UNIVERSITY OF CALIFORNIA
RIVERSIDE

# Essays on Monetary Linkage

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Zhi Zhao

December 2018

Dissertation Committee:

Dr. Marcelle Chauvet, Chairperson
Dr. Aman Ullah
Dr. Tae-hwy Lee

The Dissertation of Zhi Zhao is approved:

Committee Chairperson

University of California, Riverside

## Acknowledgments

I am grateful to my advisor Dr. Marcelle Chauvet, without whose help, I would not have been here. I would also like to thank Dr. Aman Ullah and Dr. Tae-hwy Lee for being in my dissertation committee and giving me all the support during the five years.

I thank my parents for all the support. I am grateful to all the professors and stuff at Economics department in University of California, Riverside. I will miss you all.

ABSTRACT OF THE DISSERTATION

Essays on Monetary Linkage

by

Zhi Zhao

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, December 2018
Dr. Marcelle Chauvet, Chairperson

This dissertation attempts to explore the monetary transmission mechanism with an innovating empirical framework: mixed-frequency factor-augumented vector autoregressive model, which allows to include a large number of economic variable with different frequencies. Extensions of the model are also provided: a stacked-vector system equips the model with classic impulse response function analysis; Markov-switching feature is added to the model to study the dynamics of unobservable states. The first chapter proposes a mixed-frequency version of the factor-augmented vector autoregressive regression (FAVAR) model, which is used to construct a coincident index to measure the monetary transmission mechanism. The model divides the transmission of changes in monetary policy to the economy into three stages according to the timing and order of the impact. Indicators of each stage are measured and identified using different data frequencies: fast-moving variables (stage 1, asset returns at the weekly frequency), intermediate moving variables (stage 2, credit market data at the monthly frequency), and slow-moving variables (stage 3, macroeconomic variables at the quarterly frequency). The resulting coincident index exhibits leading signal

for all recessions in the sample period and provides implications on the dynamics of the monetary transmission mechanism. The second chapter extend the analysis to unconventional monetary policy. The FAVAR with mixed frequency model is used to account for monetary transmission mechanism when not only the federal fund rate is used as a monetary tool but also the Large-Scale Asset Purchase program. The impact of unconventional monetary policy is captured by "shadow rate" constructed using one-month forward rates. This allows a more complete analysis of the impact of monetary policy on the economy also during the more recent years since the 2008 financial crisis. The third chapter aims to examine the lead-lag relationship between channels in monetary transmission mechanism. A two-state Markov-switching mixed-frequency factor-augumented vector autoregressive model that allows each channel to switch states individually is estimated to analyze lead-lag relationship between different channels over time. This will shed light on how to track the impact of monetary policy shocks in real time.

# Contents

# List of Figures

# List of Tables

# Chapter 1

# Introduction

Monetary linkage is the relationship between macro economic variables and measures of monetary policy instruments, also known as monetary transmission mechanism. There is a growing literature of both theoretical and empirical work seeking to reveal the underlying channels of monetary transmission mechanism. Theoretically, traditional views of monetary tranmission mechnism mostly focus on the interactions between investors and borrowers in the credit market. After 2008 financial crisis, financial frictions has drawn a lot of attention as researchers came to realize that the state of banking system is playing a more important role in the monetary transsmission mechanism than people think. Taking the financial intermediates as the "missing link" in monetary transmission mechanism provided a new perspective for researchers to explore the potential channals of monetary transmission mechanism.

The traditional view of monetary transmission mechnism focused on the interest rate channel, which is described as the inverstors adjust their asset according to the changes in interest rate as discussed in Tobin (1969). In this paper, the user cost of capital is the key factor that determines the demand for capital. The user cost of capital ($u_c$) can be written as

$$u_c = p_c\{[(1-\tau)i - \pi^e] - (\pi_c^e - \pi^e)\}$$

where $p_c$ is the relative price of new capital, $\tau$ is the marginal tax rate, $i$ is the norminal interest rate, $\pi^e$ is the expected inflation and $\pi_c^e$ is the expected price appreciation of the capital asset. Therefore, the formula can be interpreted as the user cost of capital equals the difference between the after-tax real term interest rate and the expected real appreciation rate adjusted by the relative price. With a rise in short-term interest rate, the user cost of capital also rises. Naturally, investotrs adjust their demand for these capital assets and consequently decease the consumption.

Another view of monetary transmission mechnism is known as credit channel, which take the amplification effect of loan supply in credit market due to a change in interest rate as studied by Bernanke and Gertler (1989). The rich literature on credit channel is biult on the assumption that the credit market is imperfect as a result of government intervention, asymmetric information and agency problems. The credit channel can be further divided into two mechanisms, bank lending channel and firm balance sheet channel. In bank lending channel, banks are playing an important and unique role as they are the only source of finance for certain borrowers who has no access to the credit market, such as small firms. Therefore, under expansionary monetary policy, bank reserve and deposits increase, resulting the quantity of loan available for those certain borrowers increases, followed by increase in investment and consumption in real economy. The balance sheet channel, on the other hand, refers to the effect that change in interest rate directly change the borrower's ability of borrowing due to change in asset value and profitability. Lower net worth, on the one hand, means less collateral for loans and more loss from adverse selection. On the other hand, it also means less equity stake which increases the incentive for risk-taking behaviors. As a result, lenders will be more conservative making loans, leading to a decrease in investment and consumption in real economy. The condition for the credit channel to work is that: reservable and non-reservable liabilities are not perfect substitutes for banks; bank and non-bank funding are not perfect substitutes to firms and consumers.

More recently, aside from the supply side of credit, Nicolo et al. (2010) discussed about the three different risk-taking channel through which expansionary monetary policy could lead to risk-taking behavior. The first channel is that banks have incentive to substitute low yield safe asset with high yield riskier asset. The second channel is through a "search for yield." Low interest rate give financial instititutes with long-term commitment incentive to switch to risky asset in order to have a higher probability to match the yield they promised. The last channel is based on banks always tend to maintain a constant leverage ratio. The leverage ratio will drop under monetary policy easing as risky asset weight falls. This could again lead banks to a switch towards risky assets.

Empirically, one stream of the literture focuses on providing evidence for the theoretical model. The rest employs various econometric models to identify the monetary policy effect on real economy variables. However, very few attempts have been made to measure monetary transmission mechanism itself. It is well-accepted that aggragate economy could be measured by business cycles indicators, for instance, Real GDP. There also exists some other popular coincident indices considered as alternatives, such as Stock-Watson Experimental Coincident Index. Therefore, it is also possible to measure monetary transmission mechnism by constructing coincident index.

The methodology of empirical studies is developing over recent years. Vector Antoregressive regression (VAR) has been the standard approach to measure monetary policy shocks in the literature. A standard VAR model of a k-dimensional vector $Z_t$ can be represented by

$$Z_t = \sum_{j=1}^{l} B_j Z_{t-j} + u_t \tag{1.1}$$

where

$$E(u_t u_t') = V$$

In most context, $Z_t = \begin{bmatrix} S_t \\ X_t \end{bmatrix}$ where $S_t$ stands for the Federal Reserve's instrument of monetary policy, i.e. federal fund rate in most cases, and $X_t$ is other informational economic variables. To proceed the model with monetary policy shocks, we need to further assume the relationship between the VAR innovation $u_t$ and monetary policy shock $\varepsilon_t$is given by $A_0 u_t = \varepsilon_t$, where $A_0$ is invertible and $E(\varepsilon_t \varepsilon_t') = D$ and $D$ is positively definite. Premultiplying (1.1) by $A_0$ yields

$$A_0 Z_t = \sum_{j=1}^{l} A_j Z_{t-j} + \varepsilon_t \tag{1.2}$$

where $B_i = A_0^{-1} A_i$ and $V = A_0^{-1} D (A_0^{-1})'$ . To estimate the model, several assumptions and restrictions should be imposed and this may lead to identification issues.

Although VAR has significant contribution to the analysis of monetary policy shocks, there are still some identification issues that researchers disagree about. The identification problem comes from the fact that we can not use the observations of policy maker's action because it could be a reaction to "nonmonetray developments" in the economy while our primary focus is the effect of monetary policy shock. Therefore in literature, there are three approaches for isolating the monetary policy shocks. The first approach is to assume the functional form and relevant variables in the Federal Reserve's Feedback rule. Additional assumptions include these variables are orthogonal to the policy shocks and the recursiveness assumption which justifies the two-step estimating procedure of VAR. The economic interpretation of recursiveness assumption is that at time t the variables in the Federal Reserve's information set doesn't respond to the realizations of monetary shock at time t. Under the recursiveness assumption, the two-step estimation can be implement as follows: first, run least squares regression and get the fitted residual as the estimated monetary shocks; second, run estimate the parameters base on the current and lagged values of monetary policy shocks calculated in step 1. The second approach of identification is implemented by making assumptions about the relationship between observable signals and actual exogenous monetary policy shock. A simple example will be King (1991) assuming the all movements in money reflect shocks in monetary policy. The third approach is assuming the monetary shocks do not have effect on economy in the long run.

Dynamic Factor Model is developed to construct and make use of diffusion indexes which are consist of contemporaneous values of a large number of time series data as mentioned in Stock and Watson (1998). Since the indexes are weighted average of a large set of economic variables, they summarize the information of many aspects of economic activities. Classic use of diffusion indexes includes recession indicator and coincident economic indicators. Let $y_t$ be the economic variable of interest and $X_t$ be the N-dimensional vector of informational economic variables. Then a dynamic factor model of $(X_t, y_{t+1})$ with $\bar{r}$-dimensional common factor $f_t$ is given by

$$X_{it} = \bar{\lambda}_i(L)f_t + e_{it} \tag{1.3}$$

$$y_{t+1} = \bar{\beta}(L)f_t + \epsilon_{t+1} \tag{1.4}$$

where $e_t = (e_{1t}, ...e_{Nt})$ is the corresponding N-dimensional innovation and $\bar{\lambda}_i(L)$ and $\bar{\beta}(L)$ are polynomials of lag operator $L$ with nonnegative power. It's standard to assume both $\bar{\lambda}_i(L)$ and $\bar{\beta}(L)$ are polynomials with finite order at most $q$. Suppose $\bar{\lambda}_i(L) = \sum_{j=1}^{q} \bar{\lambda}_{ij} L^j$ and $\bar{\beta}(L) = \sum_{j=1}^{q} \bar{\beta}_j L^j$, then we can rewrite the model in static form

$$X_t = \Lambda F_t^0 + e_t \tag{1.5}$$

$$y_{t+1} = \beta' F_t^0 + \epsilon_{t+1} \tag{1.6}$$

where $F_t^0 = (f_t, ..., f_{t-q})$ is $r \times 1$ and $r = (q+1)\bar{r}$. In addition, the i-th row of $\Lambda$ is given by $(\bar{\lambda}_{i0}, ..., \bar{\lambda}_{iq})$ and $\beta = (\bar{\beta}_0, ..., \bar{\beta}_q)'$.

The standard estimation of dynamic factor model is a two-step procedure. However, since $N$ is large in most cases, it's better to estimate it nonparamatrically. Consider the nonlinear least squares objective function

$$V(F, \Lambda) = (NT)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} (X_{it} \lambda_i' F_t)^2 \tag{1.7}$$

where $X_{it}$ is the i-th observation at time $t$ and $\lambda_i$ is the i-th row of $\Lambda$. Concentrating out F make the minimizing problem equivalent to

$$\min tr[\Lambda'(X'X)\Lambda] \tag{1.8}$$

subject to

$$\Lambda'\Lambda/N = I_r \tag{1.9}$$

where the t-th row of $X$ is $X_t'$. Then the principle components estimator of $F$ is given by

$$\hat{F} = X\hat{\Lambda}/N$$

where $\hat{\Lambda}$ is set to $N^{1/2}$ times the eigenvectors of $X'X$'s first $r$ largest eigenvalues.

Alternatively, when $N > T$, a computational simpler approach could be concentrating $\Lambda$ out such that the minimizing problem is equivalent to

$$\min tr[F'(X'X)F] \tag{1.10}$$

subject to

$$F'F/T = I \tag{1.11}$$

then the estimator $\tilde{F}$ can be expressed as $T^{1/2}$ time the the eigenvectors of $XX'$'s first $r$ largest eigenvalues.

There are a few methods to determine (or estimate) the number of static factors $r$ and dynamic factors $q$. For static factors, Bai and Ng (2002) developed an information criteria motivated by that in model selection. They proposed the following penalized sum of lost function

$$IC(r) = \ln V_r(\hat{\Lambda}, \hat{F}) + rg(N, T)$$

where $V_r(\hat{\Lambda}, \hat{F})$ is the least squares objective function of the principal component estimation evalutated at the principal component estimator $(\hat{\Lambda}, \hat{F})$ and $g(N, T)$ is an penalty function such that $g(N, T) \to 0$ as $N, T \to \infty$. The information criteria makes a trade off between the benefit of increasing another factor and the cost of estimating another parameter which is realized by the penalty function $rg(N,T)$. As mentioned in Bai and Ng (2002), by choosing $g(N,T) = (N+T)\ln(\min(N,T))/(NT)$, in the special case when $N = T$, the penalty term of the information criteria will just collapse to 2 times BIC penalty factor.

As for dynamic factors, Bai and Ng (2007) propsoed an estimation based on the innovation variance matrix. The estimatino has two steps. The first step is to run VAR regression of the lags of principal components on the principal components estimator itself. In the following step, we can compare the eigenvalues of the residual variance matrix to a shrinking bound depending on $(N,T)$. Stock and Watson's simulation shows that Bai-Ng (2007) procedure shows a better performance in finite sample than others'.

Factor-augment vector autoregresssive regression (FAVAR) was introduced by Bernanke, Boivin and Eliasz (2005) to solve the dimensionality problem. There are two distinct advantages of using FAVAR instead of VAR. First, in most VAR, researchers only include handful number of variables in the regression while in reality the monetary policy makers in central bank making their decisions base on the observation of a large number of economic variables. Therefore, it's natural to expect mismeasurement problem in VAR. By contrast, FAVAR summarize the information of this large number of economic variables in the set of factors (indices). Second, when we analyze the impulse response of the model, by the same reason, VAR has a limitation on the number of variables while we're interested in "a list of variables". In addition, it's hard to find some existing economic variables which can represent a general economic concept (economic activity, monetary transmission mechanism).

Let $Y_t$ denotes a $M \times 1$ vector of observable economic variables of interest. In the baseline model, they only include the federal fund rate in $Y_t$ (,hence $M = 1$). A $K \times 1$ unobservable factor vector $F_t$ is summarize the additional information other $N$ economic variables, where $K$ is assumed to be small. Then the joint dynamics of the common components $C_t = (F_t', Y_t')'$ will be given by

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \tag{1.12}$$

where $\Phi(L)$ is a lag polynomial of order $d$ and the $v_t\tilde{}NID(0, Q)$.

Note that the federal fund rate in the last row of the vector, which is standard in literature. The underlying assumption is the monetary shocks do not have instant effect on the latent factors. They further assume that the terms of $\Phi(L)$ is restricted such that $Y_t$ and $F_{t-1}$ are related. Otherwise this model will reduce to simple VAR. Due to the fact that the factor vector $F_t$ is not observable, (1.12) can not be directly estimated. Let $X_t$ be a $N \times 1$ vector which contains other informative economic variables as we mentioned. Here we assume $N$ is large such that $K + M << N$. The dynamic of the informational vector $X_t$ is given by

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \tag{1.13}$$

where $\Lambda^f$ and $\Lambda^y$ are $N \times K$ and $N \times M$ matrix of factor loading for unobservable factors and observable economics variables respectively. The error term $e_t$ is $N \times 1$ vector which is assumed to be either weakly correlated or uncorrelected. (1.13) has a underlying assumption: the informational economic variables only depend on the current values of factors but not any lag terms. However, this assumption can be relaxed later. In Bernanke, Boivin and Eliasz (2005), they classified the variables in $X_t$ into "slow-moving variables" and "fast-moving variables". In their definition, "A slow-moving variable is one that is largely predetermined as of the current period, while a fast moving variable – think of an asset-price – is highly sensitive to contemporaneous economic news and shocks".

There are two ways to estimate the model. The first approach is a two-step approach. The two step approach has the advantage of computational simplicity and it is implemented in the following way:

Step 1: Estimate slow-moving factors $F_t^s$ are estimated using the principle component of "slow-moving variables". Then estimate the regression

$$\hat{C}_t = b_{F^s}\hat{F}_t^s + b_Y Y_t + e_t$$

and construct $\hat{F}_t$ from $\hat{C}_t - \hat{b}_Y Y_t$.

Step 2: Estimate the VAR model of $\hat{F}_t$ and $Y_t$ in (1.12) recursively

Despite the clear advantage of this two-step approach, it suffers from the "generated regressor" problem in the second step, therefore the confidence interval must be calculated based on a bootstrap procedure by Kilian (1998).

The alternative approach is a single-step Bayesian likelihood approach implemented by Gibbs Sampling.

In real-time, researchers often deal with unbalanced dataset. The most common problem is "ragged-data" problem which refers to the missing values at the end of the sample caused by publication delays. Mixed-frequency datasets have indicators sampled in different frequency, could be considered as a special case of missing values where the missing values appear regularly. Baffigi, Golinelli and Parigi (2004) first proposed the bridge equations that create a linkage between low-frequency data with time-aggregated data. The bridge equation model could be written as follows

$$y_{t_q} = \alpha + \sum_{i=1}^{j} \beta_i(L) x_{it_q} + u_{t_q} \tag{1.14}$$

where $y_{t_q}$ denotes the low-frequency data, $\beta_i(L)$ is a lag polynomial of length k, $x_{it_q}$ is the selected high-frequency data aggregated into low frequency. The process is of two steps. Firstly, use VAR to obtain forcast of high-frequency data and aggregate the selected indicators (usually based on information criteria or RMSE performance) into low frequency. Secondly, use the aggregated low-frequency value as regressor in the brige equation to forcast the low-frequency indicator.

Another approach to deal with mixed-frequency data is Mixed-Data Sampling (MIDAS) approach first introduced by Ghysels et al. (2004). Consider a time series $\{y_{t_q}\}$, $t_q = 1, ..., T_q$ being the low-frequency indicator. Let $m$ be the number of higher frequency series $\{x_{t_m}\}$, $t_m = 1, ...T_m$ that are observed in one period of $t_q$. For example, $m = 3$ if $\{y_{t_q}\}$ is quarterly and $\{x_{t_m}\}$. Naturally we can fit the low-frequency indicator $\{y_{t_q}\}$ in to high frequency by setting $y_{t_m} = y_{t_q}, \forall t_m = mt_q$. Therefore, a basic linear MIDAS model is given by

$$y_{t_q} = \beta_0 + B(L^{1/m}) x_{mt_q} + \epsilon_t^{(m)} \tag{1.15}$$

where $B(L^{1/m}) = \sum_{k=0}^{K} c(k;\theta) L^{j/m}$ is a polynomial with length $K$ and $c(k;\theta)$ is the coefficient of the lag operator. $L^{j/m} x_{t_q} = x_{mt_q - j}$ since $x_{t_q}$ is updated in a higher frequency.

One of the key features of MIDAS model is that the parameterization of the coefficient of $c(k;\theta)$ could be quite flexible. A linear scheme could just set $c(k;\theta) = 1/K$, leaving no parameter of $\theta$ needs to be estimated. A Geomtric scheme with $c(k;\theta) = \theta^k / \sum_{k=1}^{K} \theta^k$, where $|\theta| < 1$ so that the sum of the coefficients is normalized to equal to one.

A exponential scheme expressed as

$$c(k;\theta) = \exp(\sum_{q=1}^{Q} \theta_q k^q) / \sum_{k=1}^{K} \exp(\sum_{q=1}^{Q} \theta_q k^q)$$

A "Beta lag" scheme with

$$c(k;\theta_1.\theta_2) = f(\frac{k}{K};\theta_1,\theta_2) / \sum_{k=1}^{K} f(\frac{k}{K};\theta_1,\theta_2)$$

where $f(\frac{k}{K};\theta_1,\theta_2) = [(\frac{k}{K})^{\theta_1-1}(1-\frac{k}{K})^{\theta_2-1}\Gamma(\theta_1+\theta_2)]/[\Gamma(\theta_1)\Gamma(\theta_2)]$ and $\Gamma(\theta) = \int_0^\infty e^{-x}x^{\theta-1}dx$.

A hyperbolic scheme with

$$c(k;\theta) = g(\frac{k}{K},\theta) / \sum_{k=1}^{K} g(\frac{k}{K},\theta)$$

where $g(k,\theta) = \Gamma(k+\theta)/[\Gamma(k+1)\Gamma(\theta)]$.

The above two approaches are mainly used for univariable case. The mixed-frequency autoregressive regression method introduced by Mariano and Murasawa (2010) could be used to analyse a system of variables with different frequencies.

Consider a example of GDP growth $x_{1,t_m}$ that could only be observed every three months and N other indicators $x_{2,t_m}$ that are observed monthly. The mixed-frequency VAR model is based on the key assumption that the quarterly observed GDP $x_{1,t_m}$ is equal to the geometric mean of monthly latent variable of the GDP $x^*_{1,t_m}$ of this month and the previous two months. That is

$$\ln x_{1,t_m} = \frac{1}{3}(\ln x^*_{1,t_m} + \ln x^*_{1,t_{m-1}} + \ln x^*_{1,t_m-2}) \tag{1.16}$$

Define the growth rate of quarterly GDP to be $y_{1,t_m} = \Delta_3 \ln x_{1,t_m}$, growth rate of monthly latent variable to be $y^*_{1,t_m} = \Delta \ln x^*_{1,t_m}$ and growth rate of other monthly indicator to be $y_{2,t_m} = \Delta \ln x_{2,t_m}$. Then

$$\begin{aligned}
\ln x_{1,t_m} - \ln x_{1,t_m-3} &= \frac{1}{3}[(\ln x^*_{1,t_m} - \ln x^*_{1,t_m-3}) + (\ln x^*_{1,t_m-1} - \ln x^*_{1,t_m-4}) \\
&\quad +(\ln x^*_{1,t_m-2} - \ln x^*_{1,t_m-5})] \\
y_{1,t_m} &= \frac{1}{3}[(y^*_{1,t_m} + y^*_{1,t_m-1} + y^*_{1,t_m-2}) + (y^*_{1,t_m-1} + y^*_{1,t_m-2} + y^*_{1,t_m-3}) \\
&\quad +(y^*_{1,t_m-2} + y^*_{1,t_m-3} + y^*_{1,t_m-3})] \\
&= \frac{1}{3}y^*_{1,t_m} + \frac{2}{3}y^*_{1,t_m-1} + y^*_{1,t_m-2} + \frac{2}{3}y^*_{1,t_m-3} + \frac{1}{3}y^*_{1,t_m-4} \qquad (1.17)
\end{aligned}$$

Let the mixed-frequency vectors be

$$\begin{aligned}
\mathbf{y}_{t_m} &= \begin{pmatrix} y_{1,t_m} \\ y_{2,t_m} \end{pmatrix} \\
\mathbf{y}^*_{t_m} &= \begin{pmatrix} y^*_{1,t_m} \\ y^*_{2,t_m} \end{pmatrix}
\end{aligned}$$

and define $\boldsymbol{\mu} = E(\mathbf{y}_{t_m})$ and $\boldsymbol{\mu}^* = E(\mathbf{y}^*_{t_m})$. Then the relationship between demeaned $y_{t_m}$ and $y^*_{t_m}$ could be written as

$$\mathbf{y}_{t_m} - \boldsymbol{\mu} = \mathbf{H}(L)(\mathbf{y}^*_{t_m} - \boldsymbol{\mu}^*) \qquad (1.18)$$

where

$$\begin{aligned}
\mathbf{H}(L) &= \begin{bmatrix} \frac{1}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{I}_N \end{bmatrix} + \begin{bmatrix} \frac{2}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L + \begin{bmatrix} \mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^2 \\
&\quad + \begin{bmatrix} \frac{2}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^3 + \begin{bmatrix} \frac{1}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^4
\end{aligned}$$

Further assume $\{\mathbf{y}^*_{t_m}\}$ follows VAR(p) model:

$$\Phi(L)(\mathbf{y}^*_{t_m} - \boldsymbol{\mu}^*) = \boldsymbol{\omega}_t \qquad (1.19)$$

where $\boldsymbol{\omega}_t \tilde{} IN(\mathbf{0}, \boldsymbol{\Sigma})$.

There are two cases for the state-space representation of the mixed-frequency VAR model. If $p \leq 5$, then the state variable is defined as

$$\mathbf{s}_t = \begin{pmatrix} \mathbf{y}^*_{t_m} - \boldsymbol{\mu}^* \\ \vdots \\ \mathbf{y}^*_{t_m-4} - \boldsymbol{\mu}^* \end{pmatrix}$$

Then the state-space representation could be written as follows:

$$\mathbf{s}_{t_m+1} = \mathbf{A}\mathbf{s}_{t_m} + \mathbf{B}\mathbf{z}_{t_m} \tag{1.20}$$

$$\mathbf{y}_{t_m} = \boldsymbol{\mu} + \mathbf{C}\mathbf{s}_{t_m} \tag{1.21}$$

$$\{\mathbf{z}_{t_m}\} \tilde{} IN(\mathbf{0}, \mathbf{I}_{N+1})$$

where

$$\mathbf{A} = \begin{bmatrix} \Phi_1 & \cdots & \Phi_p & \mathbf{O}_{(N+1)\times(5-p)(N+1)} \\ & \mathbf{I}_{4(N+1)} & & \mathbf{O}_{4(N+1)\times N} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \boldsymbol{\Sigma}^{1/2} \\ \mathbf{O}_{4(N+1)\times(N+1)} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{H}_0 & \cdots & \mathbf{H}_4 \end{bmatrix}$$

When $p \geq 5$, the state variable is instead defined as

$$\mathbf{s}_t = \begin{pmatrix} \mathbf{y}^*_{t_m} - \boldsymbol{\mu}^* \\ \vdots \\ \mathbf{y}^*_{t_m-p+1} - \boldsymbol{\mu}^* \end{pmatrix}$$

and $A$, $B$, $C$ matrix are adjusted accordingly:

$$\mathbf{A} = \begin{bmatrix} \Phi_1 & \cdots & \Phi_{p-1} & \Phi_p \\ & \mathbf{I}_{(p-1)(N+1)} & & \mathbf{O}_{(p-1)(N+1)\times N} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{\Sigma}^{1/2} \\ \mathbf{O}_{(p-1)(N+1)\times(N+1)} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{H}_0 & \cdots & \mathbf{H}_4 & \mathbf{0}_{(N+1)\times(p-5)(N+1)} \end{bmatrix}$$

The mixed-frequency VAR model is estimated by quasi-Newton method, which requires good starting value when the model has so many parameters to estimate. So it is suggested to use EM algorithm first to find a good starting value before implement quasi-Newton method.

Mixed-frequency factor model is proposed by Mariano and Murasawa (2003) to construct a new coincident index of business cycle as an extension of S-W coincident index. Follow the same set up and notation as above, a dynamic one-factor is given by

$$\begin{pmatrix} \mathbf{y}_{1,t_m} \\ \mathbf{y}_{2,t_m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix} + \begin{pmatrix} \beta_1(\frac{1}{3}f_{t_m} + \frac{2}{3}f_{t_m-1} + f_{t_m-2} + \frac{2}{3}f_{t_m-3} + \frac{1}{3}f_{t_m-4}) \\ \beta_2 f_t \end{pmatrix} + \begin{pmatrix} \frac{1}{3}u_{1,t_m} + \frac{2}{3}u_{1,t_m-1} + u_{1,t_m-2} + \frac{2}{3}u_{1,t_m-3} + \frac{1}{3}u_{1,t_m-4} \\ u_{2,t_m} \end{pmatrix} \quad (1.22)$$

And again we need to assume AR process for $f_{t_m}$ and $u_{t_m}$:

$$\phi_f(L)f_{t_m} = v_{1,t_m} \quad (1.23)$$

$$\Phi_u(L)u_{t_m} = v_{2,t_m} \quad (1.24)$$

where $\begin{pmatrix} v_{1,t_m} \\ v_{2,t_m} \end{pmatrix} \tilde{} NID(0, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \mathbf{\Sigma}_{22} \end{bmatrix})$, $\phi_f(L)$ and $\Phi_u(L)$ are of $p$th-order and $q$th-order polynomial respectively.

When $p, q \leq 4$, define the state variable as

$$\mathbf{s}_{t_m} = \begin{pmatrix} f_{t_m} \\ \vdots \\ f_{t_m-4} \\ u_{t_m} \\ \vdots \\ u_{t_m-4} \end{pmatrix}$$

The state-space representation of the model could be given by

$$\mathbf{s}_{t_m} = \mathbf{F}\mathbf{s}_{t_m-1} + \mathbf{G}\mathbf{v}_{t_m} \tag{1.25}$$

$$\mathbf{y}_{t_m} = \boldsymbol{\mu} + \mathbf{H}\mathbf{s}_{t_m} \tag{1.26}$$

where

$$\mathbf{F} = \begin{bmatrix} \phi_{f,1} & \cdots & \phi_{f,p} & \mathbf{o}'_{5-p} & & & & \\ 1 & & 0 & 0 & & \mathbf{O}_{5\times 5(N+1)} & & \\ & \ddots & & \vdots & & & & \\ 0 & & 1 & 0 & & & & \\ & & & & \Phi_{u,1} & \cdots & \Phi_{u,q} & \mathbf{O}_{(N+1)\times(5-q)} \\ & & & & \mathbf{I}_N & & 0 & \mathbf{O}_{(N+1)\times(N+1)} \\ & \mathbf{O}_{5(N+1)\times 5} & & & & \ddots & & \vdots \\ & & & & 0 & & \mathbf{I}_N & \mathbf{O}_{(N+1)\times(N+1)} \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 1 & \mathbf{o}'_{N+1} \\ 0 & \mathbf{o}'_{N+1} \\ \vdots & \vdots \\ \mathbf{o}_{N+1} & \mathbf{I}_{N+1} \\ \mathbf{o}_{N+1} & \mathbf{O}_{(N+1)\times(N+1)} \\ \vdots & \vdots \end{bmatrix}$$

$$\mathbf{H} = \begin{pmatrix} \frac{1}{3}\beta_1 & \frac{2}{3}\beta_2 & \beta_3 & \frac{2}{3}\beta_4 & \frac{1}{3}\beta_5 & \frac{1}{3} & \mathbf{O}_{1\times N} & \frac{2}{3} & \mathbf{O}_{1\times N} & \cdots \\ \beta_2 & & \mathbf{O}_{N\times 4} & & & \mathbf{O}_{N\times 1} & I_N & & \mathbf{O}_{N\times(N+1)} & \cdots \end{pmatrix}$$

where $o_n$ is $n \times 1$ zero vector and $O_{m\times n}$ is the $m \times n$ zero matrix.

The estimation of mixed-frequency factor model is very similar to that of mixed-frequency VAR except that the EM algorithm doesn't apply in this case. The reason is that there is no error term in the measurement equation so that the unknown parameters in measurement could never get into the likelihood function.

In this dissertation, I attempt to explore the monetary transmission mechanism with an innovating empirical framework: mixed-frequency factor-augumented vector autoregressive model, which allows to include a large number of economic variable with different frequencies. Extensions of the model are also provided: a stacked-vector system equips the model with classic impulse response function analysis; Markov-switching feature is added to the model to study the dynamics of unobservable states. The first chapter studies the monetary transmission mechanism in the U.S. It proposes a mixed-frequency version of the factor-augmented vector autoregressive regression (FAVAR) model, which is used to construct a coincident index to measure the monetary transmission mechanism. The model divides the transmission of changes in monetary policy to the economy into three stages according to the timing and order of the impact. Indicators of each stage are measured and identified using different data frequencies: fast-moving variables (stage 1, asset returns at the weekly frequency), intermediate moving variables (stage 2, credit market data at the monthly frequency), and slow-moving variables (stage 3, macroeconomic variables at the quarterly frequency). The resulting coincident index exhibits leading signal for all recessions in the sample period and provides implications on the dynamics of the monetary transmission mechanism. The proposed coincident index also indicates that monetary transmission mechanism is changing over time. The second chapter extends the analysis to overall monetary policy that includes unconventional monetary policy. When the U.S. federal fund rate hit the Zero Lower Bound in 2008, conventional monetary policy became ineffective. Unconventional policies such as the Large-Scale Asset Purchase were then implemented in order to conduct monetary policy. In this chapter I extend the FAVAR with mixed frequency model to account for monetary transmission mechanism when not only the federal fund rate is used as a monetary tool but also the Large-Scale Asset Purchase program. The impact of unconventional monetary policy is captured by "shadow rate" constructed using one month forward rates. This allows a more complete analysis of the impact of monetary policy on the economy also during the more recent years since the 2008 financial crisis. The third chapter focus more on examing the lead-lag relationship between channels in monetary transmission mechanism. As well studied in the literature, monetary transmission mechanism has a number of channels through which the monetary policy can affect the real economy. However, the speed of the transmission of the channel draws little attention from researchers. This chapter proposes to disentangle the monetary transmission mechanism into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. A two-state markov-switching mixed-frequency factor-augumented vector autoregressive model that allows each channel to switch states individually is estimated to analyze lead-lag relationship between different channels over time. This will shed light on how to track the impact of monetary policy shocks in real time.

# Chapter 2

# Quantifying the Monetary Transmission Mechanism: A Mixed-Frequency Factor-Augmented VAR Approach

## 2.1 Introduction

Monetary transmission mechanism describes how monetary policy shock affects real variables in the economy such as aggregate output and employment rate. As monetary policy impacts many real variables in the economy in the short run, it is important for policy makers to have an assessment of the timing and scale of such effects. This requires understanding of the underlying connections between monetary policy and real variables.

However, there exist many channels through which the monetary transmission mechanism takes place, which makes it a more complex and yet enticing research question.

There is a growing literature, both theoretical and empirical, which aims to unveil the underlying channels of the monetary transmission mechanism. Theoretically, the traditional view of monetary transmission mechanism is known as the interest rate channel, in which investors adjust their assets according to changes in interest rate (Tobin 1969). Another view of the monetary transmission mechanism is through the credit channel, related to the amplification effect of changes in interest rate on loan supply in credit markets (Bernanke and Gertler 1989). The credit view is built on the assumption that the credit market is imperfect because of government intervention, asymmetric information, and agency problems. The credit channel can be further divided into two mechanisms: bank lending channel and firm balance sheet channel. In the bank lending channel, banks play an important and unique role as they are the only source of finance for certain borrowers who have no access to the credit market, such as small firms. The balance sheet channel, on the other hand, refers to the direct effect of interest rate changes on agents' ability to borrow due to changes in asset value and profitability.

While the credit channels focus on the supply side of the credit market, there are also the risk-taking channel, which is related to the demand side of the credit market. Nicolo et al. (2010) discuss three different risk-taking channels through which expansionary monetary policy could lead to risk-taking behavior. The first channel is that banks have incentive to substitute low yield safe asset with high yield riskier asset. The second channel is through a "search for yield," that is, low interest rate gives financial institutions with long-term commitment an incentive to switch to risky asset in order to attain a higher probability of matching their promised yield. The last channel refers to the fact that banks always tend to maintain a constant leverage ratio. The leverage ratio tends to drop with monetary policy easing as risky asset weight falls, and this could lead banks to switch towards risky assets.

Empirically, one stream of the literature focuses on providing evidence and testing the theoretical models. A main identification obstacle these empirical studies face is the difficulty of distinguishing the impact of changes in monetary policy on the loan supply from the impact on loan demand since both will be affected. The other stream employs econometric models to measure the effects of monetary policy shocks on real economy variables. For example, structural vector autoregressive models (SVAR) with a few plausible identification restrictions could provide impulse response functions that describe the impact of monetary policy shocks on specific variables. Most SVAR models include interest rate and real macroeconomic variables only, which may be sufficient to measure the impact of monetary policy shocks, but provide little information on the monetary transmission mechanism.

While there is a huge literature on the effects of monetary policy shocks, very few attempts have been made to measure monetary transmission mechanism itself. One strategy may be to consider the weighted measures of the several individual channels. This could be difficult to implement empirically, though, since different channels arise from different economic models and assumptions, which makes it hard to determine the weight for each measure in aggregate. Another strategy could be to include in one model intermediate variables through which the mechanism is transmitted. This is also challenging since the monetary transmission mechanism has many channels, and there is an array of intermediate variables to be included at different frequencies in the model. However, this can be accomplished with innovating empirical models, as implemented in this paper. We extend the factor-augmented vector autoregressive regression (FAVAR) to a mixed-frequency version to construct a high-frequency coincident index of monetary transmission mechanism in U.S. We divide the monetary transmission mechanism in three stages according to the timing and order of the effect. The first stage has the fast-moving variables such as asset return measured with high frequency data. The second stage has medium-moving intermediate variables that measure the credit market changes in medium-run frequency. The last stage is the slow-moving real macro variables in low frequency. We propose a baseline model and an alternative model that include many variables and yet reduce the number of parameters to be estimated. The mixed-frequency FAVAR model is estimated with a two-stage maximum likelihood estimation process and yields as output a coincident index that measures the monetary transmission mechanism in U.S.

There are three papers that are closely related to this paper. One is Bernanke, Boivin and Eliasz (2005), who introduce the Factor-Augmented Vector Autoregressive Regression (FAVAR) model. The main advantage of the FAVAR model is that it does not require restrictions on the number of informational variables as the traditional VAR, and still maintains the general framework of VAR analysis. Including large number of informational variables in the model minimizes the mismeasurement problem. Additionally, it makes it closer to the situation faced by central bank or policy makers. Bernanke, Boivin and Eliasz (2005) apply the FAVAR model to 120 monthly U.S. macroeconomic and financial series. The effect of monetary policy shocks is measured by impulse response functions of these variables. A second closely related paper is Mariano and Murasawa (2003), who extent Stock and Watson monthly coincident index by including a variable at the quarterly frequency, real GDP. They proposed a mixed-frequency one-factor model by filling the missing observations in quarterly data with random draws from standard normal distribution with zero mean. The resulting coincident index is an estimated latent monthly real GDP. Mariano and Murasawa (2010) introduced a mixed-frequency VAR model and a mixed-frequency dynamic K-factor model to estimate a new coincident index of monthly real GDP. While maintaining the same mixed frequency methods in their 2003 paper, they select the number of lags in the VAR model and the number of factors for the dynamic factor model according to model selection criteria, and the resulting coincident indices differ substantially from their previous version.

This paper has three main contributions. First, as most literature on mixed-frequency data have focused on data with two frequencies, we propose a mixed-frequency version of FAVAR model that combines three different frequencies in the same model. Second, the estimation of the proposed model is very time-consuming even with limited number of variables and sample period. Therefore, we developed an alternative approach to largely reduce the number of parameters and simplify the estimation of the original version of MF-FAVAR model. Finally, we construct a high-frequency coincident index from the model, which measures monetary transmission mechanism in U.S. and provides leading signal for recessions.

The mixed-frequency FAVAR model proposed in this paper is a combination of the original FAVAR model and the mixed-frequency factor model. The proposed model not only allows for a large number of indicators as in the standard FAVAR model, it is also compatible with data from several different frequencies.

The model estimation is implemented using a similar two-step procedure as in standard FAVAR. For the baseline model, in the first step stage factors of different frequencies are estimated individually using the mixed-frequency factor model. The second step uses the estimated factors from step one on standard recursive VAR. However, the fact that the baseline model has too many parameters makes the estimation very time-consuming. We, therefore, propose an alternative method, which replaces some high-frequency data with skip-sampled lower-frequency data in the first step. The second step is adjusted accordingly to be the mixed-frequency VAR model. The resulting coincident index is the estimated latent high-frequency common factor of federal fund rate and real macro variables.

The proposed coincident index of U.S. monetary transmission mechanism measures the effectiveness of the impact of monetary policy on the economy. The dynamics of the index depict the evolution of the U.S. monetary transmission mechanism over the last two decades. There are two major peaks, in 2000 and 2007, indicating the large impact of monetary transmission mechanism driven by rapid expansion of credit market as well as financial innovations. The index also exhibits a clear pattern, in which it reaches local peaks right before the recessions, and it declines during the recessions. This implies that the effectiveness of monetary transmission mechanism could be lessen during recessions.

As far as we know, this is the first paper quantifying the monetary transmission mechanism, thus there are no other comparable indices available. We, thereby, use for comparison, a simple time-varying parameter model to estimate the coefficient of the the first difference of federal fund rate to growth rate of real GDP. The result is consistent with the proposed coincident index.

The rest of the paper is structured as follows. Section two presents the proposed mixed-frequency version of factor-augmented vector autoregressive regression model, the alternative model and their corresponding two-stage ML estimation processes. Section three applies the model to U.S. macroeconomic and financial data to construct the coincident index of monetary transmission mechanism, and discuss the empirical results. Section four concludes.

## 2.2 The model

### 2.2.1 The Baseline Model

Consider $Y_t$ to be a $M \times 1$ vector of observable economic variables of interest that is driving the economy. In our application, it has federal fund rate in weekly frequency only. After a policy change, due to the nature of the existing economic structure, the impact will go through the variables in a certain order. In the context of monetary policy change, we consider two possible scenarios to track the shockwaves. The first scenario simply follows the classic interest rate channel and bank-lending channel of monetary transmission mechanism in the literature. An increase in interest rate leads to a drop in the amount of credit available to firms and consumers (loan supply), leading to a decrease in investment by firms and consumption by consumers. In the second scenario, we consider balance sheet channel and risk-taking channel in a expectation perspective, as interest rate rises, the amount of credit available to firms and consumers are expected to drop, causing a drop in firms' profitability which is shown in the balance sheet. The asset price and value in financial markets will drop resulting firms and consumers adjust the investment and consumption accordingly. The main difference between the scenarios described above, is the timing (or spreading speed) of the real effect of one monetary policy change. This is not a problem in the standard VAR or FAVAR model since the variables are of the same frequency and the time length for one period is implicitly determined by the frequency of the data. The common assumption in the literature that the variables are not contemporaneously affected by monetary policy shock also implies that the spreading speed is the same for all variables, which may not be a plausible assumption when the frequency of the data is too low.

The solution we propose in this paper is that we measure different variables in different frequency. More specifically, in the context of monetary transmission mechanism, the transmission mechanism is divided into three stages: stage 1 of fast-moving variables such as asset returns are measured in high frequency; stage 2 of medium-moving variables in credit market are measured in medium frequency; stage 3 of real macro variables are considered slow-moving variables measured in low frequency. We extend the standard factor-augmented vector autoregressive regression (FAVAR) model to a mixed-frequency version by assuming three unobservable factors $f_{1,t}$, $f_{2,t}$ and $f_{3,t}$ of low, medium and high frequency respectively that summarize the information of different stages in monetary transmission mechanism. Formally, a three-stage mixed-frequency FAVAR model is given by

$$\begin{bmatrix} f_{1,t} \\ f_{2,t} \\ f_{3,t} \\ y_t \end{bmatrix} = \phi(L) \begin{bmatrix} f_{1,t-1} \\ f_{2,t-1} \\ f_{3,t-1} \\ y_t \end{bmatrix} + \mathbf{v}_t \tag{2.1}$$

where $\phi(L)$ is a lag polynomial of order $d$ and $v_t \tilde{} NID(0, Q)$. Note that the factors are ordered from low to high frequency and federal fund rate is placed at the bottom as in standard literature.

The unobservable factors are interpreted as the indicators of different stages in monetary transmission mechanism, which are extracted from various related informational economic variables. Let $X_{1,t}$, $X_{2,t}$, $X_{3,t}$ be $N_1 \times T_1$, $N_2 \times T_2$, $N_3 \times T_3$ informational data we observe at low, medium and high frequency respectively with $N_i$ being the number of variables and $T_i$ the number of observations $i = 1, 2, 3$. The time length of one period in the model is set to be consistent with that of the highest frequency data, namely $X_{3,t}$. Therefore, $X_{3,t}$ is observed every period, while $X_{2,t}$ and $X_{1,t}$ is observed every $n$ and $m$ period where $m > n > 1$. In the case of $X_{3,t}$ being weekly data, $X_{2,t}$ being monthly data and $X_{1,t}$ being quarterly data, we can set $m = 12$ and $n = 4$.

Following Mariano and Murasawa (2003), let $X^*_{1,t}$ and $X^*_{2,t}$ be the underlying latent variable in highest frequency such that the observed variable is equal to the geometric average of the last three periods' latent variable. Formally,

$$\begin{aligned} \ln x_{1,t} &= \frac{1}{3}(\ln x^*_{1,t} + \ln x^*_{1,t-1} + \ln x^*_{1,t-2}) \\ \ln x_{2,t} &= \frac{1}{3}(\ln x^*_{2,t} + \ln x^*_{2,t-1} + \ln x^*_{2,t-2}) \end{aligned}$$

Let $y_{1,t} = \Delta_{12} \ln x_{1,t},\ y^*_{1,t} = \Delta \ln x^*_{1,t},\ y_{2,t} = \Delta_4 \ln x_{2,t},\ y^*_{2,t} = \Delta \ln x^*_{2,t}$ and $y_{3,t} = \Delta \ln x_{3,t}$. We have

$$y_{1,t} = \frac{1}{3}y^*_{1,t} + \frac{2}{3}y^*_{1,t-1} + y^*_{1,t-2} + ... + y^*_{1,t-11} + \frac{2}{3}y^*_{1,t-12} + \frac{1}{3}y^*_{1,t-13} \quad (2.2)$$

$$y_{2,t} = \frac{1}{3}y^*_{2,t} + \frac{2}{3}y^*_{2,t-1} + y^*_{2,t-2} + y^*_{2,t-3} + \frac{2}{3}y^*_{2,t-4} + \frac{1}{3}y^*_{2,t-5} \quad (2.3)$$

Let

$$\mathbf{y}_{1,t} = \begin{pmatrix} y_{1,t} \\ y_t \end{pmatrix}$$

$$\mathbf{y}^*_{1,t} = \begin{pmatrix} y^*_{1,t} \\ y_t \end{pmatrix}$$

Define $\mu_i = E(y_{i,t}), i = 1, 2,\ \mu_y = E(y_t)$ and

$$\boldsymbol{\mu}_i = \begin{pmatrix} \mu_i \\ \mu_y \end{pmatrix}$$

$$\boldsymbol{\mu}^*_i = \begin{pmatrix} \mu^*_i \\ \mu_y \end{pmatrix}$$

Then relationship between $\mathbf{y}_{1,t}$ and $\mathbf{y}^*_{1,t}$ could be written as

$$\mathbf{y}_{1,t} - \boldsymbol{\mu}_1 = \mathbf{J}_1(L)(\mathbf{y}^*_{1,t} - \boldsymbol{\mu}^*_1) \quad (2.4)$$

where

$$\begin{aligned}\mathbf{J}_1(L) = & \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^2 + ... \\ & + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^{11} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^{12} + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^{13}\end{aligned}$$

Similarly, for $\mathbf{y}_{2,t}$ and $\mathbf{y}^*_{2,t}$

$$\mathbf{y}_{2,t} - \boldsymbol{\mu}_2 = \mathbf{J}_2(L)(\mathbf{y}^*_{2,t} - \boldsymbol{\mu}^*_2) \tag{2.5}$$

where

$$\begin{aligned}\mathbf{J}_2(L) &= \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^2 + \\ &+ \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^3 + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^4 + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^5\end{aligned}$$

For each stage, we extract the common factor between monetary policy and informational variables in the stage, which is interpreted as linkage between monetary policy and economic variables of corresponding stage. For stage 2 and stage 3, the factor is related to informational data using a mixed-frequency dynamic factor model because of frequency difference. In stage 1, we maintain the factor model as in standard FAVAR:

$$\begin{aligned}\begin{pmatrix} y_{1,t} \\ y_t \end{pmatrix} &= \begin{pmatrix} \mu_1 \\ \mu_y \end{pmatrix} + \begin{pmatrix} \beta_{11}(\frac{1}{3}f_{1,t} + \frac{2}{3}f_{1,t-1} + \sum_{j=2}^{11} f_{1,t-j} + \frac{2}{3}f_{1,t-12} + \frac{1}{3}f_{1,t-13}) \\ \beta_{12}f_{1,t} \end{pmatrix} \\ &+ \begin{pmatrix} \frac{1}{3}e_{1,t} + \frac{2}{3}e_{1,t-1} + \sum_{j=2}^{11} e_{1,t-j} + \frac{2}{3}e_{1,t-12} + \frac{1}{3}e_{1,t-13} \\ e_t \end{pmatrix} \end{aligned} \tag{2.6}$$

$$\begin{aligned}\begin{pmatrix} y_{2,t} \\ y_t \end{pmatrix} &= \begin{pmatrix} \mu_2 \\ \mu_y \end{pmatrix} + \begin{pmatrix} \beta_{21}(\frac{1}{3}f_{2,t} + \frac{2}{3}f_{2,t-1} + f_{2,t-2} + f_{2,t-3} + \frac{2}{3}f_{2,t-4} + \frac{1}{3}f_{2,t-5}) \\ \beta_{22}f_{2,t} \end{pmatrix} \\ &+ \begin{pmatrix} \frac{1}{3}e_{2,t} + \frac{2}{3}e_{2,t-1} + e_{2,t-2} + e_{2,t-3} + \frac{2}{3}e_{2,t-4} + \frac{1}{3}e_{2,t-5}) \\ e_t \end{pmatrix} \end{aligned} \tag{2.7}$$

$$y_{3,t} = \Lambda_3 f_{3,t} + \Lambda_i^y y_t + e_{3,t} \tag{2.8}$$

where $\beta_{ij}$ are corresponding factor loading vectors, $\boldsymbol{\beta}_i = (\beta'_{i1}, \beta'_{i2})', i = 1, 2$; $\Lambda_3$ is $N_3 \times K$ factor loading matrix; $e_{3,t}$ is $N_3 \times 1$ error term vectors.

### 2.2.2 Estimation

The model could be estimated using a similar two-step procedure as in standard FAVAR literature. The first step is to estimate factors $\hat{f}_i, i = 1, 2, 3$ individually. To estimate (2.6), first we need to assume AR process for $f_{1,t}$ and $e_{1,t}$:

$$\phi_1^f(L) f_{1,t} = v_{1,t} \tag{2.9}$$

$$\Phi_1^e(L) e_{1,t} = v_{2,t} \tag{2.10}$$

$$\begin{pmatrix} v_{1,t} \\ v_{2,t} \end{pmatrix} \tilde{\ } NID(0, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \Sigma_{22} \end{pmatrix})$$

where $\phi_1^f(L)$ is a lag operation polynomial of $p_1$th-order and $\Phi_1^e(L)$ is a lag operation polynomial of $q_1$th order. The variance-covariance matrix is assumed to be diagonal with the first element equals 1, which is a standard identification strategy in factor model literature.

Define the state vector to be

$$\mathbf{s}_{1,t} = \begin{pmatrix} f_{1,t} \\ \vdots \\ f_{1,t-13} \\ u_{1,t} \\ \vdots \\ u_{1,t-13} \end{pmatrix}$$

where $u_{1,t} = (e'_{1,t}, e'_t)'$. The state-space representation when $p_1, q_1 \leq 14$ could be written as

$$\mathbf{s}_{1,t} = \mathbf{F}_1 \mathbf{s}_{1,t-1} + \mathbf{G}_1 \mathbf{z}_{1,t} \tag{2.11}$$

$$\mathbf{y}_{1,t} = \boldsymbol{\mu}_1 + \mathbf{H}_1 \mathbf{s}_t \tag{2.12}$$

$$\{\mathbf{z}_{1,t}\} \tilde{} IN(\mathbf{0}, \mathbf{I}_{N_1+1})$$

where

$$\mathbf{F}_1 = \begin{bmatrix} \phi_1^f(1) \cdots \phi_1^f(p_1) & \mathbf{o}'_{14-p_1} & \mathbf{O}_{14\times 14(N_1+1)} & \\ \mathbf{I}_{13} & \mathbf{o}_{13} & & \\ & & \Phi_1^e(1) \cdots \Phi_1^e(q_1) & \mathbf{O}_{(N_1+1)\times(14-q_1)(N_1+1)} \\ \mathbf{O}_{14(N_1+1)\times 14} & & \mathbf{I}_{13(N_1+1)} & \mathbf{O}_{13(N_1+1)\times(N_1+1)} \end{bmatrix}$$

$$\mathbf{G}_1 = \begin{bmatrix} \sigma_1 & \mathbf{o}_{(N_1+1)} \\ \mathbf{o}_{13} & \mathbf{O}_{13\times(N_1+1)} \\ \mathbf{o}_{(N_1+1)} & \mathbf{\Sigma}_{22}^{1/2} \\ \mathbf{o}_{13(N_1+1)} & \mathbf{O}_{13(N_1+1)\times(N_1+1)} \end{bmatrix}$$

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{J}_1(0)\boldsymbol{\beta}_1 & \cdots & \mathbf{J}_1(13)\boldsymbol{\beta}_1 & \mathbf{J}_1(0) & \cdots & \mathbf{J}_1(13) \end{bmatrix}$$

Similarly, we can write the state-space representation of (2.7) when $p_2, q_2 \leq 6$ as

$$\mathbf{s}_{2,t} = \mathbf{F}_2 \mathbf{s}_{2,t-1} + \mathbf{G}_2 \mathbf{z}_{2,t} \tag{2.13}$$

$$\mathbf{y}_{2,t} = \boldsymbol{\mu}_2 + \mathbf{H}_2 \mathbf{s}_t \tag{2.14}$$

$$\{\mathbf{z}_{2,t}\} \tilde{} IN(\mathbf{0}, \mathbf{I}_{N_2+1})$$

where

$$\mathbf{s}_{2,t} = \begin{pmatrix} f_{2,t} \\ \vdots \\ f_{2,t-5} \\ u_{2,t} \\ \vdots \\ u_{2,t-5} \end{pmatrix}$$

$$\begin{aligned}
\mathbf{F}_2 &= \begin{bmatrix} \phi_2^f(1)\cdots\phi_2^f(p_2) & \mathbf{o}'_{6-p_2} & \mathbf{O}_{6\times 6(N_2+1)} & \\ \mathbf{I}_5 & \mathbf{o}_5 & & \\ & & \Phi_2^e(1)\cdots\Phi_2^e(q_2) & \mathbf{O}_{(N_2+1)\times(6-q_2)(N_2+1)} \\ \mathbf{O}_{6(N_1+1)\times 6} & & I_{5(N_2+1)} & \mathbf{O}_{5(N_2+1)\times(N_2+1)} \end{bmatrix} \\
\mathbf{G}_2 &= \begin{bmatrix} \sigma_2 & \mathbf{o}_{(N_2+1)} \\ \mathbf{o}_5 & \mathbf{O}_{5\times(N_2+1)} \\ \mathbf{o}_{(N_2+1)} & \mathbf{\Sigma}_{22}^{1/2} \\ \mathbf{o}_{5(N_2+1)} & \mathbf{O}_{5(N_2+1)\times(N_2+1)} \end{bmatrix} \\
\mathbf{H}_2 &= \begin{bmatrix} \mathbf{J}_2(0)\boldsymbol{\beta}_2 & \cdots & \mathbf{J}_2(5)\boldsymbol{\beta}_2 & \mathbf{J}_2(0) & \cdots & \mathbf{J}_2(5) \end{bmatrix}
\end{aligned}$$

The above two models could be estimated using the standard method to get the estimated factors $\hat{f}_{1,t}$ and $\hat{f}_{2,t}$. For (2.8), $\hat{f}_{3,t}$ could be estimated using principal component method as in standard FAVAR model.

Note that the estimates of the factors are all of the highest frequency, the second step could just follow the standard FAVAR model to estimate (2.1) under the recursive VAR environment.

### 2.2.3 The Alternative Model

One disadvantage of the baseline model is that the time of estimation is too long as we can observe the number of parameters in (2.6) is too large. In our application, we used an alternative model that could greatly reduce the number of lags and coefficients to simplify the estimation.

The alternative model replace the high-frequency series $y_t$ in (2.6) with medium-frequency series $y_t^m$ which is skip-sampled from $y_t$. Accordingly, we have to make following adjustments.

Let $y_{1,t}^m = \Delta_3 \ln x_{1,t}$, we have

$$y_{1,t}^m = \frac{1}{3}y_{1,t}^* + \frac{2}{3}y_{1,t-1}^* + y_{1,t-2}^* + \frac{2}{3}y_{1,t-3}^* + \frac{1}{3}y_{1,t-4}^* \tag{2.15}$$

Define $\mu_1^m = E(y_{1,t}^m)$, $\mu_y^m = E(y_t^m)$ and

$$\boldsymbol{\mu}_1^m = \begin{pmatrix} \mu_1^m \\ \\ \mu_y^m \end{pmatrix}$$

Thus the relationship between $y_{1,t}^m$ and $y_{1,t}^*$ is given by

$$\mathbf{y}_{1,t}^m - \boldsymbol{\mu}_1^m = \mathbf{J}_1(L)(\mathbf{y}_{1,t}^* - \boldsymbol{\mu}_1^*)$$

where

$$\mathbf{y}^m_{1,t} = \begin{pmatrix} y^m_{1,t} \\ y^m_t \end{pmatrix}$$

$$\mathbf{J}^m_1(L) = \begin{bmatrix} \frac{1}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & 1 \end{bmatrix} + \begin{bmatrix} \frac{2}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L + \begin{bmatrix} \mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^2 + \begin{bmatrix} \frac{2}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^3 + \begin{bmatrix} \frac{1}{3}\mathbf{I}_1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^4$$

The alternative model is given by

$$\begin{pmatrix} y^m_{1,t} \\ y^m_t \end{pmatrix} = \begin{pmatrix} \mu^m_1 \\ \mu^m_y \end{pmatrix} + \begin{pmatrix} \beta_{11}(\frac{1}{3}f_{1,t} + \frac{2}{3}f_{1,t-1} + f_{1,t-2} + \frac{2}{3}f_{1,t-3} + \frac{1}{3}f_{1,t-4}) \\ \beta_{12}f_{1,t} \end{pmatrix} + \begin{pmatrix} \frac{1}{3}e_{1,t} + \frac{2}{3}e_{1,t-1} + e_{1,t-2} + \frac{2}{3}e_{1,t-3} + \frac{1}{3}e_{1,t-4} \\ e^m_t \end{pmatrix} \tag{2.16}$$

The state-space representation when $p_1, q_1 \leq 5$ is given by

$$\mathbf{s}^m_{1,t} = \mathbf{F}^m_1\mathbf{s}^m_{1,t-1} + \mathbf{G}^m_1\mathbf{z}_{1,t} \tag{2.17}$$

$$\mathbf{y}^m_{1,t} = \boldsymbol{\mu}^m_1 + \mathbf{H}^m_1\mathbf{s}^m_t \tag{2.18}$$

$$\{\mathbf{z}_{1,t}\}\tilde{\ } IN(\mathbf{0}, \mathbf{I}_{N_1+1})$$

where

$$\mathbf{s}^m_{1,t} = \begin{pmatrix} f_{1,t} \\ \vdots \\ f_{1,t-4} \\ u_{1,t} \\ \vdots \\ u_{1,t-4} \end{pmatrix}$$

$$\mathbf{F}^m_1 = \begin{bmatrix} \phi^f_1(1)\cdots\phi^f_1(p_1) \quad \mathbf{o}'_{5-p_1} & \mathbf{O}_{5\times 5(N_1+1)} & \\ \mathbf{I}_4 \quad \mathbf{o}_4 & & \\ & \Phi^e_1(1)\cdots\Phi^e_1(q_1) & \mathbf{O}_{(N_1+1)\times(5-q_1)(N_1+1)} \\ \mathbf{O}_{5(N_1+1)\times 5} & \mathbf{I}_{4(N_1+1)} & \mathbf{O}_{4(N_1+1)\times(N_1+1)} \end{bmatrix}$$

$$\mathbf{G}^m_1 = \begin{bmatrix} \sigma_1 & \mathbf{o}_{(N_1+1)} \\ \mathbf{o}_4 & \mathbf{O}_{4\times(N_1+1)} \\ \mathbf{o}_{(N_1+1)} & \boldsymbol{\Sigma}^{1/2}_{22} \\ \mathbf{o}_{4(N_1+1)} & \mathbf{O}_{4(N_1+1)\times(N_1+1)} \end{bmatrix}$$

$$\mathbf{H}^m_1 = \begin{bmatrix} \mathbf{J}^m_1(0)\boldsymbol{\beta}_1 & \cdots & \mathbf{J}^m_1(4)\boldsymbol{\beta}_1 & \mathbf{J}^m_1(0) & \cdots & \mathbf{J}^m_1(4) \end{bmatrix}$$

Note that in the alternative model, the estimated factor $\hat{f}_{1,t}$ is in medium frequency, which make the standard VAR in second step no longer applicable. Instead, we adopt mixed-frequency VAR in the second step. Recall (2.1) with the estimated factors plugged in:

$$\begin{bmatrix} \hat{f}_{1,t} \\ \hat{f}_{2,t} \\ \hat{f}_{3,t} \\ y_t \end{bmatrix} = \phi(L) \begin{bmatrix} \hat{f}_{1,t-1} \\ \hat{f}_{2,t-1} \\ \hat{f}_{3,t-1} \\ y_t \end{bmatrix} + \mathbf{v}_t \tag{2.19}$$

Let $y^f_{1,t} = \Delta_4 \ln \hat{f}_{1,t}$ and $y^{f*}_{1,t} = \Delta \ln \hat{f}^*_{1,t}$, where $\hat{f}^*_{1,t}$ is the weekly latent variable of $\hat{f}_{1,t}$ and $y^f_{2,t} = \Delta \ln \begin{pmatrix} \hat{f}_{2,t} \\ \hat{f}_{3,t} \\ y_t \end{pmatrix}$. Then we have

$$y^f_{1,t} = \frac{1}{3} y^{f*}_{1,t} + \frac{2}{3} y^{f*}_{1,t-1} + y^{f*}_{1,t-2} + y^{f*}_{1,t-3} + \frac{2}{3} y^{f*}_{1,t-4} + \frac{1}{3} y^{f*}_{1,t-5} \tag{2.20}$$

Let

$$\mathbf{y}_t = \begin{pmatrix} y^f_{1,t} \\ y^f_{2,t} \end{pmatrix}$$

$$\mathbf{y}^*_t = \begin{pmatrix} y^{f*}_{1,t} \\ y^f_{2,t} \end{pmatrix}$$

Define $\boldsymbol{\mu} = E(\mathbf{y}_t)$, $\boldsymbol{\mu}^* = E(\mathbf{y}^*_t)$the relationship between $\mathbf{y}_t$ and $\mathbf{y}^*_t$ is given by

$$\mathbf{y}_t - \boldsymbol{\mu} = \mathbf{J}(L)(\mathbf{y}^*_t - \boldsymbol{\mu}^*) \tag{2.21}$$

where

$$\begin{aligned} \mathbf{J}(L) &= \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^2 + \\ &\quad + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^3 + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^4 + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^5 \end{aligned}$$

Assume $\mathbf{y}_t^*$ follows Gaussian VAR($p$)

$$\boldsymbol{\phi}(L)(\mathbf{y}_t^* - \boldsymbol{\mu}^*) = \mathbf{w}_t, \mathbf{w}_t \tilde{} IN(\mathbf{0}, \boldsymbol{\Sigma}) \tag{2.22}$$

Let the state variable be

$$\mathbf{s}_t = \begin{pmatrix} \mathbf{y}_t^* - \boldsymbol{\mu}^* \\ \vdots \\ \mathbf{y}_{t-5}^* - \boldsymbol{\mu}^* \end{pmatrix}$$

The state-space representation when $p \leq 6$ is given by

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{z}_t \tag{2.23}$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}\mathbf{s}_t \tag{2.24}$$

$$\{\mathbf{z}_t\} \tilde{} IN(\mathbf{0}, \mathbf{I}_{K+3})$$

where

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\phi}_1 & \cdots & \boldsymbol{\phi}_p & \mathbf{O}_{(K+3)\times(6-p)(K+3)} \\ & \mathbf{I}_{5(K+3)} & & \mathbf{O}_{5(K+3)\times(K+2)} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \boldsymbol{\Sigma}^{1/2} \\ \mathbf{O}_{5(K+3)\times(K+3)} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{J}(0) & \cdots & \mathbf{J}(5) \end{bmatrix}$$

For all the mixed-frequency models above, the lower frequency series are not always observable. Follow Mariano and Murasawa (2003), we replace the missing observations with random variable $\epsilon_t \tilde{} N(0,1)$ which has a realization of 0 and adjust the rest of the measurement equation accordingly. For example, the measurement equation (2.24) can be written as

$$\begin{pmatrix} y^f_{1,t} \\ \\ y^f_{2,t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}(1) \\ \\ \boldsymbol{\mu}(2) \end{pmatrix} + \begin{pmatrix} \mathbf{C}(1) \\ \\ \mathbf{C}(2) \end{pmatrix} \mathbf{s}_t \tag{2.25}$$

With the missing values replaced, the measurement equation becomes

$$\mathbf{y}^+_t = \boldsymbol{\mu}_t + \mathbf{C}_t\mathbf{s}_t + \mathbf{D}_t\epsilon_t \tag{2.26}$$

where

$$\begin{aligned} \mathbf{y}^+_t &= \begin{pmatrix} y^{f+}_{1,t} \\ \\ y^f_{2,t} \end{pmatrix}, \boldsymbol{\mu}_t = \begin{pmatrix} \boldsymbol{\mu}_t(1) \\ \\ \boldsymbol{\mu}_t(2) \end{pmatrix} \\ \mathbf{C}_t &= \begin{pmatrix} \mathbf{C}_t(1) \\ \\ \mathbf{C}_t(2) \end{pmatrix}, \mathbf{D}_t = \begin{pmatrix} \mathbf{D}_t(1) \\ \\ \mathbf{O} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} y^{f+}_{1,t} &= \{ \begin{matrix} y^f_{1,t} \text{ if } \mathbf{y}_t \text{ observed} \\ \\ \epsilon_t \text{ if } \mathbf{y}_t \text{ not observed} \end{matrix}, \boldsymbol{\mu}_t(1) = \{ \begin{matrix} \boldsymbol{\mu}(\mathbf{1}) \text{ if } \mathbf{y}_t \text{ observed} \\ \\ \mathbf{0} \text{ if } \mathbf{y}_t \text{ not observed} \end{matrix} \\ \mathbf{C}_t(1) &= \{ \begin{matrix} \mathbf{C}(\mathbf{1}) \text{ if observed} \\ \\ \mathbf{O} \text{ if not observed} \end{matrix}, \mathbf{D}_t(1) = \{ \begin{matrix} \mathbf{D}(1) \text{ if observed} \\ \\ 1 \text{ if not observed} \end{matrix} \end{aligned}$$

The state-space representation with missing values replaced is given by

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{z}_t \tag{2.27}$$

$$\mathbf{y}^+_t = \boldsymbol{\mu}_t + \mathbf{C}_t\mathbf{s}_t + \mathbf{D}_t\epsilon_t \tag{2.28}$$

$$\left\{ \begin{pmatrix} \mathbf{z}_t \\ \\ \epsilon_t \end{pmatrix} \right\} \tilde{}\, IN(\mathbf{0}, \mathbf{I}_{K+4})$$

Now that $\left\{\mathbf{y}^+_t\right\}$ has no missing values, the Kalman filter can apply directly.

## 2.3 Application

### 2.3.1 Data

We apply the mixed-frequency FAVAR model to US economy data to construct a measurement of monetary transmission mechanism. The series we use are of three frequencies sampled from July 1987 to December 2015. Quarterly indicators are "slow-moving" variables that measure the real economy activities. Monthly indicators are chosen to be variables that reflects the loan and credit change in financial intermediates. Weekly indicators are mostly "fast-moving" return rates that reflects the financial market movement. Note that for each month we may have either four or five weekly observations, we made the following adjustment for months of the latter case. Let $\{x_t, ...x_{t+4}\}$ be the five weekly observations in a month. The adjusted observations are given by

$$\{\frac{1}{2}(x_t + x_{t+1}), \frac{1}{2}(x_{t+1} + x_{t+2}), \frac{1}{2}(x_{t+2} + x_{t+3}), \frac{1}{2}(x_{t+3} + x_{t+4})\}$$

Since weekly indicators are return rates in level, the average of adjacent observations could be considered as pseudo observation over the this time period. This will guarantee us four weekly observations every month.

All the series are selected based on three criterion: whether the movement speed of variable matches its frequency of being observed; whether the variable fits in the potential process of monetary transmission mechanism; whether the variable is available within the sample period. Since the estimation is very time-consuming, we carefully restrict our number of variables and time window within an acceptable range. Table 1 summarizes the detailed descriptions of the series. "SA" stands for "seasonally adjusted", "NSA" stands for "not seasonally adjusted" and "AR" stands for "annual rate". All data are directly downloaded from FRED, except Divisia M4 is provided by Center for Financial Stability.

Table 2 summarizes the descriptive statistics of the standardize indicators. The transformation codes are: 1 – no transformation; 2 – first difference; 5 – first difference of logarithm. Since some of the series experienced structural break for regulation reasons, we adopted 1% winsorization to maintain stationarity. As mentioned above, we use the alternative model in our application. The monthly federal fund rate are skip-sampled from original weekly series.

### 2.3.2 Estimated Results

The number of lags for AR process are determined using information criterion AIC and SBIC same in Mariano and Murasawa (2003):

$$AIC = -\frac{1}{T}\{\ln L(\hat{\theta}) - [(N-1) + p + 1 + N(q+1)]\} \tag{2.29}$$

$$SBIA = -\frac{1}{T}\{\ln L(\hat{\theta}) - \frac{\ln T}{2}[(N-1) + p + 1 + N(q+1)]\} \tag{2.30}$$

The selected model are $(p_1, q_1) = (2, 1)$, $(p_2, q_2) = (1, 1)$ and $p = 1$. The number of factors are in the principal component is chosen to be $K = 2$.

We followed the standard literature to have the series demeaned so all the constant terms in the models are deleted. Using Ox 7.10 and code modified from Mariano and Murasawa (2003, 2010), the approximate ML estimator could be estimated using quasi-Newton method. Table 3 and 4 summarized the estimated result of the mixed-frequency factor model in the first step.

The estimated factors from quarterly indicators and monthly indicators are shown in Figure 1 and 4 respectively. We can observe that factor estimated from quarterly indicators is more active during the recession while factor estimated from monthly indicator is more active prior to the recession. This is consistent with our assumption that credit market variables in stage 2 react faster to monetary policy change than real macro variables in stage 3. We can construct the coincident index at this stage by taking the partial sum for each series, shown in Figure 2 and 5. With the shaded area being NBER recession dates, we can see the quarterly index has significant leading signal on the recessions in 2001 and 2007. Monthly index, on the other hand has a spike during the 2009 recession. Figure 3 and 6 are smoothed coincident indices from Figure 2 and 5. Figure 7 and 8 show the first two principal components of weekly indicators. Both principal components have greater deviation from average during the recession periods.

The having all the estimated factors plugged in, the monetary linkage factor is estimated in the second stage, which is the weekly latent factor estimated from quarterly indicators, shown in figure 9. Again by taking the partial sum, the constructed coincident index is shown in figure 10. The coincident index is meant to capture the common factor component of real economy variables and federal fund rate in weekly frequency. The index exhibits clear leading signal for all the recessions in our sample period. Note that at the end of 2007 recession, the index is almost flat by the fact that the effective federal fund rate was approaching the Zero Lower Bound. Figure 11 is the smoothed verion of Figure 10 to show the trend in a neat way.

To our best knowledge, there is no other coincident index that we know of that we can compare our coincident index with. Therefore, we run a simple exercise following time-varing-parameter model by Kim and Nelson (1989). Consider the following model

$$d\log(GDP_t) = \beta_{0,t} + \beta_{1,t} dffr + \beta_{2,t} d\log(GDP_{t-1}) + e_t \tag{2.31}$$

$$\beta_{i,t} = \beta_{i,t-1} + v_{i,t}, i = 0, 1, 2 \tag{2.32}$$

$$\left\{ \begin{pmatrix} e_t \\ v_t \end{pmatrix} \right\} \tilde{} IN(\mathbf{0}, \mathbf{\Sigma})$$

where $d\log(GDP_t)$ denotes the growth rate of real GDP and $dffr$ is the first difference of effective federal fund rate. All the data are of quarterly frequency. The estimated time-varying parameter of our interest $\beta_{1,t}$ is shown in figure 12. The coefficient of federal fund rate appear to deviate more from period average during the recessions. Although this model is simple and far from being complete and correct, it shows some strange behavior of monetary transmission mechanism during the recessions which is consistent with our result to some degree.

### 2.3.3 Policy Implications

There are two implications from our coincident index that we could think of. Firstly, our index supports the view that the monetary transmission mechanism has been changing over time. Over the last two decades, the way in which financial market operates, monetary policy implements and information being processed has experienced significant revolutions. The two major peaks in our index around 1999 and 2007 could be explained by the rapid expansion of financial markets as the fluctuations before 1995 are too small to be comparable. The index stays low after 2008, implying that some major traditional channels of monetary transmission mechanism have been shutting down when the effective federal fund rate stays around Zero Lower Bound. Secondly, the clear pattern that the index reaches local peaks before the recessions and declines during the recessions shows that during the recession the monetary transmission mechanism may not function well as they do in normal times. Various reasons could provide intuition for that: the spread of panic sentiment; overconservative behavior of financial intermediates for example.

In conclusion, the coincident index we construct measures the linkage between federal fund rate and real economy variables in weekly frequency. The most significant feature is that our coincident index provides some leading signal for the recessions in our sample period. Such index could be considered as a measure of monetary transmission mechanism for policy makers.

## 2.4 Conclusion

This paper studies the monetary transmission mechanism in the U.S. It proposes a mixed-frequency version of the factor-augmented vector autoregressive regression (FAVAR) model, which is used to construct a coincident index to measure the monetary transmission mechanism. The model divides the transmission of changes in monetary policy to the economy into three stages according to the timing and order of the impact. Indicators of each stage are measured and identified using different data frequencies: fast-moving variables (stage 1, asset returns at the weekly frequency), intermediate moving variables (stage 2, credit market data at the monthly frequency), and slow-moving variables (stage 3, macroeconomic variables at the quarterly frequency). The resulting coincident index exhibits leading signal for all recessions in the sample period and provides implications on the dynamics of the monetary transmission mechanism. The proposed coincident index also indicates that monetary transmission mechanism is changing over time.

# Bibliography


[1] Baffigi, Alberto, Roberto Golinelli, and Giuseppe Parigi. "Bridge models to forecast the euro area GDP." International Journal of forecasting 20.3 (2004): 447-460.

[2] Bai, Jennie, Eric Ghysels, and Jonathan H. Wright. "State space models and MIDAS regressions." Econometric Reviews 32.7 (2013): 779-813.

[3] Bai, Jushan, and Serena Ng. "Determining the number of factors in approximate factor models." Econometrica 70.1 (2002): 191-221.

[4] Bai, Jushan, and Serena Ng. "Confidence intervals for diffusion index forecasts and inference for factor-Augmented regressions." Econometrica 74.4 (2006): 1133-1150.

[5] Bai, Jushan, and Serena Ng. "Determining the number of primitive shocks in factor models." Journal of Business & Economic Statistics 25.1 (2007): 52-60.

[6] Barnett, William A., Marcelle Chauvet, and Danilo Leiva-Leon. Real-time nowcasting of nominal GDP under structural breaks. No. 2014-39. Bank of Canada Working Paper, 2014.

[7] Beck, Thorsten, Andrea Colciago, and Damjan Pfajfar. "The role of financial intermediaries in monetary policy transmission." Journal of Economic Dynamics and Control 43 (2014): 1-11.

[8] Bernanke, B., and Gertler, M., 1989. Agency Costs, Net Worth and Business Fluctuations. American Economic Review 79, 14-31.

[9] Bernanke, Ben S., Jean Boivin, and Piotr Eliasz. Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. No. w10220. National Bureau of Economic Research, 2004.

[10] Boivin, Jean, Michael T. Kiley, and Frederic S. Mishkin. How has the monetary transmission mechanism evolved over time?. No. w15879. National Bureau of Economic Research, 2010.

[11] Bry, Gerhard, and Charlotte Boschan. "Front matter to" Cyclical Analysis of Time Series: Selected Procedures and Computer Programs"." Cyclical Analysis of Time Series: Selected Procedures and Computer Programs. NBER, 1971. 13-2.

[12] Buch, Claudia M., Sandra Eickmeier, and Esteban Prieto. "In search for yield? Survey-based evidence on bank risk taking." Journal of Economic Dynamics and Control 43 (2014): 12-30.

[13] Carpenter, S.B., Demiralp, S., Eisenschmidt, J., 2014. The effectiveness of nonstandard monetary policy in addressing liquidity risk during the financial crisis: The experiences of the Federal Reserve and the European Central Bank. J. Econ. Dyn. Control

[14] Christiano, Lawrence J., and Martin Eichenbaum. Liquidity effects and the monetary transmission mechanism. No. w3974. National Bureau of Economic Research, 1992.

[15] Christiano, Lawrence J., and Martin Eichenbaum. "Liquidity effects, monetary policy, and the business cycle." Journal of Money, Credit and Banking (1995): 1113-1136.

[16] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Monetary policy shocks: What have we learned and to what end?." Handbook of macroeconomics 1 (1999): 65-148.

[17] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. Two reasons why money and credit may be useful in monetary policy. No. w13502. National Bureau of Economic Research, 2007.

[18] Frale, Cecilia, et al. "EUROMIND: a monthly indicator of the euro area economic conditions." Journal of the Royal Statistical Society: Series A (Statistics in Society) 174.2 (2011): 439-470.

[19] Foroni, Claudia, and Massimiliano Giuseppe Marcellino. "A survey of econometric methods for mixed-frequency data." (2013).

[20] Ghysels, Eric. "Macroeconomics and the reality of mixed frequency data." Journal of Econometrics 193.2 (2016): 294-314.

[21] Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov. "The MIDAS touch: Mixed data sampling regression models." Finance (2004).

[22] Harding, Don, and Adrian Pagan. "Synchronization of cycles." Journal of Econometrics 132.1 (2006): 59-79.

[23] He, Zhiguo, and Arvind Krishnamurthy. A macroeconomic framework for quantifying systemic risk. No. w19885. National Bureau of Economic Research, 2014.

[24] Kashyap, A.K., Stein, J.C., 1995. The impact of monetary policy on bank balance sheets. Carnegie-Rochester Conf. Ser. Public Policy 42(1), 151–195.

[25] Killian, Lutz (1996), 'Small - Sample Conference Intervals for Impulse Response Functions', manuscript, University of Pennsylvania.

[26] Kim, Chang-Jin, and Charles R. Nelson. "The time-varying-parameter model for modeling changing conditional variance: The case of the lucas hypothesis." Journal of Business & Economic Statistics 7.4 (1989): 433-440.

[27] Kim, Chang-Jin, and Charles R. Nelson. "Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching." The Review of Economics and Statistics 80.2 (1998): 188-201.

[28] Kim, Chang-Jin, and Charles R. Nelson. "State-space models with regime switching." (1999).

[29] King, Robert, et al. "Stochastic trends and economic fluctuations." (1987).

[30] King, Robert G. (1991), 'Money and Business Cycles', manuscript, University of Rochester.

[31] Mariano, Roberto S., and Yasutomo Murasawa. "A new coincident index of business cycles based on monthly and quarterly series." Journal of applied Econometrics 18.4 (2003): 427-443.

[32] Mariano, Roberto S., and Yasutomo Murasawa. "A coincident index, common factors, and monthly real GDP." Oxford Bulletin of Economics and Statistics 72.1 (2010): 27-46.

[33] Nicolo G.D., Dell'Ariccia, G., Laeven, L., Valencia, F., 2010. Monetary Policy and Bank Risk-Taking. IMF Staff Position Notes 2010/09, International Monetary Fund.

[34] Schorfheide, Frank, and Dongho Song. "Real-time forecasting with a mixed-frequency VAR." Journal of Business & Economic Statistics 33.3 (2015): 366-380.

[35] Sims, Christopher A. "Macroeconomics and reality." Econometrica: Journal of the Econometric Society (1980): 1-48.

[36] Stock, James H., and Mark W. Watson. "A probability model of the coincident economic indicators." (1988).

[37] Stock, James, and Mark Watson, "New Indexes of Coincident and Leading Economic Indicators", NBER Macroeconomics Annual (4), Cambridge MA: MIT Press, 1989.

[38] Stock, James H., and Mark W. Watson. Diffusion indexes. No. w6702. National Bureau of Economic Research, 1998.

[39] Stock, James H., and Mark W. Watson. "Estimating turning points using large data sets." Journal of Econometrics 178 (2014): 368-381.

[40] Taylor, John B. "The monetary transmission mechanism: an empirical framework." The Journal of Economic Perspectives 9.4 (1995): 11-26.

[41] Tobin, J., 1969. A general equilibrium approach to monetary theory. J. Money Credit Bank. 1 (1), 15–29.

## 2.5 Tables and Figures

Table 2.1: US MonetaryTtransmission Mechanism Indicators

| Indicator | Description |
|---|---|
| | Quarterly |
| outbs | Business Sector: Real Output (2009=100,SA) |
| outmdgs | Manufacturing Durable Goods Sector: Real Output (2009=100,SA) |
| outms | Manufacturing Sector: Real Output (2009=100,SA) |
| outnfb | Nonfarm Business Sector: Real Output (2009=100,SA) |
| pcec | Personal Consumption Expenditures (SAAR) |
| pcdg | Personal Consumption Expenditures: Durable Goods (SAAR) |
| pcend | Personal Consumption Expenditures: Nondurable Goods (SAAR) |
| pcesv | Personal Consumption Expenditures: Services ( BIL 09$, SAAR) |
| gdpc | Real Gross Domestic Product (BIL 09$, SAAR) |
| | Monthly |
| busloans | Commercial and Industrial Loans, All Commercial Banks (BIL$, SA) |
| consumer | Consumer Loans at All Commercial Banks (BIL$, SA) |
| iblacbm027sbog | Interbank Loans, All Commercial Banks (BIL$, SA) |
| invest | Securities in Bank Credit at All Commercial Banks (BIL$, SA) |
| lcbacbm027sbog | Loans to Commercial Banks , All Commercial Banks (BIL$, SA) |
| loaninv | Bank Credit at All Commercial Banks (BIL$, SA) |
| loans | Loans and Leases in Bank Credit, All Commercial Banks (BIL$, SA) |
| dm4 | CFS Divisia M4 (PERCENTAGE GROWTH RATE) |
| ollacbm027sbog | Other Loans and Leases, All Commercial Banks (BIL$, SA) |
| olracbm027sbog | Other Loans and Leases: Fed Funds and Reverse RPs with Nonbanks All Commercial Banks (BIL$, SA) |
| realln | Real Estate Loans, All Commercial Banks (BIL$, SA) |
| | Weekly |
| wgs10yr | 10-Year Treasury Constant Maturity Rate (PERCENT, NSA) |
| wgs5yr | 5-Year Treasury Constant Maturity Rate (PERCENT, NSA) |
| wgs1yr | 1-Year Treasury Constant Maturity Rate (PERCENT, NSA) |
| wgs6mo | 6-Month Treasury Constant Maturity Rate (PERCENT, NSA) |
| wgs3mo | 3-Month Treasury Constant Maturity Rate (PERCENT, NSA) |
| waaa | Moody's Seasoned Aaa Corporate Bond Yield (PERCENT, NSA) |
| wbaa | Moody's Seasoned Baa Corporate Bond Yield (PERCENT, NSA) |
| ff | Effective Federal Funds Rate (PERCENT, NSA) |

Table 2.2: Descriptive Statistics for Standarized Indicators

| Indicator | Transformation Code | Mean | Std. Dev. | Max. | Min. |
|---|---|---|---|---|---|
| | | | Quarterly | | |
| outbs | 5 | 0.006945 | 0.007965 | 0.021729 | -0.029583 |
| outmdgs | 5 | l 0.006356 | 0.023152 | l 0.048905 | -0.121451 |
| outms | 5 | 0.005168 | 0.018717 | 0.044756 | -0.101291 |
| outnfb | 5 | 0.006943 | 0.008171 | 0.022858 | -0.031514 |
| pcec | 5 | 0.012201 | 0.006499 | 0.025373 | -0.026552 |
| pcdg | 5 | 0.009660 | 0.021472 | 0.075043 | -0.082049 |
| pcend | 5 | 0.010427 | 0.013082 | 0.032440 | -0.077902 |
| pcesv | 5 | 0.013379 | 0.005305 | 0.027369 | -0.005832 |
| gdpc | 5 | 0.006228 | 0.006114 | 0.018709 | -0.021352 |
| | | | Monthly | | |
| busloans | 5 | 0.003694 | 0.008671 | 0.035458 | -0.029041 |
| consumer | 5 | 0.004061 | 0.016825 | 0.270704 | -0.028779 |
| iblacbm027sbog | 5 | -0.002961 | 0.049922 | 0.233067 | -0.286793 |
| invest | 5 | 0.005275 | 0.008864 | 0.053845 | -0.027328 |
| lcbacbm027sbog | 5 | -0.004149 | 0.107698 | 0.327584 | -1.204621 |
| loaninv | 5 | 0.004946 | 0.005151 | 0.037096 | -0.011758 |
| loans | 5 | 0.004835 | 0.006042 | 0.046561 | -0.016713 |
| dm4 | 1 | 0.003669 | 0.004590 | 0.022875 | -0.017254 |
| ollacbm027sbog | 5 | 0.005503 | 0.017676 | 0.065324 | -0.075375 |
| olracbm027sbog | 5 | 0.007354 | 0.054965 | 0.242955 | -0.169333 |
| realln | 5 | 0.005720 | 0.006732 | 0.046481 | -0.015771 |
| | | | Weekly | | |
| wgs10yr | 2 | 5.190195 | 2.079093 | 10.11000 | 1.470000 |
| wgs5yr | 2 | 4.638075 | 2.369535 | 9.730000 | 0.590000 |
| wgs1yr | 2 | 3.675740 | 2.643454 | 9.780000 | 0.090000 |
| wgs6mo | 2 | 3.530155 | 2.631128 | 9.620000 | 0.030000 |
| wgs3mo | 2 | 3.380424 | 2.585795 | 9.370000 | 0.000000 |
| waaa | 2 | 6.544966 | 1.754359 | 10.73000 | 3.260000 |
| wbaa | 2 | 7.513075 | 1.727848 | 11.78000 | 4.360000 |
| ff | 1 | 3.580639 | 2.737047 | 9.950000 | 0.050000 |

Table 2.3: Estimation Result of Quarterly Indicators

| Indicators | $\beta_1$ | $\phi_1^f(1)$ | $\phi_1^f(2)$ | $\sigma_1^2$ | $\Phi_1^e$ | $\Sigma_{22}$ |
|---|---|---|---|---|---|---|
| $\Delta \ln pcdg$ | 0.29229 | | | | -0.83038 | 1.117 |
| $\Delta \ln pcend$ | 0.54445 | | | | -0.75218 | 0.70763 |
| $\Delta \ln pcesv$ | 0.31338 | | | | -0.59187 | 0.81089 |
| $\Delta \ln gdpc$ | 0.88695 | | | | -0.54397 | 0.011948 |
| $\Delta \ln outbs$ | 0.87986 | -0.53364 | 0.23524 | 1 | -0.60101 | 0* |
| $\Delta \ln outms$ | 0.28227 | | | | 0.7742 | 0.040229 |
| $\Delta \ln outnfb$ | 0.87501 | | | | -0.74225 | 0.016328 |
| $\Delta \ln pcec$ | 0.59798 | | | | -0.72114 | 0.67104 |
| $\Delta \ln outmdgs$ | 0.314 | | | | 0.65882 | 0.061898 |
| $\Delta \ln ff^m$ | 0.0013241 | | | | 0.99704 | 0.010551 |

Table 2.4: Estimation Result of Monthly Indicators

| Indicators | $\beta_2$ | $\phi_2^f$ | $\sigma_1^2$ | $\Phi_2^e$ | $\Sigma_{22}$ |
|---|---|---|---|---|---|
| $\Delta \ln busloans$ | 0.41631 | | | 0.80942 | 0.018899 |
| $\Delta \ln cons$ | 0.48247 | | | 0.58234 | 0.061351 |
| $\Delta \ln iblac$ | 0.14838 | | | -0.64904 | 0.79852 |
| $\Delta \ln invest$ | 0.07205 | | | -0.69312 | 0.83323 |
| $\Delta \ln lcbac$ | -0.017592 | | | -0.44282 | 0.63923 |
| $\Delta \ln loaninv$ | 0.93497 | -0.99857 | 1 | -0.99859 | 0* |
| $\Delta \ln loans$ | 1.0237 | | | -0.99915 | 0.021859 |
| $\Delta \ln ollac$ | 0.62066 | | | -0.17756 | 0.28383 |
| $\Delta \ln olrac$ | 0.39323 | | | -0.70792 | 0.766 |
| $\Delta \ln \text{real}$ | 0.75587 | | | 0.6312 | 0.036107 |
| $\Delta \ln m4$ | -0.017237 | | | -0.54596 | 0.63936 |
| $\Delta \ln ff$ | 0.00033159 | | | 0.99906 | 0.0028935 |

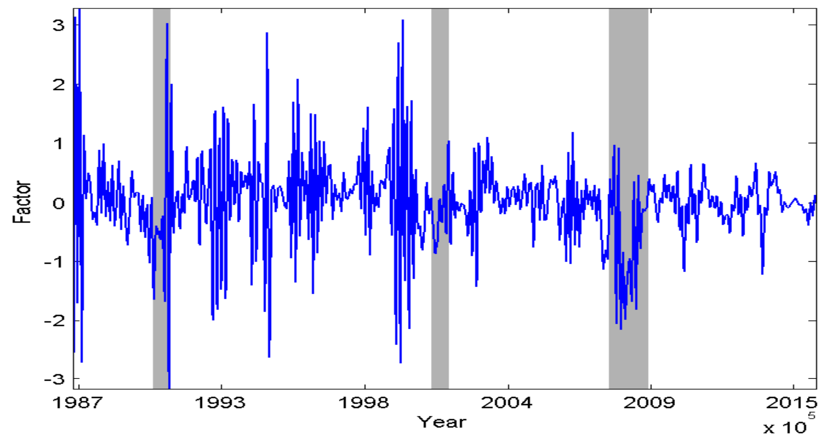


Figure 2.1: Estimated factor from quarterly indicators.

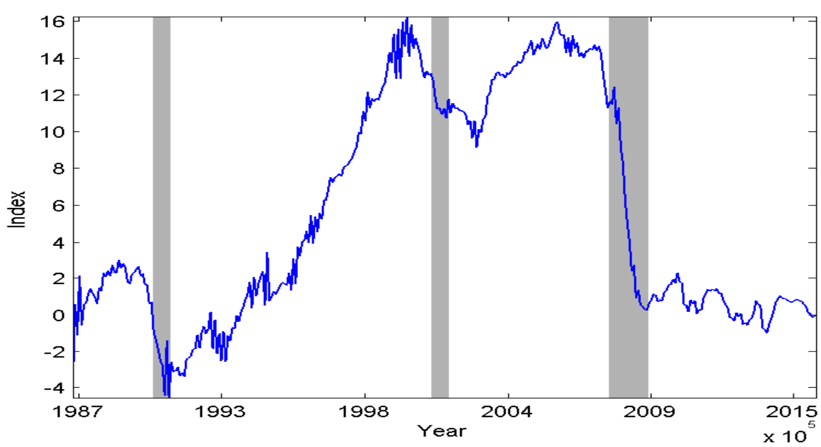


Figure 2.2: Coincident index constructed using quarterly indicator factor.

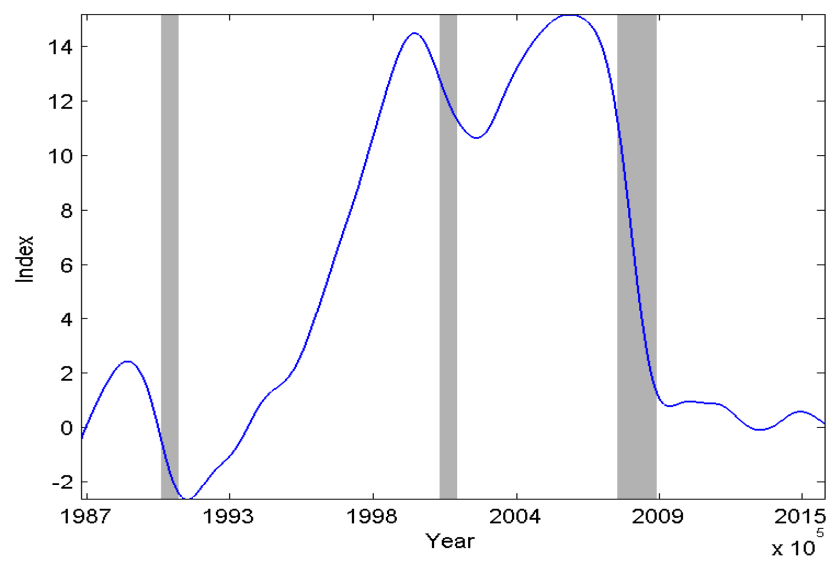


Figure 2.3: Smoothed coincident index constructed using quarterly indicator factor.

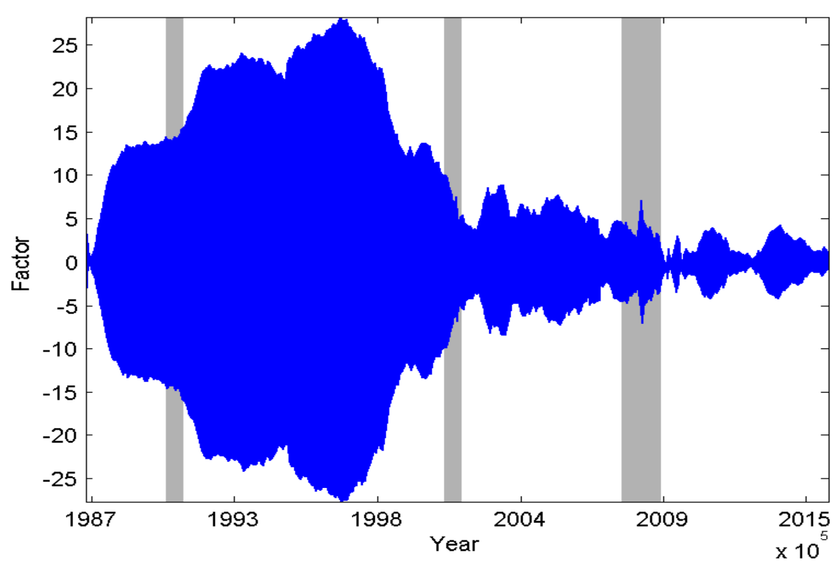


Figure 2.4: Estimated factor from monthly indicators.

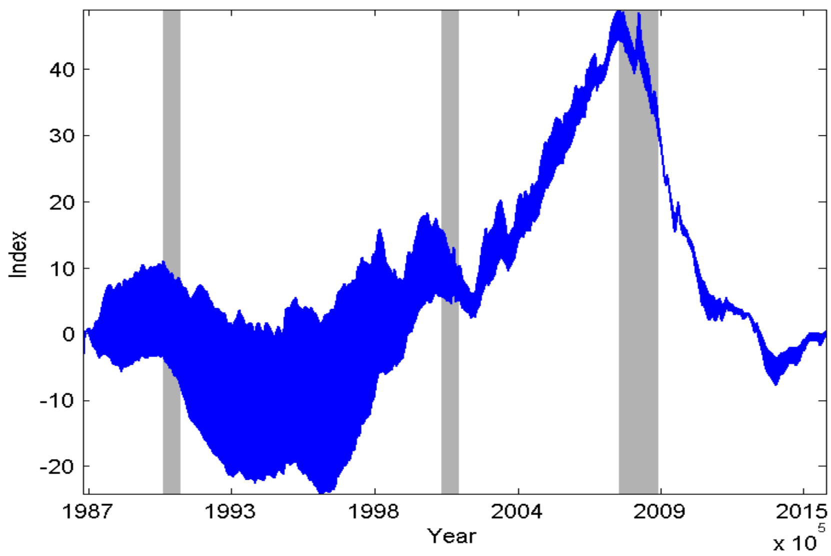


Figure 2.5: Coincident index constructed using the monthly indicator factor.

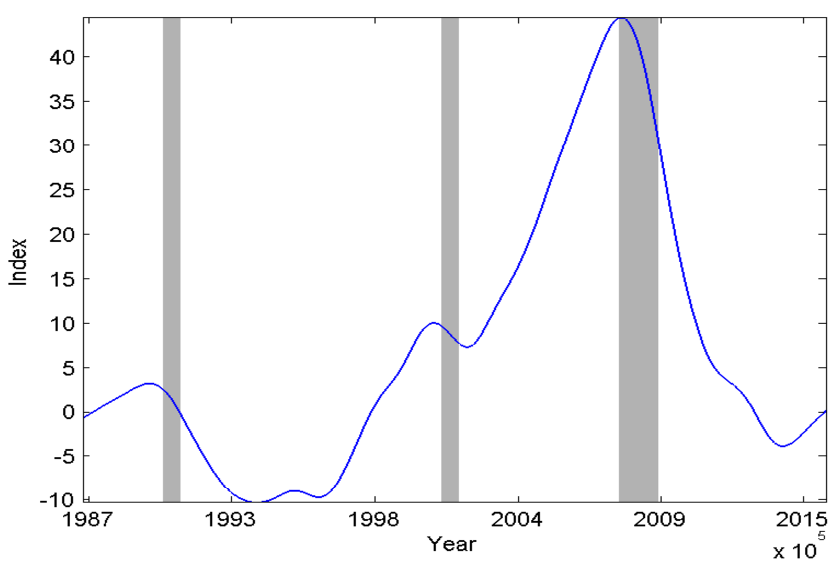


Figure 2.6: Smoothed coincident index constructed using the monthly indicator factor.

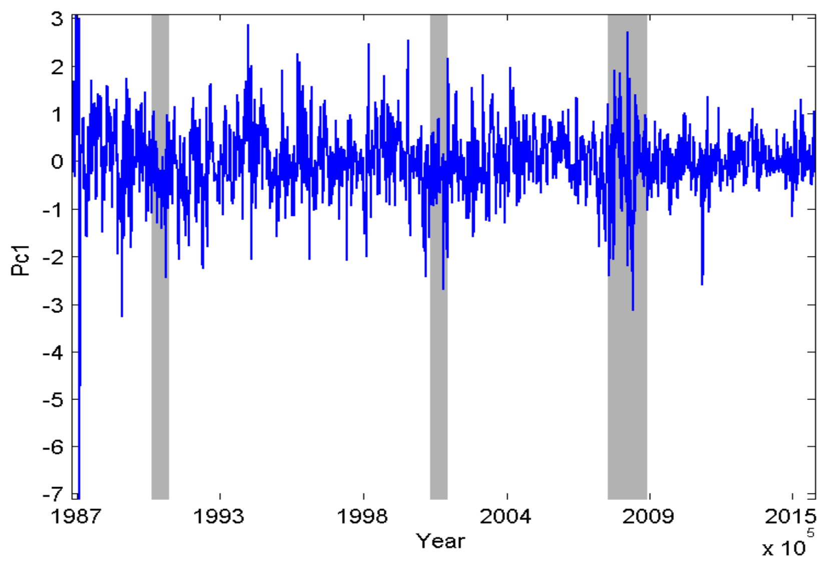


Figure 2.7: First principal component of weekly indicators.

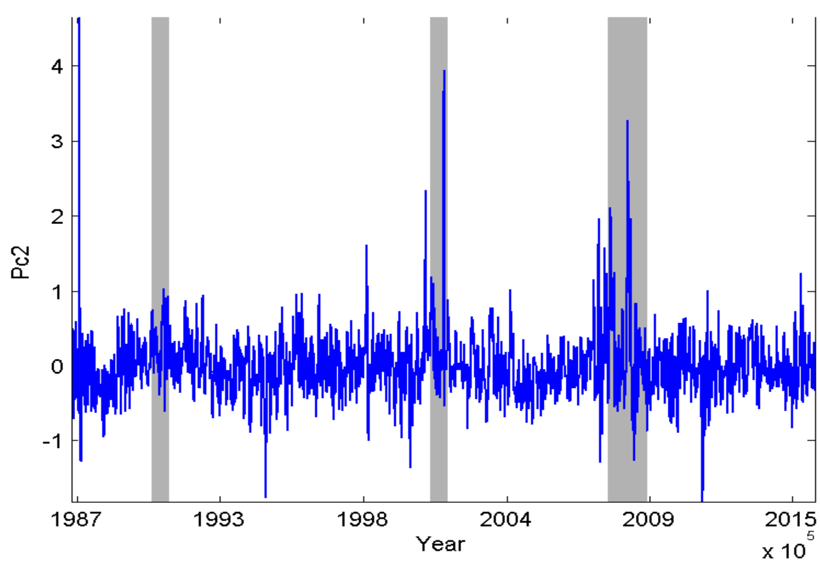


Figure 2.8: Second principal component of weekly indicators.

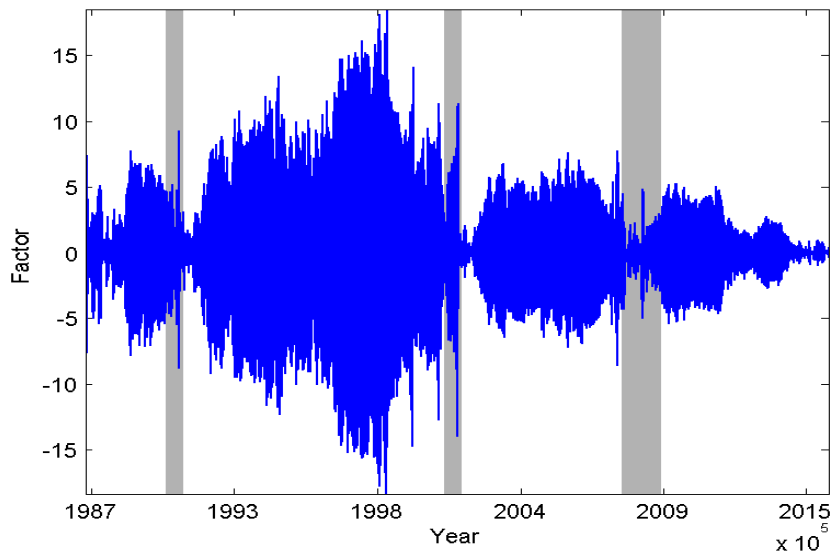


Figure 2.9: Estimated factor of monetary linkage.

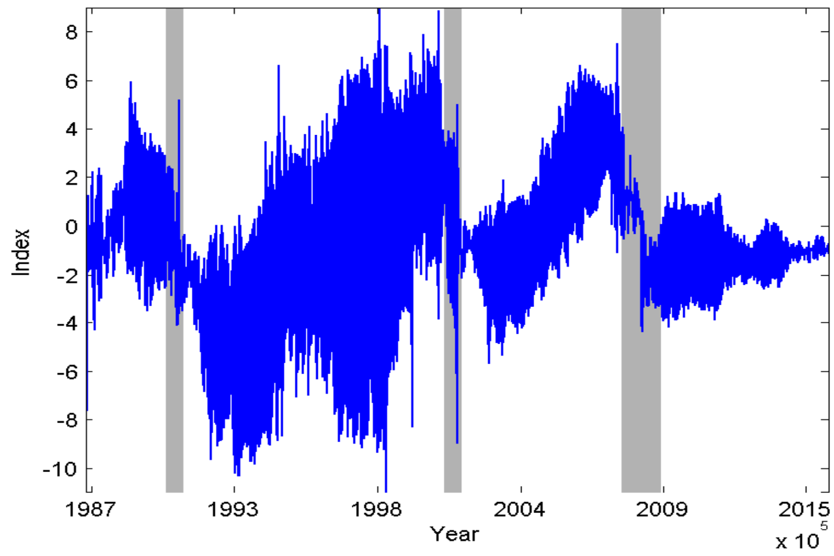


Figure 2.10: Coincident index constructed using estimated monetary linkage factor.

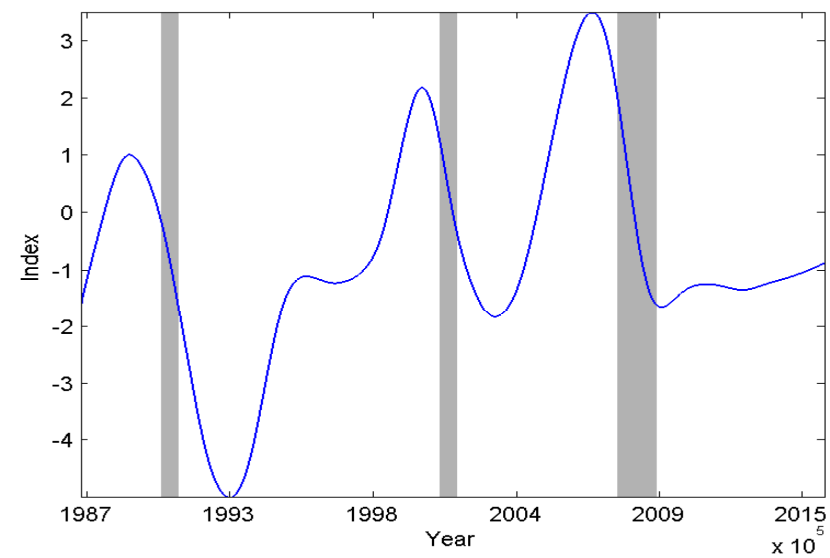


Figure 2.11: Smoothed coincident index constructed using estimated monetary linkage factor.

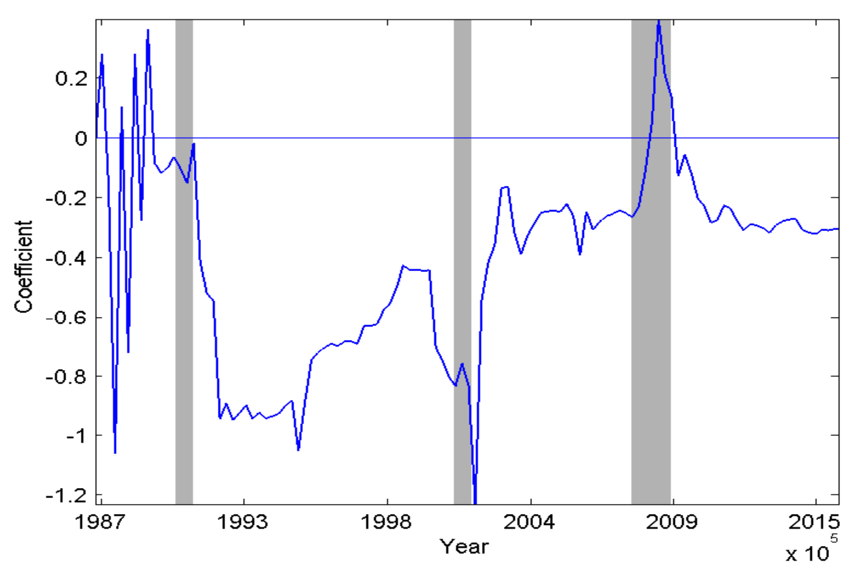


Figure 2.12: Estimated time-varing coefficient of monetary policy tool.

# Chapter 3

# Disentangling Monetary Transmission Mechanism at Zero Lower Bound

## 3.1 Introduction

Federal fund rate has been considered as the primary monetary policy instrument until December 2008 when it became close to zero. Unconventional monetary policy such as large-scale asset purchases was then introduced to serve as an alternative of conventional monetary policy. Therefore, understanding the transmission mechanism of unconventional monetary policy becomes crucial to the policy makers. As there exist many channels in the transmission mechanism of conventional monetary policy, the transmission mechanism of unconventional monetary policy is also not simple.

Unconventional monetary policy takes a variety of forms in reality. One of the most common form of it is central bank purchasing massive government bond, resulting an expansion of central bank's balance sheet. The central bank holding more assets means the economy getting more liquidity. One typical example is quantitative easing program (QE) implemented by Federal Reserve since 2008 financial crisis. Another form of unconventional monetary policy is referred as "Operation Twist", which is implemented by central bank selling short-term government bond and purchasing long-term government bond at equal amount. In this case the balance sheet of central bank remains unchanged, while this will lead to a compression in the difference between short-term and long-term interest rates.

There are two major channels of the transmission mechanism of unconventional monetary policy summarized in the literature. How these two channels work in the context of Bank of England purchasing gilts is well explained by Miles (2011, 2012). The first channel is portfolio substitution channel. It starts with central banking purchasing long-term assets from the market. The investors who sell long-term assets to the central bank get a relatively short-term asset, bank deposits, in return, which will largely reduce the duration of their original portfolio. Therefore, most investors would like to purchase some other relatively long-term assets, such as corporate bond, to rebalance the duration of assets in their portfolio. An increase in supply side of corporate bond market leads to a rise in the asset prices and a decline in bond yield. This will create a rather friendly environment for the firms to get financed in the credit market, leading to an increase in investment of firms, consumption of consumers.

The second channel is bank funding channel. When central bank purchase long-term asset from the market, for a typical individual band its bank deposits and reserve balance are likely to rise. This will lead to an expand in the bank loan to firms and therefore increase investment in of firms and consumption of consumers.

In the literature, numbers of studies have been focusing on how unconventional monetary policy affects the yield curve or yields on assets of different maturities. For instance, Hamilton and Wu (2012) adopted a model of risk-averse arbitrageurs to describe the relationship between debt hold by public and level, slope and curvature of yield curve; Wright (2012) used a structural VAR model and found stimulative monetary shocks lower bond yields, but such impacts are not very persistent. The introduction of shadow interest rate model allow the researchers to analyze the overall impact of unconventional monetary policy on the economy. Wu and Xia (2015) proposed a analytical representation of bond price in the original shadow rate term structure model (SRTSM) and use it in a factor-augumented vector autoregressive (FAVAR) model to extend the previous literature to the period during Zero Lower Bound period. Lomvardi and Zhu (2015) included a large number of monetary policy indicators in a dynamic factor model and constructed a more data-driven version of shadow rate.

While there are a lot of studies focusing on the measuring the impact of unconventional monetary policy on the macro economic variables, very few attempts have been made to the measure the transmission mechanism itself. Plugging shadow interest rate in into standard structure VAR or FAVAR model may be sufficient to measure the impact of monetary policy shocks. However, as these model generally only include real macro economic variables and policy rate only, information on the underlying transmission mechanism is very limited. In contrast, analysis of the transmission mechanism of unconventional monetary policy generally requires to include a large array of intermediate variables measured in different frequency in one model.

In this paper, I incorporate the mixed-frequency factor-augumented vector autoregressive model with shadow rate term structure model and construct a set of coincident indices that measures the overall monetary transmission mechanism as well as individual channels in the mechanism. I propose to disentangle the monetary transmission mechanism into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. In the model I reestimate the shadow rate term structure model in a higher frequency and plug the result into a extended version of fixed-frequency factor-augument vector autoregressive model that yields as result not only a coincident index that measures the overall monetary transmission mechanism but also a map of impulse response functions that depicts the transmission process through different channels.

The proposed empirical model is a combination of several models in the existing literature. Factor-augumented vector autoregressive model by Bernanke, Boivin and Eliasz (2005) allows the model for large number of variables while maintain the general framework of VAR analysis. Mixed-frequency dynamic factor model and mixed-frequency VAR model by Mariano and Murasawa(2003, 2010) are featured with a assumption on the dynamic of the latent variable of low-frequency indicator and result in a high-frequency estimate of low-frequency indicator. Mixed-frequency (stacked) VAR by Ghysels (2016) adopts a stacked vector system and incorporates the standard impulse response function analysis into a mixed-frequency context.

The mixed-frequency factor autoregressive regression model is estimated in similar two-step procedure as in standard FAVAR model. In the first step, coincident indices of each channel are estimated using mixed-frequency dynamic factor model individually. The estimation in second step takes two forms, both include the policy rate as well as the coindent indices from the first stage. The first form is mixed-frequency vector autoregressive model that results in a high-frequency estimate of the low-frequency index that measures the overall monetary transmission mechanism in U.S. The second form takes a stacked vector autoregressive model that produce a map of impulse response functions of the indices of different channels to shocks in policy rates.

The rest of the paper is structured as follows. Section two presents the empirical model we use: shadow rate term structure model used to construct the high-frequency shadow rates, mixed-frequency factor-augmented vector autoregressive regression model and the corresponding two-stage ML estimation processes. Section three applies the model to U.S. macroeconomic and financial data to construct the coincident index of overall monetary transmission mechanism and obtain the impulse response functions of the indices of different channels to shocks in policy rates. Section four concludes.

## 3.2 The Model

### 3.2.1 The Shadow Rate Term Structure Model

In order to consider unconventional monetary policy during the Zero Lowere Bound period, I need to construct shadow interest rate as policy rate that measures the overall monetary policy. I choose to follow Wu and Xia (2015) shadow rate term structure model for the reason that the model could be reestimated so that resulting shadow rate can have higher frequency.

Following the standard Shadow Rate Term Structure Model, assume nominal short-run interest rate $r_t$ is equal to shadow rate $s_t$ when shadow rate is above the lower bound $\underline{r}$ and nominal short-run interest rate is set to stay at its lower bound when shadow rate is below the lower bound. Formally,

$$r_t = \max(\underline{r}, s_t)$$

The shadow rate $s_t$ is assumed to be a function of some state variable $X_t$, which follows a AR(1) process:

$$s_t = \delta_0 + \delta_1' X_t$$

where

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}$$

$$\varepsilon_{t+1} \tilde{} N(0, I)$$

At time t, the forward rate of a loan starting at t+n and maturing at t+n+1 is defined as

$$f_{n,n+1,t} = (n+1)y_{n+1,t} - ny_{n,t}$$

where $y_{n+1,t}$ and $y_{n,t}$ are yields on risk-free pure discount bonds at $n+1$ and $n$ period respectively.

Wu and Xia (2015) proposed the following analytical approximation for the forward rate in the shadow rate term structure model:

$$f^{SRTSM}_{n,n+1,1} = \underline{\mathrm{r}} + \sigma^Q_n g(\frac{a_n + b'_n X_t - \underline{\mathrm{r}}}{\sigma^Q_n})$$

where

$$\begin{aligned}
(\sigma^Q_n)^2 &= var^Q_t(s_{t+n}) \\
\bar{a}_n &= \delta_0 + \delta'_1(\sum_{j=0}^{n-1}(\rho^Q)^j)\mu^Q \\
a_n &= \bar{a}_n - \frac{1}{2}\delta'_1(\sum_{j=0}^{n-1}(\rho^Q)^j)\Sigma\Sigma'(\sum_{j=0}^{n-1}(\rho^Q)^j)'\delta_1 \\
b'_n &= \delta'_1(\rho^Q)^n
\end{aligned}$$

The state space representation of forward rate in shadow rate term structure model is straight forward. The measurement equation is

$$\begin{aligned}
f^o_{n,n+1,1} &= \underline{\mathrm{r}} + \sigma^Q_n g(\frac{a_n + b'_n X_t - \underline{\mathrm{r}}}{\sigma^Q_n}) + \eta_{n,t} \\
& \eta_{n,t} \tilde{} N(0,\omega)
\end{aligned}$$

where $f^o_{n,n+1,1}$ denotes the observed forward rate and $\eta_{n,t}$ is measurement error. The transition equation of the state variable is

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}$$
$$\varepsilon_{t+1} \tilde{} N(0, I)$$

The model could be estimated using extended Kalman Filter after linearizing function $g(.)$ around its current estimates.

### 3.2.2 The Mixed-Frequency Factor-augumented Vector Autoregressive Model

#### The General Set-up

The mixed-frequency factor-augumented vector autoregressive model allows a large number of variables with different frequencies in the same model. In our context of monetary transmission mechanism, consider $Y_t$ to be a $M \times 1$ vector of observable economic variables of interest that is driving the economy. In our application, it is the weekly frequency shadow rate constructed from shadow rate term structure model that measures the overall monetary policy. The monetary transmission mechanism is disentangled into three channels according to the transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. Each of the channel is summarized by unobservable factors of its own frequency. Let $f_{s,t}$, $f_{m,t}$ and $f_{f,t}$ denote the unobservable factors for slow-moving, medium-moving and fast-moving channel respectively, a three-stage mixed-frequency FAVAR model is given by

$$\begin{bmatrix} f_{s,t} \\ f_{m,t} \\ f_{f,t} \\ y_t \end{bmatrix} = \phi(L) \begin{bmatrix} f_{s,t-1} \\ f_{m,t-1} \\ f_{f,t-1} \\ y_t \end{bmatrix} + \mathbf{v}_t \tag{3.1}$$

where $\phi(L)$ is a lag polynomial of order $d$ and $v_t\tilde{}NID(0, Q)$.

Let $X_{s,t}$, $X_{m,t}$, $X_{f,t}$ be $N_s \times T_s$, $N_m \times T_m$, $N_f \times T_f$ informational data we observe in slow-moving, medium-moving and fast-moving channel respectively with $N_i$ being the number of variables and $T_i$ the number of observations $i = s, m, f$. The time length of one period in the model is set to be consistent with that of the highest frequency data, namely $X_{f,t}$. Therefore, $X_{f,t}$ is observed every period, while $X_{m,t}$ and $X_{s,t}$ is observed every $n$ and $m$ period where $m > n > 1$. In the case of $X_{f,t}$ being weekly data, $X_{m,t}$ being monthly data and $X_{s,t}$ being quarterly data, we can set $m = 12$ and $n = 4$.

To deal with data of different frequencies, we follow the method proposed by Mariano and Murasawa (2003). Let $X^*_{s,t}$ be the latent variable of monthly frequency and $X^*_{m,t}$ be the latent variable in weekly frequency respectively such that the observed variable is equal to the geometric average of the last three periods' latent variables. Formally,

$$\ln x_{s,t} = \frac{1}{3}(\ln x^*_{s,t} + \ln x^*_{s,t-4} + \ln x^*_{s,t-8}) \tag{3.2}$$

$$\ln x_{m,t} = \frac{1}{3}(\ln x^*_{m,t} + \ln x^*_{m,t-1} + \ln x^*_{m,t-2}) \tag{3.3}$$

Let $y_{s,t} = \Delta_{12} \ln x_{s,t}$, $y^*_{s,t} = \Delta_4 \ln x^*_{s,t}$, $y_{m,t} = \Delta_4 \ln x_{m,t}$, $y^*_{m,t} = \Delta \ln x^*_{m,t}$ and

$y_{f,t} = \Delta \ln x_{f,t}$. We have

$$y_{s,t} = \frac{1}{3}y^*_{s,t} + \frac{2}{3}y^*_{s,t-4} + y^*_{s,t-8} + +\frac{2}{3}y^*_{s,t-12} + \frac{1}{3}y^*_{s,t-16} \quad (3.4)$$

$$y_{f,t} = \frac{1}{3}y^*_{m,t} + \frac{2}{3}y^*_{m,t-1} + y^*_{m,t-2} + y^*_{m,t-3} + \frac{2}{3}y^*_{m,t-4} + \frac{1}{3}y^*_{m,t-5} \quad (3.5)$$

Let

$$\mathbf{y}_{s,t} = \begin{pmatrix} y_{s,t} \\ y_t \end{pmatrix} \quad \mathbf{y}_{m,t} = \begin{pmatrix} y_{m,t} \\ y_t \end{pmatrix}$$

$$\mathbf{y}^*_{s,t} = \begin{pmatrix} y^*_{s,t} \\ y_t \end{pmatrix} \quad \mathbf{y}^*_{m,t} = \begin{pmatrix} y^*_{m,t} \\ y_t \end{pmatrix}$$

Define $\mu_i = E(y_{i,t}), i = s, m$, $\mu_y = E(y_t)$ and

$$\boldsymbol{\mu}_i = \begin{pmatrix} \mu_i \\ \mu_y \end{pmatrix}$$

$$\boldsymbol{\mu}^*_i = \begin{pmatrix} \mu^*_i \\ \mu_y \end{pmatrix}$$

Then relationship between $\mathbf{y}_{i,t}$ and $\mathbf{y}^*_{i,t}$, $i = s, m$ could be written as

$$\mathbf{y}_{i,t} - \boldsymbol{\mu}_i = \mathbf{J}_i(L)(\mathbf{y}^*_{i,t} - \boldsymbol{\mu}^*_i) \quad (3.6)$$

where

$$\mathbf{J}_s(L) = \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^4 + \begin{pmatrix} \mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^8 + ... \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^{12} + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^{16} \quad (3.7)$$

$$\mathbf{J}_m(L) = \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L + \begin{pmatrix} \mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^2 + $$
$$+ \begin{pmatrix} \mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^3 + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^4 + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_m} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^5 \quad (3.8)$$

For each channel, we extract the common factor between monetary policy and informational variables individually, which is interpreted as linkage between monetary policy and economic variables of corresponding channel. For slow-moving channel and medium-moving channel, the factors are related to informational data using a mixed-frequency dynamic factor model because of frequency difference. In fast-moving channel, we maintain the principal component model as in standard FAVAR:

$$\begin{pmatrix} y_{s,t} \\ y_t \end{pmatrix} = \begin{pmatrix} \mu_s \\ \mu_y \end{pmatrix} + \begin{pmatrix} \beta_{s1}(\frac{1}{3}f_{s,t} + \frac{2}{3}f_{s,t-4} + f_{s,t-8} + \frac{2}{3}f_{s,t-12} + \frac{1}{3}f_{s,t-16}) \\ \beta_{s2}f_{s,t} \end{pmatrix}$$
$$+ \begin{pmatrix} \frac{1}{3}e_{s,t} + \frac{2}{3}e_{s,t-4} + e_{s,t-8} + \frac{2}{3}e_{s,t-12} + \frac{1}{3}e_{s,t-16} \\ e_t \end{pmatrix} \quad (3.9)$$

$$\begin{pmatrix} y_{m,t} \\ y_t \end{pmatrix} = \begin{pmatrix} \mu_m \\ \mu_y \end{pmatrix} + \begin{pmatrix} \beta_{m1}(\frac{1}{3}f_{m,t} + \frac{2}{3}f_{m,t-1} + f_{m,t-2} + f_{m,t-3} + \frac{2}{3}f_{m,t-4} + \frac{1}{3}f_{m,t-5}) \\ \beta_{m2}f_{m,t} \end{pmatrix}$$
$$+ \begin{pmatrix} \frac{1}{3}e_{m,t} + \frac{2}{3}e_{m,t-1} + e_{m,t-2} + e_{m,t-3} + \frac{2}{3}e_{m,t-4} + \frac{1}{3}e_{m,t-5}) \\ e_t \end{pmatrix} \quad (3.10)$$

$$y_{f,t} = \Lambda_f f_{f,t} + \Lambda^y y_t + e_{f,t} \quad (3.11)$$

where $\beta_{ij}$, $i = s, m$, $j = 1, 2$ are corresponding factor loading vectors; $\Lambda_f$ is $N_f \times K$ factor loading matrix; $e_t$ is error term for constructed shadow rate; $e_{f,t}$ is $N_f \times 1$ error term vectors.

**Estimation**

The model could be estimated using a similar two-step procedure as in standard FAVAR literature. The first step is to estimate factors $\hat{f}_i, i = s, m, f$ individually. For computational simplicity, the weekly constructed shadow rate $y_t$ in (3.9) is replaced with skipped sampled monthly constructed shadow rate $y_t^m$ so it matches the frequency of the latent variable of slow-moving channel. Therefore, equation (3.9) degenerates into the following monthly frequency model:

$$\begin{pmatrix} y_{s,t} \\ y_t^m \end{pmatrix} = \begin{pmatrix} \mu_s \\ \mu_y^m \end{pmatrix} + \begin{pmatrix} \beta_{s1}(\frac{1}{3}f_{s,t} + \frac{2}{3}f_{s,t-1} + f_{s,t-2} + \frac{2}{3}f_{s,t-3} + \frac{1}{3}f_{s,t-4}) \\ \beta_{s2}f_{s,t} \end{pmatrix} + \begin{pmatrix} \frac{1}{3}e_{s,t} + \frac{2}{3}e_{s,t-1} + e_{s,t-2} + \frac{2}{3}e_{s,t-3} + \frac{1}{3}e_{s,t-4} \\ e_t^m \end{pmatrix} \tag{3.12}$$

where $e_t^m$ is error term for monthly constructed shadow rate.

Accordingly, (3.6) and (3.7) are adjusted as follows:

$$\mathbf{y}_{s,t} - \boldsymbol{\mu}_s^m = \mathbf{J}_s^m(L)(\mathbf{y}_{s,t}^* - \boldsymbol{\mu}_s^*) \tag{3.13}$$

$$\mathbf{J}_s^m(L) = \begin{bmatrix} \frac{1}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{1} \end{bmatrix} + \begin{bmatrix} \frac{2}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L + \begin{bmatrix} \mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^2 + \begin{bmatrix} \frac{2}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^3 + \begin{bmatrix} \frac{1}{3}\mathbf{I}_{N_s} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^4 \tag{3.14}$$

To estimate (3.12) we need to assume AR process for $f_{s,t}$ and $e_{s,t}$:

$$\phi_s^f(L) f_{s,t} = v_{s,t} \tag{3.15}$$

$$\Phi_s^e(L) e_{s,t} = v_{e,t} \tag{3.16}$$

$$\begin{pmatrix} v_{sf,t} \\ v_{e,t} \end{pmatrix} \tilde{} NID(0, \begin{pmatrix} \sigma_s^2 & 0 \\ 0 & \Sigma_e \end{pmatrix})$$

where $\phi_s^f(L)$ is a lag operation polynomial of $p_s$th-order and $\Phi_s^e(L)$ is a lag operation polynomial of $q_s$th order. The variance-covariance matrix is assumed to be diagonal with the first element equals 1, which is a standard identification strategy in factor model literature.

A state space representation of (3.12) when $p_s, q_s \leq 5$ is given by

$$\mathbf{s}_{s,t} = \mathbf{F}_s \mathbf{s}_{s,t-1} + \mathbf{G}_s \mathbf{z}_{s,t} \tag{3.17}$$

$$\mathbf{y}_{s,t}^m = \boldsymbol{\mu}_s^m + \mathbf{H}_s \mathbf{s}_s \tag{3.18}$$

$$\{\mathbf{z}_{1,t}\} \tilde{} IN(\mathbf{0}, \mathbf{I}_{N_1+1})$$

where

$$\mathbf{s}_{s,t} = \begin{pmatrix} f_{s,t} \\ \vdots \\ f_{s,t-4} \\ e_{s,t} \\ \vdots \\ e_{s,t-4} \end{pmatrix}$$

$$\mathbf{F}_s = \begin{bmatrix} \phi_s^f(1)\cdots\phi_s^f(p_s) & \mathbf{o}'_{5-p_s} & \mathbf{O}_{5\times 5(N_s+1)} & \\ \mathbf{I}_4 & \mathbf{o}_4 & & \\ & & \Phi_s^e(1)\cdots\Phi_s^e(q_s) & \mathbf{O}_{(N_s+1)\times(5-q_s)(N_s+1)} \\ \mathbf{O}_{5(N_s+1)\times 5} & & \mathbf{I}_{4(N_s+1)} & \mathbf{O}_{4(N_s+1)\times(N_s+1)} \end{bmatrix}$$

$$\mathbf{G}_s = \begin{bmatrix} \sigma_s & \mathbf{o}_{(N_s+1)} \\ \mathbf{o}_4 & \mathbf{O}_{4\times(N_s+1)} \\ \mathbf{o}_{(N_s+1)} & \boldsymbol{\Sigma}_e^{1/2} \\ \mathbf{o}_{4(N_s+1)} & \mathbf{O}_{4(N_s+1)\times(N_s+1)} \end{bmatrix}$$

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{J}_s^m(0)\boldsymbol{\beta}_s & \cdots & \mathbf{J}_s^m(4)\boldsymbol{\beta}_s & \mathbf{J}_s^m(0) & \cdots & \mathbf{J}_s^m(4) \end{bmatrix}$$

$$\boldsymbol{\beta}_s = (\beta'_{s1}, \beta'_{s2})'$$

The mixed-frequency dynamic factor model can then be estimated using Kalman Filter. Note that (3.12) is a monthly frequency model, therefore, the estimated unobservable factor $\{\hat{f}_{s,t}\}$ is of monthly frequency also.

Similarly, a state space representation of (3.10) when $p_m, q_m \leq 6$ can be written as

$$\mathbf{s}_{m,t} = \mathbf{F}_m \mathbf{s}_{m,t-1} + \mathbf{G}_m \mathbf{z}_{m,t} \tag{3.19}$$

$$\mathbf{y}_{m,t} = \boldsymbol{\mu}_m + \mathbf{H}_m \mathbf{s}_t \tag{3.20}$$

$$\{\mathbf{z}_{m,t}\}\tilde{\ } IN(\mathbf{0}, \mathbf{I}_{N_m+1})$$

where

$$\mathbf{s}_{m,t} = \begin{pmatrix} f_{m,t} \\ \vdots \\ f_{m,t-5} \\ e_{m,t} \\ \vdots \\ e_{m,t-5} \end{pmatrix}$$

$$\begin{aligned}
\mathbf{F}_m &= \begin{bmatrix} \phi^f_m(1)\cdots\phi^f_m(p_m) & \mathbf{o}'_{6-p_m} & \mathbf{O}_{6\times 6(N_m+1)} & \\ \mathbf{I}_5 & \mathbf{o}_5 & & \\ & & \Phi^e_m(1)\cdots\Phi^e_m(q_m) & \mathbf{O}_{(N_m+1)\times(6-q_m)(N_m+1)} \\ \mathbf{O}_{6(N_m+1)\times 6} & & I_{5(N_m+1)} & \mathbf{O}_{5(N_m+1)\times(N_m+1)} \end{bmatrix} \\
\mathbf{G}_m &= \begin{bmatrix} \sigma_m & \mathbf{o}_{(N_m+1)} \\ \mathbf{o}_5 & \mathbf{O}_{5\times(N_m+1)} \\ \mathbf{o}_{(N_m+1)} & \mathbf{\Sigma}_e^{1/2} \\ \mathbf{o}_{5(N_m+1)} & \mathbf{O}_{5(N_{2m}+1)\times(N_m+1)} \end{bmatrix} \\
\mathbf{H}_m &= \begin{bmatrix} \mathbf{J}_m(0)\boldsymbol{\beta}_m & \cdots & \mathbf{J}_m(5)\boldsymbol{\beta}_m & \mathbf{J}_2(0) & \cdots & \mathbf{J}_m(5) \end{bmatrix} \\
\boldsymbol{\beta}_m &= (\beta'_{m1}, \beta'_{m2})'
\end{aligned}$$

The mixed-frequency dynamic factor model can then be estimated using Kalman Filter and the estimated unobservable factor $\{\hat{f}_{m,t}\}$ is of weekly frequency.

Equation (3.11) could be estimated using principal component method as in standard FAVAR model. The estimated unobservable factor $\{\hat{f}_{f,t}\}$ is of weekly frequency.

Following standard FAVAR literature, The second step is to plug all the estimated unobservable factors into (3.1):

$$\begin{bmatrix} \hat{f}_{s,t} \\ \hat{f}_{m,t} \\ \hat{f}_{f,t} \\ y_t \end{bmatrix} = \phi(L) \begin{bmatrix} \hat{f}_{s,t-1} \\ \hat{f}_{m,t-1} \\ \hat{f}_{f,t-1} \\ y_t \end{bmatrix} + \mathbf{v}_t \tag{3.21}$$

Note that $\{\hat{f}_{s,t}\}$ is of monthly frequency while $\{\hat{f}_{m,t}\}$, $\{\hat{f}_{f,t}\}$ and $\{y_t\}$ are of weekly frequency, (3.1) is a mixed-frequency VAR model. The estimation takes two possible forms depending on the expected outcome.

The first form is to follow the estimation of Mariano and Murasawa (2010) and yield estimated weekly frequency latent variable of $\{\hat{f}_{m,t}\}$, which could be used to construct a coincident index of the linkage between overall monetary policy and macro economy.

Let $y^f_{1,t} = \Delta_4 \ln \hat{f}_{s,t}$ and $y^{f*}_{s,t} = \Delta \ln \hat{f}^*_{s,t}$, where $\hat{f}^*_{s,t}$ is the weekly latent variable of $\hat{f}_{s,t}$ and $y^f_{2,t} = \Delta \ln \begin{pmatrix} \hat{f}_{m,t} \\ \hat{f}_{f,t} \\ y_t \end{pmatrix}$. Then we have

$$y^f_{1,t} = \frac{1}{3}y^{f*}_{s,t} + \frac{2}{3}y^{f*}_{s,t-1} + y^{f*}_{s,t-2} + y^{f*}_{s,t-3} + \frac{2}{3}y^{f*}_{s,t-4} + \frac{1}{3}y^{f*}_{s,t-5} \tag{3.22}$$

Let

$$\mathbf{y}_t = \begin{pmatrix} y^f_{1,t} \\ y^f_{2,t} \end{pmatrix}$$

$$\mathbf{y}^*_t = \begin{pmatrix} y^{f*}_{1,t} \\ y^f_{2,t} \end{pmatrix}$$

Define $\boldsymbol{\mu} = E(\mathbf{y}_t)$, $\boldsymbol{\mu}^* = E(\mathbf{y}_t^*)$the relationship between $\mathbf{y}_t$ and $\mathbf{y}_t^*$ is given by

$$\mathbf{y}_t - \boldsymbol{\mu} = \mathbf{J}(L)(\mathbf{y}_t^* - \boldsymbol{\mu}^*) \tag{3.23}$$

where

$$\begin{aligned}\mathbf{J}(L) \quad = \quad & \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & 1 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^2 + \\ & + \begin{pmatrix} \mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^3 + \begin{pmatrix} \frac{2}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^4 + \begin{pmatrix} \frac{1}{3}\mathbf{I}_{N_1} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{pmatrix} L^5\end{aligned}$$

Assume $\mathbf{y}_t^*$ follows Gaussian VAR$(p)$

$$\boldsymbol{\phi}(L)(\mathbf{y}_t^* - \boldsymbol{\mu}^*) = \mathbf{w}_t, \mathbf{w}_t \tilde{\ } IN(\mathbf{0}, \boldsymbol{\Sigma}) \tag{3.24}$$

Let the state variable be

$$\mathbf{s}_t = \begin{pmatrix} \mathbf{y}_t^* - \boldsymbol{\mu}^* \\ \vdots \\ \mathbf{y}_{t-5}^* - \boldsymbol{\mu}^* \end{pmatrix}$$

The state-space representation of (3.21) when $p \leq 6$ is given by

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{z}_t \tag{3.25}$$

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{C}\mathbf{s}_t \tag{3.26}$$

$$\{\mathbf{z}_t\} \tilde{\ } IN(\mathbf{0}, \mathbf{I}_{K+3})$$

where

$$
\begin{aligned}
\mathbf{A} &= \begin{bmatrix} \boldsymbol{\phi}_1 & \cdots & \boldsymbol{\phi}_p & \mathbf{O}_{(K+3)\times(6-p)(K+3)} \\ & \mathbf{I}_{5(K+3)} & & \mathbf{O}_{5(K+3)\times(K+2)} \end{bmatrix} \\
\mathbf{B} &= \begin{bmatrix} \boldsymbol{\Sigma}^{1/2} \\ \mathbf{O}_{5(K+3)\times(K+3)} \end{bmatrix} \\
\mathbf{C} &= \begin{bmatrix} \mathbf{J}(0) & \cdots & \mathbf{J}(5) \end{bmatrix}
\end{aligned}
$$

The mixed-frequency VAR then could be estimated using Kalman Smoother.

The second form is to apply a stacked vector system following Ghysels (2016) and obtain the impulse response functions of indices of different channels to shocks in constructed shadow rate.

Consider a structural VAR(p) of monthly frequency $t^m = 1, ..., T^m$ as follows:

$$
\begin{pmatrix} y(t^m, 1) \\ \hat{f}_f(t^m, 1) \\ \hat{f}_m(t^m, 1) \\ \vdots \\ y_t(t^m, 4) \\ \hat{f}_f(t^m, 4) \\ \hat{f}_m(t^m, 4) \\ \hat{f}_{s,t}(t^m) \end{pmatrix} = A_0 + \sum_{j=1}^{p} A_j \begin{pmatrix} y(t^m - 1, 1) \\ \hat{f}_f(t^m - 1, 1) \\ \hat{f}_m(t^m - 1, 1) \\ \vdots \\ y_t(t^m - 1, 4) \\ \hat{f}_f(t^m - 1, 4) \\ \hat{f}_m(t^m - 1, 4) \\ \hat{f}_{s,t}(t^m - 1) \end{pmatrix} + \varepsilon(t^m) \tag{3.27}
$$

where the intra-month indicators are stacked in the order of being observed (or released).

Let $L_m$ is the monthly lag-operator, rewrite (3.27) into the following form:

$$A(L_m)(\underline{\mathrm{y}}(t^m) - \mu_{\underline{\mathrm{y}}}) = \varepsilon(t^m) \tag{3.28}$$

where

$$\begin{aligned}
\underline{\mathrm{y}}(t^m) &= \begin{pmatrix} y(t^m, 1) \\ \hat{f}_f(t^m, 1) \\ \hat{f}_m(t^m, 1) \\ \vdots \\ y_t(t^m, 4) \\ \hat{f}_f(t^m, 4) \\ \hat{f}_m(t^m, 4) \\ \hat{f}_{s,t}(t^m) \end{pmatrix} \\
A(L^m) &= I - \sum_{j=1}^{p} A_j L_m^j \\
\mu_{\underline{\mathrm{y}}} &= (I - \sum_{j=1}^{p} A_j)^{-1} A_0
\end{aligned}$$

Therefore, the impulse response function for the stacked system is given by

$$(\underline{\mathrm{y}}(t^m) - \mu_{\underline{\mathrm{y}}}) = (I - \sum_{j=1}^{p} A_j L_m^j)^{-1} \varepsilon(t^m) \tag{3.29}$$

and it can be estimated in standard structure VAR estimation procedures.

### Missing Values

For all the mixed-frequency models above, the lower frequency series are not always observable. Follow Mariano and Murasawa (2003), we replace the missing observations with random variable $\epsilon_t\tilde{}N(0,1)$ which has a realization of 0 and adjust the rest of the measurement equation accordingly. For example, the measurement equation (3.26) can be written as

$$\begin{pmatrix} y^f_{1,t} \\ y^f_{2,t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}(1) \\ \boldsymbol{\mu}(2) \end{pmatrix} + \begin{pmatrix} \mathbf{C}(1) \\ \mathbf{C}(2) \end{pmatrix} \mathbf{s}_t \tag{3.30}$$

With the missing values replaced, the measurement equation becomes

$$\mathbf{y}^+_t = \boldsymbol{\mu}_t + \mathbf{C}_t\mathbf{s}_t + \mathbf{D}_t\epsilon_t \tag{3.31}$$

where

$$\begin{aligned} \mathbf{y}^+_t &= \begin{pmatrix} y^{f+}_{1,t} \\ y^f_{2,t} \end{pmatrix}, \boldsymbol{\mu}_t = \begin{pmatrix} \boldsymbol{\mu}_t(1) \\ \boldsymbol{\mu}_t(2) \end{pmatrix} \\ \mathbf{C}_t &= \begin{pmatrix} \mathbf{C}_t(1) \\ \mathbf{C}_t(2) \end{pmatrix}, \mathbf{D}_t = \begin{pmatrix} \mathbf{D}_t(1) \\ \mathbf{O} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} y^{f+}_{1,t} &= \{ \begin{matrix} y^f_{1,t} \text{ if } \mathbf{y}_t \text{ observed} \\ \epsilon_t \text{ if } \mathbf{y}_t \text{ not observed} \end{matrix}, \boldsymbol{\mu}_t(1) = \{ \begin{matrix} \boldsymbol{\mu}(\mathbf{1}) \text{ if } \mathbf{y}_t \text{ observed} \\ \mathbf{0} \text{ if } \mathbf{y}_t \text{ not observed} \end{matrix} \\ \mathbf{C}_t(1) &= \{ \begin{matrix} \mathbf{C}(\mathbf{1}) \text{ if observed} \\ \mathbf{O} \text{ if not observed} \end{matrix}, \mathbf{D}_t(1) = \{ \begin{matrix} \mathbf{D}(1) \text{ if observed} \\ 1 \text{ if not observed} \end{matrix} \end{aligned}$$

The state-space representation with missing values replaced is given by

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \mathbf{B}\mathbf{z}_t \tag{3.32}$$

$$\mathbf{y}^+_t = \boldsymbol{\mu}_t + \mathbf{C}_t\mathbf{s}_t + \mathbf{D}_t\epsilon_t \tag{3.33}$$

$$\left\{ \begin{pmatrix} \mathbf{z}_t \\ \\ \epsilon_t \end{pmatrix} \right\} \tilde{\ } IN(\mathbf{0}, \mathbf{I}_{K+4})$$

Now that $\{\mathbf{y}_t^+\}$ has no missing values, the Kalman filter can apply directly.

## 3.3 Application

### 3.3.1 Data

The data I use sampled from July 1987 to December 2013. For shadow rate term structure model, forward rates for different maturities are constructed using the end-of-week observations from Gurkaynak, Sack and Wright (2007) data, following Wu and Xia (2015). For mixed-frequency factor-augumented vector autoregressive model, I selected series of weekly, monthly and quarterly frequencies. Weekly indicators are mostly "fast-moving" return rates that reflects the financial market movement. Monthly indicators are chosen to be variables that reflects the loan and credit change in financial intermediates. Quarterly indicators are "slow-moving" variables that measure the real economy activities. All the series are selected based on three criterion: whether the movement speed of variable matches its frequency of being observed; whether the variable fits in the potential process of monetary transmission mechanism; whether the variable is available within the sample period. Since the estimation is very time-consuming, we carefully restrict our number of variables and time window within an acceptable range. Table 1 summarizes the detailed descriptions of the series. "SA" stands for "seasonally adjusted", "NSA" stands for "not seasonally adjusted" and "AR" stands for "annual rate". All data are directly downloaded from FRED, except Divisia M4 is provided by Center for Financial Stability.

Note that for each month we may have either four or five weekly observations, we made the following adjustment for months of the latter case. Let $\{x_t, ...x_{t+4}\}$ be the five weekly observations in a month. The adjusted observations are given by

$$\{\frac{1}{2}(x_t + x_{t+1}), \frac{1}{2}(x_{t+1} + x_{t+2}), \frac{1}{2}(x_{t+2} + x_{t+3}), \frac{1}{2}(x_{t+3} + x_{t+4})\}$$

Since weekly indicators are return rates in level, the average of adjacent observations could be considered as pseudo observation over the this time period. This will guarantee us four weekly observations every month.

### 3.3.2 Estimated Results and Discussions

The shadow rate term structure model is estimated using code modified from Wu and Xia (2015). The estimated shadow rate along with effective federal fund rate in weekly frequency are shown in figure 1. The shadow rate and effective federal fund rate move together until end of 2008. When federal fund rate hit the Zero lower Bound, effective federal fund rate stay constant while the estimated shadow rate continue to drop below zero. We can also observe declines in the shadow rate during the periods of Quantitative Easing program (QE). It is also worth mentioning that estimated shadow rate is more volatile than federal fund rate even before the Zero Lower Bound period.

The mixed-frequency factor-augumented vector autoregressive model is estimated Using Ox 7.10 and code modified from Mariano and Murasawa (2003, 2010).The number of lags for AR process are determined using information criterion AIC and SBIC same in

Mariano and Murasawa (2003):

$$AIC = -\frac{1}{T}\{\ln L(\hat{\theta}) - [(N-1) + p + 1 + N(q+1)]\} \quad (3.34)$$

$$SBIA = -\frac{1}{T}\{\ln L(\hat{\theta}) - \frac{\ln T}{2}[(N-1) + p + 1 + N(q+1)]\} \quad (3.35)$$

The selected model are $(p_1, q_1) = (2, 1)$, $(p_2, q_2) = (1, 1)$ and $p = 1$. The number of factors are in the principal component is chosen to be $K = 2$.

Figure 2 illustrates the estimated common factor of quarterly indicators and estimated shadow rate that measures the direct linkage between monetary policy and real economic activities. Some activities could be observed even after Dec. 2008. Figure 3 is the coincident index constructed using the common factors estimated from quarterly indicators by taking the partial sum. The coincident index essentially measures how effective monetary policy is over the time. It has a nice property that it reaches local peaks right before the recession and declines rapidly during the recessions, suggesting that the impact of monetary policy lessens during the periods of recessions. We can also observe three small peak after Dec 2008 which coincident with quantitative easing program (QE) conducted by the Federal Reserve which means the unconventional monetary policy is effective affecting the economy. Figure 4 shows the estimated common factor of monthly indicators and estimated shadow rate which measure the linkage between monetary policy and credit market activities. Figure 5 is the coincident index constructed using the common factor of monthly indicators by taking the partial sum. The coincident index shares the same properties as that in figure 3. However, we can see credit market is more reactive to monetary policy that the macro economy.

Figure 6 is the estimated factor of the second step mixed-frequency VAR model.

The estimated high-frequency factor measures the linkage between overall monetary policy and real economic activities taking all three channels into consideration. Figure 7 and 8 are the corresponding coincident index constructed using the same method and its smoothed version. The coincident index rises during the recession of 1990 and 2001 while declines during the 2008 crisis. One interesting observation people may ignore is that there is a sharp drop in the index followed by a bounce back right before each recession in our sample. This indicates that the resulting coincident index is actually leading the recessions substantially. One observation that supports this view is the to look at the duration of the recessions. During the recession of 1990 and 2001, the coincident index start to rise at the beginning of the recession, and as a result these two recessions last shorter. In contrast, after a small bounce, the index continue to drop during the 2008 recession, and the recession last longer. This could be a result of estimating shadow rate using one month forward rate of bond of different maturity. Therefore, the shadow rate is leading the effective federal fund rate by construction. This effect is somehow amplified by our second step estimation of mixed-frequency VAR model.

I also report the impulse response analysis from the stacked system of mixed-frequency VAR model. Figure 9 shows the impulse response functions of fast-moving channel index to a standard shock in shadow rate. We can observe that monetary policy shocks have a short-run effect on the financial market that decays within four periods. Figure 11 shows the impulse response function of medium-moving channel index. In contrast with fast-moving channel index, monetary policy shocks have more persistent impact that lasts for more than 12 periods. Figure 12 is the impulse response function of slow-moving channel index. Recall that within one month shadow rate is observed four times while the index is observed once at the end of the month, monetary policy shocks at the beginning and end of the month have relatively larger impact than intra-month shocks. It is also worth noticing the index cross 0 several times, indicating the impact of monetary policy shock on the real economy is not monotonic.

## 3.4 Conclusion

This paper studies the monetary transmission taking unconventional monetary policy in consideration. The impact of unconventional monetary policy is reflected on the dynamics of constructed shadow rate which is estimated using one month forward rate of bond of different maturities. The monetary transmission mechanism is disentangled into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. A set of coincident indices are constructed using the mixed-frequency factor-augumented autoregressive model. Impulse response analysis are also applied by imposing the stacked vector system in the second step of the estimation. The resulting coincident index of overall monetary transmission mechanism captures major unconventional monetary policy during the Zero Lower Bound and exhibits a substaintial lead on the recessions in the sample period, which is due to the construction process of shadow rate. The impulse response functions show that monetary policy shocks have relatively persistent impact on medium-moving channel (or credit market). Also, we find the slow-moving channel is more responsive to monetary policy shocks in the beginning and end of a month compared to those in the middle.

# Bibliography

[1] Baffigi, Alberto, Roberto Golinelli, and Giuseppe Parigi. "Bridge models to forecast the euro area GDP." International Journal of forecasting 20.3 (2004): 447-460.

[2] Bai, Jennie, Eric Ghysels, and Jonathan H. Wright. "State space models and MIDAS regressions." Econometric Reviews 32.7 (2013): 779-813.

[3] Bai, Jushan, and Serena Ng. "Determining the number of factors in approximate factor models." Econometrica 70.1 (2002): 191-221.

[4] Bai, Jushan, and Serena Ng. "Confidence intervals for diffusion index forecasts and inference for factor-Augmented regressions." Econometrica 74.4 (2006): 1133-1150.

[5] Bai, Jushan, and Serena Ng. "Determining the number of primitive shocks in factor models." Journal of Business & Economic Statistics 25.1 (2007): 52-60.

[6] Barnett, William A., Marcelle Chauvet, and Danilo Leiva-Leon. Real-time nowcasting of nominal GDP under structural breaks. No. 2014-39. Bank of Canada Working Paper, 2014.

[7] Beck, Thorsten, Andrea Colciago, and Damjan Pfajfar. "The role of financial intermediaries in monetary policy transmission." Journal of Economic Dynamics and Control 43 (2014): 1-11.

[8] Bernanke, B., and Gertler, M., 1989. Agency Costs, Net Worth and Business Fluctuations. American Economic Review 79, 14-31.

[9] Bernanke, Ben S., Jean Boivin, and Piotr Eliasz. Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. No. w10220. National Bureau of Economic Research, 2004.

[10] Boivin, Jean, Michael T. Kiley, and Frederic S. Mishkin. How has the monetary transmission mechanism evolved over time?. No. w15879. National Bureau of Economic Research, 2010.

[11] Bry, Gerhard, and Charlotte Boschan. "Front matter to" Cyclical Analysis of Time Series: Selected Procedures and Computer Programs"." Cyclical Analysis of Time Series: Selected Procedures and Computer Programs. NBER, 1971. 13-2.

[12] Buch, Claudia M., Sandra Eickmeier, and Esteban Prieto. "In search for yield? Survey-based evidence on bank risk taking." Journal of Economic Dynamics and Control 43 (2014): 12-30.

[13] Carpenter, S.B., Demiralp, S., Eisenschmidt, J., 2014. The effectiveness of nonstandard monetary policy in addressing liquidity risk during the financial crisis: The experiences of the Federal Reserve and the European Central Bank. J. Econ. Dyn. Control

[14] Christiano, Lawrence J., and Martin Eichenbaum. Liquidity effects and the monetary transmission mechanism. No. w3974. National Bureau of Economic Research, 1992.

[15] Christiano, Lawrence J., and Martin Eichenbaum. "Liquidity effects, monetary policy, and the business cycle." Journal of Money, Credit and Banking (1995): 1113-1136.

[16] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Monetary policy shocks: What have we learned and to what end?." Handbook of macroeconomics 1 (1999): 65-148.

[17] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. Two reasons why money and credit may be useful in monetary policy. No. w13502. National Bureau of Economic Research, 2007.

[18] Frale, Cecilia, et al. "EUROMIND: a monthly indicator of the euro area economic conditions." Journal of the Royal Statistical Society: Series A (Statistics in Society) 174.2 (2011): 439-470.

[19] Foroni, Claudia, and Massimiliano Giuseppe Marcellino. "A survey of econometric methods for mixed-frequency data." (2013).

[20] Ghysels, Eric. "Macroeconomics and the reality of mixed frequency data." Journal of Econometrics 193.2 (2016): 294-314.

[21] Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov. "The MIDAS touch: Mixed data sampling regression models." Finance (2004).

[22] Harding, Don, and Adrian Pagan. "Synchronization of cycles." Journal of Econometrics 132.1 (2006): 59-79.

[23] He, Zhiguo, and Arvind Krishnamurthy. A macroeconomic framework for quantifying systemic risk. No. w19885. National Bureau of Economic Research, 2014.

[24] Kashyap, A.K., Stein, J.C., 1995. The impact of monetary policy on bank balance sheets. Carnegie-Rochester Conf. Ser. Public Policy 42(1), 151–195.

[25] Killian, Lutz (1996), 'Small - Sample Conference Intervals for Impulse Response Functions', manuscript, University of Pennsylvania.

[26] Kim, Chang-Jin, and Charles R. Nelson. "The time-varying-parameter model for modeling changing conditional variance: The case of the lucas hypothesis." Journal of Business & Economic Statistics 7.4 (1989): 433-440.

[27] Kim, Chang-Jin, and Charles R. Nelson. "Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching." The Review of Economics and Statistics 80.2 (1998): 188-201.

[28] Kim, Chang-Jin, and Charles R. Nelson. "State-space models with regime switching." (1999).

[29] King, Robert, et al. "Stochastic trends and economic fluctuations." (1987).

[30] King, Robert G. (1991), 'Money and Business Cycles', manuscript, University of Rochester.

[31] Mariano, Roberto S., and Yasutomo Murasawa. "A new coincident index of business cycles based on monthly and quarterly series." Journal of applied Econometrics 18.4 (2003): 427-443.

[32] Mariano, Roberto S., and Yasutomo Murasawa. "A coincident index, common factors, and monthly real GDP." Oxford Bulletin of Economics and Statistics 72.1 (2010): 27-46.

[33] Nicolo G.D., Dell'Ariccia, G., Laeven, L., Valencia, F., 2010. Monetary Policy and Bank Risk-Taking. IMF Staff Position Notes 2010/09, International Monetary Fund.

[34] Schorfheide, Frank, and Dongho Song. "Real-time forecasting with a mixed-frequency VAR." Journal of Business & Economic Statistics 33.3 (2015): 366-380.

[35] Sims, Christopher A. "Macroeconomics and reality." Econometrica: Journal of the Econometric Society (1980): 1-48.

[36] Stock, James H., and Mark W. Watson. "A probability model of the coincident economic indicators." (1988).

[37] Stock, James, and Mark Watson, "New Indexes of Coincident and Leading Economic Indicators", NBER Macroeconomics Annual (4), Cambridge MA: MIT Press, 1989.

[38] Stock, James H., and Mark W. Watson. Diffusion indexes. No. w6702. National Bureau of Economic Research, 1998.

[39] Stock, James H., and Mark W. Watson. "Estimating turning points using large data sets." Journal of Econometrics 178 (2014): 368-381.

[40] Taylor, John B. "The monetary transmission mechanism: an empirical framework." The Journal of Economic Perspectives 9.4 (1995): 11-26.

[41] Tobin, J., 1969. A general equilibrium approach to monetary theory. J. Money Credit Bank. 1 (1), 15–29.

## 3.5 Tables and Figures

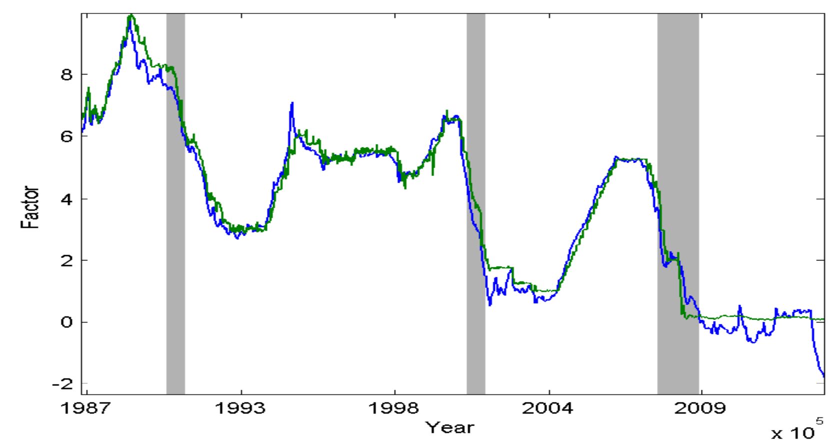


Figure 3.1: The estimated shadow rate and effective federal fund rate.

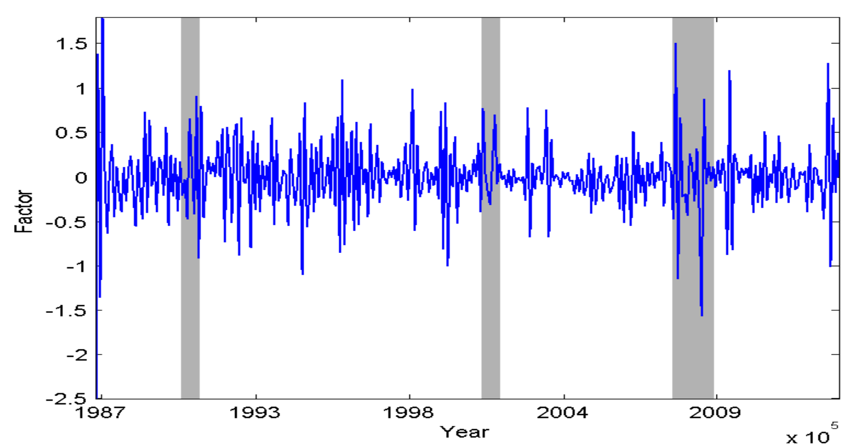


Figure 3.2: Estimated factor from quarterly indicators.

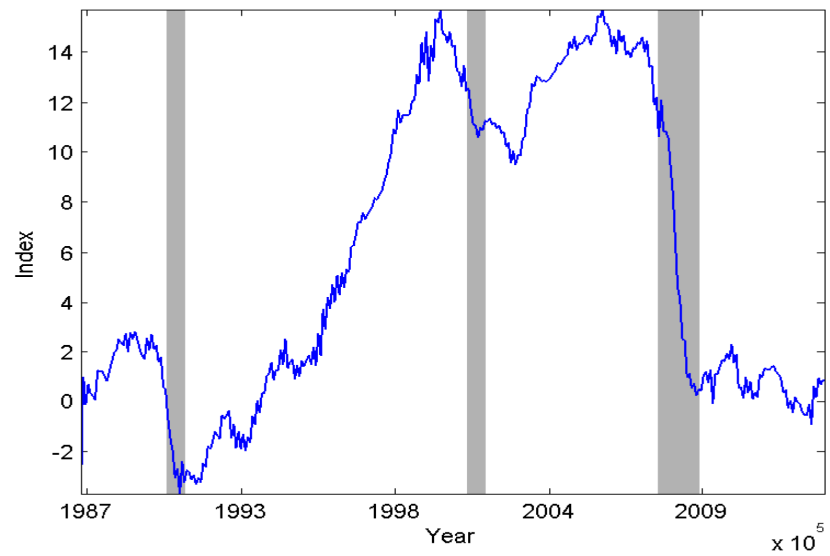


Figure 3.3: Coincident index constructed using quarterly indicator factor.

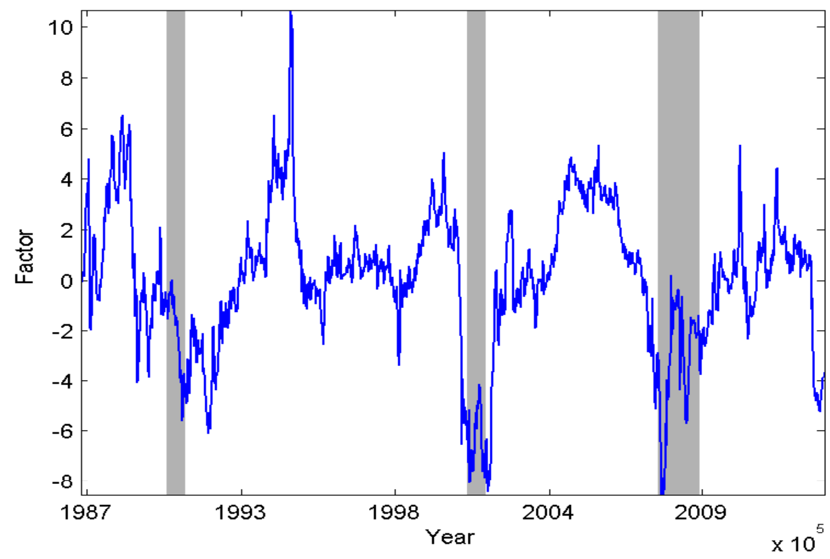


Figure 3.4: Estimated factor from monthly indicators.

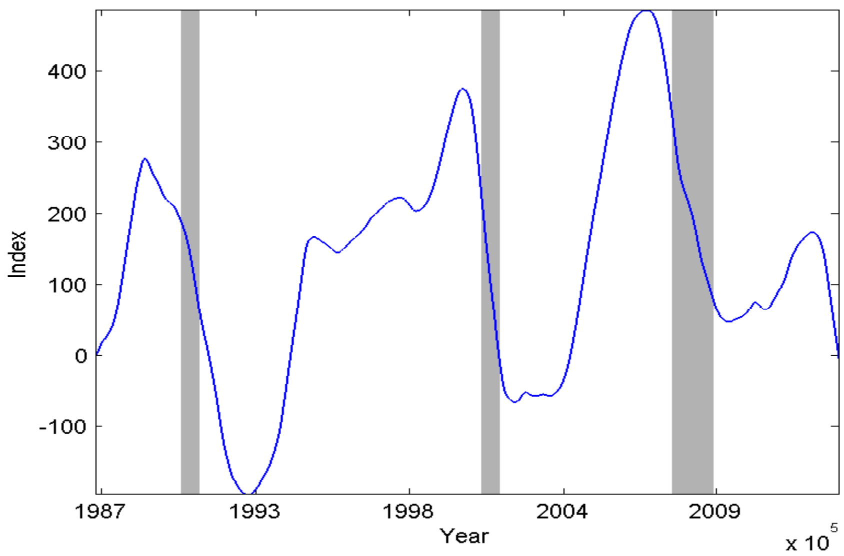


Figure 3.5: Coincident index constructed using the monthly indicator factor.

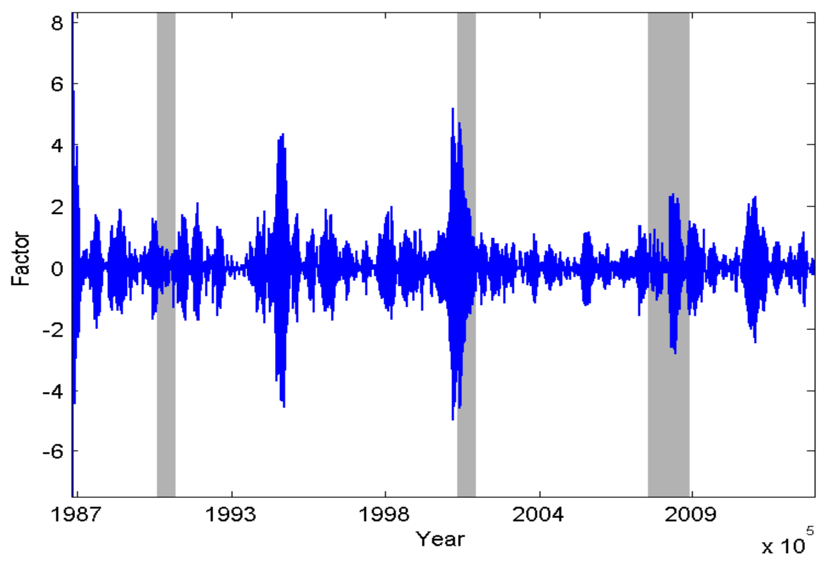


Figure 3.6: Estimated factor of monetary linkage.

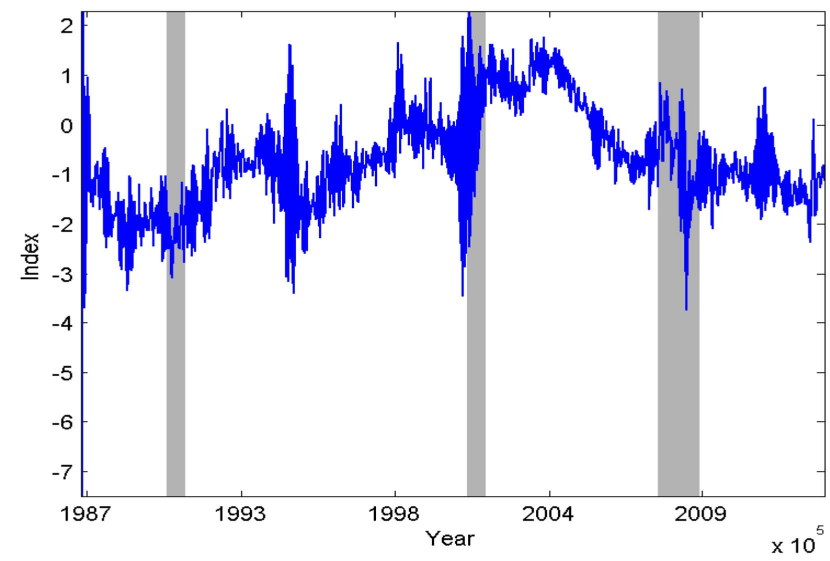


Figure 3.7: Coincident index constructed using estimated monetary linkage factor.

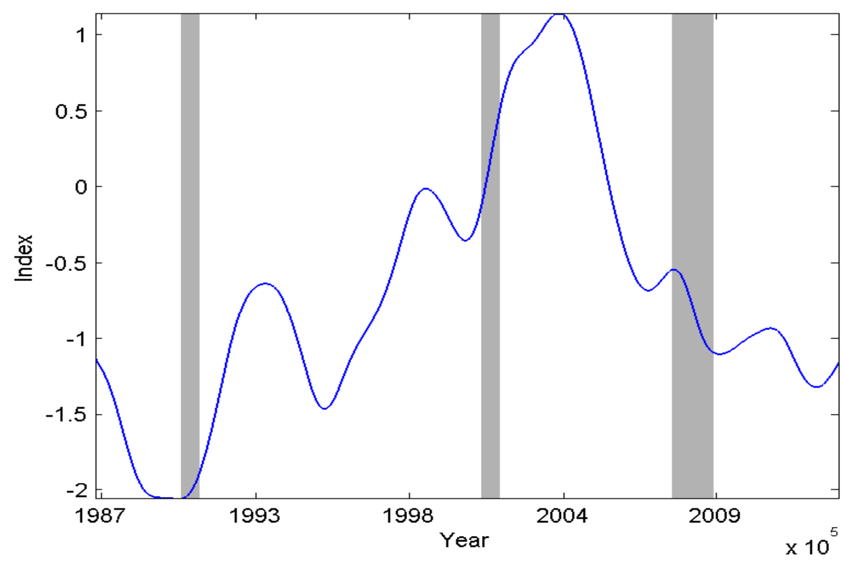


Figure 3.8: Smoothed coincident index constructed using estimated monetary linkage factor.

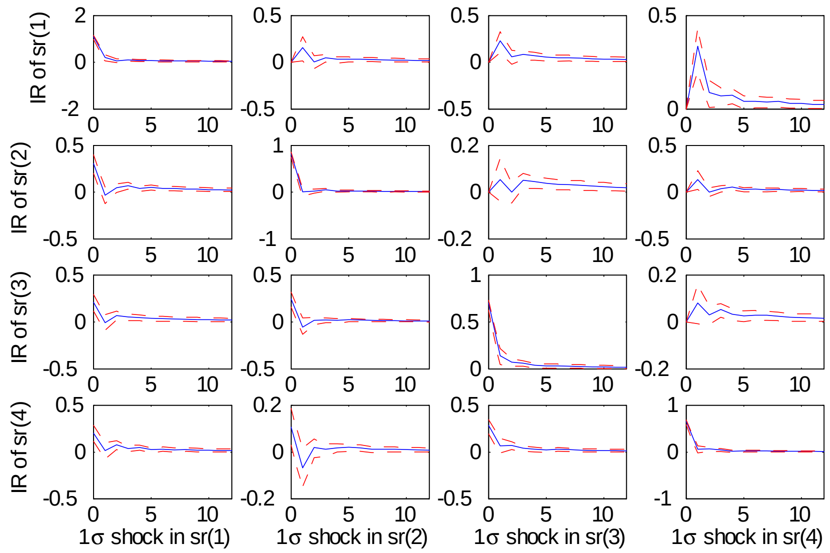


Figure 3.9: Impulse response functions of shadow rate to a standard shock in shadow rate.

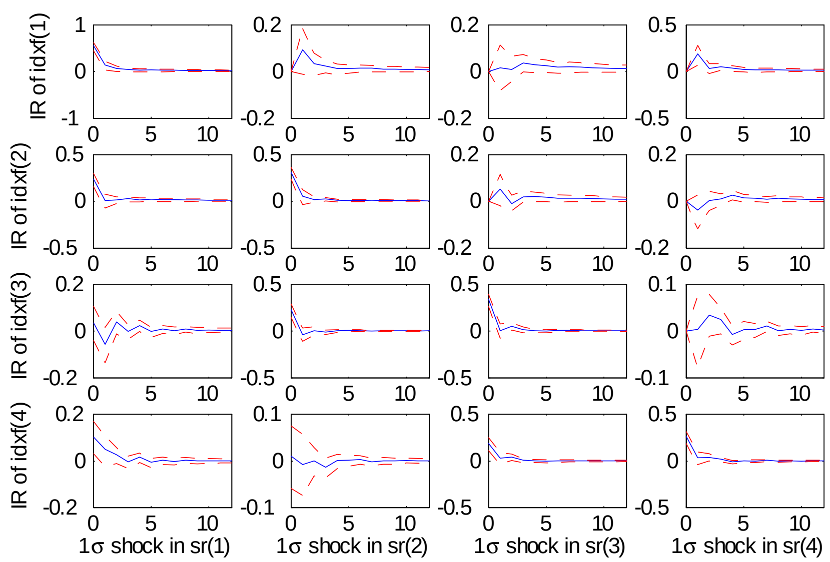


Figure 3.10: Impulse response functions of fast-moving channel index to a standard shock in shadow rate.

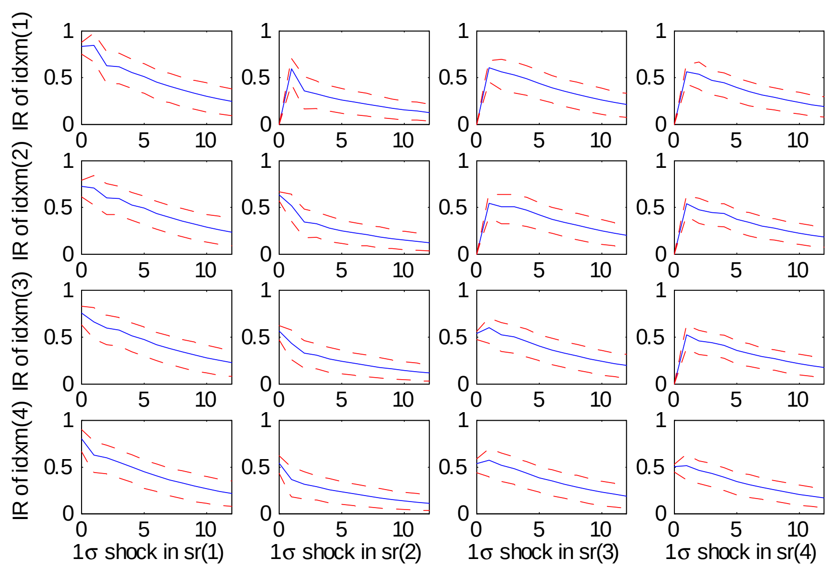


Figure 3.11: Impulse response functions of medium-moving channel index to a standard shock in shadow rate.

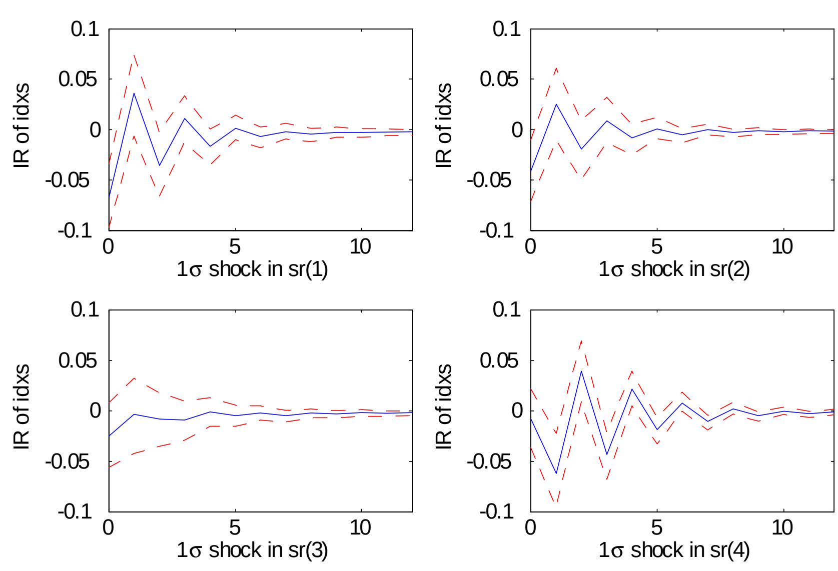


Figure 3.12: Impulse response functions of slow-moving channel index to a standard shock in shadow rate.

# Chapter 4

# Examining the Lead-Lag Relationship of Channels in Monetary Transmission Mechanism

## 4.1 Introduction

Monetary transmission mechanism describes how monetary policy shock affects real variables in the economy such as aggregate output, employment rate and inflation. There is a large literature that focus on unveil different channels through which the monetary policy shock can affect the real economy. Since monetary policy shock could affect many economic variables in the economy, it is hard to track the impact transits from time to time.

Theoretical models about monetary transmission mechanism started from Tobin (1969). The interest rate channel says investors would adjust their asset portfolio according to changes in interest rate, consumption will also change as a result. The credit channel is a more recent view proposed by Bernanke and Gertler (1989). The credit view is built on the assumption that the credit market is imperfect due to government intervention, asymmetric information, and agency problems. The bank lending channel refers to the situation where some small firms that have no direct access to credit market consider bank loan is their only source of finance. As a result of monetary policy shock, the credit available in bank will be adjusted accordingly, which most likely will have impact on investment of such small firms. The balance sheet channel considers the direct effect of monetary policy shock on agents' ability to borrow due to changes in asset value and profitability. The latest risk-taking view suggests that expansionary monetary policy could lead to risk-taking behavior. Nicolo et al. (2010) discuss three different channels of it. The first channel is that banks always have incentive to substitute low yield safe asset with high yield riskier asset. The second channel is through a "search for yield," that is, low interest rate gives financial institutions with long-term commitment an incentive to switch to risky asset in order to attain a higher probability of matching their promised yield. The last channel refers to the fact that banks always tend to maintain a constant leverage ratio. The leverage ratio tends to drop with monetary policy easing as risky asset weight falls, and this could lead banks to switch towards risky assets.

Despite all the channels studied in the literature, transition speed of different channels is often ignore by researchers. It is intuitive to see that the impact transits through interest rate channel should take place slower than one that goes by balance sheet channel as the later basicly only include changes in information and expectation which merely takes any time. The difference in the speed of transition of different channels also increases the complexity of the impact on the economy from a monetary policy shock.

In contrast with most of the literature that focusing on the "real" impact of monetary policy, I need to include not only policy rate and real macroeconomic variables, but also variables from financial and credit market in the model to investigate the transmission mechanism. In this paper, I propose to disentangle the monetary transmission mechanism into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. A coincident index of each of the channel could be constructed using a mixed-frequency dynamic factor model, as the indicators are sampled in different frequencies. Intuitively, if there is difference in speed of transition between channels in the monetary transmission mechanism, there should be a clear lead-lag relationship between the coincident index of each of the channel.

One potential problem is that as the coincident indices are estimated individually, there exist a scale difference between the indices of each channel. Although a classic turning-point analysis could still be applied, this will cause problems in potential further extensions. The problem is resolved by introducing a Markov-switching feature to the model. In this paper, I adopt a extended Markov-switching mixed-frequency vector autoregressive model to the coincident indices of all three channels. Each channel is assumed to switch independently between two states: "good state" during expansions and "bad state" for recessions. As a result, instead of looking for lead-lag relationship between the coincident indices, we can expect the lead-lag relationship in the probability of "bad state".

This is not the first paper that studies Markov-switching mixed-frequency VAR models. Camacho (2012) extended Markov-switching mixed- frequency VAR model to a mixed-frequency version following Mariano and Murasawa (2003) method and successfully captured NBER business cycle. Foroni, Guerin and Marcellino (2015) summarized general estimation and inference of a family of Markov-switching mixed-frequency VAR models and found the model is extremely useful to estimate and predict status of economic activities. However, to my best knowledge, this is the first paper to allow variables to switching individually and study the relationship of the indicators within the model.

The rest of the paper is structured as follows. Section two presents the mixed-frequency dynamic factor model used to construct the coincident indices and the extended Markov-switching mixed- frequency VAR model used to estimate the probability of "bad state". Section three applies the model to U.S. macroeconomic and financial data to the model and discuss the empirical results. Section four concludes.

## 4.2 The Model

### 4.2.1 Mixed-Frequency Dynamic Factor Model

Consider $Y_t$ to be a $M \times 1$ vector of observable economic variables of interest that is driving the economy. In our application, it has federal fund rate in weekly frequency only. The monetary transmission mechanism is disentangled into three channels according to the transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. A coincident index of each of the channel could be constructed using the a mixed-frequency dynamic factor model following Mariano and Murasawa (2003).

Let $X_{s,t}$, $X_{m,t}$, $X_{f,t}$ be $N_s \times T_s$, $N_m \times T_m$, $N_f \times T_f$ informational data we observe in slow-moving, medium-moving and fast-moving channel respectively with $N_i$ being the number of variables and $T_i$ the number of observations $i = s, m, f$. The time length of one period in the model is set to be consistent with that of the highest frequency data. For instance, in a model that has $X_{m,t}$ and $Y_t$, $Y_t$ is observed every period, while $X_{m,t}$ is observed every $n$ where $n > 1$. In the case of $Y_t$ being weekly data, $X_{m,t}$ being monthly data, $n = 4$.

Take the slow-moving channel as an example, I illustrate the construction of the corresponding coincident index. For computational simplicity, I include indicators in $X_{s,t}$ in quarterly frequency and skip-sampled federal fund rate $y_t^m$ in monthly frequency instead of original $y_t$ in weekly frequency to reduce number of lags in the model. Following Mariano and Murasawa (2003), assume the observed lower frequency indicator $X_{s,t}$ is equal to the geometric average of the last three periods' latent variables $X_{s,t}^*$ in higher frequency. Formally,

$$\ln x_{s,t} = \frac{1}{3}(\ln x_{s,t}^* + \ln x_{s,t-1}^* + \ln x_{s,t-2}^*)$$

Let $y_{s,t} = \Delta_3 \ln x_{s,t}$ and $y_{s,t}^* = \Delta \ln x_{s,t}^*$, we have

$$y_{s,t} = \frac{1}{3}y_{s,t}^* + \frac{2}{3}y_{s,t-1}^* + y_{s,t-2}^* + + \frac{2}{3}y_{s,t-3}^* + \frac{1}{3}y_{s,t-4}^*$$

Define

$$\mathbf{y}_{s,t} = \begin{pmatrix} y_{s,t} \\ y_t^m \end{pmatrix} \quad \boldsymbol{\mu}_s = \begin{pmatrix} \mu_s \\ \mu_{y^m} \end{pmatrix}$$

$$\mathbf{y}^*_{s,t} = \begin{pmatrix} y^*_{s,t} \\ y_t^m \end{pmatrix} \quad \boldsymbol{\mu}^*_s = \begin{pmatrix} \mu^*_s \\ \mu_{y^m} \end{pmatrix}$$

where $\mu_s = E(y_{s,t})$, $\mu_{y^m} = E(y_t^m)$.

Then relationship between $\mathbf{y}_{s,t}$ and $\mathbf{y}^*_{s,t}$ is given by

$$\mathbf{y}_{s,t} - \boldsymbol{\mu}_s = \mathbf{J}_s(L)(\mathbf{y}^*_{s,t} - \boldsymbol{\mu}^*_s)$$

where

$$\begin{aligned} \mathbf{J}_s(L) &= \begin{bmatrix} \frac{1}{3}\mathbf{I}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{1} \end{bmatrix} + \begin{bmatrix} \frac{2}{3}\mathbf{I}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L + \begin{bmatrix} \mathbf{I}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^2 \\ &\quad + \begin{bmatrix} \frac{2}{3}\mathbf{I}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^3 + \begin{bmatrix} \frac{1}{3}\mathbf{I}_s & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^4 \end{aligned}$$

Let $f_{s,t}$ denotes the common factor that links the policy rates to low frequency indicators in the slow-moving channel. The mixed-frequency dynamic factor model of $y_{s,t}$ and $y_t^m$ is could be written as

$$\begin{aligned} \begin{pmatrix} y_{s,t} \\ y_t^m \end{pmatrix} &= \begin{pmatrix} \mu_s \\ \mu_{y^m} \end{pmatrix} + \begin{pmatrix} \beta_{s1}(\frac{1}{3}f_{s,t} + \frac{2}{3}f_{s,t-1} + f_{s,t-2} + \frac{2}{3}f_{s,t-3} + \frac{1}{3}f_{s,t-4}) \\ \beta_{s2}f_{s,t} \end{pmatrix} \\ &\quad + \begin{pmatrix} \frac{1}{3}e_{s,t} + \frac{2}{3}e_{s,t-1} + e_{s,t-2} + \frac{2}{3}e_{s,t-3} + \frac{1}{3}e_{s,t-4} \\ e_t^m \end{pmatrix} \end{aligned}$$

where $e_t^m$ is error term for monthly constructed shadow rate.

The model is estimated with Kalman filter.

Assume AR process for $f_{s,t}$ and $e_{s,t}$:

$$\phi_s^f(L)f_{s,t} = v_{s,t} \tag{4.1}$$

$$\Phi_s^e(L)e_{s,t} = v_{e,t} \tag{4.2}$$

$$\begin{pmatrix} v_{sf,t} \\ v_{e,t} \end{pmatrix} \tilde{} NID(0, \begin{pmatrix} \sigma_s^2 & 0 \\ 0 & \Sigma_e \end{pmatrix})$$

where $\phi_s^f(L)$ is a lag operation polynomial of $p_s$th-order and $\Phi_s^e(L)$ is a lag operation polynomial of $q_s$th order. The variance-covariance matrix is assumed to be diagonal with the first element equals 1, which is a standard identification strategy in factor model literature.

Define the state vector as

$$\mathbf{s}_{s,t} = \begin{pmatrix} f_{s,t} \\ \vdots \\ f_{s,t-4} \\ e_{s,t} \\ \vdots \\ e_{s,t-4} \end{pmatrix}$$

The state space representation $p_s, q_s \leq 5$ follows by

$$\begin{aligned} \mathbf{s}_{s,t} &= \mathbf{F}_s \mathbf{s}_{s,t-1} + \mathbf{G}_s \mathbf{z}_{s,t} \\ \mathbf{y}_{s,t} &= \boldsymbol{\mu}_s + \mathbf{H}_s \mathbf{s}_s \end{aligned}$$

where

$$
\mathbf{F}_s = \begin{bmatrix} \phi_s^f(1)\cdots\phi_s^f(p_s) & \mathbf{o}'_{5-p_s} & \mathbf{O}_{5\times 5(N_s+1)} & \\ \mathbf{I}_4 & \mathbf{o}_4 & & \\ & & \Phi_s^e(1)\cdots\Phi_s^e(q_s) & \mathbf{O}_{(N_s+1)\times(5-q_s)(N_s+1)} \\ \mathbf{O}_{5(N_s+1)\times 5} & & \mathbf{I}_{4(N_s+1)} & \mathbf{O}_{4(N_s+1)\times(N_s+1)} \end{bmatrix}
$$

$$
\mathbf{G}_s = \begin{bmatrix} \sigma_s & \mathbf{o}_{(N_s+1)} \\ \mathbf{o}_4 & \mathbf{O}_{4\times(N_s+1)} \\ \mathbf{o}_{(N_s+1)} & \boldsymbol{\Sigma}_e^{1/2} \\ \mathbf{o}_{4(N_s+1)} & \mathbf{O}_{4(N_s+1)\times(N_s+1)} \end{bmatrix}
$$

$$
\mathbf{H}_s = \begin{bmatrix} \mathbf{J}_s^m(0)\boldsymbol{\beta}_s & \cdots & \mathbf{J}_s^m(4)\boldsymbol{\beta}_s & \mathbf{J}_s^m(0) & \cdots & \mathbf{J}_s^m(4) \end{bmatrix}
$$

$$
\boldsymbol{\beta}_s = (\beta'_{s1}, \beta'_{s2})'
$$

Recall that the higher frequency variable in the model is $\mathbf{y}_{s,t}^m$, which is of monthly frequency. Therefore, the estimated common factor $\hat{f}_{s,t}$ will also be of monthly frequency.

### Extended Markov-Switching Mixed-Frequency Vector Autoregressive Model

A extended Markov-switching mixed-frequency VAR model includes

$$
\mathbf{Y}_t = \begin{bmatrix} \hat{f}_{s,t} \\ \hat{f}_{m,t} \\ \hat{f}_{f,t} \\ y_t \end{bmatrix}
$$

where $\hat{f}_{s,t}$ is the estimated common factor in monthly frequency from slow-moving channel; $\hat{f}_{m,t}$ is the estimated common factor in weekly frequency from medium-moving channel; $\hat{f}_{f,t}$

is the principal component in weekly frequency estimated from medium-moving channel.

Follow the same process of mixed-frequency method, let $y^f_{s,t} = \Delta_4 \ln \hat{f}_{s,t}$, $y^{f*}_{s,t} = \Delta \ln \hat{f}^*_{s,t}$ and $y^f_{2,t} = \Delta \ln \begin{pmatrix} \hat{f}_{m,t} \\ \hat{f}_{f,t} \\ y_t \end{pmatrix}$, where $\hat{f}^*_{s,t}$ is the weekly latent variable of $\hat{f}_{s,t}$we have

$$y^f_{s,t} = \frac{1}{3} y^{f*}_{s,t} + \frac{2}{3} y^{f*}_{s,t-1} + y^{f*}_{s,t-2} + y^{f*}_{s,t-3} + \frac{2}{3} y^{f*}_{s,t-4} + \frac{1}{3} y^{f*}_{s,t-5}$$

and accordingly

$$\mathbf{y}^f_t - \boldsymbol{\mu}^f = \mathbf{J}_s(L)(\mathbf{y}^{f*}_t - \boldsymbol{\mu}^{f*})$$

where

$$\mathbf{y}^f_t = \begin{pmatrix} y^f_{s,t} \\ y^f_{2,t} \end{pmatrix} \quad \boldsymbol{\mu}^f = \begin{pmatrix} \mu^f_s \\ \mu_{y^f_2} \end{pmatrix}$$

$$\mathbf{y}^{f*}_t = \begin{pmatrix} y^{f*}_{s,t} \\ y^f_{2,t} \end{pmatrix} \quad \boldsymbol{\mu}^{f*} = \begin{pmatrix} \mu^{f*}_s \\ \mu_{y^f_2} \end{pmatrix}$$

$$\mu^f_s = E(y^f_{s,t}) \quad \mu_{y^f_2} = E(y^f_{2,t})$$

$$\begin{aligned} \mathbf{J}(L) &= \begin{bmatrix} \frac{1}{3} & \mathbf{O} \\ \mathbf{O} & \mathbf{I}_3 \end{bmatrix} + \begin{bmatrix} \frac{2}{3} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L + \begin{bmatrix} 1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^2 \\ & \begin{bmatrix} 1 & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^3 + \begin{bmatrix} \frac{2}{3} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^4 + \begin{bmatrix} \frac{1}{3} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} \end{bmatrix} L^5 \end{aligned}$$

I assume independent two-state Markov-switching process in mean, autoregressive coefficient matrix and covariance matrix for each of the channel:

$$
\begin{aligned}
y_{s,t}^{f*} &= \mu_{s_{s,t}} + \phi_{s_{s,t}}(L) y_{s,t}^{f*} + \varepsilon_{s,t} \\
\hat{f}_{m,t} &= \mu_{s_{m,t}} + \phi_{s_{m,t}}(L) \hat{f}_{m,t} + \varepsilon_{m,t} \\
\begin{bmatrix} \hat{f}_{f,t} \\ y_t \end{bmatrix} &= \begin{bmatrix} \mu_{s_{f,t}} \\ \mu_{s_{f,t}} \end{bmatrix} + \phi_{s_{f,t}}(L) \begin{bmatrix} \hat{f}_{f,t} \\ y_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{f,t} \\ \varepsilon_{y,t} \end{bmatrix}
\end{aligned}
$$

where $\varepsilon_{i,t}\tilde{}N(0, \Sigma_{s_{i,t}}), i = s, m, f, y$ are the corresponding error term and $\phi_{s_{i,t}}(L), i = s, m, f$ are lag polynomial of $p_i$th order.

The transition probabilities follow a hidden Markov chain given by

$$
\begin{aligned}
p_s^{ij} &= p_s(s_{s,t} = j | s_{s,t-1} = i, \varphi_{t-1}) \\
&= p_s(s_{s,t} = j | s_{s,t-1} = i) \\
p_m^{ij} &= p_m(s_{m,t} = j | s_{m,t-1} = i, \varphi_{t-1}) \\
&= p_m(s_{m,t} = j | s_{m,t-1} = i) \\
p_f^{ij} &= p_f(s_{f,t} = j | s_{f,t-1} = i, \varphi_{t-1}) \\
&= p_f(s_{f,t} = j | s_{f,t-1} = i)
\end{aligned}
$$

where $i, j$ are either 0 or 1 and $\varphi_{t-1}$ denotes all the information available until $t-1$ period.

Define the state vector to be

$$\boldsymbol{\beta}_t = \begin{bmatrix} \mathbf{y}_t^{f*} \\ \mathbf{y}_{t-1}^{f*} \\ \mathbf{y}_{t-2}^{f*} \\ \mathbf{y}_{t-3}^{f*} \\ \mathbf{y}_{t-4}^{f*} \\ \mathbf{y}_{t-5}^{f*} \end{bmatrix}$$

A state space representation of a extended Markov-switching mixed-frequency VAR model when $p \leq 6$ is given by

$$\begin{aligned} \boldsymbol{\beta}_t &= \boldsymbol{\mu}_{s_t} + F_{s_t}\boldsymbol{\beta}_t + Q_{\mathbf{s}_t}\mathbf{z}_t \\ \mathbf{y}_t^f &= \boldsymbol{\mu}_{s_t}^f + H\boldsymbol{\beta}_t \\ & \{\mathbf{z}_t\}\tilde{\ } IN(\mathbf{0}, \mathbf{I}) \end{aligned}$$

where

$$\boldsymbol{\mu}_{s_t} = \begin{bmatrix} \boldsymbol{\mu}^f_{s_t} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \end{bmatrix}$$

$$F_{s_t} = \begin{bmatrix} \phi_{s_{s,t}}(1) & \cdots & \phi_{s_{s,t}}(p_s) & & \\ \phi_{s_{m,t}}(1) & \cdots & \phi_{s_{m,t}}(p_m) & & \mathbf{O} \\ \phi_{s_{f,t}}(1) & \cdots & \phi_{s_{f,t}}(p_f) & & \\ \phi_{s_{f,t}}(1) & \cdots & \phi_{s_{f,t}}(p_f) & & \\ I_4 & & & & \\ & I_4 & & & \\ & & I_4 & & \\ & & & I_4 & \mathbf{O} \end{bmatrix}$$

$$Q_{\mathbf{s}_t} = \begin{bmatrix} \Sigma^{1/2}_{s_{s,t}} & & & \\ & \Sigma^{1/2}_{s_{m,t}} & & \\ & & \Sigma^{1/2}_{s_{f,t}} & \\ & & & \Sigma^{1/2}_{s_{f,t}} \\ & \mathbf{O} & & \mathbf{O} \end{bmatrix}$$

$$H = \begin{bmatrix} \mathbf{J}(0) & \cdots & \mathbf{J}(5) \end{bmatrix}$$

The model is estimated using Kalman filter as in standard Markov-switching models.

## 4.3 Application

### 4.3.1 Data

We use US economy data to construct the coincident indices for each channel. The series we use are of three frequencies sampled from July 1987 to December 2015. Quarterly indicators are "slow-moving" variables that measure the real economy activities. Monthly indicators are chosen to be variables that reflects the loan and credit change in financial intermediates. Weekly indicators are mostly "fast-moving" return rates that reflects the financial market movement. Note that for each month we may have either four or five weekly observations, we made the following adjustment for months of the latter case. Let $\{x_t, ...x_{t+4}\}$ be the five weekly observations in a month. The adjusted observations are given by

$$\{\frac{1}{2}(x_t + x_{t+1}), \frac{1}{2}(x_{t+1} + x_{t+2}), \frac{1}{2}(x_{t+2} + x_{t+3}), \frac{1}{2}(x_{t+3} + x_{t+4})\}$$

Since weekly indicators are return rates in level, the average of adjacent observations could be considered as pseudo observation over the this time period. This will guarantee us four weekly observations every month.

Table 1 summarizes the detailed descriptions of the series. "SA" stands for "seasonally adjusted", "NSA" stands for "not seasonally adjusted" and "AR" stands for "annual rate". All data are directly downloaded from FRED, except Divisia M4 is provided by Center for Financial Stability. Table 2 summarizes the descriptive statistics of the standardize indicators. The transformation codes are: 1 – no transformation; 2 – first difference; 5 – first difference of logarithm. Since some of the series experienced structural break for regulation reasons, we adopted 1% winsorization to maintain stationarity.

### 4.3.2 Estimated Results and Discussions

When estimating the coincident indices using mixed-frequency dynamic factor model, we followed the standard literature to have the series demeaned so all the constant terms in the models are deleted. Using Ox 7.10 and code modified from Mariano and Murasawa (2003, 2010), the approximate ML estimator could be estimated using quasi-Newton method. Table 3 and 4 summarized the estimated result of the mixed-frequency factor model in the first step. The number of lags for AR process are determined using information criterion AIC and SBIC same in Mariano and Murasawa (2003):

$$AIC = -\frac{1}{T}\{\ln L(\hat{\theta}) - [(N-1) + p + 1 + N(q+1)]\} \quad (4.3)$$

$$SBIA = -\frac{1}{T}\{\ln L(\hat{\theta}) - \frac{\ln T}{2}[(N-1) + p + 1 + N(q+1)]\} \quad (4.4)$$

The selected model are $(p_s, q_s) = (2, 1)$, $(p_m, q_m) = (1, 1)$ and $p = 1$.

Figure 1 illustrates the estimated common factor of quarterly indicators and estimated shadow rate that measures the direct linkage between monetary policy and real economic activities. Figure 2 is the coincident index constructed using the common factors estimated from quarterly indicators by taking the partial sum. The coincident index essentially measures how effective monetary policy is over the time. It has a nice property that it reaches local peaks right before the recession and declines rapidly during the recessions, suggesting that the impact of monetary policy lessens during the periods of recessions. Figure 4 shows the estimated common factor of monthly indicators and estimated shadow rate which measure the linkage between monetary policy and credit market activities. Figure 5 is the coincident index constructed using the common factor of monthly indicators by taking the partial sum. The coincident index shares the same properties as that in figure 2. However, we can see credit market is more reactive to monetary policy that the macro economy. Since the estimated coincident indices for slow-moving and medium-moving channel are too noisy, we apply HP filter to extract the trend and smooth the indices. Figure 3 and 6 are the smoothed smoothed coincident indices for slow-moving and medium-moving channels. Figure 7 shows the principal component extracted from indicators in fast-moving channel.

The Markov-switching mixed-frequency VAR is estimated with number of lag chosen to equal to 1. The first 300 observations are used to calibrate the model. Figure 8 illustrates the smoothed probability of "bad state" in slow-moving, medium-moving and fast-moving channels respectively. Notice that the probability for bad state is most volatile for fast-moving channel and least volatile for slow-moving channel, which is not surprising as it is guarantee by construction when we select the data. Although we can observe some sychronicity between probabilities of "bad state" in slow-moving channel and medium-moving channel, their relationship with probability of fast-moving channel is unclear. Figure 9, 10 and 11 show the probabilities of "bad state" in three channels during the periods of recessions in our sample period. Although probability of "bad state" in slow-moving channel is relatively flat, the fact that probability of "bad state" in fast-moving channel is leading that in medium-moving channel become much more clear. In addition, the fact that probability of "bad state" in medium-moving channel stay close to 1 most of the time during the periods recessions indicates the monetary policy is most likely to lose its influence in credit market within the transmission mechanism.

Table 3 shows the pairwise Granger causality tests of the fitted probabilities of "bad state" in three channels with lag horizon equal to 1. The statistics supports the probability of "bad state" in fast-moving channel is causing that in medium-moving channel and slow-moving channel. However, the causality between medium-moving channel and slow-moving channel is still unclear.

## 4.4 Conclusion

This paper aims to examine the lead-lag relationship of different channels in monetary transmission mechanism. I propose to disentangle the monetary transmission mechanism into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. I employed a mixed-frequency dynamic factor model to construct coincident indices for each of the channel from US economy data sampled from July 1987 to December 2015. The constructed coincident indices of three channels are estimated in a extended Markov-switching mixed-frequency vector autoregressive model that allows the indices of each channel to switch independently between two channels. The result provide empirical evidence for fast-moving channel leading medium-moving and slow-moving channel. However, the lead-lag relationship between medium-moving channel and slow-moving channel is still unclear.

# Bibliography


[1] Baffigi, Alberto, Roberto Golinelli, and Giuseppe Parigi. "Bridge models to forecast the euro area GDP." International Journal of forecasting 20.3 (2004): 447-460.

[2] Bai, Jennie, Eric Ghysels, and Jonathan H. Wright. "State space models and MIDAS regressions." Econometric Reviews 32.7 (2013): 779-813.

[3] Bai, Jushan, and Serena Ng. "Determining the number of factors in approximate factor models." Econometrica 70.1 (2002): 191-221.

[4] Bai, Jushan, and Serena Ng. "Confidence intervals for diffusion index forecasts and inference for factor-Augmented regressions." Econometrica 74.4 (2006): 1133-1150.

[5] Bai, Jushan, and Serena Ng. "Determining the number of primitive shocks in factor models." Journal of Business & Economic Statistics 25.1 (2007): 52-60.

[6] Barnett, William A., Marcelle Chauvet, and Danilo Leiva-Leon. Real-time nowcasting of nominal GDP under structural breaks. No. 2014-39. Bank of Canada Working Paper, 2014.

[7] Beck, Thorsten, Andrea Colciago, and Damjan Pfajfar. "The role of financial intermediaries in monetary policy transmission." Journal of Economic Dynamics and Control 43 (2014): 1-11.

[8] Bernanke, B., and Gertler, M., 1989. Agency Costs, Net Worth and Business Fluctuations. American Economic Review 79, 14-31.

[9] Bernanke, Ben S., Jean Boivin, and Piotr Eliasz. Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. No. w10220. National Bureau of Economic Research, 2004.

[10] Boivin, Jean, Michael T. Kiley, and Frederic S. Mishkin. How has the monetary transmission mechanism evolved over time?. No. w15879. National Bureau of Economic Research, 2010.

[11] Bry, Gerhard, and Charlotte Boschan. "Front matter to" Cyclical Analysis of Time Series: Selected Procedures and Computer Programs"." Cyclical Analysis of Time Series: Selected Procedures and Computer Programs. NBER, 1971. 13-2.

[12] Buch, Claudia M., Sandra Eickmeier, and Esteban Prieto. "In search for yield? Survey-based evidence on bank risk taking." Journal of Economic Dynamics and Control 43 (2014): 12-30.

[13] Carpenter, S.B., Demiralp, S., Eisenschmidt, J., 2014. The effectiveness of nonstandard monetary policy in addressing liquidity risk during the financial crisis: The experiences of the Federal Reserve and the European Central Bank. J. Econ. Dyn. Control

[14] Christiano, Lawrence J., and Martin Eichenbaum. Liquidity effects and the monetary transmission mechanism. No. w3974. National Bureau of Economic Research, 1992.

[15] Christiano, Lawrence J., and Martin Eichenbaum. "Liquidity effects, monetary policy, and the business cycle." Journal of Money, Credit and Banking (1995): 1113-1136.

[16] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Monetary policy shocks: What have we learned and to what end?." Handbook of macroeconomics 1 (1999): 65-148.

[17] Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. Two reasons why money and credit may be useful in monetary policy. No. w13502. National Bureau of Economic Research, 2007.

[18] Frale, Cecilia, et al. "EUROMIND: a monthly indicator of the euro area economic conditions." Journal of the Royal Statistical Society: Series A (Statistics in Society) 174.2 (2011): 439-470.

[19] Foroni, Claudia, and Massimiliano Giuseppe Marcellino. "A survey of econometric methods for mixed-frequency data." (2013).

[20] Ghysels, Eric. "Macroeconomics and the reality of mixed frequency data." Journal of Econometrics 193.2 (2016): 294-314.

[21] Ghysels, Eric, Pedro Santa-Clara, and Rossen Valkanov. "The MIDAS touch: Mixed data sampling regression models." Finance (2004).

[22] Harding, Don, and Adrian Pagan. "Synchronization of cycles." Journal of Econometrics 132.1 (2006): 59-79.

[23] He, Zhiguo, and Arvind Krishnamurthy. A macroeconomic framework for quantifying systemic risk. No. w19885. National Bureau of Economic Research, 2014.

[24] Kashyap, A.K., Stein, J.C., 1995. The impact of monetary policy on bank balance sheets. Carnegie-Rochester Conf. Ser. Public Policy 42(1), 151–195.

[25] Killian, Lutz (1996), 'Small - Sample Conference Intervals for Impulse Response Functions', manuscript, University of Pennsylvania.

[26] Kim, Chang-Jin, and Charles R. Nelson. "The time-varying-parameter model for modeling changing conditional variance: The case of the lucas hypothesis." Journal of Business & Economic Statistics 7.4 (1989): 433-440.

[27] Kim, Chang-Jin, and Charles R. Nelson. "Business cycle turning points, a new coincident index, and tests of duration dependence based on a dynamic factor model with regime switching." The Review of Economics and Statistics 80.2 (1998): 188-201.

[28] Kim, Chang-Jin, and Charles R. Nelson. "State-space models with regime switching." (1999).

[29] King, Robert, et al. "Stochastic trends and economic fluctuations." (1987).

[30] King, Robert G. (1991), 'Money and Business Cycles', manuscript, University of Rochester.

[31] Mariano, Roberto S., and Yasutomo Murasawa. "A new coincident index of business cycles based on monthly and quarterly series." Journal of applied Econometrics 18.4 (2003): 427-443.

[32] Mariano, Roberto S., and Yasutomo Murasawa. "A coincident index, common factors, and monthly real GDP." Oxford Bulletin of Economics and Statistics 72.1 (2010): 27-46.

[33] Nicolo G.D., Dell'Ariccia, G., Laeven, L., Valencia, F., 2010. Monetary Policy and Bank Risk-Taking. IMF Staff Position Notes 2010/09, International Monetary Fund.

[34] Schorfheide, Frank, and Dongho Song. "Real-time forecasting with a mixed-frequency VAR." Journal of Business & Economic Statistics 33.3 (2015): 366-380.

[35] Sims, Christopher A. "Macroeconomics and reality." Econometrica: Journal of the Econometric Society (1980): 1-48.

[36] Stock, James H., and Mark W. Watson. "A probability model of the coincident economic indicators." (1988).

[37] Stock, James, and Mark Watson, "New Indexes of Coincident and Leading Economic Indicators", NBER Macroeconomics Annual (4), Cambridge MA: MIT Press, 1989.

[38] Stock, James H., and Mark W. Watson. Diffusion indexes. No. w6702. National Bureau of Economic Research, 1998.

[39] Stock, James H., and Mark W. Watson. "Estimating turning points using large data sets." Journal of Econometrics 178 (2014): 368-381.

[40] Taylor, John B. "The monetary transmission mechanism: an empirical framework." The Journal of Economic Perspectives 9.4 (1995): 11-26.

[41] Tobin, J., 1969. A general equilibrium approach to monetary theory. J. Money Credit Bank. 1 (1), 15–29.

## 4.5 Tables and Figures

Table 4.1: Pairwise Granger Causality Tests

| Null Hypothesis: | Obs | F-Statistic | Prob. |
|---|---|---|---|
| Medium-moving not Cause slow-moving | 1357 | 17.2588 | 3.E-05 |
| Slow-moving not Cause medium-moving | | 12.6851 | 0.0004 |
| Fast-moving not Cause slow-moving | 1357 | 74.0735 | 2.E-17 |
| Slow-moving not Cause fast-moving | | 0.34663 | 0.5561 |
| Fast-moving not Cause medium-moving | 1357 | 94.8806 | 1.E-21 |
| Medium-moving not Cause fast-moving | | 0.88612 | 0.3467 |

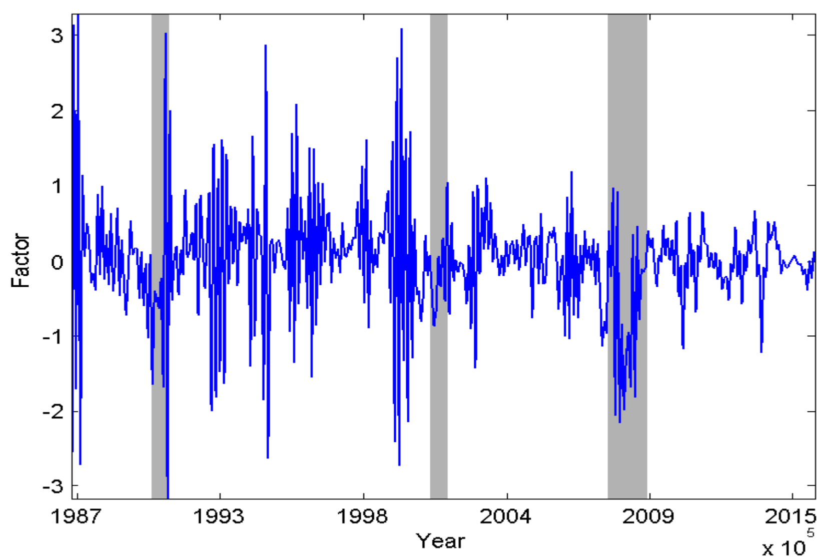


Figure 4.1: Estimated factor from quarterly indicators.

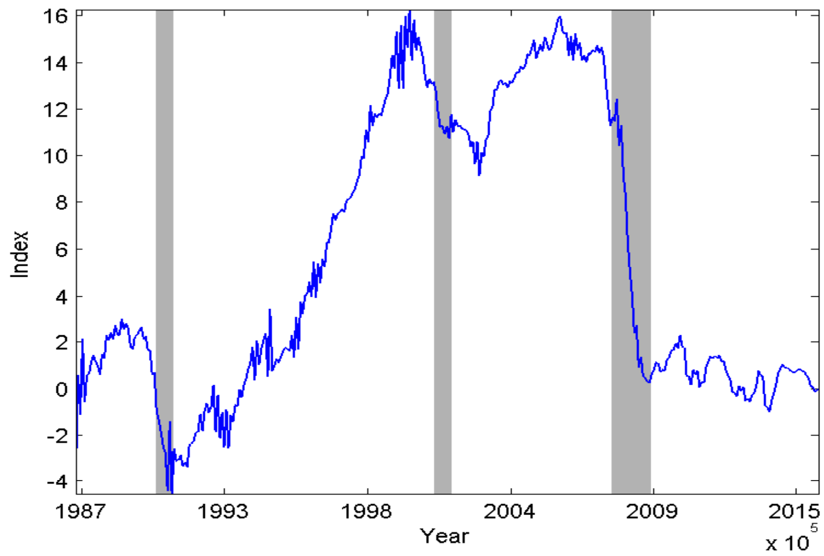


Figure 4.2: Coincident index constructed using quarterly indicator factor.

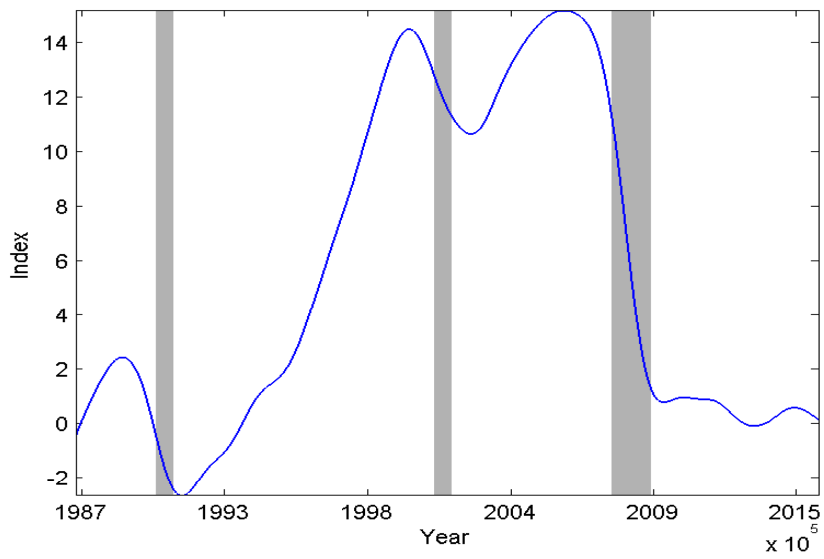


Figure 4.3: Smoothed coincident index constructed using quarterly indicator factor.

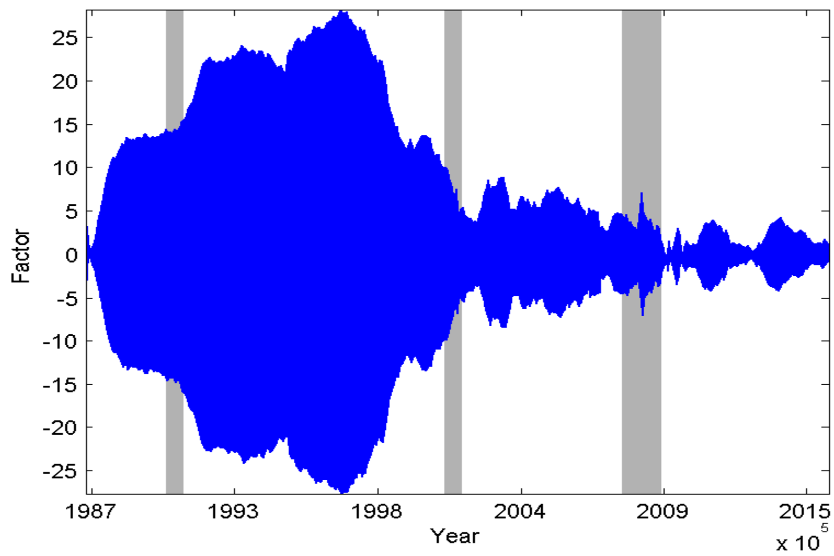


Figure 4.4: Estimated factor from monthly indicators.

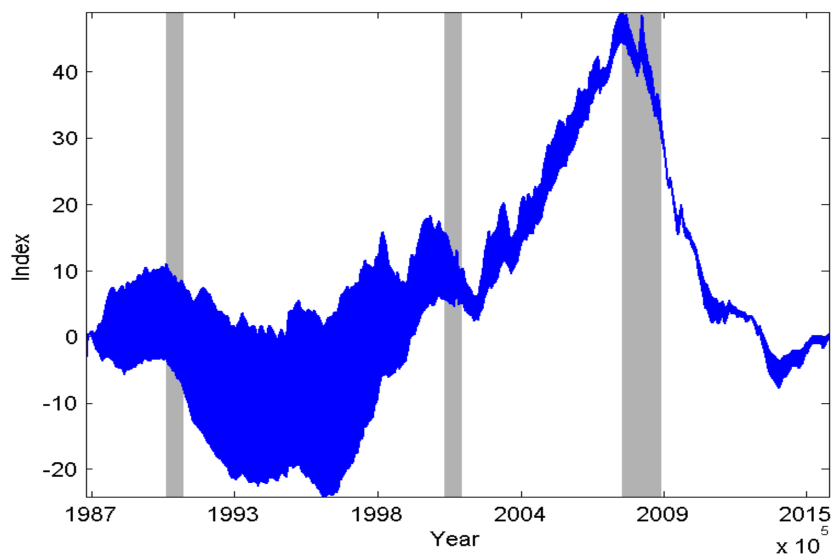


Figure 4.5: Coincident index constructed using the monthly indicator factor.

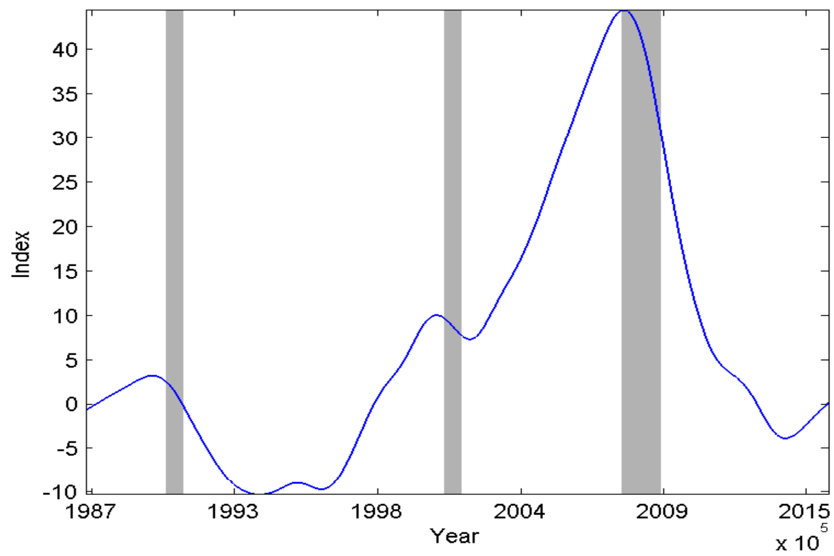


Figure 4.6: Smoothed coincident index constructed using the monthly indicator factor.

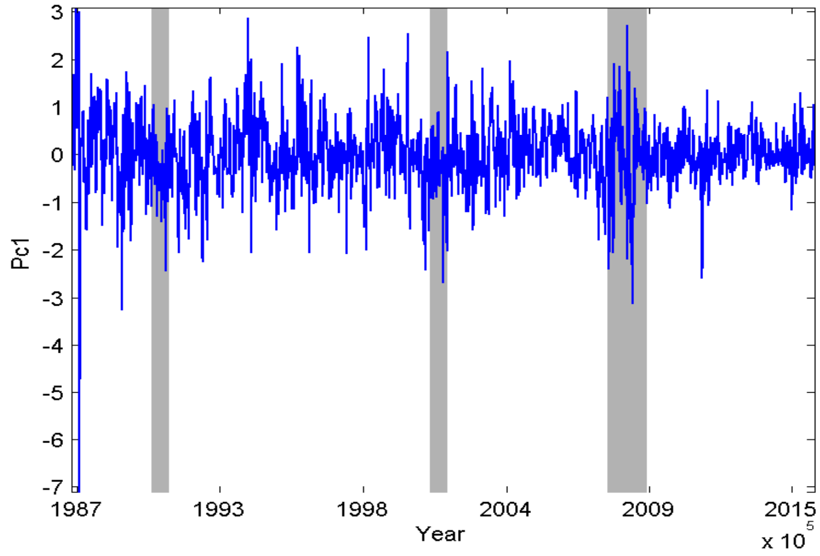


Figure 4.7: First principal component of weekly indicators.

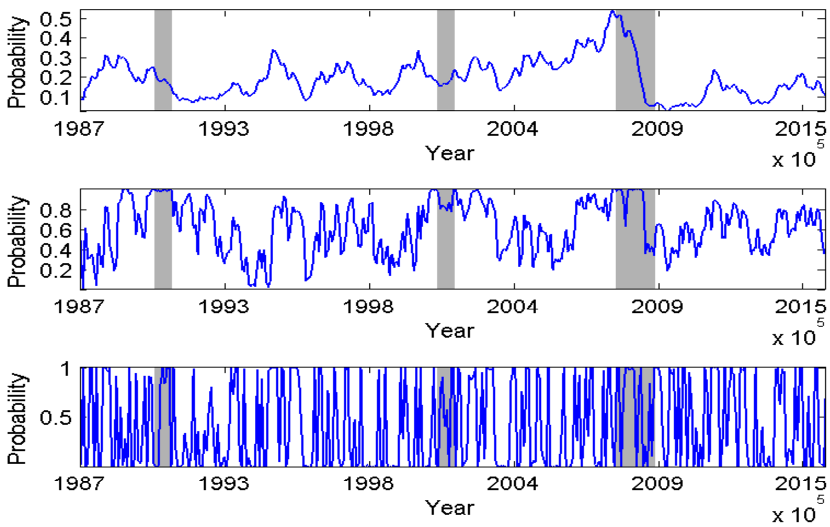


Figure 4.8: Smoothed probability of "bad state" in slow-moving, medium-moving and fast-moving channels.

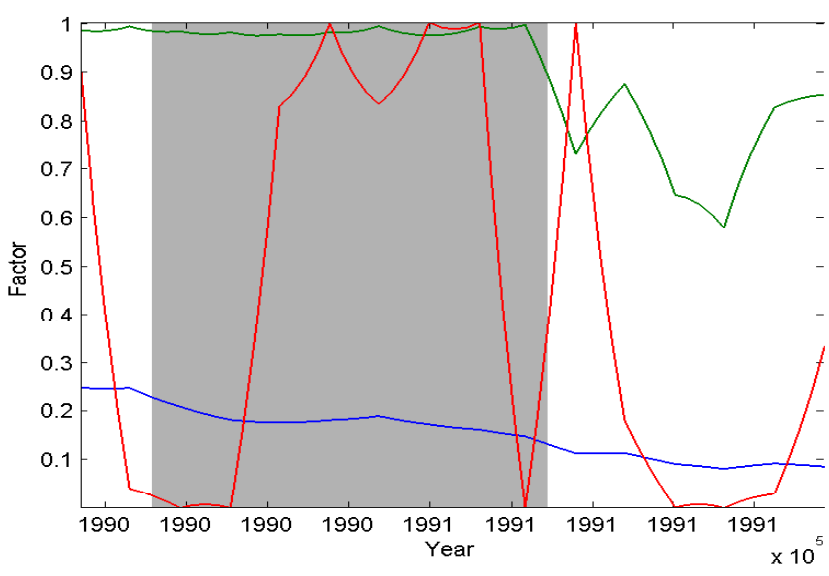


Figure 4.9: Smoothed probability of "bad state" in slow-moving, medium-moving and fast-moving channels during 1990 recession.

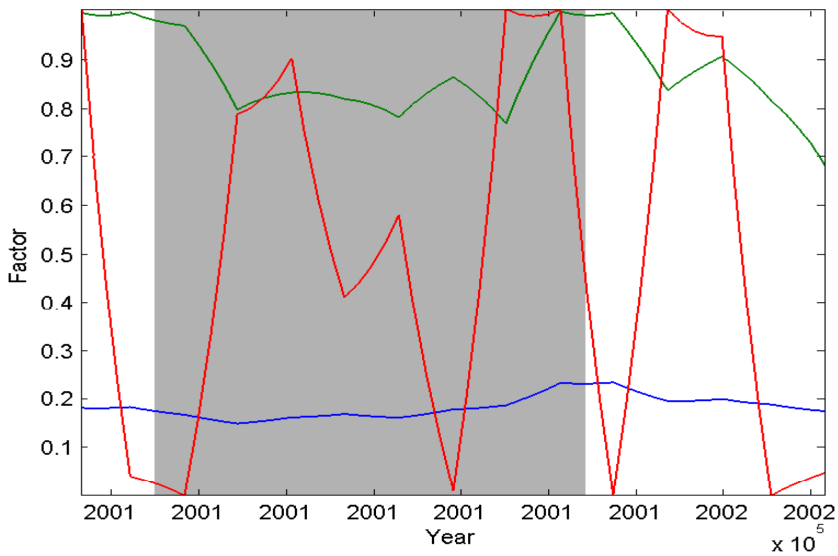


Figure 4.10: Smoothed probability of "bad state" in slow-moving, medium-moving and fast-moving channels during 2001 recession.

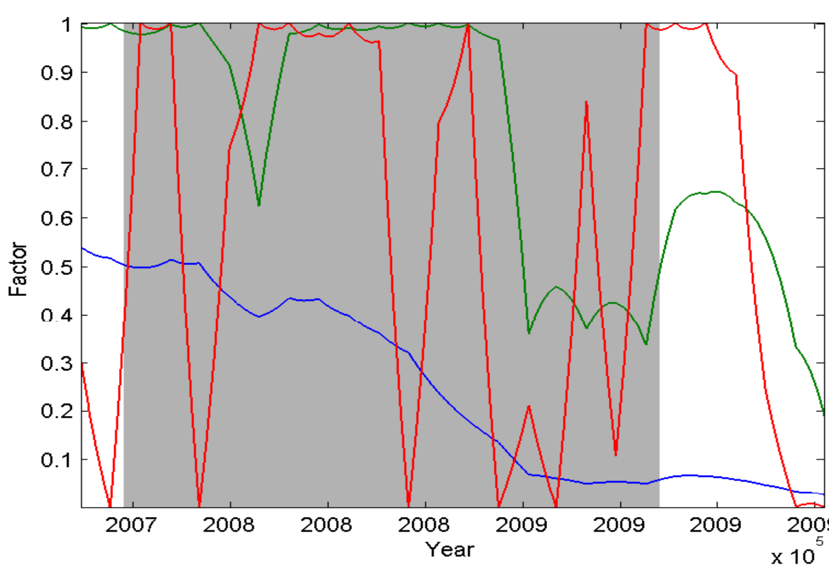


Figure 4.11: Smoothed probability of "bad state" in slow-moving, medium-moving and fast-moving channels during 2008 recession.

# Chapter 5

# Conclusion

This dissertation attempts to explore the monetary transmission mechanism with an innovating empirical framework: mixed-frequency factor-augumented vector autoregressive model, which allows to include a large number of economic variable with different frequencies. Extensions of the model are also provided: a stacked-vector system equips the model with classic impulse response function analysis; Markov-switching feature is added to the model to study the dynamics of unobservable states.

The first chapter studies the monetary transmission mechanism in the U.S. It proposes a mixed-frequency version of the factor-augmented vector autoregressive regression (FAVAR) model, which is used to construct a coincident index to measure the monetary transmission mechanism. The model divides the transmission of changes in monetary policy to the economy into three stages according to the timing and order of the impact. Indicators of each stage are measured and identified using different data frequencies: fast-moving variables (stage 1, asset returns at the weekly frequency), intermediate moving variables (stage 2, credit market data at the monthly frequency), and slow-moving variables (stage 3, macroeconomic variables at the quarterly frequency). The resulting coincident index exhibits leading signal for all recessions in the sample period and provides implications on the dynamics of the monetary transmission mechanism. The proposed coincident index also indicates that monetary transmission mechanism is changing over time.

The second studies the monetary transmission taking unconventional monetary policy in consideration. The impact of unconventional monetary policy is reflected on the dynamics of constructed shadow rate which is estimated using one-month forward rate of bond of different maturities. The monetary transmission mechanism is disentangled into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro economic variables measured in low frequency. A set of coincident indices are constructed using the mixed-frequency factor-augumented autoregressive model. Impulse response analysis are also applied by imposing the stacked vector system in the second step of the estimation. The resulting coincident index of overall monetary transmission mechanism captures major unconventional monetary policy during the Zero Lower Bound and exhibits a substaintial lead on the recessions in the sample period, which is due to the construction process of shadow rate. The impulse response functions show that monetary policy shocks have relatively persistent impact on medium-moving channel (or credit market). Also, we find the slow-moving channel is more responsive to monetary policy shocks in the beginning and end of a month compared to those in the middle.

The third chapter aims to examine the lead-lag relationship of different channels in monetary transmission mechanism. I propose to disentangle the monetary transmission mechanism into three channels according to transmission speed of the impact: fast-moving channel that links the policy rates to asset returns in the financial market measured in high frequency; medium-moving channel that links the policy rates to loan and credit data in credit market measured in medium frequency; slow-moving channel that links the policy rates to real macro-economic variables measured in low frequency. I employed a mixed-frequency dynamic factor model to construct coincident indices for each of the channel from US economy data sampled from July 1987 to December 2015. The constructed coincident indices of three channels are estimated in a extended Markov-switching mixed-frequency vector autoregressive model that allows the indices of each channel to switch independently between two channels. The result provide empirical evidence for fast-moving channel leading medium-moving and slow-moving channel. However, the lead-lag relationship between medium-moving channel and slow-moving channel is still unclear.