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The Impact of Pricing, and Targeting on Consumer Choice

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The Impact of Pricing, and Targeting on Consumer Choice

DISSERTATION

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for the degree of

DOCTOR OF PHILOSOPHY

in Management

by

Ran Zhang

Dissertation Committee:
Professor Vidyanand Choudhary, Chair
Professor Shivendu Shivendu
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2017

DEDICATION

This dissertation is dedicated to my parents

Who inspired me by practicing the virtues of hard work, honesty, commitment, and passion

in their life.

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Economic impact of pricing, bundling, and targeting strategy on consumer choice

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ABSTRACT OF THE DISSERTATION

By

Ran Zhang

Doctor of Philosophy in Management

University of California, Irvine, 2017

Professor Vidyanand Choudhary, Chair

Firms offer products or services through different strategies. How to optimally price and bundle products is one of the most fundamental questions facing firms when consumers in the market are heterogeneous in their valuations for products. The pricing and bundling problems become more complicated when competition exists or the product is offered through different platforms. Chapter 1 of my dissertation focuses on the quality-differentiated firms' bundling strategy for their core and add-on products. Motivated by firms' distinct bundling practices, I build an analytical model to explain the firms' optimal strategy. I find the critical role of competition and cost-to-quality ratio in driving the asymmetric firm's bundling and pricing strategy. When there is more competition from the inferior firm, the superior firm has stronger incentive to bundle its add-on, even when the add-on product is costly.

Further in this direction, to study the firm's pricing and bundling strategy when the product is offered through different mediums, chapter 2 of my dissertation focuses on the content publisher's optimal strategy when the content can be offered in digital, physical, or a bundle of mediums. I develop an analytical model and find that offering digital medium only or offering bundle and digital medium can be optimal under different market

conditions. Surprisingly, I find consumer surplus and social welfare may decrease as the proportion of digital-savvy consumers increases.

To further study the complementarity and substitutability of platforms, chapter 3 of my dissertation studies how firms should run their targeted advertising on different online platforms, i.e. social media and traditional platform. Specifically, I measure the effectiveness of targeted advertising on social media, relative to that on a traditional platform, on consumers' ultimate conversions. I measure the interaction effects of the two platforms by using case-control design and post-regularized choice models. I find that targeting across platforms is positively associated with the ultimate conversion for the lower funnel, but there is no measurable synergistic effect for the upper funnel consumers.

INTRODUCTION

My dissertation focuses on the general question of the impact of firms' strategic offerings on consumer choice. Firms impact consumer choice (e.g. purchase) through an assortment of offering strategies. For example, firms can offer their products (or services) separately, or in a bundle, or in a bundle as well as in *à la carte*. For each strategy, firms also need to consider their pricing strategy. Firms' bundling and pricing strategy impact consumers' choice of purchase. Consumers' type and market conditions in turn affect firms' bundling and pricing strategy. Besides bundling and pricing products, firms often use marketing tools (e.g. advertising) through different platforms to influence consumers' purchase decision. A widely adopted way for such marketing is through targeted advertising on multiple online platforms, the effectiveness of which, however, has yet to been measured thoroughly. Moreover, the complementarity and substitutability of targeting on different online platforms are yet to be examined. My dissertation seeks to systematically address these questions in the following chapters.

In chapter 1 and chapter 2, I consider the impact of firms' pricing and bundling strategy on consumer choice in different contexts. In chapter 1, I study how firms should optimally bundle and price their core product with their add-on product, and how competition can impact firms' optimal bundling decisions. In addition, I examine how public policy on add-on pricing can impact firms' profitability and consumer welfare. In chapter 2, I study how publishers should offer their contents through different media in the content industry. I focus on how publishers should bundle and price the same content in different media. In addition, I examine the impact of digitization on content markets, i.e. how digital medium access can affect consumer surplus and social welfare.

In chapter 2 and chapter 3, I study the tradeoff effects between platforms or mediums on consumer choice. Specifically, I examine the complementary and substitutability of two platforms or mediums in different industries. In chapter 2, I focus on the substitutability between the digital medium and the physical medium, and how such tradeoff effects impact publisher's optimal offering strategies. In chapter 3, I analyze the relationship between targeting on social media and targeting on the traditional online platform for consumers at different purchasing stages. For different type of consumers, the effectiveness of targeting on social media may either complement with or substitute to that of targeting on the traditional platform. I discuss more details below.

In chapter 1, in an effort to examine firms' strategic interactions with consumers, I study firms' bundling strategy on add-on products in a vertically differentiated competitive market. Firms often offer a variety of add-on products (e.g. technical support) in addition to their core products (e.g. software). How should firms offer such add-on products? Should they offer them as a bundle or à la carte? How does competition impact firms' bundling choice?

I develop an analytical model and identify the critical role of competition and cost-to-quality ratio in firm's bundling decisions. I find that when there is more competition from the inferior firm, the superior firm will have stronger incentive to bundle its add-on, even when the add-on product is costly. I show that consumers are never better off when add-on pricing is prohibited by regulators. Counter-intuitively, I find the inferior firm's optimal price and profit can decrease with its core good quality. These results help explain the different bundling and pricing strategies adopted for quality-differentiated firms in IT and travel industries.

This study yields important managerial implications. First, when facing competition from an inferior firm, the superior firm should decide its bundling strategy based on the intensity of the competition. When the inferior firm has relatively low quality, the superior firm should unbundle its add-on to gain a larger total demand, i.e. only high valuation consumers buy both its core and add-on, and consumers who have relatively low valuation buy only the core product.

In chapter 2, I study the impact of digitization on the content markets. The digitization has enabled publishers across content industries to offer their information goods in physical medium as well as in digital medium. The two media are considered partial substitutes because the same content can be offered in either medium.

It is unclear how digitization impacts the publisher's optimal prices, profit as well as social welfare. On one hand, digitization can expand the consumer base by providing additional convenience through anytime-anywhere access (Kouikova et al. 2008). On the other hand, online access to information goods may substitute for access through a physical medium, thereby cannibalizing print sales (Kannan et al. 2009). In practice, publishers adopt different pricing strategies, ranging from offering only digital medium, to offering bundle, to offering a choice of bundle or digital medium. Prior research has not studied the publisher's optimal pricing strategy when consumers are heterogeneous in both valuation for the content and preference for the medium. This study fills this void. I use a game theoretic model to develop optimal pricing strategies and identify the interactive and competing roles of different market parameters on the market outcome.

I find that offering only digital access is optimal under some market conditions, while offering a choice of a digital-print bundle or digital-alone is optimal under other market conditions. Interestingly, while the bundle price increases with the marginal cost of the physical medium, the price of the digital medium may decrease with marginal cost. Counter to intuition, whereas the price of digital-only weakly increases with the proportion of digital-savvy consumers, the bundle price, market coverage, consumer surplus, and social welfare can decrease when more consumers prefer the digital medium.

These findings offer managerial guidance to publishers regarding the optimal pricing strategies under different market conditions, and draw managerial implications for regulatory bodies like FCC to design an appropriate policy framework that enhances publisher profits together with consumer surplus.

In chapter 3, it is controversial as to whether and how targeted ads on social media can be effective in converting potential consumers, relative to that on the traditional platform. I focus on the two platforms, i.e. social media and traditional platform, and examine whether and how the effects of targeted advertising on social media differ from that on the traditional media for consumers at different purchasing stage (funnel). To measure the complementarity and substitutability effects of the two platforms, I use robust Subspace K-means clustering techniques to form retrospective case-control designs. I then analyze the effects of targeting across platforms on purchasing conversions by using post-regularized choice models.

I find that first, clustering based on the similarity of consumers, compared with no clustering, mitigates heterogeneity and offer more meaningful insights on platform effects. Second, I show that targeting across platforms has synergistic effects with the ultimate conversion for consumers at the lower funnel, but does not appear to provide any interaction effect for the upper funnel consumers. Third, my result shows that the main effect of targeting on social media is positively associated with the odds ratio of purchase for the upper funnel consumers, but has no impact about the traditional platform for consumers at the lower funnel. Lastly, my finding indicates that the commonly implemented "retargeting ads" are more effective than other more sophisticated targeting strategies, and that retargeting may have a positive and significant association with the ultimate conversions for consumers at the lower funnel. These findings will help practitioners derive more efficient targeting strategies across different platforms for different customers, and thereby lending to better allocation of their advertising budget.

CHAPTER 1

Bundling Add-on Products in a Vertically Differentiated Duopoly

Abstract

Firms often offer a variety of add-on products or services in addition to their core products. How should firms offer such add-on products? Should they offer them as a bundle or à la carte? How does competition impact firms' bundling and pricing strategy? What is the impact of regulators' decision to limit add-on pricing on firms' profitability and consumer surplus? Motivated by these questions and different pricing practices for add-on products across industries, we develop an analytical model to examine firms' bundling and pricing strategy when competing firms are quality-differentiated. We identify the critical role of competition in firm's bundling decision. When there is more competition from the inferior firm, the superior firm has stronger incentive to bundle its add-on, even when the add-on product is costly. When facing intense competition from the inferior firm, the superior firm will always bundle. We show that consumers are never better off when add-on pricing is prohibited by regulators. Counter-intuitively, we find that the inferior firm's optimal price and profit may decrease as its core product quality increases, and the superior firm's demand can increase as its rival's core product quality increases. In addition, we find that consumer surplus can decrease as the superior firm's add-on quality increases when there is intense competition.

1. Introduction

Firms often provide add-on products or services in addition to their core products. For example, software vendors offer technical support and training besides software. Hotels provide Internet access and breakfast in addition to lodging. Banks offer debit card and overdraft protection in addition to the basic checking service. Telecommunication providers promote additional data plans besides voice calling service. How should firms offer such add-on services? Should they charge each of these services separately as many airlines have done recently? Or should they offer them as a bundle? How does competition impact a firm's pricing decision? How does the change of a firm's quality impact its profit and its rival's market share? What is the impact of public policy regarding add-on pricing on firms' profitability and consumer surplus?

In the software industry, a high-quality software vendor such as *Adobe Systems Inc*. charges Adobe Acrobat software and real-time technical support separately, whereas a lower-featured software vendor such as *Nitro Corp.* offers their customer support and the software for a bundled price. In the IT security industry, whereas low-quality vendors like *PRTG network monitor* offer all-in, no add-ons price for both its core network monitoring functions and add-on features such as cluster failure solution and security SSL encryption, the high-quality vendors bundle their add-ons. For example, *SolarWinds* offers the network performance monitor v11 for a bundled price including additional features such as route monitoring and packet analysis[1]. In the hotel industry, high-end hotels like *Hilton hotels* unbundle their product offering so that the quoted prices include only the room while amenities such as Internet access are available for additional charges. In contrast, many economy hotels such as *Days Inn* bundle their Internet access and breakfast with the lodging service. The anecdotal examples cited above are counter to intuition. How can we

[1] Based on prices and reviews, they are considered as high-quality firms.

explain this phenomenon? Prior research has not explained the differences in the high-quality firm's bundling strategies across industries and this is the first focus of this study.

Our second focus is to examine the impact of public policy regarding add-on pricing on firms' profitability and consumer surplus. Add-on pricing has been widely adopted across industries and generates a fair amount of revenue. For example, airlines in the US took in $23.7 billion in a-la-carte fees for food, baggage, seat preference, and other services (Ellis 2014). In the UK, airlines impose a variety of add-on fees associated with bookings and make around £300 million from them (Ensor 2013). In the banking industry, the Consumer Financial Protection Bureau (CFPB) reported that in 2012, banks took in $32 billion in overdraft fees (Touryalai 2013). Given the large amount of revenues generated from add-on services, general concerns prevail in public and among consumer groups that companies are over charging through various add-on services. Hence, government and regulators have acted to regulate the add-on services market to protect consumers, sometimes even to prohibit the add-on pricing practices. For example, the Consumer Financial Protection Bureau (CFPB) has proposed regulatory scrutiny on add-on sales such as extended warranties and tire plans in the auto industry (Willis 2013); Senate Richard Durbin criticized debit card fees in the bank industry and called for regulations to prevent onerous fees in future (Mui 2011). The British government has banned travel companies and retailers from charging additional fees when consumers use their credit cards to pay online (Ensor 2013). The financial conduct authority (FCA) intervened the £1B general insurance add-on market and proposed to ban separate charges of insurance add-on products (FCA Press 2014). Although regulation of add-on pricing seems necessary in some circumstances, it is not clear whether prohibiting add-on pricing benefits consumers. In this study, we explicitly examine the implications of add-on pricing policy on firm profitability and consumer surplus. We provide insights on the impact of well-intentioned regulators' intervention on add-on pricing under different conditions.

We study these questions by developing an analytical model in a duopoly competition setting. While prior literature on add-ons studies symmetric firms that offer the same quality core and add-on products (Ellison 2005, Gabaix and Laibson 2006), we study the case where the competing firms offer asymmetric qualities of core products to reflect the industry context. Moreover, we allow asymmetry of add-on product quality for the duopolistic firms, i.e. the high-quality (superior) firm offers higher-quality add-on than the low-quality (inferior) firm. We also incorporate asymmetric marginal cost of the add-ons and identify the key role the competition plays in firms' bundling decision. On the consumer side, we model consumers that are continuously differentiated in their taste of qualities for both the core and the add-on products, and we study the impact of such heterogeneity in the taste of quality on firms' pricing strategy and consumer surplus. Also, we assume consumers' willingness to pay (WTP) for the core product to be positively correlated with their WTP for the add-on. For example, consumers who appreciate higher quality of an ERP system are also likely to appreciate higher quality of the after-sales support. In addition, whereas some prior literature assumes a segment of boundedly rational consumers (i.e. consumers who are not aware of the price of add-on) and their optimal results is based on this segment of boundedly rational consumers, we focus on the case in which all consumers have full information about the prices of both the core and the add-on products. Therefore, our results are not dependent on consumers' irrationality but are derived based on fully informed, rational consumers.

Our analysis has produced several interesting results. First, we identify the critical role of competition in firm's add-on bundling strategy. Specifically, when there is more competition from the inferior firm, the superior firm has stronger incentive to bundle its add-on, even when the add-on product is costly. Second, while in the benchmark case, the monopolistic firm unbundles the add-on when the marginal cost of the add-on is positive, we show in duopoly that the superior firm bundles even if the marginal cost of add-on is positive, as long as the ratio of its marginal cost to quality is not too high. Third, surprisingly, we find that consumers are never better off when regulators prohibit the add-on pricing practice; The superior firm may lose profit from such prohibition policy. Fourth, counter-intuitively, our results show that the inferior firm's price and profit may decrease as its core product quality increases, and the superior firm's demand may increase with the inferior firm's core quality. Furthermore, consumer surplus can decrease as the superior firm's add-on quality increases when there is strong competition.

The rest of the paper is organized as follows. We review the relevant literature in §2. In §3, we analyze the benchmark monopoly case, followed by the model setup for the duopoly case in §4. We present our analysis and results in §5, and report the analysis for the comparative statics in §6. In §7, we discuss the impact of public policy regarding add-on pricing on firms and consumers. We discuss the contributions to the literature, managerial implications, and extensions for the study in §8.

2. Literature Review

One stream of add-on literature studies the profit implications for a monopoly that offers core and add-on products. Fruchter et al. (2010) find that it is profit-equivalent for the monopoly to offer the add-on for a fee or for free when both types of consumers value the add-on similarly, and it is optimal to provide a free add-on when only the low type of consumers value the add-on. Adachi et al (2011) show that the monopoly should unbundle the add-on from the core product when the range of the add-on product valuation exceeds a threshold value. Shugan et al. (2016) study whether a monopoly should offer a product line, i.e., two core products, and if so, whether the monopoly should bundle the add-on with low-end core or high-end core under different constraints. We use monopoly setting as the benchmark and show that when the monopoly has positive marginal cost for the add-on, it is optimal for the monopoly to unbundle and offer add-on pricing. It is optimal to bundle when the monopoly has negligible marginal cost for the add-on.

Another stream of literature examines price and profit implications of add-on pricing in duopolistic competition (Ellison 2005, Shulman and Geng 2013, Geng and Shulman 2015). Add-on pricing is profit enhanced for symmetric firms when there is sufficient asymmetry of price sensitivity between the two types of consumers (Ellison 2005). On the other hand, add-on pricing does not affect profit for either firm when firms have symmetric add-on qualities and all consumers know the add-on prices (Lal and Matutes 1994, Verbon 1999, Shulman and Geng 2013). When there is asymmetry in add-on qualities, the superior firm is better off and the inferior firm is worse off when add-on pricing is allowed than the case when add-on pricing is prohibited (Shulman and Geng 2013). An elegant model by Geng and Shulman (2015) characterizes horizontal competition between symmetric firms. They find that both firms bundle in equilibrium when consumers have the same transportation cost and firms' add-on marginal costs are zero. Firms have asymmetric bundling strategies when consumers have different transportation cost and firms' add-on marginal cost is relatively small. Our research is different from these studies in that we model vertical competition between firms with asymmetric core and add-on products. We assume consumers are continuously differentiated in their taste for qualities, and our model allows consumers to buy products from either firm or not buy at all. Our results show that the superior firm either bundles or unbundles in equilibrium, depending on the intensity of competition and the ratio of its marginal cost of add-on to quality, and the inferior firm bundles. When there is more competition from the inferior firm, the superior firm will have stronger incentive to bundle, even when the marginal cost of add-on is positive. When competition is strong, the superior firm always bundles, irrespective of its marginal cost. Moreover, we find that the inferior firm's price and profit may decrease while the superior firm's demand may increase when the inferior firm's core quality increases. When competition is strong, consumer surplus may decrease as the superior firm's add-on quality increases.

Regarding consumer welfare implications, Ellison (2005) find that when there is significant asymmetry in price sensitivity between the two types of consumers, consumer welfare is better off if the core product cannot be sold separately with the add-on than the case if the core product can be sold separately. Our result shows that when consumers have continuously heterogeneous valuation for the core and the add-on, consumer welfare is either worse off or indifferent if core product cannot be offered separately, i.e. when add-on pricing is prohibited by regulation.

Prior add-on literature studies the impact of the existence of boundedly rational consumers on profit implications of add-on pricing (Lal and Matutes 1994, Verboven 1999, Gabaix and Laibson 2006). Our research differs with this stream of research by assuming that all consumers are rational and fully informed. Therefore, our bundling results and profit implications don't rely on the existence of bounded rational consumers. We identify the key role of competition in impacting firm's bundling strategy and the implication of regulatory policy on add-on pricing.

Our study is broadly related to the literature on mixed bundling. The early work (Stigler 1963, Adams and Yellen 1976) illustrates that bundling can serve as a useful price discrimination technique, and mixed bundling can be more profitable than pure component strategy. While Adams and Yellen (1976) suggest that bundling is profitable when valuations for the two products are negatively correlated, Schmalensee (1984) illustrate that bundling can increase profits when the valuations of the two products are independent, or even positively but not perfectly correlated. McAfee et al. (1989) analyze the bundling strategy for the multiproduct monopolist when the products have positive marginal costs and consumers' valuations are independently distributed, and derive conditions under which mixed bundling either dominates pure component pricing or weakly dominates pure bundling. In addition, Venkatesh and Kamakura (2003) study monopolist's bundling strategy and compare pure bundling with mixed bundling when the two products are either substitutes or compliments. They use numerical methods and find pure bundling should be optimal if the two products are strong complements.

Prior literature studies firms' bundling strategies in a competitive market (e.g., Chen 1997, Ghosh and Balachander 2007, Huang et al. 2013, Nalebuff 2004). Chen (1997) study the case where a primary product is produced in a duopoly market and one or more other products are produced under perfectly competitive conditions. They find that bundling is optimal for one or both of the duopolists. Reisinger (2006) study the strategies for the duopolist to sell their products as a bundle. They show that contrary to the monopoly case, bundling reduces profits for the duopolist if the correlation of consumers' reservation values is highly negative, because negative correlation bundling reduces consumer heterogeneity and makes price competition more aggressive. Ghosh and Balachander (2007) model the competition between a multiproduct generalist firm and two single-product specialist firms in two product categories. Gumus et al. (2013) examine whether online retailers should bundle or unbundle the shipping and handling with the product offering, and provide insights into how the retailer type and product characteristics drive a retailer's bundling strategy.

While the bundling research informs our study, the results from the bundling literature are not readily applicable to the context of add-on products bundling. First, add-on product is typically purchased and consumed contingent on the purchase of the core product. But the core product can be purchased without the add-on. Second, the valuation of the add-on is typically smaller than that of the core product. Put differently, there is an inherent asymmetry between the valuation of the core and add-on that imposes constraints on the add-on bundling problem. In addition, because the add-on product is by nature complementary to the core product, consumers have positively correlated demand for the core and the add-on products. Our research allows for the continuous heterogeneity in consumers' WTP for both core and add-on products. The unique relationship of core and add-on products and positively correlated demand allow us to examine add-on pricing and bundling strategy that may be different from the traditional bundling literature.

Unbundling the add-on with the core product can be seen as a form of second-degree price discrimination, therefore, our paper is also related to the price discrimination literature for asymmetric firms. Corts (1998) study competition between quality differentiated firms with two discrete type of consumers, and find that the strategic commitment not to price discriminate may raise both firms' profits by softening price competition. Shaffer and Zhang (2002) study asymmetric firms' price discriminations through one-to-one promotions. They show that when consumers have heterogeneous brand loyalty, the high-quality firm may be better off from one-to-one promotions. Prior research has analyzed competing firms' price discrimination based on consumers' past purchases. Chen (2008) find that price discrimination based on purchase histories may benefit consumers if it does not cause the low-quality firm to exit the market. Pazgal and Soberman (2008) argue that the firm with capacity to add more benefits to its offer to the past consumers will price discriminate between new and old consumers, while the firm with low capacity to add benefits to its offer may be better off by adopting uniform price. Our research is different in that we study whether firms should price discriminate by unbundling the add-on product with the core product when consumers are continuously differentiated by their heterogeneous taste for both the core and the add-on product.

3. Benchmark Monopoly Case

We first analyze a benchmark case of the monopoly, and then we analyze duopolistic firms' pricing strategies and examine the effect of competition on a firm's bundling and pricing strategy in section 4.

A monopolistic firm offers a core product with quality q_c and an add-on product (or service) with quality q_a ($q_c > q_a$). He needs to decide whether to offer them as a bundle or à la carte. To simplify the notation in the analysis, we normalize the firm's core quality to 1 ($q_c = 1$). The monopoly incurs negligible marginal cost for the core product and non-negative marginal cost c_a for add-on. All qualities and cost are exogenous. The market consists of consumers who have heterogeneous taste for quality. The taste parameter θ is uniformly distributed over [0, 1] (Choudhary et al. 2005). The distribution is known while each consumer's valuation is private information. The model follows a two-stage game. First, the monopoly decides whether to bundle or unbundle. Secondly, the monopoly decides what price(s) to offer, and consumers decide whether and what to purchase. Consumers can buy the core product only or both the core and the add-on or nothing. The consumer's generic utility function is $U = \theta q - p$. Consumers buy at most one unit of core product and one unit of add-on (Ellison 2005, Shulman and Geng 2013, Geng and Shulman 2015).

The monopoly's problem is whether to bundle the add-on with the core product. If the monopoly bundles and offers p_B, then a consumer buys the bundle if her individual rationality (IR) constraint is met: $\theta_B(1+q_a) - p_B \geq 0$. The firm's profit function is $\pi_B = (p_B - c_a)(1 - \frac{p_B}{1+q_a})$. If the firm unbundles and offers prices p_c for the core and p_a for the add-on, then consumers buy the core good if her IR constraint is met: $\theta_c - p_c \geq 0$, or buy both core and add-on if the incentive compatibility (IC) constraint is met: $\theta_a(1+q_a) - p_c - p_a \geq \theta_a - p_c$. We can derive the demand based on these constraints. The firm's profit function is $\pi_N = p_c(\frac{p_a}{q_a} - p_c) + (p_c + p_a - c_a)(1 - \frac{p_a}{q_a})$. The monopoly optimizes the profit functions to get optimal price and profit for each case. Comparing results from the two cases, we get the equilibrium for the monopoly.

LEMMA 1. *In equilibrium, the monopoly unbundles if* $c_a > 0$*, and bundles if* $c_a = 0$.

Lemma 1 shows that the monopoly unbundles as long as the marginal cost is positive. The unbundling strategy serves as a price discrimination mechanism such that only the high-valuation consumers are willing to buy the add-on. While a prior study finds that the monopoly should unbundle when the range of add-on valuation is large (Adachi et al. 2011), we find that even when consumers' valuations for the add-on are small, it is optimal for the monopoly to unbundle with positive marginal cost. We now turn to our main analysis of duopoly competition, and compare the results with the benchmark case.

4. Duopoly Case

4.1. Firms

There are two competing firms with heterogeneous qualities for both the core and the add-on product. We denote the high-quality firm as the superior firm (H) and the low-quality firm as the inferior firm (L). The superior firm offers its core product with quality q_c^H and add-on with quality q_a^H, and the inferior firm offers its core product q_c^L ($< q_c^H$) and add-on q_a^L ($\leq q_a^H$). Given the nature of add-on relative to the core product, we assume that each firm's add-on quality is strictly lower than its core quality ($q_a^L < q_c^L$, $q_a^H < q_c^H$). In addition, we assume that the inferior firm's sum quality of core and add-on is no greater than the superior firm's core quality ($q_c^L + q_a^L \leq 1$). Note that our equilibrium results do not rely on this assumption.

Both firms have negligible marginal cost for their core product. To simplify the analysis while maintaining the asymmetric cost structure of duopoly, we normalize the marginal cost of the inferior firm's add-on to be zero while keeping the marginal cost of the superior firm's add-on to be non-negative ($c_a^H \geq 0$). This abstraction is consistent with anecdotal evidence. For example, in the software industry, *Adobe Systems Inc.* offers digital document editing software and associated real time expert support over phone and on-site. This technical support incurs labor cost and transportation cost. In contrast, *Adobe's* competitors such as *Nitro Corp.* offers lower quality versions of pdf editing software with technical support via discussion forum and online Q&A. This technical support is likely to have negligible marginal cost. In the hotel industry, luxury hotels generally provide better quality for add-on services such as sit-down breakfast and valet parking, comparing to economy hotels which provide self-parking or self-service continental breakfast. The marginal cost for providing an additional room is negligible for both types of hotels, as long as a room is available. However, luxury hotels have non-negligible marginal cost for serving an additional sit-down breakfast or valet parking, since labor cost for waiter, kitchen staff, and parking is significant, whereas economy hotels have negligible marginal cost for serving a simple self-service breakfast or self-parking.

4.2. Consumers

The market consists of one unit mass of consumers. Consumers' taste θ follows the same assumption as in the monopoly case. A consumer's utility function is the same as in the monopoly setting. We model a two-stage game. In stage 1, firms choose their bundling strategies, respectively. In stage 2, firms simultaneously decide prices. They will decide the bundle prices if they choose to bundle in the first stage, or the separate prices for the core and the add-on if they choose to unbundle in the first stage. Consumers observe the quality and prices of the offerings and decide whether to buy, and if buy, from which firm, to maximize their utilities.

5. Analysis and Results

Each firm needs to decide whether to bundle its core and add-on product. Hence, four cases may occur: both firms bundle (BB), the superior firm bundles and the inferior firm unbundles (BN), the superior firm unbundles and the inferior firm bundles (NB), and both firms unbundle (NN). We adopt the backward induction approach. Specifically, we first derive the optimal prices firms should adopt within each bundling case. Then for each firm, we compare its optimal profits derived from each of the four bundling cases, given the feasibility conditions are met, and then decide its optimal bundling strategy, given the rival firm's bundling decision.

5.1. Both firms bundle

When both firms bundle (BB), a consumer buys the inferior firm's bundle if her individual rationality (IR) constraint is satisfied: $\theta^{L}_{BB}(q^{L}_{c}+q^{L}_{a})-p^{L}_{BB}>0$. A consumer chooses to buy the superior firm's bundle over the inferior firm's bundle if her incentive compatibility (IC) constraint is satisfied: $\theta^{H}_{BB}(1+q^{H}_{a})-p^{H}_{BB}>\theta^{H}_{BB}(q^{L}_{c}+q^{L}_{a})-p^{L}_{BB}$. Hence, for consumers who buy the inferior firm's bundle, the marginal consumer's IR constraint must be binding, i.e. $\theta^{L}_{BB}(q^{L}_{c}+q^{L}_{a})-p^{L}_{BB}=0$. For consumers who buy the superior firm's bundle, the marginal consumer's IC must be binding, i.e. $\theta^{H}_{BB}(1+q^{H}_{a})-p^{H}_{BB}=\theta^{H}_{BB}(q^{L}_{c}+q^{L}_{a})-p^{L}_{BB}$.

Accordingly, the superior firm's profit is $\pi^{H}_{BB}=(p^{H}_{BB}-c^{H}_{a})(1-\theta^{H}_{BB})$ and the inferior firm's profit is $\pi^{L}_{BB}=p^{L}_{BB}(\theta^{H}_{BB}-\theta^{L}_{BB})$. We derive the optimal prices by taking the first order condition (FOC) for the profit functions and solving them simultaneously (p^{H*}_{BB}, p^{L*}_{BB}), and then we get the optimal profit for each firm (π_{BB}^{H*}, π_{BB}^{L*}). The analytical expressions for the optimal prices and profits are reported in lemma 2 in the Appendix. The conditions for this case to be feasible are: $0<\theta_{BB}^{L*}<\theta_{BB}^{H*}<1$ (see Figure 1(i)).

5.2. The superior firm bundles, the inferior firm unbundles

When the superior firm (H) bundles and the inferior firm (L) unbundles (BN), for consumers who buy firm L's core product, the marginal consumer's IR constraint must be binding: $\theta_{BN}^{Lc}q_c^L - p_{BN}^{Lc} = 0$; For consumers who buy firm L's core and add-on product, the marginal consumer who is indifferent to buy firm L's core and add-on or firm L's core has valuation that satisfies $\theta_{BN}^{La}(q_c^L + q_a^L) - p_{BN}^{Lc} - p_{BN}^{La} = \theta_{BN}^{La}q_c^L - p_{BN}^{Lc}$; For consumers who buy firm H's bundle, the marginal consumer who is indifferent to buy firm H's bundle or firm L's core and add-on has valuation that meets $\theta_{BN}^{H}(1+q_a^H) - p_{BN}^{H} = \theta_{BN}^{H}(q_c^L + q_a^L) - p_{BN}^{Lc} - p_{BN}^{La}$.

The inferior firm's profit is $\pi_{BN}^{L} = p_{BN}^{Lc}(\theta_{BN}^{La} - \theta_{BN}^{Lc}) + (p_{BN}^{Lc} + p_{BN}^{La})(\theta_{BN}^{H} - \theta_{BN}^{La})$ and the superior firm's profit is: $\pi_{BN}^{H} = (p_{BN}^{H} - c_a^H)(1-\theta_{BN}^{H})$. The feasibility conditions for this case are: $0<\theta_{BN}^{La*}<\theta_{BN}^{Lc*}<\theta_{BN}^{H*}<1$ (see Figure 1(ii)). The optimal prices and profits are reported in lemma 3 in the Appendix. We find in this case that the optimal demand for the inferior firm's core product is zero ($\theta_{BN}^{La*}=\theta_{BN}^{Lc*}$). Therefore, given that the superior firm bundles, the inferior firm's optimal strategy is to bundle its core and add-on product.

5.3. The superior firm unbundles, the inferior firm bundles

When the superior firm unbundles and the inferior firm bundles (NB), we follow the similar analysis as in the previous case. We derive the marginal consumer's valuation based on IR or IC constraints, and derive the demand for the inferior firm's bundle, the superior firm's core product, and the superior firm's core and add-on, respectively. The inferior firm's profit is:

$\pi_{NB}^{L} = p_{NB}^{L}(\theta_{NB}^{Hc} - \theta_{NB}^{L})$, and the superior firm's profit function can be written as the following: $\pi_{NB}^{H} = p_{NB}^{Hc}(\theta_{NB}^{Ha} - \theta_{NB}^{Hc}) + (p_{NB}^{Hc} + p_{NB}^{Ha} - c_{a}^{H})(1 - \theta_{NB}^{Ha})$ (see Figure 1(iii)). The feasibility conditions for this case are: $0 < \theta_{NB}^{L*} < \theta_{NB}^{Hc*} < \theta_{NB}^{Ha*} < 1$. The conditions are met when $c_{a}^{H} < q_{a}^{H}$. The optimal prices and profits are reported in lemma 4 in the Appendix.

5.4. Both firms unbundle

When both firms unbundle (NN), we derive the marginal consumers' valuations based on IR and IC constraints, and then we obtain the demand for each firm. The inferior firm's profit function is: $\pi_{NN}^{L} = p_{NN}^{Lc}(\theta_{NN}^{La} - \theta_{NN}^{Lc}) + (p_{NN}^{Lc} + p_{NN}^{La})(\theta_{NN}^{Hc} - \theta_{NN}^{La})$, and the superior firm's profit function is: $\pi_{NN}^{H} = p_{NN}^{Hc}(\theta_{NN}^{Ha} - \theta_{NN}^{Hc}) + (p_{NN}^{Hc} + p_{NN}^{Ha} - c_{a}^{H})(1 - \theta_{NN}^{Ha})$. The feasibility conditions for this case are: $0 < \theta_{NN}^{Lc*} < \theta_{NN}^{La*} < \theta_{NN}^{Hc*} < \theta_{NN}^{Ha*} < 1$. The conditions are met when $c_{a}^{H} < q_{a}^{H}$. The optimal prices and profits are reported in lemma 5 in the Appendix.

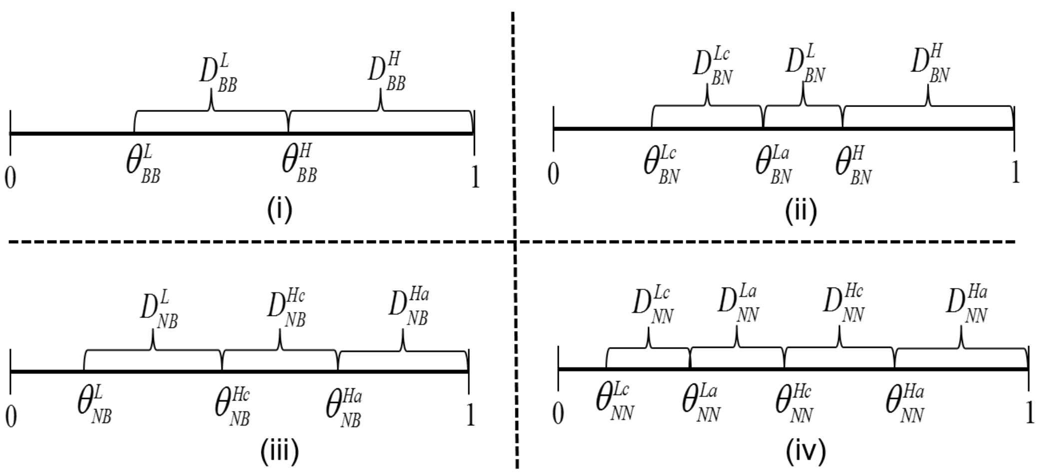

Figure 1: Four Demand Profiles. (i) both firms bundle; (ii) the superior firm bundles, the inferior firm unbundles; (iii) the superior firm unbundles, the inferior firm bundles; (iv) both firms unbundle

We find in this case that the optimal demand for the inferior firm's core product only is zero ($\theta_{BN}^{La*} - \theta_{BN}^{Lc*} = 0$) and the optimal sum price for its core and add-on is the same as its optimal price of bundle in NB case. Therefore, we conclude that it is not optimal for both firms to unbundle. Given that the superior firm unbundles, the inferior firm's optimal strategy is to bundle.

5.5. Bundling cases in equilibrium

From the analysis above, we know that the optimal strategy for the inferior firm is to bundle its core and add-on product. Hence, we focus only the on BB and NB cases in the following analysis to derive the equilibrium. Given that the inferior firm bundles, the superior firm compares the profit he will gain from bundling and unbundling to decide his optimal strategy.

LEMMA 6. *In equilibrium both firms bundle (BB) when* $c_a^H \le \tilde{q}$. *The superior firm unbundles and the inferior firm bundles (NB) when* $c_a^H > \tilde{q}$.

The optimal prices and profits are reported in Lemma 2 for BB case and Lemma 4 for NB case in the Appendix. The threshold ($\tilde{q}$) is a function of all qualities and increases with the inferior firm's bundle quality ($q_c^L + q_a^L$). Comparing the equilibrium strategy in the duopoly case with that in the benchmark case, we can see how the superior firm's bundling strategy changes when the inferior firm enters into the market. The results are in the following proposition:

PROPOSITION 1. *Whereas the superior firm with positive marginal cost always unbundles in monopoly, in duopoly competition, the superior firm bundles even when the marginal cost is positive, as long as* $c_a^H \le \tilde{q}$; *and the superior firm unbundles if* $c_a^H > \tilde{q}$. *The inferior firm bundles in both monopoly and duopoly.*

Proposition 1 highlights the changes in the superior firm's bundling strategy when facing competition from the inferior firm. On the contrary to the monopoly case wherein the superior firm unbundles, when the inferior firm competes in the market, the superior firm bundles if his marginal cost is not too large. The intuition is that when competition exists and add-on is not costly to offer, the superior firm is better off bundling its core and add-on to have a single high quality product. The increased quality increases differentiation and softens price competition and so that each firm obtains a higher profit margin. When the marginal cost is sufficiently large, the add-on becomes more costly to offer. Then it is better for the superior firm to charge separate prices such that only consumers with high valuation buy both core and add-on products. Consumers with relatively low valuation buy only the core product from the superior firm. The higher savings on the marginal cost dominates the competition effect, therefore, the superior firm is better off by unbundling to avoid serving costly add-on product to low-valuation consumers. Next we examine a special case when competition is strong.

Proposition 1 contributes to the add-on literature by identifying the superior firm's bundling strategy in duopoly setting is different from that in the monopoly setting, and highlighting the effects of competition on the superior firm's bundling strategy. Shulman and Geng (2013) find that with asymmetry in quality for both the core and the add-on, the superior firm always unbundles if there is significant asymmetry in add-on quality; In contrast, we find that the superior firm will either bundle or unbundle, depending on the intensity of competition and ratio of the marginal cost to quality. The underlying reason for the difference in results is that the add-on pricing strategy in Shulman and Geng (2013) is derived based on the two exogenous consumer segments, the core segment of consumers who has no valuation for the add-on and the knowledgeable segment of consumers who has value for the add-on. In equilibrium, the knowledgeable segment always buys add-on while the core segment never buys add-on. In contrast, we allow all consumers to have valuation for the add-on and consumes' purchase decision for the add-on is endogenous. Our results of asymmetric bundling strategy are based on consumers' continuously heterogenous valuation for qualities.

COROLLARY 1. *When competition is strong, i.e. the inferior firm's bundle quality is close to the superior firm's core product quality, then we have* $\tilde{q} > q_a^H$ *, and the superior firm always bundles.*

Corollary 1 implies that when facing strong competition from the inferior firm, the superior firm bundles to maximally differentiate from its rival's quality. Even if the marginal cost of the add-on is high, as long as the feasibility condition is met ($c_a^H \le q_a^H$), the superior firm's gain from softening the price competition outweighs his gain from saving on the marginal cost, so the superior firm bundles in equilibrium.

To illustrate how the equilibrium changes with different parameters, we show the optimal regions for the equilibrium outcomes (see Figure 2). Specifically, to see the impact of the inferior firm's core quality on the superior firm's bundling strategy, given a certain value of the superior firm's add-on quality, the superior firm unbundles when the competition is weak (i.e. q_c^L is relatively small); The superior firm bundles when the competition is intensified (i.e. q_c^L is sufficiently large); When competition is very strong (i.e. q_c^L is close to 1), the superior firm always bundles even when its add-on quality is small (the ratio of cost to quality is large).

To see how the superior firm's add-on quality (q_a^H) impacts its optimal bundling strategy, given that the competition from inferior firm is moderate (i.e. q_c^L is moderate, $q_c^L \in (0.49,\ 0.72)$), if q_a^H is relatively small, then the ratio of the marginal cost to quality is large, and the superior firm's add-on is costly to serve. The superior firm is better off by unbundling the add-on so that only high valuation consumers will buy its add-on together with the core product. If q_a^H is relatively large, then consumers' WTP for the superior firm's add-on is higher. The superior firm is better off bundling its add-on so that more consumers will buy both its add-on and core product.

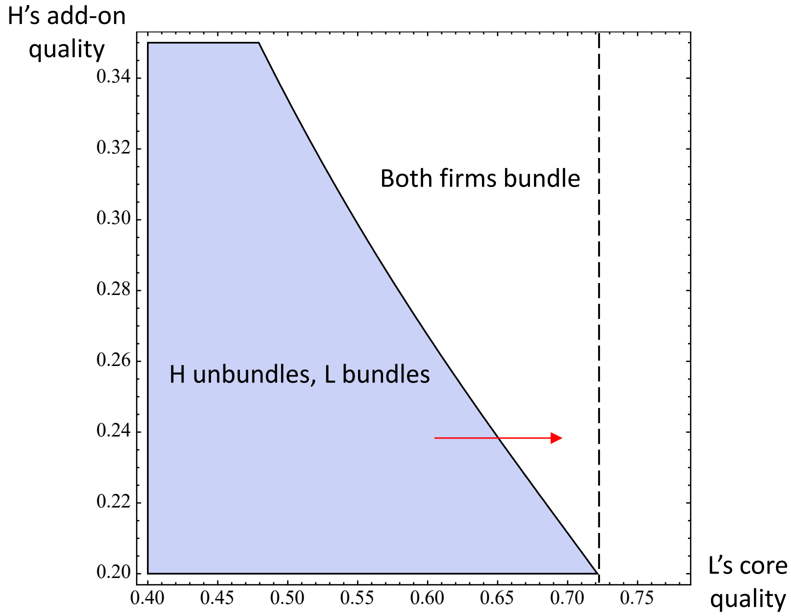

Figure 2: The optimal regions for the equilibrium cases ($q_a^L = 0.2, c_a^H = 0.2$ **)**

6. Comparative Statics for the Equilibrium Cases

It is not clear how firms' optimal prices, demands, and profits change with the inferior firm's core quality. One might expect the inferior firm's optimal price and profit will increase with its core quality because of consumers' increased WTP for its core product. And intuitively, the superior firm's optimal demand will decrease as the inferior firm's core quality increases, because consumers' WTP for the inferior firm's product has increased and the relative attractiveness of the superior firm's product will decrease. We now examine the impact of the inferior firm's core quality on both firms' optimal prices, demand, and profits, and explain the intuition.

6.1. The inferior firm's optimal price, profit, and demand with its core quality

We first focus on the inferior firm and examine the impact of its core quality on its optimal price, profit, and demand, and explain the intuition behind the results.

PROPOSITION 2. *In equilibrium, the inferior firm's price and profit can be decreasing as its core quality increases, while the inferior firm's demand increases with its core quality.*

Proposition 2 shows the interesting results that in either equilibrium case (NB or BB), the inferior firm's price of the core product and profit can decrease with its core quality.

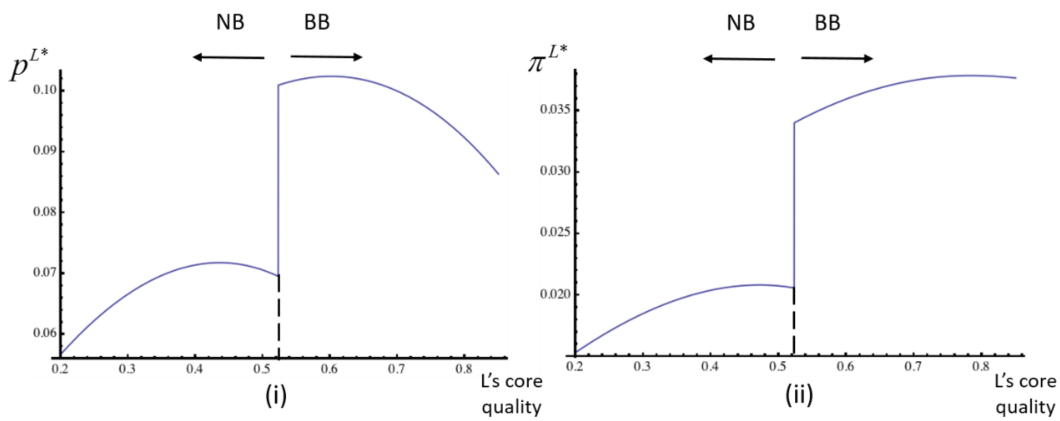

Figure 3: Comparative Statics for Inferior Firm. (i) the inferior firm's bundle price (p^{L^*}) with its core quality (q_c^L); (ii) the inferior firm's profit (π^{L^*}) with q_c^L .

$$q_a^H = 0.2,\ q_a^L = 0.1,\ c_a^H = 0.1$$

Figure 3 describes the non-monotone relationship between the inferior firm's optimal price, profit, and his core quality. Specifically, the inferior firm's price and profit first increase and then decrease with q_c^L in each case in equilibrium, and have a discrete increase at the threshold. The intuition is that as its core quality (q_c^L) increases, consumers' WTP for the inferior firm's bundle increases, thus the inferior firm's optimal price increases. When q_c^L is sufficiently large, the quality differentiation between the inferior firm's bundle and the superior firm's core product becomes sufficiently small, leading to intense price competition, i.e. the superior firm's optimal price decreases. The competition effect becomes more salient and outweighs the effect of consumers' increased WTP, leading the inferior firm to lower the price to compete with the superior firm. So the inferior firm begins to reduce price to keep consumers from switching to the superior firm's core product.

The discrete increase of the inferior firm's optimal price occurs when the superior firm changes strategy from unbundling to bundling. This is because when the superior firm start to bundle, the quality differentiation between the competing firms is larger, this in turn softens the price competition. Hence, the inferior firm's optimal price exhibits a discrete increase. When q_c^L increases further while the superior firm still bundles its core with the add-on, the quality differentiation between the two firm's bundle again decreases. So the inferior firm has to reduce price again to compete with the superior firm. The inferior firm's optimal profit has a similar non-monotone relationship with q_c^L, and the underlying reason follows the same logic as described above.

6.2. The superior firm's optimal price, profit, and demand with the inferior firm's core quality

We now examine the impact of the inferior firm's core quality on the superior firm's optimal price, profit, and demand. One may expect the superior firm's demand will decrease as its competitor's core quality increases, due to the increased attractiveness of its competitor's bundle. The following Proposition shows the surprising result.

PROPOSITION 3. *In equilibrium, the superior firm's demand can increase with the inferior firm's core quality, while the superior firm's prices and profit decrease as the inferior firm's core quality increases.*

Proposition 3 shows the counter-intuitive result that the superior firm's demand first increases and then decreases as the inferior firm's core quality (q_c^L) increases (see Figure 4).

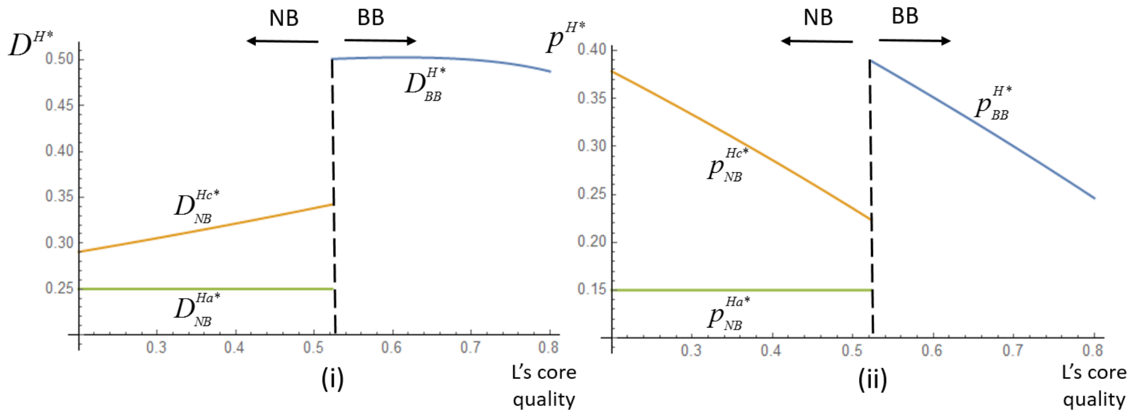

Figure 4: Comparative Statics for Superior Firm. (i) The superior firm's demand with the inferior firm's core quality (q_c^L); (ii) the superior firm's prices with q_c^L ($q_a^H = 0.2,\ q_a^L = 0.1,\ c_a^H = 0.1$).

Specifically, when the superior firm unbundles in equilibrium, the demand of its core product increases with q_c^L; after the superior firm starts to bundle, its bundle demand first increases and then decreases as q_c^L increases. The intuition for this surprising result is that when the superior firm unbundles, the demand for the superior firm's core product is a function of q_c^L ($D_c^H = -\frac{p_c^H}{1-(q_c^L+q_a^L)} + \frac{p_b^L}{1-(q_c^L+q_a^L)} + \frac{p_a^H}{q_a^H}$). As q_c^L increases, the intercept of demand function is larger, so the demand curve will shift upward. Meanwhile, the absolute value of the price elasticity of demand will increase with q_c^L ($\frac{\partial e_h}{\partial q_c^L} = \frac{-p_c^H p_a^H q_a^H}{(q_a^H(p_c^H - p_b^L) - p_a^H(1-q_c^L-q_a^L))^2} < 0$). Hence, the tradeoff between the profit margin and the demand has changed. The superior firm will reduce its price of core product quickly so that its demand increases to balance the loss in profit margin. Therefore, the optimal demand for the superior firm's core product increases with q_c^L when q_c^L is relatively small.

The demand for the superior firm's bundle first increases and then decreases after the superior firm starts to bundle. The intuition is that when q_c^L increases but remains not large, the superior firm reduces price quickly to take advantage of the up-shifting demand and the increased price elasticity so that the superior firm's demand will increase to mitigate the loss of profit margin.

When q_c^L is large, the competing firms' bundle qualities are sufficiently close, so the price competition becomes more intensified. This causes the superior firm's optimal demand to decrease. When the ratio of the marginal cost to quality for the superior firm's add-on is sufficiently large, such as in Figure 4, then the superior firm does not reduce the bundle price significantly enough given the increased price elasticity of demand. Hence, the relative attractiveness of the superior firm's bundle will decrease vis-à-vis that of the inferior firm's bundle. Therefore, the superior firm's optimal demand will start to decrease.

6.3. Comparative Statics for Consumer Surplus

It is not clear how consumer surplus changes with the superior firm's add-on quality because the increased quality of the add-on will increase consumers' WTP but may also increase the quality differentiation between firms when the superior firm bundles.

PROPOSITION 4. *When there is strong competition, consumer surplus can decrease with the increase in the superior firm's add-*on quality.

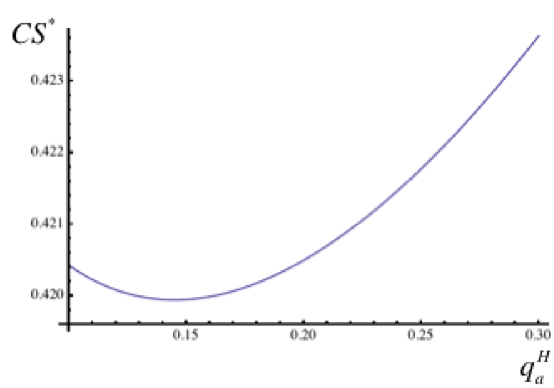

Figure 5: Comparative statics for consumer surplus with the superior firm's add-on quality. $q_c^L = .8,\ q_a^L = .15,\ c_a^H = .1$

Proposition 4 shows that consumer surplus first decreases and then increases with the superior firm's add-on quality (q_a^H) when there is strong competition. The underlying logic is that when the inferior firm's bundled quality is close to the superior firm's core quality, price competition is strong. This causes both firms to charge a relatively low bundle price, and consumers benefit from such low prices. As the superior firm's core quality increases, the superior firm's bundle quality is larger, so the quality difference between the competing firms is larger. This softens price competition, and both firms' prices increase significantly. As a result, consumer surplus decreases because of (1) fewer consumers purchase and (2) consumers who purchase get lower surplus. As q_a^H continues to increase, however, the marginal loss of consumer surplus from the increased firms' prices is less. At the same time, the marginal gain of consumer surplus from increased WTP is at a constant rate because of consumers' linear utility function. Hence, as q_a^H further increases, beyond a threshold, the positive effect of increased WTP is greater than the negative effect of increase in prices so that consumer surplus increases for each consumer on average. Consumer surplus will increase with the superior firm's add-on quality.

7. Public Policy on add-on pricing

7.1. The impact of prohibiting add-on pricing on firms

Following the add-on literature, we examine the implications of industrial regulations and government intervention aimed to limit the firms' ability to charge for their add-ons. For instance, the Consumer Financial Protection Bureau has proposed regulatory scrutiny on add-on sales such as extended warranties and tire plans in auto industry (Willis 2013); the British government has banned travel companies and retailers from charging additionally when consumers use their credit cards to pay online (Ensor 2013). We compare price, market share, and profit for firms as well as consumer surplus between the case in which add-on pricing is allowed and the case in which add-on pricing is prohibited. When add-on pricing is prohibited, both firms have to bundle; when add-on pricing is feasible, firms will play their b equilibrium bundling strategy. We report the comparison results in the following proposition:

PROPOSITION 5. *Prohibiting add-on pricing changes the equilibrium if* $c_a^H > \tilde{q}$ *. Under these conditions, prohibition increases the inferior firm's profit but decreases the superior firm's profit* $(\overline{\pi}^{L*} > \pi_{NB}^{L*},\ \overline{\pi}^{H*} < \pi_{NB}^{H*})$ *. The higher* c_a^H *, the greater the gain of the inferior firm, and the greater the loss of the superior firm due to the prohibition. Both firms' prices for their core and add-on products increase (* $\overline{p}^{H*} > p_{NB}^{Hc*} + p_{NB}^{Ha*}$, $\overline{p}^{L*} > p_{NB}^{L*}$ *).*

Proposition 5 highlights the impact of prohibiting add-on pricing on firms' price and profit. When add-on pricing is feasible and $c_a^H > \tilde{q}$, the optimal strategy for the superior firm is to unbundle; when add-on pricing is prohibited and $c_a^H > \tilde{q}$, the superior firm has to bundle, which is a suboptimal strategy. As a result, both firms charge a higher price for their core and add-on than the case where add-on pricing is feasible. This result is different compared to that in Shulman and Geng (2013), wherein they show both firms charge a lower bundle price when add-on pricing is prohibited than the case when add-on pricing is allowed. Their result is driven by the exogenous segments of customers and increased quality difference between firms' add-on products, while our result occurs because when add-on pricing is prohibited, the superior firm bundles. The increased quality differentiation softens price competition, and finally softened competition leads to prices increase from both firms.

In terms of profit implications, Proposition 5 shows that the inferior firm benefits and the superior firm loses when add-on pricing is prohibited and $c_a^H > \tilde{q}$. Moreover, we find that as the marginal cost of the superior firm's add-on increases, the inferior firm gains more profit while the superior firm incurs greater loss when add-on pricing is prohibited, suggesting that when marginal cost of the superior firm's add-on is large, while the inferior firm gains more from the policy of prohibiting add-on pricing, the superior firm loses more from such a policy.

7.2. The impact of prohibiting add-on pricing on consumers

We now examine the impact of prohibiting add-on pricing on consumer surplus. In the following Proposition, we show that consumers are never better off when add-on pricing is prohibited.

PROPOSITION 6. *Prohibiting add-on pricing reduces consumer surplus when* $c_a^H > \tilde{q}$, *and it has no impact on consumer surplus when* $c_a^H \leq \tilde{q}$.

Proposition 6 highlights the negative impact of regulating add-on pricing on consumer surplus. We know from Proposition 5 that both firms prices' increase when add-on pricing is prohibited and $c_a^H > \tilde{q}$. However, consumers' WTP for each firm's core and add-on remains the same, and the total demand is lower, thereby consumer surplus is lower than the case when add-on pricing is feasible.

Ellison (2005) find that consumer surplus may be higher when offering only core product is not feasible. The result in Ellison (2005) is driven by symmetric firms and two type of consumers with discrete marginal utility of income. When offering only the core product is feasible and consumers' marginal utilities of income are sufficiently different, each firm unbundles by offering the core product, and both the core and the add-on. In equilibrium, the low valuation consumers will buy only the core product and the high valuation consumers will buy both the core and the add-on. More consumer surplus will be extracted than the case in which offering only the core product is infeasible. In our study, the result of consumer surplus comparison is driven by the competition effect and firms' asymmetric quality and cost. When add-on pricing is allowed and the marginal cost of the superior firm's add-on is sufficiently large, the superior firm unbundles and the inferior firm bundles. The inferior firm's bundle quality is close to the superior firm's core quality, leading to relatively strong price competition. When add-on pricing is prohibited, the superior firm is forced to bundle, so the competing firms are more quality differentiated. Now the competition is softened and each firm increases its price. Hence, consumer surplus is less than the case when add-on pricing is allowed.

8. Discussion and Conclusion

This study extends prior literature in the following respects. First, this is one of the few papers to explicitly examine the asymmetric firms' add-on bundling and pricing problem. Our model allows consumers to have heterogeneous taste for qualities of both the core and the add-on products. We examine the impact of heterogeneity in consumers' taste of qualities on firms' bundling and pricing strategies. Second, different from prior literature (Shulman and Geng 2013) that shows the superior firm would always unbundle if there is significant asymmetry in add-on quality between the two firms, we find the superior firm would either bundle or unbundle, depending on the intensity of competition and the marginal cost of the add-on relative to quality.

Third, whereas prior literature finds that for symmetric firms, both firms bundle when consumers have the same transportation cost and firms' add-on marginal costs are zero; and one firm bundles and the other firm unbundles when consumers have different transportation cost and firms' add-on marginal cost is relatively small (Geng and Shulman 2015). We identify the role of competition and the marginal cost to quality ratio on asymmetric firms' bundling strategies. Comparing to the benchmark monopoly case where the monopoly unbundles when the marginal cost of add-on is positive, in duopoly the superior firm bundles when the competition is sufficiently strong, even if there is significant marginal cost. When there is strong competition from the inferior firm, the superior firm will always bundle irrespective of the marginal cost.

Fourth, while prior research suggests that the low-quality firm is more likely to price discriminate than high-quality firm in forms of price discrimination such as rebates (Dogan et al. 2010), we show the opposite result where the high-quality firm rather than the low-quality firm will price discriminate for their core and add-on products when competition is weak or the superior firm's marginal cost to quality ratio is large. Moreover, the inferior firm would have to compete more aggressively when the superior firm switches from bundling to unbundling. Hence, the superior firm's unbundling decision will reduce the inferior firm's profit because of the intensified price competition.

In addition, prior literature shows that as the quality of a firm's product increases, the firm's price and the profit also increases, and the rival firm's demand decreases (Tirole 1998, Laffont and Martimort 2009). Interestingly, our results show that the inferior firm's price and profit may decrease as its core quality increases, and the superior firm's demand may increase with the inferior firm's core quality. Our results are driven by the intensified competition between quality-differentiated firms, heterogeneous consumers' tastes for both the core and the add-ons, and the increased price elasticity of the superior firm.

Lastly, our results contribute to the literature by studying the impact of public policy on firms' prices, profits, and consumer surplus. Prior research implies that given exogenous segments of customers and increased quality difference between the add-ons, both asymmetric firms will charge a lower bundle price and consumer surplus will be higher when add-on pricing is infeasible (Ellison 2005). Surprisingly, our result shows that government prohibition on add-on pricing may cause both asymmetric firms to charge a higher bundle price, leading to reduced consumer surplus. The prohibition of add-on pricing may have asymmetric impacts on firms' profitability, i.e. the inferior firm benefits from the prohibition policy while the superior firm loses from such policy. Our result is significant when the competition between quality-differentiated firms is not strong and there is significant asymmetry in firms' marginal cost of add-ons. Our results suggest that well-intentioned government intervention to prohibit add-on pricing does not benefit consumers and may even reduce consumer surplus.

Our research yields important managerial implications. First, when facing competition from an inferior firm, the superior firm should decide its bundling strategy based on the intensity of the competition. When the inferior firm has relatively low quality, the superior firm should unbundle its add-on to gain a larger total demand, i.e. only high valuation consumers buy both its core and add-on, and consumers who have relatively low valuation buy only the core product. When the inferior firm has high quality, the superior firm should bundle its add-on to soften competition, so that the superior firm can charge a high bundle price and gain more profit. This bundling result helps explain the industrial examples we discussed in the introduction. For example, in hotel industry, the room quality of an economy hotel is generally much lower than that of an up-scale hotel, and we see that economy hotels bundle while the up-scale hotels unbundle. In contrast, in IT security industry, the quality of the network monitoring service (core service) from a low-quality vendor is not much lower than that of a high-quality vendor, in this case, both low-quality vendor and high-quality vendor bundle their add-on services with the core service.

Secondly, when competing with a superior firm, the inferior firm has incentive to improve its core quality through different means. However, the inferior firm should not increase its core quality all the way to be close to the superior firm's core quality, because if so, the reduced quality-differentiation will result in strong price competition, and the inferior firm's profit will decrease. Instead, the inferior firm should increase its core quality to a level that entice the superior firm to change its strategy from unbundling to bundling, so that the quality differentiation becomes large, competition is softened, and both firms gain a higher profit. Third, government and regulators should be more cautious in imposing public policy of prohibiting add-on pricing. Though it is well-intentioned, it may reduce consumer surplus because of quality-differentiated firms' pricing strategies.

This study can be extended in several respects. First, in the current model we have assumed consumers' valuation for add-on product is positively correlated with core product. It may be interesting to examine the case where consumers have independent valuations for core and add-on products. The results can be compared with the current case to see how firms' competition and pricing strategy will change. Second, a potential direction for future research is to examine a multi-period case where firms face a segment of repeated purchasing consumers. Firms will consider their bundling and pricing strategy to maximize their total profit for all periods.

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Appendix A: Summary of Notations

Notation	Definition	Notation	Definition
q_c^H	Quality of firm H’s core	q_c^L	Quality of firm L’s core
q_a^H	Quality of firm H’s add-on	q_a^L	Quality of firm L’s add-on
p_{BB}^H	Price of H’s bundle in BB case	p_{BB}^L	Price for firm L’s bundle in BB case
p_{NB}^{Hc}	Price of H’s core in NB case	p_{NB}^L	Price of L’s bundle in NB case
p_{NB}^{Ha}	Price of H’s add-on in NB case	p_{BN}^{Lc}	Price of L’s core in BN case
p_{BN}^H	Price of H’s bundle in BN case	p_{BN}^{La}	Price of L’s add-on in BN case
p_{BB}^H	Profit of firm H in BB case	p_{BB}^L	Profit of firm L in NB case
p_{NB}^H	Profit of firm H in NB case	p_{NB}^L	Profit of firm L in NB case
p_{BN}^H	Profit of firm H in BN case	π_{BN}^L	Profit of firm L in BN case

Note: a. notations for NN case follows a similar manner as the other three cases.

b. Variables with an upper bar refer to the case where add-on pricing is prohibited, i.e. $\overline{CS}^*$ refers to the optimal consumer surplus when the add-on pricing is prohibited; Variables without an upper bar refer to the regular case where add-on pricing is allowed, i.e., CS^*_{NB} refers to the optimal consumer surplus when the superior firm unbundles and the inferior firm bundles, and add-on pricing is feasible.

c. q_c^H is normalized to 1.

Appendix B: Proof for lemmas and propositions

LEMMA 1. *In equilibrium, the monopoly unbundles if* $c_a > 0$ *, and bundles if* $c_a = 0$.

Proof for lemma 1. When the monopoly bundles, the demand for monopoly bundle is $1-\frac{p_B}{1+q_a}$ and profit is $\pi_B = (p_B - c_a)(1-\frac{p_B}{1+q_a})$. By solving the F.O.C of the profit function for price, we got the optimal price: $p_B^* = \frac{1}{2}(1+q_a+c_a)$. We plug the optimal price into the profit function, we got the optimal profit: $\pi_B^* = \frac{(1+q_a-c_a)^2}{4(1+q_a)}$.

When the monopoly unbundles, the demand for the core only is $\frac{p_a}{q_a}-q_c$, and the demand for both the core and the add-on is $1-\frac{p_a}{q_a}$. So the profit function is:

$p_c(\frac{p_a}{q_a}-q_c)+(p_c+p_a-c_a)(1-\frac{p_a}{q_a})$. By solving the F.O.C of the profit function for prices, we got the optimal prices: $p_c^* = \frac{1}{2}$ and $p_a^* = \frac{1}{2}(c_a+q_a)$. The corresponding the optimal profit is:

$\pi_N^* = \dfrac{c_a^2 - 2c_a q_a + q_a(1+q_a)}{4q_a}$. It can be seen that when $c_a = 0$, $\pi_N^* = \pi_B^*$; when $c_a > 0$, we have

$\pi_N^* - \pi_B^* = \dfrac{{c_a}^2}{4q_a(1+q_a)} > 0$.

Proof for lemma 2 – 5

LEMMA 2. *When both firms bundle their core and add-on products (BB), the optimal prices, demand, and profits for both firms are:*

$$p_{BB}^{L*} = \frac{(q_c^L + q_a^L)(-1 - c_a^H - q_a^H + q_c^L + q_a^L)}{-4 - 4q_a^H + q_c^L + q_a^L} \qquad p_{BB}^{H*} = -\frac{2(1+q_a^H)(1+c_a^H + q_a^H - q_c^L - q_a^L)}{-4 - 4q_a^H + q_c^L + q_a^L}$$

$$\rho_{BB}^{L*} = \frac{(1+q_a^H)(1+c_a^H + q_a^H - q_c^L - q_a^L)^2(q_c^L + q_a^L)}{(1+q_a^H - q_c^L - q_a^L)(-4 - 4q_a^H + q_c^L + q_a^L)^2} \quad \rho_{BB}^{H*} = \frac{(2(1+q_a^H)(1+q_a^H - q_c^L - q_a^L) + c_a^H(-2 - 2q_a^H + q_c^L + q_a^L))^2}{(1+q_a^H - q_c^L - q_a^L)(-4 - 4q_a^H + q_c^L + q_a^L)^2}$$

$$d_{BB}^{L*} = \frac{1}{3}\left(\frac{c_a^H}{1+q_a^H - q_c^L - q_a^L} + \frac{c_a^H - 3(1+q_a^H)}{-4 - 4q_a^H + q_c^L + q_a^L}\right) \quad d_{BB}^{H*} = \frac{1}{3}\left(-\frac{c_a^H}{1+q_a^H - q_c^L - q_a^L} + \frac{2(c_a^H - 3(1+q_a^H))}{-4 - 4q_a^H + q_c^L + q_a^L}\right)$$

$$q_{BB}^{L*} = 1 + \frac{3 - c_a^H + 3q_a^H}{-4 - 4q_a^H + q_c^L + q_a^L} \qquad q_{BB}^{H*} = 1 + \frac{c_a^H}{3 + 3q_a^H - 3q_c^L - 3q_a^L} + \frac{2(-c_a^H + 3(1+q_a^H))}{3(-4 - 4q_a^H + q_c^L + q_a^L)}$$

Proof for Lemma 2 (BB case).

As shown in the paper, two indifferent points: $\theta^L(q_c^L + q_a^L) - p_{BB}^L \geq 0$ and $\theta^H(1+q_a^H) - p_{BB}^H \geq \theta^H(q_c^L + q_a^L) - p_{BB}^L$, so we got $\theta^L = \dfrac{p_{BB}^{LB}}{q_c^L + q_a^L}$ and $\theta^H = \dfrac{p_{BB}^{HB} - p_{BB}^{LB}}{1+q_a^H - q_c^L - q_a^L}$.

Profit functions for L and H: $\pi_{BB}^L = p_{BB}^L(\theta_{BB}^H - \theta_{BB}^L)$ and $\pi_{BB}^H = (p_{BB}^H - c_a^H)(1-\theta_{BB}^H)$. By taking the F.O.C of the two profit functions for prices and solving the prices simultaneously, we got the optimal prices p_{BB}^{L*} and p_{BB}^{H*}. We plug the optimal prices into the demand and profit functions; we got the results for the optimal demands and profits. The feasibility conditions are

$0 < \theta_{BB}^{L*} < \theta_{BB}^{H*} < 1$, plug the optimal prices into θ_{BB}^{L*}, and θ_{BB}^{H*} , we find the feasibility conditions are met, there is no additional condition required.

LEMMA 3. *When the superior firm bundles and the inferior firm unbundles, the optimal prices and profits are*:

$$p_{BN}^{Lc*} = \frac{q_c^L(-1 - c_a^H - q_a^H + q_c^L + q_a^L)}{-4 - 4q_a^H + q_c^L + q_a^L} \quad p_{BN}^{La*} = -\frac{q_a^L(1 + c_a^H + q_a^H - q_c^L - q_a^L)}{-4 - 4q_a^H + q_c^L + q_a^L} \quad p_{BN}^{H*} = -\frac{2(1+q_a^H)(1+c_a^H + q_a^H - q_c^L - q_a^L)}{-4 - 4q_a^H + q_c^L + q_a^L}$$

$$\pi_{BN}^{L*} = \frac{(1+q_a^H)(1+c_a^H + q_a^H - q_c^L - q_a^L)^2(q_c^L + q_a^L)}{(1+q_a^H - q_c^L - q_a^L)(-4 - 4q_a^H + q_c^L + q_a^L)^2} \quad \pi_{BN}^{H*} = \frac{(2(1+q_a^H)(1+q_a^H - q_c^L - q_a^L) + c_a^H(-2 - 2q_a^H + q_c^L + q_a^L))^2}{(1+q_a^H - q_c^L - q_a^L)(-4 - 4q_a^H + q_c^L + q_a^L)^2}$$

$$\theta_{BN}^{La*} = \theta_{BN}^{Lc*} = 1 + \frac{3 - c_a^H + 3q_a^H}{-4 - 4q_a^H + q_c^L + q_a^L} \qquad \theta_{BN}^{H*} = 1 + \frac{c_a^H}{3 + 3q_a^H - 3q_c^L - 3q_a^L} + \frac{2(-c_a^H + 3(1+q_a^H))}{3(-4 - 4q_a^H + q_c^L + q_a^L)}$$

Proof for Lemma 3 (BN case). As shown in the paper, we have three indifferent points: θ_{BN}^{Lc}, θ_{BN}^{Lc}, and θ_{BN}^{H} , and the profit functions are given in the paper. By taking the F.O.C of the two profit functions for prices and solving the prices simultaneously, we got the optimal p_{BN}^{Lc*}, p_{BN}^{La*} and p_{BN}^{H*} . Plugging the optimal prices in the demand and profit functions, then we got the optimal demand and profits shown in the Lemma 3. We plug the optimal prices into the indifferent points: θ_{BN}^{Lc}, θ_{BN}^{Lc}, and θ_{BN}^{H} , we got $\theta_{BN}^{Lc*} = \theta_{BN}^{La*}$. This implies that consumers who buy the inferior firm's core product will always buy the inferior firm's add-on product also. No consumers buy the inferior firm's core product only.

LEMMA 4. *When the superior firm unbundles and the inferior firm bundles, the optimal prices and profits are:*

$$p_{NB}^{L*}=3+q_a^L+q_c^L+\frac{12}{-4+q_a^L+q_c^L}\quad p_{NB}^{Hc*}=2+\frac{6}{-4+q_a^L+q_c^L}\quad p_{NB}^{Ha*}=\frac{1}{2}\left(c_a^H+q_a^H\right)$$

$$\rho_{NB}^{L*}=-\frac{\left(-1+q_a^L+q_c^L\right)\left(q_a^L+q_c^L\right)}{\left(-4+q_a^L+q_c^L\right)^2}\quad \rho_{NB}^{H*}=\frac{\left(c_a^H-q_a^H\right)^2}{4q_a^H}-\frac{4\left(-1+q_a^L+q_c^L\right)}{\left(-4+q_a^L+q_c^L\right)^2}$$

$$q_{NB}^{L*}=1+\frac{3}{-4+q_a^L+q_c^L}\quad q_{NB}^{Hc*}=1+\frac{2}{-4+q_a^L+q_c^L}\quad q_{NB}^{Ha*}=\frac{1}{2}\left(1+\frac{c_a^H}{q_a^H}\right)$$

Proof for lemma 4 (NB case). In NB case, we have three indifferent points: θ_{NB}^{L}, θ_{NB}^{Hc}, and θ_{NB}^{Ha}, the profit functions for L and H are: $\pi_{NB}^{L}=p_{NB}^{L}(\theta_{NB}^{Hc}-\theta_{NB}^{L})$ and $\pi_{NB}^{H}=p_{NB}^{Hc}(\theta_{NB}^{Ha}-\theta_{NB}^{Hc})+(p_{NB}^{Hc}+p_{NB}^{Ha}-c_a^H)(1-\theta_{NB}^{Ha})$, using the same technique as described in the lemma 3, we got the result of optimal prices, demand, and profit in Lemma 4. The feasibility conditions for this case are: $0<\theta_{NB}^{L*}<\theta_{NB}^{Hc*}<\theta_{NB}^{Ha*}<1$. The conditions are met when $c_a^H<q_a^H$.

LEMMA 5. *When both firms unbundle, the optimal prices and profits for both firms are:*

$$p_{NN}^{Lc*}=q_c^L\left(1+\frac{3}{-4+q_a^L+q_c^L}\right)\quad p_{NN}^{La*}=\frac{q_a^L\left(-1+q_a^L+q_c^L\right)}{-4+q_a^L+q_c^L}\quad p_{NN}^{Hc*}=2+\frac{6}{-4+q_a^L+q_c^L}\quad p_{NN}^{Ha*}=\frac{1}{2}\left(c_a^H+q_a^H\right)$$

$$\rho_{NN}^{L*}=-\frac{\left(-1+q_a^L+q_c^L\right)\left(q_a^L+q_c^L\right)}{\left(-4+q_a^L+q_c^L\right)^2}\quad \rho_{NN}^{H*}=\frac{1}{4}\frac{\left(c_a^H-q_a^H\right)^2}{q_a^H}-\frac{4\left(-1+q_a^L+q_c^L\right)}{\left(-4+q_a^L+q_c^L\right)^2}$$

$$q_{NN}^{Lc*}=1+\frac{3}{-4+q_a^L+q_c^L}\quad q_{NN}^{La*}=1+\frac{3}{-4+q_a^L+q_c^L}\quad q_{NN}^{Hc*}=1+\frac{2}{-4+q_a^L+q_c^L}\quad q_{NN}^{Ha*}=\frac{1}{2}\left(1+\frac{c_a^H}{q_a^H}\right)$$

Proof for lemma 5 (NN case). As shown in the NN case in paper, we have four indifferent points: θ_{NN}^{Lc}, θ_{NN}^{La}, θ_{NN}^{Hc}, and θ_{NN}^{Ha}, the profit functions for L and H are: $\pi_{NN}^{L}=p_{NN}^{Lc}(\theta_{NN}^{La}-\theta_{NN}^{Lc})+(p_{NN}^{Lc}+p_{NN}^{La})(\theta_{NN}^{Hc}-\theta_{NN}^{La})$ and $\pi_{NN}^{H}=p_{NN}^{Hc}(\theta_{NN}^{Ha}-\theta_{NN}^{Hc})+(p_{NN}^{Hc}+p_{NN}^{Ha}-c_a^H)(1-\theta_{NN}^{Ha})$, using the same technique as described in the lemma 3, we got the optimal prices, demand, and profit in Lemma 5. It is also easy to see that

$\theta_{NN}^{Lc*} = \theta_{NN}^{La*}$. The feasibility conditions for this case are: $0 < \theta_{NN}^{Lc*} < \theta_{NN}^{La*} < \theta_{NN}^{Hc*} < \theta_{NN}^{Ha*} < 1$. The conditions are met when $c_a^H < q_a^H$.

LEMMA 6. *In equilibrium both firms bundle (BB) when $c_a^H \le \tilde{q}$. The superior firm unbundles and the inferior firm bundles (NB) when $c_a^H > \tilde{q}$. The optimal prices and profits are reported in Lemma 2 for BB case and Lemma 4 for NB case in the Appendix.*

Proof for Lemma 6. We compare the optimal profit for the superior firm in BB and NB cases. π_{BB}^{H*} and π_{NB}^{H*} are given in Lemma 2 and Lemma 4, respectively. We solve the equation $\pi_{NB}^{H*} - \pi_{BB}^{H*} = 0$ as a function of c_a^H, which is a quadratic function of c_a^H. We got that a negative root, and the other root is positive (denoted as $\tilde{q}$), and the intercept is negative. The expression of $\tilde{q}$ is as follows. Note that $\tilde{q}$ is a function of all qualities (q_a^H, q_c^L, q_a^L). Given the results of the sign of the two roots and the intercept, we get the result in Lemma 6.

$$\tilde{q} = \frac{q_a^H(4-q_c^L-q_a^L)(q_c^L+q_a^L)^2(1+q_a^H-q_c^L-q_a^L)+2q_a^H(q_c^L+q_a^L)(4+4q_a^H-q_c^L-q_a^L) - \sqrt{(1+q_a^H-q_c^L-q_a^L)[q_a^H(-10+q_c^L+q_a^L)(-2+q_c^L+q_a^L)+4(-5+q_c^L+q_a^L)(-1+q_c^L+q_a^L)]}}{(4-q_c^L-q_a^L)(8(q_a^H)^2(2-q_c^L-q_a^L)+(-4+q_c^L+q_a^L)^2(1-q_c^L-q_a^L)+q_a^H(32(1-q_c^L-q_a^L)+5(q_c^L+q_a^L)^2))}$$

PROPOSITION 1. *Whereas the superior firm with positive marginal cost always unbundles in monopoly, in duopoly competition, the superior firm bundles even when the marginal cost is positive, as long as $c_a^H \le \tilde{q}$; and the superior firm unbundles if $c_a^H > \tilde{q}$. The inferior firm bundles in both monopoly and duopoly.*

Proof for Proposition 1: from Lemma 1, we know the monopoly unbundles when the marginal cost is positive. From Lemma 6, we know the superior firm bundles when $c_a^H \le \tilde{q}$, and unbundles when $c_a^H > \tilde{q}$. So we get the conclusion in Proposition 1 about the changes of the superior firm's bundling strategy in duopoly compared to that in monopoly. The inferior firm bundles in duopoly, and the monopoly with zero marginal cost also bundles.

COROLLARY 1. *When the competition is strong, i.e. the inferior firm's bundle quality is close to the superior firm's core good quality, then we have* $\tilde{q} > q_a^H$ *, and the superior firm always bundles.*

Proof for Corollary 1: We numerically show the existence that for the inferior firm's bundle quality is large and close to the superior firm's core quality (i.e. $q_c^L + q_a^L \in (0.92,\ 1)$), we have $\tilde{q} > q_a^H$. Because $q_a^H > c_a^H$, we got $\tilde{q} > c_a^H$. From Lemma 6, we know when $c_a^H < \tilde{q}$, the superior firm bundles.

PROPOSITION 2. *In equilibrium, the inferior firm's price and profit can be decreasing as its core quality increases, while the inferior firm's demand increases with its core quality.*

Proof for Proposition 2. For the inferior firm's bundle price:

$\frac{\partial p_{BB}^{L*}}{\partial q_c^L} = 1 + \frac{4\left(1+q_a^H\right)\left(c_a^H - 3\left(1+q_a^H\right)\right)}{\left(-4-4q_a^H + q_c^L + q_a^L\right)^2}$, we transform the expression by letting $q_c^L + q_a^L = dq$, and solve it as a function of dq , the two roots are both positive, intercept is also positive, the two roots are $4+4q_a^H - 2\sqrt{\left(1+q_a^H\right)\left(3-c_a^H+3q_a^H\right)},\ 4+4q_a^H + 2\sqrt{\left(1+q_a^H\right)\left(3-c_a^H+3q_a^H\right)}$, respectively. The second root is greater than 1. We denote the first root as q_1. So we got:

$$\begin{cases} \frac{\partial p_{BB}^{L*}}{\partial q_c^L} > 0, & \text{if } q_c^L < q_1 - q_a^L \\ \frac{\partial p_{BB}^{L*}}{\partial q_c^L} < 0, & \text{if } q_c^L > q_1 - q_a^L \end{cases}$$

Where $q_1 = 4 + 4q_a^H - 2\sqrt{(1+q_a^H)(3-c_a^H+3q_a^H)}$.

For the inferior firm's demand: $\frac{\partial d_{BB}^{L*}}{\partial q_c^L} = \frac{1}{3}\left(\frac{3 - c_a^H + 3q_{HA}}{\left(-4 - 4q_a^H + q_c^L + q_a^L\right)^2} + \frac{c_a^H}{\left(-1 - q_a^H + q_c^L + q_a^L\right)^2} \right) > 0.$

For the inferior firm's profit:

$$\frac{\partial \pi_{BB}^{L*}}{\partial q_c^L} = \frac{-\left(1+q_a^H\right)\left(1+c_a^H+q_a^H-q_a^L-q_c^L\right)\left(\begin{array}{l}\left(1+q_a^H\right)\left(4+4q_a^H-7q_a^L-7q_c^L\right)\left(1+q_a^H-q_a^L-q_c^L\right)+\\ c_a^H\left(4+4(q_a^H)^2+q_a^L+q_c^L-2\left(q_a^L+q_c^L\right)^2+q_a^H\left(8+q_a^L+q_c^L\right)\right)\end{array}\right)}{\left(-4-4q_a^H+q_a^L+q_c^L\right)^3\left(-1-q_a^H+q_a^L+q_c^L\right)^2},$$

We simplify the expression by letting $q_c^L + q_a^L = dq$, then solve the expression as a function of dq, we got the following two roots:

$$\frac{\left(\begin{array}{l}-11+c_a^H\left(1+q_a^H\right)-11q_a^H\left(2+q_a^H\right)+\\ \sqrt{3}\sqrt{\left(-1+11c_a^H-q_a^H\right)\left(1+q_a^H\right)^2\left(c_a^H-3\left(1+q_a^H\right)\right)}\end{array}\right)}{2\left(2c_a^H-7\left(1+q_a^H\right)\right)}, \quad -\frac{\left(\begin{array}{l}11-c_a^H\left(1+q_a^H\right)+11q_a^H\left(2+q_a^H\right)+\\ \sqrt{3}\sqrt{\left(-1+11c_a^H-q_a^H\right)\left(1+q_a^H\right)^2\left(c_a^H-3\left(1+q_a^H\right)\right)}\end{array}\right)}{2\left(2c_a^H-7\left(1+q_a^H\right)\right)}$$

Both roots are positive and the second root is greater than 1, the intercept of the function is positive (i.e. $4\left(1+q_{HA}\right)^2\left(1+c_{HA}+q_{HA}\right)$). We denote the first root as q_2, we got:

$$\begin{cases} \frac{\partial \pi_{BB}^{L*}}{\partial q_c^L} > 0, & \text{if } q_c^L + q_a^L < q_2 \\ \frac{\partial \pi_{BB}^{L*}}{\partial q_c^L} < 0, & \text{if } q_c^L + q_a^L > q_2 \end{cases}$$

Where $q_2 = \frac{-11 + c_a^H(1+q_a^H) - 11q_a^H(2+q_a^H) + \sqrt{3}(1+q_a^H)\sqrt{(-1+11c_a^H-q_a^H)(c_a^H-3(1+q_a^H))}}{2(2c_a^H-7(1+q_a^H))}$.

If $q_c^L + q_a^L < q_2$, then π_{BB}^{L*} increases with q_c^L; if $q_c^L + q_a^L > q_2$, then π_{BB}^{L*} decreases as q_c^L increases. ■

For NB case,

$\frac{\P p_{NB}^{L*}}{\P q_c^L} = 1 - \frac{12}{\left(-4 + q_a^L + q_c^L\right)^2}$, let $q_c^L + q_a^L = dq$, solve as a function of dq, we got the two roots ($4-2\sqrt{3},\ 4+2\sqrt{3}$), so $\frac{\partial p_{NB}^{L*}}{\partial q_c^L} > 0$ if $q_c^L + q_a^L < 4-2\sqrt{3}$; $\frac{\partial p_{NB}^{L*}}{\partial q_c^L} < 0$ if $q_c^L + q_a^L > 4-2\sqrt{3}$ and $q_c^L + q_a^L$ is not too close to 1 such that $\tilde{q} < q_a^H$.

$$\frac{\partial d_{NB}^{LB*}}{\partial q_c^L} = \frac{1}{\left(-4 + q_a^L + q_c^L\right)^2} > 0.$$

$\frac{\partial \pi_{NB}^{L*}}{\partial q_c^L} = \frac{-4 + 7q_a^L + 7q_c^L}{\left(-4 + q_a^L + q_c^L\right)^3}$, so we got $\frac{\partial \pi_{NB}^{L*}}{\partial q_c^L} > 0$ if $q_c^L + q_a^L < \frac{4}{7}$; $\frac{\partial \pi_{NB}^{L*}}{\partial q_c^L} < 0$ if $q_c^L + q_a^L > \frac{4}{7}$ and $q_c^L + q_a^L$ is not too close to 1 such that $\tilde{q} < q_a^H$. ■

PROPOSITION 3. *In equilibrium, the superior firm's demand can increase with the inferior firm's core quality, while the superior firm's prices decreases as the inferior firm's core quality increases.*

Proof for Proposition 3. For BB case, for the superior firm's prices:

$\frac{\partial p_{BB}^{H*}}{\partial q_c^L} = \frac{2\left(1+q_{HA}\right)\left(c_{HA} - 3\left(1+q_{HA}\right)\right)}{\left(-4 - 4q_{HA} + q_L + q_{LA}\right)^2}$. The numerator is negative, so $\frac{\partial p_{BB}^{H*}}{\partial q_c^L} < 0$.

For the superior firm's demand:

$$\frac{\partial d_{BB}^{H*}}{\partial q_c^L}=\frac{1}{3}\left(\frac{6-2c_a^H+6q_a^H}{\left(-4-4q_a^H+q_c^L+q_a^L\right)^2}-\frac{c_a^H}{\left(-1-q_a^H+q_c^L+q_a^L\right)^2}\right)$$, we solve it as a function of c_a^H, and

we got one root (q_1^H): $\frac{2\left(1+q_a^H\right)\left(-1-q_a^H+q_c^L+q_a^L\right)^2}{6+6(q_a^H)^2-4q_c^L-4q_a^L-4q_a^H\left(-3+q_c^L+q_a^L\right)+\left(q_c^L+q_a^L\right)^2}$, which is

positive. The intercept is positive. Therefore, we got:

$$\begin{cases}\frac{\partial d_{BB}^{H*}}{\partial q_c^L}>0 \text{ if } c_a^H<q_1^H \\ \frac{\partial d_{BB}^{H*}}{\partial q_c^L}<0 \text{ if } c_a^H>q_1^H\end{cases}$$

Numerically, we show the existence that q_1^H could be greater or smaller than $\tilde{q}$.

Note that q_1^H is a function of q_c^L , and $\frac{\partial q_1^H}{\partial q_c^L}<0$, so we know $c_a^H<q_1^H$ is an implicit condition for q_c^L to be small. As q_c^L increases and beyond a threshold, q_1^H becomes less than c_a^H , thereby we have $c_a^H>q_1^H$ is an implicit condition for q_c^L to be sufficiently large.

Now, we proceed to prove $\frac{\partial \pi_{BB}^{H*}}{\partial q_c^L}<0$

$$\frac{\partial \pi_{BB}^{H*}}{\partial q_c^L}=\frac{\begin{pmatrix}c_{HA}\left(2+2q_{HA}-q_L-q_{LA}\right)\\ +2\left(1+q_{HA}\right)\left(-1-q_{HA}+q_L+q_{LA}\right)\end{pmatrix}\begin{pmatrix}2\left(1+q_{HA}\right)\left(1+q_{HA}-q_L-q_{LA}\right)\left(2+2q_{HA}+q_L+q_{LA}\right)\\ +c_{HA}\left(4q_{HA}^2-2q_{HA}\left(-4+q_L+q_{LA}\right)-2\left(-2+q_L+q_{LA}\right)+\left(q_L+q_{LA}\right)^2\right)\end{pmatrix}}{\left(\left(4+4q_{HA}-q_L-q_{LA}\right)^3\left(-1-q_{HA}+q_L+q_{LA}\right)^2\right)}$$

The only part of the expression that has uncertain sign is:

$c_a^H\left(2+2q_a^H-q_a^L-q_c^L\right)+2\left(1+q_a^H\right)\left(-1-q_a^H+q_a^L+q_c^L\right)$, we solve it as a function of c_a^H , we got the root is greater than q_a^H. The intercept is less than 0, so the uncertain part is negative, so we got $\frac{\partial \pi_{BB}^{H*}}{\partial q_c^L} < 0$. ■

For the superior firm in NB case,

$\frac{\partial p_{NB}^{Hc*}}{\partial q_c^L} = -\frac{6}{\left(-4+q_a^L+q_c^L\right)^2}$, so $\frac{\partial p_{NB}^{Hc*}}{\partial q_c^L} < 0$. We can easily see $\frac{\partial p_{NB}^{Ha*}}{\partial q_c^L} = 0$.

$\frac{\partial d_{NB}^{Hc*}}{\partial q_c^L} = \frac{2}{\left(-4+q_a^L+q_c^L\right)^2} > 0$. $\frac{\partial d_{NB}^{HB*}}{\partial q_c^L} = 0$

$\frac{\partial \pi_{NB}^{H*}}{\partial q_c^L} = \frac{4\left(2+q_a^L+q_c^L\right)}{\left(-4+q_a^L+q_c^L\right)^3} < 0$, so we got $\frac{\partial \pi_{NB}^{L*}}{\partial q_c^L} < 0$.

PROPOSITION 4. *When there is strong competition, consumer surplus can decrease with the increase in the superior firm's add-on quality.*

Proof for Proposition 4. When there is strong competition, i.e., the parameter values of the inferior firm's qualities in figure 5, the superior firm is better off by bundling the add-on with the core good. Under the case that both firms bundle, we have

$$\begin{aligned} CS_{BB}^{*} &= \int_{\theta_{BB}^{L*}}^{\theta_{BB}^{H*}} (\theta(q_c^L+q_a^L)-p_{BB}^{L*}) + \int_{\theta_{BB}^{H*}}^{1} (\theta(1+q_a^H)-p_{BB}^{H*}) \\ &= \frac{1}{2}(q_c^L+q_a^L)(\theta_{BB}^{H*}+\theta_{BB}^{L*})(\theta_{BB}^{H*}-\theta_{BB}^{L*}) - p_{BB}^{L*}(\theta_{BB}^{H*}-\theta_{BB}^{L*}) + \frac{1}{2}(1+q_a^H)(1+\theta_{BB}^{H*})(1-\theta_{BB}^{H*}) - p_{BB}^{H*}(1-\theta_{BB}^{H*}) \end{aligned}$$

$$CS_{NB}^{*} = \int_{\theta_{NB}^{L*}}^{\theta_{NB}^{Hc*}} (\theta(q_c^L+q_a^L)-p_{NB}^{L*}) + \int_{\theta_{NB}^{Hc*}}^{\theta_{NB}^{Ha*}} (\theta-p_{NB}^{Hc*}) + \int_{\theta_{NB}^{Ha*}}^{1} (\theta(1+q_a^H)-p_{NB}^{Hc*}-p_{NB}^{Ha*})$$

By plug in θ_{BB}^{H*} and θ_{BB}^{L*} into CS_{BB}^{*}, we got the results for the optimal consumer surplus under the case where both firms bundle.

$$\frac{\partial CS_{BB}^*}{\partial q_a^H} = \frac{32(1+q_a^H)(c_a^H - 3(1+q_a^H))^2}{3(-4-4q_a^H + q_a^L + q_c^L)^3} + \frac{2(7c_a^H - 69(1+q_a^H))(c_a^H - 3(1+q_a^H))}{9(-4-4q_a^H + q_a^L + q_c^L)^2} + \frac{5 - c_a^H + 5q_a^H}{-4-4q_a^H + q_a^L + q_c^L} - \frac{c_a^{H\,2}}{18(-1-q_a^H + q_a^L + q_c^L)^2}$$

, Solve the equation

$\frac{\partial CS_{BB}^*}{\partial q_a^H} = 0$ as a function of c_a^H, we got the two roots, denoted as cs_1, cs_2.

Further, we got that $\frac{1}{2}(cs_1 + cs_2) > 0$, and $\left.\frac{\partial CS_{BB}^*}{\partial q_a^H}\right|_{c_a^H = \frac{1}{2}(cs_1 + cs_2)} > 0$. Hence, we got:

$$\begin{cases} \frac{\partial CS_{BB}^*}{\partial q_a^H} < 0, & \text{if } c_a^H < cs_1 \text{ or } c_a^H > cs_2 \\ \frac{\partial CS_{BB}^*}{\partial q_a^H} > 0, & \text{if } cs_1 < c_a^H < cs_2 \end{cases}$$

. In addition, we got $\frac{\partial cs_1}{\partial q_a^H} < 0$. Given the numerical parameter values in figure 5, we show that as q_a^H increases, the value of cs_1 is first greater and then less than c_a^H. Therefore, CS^* first decreases and then increases with q_a^H. ■

PROPOSITION 5. *Prohibiting add-on pricing changes the equilibrium if* $c_a^H > \tilde{q}$ *. Under these conditions, the prohibition increases the inferior firm's profit but decreases the superior firm's profit* $(\bar{\pi}^{L*} > \pi_{NB}^{L*},\ \bar{\pi}^{H*} < \pi_{NB}^{H*})$*. The higher* c_a^H*, the greater the gain of the inferior firm, and the greater the loss of the superior firm due to the prohibition. Both firms' prices for their core and add-on products increase (* $\bar{p}^{H*} > p_{NB}^{Hc*} + p_{NB}^{Ha*}$, $\bar{p}^{L*} > p_{NB}^{L*}$*).*

Proof for Proposition 5:

When prohibiting add-on pricing, both firms have to bundle. This is equivalent to both bundle (BB) case. $\bar{p}^{H*} = p_{BB}^{H*}$, $\bar{p}^{L*} = p_{BB}^{L*}$, and $\bar{\pi}^{H*} = \pi_{BB}^{H*}$, $\bar{\pi}^{L*} = \pi_{BB}^{L*}$. When add-on pricing is feasible, and $c_a^H > \tilde{q}$, then the superior firm unbundles and the inferior firm bundles in equilibrium (NB).

Therefore, we have $\bar{p}^{H*} - (p_{NB}^{Hc*} + p_{NB}^{Ha*}) = \dfrac{(q_c^L + q_a^L)(c_a^H(4 - q_c^L - q_a^L) + 3q_a^H(q_c^L + q_a^L))}{2(4 - q_c^L - q_a^L)(4 + 4q_a^H - q_c^L - q_a^L)} > 0$; and

$$\bar{p}^{L*} - p_{NB}^{L*} = \frac{(q_c^L + q_a^L)(c_a^H(4 - q_c^L - q_a^L) + 3q_a^H(q_c^L + q_a^L))}{(4 - q_c^L - q_a^L)(4 + 4q_a^H - q_c^L - q_a^L)} > 0.$$

When $c_a^H > \tilde{q}$, we know the superior firm is better off by unbundling the add-on, so we got

$\bar{\pi}^{H*} - \pi_{NB}^{H*} = \bar{\pi}_{BB}^{H*} - \pi_{NB}^{H*} < 0$. Furthermore, we have

$$\bar{\pi}^{H*} - \pi_{NB}^{H*} = \frac{(2(1+q_a^H)(1+q_a^H - q_c^L - q_a^L) - c_a^H(2 + 2q_a^H - q_c^L - q_a^L))^2}{(1+q_a^H - q_c^L - q_a^L)(4 + 4q_a^H - q_c^L - q_a^L)^2} - \frac{4(1 - q_c^L - q_a^L)}{(4 - q_c^L - q_a^L)^2} - \frac{(q_a^H - c_a^H)^2}{4q_a^H}$$

, which is a quadratic function of c_a^H. from the proof of Lemma 6, we know that solving

$\bar{\pi}^{H*} - \pi_{NB}^{H*} = 0$ for c_a^H, we got one negative root, one positive root, and a positive intercept.

Therefore, in the feasible region of c_a^H ($c_a^H \in (0,\ q_a^H)$), we got $\dfrac{\partial(\bar{\pi}^{H*} - \pi_{NB}^{H*})}{\partial c_a^H} < 0$.

For the inferior firm's profit, we have

$\bar{\pi}^{L*} - \pi_{NB}^{L*} = \dfrac{(1+q_a^H)(1+q_a^H + c_a^H - q_c^L - q_a^L)^2(q_c^L + q_a^L)}{(1+q_a^H - q_c^L - q_a^L)(4 + 4q_a^H - q_c^L - q_a^L)^2} - \dfrac{(1 - q_c^L - q_a^L)(q_c^L + q_a^L)}{(4 - q_c^L - q_a^L)^2}$, which is a

quadrative function of c_a^H. Solving $\bar{\pi}^{L*} - \pi_{NB}^{L*} = 0$ as a function of c_a^H, we got two negative roots, and a

positive intercept. So in the feasible region of c_a^H, we got $\dfrac{\partial(\bar{\pi}^{L*} - \pi_{NB}^{L*})}{\partial c_a^H} > 0$. ■

PROPOSITION 6. *Prohibiting add-on pricing reduces consumer surplus when* $c_a^H > \tilde{q}$, *and it has no impact on consumer surplus when* $c_a^H \le \tilde{q}$.

Proof for Proposition 6. From the proof of Proposition 5, we know that when $c_a^H > \tilde{q}$, we know the superior firm is better off by unbundling the add-on. When add-on pricing is prohibited, both firms have to bundle their add-on.

When $c_a^H > \tilde{q}$, we have

$$\overline{CS}^* - CS_{NB}^* = \frac{1}{8}[-\frac{(q_a^H - c_a^H)^2}{q_a^H} - \frac{4(4+5q_c^L+5q_a^L)}{(4-q_c^L-q_a^L)^2} + \frac{4(1+q_a^H)((c_a^H)^2(4+4q_a^H-3q_c^L-3q_a^L) - 2c_a^H(1+q_a^H-q_c^L-q_a^L)(4+4q_a^H+q_c^L+q_a^L) + (1+q_a^H)(1+q_a^H-q_c^L-q_a^L)(4+4q_a^H+5q_c^L+5q_a^L))}{(1+q_a^H-q_c^L-q_a^L)(-4-4q_a^H+q_c^L+q_a^L)^2}]$$

Solve it as a function of c_a^H, we got that the intercept is negative. Furthermore,

$\frac{\partial(\overline{CS}^* - CS_{NB}^*)}{c_a^H} < 0$, so we got $\overline{CS}^* < CS_{NB}^*$.

When $c_a^H < \tilde{q}$, the superior firm is better off by bundling, and both firm bundles in equilibrium. So prohibiting add-on pricing does not change firms' bundling strategy. Therefore, the consumer surplus remains unchanged.

CHAPTER 2

The Impact of Digitization on Content Markets: Prices, Profit, and Social Welfare

Abstract

The pervasiveness of the Internet and digitization has revolutionized the delivery and consumption of information goods. This research studies the impact of digitization and shift in consumers' preference for digital medium on outcomes in content markets including social welfare. We consider a publisher who offers information goods in the physical and digital mediums and also in a bundle of physical and digital mediums in a market where consumers are heterogeneous in both their valuations for content and their preferences for mediums. We find that the publisher's optimal medium-pricing strategy is to offer content only in the digital medium under some market conditions, while under other market conditions the publisher's best strategy is to offer content in a menu of a bundle of mediums and the digital medium. Interestingly, while the price of the bundle of mediums increases with the marginal cost of the physical medium, the price of the digital medium may decrease with the marginal cost. Surprisingly, we find that consumer surplus and social welfare may decrease as the proportion of digital-savvy consumers increases. Counter to intuition, while the digital price increases with the proportion of *digital-savvy* consumers, the price of the bundle may decrease when more consumers prefer the digital medium.

1. Introduction

The advent of the Internet and information technology has led to the digitization of content industries and, this in turn, has transformed the distribution and the consumption of information goods. While traditionally consumers purchased the physical medium to access information goods (print newspapers, books, CDs, DVDs, etc.), in the digital era, an increasing number of consumers buy and consume information good or content in the digital medium (digital newspapers, eBooks, album downloads, video downloads, etc.) (Sporkin 2011).

Digitization can potentially expand the consumer base by providing additional convenience and ease of use through anytime-anywhere access (Kouikova et al. 2008). On the other hand, the digital access to information goods may serve as a substitute for access through the physical medium, thereby cannibalizing physical sales (Kannan et al. 2009). In addition, pricing of content over the two mediums varies both within and across content industries. For example, both *The Wall Street Journal* and *The New York Times* offer a choice of home delivery + digital or digital-only access, but they do not offer a home delivery-only option. *Game Informer* magazine offers print-only and digital-only options but does not offer a bundle of the two. *Warner Music* sells digital-only albums as well as CDs that come with a digital copy. Independent record labels such as *Soulection* and *Triple Pop* offer only digital albums and tracks (Droppo 2014). An important question for content publishers is to determine the optimal content pricing strategies over dual-medium access under different market conditions.

While the physical and digital mediums differ significantly in costs to produce and distribute, publishers also must consider consumers' heterogeneous preferences for one medium over the other. For example, a recent consumer report finds that 38% of consumers preferred digital access to video games, while 62% of consumers still preferred having a physical CD of the games

(NPD Group 2015). Moreover, the growth of consumption of digital content by 157% from 2010 to 2014 indicates that the proportion of consumers who prefer digital medium is growing over the time, and this trend is likely to continue (comScore 2015). The heterogeneous and evolving consumer preferences toward the two mediums raises new challenges for publishers on how to price information goods in both physical and digital mediums. Moreover, it's not clear how this wave of digitization of content and the shift in consumers' preferences for the digital medium will impact the publishers, consumers and society.

Though there is a growing literature in IS on information goods pricing (Varian 1995, Choudhary et al. 2005, Dou et al. 2013, Niculescu and Wu 2014), the impact of digitization of content and the shift in consumer preferences towards the digital medium on the market outcomes has not received much attention. In this paper, we bridge this gap in literature by studying the following research questions: (a) What are the publisher's optimal content-medium pricing strategies? (b) Are there conditions under which offering content only in the digital medium is optimal? (c) What is the impact of shift in consumers' preferences towards the digital medium on profit, market coverage, consumer surplus, and social welfare? (d) How do the heterogeneity in consumers' preferences for the mediums, the marginal cost of the physical medium, and substitutability between the physical and the digital mediums impact the market outcomes?

In our analytical model, a monopolist publisher has the infrastructure to offer information goods in the physical as well as in the digital mediums. The marginal cost of the physical medium is non-negligible, but that of the digital medium is negligible (Bakos and Brynjolfsson 1999, Cusumano 2007). Consumers are vertically differentiated in their valuations for information goods or content (Choudhary et al. 2005, Lahiri and Dey 2013) and have heterogeneous preferences for mediums (Venkatesh and Chatterjee 2006). In the market, some consumers prefer the digital medium (we refer to this segment as *digital-savvy* consumers) while others prefer the physical medium (we refer to this segment as *traditional* consumers). Specifically, if a consumer gets the information goods in the medium he prefers[2], then his willingness to pay (WTP) is the same as his valuation for the information goods, but if he gets the information goods in the un-preferred medium, then he incurs disutility and his WTP is lower than his valuation for the information goods. We abstract this medium-mismatch disutility through a medium mismatch cost parameter. Furthermore, in our setup, the two mediums are partial substitutes (Venkatesh and Kamakura 2003, Armstrong 2013) which implies that a consumer's WTP for the content in a bundle of physical and digital mediums is greater than his WTP for content in any individual medium but less than the sum of his WTP for content in each of the two mediums. We abstract this partial substitutability of the two mediums through a sub-additive parameter.

We identify two optimal content-medium pricing strategies: (i) the publisher offers information goods only in the digital medium under some market conditions, and (ii) under some other market conditions, the publisher offers a choice of a bundle of the digital and physical mediums or the digital-only medium. Our closed-form solution for the optimal pricing strategy enables us to identify the interactive role of the marginal cost with the proportion of *digital-savvy* consumers in the market, the medium mismatch cost parameter, and the sub-additive parameter on the market outcomes.

Counterintuitively, we find that the price of the digital medium can decrease as the marginal cost of the bundle increases, though the bundle price always increases with the marginal cost. In addition, we find that under some conditions, as the proportion of *digital-savvy* consumers increases, the price of bundle and the total market coverage decrease though the price of the digital-

[2] Throughout this paper, our publisher is 'she' and consumer is 'he'.

only increases. Interestingly, while the publisher's optimal profit always increases as the proportion of *digital-savvy* consumers in the market increases, consumer surplus and social welfare can decrease when more consumers prefer to consume content in the digital medium.

Our work contributes to the literature in several streams. This is one of the few papers in the content pricing literature that develops an analytical model to study the optimal pricing strategies and the impact of digitization of content on the market outcomes, when the publisher offers content in the physical, the digital, and a bundle of mediums. We contribute the extant content pricing literature by showing that offering content only in the digital medium is optimal under some conditions, while offering content in a bundle of mediums as well as in the digital medium is optimal under other conditions (Venkatesh and Chatterjee 2006, Simon and Kadiyali 2007). Moreover, our work also contributes to the bundling literature by identifying conditions under which single component strategy (offering content only in the digital medium) is optimal (McAfee et al. 1989, Venkatesh and Kamakura 2003, Armstrong 2013; Bhargava 2014) [3].

Second, our work contributes to the literature on the pricing of partial substitutes. While the literature on partially substitutable goods suggests that when the price of one good increases, the price of the substitute good also increases (Bakos and Brynjolfsson 1999, Milgrom and Strulovici 2009), we identify conditions under which the opposite is true: in the context of content-medium markets, under some conditions, as the price of the bundle of mediums increases, the price of the digital medium decreases. Our result is driven by the characteristics of content markets wherein the marginal cost of offering content in different mediums is asymmetric and consumers have heterogeneous preferences (and thus heterogeneous WTP) for the two mediums.

[3] See Stremersch and Tellis (2002), Venkatesh and Mahajan (2009) for a comprehensive review of bundling literature.

Third, our work contributes to a growing literature on the impact of digitization of content on prices, market coverage, and profitability (Rob and Waldfogel 2006, Li 2015). We show that the digital price is closer to the bundle price as more consumers embrace the digital medium. Moreover, as more consumers become *digital-savvy*, the digital price increases, leading to the decrease in the total market coverage. In addition, we find that offering digital medium is always profit enhancing. Whereas prior literature and trade articles (Venkatesh and Chatterjee 2006, Harkaway 2012) find that offering the digital medium together with the physical medium is profit enhancing, we extend this result by showing that under some conditions offering only the digital medium improves profits.

Fourth, this research contributes to the debate on the impact of digitization on content markets and society (Esterl 2005, Knight 2015). While prior literature shows consumer surplus increases when the publisher offers information goods in both physical and digital mediums (Gentzkow 2007), we extend this result by showing that under some conditions, consumer surplus decreases when the proportion of *digital-savvy* consumers in the market is relatively large. Moreover, while the popular press (*Forbes* 2013) suggests that the widespread adoption of the digital medium is likely to lead to the increase in social welfare, we find that, under some conditions, social welfare may decrease as the proportion of *digital-savvy* consumer increases. This is because as more consumers prefer the digital medium, the price of the digital medium may increase, leading to the decrease in the market coverage as well as social welfare.

1.1 Related Literature

In the content pricing literature, empirical as well as analytical research informs our paper. For example, Simon and Kadiyali (2007) find that offering digital medium cannibalizes the demand for print media and reduces print sales by 9%. Kannan et al. (2009) examine digital content pricing and find that offering a bundle of print and PDF can increase a book publisher's profit when the two forms are viewed as imperfect substitutes. On the other hand, if the two forms are viewed as almost-perfect substitutes, then offering the bundle is not profit enhancing. This stream of research posits that physical and digital mediums are partial substitutes.

In addition, Kouikova et al. (2008) find that physical and digital formats each have advantages in specific usage situations. They employ experimental method and show that consumers' increased awareness of each format's advantages can increase demand for the bundle of the two formats. Along these lines, Kouikova et al. (2012) demonstrate that different product formats have distinct attributes (e.g. display ability for print and search ability for PDF), and therefore, consumers may have higher valuation for the bundle.

Literature in this stream has also studied the changes in the publisher's market coverage, and consumer surplus when the publisher introduces digital medium along with physical medium. Li (2015) study the impact of e-Books sales on changes in market coverage, and find that the total market expands when the publisher offers e-books together with print books. Gentzkow (2007) examine the welfare impact of the introduction of digital medium and find that consumer surplus increases when the publisher offers both physical and digital medium.

Although these empirical studies inform some of our theoretical basis, they only compare offering the bundle of the two mediums (or offering the two mediums separately) with offering only physical medium. They do not provide insights for other pricing strategies such as offering content in a bundle of mediums as well as in digital medium or offering content in only digital medium. Our study analyzes the publisher's all possible content-medium pricing strategies by developing an analytical model that abstracts consumers' heterogeneous preferences over mediums and other consumer characteristics.

An analytical study by Venkatesh and Chatterjee (2006) examines a monopolist publisher's profitability of offering content in either the physical medium only or the physical as well as the digital mediums. Our paper is distinct from their paper in the following two aspects. First, they do not examine the optimality of the offering of bundled mediums, a frequently observed content-medium pricing strategy (Benedict et al. 2011, Kouikova et al. 2012, Pew 2012). Since they do not consider bundle of mediums, their model does not account for the additional convenience perceived by consumers due to bundled medium access (Kouikova et al. 2008, Kannan et al. 2009). Second, we provide closed-form analytical solutions for the publisher's optimal pricing schemes which enables us to analyze the impact of changes in market characteristics on prices, profit, market coverage, as well as on consumer surplus and social welfare.

Our study is also broadly related to literature on bundling in general and bundling of two mediums in particular. Though there is a vast literature on bundling, we limit our attention to bundling research that is relevant in the context of content markets. Some early works in bundling (Stigler 1963, Adams and Yellen 1976) illustrate that bundling can serve as a useful price discrimination technique, and mixed bundling can be more profitable than pure component strategy. While Adam and Yellen (1976) suggest that bundling is profitable when valuations for the two products are negatively correlated, Schmalensee (1984) illustrate that bundling can increase profits when the valuations of the two products are independent, or even positively but not perfectly correlated. McAfee et al. (1989) analyze the bundling strategy for the multiproduct monopolist when the products have positive marginal costs and consumers' valuations are independently distributed, and derive conditions under which mixed bundling either dominates pure component pricing or weakly dominates pure bundling.

Although these studies provide insightful results about the optimality of mixed bundling, these results cannot be readily applied to the context of bundling of mediums. While in McAfee et al. (1989) consumer valuations for the two products are independently distributed, in the context of content pricing under dual medium access, bundling of two mediums with the same content suggests that consumers' valuations for the two mediums are correlated. In addition, these studies (Schmalensee 1984, McAfee et al. 1989) consider only the additive valuation for a bundle of the two goods, while in the context of dual-medium access, it is more likely to be sub-additive WTP for the bundle of the two mediums because the same content is accessed in both mediums.

Another stream of literature studies firms' bundling strategy for multiple products (Bakos and Brynjolfsson 1999, 2000, Armstrong 1999). When a monopolist offers a large number of information goods, and consumers have independent valuations of the individual goods, asymptotic results show that if the marginal cost of goods is negligible, then selling a bundle of all information goods can be superior to selling them separately, and if the marginal cost is positive, then selling the goods separately is optimal (Bakos and Brynjolfsson 1999). However, in a competitive market bundling can create "economies of aggregation" for information goods if their marginal costs are very low (Bakos and Brynjolfsson 2000). Armstrong (1999) study the optimal selling strategy for a monopolist who offers a large number of physical products with positive marginal cost for each product. If consumers' tastes are correlated across products, the monopolist can implement a close-to-optimal tariff as a menu of two-part tariffs. Our study is different from this stream of literature because we study the pricing strategy of a monopolist who can offer content in dual mediums wherein marginal cost is zero for the digital medium and is positive for the physical medium.

Prior research has studied the bundling strategy of a content provider who offers digital content on multiple devices (Bhargava 2014). They find that when consumers' valuations for devices are vertically differentiated and positively correlated, full mixed bundling, partial mixed bundling, or pure bundling can be optimal under different conditions. On the other hand, when consumers' valuations for devices are horizontally differentiated and negatively correlated, full mixed bundling is optimal. Our model is different because in our setup, consumers' valuations for the content are vertically differentiated, and consumers' preferences for the physical and digital mediums are horizontally differentiated. In our setting, some proportion of consumers prefer the digital medium while other proportion of consumers prefer the physical medium. In addition, whereas in Bhargava (2014), consumers have a higher range of valuations for one device relative to that for the other device, in our model, some proportion of consumers (*digital-savvy* consumers) has higher range of WTP for the digital medium compared to the physical medium while the other proportion of consumers (*traditional* consumers) has higher range of WTP for the physical medium compared to the digital medium.

Due to the key differences in the abstraction of consumer valuations and preferences, while Bhargava (2014) find full mixed bundling to be optimal when devices are horizontally differentiated, we find that when mediums are horizontally differentiated, either offering only the digital (single component) or offering the bundle and the digital medium (partial mixed bundling) is optimal. Furthermore, our analytical approach lends to the closed-form solutions which allow us to study the impact of changes in consumers' preference for the digital medium on the market outcomes and social welfare.

Prior literature also examines bundling of complementary or substitutable products. For example, Lewbel (1985) suggest that because the components of a bundle can be either complementary or substitutable, consumers may have either super-additive or sub-additive valuation for the bundle. They numerically prove that bundling can be optimal even when components are substitutable. Venkatesh and Kamakura (2003) study a monopolist's bundling strategy and compare pure bundling with mixed bundling when the two products are either substitutes or complements. They numerically find that pure bundling is optimal if the two products are strong complements. Armstrong (2013) find that when the component goods are partially substitutable, a firm has incentive to offer a bundle discount in at least as many cases as with the model with an additive-value of the bundle. Though these studies offer valuable insights regarding bundling of complementary or substitutable goods, none of them considers positively correlated demand of the component goods or examines partial mixed bundling and single component strategies.

The rest of paper is organized as follows. §2 describes the model set up and §3 presents the optimal content-medium pricing strategies. §4 analyzes comparative statistic, §5 presents numerical analyses, in §6, we discuss the contributions, implications, limitations and conclusion.

2. Model Setup

The market consists of a publisher of information good such as a newspaper publisher, a music label, a video game developer, a movie studio or a book publisher who has the ownership rights over the content. Given that each publisher has unique content and editorial style, following prior literature (Chen and Png 2003, Venkatesh and Chatterjee 2006, Wei and Nault 2014), we model the content publisher as a monopolist. Moreover, we assume that the cost of acquiring or developing content is sunk (Wu and Chen 2008). Consumers derive value from consumption of the content, but they can consume the content only if it is provided in some medium (Nielsen 2014). The publisher can provide the content either in the physical medium such as the paper-edition of a newspaper, magazine, CD, video game in a box, or book, or in the digital medium such as digital access to the newspaper or magazine, a downloadable video game or music, or an eBook. The publisher can also offer the content in both mediums as a bundle, so that consumers can consume it in either medium.

The publisher incurs a marginal cost $c_p \in (0,1)$ to serve a consumer to whom she provides content in the physical medium. The marginal cost of the physical medium includes material cost, handling and shipping cost, and labor cost. On the other hand, the publisher incurs zero marginal cost to serve a consumer to whom she provides content in the digital medium. When the publisher offers content in a bundle of the digital and the physical mediums to a consumer, then she incurs the same marginal cost as in the case of offering the content in only physical medium, i.e., c_p. We denote the price charged by the publisher for content in a physical medium as p_p, the price for content in a digital medium as p_d and the price for content in a bundle of mediums as p_b. To keep the focus of this research on optimal content-medium pricing strategies, we assume that all infrastructural costs for providing content in physical or digital medium are sunk, and the publisher has no supply-side constraints.

2.1 Consumers

The market consists of a unit mass of consumers. Consumers' valuations (v) for the content are independent and are uniformly distributed, i.e. $v \in U[0,1]$ (Lang and Vragov 2005, Dou et al. 2013). A recent study by Newspaper Association of America (NAA) finds that around 60% of readers read content in print, 48% of readers read content in digital formats and 34% read content in print as well as in digital mediums. The NAA study finds that each medium "provides somewhat different functional and experiential qualities fitting some usage occasions better than others."[4] A recent consumer report finds that while some consumers prefer video games in digital medium, others prefer video games in a physical medium compared to digital downloads (NPD Group 2015). These studies indicate that a segment of consumers prefers the digital medium over the physical medium while the other segment of consumers prefers the physical medium over the digital medium.

We model consumers preferences for mediums by segmenting the market wherein r proportion of consumers prefers the digital medium (*digital-savvy* consumers) and $1-r$ proportion of consumers prefers the physical medium (*traditional* consumers) (Kannan et al. 2009). A consumer who has valuation v for the content has WTP v for the access to the content if he is offered the content in the medium he prefers. On the other hand, if a consumer is offered the content in the medium that he does not prefer, then he incurs a medium mismatch cost $\theta \in (0,\ 1)$. This implies that a consumer who has valuation v for the content has a lower WTP for the access to the content, i.e., $v(1-\theta)$, if he is offered the content in his un-preferred medium.

This modeling approach is similar to one adopted in operations research. Hsiao and Chen (2014) categorize consumers into one of two segments based on their preference for purchasing online or purchasing in a retail store. Consumers in the first segment have higher utility from purchasing in retail channel (utility v for purchasing in a physical store, and utility $\beta_1 v$ for purchasing online, where $\beta_1 < 1$), whereas consumers in the second segment obtain higher utility from purchasing online (utility v for purchasing in a physical store, and utility $\beta_2 v$ for purchasing online, where $\beta_2 > 1$). Tan and Carrillo (2014) utilize a non-negative consumer acceptance level

[4] "2012 Newspaper Multiplatform Usage Study", http://www.naa.org/docs/NewspaperMedia/data/NAA-Multiplatform-Usage-Study.pdf

parameter to capture consumers' perceptions of digital goods relative to *traditional* goods. If the consumer acceptance level parameter is less than 1, it represents the situation in which consumers prefer traditional goods to digital goods and vice versa. Our conceptualization of two segments of consumers with a medium mismatch cost (θ) is similar to this strand of literature. Specifically, in our model, if the publisher offers content in a physical medium, then *traditional* consumers' WTP is v and *digital-savvy* consumers' WTP is $v(1-\theta)$. On the other hand, if the publisher offers content in a digital medium, then *traditional* consumers' WTP is $v(1-\theta)$ and *digital-savvy* consumers' WTP is v.

When the publisher offers the content in both mediums as a bundle, consumers' WTP for the content in the bundled medium is sub-additive. This implies that consumers view the content offering in a physical medium and a digital medium as partial substitutes (PWC 2008). This abstraction of dual-medium access of content is similar to the consumers' degree of contingency modeled in Venkatesh and Kamakura (2003) where the reservation price of a bundle of two products is less than the sum of the stand-alone reservation price of the two products, if the products are substitutes. This abstraction is further supported by empirical evidence. For example, Gentazkow (2007) show that raising the price of the physical newspaper increases the viewership of the digital newspaper. Li (2015) verify the partial substitutability of two content mediums by showing that sales of digital book increase at the expense of cannibalizing print book sales.

We abstract this sub-additive characteristic of mediums through a parameter $\alpha \in (0,\ 1-\theta)$ such that a consumer's WTP for content in both mediums is $v + \alpha v = v(1+\alpha)$. This conceptualization is similar to the modeling approach of bundling of substitutes in prior literature (Lewbel 1985, Venkatesh and Kamakura 2003, Armstrong 2013). Parameter α can also be interpreted as the additional convenience that consumers experience, if they have a choice of consuming the content in either medium. For analytical tractability, we assume that the sub-additive parameter α is homogeneous across all consumers. While Venkatesh and Kamakura (2003) make this assumption to get a closed-form solution, this abstraction is also similar to the constant value dependence assumption in McGuire and Staelin (1983) and to the analysis of substitute goods in Bakos and Brynjolfsson (1999). Note that if the content is offered in both mediums (bundle), then *digital-savvy* as well as *traditional* consumers have no medium-related heterogeneity because they get content in both. Note that $\theta + \alpha < 1$, because WTP for content in a bundle is sub-additive, i.e., $v(1+\alpha) < v + v(1-\theta)$.

2.2. The possible pricing strategies and market coverage profiles[5]

The publisher's optimal content-medium pricing strategy takes into account consumer heterogeneities and costs of offering the content in either of the mediums or in a bundle of mediums. The publisher has seven possible pricing strategies: offer content in (1) physical medium only; (2) digital medium only; (3) physical as well as in digital medium; (4) a bundle; (5) a bundle as well as in physical and digital mediums; (6) a bundle and in physical medium and (7) a bundle and in digital medium.

In order to focus our analysis on only those pricing strategies that may be optimal under different values of parameters and marginal cost, we first rule out those pricing strategies that are suboptimal.

LEMMA 1: *(a) Offering content only in the physical medium is dominated by offering the content in a bundle. (b) The pricing strategy of offering content in a bundle and in the digital medium dominates the pricing strategy of offering content only in a bundle.*

[5] We use Profile to denote market coverage profile in the rest of the paper.

The logic of Lemma 1(a) is that while all consumers' WTP for content in a bundle of mediums is strictly higher than that for content in the physical medium only, the publisher incurs no additional marginal cost by bundling the digital medium with the physical medium. So by offering a bundle of the mediums rather than offering the physical medium only, the publisher gains more profits. The intuition for Lemma 1(b) is that by offering content in the digital medium in addition to the bundle, the publisher can make more profit even if she keeps the same bundle price and offers the digital at price slightly lower than $p_b - c_p$. This is so because some low valuation *digital-savvy* consumers buy the digital medium but no consumer shifts from buying the bundle to buying the digital. This leads to the increased market coverage. In other words, under some conditions, by offering the digital medium, the publisher can gain additional revenue without cannibalizing the revenue from the bundle. Since the marginal cost of the digital medium is zero, under these conditions, the publisher is strictly better off.

From the analysis above, we need to focus on the two possible optimal content-medium pricing strategies, that is, offering the digital medium and the bundle, and offering only the digital medium. When the publisher offers content in a bundle as well as in digital medium only, the consumers' purchase decisions are based on their surplus from buying either of the offerings. We derive all possible market coverage profiles from the incentive compatibility (IC) and individual rationality (IR) constraints for each of the consumer types. A *traditional* consumer buys the bundle if the following IR constraint is met:

$$v(1+\alpha) - p_b \geq 0 \qquad \text{(IR: T-b)}$$

and he buys the digital medium if the following IR constraint is met:

$$v(1-\theta) - p_d \geq 0 \qquad \text{(IR: T-d)}$$

A *traditional* consumer buys the bundle over the digital medium if the following IC constraint is satisfied:

$$v(1+\alpha)-p_b \geq v(1-\theta)-p_d \qquad \text{(IC: T-b)}$$

Similarly, a *digital-savvy* consumer buys the bundle if the following IR constraint is met:

$$v(1+\alpha)-p_b \geq 0 \qquad \text{(IR: DS-b)}$$

and he buys the digital medium if the following IR constraint is met:

$$v-p_d \geq 0 \qquad \text{(IR: DS-d)}$$

A *digital-savvy* consumer buys the bundle over the digital medium if the following IC constraint is met:

$$v(1+\alpha)-p_b \geq v-p_d \qquad \text{(IC: DS-b)}$$

From these IC and IR constraints, we derive four possible Profiles for each type of consumers. The Profiles for *traditional* consumers are, 1) some buy the bundle, 2) some buy the bundle, and some buy the digital, 3) some buy the digital, and 4) none buys the bundle or digital. The Profiles for *digital-savvy* consumers are similar. Combining the Profiles for the two types of consumers, we derive overall Profiles. Among them, it is easy to see that any Profile in which one type of consumers buys nothing is suboptimal. Hence, we now need to focus only on eight Profiles in which both consumer segments participate in the market.

PROPOSITION 1: *Any Profile in which digital-savvy consumers only buy the bundle is suboptimal.*

Proposition 1 implies that it can never be optimal for the publisher to set prices p_d and p_b such that *digital-savvy* consumers buy only the bundle. The intuition is as follows. Suppose there is some Profile in which *digital-savvy* consumers only buy bundle is optimal. *Traditional* consumers will also buy only the bundle because they have the same WTP for the bundle but a lower WTP for the digital than that of the *digital-savvy* consumers. So consumers in both segments will buy only the bundle. We have shown that offering only the bundle is suboptimal. Therefore, any Profile in which *digital-savvy* consumers buy only the bundle is suboptimal. From Proposition 1, we rule out the three Profile in which *digital-savvy* consumers buy only the bundle. Hence, only the following five possible Profiles can potentially be optimal.

Profile 1 ($T-b \ \& \ DS-b \ \& \ d$): While *traditional* consumers buy the bundle, some *digital-savvy* consumers buy the bundle and some buy the digital. In order for Profile 1 to be feasible, the following two conditions must be met. First, the valuation of the marginal *traditional* consumer who is indifferent to the choice between buying the digital and not buying it (derived from IR: T-d) must not be less than that of the marginal *traditional* consumer who is indifferent to the choice between buying the bundle and not buying it (derived from IR: T-b), i.e. $p_b/(1+\alpha) \le p_d/(1-\theta)$. (2) The marginal valuation of a *digital-savvy* consumer who is indifferent to the choice between buying the bundle and buying the digital (derived from IC: DS-b) must not be less than that of the marginal *digital-savvy* consumer who is indifferent to the choice between buying the digital and not buying it (derived from IR: DS-d) and must be less than the highest valuation (which is 1), i.e. $p_d \le (p_b - p_d)/\alpha < 1$ (see Figure 1a).

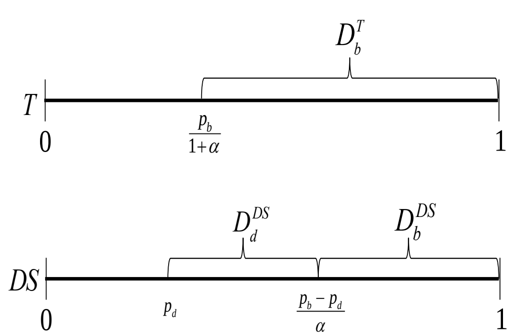

Figure 1a: Market coverage profile 1: *traditional* consumers buy the bundle ($T-b$), and some *digital-savvy* consumers buy the bundle, some buy the digital ($DS-b\,\&\,d$)

Now we can write the publisher's optimization problem under Profile 1 as:

$$\max_{p_b,\ p_d} \ \pi_1 = (p_b - c_p)(r(1-\frac{p_b - p_d}{\alpha}) + (1-r)(1-\frac{p_b}{1+\alpha})) + r(\frac{p_b - p_d}{\alpha} - p_d)p_d \qquad (1)$$

$$\text{s.t. } p_b/(1+\alpha) \le p_d/(1-\theta), \text{ and } p_d \le (p_b - p_d)/\alpha < 1$$

Under Profile 2, *traditional* consumers buy only the bundle and *digital-savvy* consumers buy only digital; Under profile 3, some *traditional* consumers buy the bundle and some buy the digital, and *digital-savvy* consumers buy only digital. Under profile 4, both types of consumers buy the bundle and the digital; under profile 5, both types of consumers buy the digital medium only (see Figure 1b). The detailed IR and IC constraints and profit function for each Profile are provided in the Appendix A.

We derive the publisher's optimal equilibrium pricing strategies by adopting a two-step approach. First, we derive the optimal prices under each Profile keeping in view the conditions for that Profile to be feasible. Second, we compare the feasible conditions for each Profile to determine if there is any overlapping region, and compare the optimal profit in the overlapping region to determine the optimal prices in that region. We find that Profile 4 is dominated by either Profile, 1 or 2, or 3 under a different parameter space. (The analytical proof is in the Appendix B).

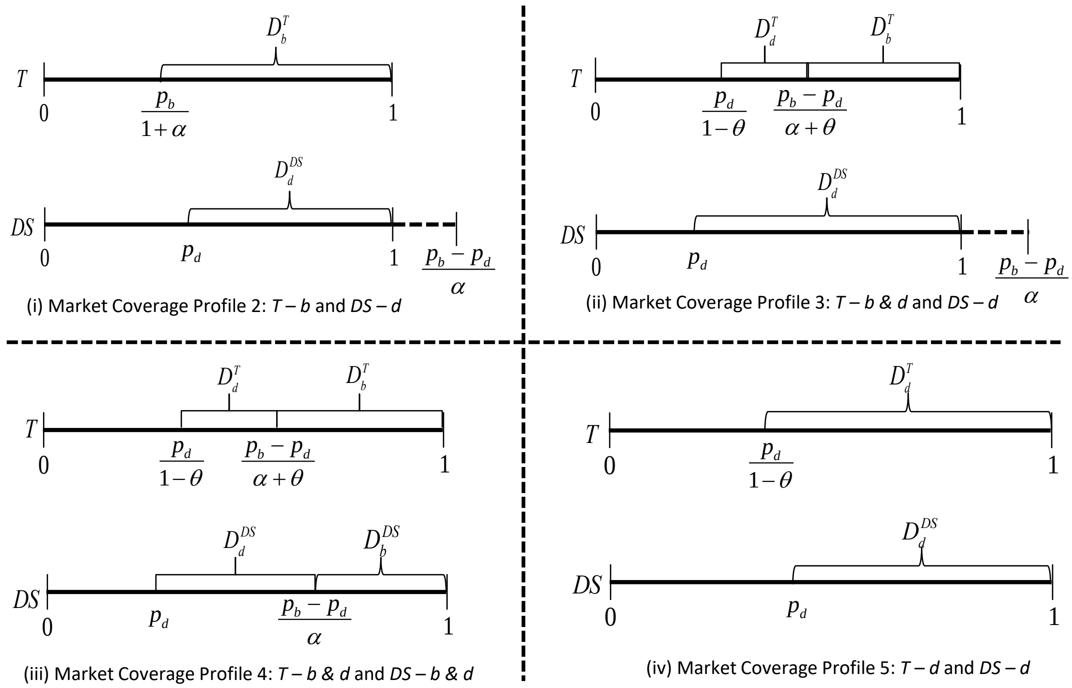

Figure 1b: Market coverage profile 2 - 5: (i) Profile 2: $T-b$ and $DS-d$; (ii) Profile 3: $T-b\,\&\,d$ and $DS-d$; (iii) Profile 4: $T-b\,\&\,d$ and $DS-b\,\&\,d$; and (iv) Profile 5: $T-d$ and $DS-d$

3. The optimal Content-medium pricing strategy

When the publisher offers content in the digital medium only, she incurs zero cost and her profit optimization problem reduces to a revenue maximization problem.

LEMMA 2: *When the publisher offers only the digital medium, the optimal price is* $p_d^* = (1-\theta)/(2-2r\theta)$, *and the optimal profit is* $\pi_d^* = (1-\theta)/(4-4r\theta)$.

It is easy to see that when content is offered in digital medium only, both optimal price (p_d^*) and profit (π_d^*) are increasing with the proportion of digital-*savvy* consumers (r) in the market, and decreasing with mismatch cost parameter (θ). If all consumers in the market are *traditional* ($r=0$), then the optimal price is $(1-\theta)/2$, and if all consumers in the market are *digital-savvy* ($r=1$), then the optimal price is $1/2$ ($>(1-\theta)/2$).

LEMMA 3: *When the publisher offers content in a bundle, and in the digital medium (partial mixed bundling) or in the digital medium only (single component), for any feasible parameter space, the optimal pricing scheme is from the set* $\left\{\left\{P_I^*\right\},\ \left\{P_{II}^*\right\},\ \left\{P_{III}^*\right\}\right\}$*, where*

$$P_I^* = \left\{p_I^{d*} = 1/2,\ p_I^{b*} = (1+c_p+\alpha)/2\right\},\ P_{II}^* = \left\{p_{II}^{d*} = \frac{1-\theta}{2-2r\theta},\ p_{II}^{b*} = \frac{\theta+\alpha+c_p}{2} + \frac{1-\theta}{2-2r\theta}\right\},\ and$$

$$P_{III}^* = \left\{p_{III}^{d*} = \frac{1-\theta}{2-2r\theta}\right\}.$$

Under optimal pricing scheme P_I^*, the Profile is either 1 or 2 depending upon the parameter space. On the other hand, under the optimal pricing scheme P_{II}^*, the Profile is 3, and under the optimal pricing scheme P_{III}^*, the Profile is 5. It is easy to see from Lemma 3 that while the price of the digital medium in pricing scheme P_I^* is higher than that in pricing schemes P_{II}^* and in P_{III}^*. On the other hand, the price of the bundle in pricing scheme P_I^* is lower than that in pricing scheme P_{II}^*. Note that in pricing scheme P_{III}^*, the publisher does not offer a bundle. Now we compare the conditions given in the equations (1) to (5) to figure out the parameter space in which the publisher should offer one of the three optimal pricing schemes.

3.1. Optimal pricing schemes when θ is larger than $1/(1+r)$

The condition in 3.1 implies that the relatively large value of the mismatch cost parameter (θ) should be viewed in relation to the proportion of *digital-savvy* consumers (r). Note that if $\theta \le 0.5$, then the results of the Proposition 2 are not applicable for any r.

PROPOSITION 2: *If the proportion of digital-savvy consumers (r) is such that* $\theta > 1/(1+r)$, *then the publisher adopts the pricing scheme* P_I^* *for the entire feasible parameter space.*

The economic reasoning for Proposition 2 is as follows. We know that the value of the mismatch cost parameter (θ) affects only *traditional* consumers' WTP for digital but does not affect *digital-savvy* consumers' WTP for digital. When θ large, traditional consumers' WTP for digital is low. Moreover, from Lemma 3, we know that the price of digital decreases with θ and is lower in optimal pricing schemes P_{II}^* and P_{III}^* than that in P_I^*, for any $\theta \in (0, 1)$. This implies that given a large θ, if the publisher were to adopt P_{II}^* or P_{III}^*, then the price of digital has to be lower.

When the publisher considers adopting pricing scheme P_{II}^* or P_{III}^* instead of P_I^*, her tradeoff is between reducing the cost by selling the digital instead of a bundle to *traditional* consumers and reducing revenue from *digital-savvy* consumers by reducing the digital price and from *traditional* consumers by incentivizing them to switch to the digital medium for which they have lower WTP. When θ is large, the reduction in revenue from both segments of consumers outweighs the reduction in cost by selling digital to *traditional* consumers. Hence, the publisher is better off by adopting P_I^*, rather than P_{II}^* or P_{III}^*, even when the marginal cost is high.

The condition in Proposition 2 also implies that pricing scheme P_I^* can be optimal for either small or large r as long as this proportion is above the threshold, i.e. $r > (1-\theta)/\theta$. This is because when the mismatch cost parameter is very large, even if the proportion of *digital-savvy* consumers is small and offering bundle is costly, the publisher is better off by offering pricing scheme P_I^* under which the *traditional* consumers buy only the bundle for which they have higher WTP, rather than the digital for which they have lower WTP.

3.2. Optimal pricing schemes when θ is not larger than $1/(1+r)$

Now, we describe the optimal pricing schemes when the medium mismatch cost parameter is not larger than $1/(1+r)$. This condition also implies that θ can take any value between 0 and 1. We group the results around the value of the mismatch cost parameter and describe the optimal pricing scheme in the entire feasible parameter space. We first report some technical thresholds which characterize these regions.

LEMMA 4: *The optimal pricing scheme regions in the parameter space are characterized by (i) three thresholds for the proportion of digital-savvy consumers (that is for* r *),* r_0 *,* r_1 *and* r_2 *where* $0 < r_0 < r_1 < r_2$*; (ii) three thresholds for the sub-additive parameter (that is for* α *),* α_1*,* α_2 *and* α_3*; and (iii) five thresholds for the marginal cost (that is* c_p *),* a *,* $\alpha+\theta$ *,* c_{p1}*,* c_{p2}*, and* c_{p3}. (Analytical expressions for threshold values are in the Appendix B).

Note that the ordering of thresholds for the proportion of *digital-savvy* consumers (r) is maintained for all parameter values; the ordering of thresholds for sub-additive parameter (α) depends on mismatch cost parameter (θ). If $\theta > 0.5$, then α thresholds are infeasible and are not required in characterizing optimal pricing schemes (see Proposition 2). If $\theta \in (0.445,\ 0.5]$, we have $\alpha_1 < \alpha_2 < \alpha_3$; if $\theta \in (0.333,\ 0.445]$, we have $\alpha_2 < \alpha_1 < \alpha_3$; and if $\theta \le 0.333$, then we have $\alpha_2 < \alpha_3 < \alpha_1$. The ordering of the thresholds of the marginal cost depends on values of parameters r, θ, and α.

3.2.1. Optimal pricing schemes when the mismatch cost parameter is large ($0.445 < \theta \le 1/(1+r)$)

When the mismatch cost parameter θ is large but the proportion of *digital-savvy* consumers is such that $1/(1+r)$ is not smaller than θ, the publisher may be better off by offering P_I^*, P_{II}^* or P_{III}^* under different parameter values.

PROPOSITION 3a: *If (I)* $0.5 < \theta \le 1/(1+r)$ *or (II)* $0.445 < \theta \le 0.5$ *and* $\alpha > \alpha_1$*, then offering digital-only is optimal when* $r \le r_1$ *and* $c_p \ge \alpha + \theta$ *or when* $r_1 \le r \le r_2$ *and* $c_p \ge c_{p3}$*; offering a menu of bundle and digital is optimal under other conditions. The publisher's optimal content-medium strategies are characterized in the following table:*

<table>
<tr><th>r \ c_p</th><th>$(0,\ c_{p1}]$</th><th>$(c_{p1},\ c_{p2}]$</th><th>$(c_{p2},\ \alpha+\theta]$</th><th>$(\alpha+\theta,\ c_{p3}]$</th><th>$(c_{p3},1]$</th></tr>
<tr><td>$(0,\ r_0]$</td><td>P_I^*</td><td colspan="2">P_{II}^*</td><td colspan="2">P_{III}^*</td></tr>
<tr><td>$(r_0,\ r_1]$</td><td colspan="2">P_I^*</td><td>P_{II}^*</td><td colspan="2">P_{III}^*</td></tr>
<tr><td>$(r_1,\ r_2]$</td><td colspan="4">P_I^*</td><td>P_{III}^*</td></tr>
<tr><td>$(r_2,\ 1]$</td><td colspan="5">P_I^*</td></tr>
</table>

Proposition 3a highlights the finding that offering digital-only is optimal under some market conditions (shaded regions where pricing scheme P_{III}^* is optimal) while under other market conditions, offering a menu of the bundle and the digital is optimal.

Specifically, when the mismatch cost is large ($0.5 < \theta \le 1/(1+r)$) or mismatch cost is relatively moderate and the sub-additive parameter is large ($0.445 < \theta \le 0.5$ and $\alpha > \alpha_1$), if the proportion of *digital-savvy* consumers is relatively small ($r \le r_1$) and the marginal cost is relatively large ($c_p \in (\alpha+\theta,\ 1]$), or r is moderate ($r_1 \le r \le r_2$) and c_p is large ($c_p \in (c_{p3},\ 1]$), then offering content in digital medium only is optimal. The intuition for this new finding is the following. When the proportion of *digital-savvy* consumers is relatively small or moderate, the publisher does not gain much profit by charging a high digital price (p_I^{d*}). Moreover, because the marginal cost is large, the incentive for the publisher to avoid offering costly bundle to any consumers is strong. However, since the mismatch cost is large, *traditional* consumers have a low WTP for the digital medium. In order to incentivize the *traditional* consumers to buy the digital rather than the bundle, the publisher has to offer a low digital price (p_{III}^{d*}). Therefore, due to the combined effect of the relatively small proportion of *digital-savvy* consumers, the large marginal cost of the bundle, and the large mismatch cost of *traditional* consumers, the publisher is better off by shutting down the bundle, and offering a low digital price so that all consumers buy the digital medium only.

On the other hand, under the same conditions for the mismatch cost and the sub-additivity parameter, if the proportion of *digital-savvy* consumers is large ($r > r_2$), then offering a menu of the bundle and the digital is optimal, even when the marginal cost is large. The reason is that because of the large proportion of *digital-savvy* consumers in the market, the publisher can gain larger profit from *digital-savvy* consumers by charging them a high digital price, but the higher digital price makes the digital medium less attractive compared to the bundle. Hence, the publisher is better off by offering a menu of the bundle and the digital which allows some *traditional* consumers to buy the bundle instead of not buying anything due to the high price of the digital. Therefore, offering a menu of the bundle and the digital with a high digital price becomes optimal (P_I^*).

In the parameter regions where it is optimal to offer a menu of the bundle and the digital, the publisher offers P_I^* or P_{II}^* under different conditions. When the mismatch cost is large or relatively large and the sub-additivity is large, the difference of valuation between the bundle and the digital for *traditional* consumers is large. When the marginal cost is moderate ($c_p \in (c_{p2},\ \alpha + \theta]$), if the proportion of *digital-savvy* consumers is low ($r \le r_0$), the publisher's optimal pricing scheme is P_{II}^* wherein *traditional* consumers buy the bundle and digital but *digital-savvy* consumers buy only digital (Profile 3, Figure 2b). This is because the publisher's gain from saving on the marginal cost by incentivizing both consumer segments to buy the digital is higher than her loss from *digital-savvy* consumers who are offered the digital at a lower price. When the marginal cost is moderate ($c_p \in (c_{p2},\ \alpha+\theta]$), if the proportion of *digital-savvy* consumers is low ($r \le r_0$), the publisher is better off by switching to P_I^* wherein she gains more profit from the larger proportion of *digital-savvy* consumers by charging them a higher digital price ($p_I^{d*} > p_{II}^{d*}$).

PROPOSITION 3b: *If* $0.445 < \theta \le 0.5$ *and* $\alpha < \alpha_1$, *then (a) when* $r \in (0, r_1]$, *the publisher's optimal pricing strategies remain the same as presented in the first two rows of the table in Proposition 3a; and (b) when* $r \in (r_1, 1]$, *the publisher adopts* P_I^* *if* $c_p \in (0, c_{p3}]$, *and adopts* P_{III}^* *if* $c_p \in (c_{p3}, 1]$.

Proposition 3b describes the optimal pricing schemes when the mismatch cost parameter is less than 0.5 ($0.445 < \theta \le 0.5$) and the sub-additive parameter is relatively small ($\alpha < \alpha_1$). In this scenario, the threshold r_2 is irrelevant, i.e. $r_2 > 1$. When r is relatively large ($r \in (r_1,\ 1]$), the publisher's optimal pricing scheme is P_I^* if the marginal cost is low, and is P_{III}^* if the marginal cost is high. Pricing scheme P_{II}^* is not optimal. This is because when both θ and α are not sufficiently large, *traditional* as well *digital-savvy* consumers' difference in WTP for the bundle and for digital is not large. Therefore, when r is relatively large, the publisher is better off by offering P_I^* wherein she gains more revenue from *digital-savvy* consumers, if c_p is not large. On the other hand, if c_p is large, then the publisher switches to P_{III}^* and shuts down the bundle to economize on cost.

3.2.2. Optimal pricing schemes when the mismatch cost parameter is moderate to small ($0 \le \theta \le 0.445$)

In this scenario, we identify the regions in which pricing scheme P_I^*, P_{II}^*, or P_{III}^* is optimal when (i) the mismatch cost parameter (θ) is moderate ($\theta \in ((1.5-\sqrt{1.25}),\ 0.445)$), and the sub-additive parameter (α) is large ($\alpha > \alpha_2$), or (ii) θ is relatively small ($\theta \in (0.333,\ 1.5-\sqrt{1.25})$), or (iii) θ is small ($\theta \le 0.333$) and α is relatively small ($\alpha \le \alpha_3$). Note that unlike the Propositions 3a and 3b, in this case we have all three pricing schemes across different parameter spaces of marginal cost and the proportion of *digital-savvy* consumers. The intuition is as follows. When θ is moderate or small, it implies that the difference between *traditional* consumers' WTP for digital and *digital-savvy* consumers' WTP for digital is relatively smaller than in the case described in Proposition 3a and 3b. In other words, the two types *of* consumers become more similar in terms of their WTP. This implies that the proportion of one type of consumers, though it continues to play a role in the choice of optimal pricing schemes, is not dominant enough to rule out P_{II}^* or P_{III}^* under some parameter space. The detailed analysis and proof is in the Appendix B.

3.3. Optimal pricing schemes under feasible parameter regions

We now use two dimensional region plots to provide further insights and intuition for regions in which offering digital only or offering a menu of bundle and digital only is optimal along with the optimal pricing schemes in each of these regions. Note that we use conditions described in Propositions 2 and 3 to draw these region plots.

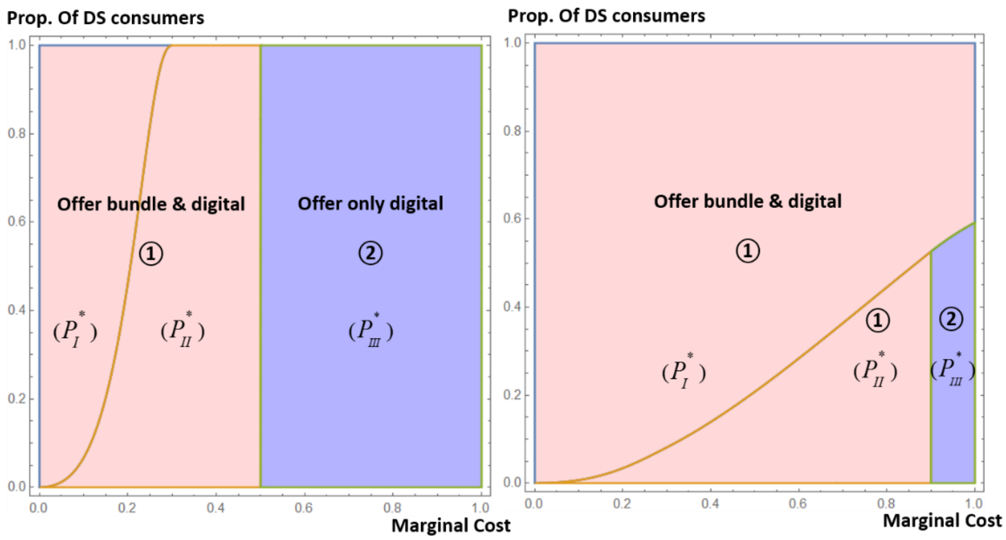

Figure 2a: Optimal pricing schemes in $r-c_p$ space ($\alpha = 0.3$) (i) Left plot: when the medium mismatch parameter is small ($\theta = 0.2$) (ii) Right plot: when θ is large ($\theta = 0.6$)

Figure 2a (i) illustrates optimal pricing schemes in different regions of r - c_p space when the mismatch cost parameter is small ($\theta = 0.2$). Irrespective of the proportion of *digital-savvy* consumers (r), if the marginal cost is moderate to large, then the publisher is better off by offering only the digital medium (P_{III}^*) when marginal cost is high ($c_p \in (0.5,\ 1)$), and is better off by offering a menu of the bundle and the digital when the marginal cost is small to moderate ($c_p \in (0,\ 0.5)$).

Figure 2a (ii) illustrates optimal pricing schemes when the mismatch cost parameter is large ($\theta = 0.6$). In this case, the publisher is better off to offer both the bundle and the digital even when the marginal cost is large ($c_p \in (0.5,\ 0.9)$). This is mainly because a proportion of consumers in the market (*traditional* consumers) does not prefer the digital medium, and they have low WTP for the digital. The relative attractiveness of bundle is much higher than that of digital for *traditional* consumers. Hence, the publisher is better off by offering both the bundle and the digital medium so that at least some *traditional* consumers buy bundle. When the marginal cost is very large ($c_p \in (0.9,\ 1)$), and if the proportion of *digital*-*savvy* consumers is large ($r \in (0.55,\ 1)$), the publisher is still better off by offering a choice of the digital medium and the bundle. On the other hand, when the marginal cost is very large ($c_p \in (0.9,\ 1)$), but the proportion of *digital*-*savvy* consumers is relatively small ($r \in (0,\ 0.55)$), the publisher is better off by offering content in digital medium only (P_{III}^*). This may seem counter-intuitive because one may expect that the publisher is better off by offering digital medium only when the proportion of *digital*-*savvy* consumers is large. The intuition is as follows. We know that the publisher charges a high digital price when offering pricing scheme P_I^*. This implies that the profit margin of the publisher is higher in offering the digital medium under pricing scheme P_I^*. When the proportion of *digital*-*savvy* consumers is large, the profit gain from the large proportion of *digital*-*savvy* consumers outweighs the loss in the marginal cost from offering costly bundle to *traditional* consumers under pricing scheme P_I^*. Therefore, offering both the bundle and the digital medium is better than offering only the digital medium when the proportion of *digital*-*savvy* consumers is large, and the publisher is better off by offering digital-only when the proportion of *digital*-*savvy* consumers is relatively small.

The optimal pricing schemes in θ - c_p space are illustrated in Figure 2b. Figure 2b (i) illustrates the optimal pricing schemes when r is small. If c_p is small, then P_I^* is optimal for all θ; if c_p is moderately small, then P_{II}^* is optimal when θ is small and P_I^* is optimal when θ is sufficiently large; and if c_p is relatively large, then P_{III}^* is optimal when θ is small, and P_{II}^* is optimal when θ is large.

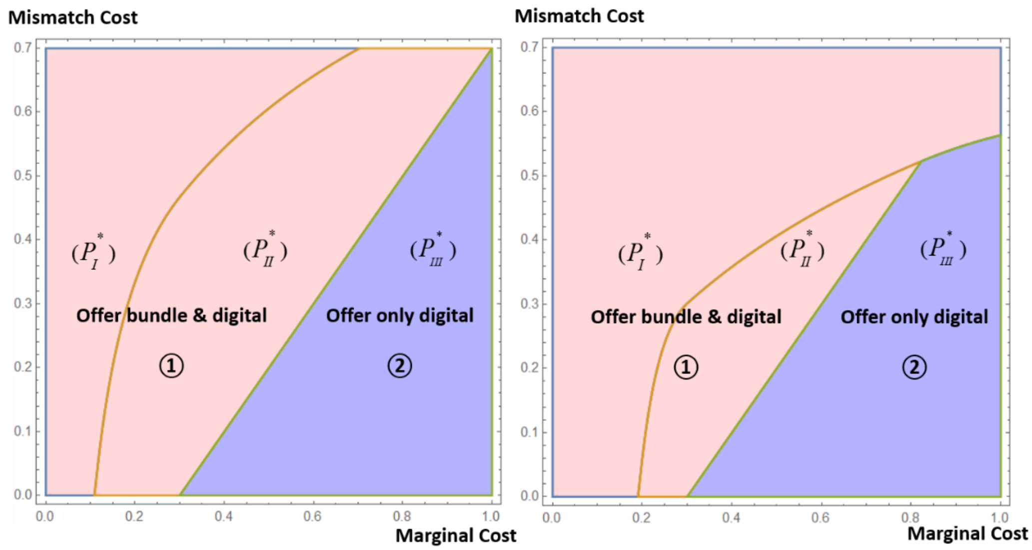

Figure 2b: Optimal pricing schemes in $\theta - c_p$ space ($\alpha = 0.3$) (i): when the proportion of *digital-savvy* consumers is small ($r = 0.2$), (ii): when r is large ($r = 0.7$).

Figure 2b (ii) illustrates the optimal pricing schemes when r is large. By comparing Figure 2b (i) and Figure 2b (ii), we can see that as r increases, the region in θ - c_p space in which the publisher implements optimal pricing scheme P_{II}^* becomes smaller, the region in which optimal pricing scheme P_{III}^* is implemented moves downward. The reason is that when r is large, if the publisher implements P_{II}^* or P_{III}^*, then the loss from *digital-savvy* consumers is large and increases with θ (because the digital price is lower and decreases as θ increases). Hence, the optimality of P_{II}^* becomes more restrictive and that region shrinks, and P_{III}^* remains optimal only when c_p is large and θ is not large.

4. Comparative Statics

In this section, we analyze the impact of changes in the marginal cost, the proportion of *digital-savvy* consumers, on a publisher's optimal prices, market coverage, profit, consumer surplus and social welfare.

4.1 Impact of the marginal cost on optimal prices

PROPOSITION 4a: *While the optimal price of a bundle increases with the marginal cost, the optimal price of digital can decrease when the marginal cost is sufficiently large.*

Proposition 4a highlights the impact of the marginal cost on the digital price as well the bundle price (Figure 3). Note that when the publisher shifts from the pricing scheme P_I^* to P_{II}^*, (i) the price of the bundle has a discrete increase and its rate of increase with the marginal cost remains the same, and (ii) the digital price has a discrete decrease and remains constant with c_p .

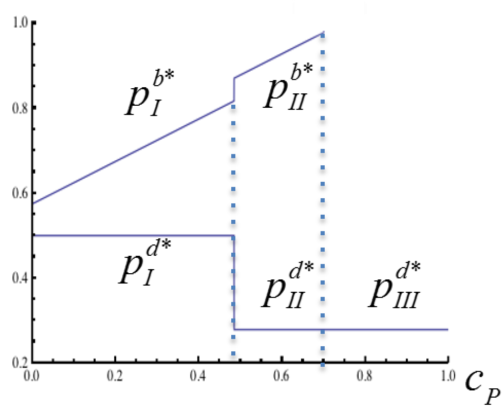

Figure 3: Optimal prices with the marginal cost of the bundle $\alpha = 0.15,\ r = 0.35,\ \theta = 0.55$

This result is counter-intuitive because prior literature (Bakos and Brynjolfsson 1999, Milgrom and Strulovici 2009) suggests that as the price of one good increases, the price of the substitutable good also increases. So the increase in the marginal cost of the bundle may lead to an increase in the digital price because the digital medium and the bundle are partial substitutes (Kannan et al. 2009, Kouikova et al. 2008). The intuition for this result is as follows. Note that the marginal cost of the digital medium is zero, and *traditional* consumers have a lower WTP for the digital medium. Therefore, when the marginal cost of the bundle is sufficiently large and the proportion of *traditional* consumers in the market is relatively large, the gain from saving on the marginal cost and increasing the market coverage from *traditional* consumers outweighs the revenue loss from *digital-savvy* consumers. Hence, the publisher is better off by raising the bundle price and at the same time reducing the digital price. This leads to some low-valuation *traditional* consumers to switch from the costly bundle to the digital medium, and only high-valuation *traditional* consumers buy the bundle.

4.2. Impact of the proportion of *digital-savvy consumers* on the optimal prices

PROPOSITION 4b: *When the proportion of digital-savvy consumers is sufficiently large, while the digital price is higher under* P_I^* *compared to that under* P_{II}^**, the bundle price can be lower under* P_I^* *compared to that under* P_{II}^*.

Figure 3b (a) illustrates two cases: (i) when r is small to moderate, the publisher adopts P_{II}^* , and (ii) when r is relatively large, the publisher switches to P_I^* . Interestingly, this implies that when the proportion of *digital-savvy* consumers is sufficiently large, while the digital price increases, the bundle price has a discrete decrease.

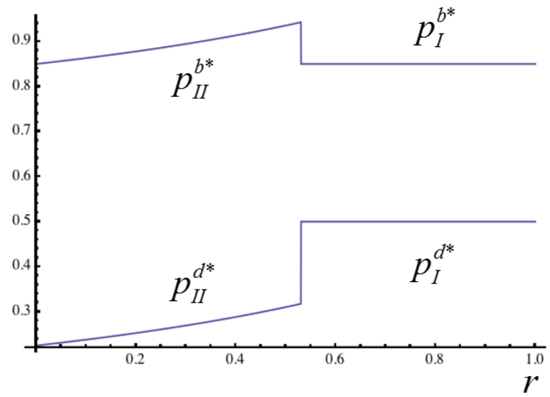

Figure 3b: The optimal prices with the proportion of *digital-savvy* consumers:

$$\{\alpha = 0.1,\ c_p = 0.6,\ \theta = 0.55\}$$

The economic intuition for the first case is as follows. When r is small and the marginal cost is large, the publisher offers pricing scheme P_{II}^*. This is because given the large marginal cost, the publisher is better off by incentivizing some *traditional* consumers to buy the digital rather than the bundle (Profile 3, Figure 1b (ii)). Further, in pricing scheme P_{II}^* , both the digital price and the bundle price increase with r . This is because as r increases, the publisher is better off by increasing the digital price to increase the revenue from *digital-savvy* consumers who buy the digital medium. Moreover, the relative attractiveness of the bundle increases as the digital price increases, so the publisher increases the bundle price to gain a higher profit from *traditional* consumers who buy the bundle.

The second case where r is large, highlights an interesting result. As r increases, the publisher is better off by switching to the pricing scheme P_I^* from P_{II}^*. This implies that the publisher charges an even higher digital price (p_I^{d*} , discrete increase) and at the same time lowers the bundle price (discrete decrease). This allows the publisher to gain the maximum profit from *digital-savvy* consumers. Since the digital price is high, the publisher is better off by lowering the bundle price such that no *traditional* consumer buys digital and they only buy the bundle for which they have a higher WTP than that for the digital medium. This result also implies that as more consumers become *digital-savvy*, the difference between the digital price and the bundle price may be smaller.

4.3. Impact of the proportion of *digital-savvy* consumers on the market coverage

PROPOSITION 5: *(i) Under pricing schemes* P_{II}^* *and* P_{III}^* *, the market coverage is* $1/2$ *(ii) under pricing scheme* P_I^* *, the market coverage is lower than* $1/2$ *but increases with the proportion of digital-savvy consumers.*

Proposition 5 describes the impact of the proportion of *digital-savvy* consumers on the market coverage. Note that under P_{III}^* , the publisher offers only digital to all consumers such that the total market coverage is $1/2$. When the publisher offers pricing scheme P_{II}^{*}, the digital price remains the same as in P_{III}^{*} (Lemma 3), and *digital-savvy* consumers buy only the digital. Hence, the market coverage of *digital-savvy* consumer segment remains the same under P_{II}^{*} and P_{III}^{*}. Under pricing scheme P_{II}^{*}, the Profile 3 describes the market (Figure 1b); the market coverage from *traditional* consumer is determined by the marginal consumer who buys the digital. Since the digital price under P_{II}^{*} is the same as that under P_{III}^{*}, the market coverage from *traditional* consumers under P_{II}^{*} remains the same as that under P_{III}^{*}, though some *traditional* consumers buy the bundle. Therefore, under pricing scheme P_{II}^{*}, the total market coverage remains $1/2$.

Under pricing scheme P_{I}^{*}, the market coverage of the *traditional* consumer segment (not multiplied by $1-r$) is less than $1/2$ (i.e., $(1+\alpha-c_p)/2(1+\alpha)$), and that of the *digital-savvy* consumer segment is $1/2$. Note that the market coverage of each consumer segment is independent of the proportion of *digital-savvy* consumers (r) in the market. As r increases, the market coverage of *digital-savvy* consumers increases at a higher rate (i.e., $1/2$) than the decrease in the market coverage of *traditional* consumers (i.e., $(1+\alpha-c_p)/2(1+\alpha)$). Therefore, the total market coverage under pricing scheme P_{I}^{*} increases with r.

When the publisher's is switches from pricing scheme P_{II}^{*} (or P_{III}^{*}) to P_{I}^{*}, at the critical value of r where the publisher switches the pricing schemes, the market coverage decreases (Figure 4). This result is surprising because one might expect the total market coverage to increase monotonically with r because *digital-savvy* consumers have a higher WTP for the digital compared to *traditional* consumers and all consumers have the same WTP for the bundle.

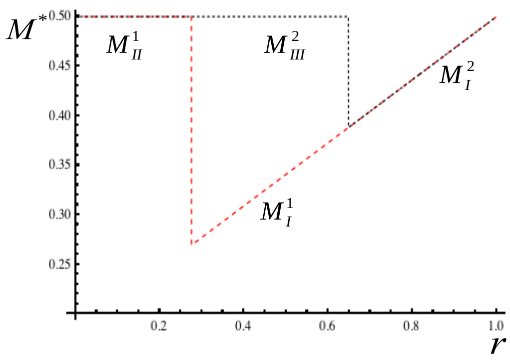

Figure 4: Total market coverage with r. M_{II}^1 and M_I^1: publisher switches pricing from P_{II}^* to P_I^*, $\{\alpha = 0.1,\ c_p = 0.7,\ \theta = 0.7\}$; M_{III}^2 and M_I^2: from P_{III}^* to P_I^*, $\{\alpha = 0.1,\ c_p = 0.7,\ \theta = 0.55\}$

The intuition for this result is that when r becomes sufficiently large, the publisher is better off either by raising the price for the digital medium and reducing the bundle price (switching from P_{II}^* to P_I^*, Figure 3b) or by offering the bundle along with the digital (switching from P_{III}^* to P_I^*). Under pricing scheme P_I^*, the market coverage profile 1 or 2 emerges wherein *traditional* consumers only buy the bundle (Figure 1a and 1b) because they have a higher WTP for the bundle than for the digital. This leads to a lower market coverage for the *traditional* consumers segment. Moreover, since the digital price increases under pricing scheme P_I^*, the market coverage for the *digital-savvy* consumers segment also decreases. Hence, at the threshold value of r where the publisher switches to pricing scheme P_I^*, the total market coverage decreases (Figure 4).

4.4. Comparative statics for optimal profit, consumer surplus, and social welfare

PROPOSITION 6*: The publisher's optimal profit increases with the proportion of digital-savvy consumers under all pricing schemes.*

The publisher's profit increases with the proportion of *digital-savvy* consumers (r) in pricing scheme P_I^*. This is because the total market coverage increases with r while profit margins from the bundle ($p_I^{b*} - c_p$) and digital (p_I^{d*}) are independent of r. The profit gain from *digital-savvy* consumers outweighs the profit loss from *traditional* consumers leading to a higher profit. The publisher's profit increases with r in pricing scheme P_{II}^{*} because 1) the profit margin from both the bundle ($p_{II}^{b*} - c_p$) and the digital (p_{II}^{d*}) increase with r since the bundle and digital prices increases with r and 2) the increase in market coverage from *digital-savvy* consumers is greater than the decrease in market coverage from *traditional* consumers. The publisher's profit increases with r under pricing scheme P_{III}^{*} because the digital price increases with r leading to the increase in profit margin from the digital even though the total market coverage under pricing scheme P_{III}^{*} is independent of r.

PROPOSITION 7: *Consumer surplus increases with the proportion of digital-savvy consumers (r) when pricing scheme P_{I}^{*} is optimal and first increases and then decreases with r when pricing scheme P_{II}^{*} or P_{III}^{*} is optimal. Social welfare can decrease when r is sufficiently large.*

One may expect that consumer surplus monotonically decreases with r under pricing scheme P_{II}^{*} or P_{III}^{*} since both the digital price and the bundle price increase with r and the total market coverage is independent of r. Surprisingly, we find consumer surplus first increases and then decreases with r under pricing scheme P_{II}^{*} or P_{III}^{*}. In addition, we find that, under some market conditions, consumer surplus as well as social welfare has a discrete decrease when the publisher switches either from P_{II}^{*} or P_{III}^{*} to pricing scheme P_{I}^{*}.

Figure 5(a) shows that in the feasible region of r where first P_{II}^{*} and then P_{I}^{*} is optimal, consumer surplus (CS_{II}^{1} , CS_{I}^{1} in Scenario 1) first increases, and then decreases by a discrete value when r reaches a threshold, and then increase again with r. Consumer surplus (CS_{III}^{2} , CS_{I}^{2} in Scenario 2) has a similar pattern with r when pricing scheme P_{III}^{*} or P_{I}^{*} is optimal. The logic for the discrete decrease in consumer surplus is as follows. When the publisher switches to P_I^*, the digital price increases, thus each consumer who buys the digital gets lower surplus. At the same time, as the digital price increases, the total demand decreases. This leads to a decrease in the total consumer surplus. On the other hand, the bundle price under P_I^* is lower than that under P_{II}^*; this leads to a gain in consumer surplus for those who buy the bundle. However, the loss of consumer surplus from those who buy the digital outweighs the surplus gain from consumers who purchase the bundle. Hence, consumer surplus has a discrete drop. Lastly, the intuition for the increase of consumer surplus with r following the discrete drop can be understood as follows. When P_I^* is optimal, both the digital price and the bundle price are independent of r. When r is large, the market coverage from *digital-savvy* consumers is large and the market coverage from *traditional* consumers is small. Since the average consumer surplus of *digital-savvy* consumers is larger than that of *traditional* consumers under P_I^*. This implies that the total consumer surplus increases as r increases.

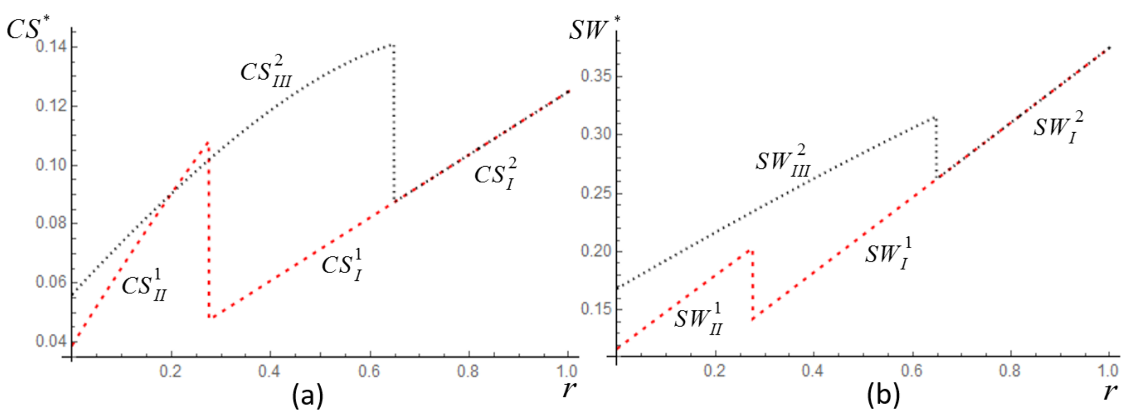

Figure 5 (a): consumer surplus with r. (b): social welfare with r. Subscript: pricing schemes. Superscript: scenarios. Scenario 1: $\{\alpha = 0.1,\ c_p = 0.7,\ \theta = 0.7\}$; Scenario 2: $\{\alpha = 0.1,\ c_p = 0.7,\ \theta = 0.55\}$

Figure 5(b) shows that social welfare has a discrete decrease when the publisher switches from pricing scheme P_{II}^{*} or P_{III}^{*} to P_{I}^{*}. This pattern is similar to that for consumer surplus. On the other hand, social welfare under any particular pricing scheme increases with r. The economic intuition for a discrete decrease in social welfare when the publisher switches to P_{I}^{*} is similar to the intuition for the discrete decrease in consumer surplus. The social welfare increases with r for any particular pricing scheme because the increase of the publisher's profit is higher than the decrease in consumer surplus in the region where consumer surplus decreases with r.

5. Numerical analysis for the generalized model

In this section, we generalize our model by relaxing the assumption of a homogeneous medium mismatch cost parameter (θ) for all consumers to illustrate the robustness of our results. The generalized model becomes analytically intractable and hence, we present numerical analysis here.

In our generalized model, consumers have heterogeneous valuations for the content as well as heterogeneous medium mismatch costs, which are independent of valuations. In the generalized model consumers also have heterogeneous preferences for a medium, where some consumers prefer a digital medium (*digital-savvy*) while others prefer a physical medium (*traditional*). Note that the setup with the heterogeneous medium mismatch cost within each consumer segment is more general than the setup with only a heterogeneous medium mismatch cost for all consumers. Consistent with prior literature (Venkatesh and Chatterjee 2006), we assume that the medium mismatch cost parameter follows the uniform distribution, i.e. $\theta \sim U[0.2, 0.7]$[6]. Under the generalized model, the tradeoffs of the publisher in choosing the optimal content-medium pricing strategies are the same as described in §2. Following the logic described in §2, it is easy to see

[6] We set the support for θ from 0.2 to 0.7 to have better visualization of our results. We have checked that our results hold for the support for θ in any range from 0 to 1.

that the possible optimal content-medium strategies are offering the bundle and the digital, or offering the digital medium only.

Now we describe our numerical strategy. First, for each consumer type (*digital-savvy*, or *traditional*), we generate 5151 synthetic consumers based on uniformly distributed points in the two dimension space (v and θ). A consumer i is represented by a duplet (v_i , θ_i). Then we create 10,201 different scenarios wherein market parameters c_p and r range from 0 to 1 with an increment of 0.01. For each scenario, we determine the optimal bundling strategy by comparing the optimal profit from offering the bundle and digital with that from offering the digital medium only. We adopt a grid search procedure to search the optimal prices that generate maximum profit for each case of offering. Note that in computation of profit, we make sure the IC and IR constraints are satisfied. Once we have the optimal prices, we compute the optimal demand, consumer surplus, and social welfare for each scenario. We present our results of the numerical analysis as follows:

Result 1: *The publisher's optimal bundling strategies are the same as reported in Proposition 3.*

Figure 6 presents the region plot of optimal bundling strategies in the marginal cost and the proportion of *digital-savvy* consumers space. When the marginal cost is high and the proportion of *digital-savvy* consumers is relatively small, offering the digital medium only is the optimal pricing strategy. Hence, under the generalized model set up, our finding that offering only digital medium is optimal under some conditions is valid under a generalized set up.

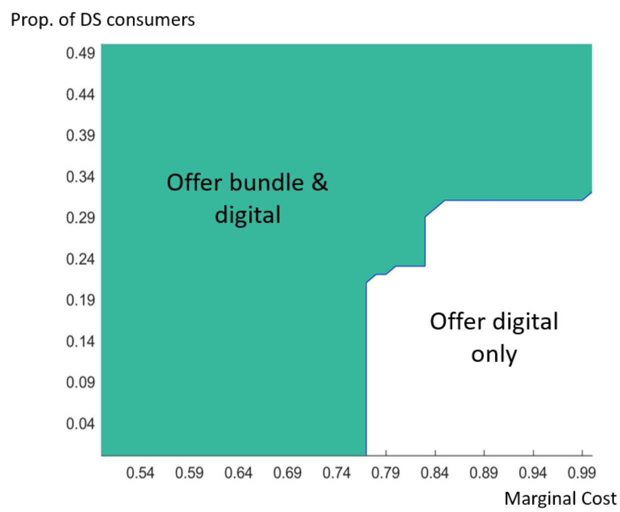

Figure 6: Region plots for optimal bundling strategies with marginal cost and the proportion of *digital-savvy* consumers, $\alpha = 0.1$

Result 2: *The price of the digital medium can decrease as the marginal cost of the bundle increases (consistent with Proposition 4a). The price of the bundle can decrease as the proportion of digital-savvy consumers increases (consistent with Proposition 4b).*

We examine whether the optimal price of the digital can decrease as the marginal cost increases, and whether the optimal price of bundle can decrease as the proportion of *digital-savvy* consumers increases (see Figure 7).

Figure 7(a) shows that as the marginal cost of the bundle increases, while the bundle price increases, the price of the digital decreases. Figure 7(b) shows that while the price of the digital can increase, the price of the bundle may decrease, as the proportion of *digital-savvy* consumers increases.

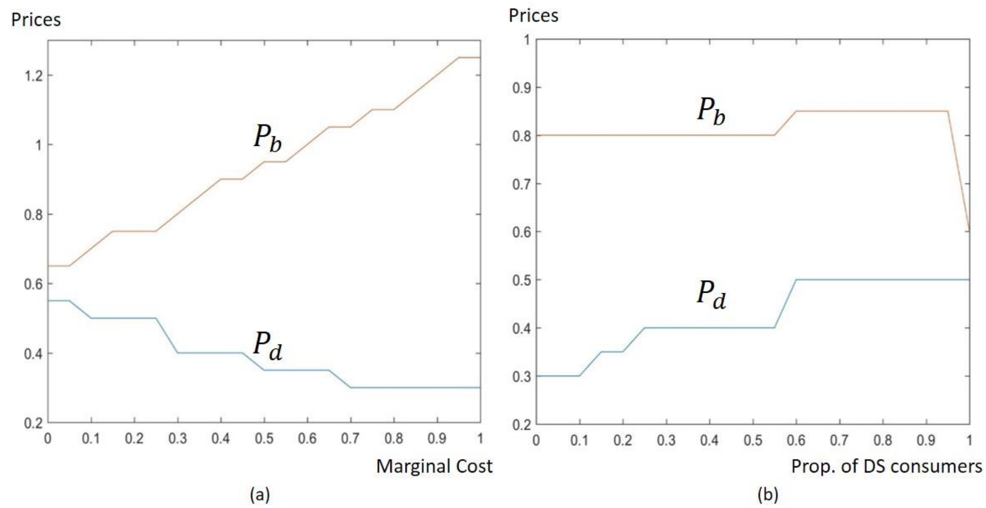

Figure 7: Optimal prices with (a) marginal cost, $r = 0.2,\ \alpha = 0.3$; (b) the proportion of *digital-savvy* consumers, $c_p = 0.4,\ \alpha = 0.1$

Result 3: *The market coverage can decrease as the proportion of digital-savvy consumers increases. This result is consistent with Proposition 5.*

We validate whether the market coverage can decrease as the proportion of digital-savvy consumers increases. Figure 8 shows that as the proportion of *digital-savvy* consumers increases, the market coverage can decrease in the feasible region.

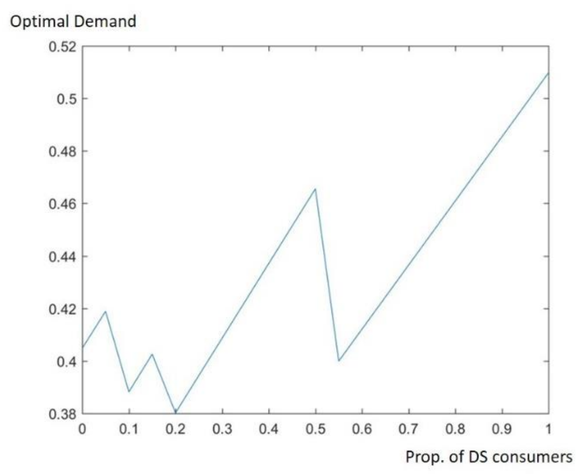

Figure 8: Optimal demand with the proportion of *digital-savvy* consumers, $c_p = 0.7,\ \alpha = 0.1$

Result 4: *Consumer surplus and social welfare can decrease as the proportion of digital-savvy consumers increases. This result is consistent with Proposition 7.*

Lastly, we validate that the consumer surplus and social welfare are decreasing under some conditions, as the proportion of *digital-savvy* consumers increases (see Figure 9).

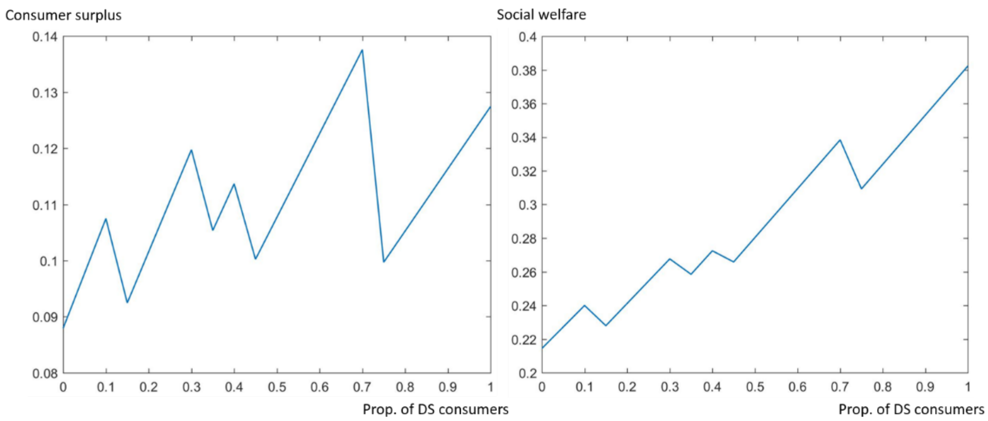

Figure 9: Optimal consumer surplus, social welfare with the proportion of *digital-savvy* consumers, $c_p = 0.8, \alpha = 0.1$

6. Discussion and Conclusion

The impact of digitization on publishers and consumers is an imperative question for content industries in the digital era. Prior analytical studies on information goods pricing over dual-medium access provide limited insights because they either have not abstracted the unique market characteristics and provided analytically tractable pricing solutions, or have not analyzed the impact of key market changes on the publisher's prices, market coverage, profit, and on the consumers' surplus and social welfare. In this paper, we build an analytical model with heterogeneous consumers' valuations and preferences, and an asymmetric cost structure for partially substitutable mediums. Our model lends to closed-form analytical solutions for optimal pricing schemes under various market conditions, and enables us to analyze the impact of changes in market characteristics on a publisher's optimal prices, profit, and market coverage, as well as on consumer surplus and social welfare.

We find that offering the digital-medium only is optimal under some market conditions, while offering a bundle of mediums and the digital medium is optimal under other market conditions. Offering the physical and digital mediums separately or offering a bundle as well as physical and digital mediums is not optimal when the two mediums are partial substitutes. Moreover, we find that offering only the bundle of mediums is not optimal because the physical medium has non-negligible marginal costs. This result of the optimal content-medium strategy is new in literature on content pricing over dual mediums (Venkatesh and Chatterjee 2006, Simon and Kadiyali 2007), and may explain the increasingly popular pricing strategy of offering a bundle and digital medium or digital medium only in content industries (Yang 2012, Hiers 2014). This result contributes to the bundling literature (Schmalensee 1984, McAfee et al 1989, Venkatesh and Kamakura 2003, Armstrong 2013) by identifying conditions under which either partial mixed bundling or a single component strategy is optimal.

Our analysis of optimal prices shows that while the price of bundle increases with the marginal cost, the price of digital can decrease when the marginal cost is relatively large. This surprising result is driven by the asymmetric marginal cost of offering information goods in different mediums and consumers' heterogeneous preferences over the two mediums. Our result recommends that when the marginal cost increases, the publisher can be better off by increasing the bundle price while reducing the digital price, or by shutting down the bundle and offering only the digital medium under some conditions.

Furthermore, we find that when the proportion of *digital-savvy* consumers is sufficiently large, while the optimal price of digital can be higher, the optimal bundle price can decrease. This finding suggests that as more consumers become *digital-savvy*, the publisher may be better off by increasing the price of content in the digital medium and decreasing the bundle price, so the difference between the bundle price and the digital price may become smaller. Pricing practices adopted by publishers such as *The Wall Street Journal* and *The New York Times* over the last decade indicate that the difference between the digital and bundle prices has indeed narrowed. We speculate that other content publishers will also move towards offering content in bundles and the digital with the bundle price being higher than the digital price only by a small amount.

In addition, we show that, under some market conditions, the total market coverage is lower when the proportion of *digital-savvy* consumers is relatively large. This implies that as more consumers prefer content in a digital medium, the price for the digital may increase, leading to a decrease in market coverage. This surprising result has support from the changes in the demand for content publishers since the advent of digitization. While Leggatt (2013) finds an uneven impact on demand of different publishers due to digitization, prestigious publishers like the *Guardian* and *Financial Times* have seen their circulation drop by more than 30% since 2004 (Turvill 2014).

In addition, our finding that offering the digital medium is always optimal for the publisher even when most consumers are *traditional* consumers and the marginal cost of the physical medium is low, informs the debate about adoption of digital formats in content industries (Seelye 2005, Harkaway 2012). We show that the publishers' fear of the negative impact of the adoption of digital technologies on their profits due to the cannibalization of physical medium sales is rather untenable. Data from American Press Institute shows that the percentage of newspaper publishers that adopt digital subscription increased from 3% to 79% between 2001 and 2015 (Williams 2016).

Our analytical model allows us to study the impact of an increasingly larger proportion of consumers adopting digital technologies, or the advent of "*digital natives*" generation. We find that as the proportion of *digital-savvy* consumers increases, consumer surplus and social welfare can decrease, even though the publisher's profit increases. This result informs the ongoing debate in popular press which posits that digital consumption increases social welfare (Booz and Company 2012). Further, this result has policy implications for regulatory bodies like FCC, because the prevalence of digitization may be driven by publisher profit enhancing initiatives and may hurt consumer surplus and society as a whole. This may call for design of an appropriate policy framework that enhances publisher profits together with consumer surplus.

Our analytical framework has some limitations. For analytical tractability, we assume that *traditional* and *digital-savvy* consumers have the same medium mismatch cost parameter. However, through numerical analysis we show that the directionality of the optimal pricing strategy for content-medium and our other key results do not change even when the medium mismatch cost is heterogeneous for consumers.

Our research has several avenues for future extensions. First, when the valuation of content is unknown to consumers, publishers may want to offer a free introductory period for subscription or free samples of digitized content. It would be interesting to examine how the introductory period or free samples signal the quality of content and how that affects a publisher's optimal pricing strategy over dual mediums. Second, in the current study we focus on content pricing over two mediums, physical and digital. It would be interesting to see how a publisher's pricing strategy would change when content is accessed via a physical medium as well as multi-digital mediums such as computer and mobile phone. Third, future work can abstract a market wherein some proportion of the market does not have access to one of the mediums; e.g. some fraction of seniors lack the necessary computer skills to consume content online, or some content provider does not offer physical medium (home delivery) in all geographical areas. It may be interesting to analyze how this market setting would affect a publisher's pricing and bundling policy. In addition, an important source of revenue for medium publishers is advertising. The advertising revenue may vary greatly across mediums, which may in turn affect the publishers' decisions on content-medium pricing. Extending our model to include advertising may be a fruitful avenue for future research.

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APPENDIX A: Profit functions for the rest four market coverage profiles

IR and IC constraints and the profit function for each of the five market coverage profiles (Profiles):

Under Profile 1, where *traditional* consumers buy only the bundle and *digital-savvy* consumers buy the bundle and digital, the publisher's optimization problem is as follows:

$$\max_{p_b,\ p_d} \ \pi_1 = (p_b - c_p)(r(1-\frac{p_b - p_d}{\alpha}) + (1-r)(1-\frac{p_b}{1+\alpha})) + r(\frac{p_b - p_d}{\alpha} - p_d)p_d \tag{1}$$

s.t. $p_b/(1+\alpha) \le p_d/(1-\theta)$, and $p_d \le (p_b - p_d)/\alpha < 1$

Under Profile 2, where *traditional* consumers buy only the bundle and *digital-savvy* consumers buy only the digital, the publisher's optimization problem is as follows:

$$\max_{p_b,\ p_d} \ \pi_2 = (1-r)(1-\frac{p_b}{1+\alpha})(p_b - c_p) + r(1-p_d)p_d \tag{2}$$

s.t. $p_b/(1+\alpha) \le p_d/(1-\theta)$, and $(p_b - p_d)/\alpha > 1$

Under Profile 3, where *traditional* consumers buy both the bundle and the digital, and *digital-savvy* consumers buy only the digital, the publisher's optimization problem is as follows:

$$\max_{p_b, p_d} \ \pi_3 = (1-r)(1-\frac{p_b - p_d}{\alpha+\theta})(p_b - c_p) + ((1-r)(\frac{p_b - p_d}{\alpha+\theta} - \frac{p_d}{1-\theta}) + r(1-p_d))p_d \tag{3}$$

s.t. $p_d/(1-\theta) \le (p_b - p_d)/(\alpha+\theta) < 1$, and $(p_b - p_d)/\alpha > 1$

Under Profile 4, where both type of consumers buy the bundle and the digital, the publisher's optimization problem is as follows

$$\max_{p_b, p_d} \ \pi_4 = ((1-r)(1-\frac{p_b - p_d}{\alpha+\theta}) + r(1-\frac{p_b - p_d}{\alpha}))(p_b - c_p) + ((1-r)(\frac{p_b - p_d}{\alpha+\theta} - \frac{p_d}{1-\theta}) + r(\frac{p_b - p_d}{\alpha} - p_d))p_d \tag{4}$$

s.t. $p_d/(1-\theta) \le (p_b - p_d)/(\alpha+\theta) < 1$, and $(p_b - p_d)/\alpha > 1$

Under Profile 5, where both type of consumers buy the digital medium only, the publisher's optimization problem is as follows:

$$\max_{p_b, p_d} \ \pi_5 = (1-r)(1-p_b/(1-\theta))(p_b - c_p) + r(1-p_d)p_d \tag{5}$$

s.t. $p_d / (1-\theta) \le (p_b - p_d)/(\alpha+\theta) < 1$, and $(p_b - p_d)/\alpha > 1$

APPENDIX B: Proofs of Lemmas and Propositions

LEMMA 1: *(a) Offering content only in the physical medium is dominated by offering the content in a bundle. (b) The pricing strategy of offering content in a bundle and in the digital medium dominates the pricing strategy of offering content only in a bundle.*

Proof of Lemma 1(a): We know that both types of consumers' willingness to pay (WTP) for the bundle is $v(1+\alpha)$, *traditional* consumer's WTP for the physical medium is v, and *digital-savvy* consumer's WTP for the physical medium is $v(1-\theta)$. Also, we know $v(1+\alpha) > v$, and $v(1+\alpha) > v(1-\theta)$. Moreover, the bundle and the physical mediums have the same marginal cost. Since for all consumers, the willingness to pay for the bundle is always higher than that for the physical, the bundle is a strictly better offering for publisher than the physical medium. Therefore, the publisher is never better off by offering the physical medium along with the bundle. The publisher is strictly better off by offering the bundle than offering the physical medium. ■

Proof of Lemma 1(b): Suppose the publisher offers only the bundle at price p_b. The publisher's profit is $\pi_1 = (p_b - c_p)(1 - \frac{p_b}{1+\alpha})$, we get $p_b^* = (1+\alpha+c_p)/2$, $\pi_1^* = (1+\alpha-c_p)^2/4(1+\alpha)$. Now, suppose the publisher also offers the digital medium along with the bundle at a lower price than $1/2$, then the publisher's profit is $\pi_2 = (1-r)(p_b - c_p)(1 - p_b/(1+\alpha)) + r((p_b - c_p)(1-(p_b - p_d)/\alpha) + ((p_b - p_d)/\alpha - p_d)p_d)$, we get optimal profit $\pi_2^* = (1+\alpha+c_p(-2+(r+\alpha)c_p/\alpha(1+\alpha)))/4$. Since $\pi_2^* > \pi_1^*$, it implies that by offering the digital medium along with the bundle, the publisher gets strictly higher profit. Therefore, the pricing strategy of offering content in a bundle, and in the digital medium dominates pricing strategy of offering content only in a bundle. ■

PROPOSITION 1: *Any Profile in which the digital-savvy consumers only buy the bundle is suboptimal.*

Proof of Proposition 1: Suppose some Profile in which *digital-savvy* consumers only buy the bundle is optimal. *Traditional* consumers will also only buy the bundle because they have the same WTP for the bundle but lower WTP for the digital than *digital-savvy* consumers. So consumers in both segments will only buy the bundle. We have shown in Lemma 1(b) that offering the bundle only is suboptimal. Therefore, any Profile in which *digital-savvy* consumers buy only the bundle is never optimal. ■

LEMMA 2: *When the publisher offers content only in the digital medium, the optimal price is* $p_d^* = (1-\theta)/(2-2r\theta)$, *and the publisher's optimal profit is* $\pi_d^* = (1-\theta)/(4-4r\theta)$.

Proof of Lemma 2: When the publisher offers only the digital medium at price p_d, the publisher's profit is $\pi_d = rp_d(1-p_d)+(1-r)p_d(1-p_d/(1-\theta))$, take the first order condition (F.O.C) with p_d, we get $p_d^* = (1-\theta)/2(1-r\theta)$, plug p_d^* into the profit function, we get $\pi_d^* = (1-\theta)/4(1-r\theta)$. ■

LEMMA 3: *When the publisher offers content in a bundle, and in the digital medium (partial mixed bundling) or in the digital medium only (single component), for any feasible parameter space, the optimal pricing scheme is from set* $\left\{\left\{P_I^*\right\},\ \left\{P_{II}^*\right\},\ \left\{P_{III}^*\right\}\right\}$, *where* $P_I^* = \left\{p_I^{d*} = 1/2,\ p_I^{b*} = (1+c_p+\alpha)/2\right\}$,

$$P_{II}^* = \left\{p_{II}^{d*} = \frac{1-\theta}{2-2r\theta},\ p_{II}^{b*} = \frac{\theta+\alpha+c_p}{2} + \frac{1-\theta}{2-2r\theta}\right\}, \textit{ and } \ P_{III}^* = \left\{p_{III}^{d*} = \frac{1-\theta}{2-2r\theta}\right\}.$$

Proof of Lemma 3: From the F.O.C of the profit function π_{bd}^1 with p_d and p_b in (1) (see Appendix A), we get the optimal prices for the digital and the bundle as $p_I^{d*} = 1/2$, and $p_I^{b*} = (1+c_p+\alpha)/2$. Similarly, from the F.O.C. of the profit function π_2 in (2), we get the same optimal prices p_I^{d*} and p_I^{b*}.

From the F.O.C. of the profit function π_3 in (3), we get the optimal prices for the digital medium and the bundle as $p_{II}^{d*} = (1-\theta)/(2-2r\theta),\ p_{II}^{b*} = (\theta+\alpha+c_p+(1-\theta)/(1-r\theta))/2$. Similarly, from the F.O.C. of the profit function π_4 in (4), we get $\pi_4 = (\frac{(1+\alpha)(r\theta - r\theta^2 + \alpha - \alpha r\theta)}{(1-r\theta)(\alpha + r\theta)} + c_p)/2$. From the F.O.C. of the profit function π_5 in (5), we get $p_{III}^{d*} = (1-\theta)/(2-2r\theta)$.

For Profile 1 to be feasible, we have the constraints: $\frac{p_b}{1+\alpha} \le \frac{p_d}{1-\theta}$, and $p_d \le \frac{p_b - p_d}{\alpha} < 1$ (see equation (1)).

Plugging the optimal prices into the constraints, we get the feasible conditions (regions) for Profile 1 as:

$$\begin{cases} c_p < \frac{\theta}{1-\theta}(1+\alpha), \text{ when } \theta < 1/3 \text{ and } \alpha > \theta/(1-2\theta) \\ c_p < \alpha, \text{ when } \theta < 1/3 \text{ and } \alpha < \theta/(1-2\theta) \text{ or } \theta > 1/3 \end{cases}.$$

Using a similar approach, we get the feasible conditions for Profile 2 as: $c_p \in (\alpha, \frac{\theta}{1-\theta}(1+\alpha))$, where $\theta > 1/3$, or $\theta > 1/3$ and $\alpha < \theta/(1-2\theta)$.

The feasible conditions for Profile 3 are: $c_p \in (\max\{\alpha - \theta, \frac{r\theta}{1-r\theta}(\alpha+\theta)\}, \alpha+\theta)$. Specifically, the feasible conditions for Profile 3 can be expressed as: $\begin{cases} \frac{r\theta}{1-r\theta}(\alpha+\theta) < c_p < \alpha+\theta, \text{ when } r\theta < 1/2 \text{ and } \alpha < \theta/(1-2r\theta) \\ \alpha - \theta < c_p < \alpha + \theta, \text{ when } 2r\theta + \theta < 1 \text{ and } \alpha > \theta/(1-2r\theta) \end{cases}$.

The feasible conditions for Profile 4 are: $c_p \in (\frac{r\theta(\alpha+\theta)(1+\alpha)}{(1-r\theta)(\alpha+r\theta)}, \frac{\alpha}{\alpha+r\theta}(\alpha - \theta + 2r\theta))$ where $r < (1-\theta)/2\theta$ and $\alpha > \theta/(1-2r\theta)$.

We rule out Profile 4 because π_{bd}^{4*} is dominated either by π_{bd}^{1*} or by π_{bd}^{3*} in the region in which Profile 4 is feasible. Note that the feasible region of Profile 1 and that of Profile 4 overlap. The overlap region is: (a) $\frac{r\theta(\alpha+\theta)(1+\alpha)}{(1-r\theta)(\alpha+r\theta)} < c_p < \frac{\alpha}{\alpha+r\theta}(\alpha-\theta+2r\theta)$ when (i) $\theta/(1-2r\theta) < \alpha < \theta/(1-2\theta)$ and $\theta < 1/3$, or (ii) $r < (1-\theta)/2\theta$ and $\alpha > \theta/(1-2r\theta)$ and $\theta > 1/3$; or (b) $\frac{r\theta(\alpha+\theta)(1+\alpha)}{(1-r\theta)(\alpha+r\theta)} < c_p < \theta(1+\alpha)/(1-\theta)$ when $\alpha > \theta/(1-2\theta)$ and $\theta < 1/3$. Comparing the optimal profit for Profile 1 and that for Profile 4, we get $\pi_{bd}^{1*} > \pi_{bd}^{4*}$ in the overlap region, i.e. condition (a) or (b) holds.

The feasible region of Profile 3 and that of Profile 4 also overlap. The overlap region is: $\min(\alpha-\theta,\ \frac{r\theta(\alpha+\theta)(1+\alpha)}{(1-r\theta)(\alpha+r\theta)}) < c_p < \frac{\alpha}{\alpha+r\theta}(\alpha-\theta+2r\theta)$ when $r<(1-\theta)/2\theta$ and $\alpha>\theta/(1-2r\theta)$. Furthermore, the feasible region of Profile 3 and that of Profile 4 have no overlap when $r<(1-\theta)/2\theta$ and $\alpha<\theta/(1-2r\theta)$. We get $\pi_{bd}^{3*}>\pi_{bd}^{4*}$ in the overlap region.

Therefore, given the results above, we have three possible optimal pricing schemes as in Lemma 3. ■

PROPOSITION 2: *If the proportion of digital-savvy consumers (r) is such that $\theta>1/(1+r)$, then the publisher adopts the pricing scheme P_I^* for the entire feasible parameter space.*

Proof of Proposition 2: We know $\theta>r\theta$, since $r<1$. If $r\theta+\theta>1$, then it implies that $\theta>1/2$. From the assumption of the model, we have $\alpha+\theta<1$. Hence, under the condition that $r\theta+\theta>1$, the condition for Profile 3 to be feasible is violated. Hence, Profile 3 is ruled out. Moreover, it is easy to see that when $r\theta+\theta>1$, $\pi_{bd}^{4*}-\pi_{bd}^{5*}>0$. So Profile 4 dominates Profile 5 when $r\theta+\theta>1$. We know from the previous proof that Profile 4 is dominated by either Profiles 1, 2 or 3, and Profile 3 has been ruled out. Therefore, when $\theta>1/(1+r)$, only Profile 1 and 2 are optimal, which implies that the optimal pricing scheme is P_I^*.

LEMMA 4: *The optimal pricing scheme regions in the parameter space are characterized by (i) three thresholds for proportion of digital-savvy consumers (r), $r_0=\frac{(1-\theta)\alpha^2}{\theta(\alpha^2+\theta^2+\alpha\theta+\alpha\theta^2)}$, $r_1=\frac{1}{\theta}-\frac{1+\alpha}{\alpha+\theta(2-\theta)}$, and $r_2=\frac{1}{\theta}-\frac{1+\alpha}{1+\alpha(1-\alpha)}$ where $0<r_0<r_1<r_2$; (ii) three thresholds for the sub-additive parameter (α), $\alpha_1=(1-2\theta+\sqrt{(1-2\theta)(5-6\theta)})/2(1-\theta)$, $\alpha_2=\theta(3\theta-\theta^2-1)/(1-2\theta)$, and $\alpha_3=\theta/(1-2\theta)$; and (iii) five thresholds for marginal cost (c_p), that is, α, $\alpha+\theta$,*

$$c_{p1}=\frac{\sqrt{r\alpha(1-r)(1+\alpha)(\alpha+\theta)(1-r\theta)\left(\alpha\theta^2(1-2r-\theta)+\alpha^2(1-\theta-r\theta)-r\theta^3\right)}-r\alpha(1+\alpha)(\alpha+\theta)(1-r\theta)}{(1-r\theta)\left(\alpha(1-2r-r\alpha)-\theta(r-\alpha)\right)},$$

$c_{p2}=\theta\sqrt{r(1+\alpha)(\alpha+\theta)/(1-\theta)(1-r\theta)}$, *and* $c_{p3}=1+\alpha-\sqrt{(1+\alpha)(1-\theta)(1-\theta-r\theta)}/(1-r\theta)$.

PROPOSITION 3a: *If (I)* $0.5 < \theta \le 1/(1+r)$ *or (II)* $0.445 < \theta \le 0.5$ *and* $\alpha > \alpha_1$*, then in the feasible space of the proportion of digital-savvy consumers (* r *) and marginal cost (* c_p *), the publisher's optimal content-medium pricing strategies are characterized in the following table:*

<table>
<tr><th>r \ c_p</th><th>$(0,\ c_{p1}]$</th><th>$(c_{p1},\ c_{p2}]$</th><th>$(c_{p2},\ \alpha+\theta]$</th><th>$(\alpha+\theta, c_{p3}]$</th><th>$(c_{p3}, 1]$</th></tr>
<tr><td>$(0,\ r_0]$</td><td>p_I^*</td><td colspan="2">p_{II}^*</td><td colspan="2">p_{III}^*</td></tr>
<tr><td>$(r_0,\ r_1]$</td><td colspan="2">p_I^*</td><td>p_{II}^*</td><td colspan="2">p_{III}^*</td></tr>
<tr><td>$(r_1,\ r_2]$</td><td colspan="4">p_I^*</td><td>p_{III}^*</td></tr>
<tr><td>$(r_2,\ 1]$</td><td colspan="5">p_I^*</td></tr>
</table>

PROPOSITION 3b: *If* $0.445 < \theta \le 0.5$ *and* $\alpha < \alpha_1$*, then (a) when* $r \in (0, r_1]$*, the publisher's optimal pricing strategies remain the same as presented in the table in Proposition 4a; and (b) when* $r \in (r_1, 1]$*, the publisher adopts* p_I^* *if* $c_p \in (0, c_{p3}]$*, and adopts* p_{III}^* *if* $c_p \in (c_{p3}, 1]$.

Proof for Lemma 4, Propositions 3a and 3b, and results in §3.2.2:

Note that the condition $r\theta + \theta < 1$ is common for Propositions 3. We will use the publisher's profit functions and conditions for each of the five Profiles discussed in §2.2 and in Appendix A.

The publisher's optimal profit under Profile 1 is $\pi_1^* = (1+\alpha - c_p(2-(rc_p + \alpha c_p)/(\alpha+\alpha^2)))/4$. We obtain it by plugging the optimal prices from P_I^* into the profit function (1). Similarly, the optimal profit of the publisher under Profile 2 is: $\pi_2^* = (r + (1-r)(1+\alpha - c_p)^2/(1+\alpha))/4$. The optimal profit of the publisher under Profile 3 is: $\pi_3^* = (\frac{1 - r\theta(1+\theta - r\theta) + \alpha(1-r)(1-r\theta)}{1-r\theta} - \frac{(1-r)(2\alpha + 2\theta - c_p)c_p}{\alpha+\theta})/4$. The optimal profit of the publisher under Profile 5 is: $\pi_5^* = (1-\theta)/4(1-r\theta)$.

We compare the feasible conditions for Profile 1, 2, 3, and 5 (pair-wise comparison) to determine if there is any overlap region between feasible regions of any of the two Profiles, and then compare the optimal profits to determine the optimal prices in the overlap region. In this way, we get the thresholds in Lemma 4 and optimal pricing schemes under different parameter space.

Solving $\pi_4^* - \pi_5^* = 0$ for c_p, we get a positive intercept, and two positive roots. One root is greater than 1, and we denote the other root as c_{p3}, ($c_{p3} = 1+\alpha - \sqrt{(1+\alpha)(1-\theta)(1-\theta-r\theta)} / (1-r\theta)$, see Lemma 4). Compare c_{p3} with 1, we get that if $r > r_2$ ($r_2 = \dfrac{1}{\theta} - \dfrac{1+\alpha}{1+\alpha(1-\alpha)}$, see Lemma 4), then $c_{p3} > 1$, therefore, $\pi_4^* > \pi_5^*$. When $r < r_2$, we get if $c_p < c_{p3}$, then $\pi_4^* > \pi_5^*$; if $c_p > c_{p3}$, then $\pi_4^* < \pi_5^*$.

Also, whenever the conditions for Profile 3 to be feasible are met, we get $\pi_3^* > \pi_5^*$. In addition, solving $\pi_4^* - \pi_3^* = 0$ for c_p, we get that if $c_p < c_{p2}$ ($c_{p2} = \theta\sqrt{r(1+\alpha)(\alpha+\theta) / (1-\theta)(1-r\theta)}$, see Lemma 4), then $\pi_4^* > \pi_3^*$; and if $c_p > c_{p2}$, then $\pi_4^* < \pi_3^*$.

Note that the upper bound of the condition for Profile 3 to be feasible is $c_p < \alpha + \theta$. Comparing c_{p2} with $\alpha + \theta$, we get (i) $c_{p2} > \alpha + \theta$, if $r > r_1$ ($r_1 = \dfrac{1}{\theta} - \dfrac{1+\alpha}{\alpha + \theta(2-\theta)}$, see Lemma 4); and (ii) $c_{p2} < \alpha + \theta$, if $r < r_1$. Hence, we have that if $r < r_1$ and $c_p < c_{p2}$, or $r > r_1$, then $\pi_4^* > \pi_3^*$; and if $r < r_1$ and $c_p > c_{p2}$, then $\pi_4^* < \pi_3^*$.

Note that the upper bound of the condition for Profile 1 to be feasible is: $c_p < \alpha$. The lower bound of the condition for Profile 4 to be feasible is: $c_p > \alpha$. Therefore, we need not compare π_1^* with π_2^*.

Now we focus on π_1^* and π_3^*. We have that if $r < r_0$ ($r_0 = \dfrac{(1-\theta)\alpha^2}{\theta(\alpha^2+\theta^2+\alpha\theta+\alpha\theta^2)}$, see Lemma 4), then $r\theta(\alpha+\theta)/(1-r) < \alpha$, which implies that the feasible conditions for Profile 1 and for Profile 3 overlap, therefore, we must compare π_1^* with π_3^*. On the other hand, if $r > r_0$, then feasible conditions for Profile 1 and for Profile 3 do not overlap, hence, we do not need to compare π_1^* with π_3^*. Solving $\pi_1^* - \pi_3^* = 0$ for c_p, we get a positive intercept and two positive roots. Moreover, if $r < r_0$, then $\pi_1^* - \pi_3^* < 0$ when $c_p = \alpha$. Therefore, we know one root is smaller than α and the other root is greater than α. Therefore, if $r < r_0$ and $c_p < c_{p1}$, then $\pi_1^* > \pi_3^*$; and if $r < r_0$ and $c_p > c_{p1}$, then $\pi_1^* < \pi_3^*$.

It is easy to see that the thresholds for r (r_0, r_1, and r_2) are such that $r_0 < r_1 < r_2$. In addition, we have that $r_2 < 1$, if (i) $0.5 < \theta \le 1/(1+r)$, or (ii) $0.445 < \theta < 0.5$ and $\alpha > \alpha_1$. Further, we have $r_1 < 1$ but $1 < r_2$, if (i) $0.445 < \theta < 0.5$ and $\alpha < \alpha_1$, or (ii) $1.5 - \sqrt{1.25} < \theta < 0.445$ and $\alpha < \alpha_2$. Furthermore, $r_0 < 1$ but $1 < r_1 < r_2$, if (i) $1.5 - \sqrt{1.25} < \theta < 0.445$ and $\alpha > \alpha_2$, or (ii) $0.333 < \theta < 1.5 - \sqrt{1.25}$, or (iii) $\theta \le 0.333$ and $\alpha \le \alpha_3$.

Organizing the results to identify the regions, we have Lemma 4, Propositions 3, and results in §3.2.2. ■

PROPOSITION 4a: *While the optimal price of a bundle increases with the marginal cost, the optimal price of digital can decrease when the marginal cost is sufficiently large.*

Proof for Proposition 4a: Since $\partial p_I^{b*} / \partial c_p = \partial p_{II}^{b*} / \partial c_p = 1/2 > 0$, hence both p_I^{b*} and p_{II}^{b*} increase with c_p at a constant rate, i.e. $1/2$. And it is easy to see p_I^{d*}, p_{II}^{d*}, and p_{III}^{d*} are independent of c_p, and we know $p_I^{d*} > p_{II}^{d*} = p_{III}^{d*}$. Under the market conditions (i.e. the parameter values for Figure 3a (i) and (ii)) where the publisher's optimal pricing strategy switches from P_I^* to P_{II}^*, or from P_I^* to P_{III}^*, the digital price will have a discrete drop, and the bundle price will incur a discrete increase at the threshold point when the publisher switches pricing schemes. ■

PROPOSITION 4b: *When the proportion of digital-savvy consumers is sufficiently large, while the price of digital is higher under P_I^* compared to that under P_{II}^*, the price of the bundle can be lower under P_I^* compared to that under P_{II}^*.*

Proof for Proposition 4b: Taking derivative of the optimal prices with the proportion of *digital-savvy* consumers (r), we got $\partial p_{II}^{d*} / \partial r = \partial p_{III}^{d*} / \partial r = \theta(1-\theta)/2(1-r\theta)^2 > 0$, and $\partial p_{II}^{b*} / \partial r = \theta(1-\theta)/2(1-r\theta)^2 > 0$, and it is easy to see p_I^{d*} and p_I^{b*} are independent of r, and we know $p_I^{d*} > p_{II}^{d*}$ and $p_I^{b*} < p_{II}^{b*}$. Under the market conditions (i.e. the parameter values for Figure 3b) where the publisher's optimal pricing strategy switches from P_{II}^* or P_I^*, the digital price will have a discrete increase, and the bundle price will incur a discrete drop at the threshold point when the publisher switches pricing schemes. ■

PROPOSITION 5: *(i) Under pricing schemes* P_{II}^* *and* P_{III}^**, the market coverage is* $1/2$ *(ii) under pricing scheme* P_I^**, and the market coverage is lower than* $1/2$ *but increases with the proportion of digital-savvy consumers.*

Proof for Proposition 5: For market coverage under optimal pricing scheme P_{II}^*, we get market coverage $M_{II} = (1-r)(1-\frac{p_{II}^b - p_{II}^d}{\alpha+\theta}) + r(1-p_{II}^d) + (1-r)(\frac{p_{II}^b - p_{II}^d}{\alpha+\theta} - \frac{p_{II}^d}{1-\theta})$. Plugging in optimal prices p_{II}^{b*} and p_{II}^{d*} from Lemma 3, we get $M_{II}^* = 1/2$. Under the optimal pricing scheme P_{III}^*, we get the market coverage $M_{III} = r(1-p_{III}^d) + (1-r)(1-\frac{p_{III}^d}{1-\theta})$. Plugging in p_{III}^{d*} into the function, we get $M_{III}^* = 1/2$.

For pricing scheme P_I^*, the total market coverage is same for Profile 1 and 2. So for market coverage under pricing scheme P_I^*, we get $M_I = r(1-p_I^d) + (1-r)(1-\frac{p_I^b}{1+\alpha})$. Plugging in optimal prices p_I^{d*} and p_I^{b*} from Lemma 3, we get $M_I^* = (1+\alpha-(1-r)c_p)/(2+2\alpha) < 1/2$. It is easy to see M_I^* increases with r. ■

PROPOSITION 6: *The publisher's optimal profit increases with the proportion of digital-savvy consumers (* r *) under all pricing schemes.*

Proof for Proposition 6: The optimal profit for pricing scheme P_I^* is:

$\pi_1^* = (1+\alpha-c_p(2-(rc_p+\alpha c_p)/(\alpha+\alpha^2)))/4$, $\pi_2^* = (r+(1-r)(1+\alpha-c_p)^2/(1+\alpha))/4$. The optimal profit for pricing scheme P_{II}^* is $\pi_3^* = (\frac{1-r\theta(1+\theta-r\theta)+\alpha(1-r)(1-r\theta)}{1-r\theta} - \frac{(1-r)(2\alpha+2\theta-c_p)c_p}{\alpha+\theta})/4$. The optimal profit for pricing scheme P_{III}^* is $\pi_5^* = (1-\theta)/4(1-r\theta)$.

It is easy to see $\partial \pi_1^* / \partial r > 0$, $\partial \pi_2^* / \partial r > 0$, and $\partial \pi_5^* / \partial r > 0$. Solving $\pi_3^* = 0$ for c_p, we get a negative intercept and two positive roots : $(\alpha+\theta)(1+\sqrt{(\theta-\theta^2)/(\alpha+\theta)}/(1-r\theta))$, and $(\alpha+\theta)(1-\sqrt{(\theta-\theta^2)/(\alpha+\theta)}/(1-r\theta))$). Given the conditions under which market profile 3 is feasible, the feasible c_p is between the two roots, so we get $\partial \pi_3^* / \partial r > 0$. ■

PROPOSITION 7: *Consumer surplus increases with the proportion of digital-savvy consumers (r) when the pricing scheme P_I^* is optimal, and consumer surplus first increases and then decreases with r when pricing scheme P_{II}^* or P_{III}^* is optimal. Social welfare can decrease when r is sufficiently large.*

Proof for Proposition 7: Under pricing scheme P_I^*, we get $CS_I^* = (1+\alpha+c_p(-2+(r+\alpha)c_p/\alpha(1+\alpha)))/8$, and $SW_I^* = 3(1+\alpha+c_p(-2+(r+\alpha)c_p/\alpha(1+\alpha)))/8$. We get $\partial CS_I^* / \partial r > 0$, $\partial SW_I^* / \partial r > 0$.

Under pricing scheme P_{II}^*, $CS_{II}^* = (\frac{1-r\theta(1-3\theta+3r\theta)+\alpha(1-r)(1-r\theta)}{1-r\theta} - \frac{(1-r)(2\alpha+2\theta-c_p)c_p}{\alpha+\theta})/8$. We have $\partial CS_{II}^* / \partial r = ((1-r\theta)^2(2\alpha+2\theta-c_p)c_p - (\alpha+\theta)(\alpha(1-r\theta)^2 - 3\theta^2(1-2r+r^2\theta)))/8(\alpha+\theta)(1-r\theta)^2$. By solving $\partial CS_{II}^* / \partial r = 0$ for c_p, we get a positive intercept and two positive roots, i.e. $\alpha+\theta-\sqrt{\theta(\alpha+\theta)(1+3\theta-8r\theta+4r^2\theta^2)}/(1-r\theta)$, and $\alpha+\theta+\sqrt{\theta(\alpha+\theta)(1+3\theta-8r\theta+4r^2\theta^2)}/(1-r\theta)$. We know that c_p is less than the larger root (the second one), and can be either greater or less than the smaller root (the first one), which decreases as r increases. Hence, we get $\partial CS_{II}^* / \partial r > 0$ when r is small ($\alpha+\theta-\sqrt{\theta(\alpha+\theta)(1+3\theta-8r\theta+4r^2\theta^2)}/(1-r\theta) > c_p$); $\partial CS_{II}^* / \partial r < 0$ when r is relatively large ($\alpha+\theta-\sqrt{\theta(\alpha+\theta)(1+3\theta-8r\theta+4r^2\theta^2)}/(1-r\theta) < c_p$).

$SW_{II}^* = (\frac{3-r\theta(3-\theta+r\theta)+3(1-r)\alpha(1-r\theta)}{1-r\theta} - 6(1-r)c_p + \frac{3(1-r)c_p^2}{\alpha+\theta})/8$. Taking F.O.C with r, we have

$$\partial SW_{II}^* / \partial r = \frac{-3\alpha(-1+r\theta)^2 + \theta^2(1+r(-2+r\theta))}{8(-1+r\theta)^2} + \frac{3c_p}{4} - \frac{3c_p^2}{8(\alpha+\theta)}.$$

Solving $\partial SW_{II}^{*} / \partial r = 0$ for c_p, we get a negative intercept and two positive roots, the smaller root is $(\alpha+\theta)(1-\sqrt{\theta(3+\theta-8r\theta+4r^2\theta^2)/3(\alpha+\theta)}/(1-r\theta))$. For the feasible region of c_p under optimal pricing scheme P_{II}^{*}, we get $c_P > (\alpha+\theta)(1-\sqrt{\theta(3+\theta-8r\theta+4r^2\theta^2)/3(\alpha+\theta)}/(1-r\theta))$. Hence, we get $\partial SW_{II}^{*} / \partial r > 0$.

Under pricing scheme P_{III}^{*}, we get $CS_{III}^{*} = (1-\theta+4r(1-r)\theta^2)/8(1-r\theta)$, and correspondingly, $SW_{III}^{*} = (3+\theta(-3+4r\theta-4r^2\theta))/8(1-r\theta)$. Further, we have $\partial CS_{III}^{*} / \partial r = \theta(1+3\theta-8\theta r+4r^2\theta^2)/8(1-r\theta)^2$. Solving $\partial CS_{III}^{*} / \partial r = 0$ as a function of r, we got a positive intercept and two positive roots: $(2-\sqrt{3(1-\theta)})/2\theta$, and $(2+\sqrt{3(1-\theta)})/2\theta$. Because $(2+\sqrt{3(1-\theta)})/2\theta > 1$, $r < (2+\sqrt{3(1-\theta)})/2\theta$, r can either be greater or less than $(2-\sqrt{3(1-\theta)})/2\theta$. Therefore, $\partial CS_{III}^{*} / \partial r < 0$ if $r < (2-\sqrt{3(1-\theta)})/2\theta$; and $\partial CS_{III}^{*} / \partial r > 0$ if $r > (2-\sqrt{3(1-\theta)})/2\theta$. In addition, $\partial SW_{III}^{*} / \partial r = \theta(3+\theta(1+4r(-2+r\theta)))/8(-1+r\theta)^2$. Solving $\partial SW_{III}^{*} / \partial r = 0$ as a function of r, we get two positive roots: $(2-\sqrt{(1-\theta)})/2\theta$, and $(2+\sqrt{(1-\theta)})/2\theta$. Since $r\theta < 1/2$ under P_{III}^{*}, we get $r < (2-\sqrt{(1-\theta)})/2\theta$, and since the intercept is positive, i.e. $\theta(3+\theta)/8 > 0$, we have $\partial SW_{III}^{*} / \partial r > 0$. Under some market conditions such as the parameter values for Figure 5(b), when the publisher switches from P_{II}^{*} or P_{III}^{*} to P_{I}^{*} at the threshold values of r (i.e., 0.26, 0.67), we have $SW_{I}^{*} < SW_{II}^{*}$, and $SW_{I}^{*} < SW_{III}^{*}$. ■

CHAPTER 3

An Interaction Analysis of Social Media and Traditional Platform Effects in the Consumer Purchasing Funnel[7]

Abstract

We advance an empirical strategy aiming at measuring synergistic effects of online platforms for targeted advertising along the consumer-purchasing funnel. Gauging meaningful interaction effects between activities on different platforms and within different parts of the purchasing funnel is very challenging. This is due to (a) the presence of the potential "activity biases" (Lewis et al., 2011) where the most active users end up being targeted more frequently on different platforms and (b) "rare outcomes" indicating that ultimate conversion rates are negligible. We tackle these issues by a combination of tools in the epidemiology and machine learning literature comprising (a) case-control design to match retrospectively users showing a similar level of activity and (b) post-regularized choice models, proved to be effective even in the presence of rare outcomes. Our empirical analysis finds that segmenting customers based on the similarity of their browsing activities mitigates "path-to-purchase" heterogeneity and offers more accurate associational measures related to platform effects. Second, targeting across platforms is positively associated with ultimate conversion for consumers at the lower funnel, but there is no measurable synergistic effect for the upper funnel consumers. Also, we find that the main effect of social media is positively related to the ultimate conversions for users in the early stage but has no incremental

[7] We acknowledge the support from the Marketing Science Institute for this project.

impact when consumers move down to the lower funnel. These point to the presence of an intricate pattern of complementarities between social media and traditional platforms along the purchasing funnel that, at the best of our knowledge, we are the first able to investigate.

1. Introduction

The widespread adoption of the Internet and digital technologies has profoundly changed the advertising industry. Within digital advertising spending, display ad spending is projected to grow 47.5% by the end of 2016 and will surpass search ad spending in the US for the first time (eMarketer 2016). Advertisers invest heavily on display ads that run on various general sites (traditional platform) as well as social media. Social media is an increasingly popular platform with more than $23 billion spending on advertising in 2015 alone (LePage 2015).

Our research attempts at answering whether and how to display ads on social media can be useful in converting potential consumers, relative to that on the traditional platform (Portal website, major media, lifestyle site, etc.). People naturally connect on social media platform to stay up to date with their social life, e.g. interacting with their families and friends. Thus they may have little interest in finding advertising useful (Sibley 2015). On the other hand, social media can provide advertisers detailed user profile information including demographics, stated preferences, and interests. The micro-level information becomes a great asset for advertisers allowing them to design and conduct more efficient targeting strategies, leading to higher rates of ultimate conversions (Ganguly 2015). Besides, consumers may be more likely to purchase based on social referrals, and may consult with their friends before making purchasing decisions (Ahmed 2015).

From advertisers' perspective, it is important to understand whether and how the effects of targeted advertising on social media differ from that on the traditional media. Moreover, it is not clear whether social media complements with or substitutes to the traditional platform. It may be likely that exposures across the two platforms may have synergistic effect that is greater than the sum effect of exposures on each platform; Or, it is also conceivable that the two platforms may be likely to substitute to each other, so that consumers exposed on both may end up wearing out their interest in the product faster than those exposed on just one platform. In more formal statistical terms, the interaction effect of the two platforms may be different for various types of consumers.

Moreover, both psychological and rational theories of economic behavior postulate the presence of tradeoffs when information, in the form of digital ads, is provided in a repeated way. Therefore, we posit that the presence of information across different platforms may either (a) complement the stock of information already accumulated by the user or (b) cannibalize it. Therefore, this research is concerned with understanding the effectiveness of targeted advertising on social media, and the complementarity or substitutability of platforms for targeted advertising, along the so-called consumer purchasing funnel (CPF). Specifically, our research questions are in the following:

1. What is the effectiveness of targeted advertising on social media, relative to that on traditional platform, along the consumer-purchasing funnel?

2. Does targeting on social media complement with or substitute to targeting on the traditional platform in impacting the ultimate conversions for consumers at different purchasing stages?

3. Which targeting strategy is the most effective for consumers at a certain purchasing funnel level?

The consumer purchasing funnel is thought to consist of two distinct and sequential phases: the upper funnel where users may have some engagement with the firm showing some awareness of their product, and the lower funnel where consumers have more frequent interaction with the company showing more interest beyond the general awareness. Figure 1 exemplifies these two sequential phases showing the presence of different “touchpoints” derived from consumer browsing behavior, as a result of targeted ads; these may happen on either traditional or social media platform.

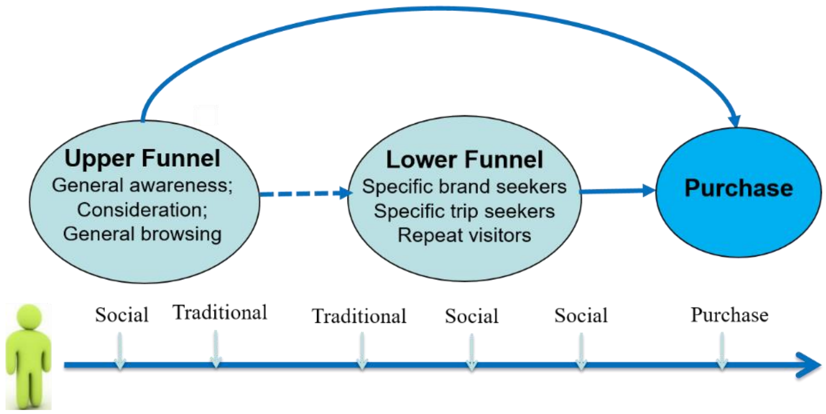

Figure 1: a consumer's "journey" to purchase

From a marketing perspective, customers may be categorized in different purchasing stages depending on the prior history and interaction with the firm. It is important to note that consumers at either funnel stage may be likely to purchase or drop out without ultimately buying. A consumer in the upper purchasing funnel would also be likely to move down to the lower funnel as she interacts more with the brand and product.

This paper helps understand three of the most important questions in targeted advertising industry, namely (a) where to target and (b) whom to target, and (c) what to target. Specifically, this study aims at providing answers to managerial questions regarding the effectiveness of targeted advertising on different platforms, whether targeting across platforms is beneficial for advertisers, and if so, to which group of consumers and through what type of targeting strategy. Answering these questions will help practitioners deriving more efficient targeting strategies across different platforms for different customers, and thereby allocating their advertising budget more wisely.

The rest of the paper is organized as follows. We first present a brief literature review by thematic areas relevant to our work; we then describe our data and introduce our methodology based on case-control studies. Following that, we present our empirical analysis and results, and we conclude by discussing our findings, managerial implications, and future study.

2. Literature review

This research topic is related to several emerging and established areas of research on online advertising.

Multichannel Attribution. The first is related to the general problem of " digital attribution" or how to proportionally split the contribution of each platform and touchpoint in the scenario of an ultimate conversion. Several studies examine the attribution problem based on the funnel framework also adopted in this work (Abhishek et al. 2012, Li and Kannan 2014, Li et al. 2015, Song et al. 2015). Abhishek et al. (2012) map observed consumer behavior to unobserved consumer purchase funnel and developed a hidden Markov model to measure how the change in the previous stage affects the probability of moving to the next stage and the likelihood of conversion. Li and Kannan (2014) study the carryover and spillover effects of prior touches through consumer purchase funnel and measure the incremental contribution of multiple channels to conversions. Based on multi-stages of consumer purchase funnel, Li et al. (2015) analyze the effects of touches on consumers' purchase choice among competing advertisers.

The effectiveness of Display Advertising. This study is also concerned with the measurement of effectiveness of display advertising (Manchanda et al. 2006, Ghose and Todri 2015). Manchanda et al. (2006) develop a survival model cast in a hierarchical Bayesian framework to measure the impact of banner advertising on consumers' probabilities of repurchase. Ghose and Todri (2015) employ a smart identification strategy based on a natural experiment, in the context of display ads, and demonstrate that more exposure to display advertising can increase users' propensity to search. Interestingly, they find that the longer the duration of exposure, the more likely consumers engage in direct search. Goldfarb and Tucker (2011) employ a large-scale field experiment and find that matching an ad to website content and increasing an ad's obtrusiveness independently increase purchase intent, but the combination of these two strategies is ineffective.

The effectiveness of Retargeting Strategies. Another related stream of literature is on the effectiveness of re-targeting. Prior research examines how the effectiveness of re-targeting is affected by information specificity (Lambrecht and Tucker 2013), timing and contextual factors (Bleier and Eisenbeiss 2015), and restricting intrusive privacy information (Aziz and Telang 2015).

Complementarity and Substitutability of Channels. This less explored body of research relates to the literature on trade-offs across different channels, i.e. the complementarity or cannibalization effects of digital and physical media. For example, Simon and Kadiyali (2007) study how offering digital content cannibalizes demand of print circulation. Gentzkow (2007) examine the impact of the introduction of digital medium on consumer welfare. Li (2015) study the impact of e-books sales on changes in market coverage and find total market expands when the publisher offers e-books together with print books. Other studies have examined the impact of online word-of-mouth on offline sales of movies (Dellarocas et al. 2010), and the effect of the closure of digital distribution channel on DVD sales (Danaher et al. 2010).

Measuring interaction effects in marketing response models. One stream of the classic marketing research studies the interaction effects in (conditionally) non-linear marketing response patterns. In a remarkable survey work, Dhar and Weinberg (2015) point out that it is often problematic to use the estimate of the parameter of the interaction term to formally tests for the presence and the extent of the interaction effect as usually thought in a linear model (i.e.

interactions affect the “slopes” in linear regression models). They clearly explain that for the commonly used choice models, such as the logit one, the coefficient of the interaction implies the effects of the interaction on the log of the odds ratio, rather than the interaction effects on the probability of choice. Kolsarici and Vakratsas (2011) model the multi-media effects of online advertising, and find media increase inefficient budget allocation, suggesting an adverse cross-media effect. Xu et al. (2014) find evidence of the spillover effects of touch points by estimating a self-exciting Poisson process.

3. Data Description, Summary Statistics and Motivating Analysis

The data analyzed in this study is provided by a top four advertising agency, the client of which is a major international travel and tourism company offering an expensive and highly considerate product. The advertiser on behalf of the travel firm runs multiple campaigns and ads on an assortment of websites classified to two platforms, i.e. social media and the traditional platform. Each ad is associated with one unique campaign and one particular targeting strategy. The advertiser adopts different targeting strategies based on previous user behavior. For example, a retargeting ad is shown on the website a user currently browses if that user has visited the firm's website before. Every time a user browses a website that belongs to the advertiser's ad network, a cookie embedded in the website places a unique identifier in the user's browser. The cookie then tracks the user's viewing and clicking on ads across all websites within the agency's ad networks. If the user visits the firm's website or makes a purchase, the information is also recorded. The agency labels a user's position at either upper or lower funnel at a given time, based on some pre-defined proprietary algorithm that accounts for the user's prior history with the brand. The general definitions for the upper and lower funnel from the advertiser are 1) upper funnel: broad awareness/perception, potential vacation seekers, and 2) lower funnel: in brand market seekers, particular product seekers, repeated visitors, etc.

Specifically, our individual-level data consists of time information of a user's impressions, visits, and purchases (if any) over a period of two months from December 2014 to January 2015. For each ad, we have information about targeting strategy, platform type, ad network, and type of publishers.

3.1. Descriptive Statistics

Our dataset consists of over 19 million users whose information about their online touch points were recorded, i.e. type of targeting, on which platform, type of ad networks visited, etc[8]. They have generated over 101 million impressions during the time window. Among these users, we only have 1555 consumers who made a purchase and their online touch points were recorded completely: this provides an effective conversion ratio of less than 0.01 percent per cookie chain. We note that this is in line with industry standard and present a methodological challenge that we plan to address in this work.

We present summary statistics in Table 1 and 2 for consumers in the upper funnel and the lower funnel, respectively. Specifically, the tables below show statistics for each targeting strategy (behavioral, contextual, geo, retargeting, etc), each platform (desktop, social), the total time of touches, and inter-time between impressions, platform for the first touch (fDesktop, etc). In general, we find that there is a lot of heterogeneity within the current data but also some distinctive patterns. Interestingly, for consumers in the upper funnel, the traditional platform (denoted as desktop in the data) is the mostly used for targeted ads, while social media platform is most frequently targeted platform for consumers in the lower funnel. Also, while consumers in the upper

[8] We thank Wharton Customer Analytics Initiative for providing us the data.

funnel on average are less exposed to retargeting ads than to behavioral or contextual targeting ads, consumers in the lower funnel receives more retargeting ads than any other targeting type.

Table 1: Descriptive statistics for consumers in the upper funnel

	Sum	Quan_.5	Quan_.975	Mean	St.dev	Max
No. of Touches	206109	2	19	4.089	5.057	87
Behavioral	69489	0	8	1.379	2.768	52
Contextual	31735	0	4	0.630	1.442	33
Geo	8283	0	0	0.164	1.763	49
Looklike	0	0	0	0.000	0.000	0
Predictive	0	0	0	0.000	0.000	0
Prospecting	47338	0	6	0.939	2.649	67
Retargeting	20500	0	4	0.407	2.554	58
Desktop	136012	2	13	2.698	4.018	86
Social	41576	0	6	0.825	2.628	58
Time Length	224076.1	0.755	25.815	4.446	7.564	61.213
Inter-time	68565.69	0.212	8.163	1.360	2.404	38.317
fDesktop	38129	1	1	0.756	0.429	1
fSocial	11566	0	1	0.229	0.420	1

Table 2: Descriptive statistics for consumers in the lower funnel

	Sum	Quan_.5	Quan_.975	Mean	St.dev	Max
No. of Touches	352913	3	28	5.705	6.990	86
Behavioral	91978	0	12	1.487	3.973	49
Contextual	32236	0	5	0.521	2.257	49
Geo	84771	0	11	1.370	3.739	50
Looklike	203	0	0	0.003	0.075	4
Predictive	529	0	0	0.009	0.135	9
Prospecting	2996	0	1	0.048	0.386	15

Retargeting	118665	0	18	1.918	5.354	86
Desktop	101536	0	11	1.641	3.600	50
Social	183634	0	23	2.968	6.344	86
Time Length	241564	0.190	22.302	3.905	6.906	60.849
Inter-time	62712.9	0.053	8.003	1.014	2.506	46.122
fDesktop	25153	0	1	0.407	0.491	1
fSocial	26151	0	1	0.423	0.494	1

3.2. Preliminary Analysis

To further motivate the need for more sophisticated methods to deal with the statistical and managerial problems of consumers targeted in the purchasing funnel, we have performed some preliminary analysis. Specifically, to explore the associations between possible predictors and the probability of purchasing, we perform a logistic regression including as covariates on all predictors. These include each type of targeting, platforms, the number of touches, etc. We split the analysis for upper and lower funnel consumers separately, consistently with the funnel labeling described earlier.

Table 3a: "Kitchen sink" logistic regression for all predictors for the upper funnel

Upper Funnel			
	Estimate	SE	tStat
(Intercept)	-4.308	0.16554	-26.024
Num.Act.	-0.054287	0.013728	-3.9546
Contextual	1.8442	0.27258	6.7658
Geo	-0.89346	0.52298	-1.7084
Prospecting	-0.65282	0.13276	-4.9175
Retargeting	1.3876	0.1679	8.2643
Int.Desk.Soc.	0.24361	0.27424	0.88829

Social	-0.3319	0.23569	-1.4082
LEN	-0.0051295	0.011004	-0.46614
INTER	-0.1148	0.033457	-3.4311
TFIRST	0.021214	0.0028403	7.4691
Network 1	-2.1513	0.14525	-14.811
Network 2	-2.3377	0.33918	-6.8922
Network 3	-1.9176	0.37955	-5.0522
Network 4	-1.0265	0.12656	-8.1111

Table 3b: "Kitchen sink" logistic regression for all predictors for the lower funnel

Lower Funnel			
(Intercept)	-0.83924	0.24702	-3.3974
Num.Act.	-0.050975	0.0097911	-5.2063
Contextual	0.028746	0.36835	0.078039
Geo	-1.0985	0.22348	-4.9156
Lookalike	1.337	0.39265	3.405
Predictive	1.5339	0.39318	3.9012
Prospecting	0.32798	0.43182	0.75953
Retargeting	2.5006	0.22843	10.947
Int.Desk.Soc.	**-0.69259**	**0.33583**	**-2.0623**
Social	**-1.5416**	**0.28374**	**-5.4331**
LEN	0.045341	0.0087003	5.2115
INTER	-0.075563	0.026298	-2.8733
TFIRST	-0.051193	0.0029563	-17.316
Network 1	0.43662	0.40772	1.0709
Network 2	-0.58041	0.33488	-1.7332

Network 3	-1.744	0.26778	-6.5129
Network 4	-1.3859	0.25311	-5.4755

The results in the Table 3 above show, unsurprisingly in "large samples," almost all predictors related to targeting variables, activities, platforms are significant in the upper funnel as well as in the lower funnel. However, it is possible to notice that social media appears to hurt conversions for consumers in the lower part of the funnel, while there is no significant effect for consumers in the upper funnel. On top, there is no evidence of synergy between channels across the funnels; at the contrary, it appears an antagonistic effect takes place in the lower funnel. These results are clearly disappointing and if taken at face value, would lead the firm to conclude that partying with the agency did not bring any substantial lift in conversions. Recall, for instance, that the large portion of retargeting on social media takes place for customers labeled as "lower funnel". If both the main effect and the interaction effects for the lower funnel consumers are negative, it seems the agency ad placements not only do not work but also are detrimental towards ultimate conversion. One potential explanation for this is that the agency is targeting "too much" with the effect of wearing out customers.

However, a quick inspection of the design matrix can easily reveal collinearity among predictors, i.e. targeting types and platforms; this is because advertisers are most likely to run platform-specific targeting strategies. This hints at the problem of the endogenous targeting assignment to the user likely to create "activity biases" as described in Lewis et al. (2011). The following section presents our methodology to address collinearity, heterogeneity and "activity biases" issue based on the case-control method described earlier in combination with modern machine learning tools.

In sum, the results in Table 3 should be taken with a grain of salt. More accurate ways of analyzing the data are needed, even if the aim is just to measure the association between treatment and outcomes. We will proceed to the Section 4 providing a detailed derivation of our empirical strategy.

4. Methodology

4.1. Relevant Quantities of Interest for Advertising Effectiveness

Consider a hypothetical marketing response model, where we have collapsed all the relevant variables into X, for notational simplicity:

$$\Pr(Y = 1|\mathrm{X}) = \pi(X) \tag{1}$$

At this stage we do not specify any functional form for $\pi(X)$, namely probabily functionals. We will rather derive conditions under which transformations of $\pi(X)$ are learnable from the data. We will then introduce the parametric assumptions after having established these identification properties. The material presented here is standard in the epidemiological literature (Manski, 1999, Rothman et. al, 2008, King and Zeng, 2001).

We define the risk as the probability that a conversion occurs given a set of values of the explanatory variables, $\Pr(\mathrm{Y} = 1|\mathrm{X} = \mathrm{F})$. The relative risk (RR) is the probability of risk relative to the probability of an event given by some counterfactual or observational value of X, denoted as F':

$$\mathrm{RR} = \frac{\Pr(\mathrm{Y}=1|\mathrm{X}=\mathrm{F})}{\Pr(\mathrm{Y}=1|\mathrm{X}=\mathrm{F}')} \tag{2}$$

The relative risk quantity is frequently reported in the popular media (e.g., the probability of suffering for a heart attack doubles if the person is severely overweight and also a smoker). In the advertising ecosystem, it can be interpreted as the relative lift of strategy F_i relative to F_i'. A related quantity is the risk difference (RD). This is defined as:

$$RD = \Pr(Y=1|\mathrm{X}=F) - \Pr(Y=1|X=F'). \tag{3}$$

It is worth mentioning that major publishers, such as Facebook[9], advance a notion of "lift" that is implied by the RD and the RR.

$$Lift = \frac{\Pr(Y=1|X=F) - \Pr(Y=1|X=F')}{\Pr(Y=1|X=F')} = \frac{\mathrm{RD}}{\pi(X=F')} = RR - 1 \tag{4}$$

The relative risk (and hence the lift) has some advantages. First, it is dimensionless, so it is easier to compare across time periods, targeting strategies, and the portion of the funnels[10].

Second, a more intriguing property of the relative risk is its behavior under rare outcomes. One can easily see that (see Cornfield, 1951 for the original derivation):

$$\lim_{\pi_i \to 0} RR = \frac{\Pr(Y=1|X=F)}{\Pr(Y=1|X=F')} = \frac{\Pr(X=F|Y=1)\Pr(X=F'|Y=0)}{\Pr(X=F'|Y=1)\Pr(X=F|Y=0)} = OR \tag{5}$$

The expression on the right side is the well-known odds ratio (OR) because it is the ratio of the odds of the customer converting for scenarios with covariates F and F'. The variation in X across the population, conditioning on the outcomes, can reveal the quantities on the right side of equation (5), so the rare-outcome assumption allows to "learn" the relative risk through the odds ratio. In other words, the RR can be learned from the data just by considering the variation of the odd ratios in a cross-sectional setup.

4.2. Rare Outcomes Biases and Response-Based Sampling

Equation (5) provides a connection with the literature on logistic regression where the log-odds are linked parametrically and log-linearly to the relevant covariates. Thus, a research analyst could safely posit a logistic functional form[11] for cookie chain i:

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[9] See https://www.facebook.com/business/news/conversion-lift-measurement and Gordon et al. (2016) for additional details.
[10] Several scholars often argue whether it is better to report the RR or the RD (Manski 1999); in this work we focus on the RR for the reasons explained above.
[11] McFadden (1981, Ch. 4), shows that the logit model targets correctly the odds ratio in large samples when the covariates are bounded.

$$\Pr(Y_i = 1|X_i) = \frac{\exp(X_i'\beta)}{1+\exp(X_i'\beta)} \tag{6}$$

where X_i, again contains all the relevant covariates.

The same analyst, if not careful, may try to estimate (6) using all the information available in the sample as in the preliminary analysis reported in the previous section. This is going to produce several complications. The first is computational: optimizing the log-likelihood function of the logit model is very costly with large sample size and the number of covariates. This is because there are not closed-form updates for the gradient of the log-likelihood. Second, even if computational time is not a requirement, a more subtle problem is driven by the small fraction of purchases, denoted as π_n. This determines a variety of small sample bias that is not related to the dimension of the sample per se, but to the small fraction of purchases. These "rare outcomes biases" clearly compound while trying to evaluate measures like the RR, especially if the baseline is a strategy with zero exposures and so the denominator is underestimated as well.

Thus, with the assumption of rare outcomes, even in the presence of exogenous variation in Xs, we may end up overestimating the contribution of the advertising variables. On the other hand, statistical significance may also be challenging. The variance of the point estimates can be shown to depend on the fraction of purchases, π_n:

$$Var(\hat{\beta}) = (\sum_{i=1}^{n} \pi_n(1-\pi_n)X_i'X_i') \tag{7}$$

The component of the covariance matrix affected by rare events is the factor $\pi_n(1-\pi_n)$ and so it is easy to see that as π_n gets smaller, the variance larger and the likelihood flatter.

The lack of concavity, as driven by (7) is "silently" solved in many black-box statistical software packages by a combination of regularization tools and approximations (Firth 1993) bounding the variance through the inclusion of penalties or regularizers in the log-likelihood. These, in turn, may add additional biases.

A clever way to bypass these complications is to sub-sample the data based on the known responses. This approach is referred as the case-control method in biostatistics and response-based sampling in economics. Specifically, we introduce an assignment indicator S_i that is one if cookie chain i is included in the sample and zero otherwise. The idea is that we may select asymmetrically the purchases and non-purchases. Thus, the conditional marketing response model of interest is given by:

$$\Pr(Y_i = 1|X_i, S_i = 1). \tag{8}$$

Using Bayes theorem:

$$\Pr(Y_i = 1|X_i) = \frac{\exp(X_i'\beta)}{1+\exp(X_i'\beta)} = \frac{\Pr(S_i=1|y_i=1,X_i)P(Y_i=1|X_i)}{\Pr(S_i=1|X_i)} \tag{9}$$

Note that the denominator, by the law of total probability, can be expressed as:

$$\Pr(S_i = 1|X_i) = \Pr(S_i = 1|y_i = 1, X_i)\Pr(y_i = 1|X_i) + \tag{10}$$

$$+\Pr(S_i = 1|y_i = 0, X_i)\Pr(y_i = 0|X_i)$$

Now we assume that the case assignment mechanism does not depend on X_i. This is also referred as exogenous response-based sampling.

Let the probability that a successful cookie chain ($Y_i = 1$) was sampled denoted by p, and let the probability that a non-conversion cookie chain ($Y_i = 0$) was sampled denoted by q. Then

$$\Pr(S_i = 1|X_i) = q\,\frac{\exp(X_i'\beta)}{1+\exp(X_i'\beta)} + p\,\frac{1}{1+\exp(X_i'\beta)} \tag{11}$$

This leads to the following simplification:

$$\Pr(Y_i = 1|X_i, S_i = 1) = \frac{q\exp(X_i'\beta)}{p+q\exp(X_i'\beta)} = \frac{\exp\left(\log(\frac{q}{p})+X_i'\beta\right)}{1+\exp\left(\log(\frac{q}{p})+X_i'\beta\right)} \tag{12}$$

This means that the retrospective sampling process does not alter the slope of the putative logistic response model. The effect is simply a shift in the intercept. In the presence of rare outcomes, one could safely assume $p = 1$, so to make the most use of the available conversions, while taking q at a fraction of the original sample of non-purchases.

A simple extension of the approach described above is to let p and q depend on some additional covariates Z_{it}, those related, for example, to customer activities or demographics. This is commonly referred as match-based sampling, where we match cookie chains based on the similarity in their activities, Z_{it}. If these are orthogonal to X_{it}, then response-based sampling is unaffected by the matching process. In other words, exclusion restrictions bind, and it is possible to retrospectively obtain case-control designs without biasing the estimates of the marketing response activities.

Unfortunately, we know that is hardly the case. Activities and exposures in the purchasing funnel are very correlated and determine "activity biases". We will proceed by first describing how these arise and then we will expound upon the idea presented in (12) and how to deal with the lack of orthogonality between the exposures and the activities.

4.3. Activity Biases

In the epidemiological and causal inference literature, "activity biases" as coined by Lewis, Rao and Reily (2011) can be thought in terms "confounders." These define a situation in which the association between an exposure and outcome is distorted by the presence of other (observed or unobserved) variables: in this case customer "activities." One could posit the presence of an observed or unobserved variable, Z, denoting a general level of browsing activities. Z lies on the causal path between the treatment, the online advertising activity, and the outcome variable. Z can be positively correlated with the treatment X and the outcome variable Y. Hence, the correlation between Y and X cannot be used as a measure of causal effectiveness because of the presence of Z independently altering the relationship between Y and X.

The "gold standard" to eliminate confounding is the execution of a randomized experiment. This assigns treatment and controls randomly and irrespective of their past and current activity patterns. However, when the aim is to measure the effectiveness regarding the ultimate purchases, as it is advisable from the firm's side, an experiment randomizing assignments for all possible combinations of treatments, is very difficult to execute. This is because online activities in the purchasing funnel happen on different ad-networks where publishers, unfortunately, do not share incentives to jointly perform online experiments. Additionally, experiments are often difficult to operationalize in the advertising ecosystem (Aziz and Telang 2016). For instance, retargeting campaigns are on averaged optimized every week, while purchases could take place in a longer time horizon.

When experimental designs are impractical, researchers can still rely on statistical methods to adjust for potentially confounding effects. These are based on a combination of (a) control variables (b) matching and stratification and (c) regularization. We will make use of these in the context presented in this work. We will now introduce a formal model of cookie chain dynamics, connecting the discussion presented in the previous sections. We will use this as a basis for the selection of case-control designs with which we will attempt to control for confounding based on customer activities.

4.4. Subspace K-Means Clustering

A clever choice is to *match* i with a j, and more generally with $m-1$ other observations, based on their confounding information Z_{it} and Z_{jt}. This procedure would ensure an equal distribution among cases and controls of the variables believed to be confounding the relationship between marketing exposures and outcomes. The matching procedure stratifies the data, where the pooled within R_I effects of advertising are estimated while keeping the confounding as constant as possible (Frangakis and Rubin, 2003). This stratification will not in general remove confounding, but will make the estimation more efficient.

However, while clustering observations lying in multi-dimensional spaces, formulating an appropriate measure of "similarity" is problematic for two reasons: (a) in high dimensional spaces, especially when the information is sparse (many zeros in the Z_{it}) computing the distance while searching at all the n-dimensions is inefficient. To provide an example, it is not uncommon that in these situations, the farthest "neighbor" of an observation is expected to be almost as close as its nearest neighbor. (b) due to the "activity bias" phenomenon (cookie chains being targeted more frequently if they are more active), a case and control groups may become too similar not only in the distribution of the confounder but also in the distribution of the exposures, rendering the procedure incapable of disentangling the effect of the exposure from the one of the confounders. Therefore, weighing all the dimensions of Z equally while computing an euclidean-type distance offers no real advantage in terms of mitigating confounding effects and positioning cases and control in same risk set.

To overcome these problems, a solution is to cluster in lower-dimensional subspaces. This is a theme of research that has become prevalent in the last decade in the machine learning literature. The general idea is that ideally, we should discard regions of the covariates space (a) where the information is sparse (it's not useful to cluster zeros with zeros) and (b) where the Zs are highly correlated with the Xs. Here we adapt to our purpose a methodology called "Subspace K-Means Clustering" (Agarwal and Mustafa, 2004). This takes as input an unweighted set of points, our Z, and a set of directions used to look for the best clusters. A penalty is applied to deal with sparsity, so that subspaces with many zeros are not pursued by the algorithm. As a search direction we specify the first two eigenvectors, i.e. principal components, of the null space of X, $N_X = I -$

$X(X'X)^{-1}X$. This allows the very efficient K-means algorithm to search for clusters along dimensions where the Z are most similar to each other, given that their level of the exposures as orthogonal (e.g. conditional independent) as possible. The number of clusters, or risk set, is based on the average similarity within each risk set, as it is with traditional K-means.

4.5. Integrative Discussion on the Methodology and Implementation Notes

We summarize the methodological development thus far. Given that we have very small number of consumers who purchased (rare events) in our observational data, as discussed earlier, we adopt a retrospectively matched case-control method to measure the odds ratio, equivalent to the relative risk, to assess the effectiveness of targeting on social platform relative to the traditional platform in both parts of the funnel.

In our dataset, we have a very small number of consumers who have moved from upper to lower funnel, and even fewer consumers among them have made purchases. The vast majority of consumers are in either upper or lower funnel in our data time window, we focus on these consumers in this study and ignore those who have experienced both funnels. Because consumers in the upper funnel may have quite different characteristics and prior history from those of consumers in the lower funnel (Li and Kannan 2014, Li et al. 2015), we differentiate upper funnel consumers and lower funnel consumers, and form them into two separate groups. Note in each group a portion of consumers have made purchases.

To find the "matched" control group, we use subspace K-means clustering technique to retrospectively identify "similar" consumers who did not purchase, based on the characteristics of those who purchased. We use consumer-initiated actions, what we denoted as Z as the similarity measures to form clusters of "similar" consumers for the upper funnel consumers and the lower funnel consumers, respectively. Consumer-initiated actions include timing variables, i.e. starting time and ending time of online path, inter-activity time. These general timing variables are characteristics of consumers and not affected by the advertiser's targeting strategy. Another consumer-initiated action is which ad network a consumer has visited. We assume that a consumer certainly knows which website she is currently visiting, but she ignores which ad-network this belongs to. An "orthogonality condition" is enforced by the subspace k-means algorithm so that clusters are formed in a way to minimize potential confounding effects.

In order to identify important predictors and address the collinearity issues between different targeting strategies and platforms, we advocate for regularization methods, particularly the *Elastic Net* logistic regression. We then adopt a "post-regularized model" producing consistent estimates of the odds ratios given the set of selected covariates by the *Elastic Net*. The derived coefficients, βs, measure the sensitivity of the customers, and can be interpreted for counterfactual within the cluster and for comparisons between risk sets. The following diagram shows the procedure of our clustering and estimation.

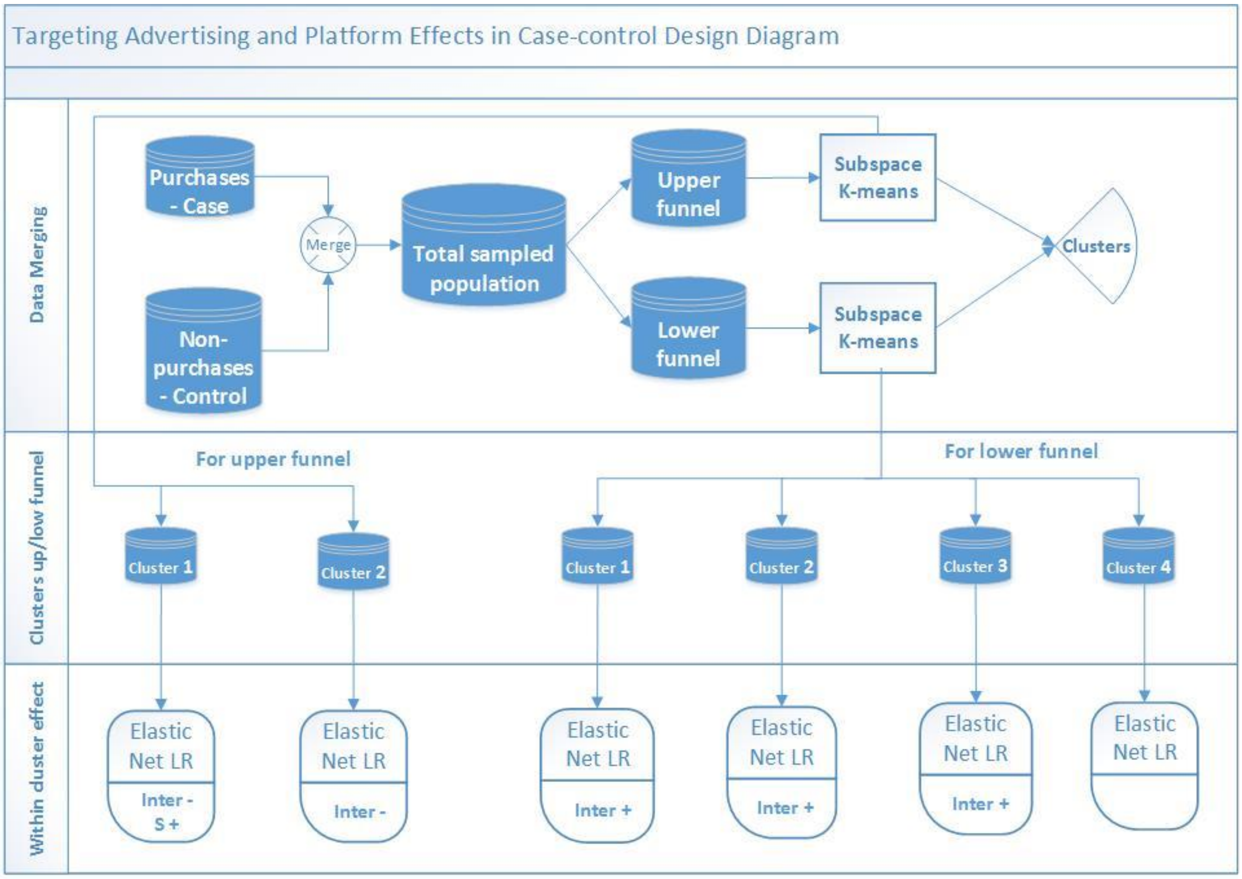

Figure 2: Targeting advertising and platform effects in case-control design

The Figure 2 shows that we apply subspace K-means clustering technique for consumers at upper funnel and lower funnel, respectively. Then we fit *regularized* logistic model for each cluster. The post-regularized model contains selected variables we want to attribute to purchases, i.e. platforms and retargeting effects.

5. Empirical analysis

We establish the different numbers of clusters for upper and lower funnel, based on the average similarity between cases and controls in the same risk set. So we experimented with different number of clusters and eventually we obtained an optimized number of clusters for each funnel, i.e. 2 clusters for the upper funnel, and 4 clusters for the lower funnel. We report the post-regularized logit model and results for each these clusters. We then present an integrative analysis quantifying tradeoff effects between the different platforms.

5.1. Risk sets analysis in the upper funnel

Upper Funnel			
First Risk Set			
	Estimate	SE	tStat
(Intercept)	-3.3487	0.087553	-38.248
Retarg.	1.2284	0.26137	4.6999
Social	**0.89346**	**0.15194**	**5.8804**
Length	-0.044312	0.0085576	-5.1781
N1	-2.1311	0.29482	-7.2287
N3	-1.5797	0.7137	-2.2134
N4	-1.4018	0.19967	-7.0209
Second Risk Set			
(Intercept)	-3.8212	0.42919	-8.9032
Num.Act.	-0.16986	0.043603	-3.8956
Int.Desk.Soc.	-0.032839	0.44752	-0.07338
Social	**1.1763**	**0.42775**	**2.75**
Length	-0.21591	0.033359	-6.4723
N1	-1.3997	0.15524	-9.0161
N3	-0.87101	0.40909	-2.1292

Table 4: Results of Estimation for the Upper Funnel

The estimation results are in Table 4. An interesting result is a reversal of what we have found earlier in Section 3. This implies that advertising strategies on social media may have a positive impact on consumers in the upper funnel and so the agency advertising activities on social media appeared to have acted proficiently. However, there is no evidence of synergistic effects in the upper funnel. Strategies like retargeting appear to work only for a subset of customers. Also, consistent with common sense, the longer the experience in the upper funnel the less likely are customers to ultimately convert (decreasing hazard hypothesis).

5.2. Lower funnel analysis

Consumers in the lower funnel are generally more experienced with the product or have more prior interaction with the firm.

Interestingly, targeting on social media alone is not significant in the lower funnel, suggesting that as consumers have moved to the lower funnel, they may become more sophisticated and might actively search for the product, so targeting on social media alone may not be helpful for moving these consumers to final purchase. However, targeting across platforms is positively associated with the odds ratio of purchase, suggesting that more targeting across platforms may be helpful in providing personalized information and in converting consumers in the lower funnel. Retargeting is significant and positively associated with the odds ratio of purchases in three risk sets. It is also interesting to notice the presence of an “empty” risk set. This means that for the people in that risk set, it was not possible to measure any significant marketing activities determining their purchases. One could speculate that it is possible that the cases and controls in this bucket did not receive much social media targeting.

Lower Funnel			
First Risk Set			
First Risk Set	**Estimate**	**SE**	**tStat**
(Intercept)	-4.9486	0.42633	-11.608
Num.Act.	-0.16417	0.061372	-2.675
Retarg.	**5.1171**	**0.52417**	**9.7623**
Int.Desk.Soc.	**1.7896**	**0.38132**	**4.6931**
Social	-0.43132	0.53663	-0.80376
Length	-0.68663	0.055093	-12.463
Inter.Act.	0.65813	0.08982	7.3273
N1	1.3421	0.67937	1.9755
N2	-0.14219	0.31657	-0.44916
Second Risk Set			
(Intercept)	-5.5767	0.54796	-10.177
Num.Act.	-0.34692	0.093158	-3.724
Retarg.	**2.644**	**0.30403**	**8.6964**
Int.Desk.Soc.	**3.2082**	**0.54635**	**5.8721**
Social	-0.028733	0.4934	-0.058235
Length	-0.89775	0.20814	-4.3132
Inter.Act.	0.85018	0.22947	3.705
N1	0.47917	0.58695	0.81638
N3	-2.8003	0.40481	-6.9174
N4	-1.678	0.33127	-5.0654
Third Risk Set			
(Intercept)	-2.7904	0.63873	-4.3687
Num.Act.	-0.074903	0.057785	-1.2962

Retarg.	**2.892**	**0.53387**	**5.417**
Int.Desk.Soc.	0.063801	0.49918	0.12781
Social	-0.40644	0.51266	-0.79281
Length	-0.128	0.016388	-7.8106
Inter.Act.	0.042379	0.029368	1.443
N1	-0.72177	0.77069	-0.93653
N2	0.14652	0.52787	0.27757
N3	-1.6206	0.66848	-2.4244
N4	-0.84134	0.54769	-1.5362
Fourth Risk Set			
(Intercept)	-4.5001	0.15899	-28.304

Table 5: Analysis for the Lower Funnel Risk Sets

5.3. Tradeoff measures between social media and the traditional platform

In the context of targeted advertising, consumers visit different sites and thereby may be exposed to targeted ads on both social media and the traditional platform at different times before making a purchasing decision. For example, a consumer might first receive targeted ads on the traditional platform, and then receive targeted ads on social media, and get targeted again later on the traditional platform. As is known in literature, there is an interaction effect of two independent variables on the dependent variable, if the effects of the two independent variables is more (or less) than the sum of the parts. The interaction of the independent variables also underlies moderation effects (Dhar and Weinberg 2015). In our context, the interaction effect of social media and the traditional platform implies that the effect of targeting on social media on the log odds ratio of purchase is moderated by the effect of targeting on the traditional platform, and vice versa. Estimation of the interaction term in also at the center of our analysis as we wish to understand whether, ex post, if it is possible to establish as to whether the multichannel targeting strategies delivered by the agency were effective in delivering ultimate conversions.

Interpreting moderation effects in nonlinear models is often non-immediate. To examine the relationship between targeting on social media and on the traditional platform, however, we do need to interpret the interaction effects in a more qualitative and insightful manner (see Dhar and Weinberg 2015, also mentioned in the previous section). As said earlier, our logistic regression targets the "relative risk" (RR) framework for assessing the importance of different risk factors from well-established epidemiological literature.

It is well known in the statistical literature that approximations for interaction analysis exist under rare outcomes assumption (Richardson et al. 2009). This allows us to estimate interaction effects and interpret interactions in a "linear" probability scale and leverage about the notion of relative risk described above. In particular we consider the Relative Excessive Risk due to Interaction (RERI)[12]. We calculate RERI based on the following:

$$RERI = RR_{11} - RR_{10} - RR_{01} + 1$$

Subscript 11 refers to activating targeting on both social media and the traditional platform, 10 refers to putting social media but shutting down the traditional platform, and 01 refers to shutting down social media while activating the traditional platform. RERI is presented in a more familiar linear and additive form (thus avoiding the cumbersome problem of inverting log-odds), and can be interpreted qualitatively as the "extra lift" of the probability of purchase due to presence of the ads on both platforms. Specifically, If $RERI > 0$, social media and traditional platform are

[12] See Vanderweele, 2015. We also note that we could call the "RERI" as the "interaction lift", given that this is dimensionally equivalent to the lift definition we described earlier.

considered complement; If $RERI < 0$, social media and traditional platform are considered substitute.

Table 6: $RERI$ table

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Upper Funnel	**-0.01**	**0**	**N/A**	**N/A**
Lower Funnel	**0.04***	**0.05***	**0.02***	**0**

***p < 0.05**

It is easy to notice that social media and traditional platform is the lack of synergistic effects in the upper funnel, and more as complement for consumers in the lower funnel. At the best of our knowledge, these results are the first of their kind highlighting the possibility of the complex complementarity patterns that could be better exploited by the firm when delivering the ads.

5.4. Robustness Checks

Our identified platform effects should also emerge even without clustering and matching. We do regularized model for each funnel stage consumers respectively to examine if we get the qualitatively same results.

Table 7a: Regularized model for all consumers at the upper funnel

For Consumers at Upper Funnel			
	Estimate	SE	tStat
Intercept	-3.9271	0.1712	-22.9394
Desk*Soc	-0.0866	0.1704	-0.5081
Social	**1.3761**	**0.1719**	**8.0048**
Length	-0.0396	0.0065	-6.0951
N1	-1.7412	0.1374	-12.6729
N4	-0.3963	0.0799	-4.9568

Table 7b: Regularized model for all consumers at the lower funnel

For Consumers at Lower Funnel

	Estimate	SE	tStat
Intercept	-5.1582	0.1118	-46.1525
Num	-0.2199	0.0328	-6.6961
Retarg	**2.7006**	**0.1223**	**22.0784**
Desk*Soc	**1.8848**	**0.1667**	**11.3095**
Length	-0.0110	0.0057	-1.9332
N3	-1.7443	0.1760	-9.9091
N4	-1.3023	0.1189	-10.9572

From table 7a, we can see that the main effect of targeting on social media is significant and positively associated with the ultimate conversion for the upper funnel consumers. This result is qualitatively the same as the result derived from clustering the consumers. Similar to the previous analysis for the lower funnel consumers, table 7b shows that targeting across both platforms (interaction term) is significantly and positively associated with the final conversion; retargeting has a significant and positive impact on the final conversion.

In our main analysis, our measurements of consumer similarity include number of ad exposures, ad networks consumers have visited, and four timing variables (start time, end time, inter-time between ad exposures, and total time of exposures). Arguably, consumers' timing variables can be correlated with the targeting strategies. For example, the advertiser's targeting decision and targeting strategy could affect a consumer's inter-time exposures between two ads. To test the robustness of our main analysis result, we drop the four timing variables from the similarity measures, and use only the number of ad exposures, and the ad networks consumers visited as the similarity measures for our sub-space K-medoids clustering. We report the results as the following:

Table 8a: Sub-space K-means with no timing variables for the upper funnel

Upper Funnel

First Risk Set			
	Estimate	SE	tStat
Intercept	-2.8356	0.1475	-19.2226
Num	-0.5251	0.0646	-8.1290
Social	-0.1944	0.2175	-0.8940
Length	-0.0022	0.0219	-0.1005
Inter	-0.0664	0.0489	-1.3570
N1	-1.8060	0.1513	-11.9353
N3	-1.8911	0.3795	-4.9837
N4	-0.3649	0.0892	-4.0920
Second Risk Set			
Intercept	-3.6554	0.4032	-9.0655
Num	-0.0044	0.0135	-0.3227
Retarg	**1.0014**	**0.3965**	**2.5259**
Desk*Soc	-0.1761	0.4158	-0.4236
Social	**0.8075**	**0.3827**	**2.1099**
Length	-0.0297	0.0114	-2.6036
Inter	0.0759	0.0464	1.6370
N1	-1.6458	0.3607	-4.5626
N2	-0.6616	0.3494	-1.8936
N3	-2.3302	1.0230	-2.2779
N4	-1.4109	0.2790	-5.0578

Table 8b: Sub-space K-means with no timing variables for the lower funnel

Lower Funnel			
First Risk Set			
First Risk Set	**Estimate**	**SE**	**tStat**
Intercept	-4.7543	0.1004	-47.3580
Retarg	**2.4480**	**0.1270**	**19.2753**
Length	-0.0595	0.0119	-5.0161
N3	-0.8557	0.2225	-3.8456
Second Risk Set			
Intercept	-5.8033	0.4287	-13.5377
Num	-1.7255	0.1761	-9.7965
Retarg	**2.1956**	**0.1863**	**11.7869**

Desk*Soc	**6.0854**	**0.6025**	**10.0995**
Social	-0.4507	0.3631	-1.2414
Length	-0.0546	0.0200	-2.7383
N2	-0.9545	0.2076	-4.5974
N3	-2.8053	0.3646	-7.6939
N4	-2.1890	0.1863	-11.7496
Third Risk Set			
Intercept	-5.3217	0.2684	-19.8280
Num	-0.0353	0.0243	-1.4560
Retarg	**1.7755**	**0.3644**	**4.8722**
Social	0.0903	0.4152	0.2175
Inter	0.0823	0.0378	2.1775
Fourth Risk Set			
(Intercept)	-4.4407	0.1676	-26.4885

From table 8a and 8b, we verify that our key results about platform effects still hold when we exclude all timing variables as the similarity measures. For upper funnel consumers, the main effect of targeting on social media has additional positive and significant impact on the final conversion, relative to that on the traditional platform. Targeting across the two platforms, however, has insignificant impact on the final conversions. In contrast, for the lower funnel consumers, targeting across the two platforms have a positive and significant impact on the final conversions, which has been shown for the consumers in the second risk set. Retargeting, comparing to other targeting strategies, has a positive and significant impact on the ultimate conversions. The positive effect has been observed across three risk sets for the lower funnel consumers.

6. Discussion and Conclusion

We have developed an empirical strategy with the aim of identifying interaction effects between activities performed on different platforms with and within different parts of the funnel. First, we find clustering based on the similarity of consumers, compared with no clustering, mitigates heterogeneity and offer more meaningful insights on platform effects. Second, our results indicate that targeting across platforms has synergistic effects with the ultimate conversion for consumers at the lower funnel, but does not appear to provide any interaction effect for the upper funnel consumers. Third, our results show that the main effect of targeting on social media is positively associated with the odds ratio of purchase for the upper funnel consumers, but has no impact relative to the traditional platform for consumers at the lower funnel. Lastly, our findings indicate that the commonly implemented "retargeting ads" are more effective than other more sophisticated targeting strategies, and that retargeting may have a positive and significant association with the ultimate conversions for consumers at the lower funnel.

Finally, our study draws managerial implications by measuring the trade-off effects between social media platform and the traditional platform for digital advertising. Our findings help answer the important managerial questions regarding what platform(s) the advertiser should run their advertisement on, through what targeting strategy, and for which type of consumers. Specifically, targeting on social media may be more helpful and can bring incremental informational value when consumers are at early stages. However, there is no synergistic effect for targeting across platforms when consumers are not experienced or familiar with the product/brand. We speculate that too much personalized ads across platform may not be helpful, or bring negative psychological impact on early stage consumers, even though the informational value is positive. When customers move to a more mature purchasing stage, targeting across platforms appears to be very beneficial. Finally, based on our findings and suggestions, advertisers may allocate more budget on retargeting than on other targeting strategies (often involving complex negotiations and complicated implementations across different ad networks) and it may be more efficient to retarget consumers who are at the lower purchasing funnel.

Our study can be extended in several aspects. First, we have ignored consumers who experienced both funnels, due to the small number of consumers with purchases. It may be interesting to include this group of consumers in future study to examine how targeting on different platforms affects the probability of consumers transitioning in purchasing funnels. Second, we may need to characterize selected clusters in a more policy interpretable manner. Third, we could potentially extend our approach to account for the effects of mobile platform on conversions, and to measure the associational effects of different types of ads across platforms.

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