Title
Asymptotic and Bayesian Confidence Intervals for Sharpe Style Weights

Authors
Kim, Tae-Hwan
White, Halbert
Stone, Douglas

Publication Date
2000-10-01

2000-27

UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

ASYMPTOTIC AND BAYESIAN CONFIDENCE INTERVALS FOR SHARPE STYLE WEIGHTS

BY

TAE-HWAN KIM

DOUGLAS STONE

AND

HALBERT WHITE

DISCUSSION PAPER 2000-27
OCTOBER 2000

Asymptotic and Bayesian Confidence Intervals for Sharpe Style Weights

Tae-Hwan Kim
School of Economics
The University of Nottingham
University Park
Nottingham NG7 2RD
United Kingdom
(Phone) 44-115-951-5466 (Fax) 44-115-951-4159
Tae-Hwan.Kim@nottingham.ac.uk

Douglas Stone
Nicholas Applegate Capital Management
600 West Broadway, 29th Floor
San Diego, CA 92101
(Phone) 619-687-2731 (Fax) 619-687-8099
doug_stone@nacm.com

Halbert White
Department of Economics, 0508
The University of California, San Diego
9500 Gilman Drive
La Jolla, CA 92093-0508
(Phone) 619-534-3502 (Fax) 619-534-7040
hwhite@weber.ucsd.edu

December 1999

Asymptotic and Bayesian Confidence Intervals for Sharpe Style Weights

Abstract: Sharpe style regression has become a widespread analytic tool in the financial community. The style regression allows one to investigate such interesting issues as style composition, style sensitivity, and style change over time. All previous methods to obtain the distribution and confidence intervals of the style coefficients are statistically valid only in the special case in which none of the true style weights are zero or one. In practice it is quite plausible to have zero or one for the values of some style weights. In this paper we apply new results of Andrews (1997a, 1999) and develop a comparable Bayesian method to obtain statistically valid distributions and confidence intervals regardless of the true values of style weights.

Key Words: Sharpe Style Regression, Non-negativity, Linear-Quadratic Optimization, Prior Density, Bayesian Highest Posterior Density Interval.

1. Introduction

Following Sharpe's (1998, 1992) seminal work, the analysis of an investment fund's style using Sharpe style regression has become widespread in the financial community. Among the interesting applications of style regression are: short term risk assessment of a fund manager [Sortino, Miller and Messina (1997)]; mutual fund style and classification [Brown and Goetzmann (1997), DiBartolomeo and Witkowski (1997)]; and hedge fund style [Fung and Hsieh (1997)]. Style regression analysis is particularly useful in situations where one wants to understand the style of a fund manager without knowing in detail which securities are held or in what proportion in the fund. Style regression analysis requires knowing only the final outcome, i.e., fund returns, over a period of time from a fund manager and regressing these on various index returns with certain coefficient restrictions (summing-to-one and non-negativity restrictions). This allows one to investigate such interesting issues as style composition, style sensitivity, and style change over time.

Despite these appealing advantages of style regression analysis, there are some limitations to its practical implementation. Chief among these is the difficulty of obtaining the correct sampling distribution of the estimated style regression coefficients because of the non-negativity restrictions imposed on the style weights. When the parameter space is restricted by inequality constraints (e.g., by non-negativity), it can be shown using the argument in Judge and Takayama (1966) that the

style regression coefficient estimator has a truncated normal distribution if the regression error is normally distributed. These authors also demonstrated that when there are more than two independent variables, it can be very difficult to obtain the desired sampling distributions using standard methods.

Lobosco and DiBartolomeo (1997) pointed out the problem of the lack of a precision measure for the style regression coefficients and proposed a convenient method to approximate such a precision measure, based on a Taylor expansion. Unfortunately, this method is valid only in the special case in which none of the true style coefficients are zero or one, and one often sees cases in practice where zero or unit coefficient values appear plausible. When some true style weights are zero or one, we are in the situation where some true regression parameters are on the boundary of the parameter space, which prevents us from using the classical Taylor expansion to obtain the asymptotic distribution of the style regression coefficients. In this paper, we apply new results of Andrews (1997a, 1999) for obtaining confidence intervals in constrained regression that permit us to obtain statistically valid asymptotic precision measures for style coefficients, regardless of their true values.

In addition, we explore the use of a Bayesian approach to style regression, related to a method proposed by Geweke (1986). Geweke developed a Bayesian approach to the inequality constrained normal linear regression model using a prior density representing the inequality constraints. We generalize his method in such a way that we can include the additional requirement in the Sharpe Style regression that the sum of all style weights must be equal to one.

Obtaining the correct distribution and confidence intervals of the style coefficients permits us to conduct statistically valid inference. Given a valid confidence interval, we can test if an index return should be in the fund's effective asset mix. For example, Sortino, Miller and Messina (1997) demonstrated that some index returns are riskier than others. In particular, the small cap growth index has the highest downside risk. Hence, a statistically valid hypothesis test allows us to correctly assess the risk which a fund manager takes. Morever, knowing the distribution of the style coefficients makes it possible to test if the fund manager's style has changed at some point of time. This information is useful to both the fund manager himself as well as to investors.

The paper is organized as follows. In Section 2 we briefly discuss Sharpe Style regression analysis. Motivation for our proposed methods is provided in Section 3. Detailed discussions of the new procedures (Andrews and Bayesian) are given in Section 4 and Section 5 respectively. Section 6 presents a Monte Carlo simulation study designed to investigate the finite sample properties of the new procedures. In Section 7 we apply our new methods to two actual fund returns (the Fidelity Magellan Fund and the Minicap Fund). Section 8 contains some concluding remarks.

2. Sharpe Style Regression Analysis

Sharpe (1988, 1992) specifies the relationship between fund returns and a collection of index returns as:

$$R_t = b_1^* F_{t1} + b_2^* F_{t2} + \ldots + b_k^* F_{tk} + e_t \qquad t = 1,2,\ldots,T, \tag{1}$$

where R_t denotes fund return in period t, F_{ti} is the ith style index return in period t, b_i^* is the "true" unknown style weight for the ith style index returns, and e_t is the fund's idiosyncratic return, orthogonal to the style indices in the sense that $E(F_{ti}e_t) = 0$. The style weights satisfy two key conditions: they must sum to one ($i'b^* = 1$ where i is the $k\times 1$ vector of ones) and they must be non-negative ($b^* \geq 0$). These restrictions ensure that we can interpret $F_t'b^*$ as the return on a "style portfolio" in which short positions in the style indices are not permitted. (See the Appendix for the list of assumptions on fund returns and indices used throughout the paper and some discussion about the statistical interpretation of the Sharpe style weights.)

This relation has the form of a constrained linear regression. There is an implicit constraint that the regression does not include a constant (*i.e.*, there is no element of F_t that equals 1 for all t), and we have the explicit portfolio constraints $(i'b^* = 1, b^* \geq 0)$, which restrict b^* to a $k-1$ dimensional (compact) simplex.

A primary goal of Sharpe style regression is to determine the unknown style weights b^*. This can be done using standard constrained least squares regression methods. Estimated style weights

satisfying the necessary constraints can be obtained by solving the constrained least squares problem

$$\min_{b} \; T^{-1}\sum_{t=1}^{T}(R_t - F_t'b)^2$$

$$s.t. \quad i'b = 1, \;\; b \geq 0 .$$

This is a convex linear-quadratic programming problem. Under general conditions (*e.g.*, Andrews (1997a, 1999)), the resulting estimator $\hat{b}_T$ can be shown to be strongly consistent for b^*. This consistency result holds even if some components of b^* are on the boundary of the parameter space. See Andrews (1997a, 1999) for a proof.

3. Random Variation in Sharpe Style Weight Estimators

Once we obtain a point estimator $\hat{b}_T$, a natural question to ask is how much confidence we can have in the point estimator. In the usual regression situation this can be answered by computing standard errors for the point estimator. However, the inequality restrictions imposed on style weights can make it complicated to compute standard errors. It can be shown that if the "true" style weights b_i^* are in the interior of the compact parameter space, then one can apply the standard techniques to obtain the appropriate asymptotic distribution even if the inequality restrictions are imposed (Amemiya 1985, Chap. 4). This is essentially what is done by Lobosco and DiBartolomeo (1997). However, if some of the "true" style weights b_i^* are on the boundary (*i.e.* if some of them are equal to zero or one), then the conventional method becomes invalid in the sense that it produces a biased sampling distribution. Indeed, this is just what Lobosco and DiBartolomeo (1997) encounter when they report that "the predictive formula … overestimates the standard errors if the true style weight is very close to either 0 or 1." Nor does the bootstrap provide an easy way out. Andrews (1997b, 2000) gives an example in which the bootstrap method is not valid when some true parameters are on the boundary.

Nevertheless, Andrews (1997a, 1999) develops an elegant and general theory that can be applied to construct confidence intervals when the parameters lie on the boundary. Since we never

know whether the "true" style weights are at the boundary or not, Andrews's method is an appealing one for Sharpe style regression analysis. An interesting aspect of Andrews's results is that the limiting distribution of the constrained estimates is not necessarily normal, so that the usual confidence intervals constructed using only standard errors are no longer necessarily appropriate. Furthermore, Andrews's method requires only mild reguarlity conditions on the error (e_t) in (1) -- normality is not necessary. Andrews's method is asymptotically valid; however, its finite sample properties are unknown. We assess finite sample properties in our Monte Carlo study in Section 6.

On the other hand, our extension of Bayesian method requires us to impose some distributional assumption on the error term (e_t). As is standard, we use a normal distribution. The payoff of imposing the normality assumption is that it provides an exact distribution for the Sharpe style weight estimator. A risk is that if the errors are in fact non-normal, as is plausible for manager returns, then the resulting posterior densities may be inaccurate.

4. Asymptotic Confidence Intervals

If we do not have inequality restrictions or we have non-binding inequality restrictions, then standard theory dictates that the asymptotic distribution of the (constrained) least squares estimator for the Sharpe style weights will have the form

$$T^{1/2}(\hat{b}_T - b^*) \xrightarrow{d} N(0, \Omega),$$

where Ω is the asymptotic covariance matrix. When inequality restrictions are imposed, one can show using the Andrews (1997a, 1999) results that

$$T^{1/2}(\hat{b}_T - b^*) \xrightarrow{d} \hat{l}.$$

The limiting random variable $\hat{l}$ is the solution to the linear-quadratic optimization problem

$$\min_l \ (l - Z)'M(l - Z)$$

$$s.t. \quad i'l = 0, Ql \le 0,$$

where $M \equiv E(F_t F_t')$, $Z \equiv M^{-1}G$, G is a multivariate normal random vector, $G \sim N(0, V)$, $V \equiv E(e_t^2 F_t F_t')$, $e_t \equiv R_t - F_t'b^*$, and $Q = [q_{ij}]$ is a matrix identifying the elements of b^* that are

"known" to satisfy the boundary condition, $b_j^* = 0$. The matrix Q has l rows (one for each element of b^* "known" to be zero) and k columns (one for each element of b^*). The elements q_{ij} of Q are zero, except when the ith "zeroed" element of b^* has index j, in which case q_{ij} = -1. Fortunately, we are not required to have exact *a priori* knowledge about which elements of b^* are zero. We can acquire this knowledge by running a preliminary unconstrained least squares regression and identifying elements b_j^* satisfying the boundary condition as those whose associated $t-$statistics are insignificant (at a level a tending to zero as $T \to \infty$).

Because we can compute a large number of Monte Carlo realizations of $\hat{l}$ by solving the linear-quadratic optimization for a large number of Monte Carlo realizations of G, we can build up a Monte Carlo estimate of the distribution of $\hat{l}$. The Monte Carlo method proceeds as follows:

1) Compute $\hat{M}_T \equiv T^{-1}\sum_{t=1}^{T} F_t F_t'$.

2) Compute $\hat{V}_T \equiv T^{-1}\sum_{t=1}^{T} \hat{e}_t^2 F_t F_t'$ where $\hat{e}_t \equiv R_t - F_t'\hat{b}_T$, where $\hat{b}_T$ is the Sharpe style constrained least squares estimator.

3) For each i=1, …, k and a given level a, $0 < a < 1$, "pre-test" the hypothesis

$$H_o : b_i^* = 0 \quad \text{vs} \quad H_a : b_i^* > 0 .$$

We discuss this pre-test in greater detail below.

4) Construct the Q matrix according to the outcome of the pre-test in 3). For example, suppose $k = 5$, and we fail to reject the null hypothesis for $i = 2, 5$. Then the matrix Q is

$$Q = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

5) Draw a large number of independent random vectors $\{\hat{G}_{T\,j} : j = 1, 2, ..., N\}$ from the distribution $N(0, \hat{V}_T)$. For each $\hat{G}_{T\,j}$, solve the following linear-quadratic optimization problem to obtain the solution $\hat{l}_{T\,j}$:

$$\min_{l} \; (l - \hat{M}_T^{-1}\hat{G}_{T\,j})'\hat{M}_T(l - \hat{M}_T^{-1}\hat{G}_{T\,j})$$

$$s.t. \quad i'l = 0, Ql \le 0$$

6) Build a histogram of $\{\hat{l}_{Tj} : j = 1,2,...,N\}$ to obtain desired precision measures, such as standard errors or confidence intervals. In the simulations discussed below, we set N, the number of Monte Carlo draws, to be 5000.

In particular, to obtain confidence intervals, proceed as follows. Recall that $T^{1/2}(\hat{b}_T - b^*) \xrightarrow{d} \hat{l}$. Because we can approximate the distribution of $\hat{l}$ by building up a Monte Carlo histogram, we can compute any percentile of the distribution of $\hat{l}$. Let $(1-a^*)$ be the confidence level, and let z_L be the $a^*/2$ percentile and z_U be the $1-a^*/2$ percentile. Then,

$$\begin{aligned} 1-a^* &\approx P[z_L \le \hat{l} \le z_U] \\ &\approx P[z_L \le T^{-1/2}(\hat{b}_T - b^*) \le z_U] \\ &= P[\hat{b}_T - z_U T^{-1/2} \le b^* \le \hat{b}_T - z_L T^{-1/2}] \end{aligned}$$

Hence, the $(1-a^*)\%$ confidence interval is $(\hat{b}_T - z_U T^{-1/2}, \hat{b}_T - z_L T^{-1/2})$. Because z_L and z_L are each associated with the same tail area, $a/2$, we have an "equal-tailed" confidence interval.

The pre-test procedure of step 3 above can be conducted in different ways. For example, one can run an unconstrained OLS regression for the Sharpe style equation (1). One would then fail to reject the null $H_o : b_i^* = 0$ whenever the associated asymptotic t-statistic is not significant at the $a \times 100\%$ level, where a is a pre-specified level for the pre-test. Unconstrained OLS is not efficient, however, because it ignores the easily imposed summing-to-one restriction. Because the power of the pre-test plays a key role in determining the properties of our procedures, it is important to exploit this information.

Consequently, our pre-test procedure in step 3 is implemented as follows:

(a) Fix a pre-specified level a, $0 < a < 1$ for the pretest;

(b) compute the constrained least squares regression coefficients obtained by imposing only the summing-to-one constraint:

$$\tilde{b}_T = \ddot{b}_T - \hat{M}_T^{-1} i (i' \hat{M}_T^{-1} i)^{-1} (i' \ddot{b}_t - 1)$$

where $\ddot{b}_T$ is the unconstrained OLS estimator, $\ddot{b}_T = \hat{M}_T^{-1}(T^{-1}\Sigma_{t=1}^{T} F_t R_t)$;

(c) compute the standard errors of the constrained LS estimator $\tilde{b}_T$ from the diagonal elements of the (scaled) asymptotic variance-covariance matrix estimator:

$$\tilde{C}_T = T^{-1}(\tilde{D}_T + \hat{M}_T^{-1} i (i'\hat{M}_T^{-1} i)^{-1} i'\tilde{D}_T i (i'\hat{M}_T i)^{-1} i'\hat{M}_T^{-1} - 2\,\tilde{D}_T i (i'\hat{M}_T^{-1} i)^{-1} i'\hat{M}_T)$$

where $\tilde{D}_T \equiv \hat{M}_T^{-1}\tilde{V}_T\hat{M}_T^{-1}$, $\tilde{V}_T \equiv T^{-1}\Sigma_{t=1}^{T}\tilde{e}_t^{\,2} F_t F_t'$, and $\tilde{e}_t = R_t - F_t'\tilde{b}_T$;

(d) Fail to reject $H_o : b_i^* = 0$ at level a (and set the corresponding element of Q to –1 in step 4) if the corresponding asymptotic *t*-statistic is not significant at level a, that is, if

$$\tilde{b}_{T,i} / \sqrt{\tilde{C}_{T,ii}} < z_a ,$$

where $\tilde{b}_{T,i}$ is the *i*th component of $\tilde{b}_T$, $\tilde{C}_{T,ii}$ is the *i*th diagonal element of $\tilde{C}_T$, and z_a is the critical value from the normal distribution for a test of level a, $\Phi(z_a) = 1 - a$, where Φ is the standard normal cumulative distribution function.

When implementing this Monte Carlo procedure using a level of a for the pre-test in this way, we will refer to our procedure as "Andrews ($a \times 100\%$)". Thus, Andrews (5%) is the procedure just described using a pre-test level of 5% to determine whether or not an element of b^* is zero.

5. Bayesian Confidence Intervals

In the Bayesian literature, there is a well developed literature on the issue of estimation with a restricted parameter space. Among the key articles are those of Lindley (1961), O'Hagan (1973), Davis (1978), and Geweke (1986). Davis (1978) considered a general case where the parameter space is restricted by multiple inequality constraints. His theory, however, requires one to know which constraints are binding, which is not plausible in the Sharpe style regression. One the other hand, Geweke (1986) does not require such knowledge about the binding restrictions. Following Geweke (1986), we propose to use as our prior an indicator function representing the two restrictions: $i'b = 1$, $b \geq 0$. This prior, however, is not properly defined because the integral of the prior over its domain (a positive section of a hyperplane) will be zero due to the summing-to-one restriction. This problem was also recognized in Davis (1978). To solve this problem, we reduce

the dimension of the domain by transforming the positive section of the hyperplane into a simplex in a lower dimensional space. For this, the Style equation in (1) can be written as

$$\tilde{R}_t = b_1^* \tilde{F}_{t1} + b_2^* \tilde{F}_{t2} + ... + b_{k-1}^* \tilde{F}_{tk-1} + e_t \quad t = 1,2,...,T,$$

where $\tilde{R}_t \equiv R_t - F_{tk}$, $\tilde{F}_{ti} \equiv F_{ti} - F_{tk}$ and the two restrictions $i'b^* = 1$, $b^* \geq 0$ can now be written as $i'b^*_{(k-1)} \leq 1$, $b^*_{(k-1)} \geq 0$ where $b^*_{(k-1)} \equiv (b_1^*, b_2^* ..., b_{k-1}^*)'$. Therefore, the constrained least squares problem is now given by

$$\min_{b_{(k-1)}} T^{-1} \sum_{t=1}^{T} (\tilde{R}_t - \tilde{F}_t' b_{(k-1)})^2$$

$$s.t.\ i'b_{(k-1)} \leq 1,\ b_{(k-1)} \geq 0.$$

Once we obtain the solution $\hat{b}_{(k-1)}$, then the last coefficient estimator can be computed as $\hat{b}_k = 1 - i'\hat{b}_{(k-1)}$. For notational convenience we will denote $b_{(k-1)}$ by b when there is no confusion. As is typical in the Bayesian literature, we impose the following assumptions.

Assumption 1. The error e_t is independent normal with mean 0 and variance s^2.

Assumption 2. The vector of independent variables $\tilde{F}_{t(k-1)}$ is independent of e_t, b and s^2.

Using Assumption 2, the posterior of (b, s) given the data $(\tilde{R}, \tilde{F})$ can be expressed as a product of the prior and the likelihood function:

$$P(b, s \mid \tilde{R}, \tilde{F}) \propto P(b, s) P(\tilde{R} \mid \tilde{F}, b, s).$$

One can show using Assumption 1 that the likelihood function is given by

$$P(\tilde{R} \mid \tilde{F}, b, s) \propto s^{-T} \exp[-(2s^2)^{-1}(ns^2 + (b - \hat{b})'\tilde{F}'\tilde{F}(b - \hat{b})]$$

where $n \equiv T - (k-1)$ and $s^2 \equiv v^{-1}(\tilde{R} - \tilde{F}\hat{b})'(\tilde{R} - \tilde{F}\hat{b})$. We specify our prior to reflect our two restrictions as follows:

$$P(b, s) \equiv s^{-1} 1[s > 0] q(b)$$

where $q(b) \equiv 1[i'b \leq 1, b \geq 0]$. By integrating out s, we can obtain the posterior of b as

$$P(b \mid \tilde{R}, \tilde{F}) \propto \int_0^\infty P(b, s \mid \tilde{R}, \tilde{F}) ds$$

$$\propto [n + (b - \hat{b})' V (b - \hat{b})]^{-0.5(n+k-1)} q(b)$$

where $V \equiv s^{-2} \tilde{F}' \tilde{F}$. Therefore, the posterior of b is a multivariate student-t distribution with mean 0 and variance $n(n-2)^{-1} V^{-1}$.

The point estimator of $g(b)$ for any function $g(\cdot)$ is given by

$$\tilde{g}(b) \equiv E[g(b)]$$

$$= \frac{\int_b g(b) P(b \mid \tilde{R}, \tilde{F}) db}{\int_b P(b \mid \tilde{R}, \tilde{F}) db}.$$

We approximate the expectation using the Monte Carlo integration procedures of Kloek and Van Dijk (1978) with the importance function $I(b)$ specified as a multivariate student-t distribution with mean 0 and variance $n(n-2)^{-1} V^{-1}$. It can then be shown that

$$\hat{g}_N(b) \equiv \frac{\sum_{n=1}^{N} g(b_n) q(b_n)}{\sum_{n=1}^{N} q(b_n)} \xrightarrow{p} \tilde{g}(b)$$

as $N \to \infty$ where b_n is the n^{th} random draw from $I(b)$. In order to compute the mean and variance of the i^{th} Sharpe style weight, we simply set $g(b) = b_i$ for the mean and $g(b) = (b_i - E(b_i))^2$ for the variance.

To compute the coverage rate of the Bayesian method, we consider two types of confidence intervals: the equal-tailed confidence interval and the Bayesian Highest Posterior Density (HPD) interval. By construction, the equal-tailed confidence interval is likely to be inside the unit interval, so that its coverage rate can be close to zero for the binding parameters. The HPD interval is the shortest interval over which the area of the posterior is equal to a pre-specified confidence level. Since the support of the posterior of the i^{th} Sharpe style weight b_i is restricted to the unit interval, the HPD interval denoted by (c_L, c_U) is derived from the following inequality constrained minimization problem[1]:

[1] In our simulation all binding parameters are set to zero. Hence, imposing only $0 \le c_L$ in the minimization problem is sufficient for our purpose and this will simplify the numerical search procedure. The correct set of constraints, however, should include $c_U \le 1$ as well.

$$\min_{c_L, c_U} \; c_U - c_L$$

$$s.t.\ (1) \int_{c_L}^{c_U} P(b_i \mid \tilde{R}, \tilde{F}) db_i = 1 - a \quad (2)\ 0 \le c_L$$

where $P(b_i \mid \tilde{R}, \tilde{F})$ is the marginal posterior of b_i and a is a pre-specified test size which is set to 5% in our simulation. The Kuhn-Tucker conditions are given by

(1) $P(c_L \mid \tilde{R}, \tilde{F}) \ge P(c_U \mid \tilde{R}, \tilde{F})$

(2) $c_L [P(c_L \mid \tilde{R}, \tilde{F}) - P(c_U \mid \tilde{R}, \tilde{F})] = 0$,

together with the two constraints of the minimization problem. According to the Kuhn-Tucker conditions, $P(c_L \mid \tilde{R}, \tilde{F}) > P(c_U \mid \tilde{R}, \tilde{F})$ implies that $c_L = 0,\ c_U = (1-a)^{th}$ percentile, whereas $c_L > 0$ implies that $P(c_L \mid \tilde{R}, \tilde{F}) = P(c_U \mid \tilde{R}, \tilde{F})$. Therefore, one implementation for finding the HPD interval is as follows:

(1) Check if $P(0 \mid \tilde{R}, \tilde{F}) > P(q_{(1-a)} \mid \tilde{R}, \tilde{F})$ where $q_{(1-a)} = (1-a)^{th}$ percentile. If the condition is satisfied, then $c_L = 0,\ c_U = q_{(1-a)}$.

(2) Otherwise, we numerically find the shortest interval (c_L, c_U) such that $P(c_L \mid \tilde{R}, \tilde{F}) = P(c_U \mid \tilde{R}, \tilde{F})$.

The two step procedure can be done using the marginal posterior $P(b_i \mid \tilde{R}, \tilde{F})$ directly. This method, however, is computationally intensive since a numerical integration over a multi-dimensional space must be carried out for each point in the search space. Alternatively, we can draw a large number of Monte Carlo draws $(b_1, b_2, ..., b_{k-1})$ from the joint posterior and use the empirical histogram to implement Steps (1) and (2). In our simulation, we use the second method since it is more convenient to implement. The number of Monte Carlo draws of $(b_1, b_2, ..., b_{k-1})$ is set to 1,500. The Monte Carlo simulation to generate $(b_1, b_2, ..., b_{k-1})$ is placed inside the main simulation whose number of replications is set to 1,000. Note that once we have a random draw $(b_1, b_2, ..., b_{k-1})$ from the joint posterior, the last style coefficient b_k is obtained by $b_k = 1 - \sum_{j=1}^{k-1} b_j$.

6. Finite Sample Coverage Rates

In this section we investigate the finite sample properties of asymptotic and Bayesian confidence intervals for style weights via a Monte Carlo study.

In order to inject some reality into the simulation study, we use real style indices as explanatory variables. The style indices included in the study are the Russell 2000 growth (r2growth), the Russell 2000 value (r2value), the Russell 1000 growth (r1growth), the Russell 1000 value (r1value) and the 30 day T-bill rate (tbill). The indices are all monthly percent returns over the sample period from January 1979 through July 1997. The total number of observations is T = 233. Table 1 provides summary statistics for these style indices. We generate artificial fund returns, used as the dependent variable in the simulation, as follows.

$$R_t = b_1^* F_{t1} + b_2^* F_{t2} + b_3^* F_{t3} + b_4^* F_{t4} + b_{t5}^* F_{t5} + s e_t \quad t = 1,2,\ldots,T,$$

where F_{t1} = the Russell 2000 growth return, F_{t2} = the Russell 2000 value return, F_{t3} = the Russell 1000 growth return, F_{t4} = the Russell 1000 value return, F_{t5} = the 30 day T-bill rate, s is a scaling factor controlling the relative contribution of idiosyncratic returns, and $e_t \sim$ identical and independent $N(0,1)$. By repeatedly drawing errors from the normal distribution we can generate as many sets of fund returns as we like. The simulation was carried out on a 450MHz Pentium PC using GAUSS and the Quadratic Programming Subroutine for GAUSS.

First we set the true style weights to $b_1^* = b_2^* = b_3^* = b_4^* = b_5^* = 0.2$, which corresponds to the case where all true style weights are in the interior of the compact parameter space. In the second experiment, we set the true style weights to $b_1^* = 0.4$, $b_2^* = 0$, $b_3^* = 0.2$, $b_4^* = 0.4$, and $b_5^* = 0$, which corresponds to the case where some of true style weights are on the boundary. The criteria we use to evaluate finite sample performance are the coverage rate of the confidence intervals and their width. The nominal coverage rate is set to 95%. Hence, the coverage rate measures how often the confidence intervals constructed based on the nominal 95% coverage rate contain the true style weights. For comparison purposes, we also study the confidence intervals based on the unconstrained Ordinary Least Squares (OLS) estimator, the OLS estimator with the summing-to-

one restriction imposed (but not non-negativity), and the method proposed by Lobosco and DiBartolomeo (1997), henceforth "LDB".

The scaling factor s controls the R^2 of the style regression. For example, for a small value of s, the R^2 is close to one. Because it is interesting to investigate the effects of s on coverage rates, we consider a set of three values for s, corresponding to R^2's of 0.8, 0.9 and 0.95, bracketing a range of empirically relevant R^2 values. For the first experiment, the corresponding values for s are 1.9, 1.2 and 0.9. For the second experiment, they are given by 2.4, 1.6 and 1.1.

Table 2 and Table 3 show the coverage rates for the first experiment and for the second experiment respectively. Since all true style weights are in the interior of the parameter space in the first experiment, the Andrews method and LDB are asymptotically equivalent. However, as shown in Table 2, their finite sample performance is quite different. For the Andrews method with a pre-test level of 5% (under the heading "Andrews (5%)"), corresponding to the 5% significance level commonly applied, the coverage rates depend sensitively on the value of s (hence R^2). On the other hand, LDB and the Bayesian methods (equal tailed and HPD intervals) have fairly good coverage rates (sufficiently close to the nominal coverage rate) over all three values of s, as reported under the headings "LDB", "Bayes (Equal Tailed)" and "Bayes (HPD)".

In Table 3 we examine the performance for the second experiment. As explained in Lobosco and DiBartolomeo (1997), their method overestimates standard errors, which makes their confidence intervals wider. As a result, the coverage rates are consistently greater (0.97 – 0.99) than the nominal rate. It appears that the coverage rates based on Andrews (5%) converge to the nominal rate faster in the second experiment than in the first experiment, indicating that Andrews' method has better performance when some of parameters are on the boundary. It is interesting to note that even when s = 2.4 (R^2 = 0.8), the coverage rates for the binding parameters ($b_2^* = 0$, $b_5^* = 0$) are fairly close to the nominal rate. However, the performance of Andrews (5%) is again sensitive to the value of s. As predicted from the theory, the Bayesian equal tailed confidence intervals have zero coverage rates for the binding parameters. The Bayesian HPD intervals perform generally better than Andrews (5%) for all coefficients except that for the Russell 1000 value index.

When s is large, Andrews' (5%) confidence intervals are too narrow and do not bracket the true parameters sufficiently often (See Table 4 and Table 5 for averaged 95% confidence intervals for both experiments, and compare the confidence intervals under the headings Andrews (5%) and LDB). The reason why Andrews (5%) has too narrow confidence intervals can be found in Table 6 and Table 7 which show averaged standard errors for both experiments. In order to compute the "true" standard errors, we generate 100,000 sets of fund returns and compute 100,000 style coefficient estimates. The "true" standard errors in Table 6 and 7 are the sample standard errors of the 100,000 style coefficient estimates, whose histograms are displayed in Figure 1 and Figure 2. The Andrews (5%) method tends to underestimate the true standard errors when s is large.

Nonetheless, for the Andrews (5%) method, as the value of s decreases, the coverage rates approach the nominal rate. We conjecture that the sensitivity of Andrews' method declines as the number of observations increases because this increases the power of the pre-test. This conjecture is partially verified by examining the relation between coverage rates and the performance of the pre-test for identifying binding parameters shown in Table 8 and Table 9. When s is large, the power of the pre-test (one minus acceptance rate) for Andrews (5%) is small, but as s decreases, the power approaches one and coverage rates converge to the nominal rate. If we increase the number of observations, the power of the pre-test will increase and converge to one eventually. Nevertheless, we do not explore this dimension in the current study since the number of observations is fixed at the empirically plausible value 223 in both experiments. Instead, we find the optimal pre-test level for 95% confidence intervals when the sample size is fixed. We do this by minimizing the mean absolute deviation (across the 5 coefficients) of the coverage rate from the nominal rate over a range of pre-test levels. For this, we use a grid search over the interval [1%, 99%] with 1% increment. Due to the extended search process involved, the number of replications for the main loop and the inside linear-quadratic optimization loop is reduced to 1000. The Appendix contains a detailed explanation of this procedure.

The results using the optimal pre-test level are reported in Table 2 and Table 3 under the heading "Andrews (Optimal)". The optimal pre-test levels are given by 66% for R^2 = 0.8, 35% for R^2 = 0.9 and 7% for R^2 = 0.95 for the first experiment and 61% for R^2 = 0.8, 55% for R^2 = 0.9 and 52% for R^2 = 0.95 for the second experiment. The coverage rates are very close to the nominal

rate over all values of R^2 for both experiments. The observed dependence of the optimal level on R^2 confirms our previous conjecture that we need better power when s is large.

Of course, the optimal pre-test level is not available when using real data. To investigate the performance of other pre-test levels, we show in Figure 3 how the mean absolute deviation of nominal versus actual confidence interval limits depends on the pre-test level as it varies from 1% to 99%. An interesting fact emerging from these graphs is that the performance of Andrews method can be greatly improved by increasing the power of the pre-test (*i.e.* by decreasing the Type *II* error). More interestingly, Andrews' method is not very sensitive to the Type *I* error. When boundary parameters are absent (Figure 3), it is not sensitive to the Type *I* error at all. On the other hand, the performance begins to deteriorate slightly when the Type *I* error exceeds about 60% when boundary parameters are present (Figure 3). One implication of this observation is that we can obtain a feasible method by setting the pre-test level to a fixed value, utilizing the trade-off between the Type *I* error and the Type *II* error effects. Hence, we investigate the Andrews' method when the pre-test level is set to 50%.

The results are reported in Table 2 and Table 3 under the heading "Andrews (50%)". The coverage rates are reasonably close to the 95% nominal rate over all values of R^2 in both experiments. In particular, Andrews (50%) does not exhibit the consistent pattern of overly broad confidence intervals evidenced by LDB. When there is no boundary parameter, Andrews (50%) is quite comparable with the Bayesian HPD intervals. On the other hand, Andrews (50%) exhibits better coverage rates than the Bayesian HPD intervals when there are some boundary parameters. Thus, our simulations indicate that Andrews (50%) is prefered to LDB and Bayesian methods when we do not know if there are some boundary parameters and when the number of observations is comparable to the number of observations used in our simulation.

Our simulations also make clear that the Bayesian method is very computationally intensive when some parameters are on the boundary. Each HPD confidence interval is constructed based on 1,000 random samples of $b_{(k-1)}$ which are drawn from the intersection of a multivariate student-t distribution and the simplex determined by $i'b_{(k-1)} \le 1,\ b_{(k-1)} \ge 0$. Therefore, the total number of random samples to generate 1,000 such $b_{(k-1)}$ is inversely related to the probability over the simplex (POS). We found that the POS is on average 78%, 97% and 99% for $R^2 = 0.8,\ 0.9$ and

0.95 respectively in the first experiment and 17%, 19% and 18% for $R^2 = 0.8$, 0.9 and 0.95 respectively in the second experiment. Although we do not show this in our simulation results, we also found that (as should be expected) the POS becomes even smaller when we increase the number of binding parameters.

In Table 4 and Table 5 we see that Andrews (50%) gives shorter confidence intervals than LDB for all values of R^2 in both experiments. For comparison purposes, we also include the ideal case where we know which parameter is binding or not (*i.e*. we know the Q-matrix). The results are reported under the heading "Andrews (Q known)". The three methods Andrews (50%), Andrews (Optimal) and Andrews (Q known) behave very similarly.

The performances of the unconstrained OLS method and OLS with the summing-to-one restriction are reported in Table 2 and Table 3 under the headings "U-OLS" and "C-OLS" respectively. The unconstrained OLS method exhibits the best performance in terms of coverage rate. This might at first seem surprising. However, since the regression error is normal in both experiments, the finite sampling distribution of the unconstrained OLS estimator is also normal. Therefore, the coverage rates based on the unconstrained OLS estimator and the normal table should be quite close to the nominal coverage rate, given that we have 5000 replications and because this method is using the correct finite sampling distribution. Nevertheless, because this method does not utilize useful information (the summing-to-one restriction and non-negativity) it is not efficient, as reflected in the wider confidence regions. For $R^2 = 0.95$, where the Andrews method is comparable to unconstrained OLS in terms of coverage rate, the confidence intervals based on the Andrews method are consistently tighter than the confidence intervals based on the unconstrained OLS method.

Of particular interest are the sampling distributions in the second experiment. The histograms of the 50,000 estimated style weights for each b_i^* appear in Figure 2. The distribution of the second and fifth style weights (Russell 2000 value and T-bill) are piled up at zero, as we should expect, since the true style weight is zero. Further, the sampling distribution for the first and fourth style weights seem to be asymmetric (skewed to the left) rather than normal, despite the fact that the associated true style weights are in the interior of the parameter space.

Before proceeding, we must note that in the absence of any intervention, it turns out that the Andrews confidence intervals for the second variable (r2value) and the fifth variable (tbill) in the second experiment can have negative lower bounds. Having a negative lower bound appears counter-intuitive, because the true parameters (b_2^*, b_5^*) are zero and we restrict the estimates to be greater than zero. Hence, one might think that zero would be the smallest possible value for the lower bound. However, when applying the Andrews method in finite samples, we are bound to observe negative values in the confidence interval whenever the pre-test fails to reject the null hypothesis $H_o : b_i^* = 0$. The reason is as follows. If the pre-test fails to reject $H_o : b_i^* = 0$, then the "inside" linear-quadratic optimization imposes the binding condition $l_i \geq 0$ for all replications to build up the sampling distribution of $\hat{l}_i$. As a result, the sampling distribution of $\hat{l}_i$ is piled up at zero with part of the distribution in the positive area. Let z_L be the $a^*/2$ percentile and z_U be the $1-a^*/2$ percentile of the sampling distribution of $\hat{l}_i$. Then z_L is most likely to be zero and z_U is some positive number. Note that the upper bound of the $(1-a^*)\%$ confidence interval depends on z_L and the lower bound of the $(1-a^*)\%$ confidence interval depends on z_U as derived previously:

$$1-a^* \approx P[\hat{b}_T - z_U T^{-1/2} \leq b^* \leq \hat{b}_T - z_L T^{-1/2}].$$

Hence, the $(1-a^*)\%$ confidence interval is $(\hat{b}_T - z_U T^{-1/2}, \hat{b}_T - z_L T^{-1/2})$. Since z_L is virtually zero, the $(1-a^*)\%$ confidence interval is approximately $(\hat{b}_T - z_U T^{-1/2}, \hat{b}_T)$. The estimate for $\hat{b}_T$ is also very close to zero in this case. Consequently, the lower bound can easily be a negative number and the upper bound is very close to zero. This effect arises because we are applying the Andrews method in finite samples. The negative lower bound will converge to zero as the sample size increases. A simple "fix" for this phenomenon is to move any probability mass below zero to zero; note that this has no effect on coverage probabilities. Our graphics and tables embody this adjustment.

7. Application

In this section we use our new procedures to investigate the Sharpe style of two fund returns, the Fidelity Magellan Fund and the Minicap Fund. Table 1 provides summary statistics and Figure 4 shows time-series plots of the performance of the two funds. The sample period is January 1979 through July 1997 for the Fidelity Magellan Fund and August 1991 through March 1998 for the Minicap Fund. We use the Russell 2000 growth, the Russell 2000 value, the Russell 1000 growth, the Russell 1000 value and the 30 day T-bill rate as explanatory variables in both style regressions. We set the number of Monte Carlo replications for linear-quadratic optimization to be $N = 50{,}000$. For comparison purposes we also show the results based on Lobosco and DiBartolomeo's method, the unconstrained OLS method and the OLS method with the summing-to-one restriction.

The results of the style regression analysis for the Fidelity Magellan Fund are reported in Table 10. It turns out that both Andrews (5%) and Andrews (50%) give exactly the same results for this data set, so we report them in a single column. The proportion of the variance of the Fidelity Magellan Fund explained by the four style indices is 90% (Andrews, LDB, C-OLS) and 92% (U-OLS). The analysis indicates that the fund was oriented toward large cap-growth during the sample period. All style weights except for cash (tbill) are statistically different from zero and one based on inversion of the confidence intervals. Standard errors based on the Andrews method and Bayesian methods are consistently smaller than those based on LDB. Similarly, the Andrews 95% confidence intervals and Bayesian confidence intervals are shorter than LDB. The confidence interval for the style weight for the 30 day T-bill includes zero in all methods except the unconstrained OLS method. Hence, the style weight for the 30 day T-bill is insignificant at the 5% level. The length of Andrews' confidence interval for the 30 day T-bill is smaller than half that of LDB. This is because the corresponding style weight is presumed to be zero in the pre-test for Andrews' method. The sampling distributions of style weights based on the Andrews method are displayed in Figure 5. All distributions except for the 30 day T-bill rate appear symmetric, and the sampling distribution of the 30 day T-bill coefficient displays the sample pattern for boundary parameters found in the simulation.

Table 11 shows the style regression for the Minicap Fund. The Minicap Fund was heavily oriented toward small cap-growth for the sample period, as is evident from the style weight for the Russell 2000 growth being equal to one. According to the 95% confidence intervals from Andrews (5%) and Andrews (50%), all style weights for the Russell 2000 value, the Russell 1000 growth, the Russell 1000 value and the 30 day T-bill rate are insignificant at the 5% level. Thus, there may be four binding parameters, which makes it difficult in the extreme to implement the Bayesian method: the POS is very close to zero. In fact, because of its extreme computational cost, we have not applied the Bayesian method to this data set. Andrews (5%) and Andrews (50%) give different 95% confidence intervals, but both methods produce tighter confidence intervals than that of LDB. The R^2 is 0.84 for Andrews, LDB, and C-OLS and 0.90 for U-OLS which are smaller than the corresponding R^2's for the Fidelity Magellan Fund.

Figure 6 shows the sampling distributions of the style weight coefficients for the Minicap Fund using Andrews (50%). We see that the style weights are near the boundary in every case, with the distributions displaying substantial asymmetry.

8. Conclusion

We apply results of Andrews (1997a, 1999) to propose a statistically valid procedure for obtaining large sample standard errors and confidence intervals for the Sharpe style weights constrained least squares estimator. We also extend the results of Geweke (1986) to obtain comparable Bayesian confidence intervals. Although the validity of the Andrews procedure has been proved only for large samples, our simulations suggest that the Andrews method with a 50% pre-test level has good coverage rates with relatively short confidence intervals over empirically relevant values of R^2, regardless of the presence or absence of binding parameters. In particular, the method does not exhibit the consistent pattern of overly broad confidence intervals exhibited by Lobosco and Di Bartolomeo's method when boundary parameters are present. The message of our simulation study is that the Andrews method is comparable with Bayesian methods and with Lobosco and Di Bartolomeo's method in terms of coverage rates when there is no boundary parameter, but it is better than the other methods when some boundary parameters are present. We thus recommend

that when there is no prior knowledge about boundary parameters being absent, Andrews (50%) be used. Moreover, the Andrews method is straightforward to implement and thus should prove useful to practitioners.

Appendix

1. We give a set of sufficient conditions for the validity of the proposed procedure:

   1) (R_t, F_t') $t = 1, \ldots, T$ are identically and independently distributed.

   2) $E(R_t^2) < \infty, \quad E((F_t' F_t)^2) < \infty.$

   3) $\det(V) > 0$ where $V \equiv E(e_t^2 F_t F_t')$, $e_t \equiv R_t - F_t' b^*$ for b^* defined below.

   4) $\det(M) > 0$ where $M \equiv E(F_t F_t')$.

The first assumption can be relaxed substantially to allow dependent and serially correlated returns. In that case we would need to use an HAC (heteroskedasticity and autocorrelation consistent covariance matrix) estimator in estimating V. Here we maintain the i.i.d. assumption for simplicity. The statistical definition of the vector of Sharpe style weights is

$$b^* = E(F_t F_t')^{-1} E(F_t R_t),$$

which is the solution to the minimization problem

$$\min_b \; E((R_t - F_t' b)^2).$$

We assume that $i' b^* = 1$ and $b^* \geq 0$. The solution b^* is what we call the "true" Sharpe style weight vector. Note that we do not require the model to be correctly specified, beyond the constraints on b^*. It is straightforward to show that the definitions of b^* and e_t imply the orthogonality condition $E(F_t e_t) = 0$, which is weaker than the correct specification condition: $E(F_t \mid e_t) = 0$. Without assuming that the model is correctly specified and that there is no conditional heteroscedasticity in e_t, the covariance matrix V generally does not simplify to the product of s_0^2 and Q.

2. In order to find the optimal pre-test level, we carry out the following procedure.

   (1) Fix s.

   (2) Specify a parameter space $\Theta \equiv \{1\%, 2\%, \ldots, 99\%\}$.

   (3) For each level $s \in \Theta$ do the following:

      a) Conduct the pre-test to identify binding parameters.

      b) Carry out the Monte Carlo Simulation based on the pre-test results.

c) Obtain nominal 95% confidence intervals for $b_i^*, \quad i = 1, \ldots, k$ based on 5000 replications.

d) Let $C_i(s)$ be the observed coverage rate for the *i*th confidence interval constructed in 3)

e) Let $MAD(s) \equiv \frac{1}{k} \sum_{i=1}^{k} |C_i(s) - 0.95|$ where $k = 5$.

(4) Solve the following minimization problem.

$$\min_s \; MAD(s)$$

$$s.t. \quad s \in \Theta.$$

Let s^* be the solution. We call s^* the "optimal pre-test level". The corresponding coverage rate is given by $C_i(s^*)$.

References

Andrews, D.W.K. (1997a). Estimation When a Parameter Is on a Boundary: Theory and Application, *Yale: Cowles Foundation Discussion Paper*.

Andrews, D.W.K. (1997b). A Simple Counterexample to the Bootstrap, *Yale: Cowles Foundation Discussion Paper*.

Andrews, D.W.K. (1999). Estimation When a Parameter Is on a Boundary, *Econometrica*, 67, 1341-1383.

Andrews, D.W.K. (2000). Inconsistency of the Bootstrap When a Parameter Is on the Boundary of the Parameter Space, *Econometrica*, 68, forthcoming.

Amemiya, T. (1985). *Advanced Econometrics*. Cambridge, Mass.: Harvard University Press.

Brown, S.J. and W.N. Goetzmann (1997). Mutual Fund Styles, *Journal of Financial Economics*, 43, 373-399.

Davis, W.W. (1978). Bayesian Analysis of the Linear Model Subject to Linear Inequality Constraints, *Journal of the American Statistical Association*, 78, 573-579.

DiBartolomeo, D. and E. Witkowski (1997). Mutual Fund Misclassification: Evidence Based on Style Analysis, *Financial Analyst Journal*, September, 32-43.

Fung, W. and D.A. Hsieh (1997). Empirical Characteristics of Dynamic Trading Strategies: The Case of Hedge Funds, *The Review of Financial Studies*, 10, 275-302.

Geweke, J. (1986). Exact Inference in The Inequality Constrained Normal Linear Regression Model, *Journal of Applied Econometrics*, 1, 127-141.

Judge, G.G. and T. Takayama (1966). Inequality Restrictions in Regression Analysis, *Journal of American Statistical Association*, 61, 166-181.

Kloek, T. and H.K Van Dijk (1978). Bayesian Estimates of Equation System Parameters: An Application of Integration by Monte Carlo, *Econometrica*, 46, 1-19.

Lobosco, A. and D. DiBartolomeo (1997). Approximating the Confidence Intervals for Sharpe Style Weights, *Financial Analyst's Journal*, July/August, 80-85.

Lindley, D.V. (1961). Discussion of Bartholomew, D.J., A Test of Homogeneity of Means under Restricted Alternatives, *Journal of the Royal Statistical Society*, Ser. B, 23, 272-274.

O'Hagan, A. (1973). Bayes Estimation of a Convex Quadratic, *Biometrika*, 60, 565-571.

Sharpe, W.F. (1988). Determining a Fund's Effective Asset Mix, *Investment Management Review*, November/December, 59-69.

Sharpe, W.F. (1992). Asset Allocation: Management Style and Performance Measurement, *Journal of Portfolio Management*, 18, 7-19.

Sortino, F.A., G.A. Miller and J.M. Messina (1997). Short-Term Risk-Adjusted Performance: A Style-Based Analysis, *The Journal of Investing*, 18, 7-19.

Table 1. Summary Statistics

	Sample period: January 1979 - July 1997 (223 Observations)						Sample period: August 1991 - March 1998 (80 Observations)					
Statistics	fidelity	r2growth	r2value	r1growth	r1value	tbill	Minicap	r2growth	r2value	r1growth	r1value	tbill
Mean	2.008	1.275	1.502	1.413	1.437	0.611	2.137	1.276	1.677	1.538	1.634	0.376
Median	1.965	1.977	2.082	1.558	1.526	0.590	3.100	1.964	2.122	1.617	1.909	0.415
Maximum	14.384	16.518	12.192	14.342	13.671	1.254	17.290	15.030	8.365	14.134	8.381	0.494
Minimum	-25.805	-32.950	-28.285	-23.231	-20.161	0.233	-14.350	-12.208	-5.316	-5.859	-5.132	0.233
Standard Deviation	5.136	6.289	4.718	4.766	3.964	0.246	6.104	4.816	2.973	3.595	2.966	0.082
Skewness	-0.708	-0.876	-1.418	-0.550	-0.683	0.715	-0.052	-0.138	-0.290	0.366	-0.231	-0.452
Excess Kurtosis	3.196	3.410	6.911	2.808	3.792	-0.007	-0.022	0.066	-0.327	0.783	-0.381	-1.340
Correlation												
Fund	1.000						1.000					
r2growth	0.903	1.000					0.942	1.000				
r2value	0.896	0.929	1.000				0.774	0.853	1.000			
r1growth	0.923	0.872	0.820	1.000			0.634	0.670	0.628	1.000		
r1value	0.904	0.804	0.860	0.907	1.000		0.516	0.569	0.724	0.839	1.000	
tbill	0.043	-0.046	-0.050	-0.063	-0.065	1.000	0.184	0.091	0.028	0.239	0.166	1.000

Note: (1) Frequency: monthly

(2) Unit: percent return

Table 2. Coverage Rates for Experiment 1 $(b_1^* = 0.2, b_2^* = 0.2, b_3^* = 0.2, b_4^* = 0.2, b_5^* = 0.2)$

R^2	s	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes (Equal Tailed)	Bayes (HPD)	LDB	C-OLS	U-OLS
0.8	1.9	r2growth	0.680	0.936	0.941	0.948	0.972	0.949	0.960	0.942	0.949
		r2value	0.547	0.944	0.950	0.949	0.974	0.957	0.964	0.944	0.951
		r1growth	0.584	0.931	0.951	0.948	0.975	0.961	0.960	0.942	0.950
		r1value	0.475	0.934	0.959	0.958	0.986	0.969	0.975	0.949	0.955
		tbill	0.922	0.941	0.934	0.943	0.944	0.939	0.955	0.941	0.943
0.9	1.2	r2growth	0.897	0.948	0.939	0.942	0.952	0.941	0.954	0.942	0.949
		r2value	0.896	0.949	0.948	0.944	0.960	0.943	0.956	0.944	0.951
		r1growth	0.893	0.941	0.950	0.942	0.966	0.955	0.953	0.942	0.950
		r1value	0.874	0.945	0.958	0.948	0.967	0.945	0.959	0.949	0.955
		tbill	0.939	0.941	0.934	0.942	0.953	0.948	0.954	0.941	0.943
0.95	0.9	r2growth	0.939	0.948	0.939	0.942	0.948	0.941	0.954	0.942	0.949
		r2value	0.942	0.949	0.948	0.944	0.944	0.938	0.956	0.944	0.951
		r1growth	0.940	0.941	0.950	0.942	0.954	0.946	0.953	0.942	0.950
		r1value	0.948	0.945	0.958	0.948	0.959	0.952	0.959	0.949	0.955
		tbill	0.942	0.941	0.934	0.942	0.955	0.952	0.954	0.941	0.943

Note: Andrews' optimal level is given as follows.

(1) 66% when R^2 = 0.8.

(2) 35% when R^2 = 0.9.

(3) 7% when R^2 = 0.95.

Table 3. Coverage Rates for Experiment 2 $(b_1^* = 0.4, b_2^* = 0, b_3^* = 0.2, b_4^* = 0.4, b_5^* = 0)$

R^2	s	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes (Equal Tailed)	Bayes (HPD)	LDB	C-OLS	U-OLS
0.8	2.4	r2growth	0.883	0.945	0.955	0.938	0.911	0.915	0.986	0.942	0.949
		r2value	0.912	0.963	0.969	0.943	0	0.961	0.983	0.944	0.951
		r1growth	0.546	0.929	0.932	0.942	0.956	0.943	0.974	0.942	0.950
		r1value	0.787	0.941	0.950	0.949	0.795	0.714	0.974	0.949	0.955
		tbill	0.945	0.961	0.966	0.951	0	0.974	0.987	0.941	0.943
0.9	1.6	r2growth	0.911	0.949	0.941	0.937	0.927	0.926	0.985	0.942	0.949
		r2value	0.941	0.958	0.950	0.945	0	0.971	0.983	0.944	0.951
		r1growth	0.822	0.944	0.951	0.940	0.936	0.930	0.966	0.942	0.950
		r1value	0.887	0.952	0.967	0.947	0.791	0.788	0.974	0.949	0.955
		tbill	0.942	0.947	0.950	0.942	0	0.970	0.987	0.941	0.943
0.95	1.1	r2growth	0.933	0.949	0.940	0.937	0.933	0.941	0.985	0.942	0.949
		r2value	0.935	0.952	0.950	0.936	0	0.968	0.983	0.944	0.951
		r1growth	0.937	0.944	0.950	0.940	0.939	0.942	0.966	0.942	0.950
		r1value	0.944	0.954	0.966	0.947	0.782	0.793	0.974	0.949	0.955
		tbill	0.922	0.935	0.916	0.922	0	0.972	0.987	0.941	0.943

Note: Andrews' optimal level is given as follows.

(1) 61% when R^2 = 0.8.

(2) 55% when R^2 = 0.9.

(3) 52% when R^2 = 0.95.

Table 4. Averaged 95% Confidence Intervals for Experiment 1 $(b_1^* = 0.2, b_2^* = 0.2, b_3^* = 0.2, b_4^* = 0.2, b_5^* = 0.2)$

R^2	s	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes (Equal Tailed)	Bayes (HPD)	LDB
0.8	1.9	r2growth	0.088 0.313	0.060 0.343	0.056 0.342	0.057 0.343	0.074 0.321	0.076 0.319	0.049 0.350
		r2value	0.049 0.325	0.014 0.382	0.019 0.391	0.017 0.388	0.053 0.365	0.051 0.356	0.007 0.398
		r1growth	0.063 0.321	0.030 0.369	0.032 0.375	0.030 0.372	0.057 0.346	0.056 0.340	0.023 0.378
		r1value	0.039 0.319	0.005 0.395	0.000 0.392	0.000 0.396	0.048 0.372	0.046 0.363	-0.006 0.402
		tbill	0.140 0.266	0.135 0.264	0.137 0.266	0.137 0.265	0.133 0.264	0.134 0.263	0.132 0.269
0.9	1.2	r2growth	0.112 0.288	0.111 0.291	0.109 0.290	0.110 0.290	0.107 0.286	0.108 0.285	0.105 0.295
		r2value	0.087 0.313	0.082 0.317	0.085 0.321	0.084 0.319	0.090 0.321	0.091 0.319	0.078 0.325
		r1growth	0.096 0.305	0.091 0.308	0.094 0.311	0.092 0.308	0.095 0.308	0.096 0.307	0.088 0.313
		r1value	0.077 0.314	0.075 0.326	0.071 0.322	0.073 0.323	0.076 0.319	0.076 0.315	0.069 0.327
		tbill	0.160 0.241	0.159 0.240	0.161 0.242	0.160 0.241	0.158 0.242	0.158 0.241	0.157 0.244
0.95	0.9	r2growth	0.132 0.267	0.133 0.268	0.132 0.267	0.132 0.268	0.131 0.269	0.132 0.269	0.129 0.271
		r2value	0.113 0.289	0.111 0.288	0.114 0.291	0.113 0.289	0.109 0.288	0.109 0.287	0.109 0.294
		r1growth	0.120 0.281	0.119 0.281	0.120 0.283	0.119 0.281	0.116 0.281	0.117 0.280	0.116 0.284
		r1value	0.105 0.292	0.106 0.294	0.103 0.291	0.105 0.293	0.106 0.297	0.107 0.296	0.102 0.296
		tbill	0.170 0.231	0.169 0.230	0.170 0.231	0.170 0.231	0.169 0.232	0.169 0.232	0.168 0.233

Table 4 Continued

R^2	s	Index	C-OLS		U-OLS	
0.8	1.9	r2growth	0.056	0.342	0.052	0.347
		r2value	0.016	0.388	0.010	0.394
		r1growth	0.029	0.372	0.026	0.375
		r1value	-0.001	0.395	-0.005	0.399
		tbill	0.137	0.265	-0.208	0.607
0.9	1.2	r2growth	0.109	0.290	0.106	0.293
		r2value	0.084	0.319	0.080	0.323
		r1growth	0.092	0.308	0.090	0.311
		r1value	0.073	0.323	0.071	0.326
		tbill	0.160	0.241	-0.058	0.457
0.95	0.9	r2growth	0.132	0.267	0.130	0.270
		r2value	0.113	0.289	0.110	0.292
		r1growth	0.119	0.281	0.117	0.283
		r1value	0.105	0.292	0.103	0.294
		tbill	0.170	0.231	0.007	0.393

Table 5. Averaged 95% Confidence Intervals for Experiment 2 $(b_1^* = 0.4, b_2^* = 0, b_3^* = 0.2, b_4^* = 0.4, b_5^* = 0)$

R^2	s	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes (Equal Tailed)	Bayesian (HPD)	LDB
0.8	2.4	r2growth	0.293 0.538	0.245 0.552	0.237 0.559	0.286 0.540	0.215 0.469	0.220 0.472	0.189 0.569
		r2value	0.000 0.055	0.000 0.158	0.000 0.185	0.000 0.042	0.005 0.248	0.001 0.218	-0.205 0.288
		r1growth	0.012 0.321	0.002 0.410	0.001 0.415	0.002 0.395	0.073 0.435	0.065 0.417	-0.005 0.444
		r1value	0.245 0.586	0.155 0.604	0.141 0.603	0.219 0.606	0.092 0.454	0.093 0.447	0.087 0.604
		tbill	0.000 0.019	0.000 0.055	0.000 0.066	0.000 0.015	0.002 0.089	0.000 0.078	-0.072 0.102
0.9	1.6	r2growth	0.323 0.494	0.296 0.501	0.290 0.502	0.324 0.494	0.274 0.450	0.278 0.452	0.260 0.513
		r2value	0.000 0.037	0.000 0.105	0.000 0.119	0.000 0.028	0.004 0.173	0.001 0.151	-0.137 0.192
		r1growth	0.068 0.314	0.067 0.343	0.069 0.348	0.068 0.330	0.099 0.364	0.094 0.354	0.063 0.362
		r1value	0.282 0.537	0.235 0.536	0.226 0.534	0.279 0.538	0.179 0.444	0.183 0.446	0.191 0.536
		tbill	0.000 0.013	0.000 0.037	0.000 0.042	0.000 0.010	0.001 0.060	0.000 0.052	-0.048 0.068
0.95	1.1	r2growth	0.346 0.465	0.328 0.470	0.326 0.470	0.348 0.464	0.314 0.435	0.317 0.437	0.303 0.478
		r2value	0.000 0.025	0.000 0.072	0.000 0.077	0.000 0.019	0.002 0.117	0.000 0.102	-0.094 0.132
		r1growth	0.109 0.289	0.109 0.299	0.110 0.301	0.109 0.289	0.129 0.314	0.127 0.311	0.106 0.312
		r1value	0.314 0.494	0.286 0.494	0.282 0.492	0.317 0.495	0.247 0.431	0.250 0.433	0.257 0.493
		tbill	0.000 0.009	0.000 0.025	0.000 0.028	0.000 0.007	0.001 0.041	0.000 0.036	-0.033 0.047

Table 5 Continued

R^2	s	Index	C-OLS		U-OLS	
0.8	2.4	r2growth	0.219	0.580	0.213	0.586
		r2value	-0.232	0.237	-0.240	0.246
		r1growth	-0.015	0.417	-0.020	0.422
		r1value	0.146	0.646	0.141	0.651
		Tbill	-0.080	0.082	-0.515	0.515
0.9	1.6	r2growth	0.279	0.520	0.275	0.524
		r2value	-0.155	0.158	-0.160	0.164
		r1growth	0.056	0.344	0.053	0.348
		r1value	0.231	0.564	0.228	0.568
		Tbill	-0.053	0.055	-0.344	0.343
0.95	1.1	r2growth	0.317	0.482	0.314	0.485
		r2value	-0.106	0.109	-0.110	0.113
		r1growth	0.101	0.299	0.099	0.302
		r1value	0.284	0.513	0.281	0.515
		Tbill	-0.037	0.038	-0.236	0.236

Table 6. Averaged Standard Errors for Experiment 1 $(b_1^* = 0.2, b_2^* = 0.2, b_3^* = 0.2, b_4^* = 0.2, b_5^* = 0.2)$

R^2	s	Index	True	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes	LDB	C-OLS	U-OLS
0.8	1.9	r2growth	0.072	0.058	0.072	0.073	0.073	0.064	0.077	0.073	0.075
		r2value	0.094	0.073	0.094	0.095	0.095	0.081	0.100	0.095	0.098
		r1growth	0.086	0.067	0.086	0.087	0.087	0.075	0.091	0.087	0.089
		r1value	0.099	0.074	0.100	0.101	0.101	0.085	0.104	0.101	0.103
		tbill	0.034	0.032	0.033	0.033	0.033	0.033	0.035	0.033	0.208
0.9	1.2	r2growth	0.047	0.045	0.046	0.046	0.046	0.046	0.048	0.046	0.048
		r2value	0.061	0.058	0.060	0.060	0.060	0.059	0.063	0.060	0.062
		r1growth	0.056	0.053	0.055	0.055	0.055	0.054	0.057	0.055	0.056
		r1value	0.065	0.061	0.064	0.064	0.064	0.062	0.066	0.064	0.065
		tbill	0.021	0.021	0.021	0.021	0.021	0.021	0.022	0.021	0.131
0.95	0.9	r2growth	0.035	0.035	0.035	0.035	0.035	0.035	0.036	0.035	0.036
		r2value	0.046	0.045	0.045	0.045	0.045	0.046	0.047	0.045	0.046
		r1growth	0.042	0.041	0.041	0.042	0.041	0.042	0.043	0.041	0.042
		r1value	0.049	0.048	0.048	0.048	0.048	0.049	0.049	0.048	0.049
		tbill	0.016	0.016	0.016	0.016	0.016	0.016	0.017	0.016	0.099

Table 7. Averaged Standard Errors for Experiment 2 $(b_1^* = 0.4, b_2^* = 0, b_3^* = 0.2, b_4^* = 0.4, b_5^* = 0)$

R^2	s	Index	True	Andrews (5%)	Andrews (50%)	Andrews (Optimal)	Andrews (Q known)	Bayes	LDB	C-OLS	U-OLS
0.8	2.4	r2growth	0.065	0.062	0.078	0.082	0.064	0.065	0.097	0.092	0.095
		r2value	0.065	0.066	0.093	0.100	0.064	0.066	0.126	0.12	0.124
		r1growth	0.100	0.083	0.104	0.106	0.100	0.094	0.115	0.11	0.113
		r1value	0.099	0.087	0.114	0.118	0.098	0.093	0.132	0.128	0.13
		tbill	0.022	0.023	0.032	0.035	0.022	0.024	0.044	0.041	0.263
0.9	1.6	r2growth	0.043	0.043	0.052	0.054	0.043	0.045	0.065	0.061	0.063
		r2value	0.043	0.045	0.062	0.065	0.043	0.046	0.084	0.08	0.083
		r1growth	0.068	0.064	0.070	0.071	0.067	0.068	0.076	0.073	0.075
		r1value	0.066	0.065	0.077	0.079	0.066	0.068	0.088	0.085	0.087
		tbill	0.015	0.015	0.021	0.022	0.015	0.016	0.030	0.028	0.175
0.95	1.1	r2growth	0.030	0.030	0.036	0.036	0.029	0.031	0.044	0.042	0.044
		r2value	0.030	0.031	0.043	0.044	0.029	0.031	0.058	0.055	0.057
		r1growth	0.047	0.046	0.048	0.049	0.046	0.047	0.053	0.051	0.052
		r1value	0.046	0.046	0.053	0.054	0.045	0.047	0.060	0.058	0.060
		tbill	0.010	0.011	0.015	0.015	0.010	0.011	0.020	0.019	0.120

Table 8. Acceptance Rates for Experiment 1 $(b_1^* = 0.2, b_2^* = 0.2, b_3^* = 0.2, b_4^* = 0.2, b_5^* = 0.2)$

R^2	s	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)
0.8	1.9	r2growth	0.143	0.002	0.001
		r2value	0.319	0.018	0.005
		r1growth	0.262	0.012	0.006
		r1value	0.379	0.021	0.009
		tbill	0.000	0.000	0.000
0.9	1.2	r2growth	0.004	0.000	0.000
		r2value	0.049	0.000	0.001
		r1growth	0.026	0.000	0.000
		r1value	0.076	0.001	0.002
		tbill	0.000	0.000	0.000
0.95	0.9	r2growth	0.000	0.000	0.000
		r2value	0.002	0.000	0.001
		r1growth	0.001	0.000	0.000
		r1value	0.006	0.000	0.003
		tbill	0.000	0.000	0.000

Table 9. Acceptance Rates for Experiment 2 $(b_1^* = 0.4, b_2^* = 0, b_3^* = 0.2, b_4^* = 0.4, b_5^* = 0)$

R^2	S	Index	Andrews (5%)	Andrews (50%)	Andrews (Optimal)
0.8	2.4	r2growth	0.004	0.000	0.000
		r2value	0.945	0.508	0.393
		r1growth	0.433	0.035	0.025
		r1value	0.076	0.001	0.000
		tbill	0.945	0.503	0.387
0.9	1.6	r2growth	0.000	0.000	0.000
		r2value	0.945	0.508	0.473
		r1growth	0.147	0.004	0.005
		r1value	0.002	0.000	0.000
		tbill	0.945	0.503	0.451
0.95	1.1	r2growth	0.000	0.000	0.000
		r2value	0.945	0.508	0.499
		r1growth	0.011	0.000	0.000
		r1value	0.000	0.000	0.000
		tbill	0.945	0.503	0.480

Table 10. Style Regression Analysis for Fidelity Magellan Fund

Index	Style Weights				Standard Error				
	Andrews (5%) (50%)	Bayes	C-OLS	U-OLS	Andrews (5%) (50%)	Bayes	LDB	C-OLS	U-OLS
r2growth	0.212	0.218	0.129	0.200	0.059	0.057	0.066	0.058	0.057
r2value	0.190	0.182	0.277	0.192	0.078	0.075	0.086	0.079	0.074
r1growth	0.378	0.377	0.414	0.373	0.073	0.076	0.078	0.070	0.067
r1value	0.221	0.216	0.289	0.309	0.083	0.083	0.090	0.082	0.078
tbill	0.000	0.007	-0.109	0.988	0.015	0.006	0.031	0.027	0.157

Note: (1) R^2 = 0.90 (Andrews, Lobosco, C-OLS), R^2 = 0.92 (U-OLS)

(2) Seed for Random Number Generator: 1338013

(3) Number of Replications for Quadratic Optimization: 20000

(4) Sample period: January 1979 - July 1997, Number of Observations: 223

(5) Computation Time: 12 seconds

Table 10 Continued

Index	95% Confidence Interval					
	Andrews (5%) (50%)	Bayes (Equal Tailed)	Bayes (HPD)	LDB	C-OLS	U-OLS
r2growth	0.09 0.32	0.10 0.33	0.08 0.81	0.08 0.34	0.02 0.24	0.09 0.31
r2value	0.05 0.35	0.03 0.34	0.01 0.30	0.02 0.36	0.12 0.43	0.05 0.34
r1growth	0.24 0.53	0.24 0.52	0.24 0.52	0.23 0.53	0.28 0.55	0.24 0.51
r1value	0.07 0.39	0.06 0.38	0.07 0.38	0.04 0.40	0.13 0.45	0.16 0.46
tbill	0.00 0.00	0.00 0.02	0.00 0.02	-0.06 0.06	-0.16 -0.06	0.68 1.30

Table 11. Style Regression Analysis for Minicap Returns

Index	Style Weights			Standard Error					95% Confidence Interval				
	Andrews (5%) (50%)	C-OLS	U-OLS	Andrews (5%)	Andrews (50%)	LDB	C-OLS	U-OLS	Andrews (5%)	Andrews (50%)	LDB	C-OLS	U-OLS
r2growth	1	1.176	1.308	0.062	0.110	0.146	0.100	0.116	1.00 1.21	0.74 1.17	0.71 1.29	0.98 1.37	1.08 1.54
r2value	0	0.010	-0.234	0.041	0.146	0.254	0.179	0.201	0.00 0.00	0.00 0.40	-0.50 0.50	-0.34 0.36	-0.63 0.16
r1growth	0	0.111	0.011	0.040	0.154	0.187	0.156	0.150	0.00 0.00	0.00 0.41	-0.37 0.37	-0.20 0.42	-0.28 0.30
r1value	0	-0.056	-0.013	0.026	0.130	0.245	0.184	0.194	0.00 0.00	0.00 0.00	-0.48 0.48	-0.42 0.31	-0.39 0.37
Tbill	0	-0.240	2.506	0.029	0.062	0.114	0.086	0.734	0.00 0.00	0.00 0.00	-0.22 0.22	-0.41 -0.07	1.07 3.95

Note: (1) R^2 = 0.844 (Andrews, Lobosco-DiBartolomeo. C-OLS), R^2 = 0.895 (U-OLS)

(2) Seed for Random Number Generator: 1338013

(3) Number of Replications for Quadratic Optimization: 20000

(4) Sample period: August 1991 - March 1998

(5) Number of Observations: 80

(6) Computation Time: 18 seconds

Figure 1. Sampling Distribution of Style Weights: Boundary Parameters Absent

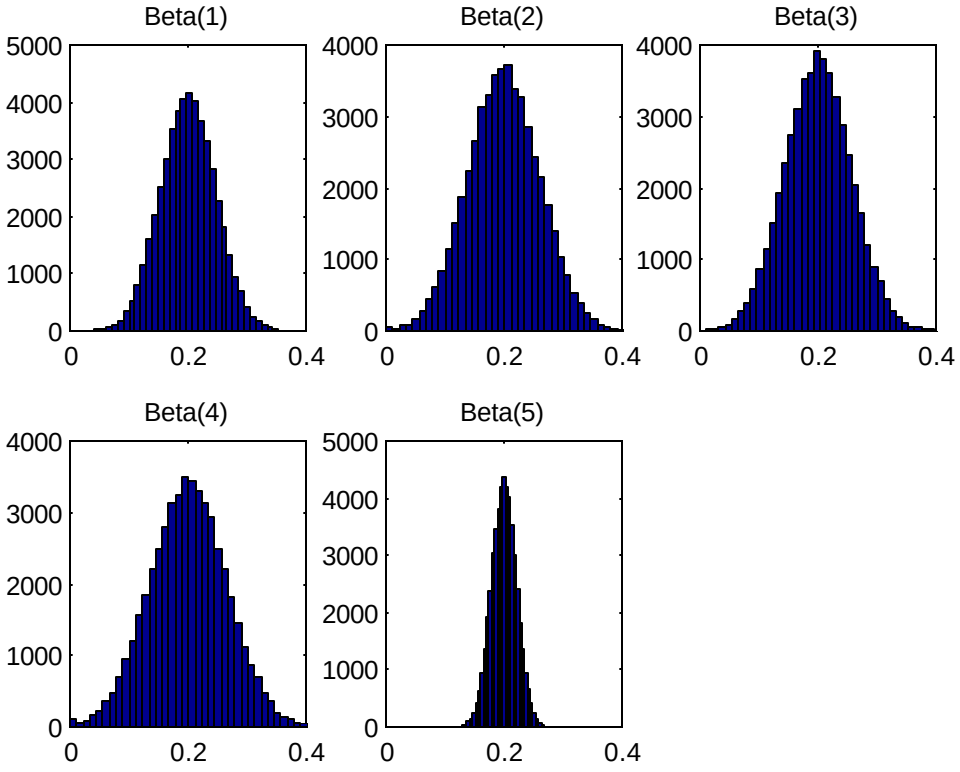

Figure 2. Sampling Distribution of Style Weights: Boundary Parameters Present

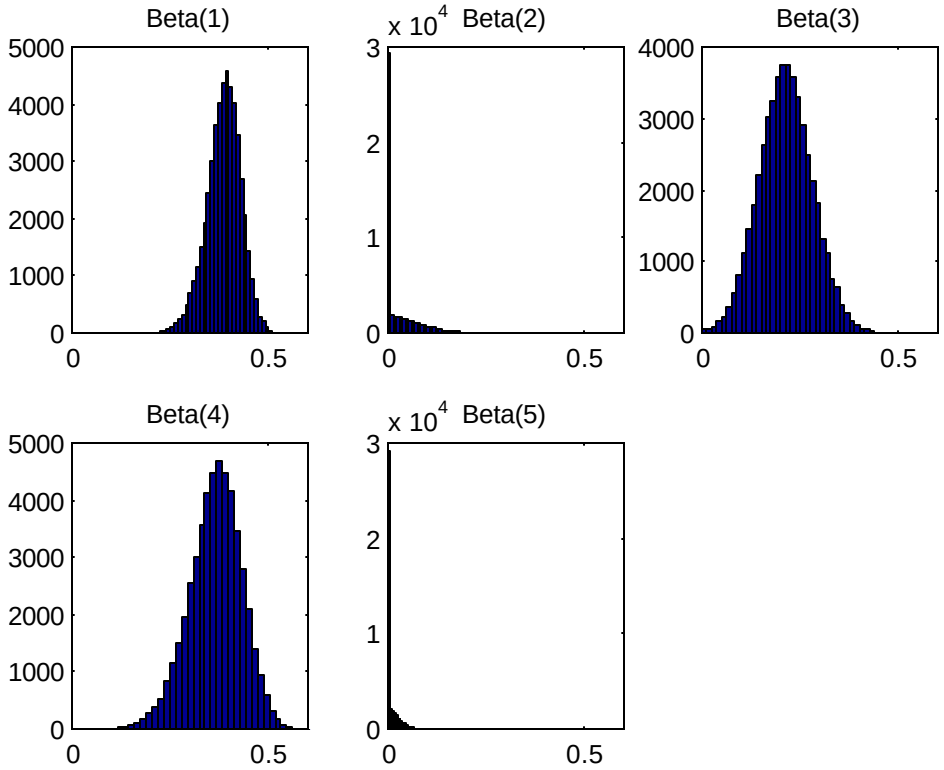

Figure 3. Mean Absolute Deviation of Nominal vs. Actual Confidence Intervals Limits : R^2 = 0.8 (*), 0.9 (o), 0.95 (+)

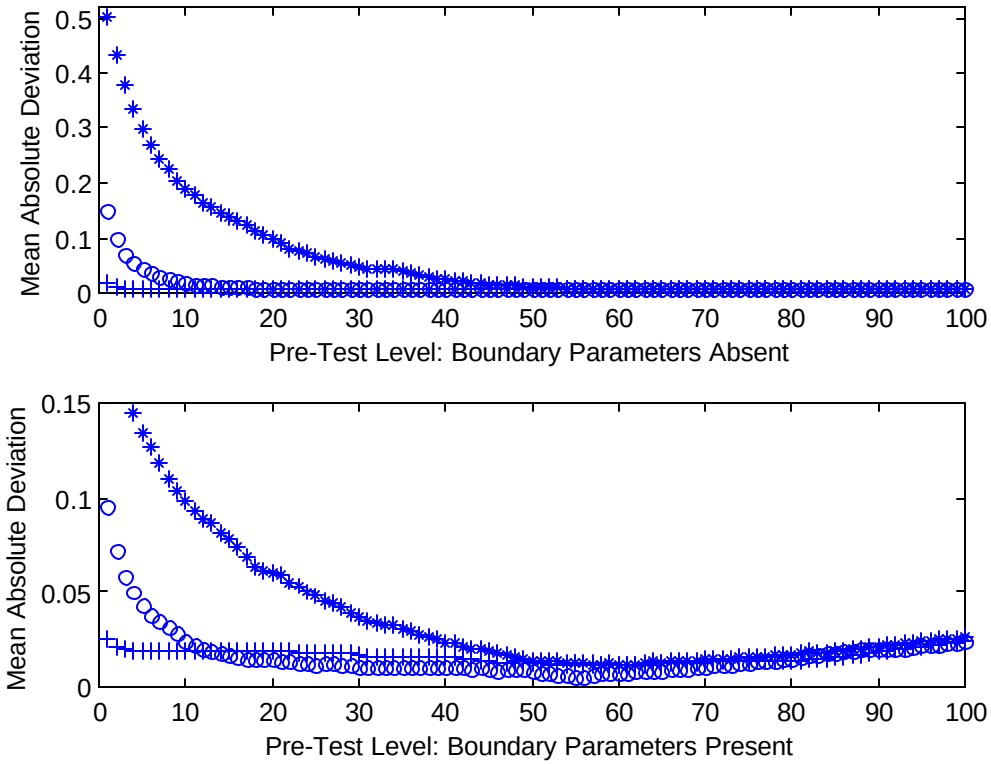

Figure 4. Time-series Plot of the Fidelity Magellan Fund and the Minicap Fund Monthly Returns

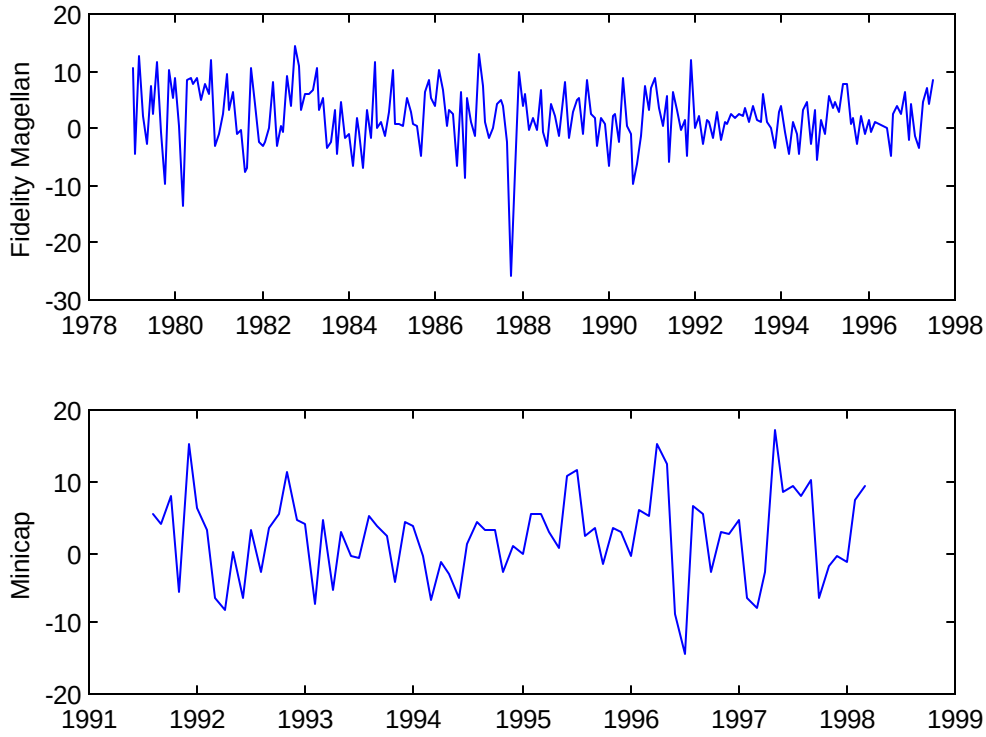

Figure 5. Sampling Distribution of Style Weights: Fidelity Magellan Fund (Andrews (5%, 50%))

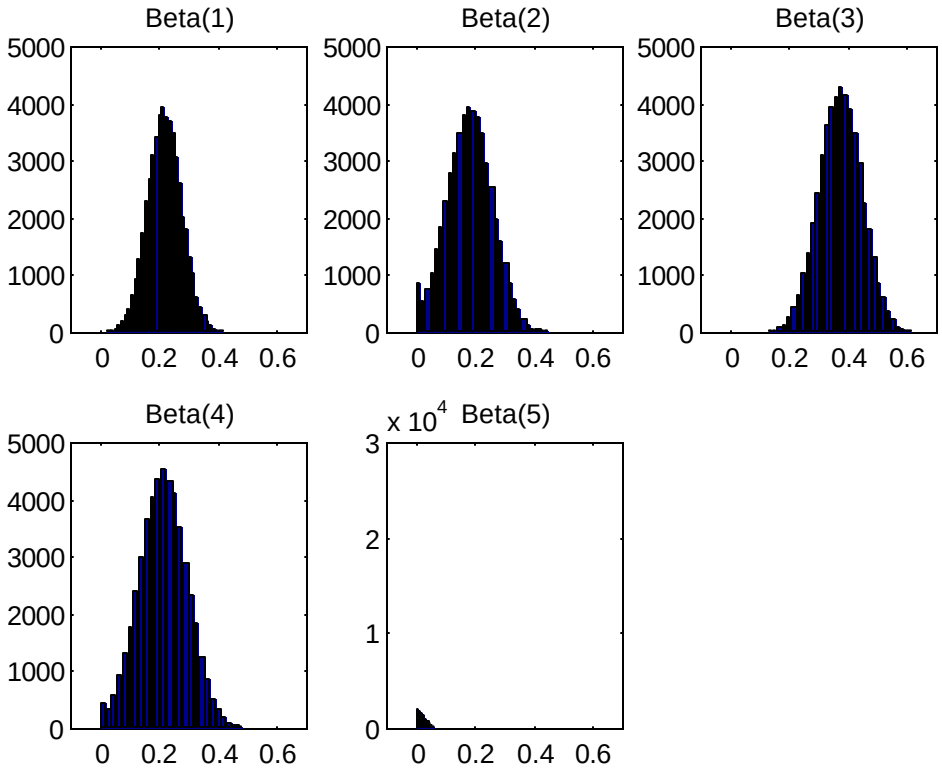

Figure 6. Sampling Distribution of Style Weights: Minicap Fund (Andrew (50%))