Title
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Separating the Hawks from the Doves *

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Abstract

Human players in our laboratory experiment converge closely to the symmetric mixed Nash equilibrium when matched in a single population version of the standard Hawk-Dove game. When matched across two populations, the same players show clear movement towards an asymmetric (and very inequitable) pure Nash equilibrium of the same game. These findings support a distinctive prediction of evolutionary game theory.

Keywords: Evolutionary dynamics, Hawk-Dove game, game theory, laboratory experiment, continuous time game.

JEL codes: C73, C92, D74

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1 Introduction

Evolutionary game theory predicts that the matching protocol can affect equilibrium selection as well as transitory dynamics. A leading example is the Hawk-Dove game (Maynard Smith, 1982), which has a symmetric mixed Nash equilibrium (NE) and two pure aysmmetric NE. Evolutionary game theory predicts convergence to the mixed NE with a single population matching protocol, but with a standard two population matching protocol it predicts convergence to one of the pure NE.

In this paper we report the first laboratory test of this distinctive prediction. The test is quite stringent because the payoffs in our asymmetric pure NE are very unequal—members of one population get a payoff five times larger than members of the other population. Thus the evolutionary prediction must overcome equity considerations and somehow resolve a fierce multilateral war of attrition.

Nevertheless, in four laboratory sessions, each with 10-12 human players, we observe close convergence to the mixed NE in 10 one-population periods. Then, in 10 subsequent two-population periods with the same subjects and payoff matrix, we see clear movement towards an asymmetric pure NE in three of the four sessions.

Payoff Bimatrix	NE profile	NE payoffs
$\begin{array}{c} H \\ D \end{array} \begin{pmatrix} 0,0 & 15,3 \\ 3,15 & 9,9 \end{pmatrix}$	$\frac{2}{3}H + \frac{1}{3}D,\ \frac{2}{3}H + \frac{1}{3}D$	5,5
	H,D	15,3
	D,H	3,15

Table 1: The Hawk-Dove Game

2 Theoretical Predictions

We use the Hawk-Dove payoff matrix in Table 1. In a one-population game, the fraction of Hawk players is denoted s_H, the fraction of Doves is $1 - s_H$, and their expected payoffs are respectively $W_H = 0(s_H)+15(1-s_H) = 15-15s_H$ and $W_D = 3(s_H)+9(1-s_H) = 9-6s_H$. By solving $W_{H-D} = W_H - W_D = 0$, we obtain the symmetric mixed NE share $s_H^* = \frac{2}{3}$ with payoffs $W_H^* = W_D^* = 5$. Since $\frac{dW_{H-D}}{ds_H} < 0$, this equilibrium is stable under replicator dynamics or any other sign-preserving dynamics (Weibull, 1995).

Now consider the two population game. Let s_{iH} denote the shares of Hawks in population $i = 1, 2$.

Since the standard protocol here is to match only across populations, we have fitness functions $W_{1H} = 0(s_{2H}) + 15(1 - s_{2H})$, $W_{1D} = 3(s_{2H}) + 9(1 - s_{2H})$ and $W_{1H-1D} = 6 - 9s_{2H}$. Hence $W_{1H-1D} \gtreqless 0$ as $s_{2H} \lesseqgtr \frac{2}{3}$. Likewise, in the other population, $W_{2H-2D} = 6 - 9s_{1H} \gtreqless 0$ as $s_{1H} \lesseqgtr \frac{2}{3}$. Thus for any sign preserving dynamics, $\frac{s_{iH}}{dt} \gtreqless 0$ as $s_{-iH} \lesseqgtr \frac{2}{3}$. That is, Hawk is the fitter strategy (and therefore becomes more prevalent over time) in a given population whenever the fraction of Hawks in the other population is less than 2/3 (Friedman, 1991; Weibull, 1995, pp. 183-186; Friedman, 1996, pp. 7).

Figure 1 shows the resulting phase portrait. The symmetric interior NE is a saddle point and hence unstable. The two corner equilibria are stable, and specify one population playing all Hawk and the other population playing all Dove. The saddle path $s_{1H} = s_{2H}$ separates the basins of attraction of these two pure asymmetric equilibria. In the single population case, play is by definition restricted to the saddle path, so in that case the symmetric interior equilibrium is stable.

The theory so far assumes large populations in which sampling error is negligible. It applies as an approximation to finite populations, resulting in the following predictions:

Prediction 1 *In a one-population matching protocol, the average fraction of Hawk play will approximate $s^*_H = \frac{2}{3}$.*

Prediction 2 *In a two-population matching protocol with equal populstion sizes, the average fraction of Hawk play will approach $s^*_{iH} = 1$ in one population and $s^*_{-iH} = 0$ in the other, resulting in an overall average of approximately $\frac{1}{2}$.*

Finally, since the basins of attraction are separated by the diagonal $s_{1H} = s_{2H}$, we obtain

Prediction 3 *The population i that more nearly converges to $s^*_{iH} = 1$ in a 2 population matching protocol will usually be the one that initially has a larger fraction of Hawk players.*

3 Experimental Design

Each laboratory session involved $n = 10$ or 12 human subjects using a new software package called ConG (Continuous Games). Subjects earn flow payoffs based on the matrix in Table 1, using the the neutral labels A and B for the two alternative strategies instead of Hawk and Dove.

In the one population treatment, denoted T1P, each player j is matched each period with each of the $n - 1$ other players. Her instantaneous payoff when n_{-jH} of the other players are choosing A

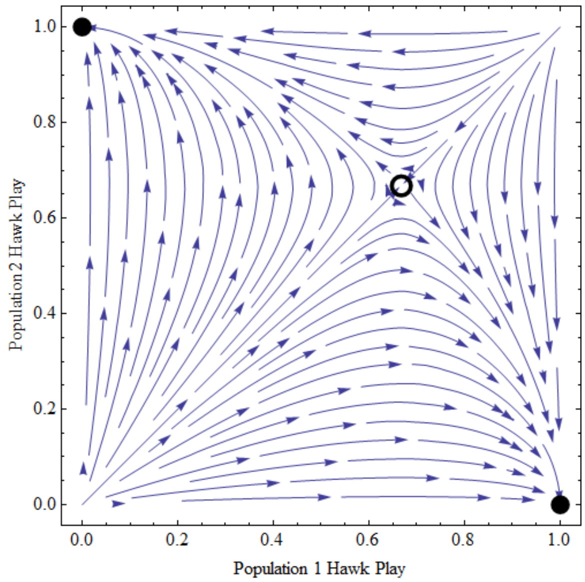

Figure 1: Phase portrait of evolutionary dynamics.

(or Hawk) is $15 - 15\frac{n_{-jH}}{n-1}$ if she plays A and is $9 - 6\frac{n_{-jH}}{n-1}$ if she plays B. In the two population treatment, denoted T2P, each subject j in population $i(j)$ is matched only with the $n/2$ subjects in the other population $-i$ and earns instantaneous payoff $15 - 15\frac{n_{-iH}}{n/2}$ for A or $9 - 6\frac{n_{-iH}}{n/2}$ for B, where n_{-iH} is the current number of Hawks in the other population $-i$.

The instantaneous payoffs are accumulated over each two minute period. For example, if all players independently spend half their time as Hawks (A), then each player earns $\frac{1}{2}(15-15\cdot\frac{1}{2})+\frac{1}{2}(9-6\frac{1}{2}) =$ 6.75 points per period.

Subjects were assigned (without their knowledge) either to population $i = 1$ or to population $i = 2$ at the beginning of the session. During the first 10 periods subjects played under the one population treatment, T1P. Then, with no announcement or other indication, we switched to the two population treatment, T2P, for the rest of the session, periods 11-20.

We ran four sessions using undergraduate subjects at the University of California, Santa Cruz. Twelve subjects participated in each of the first three sessions and ten participated in the fourth. Subjects were informed only that they would sometimes be matched with the entire group and sometimes with the other of two groups; see Appendix A for a copy of all instructions. Sessions lasted roughly 75 minutes and earnings, including a five dollar show up fee, averaged 14 dollars per hour.

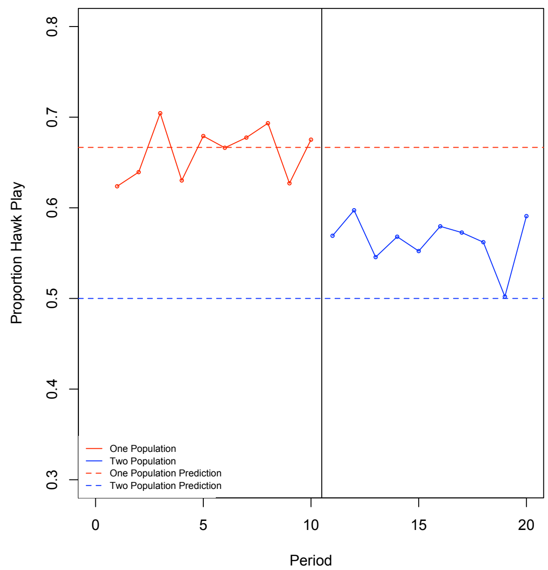

Figure 2: Mean rates of Hawk play.

3.1 Results

We begin with a qualitative overview. Figure 2 shows that, averaging within each period and over the four sessions, the fraction of total time in which players chose Hawk fluctuates narrowly around the equilibrium value of 2/3 in the T1P treatment (periods 1-10). It also shows that the Hawk fraction drops sharply in periods 11-20, when we impose the two population treatment T2P. Except for period 19, the Hawk fraction remains above the predicted level of 1/2.

Figure 3 plots the Hawk fraction separately for each of the two populations within each period and session. In T1P, the population index i is only an arbitrary labelling convention, and indeed we see in periods 1-10 no trend towards separation of the populations. In T2P, the index i becomes meaningful and here we see apparent separation in three of the four sessions. In session 3, first one population and then the other moves towards mostly Hawk, but neither prevails in the end. In sessions 1,2 and 4 the blue population plays almost exclusively Hawk by the end, while the Hawk fraction shrinks in the red population. Indeed, in Sessions 2 and 4, Hawk play in the red population drops to nearly zero by the last period.

In the T2P periods, one can see two forces at work. In the first few 20-second time intervals in most periods, the Hawk fraction is relatively large in both populations. This leads to low profits, and in the remaining 20-second time intervals the fraction usually decreases in the population with

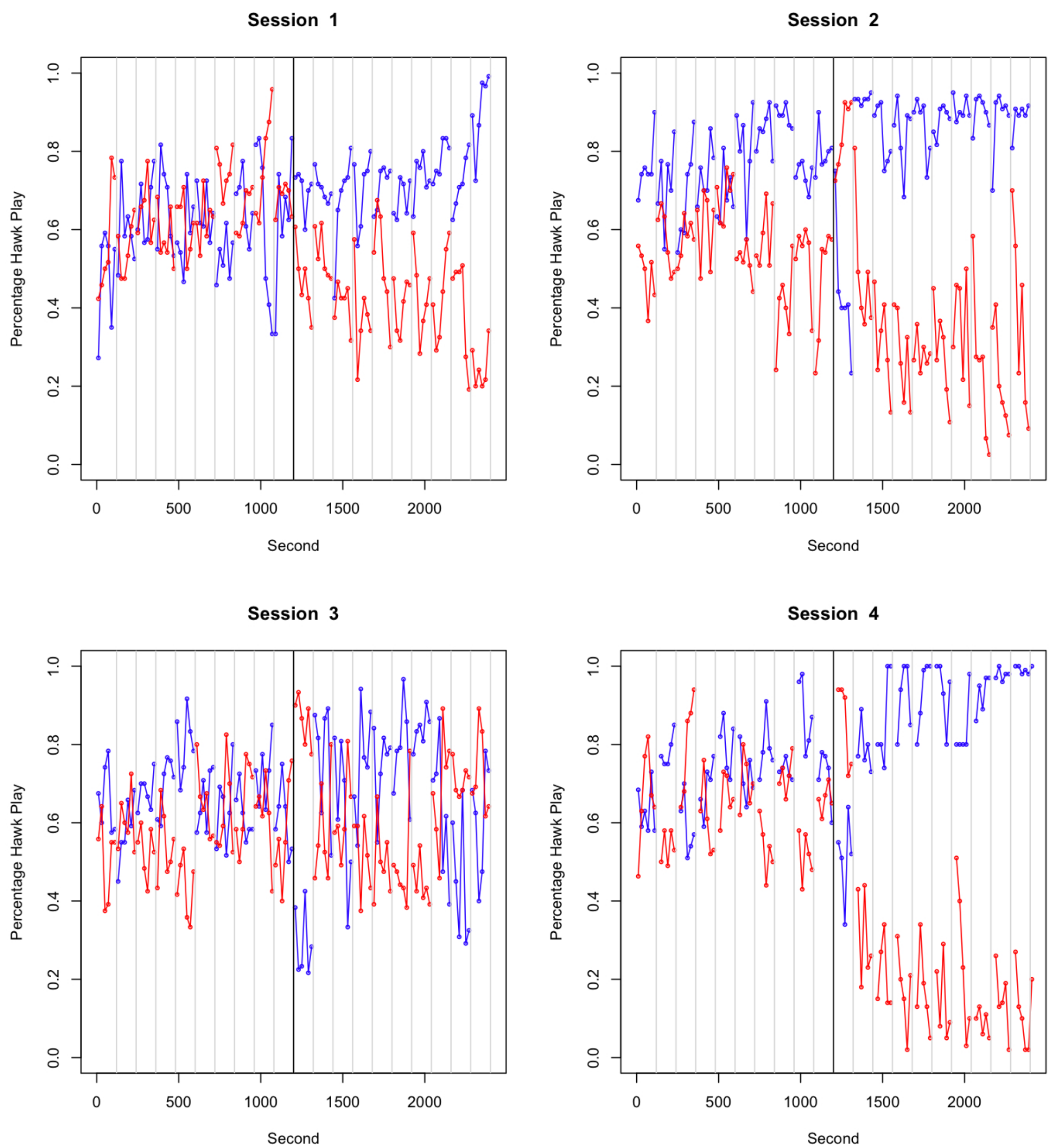

Figure 3: Hawk Play Over Time. Graphs show average fraction of Hawks in each 20 second time interval of each session among members of population 1 (in blue) and population 2 (in red). Periods are separated by vertical gray lines, and the vertical black line separates the T1P periods 1-10 from the T2P periods 11-20.

the smaller Hawk fraction. Thus within periods, we see evidence of a multilateral war of attrition that is often resolved by the end of the period.

Figure 4 shows an apparent pattern to the resolutions. It plots the Hawk fraction only in the last 20 second time interval of each period. Except in Session 3, we see that the separation trend builds over time, and that we approach an asymmetric Nash equilibrium. The same data are replotted in Figure 5, using the conventions of Figure 1 and a trianglar smoother to help detect trends. In sessions 2 and 4 we observe near convergence to a corner pure NE, while in session 1 we observe decisive movement only after period 16.

We now turn to formal tests of Predictions 1-3. We first run the random effects regression

$$s_{Hpk} = \alpha + \beta \cdot T2P_p + \psi_k + \epsilon_{pk}, \tag{1}$$

where s_{Hpk} is the observed Hawk fraction in the final 20 seconds of period p in session k, $T2P_p$ is the indicator variable for T2P treatment, ψ_k is a session level random effect and ϵ_{pk} is a normally distributed error term. The intercept α measures the Hawk fraction in T1P periods. The estimate of 0.662 (p=0.000) is almost exactly the Prediction 1 value of 0.667.

Prediction 2 calls for a decline of approximately 0.167 in the overall Hawk fraction when we switch to the 2 population protocol T2P. The $T2P$ coefficient estimate is -0.097 (p=0.000). Thus the drop is not quite as strong as predicted, but it is in the right direction and of the right order of magnitude.

Prediction 2 calls for separation in Hawk fractions across populations as well as an overall drop. To test this, we run the following random effects regression on the last 20-second data each period:

$$|s_{1Hpk} - s_{2Hpk}| = \alpha + \beta \cdot T2P_p + \psi_k + \epsilon_{pk}, \tag{2}$$

where s_{iHpk} is the average level of Hawk play for population i in session k over the last 20 seconds of period p. The intercept estimate 0.139 (p=0.0331) indicates a nearly 14 percent average absolute separation between the two populations with the T1P protocol, presumably due to sampling variance. The $T2P$ coefficient estimate 0.293 (p-0.000) indicates that the two population protocol more than triples the separation on average. Of course, Figures 3-5 show that separation increases over time, so its average probably is far larger than $0.139 + 0.293$ by the last period.

Figures 3-5 also reveal that one population tends to consistently maintain a higher Hawk fraction across T2P periods. Exact binomial tests confirm that in 3 out 4 sessions (sessions 1,2 and 4) one

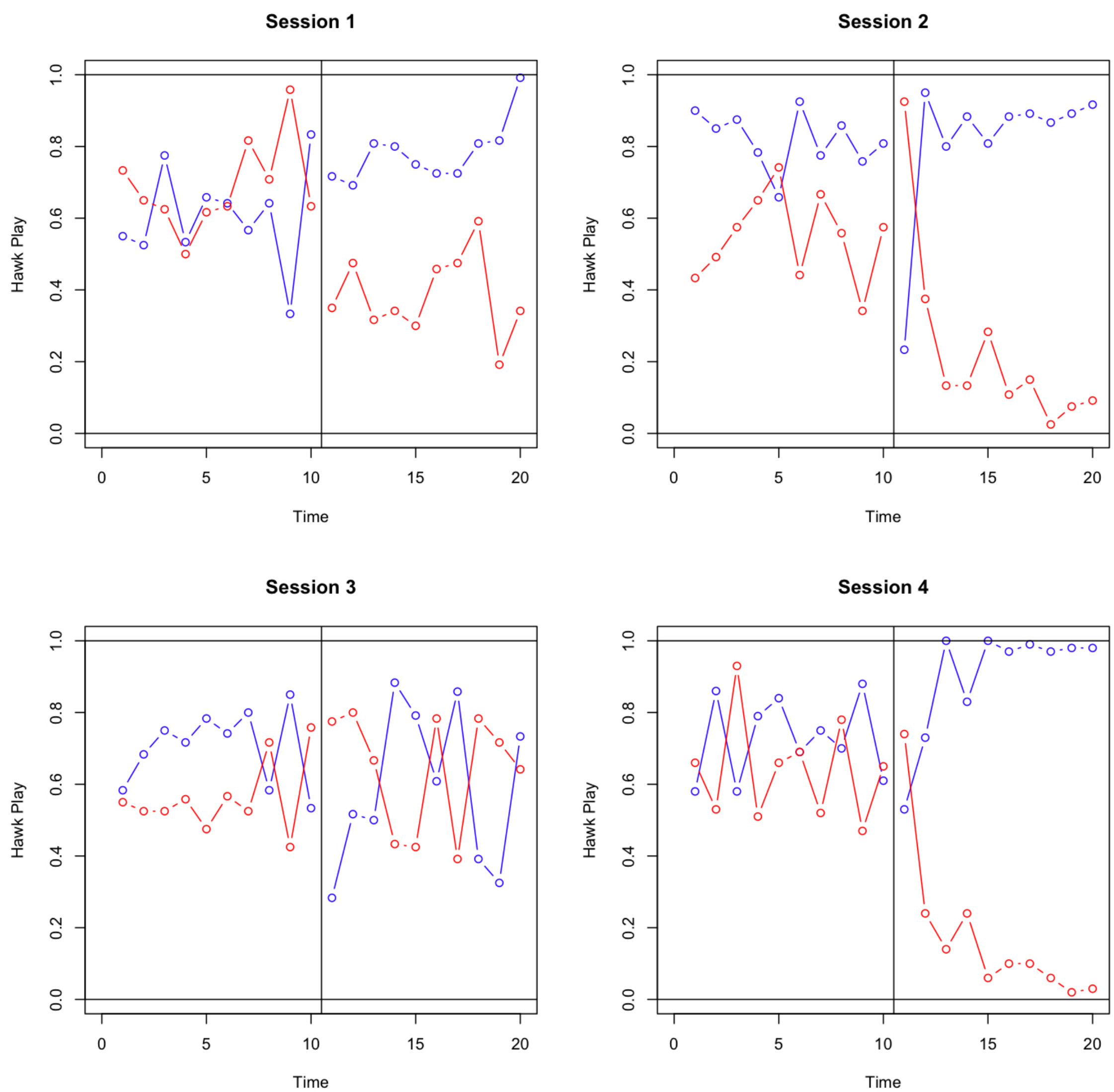

Figure 4: Hawk Play at Period End. Graphs show average fraction of Hawks in last 20 second time interval of each period among members of population 1 (in blue) and population 2 (in red). The vertical black line separates the T1P periods 1-10 from the T2P periods 11-20.

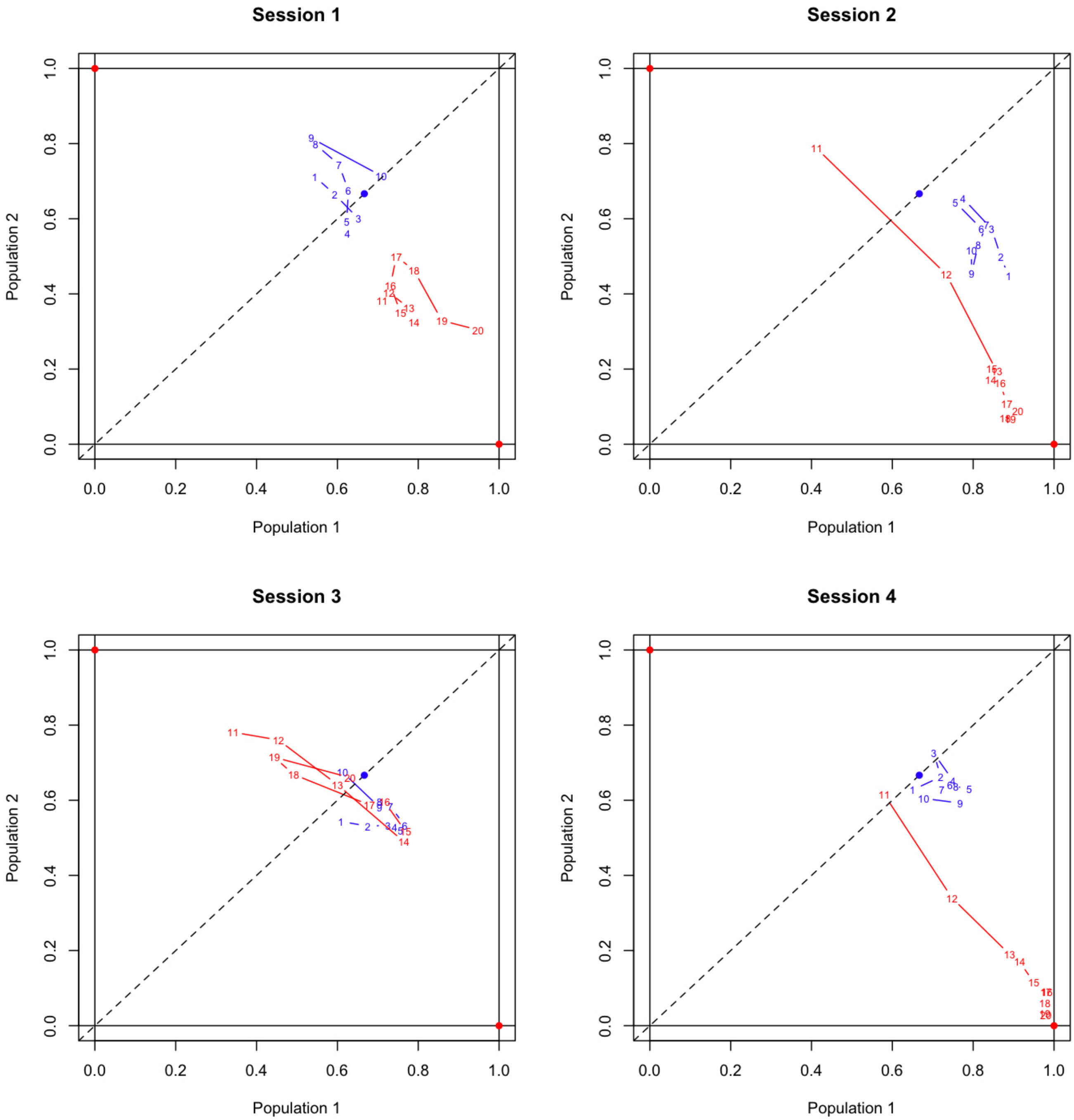

Figure 5: Smoothed period to period dynamics in $(s_{pk}^{1H}, s_{pk}^{2H})$ space using mean behavior from the final 20 seconds of each period. Blue series correspond to the one population treatment while red series correspond to two population treatments.

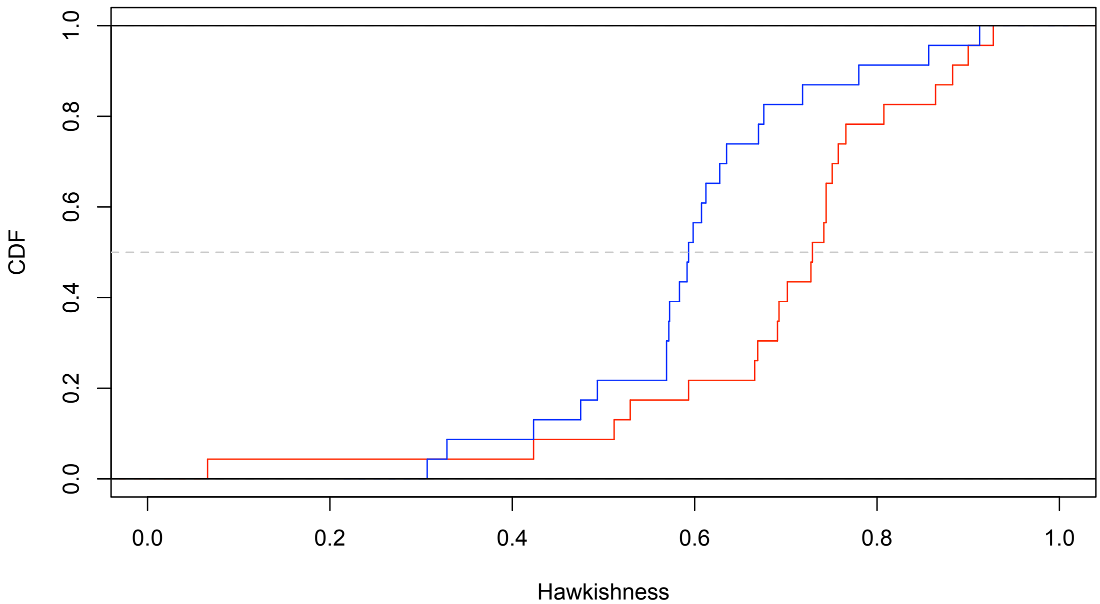

Figure 6: Cumulative distribution functions for the fraction of time each player used the Hawk strategy in periods 1-10 Players assigned to population with the larger Hawk fraction in periods 11-20 are shown in red and those assigned to the other population are shown in blue.

population is significantly more likely than the other to play Hawk across periods.

Our final test of Prediction 2 examines the Euclidean distance d_{pk} between the observed Hawk fraction in the two populations over the last 20 seconds of the period and the closer of the two pure strategy equilibria. To test the prediction that this distance is significantly smaller under the T2P treatment than under the T1P treatment, we run the following random effects regression

$$d_{pk} = \alpha + \beta \cdot T2P_p + \psi_k + \epsilon_{pk}. \tag{3}$$

The intercept estimate is 0.60 (p=0.000) and the treatment coefficient estimate is -.268 (p=0.000), indicating that the average T2P period is about half as far from a corner equilibrium as an average T1P period. Thus Prediction 2 does a good job of organizing several aspects of our data.

We turn finally to Prediction 3. Recall from Figures 4-5 that even in sessions 2 and 4, the Hawk factions jump across the saddle path in period 11 before converging towards the appropriate asymmetric pure NE in the remaining periods. Figure 3 also suggests ongoing multi-player wars of attrition, especially early on in the period. Hence Prediction 3 at best describes broad tendencies.

Our test therefore simply calculates the Hawk fraction for each subject over the T1P periods, and separates them by their population's fate in the T2P periods. Prediction 3 is that the population that more closely evolves in T2P towards $s^*_{iH} = 1$ (call it h) will be the one that had more Hawk play in the prior T1P periods. Figure 6 plots the relevant cumulative distribution functions. As predicted, the h populations indeed had more Hawkish members in periods 1-10, an impression confirmed via the standard Kolmogorov-Smirnov test (p=0.003).

To check the robustness of results obtained in the four sessions just analyzed, we ran a second set of three sessions. The second set differed from the first only in that subjects were randomly reassigned to a new population in each T2P period. Results were qualitatively similar. During T1P periods Hawk proportions in the final 20 seconds of periods averaged 0.694, again close to mixed strategy equilibrium. In the T2P treatment the overall average Hawk fraction again dropped nearly ten percent ($p = 0.000$). Moreover, within period, we observed significant separation between the populations, with the average Hawk fraction 45 percentage points larger in one population than the other. The T2P periods of the new sessions were generally noisier than in the main set of sessions, but they all showed a greater degree of separation than our outlier Session 3.

4 Discussion

Our findings are broadly consistent with all three predictions from evolutionary game theory. In the single population treatment we find the the average Hawk fraction late in the period is almost exactly the symmetric mixed NE value of 2/3. When we subsequently divide subjects into two populations, the overall rate of Hawk play drops significantly, though not quite as far as predicted. More importantly, the Hawk fractions separate substantially in the two population treatment and (in 3 of 4 sessions) clearly approach an asymmetric pure NE. Finally, we find evidence that the initial level of Hawkishness is higher in the population that converges towards all-Hawk in the two population treatment.

These findings are particularly striking given the relatively small populations we study and the extreme inequity of the asymmetric pure equilibria. Far from passively acceding to dynamics, the subjects in both populations apparently engage in multi player wars of attrition as they attempt to capture the higher payoffs for their own population. However, such attempts fail decisively for one of the populations in three of the four sessions, and the evolutionary forces predominate.

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Appendix: Experimental Instructions

Instructions (C)

Welcome. This is an experiment in the economics of decision-making. If you pay close attention to these instructions you may earn a significant amount of money that will be paid to you in cash at the end of the experiment.

The Basic Idea

	a	b
A	**0**, 0	**15**, 3
B	**3**, 15	**9**, 9

In each of several periods, you will be able to choose one of two actions: A or B. Each period you will be matched with other players. Your earnings depend on the combination of your action and the other players actions that period.

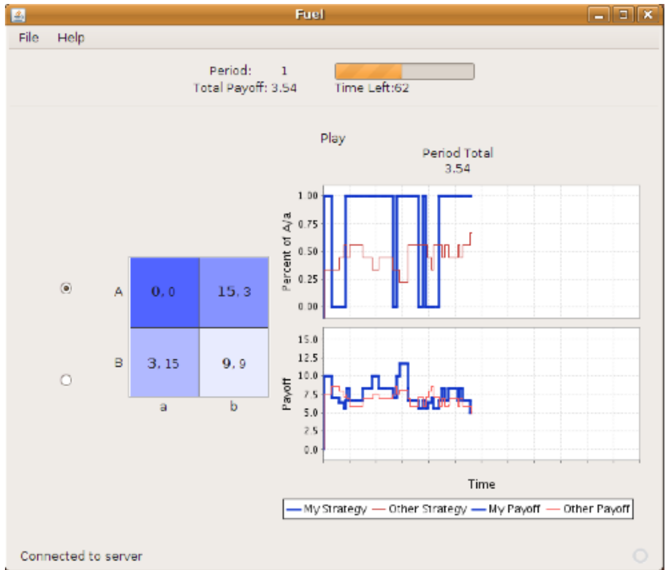
The earnings possibilities will be represented in a GAME MATRIX like the one above. Your action will determine the row of the matrix (A or B) and each other players action will determine a column of the matrix (a or b). The cell corresponding to this combination of actions will determine your EARNINGS. In each cell are two numbers. The first of the two numbers (shown in bold) is your earnings from this action combination. The second is the other player's earnings. You earn points from each match, and the points are scaled down by the number of other players.

For example, if there are 7 other players and 4 of them chose A and 3 chose B, then your payoff would be $(4*0 + 3*15)/7 = 45/7 = 6.43$ if you chose A, and it would be $(4*3 + 3*9)/7 = 39/7 = 5.57$ if you chose B.

You will not have to do this arithmetic yourself. The computer does the calculations and, as explained below, the bottom graph on your screen will display your earnings as you go along.

How to Play

There will be several periods. Each period will last 120 seconds and a counter at the top of the screen will show how much time is left. The computer randomly chooses the initial action, but



you can change your action at any time by clicking the two radio buttons or by using the up and down arrow keys. The row corresponding to your chosen action be highlighted in blue as in the figure, and the columns will be shaded in blue according to the number of players currently choosing that action. You and the other players may change your actions as often as you like each period.

The numbers in the payoff matrix are the payoffs you would earn if you maintained the same action throughout the period. For instance if you played B for the entire period and all other players played b in the example above, then you would earn 9 points and the other players also would earn 9 points each.

If you played A for the first half of the period and B for the second half while the other players played b for the entire period, your earnings would be $\frac{1}{2}(15) + \frac{1}{2}(9) = 12$, while the other players earnings would be $\frac{1}{2}(3) + \frac{1}{2}(9) = 6$. This is because you spent half of the period in the upper right corner and half in the lower right corner of the payoff matrix.

In general, your payoffs in the period will depend on how much time is spent in each of the cells on the payoff matrix. The more time you spend in any one cell, the closer the final payoffs will be to the payoffs in that cell.

To the right of the screen are two graphs showing outcomes over the course of the period. The top graph shows your action (in blue) and the average action of all other players (in red) over the period. The graph is labeled Percentage of A If this now reads 100 it means that at the moment you chose A. If it is 0 it means at that moment you chose B, and it switches between 0 and 100 as you switch actions.

The bottom graph shows your earnings over the course of the period in blue. The more area below your earnings curve, the more you have earned. In other words, the higher the blue line the more you are currently earning. The red line shows the corresponding average payoff for the other players.

Earnings

You will be paid at the end of the experiment based on the sum of point earnings throughout the experiment. These total earnings are displayed as the Total Payoff at the top of the screen.

Frequently Asked Questions

Q1. Is this some kind of psychology experiment with an agenda you haven't told us?

Answer. No. It is an economics experiment. If we do anything deceptive or don't pay you cash as described then you can complain to the campus Human Subjects Committee and we will be in serious trouble. These instructions are meant to clarify how you earn money, and our interest is in seeing how people make decisions.

Q2. If I choose the rows and the other players chooses the columns, does their screen look different than mine?

Answer. On everyone's screen, the same choices are shown as rows. For example if another player chooses row B then it shows up on your screen as a choice of column b. Of course, the payoff numbers for any choice combination are the same on both screens, but are shown in a different place.

Q3. Who am I matched with? Everyone else in the room?

Answer. Sometimes you are matched with all other players in the room. Sometimes we divide the players into two groups and we match you only with players in the other group.