Cycles and Instability in a Rock-Paper-Scissors Population Game: a Continuous Time Experiment*

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Abstract

We report laboratory experiments that use new, visually oriented software to explore the dynamics of 3×3 games with intransitive best responses. Each moment, each player is matched against the entire population, here 8 human subjects. A "heat map" offers instantaneous feedback on current profit opportunities. In the continuous slow adjustment treatment, we see distinct cycles in the population mix. The cycle amplitude, frequency and direction are consistent with standard learning models. Cycles are more erratic and higher frequency in the instantaneous adjustment treatment. Control treatments (using simultaneous matching in discrete time) replicate previous results that exhibit weak or no cycles. Average play is approximated fairly well by Nash equilibrium, and an alternative point prediction, "TASP" (Time Average of the Shapley Polygon), captures some regularities that NE misses.

JEL numbers: C72, C73, C92, D83

Keywords: experiments, learning, mixed equilibrium, continuous time.

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1 Introduction

Rock-Paper-Scissors, also known as RoShamBo, ShouShiLing (China) or JanKenPon (Japan) is one of the world's best known games. It may date back to the Han Dynasty 2000 years ago, and in recent years has been featured in international tournaments for computerized agents and humans (Fisher, 2008).

The game is iconic for game theorists, especially evolutionary game theorists, because it provides the simplest example of intransitive dominance: strategy 1 (Rock) beats strategy 3 (Sissors) which beats strategy 2 (Paper), which beats strategy 1 (Rock). Evolutionary dynamics therefore should be cyclic, possibly stable (and convergent to the mixed Nash equilibrium), or perhaps unstable (and nonconvergent to any mixture). Questions regarding cycles, stable or unstable, recur in more complex theoretical settings, and in applications ranging from mating strategies for male lizards (Sinervo and Lively, 1996) to equilibrium price dispersion with incomplete price information (e.g., Maskin and Tirole, 1988).

The present paper is an empirical investigation of behavior in RPS-like games, addressing questions such as: Under what conditions does play converge to the unique interior NE? Or to some other interior profile? Under what conditions do we observe cycles? If cycles persist, does the amplitude converge to a maximal, minimal, or intermediate level? These empirical questions spring from a larger question that motivates evolutionary game theory: To understand strategic interaction, when do we need to go beyond equilibrium theory?

Surprisingly, we were able to find only two other human subject experiments investigating RPS-like games. Cason, Friedman, Hopkins (2010) study variations on a 4x4 symmetric matrix game called RPSD, where the 4th strategy, D or Dumb, is never a best response. Using the standard laboratory software zTree, the authors conducted 12 sessions, each with 12 subjects matched in randomly chosen pairs for 80 or more periods. In all treatments the data were quite noisy, but in the most favorable condition (high payoffs and a theoretically unstable matrix), the time-averaged data were slightly better explained by TASP (see section 2 below) than by Nash equilibrium. The paper reports no evidence of cycles.

Hoffman, Suetens, Nowak and Gneezy (2012) is another zTree study begun about the same time as the present paper, and as far as we know, it is the only other human subject experiment focusing on a RPS game. The authors compare behavior with three different symmetric 3x3 matrices of the form $\begin{pmatrix} 0 & -1 & b \\ b & 0 & -1 \\ -1 & b & 0 \end{pmatrix}$, where the treatments are $b = 0.1, 1.0, 3.0$. The unique NE=(1,1,1)/3 is an ESS (hence in theory dynamically stable, see below) when $b = 3$, but not in the other two treatments. The authors report 30 sessions each with 8 human subjects matched simultaneously with all others (mean-matching) for 100 periods. They find that time average play is well approximated by NE, and that the mean distance from NE is similar to that of binomial sampling error, except in the $b = 0.1$ treatment, when the mean distance is larger. This paper also reports no evidence of cycles.

Section 2 reviews relevant theory and distills three testable hypotheses. Section 3 then lays out our experimental design. The main innovations are (a) new visually-oriented software called ConG, which enables players to choose mixed as well as pure strategies and to adjust them in essentially continuous time, and (b) asymmetric 3x3 payoff bimatrices that distinguish NE play from the centroid (1,1,1)/3. As in previous studies, we compare matrices that are theoretically stable to those that are theoretically unstable, and in the latter case we can distinguish TASP from NE as well as from the centroid. We also compare (virtually) instantaneous adjustment to continuous but gradual adjustment ("Slow"), and to the more familiar synchronized simultaneous adjustment in discrete time ("Discrete").

Section 4 reports the results. After presenting graphs of average play over time in sample periods and some summary statistics, it tests the three hypotheses. All three enjoy considerable, but far from perfect, support. Among other things, we find that cycles persist in the continuous time conditions in both the stable and unstable games, but that cycle amplitudes are consistently larger in the unstable games. In terms of predicting time average play, Nash equilibrium is better than Centroid, and when it differs from the NE, the TASP is better yet.

A concluding discussion is followed by appendices that collect mathematical details and instructions to subjects.

2 Some Theory

The games that were used in the experiments are, first, a game we call U_a

$U_a =$ (1)

	R	P	S
Rock	60, 60	0, 72	66, 0
Paper	72, 0	60, 60	30, 72
Scissors	0, 66	72, 30	60, 60

where U is for unstable because, as we will show, many forms of learning will not converge in this game. The subscript a distinguishes it from U_b that follows. Second, we have the stable RPS game,

$S =$ (2)

	R	P	S
Rock	36, 36	24, 96	66, 24
Paper	96, 24	36, 36	30, 96
Scissors	24, 66	96, 30	36, 36

Finally, we have a second unstable game U_b

$U_b =$ (3)

	R	P	S
Rock	60, 60	72,0	30, 72
Paper	0, 72	60, 60	66, 0
Scissors	72, 30	0, 66	60, 60

Notice that in U_b the best response cycle is reversed so that it is a RSP game rather than RPS.

All these games have the same unique Nash equilibrium which is mixed with probabilities (0.25, 0.25, 0.5). The equilibrium payoff is 48 in all cases.

While these games are identical in their equilibrium predictions, they differ quite substantially in terms of predicted learning behavior. Consider as in our experiments a population of players who play this game amongst themselves - one could consider either repeated random matching or playing against the average mixed strategy of the other players. Suppose they all choose a target for their mixed strategies that is (close to) a best response to the current strategies of their opponents. Then the ConG software interface would adjust their mixed strategies smoothly in that direction. Thus, we would expect that the population average mixed strategy x would move according to continuous time best response

(BR) dynamics, which assumes that the population average strategy moves smoothly in the direction of the best reply to itself. That is, formally,

$$\dot{x} \in b(x) - x \tag{4}$$

where $b(\cdot)$ is the best response correspondence.[1]

Because of the cyclical nature of the best response structure of RPS games (Rock is beaten by Paper which is beaten by Scissors which is beaten by Rock), if the evolution of play can be approximated by the best response dynamics, then there will be cycles in play. The question is whether these cycles converge or diverge.

It is easy to show that in the game S, under the best response dynamics, the average strategy would converge to the Nash equilibrium. This is because the mixed equilibrium in S is an evolutionarily stable strategy or ESS. In the games U_a and U_b, however, there will be divergence from equilibrium and play will approach a limit cycle.[2] For example, the case of U_a is illustrated in Figure 1, with the interior triangle being the attracting cycle. This cycle has been named a Shapley triangle or polygon after the work of Shapley (1964) who was the first to produce an example of non-convergence of learning in games.

More recently, Benaïm, Hofbauer and Hopkins (BHH) (2009) observe the following. If play follows the BR dynamics then, in the unstable game, play will converge to the Shapley triangle; furthermore, the time average of play will converge to a point that they name the TASP (Time Average of the Shapley Polygon), denoted "T" on Figure 1. It is clearly distinct from the Nash equilibrium of the game, denoted "N" in Figure 1.

These results can be stated formally in the following proposition. The proof can be found in the Appendix.

Proposition 1 *(a) The Nash equilibrium* $x^* = (0.25, 0.25, 0.5)$ *of the game* U_a *is unstable under the best response dynamics (4). Further, there is an attracting limit cycle, the*

[1] Because this correspondence can be multivalued we use "$\in$". However, for the RPS and RSP games we consider the BR correspondence is single valued almost everywhere.

[2] Intuitively, the instability arises in U_a and U_b because the normalized gain from winning (which ranges from 6 to 12) is much smaller than the absolute normalized loss from losing (which ranges from -30 to -60). By contrast, in the stable game S the normalized gain from winning (30 to 60) is much larger than the absolute normalized loss from losing (-6 to -12). In other words, in the unstable games draws are almost as good as wins, which pushes learning dynamics towards the corners of the simplex (see Figure 1 below) where draws are more frequent. In the stable game draws are much worse than wins and only a little better than losses, pushing the dynamics away from the corners and decreasing the amplitude of the cycles.

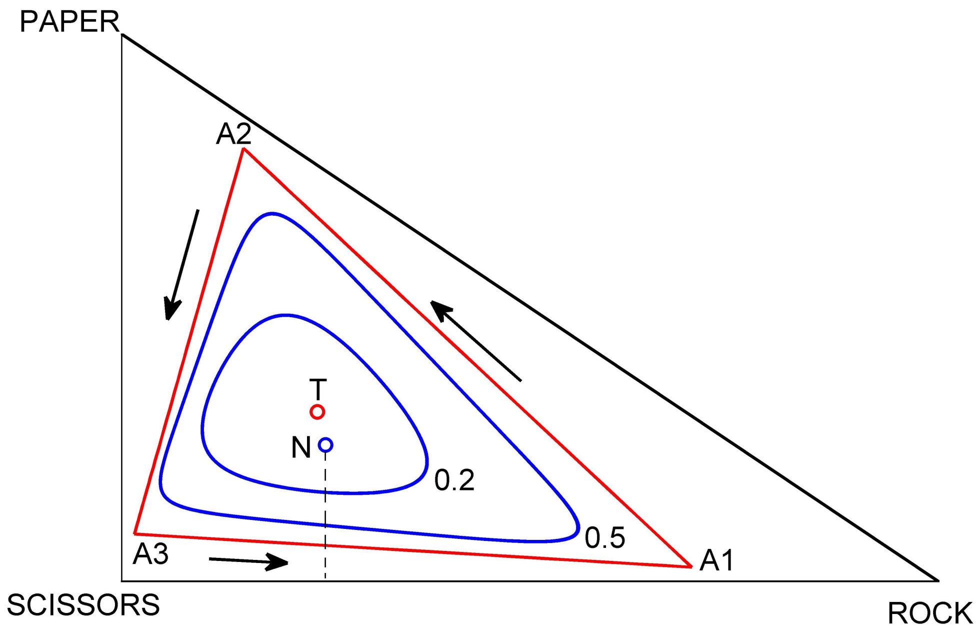

Figure 1: The Shapley triangle $A_1A_2A_3$ for game U_a with the TASP (T) and the Nash equilibrium (N). Also illustrated are orbits for the perturbed best response dynamics for precision parameter values 0.2 and 0.5.

Shapley triangle, with vertices, $A_1 = (0.694, 0.028, 0.278)$*,* $A_2 = (0.156, 0.781, 0.063)$ *and* $A_3 = (0.018, 0.089, 0.893)$ *and time average, the TASP, of* $\tilde{x} \approx (0.24, 0.31, 0.45)$*. Average payoffs on this cycle are approximately 51.1.*
(b) The Nash equilibrium $x^* = (0.25, 0.25, 0.5)$ *of the game* S *is a global attractor for the best response dynamics (4).*
(c) The Nash equilibrium $x^* = (0.25, 0.25, 0.5)$ *of the game* U_b *is unstable under the best response dynamics (4). Further, there is an attracting limit cycle, the Shapley triangle, with vertices,* $A_1 = (0.028, 0.694, 0.278)$*,* $A_2 = (0.781, 0.156, 0.063)$ *and* $A_3 = (0.089, 0.018, 0.893)$ *and time average, the TASP, of* $\tilde{x} \approx (0.31, 0.24, 0.45)$*. Average payoffs on this cycle are approximately 51.1.*

In the Appendix, we also outline the extent to which these results are robust to the possibility of experiments and mistakes. Specifically, if subjects choose best responses imprecisely, then average frequencies will evolve according to the perturbed best response (PBR) dynamics rather than the strict BR dynamics. In games U_a and U_b, the PBR dynamics also give rise to cycles, two of which for U_a are illustrated in Figure 1. How similar these are to the Shapley cycle depends on the level of precision in subjects' choices. See the Appendix for details.

These theoretical arguments lead to the following testable predictions. Note that Hypothesis 2 competes with Hypothesis 1, while Hypothesis 3(a) elaborates on Hypothesis 1 and Hypotheses 3(b,c,d) elaborate on Hypothesis 2.

Testable Hypotheses

1. **Nash Equilibrium (NE):** average play will be at the NE $(0.25, 0.25, 0.5)$ and average payoff will be 48 in all treatments.

2. **TASP:**

   (a) The population average mixed strategy further averaged over time will be closer to the TASP than to the NE in U_a and U_b.

   (b) Average payoffs will be higher in U_a and U_b than in S.

3. **BR Dynamics:**

   (a) In S, there will be counter-clockwise cycles that diminish in amplitude over time with ultimate convergence to NE.

   (b) In U_a, there will be persistent counter-clockwise cycles that approach the Shapley triangle limit cycle.

   (c) In U_b, there will be persistent clockwise cycles that approach the Shapley triangle limit cycle.

   (d) Thus the average distance from NE will be consistently higher in U_a and U_b than in S.

3 Laboratory Procedures

Figure 2 displays an example of the subjects' decision screen during an experimental session. The upper left corner indicates the game payoff matrix M, and subjects choose actions by clicking on locations on the "heat map" triangle in the lower left. They can choose a pure action by clicking on a vertex, and can choose any desired mixture by clicking an interior point. The thermometer to the right of the triangle shows how heat map colors correspond

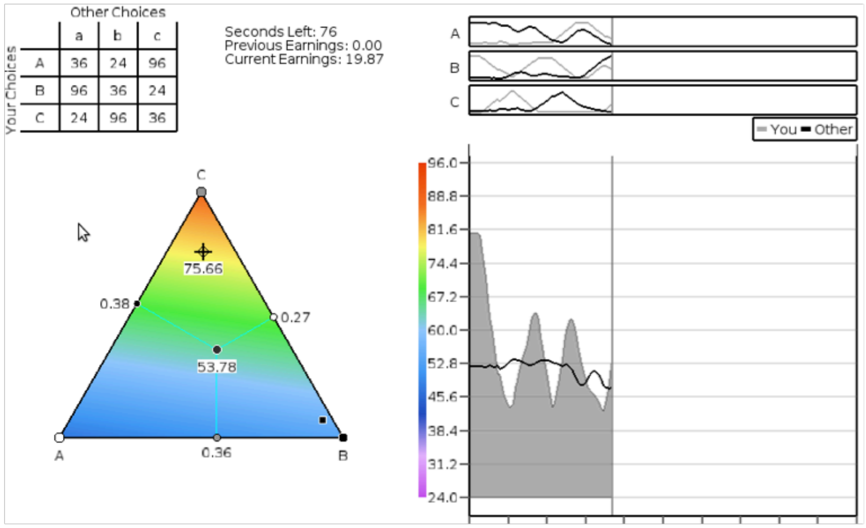

Figure 2: ConG Software: CS treatment (10 sec transit)

to current payoff flow rates, given the current average mixture $x(t)$ in the population. This hugely reduces the subjects' cognitive load, since otherwise they would continually have to approximate, for each mixture y they might choose, the matrix product $y \cdot Mx(t)$ that gives the payoff flow.

The upper right side of the screen presents in real time the dynamic time path of strategies selected by the subject and the population average. The lower right panel shows the payoff flow received by the subject and the population average; the gray area represents the player's accumulated earnings so far in the current period.

Periods lasted 180 seconds each. Each session began with one unpaid practice period, providing subjects with an opportunity to familiarize themselves with the interface and display. The written instructions that were read aloud to subjects before this practice period are included in Appendix B. Debriefings after the session was over, as well as experimenter impressions during the session, indicate that most subjects quickly became comfortable with the task, which they regarded as an enjoyable and not especially challenging video game.

Subjects participated in groups of 8 and played the three game matrices U_a, S and U_b in treatment "blocks" of 5 periods each. Treatments were held constant within blocks, while between blocks we switched the game matrix and changed the action set.

We used four different action sets: Continuous Instant, Continuous Slow, Discrete Pure and Discrete Mixed. In the Continuous conditions, subjects click mixture targets and receive payoff and population strategy updates in essentially continuous time. In the Instant case the chosen mixture adjusted to their target click instantaneously; more precisely, lags are typically less than 50 milliseconds, hence imperceptible. In the Slow case the actual mixture moves at a constant rate towards the target click; the rate is such that it would take 10 seconds to move from one vertex to another.

In the Discrete conditions the 180-second period was subdivided into 20 subperiods of 9 seconds each, and subjects received payoff and population strategy updates at the end of each subperiod. In the Pure case subjects were restricted to specify a pure strategy in each subperiod, and in the Mixed case subjects could click any mixture or pure strategy in each subperiod. (If the subject clicked several points during the subperiod, only the last click counted.) Each subperiod after the first, the heat map displayed the potential payoffs given the population mixture chosen in the previous subperiod, and therefore the heat map remained static over the 9-second interval. In the Continuous conditions the heat map (and the displays on the right side of the screen) updated every 200 milliseconds to reflect the current population mixture.

Table 1: Balanced Incomplete Block Design

	Block 1	**Block 2**	**Block 3**	**Block 4**	**Block 5**
Sess D1	U_a-DM	S-CI	U_a-DP	S-DM	U_b-CS
Sess D2	U_b-CS	U_a-CS	S-CS	U_a-CI	S-DP
Sess D3	S-CS	U_a-DM	S-CI	U_b-CS	S-DM
Sess D4	U_a-CI	S-DM	U_a-DM	S-CS	U_a-CI
Sess D5	S-DP	U_b-CS	U_a-DP	S-CI	U_a-CS
Sess D6	U_a-CS	S-DP	U_a-CI	S-DM	U_a-DP
Sess D7	S-CI	U_a-CS	U_b-CS	S-CS	U_a-DM
Sess D8	U_a-DP	S-DM	U_a-DM	S-DP	U_a-CI
Sess D9	S-CI	U_a-DP	S-DP	U_a-CS	S-CS
Sess D10	S-DM	U_a-CI	S-CS	U_a-DP	S-CI
Sess D11	U_a-CI	S-DP	U_a-CS	U_b-CS	U_a-DM

Note: Every treatment appears in Blocks 1 and 5, at least 8 out of 9 treatments appear in each of the other Blocks, and no treatment appears more than twice in any Block.

Matrices U_a and S were played in each of the four action sets, while (as a bonus treatment) U_b was played only in Continuous Slow. Thus we have 9 different combinations of matrix and action set, or treatments. Each of the 11 sessions consisted of 5 blocks of 5 periods, with the treatment sequences shown in Table 1. The design was chosen to change the matrix every block and to give 6 independent observations (i.e., from 6 different sessions) of each of the 9 treatments, while balancing treatments across block positions.[3]

Each session lasted about 100 minutes, including instruction and payment time. No subject participated in more than one session, and all were recruited broadly from the student populations at Purdue University and UC-Santa Cruz. All 25 periods in the 5 blocks were paid periods, and subject earnings averaged approximately \$25.

4 Results

We begin with graphs of the population mixtures during some sample periods in the Continuous Slow treatment. The figures show a projection of the mixture triangle into the (x, y) plane, so mixture probability for the third strategy ("Scissors") is suppressed. The vertical axis represents time remaining in the 180 second period, so moving downward in the figure corresponds to moving forward in time. The NE appears as a vertical red line at $(x, y) = (.25, .25)$. The blue line in Figure 3 shows about a dozen irregular counterclockwise cycles of the population average mix around the NE in a sample period using the stable S matrix. Many of the cycles here have amplitude less than 0.1, but a few of them reach a distance of 0.2 or more from the NE.

Figure 4 shows ten counterclockwise cycles around the NE for a sample period using the unstable matrix U_a. The first few cycles (at the top) seem centered on the centroid $(x, y) = (0.33, 0.33)$ but last few cycles center closer to the NE. The amplitude is much larger than for the S matrix, and falls only slightly by the end of the period. Figure 5 shows 11 cycles for the reverse unstable matrix U_b. They are similar to those for U_a, with one major exception: as predicted, the cycles are clockwise.

These sample periods are typical in most respects. Time graphs for other periods suggest that cycles in Continuous Slow treatments persist even with the stable matrix S, as well as with (as predicted) the unstable matrices. The cycles typically seem to converge toward an approximate limit cycle, rather than having inward or outward spirals. As we document

[3] One treatment U_a-CI was repeated in the final block of session D4, so it was excluded from the analysis dataset because it is not statistically independent of the U_a-CI data in the first block of that session.

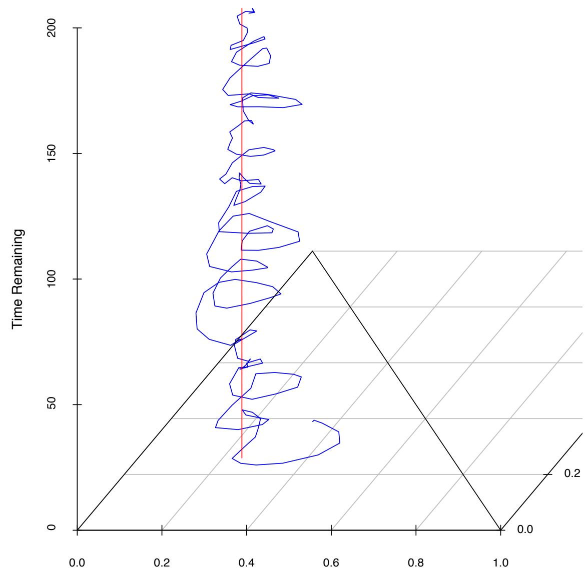

Figure 3: Session 10, period 14: S matrix, Continuous-Slow.

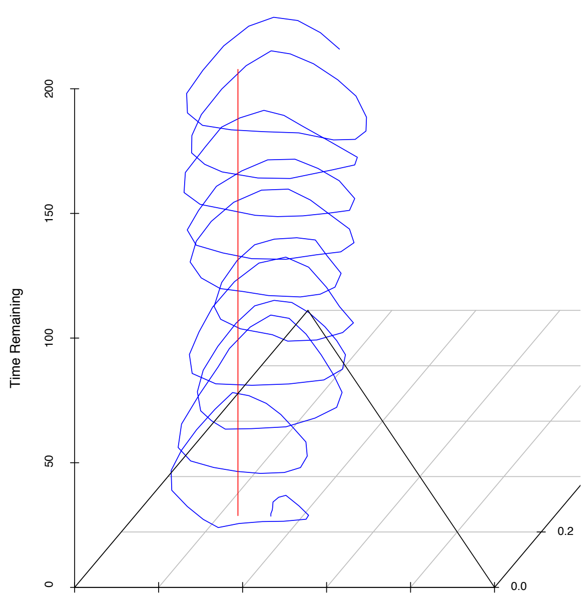

Figure 4: Session 2, period 6: U_a matrix, Continuous-Slow.

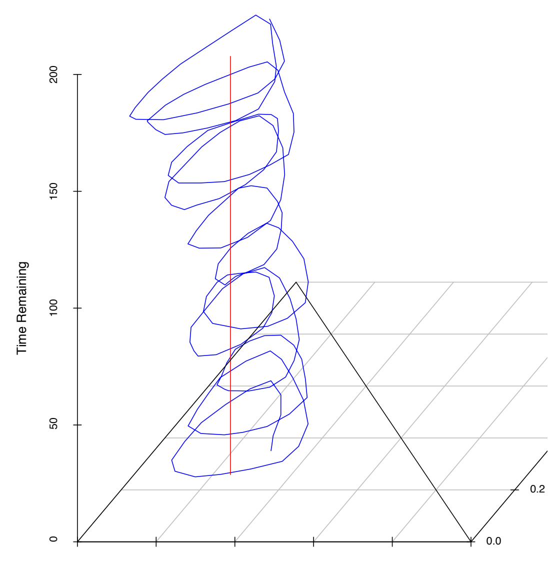

Figure 5: Session 11, period 20: U_b matrix, Continuous-Slow.

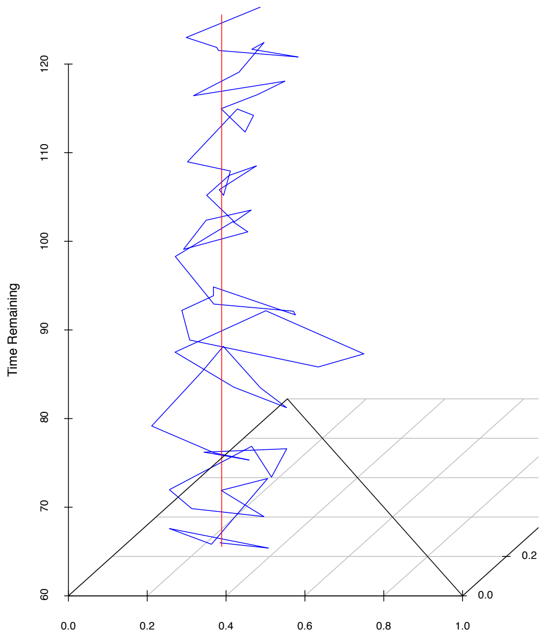

Figure 6: Session 3, period 14, Middle 1/3: S matrix, Continuous-Instant.

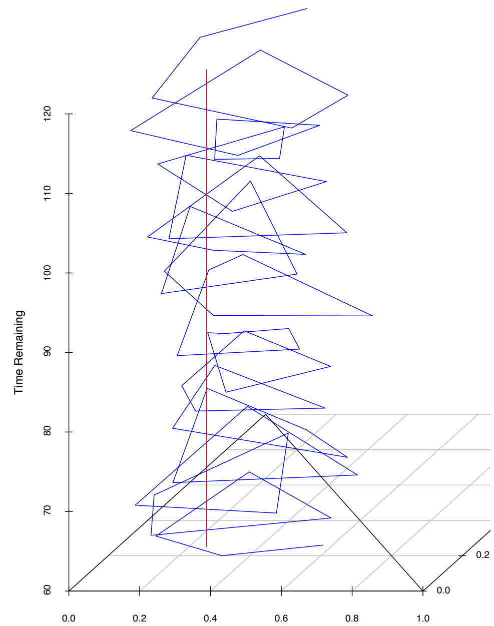

Figure 7: Session 2, period 17, Middle 1/3: U_a matrix, Continuous-Instant.

below, consistent with Hypothesis 3abc, the cycles under Continuous Slow are consistently counterclockwise for S and U_a and clockwise for U_b.

In the Instant treatment, the cycles are much more frequent and jagged, as Figures 6 and 7 illustrate, and also display greater amplitude for the U_a matrix. Note that only the middle 60 seconds are shown, but even so there are more than a dozen cycles. In the Discrete treatments, the path of population mixes by subperiod is not so obviously cyclic; see Figures 8 - 11 for typical examples.

4.1 Convergence across periods

Are there trends from one period to the next? To investigate, we plotted average population mixtures in each of the 5 periods within each block separately for each treatment in each session. The results for two of the treatments are displayed in Figures 12 and 13 for two of the nine treatments.

Figure 12 shows that behavior remains quite unsettled in the treatment featured in previous investigations — discrete time and pure strategy only — at least for the unstable matrix U_a. For example, the population mean frequency of Scissors (shown in green) in one

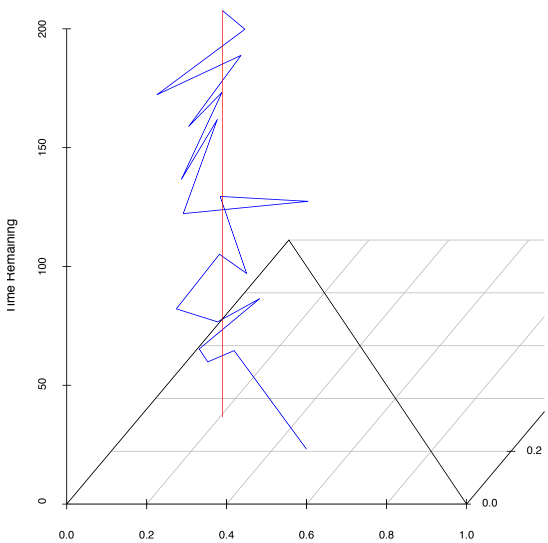

Figure 8: Session 3, period 24: S matrix, Discrete-Mixed.

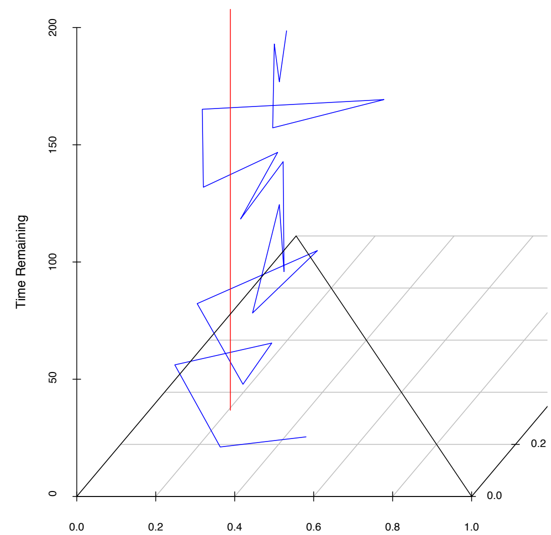

Figure 9: Session 11, period 21: U_a matrix, Discrete-Mixed.

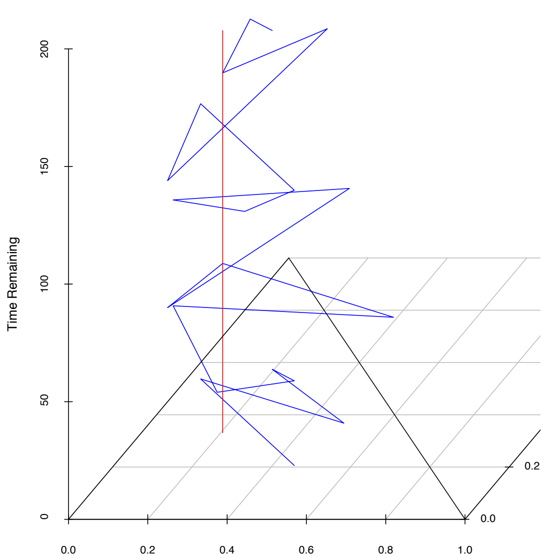

Figure 10: Session 2, period 22: S matrix, Discrete-Pure

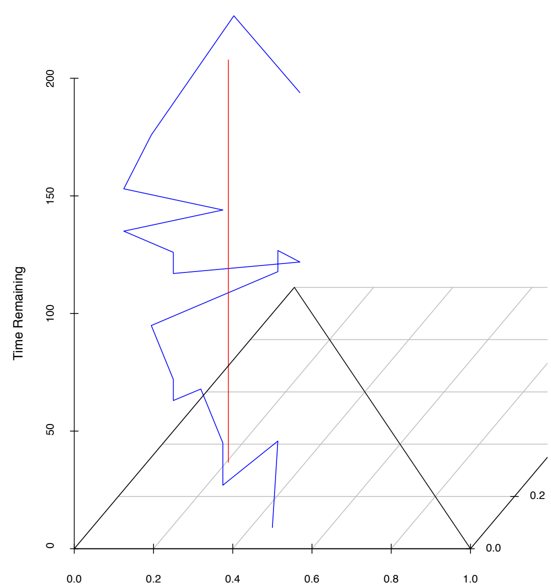

Figure 11: Session 1, period 11: U_a matrix, Discrete-Pure

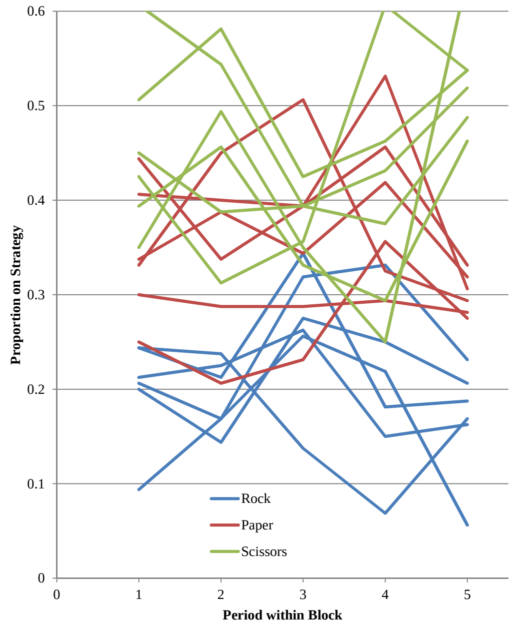

Figure 12: Mean choice by period within block for the U_a matrix, Discrete-Pure treatment.

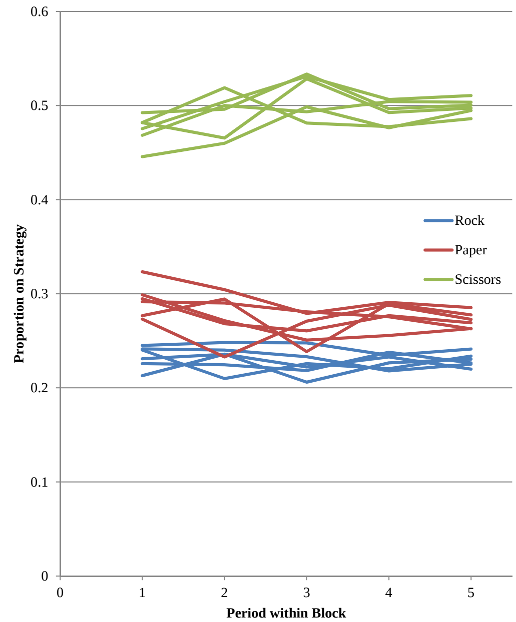

Figure 13: Mean choice by period within block for the S matrix, Continuous-Instant treatment.

session bounces from about 0.26 in period 4 to 0.61 in period 5. Although Scissors seems the most frequent strategy overall, and Rock (in blue) the least, this is reversed in some period averages.

By contrast, Figure 13 shows very consistent mean strategy choices in the Continuous-Instant treatment with the Stable matrix. The mean Rock frequency is always at or slightly below the NE value of 0.25, and the Paper frequency at or slightly above. The mean Scissors frequency is a bit below the NE value of 0.50 in period 1 but by period 5 clusters tightly around that value. The other seven treatments show behavior more settled than in Figure 12 but less than in Figure 13.

4.2 Hypothesis Tests

Table 2 displays predicted (top 3 rows) and actual (remaining 9 rows) overall mean frequency of each strategy and average payoffs. The superscripts refer to nonparametric Wilcoxon tests that conservatively treat each session as a single independent observation. We draw the following conclusions regarding the three testable Hypotheses given at the end of Section 2.

Result 1: In the Stable game, Nash Equilibrium is better than Centroid in predicting time average strategy frequencies. Average payoff is significantly lower or not significantly different than in Nash Equilibrium. Thus Hypothesis 1 finds mixed support.

Evidence: The N superscripts in Table 2 indicate that the data reject the Nash Equilibrium at the 5 percent significance level in 6 of the 12 strategy averages shown for the Stable game. The centroid can be rejected in 11 out of these 12 cases for the Stable game S. (The exception is Paper in the Discrete-Pure action set.) With only one other exception (again Paper, here in Discrete-Mixed) the observed average strategy frequency is always closer to NE than to the Centroid $(1, 1, 1)/3$. In each of the four treatments, the average payoff is at least 1.3 below the Centroid payoff of 49.33. but it is always within 0.50 of the Nash prediction of 48. The null hypothesis of Nash equilibrium payoffs is rejected (in favor of a mean payoff *lower* than in NE) in two of these four treatments.

Table 2: Time average behavior

Prediction/Treatment	*Rock*	*Paper*	*Scissors*	*Payoff*
Nash Equilibrium	0.25	0.25	0.50	48
TASP (U_a)	0.242	0.31	0.449	51.1
TASP (U_b)	0.31	0.242	0.449	51.1
S Continuous-Instant	0.226^{N}	0.269^{N}	0.504	47.59^{N}
S Continuous-Slow	0.236^{N}	0.265	0.500	48.03
S Discrete-Mixed	0.242	0.294^{N}	0.464	47.95
S Discrete-Pure	0.247	0.320^{N}	0.433^{N}	47.57^{N}
U_a Continuous-Instant	0.247	0.318^{N}	0.435^{N}	49.82^{NT}
U_a Continuous-Slow	0.228^{NT}	0.281^{N}	0.491^{T}	49.08^{NT}
U_a Discrete-Mixed	0.225	0.342^{NT}	0.433^{N}	49.70^{NT}
U_a Discrete-Pure	0.205^{N}	0.337^{N}	0.458^{N}	50.71^{N}
U_b Continuous-Slow	0.303^{N}	0.240	0.457^{N}	48.81^{NT}

Note: Superscript N denotes significantly different from Nash (5% 2-tailed Wilcoxon test), and T denotes significantly different from TASP (5% 2-tailed Wilcoxon test; conducted for U_a and U_b only).

Result 2: In the Unstable game, TASP is better than Nash Equilibrium (and *a fortiori* better than Centroid) in predicting time average strategy frequencies. The average payoff is always significantly higher than the Nash Equilibrium prediction and usually closer to the TASP prediction, albeit significantly lower than the TASP prediction in most cases. Moreover, consistent with TASP, payoffs are significantly higher in the Unstable game than the Stable game. Thus Hypothesis 2 also finds mixed support.

Evidence: The N superscripts in Table 2 indicate rejection of the NE null hypothesis in 11 of the 15 strategy frequencies for the Unstable game, and all rejections are in the direction of TASP. Rejection the TASP predictions occurs in only 3 out of these 15 cases. The average payoff always lies between the the NE and TASP predictions, but is closer to TASP in 4 of 5 cases. In all 5 cases the Nash prediction of 48 is rejected; in one case the TASP prediction is not rejected. In all 4 pairwise comparisons, average payoffs are significantly higher (at the 1% level in Mann-Whitney tests) in the Unstable than the Stable game. This is as predicted by TASP, while NE predicts no difference and Centroid predicts a difference in the wrong direction. Finally, TASP tracks the observed asymmetry between U_a and U_b in Rock and Paper time averages, while NE and Centroid predict no asymmetry. The observed asymmetry (for their shared Continuous-Slow action set) is signficant according to a Mann-Whitney test.

Result 3: Cycles are clockwise for the Unstable matrix U_b, and counter-clockwise for the other matrices, Stable S and Unstable U_a. Although the cycle amplitudes usually decrease in size across periods for the Stable matrix, they do not converge to Nash equilibrium, contrary to Hypothesis 3a. Cycles also persist and have larger amplitudes for the Unstable matrices, but less than that of the Shapley triangle limit cycle. Thus Hypotheses 3bc find mixed support.

Evidence: Define *cycle amplitude* for a period as the time average over that period of the squared Euclidean distance $A(t) = ||x(t) - x^*||^2 = (x_0(t) - x_0^*)^2 + (x_1(t) - x_1^*)^2 + (x_2(t) - x_2^*)^2$ between Nash equilibrium mix x^* and the instantaneous actual mix $x(t)$. (The average squared deviation from the period average $\bar{x}$ yields similar results.) Figures 14 and 15 display cycle amplitude period by period for each block of the Continuous conditions; each line comes from an independent session. The amplitude declines between the first and last period in 22 out of the 24 blocks; the two exceptions are both in the Unstable - Continuous Instant treatment. But the amplitudes do not decline to zero; even in later periods of the 12 Stable matrix blocks, the squared deviations remain around .01 (i.e., trajectories remain about 0.10 away NE), contrary to Hypothesis 3a.

Table 3 reports average cycle amplitude in each treatment. Numerical calculations indicate that the Shapley triangle limit cycle has amplitude 0.181 for the U_a and U_b matrices.

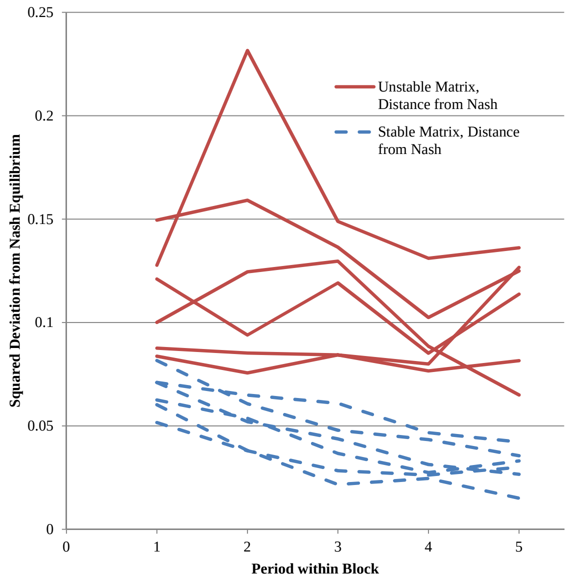

Figure 14: Mean squared deviation from NE by block in Continuous-Instant treatments

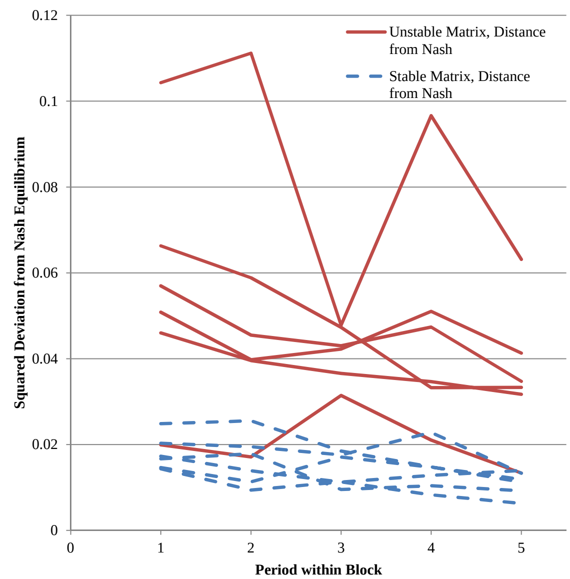

Figure 15: Mean squared deviation from NE by block in Continuous-Slow treatments.

The entries in Table 3 are all smaller than that, and Wilcoxon tests indicate that they are significantly smaller (p-value$< 5\%$) for all treatments. The observed cycle amplitudes are consistent with the perturbed best response dynamics for the low precision parameters illustrated in Figure 1.

Table 4 reports a cycle rotation index to document the clockwise or counter-clockwise rotation of the mean strategy choice cycles. To calculate this index, we first constructed a line segment in the projection of the simplex into the two-dimensional plane between the Nash equilibrium and the simplex edge, illustrated in Figure 1 as the vertical dashed line extending below the point N. This line segment $\mathcal{S}$ is the set of all mixtures in the simplex in which the Rock frequency is at its Nash equilibrium level of 0.25 and the Paper frequency is below its Nash equilibrium level of 0.25. It serves as a Poincare section (see, for example, Chapter 17 of Hofbauer and Sigmund, 1988) or "tripwire" for counting cycles.

Specifically, for each period in our data set, we counted how many times (CCT, counter-clockwise transits) the population mixture crosses $\mathcal{S}$ from left to right and how many times (CT, clockwise transits) from right to left. The average number of transits in each direction are shown in the center columns of Table 4 for each treatment. The cycle rotation index then is defined for each period as $CRI = \frac{CCT-CT}{CCT+CT} \in [-1, 1]$. Thus CRI values near 1 (-1) indicate consistently counter-clockwise (clockwise) cycles, and values near 0 indicate no

Table 3: Mean Squared Deviation from Nash Equilibrium

	Continuous-Instant	*Continuous-Slow*	*Discrete-Mixed*	*Discrete-Pure*
Stable S	0.039	0.014	0.048	0.093
Unstable U_a	0.112	0.044	0.089	0.129
Unstable U_b		0.048		
p-value for M-W: S vs. U_a	0.004	0.004	0.004	0.037
p-value for M-W: U_a vs. U_b		0.200		

Notes: Excludes first period of each 5-period block to reduce impact of hysteresis. M-W is a 2-tailed Mann-Whitney test comparing MSDs by period across the given treatments.

consistent cycles. The last column of the Table reports CRI averaged over all periods in each treatment.

The large values of CCT and CT the Continuous-Instant treatments reflect a substantially higher cycle frequency than in the Continuous-Slow treaments. The Discrete treatments have fewer transits, in part because each period has only 19 potential strategy changes, versus 179 potential strategy changes each Continuous period, where the data are sampled at one-second intervals.

The Discrete treatments for the Stable game S do not exhibit clear cyclical behavior, as indicated by CRI's not significantly different from 0. All other conditions exhibit significant cycles, with only the Unstable game U_b displaying clockwise cycles, consistent with Hypothesis 3abc. Although not shown on the table, Mann-Whitney tests with p-values below 0.05 for all four cases confirm that CRI always is larger for the Unstable matrix U_a than for the Stable matrix S.

Result 4: Cycle amplitude, and thus average distance from the Nash Equilibrium, is significantly higher in the Unstable than in the Stable game (support for Hypothesis 3d).

Evidence: The Mann-Whitney p-values shown at the bottom of Table 3 indicate that the amplitude, conditioning on the action set condition, is always significantly greater in the Unstable than the Stable game. The amplitude is not significantly different between the two Unstable games for their shared Continuous-Slow action set, and no difference is expected based on the hypotheses derived from BR dynamics.

Table 4: Mean Transits and Cycle Rotation Indexes

Game-Condition	*Number of Counter-Clockwise Transits*	*Number of Clockwise Transits*	*Cycle Rotation Index*
S Continuous-Instant	24.1	5.8	0.64*
S Continuous-Slow	9.3	0.9	0.86*
S Discrete-Mixed	2.1	1.3	0.30
S Discrete-Pure	0.5	0.7	-0.04
U_a Continuous-Instant	30.3	1.9	0.89*
U_a Continuous-Slow	8.3	0.0	1.00*
U_a Discrete-Mixed	1.8	0.3	0.78*
U_a Discrete-Pure	0.9	0.2	0.68*
U_b Continuous-Slow	0.3	8.5	-0.94*

Note: * Denotes Index significantly (p-value $< 5\%$) different from 0 according to 2-tailed Wilcoxon test.

One loose end remains. What about cycle frequency, as opposed to cycle amplitude? The last piece of evidence under Result 3 noted that the Unstable matrices produced more consistent rotation direction directions than their Stable counterparts, but the CCT and CT entries in Table 4 suggest no clear ordering on the number of cycles per period. Nor do eyeball impressions of individual period graphs. We estimated cycle frequencies each period using standard frequency domain techniques, employing the cumulative spectral distribution function to identify the most significant cycle frequencies for the strategy and payoff time series.[4] Stronger cycles are evident in the continuous treatments, but overall the frequencies are estimated with substantial noise. Nevertheless, tests show significantly higher frequencies for the U_a-CI treatment than for U_a-CS, which comes as no surprise given the time series graphs and the CCT and CT counts noted earlier. More importantly for present purposes, we find no significant differences between S and U_a (or U_b) in any action treatment. This is consistent with the Conjecture noted at the end of Appendix A.

[4]This procedure decomposes the time series into a weighted sum of sinusoidal functions to identify the principal cycle frequencies.

5 Discussion

Evolutionary game theory predicts cyclic behavior in Rock-Paper-Scissors-like population games, but such behavior has not been reported in previous work.[5] In a continuous time laboratory environment we indeed found cycles in population mixed strategies, most spectacularly for the Unstable matrices with Slow adjustment. Moreover, we consistently observed counterclockwise cycles for one Unstable matrix and clockwise cycles for another, just as predicted.

Surprisingly, we also found very persistent cycles for Stable matrices, where the theory predicted damped cycles and convergence to Nash Equilibrium. The theory was partially vindicated in that these cycles had smaller amplitude than those for corresponding Unstable matrices, but the amplitude settled down at a positive value, not at zero.

Evolutionary game theory considers several alternative dynamics. In our setting, replicator dynamics predicts that the cycles for the Unstable matrices have maximal amplitude (i.e., converge to the simplex boundary), while best response dynamics predict cycles that converge to the Shapley triangle and therefore have a particular amplitude less than maximal. The amplitude of cycles we observed with Unstable matrices varied by the action set available to each subject (instantaneous versus slow adjustment in continuous time, and pure only versus mixed strategies in discrete time), but it was always less than for the Shapley triangle. The data thus seem more consistent with perturbed best response dynamics with treatment-dependent noise.

Classic game theory predicts that, on average, play will approximate Nash equilibrium. Indeed, time averages over our three minute continuous time periods (and over 20 subperiod discrete time periods) fairly closely approximated Nash equilibrium in all treatments. However, for the Unstable matrix, evolutionary game theory provides an alternative prediction of central tendency called TASP, and it consistently outperformed Nash equilibrium.

Our results, therefore, are quite supportive of evolutionary game theory. It offers short run predictions, where classic game theory has little to say, and those predictions for the most part explained our data quite well. Its long run predictions either agreed with those of classic game theory or else were more accurate in explaining our data.

[5] In laboratory studies focusing on convergence to mixed strategy equilibrium, Friedman (1996) and Binmore et al (2001) both report some evidence of transient cyclic behavior in 2-population games. Neither study considered RPS-like games or persistent cycles.

While seeking answers to old questions, our experiment also raises some new questions. Granted that we observed very nice cyclic behavior, one now might want to know more about the necessary conditions. Does our "heat map" play a crucial role? Or is asynchronous choice in continuous time the key? Do cycles dissipate when subjects must choose simultaneously, and some choose to best respond to the previous population mix while others respond to their ("level k") anticipations of others' responses?

We hope our work inspires studies investigating such questions. It already seems clear that empirically grounded learning models and evolutionary game dynamics can help us grasp "instability," an increasingly important theme for social scientists.

Appendix

In this appendix, we state and prove some results on the behavior of the best response (BR) and perturbed best response (PBR) dynamics in the three games U_a, U_b and S.

When one considers stability of mixed equilibria under learning in a single, symmetric population, the most general criterion for stability is negative definiteness of the game matrix, which implies that a mixed equilibrium is an ESS (evolutionarily stable strategy). In contrast, mixed equilibria in positive definite games are unstable. As Gaundersdorfer and Hofbauer (1995) show, for RPS games there is a slightly weaker criterion for the stability/instability of mixed equilibria under the BR dynamics (see below). The RPS games we consider satisfy both criteria.

Proof of Proposition 1: Convergence to the Shapley triangle for the games U_a and U_b follows directly from the results of Gaunersdorfer and Hofbauer (1995). In particular, first one normalizes the payoff matrices by subtracting the diagonal payoff. Thus, for U_a, after subtracting 60, the win payoffs become (12, 12, 6) and the lose payoffs in absolute terms are (60, 60, 30). Thus clearly U_a satisfies the criterion (given in their Theorems 1 and 2) for instability that the product of the lose payoffs is greater than the product of the win payoffs.[6] One can then calculate the Shapley triangle directly from their formula (3.6, p. 286). The TASP is calculated by the procedure given in Benaïm, Hofbauer and Hopkins (2009). The average payoff can similarly be calculated. These results are easily extended to

[6]Furthermore, U_a and U_B are also positive definite with respect to the set $I\!R_0^n = \{x \in I\!R^n : \sum x_i = 0\}$ which is a stronger criterion.

U_b.

Turning to S, subtracting 36, we find the win payoffs to be (60, 60, 30) and the lose payoffs are (12, 12, 6). Thus, clearly the Nash equilibrium is globally stable because it satisfies Gaundersdorfer and Hofbauer's condition that the product of the win payoffs are greater than the product of the lose payoffs. ∎

We note that these results are largely robust to mistakes, particularly the possibility that the choice of best response is not completely accurate. Suppose that subjects chose a mixed strategy that is only an approximate best response to the current average mixed strategy. Then we might expect that the population average might evolve according to the perturbed best response dynamics

$$\dot{x} = \psi(x) - x \tag{5}$$

where the function $\psi(\cdot)$ is a perturbed choice function such as the logit.

Perturbed best response choice functions such as the logit are typically parameterized with a precision parameter λ, which is the inverse of the amount of noise affecting the individual's choice. In such models, an increase of the precision parameter λ, for learning outcomes are the following. First, it is well known that the stability of mixed equilibria under the perturbed best response dynamics (5) depend upon the level of λ. When λ is very low, agents randomize almost uniformly independently of the payoff structure and a perturbed equilibrium close to the center of the simplex will be a global attractor. This means that even in the unstable games U_a and U_b, the mixed equilibrium will only be unstable under SFP if λ is sufficiently large. For the games U_a and U_b, it can be calculated that the critical value of λ for the logit version of the dynamics is approximately 0.18. Second, in contrast, in the stable game S, the mixed equilibrium will be stable independent of the value of λ.

Proposition 2 *In U_a and U_b, the perturbed equilibrium (LE) $\hat{x}$ is unstable under the logit form of the perturbed best response dynamics (5) for all $\lambda > \lambda^* \approx 0.18$. Further, at λ^* there is a supercritical Hopf bifurcation, so that for $\lambda \in (\lambda^*, \lambda + \epsilon)$ for some $\epsilon > 0$, there is an attracting limit cycle near $\hat{x}$.*

Proof: Instability follows from results of Hopkins (1999). The linearisation of the logit PBR dynamics (5) at $\hat{x}$ will be of the form $\lambda R(\hat{x})B - I$ where R is the replicator operator and B is the payoff matrix, either U_a or U_b. Its eigenvalues will therefore be of the form $\lambda k_i - 1$ where the k_i are the eigenvalues of $R(\hat{x})B$. $R(\hat{x})B$ has only positive eigenvalues as both U_a

and U_b are positive definite. But for λ sufficiently small, all eigenvalues of $\lambda R(\hat{x})B - I$ will be negative. We find the critical value of 0.18 by numerical analysis. The existence of a supercritical Hopf bifurcation has been established by Hommes and Ochea (2012). ∎

This result is less complete than Proposition 1, in that it does not give a complete picture of PBR cycles away from equilibrium. Numeric analysis for the logit form of the PBR dynamics suggests that as for the BR dynamics there is a unique attracting limit cycle (for $\lambda > 0.18$). The amplitude of this cycle is increasing in λ and approaches that of the Shapley triangle as λ becomes large. Two sample limit cycles are illustrated in Figure 1.

The game S is negative definite and hence its mixed equilibrium is a global attractor under both the BR and PBR dynamics. This implies it is also an attractor for (stochastic) fictitious play.

Proposition 3 *The perturbed equilibrium (QRE) of the game S is globally asymptotically stable under the perturbed best response dynamics (5) for all $\lambda \geq 0$.*

Proof: It is possible to verify that in the game S is negative definite with respect to the set $\mathbb{R}^n_0 = \{x \in \mathbb{R}^n : \sum x_i = 0\}$. See e.g. Hofbauer and Sigmund (1998, p80) for the negative definiteness (equivalently ESS) condition. The result then follows from Hofbauer and Sandholm (2002). ∎

Conjecture 1 *For λ sufficiently large and for x sufficiently close to the perturbed equilibrium $\hat{x}$, cycles of the PBR dynamics, controlling for amplitude, should have the same frequency for all three games U_a, U_b and S.*

The reason behind the conjecture is the following. For λ large, the perturbed equilibrium $\hat{x}$ is close to the NE $x^* = (1/4, 1/4, 1/2)$. It is then possible to approximate the PBR dynamics by a linear system $\dot{x} = R(x^*)Bx$ where, for the logit version of the PBR dynamics, R is the replicator operator (see the proof of Proposition 2 above) and B is the payoff matrix, which could be any of U_a, U_b or S. One can calculate $R(x^*)B$, precisely for all three games and thus derive the eigenvalues for this linear system, which are exactly $6 \pm 9\sqrt{3}i$ for U_a and U_b and $-6 \pm 9\sqrt{3}i$ for S. It is the imaginary part of the eigenvalues that determines the frequency of the solutions of the linearized system $\dot{x} = R(x^*)Bx$, while the exponent of the real part determines the amplitude. The imaginary part is identical for all three games and thus one would expect similar frequencies in cycles. Admittedly, there are two approximations in making this argument. First, this is a linear approximation to the non-linear PBR dynamics and will only be valid close to equilibrium. Second, the linearization should properly be taken at $\hat{x}$ and not at the NE x^*. However, for λ large, the loss of accuracy should not be too great.

References

Benaïm, M., Hofbauer, J., and Hopkins, E. (2009). "Learning in games with unstable equilibria", *Journal of Economic Theory*, **144**, 1694-1709.

Binmore, K., Swierzbinski, J. and Proulx, C. (2001). "Does Min-Max Work? An Experimental Study," *Economic Journal*, **111**, 445-464.

Cason, T., Friedman, D. and Hopkins, E. (2010) "Testing the TASP: an Experimental Investigation of Learning in Games with Unstable Equilibria", *Journal of Economic Theory*, November, 2010, 145, 2309-2331.

Fisher, Len (2008). *Rock, paper, scissors: game theory in everyday life.* Basic Books.

Friedman, D. (1996) "Equilibrium in Evolutionary Games: Some Experimental Results," *Economic Journal*, **106: 434**, 1-25.

Gaunersdorfer, A., and J. Hofbauer (1995). "Fictitious play, Shapley Polygons, and the Replicator Equation," *Games and Economic Behavior*, **11**, 279-303.

Hofbauer, J., Sigmund K., (1998) *Evolutionary Games and Population Dynamics.* Cambridge: Cambridge University Press.

Hofbauer, J, W.H. Sandholm, (2002). "On the global convergence of stochastic fictitious play", *Econometrica*, 70, 2265-2294.

Hoffman, M., Suetens, S., Nowak, M., and Gneezy, U. (2011) "An experimental test of Nash equilibrium versus evolutionary stability", working paper.

Hommes, Cars H. and Marius I. Ochea (2012) "Multiple equilibria and limit cycles in evolutionary games with Logit Dynamics", *Games and Economic Behavior*, 74, 434-441.

Hopkins, E. (1999). "A note on best response dynamics," *Games and Economic Behavior*, **29**, 138-150.

Maskin, E., and Tirole, J. (1988), "A Theory of Dynamic Oligopoly II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles, *Econometrica*, **56**, 571-599.

Shapley, L. (1964). "Some topics in two person games," in M. Dresher et al. eds., *Advances in Game Theory*, Princeton: Princeton University Press.

Sinervo, B., and Lively, C. (1996). "The rock-paper-scissors game and the evolution of alternative male strategies," *Nature*, **380**, 240 - 243.

Appendix B: Experiment Instructions

Welcome! This is an economics experiment. If you pay close attention to these instructions, you can earn a significant sum of money, which will be paid to you in cash at the end of the last period.

Please remain silent and do not look at other participants' screens. If you have any questions, or need assistance of any kind, please raise your hand and we will come to you. If you disrupt the experiment by talking, laughing, etc., you may be asked to leave and may not be paid. We expect and appreciate your cooperation today.

The Basic Idea

Each period you will be matched anonymously with counterparts, some or all of the participants in todays experiment. You will choose mixtures of three possible actions: A, B and C. Your earnings each period will depend on your choices and those of your counterparts. At the end of the session your earnings in points will be added up over all periods, converted to US dollars at a rate shown on the whiteboard, and paid to you in cash.

The Earnings Table

In the upper part of the screen you will see a table with rows labeled by your choice (A, B and C) and columns labeled by the choice of your counterparts (a, b and c), as in Figure 1. Each table entry represents your earnings given the indicated choices. For example, if you choose action A and the counterparts choose b, then your earnings will accumulate at rate 24, the number shown in row A column b. The other 8 entries in the table show your earnings for all other combinations.

Mixtures and the Heat Map

In some periods the experiment gives you the flexibility to choose *mixtures* of your three actions. To help you see how your earnings depend on your mixture choice and your counterparties', your computer screen will show a triangular "heat map" similar to that in Figure 2.

The vertices in the triangle represent pure actions. That is, when you are at the vertex labeled A, your mixture is of 100% A and 0% B and C, when at vertex C then the mixture is of 100% action C and 0% A and B, similarly for vertex B. In some periods your choice may be restricted to these vertices. In other periods, you can choose any point in the triangle indicating a mixture of actions equal to the proportional distance to each vertex. So, if you choose to play in the middle of the triangle you will be playing a mixture with 33.33% A, 33.33% B and 33.33% C. In Figure 2 the black dot (which represents your actual mixture of strategies) is at is 27% A, 38% B and 36% C. Note that along the edge between A and B, you are choosing 0% C and varying a mixture of only A and B actions, and that percentages always have to sum to 100% (except for small rounding errors).

To adjust your own mixture, click your mouse on the desired point. This becomes the target and is displayed as a circle with crosshairs. The black dot will move steadily towards your chosen target. In some periods it may move slowly, and in other periods it may move very fast. As mentioned, your earnings rate is determined by your mixture and that of the counterparts'. For example, if your counterparts choose 100% a, and you chose the strategy shown in Fig 2, then your earnings rate would be 0.27(36) + 0.38(96) + 0.36(24) = 54.84. If your counterparts chose an interior mixture, then your earnings would be a particular weighted average of all the entries in the table.

Since it is hard to keep track of these averages yourself, the computer will use a color heat map to help you. The redder (hotter) the color, the higher would be your payoff if you were to choose that mixture; the more violet (colder) the color the less that choice would pay. A thermometer shows the payoff scale to the right of the colored triangle. In Figure 2, the player's current mixture is in a greenish-blue area, receiving a rate of pay of 53.78.

The small black square (near the B corner in Figure 2) shows the counterparts' mixture, the average mixture of all other participants with whom you are matched. As counterparts adjust their mixtures, the square will move, and that will in turn change the heat map. The map always shows your earnings rate at all of your possible mixtures, given the mixture currently chosen by your counterparts.

Accumulated Earnings

To the right of your heat map you will have a display (Figure 3) showing your accumulating earnings for the current period. Your earnings are represented by the solid gray area --- the larger the area, the greater your accumulated earnings. The height of the gray line corresponds to the color of the heat map where your dot is at that moment. So the higher the gray line, the faster your earnings are accumulating.

The black line, with no solid area under it, represents the average earnings of your counterparts. The more area under the black line, the more your counterparts have earned so far.

Your earnings at the end of the period will depend on the percentage of time you spend at each mixture combination. If, for example you spend half of the time in a mixture combination that earns you 10 and half of the time in a combination that earns you 20, then you will earn (.5)10 + (.5)20 = 15 for the period.

It is important to realize that your earnings depend not only on the mixture combination, but also on how much time you and your counterparts spend in the combination. If you spend all of your time in one mixture combination, then your payoff for the period will be the area under a flat line. If either mixture changes, then you will see the line move up or down, and the gray area accumulating faster or slower.

A small display near the top of Figure 4 keeps track of the mixes you and your counterparts have been playing during the period. In this display the gray line shows the percentage of A in your mix at each point in time, while the black line shows your counterparts' percentage of action a. Similarly, the display shows the past mixture of actions B and C during the period.

Full Screen

The full screen you will see during the experiment will look like the one in Figure 4. It contains the earnings table, heat map, the earnings chart, and the mixture chart.

The upper part of the screen shows a timer that tells you how many seconds are left in the current period. Right below the timer the screen shows your earnings so far in the current period, and the total earnings of the previous periods.

Subperiods

Some periods may be broken down into two or more subperiods. They are shown as vertical lines in the payoff display, as in Figure 5.

When you see subperiod lines, this means that your mixture choices matter only at the end of each subperiod. Your earnings for the entire subperiod are computed using your mixture and the counterparts' mixture at that point of time. Your earnings will be shown as usual as a gray area below a line. The line is automatically flat in each subperiod, so the gray area will lie below an up-and-down staircase.

Warning: during each subperiod, the heat map shows the payoffs that were available to you in the *previous subperiod*. It can't show the payoffs that actually will be available to you this subperiod, since your counterparts have not yet finalized their decisions.

Earnings

You will be paid at the end of the experiment for the total amount of points earned over all periods, after any unpaid practice periods announced by the conductor. The conversion rate from payoff points to US dollars is written on the whiteboard.

Frequently asked questions

Q1. Is this some kind of psychological experiment with an agenda you haven't told us?

Answer. No. It is an economics experiment. If we do anything deceptive or don't pay you cash as described then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are meant to clarify how you earn money; our interest is simply in seeing how people make decisions.

Q2. What changes from one period to the next?

Answer. The payoff matrix might change—be sure to look at it. When it does, of course the heat map will look differently. The adjustment speed might also be faster or slower than in the previous period. Also, some periods may be broken down into subperiods and in some periods you may only be able to choose pure actions on the three vertices. But often nothing changes between periods, and you are matched with exactly the same people in exactly the same way as in the last period.

Q3. How is the average mixture determined? Am I part of it?

Answer. The computer looks at the mixtures currently used by everyone, takes the simple average, and plots it in the triangle. In some sessions, "everyone" means everyone *except you*, but in other sessions it means everyone *including you*. The conductor will write on the white board which way the computer is doing it today. Usually we have at least 8 subjects, and the average except you is almost the same as the average including you.

Q4. If the heat map color looks about the same everywhere, how can I tell where my payoff is highest?

Answer. Occasionally this may happen, and it means that all mixtures give almost the same payoff. Even when the colors are not much of a clue, you can always rely on the payoff numbers shown by mousing over the map; they are accurate to two decimal places.

Note: If you are color blind to some or all the colors we use, please mention this to the experimenter after you are paid; it may help us adjust the color scheme later.

	Other Choices		
Your Choices	a	b	c
A	36	24	96
B	96	36	24
C	24	96	36

Figure 1

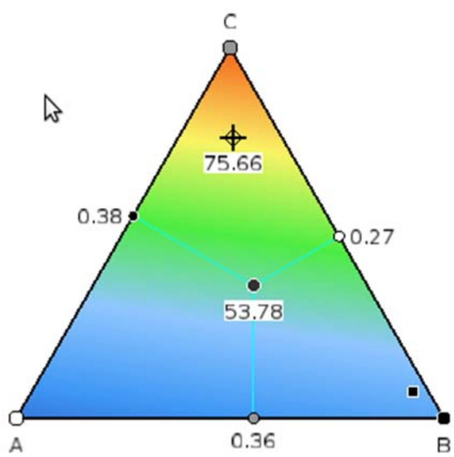

Figure 2

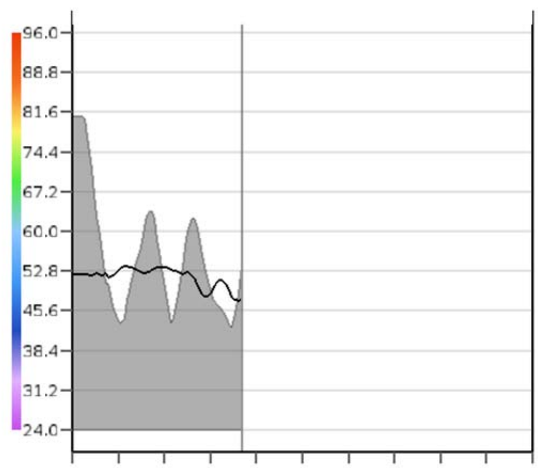

Figure 3

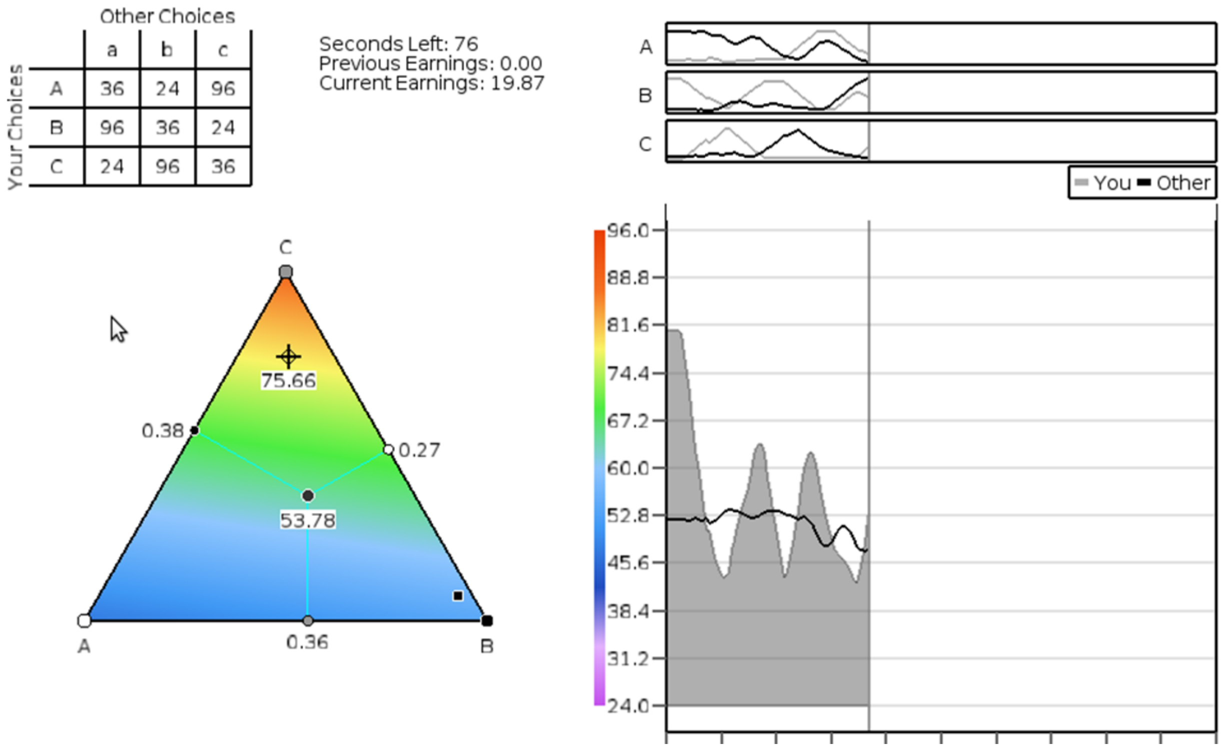

Figure 4

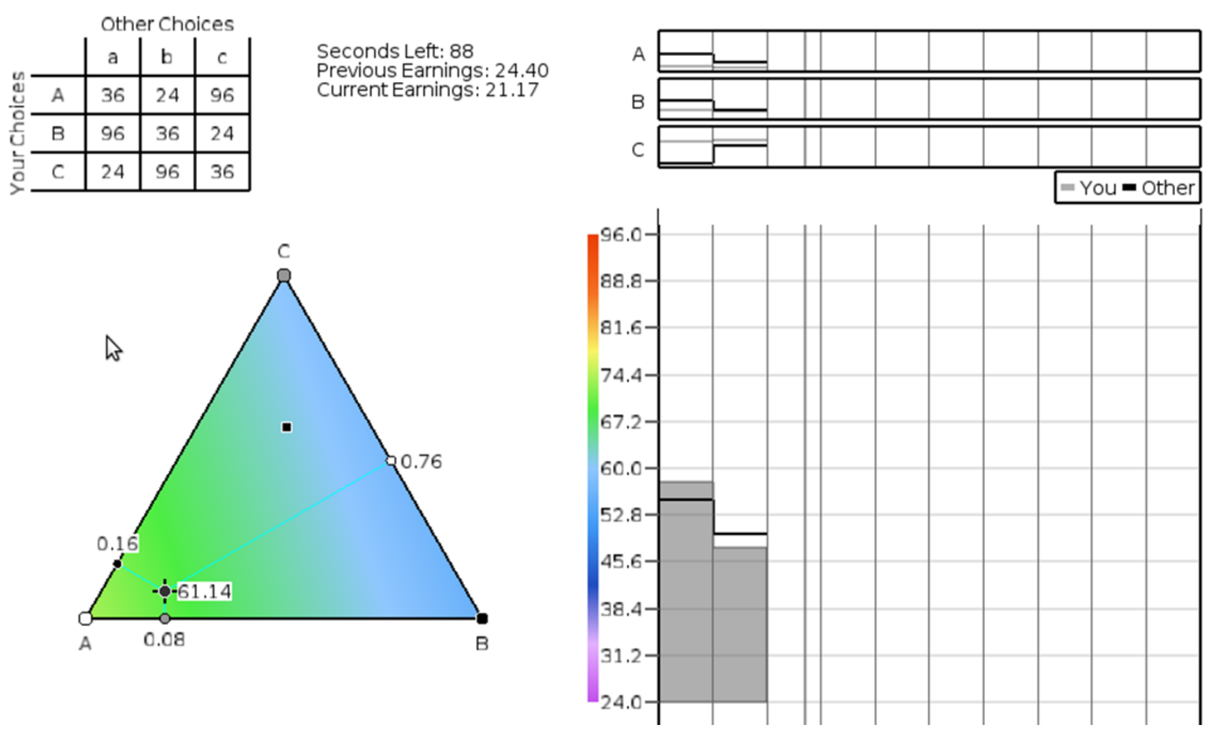

Figure 5