, x_s]$ , we know that a unique global minimum exists.

Now, we proceed to the claim in (10). By Lemmas 1 and 2, the loss function for unsophisticated agents is strictly increasing for any  $x_m > x_u^*$  and the loss function for sophisticated consumers is strictly decreasing for any  $x_m < x_s$ . If the marginal benefit

of increasing  $x_m$  to sophisticated consumers is greater than the marginal cost to the unsophisticated, the first-order condition of the aggregate loss function will be negative. Evaluating the first-order condition of the aggregate loss function at  $x_m = x_u$  yields,

$$\frac{\partial}{\partial x_m} L(x_u, \lambda_u, \lambda_s) = \frac{\lambda_u}{6} + \lambda_s \left( -1 + \frac{x_u}{x_s} \right),$$

which is negative if and only if  $\frac{\lambda_u}{\lambda_s} < 6 \left( 1 - \frac{x_u}{x_s} \right)$ , implying that the  $x_m^* > x_u$ .

Clearly, since  $\frac{\partial \mathcal{L}_u}{\partial x_m} > 0$  for all  $x_m > x_u^*$  and  $\frac{\partial \mathcal{L}_s}{\partial x_m} < 0$  for all  $x_m < x_s^*$ , the solution to the social planner's problem lies in the interval  $[x_u^*, x_s^*]$ . Moreover, it is straightforward to show that there exists an internal solution. The derivative  $\frac{\partial \mathcal{L}_s}{\partial x_m}$  is zero at  $x_s$ , whereas  $\frac{\partial \mathcal{L}_u}{\partial x_m} = \frac{2x_s}{3x_u} - \frac{1}{2}$ . Likewise, The derivative  $\frac{\partial \mathcal{L}_u}{\partial x_m}$  is zero at  $x_u^*$ , whereas  $\frac{\partial \mathcal{L}_s}{\partial x_m} = \frac{x_u}{x_s} - 1 < 0$ .

We now proceed to write an expression for  $x_m^*$  so that we may proceed with comparative statics exercises. Using the condition in (10), we consider the two possible cases for optimums. First, suppose that  $\frac{\lambda_u}{\lambda_s} \geq 6 \left( 1 - \frac{x_u}{x_s} \right)$ . First-order conditions with respect to the aggregate loss function yield

$$0 = \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m \leq x_u) \quad (\text{A4})$$

$$= \lambda_u \left[ \frac{2x_m}{3x_u} - \frac{1}{2} \right] + \lambda_s \left[ -1 + \frac{x_m}{x_s} \right]. \quad (\text{A5})$$

Recall that the ideal point for sophisticated consumers is  $x_s^* = x_s$ , and the ideal point for unsophisticated consumers, given that  $x_m \leq x_u$ , is  $x_u^* = \frac{3}{4}x_u$ . Substituting in the ideal points yields

$$0 = \lambda_u \left[ \frac{2x_m}{4x_u^*} - \frac{1}{2} \right] + \lambda_s \left[ -1 + \frac{x_m}{x_s^*} \right]. \quad (\text{A6})$$

Solving for  $x_m$  yields

$$x_m^* = \frac{x_u^* x_s^* (\lambda_u + 2\lambda_s)}{\lambda_u x_s^* + 2\lambda_s x_u^*}. \quad (\text{A7})$$

Now, consider that  $\frac{\lambda_u}{\lambda_s} < 6 \left( 1 - \frac{x_u}{x_s} \right)$ . We define the function  $g(x_m)$  to be the first-order conditions with respect to the aggregate loss function, i.e.

$$\begin{aligned} g(x_m) &\equiv \frac{\partial}{\partial x_m} L(x_m, \lambda_u, \lambda_s | x_m > x_u) \\ &= \lambda_u \left[ \frac{1}{2} - \frac{x_u^2}{3x_m^2} \right] + \lambda_s \left[ -1 + \frac{x_m}{x_s} \right]. \end{aligned} \quad (\text{A8})$$

Since, solving for an explicit solution to  $g(x_m) = 0$  is analytically intractable, we leave  $x_m^*$  as implicitly defined by  $g(x_m^*) = 0$ , and use the implicit function theorem to derive comparative statics.

Because of the piecewise construction of the aggregate loss function, we need to determine the four comparative statics for both the case when  $x_m^* \leq x_u$  and when  $x_m^* > x_u$ . First we consider the former.

The optimal level of sophistication is given by Equation A7. We start by considering the comparative static of  $x_m^*$  with respect to  $\lambda_u$  and note that  $\lambda_u + \lambda_s = 1$ ,

$$\begin{aligned}
\frac{\partial x_m^*}{\partial \lambda_u} &= \frac{\partial}{\partial \lambda_u} \frac{x_u^* x_s^* (\lambda_u + 2\lambda_s)}{\lambda_u x_s^* + 2\lambda_s x_u^*} \\
&= \frac{-x_u^* x_s^* (\lambda_u x_s^* + 2\lambda_s x_u^*) - x_u^* x_s^* (\lambda_u + 2\lambda_s) (x_s^* - 2x_u^*)}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-\lambda_u x_u^* x_s^{*2} - 2\lambda_s x_u^{*2} x_s^* - \lambda_u x_u^* x_s^{*2} + 2\lambda_u x_u^{*2} x_s^* - 2\lambda_s x_u^* x_s^{*2} + 4\lambda_s x_u^{*2} x_s^*}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-2\lambda_u x_u^* x_s^{*2} + 2\lambda_s x_u^{*2} x_s^* + 2\lambda_u x_u^{*2} x_s^* - 2\lambda_s x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{-2\lambda_u x_u^* x_s^{*2} + 2(1 - \lambda_u) x_u^{*2} x_s^* + 2\lambda_u x_u^{*2} x_s^* - 2(1 - \lambda_u) x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{2x_u^{*2} x_s^* - 2x_u^* x_s^{*2}}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2} \\
&= \frac{2x_u^* x_s^* (x_u^* - x_s^*)}{(\lambda_u x_s^* + 2\lambda_s x_u^*)^2}
\end{aligned}$$

and because  $x_u^* < x_s$ ,

$$< 0.$$

The comparative static of  $x_m^*$  with respect to  $\lambda_s$ ,

$$\frac{\partial x_m^*}{\partial \lambda_s} = \frac{\partial x_m^*}{\partial \lambda_u} \frac{\partial \lambda_u}{\partial \lambda_s},$$

and because  $\frac{\partial \lambda_u}{\partial \lambda_s} = -1$ ,

$$> 0.$$

Now we consider the comparative static of  $x_m^*$  with respect to  $x_u$ . For analytic ease, we substitute out the ideal points for both sophisticated and unsophisticated consumers, i.e.  $x_s^* = x_s$ , and given that  $x_m \leq x_u$ ,  $x_u^* = \frac{3}{4}x_u$ .

$$\begin{aligned}
\frac{\partial x_m^*}{\partial x_u} &= \frac{\partial}{\partial x_u} \frac{x_u x_s (3\lambda_u + 6\lambda_s)}{4\lambda_u x_s + 6\lambda_s x_u} \\
&= \frac{(3\lambda_u x_s + 6\lambda_s x_s)(4\lambda_u x_s + 6\lambda_s x_u) - 6\lambda_s(3\lambda_u x_u x_s + 6\lambda_s x_u x_s)}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_s^2 + 18\lambda_u \lambda_s x_u x_s + 24\lambda_u \lambda_s x_s^2 + 36\lambda_s^2 x_u x_s - 18\lambda_u \lambda_s x_u x_s - 36\lambda_s^2 x_u x_s}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_s^2 + 24\lambda_u \lambda_s x_s^2}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&> 0.
\end{aligned}$$

And lastly with the comparative static of  $x_m^*$  with respect to  $x_s$ ,

$$\begin{aligned}
\frac{\partial x_m^*}{\partial x_s} &= \frac{\partial}{\partial x_s} \frac{x_u x_s (3\lambda_u + 6\lambda_s)}{4\lambda_u x_s + 6\lambda_s x_u} \\
&= \frac{(3\lambda_u x_u + 6\lambda_s x_u)(4\lambda_u x_s + 6\lambda_s x_u) - 4\lambda_u(3\lambda_u x_u x_s + 6\lambda_s x_u x_s)}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{12\lambda_u^2 x_u x_s + 18\lambda_u \lambda_s x_u^2 + 24\lambda_u \lambda_s x_u x_s + 36\lambda_s^2 x_u^2 - 12\lambda_u^2 x_u x_s - 24\lambda_u \lambda_s x_u x_s}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&= \frac{18\lambda_u \lambda_s x_u^2 + 36\lambda_s^2 x_u^2}{(4\lambda_u x_s + 6\lambda_s x_u)^2} \\
&> 0.
\end{aligned}$$

Now we consider when  $x_m^* > x_u$ . Because we did not solve for an explicit solution for  $x_m^*$ , we utilize the Implicit Function Theorem with our characteristic equation, Equation A8, that implicitly defines  $x_m^*$ , i.e.  $g(x_m^*) = 0$ . The Implicit Function Theorem tells us how the optimal level of sophistication changes with the parameter values. For each parameter  $\theta \in \{\lambda_u, \lambda_s, x_u, x_s\}$ , the IFT gives

$$\frac{\partial x_m^*}{\partial \theta} \equiv \frac{-\left. \frac{\partial g(x_m)}{\partial \theta} \right|_{x_m=x_m^*}}{\left. \frac{\partial g(x_m)}{\partial x_m} \right|_{x_m=x_m^*}} \quad (\text{A9})$$

We begin by showing that  $\frac{\partial g(x_m)}{\partial x_m} > 0$ ,

$$\begin{aligned}\frac{\partial g(x_m)}{\partial x_m} &= \lambda_u \frac{2x_u^2}{3x_m^3} + \frac{\lambda_s}{x_s} \\ &> 0.\end{aligned}$$

Differentiating with respect to each of the parameters and recalling that  $\lambda_u + \lambda_s = 1$  yields

$$\begin{aligned}\frac{\partial g(x_m)}{\partial \lambda_u} &= \underbrace{\frac{1}{2} - \frac{x_u^2}{3x_m^2}}_{> 0 \text{ since } x_u < x_m} + \underbrace{1 - \frac{x_m}{x_s}}_{\geq 0 \text{ since } x_m \leq x_s} &> 0 \\ \frac{\partial g(x_m)}{\partial \lambda_s} &= \underbrace{-\frac{1}{2} + \frac{x_u^2}{3x_m^2}}_{< 0 \text{ since } x_u < x_m} + \underbrace{-1 + \frac{x_m}{x_s}}_{\leq 0 \text{ since } x_m \leq x_s} &< 0 \\ \frac{\partial g(x_m)}{\partial x_u} &= \frac{-2\lambda_u x_u^2}{3x_m^2} &< 0 \\ \frac{\partial g(x_m)}{\partial x_s} &= \frac{-\lambda_s x_m}{x_s^2} &< 0\end{aligned}$$

■

**Lemma A2.** *The function  $x_m^*(x_u, x_s, \lambda_u, \lambda_s)$  is continuously differentiable at  $x_u$ .*

**Proof of Lemma A2:** The proof follows directly from Lemma A1 because  $x_m^*$  is determined by the first-order condition of the aggregate loss function with respect to  $x_m$ . We know that the first and second derivatives of the aggregate loss function are continuous, therefore  $x_m^*(x_u, x_s, \lambda_u, \lambda_s)$  is continuous.

■

**Proof of Proposition 2:** We begin the proof by showing that it is a weakly dominant strategy to choose  $r_s = \bar{x}$ . The aggregate loss to sophisticated agents,  $\mathcal{L}_s$ , is a function of the true upper bound  $x_s$  and the social planner's choice of  $x_m$  based on the reports  $r_s$  and  $r_u$ . The change in the aggregate loss to sophisticated agents with respect to  $r_s$  is given by,

$$\frac{\partial}{\partial r_s} \mathcal{L}_s(x_s, x_m(r_s, r_u)) = \frac{\partial \mathcal{L}_s}{\partial x_m} \frac{\partial x_m}{r_s}$$

From Lemma 1 we know that  $\frac{\partial \mathcal{L}_s}{\partial x_m} \leq 0$ . Furthermore, because the social planner takes  $A_s$  at their word, we know from Proposition 1 that  $\frac{\partial x_m}{\partial r_s} > 0$ . Therefore,  $\frac{\partial}{\partial r_s} \mathcal{L}_s(x_s, x_m(r_s, r_u)) \leq 0$  which yields the desired result that  $r_s = \bar{x}$

Now we examine the strategy of  $A_u$ . By Lemma A2 and Proposition 1 we know that  $x_m(r_u, r_s)$  is continuously differentiable and strictly increasing in  $r_u$ . Because  $[0, \bar{x}]$  is compact, if  $x_m(0, \bar{x}) \leq x_u^* \leq x_m(\bar{x}, \bar{x})$  we know by the Intermediate Value Theorem that there exists an  $\hat{r}_u \in [0, \bar{x}]$  such that  $x_u^* = x_m(\hat{r}_u, \bar{x})$ . Furthermore, because  $x_m$  is strictly increasing in  $r_u$ ,  $\hat{r}_u$  is unique.

When  $x_u^* \notin [x_m(0, \bar{x}), x_m(\bar{x}, \bar{x})]$  it must be the case that either  $x_u^* < x_m(0, \bar{x})$  or  $x_m(\bar{x}, \bar{x}) < x_u^*$ . However, since  $x_m(\bar{x}, \bar{x}) \in [\frac{3}{4}\bar{x}, \bar{x}]$  and  $x_u^* \leq \frac{3}{4}\bar{x}$ , then it cannot be that  $x_m(\bar{x}, \bar{x}) < x_u^*$ . We know from Lemma 2 that the losses for unsophisticated agents,  $\mathcal{L}_u$ , are increasing for all  $x_m > x_u^*$ . Therefore, when  $x_u^* \notin [x_m(0, \bar{x}), x_m(\bar{x}, \bar{x})]$ , it is a dominant strategy for  $A_u$  to report the lowest possible value for  $x_u$ , that is  $r_u = 0$ .

Consequently, the reporting strategy of  $A_u$  is segregated into two cases; those when he makes a positive report and those when he reports  $r_u = 0$ . We first consider the former, when there exists an  $r_u > 0$  such that  $x_u^* = x_m(r_u, \bar{x})$ .

Because the naive planner takes  $r_u$  to be the true value of  $x_u$ , we know from Proposition 1 that the planner's choice of  $x_m$  is set according to a piecewise function. In fact,  $x_m \leq r_u$  so long as  $\frac{\lambda_u}{\lambda_s} > 6(1 - \frac{r_u}{\bar{x}})$ , and  $x_m > r_u$  otherwise. (A7) provides the planner's rule for setting  $x_m$  when it falls below  $r_u$ . The advocate chooses  $r_u$  such that it results in the planner setting  $x_m$  equal to the unsophisticated bliss point,

$$\begin{aligned} x_u^* &= x_m(r_u, \bar{x}) \\ &= \frac{r_u r_s (3\lambda_u + 6\lambda_s)}{4\lambda_u r_s + 6\lambda_s r_u} \\ &= \frac{r_u \bar{x} (3\lambda_u + 6\lambda_s)}{4\lambda_u \bar{x} + 6\lambda_s r_u}. \end{aligned}$$

A rearrangement yields,

$$4\lambda_u \bar{x} x_u^* = r_u \bar{x} (3\lambda_u + 6\lambda_s) - 6\lambda_s r_u x_u^*.$$

Solving for  $r_u$  we obtain,

$$r_u = \frac{4\lambda_u \bar{x} x_u^*}{\bar{x} (3\lambda_u + 6\lambda_s) - 6\lambda_s x_u^*},$$

and recall that  $x_u^* = (3/4)x_u$

$$r_u = x_u \left( \frac{3\lambda_u \bar{x}}{3\lambda_u \bar{x} + 6\lambda_s(\bar{x} - (3/4)x_u)} \right). \quad (\text{A10})$$

Clearly from (A10), conditional on  $x_m \leq r_u$  and  $\bar{x}$  being finite, there always exists a report  $r_u$  such that  $x_u^* = x_m(r_u, \bar{x})$ . Thus,  $r_u$  cannot be 0 if  $x_m \leq r_u$ .

Consider now the planner's rule for  $x_m > r_u$ . Although we do not have a closed-form solution, the planner chooses  $x_m$  so that it satisfies (A8). Again, the advocate will choose  $r_u$  such that  $x_u^* = x_m(r_u, \bar{x})$ ,

$$\begin{aligned} 0 &= \lambda_u \left[ \frac{1}{2} - \frac{r_u^2}{3x_m^2} \right] + \lambda_s \left[ -1 + \frac{x_m}{r_s} \right] \\ &= \lambda_u \left[ \frac{1}{2} - \frac{r_u^2}{3x_u^{*2}} \right] + \lambda_s \left[ -1 + \frac{x_u^*}{\bar{x}} \right]. \end{aligned}$$

A rearrangement yields,

$$\begin{aligned} \frac{\lambda_u r_u^2}{3x_u^{*2}} &= \frac{\lambda_u}{2} + \lambda_s \left[ -1 + \frac{x_u^*}{\bar{x}} \right] \\ r_u^2 &= \frac{3x_u^{*2}}{\lambda_u} \left( \frac{\lambda_u}{2} + \lambda_s \left[ -1 + \frac{x_u^*}{\bar{x}} \right] \right) \\ &= x_u^{*2} \left( \frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[ -3 + \frac{3x_u^*}{\bar{x}} \right] \right) \\ r_u &= x_u^* \left( \frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[ -3 + \frac{3x_u^*}{\bar{x}} \right] \right)^{1/2}, \end{aligned}$$

and recall that  $x_u^* = (3/4)x_u$

$$r_u = x_u \frac{3}{4} \left( \frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[ \frac{9x_u}{4\bar{x}} - 3 \right] \right)^{1/2}. \quad (\text{A11})$$

Equation (A11) is real valued so long as  $\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[ \frac{9x_u}{4\bar{x}} - 3 \right] \geq 0$ , or

$$x_u \geq \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right) \quad (\text{A12})$$

The unsophisticated advocate's report is characterized by either (A10) or (A11), depending on whether the planner's choice of  $x_m$  is greater or less than  $r_u$ . The specific



condition is pinned down by solving for  $r_u = x_m$ . Utilizing Equation A8 yields,

$$\begin{aligned} 0 &= \lambda_u \left[ \frac{1}{2} - \frac{r_u^2}{3x_m^2} \right] + \lambda_s \left[ -1 + \frac{x_m}{r_s} \right] \\ &= \lambda_u \left[ \frac{1}{2} - \frac{1}{3} \right] + \lambda_s \left[ -1 + \frac{r_u}{\bar{x}} \right] \\ &= \frac{\lambda_u}{6} + \lambda_s \left[ -1 + \frac{x_u^*}{\bar{x}} \right]. \end{aligned}$$

A rearrangement of this expression yields,

$$x_u^* = \bar{x} \left( \frac{6\lambda_s - \lambda_u}{6\lambda_s} \right), \quad (\text{A13})$$

or,

$$x_u = \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{9\lambda_s} \right). \quad (\text{A14})$$

We have now fully characterized when  $A_u$  reports a value greater than zero.

For  $x_u \geq \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{9\lambda_s} \right)$ , the advocate adheres to (A10) for his report. When  $x_u \in \left[ \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right), \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{9\lambda_s} \right) \right]$  the advocate will choose  $r_u$  according to (A11). An advocate is unable to make a positive report that results in  $x_m = x_u^*$  for values of  $x_u < \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right)$ . Instead, the advocate will make the smallest possible report,  $r_u = 0$ . Mathematically we define this reporting strategy,  $r_u = \sigma(x_u)$ , as

$$\sigma(x_u) = \begin{cases} 0 & \text{for } x_u < \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right) \\ x_u \left( \frac{3}{4} \sqrt{\frac{3}{2} + \frac{\lambda_s}{\lambda_u} \left[ \frac{9x_u}{4\bar{x}} - 3 \right]} \right) & \text{for } \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right) \leq x_u < \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{9\lambda_s} \right) \\ x_u \left( \frac{3\lambda_u \bar{x}}{3\lambda_u \bar{x} + 6\lambda_s(\bar{x} - (3/4)x_u)} \right) & \text{for } \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{9\lambda_s} \right) \leq x_u. \end{cases}$$

Therefore, the advocate will make a report greater than zero if, and only if,  $x_u \geq \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right)$ .

Finally, when  $r_u = 0$ , we know that  $x_m > r_u$ . Substituting into (A8) yields

$$0 = \lambda_u \left[ \frac{1}{2} - 0 \right] + \lambda_s \left[ -1 + \frac{x_m}{\bar{x}} \right].$$

Solving for  $x_m$  yields the desired result.

■

**Proof of Corollary 1:** Proposition 2 tells us that there exists a report,  $r_u > 0$ , such that  $x_u^* = x_m(r_u, \bar{x})$  if and only if  $x_u \geq \bar{x} \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right)$ . We direct our attention to Equation 15. Because both  $x_u$  and  $\bar{x}$  are greater than or equal to zero by definition, if  $0 > \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right)$  the condition is satisfied.

$$\begin{aligned} 0 &> \left( \frac{4}{3} - \frac{2\lambda_u}{3\lambda_s} \right) \\ \frac{2\lambda_u}{3\lambda_s} &> \frac{4}{3} \\ \lambda_u &> 2\lambda_s \end{aligned}$$

■

**Proof of Lemma 3:** The conditional expectation of  $x_u$ , given  $x_s$  and that it is distributed uniformly over  $[0, \bar{x}]$ , can be written as,

$$E[x_u | x_u \leq x_s] = \frac{x_s}{2}. \quad (\text{A15})$$

Similarly, the conditional expectation of  $x_s$ , given  $x_u$ , is

$$E[x_s | x_s \geq x_u] = \frac{x_u + \bar{x}}{2} \quad (\text{A16})$$

Taking expectations of both equations gives,

$$E[x_u] = \frac{E[x_s]}{2} \quad (\text{A17})$$

$$E[x_s] = \frac{E[x_u] + \bar{x}}{2} \quad (\text{A18})$$

A rearrangement yields,

$$E[x_u] = \frac{\bar{x}}{3} \quad (\text{A19})$$

$$E[x_s] = \frac{2\bar{x}}{3} \quad (\text{A20})$$

■

**Proof of Lemma 4:** Because the planner takes the midpoint of the partition, it is always a weakly dominant strategy for the sophisticated advocate to report that  $x_s$  lies in the partition that contains  $\bar{x}$ . This follows from the fact that sophisticated agents

are never harmed by having an  $x_m > x_s$ , but do incur losses if  $x_m < x_s$ . Therefore, for any realization of  $x_s$ , the probability that  $r_s = \bar{x}$  equals one.

■

**Proof of Proposition 3:** An equilibrium of this game is guaranteed by Theorem 1 of Crawford and Sobel (1982), since the utility functions of the social planner and  $A_u$  satisfy those listed on page 1433 of Crawford and Sobel (1982) for  $U^R(y, r)$  and  $U^S(y, r, b)$  respectively.

Equation 21 is derived in similar fashion as Equations (20)-(22) in from Section 4 of Crawford and Sobel (1982). We first derive the expected bias for  $A_u$ . Using Equation A7 and the results of Lemma 3, we compute

$$\begin{aligned}
E[b_u] &= E[x_m^*] - E[x_u^*] \\
&= \frac{E[x_u^*]E[x_s^*](\lambda_u + 2\lambda_s)}{\lambda_u E[x_s^*] + 2\lambda_s E[x_u^*]} - E[x_u^*] \\
&= \frac{\bar{x}^2(\lambda_u + 2\lambda_s)}{\lambda_u \frac{2\bar{x}}{3} + 2\lambda_s \frac{\bar{x}}{4}} - \frac{\bar{x}}{4} \\
&= \frac{5\lambda_s \bar{x}}{16 - 4\lambda_s} \tag{A21}
\end{aligned}$$

Evaluating Equation 21 with the unconditional expectation for  $A_u$ 's bias yields the maximum number of partitions that can be supported, which is computed as

$$\begin{aligned}
\bar{N} &= \left\langle -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{2\bar{x}}{E[b]}} \right\rangle \\
&= \left\langle -\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{32 - 8\lambda_s}{5\lambda_s}} \right\rangle \\
&= \left\langle -\frac{1}{2} + \frac{1}{2} \sqrt{\frac{32 - 3\lambda_s}{5\lambda_s}} \right\rangle \tag{A22}
\end{aligned}$$

It follows that  $\bar{N}$  is less than or equal to 1 if  $\frac{32-3\lambda_s}{5\lambda_s} \leq 9$ . Thus, if  $\lambda_s > \frac{2}{3}$  only a babbling equilibrium exists. Conversely, as  $\lambda_s \rightarrow 0$ , the number of partitions goes to infinity.

■

**Proof of Proposition 4:** The optimal levels of market sophistication under each type

of planner is denoted as

$$\begin{cases} x_m^P & \text{Perfectly Informed} \\ x_m^V & \text{Savvy} \\ x_m^N & \text{Naive,} \end{cases}$$

where  $x_m^P \neq x_m^V \neq x_m^N$ .

We begin by considering only  $x_m^P$  and  $x_m^N$ . According to Proposition 1, a perfectly informed social planner will choose  $x_m^P \in [x_u^*, x_s^*]$ . Furthermore, we know that  $\frac{\partial \mathcal{L}_u}{\partial x_m} > 0$  for all  $x_m \geq x_u^*$  and  $\frac{\partial \mathcal{L}_s}{\partial x_m} < 0$  for all  $x_m \leq x_s^*$ . Therefore, the unsophisticated agents always prefer an  $x_m < x_m^P$  and sophisticated agents always prefer an  $x_m > x_m^P$ .

From Proposition 2 we know that  $r_s = \bar{x}$  and  $r_u > 0$  if  $x_m^N(r_u, \bar{x}) = x_u^*$  and 0 otherwise. Substituting these reports into the social planner's equation for  $x_m^N$  yields two possible values,

$$x_m^N = \begin{cases} x_u^* & r_u > 0 \\ \bar{x} \left(1 - \frac{\lambda_u}{2\lambda_s}\right) & r_u = 0. \end{cases} \quad (\text{A23})$$

It follows then that if  $x_m^N < x_m^P$  then unsophisticated agents will prefer the imperfect social planner. Conversely, if  $x_m^P < x_m^N$ , sophisticated agents will prefer the imperfect social planner. Specifically, the unsophisticated agents always prefer  $\min(x_m^P, x_m^N)$  and the sophisticated prefer  $\max(x_m^P, x_m^N)$ .

It is straightforward to show that adding a third choice does not induce unanimity. If  $x_m^V < x_u^*$  and is preferred by the unsophisticated agents, it will not be preferred by sophisticated agents because  $x_m^P > x_m^V$ . If  $x_m^V > x_s^*$  and is (weakly) preferred by the sophisticated agents, it will not be preferred by unsophisticated agents because  $x_m^P < x_m^V$ . If  $x_m^V \in [x_u^*, x_s^*]$ , the argument against unanimity follows as in the two-option case above.

■

**Proof of Proposition 5:** The result for  $\lambda_u \geq \frac{2}{3}$  follows from Corollary 1. For  $\lambda_s \geq \frac{2}{3}$ , with a savvy social planner, the optimal level of market sophistication will never exceed her beliefs of  $x_s$ . That is, when sophisticated agents have a supermajority, the planner's

unconditional expectation of  $x_s$  serves as an upper bound for  $x_m^V$ , i.e.  $x_m^V \leq \frac{2\bar{x}}{3}$ .

However, sophisticated agents with a supermajority can obtain a higher level of market sophistication under a naive social planner. According to Proposition 2, unsophisticated agents will report  $x_u = 0$  if  $\lambda_u \leq 1/3$ . This leads to  $x_m^N = \bar{x} \left[ 1 - \frac{\lambda_u}{2\lambda_s} \right]$ , which obtains its minimum at  $\lambda_s = 2/3$  since it is increasing in  $\lambda_s$ . Evaluating at  $2/3$  yields  $x_m^N = \frac{3\bar{x}}{4}$ , which is strictly greater than  $x_m^V$ .

Without any further information regarding the true values of  $x_u$  and  $x_s$ , we cannot say whether sophisticated agents prefer  $x_m^P$  or  $x_m^N$ . Their choice of planner will be governed by  $\max(x_m^P, x_m^N)$ . However, the sufficient condition in the proposition can be derived as follows. We can examine whether  $x_m^N > x_m^P$  for values of  $\lambda_s \geq \frac{2}{3}$ . As mentioned above,  $x_m^N$  reaches its minimum of  $\frac{3}{4}\bar{x}$  at  $\lambda_s = \frac{2}{3}$  and is increasing in  $\lambda_s$ . Assessing  $x_m^P$ , if  $x_u = x_s = x_p$ , the perfectly informed planner sets  $x_m^P = \frac{15}{16}x_p$ . If  $x_u < x_s$ , then  $x_m^P < \frac{15}{16}x_p$ . Therefore, at  $\lambda_s = \frac{2}{3}$ , a sufficient condition for  $x_m^N > x_m^P$  is  $\bar{x} > \frac{5}{4}x_p$ . However, as  $\lambda_s$  increases,  $x_m^P$  also rises. When  $\lambda_s \rightarrow 1$ ,  $x_m^P \rightarrow x_p$ . Therefore, the sufficient condition that ensures  $x_m^N > x_m^P$  is  $\bar{x} > \frac{4}{3}x_p$  as desired.

■

**Proof of Proposition 6:** To solve for the level of market sophistication that minimizes the weighted sum of aggregate loss and agent dispersion, we begin by showing that  $|L_u - L_s| = (L_u - L_s)$  when  $x_u = x_s = x_p$ .

$$\begin{aligned}
|L_u - L_s| &= \left| \left[ \frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} \right] - \left[ \frac{x_p^2 - 2x_mx_p + x_m^2}{2x_p} \right] \right| \\
&= \left| \left[ \frac{2x_m^2}{6x_p} + \frac{-3x_mx_p + 3x_p^2}{6x_p} \right] - \left[ \frac{3x_p^2 - 6x_mx_p + 3x_m^2}{6x_p} \right] \right| \\
&= \left| \frac{2x_m^2 - 3x_mx_p + 3x_p^2 - 3x_p^2 + 6x_mx_p - 3x_m^2}{6x_p} \right| \\
&= \left| \frac{3x_mx_p - x_m^2}{6x_p} \right| \\
&= \left| \frac{x_m(3x_p - x_m)}{6x_p} \right| \\
&= \frac{x_m(3x_p - x_m)}{6x_p},
\end{aligned}$$

which is always positive since  $x_m \in [0, x_p]$ .

To find an internal solution that minimizes the aggregate loss and the disparity between agents, we need Equation 23 to be convex. To determine the conditions under which the function is convex in  $x_m$ , we take the second derivative of Equation 23,

$$\begin{aligned}
\frac{\partial^2 W}{\partial x_m^2} &= \frac{\partial^2}{\partial x_m^2} [\kappa L(x_m, \lambda_u, \lambda_s) + (1 - \kappa)D(x_m, \lambda_u, \lambda_s)] \\
&= \kappa \left[ \lambda_u \frac{2}{3x_p} + \lambda_s \frac{1}{x_p} \right] + (1 - \kappa) \left[ \frac{2}{3x_p} - \frac{1}{x_p} \right] \\
&= \kappa \left[ \frac{2\lambda_u + 3(1 - \lambda_u)}{3x_p} \right] - (1 - \kappa) \frac{1}{3x_p} \\
&= \frac{\kappa(3 - \lambda_u) - (1 - \kappa)}{3x_p}, \\
&= \frac{\kappa(4 - \lambda_u) - 1}{3x_p},
\end{aligned}$$

which is positive so long as the numerator is positive. Therefore, the welfare function is convex so long as,

$$\kappa \geq \frac{1}{4 - \lambda_u}. \tag{A24}$$

When  $\kappa$  is smaller than the condition stated in (A24), the welfare function is strictly concave in  $x_m$  over  $[0, x_p]$ . This means that the optimal level of market sophistication is a corner solution; either  $x_m^* = 0$  or  $x_m^* = x_p$ . We now show that losses are monotonically increasing in  $x_m$ , indicating that  $x_m^* = 0$  when  $\kappa < \frac{1}{4-\lambda_u}$ . The first derivative of the welfare function with respect to  $x_m$  is

$$\begin{aligned}
\frac{\partial W}{\partial x_m} &= \frac{\partial}{\partial x_m} [\kappa L(x_m, \lambda_u, \lambda_s) + (1 - \kappa)D(x_m, \lambda_u, \lambda_s)] \\
&= \kappa \left[ \lambda_u \left( \frac{2x_m}{3x_p} - \frac{1}{2} \right) + \lambda_s \left( \frac{-x_p + x_m}{x_p} \right) \right] + (1 - \kappa) \left[ \left( \frac{2x_m}{3x_p} - \frac{1}{2} \right) - \left( \frac{-x_p + x_m}{x_p} \right) \right] \\
&= \kappa \left[ \lambda_u \left( \frac{4x_m - 3x_p}{6x_p} \right) + \lambda_s \left( \frac{-6x_p + 6x_m}{6x_p} \right) \right] + (1 - \kappa) \left[ \frac{4x_m - 3x_p + 6x_p - 6x_m}{6x_p} \right] \\
&= \frac{\kappa[\lambda_u(4x_m - 3x_p + 6x_p - 6x_m) - 6x_p + 6x_m] + (1 - \kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 6x_p + 6x_m] + (1 - \kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m}{6x_p}.
\end{aligned} \tag{A25}$$

It follows that  $\frac{\partial W}{\partial x_m} > 0$  if

$$\kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m > 0$$

or

$$x_m \leq \frac{3}{2} \frac{1 - k(3 - \lambda_u)}{1 - k(4 - \lambda_u)} x_p, \tag{A26}$$

which is always the case when  $\kappa < \frac{1}{4-\lambda_u}$ . Therefore,  $x_m^* = 0$  when  $\kappa < \frac{1}{4-\lambda_u}$ .

When  $\kappa \geq \frac{1}{4-\lambda_u}$ , we can utilize the first-order condition of Equation 23 with respect to  $x_m$  since the welfare loss function is convex:

$$\begin{aligned}
0 &= \frac{\partial W}{\partial x_m} \\
&= \frac{\partial}{\partial x_m} [\kappa L(x_m, \lambda_u, \lambda_s) + (1 - \kappa)D(x_m, \lambda_u, \lambda_s)] \\
&= \kappa \left[ \lambda_u \left( \frac{2x_m}{3x_p} - \frac{1}{2} \right) + \lambda_s \left( \frac{-x_p + x_m}{x_p} \right) \right] + (1 - \kappa) \left[ \left( \frac{2x_m}{3x_p} - \frac{1}{2} \right) - \left( \frac{-x_p + x_m}{x_p} \right) \right] \\
&= \kappa \left[ \lambda_u \left( \frac{4x_m - 3x_p}{6x_p} \right) + \lambda_s \left( \frac{-6x_p + 6x_m}{6x_p} \right) \right] + (1 - \kappa) \left[ \frac{4x_m - 3x_p + 6x_p - 6x_m}{6x_p} \right] \\
&= \frac{\kappa[\lambda_u(4x_m - 3x_p + 6x_p - 6x_m) - 6x_p + 6x_m] + (1 - \kappa)[3x_p - 2x_m]}{6x_p} \\
&= \frac{\kappa[\lambda_u(3x_p - 2x_m) - 6x_p + 6x_m] + (1 - \kappa)[3x_p - 2x_m]}{6x_p} \\
&= \kappa[\lambda_u(3x_p - 2x_m) - 9x_p + 8x_m] + 3x_p - 2x_m \\
&= 3\kappa\lambda_u x_p - 2\kappa\lambda_u x_m - 9\kappa x_p + 8\kappa x_m + 3x_p - 2x_m \\
&= -3x_p(3\kappa - \kappa\lambda_u - 1) + x_m(-2\kappa\lambda_u + 8\kappa - 2) \\
x_m^* &= \frac{3}{2} \frac{1 - k(3 - \lambda_u)}{1 - k(4 - \lambda_u)} x_p \tag{A27}
\end{aligned}$$

The comparative statics of  $x_m^*$  with respect to  $\kappa$  and  $\lambda_u$  are determined by simple differentiation.

■

**Proof of Proposition 7:** The social planner's problem is given by

$$\min_{x_{m,l}, x_{m,u} \in [0, x_p], x_{m,l} \leq x_{m,u}} \{ \max\{x_{m,u}, x_p - x_{m,l}\} \}. \tag{A28}$$

Note that  $\max\{x_{m,u}, x_p - x_{m,l}\} = x_{m,u}$  if  $x_{m,u} - \frac{x_p}{2} > \frac{x_p}{2} - x_{m,l}$ . Similarly, note that  $\max\{x_{m,u}, x_p - x_{m,l}\} = x_p - x_{m,l}$  if  $x_{m,u} - \frac{x_p}{2} < \frac{x_p}{2} - x_{m,l}$ . Since  $\frac{x_p}{2}$  is fixed, the planner's choices for  $x_{m,l}$  and  $x_{m,u}$  determine both the direction of the inequality and the maximum loss. First, it is intuitive that the solution to the planner's problem requires that  $x_{m,u} - \frac{x_p}{2} = \frac{x_p}{2} - x_{m,l}$ . This implies that  $\frac{x_{m,u}^* + x_{m,l}^*}{2} = \frac{x_p}{2}$ . Second, it is straightforward to see that losses are minimized then when  $x_{m,l}^* = x_{m,u}^* = \frac{x_p}{2}$ .

■



## B Appendix

In this appendix, we explore an alternative model to the one in Section 2. There, we tethered the lower bound of the market to be zero and studied how a social planner optimal chose the upper bound. As such, we interpreted the size of  $[0, x_m]$  to be the extent of the market, where the planner could regulate how complete the market is. Here, we consider that the planner can choose both the lower and upper bounds of this continuum and show that the planner still faces the same tensions from unsophisticated and sophisticated agents. As such, the planner balances both groups' needs and chooses an internal level of market completeness.

Define  $x_{m,l} \geq 0$  as the least sophisticated product that is offered in the market and define  $x_{m,u} \geq x_{m,l}$  to be the most sophisticated product. The planner's problem is to balance the demands of the two groups to minimize the aggregate loss,

$$\min_{x_{m,l}, x_{m,u} \in [0, x_p]} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s), \quad (\text{B1})$$

where

$$L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) = \lambda_u E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] + \lambda_s E[L(\tilde{t}_s, x | x_{m,l}, x_{m,u})]. \quad (\text{B2})$$

The following proposition is the analog of Lemmas 1 and 2, which evaluates the loss to the two types of market participants.

**Proposition B1.** *The aggregate loss for sophisticated agents is*

$$\mathcal{L}_s = \frac{x_{m,l}^2 + x_p^2 + x_{m,u}^2 - 2x_m}{2x_p}, \quad (\text{B3})$$

which is increasing in  $x_{m,l}$ , decreasing in  $x_{m,u}$  and convex in both parameters respectively.

*The aggregate loss for unsophisticated agents is*

$$\mathcal{L}_u = \frac{3x_{m,l}x_{m,u} + 2(x_{m,u} - x_{m,l})^2 + 3(x_{m,u} - x_p)(x_{m,l} - x_p)}{6x_p}, \quad (\text{B4})$$

which reaches a minimum at  $x_{m,u} = x_{m,l} = \frac{1}{2}x_p$ .

**Proof of Proposition B1:** The loss for a given sophisticated agent with type  $\tilde{t}_s$  is given by

$$L(x | \tilde{t}_s, x_{m,l}, x_{m,u}) = |x - \tilde{t}_s| \quad (\text{B5})$$

Because the agent knows his type perfectly he will choose the closest product to his type. When  $\tilde{t}_s < x_{m,l}$  the agent chooses  $x = x_{m,l}$ . Similarly, when  $\tilde{t}_s > x_{m,u}$  the agent chooses  $x = x_{m,u}$ . An agent with type  $\tilde{t}_s \in [x_{m,l}, x_{m,u}]$  does not incur a loss since he will choose  $x = \tilde{t}_s$ . Since sophisticated agents are distributed uniformly, the aggregate expected loss in the population is given by,

$$\begin{aligned}
\mathcal{L}_s &= \int_0^{x_{m,l}} |x_{m,l} - \tilde{t}_s| \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,l}}^{x_{m,u}} 0 \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,u}}^{x_p} |x_{m,u} - \tilde{t}_s| \frac{d\tilde{t}_s}{x_p} \\
&= \int_0^{x_{m,l}} (x_{m,l} - \tilde{t}_s) \frac{d\tilde{t}_s}{x_p} + \int_{x_{m,u}}^{x_p} (\tilde{t}_s - x_{m,u}) \frac{d\tilde{t}_s}{x_p} \\
&= \left[ \frac{x_{m,l}\tilde{t}_s}{x_p} - \frac{\tilde{t}_s^2}{2x_p} \Big|_0^{x_{m,l}} \right] + \left[ \frac{\tilde{t}_s^2}{2x_p} - \frac{x_{m,u}\tilde{t}_s}{x_p} \Big|_{x_{m,u}}^{x_p} \right] \\
&= \left[ \frac{x_{m,l}^2}{x_p} - \frac{x_{m,l}^2}{2x_p} \right] + \left[ \frac{x_p^2}{2x_p} - \frac{x_{m,u}x_p}{x_p} - \frac{x_{m,u}^2}{2x_p} + \frac{x_{m,u}^2}{x_p} \right] \\
&= \frac{x_{m,l}^2 + x_p^2 - 2x_{m,u}x_p + x_{m,u}^2}{2x_p}
\end{aligned} \tag{B6}$$

The first-order derivative of the expected loss with respect to  $x_{m,l}$  is

$$\frac{\partial}{\partial x_{m,l}} E[L(\tilde{t}_s, x|x_{m,l}, x_{m,u})] = \frac{x_{m,l}}{x_p}, \tag{B7}$$

which is strictly positive. The first-order derivative of the expected loss with respect to  $x_{m,u}$  is

$$\frac{\partial}{\partial x_{m,u}} E[L(\tilde{t}_s, x|x_{m,l}, x_{m,u})] = \frac{x_{m,u} - x_p}{x_p}, \tag{B8}$$

which is negative since  $x_p$  serves as the maximum possible level of market sophistication.

The second-order derivative of the expected loss with respect to  $x_{m,l}$  is

$$\frac{\partial^2}{\partial x_{m,l}^2} E[L(\tilde{t}_s, x|x_{m,l}, x_{m,u})] = \frac{1}{x_p}, \tag{B9}$$

which tells us the function is convex in  $x_{m,l}$  since it is positive. Similarly, the second-order derivative of the expected loss with respect to  $x_{m,u}$  is

$$\frac{\partial^2}{\partial x_{m,u}^2} E[L(\tilde{t}_s, x|x_{m,l}, x_{m,u})] = \frac{1}{x_p}, \tag{B10}$$

which is also strictly positive.

The loss for a given unsophisticated agent with type  $\tilde{t}_u$  is given by

$$L(x|\tilde{t}_u, x_{m,l}, x_{m,u}) = \int_{x_{m,l}}^{x_{m,u}} |\tilde{x} - \tilde{t}_u| \frac{d\tilde{x}}{x_{m,u} - x_{m,l}}, \quad (\text{B11})$$

since the unsophisticated agent randomly selects a product on the continuum  $[x_{m,l}, x_{m,u}]$ . Since unsophisticated agents are distributed uniformly, the aggregate expected loss in the population is given by

$$\begin{aligned} \mathcal{L}_u &= \int_0^{x_p} \int_{x_{m,l}}^{x_{m,u}} |\tilde{x} - \tilde{t}_u| \frac{d\tilde{x}}{x_{m,u} - x_{m,l}} \frac{d\tilde{t}_u}{x_p} \\ &= \int_0^{x_p} \frac{(\tilde{x} - \tilde{t}_u)^3}{2|\tilde{x} - \tilde{t}_u|(x_{m,u} - x_{m,l})} \Big|_{x_{m,l}}^{x_{m,u}} \frac{d\tilde{t}_u}{x_p} \\ &= \frac{1}{(x_{m,u} - x_{m,l})x_p} \int_0^{x_p} \left[ \frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u, \end{aligned}$$

which expands to,

$$\begin{aligned} &= \frac{1}{(x_{m,u} - x_{m,l})x_p} \left[ \int_0^{x_{m,l}} \left[ \frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u \right. \\ &\quad + \int_{x_{m,l}}^{x_{m,u}} \left[ \frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u \\ &\quad \left. + \int_{x_{m,u}}^{x_p} \left[ \frac{(x_{m,u} - \tilde{t}_u)^3}{2|x_{m,u} - \tilde{t}_u|} - \frac{(x_{m,l} - \tilde{t}_u)^3}{2|x_{m,l} - \tilde{t}_u|} \right] d\tilde{t}_u \right] \\ &= \frac{1}{2(x_{m,u} - x_{m,l})x_p} \left[ \int_0^{x_{m,l}} [(x_{m,u} - \tilde{t}_u)^2 - (x_{m,l} - \tilde{t}_u)^2] d\tilde{t}_u \right. \\ &\quad + \int_{x_{m,l}}^{x_{m,u}} [(x_{m,u} - \tilde{t}_u)^2 + (x_{m,l} - \tilde{t}_u)^2] d\tilde{t}_u \\ &\quad \left. + \int_{x_{m,u}}^{x_p} [-(x_{m,u} - \tilde{t}_u)^2 + (x_{m,l} - \tilde{t}_u)^2] d\tilde{t}_u \right]. \end{aligned}$$

An evaluation of the integrals yields,

$$\begin{aligned}
&= \frac{1}{2(x_{m,u} - x_{m,l})x_p} \left[ \left[ x_{m,u}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,u} \tilde{t}_u^2 - x_{m,l}^2 \tilde{t}_u - \frac{\tilde{t}_u^3}{3} + x_{m,l} \tilde{t}_u^2 \right]_0^{x_{m,l}} \right. \\
&\quad + \left[ x_{m,u}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,u} \tilde{t}_u^2 + x_{m,l}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,l} \tilde{t}_u^2 \right]_{x_{m,l}}^{x_{m,u}} \\
&\quad \left. + \left[ -x_{m,u}^2 \tilde{t}_u - \frac{\tilde{t}_u^3}{3} + x_{m,u} \tilde{t}_u^2 + x_{m,l}^2 \tilde{t}_u + \frac{\tilde{t}_u^3}{3} - x_{m,l} \tilde{t}_u^2 \right]_{x_{m,u}}^{x_p} \right] \\
&= \frac{x_{m,l}x_{m,u}}{2x_p} + \frac{(x_{m,u} - x_{m,l})^2}{3x_p} + \frac{(x_{m,u} - x_p)(x_{m,l} - x_p)}{2x_p} \\
&= \frac{3x_{m,l}x_{m,u} + 2(x_{m,u} - x_{m,l})^2 + 3(x_{m,u} - x_p)(x_{m,l} - x_p)}{6x_p}. \tag{B12}
\end{aligned}$$

The first-order derivatives of the expected loss with respect to  $x_{m,l}$  and  $x_{m,u}$  are

$$\frac{\partial}{\partial x_{m,l}} E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] = \frac{4x_{m,l} + 2x_{m,u} - 3x_p}{6x_p} \tag{B13}$$

and

$$\frac{\partial}{\partial x_{m,u}} E[L(\tilde{t}_u, x | x_{m,l}, x_{m,u})] = \frac{2x_{m,l} + 4x_{m,u} - 3x_p}{6x_p}. \tag{B14}$$

At optimality (i.e., setting (B13) and (B14) equal to zero),  $x_{m,l} = x_{m,u}$ , which implies that  $x_{m,l} = x_{m,u} = \frac{1}{2}x_p$ . It is straightforward to verify that the second-order condition is satisfied so that a minimum is attained when  $x_{m,l} = x_{m,u} = \frac{1}{2}x_p$ .

■

From Proposition B1, sophisticated agents prefer the market to be complete as possible at both ends of the spectrum, whereas unsophisticated agents want the market to contract towards their median needs. The planner's problem is to balance the demands of the two groups and the following proposition solves for a socially optimal  $x_{m,l}^*$  and  $x_{m,u}^*$ .

**Proposition B2.** *There exists a unique optimal set  $\{x_{m,l}^*, x_{m,u}^*\}$ , with  $0 \leq x_{m,l}^* \leq x_{m,u}^* \leq x_p$ , that minimizes  $L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s)$ ,*

$$\{x_{m,l}^*, x_{m,u}^*\} = \left\{ \frac{\lambda_u x_p}{6 - 4\lambda_u}, x_p - \frac{\lambda_u x_p}{6 - 4\lambda_u} \right\}. \tag{B15}$$

The optimal  $x_{m,l}^*$  is

1. increasing in  $x_p$
2. increasing in the mass of unsophisticated agents.

The optimal  $x_{m,u}^*$  is

1. increasing in  $x_p$
2. decreasing in the mass of unsophisticated agents.

**Proof of Proposition B2:** Both sophisticated and unsophisticated agents' losses are convex and continuous in both  $x_{m,l}$  and  $x_{m,u}$  which means the aggregate loss function is also convex and continuous in these parameters. The first-order condition of Equation B2 with respect to  $x_{m,l}$  is,

$$\begin{aligned}
0 &= \frac{\partial}{\partial x_{m,l}} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) \\
&= \lambda_u \left[ \frac{4x_{m,l} + 2x_{m,u} - 3x_p}{6x_p} \right] + \lambda_s \left[ \frac{x_{m,l}}{x_p} \right] \\
&= \lambda_u [4x_{m,l} + 2x_{m,u} - 3x_p] + 6\lambda_s x_{m,l} \\
x_{m,l} &= \frac{3\lambda_u x_p - 2\lambda_u x_{m,u}}{4\lambda_u + 6\lambda_s}.
\end{aligned} \tag{B16}$$

The first-order condition with respect to  $x_{m,u}$  is,

$$\begin{aligned}
0 &= \frac{\partial}{\partial x_{m,u}} L(x_{m,l}, x_{m,u}, \lambda_u, \lambda_s) \\
&= \lambda_u \left[ \frac{2x_{m,l} + 4x_{m,u} - 3x_p}{6x_p} \right] + \lambda_s \left[ \frac{x_{m,u} - x_p}{x_p} \right] \\
&= \lambda_u [2x_{m,l} + 4x_{m,u} - 3x_p] + 6\lambda_s (x_{m,u} - x_p) \\
x_{m,u} &= \frac{3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s}.
\end{aligned} \tag{B17}$$

Combining Equations B16 and B17 we can solve for  $x_{m,l}^*$ ,

$$\begin{aligned}
x_{m,l} &= \frac{3\lambda_u x_p}{4\lambda_u + 6\lambda_s} - \frac{2\lambda_u(3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p)}{(4\lambda_u + 6\lambda_s)^2} \\
x_{m,l}(4\lambda_u + 6\lambda_s)^2 &= 3\lambda_u x_p(4\lambda_u + 6\lambda_s) - 2\lambda_u(3\lambda_u x_p - 2\lambda_u x_{m,l} + 6\lambda_s x_p).
\end{aligned}$$

Recall that  $\lambda_s = 1 - \lambda_u$ ,

$$\begin{aligned}
x_{m,l}(4\lambda_u + 6(1 - \lambda_u))^2 &= 3\lambda_u x_p(4\lambda_u + 6(1 - \lambda_u)) - 2\lambda_u(3\lambda_u x_p - 2\lambda_u x_{m,l} + 6(1 - \lambda_u)x_p) \\
x_{m,l}(36 - 24\lambda_u) &= 6\lambda_u x_p \\
x_{m,l}^* &= \frac{\lambda_u x_p}{6 - 4\lambda_u}.
\end{aligned} \tag{B18}$$

Now  $x_{m,u}^*$  is given by,

$$\begin{aligned}
x_{m,u} &= \frac{3\lambda_u x_p - 2\lambda_u \left( \frac{\lambda_u x_p}{6 - 4\lambda_u} \right) + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s} \\
&= \frac{3\lambda_u x_p - 2\lambda_u \left( \frac{\lambda_u x_p}{6 - 4\lambda_u} \right) + 6\lambda_s x_p}{4\lambda_u + 6\lambda_s} \\
&= \frac{(6 - 5\lambda_u)x_p}{6 - 4\lambda_u} \\
x_{m,u}^* &= x_p - \frac{\lambda_u x_p}{6 - 4\lambda_u}
\end{aligned} \tag{B19}$$

The comparative statics of  $x_{m,l}^*$  and  $x_{m,u}^*$  with respect to  $\lambda_u$  and  $x_p$  are,

$$\begin{aligned}
\frac{\partial x_{m,l}^*}{\partial \lambda_u} &= \frac{x_p(6 - 4\lambda_u) - \lambda_u x_p}{(6 - 4\lambda_u)^2} \\
&= \frac{x_p(6 - 5\lambda_u)}{(6 - 4\lambda_u)^2} && \geq 0 \\
\frac{\partial x_{m,l}^*}{\partial x_p} &= \frac{\lambda_u}{6 - 4\lambda_u} && \geq 0 \\
\frac{\partial x_{m,u}^*}{\partial \lambda_u} &= - \left( \frac{x_p(6 - 4\lambda_u) - \lambda_u x_p}{(6 - 4\lambda_u)^2} \right) \\
&= - \left( \frac{x_p(6 - 5\lambda_u)}{(6 - 4\lambda_u)^2} \right) && \leq 0 \\
\frac{\partial x_{m,u}^*}{\partial x_p} &= 1 - \frac{\lambda_u}{6 - 4\lambda_u} \\
&= \frac{6 - 5\lambda_u}{6 - 4\lambda_u} && \geq 0
\end{aligned} \tag{B20}$$

■

## C Appendix

One interesting issue that arises in discussions about regulation in financial markets is whether the social planner should limit the scope of the market to protect people who are less sophisticated versus educating them to help them protect themselves. Such education may take the form of improved literacy training, timely decision support, access to intermediaries or other resources that provide guidance, or screening mechanisms.

In our model, the key question of interest is what should a social planner do when they identify that the market is operating away from the optimum. Do they improve access to information, require standardization of products (i.e., force simplicity), or both? To address this, we need to enhance the model to allow unsophisticated agents to access information about their choice. That is, while the fractions  $\lambda_u$  and  $\lambda_s$  are still exogenously given, we need to allow unsophisticated agents to learn. Once that channel is present, the social planner can potentially intervene in two ways: via  $x_m$  and via learning.

Suppose that the social planner can exert effort to educate  $\alpha$  fraction of the unsophisticated agents. This effort is costly and the social planner incurs  $\frac{1}{2}k\alpha^2$  for some  $k > 0$ . If an unsophisticated agent becomes educated and acquires information, they are essentially the same as a sophisticated consumer. Going forward, we assume that types for all agents are uniformly distributed over  $[0, x_p]$ , where  $x_p = x_s = x_u$ .

Once the social planner chooses  $\alpha$ , the expected aggregate loss for unsophisticated agents is

$$L_u \equiv \lambda_u \left\{ (1 - \alpha) E[L(\tilde{t}_u, x|x_m)] + \alpha E[L(x|x_m, \tilde{t}_u)] \right\}. \quad (\text{C1})$$

The educated fraction,  $\alpha$ , identify the products closest to their types, which precludes them from making mistakes. As a result, their preferences mirror those of sophisticated agents in that they want markets to be complete. Given such assistance, we can now compute the aggregate loss to all agents in the market and education costs as

$$L(x_m, \lambda_u, \lambda_s, \alpha) = L_u + \lambda_s E[L(\tilde{t}_s, x|x_m)] + \frac{1}{2}k\alpha^2. \quad (\text{C2})$$

This can be written as

$$L(x_m, \lambda_u, \lambda_s, \alpha) = \lambda_u \left\{ (1 - \alpha) \left[ \frac{x_m^2}{3x_p} + \frac{x_p - x_m}{2} \right] + \alpha \left[ \frac{x_p^2 - 2x_mx_p + x_m^2}{2x_p} \right] \right\} + \lambda_s \left[ \frac{x_p^2 - 2x_mx_p + x_m^2}{2x_p} \right] + \frac{1}{2}k\alpha^2. \quad (\text{C3})$$

The next proposition calculates the marginal effects of changing  $x_m$  and altering  $\alpha$ .

**Proposition C1.** *The marginal effect on the aggregate loss function from increasing  $x_m$  is*

$$\mathcal{L}_{x_m} \equiv \lambda_u (1 - \alpha) \left[ \frac{2x_m}{3x_p} - \frac{1}{2} \right] + (\lambda_s + \alpha\lambda_u) \left[ \frac{x_m - x_p}{x_p} \right]. \quad (\text{C4})$$

*The marginal effect on the aggregate loss function from educating is*

$$\mathcal{L}_\alpha \equiv \lambda_u \left[ \frac{x_m^2 - 3x_mx_p}{6x_p} \right] + k\alpha. \quad (\text{C5})$$

*Education and standardization in the product market are strict substitutes, i.e.,*  
 $\frac{\partial^2 [L(x_m, \lambda_u, \lambda_s, \alpha)]}{\partial x_m \partial \alpha} < 0$ .

**Proof of Proposition C1:** We first consider the marginal effect with regard to  $x_m$ . Taking the derivative of (C3) with respect to  $x_m$  yields

$$\lambda_u \left\{ (1 - \alpha) \left[ \frac{2x_m}{3x_p} - \frac{1}{2} \right] + \alpha \left[ \frac{-x_p + x_m}{x_p} \right] \right\} + \lambda_s \left[ \frac{-x_p + x_m}{x_p} \right].$$

re-arranging this yields the expression in (C4).

Now, we consider the marginal effect with regard to  $\alpha$ . Taking the derivative of (C3) with respect to  $\alpha$  yields

$$-\lambda_u \left\{ \left[ \frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} - \frac{x_p^2 - 2x_mx_p + x_m^2}{2x_p} \right] \right\} + k\alpha.$$

re-arranging this yields the expression in (C5).

Taking the derivative of (C5) with respect to  $x_m$  yields

$$\frac{\lambda_u}{6x_p} (2x_m - 3x_p) < 0,$$

which implies that  $\frac{\partial^2 [L(x_m, \lambda_u, \lambda_s, \alpha)]}{\partial x_m \partial \alpha} < 0$ .

■



By inspection of Equation C4 it is clear that the expression is negative for  $x_m < \frac{3}{4}x_p$ , meaning that increasing  $x_m$  strictly enhances welfare (decreases expected losses). Conversely, the first term, which is the effect of increasing  $x_m$  on uneducated and unsophisticated agents, is strictly positive for  $x_m > \frac{3}{4}x_p$ . This is consistent with our base model where unsophisticated agents prefer simpler markets when  $x_m$  exceeds three quarters of their aggregate needs. The second term however, which represents the marginal effect on both sophisticated agents *and* the educated unsophisticated fraction, is negative for all  $x_m < x_p$ . This follows from their ability to perfectly identify optimal products, i.e. they only incur losses when underserved. This tension, between uninformed and informed agents' preferences, suggests that a planner's optimal choice of  $x_m$  falls in the interval  $\left[\frac{3}{4}x_p, x_p\right]$ . In fact, because a fraction  $\alpha\lambda_u$  essentially become sophisticated, a planner's optimal choice of  $x_m$  is likely to be higher with the ability to educate than without.

Costly education increases aggregate welfare when Equation C5 is negative. This arises when the first term, which is the expected reduction in losses from providing unsophisticated individuals full information, offsets the marginal cost of educating. As we show in the next corollary, this is likely to be the case for small values of  $k$  and when the fraction of unsophisticated agents is large.

The concluding finding of Proposition C1 is of central importance to determining optimal regulation because it tells us about the interplay between decreasing  $x_m$  and increasing  $\alpha$ . The negative sign on the cross-derivative of Equation C3 implies that clarity and simplicity are strict substitutes. That is, when the market is not welfare optimal, if the social planner chooses one type of intervention, it makes the value of the other decline. For example, if the social planner subsidizes information acquisition, i.e. *clarifying*, the benefit to limiting the scope of products in the market drops. Likewise, if the social planner enforces simplicity, the benefit of increasing access to information decreases. This relationship holds for all parameter choices, the only differences are in the magnitude.

**Corollary C1.** *In equilibrium, the optimal level of education  $\alpha^*$*

1. *is increasing in  $\lambda_u$ ,  $x_p$ , and  $x_m$*
2. *is decreasing in  $k$ .*

**Proof of Corollary C1:** First-order conditions of Equation C3 with respect to  $x_m$  and  $\alpha$  yields the following system of equations,

$$0 = -\lambda_u \left\{ \left[ \frac{x_m^2}{3x_p} + \frac{-x_m + x_p}{2} - \frac{x_p^2 - 2x_mx_p + 2x_m^2}{2x_p} \right] \right\} + k\alpha \quad (\text{C6})$$

$$0 = \lambda_u \left\{ (1 - \alpha) \left[ \frac{2x_m}{3x_p} - \frac{1}{2} \right] + \alpha \left[ \frac{-x_p + x_m}{x_p} \right] \right\} + \lambda_s \left[ \frac{-x_p + x_m}{x_p} \right]. \quad (\text{C7})$$

Define  $x_m^*$  and  $\alpha^*$  to be the optimal levels of sophistication and education that satisfy Equations C6 and C7. Because Equation C3 is strictly convex in both  $x_m$  and  $\alpha$ , we know that the set  $x_m^*$  and  $\alpha^*$  is unique. We now re-write  $\alpha^*$  in terms of  $x_m^*$ ,

$$\alpha^* = \frac{\lambda_u}{6kx_p} \left( 3x_m^*x_p - x_m^{*2} \right). \quad (\text{C8})$$

The partial derivatives of Equation C8 with respect to  $k$ ,  $\lambda_u$ ,  $x_p$ , and  $x_m^*$  are,

$$\begin{aligned} \frac{\partial \alpha^*}{\partial k} &= -\frac{\lambda_u}{6x_pk^2} \left( 3x_m^*x_p - x_m^{*2} \right) && \leq 0 \\ \frac{\partial \alpha^*}{\partial \lambda_u} &= \frac{1}{6kx_p} \left( 3x_m^*x_p - x_m^{*2} \right) && \geq 0 \\ \frac{\partial \alpha^*}{\partial x_p} &= \frac{\lambda_u x_m^2}{6kx_p^2} && \geq 0 \\ \frac{\partial \alpha^*}{\partial x_m^*} &= \frac{\lambda_u}{6kx_p} (3x_p - 2x_m^*) && \geq 0 \end{aligned} \quad (\text{C9})$$

■

Corollary C1 contains an intuitive message for planners considering the extent to educate. First, when the fraction of unsophisticated agents is large, educating becomes more efficient. A fraction  $\alpha\lambda_u$  of the entire population reaps the benefit of education. Because  $\alpha$  is independent of how many unsophisticated agents participate, education is going to have the largest impact when  $\lambda_u$  is large. Additionally, when the cost of educating is low, a planner finds it advantageous to provide more learning. In fact, as  $k$  approaches zero, all agents in the economy are provided with their type and  $x_m$  approaches  $x_p$ . Finally, as the extent of the peoples' needs increases or as the market is more complete, the optimal  $\alpha^*$  increases *ceteris paribus*.

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