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UNIVERSITY OF CALIFORNIA, IRVINE

Constant-Envelope Modulation Schemes with Turbo Coding

THESIS

submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in Electrical Engineering

by

Nistha Tandiya

Thesis Committee: Professor Ender Ayanoglu, Chair Professor Hamid Jafarkhani Professor Ahmed Eltawil

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DEDICATION

To Mummy & Papa

- - -

The oldest, shortest words - 'yes'and 'no'- are those which require the most thought.

- Pythagoras, fifth Century BC

I wish I could be more moderate in my desires. But I can't, so there is no rest.

- John Muir, 1826

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LIST OF ABBREVIATIONS

| PA | Power Amplifier |
|------------|---|
| PAPR | Peak to Average Power ratio |
| СРМ | Continuous Phase Modulation |
| PSK | Phase Shift Keying |
| OFDM | Orthogonal Frequency Division Multiplexed |
| SC-FDMA | Single Carrier Frequency Division Multiple Access |
| CE-OFDM | Constant Envelope Orthogonal Frequency Division Multiplexing |
| CE-SC-FDMA | Constant Envelope Single Carrier Frequency Division Multiple Access |
| ISI | Inter-symbol interference |
| 3GPP | 3 rd Generation Partnership Project |
| LTE | Long Term Evolution |
| PAM | Pulse amplitude modulation |
| QAM | Quadrature amplitude modulation |
| BPSK | Binary phase shift keying |
| QPSK | Quadrature phase shift keying |
| FDM | Frequency Division Multiplexing |
| СР | Cyclic prefix |
| BER | Bit error rate |
| SNR | Signal-to-noise ratio |
| DFT | Discrete Fourier Transform |
| IDFT | Inverse discrete Fourier Transform |
| FFT | Fast Fourier Transform |
| IFFT | Inverse Fast Fourier Transform |
| ICT | Information and Communication Technology |

| MMSE | Minimum mean square error equalizer |
|-------|---|
| PSD | Power spectral density |
| AWGN | Additive white Gaussian noise |
| FEC | Forward Error Correction |
| ECC | Error Control Coding |
| LDPC | Low Density Parity Check (code) |
| BCH | Bose-Chaudhuri-Hocquenghem (code) |
| RS | Reed Soloman (code) |
| VA | Viterbi Algorithm |
| ML | Maximum Likelihood |
| RSC | Recursive Systematic Convolutional (code) |
| PCCCs | Parallel Concatenated Convolutional Codes |
| SCCCs | Serial Concatenated Convolutional Codes |
| HCCCs | Hybrid Concatenated Convolutional Codes |
| LLR | Log Likelihood Ratio |
| BCJR | Bahl Cocke Jelinek Raviv (algorithm) |
| TTCM | Turbo Trellis Coded Modulation |
| ТСМ | Trellis Code Modulation |

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ABSTRACT OF THE THESIS

Constant-Envelope Modulation Schemes with Turbo Coding

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The communication infrastructure is one of the biggest energy consumers in the world. With the expected exponential growth in the demand for wireless traffic, it becomes the foremost priority to make the communication systems energy-efficient. In this work, we will explore two energy efficiency enhancement techniques: Constant-Envelope modulation and Turbo coding. In Constant-envelope modulation, the high Peak-to-Average Power Ratio (PAPR) signal is transformed into a constant-envelope phase modulated signal. Thus the PAPR of the signal reduces to 0 dB, enabling the power amplifiers at the transmitter to work at the energy-efficient operational point. The second technique known as Turbo coding, has been known to perform very close to the theoretical bounds. Thus when Turbo codes are applied on a modulation scheme, there is a significant improvement in bit-error-rate performance. Consequently, the number of retransmissions is decreased which helps to conserve power at the transmitter.

In this thesis, we will explore the application of these two established techniques for the modulation schemes used in 3GPP LTE standards: Orthogonal Frequency Division Multiplexing (OFDM) and Single Carrier-Frequency Division Multiple Access (SC-FDMA). We will also present their comparisons in terms of bit-error-rate and spectral efficiency.

Chapter 1

Introduction

Within next few years, demand for data transmitted over wireless medium is predicted to grow hundreds of folds. This requirement necessitates huge expansion of infrastructure in order to provide high speed and capacity to communication links. An immediate consequence of this will be many-fold increase in consumption of energy. With focus on improving speed and capacity, communication systems have never been designed to possess high energy efficiency. Presence of these low energy-efficient units in future infrastructure will thereby lead to huge energy loss. Therefore, there is a need to improve energy efficiency of such systems.

Power amplifier (PA) is one such low energy-efficient unit in the communication infrastructure. It is ubiquitous and is used to enhance the signal amplitude for a desired bit-error performance of a communication link. The input vs output curve for PA has two main regions: linear and nonlinear and is illustrated in Figure 1.1. When the input signal crosses the saturation level, the system suffers from spectral broadening and inter-modulation distortion, ultimately leading to loss of data. Thus, to properly reciprocate the variations in signal, PA needs to operate in the linear region which requires a constant bias voltage. Therefore, the optimum point of operation for a PA is the junction of linear and nonlinear region which enables distortion-less amplification of



Figure 1.1: Transfer function curve for power amplifier.

signal in presence of low bias voltage. Due to low bias requirement, energy consumption is the lowest at that point. However in practice, PA works much below the optimum point because of the fluctuations in the signal. If the fluctuations are high, more back-off is supplied to PA, shifting its operation point in the linear region which leads to poorer energy efficiency. Thus controlling the fluctuations of a signal is an important concern in effective energy utilization for the future.

1.1 Peak-to-Average Power Ratio

Fluctuations in a signal are characterized by the term Peak-to-Average Power ratio (PAPR). Let us consider a bandpass signal s(t) represented as:

$$s(t) = \tilde{s}(t)e^{j2\pi f_c t}.$$
(1.1)

Here $\tilde{s}(t)$ represents the low pass equivalent of s(t). Mathematically, PAPR is the ratio between the

highest amplitude square and the average amplitude square of a signal i.e.

$$PAPR_{s} \triangleq \frac{\max_{t} |\tilde{s}(t)|^{2}}{E\{|\tilde{s}(t)|^{2}\}}.$$
(1.2)

Lower the PAPR, closer the operation point of the PA could be to the optimum point. PAPR is expressed in decibels. Thus, PA can work at maximum efficiency if PAPR of the signal is 0 dB i.e. the message signal has a constant envelope.

1.2 Constant Envelope Modulation Schemes

A lot of work has been done in literature to decrease the PAPR of different modulation schemes. Continuous Phase Modulation (CPM) is one such technique with which lowest achievable PAPR i.e. 0 dB. In this technique, phase of the carrier is continuously modulated. This method is different from Phase Shift Keying (PSK) as it avoids abrupt jumps in the phase of carrier with every symbol. The abrupt jumps of MSK transfer lots of signal power in high frequency regions, which lead to low spectral efficiency in the desired bandwidth. In [1], authors have proposed Constant-Envelope Orthogonal Frequency Division Multiplexing (CE-OFDM) which can be obtained by phase modulating an Orthogonal Frequency Division Multiplexed (OFDM) signal. The same technique was adopted in [2] to construct Constant-Envelope Single Carrier Frequency Division Multiple Access (CE–SC-FDMA) signal. In this work, we will compare the performance of CE-OFDM and CE–SC-FDMA in wireless environment. Further, we will also present the comparison of their performance upon application of Turbo coding.

1.3 Thesis Overview

The remaining part of this thesis is organized as follows: In Chapter 2, the basics of OFDM and SC-FDMA are presented along with their comparison. In Chapter 3, the details of constant-envelope variants of SC-FDMA and OFDM are discussed. Their comparison is also studied. Chapter 4 covers brief summary of various coding schemes. It also discusses details, implementation and properties of Turbo coding. Finally, the performance of the two constant-envelope modulation schemes is compared after applying Turbo coding on them in Chapter 5.

Chapter 2

OFDM and SC-FDMA

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most popular modulation schemes in modern communications, with its use in systems ranging from DSL modems, DVB, 802.11 standards for Wireless LAN, WiMAX, LTE-A cellular standards, etc. OFDM was first introduced in mid '60s but, it had to wait for many years until technology and market demand made it practical. It was not until 1987 that it was proposed to be used for transmission of digital signals. Some important milestones in the development of OFDM to its present form were use of discrete Fourier transforms replacing the analog implementation, addition of cyclic extensions to ensure orthogonality among the subchannels etc. A detailed history of OFDM is presented in [3].

This preference for OFDM is due to the numerous advantages it has over other previously known modulation schemes. Some of these advantages are spectral efficiency, resilience to inter-symbol interference (ISI), immunity against selective fading, and requirement of simple equalizers; all of which make it well suited for broadband application. However, one major disadvantage of OFDM is its high signal fluctuations or peak-to-average power ratio (PAPR). This attribute of OFDM necessitates operation in a non-optimal region of the power amplifiers and thus makes it highly energy-inefficient.

The 3rd Generation Partnership Project (3GPP) is a collaboration between telecommunication organizations known as "Organizational Partners" who are responsible for standardization of third generation cellular wireless technology. They developed a standard known as Long Term Evolution (LTE) with an aim to achieve high-speed data for mobile phones and data terminals. To reduce the energy inefficiency of power amplifiers in LTE, 3GPP adopted a single-carrier variation of OFDM known as Single-Carrier frequency division multiple access (SC-FDMA) in its uplink. In this chapter, we will begin with a discussion on the implementation and properties of OFDM scheme. Next, we will present SC-FDMA and finally we will compare the performance of the two schemes.

2.1 OFDM

OFDM is a widely used multiplexing technique for broadband data transmission. In OFDM, the allocated spectrum is divided into bins of equal bandwidth. The central frequency of each of these bins, called subcarrier is modulated with a different data symbol. These data symbols belong to certain modulation schemes such as PAM, QAM, etc. The collection of these modulated subcarriers is then transmitted together and is known as a single OFDM symbol. Thus OFDM is a multi-carrier system.

OFDM is a special case of Frequency Division Multiplexing (FDM) with a more efficient way of packing data on the spectrum than in the conventional FDM techniques. A comparison between OFDM and conventional FDM is illustrated in Figure 2.1. The spectral efficiency in OFDM is possible because, the subcarriers in OFDM are overlapping and are made orthogonal by placing the spectral nulls of adjacent subchannels exactly on the central frequency of a given subchannel. This eliminates inter-carrier interference and the requirement for guard bands between subchannels.



Figure 2.1: Subcarrier spacing comparison between OFDM and conventional FDM.

2.1.1 Implementation

The block diagram for an OFDM system is shown in Figure 2.2. At the transmitter, first the information bits are mapped to data symbols (I_k) using one of the modulation schemes such as QPSK, QAM, etc. Sequence of these data symbols are then converted to M parallel streams which are mapped to N ($N \ge M$) equally spaced subcarriers. Thus an OFDM signal in a single block can be written as

$$s(t) = \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t}, \quad 0 \le t \le T_B.$$
(2.1)

In this equation, T_B is the OFDM block duration and $f_n = n/T_B$ is the central frequency of the subcarriers. If this signal is transmitted as it is; due to convolution with the channel's impulse response, OFDM symbols will extend and result in ISI. Hence a guard interval T_g needs to be inserted between different OFDM blocks. This interval is chosen to be slightly greater than the



Figure 2.2: Block diagram of an OFDM system.

longest expected delay spread of the channel. The contents of the guard interval is a cyclic prefix (CP) which is usually a signal taken from the end of an OFDM block. Thus the OFDM signal with cyclic prefix can be written as

$$s(t) = \begin{cases} \sum_{n=0}^{N-1} S_n e^{j2\pi f_n(t+T_B)} = \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t} e^{j2\pi n} = \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t} , -T_g \le t \le 0, \\ \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t} , 0 \le t \le T_B \end{cases}$$

$$= \sum_{n=0}^{N-1} S_n e^{j2\pi f_n t} , -T_g \le t \le T_B.$$

$$(2.2)$$

Figure 2.3 shows the effect of channel on an OFDM block. At the receiver, the interval occupied by the CP is distorted due to ISI. Since CP carries no new information, it is discarded and the effect of channel is isolated from the signal carrying desired data. Thus insertion of cyclic prefix helps to combat the effect of channel. Additionally, cyclic prefix also converts the signal's linear convolution with the channel impulse response to a circular convolution. To see the signal in discrete domain, let s(t) be sampled with a rate $\frac{1}{T_s}$ with $T_s = KT_B$. These samples will be

$$s[k] = \sum_{n=0}^{N-1} S_n e^{j2\pi nkT_s/T_B} = \sum_{n=0}^{N-1} S_n e^{j2\pi nk/N} = IDFT(S_n).$$
(2.4)



Figure 2.3: Effect of channel on an OFDM symbol.

This is equivalent to taking the inverse discrete Fourier transform of the data symbol sequence S_n . In practice, OFDM is implemented by taking the inverse Fast Fourier Transform (IFFT) which has low implementation complexity. The discrete signal is then converted to analog, sent to a power amplifier and finally it is transmitted across a wireless channel. While traversing the channel, the signal undergoes fading and a noise is added at the front end of the receiver. The received signal is converted back to digital domain and a discrete Fourier transform is taken. Since the symbols are already present in the transformed domain, its equalization step reduces to just multiplication with the transformed coefficients of the channel. This makes equalization in OFDM modulation simpler than single-carrier modulation, which requires use of adaptive algorithms that take time to converge. Post equalization, a hard decision is made on the soft data symbols and then they are de-mapped to obtain the transmitted bit sequence.

2.1.2 **OFDM - Advantages and Disadvantages**

OFDM is ubiquitously present in many of the high data rate wireless systems. A number of features of OFDM contribute to this popularity. In this section we will briefly discuss some of the advantages and disadvantages of OFDM modulation scheme.

Advantages

i) Resilience to fast fading

OFDM is a multicarrier modulation where the available bandwidth is divided into multiple frequency bins and the central frequency of each of these bins is modulated by a data symbol. This division of wideband spectrum into multiple bins helps OFDM to perform well even in presence of fast fading environment. This is due to the fact that even though fast fading will be observed in the available bandwidth but by properly designing the spacing between subcarriers, each of the bins will experience more or less flat fading.

ii) Simpler channel equalization

In OFDM, complexity of the equalization step is reduced because there is no need to equalize the channel as a whole. Due to flat fading experienced by the individual bins, it is sufficient to equalize individual bins which makes the step less complex.

iii) Resilience to narrow-band effects

The channel can exhibit frequency selective fading and narrow band interference. Since the data is distributed across many subcarriers, not all the data is lost at the same time due to these effects.

iv) Less intersymbol interference

In a single-carrier modulation technique let the overall data rate obtained be *R* symbols/second. If the same bandwidth is utilized using OFDM, the data rate of each subcarrier bin becomes $\frac{R}{N}$ symbols/second. Because of reduction in data rate by a factor of *N*, the OFDM symbol period is increased by the same factor. By choosing an appropriate value for *N*, i.e., the number of subcarrier bins, the length of OFDM symbol can be made much longer than the time span of the channel. Thus, the channel can distort only a small portion of the overall OFDM block. To remove the effect of channel, this part of the OFDM block, known as guard interval is filled with cyclic prefix

which do not carry any useful information and hence can be discarded at the receiver. The resultant signal is free from channel distortion. This is illustrated in Figure 2.3.

v) Spectrum efficiency

Using close-spaced overlapping subcarriers, OFDM makes efficient use of the available spectrum.

• **Disadvantages** While the above mentioned advantages favored OFDM, it still has many weaknesses which needs to be dealt for making better communication systems. Some of them are

i) High PAPR

An OFDM signal has a noise-like amplitude variation with a relatively large dynamic range. This leads to high PAPR. This impacts the RF amplifier efficiency as the amplifiers need to be linear and they need to accommodate large amplitude variations. High PAPR will inhibit the amplifier to work with a high efficiency level. In uplink, mobile user transmits data to the base station. Since a mobile has limited power and furthermore, it should be inexpensive, OFDMA is not a good choice for the uplink scenario. Figure 2.4 displays the PAPR of an OFDM signal in a single block. The PAPR can be evaluated using equation (1.1) and for this particular plot, it is 9.43 dB.

ii) Sensitivity to frequency synchronization

Carrier Frequency Offset (CFO) is one of the most common impairments found in a communication system. It is caused due to mismatch between the carrier frequencies used by the transmitter and the receiver. The spacing between subcarriers is very tight in OFDM and a slight mismatch in sampling can lead to loss of orthogonality among the subcarriers. This will cause the energy from one subcarrier to interfere with the adjacent one leading to poor bit error performance of the signal. There are three common sources for this problem. First, the frequency used by the receiver may not be same as the transmitter. Secondly, the relative motion between receiver and transmitter



Figure 2.4: Peak-to-average power ratio of an OFDM signal (9.43 dB).

can cause Doppler shifts. Finally, there could be some phase noise as well which can aggravate the problem.

2.2 SC-FDMA

Single Carrier - Frequency Division Multiple Access (SC-FDMA) is a single-carrier modulation scheme with similar throughput performance and complexity as that of an OFDM. Despite using multiple subcarriers, the term "single carrier" is used because unlike OFDM where each data symbol is carried by the individual subcarriers, in SC-FDMA the data symbols are sequentially carried over by a group of subcarriers (or frequency band) which are transmitted simultaneously. Due to this frequency allocation, the symbol duration in SC-FDMA is very small compared to that of an OFDM symbol. The difference between OFDM and SC-FDMA frames is illustrated in Figure 2.5 which is adopted from [4]. In this figure, 4 subcarriers are used and 4-QAM constellation is applied to map the bits into data symbols.



Figure 2.5: Frames of OFDM and SC-FDMA for 4 subcarriers.

This distributed frequency allocation for data symbols in SC-FDMA can be achieved by taking a discrete Fourier transform of them before feeding into the OFDM modulation blocks. This additional step helps SC-FDMA to reduce PAPR and hence become more energy-efficient in comparison with the OFDM scheme.

For a mobile user battery power is limited. Furthermore, the cost of the power amplifier is a bigger concern in mobiles compared to that of a base station. Hence, with these constraints for uplink, SC-FDMA works out as a better transmission scheme for mobile users in comparison with OFDMA. This led 3GPP to standardize it in the uplink of LTE.

SC-FDMA is a DFT-spread OFDM and its implementation is very similar to an OFDM system. The block diagram for an SC-FDMA system is presented in Figure 2.6 and on comparison with the block diagram of OFDM system (Figure 2.2), we observe that SC-FDMA requires an additional DFT and IDFT block at the transmitter and the receiver respectively.



Figure 2.6: Block diagram of an SC-FDMA system.

2.2.1 Implementation

Beginning at the modulator, the data bits are mapped according to a constellation. Sequence of these mapped symbols I_k is then divided into M parallel streams and an M point discrete Fourier transform of these streams is taken. This step transforms the time domain data symbols into frequency domain symbols. These M transformed symbols are then mapped to N ($N \ge M$) subcarriers. This assignment can be done in a number of ways, example of two of such methods: distributed subcarrier mapping and localized subcarrier mapping is shown in Figure 2.7. In distributed mapping, the symbols are equally spaced across all the subcarriers whereas in localized mode, the symbols are assigned to contiguous subcarriers. The subcarriers with no symbols (N-M) are usually assigned zero amplitude. After this step, the signal undergoes IDFT and a cyclic prefix is added to it before its transmission.

At the demodulator, the cyclic prefix of the received signal is removed and its DFT is taken to bring the signal to the frequency domain. At this step, the symbols are equalized to undo the effects of channel. Finally the N streams of signal are de-mapped to M streams whose IDFT yields input to the decoder.



Figure 2.7: Examples of subcarrier mapping modes.

2.2.2 Comparison with OFDM

Structurally, SC-FDMA is very similar to OFDM. Some of the similarities are:

- 1. Both of them are block-based modulation schemes
- 2. A cyclic prefix is added in both of the signals to counteract intersymbol interference
- 3. The transmission bandwidth is divided into many small spectral bins
- 4. Channel equalization is done in the frequency domain for both the signals

Further, if we compare the bit-error performance of SC-FDMA with OFDM, we observe that they perform very closely. In Figure 2.8, the performance of two schemes is shown in an additive white Gaussian noise (AWGN) channel. The two curves are overlapping indicating their similar bit-error performance. The waterfall curves indicating the Bit-Error-Rate (BER) performance of OFDM using different modulation schemes is compared in Figure 2.9. As expected, as the order of QAM is increased, the BER performance of the scheme degrades.



Figure 2.8: Bit Error Vs SNR performance of OFDM and SC-FDMA using 16-QAM in AWGN channel.



Figure 2.9: Bit Error performance of OFDM using different QAM modulations in AWGN channel.



Figure 2.10: Bit Error Vs SNR performance of OFDM and SC-FDMA using 16-QAM in Rayleigh fading channel.

A comparison for the BER performance of OFDM and SC-FDMA in Rayleigh fading channel is presented in Figure 2.10. Here we observe that at low SNR, both of them have a similar performance. However at high SNR values, SC-FDMA seems to combat fading better than OFDM. Figure 2.11 and Figure 2.12 show the waterfall curves for OFDM and SC-FDMA using different modulation orders.

One of the major weaknesses of OFDM is high PAPR which makes it highly energy-inefficient. In SC-FDMA, the data symbols undergo DFT transform before entering the OFDM modultation blocks. These additional processing on the signal helps it to abate the high PAPR problem of OFDM. Figure 2.13 shows the PAPR for a typical SC-FDMA signal. The difference in the maximum power level and its average value has decreased considerably when compared with OFDM. As a result of this, PAPR of SC-FDMA is approximately 4 dB better than OFDM. This allows SC-FDMA to work with less power backoff of the energy-expensive power amplifier resulting in lower energy consumption and consequently higher energy efficiency. Thus, SC-FDMA is favored



Figure 2.11: Bit Error performance of OFDM using different QAM modulations in Rayleigh fading channel.



Figure 2.12: Bit Error performance of SC-FDMA using different QAM modulations in Rayleigh fading channel.



Figure 2.13: Peak-to-average power ratio of an SC-FDMA signal (5.83 dB).

over OFDMA in uplink scenario where limited power of a mobile user is a concern.

Chapter 3

Constant Envelope OFDM and SC-FDMA

With an increasing demand for fast Internet services, Information and Communication Technology (ICT) is growing very rapidly. The reports of all the major telecommunication giants predict a large growth for demand especially for the data transmitted over wireless medium [5] [6] [7]. To cater to this growth, mobile service providers are expanding their infrastructure. But alongside this expansion comes the problem of increased energy consumption which adversely affects the environment.

According to GreenTouch – a consortium of leading ICT industries, academic and non-governmental research experts dedicated to improve energy efficiency of telecommunication networks; the communication networks consume roughly 2% of world's energy [8]. The radio access part uses more than 50% of this energy [9]. Thus, by seeking new energy-efficient technology, telecommunication companies will not only play their social responsibility of energy conservation, but they can also bring down their operational cost.

3.1 Background

Among the modules present in the radio access, the component which takes the biggest share (50 - 80%) of energy is the power amplifier [9]. As discussed in Chapter 1, the operating point of a power amplifier plays a key role in determining its efficiency. This operation point is decided by the fluctuations or the PAPR of the transmitted signal. The higher the PAPR, the lower will be the operation point in the linear region of the curve shown in Figure 1.1. This shift of operation point requires power backoff. Thus a lower operation point will require more backoff and hence will be expensive in terms of energy consumption.

As we discussed in Chapter 2, OFDM has a very high PAPR (close to 9.5 dB) and requires components in the system with a large linear range which are usually expensive. Absence of such units will cause a nonlinear distortion in the signal, leading to performance degradation. Further, since operation with a high PAPR is not efficient in terms of energy, a number of techniques have been formulated to reduce PAPR of the OFDM signal. A detailed summary of these techniques can be found in [10]. The lowest achievable PAPR (0 dB) of a signal can be achieved if the peak power of the signal equals the average power. This is possible only with a constant-envelope phase modulated signal. Based on this concept, in [11], authors proposed a constant-envelope variation of OFDM. The proposed modulation scheme yields a signal which has 0 dB PAPR and therefore PA can work at the most energy-efficient operation point.

In this chapter we will begin with a discussion on Constant-Envelope OFDM (CE-OFDM) and compare its performance with OFDM. Next, we will describe Constant-Envelope SC-FDMA (CE–SC-FDMA). Finally we will compare the performance of the two constant envelope schemes.



Figure 3.1: Mapping of OFDM signal to CE-OFDM.

3.2 CE-OFDM

For the power amplifiers to work at the optimum operation point, the input signal should have 0 dB PAPR, i.e., the peak signal power should be same as the average power. The class of signals which possess this property are the constant-envelope signals where the information is present in the phase of the signal.

In order to bring down the PAPR of OFDM signal to 0 dB, the OFDM modulated signal is embedded in the phase of a constant-envelope signal. The resultant signal is CE-OFDM and it can be thought of as a mapping of OFDM signal to a unit circle as shown in Figure 3.1. Mathematically, let the modulated baseband OFDM signal be

$$m(t) = A_m(t)e^{j2\pi\phi_m(t)}.$$
 (3.1)

For CE-OFDM, m(t) is used to phase modulate a constant envelope signal, i.e., CE-OFDM signal

should be of the form :

$$s(t) = Ae^{jCm(t)}$$
(3.2)
= $Ae^{jCA_m(t)(\cos(2\pi\phi_m(t)) + j\sin(2\pi\phi_m(t)))}$
= $Ae^{-CA_m(t)\sin(2\pi\phi_m(t))} \cdot e^{jCA_m(t)\cos(2\pi\phi_m(t))}.$ (3.3)

For 0 dB PAPR, we require s(t) to be a constant-envelope signal. This is only possible when $\phi_m(t) = 0$ or the OFDM signal m(t) is completely real.

According to equation (2.4), a baseband OFDM signal is an IDFT of the subcarrier-mapped symbols in a time period *T*. Let the total number of subcarriers be N_{dft} and the data symbols be $I_k\Big]_{k=1}^N$. Then, a real-valued OFDM symbol can be obtained by using the following subcarrier mapping before IDFT transformation

$$\mathbf{X} = [0, I_1, I_2, \cdots, I_N, 0_{1 \times N_{zp}}, 0, I_N^*, \cdots, I_1^*].$$
(3.4)

Here, $N_{zp} = N_{dft} - 2N - 2$. Taking the IDFT of **X**, we get

$$m[i] = \sum_{n=0}^{N_{dft}-1} X[n]e^{j2\pi ni/N_{dft}}$$

= $\sum_{n=1}^{N} \{I_n e^{j2\pi ni/N_{dft}} + I_n^* e^{j2\pi i} e^{-j2\pi ni/N_{dft}}\}$
= $\sum_{n=1}^{N} 2\{\mathbb{R}[I_n] \cos(\frac{2\pi ni}{N_{dft}}) - \mathbb{I}[I_n] \sin(\frac{2\pi ni}{N_{dft}})\}.$ (3.5)

This OFDM signal is completely real and therefore it is suitable for the phase modulator to generate the CE-OFDM signal. According to equation (3.2), the phase modulator also requires a normalization constant C. The purpose of this constant is to fix the angle spread of the unit circle where



Figure 3.2: Change of signal constellation of CE-OFDM due to change in *h*.

OFDM is mapped to after phase modulation. C can be mathematically written as

$$C = 2\pi h \times C_N. \tag{3.6}$$

Here $C_N = \sqrt{\frac{1}{\sigma_I^2 N}}$ is a constant used to normalize the variance of the message signal m(t). This will also ensure that the phase variance is independent of the number of symbols used (*N*). σ_I^2 is the variance of the data symbols, i.e., $\sigma_I^2 = E(|I_k|^2)$. The other constant *h* which appears in equation (3.6) is known as the modulation index. Its value decides the performance and spectral property of the CE-OFDM signal. Figure 3.2 shows the effect of changing *h* on the constellation of CE-OFDM. As the value of *h* increases, the phase spread of constellation broadens.

The generated CE-OFDM signal has 0 dB PAPR and thus it can be efficiently amplified with nonlinear power amplifiers which are not only low-cost, but can also work with good power efficiency.


Figure 3.3: Implementation of CE-OFDM system.

3.2.1 Implementation

The CE-OFDM system shares many of the blocks with a conventional OFDM system. Further, by adding a few new modules, an existing OFDM system can be modified to generate the CE-OFDM signal. The block diagram of a CE-OFDM system built over an OFDM system is shown in Figure 3.3.

The serial data bit sequence is mapped according to a fixed QAM constellation to generate data symbols I_k . A block of N of these symbols are then mapped to subcarriers according to the mapping given by equation (3.4). Next, an N_{dft} inverse discrete Fourier transform of these mapped symbols is taken which results in a real OFDM signal. This signal is normalized and a cyclic prefix is attached to each block before feeding it to the phase modulator. The transformed OFDM signal which comes out of the phase modulator has constant envelope with 0 dB PAPR and is the CE-OFDM signal. This signal can be amplified using a non-linear PA before sending it to the transmitting antenna.

This signal travels through the channel and undergoes fading. At the receiver, an additive white Gaussian noise is added to the signal. To extract out the message from this signal, first the signal

is equalized to remove the effects of multipath. Then, the cyclic prefix is removed and phase of the signal is extracted out. This phase is multiplied with the required reciprocal of the normalizing constant C and is then fed to an OFDM demodulator. Finally, the signal is passed through a threshold detector and the received symbols are de-mapped to obtain the transmitted bit sequence.

3.2.2 Performance of CE-OFDM

In order to see the performance of CE-OFDM, simulations were carried out in the MATLAB environment. The parameters used for the simulation are enumerated in Table 3.1

| M: QAM Modulation for data symbols | 16 |
|--|-------------------------------|
| N _{dft} : DFT Size | 512 |
| N_c : Number of subcarriers | 128 |
| N: Number of Symbols in a single block | 64 |
| J _{os} : Oversampling Factor | 4 |
| T_B : Block Duration | $2N \times 10^{-6}$ |
| <i>F_s</i> : Sampling Rate | $\frac{J_{os} \times N}{T_B}$ |
| T_g : Guard Interval | 10 ⁻⁵ |

Table 3.1: Simulation Parameters.

Further, two types of channels were considered: AWGN Channel and Rayleigh fading channel. We will discuss them next.

AWGN Channel

In this scenario, the signal passes through the channel unattenuated. At the front end of the receiver, an additive white Gaussian noise is added to the signal. Therefore, the low pass



Figure 3.4: Bit-Error Performance of CE-OFDM in AWGN Channel.

equivalent of the received signal is

$$r(t) = s(t) + n(t).$$
 (3.7)

Here $s(t) = Ae^{jCm(t)}$ and n(t) is a Gaussian random process with mean E[n(t)] = 0 and variance $E[n(t)n(t-\tau)] = \frac{N_0}{2}\delta(\tau)$.

The performance of CE-OFDM in AWGN channel for different modulation index (*h*) is plotted in Figure 3.4. From the graph, we observe that as we increased the modulation index from .1 to 1.5, the bit error performance improves. This is because, as we increase $2\pi h$, the phase of CE-OFDM signal spreads across a greater part of the unit circle. The spread of this phase is illustrated in Figure 3.2. However, beyond the modulation index 1.5, the bit-error performance of the signal degrades. This is because of the fact that, at a high value of modulation index, the signal can have phase beyond 2π . It would be difficult for phase modulator to decode such phases correctly, hence probability of error increases.

• Rayleigh Fading Channel

In a wireless environment, usually multiple copies of a signal arrive at the receiver after traveling through different paths. The receiver sees a summation of all these copies, combined



Figure 3.5: Bit-Error Performance of CE-OFDM in Rayleigh Fading Channel.

result of which is a highly attenuated and distorted copy of the original signal. This is known as fading. At the receiver end, an additive white Gaussian noise is added. Mathematically, the received signal can be represented as

$$r(t) = \alpha e^{j\theta} s(t) + n(t). \tag{3.8}$$

Here the channel's impulse response is taken as $h(t) = \alpha e^{j\theta} \delta(t)$. As discussed in Section 2.1.1, the duration of cyclic prefix is set according to the channel's worst delay spread. For the simulations, channel's delay spread is assumed to be 9×10^{-6} second and therefore the guard interval T_g is set to the value 10^{-5} second. Further, it is assumed that the channel's impulse response is known at the receiver. This is possible by using appropriate pilot and training symbols. Using these channel coefficients, the message signal is equalized with a Minimum Mean Square Error equalizer (MMSE). This step removes the effect of fading. The resultant signal is then fed to the CE-OFDM decoder to obtain the transmitted bits. The performance of CE-OFDM in Rayleigh fading channel is plotted in Figure 3.5. It is clear that the performance of the scheme has degraded as compared to the AWGN case. Further, it can be observed from the graph that the best performance of the scheme is obtained the modulation index 1.5 as in the case of the AWGN channel.

3.3 CE–SC-FDMA

SC-FDMA was studied in-depth in Section 2.2 and in comparison with OFDM, it turned out to be a more energy-efficient option. However, in spite of a low PAPR, its implementation would still require an expensive linear PA. Based on the concept used to generate CE-OFDM from an OFDM signal, a constant-envelope counterpart for SC-FDMA called CE–SC-FDMA was proposed in [2] for nonlinear satellite channels. Similar to CE-OFDM, it also has 0 dB PAPR and thus nonlinear power amplifiers can be used.

CE–SC-FDMA can also be mathematically represented by Equation (3.2) with m(t) being the SC-FDMA signal used to phase modulate the constant-envelope signal. However, the normalizing constant *C* will be different than that of the CE-OFDM scheme. This is because of an additional DFT step involved in the generation of SC-FDMA signal. The *C* for CE–SC-FDMA will be $2\pi h \sqrt{\frac{1}{N^2 \times \sigma_I^2}}$.

3.3.1 Implementation

The implementation for generation of CE–SC-FDMA is very similar to CE-OFDM and is illustrated in Figure 3.6. The data bits are modulated to an SC-FDMA signal by sequentially going through the steps of constellation mapping, *N*-point-DFT, subcarrier mapping, and N_{dft} -point IFT. Then, cyclic prefix is added to the signal and it is then fed to the phase modulator. The output of the phase modulator is the 0 dB PAPR CE–SC-FDMA signal. This signal is then converted into analog form before it is amplified by a power amplifier for transmission.

At the receiver, the signal is first equalized to undo the effects of channel. This equalized sig-



Figure 3.6: Implementation of CE–SC-FDMA system.

nal passes through the phase demodulator and SC-FDMA demodulator to yield the transmitted information bits.

3.3.2 Performance of CE–SC-FDMA

In order to see the performance of CE–SC-FDMA scheme, simulations were carried out in MAT-LAB environment using the parameters specified in Table 3.1. The bit-error-rate performance of CE–SC-FDMA is AWGN and Rayleigh fading channels is plotted in Figure 3.7 and Figure 3.8 respectively.

For both channels we observe that as the modulation index (h) is increased to the value 2, the performance of the scheme improves. Beyond that value, the performance degrades till it reaches an error floor.



Figure 3.7: Bit-Error Performance of CE-SC-FDMA in AWGN Channel.



Figure 3.8: Bit-Error Performance of CE–SC-FDMA in Rayleigh Fading Channel.

3.4 Performance Comparison

It is clear that the constant-envelope variants of OFDM and SC-FDMA perform much better in terms of energy efficiency than their conventional implementation. However with the exploding demand for high speed Internet, good performance and high spectral efficiency of the schemes are also important. It will be interesting to see how much degradation in performance is incurred to achieve energy-efficiency by CE schemes. In this section, we will compare the performance of different modulation schemes discussed so far.

3.4.1 OFDM/SC-FDMA vs. CE-OFDM/CE–SC-FDMA

For both CE-OFDM and CE–SC-FDMA we previously observed that their bit-error performance improves as we increase the modulation index to a certain value, after which the performance saturates to error-floor. In this section we will compare the performance of these constant envelope schemes with their parent signals.

1. OFDM vs. CE-OFDM

The bit-error performance of OFDM and CE-OFDM is plotted in Figure 3.9. From the graph we observe that the CE-OFDM signal with modulation index 0.7 performs very close to the OFDM signal. On further increasing the modulation indexes to h = 1.5, we see that CE-OFDM outperforms OFDM signal.

Next, we wish to compare the spectrum of the two schemes and analyze the effect of increasing modulation index on the spectrum of CE-OFDM. Figure 3.10 compares the power spectral density (PSD) of OFDM and CE-OFDM. In the plot, frequency is normalized with respect to the effective double sided bandwidth (*W*) which is defined as the twice of highest frequency subcarrier used, i.e., $W = \frac{N}{T_B}$ [11, p. 51]. We can notice that for OFDM, most of the power (99% [11, p. 55]) is restricted within 0.5*W* Hz. For CE-OFDM signal, we see



Figure 3.9: Bit-Error performance comparison of OFDM and CE-OFDM Signal.

that as the modulation index (h) increases, a broader spectrum is required to fully contain 99% of the power. In Section 3.2.2, we have seen that as the modulation index h is increased, the bit-error-rate curve improves. Thus, selection of h requires a trade-off between tighter spectrum and bit-error performance.



Figure 3.10: Power Spectral Density of OFDM and CE-OFDM Signal.

2. SC-FDMA vs. CE-SC-FDMA

The bit-error curves and PSD graphs are shown in Figure 3.11 and Figure 3.12 respectively. From the plots, it can be seen that they follow the same trend as the OFDM case. At h = 0.7, the performance of CE–SC-FDMA and SC-FDMA becomes comparable. Further, with higher modulation index, BER curve of CE–SC-FDMA improves at the expense of broader spectrum.

3.4.2 CE-OFDM vs. CE-SC-FDMA

OFDM and SC-FDMA have very similar bit-error performance in AWGN channel and low SNR regions of the curve for Rayleigh fading channel. However, SC-FDMA had a significant advantage over OFDM when their PAPR were compared. Both CE-OFDM and CE–SC-FDMA have 0 dB PAPR, so in terms of power efficiency none of them have an edge over the other.

The next comparison parameter is the bit-error performance. Figure 3.14 shows the performance curves of two schemes in AWGN and Rayleigh fading channel. In AWGN channel, it can be seen



Figure 3.11: Bit-Error performance comparison of SC-FDM and CE–SC-FDMA Signal.

that both of their performances are comparable. The only difference being that CE-OFDM starts to go towards the error-floor for h = 2, while CE–SC-FDMA has a good performance for that modulation index. For Rayleigh fading channel, at high SNR, the performance of CE–SC-FDMA is better than CE-OFDM. A similar behavior was observed when we compared SC-FDMA and OFDM in Figure 2.10.

Considering the similarity in performance of the schemes, it is expected that they have similar PSD as well. This is verified from the Figure 3.13. The explanation for better performance of



Figure 3.12: Power Spectral Density of SC-FDMA and CE–SC-FDMA Signal.



Figure 3.13: Power Spectral Density of CE-OFDM and CE–SC-FDMA Signal.



Figure 3.14: Bit-Error performance comparison of CE-OFDM and CE-SCFDMA Signal.

CE–SC-FDMA for h = 2 is visible in its PSD.

Thus, to conclude we summarize that CE modulation schemes have a clear advantage over conventional schemes in terms of energy efficiency. In terms of performance, there is a trade-off between the power spectral efficiency and bit-error performance. The finer the phase difference (low h), better will be the power spectral efficiency; but lower will be the bit-error performance.

Further, between CE–SC-FDMA and CE-OFDM, in AWGN channel there is no significant difference in terms of energy-efficiency, bit-error performance or spectral efficiency. However, in Rayleigh fading channel, CE–SC-FDMA performs better than CE-OFDM at high values of SNR. Thus, the extra computational blocks in CE–SC-FDMA are helpful to combat fading.

Chapter 4

Turbo Codes

In 1948, Claude Shannon referred to as the "Father of Information Theory," proposed the classical channel coding theorem in paper [12]. According to the theorem, in a noisy communication channel with bandwidth W, where the noise is additive, white, and Gaussian, information bits can be transmitted reliably with a rate R as long as $R \le C$; where C is the channel capacity or the Shannon's limit and it can be computed by the formula

$$C = W \log_2(1 + SNR) \qquad [bits/second]. \tag{4.1}$$

Further, Shannon also showed that for an $R \le C$, any bit error probability $P_b > 0$ can be achieved by incorporating appropriate encoding and decoding operations in a system as presented in Figure 4.1. However, Shannon did not specify how to implement coding to achieve this capacity.

Two years later, Hamming came up with the first error correcting code in his paper [13]. He stated that by introducing redundancy in the information bits, some of the errors can be corrected at the receiver. Based on this concept, during the next 50 years, the area of coding theory broadened with the introduction of many practical coding schemes in order to achieve performance closer to the Shannon's limit. Until 1990, the closest these practical implementations could get to the Shannon



Figure 4.1: System diagram for channel coding.

's limit was 3 dB [14, p. 376]. A detailed summary of this history is presented in [15], [16].

In 1993, a new class of codes called Turbo codes were introduced by Berrou, Glavieux, and Thitimajshima in [17]. They were based on the idea of repetitive decoding using iterative feedback. The authors surprised everyone in the community by claiming that Turbo codes can perform only half decibel away from the Shannon's limit [14, p. 376]. Over the time, many researchers verified their claim and thus Turbo codes were adopted in many systems.

Following Turbo codes, another type of code called Low-Density Parity-Check (LDPC) codes reappeared in active research. They were first invented in 1962 by Gallager in his Ph.D. thesis but were then neglected due to lack of technology required for their implementation. They also exhibited performance very similar to Turbo codes.

In this chapter, we will begin with a brief summary of some of the important coding schemes. Next, we will describe the basics of Turbo codes and present its implementation. Finally, we will verify the performance of Turbo codes on binary symbols through simulations.

4.1 Summary of coding schemes

Coding theory is based on intelligently adding redundant bits to the information bits so that at the decoder, some of the errors caused due to the noisy channel can be rectified. Designing a channel code involves a tradeoff between energy efficiency and bandwidth efficiency. Codes with lower rate (i.e., more redundancy) can in general correct more errors. Consequently, the communication system can operate with a lower transmit power, can transmit over longer distances, and can tolerate more interference. These properties make coding energy-efficient. However, low-rate codes will have large overhead and hence will consume more bandwidth. Further, decoding complexity grows with the increase in code length.

Since the errors caused by the channel in the received signal are corrected at the receiver and not by a retransmission, channel codes are called Forward Error Correction (FEC) codes. Based on their structure, FEC codes can be categorized into two groups: block codes and convolutional codes.

4.1.1 Block Codes

Block codes are a class of codes which operate on one block of data at a time. The encoder of the block code first divides the data in blocks of *k* bits. Each of these *k*-bits sized blocks $u = [u_0, u_1, \dots, u_k]$ is called a message block. This message block is then encoded to create an output block of size *n*, and is represented by $v = [v_0, v_1, \dots, v_n]$. For linear block codes, encoding can be represented in form of matrix multiplication as

$$\mathbf{u}.G_{_{k\times n}} = \mathbf{v}.\tag{4.2}$$

Here *G* is the generator matrix for a particular coding scheme. For a message of size *k*, there are (n - k) redundant bits. Further, since there are 2^k possible messages, there can be only 2^k possible encoded codewords. These codewords v along with their message block mapping are known as

(n, k) block codes. The code rate for these codes is $R = \frac{k}{n}$. Some common block codes are:

• Hamming Code

Hamming codes were the first codes to appear in the literature of coding theory. They are capable of detecting at most two errors and can correct one error in a block. Depending on the desired code length, there are infinite Hamming codes possible. For any integer $m \ge 3$, the class of Hamming codes can be represented as

| Description | Symbol | Value |
|-----------------------------|-----------|---------------|
| Code length | п | $2^m - 1$ |
| No of information bits | k | $2^m - m - 1$ |
| No of parity check bits | n-k | т |
| Minimum distance | d_{min} | 3 |
| Error Correcting capability | t | 1 |

• BCH Code

The Bose-Chaudhuri-Hocquenghem (BCH) codes are a generalization of Hamming codes. BCH codes form a class of infinite cyclic binary codes, and for any positive integer $m \ge 3$, and error-correction-capability $t \le 2^m - 1$, they can be represented by:

| Description | Symbol | Value |
|-----------------------------|-----------|----------------|
| Code length | п | $2^m - 1$ |
| Error Correcting capability | t | $\leq 2^m - 1$ |
| No of parity check bits | n-k | $\leq mt$ |
| Minimum distance | d_{min} | $\geq 2t + 1$ |

One of the key features of BCH codes is that during the code design, there is a precise control over the number of symbol errors correctable by the code. Thus, BCH codes can be

designed to suit any channel condition. Additionally, the decoding of BCH codes is based on an algebraic method known as syndrome decoding which can be implemented with low complexity.

• Reed Soloman (RS) Code

RS codes are a type of non-binary cyclic codes which are applied on symbols made of *m*bit sequences (m > 1). RS codes are suitable for errors which occur in bursts because, by correcting each symbol, many bits can be corrected at once. For a (n, k) RS code, the parameters are:

| Description | Symbol | Value |
|--|-----------|--|
| Code length (in symbols) | п | $\leq 2^m - 1$ |
| Number of parity check symbols | n-k | 2t |
| Error Correcting capability (in symbols) | t | $\left\lfloor \frac{n-k}{2} \right\rfloor$ |
| Minimum distance | d_{min} | 2t + 1 |

RS codes can achieve the largest possible code d_{min} for any linear code with the same (n, k) values. Further, the parameters of RS code can be adjusted to get a required packet size (by changing *m*), and error correcting ability (by changing *k*). However increased protection comes at the cost of increased implementation complexity and bandwidth.

4.1.2 Convolutional Codes

Like the encoder of block codes, the encoder of convolutional codes also accepts message blocks of size k bits to produce n bits of output; however, the values of k and n are relatively small. The major point of difference from the block codes lies in the computation of the n output bits which not only depends on the current k input bits, but also on the m previous message bits. Thus, an encoder of convolutional code requires memory of size m and it acts as a state machine with 2^m



Figure 4.2: Example of Convolutional code encoder.

states. Convolutional codes are represented as (n, k, m). At any point of time, the encoder inputs k bits to give out n bits and moves to a new state. The code rate for convolutional codes is $R = \frac{k}{n}$. An example of a (2, 1, 2) convolutional encoder is illustrated in Figure 4.2. The rectangular blocks in the figure represent memory or delay blocks. This encoder can be also represented by n generator polynomials. For the given example, the generator polynomials are:

$$g_1(D) = 1 + D^2$$
, and
 $g_2(D) = 1 + D + D^2$.

For decoding of convolutional codes, Viterbi algorithm (VA) is the most popular algorithm which was first published in [18]. VA is based on the maximum likelihood (ML) detection of input bits, given the soft-received bits. The complexity of the VA decoder depends on the trace-back length i.e., the number of decoding steps completed before deciding about a bit. Convolutional codes are good for correcting single bit errors, however, they are not efficient for correcting bursty errors.

4.2 Turbo Codes

Post the introduction of Turbo codes in 1993, they became very popular because they guaranteed a performance very close to the Shannon's limit. Turbo codes owe their effectiveness due to two



Figure 4.3: RSC encoder implementation for generator polynomial (4.3).

main features: a code design that produces a code with random-like property, and a decoder which utilizes soft received bits and iterative decoding. In this section, we will first present the implementation of Turbo codes. We will also verify its closeness to the Shannon 's limit through simulation results.

4.2.1 Implementation

Turbo code encoders contain two Recursive Systematic convolutional (RSC) codes separated by an interleaver. RSC encoders are essentially convolutional encoders with a feedback. An example of RSC is presented in Figure 4.3. This design of these (2, 1, 3) RSC encoders is taken from the 3GPP LTE standards [19]. The generator matrix of the encoders is:

$$G = \left[1, \frac{g_1(D)}{g_2(D)}\right],\tag{4.3}$$

where, $g_1(D) = 1 + D^2 + D^3$ and $g_2(D) = 1 + D + D^3$.

Additionally, its trellis diagram is shown in Figure 4.4.

In Turbo code encoders, the RSCs can be combined in three different ways : Parallel Concatenated



Figure 4.4: Trellis Diagram for Convolutional code given by generator polynomial (4.3).

Convolutional Codes (PCCCs), Serial Concatenated Convolutional Codes (SCCCs), and Hybrid Concatenated Convolutional Codes (HCCCs). In this work, we will discuss the PCCC implementation as specified in the specifications of 3GPP LTE standards [19].



Figure 4.5: Encoder for rate $\frac{1}{3}$ Turbo codes.

The structure of a Turbo code encoder is presented in Figure 4.5. The source bits are first divided into blocks of size *K* and each of this block is represented as $u_s = [u_0, u_1, \dots, u_K]$. During the



Figure 4.6: Decoder for rate $\frac{1}{3}$ Turbo codes.

processing of each of the block, the data bits u_s are sent sequentially to the first RSC encoder which outputs the parity bits u_{p_1} according to its generator polynomial. The data block u_s is also fed to a *K*-bit interleaver which reads the bits in pseudo-random order. These interleaved bits are inputs to the second RSC encoder which has the same transfer function as that of the first RSC encoder. The resultant is the second parity bit sequence u_{p_2} . The systematic bit u_s along with the parity bits u_{p_1} and u_{p_2} is then multiplexed and sent for modulation and transmission through the channel. Thus, Turbo codes are essentially concatenation of two convolutional codes.

The block diagram for the decoder is presented in Figure 4.6. At the decoder, first the received soft bits y are de-multiplexed to generate three parallel streams: $[y_s, y_{p_1}, y_{p_2}]$, corresponding to the systematic bits and the two parity bits. The two decoder blocks in the figure compute the Log Likelihood Ratio (LLR) of the systematic bits or its interleaved sequence. LLR is defined as:

$$LLR(u_s) = \ln\left(\frac{\operatorname{prob}(y \mid u_s = 1)}{\operatorname{prob}(y \mid u_s = 0)}\right)$$
(4.4)

The algorithm for computation of these LLR values will be discussed in the next subsection.

Each of the decoder passes an extrinsic information to the other decoder and thus there is a loop

which ensures that all the information is used while making the decision. At each decoder, both the extrinsic information passed by the other decoder and the information obtained from the received bits are used to compute the LLR values. Because of some latency involved in the exchange of extrinsic information, Turbo decoding is implemented as an iterative process. First, *K*-bit sized blocks from the streams y_s and y_{p_1} are fed to decoder-1 to yield LLR₁. Decoder-1 then passes extrinsic information from these LLR values to decoder-2. Decoder-2 uses this extrinsic information along with *K*-bit sized blocks of interleaved y_s and y_{p_2} to compute LLR and in-turn sends extrinsic information to decoder-1. This completes one iteration.

The total number of iterations required is ideally the step when both decoders converge towards the same decision with the same probabilities, for each of the data considered. This led to the name "turbo" codes. After the required number of iterations, the LLR values of both the decoders along with channel information is combined to guess the transmitted *K*-length bit sequence.

4.2.2 Decoding Algorithm: BCJR Algorithm

The Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm also known as forward-backward algorithm is a maximum a posteriori (MAP) algorithm which was initially formulated for decoding of convolutional codes [20]. However, for convolutional codes, it is not generally used because it performs almost at the same level of the Viterbi algorithm in spite of having heavier computational requirements. Nevertheless, BCJR algorithm suits the codes which require iterative decoding such as Turbo codes.

The goal of a binary-symbol MAP decoder is to determine the log likelihood ratio for input bit sequence u_k based on the soft-received bits **y** as follows:

$$L(\hat{u}_k) = \ln \frac{P(u_k = +1 | \mathbf{y})}{P(u_k = -1 | \mathbf{y})} = \ln \frac{P(u_k = +1, \mathbf{y})/P(\mathbf{y})}{P(u_k = -1, \mathbf{y})/P(\mathbf{y})}.$$
(4.5)

Thus, if $L(\hat{u}_k) > 0$, $\hat{u}_k = +1$ else it is decoded as -1. After incorporating the trellis of the RSC encoder given by Figure 4.4, the above equation can be written as:

$$L(\hat{u}_k) = \ln \frac{\sum_{S^+} P(s_{k-1} = s, \ s_k = s', \ \mathbf{y})}{\sum_{S^-} P(s_{k-1} = s, \ s_k = s', \ \mathbf{y})}, \qquad \{s, s' \in S\},$$
(4.6)

where S^+ and S^- denote the all the state transition pairs (s, s') which are caused by $u_k = +1$ and $u_k = -1$ respectively. According to BCJR algorithm, the joint probability in the above equation can be written as:

$$P(s, s', y) = \underbrace{P(s_{k-1} = s, y_{j < k})}_{P(s_{k-1} = s', y_{k} | s_{k-1} = s)} \cdot \underbrace{P(y_{j > k} | s_{k} = s')}_{P(s_{k-1} = s)}$$
(4.7)

$$= \alpha_{k-1}(s) \quad . \qquad \gamma_k(s,s') \quad . \qquad \beta_k(s'). \tag{4.8}$$

Here, $y_{j < k}$ and $y_{j > k}$ represent all the bits received before and after time k respectively.

By forward and backward recursions, $\alpha_k(s')$, $\beta_{k-1}(s)$ values can be computed as:

$$\alpha_k(s') = \sum_{s} \gamma_k(s, s') \cdot \alpha_{k-1}(s), \tag{4.9}$$

$$\beta_{k-1}(s) = \sum_{s} \gamma_k(s, s') \cdot \beta_k(s'), \qquad (4.10)$$

with initial and boundary conditions:

$$\alpha_0(s) = 1, \ \alpha_0(s \neq 0) = 0, \text{ and}$$

 $\beta_K(s = 0) = 1, \ \beta_K(s \neq 0) = 0.$

The branch transition probability can be calculated as:

$$\gamma_k(s, s') = P(s_k = s' \mid s_{k-1} = s) \cdot P(y_k \mid s_k = s', s_{k-1} = s)$$
(4.11)

$$= P(y_k \mid u_k) . P(u_k).$$
(4.12)

For BPSK modulation in AWGN channel with $SNR = \frac{E_s}{N_0}$,

$$p(y_k \mid u_k) = \frac{1}{\sqrt{\pi N_0}} exp\left\{-\frac{(y_k - \sqrt{E_s}u_k)^2}{N_0}\right\}$$
(4.13)

Thus Equation (4.6), can be written as:

$$L(\hat{u}_{k}) = \ln \frac{\sum_{S^{+}} \alpha_{k-1}(s) \cdot \gamma_{k}(s, s') \cdot \beta_{k}(s')}{\sum_{S^{-}} \alpha_{k-1}(s) \cdot \gamma_{k}(s, s') \cdot \beta_{k}(s')}.$$
(4.14)

The MAP algorithm presented so far is not practicable for implementation due to computations involving exponential functions. To overcome this disadvantage, many algorithms exist which are low-complexity modifications of the MAP algorithm. One such algorithm is the MAX-Log-MAP algorithm which although degrades the performance of decoder to a small extent, but enables significant reduction in the computation complexity. Step-by-step details of this algorithm can be found in [21].



Figure 4.7: BER performance of BPSK signal in AWGN channel with Turbo coding.



Figure 4.8: BER performance of Turbo codes with varying interleaver size K.

4.2.3 Performance of Turbo Codes

To verify the performance of Turbo codes, we implemented the system with BPSK symbols in AWGN channel using the MAX-Log-MAP BCJR algorithm for the decoder. Figure 4.7 is a plot of the simulation results. We observe that there is a significant improvement in the performance after application of Turbo coding. With Turbo codes, the BER performance gets very close to the Shannon 's limit. Further, with each iteration, the performance improves. However, after some time, the performance converges.

The interleaver size also affects the performance of Turbo codes. As the interleaver size K increases, the BER performance improves. Figure 4.8 shows this variation after 5 decoding iterations. It can be also observed that the error-floor appears first for the lower interleaver size.

Chapter 5

Turbo Codes with CE-OFDM and CE-SC-FDMA

In the previous chapter we saw Turbo code's performance on BPSK signaling. We verified through simulations, that its BER performance curve is very close to the Shannon's limit. While Turbo coding on BPSK modulated data is very simple to implement, it is not bandwidth-efficient. With BPSK modulation, only one bit is sent per signaling interval. Further, upon application of Turbo coding, due to addition of parity bits, the data rate decreases and hence the spectral efficiency deteriorates even further.

By puncturing a Turbo coded bit stream i.e., by removing few parity bits, the spectral efficiency can be improved slightly at the expense of some loss in the performance. However the data rate will be always less than one bit per period when BPSK modulation is used. In order to maximize the spectral efficiency, powerful near-capacity performing codes like Turbo codes need to be applied on higher modulation schemes.

In this chapter, we will first briefly describe the approaches developed so far to apply Turbo codes on higher constellation schemes. Next, we will use one of the methods - the pragmatic approach to apply Turbo codes on the previously discussed constant-envelope modulation schemes. Finally we will conclude the work by presenting a comparison between CE-OFDM and CE–SC-FDMA's performance with Turbo coding.

5.1 Turbo codes and Multilevel modulation schemes

The demand for wireless data is increasing exponentially, however the frequency spectrum which is serving it is limited. Therefore, spectral efficiency is an important criterion for any modulation scheme. Since higher constellation leads to better spectral efficiency than BPSK, they find their usage in present-day technologies. Turbo codes can further contribute to merits of these schemes by improving their BER performance. Following methods are present in the literature to apply Turbo codes on higher modulation schemes:

(i) **Pragmatic Turbo Coding**

The first attempt in combining Turbo codes with multilevel modulation was described in [22] and is called the "pragmatic" approach. In this scheme, the coding and modulation operations are carried out independent of each other, hence this is not exactly a coded-modulation scheme. The source bits are encoded using binary Turbo encoders and are multiplexed into a single stream. These encoded bits are then converted by a Gray mapper to a data symbol of higher constellation. If a $Q = 2^q$ point constellation is used, then the interleaver size of the encoder should be:

$$K = n \times q$$
, for some integer *n*. (5.1)

The "coded" data symbols are then transmitted through a channel. At the receiver, the symbols are demodulated and soft-bit information is extracted out of them. These soft bits are fed to a binary Turbo decoder to determine the source bit sequence.



Figure 5.1: Trellis Coded Modulation.

Although, this method is not optimal, it has many advantages. Firstly, this approach is almost at the same complexity level as the Turbo-encoded BPSK system, yet it can help in making a system significantly more spectrally-efficient. Further, the pragmatic approach can accommodate puncturing of the encoded data bits for rate matching. Additionally, this scheme can be easily adapted to work with any constellation size. With any new modulation, only the interleaver size needs to be adjusted according to Equation (5.1).

(ii) Turbo-Trellis Coded Modulation

This approach was developed by Robertson and Wörz in 1995 in their paper [23]. They used Trellis Code Modulation (TCM) instead of RSC encoders separated by an interleaver in parallel concatenation. The basic concept of TCM is illustrated in Figure 5.1. In TCM, additional parity bits are introduced by increasing the constellation size (from 2^q to 2^{q+1}) while keeping the bandwidth fixed. By using a higher constellation, the probability of symbol error increases, however it is compensated by the increased parity of the data bits. Thus TCM is an optimization combination of ECC and higher modulation. Structurally, TTCM decoder is very similar to a Turbo decoder as shown in Figure 4.6, except that it works on (q + 1)-ary symbols coming from the demodulator.

Since TCM uses an optimized approach of combination of Turbo coding and higher modulation, it performs well in AWGN channel. However, they suffer from two limitations:



Figure 5.2: System for application of Turbo codes on CE-OFDM modulation scheme.

- (i) Puncturing is not possible in TTCM, thus this method is not suitable for rate matching.
- (ii) Decoder complexity increases due to use of (q + 1)-ary symbols.

5.2 Turbo codes and Constant-envelope modulation schemes

In Chapter 3 we discussed the basics and performance of 0 dB PAPR CE-OFDM and CE–SC-FDMA modulation schemes. Their constant envelope enabled usage of energy-efficient nonlinear power amplifiers which can help in significant reduction of energy consumption.

Its not straightforward to apply Turbo codes on these schemes because they don't have discrete and fixed constellation points. The phase of the message signal is continuously varied, so in the decoder, the computation of LLR values for each received soft symbol is not possible. In this work, we take the Pragmatic approach described in the previous section to use Turbo codes on CE-OFDM and CE–SC-FDMA modulation schemes. The system diagram for the implementation is presented in Figure 5.2.

The source bits are first encoded using a Turbo encoder described in Section 4.2.1. Thus a K-length source bit sequence yields 3K bits. The size 3K should be a multiple of number of bits in a symbol



Figure 5.3: Constellation for 16-QAM with Gray mapping.

(q). After this step, q bits are grouped together and mapped to symbols such as 2^{q} -QAM. These symbols are then modulated to generate CE-OFDM/CE–SC-FDMA signals which are amplified and transmitted through a channel.

At the receiver, the signal goes through the CE-OFDM/CE–SC-FDMA demodulator to yield soft 2^{q} -QAM symbols. To generate soft-bits out of these symbols, an approach described in [24] is used. In this method, the author formulated a one-to-one relation of the real and imaginary components of QAM constellation to the constituent bits. For example, if the bit sequence $b_{3}b_{2}b_{1}b_{0}$ is represented by one 16-QAM symbol **r** according to the constellation given in Figure 5.3, then the mapping will be:

$$b_3 = \mathbb{R}(\mathbf{r}), \qquad b_2 = 2 - |\mathbb{R}(\mathbf{r})|,$$
$$b_1 = \mathbb{I}(\mathbf{r}), \qquad b_0 = 2 - |\mathbb{I}(\mathbf{r})|.$$

Once all 3K soft-bit information is determined using the above mapping, it is fed to the binary Turbo decoder. The decoder then iteratively decodes the *K* source bits.



Figure 5.4: BER Performance of CE-OFDM with and without Turbo coding.

5.3 Simulations

To compare the performance of CE-OFDM and CE–SC-FDMA with Turbo codes, simulations were carried out using the parameters listed in Table 3.1. The generator polynomial of RSC used is according to the 3GPP LTE-A specifications and is given by Equation (4.3).

Figure 5.4 shows the performance of CE-OFDM with and without coding. It can be seen that Turbo codes improve the performance of CE-OFDM for all values of *h*. Further, we observe that for modulation index up to 0.5, there is a more than 10 dB improvement at the BER 10^{-4} . However, as we increase the value of *h*, the improvement decreases. This decrement in the performance can be explained as follows

According to [25], if the received signal is

$$r(t) = Ae^{j2\pi hC_N m(t)} + n(t),$$
(5.2)

where n(t) has PSD of N_0 , then for low modulation index h, the noise added to m(t) can be ap-



Figure 5.5: BER Performance of CE–SC-FDMA with and without Turbo coding.

proximated as Gaussian noise with PSD $\approx N_0/A^2$. Thus, the decoder (which considers AWGN contamination of the symbol) works well for low modulation scheme. However, when the modulation index is increased, the noise added to m(t) no longer remains Gaussian. Additionally, according to [26], at high values of h, the phase unwrapper undergoes cycle slips more frequently. Thus CE-OFDM scheme cannot gain much with Turbo coding at high modulation index.

A comparison of BER performance of coded CE-SC-FDMA signal with its uncoded variant is plotted in Figure 5.5. From the plot, we can see the effectiveness of Turbo codes for low modulation indexes signals.

Figure 5.6 shows the comparison between coded CE-OFDM and coded CE–SC-FDMA scheme. From the graph, we can conclude that in AWGN channel, the BER performance of coded CE-OFDM and coded CE-SC-FDMA is comparable.



Figure 5.6: BER performance comparison of CE-OFDM and CE-SC-FDMA with Turbo coding.

5.4 Conclusion

In this thesis, we applied two energy efficiency improvement techniques: constant-envelope modulation and Turbo coding, on OFDM and SC-FDMA modulation schemes. We compared the performance of the two schemes and concluded that in AWGN channel, both the schemes perform almost at the same level with and without coding. However, in Rayleigh channel, the performance of uncoded CE-SC-FDMA was better than uncoded CE-OFDM at high SNR values. In future we would like to explore the performance of the two coded schemes in Rayleigh fading channel.

Bibliography

- [1] Steve C Thompson, Ahsen U Ahmed, John G Proakis, James R Zeidler, and Michael J Geile. Constant envelope ofdm. *Communications, IEEE Transactions on*, 56(8):1300–1312, 2008.
- [2] Ronald Mulinde, Talha Faizur Rahman, and Claudio Sacchi. Constant-envelope sc-fdma for nonlinear satellite channels. In *Global Communications Conference (GLOBECOM)*, 2013 *IEEE*, pages 2939–2944. IEEE, 2013.
- [3] Stephen B Weinstein. The history of orthogonal frequency-division multiplexing [history of communications]. *Communications Magazine, IEEE*, 47(11):26–35, 2009.
- [4] Difference between sc-fdma and ofdma. Available Online: http://www.rfwirelessworld.com/Articles/difference-between-SC-FDMA-and-OFDMA.html. Accessed: 2015-05-04.
- [5] Cisco visual networking index: Global mobile data traffic forecast update 20142019 white paper. http://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/white_paper_c11-520862.html. Accessed: 2015-05-15.
- [6] Ericsson mobility report. http://www.ericsson.com/res/docs/2015/ ericsson-mobility-report-feb-2015-interim.pdf. Accessed: 2015-05-15.
- [7] Alcatel lucent projects. http://www3.alcatel-lucent.com/wps/portal/!ut/p/ kcxml/04_Sj9SPykssy0xPLMnMz0vM0Y_QjzKLd4y3CAvRL8h2VAQAgDi43A!!?LMSG_ CABINET=Corporate&LMSG_CONTENT_FILE=Country_Content/India/Bell_Labs_5. xml. Accessed: 2015-05-15.
- [8] Ict sector accounts for 2 percent of global carbon emissions: Study. http://www.natureworldnews.com/articles/467/20130107/ ict-sector-account-2-percent-global-carbon.htm. Accessed: 2015-05-07.
- [9] Daquan Feng, Chenzi Jiang, Gubong Lim, Leonard J Cimini Jr, Gang Feng, and Geoffrey Ye Li. A survey of energy-efficient wireless communications. *Communications Surveys & Tutorials, IEEE*, 15(1):167–178, 2013.
- [10] Md Ibrahim Abdullah, Md Zulfiker Mahmud, Md Shamim Hossain, and Md Nurul Islam. Comparative study of papr reduction techniques in ofdm. ARPN Journal of systems and software, 1(8):263–269, 2011.
- [11] Steve C. Thompson. *Constant Envelope OFDM Phase Modulation*. PhD thesis, University of California, San Diego, 2005. Available Online: http://zeidler.ucsd.edu/ sct/thesis/.
- [12] C E Shannon. A mathematical theory of communication. *Bell System Technical Journal*, 27(3):379–423, 1948.
- [13] Richard W Hamming. Error detecting and error correcting codes. Bell System technical journal, 29(2):147–160, 1950.
- [14] Farid Dowla. Handbook of RF and wireless technologies. Newnes, 2003.
- [15] Matthew C. Valenti. The evolution to error correcting coding, 2011.
- [16] Daniel J Costello and G David Forney Jr. Channel coding: The road to channel capacity. *Proceedings of the IEEE*, 95(6):1150–1177, 2007.
- [17] C Berrou, Alain Glavieux, and Punya Thitimajshima. Near shannon limit error-correcting coding and decoding: Turbo-codes (1). In *Proc. IEEE Int. Conf. Commun*, pages 1064–1070, 1993.
- [18] Andrew J Viterbi. Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *Information Theory, IEEE Transactions on*, 13(2):260–269, 1967.
- [19] 3GPP. Evolved Universal Terrestrial Radio Access (E-UTRA); Multiplexing and channel coding. TS 36.212, 3rd Generation Partnership Project (3GPP), 03 2015. Available Online: http://www.3gpp.org/ftp/Specs/html-info/36212.htm.
- [20] LR Bahl, J Cocke, F Jelinek, and J Raviv. Optimal decoding of linear codes for minimizing symbol error rate. *Information Theory, IEEE Transactions on*, IT-20:284–287, March 1974.
- [21] William E Ryan. A turbo code tutorial, 1998.
- [22] Stephane Le Goff, Alain Glavieux, and Claude Berrou. Turbo-codes and high spectral efficiency modulation. In *Communications*, 1994. ICC'94, SUPERCOMM/ICC'94, Conference Record,'Serving Humanity Through Communications.'IEEE International Conference on, pages 645–649. IEEE, 1994.
- [23] Patrick Robertson and Thomas Wörz. Coded modulation scheme employing turbo codes. *Electronics Letters*, 31(18):1546–1547, 1995.
- [24] Amer H. Alhabsi. Spectrally Efficient Modulation and Turbo Coding for Communication Systems. PhD thesis, University of Arkansas, Little Rock, 2005. Available Online: http://www.cse.unt.edu/ rakl/Amer05.pdf.
- [25] Eng. Saleh O. Said, Ahmed A.S. Dessouki, and Mohamed S. Abo El-Soud. Performance enhancement of constant envelop ofdm using turbo code. In *1st International Conference on New Paradigms in Electronics and Information Technologies*, October 2011.
- [26] Ahsen U Ahmed, Steve C Thompson, and James R Zeidler. Constant envelope ofdm with channel coding. In *Military Communications Conference*, 2006. *MILCOM 2006. IEEE*, pages 1–7. IEEE, 2006.