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Predictability in Strategic Air Traffic Flow Management

by

Yi Liu

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

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in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:
Professor Mark Hansen, Chair
Professor Carlos Daganzo
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Professor Rhonda Righter

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Predictability in Strategic Air Traffic Flow Management

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by

Yi Liu

Abstract

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Doctor of Philosophy in Engineering – Civil and Environmental Engineering

University of California, Berkeley

Professor Mark Hansen, Chair

This dissertation investigates predictability in strategic air traffic management with a focus on ground delay programs (GDPs). Through a survey of flight operators, we confirm the proposition that flight operators care about predictability. We then develop models that incorporate predictability into GDP cost optimization after failing in finding an existing model that can serve this purpose. This is accomplished by modifying traditional GDP delay cost functions so that they incorporate predictability, and determining the sensitivities of the optimal planned capacity recovery time and associated cost to the unpredictability premiums included in the cost functions. To do this, we develop two stochastic GDP models: a GDP no-revision, or static, model; and a GDP revision, or dynamic, model considering one GDP revision. GDP scope, which matters only in the revision model, is also considered. The optimization results from the case study show that the cost of unpredictability clearly matters, particularly in the more realistic case where GDP revision is allowed. Of the two unpredictability cost parameters, the one for unplanned delay has a stronger impact than the one for planned un-incurred delay. The insights from this analysis might eventually be used to develop a decision support tool that air traffic managers could use in determining what the planned end time should be for a GDP in a manner that reflects the importance of predictability to flight operators.

Contents

List of Figures

List of Tables

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and Jingyan Ren. Though not easy, they made sure that I was emotionally stable during the entire flight. They were there to listen when I was frustrated. They were there to help me when I was lost. They were there to cheer with me when I made progress. I am so lucky to have them as my team. Amy, you are a really sweet and trustable friend. I will never forget the time that we spent together as first year transportation engineering students. Thank you for proofreading and editing my papers and my life. Xiaowei, you are my think tank. Thank you for listening to my stories and always giving me great suggestions. Hongyan and Xing, thank you for never criticizing my daring operations and being so supportive. Finally, my special thanks goes to Jingyan Ren. You are younger but you see things deeper than I do. During tough times, you helped me focus. Facing problems, you showed me the directions to go. You let me realize, for the first time, life can be different. Thank you for making my life better.

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1 Introduction

1.1 Air Traffic Flow Management

Air traffic flow management (ATFM) is the craft of managing the flow of air traffic (FAA, 2009) in the national airspace system (NAS). Unfavorable conditions, such as adverse weather and high traffic volume, may lead to imbalance between capacity and demand. The mission of ATFM is to balance demand with system capacity to ensure the maximum efficient utilization of the NAS in a safe and equitable way (FAA, 2009). When there is congestion or when congestion is foreseen, either at an airport or in some en route portion of the NAS, federal Aviation Administration (FAA) traffic specialists seek ways to ensure that there are not too many aircraft flying into or out of the congested airport or flying through the congested airspace. This is done strategically through the use of traffic management initiatives (TMIs). There are two types of TMIs: airport-specific and en route.

Airport-specific TMIs are implemented to manage flow into congested airports. Such TMIs include:

- ground delay program (GDP), implemented to balance arrival demand with available arrival capacity at the destination airport by metering takeoffs at their departure airports. GDPs are normally called at airports where capacity is reduced because of adverse weather, such as marine stratus at San Francisco International airport (SFO) and strong wind at Newark Liberty International airport (EWR). We control arrival times for the affected flights according to the available capacity and assign expect departure clearance times (EDCTs) to regulate their arrival times. If traffic specialists do not take action when capacity is reduced at the arrival airport, then incoming flights will experience airborne delay before landing which is more expensive than ground delay. Moreover, excessive number of aircraft in the air is a burden to traffic controllers and could also be a safety concern;
- ground stop (GS), similar to GDP but more restrictive. It freezes incoming flights to the GS airport on the ground at their departure airports. GSs are implemented when air traffic control is unable to safely accommodate additional aircraft in the system (FAA, 2009). For instance, GS will be called for Chicago O'Hare International airport when thunderstorm forces the airport to close; and
- traffic management advisor (TMA), designed to plan efficient flight trajectories from cruise altitude to the runway threshold (FAA, 2009). The goal is to allow the arrival airport to accept arrivals at its capacity and in the meantime prevent excessive airborne delay. Different from GDP which may affect flights from far away, TMA usually assigns EDCT to flights departing from the same or adjacent

center to the destination airport. To some degree, we may consider TMA a tactical ATFM tool since flight operators cannot find out the delay until pilots call air traffic control tower for clearance to depart.

En route TMIs are implemented to manage flow in or into congested airspace, which include:

- Airspace Flow Program (AFP), identifies constraints in the airspace and meters the demand through the congested area by delaying flights on the ground. Similar to GDP, AFP also controls the flow by assigning EDCTs to incoming flights;
- Collaborative Trajectory Options Program (CTOP), used to manage demand through constrained airspace—one or more flow constrained areas (FAA, 2014). Flights subject to CTOP are assigned either a reroute which will avoid the congested airspace or a combination of delay at the departure airports and a route through the congested area. CTOP allows flight operators to communicate their preferences with regard to both routes and delays. CTOP, introduced in 2014, is one of the new TMIs being developed within collaborative air traffic management technologies as the air transportation system progresses toward its next generation; and
- Miles-in-trail (MIT), used to apportion traffic into a manageable flow, as well as provide space for additional traffic (merging or departing) to enter the flow of traffic (FAA, 2009). To achieve this, MIT requires a separation between aircraft. MIT can be applied to aircraft departing from the same airport, over a waypoint, through an en route sector or on a specific route.

TMI decisions are made under uncertainty. Take GDPs at SFO as an example. GDPs at SFO are usually called because of marine stratus at the airport. Traffic specialists make GDP decisions and assign EDCTs to GDP affected flights based on the forecast of stratus clearing time available at the decision time. The decisions are revised when there are changes in conditions. For instance, traffic specialists may cancel the GDP and remove the constraints on the affected flights earlier if stratus clears earlier than forecasted. Given the uncertainty in the weather forecast, TMIs can be implemented differently and people with different attitudes toward risk may prefer different plans.

In the current NAS, TMI decisions are made through telephone-conferences between FAA traffic specialists and flight operators. FAA air traffic control system command center holds a strategic planning telephone-conference with flight operators every two hours. When there are active TMIs in the system, telephone-conference may be held every hour. In the telephone-conferences, flight operators express their opinions on TMI decisions verbally and traffic specialists make the decisions based on the verbal inputs and their experiences. Since the conversations focus on TMI parameter settings and not on the underlying performance objectives, it is unclear what performance can be

expected from the decisions and the decision-making process is thus subjective and ad-hoc.

To move from the current TMI decision-making process to a performance-based one, it is important for us to capture all the dimensions of the TMI performance space. Multiple performance goals can be considered for TMI performance assessment. International Civil Aviation Organization (ICAO) identified 11 key performance areas for evaluating ATFM performance (ICAO, 2005). Commonly, the community focuses on a couple of them, such as capacity and efficiency (Bradford et al., 2000; FAA, 2011; Sherali et al., 2011; FAA et al., 2014). Earlier works have shown that performance goals are often in conflict and thus differing priorities among these goals can lead to different TMI decisions (Mukherjee and Hansen, 2007; Liu and Hansen, 2013).

1.2 Motivation and Objectives

In earlier research, we conducted a retrospective performance evaluation of GDPs at SFO and EWR—two of the most congested airports in the United States. In that analysis, we develop and evaluate metrics for three performance goals: capacity utilization, efficiency, and predictability. Capacity utilization is defined as the ratio of actual arrivals during the GDP period to the count of arrivals that could have been landed assuming we knew the actual capacity at the beginning of the GDP; efficiency is quantified as the ratio of ground delay to the sum of ground delay and airborne delay; and predictability is measured as the ratio of the minimum of planned and realized delay to the maximum of the two. All the metrics fall on a 0-1 scale, where 1 implies perfect performance. We find that while capacity utilization and efficiency scores are high on average and exhibit small variability, predictability performance is weaker and more variable (Figure 1.1). On the other side, FAA has shown increasing interest in predictability (FAA, 2011) and the ATFM system users—flight operators—also consider predictability as a high priority improvement area (Aponso et al., 2015).
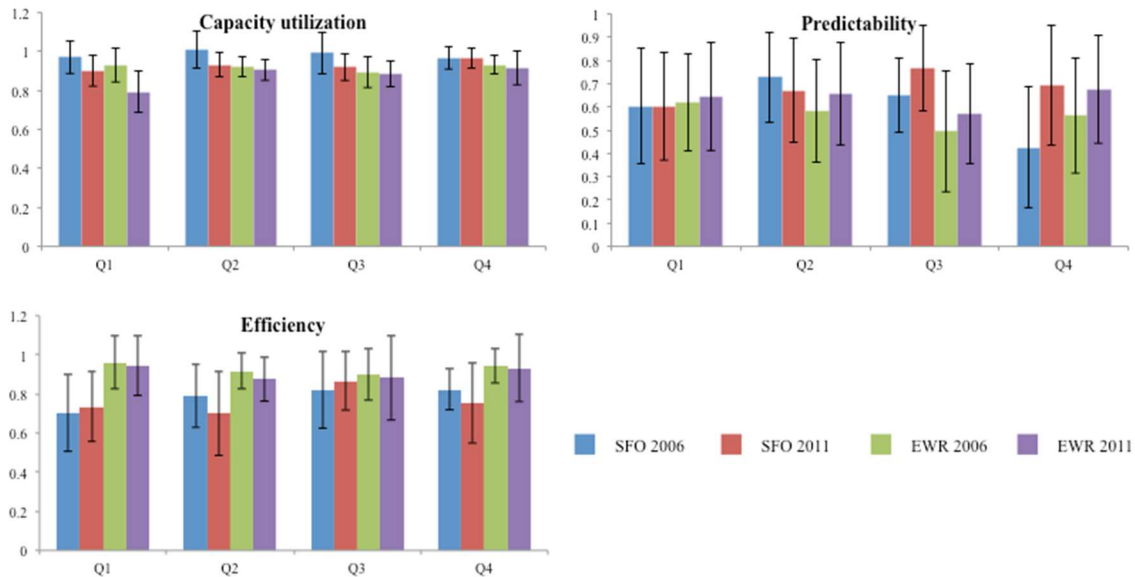
Figure 1.1: Average level and Standard Deviations of GDP Performances, Quarter Year, SFO and EWR, 2006 and 2011

The apparent discrepancy between what the community is advocating and actual predictability performance of GDP piques our interest in two questions: how the value of predictability affects GDP decisions and what is the resulting loss if predictability is undervalued. However, existing models cannot answer these questions. In virtually all the GDP cost optimization models in the literature, the total expected costs are functions of two delay components: ground delay and airborne delay. This ignores the fact that GDP decisions, which are based on weather forecast, involve great uncertainty. The initial plan for a GDP is often different—sometimes very different—from what is eventually implemented. The most common and impactful changes are extension and early cancellation. Prior research has not considered how these changes to the initial plan affect the cost of a GDP. The existing work considers the same unit cost of ground delay regardless of whether it is part of the initial GDP plan or is imposed due to an extension. In reality, however, unexpected extra delays due to the GDP extension require more flight operator dispatcher effort and reduce the set of feasible mitigation actions, which may result in extra cost compared to delays in the initial plan. In the case of an early cancellation, the existing work assumes the delay assigned in the initial plan that is not incurred to be cost-free. This ignores the efforts made and actions taken to adapt to this delay before it is de-assigned. As a result, we cannot use existing models to address our questions about the role of predictability in GDP cost optimization.

To fill the gap, we are motivated to construct GDP cost optimization models that incorporate predictability. The key ideas are that it is good to know early how late flights

will be, and that GDP decisions that recognize this can be quite different from those that do not and may also substantially reduce the cost of GDPs to flight operators. Ideally, given an amount of actual delay incurred in a GDP, the cost is least if the entire delay is accurately predicted up front; if there are subsequent modifications, the cost increases. We build our GDP cost optimization models based on deterministic queueing theory and continuous approximation and consider predictability by attaching penalties to unexpectedness in delays.

Before developing our model, we first examine the proposition that flight operators care about predictability and quantify the importance that they attach to predictability in the context of GDPs. For this purpose, we have designed and administered a survey of flight operators. Considering there has been no previous research on flight operators' views on GDP decisions, the survey also asks for their feedback on GDP decision setting and have them rate the importance of different variables in evaluating GDPs. The survey results thus will provide a comprehensive report of flight operators' perception on GDPs with emphasis on how they value predictability.

1.3 Overview of the Dissertation

The dissertation is organized as follows:

- Chapter 2 presents the survey of flight operators including survey motivation, questions, analysis methods and discussion on the results.
- Chapter 3 proposes the models that relate GDP decisions to delay components in the cost function. Using deterministic queueing theory and continuous approximation, we develop two GDP models: no-revision, where GDP revision is not considered; and one-revision, where GDP is revised once after the initial implementation. The models capture the aggregate behavior of the system without getting into details. The chapter starts with an introduction to GDP cost optimization, followed by an overview of existing GDP cost optimization models, before introducing the proposed GDP models.
- Chapter 4 defines the GDP cost function with predictability considered and present how we reach the optimal decision with both GDP models. This involves introducing so called unpredictability premiums into the cost function. We derive the closed-form expression for the optimal decision in the no-revision model. For the one-revision case, we optimize the decision numerically because of the complexity of the objective function. At the end of the chapter, we discuss about the impact of constant demand rate assumption on the cost optimization results.
- Chapter 5 illustrates our approach with a case study, in which we focus on sensitivities of the optimal planned duration of a GDP to different levels of unpredictability premiums in the cost function. We also explore how the scope decision affects the optimal planned duration of a GDP.

- Chapter 6 offers a summary of the dissertation and directions for future research.

1.4 Contributions

This dissertation contributes to the literature in three-fold:

- The survey designed and administered in this research is the first effort in making a comprehensive report of flight operator perceived value of predictability and quantifying flight operators' preferences over multiple performance goals. The insights gained from the survey results can be leveraged in driving the TMI decision-making process toward greater responsiveness to user preferences.
- This work makes the first effort in incorporating predictability into cost optimization for GDPs. While predictability has been getting increasing recognition as a vitally important aspect of operational performance by FAA and stakeholders, the existing GDP optimization models fail to consider it. By considering predictability in the cost function, we add a new dimension to the performance space in the GDP cost optimization problems. The insights from our analysis could improves how air traffic managers cope with unexpected delays and be used to develop a decision support tool that air traffic managers use to design more predictable GDPs.
- We pioneer a new technique for modeling GDP cost optimization based on continuous mathematics. To study the relationship between the enhanced GDP cost function and GDP decisions in a generic manner, we formulate the GDP optimization problem based on continuous approximation models. This is the first time that this class of models, though widely used in other areas of transport analysis, has been applied to this kind of problem. The models are based on a small number of parameters, highlighting the problem's essence without the distraction of extraneous details.

2   A Survey of Flight Operators

2.1 Survey Motivation

As part of a larger research project, we survey flight operators to ascertain their views on current GDP decision-making practices and the relative importance of different performance goals related to GDPs. One area of interest is flight operators' views on predictability. We ask respondents to rate the importance of "predictability" per se, as well as various aspects of GDP decision-making that flight operators may associate with predictability. For instance, we ask the respondents to rate the importance of accuracy of GDP end time estimation in evaluating GDP performance. The idea is that more accurate the estimation, higher the predictability. In the rating questions, we also ask them to rate the importance of other performance goals such as average flight delay and percentage of airborne delay, which are accepted measures of ATFM performance. This enables us to compare the importance of predictability-associated aspects to other aspects of GDP performance. Further, we design trade-off questions to quantify the importance that flight operators attach to predictability.

At the beginning of the survey, we present questions asking for respondents' feedback on TMI service in general and GDP decision setting. These questions are used to warm up the respondents and get them acquainted with the survey background.

2.2 The Survey

The survey consists of two types of questions: rating scale and stated preference. Two types of rating scale questions are used: semantic differential scales and Likert scales. In the semantic differential scale questions, the ends of the scale are associated with opposing statements. An example is shown in Figure 2.1. In the Likert scale questions, respondents are asked for the degree to which they agree or disagree with a statement on a seven-point scale. An example of the Likert scale question is show in Figure 2.2. We use the rating scale questions to learn respondents' assessments on TMI decisions and decision-making process. For instance, we ask respondents to rate overall quality of TMI decisions and the clarity of the logic behind the decisions. We also ask respondents to assess particular aspects of GDP decisions. For instance, we ask them whether the planned rates in the initial GDP plan are usually too low, too high, or about right. Other rating questions concern the importance of various criteria in assessing GDP performance. For instance, we ask them to rate the importance of GDP lead time (time difference between GDP report time and GDP start time) from 1 (not at all important) to 5 (extremely important). At the end of this question set, we present one open-ended question. We ask the respondents to identify other factors that they think are important in assessing GDP performance and rate their importance using the same scale accordingly.

On a scale of 1 to 7, please rate the overall quality of FAA TMI decisions:

1 2 3 4 5 6 7

Very poor ⊙ ○ ○ ○ ○ ○ ○ **Very good**

Figure 2.1: Sample Semantic Differential Scale Question

How much do you agree with the following statements, please rate from 1 (Strongly disagree) to 7 (Strongly agree):

	Strongly disagree 1	Disagree 2	Somewhat disagree 3	Neutral 4	Somewhat agree 5	Agree 6	Strongly agree 7
6. More vocal participants in TMI telecons have greater influence on TMI decisions than less vocal participants	○	○	○	○	○	○	○

Figure 2.2: Sample Likert Scale Question

The second type of questions are stated preference. We have two groups of questions in this category. An example of the first group is shown in Figure 2.3, where respondents are asked to grade GDP A against GDP B by comparing their outcomes. The GDP outcomes are described on the basis of:

- average delay per flight, calculated as minutes of arrival delay per flight caused by GDP;
- maximum flight delay, calculated as maximum minutes of delay incurred by a flight as a result of the GDP;
- unrecoverable delay per flight, calculated as the number of minutes of arrival delay per flight that could have been avoided if capacity at the GDP airport had been more fully utilized;
- change in delay per flight (or called delay change), calculated as the change in average delay between what is planned and assigned initially and what is ultimately incurred;
- lead time, calculated as the time difference in minutes between when a GDP is announced and when it starts; and

- number of revisions, which is the number of times the GDP is revised after the initial plan.

The information above is specified and shown to the respondents before the hypothetical choice scenarios. Each respondent is then presented with 16 such scenarios with the scenario sequence randomized.

Attributes	GDP A	GDP B
Average Delay per Flight (minutes)	45	50
Maximum Flight Delay (minutes)	295	250
Unrecoverable Delay per Flight (minutes)	10	15
Change in Delay per flight after Initial Plan (minutes)	-10	-5
Lead Time (minutes)	100	100
Number of Revisions	1	1

Strongly prefer A   Somewhat prefer A   No preference   Somewhat prefer B   Strongly prefer B

Figure 2.3: Sample GDP Outcome Grading Question

We determined the values for the attributes based on historical GDP data. First, we calculated actual values of the attributes for GDPs at SFO and EWR in 2011. The data source is Metron Aviation Flight Scheduler Analyzer (FSA) database (Liu and Hansen, 2014) and we use two types of information from there: GDP parameter information and individual flight data. Then, we set the levels of the attributes by referring to the actual values. The attribute levels are summarized in Table 2.1. We were mainly curious about the tradeoffs between delay change, which is viewed as a measure of unpredictability, and other outcomes. We therefore consider more levels for delay change. In the historical GDPs, we saw cases where GDPs were revised more than once. In our questions, we consider at most one revision. Finally, we ran fractional factorial analysis and selected 16 pairs of outcomes based on the analysis output. When finalizing the scenario selection, we also look at the correlations between the changes in attributes across all the scenarios. The value of each individual attribute can be measured more precisely when the correlations between its change and changes in other attributes are small. The linear correlation coefficients are controlled below 0.25 with two exceptions. First, changes in average delay and in unrecoverable delay are perfectly positively correlated. For a given situation, the minimum delay assuming perfect information is the same and thus average delay that has been incurred minus unrecoverable delay is the same in one scenario. Due to this, we cannot separate the effect of average delay from that of unrecoverable delay when analyzing the model, discussed more in Section 2.3. Second, changes in lead time and in number of revisions are highly positively correlated.

9

The logic behind this is the assumption that longer lead time involves more uncertainty in a GDP decision and thus a revision is more likely.

Table 2.1: Attribute Levels for GDP Outcome Choice Scenarios

Attributes	Levels									
Average delay (min)	35	40	45	50	55	60				
Maximum flight delay (min)	230	250	260	270	280	295				
Unrecoverable delay (min)	0	5	10	15	25					
Delay change (min)	-60	-30	-20	-15	-10	-5	5	10	30	60
Lead time (min)	30	60	90	100	150	180				
Number of revision	0	1								

In the second group of stated preference questions, we ask respondents to grade GDPs by their expected system performances. They are told to imagine a GDP is being initiated, and different plans may be made for the same situation. Each plan leads to a different set of expectations for the three system performance metrics:

- capacity utilization, the ratio of the number of total arrivals to the available capacity during the GDP period. This performance is high if most of the available capacity is used by the GDP. The range of values of this metric is from 0.6 to 1 based on our analysis of previous GDPs;
- efficiency, the ratio of ground delay at the departure airports to total arrival delay in the system, resulted from the GDP. If we are conservative in setting planned rates, the airport is likely to be capable to promptly land all airborne flights and efficiency will be high. The range of values of this metric is from 0.6 to 1 based on our analysis of previous GDPs; and
- predictability, how accurate the assigned delay and called rates in the initial GDP plan is. A value close to 1 indicates high accuracy and a value close to 0 indicates low accuracy. The range of values of this metric is from 0.4 to 1 based on our analysis of previous GDPs.

A sample GDP performance trade-off question is shown in Figure 2.4. There are 12 such questions in the survey. Based on previous research (Liu and Hansen 2014), we set four levels for each performance metrics: 0.6, 0.75, 0.9 and 1 for capacity utilization and efficiency; 0.4, 0.65, 0.9 and 1 for predictability. We finalized the questions using a fractional factorial design.

		Very poor 1	Poor 2	Somewhat poor 3	Average 4	Somewhat good 5	Good 6	Very good 7
Capacity utilization [0.6, 1]	0.6							
Efficiency [0.6, 1]	1	○	○	○	○	○	○	○
Predictability [0.4, 1]	1							
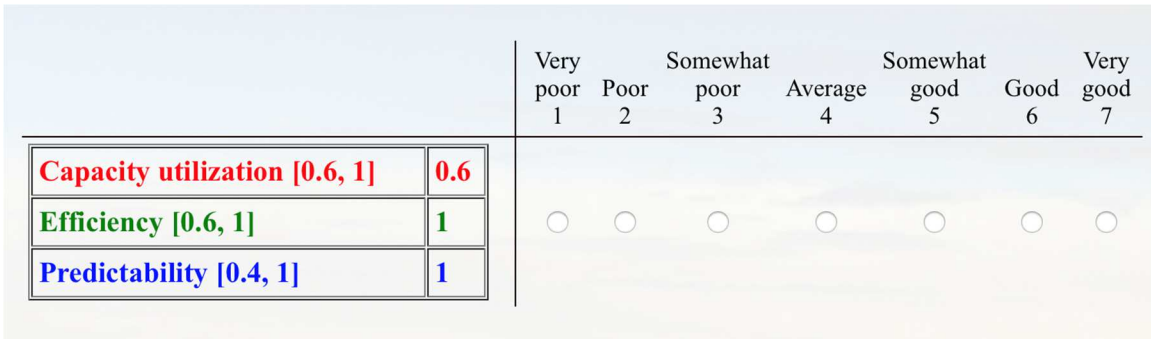
Figure 2.4: Sample GDP Expected System Performance Grading Question

2.3 Survey Results

The survey is presented to flight operators in two parts. The first part contains all the questions except the 12 trade-off questions on GDP system performance, which is in the second part. The second part of the survey also asks questions that are not relevant to the subject and thus not discussed here.

23 respondents complete the first part of the survey: 11 from legacy airlines, nine from low-cost carriers and three from cargo airlines. 17 respondents completed the second part, 11 from legacy airlines, three from low-cost carriers and three from cargo airlines. Most of Part Two respondents have over 10 years' experience in participating TMI planning telecons.

Below, in 2.3.1, we present results of the rating scale questions, including those on TMI practice, GDP decisions and GDP performance evaluation. In Section 2.3.2, we present results on the two groups of stated preference questions: one where respondents are asked to select a preferred GDP out of two according to their outcomes (Figure 2.3) and the other one where respondents are asked to grade GDPs based on their expected system performances (Figure 2.4).

2.3.1 Results of Rating Scale Questions

We present results of rating scale questions in Table 2.2 to Table 2.4. Table 2.2 summarizes general feedback on TMI decisions and TMI decision-making process. On average, flight operators have neutral attitudes towards current TMI service according to responses to the first five questions. Survey respondents do not strongly believe that TMI telecons require too much time or effort, nor that differences in decisions among FAA specialists are very different. They do agree that more vocal participants in the TMI planning telecons have greater influence on TMI decisions than less vocal ones.

11

Table 2.2: Feedback on Current TMI Practice

Questions	Range	Mean	S.D.
Overall quality of FAA TMI decisions	1 (very poor) to 7 (very good)	4.4	0.92
Clarity of the logic behind FAA TMI decisions	1 (very unclear) to 7 (very clear)	4.3	1.29
Similarity of different FAA traffic managers' TMI decisions when faced the same situation	1 (very different) to 7 (very similar)	3.8	1.35
Effective of the planning telecons in obtaining flight operators' inputs on TMI decisions:	1 (very ineffective) to 7 (very effective)	4.3	1.55
Responsiveness of the FAA TMI decisions with regard to the different needs of individual flight operators	1 (very unresponsive) to 7 (very responsive)	4.2	1.38
More vocal participants have greater influence on TMI decisions than less vocal participants	1 (strongly disagree) to 7 (strongly agree)	5.7	0.91
TMI telecons require too much time and effort	1 (strongly disagree) to 7 (strongly agree)	3.8	1.44

Table 2.3 summarizes feedback on GDP decisions. Due to uncertainty, GDP decisions, such as duration and planned rates (planned airport arrival capacity), are subject to change after they are made. The respondents report that, in the initial GDP plans, traffic specialists are conservative in setting GDP durations and its planned rates. GDP revisions are too frequent, and GDP lead time which reflects the time that flight operators have to adapt to GDPs is slightly shorter than desired. Frequent revisions and short GDP lead time are signs of unpredictability. Overall, respondents think that GDP scope, the area from which originating flights are assigned ground delays, is set about right. During briefs with respondents, we learn that responses to the GDP scope question might not be precise. GDPs are frequently implemented at different airports, such as SFO and EWR. Respondents report that the scopes set for some GDP airports are too large while for others they are too small. Therefore, the answer to this question is highly airport specific. Finally, as shown in the last row, respondents do not think airlines with large number of flights at a GDP airport have a larger influence.

Table 2.3: Feedback on GDP Decision Setting

Questions	Range	Mean	S.D.
GDP durations set in the initial plan are	1 (much too short) to 5 (much too long)	3.6	0.57
Planned rates in the initial GDP plan are	1 (much too low) to 5 (Much too high)	2.3	0.62
GDP revisions are	1 (much too infrequent) to 5 (much too frequent)	3.6	0.71
GDP lead times are	1 (much too short) to 5 (much too long)	2.6	0.65
GDP scopes are	1 (much too small) to 5 (much too large)	3.1	0.68
Influence of airlines with large numbers of flights affected by a GDP decision is	1 (much too weak) to 5 (much too strong)	3.0	0.81

Table 2.4 summarizes responses to questions regarding the importance of various GDP evaluation criteria to flight operators. These questions are designed to reveal the importance of different criteria in determining GDP performance. As reported, the most important criteria are accuracy of airport acceptance rate (AAR) estimates in the initial plan, accuracy in predicting GDP end time, and the number of AAR revisions. All these criteria are predictability-associated. It is very interesting that the percentage of airborne delay, which is an indicator of efficiency, is much less important.

As mentioned earlier, the survey also asks the respondents to add variables that are important in measuring GDP performance but not provided in the questionnaire. Those additional variables include balance between arrival and departure rates at a GDP airport and accurate prediction of demand.

Table 2.4: GDP Performance Evaluation Results

How important is each variable in evaluating GDP performance? 1 not at all important; 2 slight important; 3 important; 4 very important; 5 extremely important	Mean	S.D.
GDP lead time	3.4	0.88
GDP duration	3.6	0.82
GDP scope	4.0	0.55
Number of GDP extensions	3.7	1.13
Average flight delay of non-exempted flights	3.7	0.86
Percentage of total delay that is taken in the air	3.0	0.75
Unrecoverable delay	4.1	0.88
Maximum fight delay	2.9	1.35
Accuracy of forecast on GDP end time	4.1	0.68
Accuracy of initial delay estimates	3.5	0.88
Accuracy of airport acceptance rate (AAR) estimates in the initial plan	4.3	0.90
Number of AAR revisions	4.1	0.88

2.3.2 Results of Stated Preference Questions

This section presents results of two sets of grading questions: the ones on GDP outcome (Figure 2.3) and those on GDP expected performance (Figure 2.4). Both sets of questions share one common characteristic: the potential responses are ordered. As a result, one alternative is similar to those close to it and less similar to those further away, which violates the assumption of independent errors. We thus employ ordered choice models to analyze the responses (Train, 2009). We consider two types of ordered choice models: logit and probit. For each type, we consider three model specifications: standard, fixed-effect and random-effect. Below, we use the GDP outcome grading questions to describe the methods.

In the standard ordered logit model, utility for observation i takes the form

$$U_i = \boldsymbol{\beta}' \boldsymbol{x_i} + \varepsilon_i$$

(Eq 2.1)

where $\boldsymbol{x_i}$ is the vector of variables for observation i, $\boldsymbol{\beta}'$ is the vector of coefficients, and ε_i is the error term. For the GDP outcome grading questions, we set the variables as the

differences in the outcome attributes: the attribute values in B minus those in A. The error term is assumed to be identically and independently distributed (iid) logistic with the cumulative distribution specified as $F(\varepsilon) = \exp(\varepsilon)/(1 + \exp(\varepsilon))$.

Respondents choose a discrete response in one of five categories: strongly prefer A, somewhat prefer A, no preference, somewhat prefer B and strongly prefer B. We assume the choices are made based on the level of utility. The decision is represented as

- "strongly prefer A" if $-\infty < U < k_1$;
- "somewhat prefer A" if $k_1 < U < k_2$;
- "no preference" if $k_2 < U < k_3$;
- "somewhat prefer B" if $k_3 < U < k_4$; and
- "strongly prefer B" if $k_4 < U < +\infty$.

Following this, the probability of the answer "somewhat prefer A" for observation i is then

$$\text{Prob}_i(\text{"somewhat prefer A"}) = \text{Prob}(k_1 < U_i < k_2) = \text{Prob}(k_1 < \boldsymbol{\beta}'\boldsymbol{x_i} + \varepsilon_i < k_2)$$

$$= \text{Prob}(\varepsilon_i < k_2 - \boldsymbol{\beta}'\boldsymbol{x_i}) - \text{Prob}(\varepsilon_i < k_1 - \boldsymbol{\beta}'\boldsymbol{x_i})$$

$$= \frac{\exp(k_2 - \boldsymbol{\beta}'\boldsymbol{x_i})}{1 + \exp(k_2 - \boldsymbol{\beta}'\boldsymbol{x_i})} - \frac{\exp(k_1 - \boldsymbol{\beta}'\boldsymbol{x_i})}{1 + \exp(k_1 - \boldsymbol{\beta}'\boldsymbol{x_i})}$$

(Eq 2.2)

Probabilities for the other answers are obtained analogously. We estimate the coefficient parameters $\boldsymbol{\beta}$ and the cutoff points k_1 to k_4 by maximizing the log likelihood function

$$\text{LL}(\boldsymbol{\beta}, k_1, k_2, k_3, k_4) = \prod_i (\text{Prob}_{i,m})^{y_{i,m}}$$

(Eq 2.3)

where $y_{i,m} = 1$ if the ith response is choice m and zero otherwise and there are five possible choices in the GDP outcome grading question as listed above.

In the standard ordered probit model, we assume the error term is (iid) distributed standard normal instead of logistic. Accordingly, the probability of the answer "somewhat prefer A" is then

$$\text{Prob}_i(\text{"somewhat prefer A"}) = \Phi(k_2 - \boldsymbol{\beta}'\boldsymbol{x_i}) - \Phi(k_1 - \boldsymbol{\beta}'\boldsymbol{x_i})$$

(Eq 2.4)

where Φ is the standard cumulative normal function. Again, probabilities for the other answers can be obtained similarly.

We consider fixed-effect and random-effect versions of the models to capture potential correlations between responses from the same individual since each participant was asked repeated questions (Greene and Hensher, 2010). The utility that participant n assigns to variables in observation j is

$$U_{n,j} = \boldsymbol{\beta}' \boldsymbol{x_{n,j}} + \varphi_n + \varepsilon_{n,j}$$

(Eq 2.5)

where φ_n captures the individual specific heterogeneity and $\varepsilon_{n,j}$ is still the observation-related error term and follows the same distribution assumptions as before: iid logistic for the logit models and iid standard normal for the probit models. In the fixed-effect models, φ_n's are estimated as constants and may be correlated with $\boldsymbol{x_{n,j}}$. In the random-effect models, φ_n is assumed to be a normal random variable with mean zero and variance σ_φ^2, uncorrelated with $\boldsymbol{x_{n,j}}$ and independent of the observation-related error term $\varepsilon_{n,j}$. The mixture of a logistic error term with a normally distributed random effect is considered a bit unnatural in the random-effect ordered logit model (Greene and Hensher, 2010). This leads to more popularity of the random-effect ordered probit model.

We first tried all the model specifications. If there is no significant improvement from considering individual heterogeneity according to likelihood ratio test, then we only present the results from the standard choice models. If there is significant individual heterogeneity, then we report the results from both fixed-effect and random-effect models.

Table 2.5 summarizes the estimation results for the set of GDP outcome grading questions. Each respondent was asked 16 such questions and 23 respondents finished the questionnaire. This leads to 368 observations in total. As mentioned, differences in average delay and in unrecoverable delay are perfectly positively correlated. We only kept the former in the analysis and its utility coefficient therefore reflects the combined effect of average delay and unrecoverable delay. Actual delays at the end of a GDP are usually different from the initially assigned delay due to uncertainty. The change in delay, actual delay minus planned delay, could be positive when more delay occurs than planned or negative otherwise. To test the hypothesis that flight operators value delay saving differently from excessive delay, we distinguish difference in positive delay change from that in negative delay change in the analysis. Difference in negative delay change, ΔD_N, is calculated as

$$\Delta D_N = \Delta D_B \times I_{B,N} - \Delta D_A \times I_{A,N}$$

(Eq 2.6)

where ΔD_B (ΔD_A) is change in delay in outcome B(A) and $I_{B,N}(I_{A,N})$ is an indicator variable set to one if changes in delay in outcome B(A) is negative and zero otherwise.

Difference in positive delay change is calculated analogously. Following this, we define two variables based on the change in delay attribute. For example, in the question shown in Figure 2.3, difference in negative delay change is -5 × 1 - (-10) × 1 = 5 and difference in positive delay change is -5 × 0 - (-10) × 0 = 0.

Table 2.5: Results of GDP Outcome Grading Questions (Figure 2.3)

Variable (Option B-Option A)	Ordered probit		Ordered logit	
	Estimate	Standard error (SE)	Estimate	SE
ΔAverage delay per flight (min)	-0.076***	-10.41	-0.13***	-9.96
ΔMaximum flight delay difference (min)	0.0010	0.63	0.002	0.62
ΔNegative change in delay per flight (min)	-0.010***	-3.06	-0.019***	-3.15
ΔPositive change in delay per flight (min)	-0.012***	-2.76	-0.020***	-2.71
ΔLead time difference (min)	0.0001	0.06	-0.0003	-0.10
ΔNumber of revisions difference	-0.14	-0.58	-0.19	-0.48
k_1	-1.37***	14.05	-2.36***	-13.25
k_2	-0.17**	-2.17	-0.31**	-2.36
k_3	0.28***	3.60	0.45***	3.43
k_4	1.35***	13.78	2.32***	12.84
Log likelihood at convergence	-482.38		-482.74	
Log likelihood at constant		-570.20		
Number of observations		368		

*** Significant at 0.1% level; ** Significant at 1% level; * Significant at 5% level.

We report results from the standard ordered choice models since no significant individual heterogeneity was observed. The coefficient estimates and goodness-of-fit are similar from the ordered probit and logit models. Three variables have significant impact on grading GDP outcomes: average delay, negative change in delay and positive change in delay. Out of the three, average delay is the most important one.

The sign for the coefficient estimate of difference in negative delay change is negative, which indicates that more negative change is appreciated. In other words, all else equal (including actual average delay), flight operators would prefer an outcome where delay is overestimated in the initial plan to one where actual delay is the same as planned. However, this may not be what flight operators really mean. It is clear to us, the survey designers, that when average delay is fixed, a large magnitude of negative change does not mean more delay would be saved but rather that delay was overestimated initially. However, this idea may not carry over to the survey respondents. It is more intuitive for them to think that the actual delay will be less in an outcome where negative delay change is small (large absolute value). As a result, they appreciate negative change. The magnitude of the coefficient estimate of the negative delay change is smaller than that of average delay. This implies there is a cost associated with planned but un-incurred delay. The coefficient of the positive delay change is negative but smaller than that of average delay. If respondents interpreted the question as we intend—i.e. that total delay is fixed so that a positive delay change implies that delay was initially underestimated, then this result shows that respondents prefer to have an accurate initial delay estimate. On the other hand, if they fail to recognize that total delay is fixed, then the results implies that they attach less cost to unpredicted increases in delay than to the originally planned delay. In sum, our results suggest that there are issues with how respondents interpret this set of stated preference questions, which preclude definitive interpretation of the results. Table 2.6 summarizes results of GDP expected system performance grading questions. Fixed effects are not reported for the fixed-effect models since their magnitudes are not important. According to likelihood ratio test, there is significant individual heterogeneity and thus we are not reporting results from the standard models. It is interesting that individual heterogeneity is significant for this question set but not the previous one. The attributes in the GDP outcome questions are familiar to flight operators and different flight operator personnel may have similar sensitivities to the attributes. On the contrary, the performance metrics at the system level in this question set are new to the personnel and they may hold different standards in grading depending on how they expect these metrics to be associated with their business objectives. This may have led to significant heterogeneity in the estimation result.

In Table 2.6, all the model results show that predictability is viewed as a very important performance goal from the view of flight operator personnel, more valued than efficiency. This is consistent with the results in Table 2.4, where predictability indicators such as accuracy in setting GDP decisions and number of revisions are valued most important in assessing GDP outcomes where the percentage of airborne delay is considered less important.

Table 2.6: Results of GDP Expected System Performance Grading Questions (Figure 2.4)

| | Ordered probit | | | | Ordered logit | | | |
| | Fixed-effect[a] | | Random-effect | | Fixed-effect | | Random-effect | |
Variable	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
Capacity utilization	4.96***	0.59	4.68***	0.58	8.59***	1.08	8.04***	1.06
Efficiency	1.50***	0.52	1.42***	0.52	2.70***	0.93	2.55***	0.92
Predictability	1.98***	0.36	1.87***	0.36	3.65***	0.66	3.39***	0.65
k_1	3.47***	0.81	3.57***	0.78	6.07***	1.44	6.16***	1.40
k_2	4.60***	0.81	4.67***	0.78	8.18***	1.43	8.24***	1.39
k_3	5.40***	0.83	5.44***	0.80	9.59***	1.48	9.60***	1.44
k_4	6.09***	0.85	6.10***	0.83	10.79**	1.53	10.74***	1.50
k_5	7.07***	0.87	7.04***	0.85	12.48***	1.58	12.35***	1.55
k_6	7.94***	0.89	7.85***	0.87	13.98***	1.62	13.76***	1.59
σ_φ	—	—	0.89***	0.21	—	—	1.59***	0.35
Log likelihood at convergence	-284.33		-312.68		-284.75		-313.03	
Log likelihood at constant	-371.67							
Number of observations	204							

a Fixed effects are not reported.
*** Significant at 0.1% level; ** Significant at 1% level; * Significant at 5% level.

2.4 Summary

This chapter reports on a survey that addresses flight operators' views and concerns about predictability and quantifies the importance that flight operators attach to predictability in the context of GDPs. The survey also collects flight operators' views on TMI decisions and decision-making with a particular focus on GDP decisions.

The results show that predictability is considered very important in evaluating the performance of GDPs. The most valued GDP outcomes by flight operators are all predictability associated: accuracy of AAR estimates in the initial plan, accuracy in predicting GDP end time, and the number of AAR revisions. It is interesting that an indicator of GDP efficiency—the ratio of airborne delay to total delay—is less valued. The importance of predictability is further suggested by the results of a set of stated

preference questions, where respondents prioritize three GDP performance goals as follows: capacity utilization, predictability and efficiency. The survey results thus confirm the proposition that flight operators care about predictability and deserve attention in the design of a GDP.

3   GDP Modeling

3.1 Introduction

Most of the research on GDPs has focused on minimizing the expected cost of delay. In virtually all of these cost optimization models, the total expected costs are functions of two delay components: ground delay and airborne delay (Richetta and Odoni, 1993; Hoffman et al., 2007; Ball et al., 2003; Kotnyek and Richetta, 2006; Mukherjee and Hansen, 2007). The models recognize that the duration and degree of the capacity shortfall cannot be known with certainty at the time when the GDP plan is made. Some models represent the problem as a static one resulting in a GDP plan that cannot (from the standpoint of the model) be revised. Others recognize that revision is possible and model GDP planning as a dynamic problem.

A GDP revision can affect program duration and arrival capacity rates, with the former being much more common. A revision of GDP duration can be an extension or a cancellation. A GDP extension usually occurs when capacity remains low at the initially planned capacity recovery time. A GDP extension results in more affected flights and delay. In the existing extension models, the cost of incurred ground delay is valued the same regardless of whether it is part of the initial GDP plan or is imposed due to a GDP extension. In reality, however, unexpected extra delays due to the GDP extension require more effort from airline dispatchers and reduce the set of feasible mitigation actions, which may result in extra cost compared to expected delays in the initial plan. Conversely, a GDP may be cancelled earlier to reduce ground delay if capacity recovers earlier than initially planned. In the literature, assigned ground delay in the initial plan that is not incurred is assumed zero cost. Yet actions may have been taken to adapt to this planned but un-incurred delay, generating cost. In sum, deviations from the initial GDP plan, whether these result in additional delay or reduced delay, may carry a cost in terms of reduced predictability, which current models fail to account for.

We propose a more general GDP cost function that incorporates predictability into GDP cost optimization. Specifically, we modify the cost function in three ways. First, the unit cost of unplanned ground delay will be set higher than planned ground delay. The difference between the cost coefficients reflects the penalty due to unpredictability. Second, airborne delay will be more expensive than planned ground delay not just because of higher operating costs and safety concerns, but also because it is inherently less predictable—there is never airborne delay in the initial GDPs. Lastly, delay that is initially planned but not ultimately incurred is also considered in the cost function in recognition of the effort and actions taken to accommodate it. Following this, a GDP is most predictable when its implementation most closely follows the original plan. When the actual GDP deviates from the plan, predictability performance degrades. A natural way to measure the amount of deviation is in terms of differences between the incurred

delay and the initially planned delay. The larger the difference, the poorer the performance. For present purposes, the costs of unpredictability are assumed to be linear in the total quantities of these differences, specifically the amount of planned delay that is not incurred and the amount of extra delay over what is initially planned.

We do not consider more refined cost functions that might, for example, account for the difference between planned and incurred delay by flight, or the number of flights for which there is such a difference. One reason to focus on aggregate delay in our models is the Collaborative Decision Making (CDM) process (Ball et al., 2011), through which flight operators may re-assign slots to different flights based on their individual business objectives. These processes do not change aggregate delay but they do change delays incurred for individual flights. Slot re-assignment reduces the overall costs of delays, and may also change the relative costs of different delay components. To the extent this is true, the coefficients in our cost function should be assumed to take these effects into account. There are also cases in which flight operators make changes in their slot assignments for individual flights that in some sense reduce predictability of individual flight arrival times. We do not consider such "self-imposed unpredictability" in our analysis.

To study the relationship between the enhanced GDP cost function and GDP decision variables in a generic manner, we construct the GDP models based on deterministic queueing theory and continuous approximation using a small set of key GDP parameters (Daganzo, 1997). Unlike more widely used mathematical programming approaches, our method does not require detailed inputs about flight schedules and capacity scenarios. The output from our models—the time when the planned arrival capacity increases to reflect good weather conditions—is also much simpler. In this way, the sensitivity of the optimal planned capacity recovery time to different assumed cost functions can be easily compared. On the other hand, our approach requires a set of assumptions, which we discuss further below. The proposed GDP models can also be used to model tradeoffs between performance goals when metrics for predictability and other performance criteria are specified explicitly instead of being embedded in a cost function (Liu and Hansen, 2013). This is not discussed in this dissertation.

We use three chapters to introduce the GDP cost optimization models. In this chapter, we will give an overview of the literature on existing GDP optimization models and briefly discuss the continuous approximation method. We will also describe our GDP models in Sections 3.3 to 3.6. Two GDP models are developed: a static GDP model that does not allow revision, and a more realistic dynamic GDP model in which one GDP revision is considered. Based on the GDP models, in Chapter 4, we will propose our cost functions and show how optimal GDP decisions minimizing the expected cost can be reached for both GDP models. In Chapter 5, we will illustrate the proposed models with a case study. Research presented in Chapters 3 to 5 has been published in a journal paper by Liu and Hansen (2015).

3.2 Related Literature

Since the first study by Richetta and Odoni (1993), Integer Programming (IP) has been the primary technique for GDP stochastic optimization problems (Richetta and Odoni, 1994; Ball et al., 2003; Kotnyek and Richetta, 2006; Liu and Hansen, 2007; Mukherjee and Hansen, 2007; Ball et al., 2010; Mukherjee et al., 2012). There are two types of IP optimization models: static stochastic and dynamic stochastic. In the static GDP models (Richetta and Odoni, 1993; Ball et al., 2003; Kotnyek and Richetta, 2006), the optimal decision is made based on the available capacity scenarios forecast at the start of the planning horizon and cannot be revised in response to updated capacity information. The optimization criterion is to minimize total delay cost—ground plus airborne delay costs. The marginal cost of delaying one aircraft in the air is usually assumed as constant, whereas the functions of ground-hold cost can be linear or arbitrary. The time horizon for which the GDP is considered is discretized into equal time periods—typically 15 minutes. In the models with linear ground delay cost, the decision variables are x_i— the number of aircraft arriving at the GDP airport during period i, and y_i—the number of aircraft held on the ground from period i to $i + 1$. In the models with arbitrary ground delay cost functions, the decision variables are x_{ij}—the number of aircraft originally scheduled to arrive at the GDP airport during period i that are rescheduled to arrive during j.

In the dynamic GDP models (Richetta and Odoni, 1994; Liu and Hansen, 2007; Mukherjee and Hansen, 2007; Ball et al., 2010), uncertainty in decision-making is accommodated by allowing re-assignment of landing slots in response to updated capacity information. Richetta and Odoni (1994) modeled the dynamic evolution of the capacity forecasts and the implicit updating of the associated probabilities through a scenario tree. The model re-assigns slots to groups of aircraft based upon the updated capacity forecasts. Their model assumes that the ground delays assigned at each stage cannot be revised. Mukherjee and Hansen (2007) overcome the limitation of Richetta-Odoni model by allowing revision of ground delay assignment. Moreover, the Mukherjee-Hansen model optimizes the cost by assigning delay at the individual flight level. Both models are multistage stochastic programs, where branching points reflect new information about capacity. One shortcoming of the scenario-based models is that they assume a limited number of capacity scenarios whereas capacity may change continually rather than a few discrete branching points. In the light of this, Liu and Hansen (2007) proposed a scenario-free sequential decision model based on a value iteration algorithm.

In our study, a continuous approximation method based on deterministic queueing theory (Daganzo, 1997) will be employed to model the GDP problem. This technique is widely used in the context of ground transportation (Hendrickson and Kocur, 1981; Newell, 1987; Daganzo and Garcia, 2000; Lago and Daganzo, 2007; Yang et al., 2013).

Airport applications of this approach include Newell (1979), Hansen (2002), and Kim and Hansen (2013). The assumption is that the number of trips is sufficient that traffic can be treated as a continuous variable (Newell, 1982). One classic application is the morning commute problem, where morning commuters change their home departure times to avoid periods of high congestion at the bottleneck on their way to work (Vickrey, 1969). Commuters have ideal times when they want to pass the bottleneck. Due to congestion, the arrival and departure times at the bottleneck will be different. Continuous functions for cumulative arrivals to and departures from the bottleneck can be generated for the same set of commuters. Individual commuters choose their arrival times at the bottleneck to minimize their individual costs, including lateness, earliness and extra travel time. The GDP problem resembles the morning commute problem, since flights must change their departure times to avoid congestion at the destination airport, which acts as a bottleneck. The major differences are that the departure times are chosen by a central planner—the Federal Aviation Administration (FAA), the bottleneck results from a temporary reduction in capacity, and the duration and degree of this reduction cannot be known with certainty at the time when the central plan is created.

3.3 Basics of the GDP Models

We assume a GDP airport with a constant and continuous arrival demand rate λ, expressed in units of flights per unit time (all of the notations are summarized in Appendix). Using Aviation System Performance Metrics (ASPM) data, we find this is a reasonable assumption at airports with the most GDPs, such as EWR and SFO airports. For instance, at SFO and EWR airports, the Pearson linear correlation coefficients between the cumulative arrival counts and the scheduled arrival times are usually over 0.99 for flights involved in the initial GDPs. The origins of these flights are continuously distributed over space so that the distribution of required flight times for flights bound for the GDP airport is also continuous. For simplicity, we assume here that the flight time distribution is uniform between F_{min} and $F_{max} = F_{min} + \Delta F$, where ΔF is the range of the flight time. For a given destination airport, there are usually more flights from a closer origin airport than a further one, but the catchment area around an airport increases quadratically with distance. The joint effect of these two factors makes it plausible to assume a uniform distribution of flight time. This is further supported using flight schedule data from ASPM, which shows that uniform distribution reasonably (although by no means perfectly) approximates the empirical distribution function of flight time of GDP-delayed flights for airports with the most GDPs. We further assume that the scheduled arrival time and a required flight time are independent, and any slot reassignment resulting from CDM does not significantly change this joint distribution. The flight time for an individual flight is assumed to be fixed and deterministic. Therefore, the arrival demand rate for the subsets of flights whose required flight times

are between f_1 ($F_{min} \leq f_1 \leq F_{max}$) and f_2 ($f_1 \leq f_2 \leq F_{max}$) is $\lambda(f_2 - f_1)/\Delta F$. Under normal conditions, the GDP airport has arrival capacity $C_H > \lambda$ and thus the airport can land the arrival demand without delay. However, on some occasions, the airport operates at a reduced capacity $C_L < \lambda$. Traffic specialists anticipate these situations and implement GDPs in advance to reduce the cost of the associated delay. There are three decision variables in the design of a GDP: the scope—area from which originating flights are assigned ground delays, the time period over which the program is in effect, and the planned AAR. While the choice of scope is of fundamental importance, we will show in Section 3.6 that it is a simple extension to our basic models, in which we assume that the scope includes all origin airports. This also means that the GDP is planned at least F_{max} time units ahead of the actual start of the GDP. The other two decision variables are illustrated in the deterministic queuing diagram shown in Figure 3.1. As shown in Figure 3.1a, the scheduled arrival demand rate is assumed as a constant—λ, and thus the scheduled cumulative arrival curve— $S(t)$ can be formulated as:

$$S(t) = \lambda \cdot t$$

(Eq 3.1)

The GDP starts when the arrival capacity—known as the AAR—drops from its good-weather value, C_H, to a reduced value, C_L. This is also the time when the first plane affected by the GDP is planned to arrive at the airport. We assume that this time is known at the GDP issuance time—when the GDP is planned—and make it the origin of our time scale. We also assume that C_L and C_H are also known at the time of the plan, that $C_L < \lambda < C_H$, and that these values are used as the planned rates. Once the AAR drops, arrival capacity is insufficient to keep pace with the arrival demand and GDP starts, using a rate of C_L. The planned rate increases to C_H at time T. After that, the ground delays start to decline and finally go to zero at time T_2, which corresponds to the planned GDP end time. The planned cumulative arrival curve, $N(t,T)$, under the GDP is then a piece-wise function:

$$N(t,T) = \begin{cases} C_L \cdot t, 0 < t \leq T \\ C_L \cdot T + C_H \cdot (t - T), T < t \leq T_2 \end{cases}$$

(Eq 3.2)

where t is time and

$$T_2 = \frac{C_H - C_L}{(C_H - \lambda)} T$$

(Eq 3.3)

Note that, given a demand rate and AARs, GDP duration, T_2, is determined by the planned capacity recovery time—T—based on Eq 3.3.
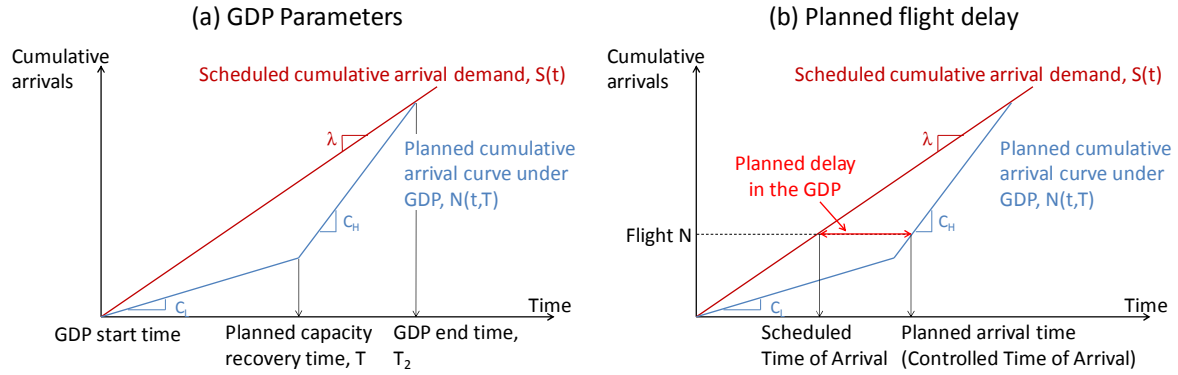
25

Figure 3.1: Cumulative Arrival Plots under the Initial Ground Delay Program Plan

While there are a number of uncertainties surrounding a GDP plan, we focus on the uncertainty about when the capacity will return to its good-weather value of C_H. Compared to the capacity recovery time, AARs can be predicted with much better accuracy, since they are based on considerable experience and understanding of how the airport operates under a given set of weather conditions. For example, at SFO, marine stratus (fog) can preclude simultaneous arrival operations on its closely spaced parallel runways, reducing the arrival capacity from 60 to 30 flights per hour (Cook and Wood 2009). The time when the capacity decreases to C_L can sometimes be a source of uncertainty, but the capacity recovery time is based on a longer term forecast and thus a larger source of uncertainty in most GDPs. According to the historical GDP data (Liu and Hansen 2014), there were rare cases of revising a GDP start time but far more cases of revising a capacity recovery time. Here, we will treat the capacity recovery time—T—as the only decision variable that has uncertainty.

Under the GDP, flights are assigned delays because their planned arrival times are later than their scheduled times of arrival due to the capacity drop. The planned arrival time is also referred to as Controlled Time of Arrival (CTA) which is determined by the ration-by-schedule principle: the available time slots are assigned to affected flights according to their arrival sequence in the schedule. Given this, the Nth arrival in the schedule will also be the Nth arrival in the GDP plan, as illustrated in Figure 3.1b. As a result, for each arrival, the horizontal difference between the CTA and the scheduled time of arrival is the delay assigned to it in the GDP plan. The total amount of planned ground delay for all the delayed flights in the GDP, d_P, is then the sum of all the flight delays and can be expressed as a function of T:

$$d_P(T) = \int_0^{T_2} [S(t) - N(t,T)]dt = \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)} \cdot T^2$$

(Eq 3.4)

26

The analysis above pertains to the plan for the GDP. The actual capacity recovery time, τ, could be earlier or later than the planned recovery time, T. When the information on actual capacity recovery time is updated, traffic specialists may make a revision to the initial GDP by cancelling the GDP earlier in the case of early clearance or extending the GDP in the case of late clearance. The impacts of revisions add considerable complexity to the models. Before considering these, it is useful to consider the case where such revisions are not allowed. Next, we will introduce the GDP no-revision model.

3.4 GDP No-revision Model

In this section, we assume that the initial GDP plan is not revised. Under this assumption, flights that are affected by the initial GDP will take off at their assigned departure time slots and flights that are not affected by the initial plan will take off at their scheduled departure times. Queueing diagrams for the GDP no-revision model are illustrated in Figure 3.2 for both the early and late capacity recovery cases. Besides the scheduled and planned cumulative arrival curves ($S(t)$ and $N(t,T)$) introduced before, there is one more curve: the ideal cumulative arrival curve— $I(t,\tau)$ which is independent of our decision on T. The ideal cumulative arrival curve is determined by the actual capacity recovery time τ, which is considered as a random variable in our analysis. If we knew τ when we were designing the GDP, then we could have allocated the time slots based on the ideal cumulative arrival curve, the slope of which shifts from C_L to C_H at time τ and becomes the same as the demand rate after time τ_2—the ideal delay clearance time:

$$I(t,\tau) = \begin{cases} C_L t, 0 < t \le \tau \\ C_L T + C_H(t - T), \tau < t \le \tau_2 \\ \lambda t, t > \tau_2 \end{cases}$$

(Eq 3.5)

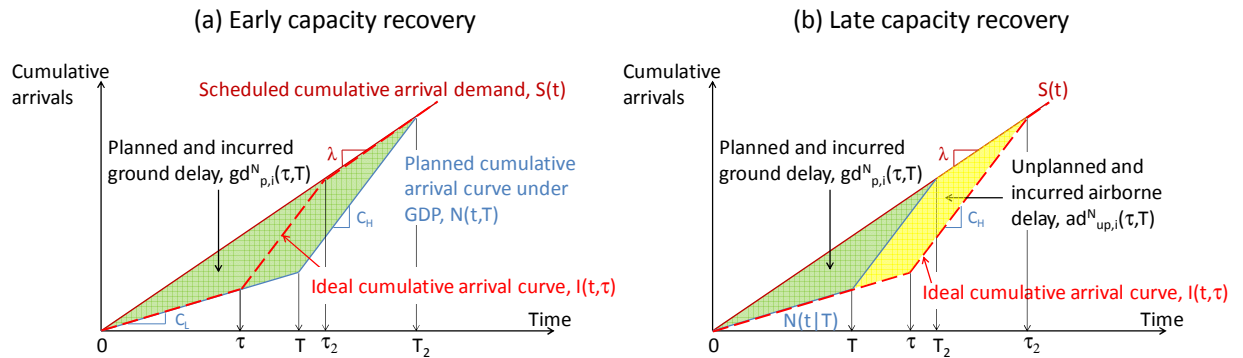(a) Early capacity recovery   (b) Late capacity recovery

Figure 3.2: GDP No-revision Model and Delay Components

In the case of early capacity recovery (Figure 3.2a), between τ and T, we should have planned the GDP on the capacity level C_H but we will actually land arrivals based on the lower rate C_L according to the GDP plan. The actual capacity is enough to carry out the planned GDP and thus the incurred delay will equal the planned delay, and be taken on the ground. The planned and incurred ground delay, $gd_{p,i}^N$, is thus formulated as:

$$gd_{p,i}^N(\tau, T) = d_P(T) = \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)} \cdot T^2, \qquad \tau \leq T$$

(Eq 3.6)

where the superscript N indicates this is a no-revision case, and the subscripts p and i indicate the delay is planned and incurred respectively. The incurred delay is larger than the ideal delay that would have been incurred in the case of early capacity recovery.

We now consider the case of late capacity recovery (Figure 3.2b). Between T and τ, we should have planned the GDP assuming the capacity level C_L but we overestimated the capacity and there will be more flights approaching the airport than can be accommodated. Because of this, flights that are affected by the initial GDP plan may experience unexpected airborne delay in addition to planned ground delay, and flights that are not affected by the initial GDP but scheduled to arrive between T_2 and τ_2 will experience unexpected airborne delay before landing. In this case, there will be two types of delay: planned ground delay—$gd_{p,i}^N$, and unplanned airborne delay—$ad_{up,i}^N$. $gd_{p,i}^N$ is calculated as in Eq 2.1. $ad_{up,i}^N$ is determined by the GDP plan and also the actual cumulative arrival curve, which is identical to the ideal cumulative arrival curve in this case:

$$ad_{up,i}^N(\tau, T) = d_R(\tau, T) - gd_{p,i}^N(\tau, T) = \int\limits_{S(t) > I(t,\tau)} [S(t) - I(t,\tau)]dt - d_P(T)$$

$$= \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)} \cdot (\tau^2 - T^2), \qquad \tau > T$$

(Eq 3.7)

where $d_R(\tau, T)$ is the total realized delay. It will be seen in the next section that $d_R(\tau, T)$ is the same with and without revision in the case of late capacity recovery. However, the delay cost is lower with revision since the extension transfers some of the airborne delay in $ad_{up,i}^N(\tau, T)$ to the ground.

3.5 GDP Revision Model

In this section, we will present the GDP revision model considering both cancellation and extension. After the initial plan, GDP could be revised more than once. In our GDP revision model, we are only considering one revision. Previous work (Liu and Hansen,

2014) has shown that GDPs at SFO airport—the airport with the most GDPs—usually have either no revision or one revision. Therefore, our model of revision, while relatively simple, is not unrealistic. The complexity in the revision model comes from the fact that the impacts of revisions are not instantaneous. In the early capacity recovery case, additional flights can be released but it takes time for them to reach the GDP airport. In the late capacity recovery, flights on the ground can be further held but flight traffic in the air already exceeds what the airport can accommodate. These effects are captured in our revision model and discussed below in Sections 3.5.1 and 3.5.2 respectively.

3.5.1 GDP Cancellation Model

Under early cancellation, flights that are ground-holding can be released earlier than their controlled times of departure as planned in the initial GDP. However, we cannot necessarily release all flights at their earliest possible take-off times, since this may still overwhelm the arrival capacity. In other words, we may still need to meter the release of flights when updating the CTA's. In the cancellation model, we assume the cumulative arrival curve is revised at time $\tau - t_{R,E}$, where $t_{R,E}$, $0 < t_{R,E} < F_{min}$, is the difference between the revision time and the actual capacity recovery time. In addition, we assume τ is known with certainty at the revision time. In the remainder of the paper, we assume $t_{R,E}$ to be 0. When $t_{R,E}$ is larger than 0, all the derived equations hold by replacing $F_{min}(F_{max})$ with $F_{min} - t_{R,E}$ $(F_{max} - t_{R,E})$.

The revised CTA's are assigned based on the revised cumulative arrival curve, which is jointly determined by the available capacity and the available cumulative arrival demand. At time τ, capacity recovers and therefore the available capacity—$C(t, \tau)$—can be written as:

$$C(t,\tau) = \begin{cases} C_L, & t \leq \tau \\ C_H, & t > \tau \end{cases}$$

(Eq 3.8)

The available cumulative arrival demand, $D(t, \tau, T)$, is obtained by releasing flights at their earliest possible take-off time and is a function of the planned and actual capacity recovery time. When capacity permits, the arrival throughput will be determined by the available demand. If capacity is insufficient for the available demand, the arrival throughput is determined by capacity. The cumulative arrival throughput, $Q(t, \tau, T)$, is thus a function of demand and capacity:

$$Q(t + \Delta t, \tau, T) = min\{D(t + \Delta t, \tau, T), Q(t, \tau, T) + C(t, \tau) \cdot \Delta t\}$$

(Eq 3.9)

which is also the revised cumulative arrival curve and the basis for assigning new GDP CTA's. Sometime after τ, the revised cumulative arrival curve goes above the planned cumulative arrival curve—$N(\tau, T)$, as shown in Figure 3.3. The area between these two

curves is then the planned un-incurred ground delay: $gd_{p,ui}$. In other words, this is the unexpected saving in ground delay due to early GDP cancellation. Because of the unexpected delay saving, planned and incurred ground delay is smaller than the ground delay assigned in the initial plan. The planned un-incurred and incurred delays are formulated in Eq 3.10 and Eq 3.11 respectively:

$$gd_{p,ui}(\tau,T) = \int\limits_{Q(t,\tau,T)>N(t,T)} [Q(t,\tau,T) - N(t,T)]dt, \qquad \tau \le T$$

(Eq 3.10)

$$gd_{p,i}(\tau,T) = d_P(T) - gd_{p,ui}(\tau,T), \qquad \tau \le T$$

(Eq 3.11)
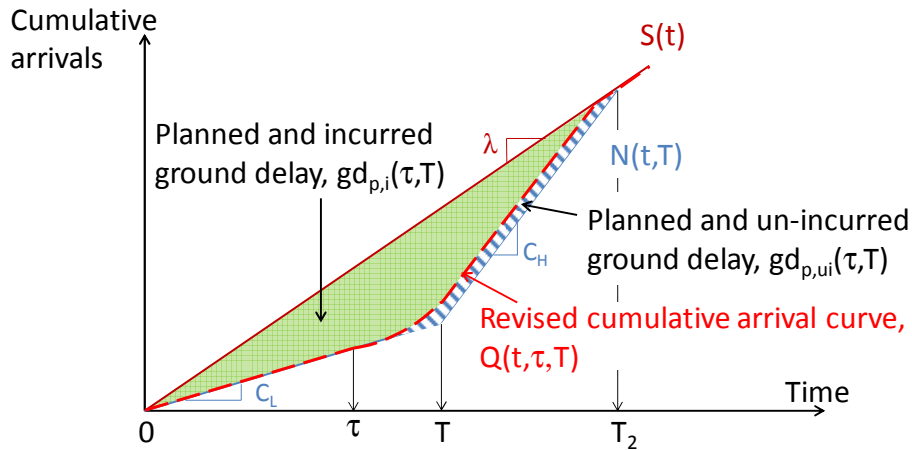
Figure 3.3: Delay Components in the Case of Early GDP Cancellation

To calculate the delay components in Figure 3.3, we must estimate the revised cumulative arrival curve $Q(t,\tau,T)$, which can be derived from revised cumulative demand curve $D(t,\tau,T)$ and the available capacity $C(t,\tau)$ as in Eq 3.9. $C(t,\tau)$ is formulated in Eq 3.8 and thus the unanswered question is the mathematical expression of $D(t,\tau,T)$. To determine the available demand, we recognize that the affected flights in the initial GDP fall into three categories, based on their status at the actual capacity recovery time τ. At this time, flights bound for the congested airport are either:

- Type I: these flights have already departed. They are scheduled to depart before τ and actually have departed before τ under the initial GDP. These flights have already incurred the ground delays assigned in the initial GDP and will arrive at their initially assigned CTA's. We denote the available cumulative demand curve, which is the same as the planned cumulative arrival curve under the original GDP, for this type of flights as $D_I(t,\tau,T)$ and its rate as $D_I'(t,\tau,T)$.

30

- Type II: these flights are being held on the ground at τ. Type II flights would have already departed in the original schedule but are waiting on the ground at time τ under the initial GDP. These flights can, in principle, depart immediately if capacity permits. The available cumulative demand curve for Type II flights, which is denoted as $D_{II}(t,\tau,T)$ with demand rate $D'_{II}(t,\tau,T)$, thus assumes that these flights all depart at τ.
- Type III: there flights are scheduled to depart after τ. Ground delays assigned in the initial GDP have not yet been incurred for flights of this type. Therefore, there would be no delay for these flights if they were allowed to take off as scheduled. Assuming they depart as scheduled, they will arrive earlier than the CTA's assigned to them under the initial GDP. The available cumulative arrival demand curve for these flights is the same as the scheduled cumulative arrival curve. The cumulative demand and its rate for Type III are denoted as $D_{III}(t,\tau,T)$ and $D'_{III}(t,\tau,T)$ respectively.

The total available cumulative arrival demand after revision, $D(t,\tau,T)$, is then the sum of the available cumulative demands of each type. The difference between $D(t,\tau,T)$ and the planned cumulative arrival demand curve in the initial GDP plan reflects the effect of GDP revision. This effect depends on the flight time distribution. If the delayed flights are concentrated in the vicinity of the affected airport, they can arrive at the airport earlier under revision, which enables the airport to utilize the extra capacity from early clearance more efficiently. If the flight time distribution is shifted toward larger values, Type II flights will take longer to reach the airport, so the extra capacity will be less fully utilized. There are four different cases for $D(t,\tau,T)$, depending on the range of flight time distribution, F_{min} and F_{max}, and the values of τ, T and T_2, as summarized in Table 3.1.

Table 3.1: Different Cases for Available Cumulative Arrival Demands after Revision, Early Cancellation

Case ID	Condition	Revision Impact on Demand
C0	$T < \tau + F_{min}$	None
C1	$\tau + F_{min} < T < T_2 < \tau + F_{max}$	Small
C2	$\tau + F_{min} < T < \tau + F_{max} < T_2$	Moderate
C3	$\tau + F_{min} < \tau + F_{max} < T < T_2$	Large

In the case of C0, the extra demand from the early cancellation could not arrive until after time T, when it was assumed in the initial plan that capacity would increase. The extra demand would arrive at a time when available capacity is being fully utilized under the initial GDP. Therefore, the GDP should not be revised in this case. In the other cases, $\tau + F_{min}$ is less than T, and a GDP revision can take advantage of the extra capacity

resulting from early clearance. The formulations for $D(t, \tau, T)$ are slightly different for each case. These formulations are discussed below, with C1 fully analyzed and C2 and C3 described in terms of their differences with C1.

3.5.1.1 Available Cumulative Arrival Demand, Case C1

A conceptual plot of available cumulative arrival demand curve for Case C1 is presented in Figure 3.4. The available demand is obtained by allowing flights to depart at their earliest possible take-off times. The earliest arrival time for these flights is $\tau + F_{min}$, which is the time when the updated arrival demand starts to be different from the planned arrivals. Even if capacity permits, delay will not vanish earlier than the planned delay clearance time T_2. This is because there will be Type I flights—flights that have been delayed on the ground and taken off before τ—arriving at the affected airport from $\tau + F_{min}$ to T_2, since $T_2 < \tau + F_{max}$. The total available demand is the sum of the available demands for the three types of flights. We now consider these three demands in turn. We don't consider flights arriving after T_2, because there is no more delay under the initial GDP after this time.
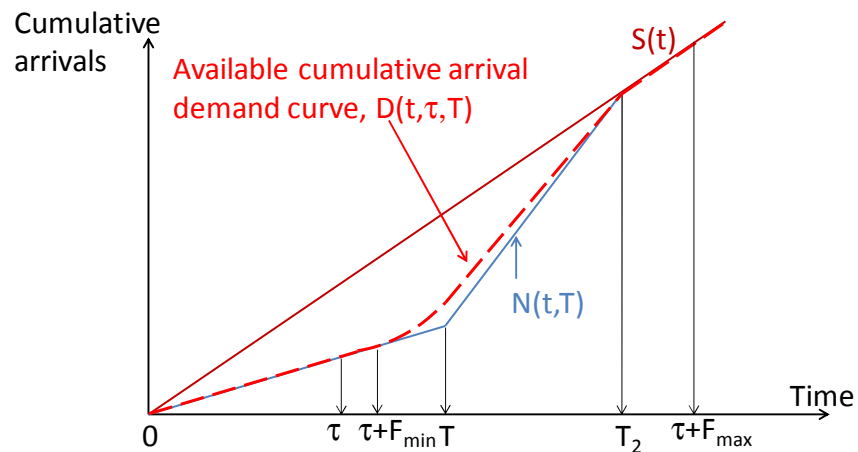
Figure 3.4: Conceptual Plot of Available Cumulative Arrival Demand Curve, Case C1

Type I flights are the flights that have been released from their departure airports at the actual capacity recovery time τ. They have arrived at the GDP airport or are in the air at this time. In either case, delay assigned to these flights in the initial GDP has already been incurred and they will arrive at their CTA's. The principles that are used to derive $D_I(t, \tau, T)$, the available demand for Type I flights, are:

- Before $\tau + F_{min}$, all planned capacity is utilized for Type I flights. All other flights take off at or after τ, making it impossible for them to arrive prior to $\tau + F_{min}$ since F_{min} is the minimum flight duration.
- For Type I flights planned to arrive between $\tau + F_{min}$ and T_2, the flight time range at any time t is between $t - \tau$ and F_{max}. If a flight arrives at t with flight

32

time less than $t - \tau$, then this flight must have taken off after τ, so it cannot be a Type I flight. Moreover, flights arriving at t with flight time in $[t - \tau, F_{max}]$ will all be Type I flights. The probability that a flight is a Type I flight given its CTA is t, $P(D_I | CTA = t)$, is then:

$$P(D_I | CTA = t) = \frac{F_{max} - (t - \tau)}{F_{max} - F_{min}}, \qquad \tau + F_{min} < t \leq T_2$$

(Eq 3.12)

Since the flight time distribution for all flights is uniform between F_{min} and F_{max}, flight time for Type I flights will be uniformly distributed between $t - \tau$ and F_{max}. Given the planned capacity rate is C_L before T and C_H afterwards, the available demand rate of Type I flights after revision, which is the same as the planned capacity rate for Type I flights under the original GDP, can be expressed as:

$$D_I'(t, \tau, T) = \begin{cases} C_L, & 0 \leq t \leq \tau + F_{min} \\ C_L \cdot \dfrac{F_{max} - (t - \tau)}{F_{max} - F_{min}}, & \tau + F_{min} < t \leq T \\ C_H \cdot \dfrac{F_{max} - (t - \tau)}{F_{max} - F_{min}}, & T < t \leq T_2 \end{cases}$$

(Eq 3.13)

With these, we integrate and express the available cumulative arrival demand curve for Type I flights as:

$$D_I(t, \tau, T) =$$

$$\begin{cases} C_L t, & 0 < t \leq \tau + F_{min} \\ -\dfrac{C_L}{2 \cdot \Delta F}[t - (\tau + F_{max})]^2 + C_L \left(\tau + F_{min} + \dfrac{\Delta F}{2}\right), & \tau + F_{min} < t \leq T \\ -\dfrac{C_H}{2 \cdot \Delta F}[t - (\tau + F_{max})]^2 + C_L \left(\tau + F_{min} + \dfrac{\Delta F}{2}\right) + \dfrac{C_H - C_L}{2 \cdot \Delta F}[T - (\tau + F_{max})]^2, & T < t \leq T_2 \end{cases}$$

(Eq 3.14)

Type II flights are the flights that should have taken off by τ if the GDP had not been initiated, but have not taken off at τ due to the GDP. All these flights have been delayed to some degree at τ. There is no delay planned for the flights arriving after T_2, and thus these flights cannot be Type II flights. The flight time range for Type II flights is thus between F_{min} and $T_2 - \tau$. Type II flights are held on the ground at τ, and can take off immediately if capacity permits. We can easily calculate the cumulative available demands for Type II flights if we know the distribution of flight time for these flights, $f(F | D_{II})$, since the new departure time will be τ for all the Type II flights. To find $f(F | D_{II})$, we use Bayes' rule, which gives

$$f(F|D_{II}) = \frac{P(D_{II}|F) \cdot f(F)}{P(D_{II})}$$

where, $P(D_{II}|F)$ is the probability that a flight is a Type II flight given that its flight time is F; $f(F)$ is the flight time distribution, assumed as a uniform distribution on F_{min} and F_{max}; and $P(D_{II})$ is the unconditional probability that a flight impacted by the GDP is a Type II flight.

To find $P(D_{II}|F)$ for a given τ, refer to Figure 3.5. For a flight with flight time F to be a Type II flight it is necessary and sufficient that it be in the virtual queue at $\tau + F$. Under ration by schedule, flights that are in the virtual queue at time $\tau + F$ correspond to flights that are scheduled to arrive between τ_s and $\tau + F$. Since scheduled arrival time and flight time are independent, the probability that a flight in the GDP is in the virtual arrival queue at $\tau + F$ given its flight time is F, is equal to the unconditional probability that a flight in the GDP is in the virtual arrival queue at this time. Therefore, $P(D_{II}|F)$ is equal to the unconditional probability that a flight in the GDP is in the virtual arrival queue at $\tau + F$. From Figure 3.5, we thus see that:

$$P(D_{II}|F) = \begin{cases} \dfrac{S(\tau + F) - N(\tau + F, T)}{S(T_2)}, & \tau + F_{min} \leq \tau + F \leq T_2 \end{cases}$$

$$= \begin{cases} \dfrac{(\lambda - C_L) \cdot (\tau + F)}{\lambda \cdot T_2}, & F_{min} \leq F \leq T - \tau \\[2mm] \dfrac{(\lambda - C_H) \cdot (\tau + F) + (C_H - C_L) \cdot T}{\lambda \cdot T_2}, & T - \tau < F \leq T_2 - \tau \end{cases}$$
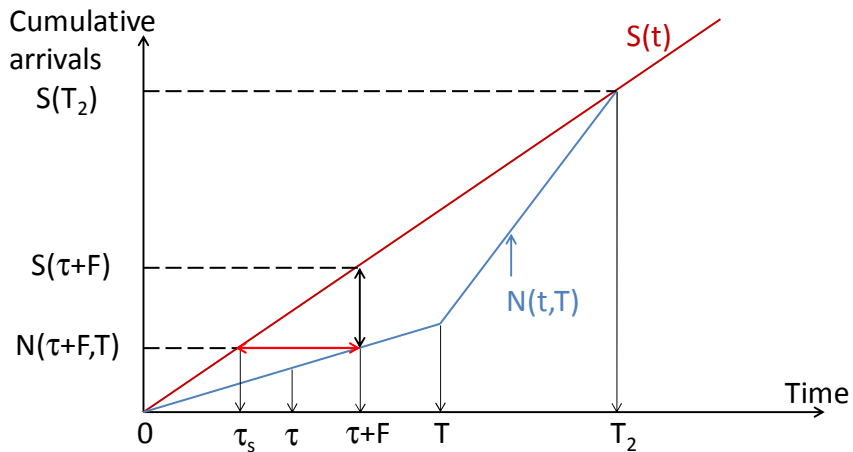
Figure 3.5: Virtual Arrival Queue Length at time $\tau + F$

To obtain $P(D_{II})$, the probability that a flight impacted by the GDP is being held on the ground at τ, we use the total probability theorem. In combination with Eq 3.16, we get

$$P(D_{II}) = \int_{F_{min}}^{F_{max}} P(D_{II}|F) f(F) dF = \int_{F_{min}}^{T_2-\tau} P(D_{II}|F) f(F) dF$$

(Eq 3.17)

Assuming F is uniformly distributed over $[F_{min}, F_{max}]$, we get

$$P(D_{II}) = \frac{1}{\lambda \cdot T_2 \cdot \Delta F} \cdot \left\{ \begin{array}{c} \int_{F_{min}}^{T-\tau} [\lambda \cdot (\tau + F) - C_L \cdot (\tau + F)] \, dF \\ + \int_{T-\tau}^{T_2-\tau} [\lambda \cdot (\tau + F) - C_L \cdot T - C_H \cdot (\tau + F - T)] \, dF \end{array} \right\}$$

(Eq 3.18)

where $\Delta F = F_{max} - F_{min}$. Integrating, we get

$$P(D_{II})$$
$$= \frac{\frac{C_H - C_L}{2}(T - \tau)^2 + \frac{\lambda - C_H}{2}(T_2 - \tau)^2 - \frac{\lambda - C_L}{2}(\tau + F_{min})^2 - \frac{\lambda - C_L}{2} \cdot \tau^2 + \frac{C_H - C_L}{C_H - \lambda} \cdot (\lambda - C_L) \cdot T^2}{\lambda \cdot T_2 \cdot \Delta F}$$

(Eq 3.19)

Applying Bayes' rule, we now obtain:

$$f(F|D_{II}) = \frac{P(D_{II}|F) \cdot f(F)}{P(D_{II})} = \left\{ \begin{array}{ll} \dfrac{\dfrac{(\lambda - C_L) \cdot (\tau + F)}{\lambda \cdot T_2 \cdot \Delta F}}{\dfrac{D_{II,total}}{\lambda \cdot T_2}}, & F_{min} \leq F \leq T - \tau \\ \\ \dfrac{\dfrac{(\lambda - C_H) \cdot (\tau + F) + (C_H - C_L) \cdot T}{\lambda \cdot T_2 \cdot \Delta F}}{\dfrac{D_{II,total}}{\lambda \cdot T_2}}, & T - \tau < F \leq T_2 - \tau \end{array} \right.$$

$$= \left\{ \begin{array}{ll} \dfrac{(\lambda - C_L) \cdot (\tau + F)}{D_{II,total} \cdot \Delta F}, & F_{min} \leq F \leq T - \tau \\ \\ \dfrac{(\lambda - C_H) \cdot (\tau + F) + (C_H - C_L) \cdot T}{D_{II,total} \cdot \Delta F}, & T - \tau < F \leq T_2 - \tau \end{array} \right.$$

(Eq 3.20)

The total number of D_{II} flights can be written as a function of $P(D_{II})$:

$$D_{II,total} = \lambda \cdot T_2 \cdot P(D_{II})$$
$$= \frac{\frac{C_H - C_L}{2}(T - \tau)^2 + \frac{\lambda - C_H}{2}(T_2 - \tau)^2 - \frac{\lambda - C_L}{2}(\tau + F_{min})^2 - \frac{\lambda - C_L}{2} \cdot \tau^2 + \frac{C_H - C_L}{C_H - \lambda} \cdot (\lambda - C_L) \cdot T^2}{\Delta F}$$

(Eq 3.21)

To obtain the cumulative demand curve by Type II flights, we require the cumulative distribution of their flight time. Integrating Eq 3.20, we obtain:

For $F_{min} \leq F \leq T - \tau$,

$$F(F|D_{II}) = \int_{F_{min}}^{F} \frac{(\lambda - C_L) \cdot (\tau + x)}{D_{II,total} \cdot \Delta F} dx = \frac{(\lambda - C_L)}{D_{II,total} \cdot \Delta F} [\left(\tau F + \frac{F^2}{2}\right) - \left(\tau F_{min} + \frac{F_{min}^2}{2}\right)]$$

(Eq 3.22)

For $T - \tau \leq F \leq T_2 - \tau$

$$F(F|D_{II}) = \int_{F_{min}}^{T-\tau} \frac{(\lambda - C_L) \cdot (\tau + x)}{D_{II,total} \cdot \Delta F} dx + \int_{T-\tau}^{F} \frac{(\lambda - C_H) \cdot (\tau + x) + (C_H - C_L) \cdot T}{D_{II,total} \cdot \Delta F} dx$$

$$= \frac{(\lambda - C_L)}{D_{II,total} \cdot \Delta F} \left[\left(\tau(T - \tau) + \frac{(T - \tau)^2}{2}\right) - \left(\tau F_{min} + \frac{F_{min}^2}{2}\right)\right]$$

$$+ \frac{(\lambda - C_H)}{D_{II,total} \cdot \Delta F} \left[\left(\tau F + \frac{F^2}{2}\right) - \left(\tau(T - \tau) + \frac{(T - \tau)^2}{2}\right)\right] + \frac{(C_H - C_L) \cdot T}{D_{II,total} \cdot \Delta F} [F - (T - \tau)]$$

(Eq 3.23)

The cumulative available demand of Type II flights, D_{II}, is obtained by assuming all these flights take off immediately at time τ. Therefore, if capacity permits, Type II flights arriving at time t after revision are the Type II flights with flight time $t - \tau$. The cumulative available demand of Type II flights at time t is then equal to the product of the total number of Type II flights times the value of the cumulative flight time distribution function at $t - \tau$:

$$D_{II}(t, \tau, T) = D_{II,total} \cdot F(t - \tau|D_{II}) =$$

$$\begin{cases} 0, & 0 \leq t \leq \tau + F_{min} \\ \frac{(\lambda - C_L)}{\Delta F} \cdot \left[\left(\tau(t - \tau) + \frac{(t - \tau)^2}{2}\right) - \left(\tau F_{min} + \frac{F_{min}^2}{2}\right)\right], & \tau + F_{min} < t \leq T \\ \frac{(\lambda - C_L)}{\Delta F}\left[\left(\tau(T - \tau) + \frac{(T - \tau)^2}{2}\right) - \left(\tau F_{min} + \frac{F_{min}^2}{2}\right)\right] + \frac{(\lambda - C_H)}{\Delta F}\left[\left(\tau(t - \tau) + \frac{(t - \tau)^2}{2}\right) - \left(\tau(T - \tau) + \frac{(T - \tau)^2}{2}\right)\right] \\ \qquad + \frac{(C_H - C_L) \cdot T}{\Delta F}(t - T), & T < t \leq T_2 \end{cases}$$

(Eq 3.24)

We now consider arrival demand from Type III flights, which are originally scheduled to take off after τ. Planned ground delay has not been incurred for these flights by τ, and thus there would be no delay for Type III flights if they could take off as scheduled. In the schedule, the first Type III flights—those with the shortest flight times—arrive just after $\tau + F_{min}$. All the scheduled arrivals are Type III flights after $\tau + F_{max}$. Between $\tau + F_{min}$ and $\tau + F_{max}$, the demand rate for Type III flights in the original schedule increases linearly at a rate $\lambda/\Delta F$. Since these flights have not been delayed at τ, the available demand for Type III is the same as the scheduled cumulative demand:

$$D_{III}(t,\tau,T) = \begin{cases} 0, & 0 \le t \le \tau + F_{min} \\ \dfrac{\lambda}{2\Delta F}(t - \tau - F_{min})^2, & \tau + F_{min} < t \le T_2 \end{cases}$$

(Eq 3.25)

Summing up the available demands for the three types of flights; we get the cumulative available arrival demand as:

$$D(t,\tau,T) = D_I(t,\tau,T) + D_{II}(t,\tau,T) + D_{III}(t,\tau,T) =$$

$$\begin{cases} C_L t, & 0 \le t \le \tau + F_{min} \\ \dfrac{\lambda - C_L}{\Delta F} \cdot t^2 - \dfrac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_L \cdot t, & \tau + F_{min} < t \le T \\ \dfrac{\lambda - C_H}{\Delta F} \cdot t^2 - \dfrac{\lambda - C_H}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_H \cdot t + \dfrac{C_H - C_L}{\Delta F} \cdot T \cdot t - \dfrac{C_H - C_L}{\Delta F} \cdot T \cdot (\tau + F_{max}), & T < t \le T_2 \end{cases}$$

(Eq 3.26)

The available demand rate can be calculated as the derivative of $D(t,\tau,T)$:

$$D'(t,\tau,T) = \begin{cases} C_L, & 0 \le t \le \tau + F_{min} \\ 2\dfrac{\lambda - C_L}{\Delta F} \cdot t - \dfrac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) + C_L, & \tau + F_{min} < t \le T \\ 2\dfrac{\lambda - C_H}{\Delta F} \cdot t - \dfrac{\lambda - C_H}{\Delta F} \cdot (\tau + F_{min}) + C_H + \dfrac{C_H - C_L}{\Delta F} \cdot T, & T < t \le T_2 \end{cases}$$

(Eq 3.27)

Between $\tau + F_{min}$ and T, the available demand rate increases with time since λ is larger than C_L. At time T, the rate increases by $(C_H - C_L)(\tau + F_{max} - T)/\Delta F$ due to the jump in demand rate for Type I flights at this time (Equation 13). The demand rate decreases with time afterwards. If capacity permits, all the held flights can be released immediately and Type III flights can take off as scheduled when the GDP is cancelled earlier. In other words, there is no need for new CTA's. Mathematically, arrival times can be determined directly from $D(t,\tau,T)$. This scenario is possible—indeed quite common—because of the natural spread of flight time and scheduled arrival time. At SFO airport, in the early clearance case, 77% of GDPs were cancelled without the need to assign new CTA's (Ball et al., 2010). Otherwise, when capacity is insufficient, the throughput rate will be jointly determined by the demand and the capacity.

3.5.1.2 *Available Cumulative Arrival Demand, Case C2*

In this case, flights that depart before τ can all arrive at the GDP airport before the planned delay clearance time T_2 and, when capacity permits, there will be no more delay in the system after $\tau + F_{max}$, as in Figure 3.6. In such a case, arrivals will follow the available cumulative arrival demand curve—the red dashed curve. The other possibility is that capacity is insufficient to handle the available demand. The revised cumulative

arrival curve—the solid black curve—is then different from the cumulative demand curve.
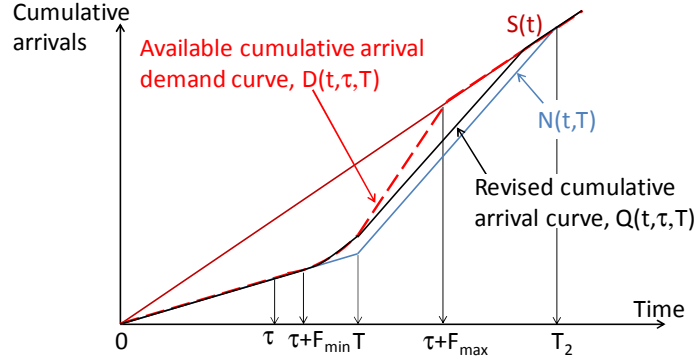

Figure 3.6: Conceptual Plot of Available Cumulative Arrival Demand Curve, Case C2

The logic used to generate the available demand curves for the three types of flights in Case C2 is in most respects the same as in Case C1. The studied period is still between 0 and T_2. The difference is that the flight time range for Type II flights is $[F_{min}, F_{max}]$ in this case. The available arrival demand curve is found to be

$$D(t, \tau, T) =$$

$$
\begin{cases}
C_L t, & 0 \le t \le \tau + F_{min} \\
\dfrac{\lambda - C_L}{\Delta F} \cdot t^2 - \dfrac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_L \cdot t, & \tau + F_{min} < t \le T \\
\dfrac{\lambda - C_H}{\Delta F} \cdot t^2 - \dfrac{\lambda - C_H}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_H \cdot t + \dfrac{C_H - C_L}{\Delta F} \cdot T \cdot t - \dfrac{C_H - C_L}{\Delta F} \cdot T \cdot (\tau + F_{max}), & T \le t \le \tau + F_{max} \\
\lambda t, & \tau + F_{max} \le t \le T_2
\end{cases}
$$

(Eq 3.28)

The first three sub-functions of the formulation are identical to the formulation for available demand in Case C1 given in Eq 3.26, with the exception of the sub-domain for the third sub-function. The fourth sub-function, which does not appear in Case C1, is when the available cumulative demand curve follows the original scheduled curve, $S(t)$.

3.5.1.3 Available Cumulative Arrival Demand, Case C3

In this case, a combination of an early τ and a small value for F_{max} enables cumulative available demand to join the scheduled demand curve prior to the planned capacity recovery time, as shown in Figure 3.7. The GDP revision should result in substantial delay saving. Since $\tau + F_{min}$ is earlier than T, there is no jump in the demand rate between $\tau + F_{min}$ and $\tau + F_{max}$ and thus the cumulative demand is quadratic in between. After this time, available arrival demand is the same as the scheduled arrival demand.
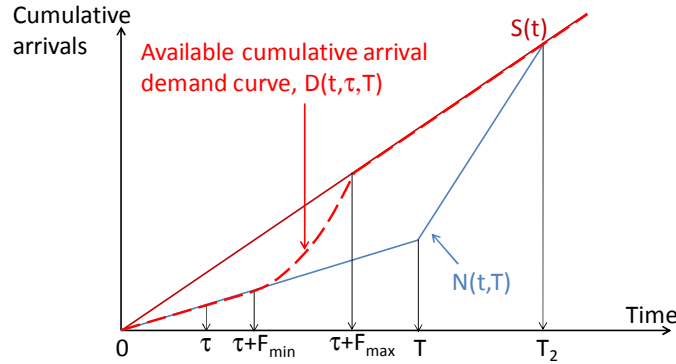
Figure 3.7: Conceptual Plot of Available Cumulative Arrival Demand Curve, Case C3

The formulation of available demand curve is simple compared to Cases C1 and C2. The major difference is that there is no jump in the available demand rate at time T, given that $\tau + F_{max} < T$. The expression is formulated as:

$$D(t,\tau,T) = \begin{cases} C_L t, & 0 \leq t < \tau + F_{min} \\ \dfrac{\lambda - C_L}{\Delta F} \cdot t^2 - \dfrac{\lambda - C_L}{\Delta F} \cdot (\tau + F_{min}) \cdot t + C_L \cdot t, & \tau + F_{min} \leq t < \tau + F_{max} \\ \lambda t, & \tau + F_{max} \leq t \leq T_2 \end{cases}$$

(Eq 3.29)

3.5.2 GDP Extension Model

If high capacity is not available at time T, there will be more aircraft approaching the airport than what the arrival capacity permits and GDP will be extended by giving priority to flights in the air and further holding other flights on the ground. The purpose of an extension, like the initial GDP, is to convert airborne delay to ground delay. For the extension model, we assume traffic managers revise the program at time $T - t_{R,L}$, where $t_{R,L}$ is the difference between the revision time $T - t_{R,L}$ and the initially planned capacity recovery time T. As mentioned, we are modeling the case of one GDP revision, and thus the actual capacity recovery time τ is known with certainty at the revision time. Further, we assume traffic managers revise the program when the planned capacity recovery time is approaching and limit $t_{R,L}$ to be between 0 and F_{min}. In the remainder of the paper, we are presenting the model by setting $t_{R,L}$ as 0. When $t_{R,L}$ is larger than 0, all the conclusions hold by replacing F_{min} (F_{max}) with $F_{min} - t_{R,L}$ ($F_{max} - t_{R,L}$). Similar to the early cancellation case, we distinguish three types of flights based on their status at time T:

- Type I: these flights have landed at time T. Type I flights have absorbed the assigned ground delays in the initial GDP, and cannot and need not be delayed further.

- Type II: these flights have taken off but not yet landed. They will experience unplanned airborne delay before landing in addition to the planned and incurred ground delay. When extending the GDP, priority is given to Type II flights in order to minimize airborne delay.
- Type III: these flights are still on the ground at time T. They either have departure time slots later than T or their scheduled arrival times are late enough that they are not involved in the initial GDP. Type III flights should be assigned additional ground delay so that they absorb as much as possible of the extra delay due to underestimating τ.

As was the case for the cancellation model, the key to constructing the extension model is updating the arrival demand. While in the cancellation case the aim is to accelerate demand, in this case the goal is to 'put on the brakes'. This is only possible for Type III flights since Type I and Type II flights have already taken off at time T. The arrival time of Type III flights can, however, be pushed back to a later time and their departure times should be postponed accordingly. In order to meter Type III departures precisely, we need to know how much capacity is available for landing Type III flights. This question is answered by comparing the planned cumulative arrival curve for Type I and Type II flights to the actual total cumulative arrival curve that can be achieved given full utilization of the available capacity. When all the flights that have taken off before T are landed, capacity can be devoted to landing Type III flights.

The GDP extension model and delay components in the case of extension are illustrated in Figure 3.8. $S(t)$ and $N(t, T)$ are the scheduled and initially planned cumulative arrival curves as defined before. The blue dotted curve in Figure 3.8a represents the planned cumulative arrival curve in the initial GDP for flights that have taken off by time T (Type I and Type II flights): $D_-(t, T)$. Before $T + F_{min}$, all the capacity is used for D_- flights in the plan and $D_-(t, T)$ overlaps with $N(t, T)$. After that, part of the capacity is also planned for flights taking off after T. Flights taking off before T last land at time $T + F_{max}$. As a result, $D_-(t, T)$ deviates from $N(t, T)$ at time $T + F_{min}$ and levels out at time $T + F_{max}$. At time T, we assume that we know τ with certainty. Therefore, the actual cumulative arrive curve is the ideal cumulative arrival curve $I(t, \tau)$—what would have been planned assuming perfect information at the GDP decision time. The time coordinate of the intersection of $D_-(t, T)$ and the actual cumulative arrival curve is denoted as t_c. Before t_c, all the available capacity should be devoted to land D_- flights to eliminate airborne delay. Since Type I flights have landed by time T, all the actual arrivals between T and t_c are Type II flights. The difference between the actual arrival time and the planned arrival time is unplanned airborne delay for these flights.
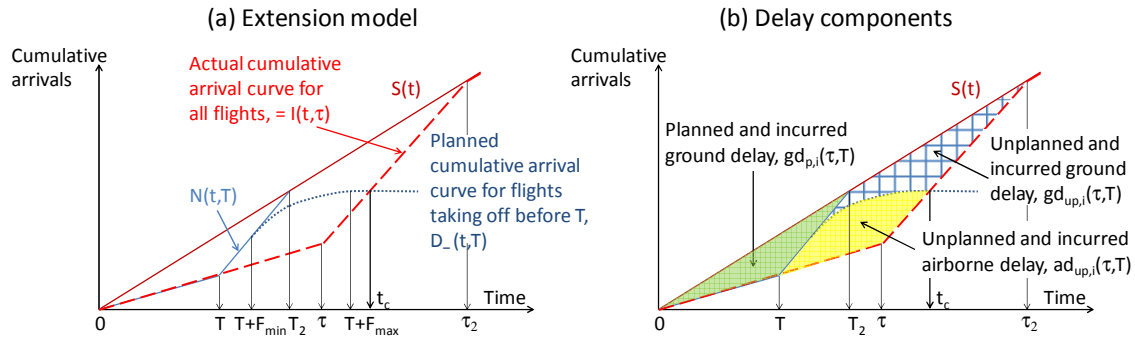
Figure 3.8: Conceptual Plot of GDP Extension Model and Delay Components

After t_c, all the available capacity is used to land Type III flights. In the initial plan, Type III flights that are arriving between $T + F_{min}$ and T_2 are delayed; Type III flights that are scheduled to arrive between T_2 and τ_2 are not involved in the original GDP and would have arrived at their scheduled times. As a result of extension, the former flights will be further delayed, while the latter flights will be delayed and take off at their assigned departure time slots in the updated program instead of their scheduled departure times.

There are three types of delay in the GDP extension model as shown in Figure 3.8b:

- Planned and incurred ground delay, $gd_{p,i}$. This delay is absorbed by Type I flights, Type II flights and Type III flights that are involved in the initial GDP.
- Unplanned and incurred airborne delay, $ad_{up,i}$. This delay is only absorbed by Type II flights.
- Unplanned and incurred ground delay, $gd_{up,i}$. This delay is only absorbed by Type III flights.

Comparing the delay components to those in Figure 3.2b, it is observed that total delay is the same with or without revision in the case of late capacity recovery. However, the delay cost with revision is smaller because part of the unplanned and incurred delay is transferred to the ground. To estimate the amount of the three types of delays, we only need to find the expression for $D_-(t,T)$ since the equations for the other curves are known. There are two expressions for $D_-(t,T)$ depending on which of the two times is earlier: T_2 or $T + F_{max}$. The principles for generating $D_-(t,T)$ are the same as for deriving the available cumulative arrival demand curve for Type I flights in the early cancellation model.

There are two expressions for $D_-(t,T)$ depending on which of the two times is earlier: T_2 or $T + F_{max}$. Figure 3.8 illustrates the case where T_2 is earlier than $T + F_{max}$. In this case:

- For planned arrival times before $T + F_{min}$, all planned capacity is utilized for D_- flights.
- For planned arrival times after $T + F_{min}$, the proportion of flow that is D_- flights at time t is:

$$P(D_-|t) = \frac{F_{max} - (t - T)}{F_{max} - F_{min}}$$

<div align="right">(Eq 3.30)</div>

Given the planned arrival rate is C_L before T, C_H between $T + F_{min}$ and T_2, and λ afterwards, the demand rate of D_- flights can be expressed as:

$$D'_-(t,T) = \begin{cases} C_L, & 0 \le t \le T \\ C_H, & T < t \le T + F_{min} \\ C_H \cdot \dfrac{F_{max} - (t - T)}{F_{max} - F_{min}}, & T + F_{min} < t \le T_2 \\ \lambda \cdot \dfrac{F_{max} - (t - T)}{F_{max} - F_{min}}, & T_2 < t \le T + F_{max} \\ 0, & t > T + F_{max} \end{cases}$$

<div align="right">(Eq 3.31)</div>

With these, we integrate and express the available cumulative arrival demand curve for D_- flights as:

$$D_-(t,T) = \begin{cases} C_L t, & 0 \le t \le T + F_{min} \\ C_L T + C_H(t - T), & T < t \le T + F_{min} \\ -\dfrac{C_H}{2 \cdot \Delta F}[t - (T + F_{max})]^2 + C_0, & T + F_{min} < t \le T_2 \\ -\dfrac{\lambda}{2 \cdot \Delta F}[t - (T + F_{max})]^2 + C_1, & T_2 < t \le T + F_{max} \\ C_1, & t > T + F_{max} \end{cases}$$

<div align="right">(Eq 3.32)</div>

where,

$$C_0 = C_H\left(F_{min} + \frac{\Delta F}{2}\right) + C_L T$$

and

$$C_1 = C_0 + \frac{\lambda - C_H}{2 \cdot \Delta F}[T_2 - (T + F_{max})]^2$$

When T_2 is later than $T + F_{max}$, $D_-(t,T)$ levels out before T_2 and a single quadratic equation governs the D_- flight arrival demand curve between $T + F_{min}$ and $T + F_{max}$:

$$D_-(t,T) = \begin{cases} C_L t, & 0 < t \leq T + F_{min} \\ C_L T + C_H(t - T), & T < t \leq T + F_{min} \\ -\dfrac{C_H}{2 \cdot \Delta F}[t - (T + F_{max})]^2 + C_0, & T + F_{min} < t \leq T + F_{max} \\ C_0, & T + F_{max} < t \end{cases}$$

(Eq 3.33)

Given the expressions of $D_-(t,T)$, we can then calculate the unplanned and incurred airborne delay:

$$ad_{up,i}(\tau,T) = \int\limits_{D_-(t,T)>I(t,\tau)} [D_-(t,T) - I(t,\tau)]dt, \qquad \tau > T$$

(Eq 3.34)

The planned and incurred delay, $gd_{p,i}$, is the same as d_P. The unplanned and incurred ground delay can then be calculated as:

$$gd_{up,i}(\tau,T) = \int\limits_{S(t)>I(t,\tau)} [S(t) - I(t,\tau)]dt - ad_{up,i}(\tau,T) - d_P(T), \qquad \tau > T$$

(Eq 3.35)

There are only two expressions for $D_-(t,T)$, but there are seven different expressions for the delays depending on where $D_-(t,T)$ intercepts the actual cumulative arrival curve, as shown in Figure 3.9. In other words, there are different extension cases depending on where t_c is. The conditions for each model are summarized in Table 3.2, where there are two categories of models depending on the GDP parameters—E1 and E2, and there are different models in each category depending also on the value of τ.
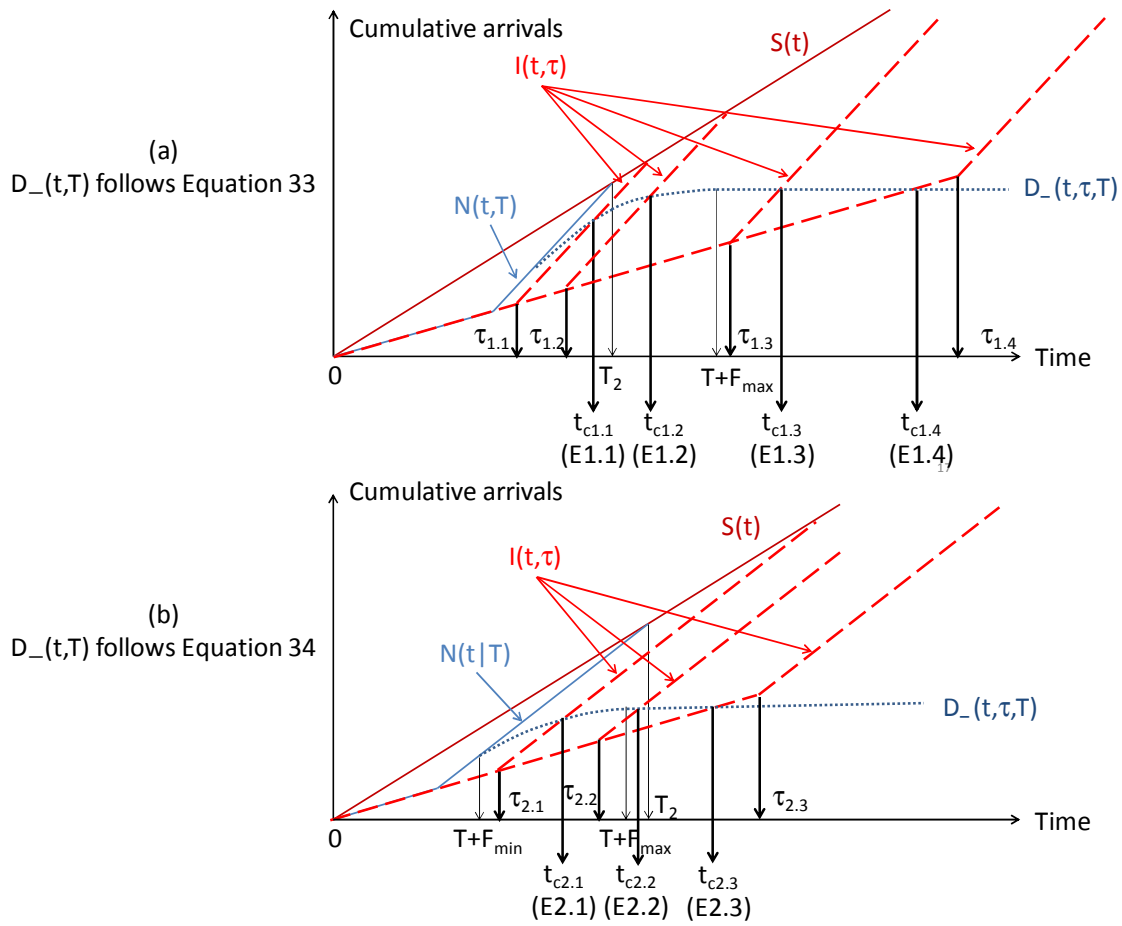
Figure 3.9: Different Cases for Extension Models

Table 3.2: Conditions for Different Cases for Extension Models

Case ID	Condition determined by GDP parameters	Condition depending on τ
E1.1	$T + F_{min} < T_2 \leq T + F_{max}$	$T + F_{min} < t_c \leq T_2$
E1.2	$T + F_{min} < T_2 \leq T + F_{max}$	$T_2 < t_c \leq T + F_{max}$
E1.3	$T + F_{min} < T_2 \leq T + F_{max}$	$T + F_{max} < t_c$ & $\tau < t_c$
E1.4	$T + F_{min} < T_2 \leq T + F_{max}$	$T + F_{max} < t_c \leq \tau$
E2.1	$T + F_{max} < T_2$	$T + F_{min} < t_c \leq T + F_{max}$
E2.2	$T + F_{max} < T_2$	$T + F_{max} < t_c$ & $\tau < t_c$
E2.3	$T + F_{max} < T_2$	$T + F_{max} < t_c \leq \tau$

3.6 Impact of GDP Scope

So far, we have assumed that all the flights heading to the affected airport with scheduled arrival times in the GDP time horizon are subject to ground delays. In practice, flights coming from longer distances are often exempt from the GDP. The scope of a GDP is defined as the region from which flights are not exempt—i.e., are subject to ground delays. In this analysis, the GDP scope is captured by the parameter F_{scope}, the maximum flight time of non-exempt flights. Flights with flight time between F_{scope} and F_{max} will be controlled by the GDP in the sense that a CTA will be assigned, but these flights are not delayed because the assigned CTA is close to its scheduled time of arrival so that no ground holding is required. The demand rate of the exempted flights, or called exemption ratio, is denoted by λ_e. By assuming a uniform distribution for flight time and that the GDP is planned at least F_{scope} ahead of the actual start of the GDP, we can obtain

$$\lambda_e = \frac{F_{max} - F_{scope}}{F_{max} - F_{min}} \cdot \lambda$$

(Eq 3.36)

Cumulative queueing diagrams of the scheduled and planned arrivals for GDP delayed flights are presented in Figure 3.10 for a non-exemption case and an exemption case with exemption rate λ_e. The non-exemption case is represented with dashed lines and the case with exempted flights is represented with solid lines. Compared to the non-exemption case, both the demand rate and the capacity rates in the exemption case are reduced by λ_e, which cannot exceed C_L. Denote the delay clearance time in the exemption case as $T_{2,e}$. It can be easily proved that $T_{2,e}$ is equal to T_2 and the total quantity of planned delays is the same in the two cases. This makes sense since there should be no delay for exempted flights. Therefore, when F_{scope} is smaller than F_{max}, the previous models are valid if we replace F_{max} with F_{scope}, λ with $\lambda - \lambda_e$, C_H with $C_H - \lambda_e$, and C_L with $C_L - \lambda_e$, where λ_e is calculated as in Eq 3.36. It should be noted that while the total assigned delay in the initial plan is the same regardless of scope, as F_{scope} decreases, this delay is concentrated among a smaller set of flights. The advantage of reduced scope is greater adaptability: demand can dialed up more quickly in the case of early cancellation and dialed down more quickly in the case of extension. Our model captures these effects.
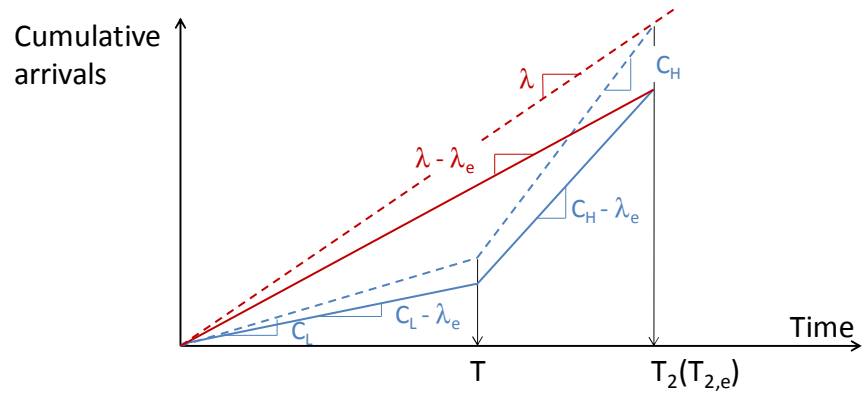
Figure 3.10: Impact of Scope on the Amount and Clearance Time of Planned Delays

4 GDP Cost Optimization

In the previous chapter, we defined different delay components in the cases of early and late capacity recovery, with and without considering GDP revision. Moreover, we showed how these delay components can be calculated from our GDP models. These lay the foundation for the discussion of GDP cost optimization.

As shown above, the delay components are functions of τ and T. τ is unknown when the decision on T must be made. Our objective is therefore to minimize the expected cost, based on the assumed distribution of τ. Our lone decision variable is T—the time at which the planned arrival rate will be increased from C_L to C_H.

GDP cost functions are defined as linear functions of the delay components weighted by their cost coefficients. Planned and incurred ground delay is set as the baseline delay and the cost coefficients for other delays are defined with respect to the cost coefficient of this baseline delay. In Section 4.1, the cost function based on no-revision GDP model is discussed and closed-form expressions are derived for the optimal T, denoted as T^*. In Section 4.2, the cost functions based on GDP revision model are presented and the algorithm used to find T^* is discussed. We have also investigated in the impact of the assumption of a constant demand rate on the optimization. We find that the assumption can be relaxed to a considerable degree without changing our results. Details on the constant demand rate assumption are provided in Section 4.3.

4.1 GDP Cost Optimization with No-revision Model

Flights are always assumed to take off at their controlled times of departure as planned in the initial GDP in the no-revision model. Two delays may occur in a GDP without considering revision: planned ground delay—$gd_{p,i}^N$, and unplanned airborne delay—$ad_{up,i}^N$. When T is selected, $gd_{p,i}^N$ is known with certainty since it is only a function of T and other known parameters, as shown in Section 3.3. $ad_{up,i}^N$ only happens in the case of late capacity recovery, and its value depends on τ. $ad_{up,i}^N$ is more expensive than $gd_{p,i}^N$ for two reasons. First, airborne delay is more expensive than ground delay due to higher operating costs and safety concerns (Mukherjee and Hansen, 2007). Second, this delay is not predictable. This leads to different cost coefficients for the two delays in the cost function. For the case of early capacity recovery, the GDP delay cost function is expressed as:

$$Cost_N(\tau, T) = C_0 \cdot gd_{p,i}^N(\tau, T) = C_0 \cdot \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)} \cdot T^2, \quad \tau \leq T$$

(Eq 4.1)

For the case of late capacity recovery, the function has two terms:

$$Cost_N(\tau, T) = C_0 \cdot gd_{p,i}^N(\tau, T) + (\beta + \Delta_{up,i}) \cdot C_0 \cdot ad_{up,i}^N(\tau, T)$$

$$= C_0 \cdot \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)} \cdot [T^2 + (\beta + \Delta_{up,i})(\tau^2 - T^2)], \qquad \tau > T$$

(Eq 4.2)

where, C_0 is the unit cost of planned and incurred ground delay; β is the cost ratio of airborne delay to ground delay; $\Delta_{up,i}$ is the additional unit cost of unplanned delay expressed as a fraction of the unit cost of planned ground delay. Assuming a uniform distribution in $[t_{min}, t_{max}]$ for τ, the expected delay cost can then be written as:

$$Z_N(T) = E[Cost_N] = \int_{t_{min}}^{t_{max}} Cost_N(\tau, T) \frac{1}{t_{max} - t_{min}} d\tau$$

$$= \frac{C}{\Delta t} \cdot [\frac{2}{3}(\beta + \Delta_{up,i})T^3 + (\Delta t - (\beta + \Delta_{up,i})t_{max})T^2 + (\beta + \Delta_{up,i})\frac{t_{max}^3}{3}]$$

(Eq 4.3)

where $C = C_0 \cdot \frac{(C_H - C_L) \cdot (\lambda - C_L)}{2 \cdot (C_H - \lambda)}$ and $\Delta t = t_{max} - t_{min}$. Given other GDP parameters such as planned AARs and scope, the objective function is only a function of T. Differentiating the objective function with respect to T, the optimal T^* is calculated as:

$$T^* = \frac{[t_{min} + (\beta + \Delta_{up,i} - 1) \cdot t_{max}]}{\beta + \Delta_{up,i}}$$

(Eq 4.4)

From Eq 4.4, we find that T^* is determined by the distribution of τ, as well as the cost coefficients β and $\Delta_{up,i}$, but does not depend on demand and capacity. The numerator in the equation is a weighted sum of the bounds of the distribution, where the weight on the upper bound is the additional unit cost of unplanned airborne delay compared to planned ground delay, expressed as a fraction of the latter. If this additional cost is zero, then the optimal solution is to plan for the weather to clear at the earliest possible time. As this cost increases, the solution, as expected, moves toward the latest possible clearance time, approaching it asymptotically as the additional cost approaches infinity.

4.2 GDP Cost Optimization with Revision Model

In the GDP revision model, flights may not take off at their controlled times of departure as planned in the initial GDP. In the case of early cancellation, some flights can take off earlier to take advantage of early recovered capacity. In the case of GDP extension, some flights may be further delayed on the ground in order to absorb unavoidable delay on the ground instead of in the air. As shown in Figure 3.3 and Figure 3.8b, four types of delay are possible in the revision model: planned and incurred ground delay, $gd_{p,i}$; planned and un-incurred ground delay, $gd_{p,ui}$; unplanned and incurred airborne delay,

$ad_{up,i}$; and unplanned and incurred ground delay $gd_{up,i}$. Given this, the GDP cost function with revision model is expressed as:

$$Cost(\tau, T) = C_0[gd_{p,i}(\tau, T) + \Delta_{p,ui} \cdot gd_{p,ui}(\tau, T) + (1 + \Delta_{up,i}) \cdot gd_{up,i}(\tau, T) + (\beta + \Delta_{up,i}) \cdot ad_{up,i}(\tau, T)]$$

(Eq 4.5)

where $\Delta_{p,ui}$ is the cost ratio of planned un-incurred delay to planned ground delay. When $\Delta_{p,ui}$ and $\Delta_{up,i}$ are set as 0, the cost functions are the same as those in prior research. Assuming a uniform distribution in $[t_{min}, t_{max}]$ for τ, the expected delay cost can then be written as:

$$Z(T) = E[Cost] = \frac{C_0}{\Delta t} \cdot \{\int_{t_{min}}^{t_{max}} gd_{p,i} d\tau + \int_{t_{min}}^{T} \Delta_{p,ui} \cdot gd_{p,ui} d\tau$$
$$+ \int_{T}^{t_{max}} [(1 + \Delta_{up,i}) \cdot gd_{up,i} + (\beta + \Delta_{up,i}) \cdot ad_{up,i}] d\tau\}$$

(Eq 4.6)

The optimal T^* is then defined as the planned capacity recovery time that minimizes the value of the objective function. The domain for T is the same as τ: $[t_{min}, t_{max}]$.

There are various methods for finding T^*. Analytical first order optimality conditions can be determined for identifying T^*. However, the conditions are complex because of the different cases (Table 3.1 and Table 3.2) and the fact that as we integrate over τ, the relevant case switches (Figure 4.1). In each plot of Figure 4.1, the delay cost—expressed in equivalent minutes of planned ground delay— is plotted as a function of τ for a given set of T and exemption ratio, where the values of the parameters are set the same as in the case study discussed in the next chapter (Table 5.1). In this case study, the range of τ is between 120 and 360 minutes. Therefore, there are only GDP extensions when T is 120 minutes and only GDP cancellations when T is 360 minutes. When T is between the two bounds, both extensions and cancellations are possible. The cost is a piece-wise function of τ and each piece is either constant or quadratic, with some of the quadratic pieces approximately linear.

Assuming uniform distribution for τ, the expected costs are equal to the areas below the cost curves divided by Δt. As seen in Figure 4.1, the controlling case, and hence the appropriate expression for the cost function, changes as we vary τ. The optimal decision T^* can be found by differentiating the expected cost function, but the piece-wise integration and differentiation lead to complexity in the analytical solutions. Moreover, the first order conditions, when they can be identified, are cubic equations whose solution is not particularly enlightening. Thus for this analysis, we optimize T numerically by means of a Golden Section Search (GSS), which does not require the calculation of the first derivatives but is nonetheless computationally efficient. The GSS

method requires the objective function to be strictly unimodal (Press et al, 1986), which we also established numerically by plotting the function for a wide range of cases.
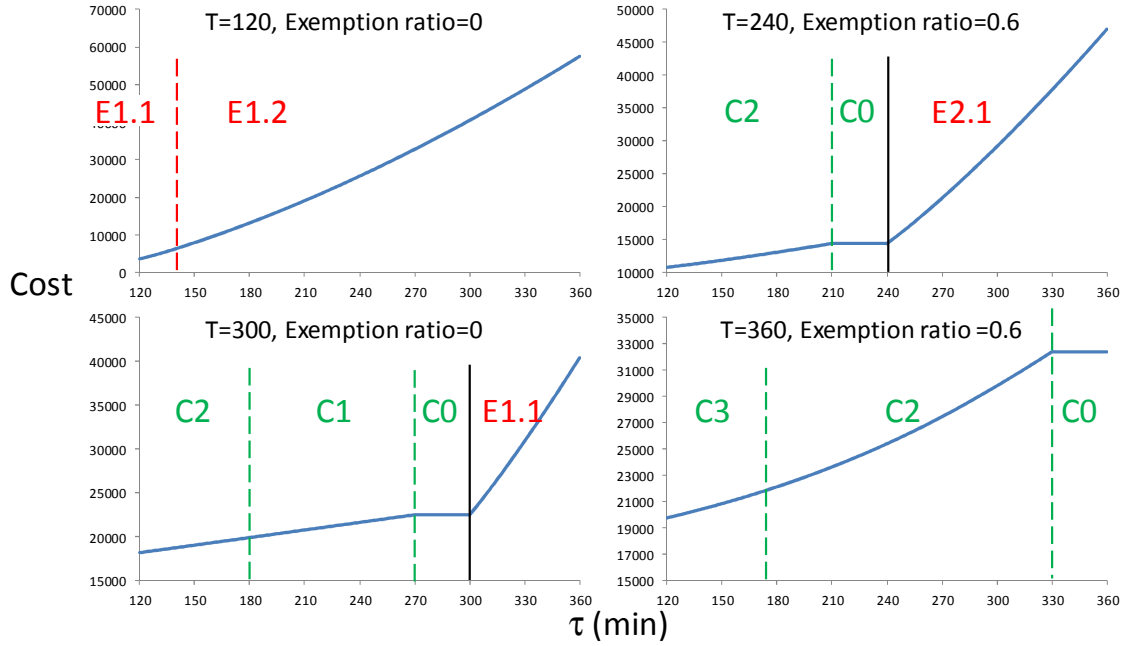

Figure 4.1: Delay Cost (in units of planned ground delay minutes) as a Function of τ, $\Delta_{up,i} = \Delta_{p,ui} = 0.5$

4.3 Impact of Constant Demand Rate Assumption

In this section, we will discuss the impact of constant demand rate assumption on our models. So far, we have assumed a constant rate for scheduled arrival demand. This is actually an unnecessary assumption for the no-revision model. Based on Equations Eq 4.1 and Eq 4.2, the objective function could be expressed as:

$$E[Cost_N] = \frac{1}{\Delta t} \int_{t_{min}}^{T} C_0 \cdot gd_{p,i}^N(\tau, T) \, d\tau + \frac{1}{\Delta t} \int_{T}^{t_{max}} C_0 \cdot gd_{p,i}^N(\tau, T) + (\beta + \Delta_{up,i}) \cdot C_0 \cdot ad_{up,i}^N(\tau, T) \, d\tau$$

$$= \frac{1}{\Delta t} \int_{t_{min}}^{t_{max}} C_0 \cdot gd_{p,i}^N(\tau, T) \, d\tau + \frac{1}{\Delta t} \int_{T}^{t_{max}} (\beta + \Delta_{up,i}) \cdot C_0 \cdot ad_{up,i}^N(\tau, T) \, d\tau$$

(Eq 4.7)

As shown in Section 3.4, planned and incurred ground delay in the no-revision model—$gd_{p,i}^N(\tau, T)$—is equal to the initially planned ground delay—$d_P(T)$; unplanned airborne delay—$ad_{up,i}^N(\tau, T)$, only applies when $\tau > T$—is equal to realized delay minus planned ground delay. The amount of realized delay in this case (late clearance case) is only a

function of τ. Here, we denote it as $d_R(\tau)$. With these, the objective function can be further written as:

$$E[Cost_N] = \frac{1}{\Delta t} \int_{t_{min}}^{t_{max}} C_0 \cdot d_P(T) \, d\tau + \frac{1}{\Delta t} \int_T^{t_{max}} (\beta + \Delta_{up,i}) \cdot C_0 \cdot (d_R(\tau) - d_P(T)) \, d\tau$$

$$= C_0 \cdot d_P(T) + \frac{(\beta + \Delta_{up,i}) \cdot C_0}{\Delta t} \int_T^{t_{max}} d_R(\tau) \, d\tau - \frac{(\beta + \Delta_{up,i}) \cdot C_0}{\Delta t} d_P(T)(t_{max} - T)$$

(Eq 4.8)

Take the first derivative with respect to T, we have

$$\frac{\partial E[Cost_N]}{\partial T} = C_0 \cdot \frac{\partial d_P(T)}{\partial T} + \frac{(\beta + \Delta_{up,i}) \cdot C_0}{\Delta t} [-d_R(T)] - \frac{(\beta + \Delta_{up,i}) \cdot C_0}{\Delta t} \cdot \frac{\partial d_P(T)}{\partial T} \cdot (t_{max} - T) - \frac{(\beta + \Delta_{up,i}) \cdot C_0}{\Delta t} \\ \cdot d_P(T) \cdot (-1)$$

$$= C_0 \cdot \frac{\partial d_P(T)}{\partial T} (1 - \frac{(\beta + \Delta_{up,i}) \cdot (t_{max} - T)}{\Delta t})$$

(Eq 4.9)

Set the derivative as 0. We obtain the optimal decision on T:

$$T^* = \frac{[t_{min} + (\beta + \Delta_{up,i} - 1) \cdot t_{max}]}{\beta + \Delta_{up,i}}$$

(Eq 4.10)

which is the same as in Eq 4.4 where a constant demand rate is assumed. Therefore, the optimal decision in the no-revision model is independent of the scheduled arrival demand function. It should though be mentioned that this applies only if $d_P(T)$ is larger than 0 for $t_{min} \leq T \leq t_{max}$.

The constant demand rate constraint can be partially relaxed in the revision models. Here, we assume delay starts to develop in the system from time 0 and does not vanish until time T_2 in the initial plan. In the GDP cancellation Models, analytical expressions are derived for available arrival demand after revision ($D(t, \tau, T)$) upon an early clearance for four different cases (Table 3.1). To derive the expressions, we group flights into three types. The expression of available demand from Type I flights is independent of the scheduled demand function ($S(t)$) but depends on the initially planned cumulative arrival curve ($N(t, T)$) and flight time distribution. The available demand expressions of Type II and Type III flights depend on the scheduled demand function. It can be proved that these expressions will remain the same if cumulative arrival demand is $\lambda \cdot (\tau + F_{min})$ at time $\tau + F_{min}$ and the demand rate is a constant λ after $\tau + F_{min}$, as illustrated in the plot (a) of Figure 4.2. For the extension models, the key is to find the expression for the cumulative arrival demand curve for flights that have already taken off before T ($D_-(t, T)$), which allows us to determine how much unexpected delay is

incurred on the ground. It can be proved that the expression for $D_-(t, T)$ remains the same if cumulative arrival demand is $\lambda \cdot T_2$ at time T_2 and the demand rate is a constant λ afterwards, as illustrated in the plot (b) of Figure 4.2. Overall, the analytical solutions we have for revision models will be valid if cumulative arrival demand is $\lambda \cdot (\tau_{min} + F_{min})$ at time $\tau_{min} + F_{min}$ and the demand rate is a constant rate λ afterwards.

If the scheduled demand rate is variable outside the ranges described above, analytical expressions may not be derivable for $D(t, \tau, T)$ in the early clearance case or $D_-(t, T)$ in the late clearance case. In this case, the problem can be analyzed numerically. The numerical approach would still be based on the same categorization of flights that we have employed in the analysis presented above.
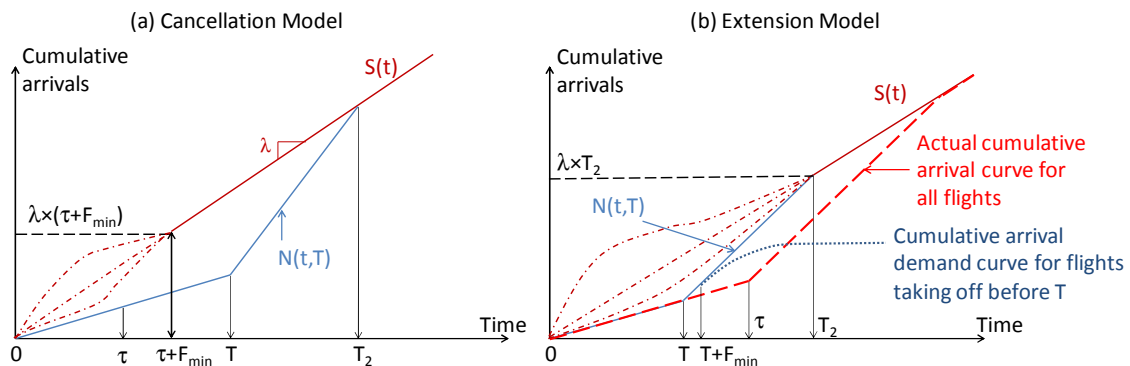

Figure 4.2: Constant Demand Rate Constraint Relaxation, Revision Models

5 Case Study

In this chapter, through a case study based loosely on SFO airport, we will show the influences of unpredictability premiums on the optimal GDP decision—T^*. Sensitivity of the optimal decision to the unpredictability coefficients is performed by varying the values of these coefficients and observing the resulting changes in T^*. In addition, we also investigate the impact of scope on T^*. Finally, we assess the cost savings from properly considering unpredictability premiums when determining T^*.

The set of parameter values in the case study is shown in Table 5.1. The AARs are chosen referring to the airport capacity benchmark report by the FAA (FAA, 2004). International flights are usually exempted in GDPs so the maximum flight time is set as 7 hours. The cost ratio of airborne delay to ground delay is set to 2 (Mukherjee and Hansen, 2007). The range of τ is different from day to day depending on the weather forecast. Here, we set the lower bound of this clearance time as 2 hours and the upper bound as 6 hours. Unpredictability premiums are not treated as known parameters. Instead, we perform sensitivity analysis to observe how these premiums affect optimal decisions and costs. We will first study the influences of the unpredictability costs on the optimal decisions by varying $\Delta_{up,i}$, assuming $\Delta_{p,ui} = 0$, and then varying $\Delta_{p,ui}$, assuming $\Delta_{up,i} = 0$. Then we look at the joint influence of both unpredictability cost parameters. T^* is generated from GSS for the revision model. For the no-revision model, T^* is calculated analytically using Eq 4.4.

Table 5.1: Parameters used in the Case Study

Notation	Description	Value	Unit
λ	Scheduled arrival demand rate	45	Arrivals per hour
C_H	High airport acceptance rate	60	Arrivals per hour
C_L	Low airport acceptance rate	30	Arrivals per hour
F_{min}	Minimum flight time	30	Minutes
F_{max}	Maximum flight time	420	Minutes
t_{min}	Lower bound for τ and T	120	Minutes
t_{max}	Upper bound for τ and T	360	Minutes
β	Cost ratio of airborne delay to ground delay	2	-

Figure 5.1 shows how the optimal decisions change with increasing unpredictability premium for unplanned and incurred delays—$\Delta_{up,i}$. In this figure, the unpredictability premium of planned un-incurred delay, $\Delta_{p,ui}$, is set as 0. The unplanned and incurred delay is always airborne delay for the no-revision model, but consists of airborne delay

and ground delay for the revision model. The dashed curve is for the no-revision model, and the solid curves are for the revision model with varied exemption ratios reflecting different GDP scopes, which matter only when there is revision. First, it is observed that T^* increases with $\Delta_{up,i}$. Unplanned delays are incurred in the case of late capacity recovery. A larger T reduces the chance of late clearance and thus decreases the probability of unplanned delays. Second, T^* is more sensitive to the premium for unplanned delays under the revision model. When $\Delta_{up,i}$ increases from 0 to 2, T^* increases by 60 minutes in the no-revision model and by 100 minutes in the revision model. In the case of early capacity recovery, some planned delay can be recovered in the revision model, whereas in the no-revision model, all the planned delay is incurred. The cost saving from early cancellation makes T^* more sensitive to $\Delta_{up,i}$. Third, we notice that for a given $\Delta_{up,i}$, T^* increases with the exemption ratio in the revision model. With a smaller GDP scope, GDP delayed flights are concentrated in the vicinity of the GDP airport and thus can arrive at the airport earlier in the case of early cancellation, enabling more efficient utilization of the unexpected extra capacity. With no penalty for early cancellation, T^* is increased to reduce the chance of GDP extension. Fourthly, T^* is usually, but not always, larger in the revision model. When $\Delta_{up,i}$ is small, the ability to revise encourages a more aggressive GDP, since the program can be extended at less cost. Lastly, we notice that values of T^* from the revision model are close to or below that from the no-revision model—240 minutes—when both unpredictability premiums are set as 0. When both premiums are set as 0, planned and un-incurred delay has no cost. Furthermore, unplanned ground delay costs the same as planned ground delay and unplanned airborne delay is more costly only because it is incurred in the air. As a result, a revision reduces the cost by saving delay in the early clearance case and transforming airborne delay to ground delay in the late clearance case. In the absence of predictability effects, the revision model yields a more optimistic planned clearance time ($\leq$240 minutes) because it is easier to transform airborne delay to the ground in the late clearance case than to recover delay in the early clearance case.
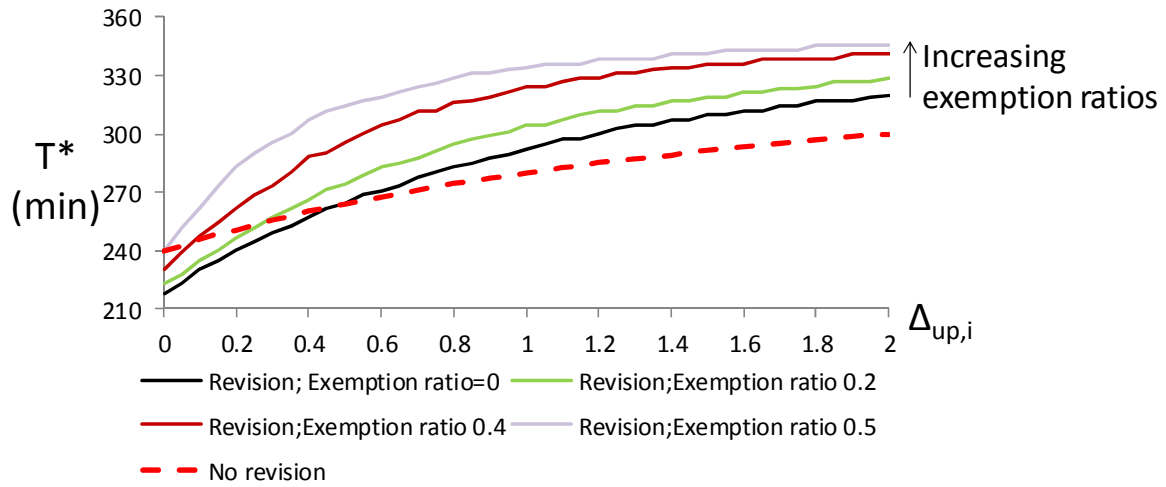
Figure 5.1: Sensitivity Analysis of $\Delta_{up,i}$ on T^*, No-revision Model and Revision Model with Different Exemption Ratios

Figure 5.2 shows how the optimal decisions change with increasing unpredictability premium for planned but un-incurred delays—$\Delta_{p,ui}$. In this figure, $\Delta_{up,i}= 0$. There is no planned delay that is not incurred in the no-revision model. Therefore, the value of T^* in the no-revision model is independent of $\Delta_{p,ui}$. For the revision model, we first observe that T^* decreases with $\Delta_{p,ui}$. This is because planned un-incurred delay happens when there is early cancellation. Thus, when the cost associated with this delay increases, the value of the recovered delay from early cancellation decreases. As a result, T^* decreases. Second, we see that when $\Delta_{p,ui}$ is low, T^* increases with exemption ratio. This is because a high exemption ratio enables more planned delay to be avoided when there is an early cancellation, which leads to substantial cost saving when $\Delta_{p,ui}$ is low. In contrast, when $\Delta_{p,ui}$ is high, the relationship flips—T^* decreases with exemption ratio. In this situation, the primary benefit of a limited scope is to reduce airborne delay in the case of a GDP extension. With a smaller scope, fewer non-exempt flights are in the air at the time of extension, and thus GDP extension is more effective in shifting airborne delay to ground delay. For this reason, a smaller scope results in an earlier T^*. Third, we see that T^* is more sensitive to $\Delta_{p,ui}$ when the exemption ratio is larger. A higher exemption ratio results in more recoverable delay when there is early cancellation. Thus increasing the value of recoverable delay—i.e. reducing $\Delta_{p,ui}$—has a more pronounced effect on T^* when the exemption ratio is high.
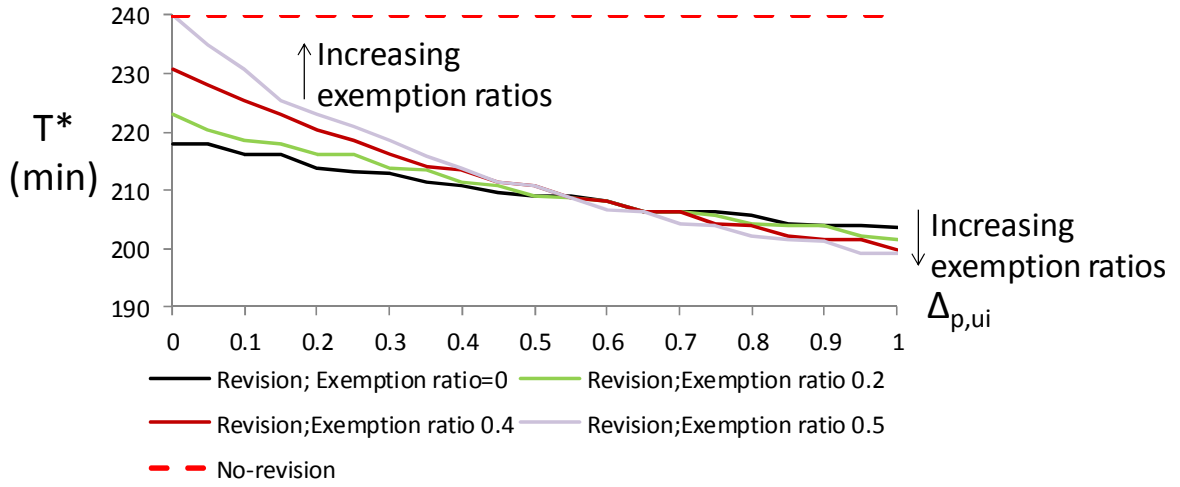
Figure 5.2: Sensitivity Analysis of $\Delta_{p,ui}$ on T^*, No-revision Model and Revision Model with Different Exemption Ratios

Figure 5.3 shows the joint influence of the two unpredictability effects on T^*. For Figure 5.3, we set the exemption ratio as 0.45, which is the average exemption ratio at SFO (Liu and Hansen, 2014). As before, we see that T^* increases with the unpredictability premium on planned un-incurred delay—$\Delta_{up,i}$—and decreases as planned un-incurred delay becomes more costly. There is also a clear interaction effect whereby $\frac{d^2 T^*}{d\Delta_{up,i} \cdot d\Delta_{p,ui}} < 0$. In other words, a high cost of planned un-incurred delay weakens the positive relationship between T^* and the cost of unplanned delay, while a high cost of unplanned delay strengthens the negative relationship between T^* and the cost of planned un-incurred delay. The values of T^* for the two most extreme cases—1-unit premium on unplanned delay and no premium on planned un-incurred delay, and no premium on unplanned delay and 1-unit premium on planned un-incurred delay—ranges from the 87th percentile of τ to the 33rd percentile. If, on the other hand, both premiums are assumed to have the same value, then T^* increases from the 47th to the 58th percentile of τ as that premium goes from 0 to 1. This observation points to a consistent pattern in our results: T^* is more sensitive to the premium on un-planned delay than that on planned un-incurred delay. This is not surprising because in most cases the magnitude of planned un-incurred delay when there is early cancellation is much smaller than that of unplanned delay when there is extension. Thus the unit cost of planned un-incurred delay has less influence on the objective function.

Finally, in Figure 5.4, we show the impact of unpredictability premiums on the optimal costs. On the vertical axis, Z^* is the optimal cost considering predictability, calculated using Equation 35 by setting T as T^*. Z_{np} is the cost considering predictability, but assuming that T is optimized ignoring predictability. The difference between these two costs is then the cost that can be saved by considering predictability in the objective

function. Figure 5.4 shows that, depending on the values of the unpredictability premiums, up to 13% of cost may be saved. Similar to the results shown in Figure 5.3, here the cost saving is also more sensitive to the premium for unplanned and incurred delay.
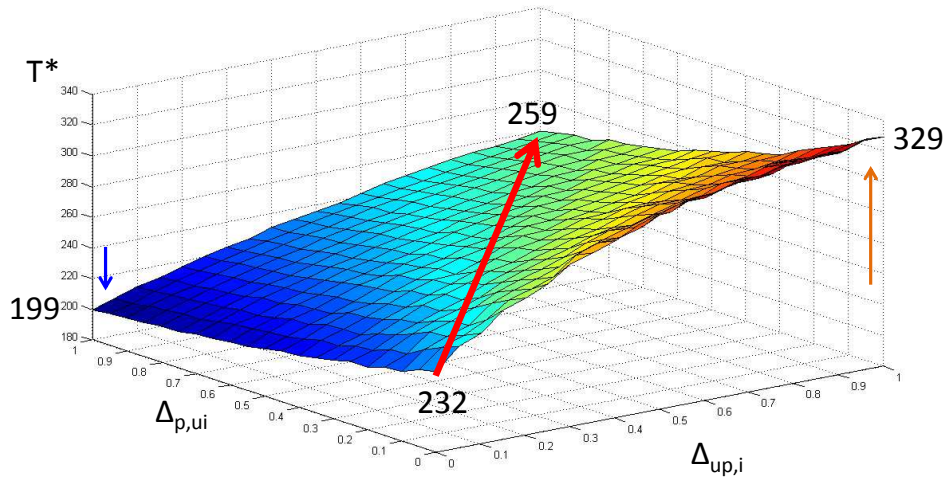

Figure 5.3: Sensitivity Analysis of $\Delta_{up,i}$ and $\Delta_{p,ui}$ on T^*, Revision Model with Exemption Ratio 0.45
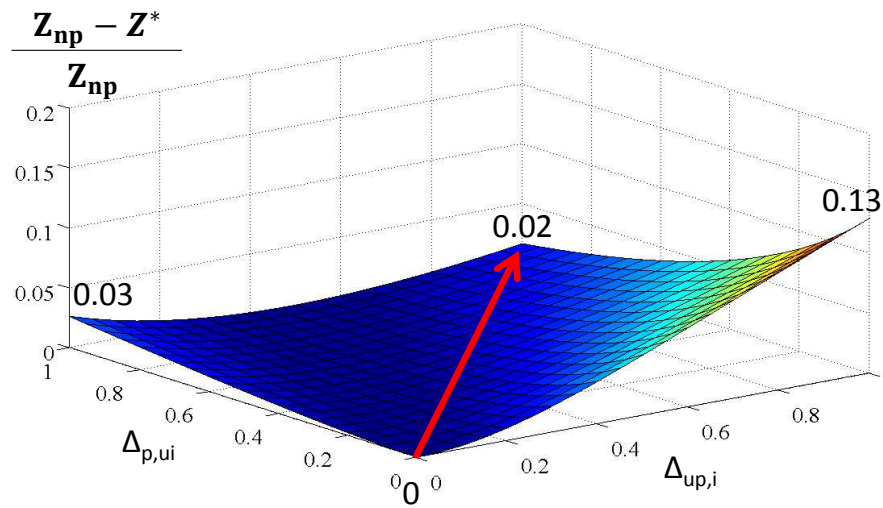

Figure 5.4: Sensitivity Analysis of $\Delta_{up,i}$ and $\Delta_{p,ui}$ on Optimal Costs, Revision Model with Exemption Ratio 0.45

6 Conclusion

6.1 Summary

In this dissertation, we investigate predictability in strategic air traffic management with a focus on GDP. We start the discussion by revealing the importance of predictability through a survey of flight operators. According to the responses, flight operators appreciate predictability, less than capacity utilization but more than efficiency. Using the survey, we have also collected flight operators' feedback on ATFM strategies in general and GDP decision setting.

We then present the new models that incorporate predictability into the GDP cost optimization problem under capacity uncertainty for a single airport case. This is accomplished by modifying traditional GDP delay cost functions so that they incorporate predictability, and determining the sensitivities of the optimal planned capacity recovery time and associated cost to the unpredictability premiums included in the cost functions. To do this, we develop two stochastic GDP models: a GDP no-revision, or static, model; and a GDP revision, or dynamic, model considering one GDP revision. GDP scope, which matters only in the revision model, is also considered.

The optimization results from the case study show that unpredictability clearly matters, particularly in the more realistic case where revision is allowed. Of the two unpredictability cost parameters, the one for unplanned delay has a stronger impact than the one for planned un-incurred delay. In general, a smaller GDP scope leads to a larger optimal planned capacity recovery time—T^*. Depending on the values of unpredictability premiums, considerable cost may be saved if the decision takes predictability into account.

The insights from this analysis might eventually be used to develop a decision support tool that traffic managers could use in determining what the planned end time for a GDP should be in a manner that reflects the importance of predictability to flight operators. Despite the idealizations used in our model, it could be adopted to a real-world setting fairly easily, with a front end that would convert conditions in a given situation into the model input parameters. Alternatively, a more general model based on the details of the flight schedule and distribution of weather clearance time might be employed. This would dispense with the analytic formulas in favor of a purely numeric algorithm informed by the same basic concepts.

More immediately, the model results can guide traffic managers about when to be optimistic, pessimistic, or neither in their assumptions about clearance times. For example, Figure 5.1 demonstrates that when there is no cost for planned, un-incurred delay, and with a high exemption ratio (small GDP scope), a planned clearance time on the late side—an hour or so after the expected value of the actual clearance time—is

appropriate. For most plausible values on the unpredictability premiums, some degree of pessimism about when the weather will improve is called for, particularly when there is a large fraction of exempt flights. Interestingly, data for SFO and EWR (Liu and Hansen, 2014) reveal that GDPs end earlier than the initial plan about 80% of the time, suggesting that air traffic managers may already know that the cost function includes a premium on unplanned delay.

More broadly, our analysis highlights the interplay of scope and unpredictability premiums in determining the appropriate risk posture to take in planning GDPs. With a high unpredictability premium for unplanned delay, the analysis suggests a more conservative plan—i.e. a later T^*—when the GDP scope is smaller. This is because unexpected extra capacity can be utilized more efficiently in the case of early clearance with a smaller scope, and thus a conservative decision should be made to reduce the chance of GDP extension. Conversely, the scope matters relatively little in the case where planned, un-incurred delay has a high cost relative to unplanned delay.

6.2 Model Limitations and Extensions

A potentially restrictive assumption in our models is that all the flights within the GDP scope can be delayed as needed. This allows the assumption of a uniform distribution of flight time for GDP affected flights. If some flights within the scope are in the air at the GDP decision time, then these flights needed to be exempted from the GDP too. In other words, two exemption rules will be applied: exemption by scope and exemption by departure status. This alters the distribution of flight time for non-exempt flights. Our current models are valid only when the maximum flight time of the GDP delayed flights is smaller than GDP lead time—GDP start time minus GDP decision time. Research is being conducted to overcome this limitation.

Another desirable extension of the models is to consider GDP decision time as a variable and explore its impact on the cost function. In the current models, we consider GDP decision time as fixed. Modeling this time as a variable will enable us to explore the tradeoffs between lead time and accuracy of information. An early decision gives a long lead time which may facilitate the mitigation actions, but probably involves large uncertainty and a greater chance of major revisions. On the contrary, a more timely decision can provide more accurate information but this will be at the expense of a shorter time window to adapt to the decision. Considering this tradeoff between timeliness and accuracy will add one more dimension to tradeoffs involving predictability.

Finally, the models can be extended to consider GDP revision time as a decision variable, particularly in the case of extension. In the extension case, the traffic managers may revise the program before the planned capacity recovery time to avoid airborne delay. It is then most likely that the actual recovery time is unknown at the GDP extension time.

A possible refinement of the current models is to update the capacity recovery time in the case of extension as the upper bound of the possible capacity recovery times. If so, the GDP will be finally cancelled earlier, as is commonly observed in practice. Enhancing the model to include the succession of a GDP extension and a subsequent early cancellation would enhance its practical value, particularly if it can also guide managers' decisions about when to extend the program.

6.3 Predictability in Strategic Air Traffic Flow Management

Through a survey of flight operators, this dissertation confirms the importance of predictability in evaluating GDP performance. Using developed models, the work further reveals that GDP optimal decisions with predictability considered in the cost function can be very different from the sub-optimal ones without considering predictability. These urge the need to consider predictability in the decision-making process of GDPs.

One may wonder whether similar stories will be told for other strategic ATFM tools, such as AFP and TMA. While further research is required to find the answer, this dissertation helps pave the way toward the broader objective of incorporating predictability in strategic ATFM.

Appendix: Summary of Notations in the GDP models

Notation	Description
t:	time; GDP start time is defined as the origin
λ:	scheduled arrival demand rate
$C_H(C_L)$:	planned high (low) airport acceptance rate (AAR)
T:	planned capacity recovery time
T_2:	planned delay clearance time
τ:	actual capacity recovery time
τ_2:	ideal delay clearance time if perfect information were available at the decision time
$F_{max}(F_{min})$:	upper (lower) bound of the flight time
λ_e:	exempted demand rate
F_{Scope}:	GDP scope, maximum flight time of the GDP delayed flights
$t_{max}(t_{min})$:	upper (lower) bound of the capacity recovery time
$S(t)$:	scheduled cumulative arrival curve, providing scheduled arrival time
$N(t,T)$:	planned cumulative arrival curve in the initial GDP, basis for assigning the initial Controlled Time of Arrival (CTA)
$I(t,\tau)$:	ideal cumulative arrival curve assuming τ were known at the decision time
$D(t,\tau,T)(D')$:	revised cumulative arrival demand (demand rate) for all flights when GDP is cancelled earlier and infinite capacity is assumed
$D_I(t,\tau,T)(D'_I)$:	revised cumulative arrival demand (demand rate) for Type I flights when GDP is cancelled earlier and infinite capacity is assumed; $D_{II}(D'_{II})$ and $D_{III}(D'_{III})$ are defined similarly
$C(t,\tau)$:	available capacity rate function
$Q(t,\tau,T)(Q')$:	revised cumulative arrival curve (arrival rate), basis for assigned the revised time slots
$D_-(t,\tau,T)(D'_-)$:	planned cumulative arrival curve in the initial GDP for flights that have taken off by time T, GDP extension case
$d_p(T)$:	planned ground delay in the initial GDP
$d_R(\tau,T)$:	realized delay at the end of the GDP
$gd_{p,i}^N(\tau,T)$:	planned and incurred ground delay, no-revision model
$ad_{up,i}^N(\tau,T)$:	unplanned and incurred airborne delay, no-revision model

$gd_{p,i}(\tau, T)$:   planned and incurred ground delay, revision model

$gd_{p,ui}(\tau, T)$:   planned un-incurred ground delay, revision model

$gd_{up,i}(\tau, T)$:   unplanned and incurred ground delay, revision model

$ad_{up,i}(\tau, T)$:   unplanned and incurred airborne delay, revision model

$Cost(\tau, T)$ $(Cost_N)$:   delay cost function, revision models (no-revision model)

$Z(T)(Z_N(T))$:   objective function, revision models (no-revision model): expected cost

C_0:   unit cost of planned and incurred ground delay

β: cost ratio of airborne delay to ground delay

$\Delta_{up,i}$:   additional cost of unplanned delay expressed as a fraction of the unit cost of planned ground delay

$\Delta_{p,ui}$:   cost ratio of planned un-incurred delay to planned ground delay

T^*:   optimal decision on planned capacity recovery time, revision model

Bibliography

Aponso B., Coppenbarger R., Jung Y., O'Connor N., Lohr G., Quon L., Engelland S. (2015) Identifying Key Issues and Potential Solutions for Integrated Arrival, Departure, Surface Operations by Surveying Stakeholder Preferences. 15th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference, Dallas, Texas, 22 June – 26 June 2015.

Ball MO, Chen CY, Hoffman R., Vossen T. (2001) "Collaborative decision making in air traffic management: current and future research directions," *New Concepts and Methods in Air Traffic Management*, Bianco, L., Dell'Olmo, P., and Odoni, A.R., Editors, Transportation Analysis, Springer Verlag, Berlin, 17 – 30.

Ball MO, Hoffman R., Odoni A., Rifkin R. (2003) A stochastic integer program with dual network structure and its application to the ground-holding problem. Operations Research 51: 167 – 171.

Ball MO, Hoffman R., Mukherjee A. (2010) Ground delay program planning under uncertainty based on the ration-by-distance principle, Transportation Science 44(1): 1 – 14.

Bradford S., Knorr D. Liang D. (2000) Performance measures for future architecture. 3rd USA/Europe Air Traffic Management Research and Development Seminar, Napoli, Italy, 13 June – 16 June 2000.

Cook LS, Wood B. (2009) A model for determining ground delay program parameters using a probabilistic forecast of stratus clearing. 8th USA/Europe Air Traffic Management Research and Development Seminar, Napa, 29 June – 2 July 2009.

Daganzo CF (1997) Fundamentals of Transportation and Traffic Operations, Elsevier Science, Inc., New York.

Daganzo CF, Garcia RC (2000) A pareto improving strategy for the time-dependent morning commute problem. Transportation Science 34(3): 303 – 311.

Federal Aviation Administration (FAA) (2004) Airport Capacity Benchmarking Report 2004. Washington, D.C..

Federal Aviation Administration (FAA) (2009) Traffic Flow Management in the National Airspace System.

Federal Aviation Administration (FAA) (2011) Report: Destination 2025.

Federal Aviation Administration (FAA) (2014) Advisory Circular on Collaborative Trajectory Options Program.

Federal Aviation Administration (FAA), European Commission, EUROCONTROL (2014) Comparison of Air Traffic Management-Related Operational Performance: U.S./Europe.

Greene, W.J., Hensher D.A. (2010) Modeling Ordered Choices: A Primer. Cambridge University Press, Cambridge.

Hansen M. (2002) Micro-level analysis of airport delay externalities using deterministic queueing models: a case study. Journal of Air Transport Management 8: 73 – 87.

Hendrickson C., Kocur G. (1981) Scheduled delay and departure time in a deterministic Model. Transportation Science 15(3): 62 – 77.

Hoffman R., Ball MO, Mukherjee A. (2007) Ration-by-distance with equity guarantees: A new approach to ground delay program planning and control. 7th USA/Europe Air Traffic Management Research and Development Seminar, Barcelona, Spain, 2 – 5 July 2007.

International Civil Aviation Organization (ICAO) (2005) Report: Global Air Traffic Management Operational Concept, 1st Edition.

Kim A., Hansen M. (2013) Deconstructing delay: a non-parametric approach to analyzing delay changes in single server queuing systems. Transportation Research Part B 58: 119 – 133.

Kotnyek B., Richetta O. (2006) Equitable models for the stochastic ground-holding problem under collaborative decision making. Transportation science 40: 133 – 146.

Lago A., Daganzo CF (2007) Spillovers, merging traffic and the morning commute. Transportation Research Part B 41: 670 – 683.

Liu P., Hansen M. (2007) Scenario-free sequential decision model for the single airport ground holding problem. 7th USA/Europe Air Traffic Management Research and Development Seminar, Barcelona, Spain, 2 – 5 July 2007.

Liu Y., Hansen M. (2015) Incorporating predictability into cost optimization for ground delay programs. Transportation Science, published online in Articles in Advance, http://dx.doi.org/10.1287/trsc.2015.0594.

Liu Y., Hansen M. (2013) Ground delay program decision-making using multiple criteria: a single airport case. 10th USA/Europe Air Traffic Management Research and Development Seminar, Chicago, Illinois, 10 – 13 June 2013.

Liu Y., Hansen M., Zou B. (2013) Aircraft gauge differences between the US and Europe and their operational impacts. Journal of Air Transport Management 29: 1 – 10.

Liu Y., Hansen M. (2014) Evaluation of the performance of Ground Delay Program. Transportation Research Record: Journal of the Transportation Research Board 2400: 54 – 64.

Mukherjee A., Hansen M. (2007) A dynamic stochastic model for a single airport ground-holding problem. Transportation Science 41: 444 – 456.

Mukherjee A., Hansen M., Grabbe S. (2012) Ground delay program planning under uncertainty in airport capacity. Transportation Planning and Technology 35(6): 611 – 628.

Newell GF (1979) Airport capacity and delays. Transportation Science 13(3): 201 – 241.

Newell GF (1982) Applications of Queuing Theory, 2nd Edition, Chapman Hall, London.

Newell GF (1987) The morning commute for non-identical travelers. Transportation Science 21(2): 74 – 88.

Press WH, Flannery BP, Teukolsky SA, Vetterling WT (1986) Numerical Recipes: the Art of Scientific Computing. 1st Edition, Cambridge University Press, Cambridge.

Richetta O., Odoni AR (1993) Solving optimally the static ground holding policy problem in air traffic control. Transportation Science 27: 228 – 238.

Richetta O., Odoni AR (1994) Dynamic solution to the ground holding problem in air traffic control. Transportation Research Part A 28: 167 – 185.

Sherali HD, Hill JM, McCrea MV, Trani AA (2011) Integrating slot exchange, safety, capacity and equity mechanisms within an airspace flow program. Transportation Science 45 (2): 271 – 284.

Train KE (2009) Discrete Choice Methods with Simulation. 2nd Edition, Cambridge University Press, New York.

Vickrey WS (1969) Congestion theory and transport investment. American Economic Review 59: 251–260.

Yang H., Liu W., Wang X., Zhang X. (2013) On the morning commute problem with bottleneck congestion and parking space constraints. Transportation Research Part B 58: 106 – 188.