Title

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Risk Sharing in a Stochastic Overlapping Generations Economy

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Abstract

This paper examines the impact of government policy on the allocation of aggregate risks in a stochastic OG model with production. The market allocation of risk depends significantly on the young generation's willingness to substitute intertemporally and on government policy. Safe government debt shifts productivity risk from old to young while wage-indexed social security is essentially neutral. I also compare the market allocation to the efficient allocation of risk. The market allocation is generally inefficient, except for the special case of wage-proportional incomes and logarithmic utility. Safe government debt seems to shift risk in the wrong direction.

1. Introduction

In economies with finitely lived agents, the government has an important role as an institution that can act on behalf of unborn generations. The redistributional effects of government debt and pay-as-you-go social security are well known. In stochastic economies, government policy also affects the allocation of risk; see Enders and Lapan (1982), Smith (1982), Fischer (1983), Stiglitz (1983), Gordon and Varian (1988), and Gale (1990). The market allocation of risk is likely inefficient (in an ex-ante sense) due to the unborn generations inability to sign insurance contracts. Hence, government intervention is potentially Pareto-improving. But not nessarily: The government may also protect current generations (voters) by shifting risks onto future generations.[1]

This paper examines the allocation of aggregate risks in a Diamond (1965) type overlapping-generations economy with production. The main objectives are to determine how the equilibrium allocation of risk depends on government policy and to compare alternative market allocations to the benchmark of a Pareto-efficient allocation. The characterization of efficient allocations in a stochastic OG economy with production may be of independent interest.

I find that policy tools with similar redistributional properties—such as debt and social security—have very different risk-shifting effects. If the government operates a wage-indexed social security system, all cohorts share the risk of uncertain future productivity growth. If the government issues safe debt, it provides safety to the old but increases

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[1] The risks at stake are huge. Just a percent per year higher growth over a generation would make the next generation much better off and substantially reduce the debt-GDP ratio. Risk-shifting also plays a key role in recent social security reform proposals (see Bohn, 1997; Advisory Council, 1997; and below).

the volatility of after-tax incomes for future generations. Future generations will have to pay a non-contingent debt service out of a stochastic income, implying a relatively high (low) tax rate whenever pre-tax incomes are unexpectedly low (high).

Production is important in this context, because it places government interventions in an environment in which the labor income of the young and the capital income of the old are naturally correlated. For the special case of Cobb-Douglas production and 100% depreciation, wage and capital incomes are proportional, so that Pareto-improving government interventions are feasible if and only a non-proportional sharing of income risk is efficient. Government debt makes after-tax incomes non-proportional and shifts risk from old to young, whereas a wage-indexed social security system provides redistribution without destroying the perfect correlation of incomes across generations.[2]

After examining the positive effects of debt and social security, I characterize the set of efficient allocations. I find that the market allocation of risk is generally not efficient, except for the special case of log-utility combined with Cobb-Douglas production, 100% depreciation, and permanent productivity shocks. In this case, wage-proportional incomes translate into wage-proportional consumption due to a constant savings rate.

For substitution elasticities below one (the empirically relevant range; see Hall 1988), the efficient allocation calls for the young

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[2] Perhaps surprisingly, none of the above authors has seriously examined a standard stochastic growth model with production. Gale and Fischer consider endowment economies. Gordon and Varian consider a deterministic wage and mean-variance optimization. Enders/Lapan have money as only store of value. Smith includes production, but focuses on demographic risk and on a numerical example. Stiglitz includes capital accumulation but with exogenous interest rates. Note that adding a standard production function removes rather than adds degrees of freedom to the model because it restricts the correlation of incomes across cohorts.

generation to bear less risk than they bear in the wage-proportional market allocation. In practice, povernment policy seems to shift risk in the "wrong" direction, because governments tend to issue substantial amounts of safe debt and only partially wage-index their social security systems. The resulting supply of safe assets to the old shifts productivity risk from old to young. For safe debt to be efficiency-improving, the young would either have to have an intertemporal elasticity of substitution above one (a counterfactual assumption) or be less risk-averse than the old (a non-standard assumption). Otherwise, if the government engages in redistribution, it should do so through risk-sensitive tools such as wage-indexed social security or nominal debt with productivity-contingent inflation rather than through safe debt.[3]

The distinction between state-contingent and safe policy tools is also relevant for social security reform. Proposals to replace social security by government bonds or to invest trust fund balances in the stock market (e.g., Feldstein, 1996; Advisory Council, 1997) would also shift productivity risk to future generations. To address social security issue, the model includes a social security trust fund with variable investment policy.

The paper is methodologically different from most of the OG literature. For reasonably general assumptions about preferences, technology, and policy, the model does not have a closed-form solution. I therefore use log-linearizations to approximate the optimal decision rules, using techniques borrowed from the business cycle and finance literature,

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[3] An extensions section explores complicating factors that might rationalize safe debt, notably negatively correlated productivity growth and CES-production with a low elasticity of factor substitution.

but applied at a much lower frequency and to obtain analytical results, not for numerical simulation.[4]

A number of simplifying assumptions are made for analytical tractability. The assumption of two period lived agents eliminates private risk sharing. With longer lived agents, private risk sharing would occur (complicating the model), but the government's role as agent of the unborn would remain. For most of the paper, I focus on productivity risk, Cobb-Douglas production, and an inelastic labor supply. In a final section about extensions, I explain why labor-leisure choices and non-productivity shocks are unlikely to be important, while deviations from Cobb-Douglas technology could be important and might be worth exploring in future research. Survival uncertainty, idiosyncratic risks, demographic uncertainty, bequests, and distortionary taxes are also left for future research.[5] Dynamic efficiency is assumed throughout, ruling out bubbles and related issues that would distract from risk sharing.

The paper is organized as follows. Section 2 describes the model and explains the positive effects of alternative policy tools. Section 3 derives the equilibrium allocation. Section 4 derives efficiency conditions, first for a general model and then with specific assumptions about the dynamics. Section 5 provides extensions. Section 6 concludes.

2. Overlapping Generations and Government Policy

This section lays out the market model and the main government institutions that affect the allocation of risk.

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[4] The model could be calibrated or simulated, but that would be beyond the scope of this theoretically oriented paper (see Bohn (1997), Rios-Rull (1996) for calibrated OG models).

[5] Idiosyncratic risks could be shared within a cohort. Tax-distortions are potentially important because the government will have to vary tax rates to execute risk sharing contracts on behalf of the unborn. Ricardian bequests would make the model uninteresting.

2.1. Individuals

Individuals live for two periods. Generation t consists of N_t individuals who work in period t and are retired in period t+1. Workers earn a wage w_t equal to the marginal product of labor, pay payroll taxes at the rate τ_t, and pay other taxes T^1_t.[6] The disposable income $w_t(1-\tau_t) - T^1_t$ is either consumed (c^1_t) or saved, either in capital (equity securities, s^k_t) or in form of bonds (s^b_t),

$$(2.1) \qquad c^1_t = w_t(1-\tau_t) - T^1_t - s^k_t - s^b_t.$$

The rates of return on equities and bonds are denoted by R^k_{t+1} and R^b_{t+1}, respectively. The old receive wage-indexed social security benefits with a replacement rate θ and pay taxes T^2_{t+1}. Their consumption is

$$(2.2) \qquad c^2_{t+1} = R^k_{t+1} s^k_t + R^b_{t+1} s^b_t + \theta w_{t+1} - T^2_{t+1}.$$

Preferences are assumed homothetic to be consistent with balanced growth. Intertemporal substitution and risk aversion are separately parametrized because of their different functions in the model. Intertemporal substitution determines the savings response to interest rate signals and is crucial for the model's dynamic structure. Risk aversion will be essentially irrelevant for the market dynamics and matters primarily for efficiency issues. To keep these concepts distinct, I assume a recursive non-expected utility function

$$(2.3) \qquad U_t = \frac{1}{1-\sigma_1}\left[(c^1_t) + \delta\left\{E_t[(c^2_{t+1})^{(1-\sigma_2)}]\right\}^{(1-\sigma_1)/(1-\sigma_2)}\right]^{(1-\sigma_1)/}$$

to capture the preferences of generation t, where δ is the rate of time preference and $E_t[\ ]$ denotes the conditional expectations at time t. This specification is similar to Epstein-Zin (1989) and Weil (1989), but generalized to allow different degrees of risk aversion for old (σ_2) and

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[6] The distortionary effect of taxation are ignored for simplicity. Each worker supplies one unit of labor. Retirement savings are assumed untaxed, implicitly assuming that such savings takes place (at least on the margin) through tax-sheltered instruments like pension plans, variable annuities, or IRA accounts.

young ($\gamma 1$); the elasticity of intertemporal substitution, $1/(1-\alpha)$, is not necessarily their inverse. The standard constant relative risk aversion (CRRA) utility is the special case of $\gamma_1 = \gamma_2 = 1-\alpha$.

In the following, I consider general preferences (any $\alpha < 1$, $\gamma_1 > 0$, $\gamma_2 > 0$). But for interpreting the solutions, I usually focus on either time-separability ($\gamma_1 = 1-\alpha$), equal risk aversion ($\gamma_1 = \gamma_2$), or the CRRA case.[7]

For positive analysis, the only relevant property of (2.3) is the implied marginal rate of substitution,

$$(2.4) \qquad MRS(c^1{}_t, c^2{}_{t+1}) = \left\{ \frac{c^2{}_{t+1}}{E_t[(c^2{}_{t+1})^{(1-\gamma_2)}]^{1/(1-\gamma_2)}} \right\}^{1-\gamma_2} \gamma_2 \left\{ \frac{c^1{}_t}{c^2{}_{t+1}} \right\}^{1-\alpha}.$$

It depends on intertemporal substitution and on the risk aversion of the old, but not on γ_1, because the young cannot participate in risk-sharing contracts before their birth; γ_1 will be important, however, for the social planner's willingness to impose risk on future generations. The optimality conditions for bond and equity holdings are then

$$(2.5) \qquad E_t[MRS(c^1{}_t, c^2{}_{t+1}) R^k{}_{t+1}] = E_t[MRS(c^1{}_t, c^2{}_{t+1}) R^b{}_{t+1}] = 1$$

Equations (2.1, 2.2, 2.5) characterize individual consumption and savings behavior for given government policy, wages, and return distributions.

2.2. Technology

All goods are produced by firms using capital K_t and labor N_t at constant returns to scale. For some of the normative analysis, it will be sufficient to write output as a general function of capital and labor, $F_t(K_t, N_t)$, where the time-dependence of $F_t(\ )$ may include arbitrary stochastic shocks. But to solve the market model and to obtain reasonably specific policy results,

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[7] Since CRRA is taken as benchmark for the interpretation, readers uncomfortable with non-expected utility should not worry that the results rely on non-expected utility; but (2.3) makes the economic intuition more transparent and allows the model to address issues that require a realistic equity premium (see Bohn, 1997). The limiting cases $\alpha = 0$, $\gamma_1 = 1$, and/or $\gamma_2 = 1$ are covered as usual by applying de l'hospital's rule.

one needs assumptions about the nature of uncertainty and to ensure balanced growth.

For most of the paper, I assume that aggregate uncertainty is due to an exogenous, labor-augmenting productivity trend A_t with stochastic i.i.d. growth rate a_t, and that the technology is Cobb-Douglas with capital coefficient α. Denoting the output of new goods (GDP) by

$$Y_t = K_t^{\alpha} (A_t N_t)^{1-\alpha},$$

the total resources available for consumption and capital investment are $Y_t + \gamma K_t$, where γ is the salvage value of old capital and $(1-\gamma)$ can be interpreted as depreciation rate. The marginal products of labor and capital are then

(2.6a) $w_t = (1-\alpha) A_t^{1-\alpha} K_t^{\alpha} N_t^{-\alpha} = (1-\alpha) A_t \left(\dfrac{k_t}{(1+a_t)(1+n)} \right)^{\alpha}$

(2.6b) $R^k_t = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha} + \gamma_t = \alpha \left(\dfrac{k_t}{(1+a_t)(1+n)} \right)^{\alpha-1} + \gamma$

where $k_t = K_t/(A_{t-1} N_{t-1})$ is the effective capital-labor ratio, $1+a_t=A_t/A_{t-1}$ is the productivity growth rate, and $1+n=N_t/N_{t-1}$ is the constant population growth rate.

If A_t is the only source of disturbances, capital and labor income are perfectly correlated (though not linearly, if $\alpha \neq 0$). This is clearly restrictive, although the cointegration results of Baxter and Jermann (1997) suggest that a high correlation is empirically plausible for long horizons. Additional uncertainty about capital income could easily be added, e.g., by making the salvage value of old capital stochastic. But this would complicate the analysis and distract from productivity risk, the main source of long-term income uncertainty. A discussion of other shocks is therefore deferred to Section 5.

In modeling the time series of total factor productivity, it seems reasonable to abstract from short-run autocorrelation, because each period

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amounts to a generational time unit of about 20-30 years. The assumption of i.i.d. productivity growth (i.e., permanent productivity shocks and a non-stationary productivity level) is potentially restrictive, however, because the young might be able to bear temporary producitivity shocks more easily than the old through consumption-smoothing. Temporary producitivity shocks are therefore examined in Section 5. Until then, I focus on permanent shocks because I consider this a better assumption for generational frequencies (keeping in mind that, say, 20 generations cover about 400-600 years). Even if a stationary trend line fits the data over a shorter horizons (say, a few decades), the likelihood of future trend breaks implies a unit root-like uncertainty in the very long.

Without government activity, individual incomes are determined entirely by technology. The young are exposed to productivity risk through their wage income. The old are exposed to productivity risk through capital income. Each member of the old generation holds $s^k_{t-1} = K_t/N_{t-1} = A_{t-1} k_t$ units of capital and earns/consumes

(2.7) $\quad c^2_{t+1} = R^k_t s^k_{t-1} = A_t \left(\dfrac{k_t}{(1+a_t)(1+n)} \right) (1+n) + k_t A_{t-1}$

$\qquad\qquad = \dfrac{}{1-} (1+n) w_t + k_t A_{t-1},$

an amount proportional to the wage rate plus the value of old capital. If $=0$, the incomes of the young and the old are both proportional to the wage rate. This sceanario of "wage-proportional" incomes provides a useful benchmark for interpreting government policies.

In general, the market economy converges to a stochastic steady state in which the capital labor ratio k_{t+1} and the ratios of consumption and income to productivity, c^1_t/A_t, c^2_t/A_t, and y^1_t/A_t are Markov processes with state variables k_t and a_t, to be examined in Section 3. The assumptions on government policy will be chosen to ensure a similar steady state behavior.

2.3. Government Spending and Social Security

I start the modeling of the government sector with government spending and social security, because these are two government activities that do not necessarily upset the proportional division of income.

Government spending is not the focus of this paper, but it should not be omitted either, because it affects the real resources available for intergenerational risk-sharing. For simplicity, I assume that government spending is a constant fraction of output, $G_t = g\,Y_t$.[8]

Government spending is financed by lump-sum taxes on the young and the old. Since GDP is proportional to wages, this can be done in a wage-proportional way by setting $\tau^1_t = \tau^1 Y_t/N_t$ and $\tau^2_t = \tau^2 Y_t/N_t$, where τ^1 and τ^2 are the tax rates on the young and the old respectively. The budget constraint requires $\tau^1 + \tau^2/(1+n) = g$.[9]

Social security can be modeled most easily by assuming a wage-indexed, pay-as-you-go (PAYG) system that collects payroll taxes $N_t w_t \tau_t$ from the young and pays benefits $N_{t-1}\,\theta_t w_t$ to the old generation $t-1$. The PAYG constraint implies that the tax rate τ_t must equal the cost rate $\theta_t/(1+n)$.[10] In the U.S., social security benefits are indexed to the average

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[8] It would be straightforward to examine the positive implications of many alternative assumptions, but that would distract from more important issues. Spending shocks and some alternative assumptions are discussed below. A constant g may be interpreted as a stylized representation of the political realities that have produced a roughly constant GDP-share of U.S. government spending in the post-war era.

[9] Assumptions about taxes are also kept simple because general state-contingent taxes would be a "too powerful" policy tool at this point. Through state-contingent taxes, the government could impose any arbitrary re-allocation of risk, making the analysis of social security, debt, and other realistic policy tools uninteresting. State-contingent taxes are therefore best interpreted in a normative context (Section 4) as a policy tool suitable to implement an efficient allocation. Income taxes are discussed in Section 5.

[10] Stochastic survival could be added at this point by assuming that only a fraction π of the young survive into old age, provided all non-social security assets are annuitized. Then the social security replacement rate could be scaled up by $1/\pi$ to $\theta_t = \tau_t N_t/(N_{t-1}\pi)$ at the same payroll tax rate, and the annuitized private returns would be R^e/π and R^b/π. Alternatively, one could follow Huggett (1996) and assume that private annuities do not exist and that accidental bequests amounting to π times private wealth are fully taxed (i.e., considered part of τ^2). If the individual rate of time preference is modified appropriately, neither of these modifications would significantly change the results.

national wage level at the time of retirement and inflation-indexed thereafter, i.e., partially wage-indexed and partially safe in real terms. Here I assume full wage-indexation for simplicity and to highlight the contrast between social security and safe debt. (A partially wage-indexed system would have intermediate properties that can be inferred from the pure cases of safe debt and fully indexed social security.)

With full wage indexation and constant population growth, both the payroll tax rate and the replacement rate can be held constant. Then the disposable income of the young

$$y^1_t = w_t(1-\ ) - \ ^2_t = [1 - \ /(1+n) - \frac{g-\ ^2/(1+n)}{1-\ }]\, w_t,$$

is again proportional to wages; and the consumption of the old,

(2.8)
$$c^2_t = R^k_t\, s^k_{t-1} + \ _t\, w_t - \ ^2_t$$
$$= [\frac{}{1-\ }(1+n) + \ - \frac{^2}{1-\ }]\, w_t + \ _k{}_t\, A_{t-1},$$

is proportional to the wage, except for the term involving the salvage value of old capital. Note that the parameter ^{2} is redundant with social security, because individual behavior depends only on real spending g and on the net transfers across generations, $-\ ^2/(1-\ )$, but not on the composition of the transfers (see Stiglitz, 1983; Kotlikoff, 1986). One may therefore assume without loss of generality that all spending is financed by the young ($^2_t\ 0$).

The government's ability to redistribute resources across generations without significantly upsetting the allocation of risk is not only conceptually noteworthy but also analytically convenient because it allows a separation risk-sharing from redistributional concerns. When examining variations in the allocation of risk, one may assume that the overall scale of intergenerational transfers is always kept at a level reflecting the government's distributional preferences.

2.4. Government Debt

Government debt has a variety of effects that depend on the type and the time-path of government debt. My key claim is that government debt generically destroys the proportional division of GDP. This point is best explained by starting with a special case: Suppose for now that all government is safe (real) debt and that the ratio of end-of-period debt D_{t+1} to output is a constant $d^Y = D_{t+1}/Y_t$.

The government budget equation

$$(2.9a) \qquad G_t + R^b_t D_t = N_t \, \tau^1_t + N_{t-1} \, \tau^2_t + D_{t+1}$$

shows that government spending and initial debt must be financed with taxes and new debt. Given $\tau^2_t = 0$ and $G_t = g\, Y_t$, the taxes on the young are

$$\tau^1_t = g\, Y_t/N_t + R^b_t \, D_t/N_t - D_{t+1}/N_t$$

and their disposable income is

$$y^1_t = (1-\alpha)\, w_t - g\, Y_t/N_t + D_{t+1}/N_t - R^b_t \, D_t/N_t$$
$$(2.9b) \qquad = \left[(1-\alpha) - \frac{g}{1-\alpha} + \frac{d^Y}{1-\alpha}\right] w_t - \frac{R^b_t}{1+n}\frac{1}{1-\alpha}\, w_{t-1},$$

using $Y_t/N_t = w_t/(1-\alpha)$. All but the last term are proportional to current wages and to GDP. But the value of old debt is proportional to last period's wage. If period-t productivity growth is high, the lagged wage term is relatively small and the young enjoy more than proportional income growth. If current productivity growth is relatively low, the lagged term remains fixed and implies a more than proportional downward movement in disposable income. Thus, safe debt increases the exposure of the young generation to productivity risk. For the old, on the other hand, safe debt represent a fixed income that reduces their relative exposure to productivity risk. Overall, safe debt shifts productivity risk from the old to the young generation.[11]

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[11] This conclusion is of course preliminary, because the general equilibrium effects of government debt have not been discussed, but it will be confirmed in Section 3 below.

More generally, debt may be state-contingent. The overall return on government debt in period t+1 can then be written as the product of a predetermined "nominal" return and an index variable P_{t+1}, $R^b_{t+1} = R^{Nom}_t P_{t+1}$, where P_{t+1} is a random variable and R^{nom}_t is determined at time t. This notation covers the case of safe real debt, if P_{t+1} 1; nominal debt, if P_{t+1} is the stochastic purchasing power of dollars at time t+1 and R^{nom}_t is literally the nominal return;[12] and wage-indexed (or equivalently, GDP-indexed) debt, if $P_{t+1}=w_{t+1}/w_t$ is the growth rate of wages. In general, (2.5) implies $R^{nom}_t = 1/\{E_t[MRS(c^1_t,c^2_{t+1}) P_{t+1}]\}$.

If new debt issues are proportional to output, the disposable income of the young is proportional to the current wage except for the term involving old debt, as in (2.9b). The burden of old debt relative to current output, $R^b_t D_t/Y_t = d^Y R^{nom}_{t-1} P_t Y_{t-1}/Y_t$, now depends on the stochastic properties of P_t in addition to productivity risk. But generically, unless $P_t Y_{t-1}/Y_t$ is deterministic, debt destroys the proportional division of GDP.

The special case of deterministic $P_t Y_{t-1}/Y_t$ is the case of wage-indexed debt. In this case, R^{nom}_t is a constant and (2.9b) reduces to

$$y^1_t = [(1-) - \frac{g}{1-} + \frac{d^Y}{1-} - \frac{R^{nom}}{1+n} \frac{d^Y}{1-}] \; w_t,$$

an amount proportional to the wage rate. For this--and only this--case, government debt is a perfect substitute for social security, and all comments about social security apply analogously.

Another general issue is the time path of debt. To obtain balanced growth, debt must grow asymptotic at the same rate as the economy, i.e., one needs a stationary debt-productivity ratio $d_t = D_t/(A_t N_t)$. A constant debt-GDP ratio yields balanced growth, because the wage-productivity ratio is stationary. But a constant debt-GDP ratio has the inconvenient

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[12] A monetary model is beyond the scope of this paper. Here, monetary policy is just a label for a randomization device that can generate a well-defined distribution for P_{t+1}.

implication that disposable income depends on lagged wages and therefore on the past productivity shocks (for but wage-index debt). This would complicate the equilibrium dynamics without adding much insight.[13] If interest rates fluctuate, it is also unclear if a constant debt-GDP ratio should be interpreted as a constant end-of-period ratio or as a constant expected debt-GDP ratio at the start of the next period.

To avoid these complications, the general equilibrium analysis will be done under the more convenient assumption that the government's choice variable is the debt-productivity ratio d_t (not d^Y) and that d_t depends at most on the current interest rate. Since state contingent debt has the same generic properties as safe debt, I further assume that all debt is safe debt, i.e., that R^b_{t+1} is known as of period t. (Otherwise, P_t would have to be added as state variable.) The taxes on the young are then

(2.10) $1_t = g/(1-)\, w_t - d_t\, A_t + R^b_t\, d_{t-1}\, A_{t-1}$,

and the disposable income is an increasing function of productivity growth and a declining function of the past return on debt, R^b_t (as before, but without involving lagged GDP). If R^b_t fluctuates over time, the government faces a choice of keeping the ratio of new debt to productivity constant ($d_t = d$), or to vary the debt supply such that $R^b_{t+1}\, d_t$ is constant, or to do something in between. I consider an elastic debt supply of the form $d_t = d^* (R^b_{t+1})^-$, where   is an elasticity parameter. This is tractable because R^b_{t+1} depends only on the current values of k_t and a_t.

In practice, U.S. government debt is mostly nominal and in part long-term, i.e., not strictly safe in real terms. In the public policy debate, government bonds are nonetheless considered prototypical safe assets. The

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[13] The capital-labor ratio would be characterized by a second-order difference equation as compared to a first-order difference equation with (2.10). The point that government debt has the potential to affect the macroeconomic dynamic is noteworthy and the implications would be straightforward to examine, but it would be a distraction here.

assumption of safe real debt above follows this tradition and it highlights the contrast to wage-indexed social security and to "risky" equity investments.[14]

2.5. The Social Security Trust Fund

The social security trust fund deserves attention because it's existence affects the net supply of government bonds and because alternative investments have potentially significant risk sharing effects. As Bohn (1997) has shown, trust funds are irrelevant if they are invested for the benefit of the generation building up the fund, but they are economically significant in the context of a defined-benefit social security system. Assuming a defined-benefit system with constant replacement rate , investment risks and returns are effectively borne by future generations of social security contributors.

To model trust fund investments, consider a mixed, partially funded social security system. Gaps between payroll tax receipts and benefit payments are invested in (or covered out of) a trust fund. The social security budget equation is then

(2.11a) $N_t w_t \tau_t + TF_t + TRS_t = N_{t-1} w_t \theta + TF^k_{t+1} + TF^b_{t+1}$

where TF^*_t is the initial trust fund, TF^k_{t+1} and TF^b_{t+1} are the new equity and bond investments, and TRS_t is a transfer from the general government to social security (to allow for this possibility). The trust fund balance at the start of period t+1 will then depend on market returns,

(2.11b) $TF_{t+1} = R^k_{t+1} \; TF^k_{t+1} + R^b_{t+1} \; TF^b_{t+1}.$

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[14] Nominal debt deserves a comment here, because nominal debt can mimic any other type of debt for an appropriately chosen inflation process. For non-stochastic money supply and non-stochastic velocity, for example, the purchasing power of money is proportional to income so that nominal debt would mimic wage-indexed debt. But without restrictions on the inflation process, nominal debt is a too vague concept to yield insightful results.

Combined with the general government budget equation (2.9a), supplemented by TRS_t as an expenditure item, this implies a unified government budget equation

(2.12) $\quad N_t w_t \tau_t + N_t \tau^1_t + N_{t-1} \tau^2_t - N_{t-1} w_t \sigma - G_t$

$$= R^b_t D_t - D_{t+1} + TF_t + TF^b_{t+1} - TF^*_{t+1}$$

$$= R^b_t D^*_t - R^k_t TF^k_t + TF^k_{t+1} - D^*_{t+1}$$

where $D^*_{t+1} = D_{t+1} - TF^b_{t+1}$ is the publicly-held (net) debt. Total revenues, new net debt issues, and initial equity holdings ($R^k_t TF^k_t$) must pay for non-interest spending, for initial net debt, and for new equity investments. The unified budget equation shows that trust fund bond holdings are equivalent to a reduction in gross Treasury debt and that regular and social security taxes are perfect substitutes. For given spending, given taxes on the old, and given social security benefits, trust fund equity holdings expose the young to the risk of low stock returns, either through variations in payroll taxes (higher τ_t) or through a social security "bailout" ($TRS_t > 0$) financed by regular taxes.

To ensure balanced growth, I assume that the security trust fund balance is, like debt, proportional to the productivity trend and that fixed shares of the trust funds are invested in stocks (θ^k) and bonds ($\theta^b = 1 - \theta^k$). That is, $TR^b_{t+1} = \theta^b \tau N_t A_t$ and $TR^k_{t+1} = \theta^k \tau N_t A_t$, where τ is the ratio of the total trust fund to the growth trend $N_t A_t$.

3. Market Equilibrium

This section examines the equilibrium allocation of risk and the macroeconomic dynamics of the OG model outlined above. The model includes government spending equal to a constant fraction of output, safe debt proportional to the productivity trend, taxes on young and old, and a social security system with trust fund.

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3.1. Equilibrium Conditions

In equilibrium, the young must hold the net supply of government bonds, $N_t s^b_t = D^*_{t+1} = D_{t+1} - TR^b_{t+1}$, and the capital stock net of social security trust fund holdings, $N_t s^k_t = K_{t+1} - TR^k_{t+1}$. The equilibrium consumption of the young is then

$$(3.1) \quad c^1_t = w_t(1-\tau_t) - \tau^1_t - \frac{K_{t+1} - TR^k_{t+1}}{N_t} - \frac{D_{t+1} - TR^b_{t+1}}{N_t} \equiv w_t - K_{t+1}/N_t - CF^1_t.$$

Its policy-dependence can be summarized by the cash flow to the government, CF^1_t. The cash flow measure CF^1_t includes involuntary payments (regular and payroll taxes) as well as voluntary payments (debt minus security sales to the trust fund). The consumption of the old can similarly be written as total capital income plus cash flows from the government,

$$(3.2) \qquad c^2_{t+1} = R^k_{t+1} \frac{K_{t+1}}{N_t} + CF^2_{t+1}, \text{ where}$$

$$(3.3) \qquad CF^2_{t+1} = \tau_{t+1} w_{t+1} - \tau^2_{t+1} + R^b_{t+1} \frac{D_t - TR^b_t}{N_t} - R^k_{t+1} \frac{TR^k_t}{N_t}$$

includes social security benefits minus general taxes, the repayment of government debt, minus the return on the securities sold to the trust fund. The unified budget equation (2.12) makes CF^1_t a function of CF^2_t and G_t, namely $N_t CF^1_t = G_t + N_{t-1} CF^2_t$. Hence, government activity can be summarized by real spending and a single summary statistic for intergenerational redistribution, CF^2_t. Combinations of government alternative tax- debt-, and social security policies that generate identical stochastic processes for CF^2_t are economically equivalent. This generalizes Stiglitz's (1983) results on the irrelevance of infra-generational tax timing and Kotlikoff's (1986) result about the irrelevance of "labeling" government transfers.

An equilibrium in this economy is characterized by the consumption equations (2.1-2.2), the Euler equations (2.5), the equilibrium conditions on bond and equity markets, and the policy rules. Under the policy assumptions of Section 2, the equilibrium allocation displays balanced

growth. The growth trend is driven by deterministic population growth, N_t, and stochastic productivity growth, A_t. The dynamics of stationary variables such as k_{t+1}, $(c^1/A)_t$, and $(c^2/A)_t$ are given by a Markov processes with state variables k_t and a_t. Throughout, I assume policies such that the allocation is dynamically efficient, to rule out a too-obvious source of inefficiency and to prevent distracting discussions about bubbles and uniqueness.

The optimal decision rules for consumption and capital investment are generally non-linear and do not have closed form solutions. I therefore follow the business cycle literature and log-linearize the relevant constraints and first-order conditions. The linearization is taken around the deterministic steady state obtained by equating the stochastic shocks to their expected values, $a_t=a$ (see King-Plosser-Rebelo, 1988a, 1988b). A log-linearization is quite appropriate here, because some of the key equations are exactly log-linear (e.g., the production and wage equations) and because the economy is exactly log-linear in interesting special cases (e.g., for log-utility, =0, and wage-proportional policies).

A linearization around a deterministic steady state is sufficient for understanding macroeconomic dynamics, but it is not necessarily sufficient for policy arguments involving uncertainty, such as questions about precautionary savings and asset pricing issues.[15] Hence, I alternatively use an approach motivated by Campbell and Viceira (1996). Campbell-Viceira use log-linearized budget equations, like King-Plosser-Rebelo, but they evaluate the exact Euler equations under the assumption of log-normal disturbances. Adapting their approach to the OG setting, I log-linearize

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[15] The Advisory Council's (1997) argument for social security equity investments is, e.g., based on a non-trivial equity premium. But the equity premium is zero if one linearizes around deterministic steady state.

the consumption equations (3.1) and (3.2) around the deterministic steady state and evaluate (2.5) for log-normal shocks.

The resulting approximations are identical to the King-Plosser-Rebelo solution, except that the log-linearized decision rules include intercept terms reflecting the mean "displacement" of the stochastic relative to the deterministic steady state. Most results below are about the slope coefficients, however, so that the King-Plosser-Rebelo approach is sufficient and log-normality is not required. The robustness with respect to the approximation method should nonetheless be reassuring for readers concerned about precautionary savings and about asset pricing issues. To be clear about the notation, let x (without subscript) be the deterministic steady state value of a stationary variable x_t, let $\hat{x}_t = \ln(x_t) - \ln(x)$ be the log-deviation from the steady state, and let

$$\hat{x}_t = {}_{x0} + {}_{xk}\hat{k}_t + {}_{xa}\hat{a}_t,$$

denote the log-linearized law of motion, where $_{xz}$ are fixed coefficients. The intercept terms $_{x0}$ are always formally included, but zero in the King-Plosser-Rebelo approximation.[16]

3.2. Equilibrium Dynamics

The key dynamic equations in the OG-model are the optimal decision rule for the capital investment of the young,

(3.4) $\hat{k}_{t+1} = {}_{k0} + {}_{kk}\hat{k}_t + {}_{ka}\hat{a}_t.$

and the implied consumption rules for the young and the old. The slope coefficients in (3.4) can be written as ratios $_{kz} = {}_{kz}/{}_k$, where

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[16] Details of all derivations are in a technical appendix available from the author (and available for downloading at http://www.econ.ucsb.edu/~bohn). The methodological innovation here is to use log-linearizations to study intergenerational risk sharing and to obtain approximate analytical (rather than numerical) solutions. The derivations are generally straightforward, but quite lengthy.

$(3.5a)$
$$\phi_k = \quad + \frac{(1-\ )(1-\ /R^k)}{1-\ } + \frac{k}{(c^1/A)} + (1-\ )\frac{(k-d(1-\ )+\ )}{(c^2/A)(1+a)} - \frac{R^k[d(\ +(1-\ )(1-\ ))-\ ]}{(c^2/A)(1+a)}$$

$(3.5b)$
$$\phi_{kk} = \quad y^1/(c^1/A) + (1-\ /R^k)(d(1-\ )-\ )/(c^1/A)$$

$(3.5c)$
$$\phi_{ka} = -\ y^1/(c^1/A) + R^k/an(1-\ )[d-\ +\ ^k(1-\ /R^k)]/(c^1/A)$$

and $an = (1+a)(1+n)$. The consumption rules are then determined by the budget equations (3.1) and (3.2). For the young, $(c^1/A)_t$ is income minus capital investment, so that the coefficients are related to the coefficients for capital; namely

$(3.5d)$
$$\phi_{c1Az} = \quad \phi_{kz}[\ \phi_k - \frac{k}{(c^1/A)}] = \quad \phi_{kz}[1 - \frac{k}{(c^1/A)}/\ \phi_k],$$

for $z=k,a$. For the old, $(c^2/A)_t$ has coefficients

$(3.6a)$
$$\phi_{c2Ak} = \quad + (1-\ )\frac{(k-d(1-\ )+\ )}{(c^2/A)(1+a)} - \frac{R^k[d(\ +(1-\ )(1-\ ))-\ ]}{(c^2/A)(1+a)}$$

$(3.6b)$
$$\phi_{c2Aa} = -\ -\ (1-\ )\frac{(k-\ ^k)+R^k(d-\ +\ ^k)}{(c^2/A)(1+a)}.$$

These equations describe parametrically how the allocation of risk depends on government policy. Three general properties of the equilibrium allocation are noteworthy.

First, the key preference parameter is the substitution parameter . Intuitively, the intertemporal elasticity of substitution $1/(1-\ )$ determines how the young generations responds to productivity shocks and to fluctuations in the capital-labor ratio. The responses are linked, because a positive productivity shock and a low lagged capital-labor ratio k_t both imply a low ratio of capital to the current efficiency-adjusted labor supply, $K_t/(A_t N_t)$. A low ratio $K_t/(A_t N_t)$ raises the expected return on capital. If the elasticity of substitution is high, this triggers a savings response that pushes the capital-labor ratio k_{t+1} back to its steady state fairly quickly, at the expense of larger fluctuations in $(c^1/A)_t$. Hence, a

high β-value is associated with relatively small (in absolute value) γ_{kk} and γ_{ka} coefficients and with relatively large γ_{c1Ak} and γ_{c1Ak} coefficients.

Second, note the glaring absence of risk aversion parameters γ_1 and γ_2 from the above formulas. Since individuals cannot privately share risks across generations, their exposure to productivity risk is determined by policy and production, except for the young generation's willingness to substitute intertemporally. This does not mean that the model somehow abstracts from precautionary savings: Using the Campbell-Viceira approximation, one can show that the intercept terms γ_{k0}, γ_{c1a0}, and γ_{c2a0} depend on γ_2. The risk aversion of the old does affect the average level of economic activity. But risk aversion and precautionary savings do not have a first-order effect on the macroeconomic dynamics (the slope coefficients).[17] The risk aversion of the young, γ_1, never matters because they cannot insure against shocks already realized at birth.

Third, to understand the effects of productivity shocks, note that <u>level</u> variables such as c^1 and c^2 have elasticities with respect to productivity growth that are one plus the elasticity value of the corresponding <u>ratio</u> variables such as (c^1/A) and (c^2/A). A negative parameter $\gamma_{c2Aa} - \rho$ (for small α, d, and δ values) for the consumption-productivity ratio in (3.6b) implies, for example, a positive response of per-capita consumption levels $c^2_t = A_t (c^2/A)_t$ of about $\eta_{c2a} \approx 1 + \gamma_{c2Aa} \approx 1 - \rho$.[18] Policy changes that increase γ_{c2Aa} in absolute value (reduce it below $-\rho$) therefore tend to <u>reduce</u> the impact of productivity shock on per-capita consumption. The same applies to c^1_t: For plausible parameters, γ_{c1Aa} is

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[17] The model also includes a reasonable asset pricing structure. The difference $\gamma_{Rk0} - \gamma_{Rb0}$ between the itercept terms for R^k and R^b can be interpreted as the equity premium. It depends on γ_2 times the covariance between R^k and c^2; see Bohn (1997) for a calibration.

[18] Since wage dynamics are given by $(\hat{w/A})_t = \alpha(\hat{k}_t - \hat{a}_t)$, a decision rule with coefficient values $\gamma_{xk} = \alpha$, $\gamma_{xa} = -\alpha$ means that the variable moves in proportion with wages.

negative while $_{c1a}=1+_{c1Aa}$ is positive, so that parameter variations that raise $|_{c1Aa}|$ actually reduce the variance of consumption. Recall that a high -value implies a high $|_{c1Aa}|$. A high subsitution elasticity therefore reduces the conditional variance of consumption levels.[19]

To summarize the general results, we find:

Result #1: <u>The market dynamics and the market allocation of risk depend on preferences only through the intertemporal substitution parameter .</u>

Result #2: <u>A high elasticity of substitution speeds up the economy's return to the steady state and reduces the (conditional) variance of the young generation's consumption.</u>

Turning to specific parametric results, the special case of wage-proportional incomes, =d= =0, serves as useful benchmark. For =d= =0, the consumption and investment coefficients above reduce to

$$_{kk} = - _{ka} = \frac{1 + k/(c^1/A)}{ + (1-)/(1-) + k/(c^1/A)} \cdot$$

$$_{c1a} = 1 - \frac{1 + k/(c^1/A)}{ + (1-)/(1-) + k/(c^1/A)} (+ \frac{1-}{1-})$$

and $_{c2a} = 1-$. Recall that $1-$ is the weight on $(A_t N_t)$ in production. Hence, $_{c2a} = 1-$ means that the consumption of the old is as volatile as output and wages. The coefficients for the young depend on the substitution parameter (as explained above) and on the steady-state capital-consumption ratio.[20] Regardless of $k/(c^1/A)$, one finds $_{c1a} \lessgtr 1-$, $_{ka} \gtrless -$ if and only if $1/(1-) \gtrless 1$; for log-utility (=0), $_{c1a} = 1-$ and $_{kk} = - _{ka}$ = . Hence, the question which generation bears more consumption risk depends entirely on the substitution parameter: For log-utility, both

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[19] Recall that the ratios (c^1/A) and (c^2/A) are stationary but not the levels. Comments about the variance of consumption levels here and below should therefore be interpreted as referring to the conditional variance at some earlier date; say, one period ahead. The conditioning date is inessential for qualitative comparisons.

[20] The steady-state capital-consumption ratio depends on time preference, government spending, and the scale of intergenerational redistribution through social security, as is well known from the literature on deterministic OG models.

generations bear equal consumption risk in proportion to income risk. For higher/lower elasticity values (positive/negative   ), a larger/smaller fraction of any productivity shock is absorbed by variations in capital investment, reducing/increasing the volatility of c^1 in absolute terms and relative to c^2.

Compared to the   =d= =0 benchmark, a positive net government debt (d- >0, ^{b}=1) reduces the dependence of old-age consumption on productivity shocks, i.e., reduces $_{c2Aa}$ and $_{c2a}$.[21] For the young, government debt increases the sensitivity of c^1_t to productivity shocks, raising $_{c1a}$. Hence, confirming the intuition from Section 2.4, we obtain <u>Result #3</u>: <u>Safe government debt shifts productivity risk from the old to the young</u>. Note that the critical variable here is the debt net of social security holdings, d- , not the gross debt.

Government debt has additional, subtle effects on the dynamics of the capital-labor ratio. A fixed, not interest-elastic volume of debt (=0) yields a variable income for the old, because R^b_{t+1} varies with the capital-labor ratio, and it raises the autocorrelation of the capital-labor ratio (reducing $_k$, increases $_{kk}$); the same applies for a low interest elasticity, 0< <1. At   =1, the return on debt $R^b_{t+1}d_t$ becomes independent of k_{t+1}, removing the dynamic effects of debt *per se*. A debt-for-social security *swap* would nonetheless raise the autocorrelation of k_t up to some critical   -value above 1, because wage-indexed social security benefits depend positively on k_t through the dependence of wages on capital.

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[21] See (3.6). An increase in debt d at a given steady state value of (c^2/A) implicitly assumes a given level of intergenerational redistribution. It is therefore best interpreted as a substitution of safe debt for wage-indexed social security. The assumption of constant redistribution is justified more formally in Section 4 below, where I show that the steady state level of intergenerational redistribution is determined by the social planner's rate of time preference. The substitution experiment represents an analytically "clean" change in risk sharing policies and it is policy-relevant in the context of social security privatization proposals (e.g., Feldstein, 1996).

A shift of social security trust fund balances from debt to equity ($k=1-b>0$) further reduces the old generation's relative exposure to productivity risk at the expense of the young. This is because such a shift increases the government's net indebtedness in bonds (less of the gross debt is held by the trust fund) and because it effectively exposes the young generation to capital income uncertainty (the k term in (3.5c)).

Undepreciated old capital (>0) also reduces the old generation's relative exposure to productivity risk, but without affecting the young. This is because old capital provides productivity-independent income.[22]

Overall, it seems easy to find reasons why the consumption variance of the old relative to the consumption variance of the young is less than in the wage-proportional allocation, but difficult to go in the other direction. The next question is how the market allocation of risk compares to the efficient allocation.

4. Efficient Risk Sharing

This section describes the efficient allocations implemented by a social planner who maximizes a weighted average of generations' utilities.

4.1. General Efficiency Conditions

A few general efficiency conditions can be obtained for general preferences $U_t(c^1_t, c^2_{t+1})$ and a general technology $F_t(K_t, N_t)$. Assume that at time $t=0$, the social planner maximizes a welfare function[23]

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[22] Incomplete depreciation also increases the capital coefficient ($c2Ak>$ for >0). Intuitively, production has a capital coefficient of while the salvage value has a unit capital coefficient.

[23] It is a philosophical question if persons born in different states of nature are the same or different individuals. Here, treating them as different (maximizing $_s$ $_t$ $_t(s)$ $E_tU_t(s)$ summed over states s) would be uninteresting, because any imposition of risk on future generations could then be rationalized trivially by an appropriate set of state-contingent welfare weights $_t(s)$; see Peled (1982). Note that if young and old are interpreted as working-age and retired, the model's next "unborn" generation are in reality children, i.e., identifiable individuals.

(4.1) $\quad W_0 = E_0[\sum_{t=-1} \alpha_t U_t]$

with welfare weights $\alpha_t > 0$, subject to the resource constraint

(4.2) $\quad K_{t+1} + N_t c^1_t + N_{t-1} c^2_t + G_t = F_t(K_t, N_t).$

Initial capital and the t=-1 consumption of the old are given; utility is increasing and concave and production satisfies the Inada conditions. Optimal consumption and investment plans are characterized by the first order conditions

(4.3) $\quad \lambda_t N_t = \alpha_t \dfrac{dU_t}{dc^1_t}, \qquad \lambda_t N_{t-1} = \alpha_{t-1} \dfrac{dU_{t-1}}{dc^2_t}, \qquad \lambda_t = E_t[\lambda_{t+1} R^k_{t+1}],$

where λ_t is the Lagrange multiplier of the resource constraint and $R^k_t = F_t / K_t$. Thus, the efficient allocation must satisfy two intuitively insightful conditions:

(i) a <u>distributional optimality condition</u> linking the consumption of old and young within a period,

(4.4) $\quad \dfrac{dU_{t-1}}{dc^2_t} \dfrac{\alpha_{t-1}/N_{t-1}}{\alpha_t/N_t} = \dfrac{dU_t}{dc^1_t}$; and

(ii) a <u>dynamic optimality condition</u>

(4.5) $\quad \dfrac{dU_t}{dc^1_t} = E_t[\dfrac{dU_t}{dc^2_{t+1}} R^k_{t+1}] \iff E_t[MRS(c^1_t, c^2_{t+1}) R^k_{t+1}] = 1,$

which shows that the social planner respects the individual optimality condition for capital investment.

The distributional condition (4.4) yields some immediate insights for the case of time-separable preferences (incl. CRRA). Separability implies that (4.4) depends only on the time-t consumption of the two generations. Then the social planning problem divides naturally into two parts, a "<u>static problem</u>" of allocating aggregate consumption to the living generations, and a "<u>dynamic problem</u>" of choosing the path of capital accumulation. If aggregate consumption differs across states of nature, marginal variations in aggregate consumption $C_t = N_t c^1_t + N_{t-1} c^2_t$ must be

allocated to the different generations according to (4.4). Regardless of how and why aggregate consumption varies, each generation's consumption must be perfectly correlated with aggregate consumption in a way that depends only on individuals' relative risk aversion and consumption shares,

$$(4.6) \qquad \frac{dc^i_t}{c^i_t} = \frac{1/\gamma_i}{\theta/\gamma_1+(1-\theta)/\gamma_2} \; \frac{dC_t}{C_t} \qquad \text{for } i=1,2$$

where $\theta = N_t\, c^1_t/C_t$ and $1-\theta = N_{t-1}\, c^2_t/C_t$ are the consumption shares of the old and young. (Neither θ nor the γ-s have to be constant here.) Equivalently, the relative volatility of the two generation's consumption must be inversely proportional to their relative risk aversion,

$$(4.7) \qquad \gamma_1 \, \frac{dc^1_t}{c^1_t} = \gamma_2 \, \frac{dc^2_t}{c^2_t}.$$

This characterization of the optimal sharing of aggregate consumption parallels the results of Gale (1990) and Stiglitz (1983). For age-independent risk aversions ($\gamma_1=\gamma_2$), it implies equal consumption volatility. Eq. (4.7) shows that age-dependent risk aversion may rationalize policies that expose one cohort to more consumption volatility than the other--e.g., safe debt, if one assumes that the old are more risk averse than the young. Eqs. (4.6-4.7) also imply that the government should never impose stochastic intergenerational transfers except to remedy a pre-existing misalignment between old and young consumption. This immediately shows the inefficiency, say, of nominal debt with "noisy" inflation (random P_t) and of "political instability" (unexpected tax changes not associated with economic fundamentals).

These general insights apply for all time-separable preferences without requiring assumptions about the stochastic structure of the model. But (4.6) and (4.7) provide only a partial solution of the optimal risk sharing problem because risks can be shared with future generations by

varying the path of capital accumulation. Assumptions about the stochastic structure of the economy are then unavoidable.

The next section therefore returns to the Markov model and balanced growth. To prepare, note that the efficient allocation displays balanced growth with homothetic preferences if and only if $(\omega_{t-1}/N_{t-1})/(\omega_t/N_t)$ in (4.4) is constant. To ensure balanced growth, I assume therefore that the planner values each generation's per-capita utility in proportion to population size and then applies a constant rate of time preference, i.e., $\omega_t = N_t \beta^t$, for some constant β.

If utility is homothetic of degree $1-\gamma$, as assumed in (2.3), the transversality condition $\lim_{T\to\infty} E_t[\omega_T K_{T+1}] = 0$ requires that the growth-adjusted rate of time preference $\beta^* = \beta(1+n)(1+a)^{1-\gamma}$ satisfies $\beta^* < 1$.[24] Equations (4.4) and (4.5) then imply that the steady state return on capital $R^k/an = 1/\beta^* > 1$ depends only on the social planner's rate of time preference; this motivates why steady state values were held constant in Section 3.

4.2. Optimal Policy in the Markov Model

This section examines the social planning problem under the preference and technology assumptions of Section 2. The planner's problem is similar to the standard infinite-horizon optimal growth problem familiar from the business cycle literature (King-Plosser-Rebelo, 1988a, 1988b), except that there are two goods and a more general utility function. Like the standard business cycle problem, the optimal risk-sharing problem generally does not have a closed form solution. A natural approach to characterizing the stochastic and dynamic properties of the optimal risk-sharing policy is

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[24] The growth-adjusted time preference can be interpreted as in representative agent models (e.g., King-Plosser-Rebelo, 1988a), except that the social planners rate of time preference matters and not the individual time preference.

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therefore to again log-linearize the economy around its deterministic
steady state. The model is transformed into stationary ratio-variables by
dividing through the growth trend $A_t N_t$. The social planning problem is then
to maximize

$$W_0 = E_0 [\sum_{t=-1} (\beta^*)^t (U/A)_t]$$

where $(U/A)_t = U_t/(A_t^{1-\gamma 1})$ and $\beta^* = \beta (1+n)(1+a)^{1-\gamma 1} < 1$, subject to the
resource constraint

$$(4.8) \qquad Y_t = (c^1/A)_t + \frac{1}{1+n}(c^2/A)_t + \frac{G_t}{A_t N_t} + k_{t+1} - \beta_t \frac{k_t}{(1+a_t)(1+n)}$$

$$= (\frac{k_t}{(1+a_t)(1+n)}) .$$

Government spending deserves some comment here. The technically
easiest assumption would be to set $G_t/(A_t N_t) = g^A$ constant, essentially
removing government spending as a dynamic variable. But that would be
inappropriate because a constant g^A implies systematic movements in the
spending-GDP ratio that would *a priori* rule out a wage-proportional
allocation and make a comparison to Section 3 impossible. A constant $G_t/Y_t =$
g is economically more reasonable, but it is technically tricky, because Y_t
is endogenous. If a constant g were interpreted as making G_t endogenous, it
would "distort" the social planner's capital accumulation decisions (just
like a proportional tax). Hence, I maintain that G_t is exogenous but at a
level that just happens to be proportional to the equilibrium path of Y_t.
Then $G_t = g Y_t$, as in Sections 2-3. This interpretation preserves both the
"undistorted" dynamic optimality condition (4.5) and prevents fluctuations
in the spending-GDP ratio that would distort the natural links between
growth and consumption opportunities.[25]

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[25] Two alternative motivations can be given. First, the assumption can be interpreted as a
reduced-form representation of a model with preferences over public goods. Given an
efficient provision of such goods, the social planner will not engage in "tax-avoidance"
to reduce public goods provision. Second, for purposes of social security policy--e.g.,
for an assessment of the Advisory Council proposals--one think of the social planner as a

The linearized solution for consumption and investment is characterized by three equations: First, there is the resource constraint

(4.9a) $k/y\ \hat{k}_t\ +\ s_1\ (\widehat{c^1/A})_t\ +s_2\ (\widehat{c^2/A})_t\ =\ [\ \ (1-g)+s\ ]\ [\hat{k}_{t-1}\ -\ \hat{a}_t]$

where $s_1=(c^1/A)/y$, $s_2=\ 1/(1+n)\ (c^2/A)/y$, and $s\ =\ /an\ k/y$. A high capital stock and a high value of old capital raise consumption and investment opportunities, while high government spending reduces them. High productivity growth $\hat{a}_t$ increases the level of aggregate income but reduces income relative to the new stochastic trend.

Second, the dynamic optimality condition (4.5) implies

(4.9b) $E_t[(\widehat{c^2/A})_{t+1}]\ -\ (\widehat{c^1/A})_t\ =\ -\quad_{Rk}\ \hat{k}_{t+1}$

where $_{Rk}\ =\ (1-\ )\ (1-\ /R^k)/(1-\ )$

reflects the dependence of returns on the capital-labor ratio,

(4.9c) $\widehat{R^k}_{t+1}\ =\ (1-\ /R^k)\ (1-\ )\ (\hat{a}_{t+1}\ -\ \hat{k}_{t+1}).$

Intuitively, individuals smooth consumption (explaining the unit coefficients on c^1 and c^2) and respond to changes in expected returns according to their elasticity of substitution.

Third, the distributional optimality condition (4.4) implies

(4.10) $2\ (\widehat{c^2/A})_t\ -\ 1\ (\widehat{c^1/A})_t\ -\ (1-\ )\ (1-\ -\ 1)\quad_{Rk}\ \hat{k}_{t+1}$

$=\ (\ 1-\ 2)\ \hat{a}_t\ +\ (\ 2-\ 1)\ (\widehat{c^1/A})_{t-1}\ +\ [\ 1-\ 2+\ (1-\ -\ 1)]\quad_{Rk}\ \hat{k}_t.$

where $(0,1)$ is a constant. This condition links the contemporaneous consumption of young and the old with weights determined by the risk aversion coefficients. Age-dependent risk aversion (1 2) implies a dependence of current consumption decisions on lagged consumption $(\widehat{c^1/A})_{t-1}$, which unfortunately increases the dimensionality of the state-space. To prevent the analysis from being side-tracked by technical complications, and because the conceptually significant point that age-dependent risk

government agency with power over redistributional policy instruments who takes government spending as given.

aversion may be a rationale for unequal consumption variances has already been made in Section 4.1, I will assume $\alpha_1=\alpha_2=\alpha$ for the remainder of this section. Since $\alpha \neq 1-\gamma$ is permitted, the preferences are still as general as Epstein-Zin's (1989).[26]

For $\alpha_1=\alpha_2=\alpha$, equation (4.10) reduces to

(4.11) $(c^{\hat{2}}/A)_t - (c^{\hat{1}}/A)_t - h_1 \hat{k}_{t+1} - h_2 \hat{k}_t = 0$

where $h_1 = (1-\alpha)(1-\gamma-\alpha)/\beta_{Rk}$ and $h_2 = \alpha(1-\gamma-\alpha))/\beta_{Rk}$ matter only for non-separable preferences $(1-\gamma \neq \alpha)$.

Equation (4.11) can be used to substitute $(c^{\hat{1}}/A)_t$ out of (4.9a-b). Equations (4.9a-b) then form a pair of expectational difference equations. For CRRA utility $(\alpha=1-\gamma)$ and no government spending (g=0), it is straightforward to show that the characteristic roots (μ_1,μ_2) satisfy $0<\mu_1<1<1/\beta^*<\mu_2$. For general $\alpha \neq 1-\gamma$ and g $\neq$ 0, I <u>assume</u> that $\mu_1<1<\mu_2$; this holds in a neighborhood of $(\alpha=1-\gamma, g=0)$ and seems satisfied for plausible parametrizations. The dynamic system is then saddle-path stable and yields optimal decision rules for $[\hat{k}_{t+1}, (c^{\hat{1}}/A)_t, (c^{\hat{2}}/A)_t]$ as function of the Markov state vector $(\hat{k}_t, \hat{a}_t)$. Specifically, one finds

(4.12a) $\hat{k}_{t+1} = \mu_1 \ \hat{k}_t + \pi^*_{ka} \ \hat{a}_t$, and

(4.12b) $(c^{\hat{2}}/A)_t = \pi^*_{c2Ak} \ \hat{k}_t + \pi^*_{c2Aa} \ \hat{a}_t$

where $\pi^*_{c2Ak}>0$, $\pi^*_{c2Aa} = -(1-1/\mu_2)\phi_{ca}<0$ depends on $\phi_{ca} = \dfrac{(1-g)+s}{s_1+s_2}>0$, and

$\pi^*_{ka} = -(1-1/\mu_2)\phi_{ka}<0$ depends on $\phi_{ka} = \dfrac{(1-g)+s}{k/y-s_1 h_1}>0$.[27]

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[26] For the time-separable case, $(1-\gamma=\alpha)$ the $\hat{k}_{t+1}$-term in (4.10) would vanish even for $\alpha_1 \neq \alpha_2$, confirming that unexpected changes in consumption levels $((c^{\hat{1}}/A)_t+\hat{a}_t)$ should be shared in the proportion to the risk aversion parameters, $\alpha_2 [(c^{\hat{2}}/A)_t+\hat{a}_t] = \alpha_1 [(c^{\hat{1}}/A)_t+\hat{a}_t] + (\alpha_2-\alpha_1) [(c^{\hat{1}}/A)_{t-1}+\beta_{Rk} \hat{k}_t]$, as in (4.7). But if $\alpha_1 \neq \alpha_2$, the dynamics are complicated by the $(c^{\hat{1}}/A)_{t-1}$ term. An analysis with $(c^{\hat{1}}/A)_{t-1}$ as state variable would be technically straightforward, but the implied higher-order dynamic system would not yield easily interpretable analytical solutions.

[27] The constants μ_1, μ_2, π^*_{c2Ak} are functions of the parameters that can be computed as in King-Plosser-Rebelo (1988a,b); derivations are in the technical appendix. For CRRA utility, the coefficients on a_t and k_t are related: $\pi^*_{c2Aa}=-\pi^*_{c2Ak}$ and $\pi^*_{ka}=-\mu_1$.

Thus, efficient capital investment and consumption are both autocorrelated, positively affected by valuation shocks, and (in ratio form) negatively affected by productivity shocks. For CRRA utility (1- =), (4.11) implies $(c^{\hat{1}}/A)_t = (c^{\hat{2}}/A)_t$, so that the consumption rules should be the same for both generations. For 1- , they are different, and the rule for $(c^{\hat{1}}/A)_t$ follows from (4.11) and (4.12a-b). The elasticity of consumption levels to productivity shocks is again given by one plus the elasticity in ratios, e.g., $^*_{c2a} = 1 - (1-1/\mu_2)_{ca}$, which is typically positive.

Recall from Section 3 that the generic market allocation implies different coefficients for the consumption rules of the young and the old. Hence, the observation that young and old consumers should have the same consumption dynamics for all CRRA utility functions, implies that <u>the market solution is generally inefficient</u>.

One notable exception is the case of wage-proportional incomes and log-utility. For the special case of log-utility (=1- =1) and full depreciation (=0), the coefficients in (4.13) reduce to $\mu_1 = - {}^*_{ka} =$ and $^*_{c2Ak} = - {}^*_{c2Aa} =$, the same coefficients as in (3.5-6). Hence, <u>the wage-proportional market allocation is efficient for log-utility and =0</u>. For this special case, any non-proportional policy tool (such as safe debt) would be a source of inefficiency.

To understand the sources of inefficiency outside this special case, first consider CRRA preferences with a lower intertemporal elasticity of substitution and higher risk aversion (<0, =1- >1; =0). Such preferences call for smaller (absolute) responses of the consumption-productivity ratio ($^*_{c2Ak} = - {}^*_{c2Aa} <$) to shocks and to initial deviations from the steady state (for both young and old consumers) and larger responses of capital investment to shocks and to initial deviations from the steady state ($\mu_1 =$

$-$ $_{ka}$ >). If old age consumption is wage-proportional, the old bear too little productivity risk ($_{c2a}$ = 1$-$ < $^*_{c2a}$) and are not sufficiently exposed to fluctuations in lagged capital ($_{c2Ak}$ = < μ_1). The consumption and investment coefficients for the young differ from in the right direction ($_{c1a}$ > 1$-$) but not by the efficient amount.

For the converse case, CRRA utility with high substitution elasticity and low risk aversion (>0, =1$-$ <1), one finds μ_1 = $-$ $^*_{ka}$ < , $^*_{c2Ak}$ = $-$ $^*_{c2Aa}$ > , so that the wage-proportional market allocation implies a too high exposure of the old to productivity shocks. The empirical evidence tends be in to favor of a low elasticity of substitution (e.g., Hall 1988), however, suggesting that the old bear too little risk.

Undepreciated old capital, >0, is likely to make the misallocation worse, because it provides a productivity-independent source of income for the old. With log-utility (as benchmark), >0 reduces the efficient consumption response to productivity shocks, 1+ $^*_{c2a}$<1$-$. The market solution yields $_{c2a}$ < $^*_{ca}$ < 1$-$ = $_{c1a}$, i.e., too much exposure to productivity risk for the young and too little for the old even with log-utility.

Redistribution through safe government debt is difficult to rationalize in this context, because it shifts additional productivity risks from the old to the young. This might be promising, if the elasticity of substitution were substantially above one, but not for empirically plausible elasticity values below one. A social security trust fund may help, if it reduces the net amount of safe debt, but not if it holds equity that imposes risk on future contributors. Wage-indexed social security, in contrast, maintains a wage-proportional distribution of incomes and therefore appears preferable to safe debt.

Epstein-Zin preferences outside the CRRA class (1-) produce a more complicated pattern of coefficients, but they do not provide a plausible justification for safe debt either. For a risk aversion parameter above the inverse elasticity of substitution (>1- , the case usually invoked to rationalize a high equity premium), the consumption of the young should be less sensitive to productivity shocks than the consumption of the old.[28] To motivate safe debt, one would need deviations from expected utility in the opposite direction, a risk aversion parameter below the inverse elasticity of substitution; but that would be inconsistent with a reasonably high equity premium.

Overall, it seems that for a wide range of reasonable parameters (0, 1- , 0), the old generation bears to little productivity risk in the market allocation. Safe government debt makes matters worse. The next section examines to what extent these conclusions are robust with respect to modifications of the model.

5. Extensions

This section generalizes the main model in several directions, discussing other aggregate shocks, elastic labor supply, CES-production, and income taxes.

5.1. Other macroeconomic risks

In the main model (Sections 3 and 4.2), permanent productivity shocks were the only source of aggregate risk. Here I consider three other sources of risk: temporary productivity shocks, an uncertain salvage value of old capital, and government spending shocks.

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[28] Eq.(4.10) implies $^*c_{1a} = {}^*c_{2a} - h_1 {}^*k_a < {}^*c_{2a}$ because $^*k_a < 0$ and $h_1 < 0$; by similar reasoning, the consumption of the young should be more sensitive to initial capital than the consumption of the old, $^*c_{1Ak} > {}^*c_{2Ak}$. Note that generally $^*c_{2Aa} - {}^*c_{2Ak}$ and $^*c_{1Aa} - {}^*c_{1Ak}$, unlike the CRRA case.

Temporary productivity shocks are worth discussing because they raise questions about the role of consumption-smoothing. If productivity shocks are temporary, the young may be able to bear them more easily than the old, because can consumption-smooth over two periods. But since a temporary productivity shock reduces interest rates while a permanent productivity shock increases them, the implications of temporary versus permanent shocks are far from obvious. The differential income effects may be offset by differential substitution effects.

To examine this issue more formally, suppose one adds a temporary productivity disturbance v_t to the model, redefines output to be

$$Y_t = K_t \quad (A_t (1+v_t) N_t)^{1-} \;,$$

and modifies the marginal products accordingly. If v_t is assumed i.i.d., both the positive model of Section 3 and social planning problem of Section 4.2 retain their Markov structures, now with the additional state variable v_t. The log-linearized decision rules gain an additional term for the v_t-shocks ($_{xv}\hat{v}_t$ in the positive model, $^*_{xv}\hat{v}_t$ in the normative model). But since the $_{xa}$ and $_{xk}$ coefficients remain unchanged, all previous results about permanent shocks and about initial capital remain unchanged.

The efficient solution for CRRA utility again requires equal coefficients for old and young consumers, $^*_{c1v}= {}^*_{c2v}$. In the market model, the distinction between permanent and temporary shocks is irrelevant for the old, i.e., $_{c2a}= {}_{c2v}$. For the young, the consumption response to temporary shocks generally differs from the response to permanent shocks because of differential consumption smoothing and intertemporal substitution effects. The difference in the coefficients on v_t and a_t shocks is

$$_{c1a} - {}_{c1v} = k/(c^1/A) \left[\frac{1+k/(c^1/A)}{k} - 1\right].$$

It is positive if and only if η_k in (3.5a) is less than $1+k/(c^1/A)$.

In the benchmark case of wage-proportional incomes and log-utility, $\eta_k = 1+k/(c^1/A)$ so that $\eta_{c1a} = \eta_{c1v}$. Then the differential interest rate movements exactly offset the differential income effects,[29] producing equal consumption coefficients and showing that the wage-proportional allocation remains optimal.

The condition $\eta_k < 1+k/(c^1/A)$ is satisfied, however, for wage-proportional incomes and a CRRA utility with elasticity of substitution below one ($\eta < 0$). For low elasticities, the differential income-effects (consumption-smoothing) dominate and the consumption response of the young is less than the consumption response of the old, $\eta_{c1v} < 1-\eta = \eta_{c2v}$. By shifting temporary productivity risk to the young, safe debt might be efficiency-improving.[30]

It would be preferable, however, to find a poliy instrument that only shifts temporary risk and not permanent risk. In practice, a government would take a huge gamble if it issues safe debt and blames low output realizations on negative temporary shocks. A negative temporary shock v_t justifies a runup in the debt-GDP ratio because the government is likely "bailed out" by above-normal productivity growth in the next period. (Temporary shocks imply negatively autocorrelated producitivity growth: $(1+a_{t+1})(1+v_{t+1})/(1+v_t)-1$ is above $E_t a_{t+1}$ in expectation, if $v_t < E_t v_{t+1}$.) But the government would be stuck with an excessive debt-output ratio if the shock turns out to be a misidentified permanent shock. Hence, one should be

[29] A positive temporary shock provides a consumption-smoothing motive to save more, but it also raises the prospective capital-labor ratio k_{t+1}, and hence depresses interest rates. In contrast, a permanent shock reduces k_{t+1} and raises interest rates, providing a substitution motive to save more. In the log-utility case, these two different mechanisms are equally strong.

[30] In an economy with both shocks, the ideal policy instrument would only reallocate temporary risk and not permanent risks. But safe debt may have a role in a second-best sense if the government cannot distinguish different shocks and temporary shocks are more common.

wary about drawing strong policy conclusions from the case of temporary productivity shocks.[31] The negative autocorrelation in productivity growth required for temporary shocks is also empirically questionable.

Second, consider <u>adding uncertainty to the return to capital</u>. Productivity shocks (permanent and temporary) imply a deterministic link between wages and the marginal product of capital, which may be considered restrictive. To see the implications of independent movements in the return to capital, suppose the salvage value of old capital is an i.i.d. random variable $_t$. In practice, the old hold a variety of long-lived capital goods of uncertain value so that one might think of a stochastic $_t$ as a general "valuation risk." The return to capital is then still correlated with productivity and wages, but contains additional "noise." The model retains its Markov structure, now with $_t$ as additional state variable and with $\hat{}_t$- terms in the log-linearized decision rules. The other coefficients remain unchanged.

In the market allocation, valuation risk is a generation-specific risk, since the old generation holds all the capital: $_{c2} > 0$, while $_{c1} = 0$ and $_k = 0$, provided $^e = 0$. Section 4.1 has shown that it is inefficient not to share such risk across generations. In any efficient allocation, valuation risk must be shared between the old, the young, and all future generations; i.e., $^*_{c2} > 0$, $^*_{c1} > 0$, and $^*_k > 0$. The proviso $^e = 0$ points to an interesting risk-sharing role of the social security trust fund. With defined benefits, the risk and return of social security equity investments is carried by future generations ($_{c1} > 0$ and $_k < 0$ if $^e > 0$), so that equity

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[31] The political temptation to identify shocks as temporary should be obvious. One may wonder if this is related to the world-wide growth in debt-GDP ratios after the post-1973 productivity slowdown.

investments are a means to share valuation risk. This may be an interesting topic for future research.

Overall, independent movements in the return to capital are another potential source of inefficiency but they do not change previous findings about productivity. One may argue that valuation risk raises the variance of the old generation's consumption, but the appropriate remedy would not be to issue safe debt that shifts all risks from old to young.

Third, <u>shocks to government spending</u> are potentially important source of risk (e.g. war spending). In the normative model, a stochastic, exogenous share of government spending $g_t = G_t/Y_t$ can be accommodated easily, because government spending reduces the resources available to consumption and capital investment in the same way as a negative productivity shock. To maintain the Markov structure, suppose the spending share g_t is i.i.d. with mean g. Then an efficient allocation requires a negative response of old and young consumption ($^*_{c1g}<0$, $^*_{c2g}<0$, with equal coefficients in case of CRRA) and a burden-sharing with future generations through variations in capital investment, $^*_{kg}<0$. In a market setting, efficient responses to spending shocks could be implemented in various ways; say, by allowing tax rates and the debt-output ratio to depend on spending shocks and lagged debt. A detailed analysis is beyond the scope of this already long paper. The main point here is that additional sources of uncertainty can be accommodated fairly easily without changing major results.

5.2. Variable labor supply

The assumed inelastic labor supply may be considered restrictive, too. One might argue, for example, that the young can bear more risk, because they

can "recover" from bad shocks by increasing their labor supply, whereas the retired old have to live with their given resources.[32]

To examine this issue, assume that individuals are endowed with one unit of time and have preferences over consumption and leisure. By assumption, the old are excluded from the labor market and use all their time for leisure. The young consume l_t units of leisure, where $0 \leq l_t \leq 1$, and provide labor supply $1-l_t$. Efficiency and individual rationality both imply that the marginal rate of substitution between young consumption and leisure equals the wage rate. In the normative analysis of Section 4.1, (4.7) must then be replaced by

$$\frac{dc^2_t}{c^2_t} \cdot \frac{1}{\gamma_2} = \frac{dc^1_t}{c^1_t} \cdot \frac{1}{\gamma_1} + \frac{U_{cl}}{U_c} \, dl_t,$$

where the subscripts in U_{cl} and U_c denote partial derivatives. Thus, the results about relative consumption volatilies remain (approximately) valid, if U_t is (approximately) separable in c^1_t and l_t. If consumption and leisure are substitutes ($U_{cl}<0$) and negatively correlated (as one may suspect in case of productivity shocks), the consumption of the old should actually be more volatile than the consumption of the young, contrary to the "recovery" argument motivating this section. In general, the relative volatility of the old and young generations' consumption depends on the correlation and the substitutability of consumption and leisure for the young. But the results based on fixed labor supply are still approximately valid, unless one is convinced that $|U_{cl}|$ is large.

To say more about the correlation of consumption and leisure, a parametrized model is again needed. Consider therefore a time-separable CRRA specification with a Cobb-Douglas aggregator over consumption and leisure, $U_t = u(c^1_t,l_t) + \beta u(c^2_t,1)$ with $u(c,l) = [c^\alpha l^{1-\alpha}]^{1-\gamma}/(1-\gamma)$, $\gamma>0$. The

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[32] This concern was often raised when I presented an earlier draft of the paper.

Cobb-Douglas aggregator implies a unit elasticity of substitution between consumption and leisure, which is necessary for a balanced growth. The efficient log-linearized decision rules can be derived as before. One finds: (a) In the special case of log-utility (1) and =0, the labor supply is constant and the wage-proportional allocation is efficient, as in the main model. (b) In the empirically most relevant case of >1 (low elasticity of substitution), negative productivity shocks induce an increase in labor supply so that consumption and leisure are negatively correlated. Since >1 implies $U_{cl}<0$, a variable labor supply implies that the consumption volatility of the young should actually be <u>less</u> than the consumption volatility of the old, contrary to the "recovery" argument. Intuitively, the "recovery" argument fails because states of nature with low income are also states of nature in which the marginal product of labor is low. Hence, it would be inefficient to ask the young to work more when aggregate income is low.

Overall, the section shows that the labor-leisure option of the young and the exclusion of the old from the labor market do NOT create a presumption that the young are better able to bear risk than the old. As shown above, the labor-leisure choice may be irrelevant (with log-utility) or even call for less risk-bearing by the young. Although other parametrizations may conceivably yield different results (I am not striving for generality here), the main model with fixed labor supply provides a reasonable approximation for the optimal allocation of risk.

5.2. CES Production

The Cobb-Douglas technology in the main model implies a unit elasticity of substitution between capital and labor. This is a significant restriction

because a non-unit elasticity changes the relative riskiness of capital and labor incomes. To see this, consider a CES-production function

$$Y_t = [\ K_t + (1-\ )(A_t N_t)\ ]^{1/}$$

with elasticity of substitution $1/(1-\ )$; Cobb-Douglas is the limiting case $=0$. Technological progress is assumed permanent and labor augmenting (to ensure balanced growth), as in the main model.

Economically most interesting is the case of an elasticity below one, <0. A positive productivity shock that raises the effective supply of labor $(N_t A_t)$ relative to the stock of capital will then reduce the labor share in output relative to the capital share. This magnifies the effect of productivity shocks on capital income and dampens the effect on labor income relative to the Cobb-Douglas case. Hence, the old generation faces a more volatile market income than the young, suggesting that the market allocation may impose too much risk on the old. This is reinforced by the fact that the efficient allocation calls for productivity shocks to be absorbed by variations in the savings rate that reduce the volatility of old and young consumption relative to the Cobb-Douglas case. The labor augmenting nature of technical progress is important here, because it makes the effective labor supply the main source of uncertainty.

Overall, a CES-technology with elasticity parameter below one may theoretically justify government interventions that shift risk from old to young such as safe debt. An elasticity of intertemporal substitution below one and a non-zero salvage value of old capital would, however, reduce the relative consumption volatility of the old. An elasticity of factor substitution below one is therefore by no means sufficient to justify government intervention.[33]

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[33] A weighting of these factors would require an empirical analysis beyond the scope of this paper. For the main model, I assume Cobb-Douglas technology because the qualitative

5.4. Income taxes as risk-sharing device

Income taxes are commonly believed to have important risk sharing effects. Throughout this paper, taxes are assumed lump-sum even if the collected amounts depend on variables like aggregate income. Tax distortions are beyond the scope of the paper because of the implied second-best considerations. The taxes on the young can nonetheless be interpreted as income taxes because of the inelastic labor supply and because the amount was assumed to be wage-proportional. The risk-sharing implications of capital income taxes are therefore the main issue.

For the old, lump-sum taxes proportional to per-capita output cannot be interpreted as income taxes, because capital income taxes would distort savings decisions and because the income of the old (as usually defined) would not be proportional to Y_t/N_t. Since the capital income of the old is $(R^k_t-1) s^k_{t-1}$, an income tax on the old at the rate CI would collect revenues

$$2_t = {}^{CI} \left[\ \left(\frac{k_t}{(1+a_t)(1+n)}\right)^{-1} + \ {}_t - 1 \right] k_t A_{t-1}.$$

The ratio of capital income revenues to GDP would then be an increasing function of the productivity and valuation shocks,

$$2_t \frac{N_{t-1}}{Y_t} = {}^{CI} \left[\ - (1-{}_t) \left(\frac{k_t}{(1+a_t)(1+n)}\right)^{1-} \ \right].$$

If the taxes on the young the residual quantity determined by the budget constraint, $^{CI}>0$ would increase the exposure of the young to productivity shocks and to valuation risk. A capital income tax is therefore another

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implications of alternative preference and policy parameters are most easily explained in a Cobb-Douglas setting (yielding wage-proportional incomes), and because Cobb-Douglas is a standard assumption in the production literature (e.g., Gomme and Greenwood, 1995). The data are difficult to interpret. In annual U.S. data, the simple correlation between the log capital share and the log output-capital ratio is actually negative (-0.31 for 1929-1996, -0.33 for 1954-1996), contrary to what one would need to rationalize safe debt. But careful production studies have found evidence for a below-unit elasiticity (e.g., Lucas, 1969); overall, this issue is best left for future research.

means of shifting risks from old to young. A uniform income tax on young and old would have similar features.[34]

Overall, it is true that income taxes on the old have potentially interesting risk-shifting effects. But the welfare benefits of such taxes are questionable, because they would clearly distort the individual first order conditions (2.5) and break the link to the planner's problem, equation (4.5). The literature on state-contingent taxation (Chari et.al, 1991; Zhu, 1992; Bohn, 1994) has shown that capital income taxes can be levied without causing distortions if and only if taxes in some states of nature are offset by negative taxes in other states of nature. But such taxes raise little revenue and are often equivalent to government debt with state-contingent returns. Hence, the comments about state-contingent debt apply analogously: They are theoretically a powerful tool for risk-shifting, but quite complicated to design and too powerful to be interesting, except perhaps for normative analysis. This explains their exclusion in the main model.

6. Conclusions

The paper has examined the intergenerational sharing of macroeconomic risk in a stochastic OG model. In the market allocation without government, the old and the young share productivity risk through its impact on capital and labor income. Safe government debt shifts productivity risk from old to young. Wage-indexed social security, in contrast, is essentially neutral with respect to the allocation of risk. A social security trust fund will,

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[34] If government spending were financed by a uniform income tax at a time-varying rate τ_t, the tax rate would have to satisfy $g Y_t = \tau_t [w_t N_t + (R^k_t-1) k_t A_{t-1} N_{t-1}]$ (for d=0 for simplicity), hence

$$\tau_t = g \left[\left(\frac{k_t}{(1+a_t)(1+n)}\right) + (\tau_t-1) \frac{k_t}{(1+a_t)(1+n)} \right]^{-1}.$$

Tax revenues from the old would be an increasing function of a_t and τ_t, which means that the tax shifts risks away from the old.

in the context of a defined benefit system, either reduce net government debt, if invested in bonds, or shift capital income risk to future generations.

A comparison of the market allocation with the set of Pareto-efficient allocations shows that the market allocation is generally inefficient. The market allocation of risk depends importantly on individuals' intertemporal elasticity of substitution--the willingness to spread risk over time--and not primarily on risk aversion. The optimal allocation requires that old and young bear consumption risk in inverse proportion to their respective relative risk aversion.

For plausible parameters, the market allocation seems to impose too much productivity risk on the young. Safe government debt is difficult to rationalize in this context because it shifts more risk from old to young. If the government engages in intergenerational redistribution, productivity-contingent transfer schemes such as wage-indexed social security seem preferable to government bonds. The widespread use of safe debt makes one wonder if politicians have been tempted to offer safe securities to current voters without considering--perhaps without recognizing--the implied risks for future generations.

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