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Essays on International Economics

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Javier Pablo Cravino

2013

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ABSTRACT OF THE DISSERTATION

Essays on International Economics

by

Javier Pablo Cravino

Doctor of Philosophy in Economics

University of California, Los Angeles, 2013

Professor Ariel T. Burstein, Chair

In these essays, I examine quantitatively some of the classic questions in the field of International Economics: What is the impact of international trade on consumption and productivity? How does international trade affect income inequality? How do exchange rates movements affect real output and productivity? This dissertation is composed of three chapters, each covering one of these topics.

The first chapter, based on my paper "Measured Gains from International Trade" with Ariel Burstein, revisits the measurement of welfare gains from trade liberalizations. Economists so far have measured these gains using one of two alternative approaches. A first approach uses structural models to infer unobservable welfare gains from changes in trade costs or in trade patterns. A second approach documents the empirical link between the level or the change in international trade and aggregate indicators of economic activity. This chapter connects these two approaches by studying the relationship between the theoretical welfare gains from trade and observable aggregate measures of economic activity, such as real GDP and real consumption, as constructed by national statistical agencies. Across a wide range of models, we find that measured real GDP and productivity rise in response to reductions in variable trade costs if GDP deflators capture the decline in trade costs. On the other hand, welfare gains from tariffs reductions are only reflected on real GDP if tariff revenues at constant prices rise.

The second chapter analyzes the impact of capital equipment imports on income inequality across 53 countries. The chapter is based on my paper "Importing Skill Biased Technology" with Ariel Burstein and Jonathan Vogel. Capital equipment, such as com-

puters and industrial machinery, is mostly operated by skilled workers and generally takes on routine tasks that are otherwise performed by unskilled workers. When a country imports capital equipment, it raises the relative demand of skilled versus unskilled workers, increasing income inequality. The chapter develops a tractable model of international trade in capital goods to quantify these effects. In doing so, it provides sufficient statistics and transparent formulas that will enable development practitioners to independently assess how trade in capital goods affects income inequality. We estimate that imports of equipment account for 16 percent of the income gap between skilled and unskilled workers in the median country in our sample, and for a much larger magnitude in economies that heavily rely on imported capital equipment. We also show that imports of capital equipment are essential to increase productivity and income in developing countries, both for skilled and unskilled workers, although my findings suggest these imports will disproportionately benefit the skilled segment of the population.

In the third chapter, "Exchange Rates, Aggregate Productivity and the Currency of Invoicing of International Trade", I use a novel dataset on prices and quantities from Chilean customs and a model of international prices with nominal rigidities to study how movements in nominal exchange rates can impact aggregate output and productivity. Empirically, I show that export prices are rigid in the currency in which exports are invoiced, so the relative price of firms invoicing in different currencies fluctuates with the exchange rate. I exploit this feature of the data to estimate how quantities of firms invoicing in different currencies selling in a common destination move with the exchange rate. I find this elasticity to be low, indicating that exchange rate movements have limited expenditure switching effects. I then ask how the observed variation in markups generated by exchange rate movements affects aggregate productivity by affecting the allocation of production across firms. Guided by a quantitative open economy model disciplined by some features of my data, I show that a 10 percent change in the exchange rate changes productivity in the tradable sector by 0.5 percent. Alternative parameterizations that do not account for the observed heterogeneity in invoicing predict changes in productivity at least five times smaller. This implies that taking heterogeneity into account is key for understanding the quantitative effects of exchange rates on productivity.

The dissertation of Javier Pablo Cravino is approved.

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2013

To my wife, Monica, her immense love and endless support make anything seem possible.

To my father, Daniel, the best role model I could have hoped for.

To my mother, Isabel, who supported me every step of the way.

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CHAPTER 1

Measured Aggregate Gains from International Trade

1.1 Introduction

What are the aggregate gains from reductions in the costs of international trade? There are two major approaches to address this question. A first approach uses structural models to infer unobservable welfare gains from changes in trade costs or in trade patterns (see e.g. Eaton and Kortum (2001) [16], Alvarez and Lucas (2007) [1], Arkolakis, Costinot and Rodriguez-Clare (2011) [4], Donaldson (2010) [16], and Waugh (2010) [48]). A second approach documents the empirical link between the level or the change in international trade and aggregate indicators of economic activity (see e.g. Frankel and Romer (1999) [23], Rodriguez and Rodrik (2001) [35], and Feyrer (2009a) [21], (2009b) [22]).

This paper connects these two approaches by studying, within a range of workhorse models of international trade, the relationship between theoretical welfare gains from trade and aggregate measures of economic activity, namely real GDP and real consumption as constructed by national statistical agencies. In doing so, we shed light on the following questions. Should we expect measured aggregate productivity and real GDP to rise with trade? Are aggregate measures of economic activity informative of theoretical gains from trade? Do different models have common sufficient statistics for the impact of trade on aggregate measures of economic activity?

The models that we base our analysis on include Armington models with perfect competition and exogenous specialization in production (e.g. Anderson (1979) [5], Ricardian models with endogenous specialization in production (e.g. Dornbusch, Fisher and Samuelson (1977) [15] and Eaton and Kortum (2001) [16]), and monopolistically competitive firm models with heterogeneous firms and constant markups (e.g. Krugman (1980) [30] and Melitz (2003) [33]). We consider extensions of the model with multiple

factors of production (but common factor intensities across producers) and with endogenous quality choice. We include international trade costs of the form of iceberg variable trade costs, fixed export costs (in the model with monopolistic competition), and import tariffs. In all of these models, reductions in international trade costs typically result in a rise in welfare for the representative consumer.

We calculate these models' implications of reductions in trade costs for real GDP and real consumption calculated following the procedures outlined by the Bureau of Labor Analysis to construct the National Income and Product Accounts (NIPA) in the United States. For many industries and components of GDP, comprehensive measures of physical quantities are difficult to obtain in practice. In such cases, real quantities are typically calculated by deflating current dollar measures of output or consumption with price indices — e.g. in most cases the producer price index (PPI) for output and the consumer price index (CPI) for consumption.

We first calculate the change in measured aggregate productivity and real GDP following a change in international trade costs. We show that, in response to a decline in variable trade costs, aggregate productivity and real GDP in any country rise only if trade costs are recorded in GDP and GDP deflators reflect the fall in trade costs. That is, measured productivity and real GDP rise when variable trade costs fall if the services and activities required to sell goods abroad (which include shipping services provided by the transportation industry and, more broadly, any other production, marketing, regulatory, and information costs that apply differentially to exported products)¹ are performed and recorded in the home country, as opposed to being performed abroad or not measured at all. This is because, under certain conditions, measured aggregate productivity in any given country only responds to shifts in its production possibility frontier as is, in principle, desirable for a measure of productivity.²

The response of real GDP also depends on the form of trade liberalization. In particular, changes in fixed trade costs (if these are expensed and hence not recorded in GDP) have no direct impact on GDP deflators and hence leave real GDP unchanged. Reduc-

¹Anderson and Van Wincoop (2004) [3] argue that these additional costs are at least as important as narrowly-defined transportation costs.

²As shown in Kohli (2004) [29] and Kehoe and Ruhl (2008) [27], the value of production at constant prices does not respond, to a first-order approximation, to changes in international prices that leave the domestic production possibility frontier unchanged.

tions in import tariffs increase real GDP from the expenditure side if tariff revenues at constant prices rise (which requires an increase in the physical quantity of imports).

Next, we compare changes in real GDP and in real consumption (in our baseline model, changes in consumption expenditures are equal to changes in total absorption). Real GDP and real consumption can differ even when trade is balanced due to movements in the price of exports relative to the price of imports (the terms of trade). We show, however, that if trade is balanced in each country, changes in world real GDP are equal, up to a first-order approximation, to changes in world real consumption (where each country is weighted by its current-dollar GDP). The equality holds at the world aggregate level because terms of trade improvements in one country are associated to terms of trade worsenings in another country. While changes in real GDP country-by-country depend critically on the patterns of specialization in the production of trade services, the equality between changes in world real GDP and world consumption does not.

Perhaps more importantly, we compare changes in measured real consumption with changes in theoretical (or welfare-based) consumption. Differences between theoretical and measured consumption arise from differences between consumption deflators and the theoretical price index. Consumption deflators in our model differ from the welfare-based price index in three respects. First, consumption deflators do not fully take into account substitution in consumption from changes in relative prices. Second, they do not take into account changes in the mass of consumed goods which, in the presence of love for variety, matter for theoretical consumption. Third, they do not take into account improvements in product quality if quality changes are measured inaccurately in consumption deflators. The report by the Boskin Commission (1996) [8] examines in detail how these and other biases in the CPI lead to an understatement of real consumption growth in the U.S.

We show how, under certain conditions, these multiple biases in consumption deflators may not result in a mismeasurement of theoretical consumption. If the set of consumed goods and product quality are fixed (so that the second and third sources of the bias are absent), then in response to any type of trade cost movement, changes in theoretical consumption are bounded between measured real consumption calculated using initial base-year prices and real consumption using end base-year prices. This implies, as is well-known (see e.g. Hausman (2003) [26]), that the substitution bias is of second order:

in each country, changes in real consumption equal changes in theoretical consumption, up to a first-order approximation.

When the set of consumed goods and product quality are not fixed, we establish the following result. In response to changes in variable trade costs, with trade balance in each country, changes in world real consumption equal changes in world theoretical consumption (defined analogously to world real consumption and world real GDP), up to a first-order approximation. That is, while changes in theoretical consumption and real consumption may differ country-by-country, these differences cancel-out when adding them across countries in the world. Under stronger assumptions (i.e. Pareto distribution of entering firms' productivity and fixed export costs paid in the destination market, as in Eaton, Kortum, and Kramarz (2010) [20]), the equality between measured consumption and theoretical consumption holds country-by-country, up to a first-order approximation, as in the model with a fixed set of consumed goods. We also show that, in response to large reductions in variable trade costs (for which we must solve the model numerically), the elasticities of theoretical consumption and real consumption can be quite close, country-by-country (and hence also at the world level), independently of whether fixed export costs are incurred domestically or abroad.

Finally, we ask whether the different models that we consider give rise to different sufficient statistics for measured gains from trade. We consider this question separately for our measures of real GDP and real consumption. Across our range of models, we obtain a common expression for the change in real GDP as an average of changes in variable trade costs weighted by export shares of continuing exporting producers. Conditional on this direct impact of changes in trade costs, reallocation of production from less productive to more productive producers, entry and exit into production and exporting, and changes in the mass of producers, have no additional effects on changes in measured aggregate productivity and real GDP.

We also show that across our range of models, changes in world real consumption and world real GDP are equal in magnitude, up to a first-order approximation, for given trade shares and for given changes in variable trade costs. This first-order equivalence in measured gains from trade in consumption across seemingly different models does not reflect an inadequacy of the aggregate measures of real consumption. Instead, this

equivalence in measured gains from trade is consistent with the underlying equivalence in the welfare implications of these models under some restrictions, as demonstrated by Arkolakis, Costinot and Rodriguez-Clare (2011) [4] and Atkeson and Burstein (2010) [6]. Note, however, that changes in fixed trade costs or foreign country size that increase trade shares (and also welfare, under the assumptions of Arkolakis, Costinot and Rodriguez-Clare (2011) [4]) may not result in measured gains from trade.

Our paper is related to a recent paper by Bajona, Gibson, Kehoe, and Ruhl (2010) [7], who ask whether the increase in welfare following a trade liberalization translates into an increase in real GDP as measured in NIPA. They conclude, as summarized in Kehoe and Ruhl (2010) [28], that “...standard trade models do not imply that opening to trade increases productivity or real GDP, but that it increases welfare”. The two main differences of our paper relative to Bajona et. al. (2010) [7] are as follows. First, while Bajona et. al. (2010) [7] focus on the implications of trade liberalization on real GDP, we also study the effects on real consumption and provide conditions under which the response of real consumption to changes in trade costs equals that of theoretical consumption. Second, Bajona et. al. (2010) [7] focus on cases in which price indices do not directly reflect changes in international trade costs, either because trade costs are fully incurred abroad or because countries are in autarky before the trade liberalization (in which case price indices of exported goods, as measured by the BLS, are not well defined since there are no continuing exported goods). In the class of models considered in both papers, this implies that measured real GDP is unchanged with trade liberalization (abstracting from changes in real tariff revenues). We show, however, that starting with positive trade levels, any reduction in trade costs that is reflected in price indices *does* result in an increase in real GDP.

Our work is also related to Feenstra (1994) [18] and Broda and Weinstein (2006) [9], who quantify the mismeasured growth in real consumption in the U.S due to the rise in the number of imported varieties that is not accounted for in the CPI, without taking a stand on the source of the growth in the number of imported varieties.³ We show in

³Relatedly, Feenstra, Reinsdorf, and Slaughter (2008) [20] and Neiman and Gopinath (2011) [24] argue that if export and import price indices are mismeasured (among other reasons, due to changes in import variety), changes in tariffs or in the terms of trade can result in changes in measured aggregate productivity.

our models that, in response to a reduction in variable trade costs that results in a rise in the number of imported varieties, to a first-order approximation there is no bias in consumption deflators at the world aggregate level or, under stronger conditions, country-by-country, when simultaneously taking into account in general equilibrium other biases in the price indices. Hence, any underestimate of real consumption growth stemming from an increase in the mass of imported varieties that is not captured in the import price index is offset by the other biases in the CPI. Finally, our paper is related to the work of Pavcnik (2002) [34] and others, that construct measures of aggregate productivity as weighted averages of productivity estimates across producers. While those measures of aggregate productivity may reflect the reallocation of production towards more productive producers induced by trade liberalization, we argue, using a range of models of trade and firm heterogeneity as a laboratory, that measures of aggregate productivity constructed from NIPA do not capture this reallocation.

The paper is organized as follows. Section 2 presents an overview of the measurement procedures that we use in our models. Section 3 presents our baseline Armington model with exogenous specialization in the set of goods that are produced and traded in each country. Section 4 derives our basic results on measured real GDP, real consumption, and theoretical consumption in the Armington model. Section 5 shows that these basic results apply in a Ricardian model with endogenous specialization and perfect competition. Section 6 extends the basic results to the version of the model with endogenous specialization and monopolistic competition. Section 7 considers two additional extensions: endogenous quality choice and multiple factors of production. Section 8 concludes. Various proofs and details are relegated to the Appendix.

1.2 Aggregate Measurement: Overview

In this section we provide a brief overview of the procedures that we use to calculate changes in aggregate quantities. We follow as closely as possible the procedures outlined by the Bureau of Economic Analysis in the United States to construct the National Income and Product Accounts (NIPA).⁴

⁴See, e.g. Concepts and Methods of the U.S. National Income and Product Accounts (2009) [45]. The procedures that we consider are broadly consistent with the recommendations by the United Nations in

To calculate aggregate measures of output such as real GDP, or aggregate measures of expenditures such as real consumption, we use a Fisher index, which is a geometric average of a Laspeyres and a Paasche quantity index. For example, real GDP in period t relative to period $t - 1$ is given by

$$\frac{RGDP_t}{RGDP_{t-1}} = \left(\frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}} \right)^{0.5} \times \left(\frac{\sum p_tq_t}{\sum p_tq_{t-1}} \right)^{0.5}, \quad (1.1)$$

where p_t and q_t denote prices and quantities in period t of the detailed components of GDP, and where the sum is calculated across all of these components. The terms $p_{t-1}q_t$ and p_tq_{t-1} represent "real" quantities of any given GDP component evaluated at constant prices. The first term in expression (1.1) is a Laspeyres quantity index (based on $t - 1$ prices), while the second term is a Paasche quantity index (based on t prices).⁵ Real GDP in period T relative to period 0 is given by

$$\frac{RGDP_T}{RGDP_0} = \prod_{t=1}^T \frac{RGDP_t}{RGDP_{t-1}}. \quad (1.2)$$

The detailed components of GDP in expression (1.1) can be industries, sectors, or groups of narrowly defined goods that jointly conform aggregate GDP or other aggregate measures of output and expenditures. While estimates of the current-dollar value of production, p_tq_t , are typically available for each of these individual components, data on physical quantities, q_t , are often not.

For those components of GDP for which data on physical output are available, real quantities are computed using either the direct valuation method (sum of quantities evaluated at constant prices) or the quantity extrapolation method (using a quantity indicator that approximates the movements of the component series). For those components of GDP for which estimates of physical quantities are not available, real quantities are estimated using the deflation method, dividing current-dollar values by appropriate price indices.⁶ In particular, for any component of GDP, $p_{t-1}q_t = (p_tq_t) / (P_t/P_{t-1})$ and

their System of National Accounts.

⁵The implicit GDP deflator is calculated as the ratio of current-dollar GDP to real GDP, $(\sum p_tq_t / \sum p_{t-1}q_{t-1}) / (RGDP_t / RGDP_{t-1})$, which is equal to a geometric average of a Laspeyres and a Paasche *price* index.

⁶The direct valuation method is used, for example, to calculate real output of autos and light trucks, while quantity extrapolation is used to calculate real output of housing and utilities services. The

$p_t q_{t-1} = (p_{t-1} q_{t-1}) \times (\mathcal{P}_t / \mathcal{P}_{t-1})$, where $\mathcal{P}_t / \mathcal{P}_{t-1}$ denotes the change in the price index between periods $t - 1$ and t . In our baseline calculations, we compute aggregate quantities using the deflation method.

To calculate real GDP from the production side using the deflation method, we deflate the current-dollar value added of production (including the value added of the activities performed at home to sell goods internationally) using the producer price index (PPI) as a deflator.⁷ The change in the PPI between periods $t - 1$ and t is a weighted average of price changes between these two periods across goods and services that are produced domestically to sell at home or to export abroad.⁸

We consider two alternative deflation procedures. The first procedure deflates the total value of production using a single aggregate price index. The second procedure deflates the value of output bound for each destination using a destination-specific price index. We show that, using disaggregated deflators by destination country, real GDP is equal to that obtained using the direct valuation method based on data on physical quantities of each commodity.

Export prices in the PPI and in the export price index (EPI) are typically measured at fob (i.e. free-on-board) values, and hence exclude shipping services incurred abroad. A critical assumption determining the impact of changes in international trade costs on measured real GDP is whether changes in measured prices in the PPI reflect, at least partly, these changes in trade costs. In addition to shipping costs (that are included in the transportation industry), international trade costs include production and marketing costs that apply differentially to exported goods, information costs, costs associated with

majority of the other subcomponents of GDP are calculated using the deflation method since physical output is not recorded across producers (see "Summary of NIPA Methodologies", p.12 for a description of the method used to estimate each subcomponent of GDP).

⁷This is the procedure used in the GDP by industry accounts published by the BEA. When intermediate inputs are used in production, real value added is calculated using the double deflation method. This consists of first deflating gross output and inputs separately (using their respective PPIs), and then computing real value added as the difference between real gross output and real intermediate inputs.

⁸To construct the PPI, the Bureau of Labor Statistics (BLS) collects prices for a sample of items that can be priced consistently through time. Price indices are then constructed by averaging price changes of individual items weighted by the value of production in some base year. The set of sampled items and the weights are updated every few years (between 5 and 7 years for the typical good in the PPI). Price changes from product replacements tend to be dropped from the index, which is equivalent to attributing to discontinued goods the rate of change in the overall price index. For more details on the construction of producer price indices and international price indices in the US, see Chapters 14 and 15 of the BLS Handbook of Methods.

the use of different currencies, contract enforcement costs, legal regulatory costs, and other time costs associated to international trade (see e.g. Anderson and Van Wincoop (2004) [3]). To understand the implications of the nature of trade costs on aggregate measurement, we consider two alternative specifications. In our baseline specification, we assume that the activities required to sell goods abroad are performed in the home country, and hence changes in the variable component of these trade costs are reflected in the home PPI. In an alternative specification, we assume that all export costs are incurred in foreign countries, in which case changes in trade costs are not reflected in the PPI.

We also calculate GDP from the expenditure side, defined as current-dollar absorption (which in our baseline model is equal to consumption), plus exports less imports. Real consumption is calculated analogously to real GDP (using expressions 1.1 and 3.5), but deflating each component of nominal consumption (when physical quantities are not available) by its consumer price index (CPI) instead of the PPI. The change in the CPI is a weighted average of consumer price changes of domestic and imported goods consumed in both time periods.⁹

In the presence of import tariffs, current-dollar GDP from the expenditure side (defined as the sum of final expenditures including tariffs) is not equal to current-dollar GDP from the production side (defined as the sum of firm value added excluding tariffs). In order to reconcile estimates of GDP from the production and expenditure sides, the BEA adds import taxes to factor payments when computing value added by industry.¹⁰ To be consistent with this procedure, in the model with tariffs we calculate real GDP from the expenditure side. In deflating consumption expenditures, the CPI is constructed using prices inclusive of tariffs. In deflating imports, the import price index (IPI) is constructed using prices exclusive of import tariffs.

⁹See *"Updated Summary of NIPA Methodologies"*, for details on the deflator used in each expenditure component of GDP. See McCully, Moyer, and Stewart (2007) [31] for a detailed comparison of the CPI and the implicit deflator for personal consumption (where the latter is constructed as the ratio of nominal and real consumption). See Feenstra, Heston, Timmer, and Deng (2009) [19] for a detailed discussion of the relation between real GDP from the production side and real GDP from the expenditure side as measured in the Penn World Tables.

¹⁰In particular, in the "Gross Domestic Product by Industry Accounts" computed by the BEA, value added is defined as the sum of: "Compensation of employees", "Taxes on production and imports less subsidies" and "Gross operating surplus". For a detailed description of the transactions that are included in value added, see "Concepts and Methods of the U.C. Input-Output Accounts", Chapter 6, under "Value-added transactions".

1.3 Model with Exogenous Specialization and Perfect Competition

In this section we present an Armington version of our model with exogenous specialization and perfect competition. The extensions of the model that follow build upon this basic setup.

The world economy is composed of I countries. The utility of the representative consumer in country n is

$$U_n = \sum_{t=0}^{\infty} \beta^t u(C_{nt}), \quad \beta \leq 1,$$

where C_{nt} denotes *theoretical* consumption of the final good at time t , given by

$$C_{nt} = \left[\int_{\Omega_{nt}} q_{nt}(\omega)^{\frac{\rho-1}{\rho}} d\omega \right]^{\frac{\rho}{\rho-1}}. \quad (1.3)$$

Here, $q_{nt}(\omega)$ denotes the consumption of good ω and Ω_{nt} denotes the set of available differentiated goods in country n . The parameter ρ denotes the elasticity of substitution across varieties. In the model with monopolistic competition below we assume $\rho > 1$. Demand for each good is $q_{nt}(\omega) = [p_{nt}(\omega) / P_{nt}]^{-\rho} C_{nt}$, where $p_{nt}(\omega)$ denotes the consumer price of good ω in country n , and $P_{nt} = \left[\int_{\Omega_{nt}} p_{nt}(\omega)^{1-\rho} d\omega \right]^{\frac{1}{1-\rho}}$ is the welfare-based price index in country n . We assume that consumption of the final good C_{nt} and the welfare-based price index P_{nt} cannot be directly observable (or similarly, that the final good is not a physically traded commodity). If C_{nt} and P_{nt} were directly observable, then measuring the gains from trade would be straightforward.

Each producer specializes in the production of a single differentiated good. Production uses labor according to the production function $y = zl$, where y and l denote output and labor of a producer with productivity z (multiple inputs are introduced in Section 7). We denote by $M_{it}(z)$ the distribution of producers, indicating the mass of producers with productivity z in country i at time t . Given the symmetry of goods in the production function of the final good (1.3), we interchangeably index goods by ω , or by their productivity z and source country i . For example, $q_{int}(z)$ and $p_{int}(z)$ denote the consumption quantity and price, respectively, in country n of good produced by z producers in country i . We assume that all prices are already expressed in a common currency (which we refer

to as dollars).

Goods can be internationally traded subject to a technology described below. We denote by Ω_{int} the set of producers (indexed by their productivity) from country i that sell a *positive* quantity to country n at time t . In the absence of international trade between countries i and n at time t , the set Ω_{int} is empty.

In the *model with exogenous specialization*, we assume that the distribution of producers, $M_{it}(z)$, is exogenously given and constant over time. We also assume that the set of goods that are internationally traded, Ω_{int} , is exogenously given and that, as long as there is any trade between countries i and n , it is constant over time. We do not make assumptions on how the set of goods Ω_{int} varies across destinations, hence not all goods sold domestically need to be exported, and vice-versa. For example, only goods with high productivity z might be traded. The case of $\Omega_{int} = \Omega_{iit}$ corresponds to the Armington model in which all goods are internationally traded (unless countries are in autarky).

Goods can be shipped across countries subject to iceberg variable international trade costs. In our baseline specification, we assume that international trade costs are incurred in each source country, as is typically assumed in the literature.¹¹ In particular, each unit of a good produced in country i with productivity z shipped to country n at time t requires $(\tau_{int} - 1)/z$ units of labor from country i , where $\tau_{int} \geq 1$ and $\tau_{iit} = 1$. International trade services could be provided by the same producer of the good, or by some third-party intermediary.¹²

Summing-up production and shipping costs, the total amount of country i labor required to deliver a unit of country i 's good in country n is τ_{int}/z . Equivalently, this technology transforms 1 unit of a good produced in country i into $1/\tau_{int}$ unit of the good for consumption in country n . Country i 's resource constraint is

$$\sum_n \int_{\Omega_{int}} \tau_{int} q_{int} / z dM_{it} = \bar{L}_i,$$

¹¹While in this formulation we assume that trade costs use factors of production in the exporting country, we can instead assume that they use factors from the importing country (or from both). This would complicate the notation without changing substantially the results.

¹²The assumption of iceberg variable international trade costs implies that producers that are more efficient at production are also more efficient at selling goods abroad. Consider an alternative formulation of the model in which goods vary by quality (as discussed in Section 7) instead of productivity. If production of higher quality goods entail higher marginal costs, the assumption of iceberg trade costs implies that higher quality goods are more expensive to sell abroad.

where \bar{L}_i denotes the labor supply in country i , integrals are evaluated with respect to z , and the dependence of q_{int} on the argument z is omitted.

In the model with perfect competition, producer prices for goods manufactured in country i and sold in country n equal $\bar{p}_{int} = W_{it}/z$, where W_{it} denotes the wage in country i . Prices for the services to sell goods from country i to country n equal $\bar{p}_{int}^s = (\tau_{int} - 1)W_{it}/z$. Consumer prices in country n equal $p_{int} = \bar{p}_{int} + \bar{p}_{int}^s = \tau_{int}W_{it}/z$. Consumption expenditures at final prices in country n are given by $E_{nt} = P_{nt}C_{nt} = \sum_n \int_{\Omega_{int}} p_{int}q_{int}dM_{it}$. GDP in current dollars from the production side (the sum of value added across all producers), is equal to GDP from the income side (total wage payments plus profits), and to GDP from the expenditure side (consumption expenditures plus exports less imports). This three-way equivalence can be expressed as:

$$\begin{aligned} GDP_{it} &= \sum_n \int_{\Omega_{int}} p_{int}q_{int}dM_{it} = W_{it}\bar{L}_i + \Pi_{it} \\ &= E_{it} + \sum_{n \neq i} \int_{\Omega_{int}} p_{int}q_{int}dM_{it} - \sum_{n \neq i} \int_{\Omega_{nit}} p_{nit}q_{nit}dM_{nt}, \end{aligned} \tag{1.4}$$

The variable Π_{it} denotes aggregate profits, which equal zero under perfect competition and constant returns to scale.

We denote by λ_{int} the share of country i 's GDP accounted for by production sold to country n ,

$$\lambda_{int} = \frac{GDP_{int}}{GDP_{it}}, \tag{1.5}$$

where $GDP_{int} = \int_{\Omega_{int}} p_{int}q_{int}dM_{it}$. Note that $1 - \lambda_{iit}$ indicates the share of total exports in country i 's GDP.

1.4 Results: Exogenous Specialization and Perfect Competition

In this section, we present our results in the basic model with exogenous specialization and perfect competition. We first calculate changes in real GDP in response to changes in variable trade costs. We then show how changes in real GDP vary if we assume that the production of international trade services is specialized in one country. We then compare changes in real consumption and theoretical consumption, and next compare changes in

world real GDP and world real consumption. Finally, we calculate the response of real GDP and consumption to changes in tariffs. We conclude this section by summarizing the results.

Real GDP

We first construct real GDP from the production side. In order to apply expressions (1.1) and (3.5), we must specify how goods are grouped into components of GDP. We consider two cases. First, we aggregate production by all producers to all destinations into a single component, and construct real quantities by deflating current-dollar GDP using a single, aggregate deflator. Second, we decompose total production by destination country, and calculate real quantities by deflating destination-specific production values using destination-specific price indices.¹³ We show that real GDP under the second case is equal to real GDP constructed using the direct valuation method in which data on physical quantities and prices of individual producers (i.e. q_{int} , p_{int}) is used.

Real GDP using aggregate deflators: We construct real quantities by deflating the total current-dollar value of production with the aggregate PPI. The PPI is a weighted average of changes in producer prices of continuing goods, based on production weights in period t_0 . We do not make assumptions on what the base-year t_0 is or how frequently it is updated with the exit of existing products or the entry of new products. The PPI in country i in period t relative to period $t - 1$ is given by¹⁴

$$\frac{PPI_{it}}{PPI_{it-1}} = \frac{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}} \right) dM_{it_0}}{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}} = \sum_n \bar{\lambda}_{int} \frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}, \quad (1.6)$$

where $\Omega_{int}^c = \Omega_{int_0} \cap \Omega_{int-1} \cap \Omega_{int}$ is the set of goods sold from country i to country n

¹³In both cases, in defining these detailed components of GDP, we are implicitly assuming that in the model there is a representative sector or industry composed of differentiated goods which aggregate according to (1.3). Extending the model to allow for heterogeneous industries or sectors, aggregated into the final good with an outer CES technology, is straightforward at the expense of extra notation, and does not substantially alter our results.

¹⁴Here we are assuming that producer prices in the PPI are the sum of manufacturing and shipping prices, \bar{p}_{int} and \bar{p}_{int}^s respectively. Alternatively, we could assume that producer prices and shipping prices are entered separately instead of summed into the PPI (because these activities are performed by distinct producers or industries). The PPIs under both assumptions are equivalent up to a first-order approximation.

with positive sales at time t_0 , $t - 1$ and t , and $\bar{\lambda}_{int}$ is the share of country i 's revenues to country n at time t_0 of these continuing goods,

$$\bar{\lambda}_{int} = \frac{\int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}}{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}}. \quad (1.7)$$

In deriving (1.6), we have used the fact that, with iceberg variable trade costs, the percentage change in prices is independent of productivity z .

Note that, if countries i, n do not trade at time t_0 , $t - 1$ or t , then $\Omega_{int}^c = \emptyset$ and the PPI excludes price changes from this pair of countries. Hence, if a country is in autarky at time t_0 , $t - 1$ or t then the PPI only takes into account changes in domestic prices.

Real GDP in period t relative to period $t - 1$, using expression (1.1) with a single aggregate component, is given by

$$\begin{aligned} \frac{RGDP_{it}}{RGDP_{it-1}} &= \left(\frac{GDP_{it}/(PPI_{it}/PPI_{it-1})}{GDP_{it-1}} \right)^{0.5} \left(\frac{GDP_{it}}{GDP_{it-1} \times (PPI_{it}/PPI_{it-1})} \right)^{0.5} \\ &= \frac{1}{\sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}}. \end{aligned} \quad (1.8)$$

Note from (1.8) that, using a single aggregate deflator, the Laspeyres and the Paasche quantity indices between periods $t - 1$ and t are equal.

From expression (1.8), we can see that if the share of exports in GDP is positive at times t_0 , $t - 1$ and t (i.e. $\sum_{n \neq i} \bar{\lambda}_{int} > 0$) and variable trade costs in country i fall between time $t - 1$ and time t (i.e. $\tau_{int} \leq \tau_{int-1}$ for $n \neq i$ with at least one strict inequality), then real GDP rises.

If trade costs are unchanged between any two consecutive periods, $\tau_{int} = \tau_{int-1}$, then real GDP remains unchanged. Therefore, if trade costs change permanently between $t = 0$ and $t = 1$, then chained real GDP in any period $T \geq 1$ relative to period $t = 0$ (using expression 3.5) is given by

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{RGDP_{i1}}{RGDP_{i0}}. \quad (1.9)$$

Intuitively, a reduction in variable trade costs entails an improvement of domestic technologies, which lowers producer prices relative to the wage, and increases real GDP.

This rise in real GDP shows up as a rise in aggregate productivity. If the PPI does not take into account changes in trade costs (or if a country is initially in autarky), then $PPI_{int} = W_{it}/W_{it_0}$, and $RGDP_t/RGDP_{t-1} = 1$.

Note that any reallocation in production towards more productive producers (due to, for example, a higher productivity of exporters relative to non-exporters) does not result in larger changes in measured aggregate productivity. To understand this implication of the model, we can rewrite the ratio of real GDP in period t to relative to period $t - 1$, using (1.4) and (1.8), as

$$\frac{RGDP_{it}}{RGDP_{it-1}} = \frac{\sum_n \int_{\Omega_{int}} \frac{l_{int}}{L_{it}} \times \frac{p_{int} q_{int}}{l_{int}} dM_{it}}{\sum_n \int_{\Omega_{int-1}} \frac{l_{int-1}}{L_{it-1}} \times \frac{p_{int-1} q_{int-1}}{l_{int-1}} dM_{it-1}} \frac{L_{it}}{L_{it-1}} \frac{1}{PPI_{it}/PPI_{it-1}}, \quad (1.10)$$

where $l_{int}(z)$ denotes production labor used by country i producers with productivity z to sell in country n , and L_{it} denotes the aggregate quantity of labor used for production in country i (equal to \bar{L}_i in this model). Note that value added per worker by individual producers, $p_{int}(z) q_{int}(z) / l_{int}(z)$, is equal to the wage, W_{it} , for all producers independent of their productivity z . Using $\sum_n \int_{\Omega_{int}} \frac{l_{int}}{L_{it}} dM_{it} = 1$ and (1.6) we obtain expression (1.8). Therefore, any reallocation of labor towards more productive producers does not result in any further increase of aggregate productivity beyond the direct effect from a reduction in variable trade costs.¹⁵

Real GDP using disaggregated deflators: We now compute real quantities by deflating destination-specific production values using destination-specific PPIs. The PPI in period t relative to period $t - 1$ for goods produced in country i and shipped to country n is

$$\frac{PPI_{int}}{PPI_{int-1}} = \frac{\int_{\Omega_{int}^c} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}} \right) dM_{it_0}}{\int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}} = \frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}. \quad (1.11)$$

Here we used the fact that percentage changes in producer prices are equal for all goods bound to a given destination. The disaggregated deflator PPI_{int}/PPI_{int-1} is well defined only when the set of continuing goods is non-empty.

Real GDP in period t relative to period $t - 1$, using equation (1.1) with destination-

¹⁵Note that if the PPI were calculated as a change in average prices (instead of an average change in prices), then reallocation of production towards more productive producers would result in a larger decline in the PPI and a higher increase in real GDP.

specific GDP components, is given by

$$\begin{aligned} \frac{RGDP_{it}}{RGDP_{it-1}} &= \left(\frac{\sum_n \frac{GDP_{int}}{PPI_{int}/PPI_{int-1}}}{\sum_n GDP_{int-1}} \right)^{0.5} \left(\frac{\sum_n GDP_{int}}{\sum_n GDP_{int-1} \times PPI_{int}/PPI_{int-1}} \right)^{0.5} \\ &= \left(\frac{\sum_n \lambda_{int} \frac{\tau_{int-1}}{\tau_{int}}}{\sum_n \lambda_{int-1} \frac{\tau_{int}}{\tau_{int-1}}} \right)^{0.5}. \end{aligned} \quad (1.12)$$

Note that, in contrast to the measures of real GDP based on aggregate deflators, the Laspeyres and the Paasche quantity indices of real GDP are not equal when we use disaggregated deflators. From expression (1.12), if trade costs fall between time $t - 1$ and t , then real GDP rises. If trade costs change permanently between $t = 0$ and $t = 1$, then chained real GDP in any period $T \geq 1$ relative to period $t = 0$, is equal to $RGDP_{i1}/RGDP_{i0}$, as in expression (1.9).

The expressions for changes in real GDP based on aggregated and disaggregated deflators, given by (1.8) and (1.12), differ in terms of the base-year in which trade shares are calculated. However, up to a first-order approximation (i.e. around $\tau_{int}/\tau_{int-1} \simeq 1$), the two measures of changes in real GDP are equal and given by

$$d \log RGDP_{it} = d \log W_{it} - \sum_n \lambda_{int} d \log PPI_{int} = - \sum_n \lambda_{int} d \log \tau_{int}. \quad (1.13)$$

Note that we can re-write the change in real GDP based on disaggregated deflators in the first line of expression (1.12) as

$$\frac{RGDP_{it}}{RGDP_{it-1}} = \left(\frac{\sum_n \int_{\Omega_{int}} p_{int-1} q_{int} dM_{it}}{\sum_n \int_{\Omega_{int-1}} p_{int-1} q_{int-1} dM_{it-1}} \right)^{0.5} \left(\frac{\sum_n \int_{\Omega_{int}} p_{int} q_{int} dM_{it}}{\sum_n \int_{\Omega_{int-1}} p_{int} q_{int-1} dM_{it-1}} \right)^{0.5}.$$

Here we used the fact that the term $GDP_{int}/(PPI_{int}/PPI_{int-1})$ in (1.12) is equal to $\sum_n \int_{\Omega_{int}} p_{int-1} q_{int} dM_{it}$, and the term $GDP_{int-1} \times PPI_{int}/PPI_{int-1}$ is equal to $\sum_n \int_{\Omega_{int-1}} p_{int} q_{int-1} dM_{it-1}$. This expression corresponds to the change in real GDP calculated according to the direct valuation method (a geometric average of Laspeyres and Paasche quantity indices), evaluating using production values at constant prices. While this procedure requires data on physical quantities and prices of individual commodities (which is typically not available in many industries and subset of goods), what we showed

is that the implied change in real GDP is equal to that using the deflation method with country specific price deflators.

Real GDP under international specialization of shipping services

We now consider an alternative specification on the nature of trade costs in which a subset of countries specializes in producing shipping services for all other countries. For concreteness (but without loss of generality for our results), we assume that the world-wide production of shipping services is concentrated in country i_s . That is, shipping one unit of a good produced in country i with productivity z to country n at time t , for all countries i and n , requires $(\tau_{nit} - 1)/z$ units of *country i_s 's labor*. Prices received by producers of these services in country i_s for the services to sell goods from country i to country n equal $\bar{p}_{int}^s = (\tau_{int} - 1)W_{ist}/z$, and consumer prices in country n equal $p_{int} = (W_{it} + (\tau_{int} - 1)W_{ist})/z$.

The resource constraint in country i is

$$\sum_n \int_{\Omega_{int}} q_{int}/z \, dM_{it} = \bar{L}_i \text{ for } i \neq i_s,$$

while in country i_s it is

$$\sum_n \int_{\Omega_{isnt}} \tau_{isnt} q_{isnt}/z \, dM_{ist} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{int}} (\tau_{int} - 1) q_{int}/z \, dM_{it} = \bar{L}_{i_s}.$$

GDP in current dollars from the production side in country $i \neq i_s$ is $GDP_{it} = \sum_n \int_{\Omega_{int}} \bar{p}_{int} q_{int} dM_{it}$, while in country $i = i_s$ it is

$$GDP_{ist} = \sum_n \int_{\Omega_{isnt}} (\bar{p}_{isnt} + \bar{p}_{isnt}^s) q_{isnt} dM_{ist} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{int}} \bar{p}_{int}^s q_{int} dM_{it}.$$

In countries $i \neq i_s$ that do not specialize in shipping services, the PPI is simply $PPI_{it}/PPI_{it-1} = W_{it}/W_{it-1}$, and the ratio of real GDP in periods t and $t - 1$ is $RGDP_{it}/RGDP_{it-1} = 1$.

In contrast to the previous specification, changes in trade costs now leave domestic technologies unchanged for those countries that do not specialize in shipping services.

Hence, in these countries changes in the PPI are equal to changes in the wage, so real GDP remains unchanged. This result holds more generally, if changes in foreign trade costs also change relative prices faced by different domestic producers (as in the Hecksher-Ohlin model, for example) but not their technologies. To see this, recall that when calculating real quantities by deflating the value of production with destination specific PPIs, real GDP is equal to the value of production evaluated at constant-year prices. From revealed production choices, the value of production falls (rises) between $t - 1$ and t when evaluated at $t - 1$ (t) prices. To a first-order approximation, real GDP remains constant. This line-of-argument cannot be used when changes in trade costs change domestic technologies, as in our baseline specification.

In Appendix B we derive the change in real GDP in country i_s that specializes in the production of shipping services. Reductions in trade costs (across any pair of countries) do improve domestic technologies in country i_s . Hence, in response to any reduction in trade costs, the PPI falls relative to the wage, and real GDP rises.

Measured real consumption and theoretical consumption in each country

We now calculate changes in real consumption, and compare them to changes in theoretical (or welfare-based) consumption of individual countries. As we did for real GDP, we use the deflation method, first using an aggregate deflator and then using country-specific deflators.

Real consumption using aggregate deflators: We calculate real consumption using the deflation method, deflating consumption expenditures with a consumer price index (CPI). We construct the CPI as a weighted average of ratios of final prices between two periods (of goods that are consumed in both periods) using t_0 weights. The CPI in country n at time t relative to time $t - 1$ is given by

$$\frac{CPI_{nt}}{CPI_{nt-1}} = \frac{\sum_i \int_{\Omega_{int}^c} (p_{int_0} q_{int_0}) \left(\frac{p_{int}}{p_{int-1}} \right) dM_{it_0}}{\sum_i \int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}}. \quad (1.14)$$

Real consumption in country n at time t relative to $t - 1$, using expression (1.1) with

a single aggregate component, is given by

$$\begin{aligned} \frac{RC_{nt}}{RC_{nt-1}} &= \left(\frac{E_{nt}/(CPI_{nt}/CPI_{nt-1})}{E_{nt-1}} \right)^{0.5} \left(\frac{E_{nt}}{E_{nt-1} \times (CPI_{nt}/CPI_{nt-1})} \right)^{0.5} \\ &= \frac{E_{nt}/(CPI_{nt}/CPI_{nt-1})}{E_{nt-1}}. \end{aligned} \quad (1.15)$$

The ratio of theoretical consumption in periods t and $t - 1$ is equal to $C_{nt}/C_{nt-1} = (E_{nt}/E_{nt-1})/(P_{nt}/P_{nt-1})$, where P_{nt} is the welfare-based price index defined above. Hence, differences between changes in real consumption and theoretical consumption stem only from differences between the CPI and the theoretical CES price index. It is straightforward to show that, to a first-order approximation, the log change in the CES price index with a fixed set of goods is equal to an expenditure-weighted average of log price changes of individual goods, as it is for the CPI defined in expression (1.14). Hence, for marginal changes in prices, changes in real consumption coincide with changes in theoretical consumption, country-by-country.

Large changes in prices give rise to the well-known substitution bias. However, if the CPI is evaluated using $t_0 = t - 1$ or $t_0 = t$ weights, we can bound this substitution bias. In particular, in Appendix A we show that, if the set of goods consumed in each period is unchanged, then the CPI with initial (final) period weights, $t_0 = t - 1$ ($t_0 = t$) overstates (understates) changes in the welfare-based price index between periods $t - 1$ and t . That is,

$$\left. \frac{CPI_{nt}}{CPI_{nt-1}} \right|_{t_0=t} \leq \frac{P_{nt}}{P_{nt-1}} \leq \left. \frac{CPI_{nt}}{CPI_{nt-1}} \right|_{t_0=t-1}. \quad (1.16)$$

Hence, changes in theoretical consumption are bounded above (below) by real-consumption calculated with the CPI based on final (initial) period weights.¹⁶ These results hold under both specifications of international trade costs.

Real consumption using disaggregated deflators: We now calculate changes in real consumption by deflating source-country specific consumption expenditures using their re-

¹⁶Inequality (1.16) does not hold in the case in which there is no trade between countries i and n at time $t - 1$ or t , so that $\Omega_{int}^c = \emptyset$. In this case, inequality (1.16) would hold if the CPI to incorporated price changes for all goods, including those that are not consumed, and assumed that unavailable goods have an infinite price. However, this is not the approach taken by the national statistics when calculating the CPI.

spective CPIs. Country specific expenditures and CPIs are defined, respectively, as

$$E_{int} = \int_{\Omega_{int}} p_{int} q_{int} dM_{it} \text{ and}$$

$$\frac{CPI_{int}}{CPI_{int-1}} = \frac{\int_{\Omega_{int}^c} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}} \right) dM_{it_0}}{\int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}}.$$

Real consumption in period t relative to period $t - 1$, using equation (1.1) with country-specific expenditure components, is given by

$$\begin{aligned} \frac{RC_{nt}}{RC_{nt-1}} &= \left(\frac{\sum_i \frac{E_{int}}{CPI_{int}/CPI_{int-1}}}{\sum_i E_{int-1}} \right)^{0.5} \left(\frac{\sum_i E_{int}}{\sum_i E_{int-1} \times CPI_{int}/CPI_{int-1}} \right)^{0.5} \\ &= \left(\frac{\sum_i \int_{\Omega_{int}} p_{int-1} q_{int} dM_{it}}{\sum_i \int_{\Omega_{int-1}} p_{int-1} q_{int-1} dM_{it-1}} \right)^{0.5} \left(\frac{\sum_i \int_{\Omega_{int}} p_{int} q_{int} dM_{it}}{\sum_i \int_{\Omega_{int-1}} p_{int} q_{int-1} dM_{it-1}} \right)^{0.5}, \end{aligned} \quad (1.17)$$

where we used the fact that percentage changes in all prices for goods coming from a common source country are equal. This expression coincides with a geometric average of Laspeyres and Paasche quantity indices using the direct-valuation method.

Up to a first-order approximation, changes in real consumption based on aggregate deflators and disaggregated deflators (as well as theoretical consumption) are equal and given by

$$d \log RC_{nt} = d \log E_{nt} - \sum_i \frac{E_{int}}{E_{nt}} d \log CPI_{int}. \quad (1.18)$$

World real GDP, consumption, and theoretical consumption

As can be observed by comparing expressions (1.13) and (1.18), differences between changes in real GDP and real consumption, country-by-country, arise from (1) differences between the current-dollar value of consumption and GDP (in the presence of trade imbalances) and (2) differences between changes in the PPI and in the CPI due to movements in relative wages and relative trade costs that change the price of exports relative to imports (i.e. the terms of trade) in each country.

We now show that, if trade is balanced in each country, a weighted-average (based on each country's current-dollar GDP) of changes in real consumption across countries is equal to the same weighted average of changes in real GDP across countries, up to a first-order approximation. Here we consider the baseline specification of trade costs in

which these are incurred using labor in each exporting country. In the Appendix we show that the equivalence between changes in world real GDP and world real consumption also holds in the model in which a subset of countries specializes in the production of shipping services.

Define s_{it} to be country i 's share in total current-dollar GDP across all countries in period t : $s_{it} = GDP_{it} / \sum_i GDP_{it}$. From expression (1.13), the world change in real GDP is, to a first-order approximation,

$$\sum_i s_{it} d \log RGDP_{it} = \frac{1}{\sum_i GDP_{it}} \left[\sum_i GDP_{it} d \log W_{it} - \sum_i \sum_n GDP_{int} d \log PPI_{int} \right]. \quad (1.19)$$

From expression (1.18), assuming balanced trade in each country (so that, as can be seen in 1.4, GDP and expenditures in current-dollars are equal, $GDP_{it} = E_{it}$), the world change in real consumption is, to a first-order approximation,

$$\sum_i s_{it} d \log RC_{it} = \frac{1}{\sum_i GDP_{it}} \left[\sum_i GDP_{it} d \log GDP_{it} - \sum_i \sum_n E_{nit} d \log CPI_{nit} \right]. \quad (1.20)$$

The first term in expression (1.19) is equal to the first term in expression (1.20) because $d \log W_{it} = d \log GDP_{it}$. The second term in expression (1.19) is equal to the second term in expression (1.20) because $GDP_{int} d \log PPI_{int} = E_{int} d \log CPI_{int}$. Intuitively, for any pair of trading countries, an improvement in the bilateral terms of trade for one country implies a worsening in the terms of trade for the other country. Hence, changes in the world CPI are equal to changes in the world PPI, and so are world real consumption and world real GDP.¹⁷

Note that, from our results on the equality of changes in real consumption and theoretical consumption country-by-country, it follows immediately that changes in world real GDP and changes in world real consumption are both equal, to a first-order approx-

¹⁷The equivalence between world changes in real GDP and real consumption also holds for large changes in trade costs if real GDP and real consumption are calculated using either Laspeyres or Paasche quantity indices (instead of using a geometric average of both, as stated in expression 1.1) based on disaggregated deflators.

imation, to a weighted average of the change in theoretical consumption across countries:

$$\sum_i s_{it} d \log RGDP_{it} = \sum_i s_{it} d \log RC_{it} = \sum_i s_{it} d \log C_{it}. \quad (1.21)$$

We can solve explicitly for the change in world real GDP and world real consumption in response to changes in variable trade costs. In particular, from (1.13) and (1.21), it follows that, to a first-order approximation,

$$\sum_i s_{it} d \log RGDP_{it} = \sum_i s_{it} d \log RC_{it} = -\frac{1}{\sum_i GDP_{it}} \times \sum_i \sum_n \text{Exports}_{int} \times d \log \tau_{in}, \quad (1.22)$$

where $\text{Exports}_{int} = \int_{\Omega_{int}} (\bar{p}_{int} + \bar{p}_{int}^s) q_{int} dM_{it}$. Changes in world real GDP and real consumption in response to changes in variable trade costs are, to a first-order approximation, equal to a weighted average of changes in bilateral variable trade costs, where the weights are simply the shares of bilateral exports in world GDP.

Tariffs and real GDP from the expenditure side

We now introduce ad-valorem import tariffs. We denote by $d_{int} \geq 1$ the gross tariff set by country n at time t for imports from country i (with $d_{iit} = 1$). Consumer prices in country n are $p_{int} = d_{int} (\bar{p}_{int} + \bar{p}_{int}^s)$. Tariffs revenues are rebated back to consumers. To simplify the notation, we calculate our aggregate statistics only for the case in which trade costs are incurred in each exporting country, but it is straightforward to extend the results to the case in which country i_s specializes in the production of shipping services.

The local equivalence, country-by-country, between changes in real consumption and theoretical consumption is immediate because, to a first-order approximation, the CPI is equal to the welfare-based price index (both of which are calculated using final prices inclusive of import tariffs). For large price changes, we still obtain the bound stated in inequality (1.16).

The relation between current-dollar GDP from the production side and current-dollar GDP from the expenditure side, provided in (1.4), must be modified by the presence of

tariffs. For example, in country $i \neq i_s$, we have

$$\begin{aligned} GDP_{it} &= \sum_n \int_{\Omega_{int}} (\bar{p}_{int} + \bar{p}_{int}^s) q_{int} dM_{it} + \Upsilon_{it} = W_{it} \bar{L}_i + \Pi_{it} + \Upsilon_{it} \\ &= E_{it} + \sum_{n \neq i} \int_{\Omega_{int}} (\bar{p}_{int} + \bar{p}_{int}^s) q_{int} dM_{it} - \sum_{n \neq i} \int_{\Omega_{nit}} (\bar{p}_{nit} + \bar{p}_{nit}^s) q_{nit} dM_{nt}, \end{aligned} \quad (1.23)$$

where $\Upsilon_{it} = \sum_n (d_{nit} - 1) \int_{\Omega_{nit}} (\bar{p}_{nit} + \bar{p}_{nit}^s) q_{nit} dM_{nt}$ denotes tariff revenues collected in country i .

Real GDP calculated from the production side excluding import tariffs from both current-dollar GDP and from price deflators is unchanged to changes in tariffs for the same reasons that real GDP in the model with only trade costs is unchanged to changes in trade costs if these are excluded from price indices.

We now calculate real GDP from the expenditure side by separately deflating each country-specific expenditure component of GDP. The export price index (EPI) for goods sold by country i to country n is given by

$$\frac{EPI_{int}}{EPI_{int-1}} = \frac{\int_{\Omega_{int}^c} (\bar{p}_{int_0} + \bar{p}_{int_0}^s) q_{int_0} \left(\frac{\bar{p}_{int} + \bar{p}_{int}^s}{\bar{p}_{int-1} + \bar{p}_{int-1}^s} \right) dM_{it_0}}{\int_{\Omega_{int}^c} (\bar{p}_{int_0} + \bar{p}_{int_0}^s) q_{int_0} dM_{it_0}}. \quad (1.24)$$

The imports price index (IPI) in country i for goods imported from country n (inclusive of trade costs incurred abroad but exclusive of tariffs) is given by $IPI_{nit}/IPI_{nit-1} = EPI_{nit}/EPI_{nit-1}$.

The Laspeyres real GDP index in country i is given by

$$\begin{aligned} \frac{RGDP_{it}}{RGDP_{it-1}} &= \frac{\sum_n \frac{E_{nit}}{CPI_{nit}/CPI_{nit-1}} + \sum_{n \neq i} \left[\frac{\text{Exports}_{int}}{EPI_{int}/EPI_{int-1}} - \frac{\text{Exports}_{nit}}{EPI_{nit}/EPI_{nit-1}} \right]}{GDP_{it-1}} = \\ &= \frac{\sum_n \int_{\Omega_{int}} (\bar{p}_{int-1} + \bar{p}_{int-1}^s) q_{int} dM_{it} + \sum_n \int_{\Omega_{nit}} (d_{nit-1} - 1) (\bar{p}_{nit-1} + \bar{p}_{nit-1}^s) q_{nit} dM_{nt}}{\sum_n \int_{\Omega_{int-1}} (\bar{p}_{int-1} + \bar{p}_{int-1}^s) q_{int-1} dM_{it-1} + \sum_n \int_{\Omega_{nit-1}} (d_{nit-1} - 1) (\bar{p}_{nit-1} + \bar{p}_{nit-1}^s) q_{nit-1} dM_{nt-1}}. \end{aligned} \quad (1.25)$$

The first term in expression (1.25) indicates the change in the constant-price value of production, and the second term represents the change in the constant price value of tariffs. The Paasche real GDP index is calculated analogously to the Laspeyres real GDP

index, but using constant period t prices and tariffs instead of period $t - 1$ prices and tariffs. The change in real GDP between period $t - 1$ and t is a geometric average of the Laspeyres and Paasche indices, as defined in expression (1.1).

Note that, in the absence of tariffs, real GDP from the expenditure side coincides with real GDP from the production side using disaggregated deflators.¹⁸ In the presence of tariffs, there is an additional source of changes in real GDP. Specifically, real GDP rises if the value of tariff revenues evaluated at base-prices and base-tariffs, $\sum_{n \neq i} \int_{\Omega_{nit}} (d_{nit_0} - 1) (\bar{p}_{nit_0} + \bar{p}_{nit_0}^s) q_{nit} dM_{nt}$ (with $t_0 = t - 1$ or $t_0 = t$), increases. That is, real GDP rises if imported physical quantities weakly increase.

Finally, consider the equivalence between world real consumption and world real GDP. Suppose that each country is under balanced trade (exclusive of tariffs), i.e. $\sum_n \int_{\Omega_{int}} (\bar{p}_{int} + \bar{p}_{int}^s) q_{int} dM_{it} = \sum_n \int_{\Omega_{nit}} (\bar{p}_{nit} + \bar{p}_{nit}^s) q_{nit} dM_{nt}$. In this case, from (1.23), current-dollar GDP (inclusive of import-tariffs) is equal to current-dollar expenditures. Define country-specific weights based on current-dollar GDP (inclusive of import-tariffs), $s_{it} = GDP_{it} / \sum_i GDP_{it}$. It is straightforward to show, following the steps above in the model without tariffs, that the change in world real GDP is equal, to a first-order approximation, to the world change in real consumption, as indicated in expression (1.21).

Summary of Results

Our central results on the implications of changes in trade costs on measures of real GDP and real consumption in our model with exogenous specialization and perfect competition can be summarized as follows:

Result 1: *In response to reductions in variable international trade costs incurred in country i that are captured in GDP and its deflators, real GDP in country i rises. If changes in variable international trade costs are not captured in country i 's GDP nor its deflators (either because producer prices in price indices exclude trade costs, or because country i starts in autarky, or because international trade services are produced in other countries), real GDP in country i is unchanged;*

Result 2: *In response to changes in physical trade costs or tariffs, the change in*

¹⁸If we use single aggregate deflators, real GDP from the production and from the expenditure side are equal up to a first-order approximation.

theoretical (welfare-based) consumption in each country lies between the changes in real consumption calculated using consumption deflators with pre- and post-trade liberalization base-year weights. To a first-order approximation, changes in real consumption and in theoretical consumption coincide country-by-country;

Result 3: *In response to changes in import tariffs that raise the value of country i 's tariff revenues at constant prices, real GDP from the expenditure side in country i rises;*

Result 4: *With balanced trade in each country, the change in world real consumption is equal, up to a first-order approximation, to the change in world real GDP (defined as cross-country weighted averages of changes in real consumption and GDP, respectively, using current-dollar GDP weights).*

Combining Results 2 and 4, we obtain the corollary that if each country is under balanced trade, changes in world real GDP equal, to a first-order approximation, changes in world theoretical consumption, independently of where are trade services produced.

1.5 Endogenous Specialization and Perfect Competition

In the model studied in the previous section, we assumed that the sets Ω_{int} , indicating the range of goods that are produced and sold in each country, were exogenously given. In this section, we briefly discuss how our previous results hold in a model that endogeneizes the set of traded goods, while keeping the assumption of perfect competition. Specifically, we consider a Ricardian version of our model, as in e.g. Dornbusch, Fisher and Samuelson (1977) [15] and Eaton and Kortum (2001) [16].

Instead of assuming that each country produces its own differentiated goods, we assume that every good ω can be produced by all countries. To incorporate this assumption in our general framework, the notation must be slightly modified as follows (see e.g. Alvarez and Lucas (2007) [1]). Each good is indexed by the vector z of productivities for this good in all countries, and $M(z)$ denotes the exogenous distribution of goods in the world. We do not make any parametric assumptions on $M(z)$. Every period, countries purchase each good from the source country with lowest marginal cost of delivering the good. These sourcing choices determine the sets Ω_{int} . With perfect competition, the final price of good z in country n is $p_{nt}(z) = \min_i \{\bar{p}_{int}(z) + \bar{p}_{int}^s(z)\}$, where $\bar{p}_{int}(z)$ and $\bar{p}_{int}^s(z)$

are equal to the marginal cost to produce and deliver, respectively, good \mathbf{z} from country i to n . We focus on the specification in which trade costs are incurred in each exporting country, but the results extend to the specification in which the production of shipping services is concentrated in a subset of countries.

Real GDP

In constructing the PPI and EPI, goods for which the identity of the producer changes over time are discontinued and hence are not included in the respective price index, as can be seen in expressions (1.6) and (1.24) with M_{it} substituted for M . All continuing producers included in the price index (i.e. those in the set Ω_{int}^c) change prices by the same percentage. Following the steps used above, we obtain the same expressions for the change in real GDP (based on aggregate deflators) as in (1.8). Hence, Result 1 remains unchanged.

Note that, while the expression for changes in real GDP is the same in the model with endogenous and in the model with exogenous specialization, the actual change in real GDP in both models can differ, for given levels of trade shares (λ_{int-1} and λ_{int}), and for given changes in trade costs (τ_{int}/τ_{int-1}). This is because changes in real GDP depend on trade shares for continuing producers $\bar{\lambda}_{int}$, which can differ from overall trade shares λ_{int} in the presence of switching in the country of origin of individual products.

The measures of real GDP based on country-specific deflators are derived in exactly the same form as in the model with exogenous specialization. Changes in real GDP are again given by expression (1.12), and are unaffected by the extent of changes over time in the source country of producers (as long as they are well defined in the sense that there is a non-zero mass of continuing producers).¹⁹

Real GDP using aggregate and disaggregated deflators now differ not only in terms of the base-year in which trade shares are calculated (as in the model with exogenous specialization), but also because the former uses trade shares for continuing producers ($\bar{\lambda}_{int}$) while the latter uses trade shares for all producers (λ_{int-1} and λ_{int}). For marginal

¹⁹To obtain an equivalence between real GDP using disaggregated deflators and real GDP calculated using the direct valuation method, we must assume that the imputed price change for newly produced (or exported) goods in a country is equal to the change in the country-specific PPI.

changes in trade costs ($\tau_{int}/\tau_{int-1} \simeq 1$), however, differences between the measures $\bar{\lambda}_{int}$, λ_{int-1} and λ_{int} have no first-order effects on real GDP (we establish this formally in the proof of Result 5 in the Appendix). Therefore, changes in real GDP based on aggregate and disaggregate deflators are equal and given by expression (1.13).

Establishing Results 3 in the model with endogenous specialization is straightforward since it was derived above using measures of real GDP based on disaggregated deflators, which are equivalent in the two models. Establishing Result 4 in this model is also straightforward since it was derived above using first-order changes in real GDP and real consumption, each of which is equal in the two models.

Real consumption

Constructing the CPI is straightforward since all goods in Ω are consumed every period. If the identity of the producer selling any given good in a particular country changes over time, we substitute the price charged by the new producer for that of the old (using the logic that the BLS looks for close substitutes if the original good is not available). That is, the CPI between periods $t - 1$ and t is given by

$$\frac{CPI_{nt}}{CPI_{nt-1}} = \frac{\int_{\Omega} (p_{nt_0} q_{nt_0}) \left(\frac{p_{nt}}{p_{nt-1}} \right) dM}{\int_{\Omega} p_{nt_0} q_{nt_0} dM}. \quad (1.26)$$

Given that all good are consumed every period, even under autarky, Result 2 on the local equivalence between real consumption and theoretical consumption applies immediately, and the counterpart of inequality (1.16) holds even if a country starts in autarky.

1.6 Endogenous Specialization and Monopolistic Competition

In this section we return to our baseline model with product differentiation, with the following two modifications. First, we assume monopolistic competition. In particular, each good is produced by a single producer that, with our CES demand, sets price as a constant markup $\rho/(\rho - 1)$ over marginal cost. Assuming that iceberg trade costs τ_{int} are incurred by the producers in their home country, and abstracting from tariffs, producer prices and final prices of goods with productivity z produced in country i and sold in

country n are²⁰

$$\bar{p}_{in}(z) + \bar{p}_{in}^s(z) = p_{in}(z) = \frac{\rho}{\rho - 1} \frac{W_{it}\tau_{int}}{z}. \quad (1.27)$$

Second, we endogeneize the distribution of producers $M_{it}(z)$ in country i , and the set of producers (indexed by their productivity z) from country i that sell in country n , Ω_{int} . To do so, we modify the technology as follows. In addition to iceberg variable trade costs, we assume that producers from country i are subject to fixed labor costs f_{int} when selling any positive amount in country n . In our baseline model, we assume that these fixed labor costs are incurred in the home country. We also consider an extension in which they are incurred in the importing country.

Every period there is an unbounded mass of potential entrants that can pay a fixed cost f_{Ei} to enter and produce a differentiated good. A measure M_{Eit} of new producers enter with a given productivity level z that remains constant throughout their life. The initial productivity is drawn from the distribution $G_i(z)$. For some of our results, we assume that $G_i(z)$ is Pareto.

Every period, producers die with probability $\delta > 0$. The distribution of producers in country i , $M_{it}(z)$, is determined by the mass of entrants, exit decisions, and the death rate. The free-entry condition implies that expected discounted profits at entry (including the fixed cost of entry) are non-positive. We assume that each period the mass of entrants is positive, $M_{Eit} > 0$, so that expected discounted profits at entry are equal to zero. Under two special cases of our model described below, our results also hold if we assume that entry is restricted so that the mass of entering firms is exogenously fixed (as in Chaney (2008) [13]).

The equivalence between GDP from the production, income, and expenditure side, in the absence of import tariffs, is given by (1.4). Current-dollar GDP from the production side is equal to aggregate revenues across all destination markets. Note that we are

²⁰This expression for final prices also results if producers and intermediaries are vertically integrated and maximize joint profits. If producers and intermediaries are not vertically integrated, then producers do not face a constant elasticity of demand (since final prices are $\bar{p} + \bar{p}^s$ and the producer chooses \bar{p}) so markups vary across producers and over time. We abstract from these complications by assuming that the producer and intermediary are vertically integrated. If the producer is vertically integrated with a foreign intermediary, and the PPI includes all costs incurred by the domestic producer (including foreign trade costs), then our results carry-through for Gross National Product, which includes profits earned abroad.

assuming that entry costs and fixed costs are expensed, and hence do not show up as output or investment in GDP. Aggregate profits Π_{it} are equal to aggregate revenues by country i producers across all destinations net of production labor, fixed labor, and entry costs:

$$\Pi_{it} = \sum_n \int_{\Omega_{int}} p_{int} q_{int} dM_{it} - W_{it} \left[\sum_n \int_{\Omega_{int}} (l_{int} + f_{int}) dM_{it} + f_{Ei} M_{Eit} \right]. \quad (1.28)$$

In what follows, we consider trade liberalization of the following form. The economy is in a steady-state at $t = 0$. Between $t = 0$ and $t = 1$, there is a permanent, unexpected change in variable and/or fixed trade costs.

We further assume that in the initial steady-state ($t = 0$) and in at least one period after the trade-liberalization ($t = T \geq 1$), aggregate profits in country i , Π_{it} , represent a constant share of aggregate revenues by country i producers. That is,

$$\Pi_{it} = \kappa_i \sum_n \int_{\Omega_{int}} p_{int} q_{int} dM_{it}, \text{ for } t = 0 \text{ and } t = T \geq 1. \quad (1.29)$$

Note from (1.4) that (1.29) also implies that aggregate profits represent a constant share current-dollar GDP. This assumption is similar to assumption R2 in Arkolakis et. al. (2011) [4].

There are three simple cases, derived in Appendix C, in which condition (1.29) is satisfied in the steady-state of our model. First, if there are no fixed costs of selling in each market (i.e. $f_{int} = 0$) so that all entering producers sell in all countries. Second, if the discount factor approaches zero ($\beta \rightarrow 1$), with or without fixed costs. In this case, aggregate profits in steady-state equal the expected discounted value of profits at entry, which are equal to zero due to the free-entry condition. Hence, $\kappa_i = 0$ in steady-state. In this case, the steady-state of our model is analogous to the equilibrium in static models with free-entry such as the ones considered in Melitz (2003) and Arkolakis et al. (2011) [4], in which aggregate profits are zero. Third, if the productivity distribution of entering producers is Pareto.

In the first and third special cases, condition (1.29) also applies if we assume that entry is restricted so that the mass of firms is exogenously fixed. Moreover, with endogenous

entry, in the first and third special cases the mass of entrants M_{Eit} does not respond to permanent changes in variable or fixed trade costs. Hence, there are no transition dynamics in response to permanent trade liberalization, and condition (1.29) holds for any time period $T \geq 1$. In all other cases with aggregate transition dynamics between steady-states, the share of profits in revenues κ_i need not be constant along the transition paths. In these cases, our results hold across steady-states.

Using (1.4) and (1.29), current-dollar GDP at time $t = 0$ and any time period $t = T$ in which condition (1.29) holds is given by

$$GDP_{it} = \frac{W_{it}\bar{L}_i}{1 - \kappa_i}. \quad (1.30)$$

We now calculate changes in real GDP and real consumption between $t = 0$ and any time period $t = T$ in which condition (1.29) holds.

Real GDP

We first calculate changes in real GDP based on aggregate deflators. The ratio of real GDP between periods $t = 0$ and $t = T$ is given by

$$\begin{aligned} \frac{RGDP_{iT}}{RGDP_{i0}} &= \prod_{t=1}^T \left(\frac{GDP_{it}/GDP_{it-1}}{PPI_{it}/PPI_{it-1}} \right) = \frac{GDP_{iT}}{GDP_{i0}} \prod_{t=1}^T \left(\frac{1}{PPI_{it}/PPI_{it-1}} \right) \\ &= \frac{1}{\sum_n \frac{\tau_{in1}}{\tau_{in0}} \bar{\lambda}_{in1}}, \end{aligned} \quad (1.31)$$

which coincides with expression (1.9) in the previous models. In deriving expression (1.31), the first step uses (3.5) and (1.8), the second step factors-out the ratios of current-dollar GDPs, and the last step uses (1.6), (1.27), and (1.30). The expression for the change in real GDP using disaggregated deflators (which, recall, is also the one resulting from using the direct valuation method) is derived in a similar fashion, and coincides with expression (1.12) in the previous models.

Note that, for given levels of trade shares by continuing producers, $\bar{\lambda}_{int}$ (which might differ from overall trade shares λ_{int} due to entry and exit by firms into individual countries) and for given changes in variable trade costs, τ_{int}/τ_{int-1} , the change in real GDP in the model with endogenous specialization and monopolistic competition is the same as

in the previous models. For given values of $\bar{\lambda}_{int}$ and τ_{int}/τ_{int-1} , reallocation of production from less productive to more productive producers (including exit by less productive producers and entry into exporting by more productive producers) does not result in an additional source of changes in aggregate productivity and real GDP. This is because value-added per production worker of individual producers, which is related to real GDP by expression (1.10), is equal to the ratio of the wage and the constant markup, independent of productivity z of individual producers.

Consider now changes in fixed costs or in the size of foreign countries when variable costs are unchanged. While these can induce changes in the volume and revenue share of trade, the ratio of PPIs is equal to W_{iT}/W_{i0} and hence does not directly reflect the changes in fixed costs. Real GDP from expression (1.9) is unchanged: $RGDP_{iT}/RGDP_{i0} = 1$. This result is summarized in the following corollary to Result 1.²¹

Corollary to Result 1: *In response to changes in fixed international trade costs between any pair of countries, real GDP in each country is unchanged.*

Real consumption and theoretical consumption

The expressions for changes in real consumption are the same as those in our baseline model: (1.15) with aggregate deflators or (1.17) with disaggregated deflators. Together with the fact that the expressions for changes in real GDP are also the same as in the previous models, Result 4 on the equivalence, to a first-order approximation, between changes in world real consumption and world GDP under trade balance holds.

What differs in this model is the comparison between real consumption and theoretical consumption, country-by-country. Changes over time in the set of consumed varieties produces differences between real consumption and theoretical consumption beyond the standard substitution bias. In particular, while the CPI between any two time periods only includes changes in prices of goods that are available for consumption in both periods, the theoretical price index also reflects changes in the mass of consumed goods.

This implies that in the model with endogenous specialization and monopolistic com-

²¹There are interactions effects from changes in variable costs and changes in fixed costs on real GDP. For example, a reduction in variable trade costs between countries i and n that is accompanied by a reduction in fixed export costs f_{int} can result in a larger trade share by continuing exporters at time t_0 and hence lead to a larger increase in real GDP.

petition, inequality (1.16) that bounds the difference between real consumption and theoretical consumption does not apply since it is derived under the assumption that the set of available goods for consumption is unchanged between time periods.²² Moreover, with changes in the mass of consumed varieties, either from changes in the set of goods supplied domestically or from changes in the set of goods imported from abroad, changes in the theoretical price index are not equal to the CPI as defined in (1.14), even to a first-order approximation. Therefore, Result 2, establishing the equality between changes in real consumption and theoretical consumption country-by-country, does not apply immediately in this version of the model. For example, an increase in the mass of consumed goods from abroad lowers the welfare-based price index (and hence increases theoretical consumption), but does not directly change the CPI (and hence does not affect measured real consumption).

We show, however, that the equivalence between changes in real consumption and theoretical consumption in response to marginal changes in variable trade costs holds at the world level. This result, which is derived in Appendix D, is summarized as follows:

Result 5: *If each country has balanced trade, then steady-state changes in world real consumption and theoretical consumption (defined as cross-country weighted averages of changes in real consumption and theoretical consumption, respectively, using current-dollar GDP weights) in response to changes in variable trade costs are equal, up to a first-order approximation, and both are given by expression (1.22).*

Results 4 and 5 combined imply that, up to a first-order approximation, steady-state changes in world real GDP and in world theoretical consumption in response to changes in variable trade costs are equal, up to a first-order approximation.

Note that, given that expression (1.22) holds in all the models that we consider, we have that for given trade shares and given marginal changes in variable trade costs, steady-state changes in world real GDP, real consumption, and theoretical consumption are all equal across these models up to a first-order approximation. This equivalence does not require any parametric assumption on the productivity distribution of entering firms, $G_i(z)$, as long as our restriction (1.29) holds.

²²Inequality (1.16) would hold if the CPI attributed a price equal to infinite to goods that are not available for consumption.

Result 5 can be understood as follows. Note that when countries are symmetric, this result states that changes in real consumption equal changes in theoretical consumption in response to marginal changes in variable trade costs. This is because, as discussed in Atkeson and Burstein (2010) [6], when countries are symmetric the indirect effect of a change in trade cost on consumption through its effect on the set of consumed goods (due to changes in the mass of entering firms and changes in exit and export thresholds, which are not captured in the CPI) is zero up to a first-order-approximation. Hence, in each country changes in the theoretical price index are approximately equal to changes in the CPI. With asymmetric countries, changes in relative country sizes alter the equivalence between real consumption and theoretical consumption, country-by-country, due to changes in the relative market size of countries. This effect, however, washes-out across countries (i.e. the gain in one country is a loss for another) when comparing steady-state changes in world real consumption and world theoretical consumption.²³

To establish the equality between real consumption and theoretical consumption, country-by-country, in response to changes in variable trade costs (as in Result 2), we must impose two additional assumptions. First, fixed export costs are paid in the importing country. Second, the distribution of productivities of entering firms, $G_i(z)$ is Pareto. These assumptions are made in Eaton, Kortum, and Kramarz (2010) [20] and for some results in Arkolakis et. al. (2011) [4].²⁴ Under these assumptions, we obtain the following result that we prove in Appendix E.

Result 6: *Suppose fixed export costs are paid in the importing country, and that the distribution of entering firms is Pareto. If each country has balanced trade, then steady-state changes in real consumption and theoretical consumption in response to changes in variable trade costs are equal country-by-country, to a first-order approximation.*

Result 6 implies that, in response to marginal changes in variable trade costs, changes

²³For this result to hold, it is important that fixed and entry costs are denominated in terms of labor. If these costs entail a combination of labor and final good, then changes in the relative wage can result in additional indirect effects from changes in the mass of consumed varieties on the welfare-based price index that are not captured in the CPI (see the related discussion for welfare in Arkolakis et. al. (2011) [4] and Atkeson and Burstein (2010) [6]).

²⁴These assumptions are required for the "ex-ante" result of Proposition 2 in Arkolakis et. al. (2011) [4]. Under these assumptions, their model responds to any global change in variable trade costs like an Armington model. Given that the welfare-based prices in the Armington model behaves, to a first-order approximation, like the CPI, we obtain the equivalence between real consumption and consumption-based welfare, country by country.

in the mass and in the composition of consumed domestic and exported goods (due to changes in exit and export thresholds) offset each other in each country's theoretical price index. Hence, changes in the CPI and in the theoretical price index coincide, up to a first-order approximation. Note that this result does not require that the mass of consumed varieties remains unchanged in each country (even though the mass of entering firms in each country does). Indeed, reductions in marginal trade costs typically result in an increase in the mass of consumed goods (which, however, does not affect the theoretical price index).

Numerical example

We illustrate how changes in real GDP, real consumption, and theoretical consumption compare in a quantitative example of our model with monopolistic competition. We consider small and large reductions in variable trade costs to evaluate the accuracy of some of our equivalence results derived using first-order approximations. We consider a two-country version of our model with trade balance, symmetric trade costs ($\tau_{12t} = \tau_{21t} = \tau_t$ and $f_{12t} = f_{21t}$), Pareto productivity distribution of entering firms with slope parameter of 5 (as in Eaton, Kortum and Kramarz (2010) [20], implying a trade elasticity equal to 5), and elasticity of substitution ρ equal to 3. Variable trade costs are fully incurred in each exporting country, and fixed export costs are incurred in either the exporting country or the importing country (in the latter case, the economy satisfies the assumptions in Result 6). We choose the initial level of variable trade costs $\tau_0 = 1.47$, and relative country sizes $\bar{L}_1/\bar{L}_2 = 2.05$, so that the goods' trade share in country 1 is $\lambda_{120} = 7\%$ and the trade share in country 2 is $\lambda_{210} = 15\%$. The share of each country in world GDP is $s_{10} = 0.68$ and $s_{20} = 0.32$, respectively. The unchanged level of fixed costs do not affect our reported results. Recall that in this specification, entry remains unchanged, so the economy immediately transits to the new steady-state (at time $t = 1$).

We consider reductions in variable trade costs, ranging from very small (corresponding to our first-order approximations) to quite large (τ falls from roughly 1.47 to 1.23 so that the trade share more than doubles). Figure 1.9.7 considers the case in which fixed export costs are paid in the exporting country and Figure 1.9.7 the case in which fixed export costs are paid in the importing country. Based on the results in Arkolakis et. al. (2010)

[4], the specification in which fixed export costs are incurred in the importing country is exactly equal to the Armington version of our model with perfect competition and exogenous specialization and to the Krugman version of our model with monopolistic competition but no fixed costs, both parameterized with $\rho = 5$.

In each figure, the x-axis displays the ratio of trade shares in the post- and pre-liberalization periods, $\lambda_{in1}/\lambda_{in0}$ and the y-axis displays the negative of the elasticity of real GDP, real consumption, and theoretical consumption with respect to the change in variable trade costs (e.g., $-\log(RGDP_1/RGDP_0)/\log(\tau_1/\tau)$). We report the measures of real GDP and real consumption calculated based on disaggregated deflators, which minimize the standard substitution bias in response to large changes in trade costs. We report separately the responses in each country and at the world level.

From Figures 1.9.7 and 1.9.7 we can observe that the higher order terms can be quite large. That is, the elasticities of each aggregate variable are largely increasing in the size of the reduction in trade costs. This implies that, for example, expression (1.21) is not a very accurate approximation for large reductions in trade costs: the elasticity of world real GDP and world real consumption is $s_{10} * \lambda_{120} + s_{20} * \lambda_{210} \simeq 0.09$ in response to a marginal reduction in trade costs, and roughly 0.15 in response to a large reduction in trade costs that doubles the trade share.

However, quite remarkably, theoretical and measured gains from trade are fairly close even for large reductions in trade costs that result in large increases in trade shares. In particular, first, the elasticity of world real GDP and the elasticity of world real consumption are almost exactly equal for any size of the reduction in trade costs (Result 4).²⁵ Second, for large reductions in trade costs, the elasticity of world real consumption is only slightly higher than the elasticity of world theoretical consumption (Result 5). Third, in each country (and especially in country 2), for any size of the reduction in trade costs the elasticity of real consumption is quite close to the elasticity of theoretical consumption. This is not only the case when fixed export costs are incurred in the importing country (Result 6) but also when fixed export costs are incurred in the exporting country (for which we do not have an analytic result). Finally, comparing the elasticity of each variable

²⁵For any change in trade costs, the increase in real GDP in country 1 (country 2) is slightly larger (smaller) than the increase in real consumption in that country, reflecting the fact that the wage in country 1 rises relative to the wage in country 2.

in Figures 1.9.7 and 1.9.7 for any given change in variable trade costs, both specifications have very similar quantitative implications for both theoretical and measured aggregate gains from trade.

1.7 Two Extensions

In this section, we consider two extensions of our model. The first extension adds endogenous quality choice by firms. The second extension introduces multiple factors of production. We introduce these extensions in our model with monopolistic competition. We provide conditions under which our previous results on the response of aggregate productivity to changes in trade costs, and on the first-order equivalence between changes in real GDP, real consumption, and theoretical consumption at the world level (or country-by-country for real and theoretical consumption under stronger conditions) hold in the extended model. Details are provided in Appendices F and G.

Endogenous quality choice

The final good is given by

$$C_{nt} = \left[\int_{\Omega_{nt}} a_{nt}(\omega)^{\frac{1}{\rho}}(\omega) q_{nt}(\omega)^{\frac{\rho-1}{\rho}} d\omega \right]^{\frac{\rho}{\rho-1}},$$

where $a_{nt}(\omega)$ denotes the quality of differentiated good ω in country n . The theoretical price index is given by $P_{nt} = \left[\int_{\Omega_{nt}} a_{nt}(\omega) p_{nt}(\omega)^{1-\rho} d\omega \right]^{\frac{1}{1-\rho}}$. Higher levels of quality decrease the price index.

Demand in country n for good z produced in country i is given by $q_{int}(z) = a_{int}(z) (p_{int}(z)/P_{nt})^{-\rho} C_{nt}$. Higher quality increases demand, given prices. We assume that each period, individual producers from country i with productivity z must employ $h(z; a_{int})$ units of labor in the home country to set quality a_{int} for sales in country n , where $h(z; \cdot)$ is increasing and convex in a .²⁶ We assume that these costs

²⁶We assume throughout that $h(z; a)$ is such that the level of a for active products is positive and bounded, and so that in steady-state there is positive entry and a stationary size distribution. All our results hold if a_{int} is constrained to be equal across destination countries, with the exception of the equivalence between real and theoretical consumption country by country, which requires that a_{int} be destination specific.

are expensed, so they are not included in GDP. Given that quality costs are independent of the volume of production, reductions in trade costs that raise the scale of exporters typically induce a higher investment in quality by exporters relative to non-exporters.

The share of profits in GDP is constant in the steady-state (condition 1.29) under the two following alternative assumptions. First, if the discount factor approaches zero ($\beta \rightarrow 1$). As $\beta \rightarrow 1$, aggregate profits, which now include the costs of quality choice, become zero from the free-entry condition, so $\kappa_i = 0$ in steady-state. Second, if $h(z; a)$ takes the form $h(z; a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^\gamma$ and either (i) there are no fixed costs of supplying individual markets or (ii) the productivity distribution of entering producers is Pareto. In Appendix F we derive κ_i for this case.

Prices set by individual producers are given by expression (1.27) as in our baseline model. A key consideration that determines the aggregate measured gains from trade is whether deflators are constructed using prices adjusted for quality (i.e. $p_{int}(z)/a_{int}(z)$) or non-adjusted for quality (i.e. $p_{int}(z)$).²⁷

If prices in the PPI do not adjust for quality changes, then the expression for changes in real GDP is equivalent to that in our baseline model without endogenous quality (expressions 1.8 and 1.12), derived using condition (1.29). If prices in the PPI do adjust for quality changes, then if average quality rises in response to a reduction in trade costs, the PPI falls relative to the scenario in which prices are not adjusted for quality changes. In this case, the increase in real GDP (conditional on trade shares and changes in trade costs) is larger than the one in expressions (1.8) and (1.12).

Consider now the response of real consumption. In Appendix F we establish the following result. If prices in the CPI do not reflect changes in product quality, then changes in world real consumption and world theoretical-consumption in response to marginal changes in variable trade costs are equal, to a first-order approximation, and given by expression (1.22). This equality also applies to world real GDP if GDP deflators do not adjust for quality changes. Intuitively, the effects on the world welfare-based price index from changes in the set of consumed goods (changes in the mass of entering firms

²⁷Product quality in this setup can be re-interpreted as producer productivity. In this case, producers innovate to improve productivity rather than product quality. This re-interpretation does not change any of the model's implications for theoretical consumption. Note, however, that changes in productivity are more likely to be captured in price indices, as when prices are adjusted for quality.

and changes in exit and export thresholds) and endogenous quality changes add up to zero, up to a first-order approximation. If prices in the CPI do not capture any of these margins (i.e. prices are not adjusted for quality changes), then the CPI coincides with the welfare-based price index.

Suppose instead that prices in the CPI do adjust for quality changes. If average quality rises in response to a reduction in trade costs, the CPI falls relative to the baseline scenario in which prices are not adjusted for quality changes, and measured gains in world real consumption exceed those in world theoretical consumption.

In Appendix F we show that if the productivity distribution of entering producers is Pareto, $h(z; a)$ takes the form $h(z; a) = \frac{\gamma_0}{\gamma} z^\mu a^\gamma$, and both fixed costs and innovation costs are incurred using labor in the importing country, then in response to marginal changes in variable trade costs the equivalence between changes in real and theoretical consumption (when prices in the CPI do not reflect changes in product quality) holds not only at the world level but also country-by-country.

Multiple factors of production

We now consider multiple factors of production, which can be accumulated or in fixed supply. The production of intermediate goods uses labor and J additional inputs, denoted by k_j , according to:

$$y = z l^{\alpha_L} \prod_{j=1}^J k_j^{\alpha_j}, \quad (1.32)$$

where we assume constant returns to scale, so $\alpha_L + \sum_{j=1}^J \alpha_j = 1$. All producers are subject to a production function with the same factor shares α_j . Fixed costs of supplying individual markets and entry costs are all denominated in terms of labor.

Without loss of generality, we assume that inputs $j \leq J_F$ can be accumulated at the aggregate level (e.g. capital), while inputs $j > J_F$ are exogenously supplied and constant over time. None of our results depend on the choice of J_F . Consumption and accumulable inputs are both produced using a final non-tradeable good defined in (1.3). The final good resource constraint in country i is $C_{it} + \sum_{j=1}^{J_F} K_{j,it} = Q_{it}$, where $K_{j,it}$ denotes the aggregate stock of input j in the economy, and Q_{it} denotes the quantity of the final good used in country i . The assumption that accumulable inputs fully depreciate every period is

without loss of generality for our results.

Letting $R_{j,it}$ denote the price of input j in country i in period t , cost minimization implies

$$\frac{R_{j,it}}{W_{it}} = \frac{\alpha_j}{\alpha_L} \frac{l_{int}}{k_{j,int}} = \frac{\alpha_j}{\alpha_L} \frac{L_{it}}{K_{j,it}}, \quad (1.33)$$

where L_{it} denotes the aggregate quantity of labor used for production in country i . The second equality follows from the assumption that factor shares and factor prices are common across firms. The optimal price of a country i producer with productivity z selling in country n is given by $p_{int}(z) = \frac{\rho}{\rho-1} \frac{\tau_{int} c_{it}}{z}$, where $c_{it} = \hat{\alpha} W_{it} \prod_{j=1}^J [R_{j,it}/W_{it}]^{\alpha_j}$ is the cost of the input bundle in country i . Using (1.33), we can rewrite c_{it} as

$$c_{it} = \hat{\alpha} W_{it} \prod_{j=1}^J \left[\frac{\alpha_j}{\alpha_L} \frac{L_{it}}{K_{j,it}} \right]^{\alpha_j}. \quad (1.34)$$

Real GDP: GDP includes output used for both consumption and accumulable inputs. We calculate real GDP using aggregate deflators. We first calculate the aggregate PPI. Note that, given that consumption and accumulable inputs use the same production technology, there is a single PPI for final goods, given by (1.6), which can be written as:

$$\frac{PPI_{it}}{PPI_{it-1}} = \frac{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}} \right) dM_{it_0}}{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} dM_{it_0}} = \frac{c_{it}}{c_{it-1}} \sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}. \quad (1.35)$$

In the Appendix, we show that in this version of the model, current dollar GDP is proportional to aggregate labor payments. Hence, the ratio of real GDP in time T to time $t = 0$ in response to a permanent trade liberalization at time $t = 1$ is

$$\begin{aligned} \frac{RGDP_{iT}}{RGDP_{i0}} &= \frac{GDP_{iT}}{GDP_{i0}} \prod_{t=1}^T \left(\frac{1}{PPI_{it}/PPI_{it-1}} \right) = \frac{W_{iT} \bar{L}_i}{W_{i0} \bar{L}_i} \prod_{t=1}^T \left(\frac{1}{\frac{c_{it}}{c_{it-1}} \sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}} \right) \\ &= \prod_{j=1}^J \left[\frac{K_{j,iT}}{K_{j,i0}} \right]^{\alpha_j} \frac{1}{\sum_n \frac{\tau_{in1}}{\tau_{in0}} \bar{\lambda}_{in1}}, \end{aligned} \quad (1.36)$$

where the last step follows from equation (1.34). Given trade shares of continuing producers and given changes in variable trade costs, the change in measured aggregate productivity coincides with that in our baseline model with a single factor of production

(i.e. expression (1.8)). Of course, growth in aggregate quantities of non-labor factors of production contributes to growth in real GDP.

Note that accumulable inputs may also be interpreted as intermediate goods. In this case, GDP differs from gross output as it excludes the use of intermediate inputs. However, our assumptions imply that the share of value added in firms' gross-output is constant, so the expression for real GDP remains unchanged.²⁸

World real GDP, consumption, and theoretical consumption: In Appendix G we derive the equivalence between world theoretical consumption, world consumption, and world GDP, up to a first order approximation, in response to marginal changes in variable trade costs (if the set of consumed products is unchanged or if the distribution of entering firms is Pareto and fixed costs are incurred in the importing country, the first-order equivalence between real consumption and theoretical consumption holds country-by-country). A key step in the analysis is that, under our assumptions, changes in trade costs do not change the steady-state ratio of consumption to final output, C/Q , in each country. The actual magnitudes of changes in world aggregates (for given trade shares and changes in trade costs) differ from those in the baseline model due to endogenous changes in aggregate quantities of non-labor factors of production.

1.8 Conclusions

In this paper we have studied the implications of trade liberalization for aggregate measures of economic activity in a widely-used class of workhorse models of international trade. We have characterized how in these models real GDP and real consumption, as calculated by statistical agencies in the United States, respond to changes in variable trade costs, fixed trade costs, and tariffs.

For the class of models that we consider, our conclusions can be broadly summarized as follows. First, aggregate output measured by real GDP and aggregate productivity constructed using data on real GDP increase in response to reductions in trade costs

²⁸We can also calculate real GDP using the double deflation method and obtain the same expression. The key is that intermediate inputs are produced using the same technology as final goods, so they are deflated using the same deflator (1.35) as that used to deflate gross output.

insofar as prices used to construct deflators reflect these changes in trade costs. Real GDP and aggregate productivity, however, do not capture the reallocation of production towards more productive producers resulting from trade liberalization. Second, gains in theoretical (welfare-based) consumption from reductions in variable international trade costs translate into measures of real consumption when aggregating these measures across countries. Under stronger but common assumptions in the literature, the equivalence between theoretical and measured consumption also holds country-by-country. Differences between consumption deflators and welfare-based price indices in response to changes in variable trade costs, that may arise from changes in the set of consumed varieties or changes in the quality of individual products wash-out when treated jointly across all countries (or country-by-country under stronger assumptions). Third, conditional on trade shares (of continuing producers) and changes in variable trade costs, all the models we consider deliver approximately the same measured aggregate gains from trade. The equivalence in measured gains from trade arises due to the equivalence in the welfare implications of these models.

Our results establish a benchmark to understand how the extensive empirical evidence on the link between trade and aggregate measures of economic activity can be interpreted through the lens of workhorse trade models, and how the theoretical link between trade and welfare in these models translates into observable aggregates. Our results should be, however, treated with caution to the extent that the measurement procedures in individual countries differ from those carried out in the United States and recommended by the United Nations. Finally, the extent to which our results carry over to richer models featuring additional sources of gains from trade to the ones we considered, such as the endogenous response of markups, remains an open research question.

1.9 Appendix

1.9.1 Substitution bias in the CPI

In this appendix we derive the well-known substitution bias on the CPI, establishing that the Laspeyres (Paasche) price index overstates (understates) changes in the welfare-based

price index. We assume through this section that the same set of goods is consumed in all periods, $\Omega_{int} = \Omega_{in}$ and $M_{it} = M_i$. Note that we can write the Laspeyres price index as:

$$\left. \frac{CPI_{nT}}{CPI_{n0}} \right|_{t_0=0} = \sum_i \Lambda_{in0} \left(\frac{CPI_{inT}}{CPI_{in0}} \right),$$

where $\Lambda_{int} = \int_{\Omega_{in}} (p_{int} q_{int} dM_i) / \sum_i \int_{\Omega_{in}} p_{int} q_{int} dM_i$ is country n 's share of expenditures on goods produced in country i at date t . Similarly, we can re-write the Paasche price index as:

$$\left. \frac{CPI_{nT}}{CPI_{n0}} \right|_{t_0=T} = \left[\sum_i \Lambda_{inT} \left(\frac{CPI_{in0}}{CPI_{inT}} \right) \right]^{-1}.$$

The welfare-based price index is defined as:

$$P_{nt} = \min_{q_{int}} \sum_i \int_{\Omega_{in}} p_{int} q_{int} dM_i : [u(C_{nt}) \geq \bar{u}].$$

Let q_{int}^* denote the solution to this problem when prices are p_{int} . The change in the welfare-based price index is given by:

$$\frac{P_{nT}}{P_{n0}} = \frac{\sum_i \int_{\Omega_{in}} p_{inT} q_{inT}^* dM_i}{\sum_i \int_{\Omega_{in}} p_{in0} q_{in0}^* dM_i} \leq \frac{\sum_i \int_{\Omega_{in}} p_{inT} q_{in0}^* dM_i}{\sum_i \int_{\Omega_{in}} p_{in0} q_{in0}^* dM_i} = \sum_i \Lambda_{in0}^* \left(\frac{CPI_{inT}}{CPI_{in0}} \right)$$

where the inequality follows from the definitions of q_{inT}^* and $\Lambda_{int}^* = \frac{\int_{\Omega_{in}} p_{int} q_{int}^* dM_i}{\sum_i \int_{\Omega_{in}} p_{int} q_{int}^* dM_i}$.

Similarly:

$$\frac{P_{nT}}{P_{n0}} = \frac{\sum_i \int_{\Omega_{in}} p_{inT} q_{inT}^* dM_i}{\sum_i \int_{\Omega_{in}} p_{in0} q_{in0}^* dM_i} \geq \frac{\sum_i \int_{\Omega_{in}} p_{inT} q_{inT}^* dM_i}{\sum_i \int_{\Omega_{in}} p_{in0} q_{inT}^* dM_i} = \left[\sum_i \Lambda_{inT}^* \left(\frac{CPI_{in0}}{CPI_{inT}} \right) \right]^{-1}. \quad (1.37)$$

If u is homothetic (so that expenditure shares only depend on relative prices and do not depend on income), then, $\Lambda_{int}^* = \Lambda_{int}$, and (1.37) implies (1.16).

1.9.2 International specialization of shipping services

We first calculate the change in real GDP (using aggregate deflators) in country i_s . The PPI in period t relative to period $t - 1$ is given by

$$\frac{PPI_{i_s t}}{PPI_{i_s t-1}} = \frac{W_{i_s t}}{W_{i_s t-1}} \left[\sum_n \bar{\lambda}_{i_s n t} \left(\frac{\tau_{i_s n t}}{\tau_{i_s n t-1}} \right) + \sum_{i \neq i_s} \sum_n \left(\frac{\tau_{i n t} - 1}{\tau_{i n t-1} - 1} \right) \bar{\lambda}_{i n t}^s \right]$$

where

$$\bar{\lambda}_{i_s n t} = \frac{\int_{\Omega_{i_s n t}^c} (\bar{p}_{i_s n t_0} + \bar{p}_{i_s n t_0}^s) q_{i_s n t_0} dM_{i_s t_0}}{\sum_n \int_{\Omega_{i_s n t}^c} (\bar{p}_{i_s n t_0} + \bar{p}_{i_s n t_0}^s) q_{i n t_0} dM_{i t_0} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{i n t}^c} \bar{p}_{i n t_0}^s q_{i n t_0} dM_{i t_0}}$$

and, for $i \neq i_s$,

$$\bar{\lambda}_{i n t}^s = \frac{\int_{\Omega_{i n t}^c} \bar{p}_{i n t_0}^s q_{i n t_0} dM_{i t_0}}{\sum_n \int_{\Omega_{i_s n t}^c} (\bar{p}_{i_s n t_0} + \bar{p}_{i_s n t_0}^s) q_{i n t_0} dM_{i t_0} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{i n t}^c} \bar{p}_{i n t_0}^s q_{i n t_0} dM_{i t_0}},$$

with $\sum_n \bar{\lambda}_{i_s n t} + \sum_{i \neq i_s} \sum_n \bar{\lambda}_{i n t}^s = 1$. The ratio of real GDP in time period t relative to $t - 1$ is

$$\begin{aligned} \frac{RGDP_{i_s t}}{RGDP_{i_s t-1}} &= \left(\frac{\frac{GDP_{i_s t}}{PPI_{i_s t}/PPI_{i_s t-1}}}{GDP_{i_s t-1}} \right)^{0.5} \left(\frac{GDP_{i_s t}}{\frac{GDP_{i_s t-1}}{PPI_{i_s t}/PPI_{i_s t-1}}} \right)^{0.5} \\ &= \frac{1}{\sum_n \bar{\lambda}_{i_s n t} \left(\frac{\tau_{i_s n t}}{\tau_{i_s n t-1}} \right) + \sum_{i \neq i_s} \sum_n \bar{\lambda}_{i n t}^s \left(\frac{\tau_{i n t} - 1}{\tau_{i n t-1} - 1} \right)}. \end{aligned} \quad (1.38)$$

Clearly, $RGDP_{i_s t}/RGDP_{i_s t-1} > 1$ if trade costs fall.

We now derive the change in world real GDP and real consumption under balanced trade. Log-differentiating (1.38),

$$d \log RGDP_{i_s t} = - \sum_n \lambda_{i_s n t} d \log \tau_{i_s n t} - \sum_{i \neq i_s} \sum_n \lambda_{i n t}^s \frac{\tau_{i n t}}{\tau_{i n t} - 1} d \log \tau_{i n t}.$$

Together with $d \log RGDP_{i t} = 0$ for $i \neq i_s$, and using the definitions of $\bar{\lambda}_{i_s n t}$ and $\bar{\lambda}_{i n t}^s$, we obtain expression (1.22), where $Exports_{i n t}$ for $i \neq i_s$ is evaluated at prices inclusive of trade services provided by country i_s .

With balanced trade in each country, the world change in real consumption is, to a

first-order approximation, given by expression (1.20), where country-specific expenditures E_{int} are calculated inclusive of trade costs provided by country i_s , and changes in country specific CPIs are given by

$$d \log CPI_{nit} = \frac{W_{nt} d \log W_{nt} + W_{i_s t} (\tau_{nit} - 1) \left[d \log W_{i_s t} + \frac{\tau_{nit}}{\tau_{nit} - 1} d \log \tau_{nit} \right]}{W_{nt} + W_{i_s t} (\tau_{nit} - 1)} \text{ for } n \neq i_s,$$

and

$$d \log CPI_{nit} = d \log W_{i_s t} + d \log \tau_{nit} \text{ for } n = i_s.$$

Substituting E_{nit} and $d \log CPI_{nit}$ into (1.20), we obtain expression (1.22).

1.9.3 Deriving the share of profits in total revenues

We now show that our assumption in equation (1.29) that aggregate profits represent a constant share of total revenues ($\Pi_{it} = \kappa_i Y_{it}$, where $Y_{it} = \sum_n \int_{\Omega_{int}} p_{int} q_{int} dM_{it}$), is satisfied in the remaining two special cases of our model described in Section 5. In the first case, there are no fixed costs of selling into individual countries so that all firms sell in each country. In the second case, there are positive fixed costs of selling in individual countries (incurred in either the exporting or importing country) and productivities are Pareto distributed. We derive equation (1.29) for the general case in which a fraction ϕ of these fixed costs are incurred in the exporting country and a fraction $1 - \phi$ of these fixed costs are incurred in the importing country. The baseline model in the body of the paper assumes $\phi = 0$. We consider the case of $\phi = 1$ in Result 6. We also show that, in these two cases, the mass of firms is unchanged following a trade liberalization. Remember that in the third special case described in Section 5, when $\beta \rightarrow 1$, it is straightforward to show that the free entry condition implies that $\kappa_i = 0$ in steady-state.

We start by deriving some preliminary equations of the model: first, note that combining (1.27) with the demand function we obtain that firm's revenues are proportional to firm's variable costs,

$$p_{int} q_{int} = \frac{\rho}{\rho - 1} W_{it} l_{int}, \quad (1.39)$$

variable labor demand is:

$$l_{int}(z) = z^{\rho-1} \tau_{int}^{1-\rho} \left[\frac{\rho}{\rho-1} W_{it} \right]^{-\rho} P_{nt}^{\rho} C_{nt} , \quad (1.40)$$

and variable profits are:

$$\pi_{int}(z) = \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^{\rho} (\rho-1)^{1-\rho}} W_{it}^{1-\rho} P_{nt}^{\rho} C_{nt} . \quad (1.41)$$

In an equilibrium with selection by firms to sell in each country, there exists a threshold \bar{z}_{int} such that only firms with $z \geq \bar{z}_{int}$ operate in destination n . That is, $\Omega_{int} = \{z : z \geq \bar{z}_{int}\}$. This threshold satisfies:

$$\pi_{int}(\bar{z}_{int}) = W_{it}^{\phi} W_{nt}^{1-\phi} f_{int} . \quad (1.42)$$

Aggregate profits in country i in period t net of fixed labor costs and entry costs are given by:

$$\Pi_{it} = Y_{it} - W_{it} L_{it} - \sum_n W_{it}^{\phi} W_{nt}^{1-\phi} f_{int} \int_{\Omega_{int}} dM_{it} - W_{it} M_{Eit} f_{Ei} ,$$

where L_{it} denotes aggregate variable labor used in production, $L_{it} = \sum_n \int_{\Omega_{int}} l_{int} dM_{it}$. Note that from expression (1.39), aggregate revenues are proportional to variable labor payments:

$$Y_{it} = \frac{\rho}{\rho-1} W_{it} L_{it} . \quad (1.43)$$

If condition (1.29) holds, then in combination with (1.30), we obtain

$$\frac{1}{1-\kappa_i} W_{it} \bar{L}_i = Y_{it} = \frac{\rho}{\rho-1} W_{it} L_{it} ,$$

which implies that variable production labor is a constant share of total labor:

$$(1-\kappa_i) \frac{\rho}{\rho-1} L_{it} = \bar{L}_i . \quad (1.44)$$

Hence, if aggregate profits represent a constant share of aggregate revenues, then aggregate variable labor represents a constant fraction of total labor.

Suppose we are on a steady-state equilibrium in which aggregate variables are con-

stant. In steady-state, the interest rate is given by $1/\beta$ and the distribution of firms is given by $M_i(z) = \frac{M_{Ei}}{\delta} G_i(z)$ (we omit time subscripts for the remainder of this section to simplify notation). The aggregate free-entry condition in steady-state is:

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1 - \beta[1 - \delta]} \left[Y_i - W_i L_i - \sum_n W_i^\phi W_n^{1-\phi} [1 - G_i(\bar{z}_{in})] f_{in} \right]. \quad (1.45)$$

In what follows, we solve for the constant of proportionality $\Pi_i/Y_{it} = \kappa_i = \kappa$ in steady-state under two special cases of our model. We then show that, in these two special cases, the aggregate response to a change in variable or fixed trade costs is immediate (i.e. there are no transition dynamics), so that κ remains constant over time.

Case 1: No fixed costs

Assume that there are no fixed costs of selling in individual countries, i.e. $f_{ii} = f_{in} = 0$, so that there is no selection. In this case, the aggregate free entry condition (1.45) is:

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1 - \beta[1 - \delta]} [Y_i - W_i L_i],$$

and using (1.43),

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1 - \beta(1 - \delta)} \frac{1}{\rho} Y_i. \quad (1.46)$$

Aggregate profits are:

$$\begin{aligned} \Pi_i &= Y_i - W_i L_i - W_i \frac{M_{Ei}}{\delta} f_{Ei} \\ &= Y_i - \frac{\rho - 1}{\rho} Y_i - \frac{\beta}{1 - \beta[1 - \delta]} \frac{1}{\rho} Y_i \\ &= \frac{1 - \beta}{\rho[1 - (1 - \delta)\beta]} Y_i, \end{aligned}$$

so $\kappa = \frac{1 - \beta}{\rho(1 - (1 - \delta)\beta)}$. Note that if $\beta < 1$, aggregate cross-sectional profits are positive even though discounted profits at entry are zero.

The steady-state mass of entering firms is given by:

$$M_{Ei} = \frac{\beta\delta}{1 - \beta(1 - \delta)} \frac{\bar{L}_i}{\rho f_{Ei} (1 - \kappa)},$$

where we used (1.43), (1.46), and equation (1.44). Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state, and $\kappa_{it} = \kappa$.

Finally, aggregate variable profits gross of entry costs are: $\Pi_i + W_i M_{Ei} f_{Ei} = \rho^{-1} Y_i$. Hence, with restricted entry (so that there are no costs incurred in entry), equation (1.29) holds with $\kappa = 1/\rho$.

Case 2: Pareto distributed productivities

Assume that there are positive fixed costs of selling in individual countries and that the distribution of entering firms G_i is Pareto with shape parameter θ , i.e. $G_i = 1 - z^{-\theta}$ for $z \geq 1$. We also assume that the productivity cutoffs are interior, $\bar{z}_{in} > 1$.

We first show that aggregate fixed labor costs are proportional to aggregate revenues. Using the Pareto form, we can rewrite the expression (1.42) that defines the cutoff \bar{z}_{in} as:

$$\frac{\bar{z}_{in}^{\rho-1-\theta} \tau_{in}^{1-\rho} W_i^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} P_n^\rho C_n = W_i^\phi W_n^{1-\phi} f_{in} \bar{z}_{in}^{-\theta}. \quad (1.47)$$

Fixed labor costs to sell in destination n are given by:

$$\frac{M_{Ei}}{\delta} W_i^\phi W_n^{1-\phi} f_{in} \bar{z}_{in}^{-\theta} = \frac{\theta + 1 - \rho}{\rho\theta} Y_{in} \quad (1.48)$$

where $Y_{in} = \int_{\Omega_{in}} p_{in} q_{in} dM_i$ denotes revenues from sales in country n . Summing across countries we obtain:

$$\frac{M_{Ei}}{\delta} \sum_n W_i^\phi W_n^{1-\phi} f_{in} [1 - G_i(\bar{z}_{in})] = \frac{\theta + 1 - \rho}{\rho\theta} Y_i, \quad (1.49)$$

Using (1.43) and (1.49), we can write the aggregate free entry condition (1.45) as:

$$M_{Ei} W_i f_{Ei} = \frac{\delta\beta}{1 - \beta[1 - \delta]} \frac{\rho - 1}{\rho\theta} Y_i, \quad (1.50)$$

Finally, combining (1.43), (1.49) and (1.50), aggregate profits are

$$\begin{aligned}\Pi_i &= Y_i - W_i L_i - \frac{M_{Ei}}{\delta} \sum_n W_i^\phi W_n^{1-\phi} [1 - G_i(\bar{z}_{in})] f_{in} - \frac{M_{Ei}}{\delta} W_i f_{Ei} \\ &= \frac{\rho - 1}{\theta \rho} \frac{1 - \beta}{1 - \beta [1 - \delta]} Y_i\end{aligned}$$

so $\kappa = \left(\frac{\rho-1}{\theta\rho}\right) \left(\frac{1-\beta}{1-\beta[1-\delta]}\right)$. The steady-state mass of entering firms is given by:

$$M_{Ei} = \frac{\delta\beta(\rho-1)}{1-\beta(1-\delta)} \frac{1}{f_{Ei}(1-\kappa)} \frac{1}{\rho\theta} \bar{L}_i,$$

where we used (1.43), (1.44), and (1.50). Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state.

Finally, aggregate variable profits gross of entry costs are $\Pi_i + W_i M_{Ei} f_{Ei} = \frac{\rho-1}{\theta\rho} Y_i$. Hence, in the model with restricted entry (in which there are no entry costs), equation (1.29) holds with $\kappa = (\rho - 1) / (\theta\rho)$.

1.9.4 Proof of Result 5

We show that the steady-state change in theoretical consumption, real GDP and real consumption in response to marginal changes in variable trade costs in the model with heterogenous firms and monopolistic competition is given by expression (1.22). We assume here that fixed costs are incurred in the exporting country (i.e. $\phi = 0$ using the notation of Appendix C).

Note first that we can re-express variable profits relative to the wage in equation (1.41) as

$$\begin{aligned}\frac{\pi_{int}(z)}{W_{it}} &= \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} W_{it}^{-\rho} P_{nt}^\rho C_{nt} \\ &= \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} \left(\frac{\bar{L}_i}{1-\kappa_i}\right)^\rho C_{it}^{1-\rho} S_{int}\end{aligned}\tag{1.51}$$

where $S_{int} = \frac{P_{nt}^\rho C_{nt}}{P_{it}^\rho C_{it}}$. In deriving this expression, we have used $\frac{W_{it}}{P_{it}} = \frac{C_{it}}{\bar{L}_i} (1 - \kappa_i)$ from (1.30) and balanced trade.

Expected variable profits (relative to the wage) per entering firms are

$$\sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} dG_i(z) = \frac{1}{\rho^\rho (\rho - 1)^{1-\rho}} \left(\frac{\bar{L}_i}{1 - \kappa_i} \right)^\rho C_{it}^{1-\rho} \sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}, \quad (1.52)$$

where $Z_{int} = \int_{\Omega_{int}} z^{\rho-1} dG_i$. Free-entry in steady-state implies:

$$\hat{\beta} \sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} dG_i(z) = f_{Ei} + \hat{\beta} \sum_n [1 - G_i(\bar{z}_{int})] f_{int},$$

where $\hat{\beta} = \frac{\beta}{1-\beta(1-\delta)}$. Log-differentiating this expression with respect to changes in τ around the initial steady-state at time t , and using (1.52) yields

$$d \log C_{it} = - \frac{\sum_n \left[d \log \tau_{int} - \frac{1}{\rho-1} d \log S_{int} \right] \tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}}. \quad (1.53)$$

Here we have used an envelope condition to obtain that changes in cutoffs \bar{z}_{int} , defined by (1.42), have no first-order effects on expected profits at entry. Using $\lambda_{int} = \frac{\tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}}$, we can re-write this expression as

$$d \log C_{it} = - \sum_n \lambda_{int} d \log \tau_{int} + \frac{1}{\rho-1} \sum_n \lambda_{int} d \log S_{int}.$$

The change in world theoretical consumption using weights $s_{it} = Y_{it} / \sum_i Y_{it}$, and using $P_{it} C_{it} = Y_{it}$ from trade balance, is given by

$$\sum_i s_{it} d \log C_{it} = \sum_i s_{it} \left[- \sum_n \lambda_{int} d \log \tau_{int} + \frac{1}{\rho-1} \sum_n \lambda_{int} d \log S_{int} \right].$$

Using balanced trade (which implies $s_{it} \sum_n \lambda_{int} = s_{nt} \sum_n \lambda_{nit}$), and $d \log S_{int} = -d \log S_{nit}$, we have $\sum_i s_{it} \sum_n \lambda_{int} d \log S_{int} = 0$, so

$$\sum_i s_{it} d \log C_{it} = - \sum_i s_{it} \sum_n \lambda_{int} d \log \tau_{int}.$$

Substituting the definition of s_{it} , we obtain expression (1.22).

We now calculate the change in world real GDP to marginal changes in variable trade

costs. The aggregate PPI defined in (1.6) between $t - 1$ and t , using (1.11), is given by

$$\frac{PPI_{it}}{PPI_{it-1}} = \sum_n \bar{\lambda}_{int} \frac{PPI_{int}}{PPI_{int-1}}.$$

Log-differentiating around $\tau = \tau_0$,

$$\begin{aligned} d \log PPI_{it} &= \sum_n \lambda_{int} d \log PPI_{int} + \sum_n d \bar{\lambda}_{int} \\ &= \sum_n \lambda_{int} d \log PPI_{int} \end{aligned}$$

where we used $\sum_n \bar{\lambda}_{int} = 1$ (which implies $\sum_n d \bar{\lambda}_{int} = 0$). Changes in trade shares by continuing producers have no first-order effects on the PPI. Hence, the change in the PPI is to a first-approximation equal to that in the model with a fixed set of producers selling in each country. Following the steps used in the model with exogenous specialization, the change in world real GDP is given by expression (1.22).

Finally, consider changes in real consumption. From equation (1.14), and the definition of CPI_{int}/CPI_{int-1} , the aggregate CPI in country i between period $t - 1$ and t is given by:

$$\frac{CPI_{it}}{CPI_{it-1}} = \sum_n \bar{\Lambda}_{nit} \frac{CPI_{nit}}{CPI_{nit-1}},$$

where $\bar{\Lambda}_{nit} = \int_{\Omega_{nit}^c} p_{nit_0} q_{nit_0} dM_{it_0} / \left[\sum_n \int_{\Omega_{nit}^c} p_{nit_0} q_{nit_0} dM_{it_0} \right]$ is the date t_0 share of country i 's expenditures on goods from country n for goods that are consumed in both periods. Note that with a constant set of consumed goods, $\bar{\Lambda}_{nit} = E_{nit_0}/E_{it_0}$. Log-differentiating around $\tau = \tau_0$,

$$\begin{aligned} d \log CPI_{it} &= \sum_n \frac{E_{nit}}{E_{it}} d \log CPI_{nit} + \sum_n d \bar{\Lambda}_{nit} \\ &= \sum_n \frac{E_{nit}}{E_{it}} d \log CPI_{nit}, \end{aligned}$$

where we used $\sum_n \bar{\Lambda}_{nit} = 1$ (which implies $\sum_n d \bar{\Lambda}_{nit} = 0$). Changes in expenditure shares due to changes in the set of consumed goods have no first-order effects on the CPI. Hence, the change in the CPI is to a first-approximation equal to that in the model with a fixed set of consumed goods. Following the steps used in the model with exogenous

specialization, the change in world real consumption under balanced trade is given by expression (1.22).

1.9.5 Proof of Result 6

We now show our Result 6 on the equivalence between real consumption and theoretical consumption, country-by-country, when fixed cost of exporting are paid in the destination country ($\phi = 1$ in the notation of Appendix C), and the productivity distribution of entering firms is Pareto ($G_i(z) = 1 - z^{-\theta}$ for $z \geq 1$). We assume that trade is balanced every period, taking into account the export of goods and the export of fixed trade costs that foreign firms incurred in the domestic economy.

We start by showing that with Pareto distributed productivities, balanced trade in any country implies balanced trade both in fixed export cost services and in goods in that country. The condition of balanced trade in country i is:

$$\sum_{n \neq i} Y_{int} + \sum_{n \neq i} \frac{M_{Ent}}{\delta} W_{it} [1 - G_{it}(\bar{z}_{nit})] f_{nit} = \sum_{n \neq i} Y_{nit} + \sum_{n \neq i} \frac{M_{Eit}}{\delta} W_{nt} [1 - G_{it}(\bar{z}_{int})] f_{int}. \quad (1.54)$$

Substituting (1.48) into (1.54) implies:

$$\sum_{n \neq i} Y_{int} = \sum_{n \neq i} Y_{nit}, \quad (1.55)$$

which is the condition of balanced trade in goods.

We now derive Result 6. Balanced trade in services implies $Y_{it} = GDP_{it}$. Then, log-differentiating equation (1.30) with respect to changes in τ around the initial steady-state at time t yields:

$$d \log GDP_{it} / W_{it} = \sum_n Y_{int} d \log Y_{int} / W_{it} = 0. \quad (1.56)$$

Using $\frac{W_{it}}{P_{it}} = \frac{C_{it}}{L_i} (1 - \kappa_i)$ from (1.30) and balanced trade in goods we can re-express Y_{int}

as:

$$\begin{aligned} Y_{int} &= \int_{\Omega_{int}} p_{int} q_{int} dM_{it} \\ &= \bar{\varphi} \frac{M_{Eit}}{\delta} W_{it} \tau_{int}^{1-\rho} S_{int} Z_{int} C_{it}^{1-\rho} , \end{aligned}$$

where $\bar{\varphi} \equiv \left(\frac{\bar{L}_i}{1-\kappa_i} \right)^\rho / [\rho^{\rho-1} [\rho-1]^{1-\rho}]$ and $S_{int} = \frac{P_{nt}^\rho C_{nt}}{P_{it}^\rho C_{it}}$ as in Appendix D. Log-differentiating we obtain:

$$\begin{aligned} d \log Y_{int}/W_{it} &= (1-\rho) d \log \tau_{int} + d \log S_{int} \\ &\quad + d \log Z_{int} + (1-\rho) d \log C_{it} , \end{aligned} \tag{1.57}$$

substituting into (1.56), we can write the change in welfare based consumption as:

$$d \log C_{it} = - \sum_n \frac{Y_{int}}{Y_{it}} \left[d \log \tau_{int} + \frac{d \log S_{int}}{1-\rho} + \frac{d \log Z_{int}}{1-\rho} \right] .$$

Log-differentiating (1.55), substituting (1.57), and some algebra gives:

$$\sum_n Y_{int} \left[d \log \tau_{int} + \frac{d \log S_{int}}{1-\rho} + \frac{d \log Z_{int}}{1-\rho} \right] = \sum_n Y_{nit} \left[d \log \tau_{nit} + d \log \frac{W_{nt}}{W_{it}} + \frac{d \log Z_{nit}}{1-\rho} \right] ,$$

then:

$$d \log C_{it} = - \sum_n \frac{Y_{nit}}{Y_{it}} \left[d \log \tau_{nit} + d \log \frac{W_{nt}}{W_{it}} + \frac{d \log Z_{nit}}{1-\rho} \right] . \tag{1.58}$$

Finally, using the Pareto form for G , we have:

$$Z_{int} = \frac{\theta}{\theta+1-\rho} \bar{z}_{int}^{\rho-1-\theta} , \tag{1.59}$$

log differentiating (1.47) and (1.59) we obtain:

$$d \log Z_{nit} = (\rho-1-\theta) [d \log \tau_{nit} + d \log W_{nt}/W_{it} + d \log C_{it}] . \tag{1.60}$$

Substituting (1.60) into (1.58) and using balanced trade in goods:

$$d \log C_{it} = - \sum_n \frac{E_{nit}}{E_{it}} [d \log \tau_{nit} + d \log W_{nt}/W_{it}] ,$$

where $E_{int} = Y_{int}$.

As shown in Appendix D, the change in the CPI is given by $d \log CPI_{it} = \sum_n \frac{E_{nit}}{E_{it}} d \log CPI_{nit}$, so the change in real consumption is given by expression (1.18). Substituting $d \log CPI_{int}$ in equation (1.18), we obtain:

$$d \log RC_{it} = \sum_n \frac{E_{nit}}{E_{it}} [-d \log \tau_{nit} - d \log W_{nt}/W_{it}], \quad (1.61)$$

which coincides with $d \log C_{it}$.

1.9.6 Endogenous quality choice

In this appendix we consider the extended model with endogenous quality choice under endogenous specialization and imperfect competition. We first derive the result that, if prices in the deflators are not adjusted for changes in quality, then changes in world real consumption and theoretical consumption are equal, to a first-order approximation, in response to marginal changes in trade costs. The logic to obtain this result is very similar to that used to obtain Result 5 in Appendix D. Next, we derive condition (1.29) in this version of our model.

Following the same steps as those used to derive expression (1.51), variable profits (relative to the wage) for a firm from country i with productivity z selling in country n are given by

$$\frac{\pi_{int}(z)}{W_{it}} = \frac{a_{int}(z) z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} \left(\frac{\bar{L}_i}{1-\kappa_i} \right)^\rho C_{it}^{1-\rho} S_{int},$$

where $a_{int}(z)$ denotes the quality choice of a firm in country i with productivity z selling in country n in period t . In an interior equilibrium with selection, the cutoff \bar{z}_{int} is given by $\frac{\pi_{int}(\bar{z}_{int})}{W_{it}} - f_{int} - h(z; a_{int}(\bar{z}_{int})) = 0$. Profits (relative to the wage) in period t across all destinations, inclusive of fixed costs and quality costs are given by

$$\sum_n \mathbb{I}(z \geq \bar{z}_{int}) \left[\frac{a_{int}(z) z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} \left(\frac{\bar{L}_i}{1-\kappa_i} \right)^\rho C_{it}^{1-\rho} S_{int} - f_{int} - h(z; a_{int}(z)) \right],$$

where $\mathbb{I}(z \geq \bar{z}_{int}) = 1$ if $z \geq \bar{z}_{int}$ and zero otherwise. The static first-order condition for

$a_{int}(z)$ is given by

$$\frac{a_{int} z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho (\rho-1)^{1-\rho}} \left(\frac{\bar{L}_i}{1-\kappa_i} \right)^\rho C_{it}^{1-\rho} S_{int} - h_2(z; a_{int}) = 0,$$

where $h_2(z; a_{int})$ denotes the derivative of h with respect to the second argument.

The free-entry condition in steady-state is given by

$$\hat{\beta} \sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} dG_i(z) = f_{Ei} + \hat{\beta} \sum_n [1 - G_i(\bar{z}_{int})] f_{int} + \int_{\Omega_{int}} h(z; a_{int}(z)) dG_i(z). \quad (1.62)$$

Log-differentiating the free-entry condition in the steady-state, using the first-order conditions for \bar{z}_{int} and $a_{int}(z)$, we obtain the same expression for the change in theoretical consumption, (1.53), as in the model without quality choice, where $Z_{int} = \int_{\Omega_{int}} a_{int}(z) z^{\rho-1} dG_i$. That is, from the envelope conditions, changes in cutoffs and quality choices have no first-order effects on expected profits of entering firms. From expression (1.53), we use the same steps as those used in Appendix D to obtain expression (1.22).

The extension of Result 6 (the equivalence between real consumption and theoretical consumption, country-by-country) under stronger assumptions, is derived in the Online Appendix.

Deriving the share of profits in total revenues

Deriving assumption (1.29) when $\beta \rightarrow 1$ is straightforward. We now show that this assumption holds if $\beta < 1$ when productivities are Pareto distributed, quality is destination-country specific, the cost of choosing quality a for a firm with productivity z is $h(z, a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^\gamma$, and a fraction ε of the innovation costs are incurred in the source country and the remaining fraction $(1 - \varepsilon)$ are incurred in the destination country. We omit time subscripts to simplify notation.

Under these assumptions, the optimal quality choice a_{in} for a firm with productivity z satisfies:

$$\pi_{in}(z) = \gamma_0 W_i^\varepsilon W_n^{1-\varepsilon} \bar{h}(z) a_{in}^\gamma(z).$$

Aggregate innovation costs, using the optimality condition and $\pi_{in} = \frac{1}{\rho-1} W_i l_{in}$, are:

$$\begin{aligned} \frac{M_{Ei}}{\delta} \sum_n \int_{\Omega_{in}} \frac{\gamma_0}{\gamma} W_i^\varepsilon W_n^{1-\varepsilon} \bar{h}(z) a_{in}^\gamma(z) dG_i(z) &= \frac{1}{\gamma(\rho-1)} \frac{M_{Ei}}{\delta} \sum_n \int_{\Omega_{in}} W_i l_{in}(z) dG_i(z) \\ &= \frac{1}{\gamma(\rho-1)} W_i L_i. \end{aligned}$$

The relation between variable and fixed labor costs is still given by equation (1.49).

Aggregate entry costs, calculated using (1.62), (1.43) and (1.49), are

$$\begin{aligned} W_i f_{Ei} M_{Ei} &= \frac{\beta\delta}{1-\beta(1-\delta)} \left[Y_i - W_i L_i - \sum_n \frac{\gamma_0}{\gamma} W_i^\varepsilon W_n^{1-\varepsilon} \int_{\Omega_{in}} a_{in}^\gamma(z) \bar{h}(z) dG_i(z) \right. \\ &\quad \left. - \sum_n W_i^\phi W_n^{1-\phi} [1 - G_i(\bar{z}_{in})] f_{in} \right] \\ &= \frac{\beta\delta}{1-\beta(1-\delta)} \frac{\gamma(\rho-1) - \theta}{\gamma\theta} \frac{1}{\rho-1} W_i L_i. \end{aligned} \quad (1.63)$$

Finally, combining (1.43), (1.49) and (1.63), aggregate profits are

$$\begin{aligned} \Pi_i &= Y_i - W_i L_i - W_i M_{Ei} f_{Ei} \\ &\quad - \frac{M_{Ei}}{\delta} \left[\sum_n \frac{\gamma_0}{\gamma} W_i^\phi W_n^{1-\phi} [1 - G_i(\bar{z}_{in})] f_{in} + \sum_n W_i^\varepsilon W_n^{1-\varepsilon} / \gamma \int_{\Omega_{in}} a_{in}^\gamma(z) \bar{h}(z) dG_i(z) \right] \\ &= \frac{\gamma(\rho-1) - \theta}{\gamma\theta\rho} \frac{1-\beta}{1-\beta(1-\delta)} Y_i \end{aligned}$$

Therefore, $\kappa_i = \left(\frac{\gamma(\rho-1)-\theta}{\gamma\theta\rho} \right) \left(\frac{1-\beta}{1-\beta(1-\delta)} \right)$. The steady-state mass of entering firms is given by

$$M_{Ei} = \frac{\beta\delta}{1-\beta[1-\delta]} \frac{\gamma[\rho-1] - \theta}{\gamma\theta} \frac{\bar{L}_i}{f_{Ei}(1-\kappa_i)\rho}$$

where we used (1.43), (1.44), and (1.63). Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state.

Finally, aggregate variable profits gross of entry costs are $\Pi_i + W_i f_{Ei} M_{Ei} = \frac{\gamma(\rho-1)-\theta}{\gamma\theta} Y_i$. Hence, in the model with restricted entry (so that there are no entry costs), equation (1.29) holds with $\kappa_i = [\gamma(\rho-1) - \theta] / \gamma\theta$.

1.9.7 Multiple factors of production

In this appendix we derive some results in the extension of the model that allows for multiple factors of production. We first show that GDP is proportional to total labor payments, and we then derive the equivalence between world real GDP, real consumption, and theoretical consumption. As in the baseline model, we assume that condition (1.29) is satisfied – it is straightforward to extend the proofs in Appendix C to this extension–.

We first show that production labor is proportional to aggregate labor supply, and that current-dollar GDP is proportional to aggregate labor payments. We also show that theoretical consumption is proportional to the aggregate production of the final good. We can write a modified version of equation (1.39) as

$$p_{int}q_{int} = \frac{\rho}{\rho-1}\alpha_L^{-1}W_{it}l_{int} ,$$

where we used (1.33) and the production function. Total revenues of active firms in country i are given by $Y_{it} = \frac{\rho}{\rho-1}L_{it}W_{it}/\alpha_L$, where L_{it} denotes aggregate labor used in variable production defined above. In the presence of intermediate inputs, total revenues are given by:

$$Y_{it} = W_{it}\bar{L}_i + \Pi_{it} + \sum_{j=1}^J R_{j,it}K_{it} .$$

In combination with (1.29), this implies, $Y_{it} = \frac{1}{1-\kappa_i} \left[W_{it}\bar{L}_i + \sum_{j=1}^J R_{j,it}K_{it} \right]$, or

$$Y_{it} = \frac{L_{it}W_{it}}{1-\kappa_i} \left[\frac{\bar{L}}{L_{it}} + \sum_i \frac{R_{j,it}K_{j,it}}{W_{it}L_{it}} \right] = \frac{L_{it}W_{it}}{1-\kappa_i} \left[\frac{\bar{L}}{L_{it}} + \frac{1-a_L}{\alpha_L} \right] .$$

In combination with $Y_{it} = \frac{\rho}{\rho-1}L_{it}W_{it}/\alpha_L$, we obtain that variable production labor is a constant share of total labor,

$$\frac{\bar{L}}{L_{it}} = \frac{\rho(1-\kappa_i) - (1-\alpha_L)(\rho-1)}{\alpha_L(\rho-1)} ,$$

and that revenues are proportional to aggregate wages,

$$Y_{it} = \frac{L_{it}W_{it}}{1-\kappa_i} \left[\frac{\bar{L}_i}{L_{it}} + \frac{1-a_L}{\alpha_L} \right] = \frac{\rho}{[1-\kappa_i]\rho - (1-a_L)(\rho-1)} W_{it}\bar{L}_i . \quad (1.64)$$

Note that intermediate inputs are also proportional to aggregate revenues:

$$[1 - \kappa_i] Y_{it} - W_{it} \bar{L}_i = \sum_{j=1}^J R_{j,it} K_{it}$$

so $Y_{it} = \frac{\rho}{(1-\alpha_L)(\rho-1)} \sum_{j=1}^J R_{j,it} K_{it}$. Finally, note that together with balanced trade this implies that consumption expenditures are proportional to aggregate labor payments and to expenditures in intermediate inputs:

$$Y_{it} = P_{it} Q_{it} = P_{it} C_{it} + \sum_{j=1}^J R_{j,it} K_{it} = \frac{\rho}{\rho - (1 - \alpha_L)(\rho - 1)} P_{it} C_{it} ,$$

so

$$Q_{it} = \frac{\rho}{\rho - (1 - \alpha_L)(\rho - 1)} C_{it} . \quad (1.65)$$

We now show the equivalence, to a first-order approximation, between world GDP, measured real consumption, and theoretical consumption. Variable profits are given by

$$\pi_{int}(z) = \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho [\rho - 1]^{1-\rho}} c_{it} \left[\frac{c_{it}}{P_{nt}} \right]^{-\rho} Q_{nt} . \quad (1.66)$$

The threshold \bar{z}_{int} satisfies $\pi_{int}(\bar{z}_{int}) = W_{it} f_{int}$. Expected profits at entry, using equation (1.66) are given by:

$$\begin{aligned} \sum_n \int_{\Omega_{int}} \pi_{int}(z) dG_i(z) &= c_{it} \sum_n \int_{\Omega_{int}} \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho [\rho - 1]^{1-\rho}} c_{it}^{-\rho} \Gamma_{nt} dG_i(z) \\ &= [c_{it}/W_{it}]^{1-\rho} \frac{W_{it} Q_{it}}{\rho^\rho [\rho - 1]^{1-\rho}} \left[\frac{W_{it}}{P_{it}} \right]^{-\rho} \sum_n \tau_{int}^{1-\rho} \int_{\Omega_{int}} z^{\rho-1} S_{int} dG_i(z) , \end{aligned}$$

where $\Gamma_{nt} = P_{nt}^\rho Q_{nt}$, and $S_{int} = \frac{\Gamma_{nt}}{\Gamma_{it}}$. Using (1.64) and (1.65) we can write this as:

$$\sum_n \int_{\Omega_{int}} \pi_{int} dG_i(z) = W_{it} \frac{[c_{it}/W_{it}]^{1-\rho} \bar{\psi} C_{it}^{1-\rho} \bar{L}_i^\rho}{\rho^\rho [\rho - 1]^{1-\rho}} \sum_n \tau_{int}^{1-\rho} \int_{\Omega_{int}} z^{\rho-1} S_{int} dG_i(z) , \quad (1.67)$$

where $\bar{\psi} = \frac{\rho}{\rho-1} \frac{[1-\kappa_i] \frac{\rho}{\rho-1} - 1 + \alpha_L}{[1/(\rho-1) + \alpha_L]^{1-\rho}}$ is a constant. Free-entry in steady-state implies:

$$\hat{\beta} \sum_n \int_{\Omega_{int}} \pi_{int}(z) dG_i(z) = W_{it} f_{Ei} + \hat{\beta} \sum_n W_{it} [1 - G_i(\bar{z}_{int})] f_{int},$$

which can be rewritten as

$$[c_{it}/W_{it}]^{1-\rho} \frac{\bar{\kappa} C_{it}^{1-\rho} \bar{L}_i^\rho}{\rho^\rho [\rho-1]^{1-\rho}} \sum_n \tau_{int}^{1-\rho} S_{int} Z_{int} = \frac{f_{Ei}}{\hat{\beta}} + \sum_n [1 - G_i(\bar{z}_{int})] f_{int}.$$

Log-differentiating this expression in steady-state at time t and using the envelope condition for the cutoffs yields

$$d \log C_{it} = - \frac{\sum_n \left[d \log \tau_{int} - \frac{1}{\rho-1} d \log S_{int} \right] \tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}} - d \log [c_{it}/W_{it}] .$$

Note that $\lambda_{int} = \frac{\tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}}$, so the change in world theoretical consumption using the weights s_{it} defined above is:

$$\sum_i s_{it} d \log C_{it} = \sum_i s_{it} \left[\begin{aligned} & - \sum_n \lambda_{int} d \log \tau_{int} \\ & + \frac{1}{\rho-1} \sum_n \lambda_{int} d \log S_{int} - d \log c_{it}/W_{it} \end{aligned} \right] .$$

Following the steps in Appendix C, we can show that $\sum_i s_{it} \sum_n \lambda_{int} d \log S_{int} = 0$, which implies:

$$\sum_i s_{it} d \log C_{it} = - \sum_i s_{it} \left[\sum_n \lambda_{int} d \log \tau_{int} + d \log c_{it}/W_{it} \right] .$$

Log differentiating the expression in (1.36) and doing the weighted sum across countries we obtain the equivalence, to a first-order approximation, between world theoretical consumption and world real GDP. To show the equivalence between world real GDP and world real consumption we follow the same steps used in our baseline model, together with the fact that current-dollar GDP is proportional to current-dollar consumption.

Note that the expression for the change in world real GDP, real consumption, and theoretical consumption differs from that in the model with only labor as a factor of production, in the presence of changes in the marginal cost to wage ratio, c_{it}/W_{it} . From expression (1.34), changes in this price ratio are driven by changes in aggregate quantities of non-labor factors of production.

The extension of Result 6 (the equivalence between real consumption and theoretical consumption, country-by-country) under stronger assumptions, is derived in an Online Appendix.

Figure 1.1: Fixed costs paid in the home country

Figure 1: Gains from reductions in variable trade costs, Fixed costs in exporting country

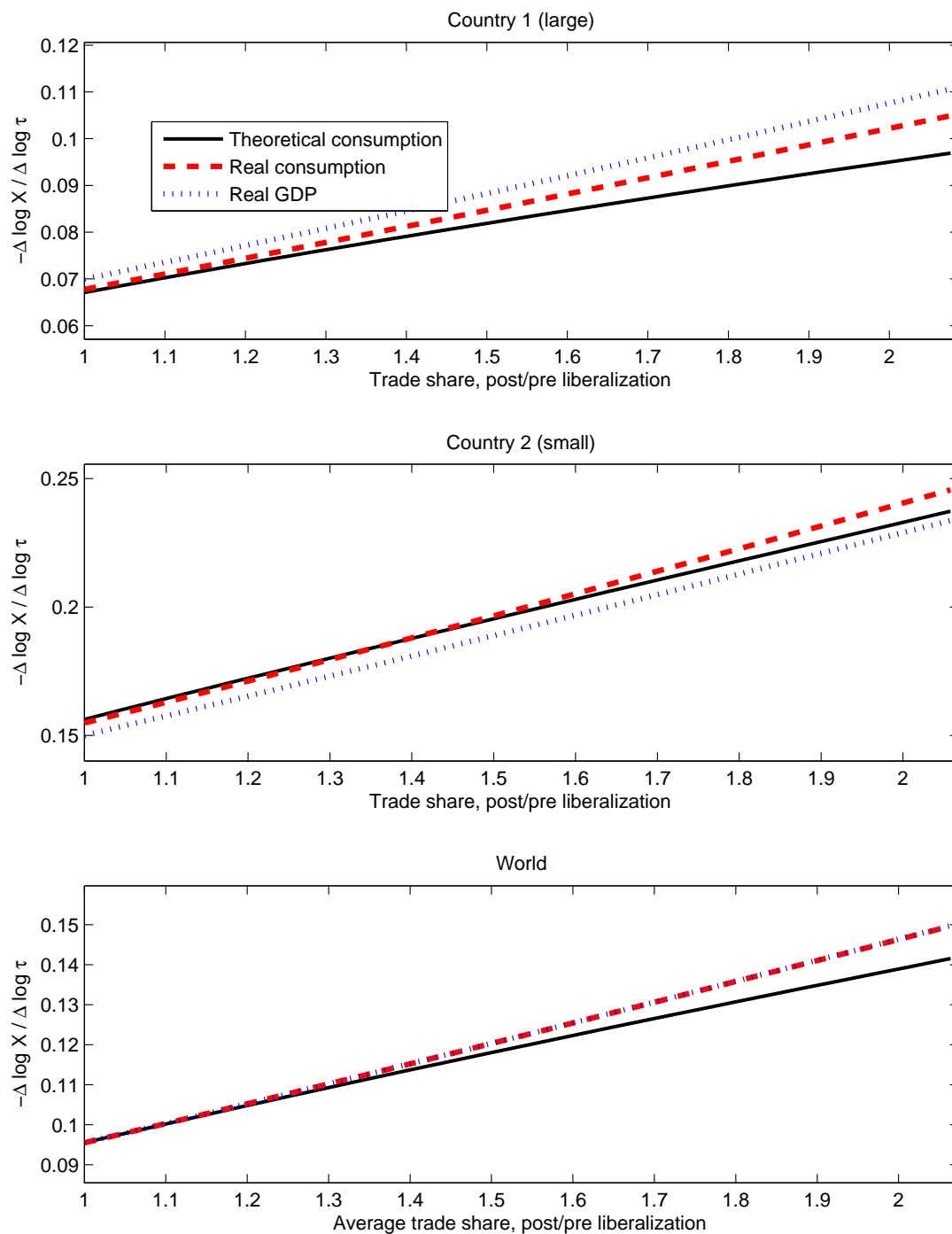
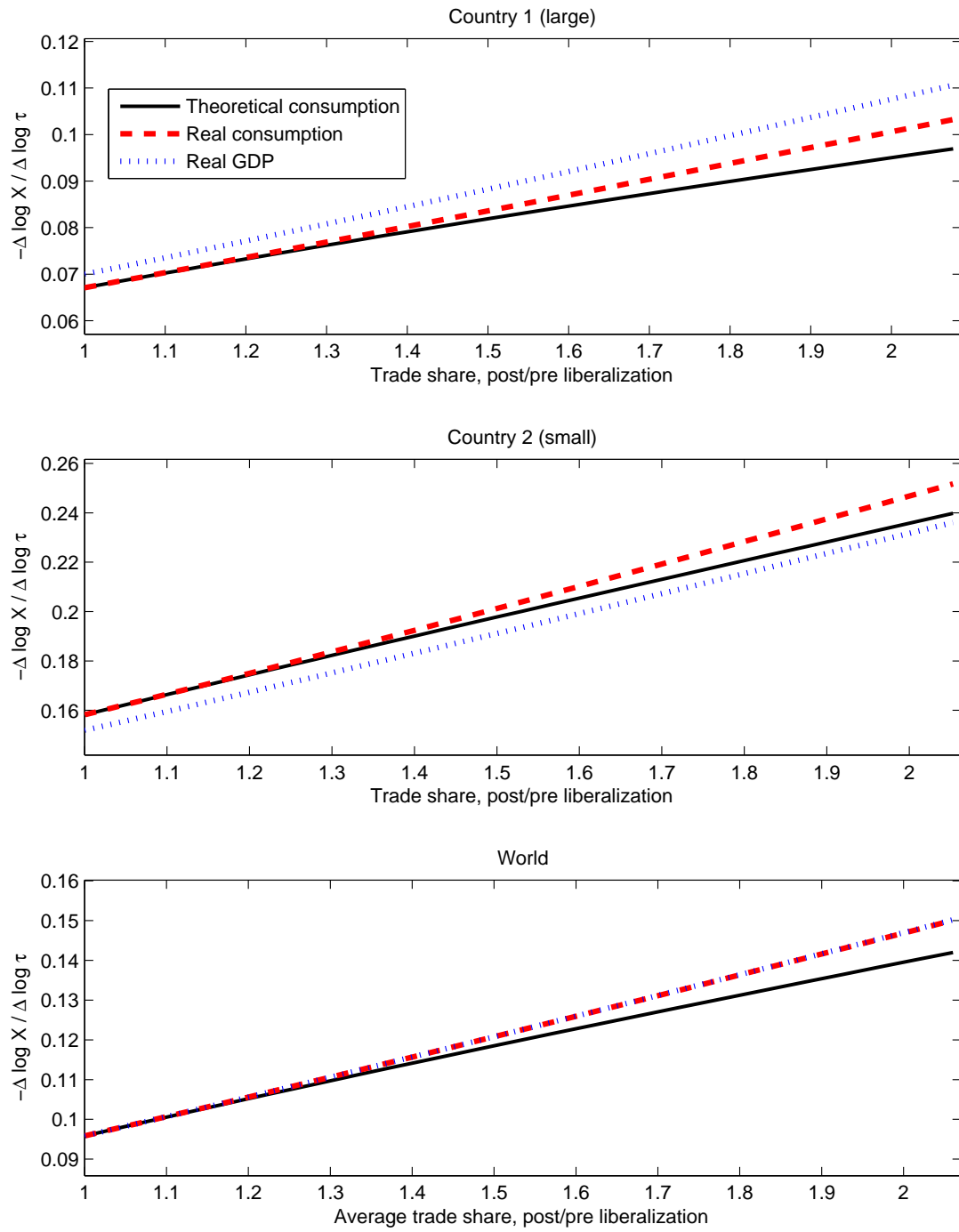


Figure 1.2: Fixed costs paid abroad

Figure 2: Gains from reductions in variable trade costs, Fixed costs in importing country



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CHAPTER 2

Importing Skill Biased Technology

2.1 Introduction

The production of capital equipment—such as computers and industrial machinery—is concentrated among a small group of countries, and many countries import a large share of their equipment;¹ see e.g. Eaton and Kortum (2001) [18]. Although the evidence is not definitive, a large body of research has argued that capital-skill complementarity is an important feature of technology.² Taking this evidence at face value, it is possible that international trade has important effects on the skill premium through its impact on the accumulation of capital equipment. The first goal of this paper is to provide a tractable framework for evaluating this effect. Given the lack of consensus on the extent of capital-skill complementarity, the second goal is to obtain a transparent analytic mapping between the extent of capital-skill complementarity and the strength of this effect. The final goal is to quantify the importance of this effect for a large set of countries.

To do so, we embed a production function that allows for capital-skill complementarity as in Krusell et al. (2000) [32], henceforth KORV, into the multi-country model of international trade developed in Eaton and Kortum (2002) [19], henceforth EK. With capital-skill complementarity, an increase in the stock of capital equipment raises the demand for skilled relative to unskilled labor. With international trade, the aggregate stock of capital equipment in one country depends on foreign and domestic productivities and labor endowments and on trade costs between every pair of countries. In our model

¹For example, 80% of the world's capital equipment production occurred in just eight countries in the year 2000: the U.S., Japan, Germany, China, France, Korea, the U.K., and Italy. The share of domestic absorption imported from abroad in the equipment sector in the year 2000 was 73% in the U.K., 81% in Australia, 84% in Chile, and 96% in Cameroon. Source: our calculation using NBER-UN world trade data described in Feenstra et. al. (2005) [21] and Unido Industrial Statistics.

²See e.g. Katz and Autor (1999) [28], who summarize the literature documenting a positive correlation between the use of computer-based technologies and employment of skilled labor within industries, firms, and plants.

as in EK, changes in all trade costs and foreign variables affect a country's steady-state stock of capital equipment only through changes in its domestic sectoral expenditure shares, i.e., the share of its sectoral absorption that is produced domestically.³ Using this result, we provide simple analytic expressions relating steady-state changes in (i) the skill premium, (ii) the real wage of skilled workers, and (iii) the real wage of unskilled workers to changes in domestic expenditure shares, domestic productivities, and domestic labor endowments.

Three parameters are key in shaping the elasticities of (i) – (iii) with respect to changes in observable domestic expenditure shares in each sector. The first is the elasticity of trade with respect to variable trade costs, which depends only on the dispersion of productivities within sectors in our Ricardian model. As in standard quantitative trade models, this parameter shapes the extent to which observable changes in domestic sectoral expenditure shares lead to changes in the domestic stock of capital equipment. The other two important parameters are production function elasticities that jointly determine the extent of capital-skill complementarity and the elasticity of substitution between skilled and unskilled labor. In equilibrium these parameters shape the response of the skill premium to a given change in the stock of capital equipment. We pursue several strategies similar to those in KORV to parameterize these elasticities using structural equations delivered by the model and calibrated using US and Chilean data.

We use our parameterized model to quantify the impact of trade, through capital-skill complementarity, on the skill premium and the real wages of skilled and unskilled workers. We conduct two counterfactuals exploiting the simple structure of our solution, which allows us to conduct these exercises country-by-country. In the first counterfactual, we hold all technologies and factor endowments fixed and raise all trade costs to infinity. Through this counterfactual we quantify how much each country's skill premium and both of its real wages would change if it were moved to autarky. In the second counterfactual, we hold a given country's technologies and factor endowments fixed and change its domestic expenditure shares from their observed levels in 2000 to those in 1963. This second counterfactual measures, up to a first-order approximation, the response of real

³This result applies in EK to a country's stock of domestic consumption (average real wage). Arkolakis, Costinot, and Rodriguez-Clare (2012) [4] show that this result holds across a wide range of quantitative trade models. In section 2.3.3 we discuss the generality of our results.

wages in a given country to all changes over this time period in technologies, endowments, and trade costs—both domestic and foreign—relative to what the response to these same changes in primitives would have been had that country been in autarky over this time period.

Given our baseline parameter values, we find that while international trade raises the real wage of both skilled and unskilled workers, it benefits skilled workers disproportionately: in our counterfactuals the log point change in the real wage is more than two times greater for skilled workers than for unskilled workers in the median country. While international trade plays an important role in shaping the skill premium through capital-skill complementarity, we find that its importance varies widely across countries in our sample. For example, moving from the trade levels observed in the year 2000 to 1963, or the first year with available data, would imply a reduction in the skill premium of 0.05 log points (about 5%) for the median country in our sample. The decrease in the skill premium is relatively small in the US (0.04 log points), which has a comparative advantage in capital equipment,⁴ and is much larger in countries that rely heavily on imports for their capital equipment, including developed countries such as Canada (0.17 log points) and developing countries such as Latvia (0.26 log points).⁵

We conduct sensitivity analyses taking advantage of our analytic results and our exact quantitative solution. In each exercise we report the elasticity of the skill premium to changes in domestic sectoral expenditure shares resulting from alternative parameter values (using our first-order approximation) as well as the median change in the skill premium for both of our counterfactual exercises (using the exact solution). We emphasize, in particular, alternative values for the parameters that control capital-skill complementarity, since our baseline calibration strategy is subject to the same set of issues that have led to an active debate on the strength of this force; see e.g. Acemoglu (2002) [1] and our discussion in section 2.4.4.

⁴The contribution of trade predicted by our model is small compared to the observed change in the US skill premium, which is roughly 0.3 log points over this period.

⁵For most countries we consider there are no consistent measures of changes in the skill premium, and producing such consistent measures for a large set of countries is out of the scope of this paper. Krueger, Perri, Pistaferri, and Violante (2010) [31] document college premium changes in 9 countries over different years. For 4 out of 9 of those countries, the skill premium fell. Hence, comparing the impact of trade to the overall change in the skill premium does not make sense for these 4 countries. According to our results, in the absence of trade the reduction in the skill premium would have been larger in these 4 countries.

Our paper builds on a growing literature empirically documenting the impact of international trade on the skill intensity of production—see e.g. Verhoogen (2008) [47], Bloom, Draca, and Van Reenen (2011) [6], Bustos (2011) [10], and Koren and Csillag (2011) [30]—using detailed firm, plant, and sector-level data. These papers provide empirical support for the hypothesis that international trade can generate skill-biased technological change, as posited by, e.g., Acemoglu (2003) [2], Thoenig and Verdier (2003) [41], and Yeaple [49] (2005). Our contribution is to embed a mechanism studied in these papers into a multi-country general equilibrium trade model.⁶

To isolate the impact of importing equipment on real wages and the skill premium in a simple and transparent way, we abstract from many other mechanisms through which trade affects relative wages. Hence, we do not view our paper as providing a full quantitative assessment of the role of international trade in shaping the skill premium.⁷

Our paper is most closely related to Parro (2012) [35], who uses a similar model that incorporates capital-skill complementarity to study the impact of trade on the skill premium.⁸ There are two main differences between these papers. First, we provide simple expressions for steady-state changes in a country’s skill premium and both of its real wages, up to first-order approximations, which yield analytic mappings from parameters to quantitative results. This is particularly useful given the extent of uncertainty regarding a number of key parameters, especially the degree of capital-skill complementarity. Second, the counterfactuals that we perform are different. Whereas we study the overall impact of given changes in trade patterns on the skill premium (which can be understood in terms of changes in primitives, as summarized above), Parro feeds into his model estimated changes in trade costs and sector-level technologies. Beyond differences in their nature, a benefit of our counterfactuals is that they can be solved country-by-country without solving the full world-wide general equilibrium; hence, our counterfactual results

⁶The approach has served as a basic building block in a number of other macroeconomic models of inequality; see e.g. Polgreen and Silos (2008) [36] and Jaimovich, Pruitt, and Siu (2009) [27]

⁷In a related paper, Burstein and Vogel (2012) [9] study the impact of international trade on the skill premium arising from two mechanisms from which we abstract: (i) the Stolper-Samuelson effect and (ii) within-sector factor reallocation in the presence of skill-biased productivity. The presence of firm heterogeneity in skill intensity allows Burstein and Vogel (2012) [9] to discipline their parameters using cross-sectional firm-level evidence at the expense of losing analytic gravity equations and, hence, simple analytic results on changes in the skill premium.

⁸For an earlier theoretical treatment of trade in skill-complementary capital in a neo-classical growth model, see Stokey (1996) [40].

for a given country are not sensitive to most of the parameter values we assign to its trading partners. A benefit of Parro's counterfactuals is that, given his estimates, he can answer a broader range of questions such as the impact on the skill premium in each country of separately changing trade costs and sector-level technologies.

2.2 The Model

Overview: We consider a world economy featuring I countries, indexed by $i = 1, \dots, I$. Within each country, a representative household acquires utility from consumption of manufactured goods and services. Each country is endowed with H_i and L_i efficiency units of skilled and unskilled labor, respectively. Heterogeneous producers of intermediate goods use labor in combination with capital equipment, capital structures, and intermediate inputs. To incorporate capital-skill complementarity, we allow for the elasticity of substitution between skilled labor and capital equipment to differ from that between unskilled labor and equipment.

Producers differ in terms of productivity and the sector in which they produce. There are three sectors, indexed by j : (i) a manufacturing sector, $j = M$, in which firms produce tradable goods that are used for consumption and as intermediate inputs; (ii) a service sector, $j = S$, in which firms produce non-tradable goods that are used for consumption, intermediate inputs, and investment in structures; and (iii) a capital equipment sector, $j = E$, in which firms produce tradable goods that are used for investment in capital equipment.⁹ Tradable goods are subject to variable iceberg international trade costs. All labor and goods markets are perfectly competitive.

Preferences: Utility of the representative household is given by

$$\sum_{t=0}^{\infty} \beta^t u \left(C_{i,t}(M)^\phi C_{i,t}(S)^{1-\phi} \right),$$

where $C_{i,t}(M)$ and $C_{i,t}(S)$ denote consumption of manufactured goods and services, respectively, $u(\cdot)$ is a concave sub-utility function defined over aggregate consumption, $\phi \in [0, 1]$ is the share of manufactured goods in consumption, and $\beta \in (0, 1)$ is the

⁹We abstract from government, agriculture, and mining.

discount rate. The household's budget constraint equates consumption and investment expenditures (investment is discussed below) with labor income, payments to capital, and the value of net exports. Given that our steady-state results do not depend on the value of the trade balance, we do not make assumptions on the availability of international financial assets. Given that we focus our attention on steady-state equilibria, in what follows we mostly abstract from time subscripts.

Sectoral output: Sector j uses a continuum of intermediate goods, each indexed by $\omega \in [0, 1]$, according to a CES production function with country- and sector-specific elasticity of substitution $\eta_i(j) > 1$,

$$Y_i(j) = \left\{ \int_0^1 q_i(\omega, j)^{[\eta_i(j)-1]/\eta_i(j)} d\omega \right\}^{\eta_i(j)/[\eta_i(j)-1]}, \quad (2.1)$$

where $q_i(\omega, j)$ is consumption of intermediate good (ω, j) in country i . Each intermediate good (ω, j) is potentially produced in every country.

Output from the manufacturing sector can be used for consumption, $C_i(M)$, and intermediate inputs, $X_i(M)$:

$$Y_i(M) = C_i(M) + X_i(M). \quad (2.2)$$

Output from the service sector can be used for consumption, $C_i(S)$, intermediate inputs, $X_i(S)$, and structures investment, $I_i(S)$:

$$Y_i(S) = C_i(S) + X_i(S) + I_i(S). \quad (2.3)$$

Output from the equipment sector is used only for equipment investment, $I_i(E)$:

$$Y_i(E) = I_i(E). \quad (2.4)$$

The aggregate law of motion of structures and equipment is

$$K_{i,t+1}(j) = [1 - \delta_i(j)] K_{i,t}(j) + I_{i,t}(j), \text{ for } j = S, E,$$

where we have re-introduced time subscripts to indicate the dynamics, and where $\delta_i(j) \in (0, 1)$ is the depreciation rate of capital of type $j = S, E$ in country i .

Production of intermediate goods: All producers of intermediate good (ω, j) in country i produce according to the following constant returns to scale production function:

$$y_i(\omega, j) = A_i(j) z_i(\omega, j) (x_S^{\varepsilon_i} x_M^{1-\varepsilon_i})^{1-\zeta_i} k_S^{\alpha_i \zeta_i} \times \left\{ \mu_i^{\frac{1}{\sigma}} l^{\frac{\sigma-1}{\sigma}} + (1 - \mu_i)^{\frac{1}{\sigma}} \left[\lambda_i^{\frac{1}{\rho}} k_E^{\frac{\rho-1}{\rho}} + (1 - \lambda_i)^{\frac{1}{\rho}} h^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho(\sigma-1)}{(\rho-1)\sigma}} \right\}^{\frac{\sigma(1-\alpha_i)\zeta_i}{\sigma-1}} \quad (2.5)$$

Producers combine intermediate inputs (of services, x_S , and manufactured goods, x_M) with structures, k_S , capital equipment, k_E , unskilled labor, l , and skilled labor h . The share of value added in gross output is given by ζ_i . As discussed in more detail below, the parameters σ and ρ determine the elasticities of substitution between capital equipment, unskilled labor, and skilled labor. A low value of ρ relative to σ implies that capital equipment is less substitutable with skilled labor than with unskilled labor. In particular, when $\sigma > \rho$ the production function exhibits capital-skill complementarity.¹⁰

Productivity of all country i producers in (ω, j) is given by the product of a country-sector-specific term, $A_i(j)$, shared by all sector j producers in country i , and a country-intermediate-good-specific productivity, $z_i(\omega, j)$, shared by all (ω, j) intermediate good producers in country i . The country-intermediate-good-specific productivity is equal to $z_i(\omega, j) = u^{-\theta(j)}$, where u is an *i.i.d* random variable that is exponentially distributed with mean and variance 1. A higher value of $\theta(j)$ increases the dispersion of productivities across producers within sector j .

The production function (2.5) extends that in KORV to include (i) intermediate inputs; (ii) differences in productivities across sectors, as in a standard Ricardian model, so that countries can have sectoral comparative advantages; and (iii) exponentially distributed country-intermediate-good-specific productivities within a sector, as in EK, so that our multi-country framework remains tractable. In an extension we allow for skill-biased technical change by incorporating exogenous trend growth in the productivity of the composite of skilled labor and capital equipment relative to unskilled labor. While

¹⁰We use a nested CES so that the elasticities are constant globally. We follow the literature in nesting equipment and skilled labor together.

our analytic results are unchanged, our parameter values depend on this trend growth.

International trade: Delivering a unit of intermediate good (ω, j) from country i to country n requires producing $\tau_{in}(j) \geq 1$ units of that good in country i , where $\tau_{ii}(j) = 1$. We assume that services are not tradable, so that $\tau_{in}(S)$ is infinite for all $i \neq n$.

Equilibrium: Producers hire unskilled and skilled labor at wages w_i and s_i , respectively, and rent structures and capital equipment at rental rates v_i and r_i , respectively. The skill premium in country i is defined as s_i/w_i . To construct prices, it is useful to define the unit cost of producers of intermediate good (ω, j) producing in country i and selling in country n , $c_{in}(\omega, j)$, where

$$c_{in}(\omega, j) = \frac{c_i \tau_{in}(j)}{A_i(j) z_i(\omega, j)}.$$

Here, c_i is the unit cost of production for the domestic market of a producer of any intermediate (ω, j) in country i with productivity $A_i(j) z_i(\omega, j) = 1$, and is given by:

$$c_i = \kappa_i [P_i(S)^{\varepsilon_i} P_i(M)^{1-\varepsilon_i}]^{1-\zeta_i} v_i^{\alpha_i \zeta_i} \times \left\{ \mu_i w_i^{1-\sigma} + (1-\mu_i) [\lambda_i r_i^{1-\rho} + (1-\lambda_i) s_i^{1-\rho}]^{\frac{1-\sigma}{1-\rho}} \right\}^{\frac{(1-\alpha_i)\zeta_i}{1-\sigma}}$$

where κ_i is a constant, and $P_i(j)$ is the aggregate price of output in sector j .¹¹

The price of intermediate good (ω, j) in country n is:

$$p_n(\omega, j) = \min_i \{c_{in}(\omega, j)\},$$

where we have used the fact that good (ω, j) is perfectly substitutable across all potential source countries that can supply the good to country n . The aggregate price of sector j output in country n is given by

$$P_n(j) = \left[\int_0^1 p_n(\omega, j)^{1-\eta_i(j)} d\omega \right]^{1/[1-\eta_i(j)]}.$$

The share of country n 's expenditure in sector j that is allocated to goods from

¹¹The constant is given by $\kappa_i = \left[(1-\zeta_i) \varepsilon_i^{\varepsilon_i} (1-\varepsilon_i)^{1-\varepsilon_i} \right]^{\zeta_i-1} \left[\zeta_i \alpha_i^{\alpha_i} (1-\alpha_i)^{1-\alpha_i} \right]^{-\zeta_i}$

country i , $\pi_{in}(j)$, is given by

$$\pi_{in}(j) = \int_0^1 p_n(\omega, j)_{in}^{1-\eta_i(j)} \mathbb{I}_{in}(\omega, j) d\omega \Big/ P_n(j)^{1-\eta_i(j)}. \quad (2.6)$$

where $\mathbb{I}_{in}(\omega, j)$ is an indicator variable that equals one if country n purchases intermediate good (ω, j) from country i , and equals zero otherwise. The domestic expenditure share is given by $\pi_{ii}(j)$. Using the assumption of exponentially distributed productivities, one can show (see e.g. EK 2002) that in equilibrium

$$\pi_{in}(j) = \left[\tau_{in}(j) \frac{c_i}{A_i(j)} \right]^{-1/\theta(j)} \Big/ \sum_{k=1}^I \left[\tau_{kn}(j) \frac{c_k}{A_k(j)} \right]^{-1/\theta(j)}. \quad (2.7)$$

In the following sections, we use Equation (2.7) to solve analytically for the change in the skill premium between any two steady states.

A competitive equilibrium is a set of prices and quantities such that all markets clear. Each producer must satisfy worldwide demand for its output. Sectoral output must satisfy the resource constraints (2.2), (2.3), and (2.4). The demand for unskilled and skilled labor across producers must equal the endowments L_i and H_i , respectively. The demand for intermediate inputs of services and manufacturing must equal $X_i(S)$ and $X_i(M)$, respectively. The demand for structures and capital equipment across producers must equal their supplies $K_i(S)$ and $K_i(E)$. The supplies of each type of capital must be consistent with the household's optimal investment decisions. The household's budget constraints must be satisfied. A steady-state equilibrium is an equilibrium in which all variables remain constant over time. We characterize the steady-state equilibrium in Appendix 2.6.1.

2.3 Analytic Results

In this section, we examine the central forces that shape changes in the skill premium and in real wages for skilled and unskilled workers in our model.

2.3.1 The Skill Premium

Cost minimization implies that producers set the ratio of the marginal product of skilled labor to unskilled labor equal to the skill premium. Equation (2.5) and the fact that producers in all sectors use the same factor intensity imply

$$\frac{s_i}{w_i} = \left(\frac{1 - \mu_i}{\mu_i} \right)^{\frac{1}{\sigma}} (1 - \lambda_i)^{\frac{1}{\rho}} \left(\frac{L_i}{H_i} \right)^{\frac{1}{\sigma}} \left[\lambda_i^{\frac{1}{\rho}} \left(\frac{K_i(E)}{H_i} \right)^{\frac{\rho-1}{\rho}} + (1 - \lambda_i)^{\frac{1}{\rho}} \right]^{\frac{\sigma-\rho}{(\rho-1)\sigma}}, \quad (2.8)$$

exactly as in KORV. From equation (2.8), changes in country i 's skill premium are fully determined by changes in country i 's endowments of skilled and unskilled labor and changes in its stock of capital equipment. All else equal, an increase in unskilled labor relative to skilled labor increases the skill premium with an elasticity of $1/\sigma$ while an increase in capital equipment relative to skilled labor increases the skill premium if and only if $\sigma > \rho$ (that is, if skilled labor is more complementary with capital equipment than is unskilled labor). This second component captures the effect on the skill premium of capital-skill complementarity.

Of course, the stock of capital equipment, $K_i(E)$, is endogenous, and changes in $K_i(E)$ potentially depend on changes in bilateral trade costs (between each pair of countries and in each sector), changes in each country-sector-specific productivity, and changes in labor endowments in each country. We can show, however, that there is a small set of sufficient statistics that fully determine the equilibrium change in the stock of capital equipment and the skill premium across steady-states. Appendix 2.6.1 presents a set of six equations from which the steady-state change in the skill premium (and the real wages of skilled and unskilled workers) can be calculated for any country i .

For given values of the elasticities of substitution (σ and ρ), the dispersion of productivities $\theta(j)$, and factor shares in the initial equilibrium, the change in country i 's skill premium depends only on: (i) changes in domestic expenditure shares, $\pi_{ii}(j)$ for all j ; (ii) changes in domestic technologies, $A_i(j)$ for all j ; and (iii) changes in domestic labor endowments, H_i and L_i . Importantly, conditional on (i) – (iii), changes in trade costs, changes in other countries' technologies and endowments, and changes in all other trade shares do not affect country i 's skill premium. That is, international trade costs, for-

eign technologies, and foreign endowments only affect country i 's skill premium through $\pi_{ii}(j)$. Moreover, for a given change in domestic expenditure shares $\pi_{ii}(j)$, we do not need to compute the multi-country general equilibrium model to calculate the change in country i 's skill premium.

First-Order Approximation for Changes in the Skill Premium: To better understand the role of changes in (i) domestic expenditure shares, (ii) domestic technologies, and (iii) domestic labor endowments in shaping changes in the skill premium, we log-linearize the steady-state equilibrium equations. In Appendix 2.6.2 we show that the change in the skill premium is, to a first-order approximation, given by

$$\widehat{s}_i - \widehat{w}_i = -\frac{\xi_i^L + \xi_i^H}{\rho\xi_i^L + \sigma\xi_i^H} \left(\widehat{H}_i - \widehat{L}_i \right) + \Theta_i \sum_j \varkappa_i(j) \left[\widehat{A}_i(j) - \theta(j) \widehat{\pi}_{ii}(j) \right], \quad (2.9)$$

where variables with hats denote log differences, ξ_i^H denotes the initial steady-state ratio of skilled labor payments to capital equipment payments, and ξ_i^L denotes the initial steady-state ratio of unskilled labor payments to the sum of all labor payments and payments to capital equipment,

$$\xi_i^H = \frac{s_i H_i}{r_i K_i(E)} \quad \text{and} \quad \xi_i^L = \frac{w_i L_i}{w_i L_i + s_i H_i + r_i K_i(E)}.$$

The elasticity of the skill premium with respect to $\widehat{A}_i(j) - \theta(j) \widehat{\pi}_{ii}(j)$ is given by $\Theta_i \varkappa_i(j)$, where

$$\Theta_i = \frac{\sigma - \rho}{\rho\xi_i^L + \sigma\xi_i^H} \quad (2.10)$$

is common across sectors, and where

$$\varkappa_i(j) = \begin{cases} \frac{(1-\zeta_i)\varepsilon_i + \zeta_i\alpha_i}{\zeta_i(1-\alpha_i)} & \text{if } j = S \\ \frac{(1-\zeta_i)(1-\varepsilon_i)}{\zeta_i(1-\alpha_i)} & \text{if } j = M \\ 1 & \text{if } j = E \end{cases} \quad (2.11)$$

depends on production function parameters and varies across sectors.

Decomposing changes in the skill premium: Equation (2.9) decomposes the change in the skill premium into four components. The first component depends on the growth of skilled labor relative to unskilled labor and captures the relative supply effect already

present in equation (2.8). All else equal, an increase in the relative supply of skilled labor reduces the skill premium with an elasticity of $(\xi_i^H + \xi_i^L) / (\rho \xi_i^L + \sigma \xi_i^H)$. Note that if $\sigma = \rho$, so that equipment is equally complementary to skilled and unskilled labor, then this elasticity reduces to $1/\rho$, exactly as in Tinbergen (1974, 1975) [42] [43] and Katz and Murphy (1992) [29], what Acemoglu and Autor (2010) [3] call the *canonical model*.

The second, third, and fourth components ($j = S, M$, and E) are all contained in the summation term in equation (2.9). Each component depends on changes in sector j 's productivity and domestic expenditure share and captures the capital-skill complementarity effect. All else equal, the elasticity of the skill premium with respect to $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)$ is $\Theta_i \varkappa_i(j)$, where $\varkappa_i(j) \geq 0$ for all j . If $\sigma > \rho$, so that $\Theta_i > 0$, then an increase in the supply of capital equipment relative to skilled labor increases the skill premium, as shown in equation (2.8). Here, we describe why $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j) > 0$ for any j tends to raise $K_i(E)$, and hence the skill premium.

Intuitively, country i 's stock of equipment rises either through increased domestic production or increased imports of equipment. All else equal, country i produces more equipment as $A_i(E)$ rises and imports more equipment as $\pi_{ii}(E)$ falls.

Country i 's supply of equipment also rises if $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j) > 0$ for $j = S, M$. Intuitively, in equilibrium $X_i(S)$ and $X_i(M)$ rise with $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)$ for $j = S$ and $j = M$, respectively, for the same reason that $K_i(E)$ rises with $\hat{A}_i(E) - \theta(j) \hat{\pi}_{ii}(E)$. Because $X_i(S)$ and $X_i(M)$ are used as inputs in the production of equipment, the stock of equipment rises as well.

The elasticity of the skill premium: Equation (2.9) provides the elasticity of a country's skill premium with respect to each of its sectoral productivities, $\Theta_i \varkappa_i(j)$, and each of its domestic sectoral expenditure shares, $-\Theta_i \varkappa_i(j) \theta(j)$. These elasticities have clear economic interpretations that highlight the roles played by different model parameters and they allow us to conduct sensitivity analyses analytically.

A higher value of within-sector technological dispersion, $\theta(j)$, tends to magnify the impact of changes in trade shares on the skill premium. This follows from the fact that for a given domestic expenditure share in the equipment sector (for example), the increase in the stock of equipment generated by trade is greater for higher values of $\theta(j)$. Intuitively,

when productivity dispersion rises, the cost differential between imported varieties and the domestic varieties they replace becomes greater, so that the same reduction in the domestic expenditure share leads to a greater reduction in the price of capital equipment and, therefore, a greater increase in its stock.

Similarly, a higher value of the elasticity $\Theta_i \varkappa_i(j)$ tends to magnify the impact of $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)$ on the skill premium. Stronger capital-skill complementarity implies a higher value of Θ_i . Inspecting equation (2.11), it is apparent that sectors that are more important in the production of capital equipment have a higher value of $\varkappa_i(j)$, and hence have a higher elasticity of the skill premium with respect to $\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)$.

Note that the equipment stock and the skill premium rise if there is growth in technology and trade in manufacturing, equipment, or services—regardless of the sector in which growth is greatest—whereas the price of equipment relative to the price of manufacturing (for instance) falls if technological and trade growth are relatively larger in the equipment sector:

$$\hat{P}_i(E) - \hat{P}_i(M) = \hat{A}_i(M) - \hat{A}_i(E) + \theta(E) \hat{\pi}_{ii}(E) - \theta(M) \hat{\pi}_{ii}(M).$$

Hence, an increase in the stock of equipment and the skill premium are not necessarily accompanied by a decline in the relative price of equipment to manufactured consumption goods, contrary to what is typically discussed in the literature. Instead, they are accompanied by a decline in the relative price of equipment to a composite of equipment, skilled labor, and unskilled labor (which we define in Appendix 2.6.1).

Summary: We summarize the previous results in the following Proposition.

Proposition 1 *In any equilibrium, the skill premium in country i is given by equation (2.8), and the change in the skill premium in country i across two steady-states is, to a first-order approximation, given by equation (2.9).*

2.3.2 Real Wages

Whereas our previous focus has been on the skill premium, most of the quantitative trade literature focuses on gains from trade. Here we show that our framework also

yields clear predictions on how changes in (i) domestic expenditure shares, (ii) domestic technologies, and (iii) domestic endowments shape changes in real wages for skilled and unskilled workers. Real wages of skilled and unskilled workers are simply $s_i/P_i(C)$ and $w_i/P_i(C)$ respectively, where

$$P_i(C) = \frac{P_i(S)^{1-\phi} P_i(M)^\phi}{\phi^\phi (1-\phi)^{1-\phi}}.$$

In Appendix 2.6.2 we show that changes in real skilled and unskilled wages are, to a first-order approximation, given by

$$\hat{s}_i - \hat{P}_i(C) = -\frac{\xi_i^L (1 + \xi_i^H)}{\rho \xi_i^L + \sigma \xi_i^H} (\hat{H}_i - \hat{L}_i) + \sum_j \nu_i(j) [\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)] \quad (2.12)$$

and

$$\hat{w}_i - \hat{P}_i(C) = \frac{(1 - \xi_i^L) \xi_i^H}{\rho \xi_i^L + \sigma \xi_i^H} (\hat{H}_i - \hat{L}_i) + \sum_j [\nu_i(j) - \Theta_i \varkappa_i(j)] [\hat{A}_i(j) - \theta(j) \hat{\pi}_{ii}(j)], \quad (2.13)$$

where

$$\nu_i(j) = \begin{cases} \frac{\sigma(1+\xi_i^H)}{\rho \xi_i^L + \sigma \xi_i^H} \frac{\zeta_i + \varepsilon_i - \zeta_i \varepsilon_i}{\zeta_i(1-\alpha_i)} - \frac{\sigma - \rho \xi_i^L}{\rho \xi_i^L + \sigma \xi_i^H} - \phi & \text{if } j = S \\ \frac{\sigma(1+\xi_i^H)}{\rho \xi_i^L + \sigma \xi_i^H} \frac{(1-\zeta_i)(1-\varepsilon_i)}{\zeta_i(1-\alpha_i)} + \phi & \text{if } j = M \\ \frac{\sigma - \rho \xi_i^L}{\rho \xi_i^L + \sigma \xi_i^H} & \text{if } j = E \end{cases}$$

depends on production function parameters and factor shares and varies across sectors. Note that equations (2.12) and (2.13) together imply equation (2.9).

Decomposing changes in real wages: Equations (2.12) and (2.13) decompose changes in real wages into four components. The first component depends on the growth of skilled labor relative to unskilled labor and captures the relative supply effect. All else equal, the real wage of a given factor is decreasing in its relative supply.

The second, third, and fourth components ($j = S, M$, and E) are all contained in the summation terms in equations (2.12) and (2.13). Each component depends on changes in sector j 's productivity and domestic expenditure share, and captures both the effects of trade and productivity growth on the real wage in standard quantitative trade

models as well as the capital-skill complementarity effect. To see the standard effects, consider the case in which capital is equally complementary with skilled and unskilled labor, $\sigma = \rho$. In this case $\Theta_i = 0$, so that $\hat{A}_i(j) - \theta(j)\hat{\pi}_{ii}(j)$ has the same effect on the real wage of skilled and unskilled workers. Specifically, in this case $\hat{A}_i(j) - \theta(j)\hat{\pi}_{ii}(j) > 0$ raises the real wage of both factors, as $\nu_i(j) > 0$ for all j . In the presence of capital-skill complementarity, however, $\hat{A}_i(j) - \theta(j)\hat{\pi}_{ii}(j)$ has different effects on skilled and unskilled workers as discussed above. Specifically, $\hat{A}_i(j) - \theta(j)\hat{\pi}_{ii}(j) > 0$ raises the real wage of skilled workers relatively more than the real wage of unskilled workers for any j .¹²

As in the section on the skill premium, equations (2.12) and (2.13) provide the elasticity of skilled and unskilled real wages in country i with respect to each of i 's sectoral productivities and each of its domestic sectoral expenditure shares.

Summary: We summarize the previous results in the following Proposition.

Proposition 2 *The changes in real wages for skilled and unskilled workers in country i across two steady-states are, to a first-order approximation, given by equations (2.12) and (2.13), respectively.*

2.3.3 Robustness

Alternative quantitative trade models: In this paper we embed capital-skill complementarity into a version of the quantitative Ricardian model of international trade pioneered in EK. In EK, changes in trade costs and in foreign technologies and labor endowments affect the domestic real wage only through their impact on the domestic expenditure share; moreover, the elasticity of the real wage to the domestic expenditure share is θ . Arkolakis, Costinot, and Rodriguez-Clare (2012) [4], henceforth ACR, show that these two results hold across a range of quantitative trade models. To what extent does this generality apply to our results?

¹²In response to increases in trade shares in any sector ($\hat{\pi}_{ii}(j) < 0$), the real wage of unskilled workers increases for any value of σ and ρ , while the real wage of skilled workers may fall if skilled labor is sufficient substitutable with capital equipment ($\sigma \ll \rho$). However, this result depends on the specific form of our production function. Reversing the nest in the production function (i.e. nesting equipment and unskilled labor together) we obtain the opposite result: the real wage of skilled workers increases in response to $\hat{\pi}_{ii}(j) < 0$ for any degree of capital-skill complementarity, while the real wage of unskilled workers may fall if unskilled labor is sufficient substitutable with equipment capital.

Consider an Armington version of our model (in which the pattern of specialization across intermediate goods is exogenous). In this model, all our results are unchanged except that the dependence of our expressions on the dispersion parameter $\theta(j)$ is substituted by the inverse of the elasticity of substitution across intermediate goods in sector j , $1/(\eta(j) - 1)$.

Next, consider a monopolistic competition version of our model in which each firm produces a differentiated intermediate good and is subject to a fixed cost (expressed in terms of the factor composite) to sell the good in each country. If firm entry is restricted in each sector and the productivity distribution of entering firms is Pareto (as in Chaney 2008 [13]), we show the following two results in the Online Addendum. First, as in our EK model, changes in variable and fixed trade costs and in foreign technologies and labor endowments affect prices and quantities in the domestic economy only through their impact on a small set of sufficient statistics in the domestic economy: domestic sectoral expenditure shares, as in our EK model, and total net exports relative to GDP in the domestic country, unlike in our EK model. Hence, given these statistics, changes in real wages and the skill premium can still be solved country-by-country without computing the multi-country general equilibrium model. Second, the expressions linking changes in the skill premium to changes in domestic sectoral expenditure shares (even when net exports are zero) differ from those in the Ricardian model because they depend on changes in the share of each sector in total absorption in the domestic economy, which in general are not constant.¹³ If, instead, firm entry into each sector is endogenous (as in Melitz 2003 [33]), then our first result above does not hold and, analogously to the results in Section 5.1 of ACR, changes in factor prices also depend on changes in employment in each sector. Hence, in the case of endogenous entry we cannot apply the simple sufficient statistics approach that we use in this paper to solve, country by country, for changes in factor prices.

Differences in factor intensities across sectors: In the Online Addendum we briefly discuss an extension of our basic environment that relaxes our assumption that factor intensities are common across sectors. In particular, we allow for the parameters of the

¹³In the Cobb-Douglas multi-sector extension of ACR, the share of each sector in total absorption is constant.

production function $\{\varepsilon, \zeta, \alpha, \mu, \lambda, \rho, \sigma\}$ to all vary across sectors. We show that changes in a country's skill premium are not only determined by changes in domestic productivities, domestic labor endowments, and domestic expenditure shares—as in our baseline model—but also by changes in the factor-content of trade (i.e., the amount of each factor embodied in a country's net exports).¹⁴ This extended model thus embeds the standard Stolper-Samuelson effect, through which international trade raises the relative return of the factor used intensively in the comparative advantage sector. We show, however, that conditional on observing changes in domestic productivities, domestic labor endowments, domestic expenditure shares, and the factor-content of trade in country i , one can still calculate changes in country i 's skill premium without actually computing the multi-country general equilibrium model. Burstein and Vogel (2012) [9] show that the Stolper-Samuelson effect is not quantitatively strong in a multi-country model. Hence, in order to isolate the role of capital-skill complementarity on the skill premium, we assume that factor intensities are common across sectors in our quantitative analysis.

2.3.4 Motivating our counterfactuals

In the next section, we use our framework to conduct two counterfactual exercises quantifying the impact of international trade on the skill premium (and real wages) through its impact on the accumulation of capital equipment. Specifically, we solve for changes in real wages and the skill premium resulting from given changes in domestic sectoral expenditure shares ($\pi_{ii}(j)$ s) using equations (2.38) – (2.43) in Appendix 2.6.1. In the first counterfactual we move countries to autarky. In the second counterfactual we move countries from their domestic sectoral expenditure shares observed in 2000 to those in 1963 (or the first year with available data).

In what follows we show that our second counterfactual provides a specific way to quantify the impact of international trade on real wages and the skill premium over a given time period. Fix the set of parameters $\{\sigma, \rho, \theta(M), \theta(E), \zeta_i, \varepsilon_i, \alpha_i, \phi_i\}$ and fix country i 's year t steady-state factor shares $\{\xi_i^H, \xi_i^L\}$. Suppose that between two steady-state years, t and t' , the primitives—worldwide trade costs, technologies, and labor endowments—

¹⁴See Burstein and Vogel (2011) [8] for a discussion of the factor content of trade in a general class of trade models.

changed in some unobserved manner. These changes in primitives cause changes in domestic sectoral expenditure shares, the skill premium, real wages of skilled workers, and real wages of unskilled workers in each country i , which we denote by $\widehat{\pi}_{ii}(j)$ for all j , $\widehat{s_i/w_i}$, $\widehat{s_i/P(C)_i}$, and $\widehat{w_i/P(C)_i}$ for all i . Now consider a counterfactual environment in which country i is in autarky between years t and t' .¹⁵ Suppose that the same percentage changes in unobserved primitives occurred, excluding the changes in country i 's trade costs, which are set to infinity in both years in this counterfactual environment. These changes in primitives cause changes in country i 's skill premium and real wages, which we denote by $\widehat{s'_i/w'_i}$, $\widehat{s'_i/P'_i(C)}$, and $\widehat{w'_i/P'_i(C)}$. The following corollary of Propositions 1 and 2 relates the steady-state implications of this change in primitives between the environment in which country i trades and the counterfactual environment in which it is in autarky.

Corollary 1 *To a first-order approximation,*

$$\begin{aligned}\widehat{s_i/w_i} - \widehat{s'_i/w'_i} &= -\Theta_i \sum_j \varkappa_i(j) \theta(j) \widehat{\pi}_{ii}(j) \\ \widehat{s_i/P_i(C)} - \widehat{s'_i/P'_i(C)} &= -\sum_j \nu_i(j) \theta(j) \widehat{\pi}_{ii}(j) \\ \widehat{w_i/P_i(C)} - \widehat{w'_i/P'_i(C)} &= -\sum_j [\nu_i(j) - \Theta_i \varkappa_i(j)] \theta \widehat{\pi}_{ii}(j),\end{aligned}$$

for a fixed set of parameters $\{\sigma, \rho, \theta(M), \theta(E), \zeta_i, \varepsilon_i, \alpha_i, \phi_i\}$ and fixed year t steady-state factor shares $\{\xi_i^H, \xi_i^L\}$ in country i .

Corollary 1 provides an answer to the following question: What are the additional effects of changes in primitives on the skill premium and real wages in an open economy relative to the effects in a closed economy? According to corollary 1, we can answer this question—up to a first-order approximation—using observable changes in domestic sectoral expenditure shares between two time periods without needing to observe the underlying changes in primitives.

Our answer to this question would be exact (instead of a first-order approximation) if

¹⁵In order to fix country i 's year t steady-state factor shares $\{\xi_i^H, \xi_i^L\}$ in this counterfactual environment (without trade) at their levels in our baseline environment (with trade), we must adjust the levels of some combination of country i 's sectoral productivities, its factor endowments, and its parameters λ_i and μ_i .

factor shares and the share of each sector in total absorption were constant across steady-states, conditions which in general do not hold in our model. However, these conditions do hold trivially in standard quantitative trade models with one factor and one sector. It is straightforward to show that in versions of such models considered in ACR, this question can be answered exactly using a result similar to that in corollary 1.

Note that changes in international trade patterns that affect relative prices—both the skill premium and sectoral price indices—may alter incentives to acquire education and engage in innovative activities that affect sectoral productivities. In our two counterfactuals, we abstract from the indirect effects of trade on the supply of skilled and unskilled labor and on sector-level productivities.

2.4 Quantitative Results

In this section we conduct the two counterfactuals described above. To conduct these counterfactuals we need information on domestic expenditure shares, $\pi_{ii}(j)$, and we need to assign values to our model’s parameters. In what follows, we first describe how we construct domestic expenditure shares and how we parameterize the model. Further details are provided in Appendix 2.6.3. We next present our baseline quantitative results. Finally, we conduct alternative parameterizations and sensitivity analyses.

2.4.1 Domestic Expenditure Shares

To construct domestic expenditure shares in equipment and manufacturing, $\pi_{ii}(E)$ and $\pi_{ii}(M)$, we use trade and production data and compute expenditures as the difference between gross output and net exports. Trade data comes from Feenstra et al. (2005) [21], which contains data by commodity, disaggregated at the 4-digit Standard International Trade Classification (SITC) level, for the 1962-2000 period. For gross output data, we use the UNIDO Industrial Statistics Database [44], which covers the 1963-2007 period and is arranged at the 2-digit level of the third revision of the International Standard Industrial Classification (ISIC Rev. 3). Recall that we abstract from trade in non-manufacturing industries (which, in our model, means that we abstract from trade in the non-manufacturing, non-equipment sectors).

We follow Eaton and Kortum (2001) [18], who group manufactured commodities into equipment goods and other manufacturing goods using input-output tables and capital flows tables of domestic transactions (OECD, 1996) for the three major capital goods producers (Germany, Japan, and the US). For trade data, we match 4 digit SITC codes to a set of industry codes used by the Bureau of Economic Analysis (BEA) [45]. Following Eaton and Kortum, we define equipment trade as the sum of BEA industry codes 20-27 and 33.

For gross output data, Eaton and Kortum identify three ISIC Rev. 2 industries as equipment producers: non-electrical equipment, electrical equipment, and instruments. We define equipment producers as the ISIC Rev. 3 industries that most closely correspond to the ISIC Rev. 2 industries identified by Eaton and Kortum.¹⁶ In particular, we define equipment commodities to be the sum of ISIC Rev. 3 codes 29-33.

After combining these datasets, we are left with 53 countries for which data both on trade and output is available until at least 1995. For each country in our sample, our counterfactuals are based on the first and last year with available data. Importantly, we do not require a balanced panel because we do not need data on changes in any country $n \neq i$ when solving for the change in the skill premium in country i in our counterfactuals.

We report the resulting domestic expenditure shares in Table 2.2. Two features are striking from the table. First, as noticed by Eaton and Kortum (2001) [18], most countries import a significant fraction of their capital equipment. For the median country in our sample, the import share of equipment in the year 2000 is roughly $1 - 0.25 = 0.75$, more than twice as large as the import share for other manufactured goods. Note that these import shares are large for countries at different stages of the development process, including developed countries such as Canada and the UK. Second, most countries experienced sizable increases in their import shares over our sample period, especially in the equipment sector. Notable exceptions are the poorest countries in the sample, which were already importing almost all of their equipment at the beginning of the sample. The median values across countries for the changes in the domestic expenditure shares in equipment and manufacturing, $\hat{\pi}_{ii}(E)$ and $\hat{\pi}_{ii}(M)$, are -0.3 and -0.15 , respectively.

The fact that $\pi_{ii}(E)$ tends to be lower in developing countries might suggest that the

¹⁶UNIDO discontinued its Industrial Statistics Database using ISIC Rev. 2.

relative price level of equipment is higher in these countries; see e.g. Eaton and Kortum (2001) [18] and Hsieh and Klenow (2007) [26]. In our model, this relative price depends on a combination of trade costs and productivities in each country. Since our parameterization does not separately identify trade costs and productivities in each country, our paper is silent on our model's implications for these relative prices.¹⁷

2.4.2 Parameterization

By inspecting the set of equations that determines the change in the skill premium and real wages in our counterfactuals (described in Appendix 2.6.1) and in the log-linearized equations above, the parameters that we must choose are those that determine the elasticities of substitution between capital equipment, unskilled labor, and skilled labor, σ and ρ ; the within-sector dispersion of productivity in manufacturing and equipment, $\theta(M)$ and $\theta(E)$; the constant share of value added in production, ζ_i ; the constant share of services in intermediate inputs, ε_i ; the constant share of structures in value added, α_i , and the constant share of manufacturing in consumption, ϕ_i . We must also assign values to relative factor shares ξ_i^L and ξ_i^H in the initial equilibrium (the year 2000). Conditional on matching these endogenous values, we do not need to assign values to the two remaining production function parameters (μ_i and λ_i) or to sectoral productivities $A_i(j)$ and labor endowments H_i and L_i . Because of data availability, we assume that all of the above parameters $\{\sigma, \rho, \theta(M), \theta(E), \zeta, \varepsilon, \alpha, \phi\}$ are common across countries. We also assume that relative factor shares in the initial equilibrium $\{\xi^L, \xi^H\}$ are common across countries.¹⁸ Given data availability it would be straightforward to run our counterfactuals allowing all parameters except $\theta(j)$ to vary across countries. We now provide an overview of our baseline procedure, the results of which are summarized in Table 2.1.

Baseline parameterization: We pick $\phi, \zeta, \varepsilon, \alpha, \sigma, \rho, \xi^L$, and ξ^H to match certain features of US data between 1963 and 2000. The share of manufacturing in households' consumption, ϕ , the share of value added in gross output, ζ , and the share of services in

¹⁷Waugh (2010) [48] shows that quantitative Ricardian models are consistent with observed differences across countries in the level of tradeable goods prices if one allows for asymmetric trade costs (e.g. $\tau_{in}(j) \neq \tau_{ni}(j)$), as we do in this paper.

¹⁸The assumption that $\{\xi^L, \xi^H\}$ are equal across countries in the initial equilibrium implies that a combination of sectoral productivities $A_i(j)$, labor endowments H_i and L_i , and parameters λ_i and μ_i vary across countries to match these relative factor shares.

intermediate inputs, ε , are set at their average shares in 1995 and 2000 from the OECD Input-Output database [34].¹⁹ We calibrate the share of structures in value added, α , and relative factor shares in the initial equilibrium (the year 2000), ξ^L and ξ^H , to match observed factor shares in the US. Annual estimates for these shares are obtained as follows. We calculate the labor share in value added from NIPA as the ratio of compensation for employees to value added less taxes, in the corporate and non-corporate business sector. We disaggregate labor payments into skilled and unskilled labor using data on quantities and prices of skilled and unskilled labor from Polgreen and Silos (2008) [36], who use detailed CPS data. We disaggregate capital payments into structures and equipment using data on the value of capital stocks and, since rental rates are not directly observable, using the steady-state Euler equations of our model for the accumulation of each type of capital, where a time period represents a year. We set α equal to the share of payments to structures capital in total factor payments on average between 1963 and 2000. We set ξ^L and ξ^H in the original equilibrium (year 2000) equal to the respective relative factor shares on average between 1996 and 2000.²⁰ This procedure implies $\alpha = 0.1$, $\xi^L = 0.44$, and $\xi^H = 1.37$.²¹ Further details are provided in Appendix 2.6.3.

From equation (2.7), $1/\theta(j)$ is the sector-level elasticity of trade with respect to trade costs (i.e., the trade elasticity). If trade costs and the trade elasticity are the same across sectors, then $1/\theta(j)$ is also the aggregate trade elasticity. Under these assumptions, we could choose $\theta(j)$ using aggregate trade elasticities. To match an aggregate elasticity of 5, for instance, we would pick $\theta(j) = 0.2$. However, we allow for variation in trade costs across sectors. In this case, even if $\theta(j)$ is constant across sectors, $1/\theta(j)$ need not

¹⁹In calculating these statistics, we only consider consumption, valued added, gross output, and intermediates of manufacturing (which includes equipment and non-equipment manufacturing in our model) and service industries in the IO tables. The resulting parameter values are $\phi = 0.2$, $\zeta = 0.54$ and $\varepsilon = 0.62$.

²⁰Consistent with our model, factor shares ξ^H and ξ^L in the U.S. changed considerably in our time period (e.g. the payments to capital equipment rise over time relative to the payments to skilled labor). While our baseline year (the initial equilibrium) is 2000, we use the average estimated shares in the period 1996-2000 to reduce measurement error. Using instead the average estimates of factor shares between 1963 and 2000, the elasticity of the skill premium to trade flows is significantly larger than in our baseline parameterization.

²¹We assume that factor shares are identical across countries because of data limitations only. If, contrary to our assumption, developing countries have lower equipment shares (or lower skill shares), then Θ_i would be lower (higher) in developing countries. Our assumption that the labor share is not systematically correlated with a country's level of development is consistent with evidence in Gollin (2002) [25]. In our model the labor share changes in response to the changes in trade shares we feed in from the data, but for our counterfactuals these changes are very small.

equal the aggregate trade elasticity. Using a technique developed in Caliendo and Parro (2011) [11], Parro (2012) [35] estimates sector-level trade elasticities in the equipment and manufacturing sectors using gravity equations that hold in our model. We use his estimates, implying $\theta(E) = 0.22$ and $\theta(M) = 0.19$.

The two final and key parameters whose values we need to pick are σ and ρ . We pursue several strategies to parameterize these. We calibrate σ and ρ so that our model reproduces the observed cumulative changes in factor shares and the skill premium in the US between 1963 and 2000, given the observed changes in the supplies of capital equipment and of skilled and unskilled labor. In particular, we use the two following equations

$$\rho^{-1} = 1 + \frac{\widehat{\xi^H}}{\widehat{K(E)/H}} \quad (2.14)$$

$$\sigma = \frac{(\rho - 1) \widehat{(H/L)} + \rho \widehat{(1 + 1/\xi^H)}}{(1 - \rho) \widehat{(s/w)} + \widehat{(1 + 1/\xi^H)}}, \quad (2.15)$$

where variables with hats denote log differences between 1963 and 2000. Equation (2.14) is obtained by log-differentiating the producers' first-order condition for capital equipment relative to skilled labor. Equation (2.15) is obtained by log-differentiating equation (2.8). In solving for ρ and σ , we use data on changes in the skill premium and on the stocks of (quality adjusted) capital equipment, skilled labor and unskilled labor from Polgreen and Silos (2008) [36]. This procedure implies $\rho = 0.63$ and $\sigma = 1.56$.

With these parameters, the elasticity of the skill premium with respect to $\widehat{A}_i(j) - \theta(j) \widehat{\pi}_{ii}(j)$ in all countries is $\Theta = 0.39$ for equipment goods and $\Theta \varkappa_i(M) = 0.39 \times 0.36 = 0.14$ for manufacturing goods, from equation (2.9).²² Together with our values of $\theta(j)$, this implies an elasticity of the skill premium with respect to domestic expenditure shares, $\Theta \varkappa(j) \theta(j)$, in equipment and manufacturing of 0.085 and 0.026, respectively.

In addition to determining the extent of capital-skill complementarity, Θ , the parameters ρ and σ also determine the elasticity of substitution between skilled and unskilled labor. Following Sato (1967) [38], the direct partial elasticity of substitution between skilled

²²Using measures of changes in labor supplies and the skill premium from Acemoglu and Autor (2010) [3] we obtain $\Theta = 0.40$. If we parameterize our model using data from 1963 to 1992 as in KORV (as opposed to 1963-2000), we obtain $\Theta = 0.35$. Using the values of the elasticities σ and ρ estimated in KORV we obtain $\Theta = 0.39$.

and unskilled labor—defined as $d \log(H/L)/d \log(w/s)$ holding output and all inputs except H and L constant—is given by $\sigma \rho (\xi^L + 1) / (\xi^L \sigma + \rho)$, which equals 1.08. We also calculate an alternative measure of the elasticity of substitution between skilled and unskilled labor: $d \log(H/L)/d \log(w/s)$ holding $H/K(E)$ constant, which equals $\sigma = 1.56$. In the context of our production function, this elasticity is equivalent to the Allen partial elasticity of substitution between skilled and unskilled labor, as defined in Sato (1967) [38].

2.4.3 Baseline Results

We now quantify the impact of international trade, through capital-skill complementarity, on real wages and the skill premium.²³ We perform the two counterfactual exercises described above using our baseline parameterization.

Counterfactual 1—Autarky: In our first counterfactual, we hold all technologies and factor endowments fixed at the baseline levels and raise all trade costs to infinity. Through this counterfactual we quantify how much each country’s skill premium and both of its real wages would change if it were moved to autarky. The counterfactual implications for real wages and the skill premium are reported in Table 2.3. The results of our first counterfactual exercise are summarized in Figures 2.1 and 2.2. Figure 2.1 plots the logarithmic change in real wages of skilled and unskilled workers in each country (y-axis). Given our emphasis on international trade in capital goods, we plot on the x-axis the log change of the domestic expenditure share in the equipment sector moving from the year 2000 to autarky.

Figure 2.1 establishes two results. First, moving to autarky, real wages fall for both skilled and unskilled workers in all countries, and, as in most standard models, fall relatively more in countries that experience a larger increase in domestic expenditure shares, both in equipment and in manufacturing, as implied by equations (2.12) and (2.13). Second, the losses from moving to autarky are unevenly distributed within countries. While both factors lose, skilled workers lose disproportionately. The ratio of the change in a skilled worker’s real wage relative to the change in an unskilled worker’s real wage,

²³In our model, real wages do not equal welfare because net exports are not zero.

$\Delta \log(s_n/P_n) / \Delta \log(w_n/P_n)$, is 2.36 in the median country.

This ratio can be expressed as a function of the log change in the skill premium

$$\frac{\Delta \log(s_n/P_n)}{\Delta \log(w_n/P_n)} = 1 + \frac{\Delta \log(s_n/w_n)}{\Delta \log(w_n/P_n)}.$$

Figure 2.2 plots the logarithmic change in the skill premium (y-axis)—i.e. the vertical distance between changes in log real wages of skilled and unskilled workers plotted in Figure 2.1—and the log change in the domestic expenditure share in the equipment sector moving from the year 2000 to autarky (x-axis). Absent international trade in both capital equipment and manufactures the skill premium falls in all countries. The log of the skill premium falls by roughly 0.14 in the median country. While the skill premium falls everywhere, the decrease is much larger for countries that are very dependent on imports of capital equipment, such as Cameroon and the Czech Republic. On the other extreme, the decline in the log of the skill premium is only 0.01 for Japan and 0.05 for the US.

Expression (2.9) provides a decomposition of changes in the skill premium induced by changes in trade shares in equipment and changes in trade shares in manufacturing (recall that we refer to non-equipment manufacturing simply as manufacturing). The line in Figure 2.2 shows the log change in the skill premium resulting from shutting down trade in equipment goods only, while keeping trade shares in the manufacturing sector constant. The skill premium falls by less when only equipment trade is shut down because manufacturing imports raise the stock of equipment and, therefore, the skill premium. The role of (non-equipment) manufacturing trade in shaping the skill premium is large for some countries such as Bulgaria, Slovakia, and Greece, which import a substantial share of their manufacturing absorption. However, for most countries, trade in equipment is significantly more important than trade in manufacturing in driving the change in the skill premium, because both the 2000 import share and the elasticity of the skill premium with respect to a change in the import share are larger for equipment than for manufacturing.

The first-order approximation of the change in the skill premium from going to autarky implied by equation (2.9) is quite accurate. Across our set of countries, the median and maximum differences between the exact and approximated changes in the skill premium

are 0.01 and 0.09 log points, respectively (which represent 8% and 21%, respectively, of the exact changes in the skill premium). Of course, the approximation error is larger for countries with lower domestic expenditures shares. The first-order approximations of the changes in the real wages of skilled and unskilled workers are similarly accurate.

Counterfactual 2—Observed changes in trade shares: In our second counterfactual, we hold a given country’s technologies and factor endowments fixed and change its domestic expenditure shares from their observed levels in 2000 to 1963, or the first year with available data. Through this counterfactual we gauge the response of real wages in a given country to all changes in technologies, endowments, and trade costs (over this time period) relative to the what the responses would have been had that country been in autarky over this time period, as stated in corollary 1.

This counterfactual change in trade disproportionately impacts skilled workers: the ratio of the change in a skilled worker’s real wage relative to the change in an unskilled worker’s real wage, $\Delta \log(s_n/P_n) / \Delta \log(w_n/P_n)$, is 2.38 in the median country. The results on the skill premium are summarized in Figure 2.3, which plots the logarithmic change in the skill premium (y-axis) and the logarithmic change in the domestic expenditure share in the equipment sector (x-axis). International trade plays an important role in shaping the skill premium through capital-skill complementarity, but its importance varies widely across countries in our sample depending on the magnitude of the changes in the domestic expenditure shares in equipment and other manufactured goods. While the counterfactual change in the skill premium is -0.05 log points for the median country of our sample and -0.04 log points for the US, the decline in the skill premium is quite large in various developing countries such as Argentina, Chile, Colombia, Greece, and Uruguay, and in some developed countries such as Canada and the UK. Note that for countries in the northwest corner of Figure 2.3, domestic expenditure shares in the equipment sector rose during our sample period, so that moving from the domestic expenditure shares in equipment observed in 2000 to those in the base year contributes to increasing the skill premium. Once again, trade in equipment plays a more significant role than trade in other manufactured goods in shaping the change in skill premium.

As in the previous counterfactual exercise, the first-order approximation of the change in the skill premium from equation (2.9) is quite accurate. The median and maximum

differences between the exact and approximated changes in the skill premium are only 0.003 and 0.04 log points, respectively (which represent 4% and 14% of the exact changes in the respective skill premia). The first-order approximations of the changes in the real wages of skilled and unskilled workers are similarly accurate.

2.4.4 Alternative Parameterizations and Sensitivity

In this section we provide a set of alternative parameterization strategies and conduct sensitivity analyses using the results. We consider alternative strategies for determining the strength of capital-skill complementarity (σ and ρ) and we pick alternative values for the dispersion of productivities ($\theta(j)$) and the share of structures (α). For each alternative parameterization Table 2.4 reports the elasticity of the skill premium with respect to changes in domestic expenditure shares in equipment and in manufacturing (using the first-order approximation), $\Theta_{\kappa(j)} \theta(j)$, as well as the median log points change in the skill premium for both of our counterfactuals (using the exact solution).

The strength of capital-skill complementarity: Because no consensus exists in the literature on the strength of capital-skill complementarity, we provide a number of alternative strategies for choosing σ and ρ .

One concern in the literature is that if there is exogenous skill-biased technical change, then estimates of σ and ρ that ignore this trend will overstate the extent of capital-skill complementarity; see e.g. Acemoglu (2002) [1]. While addressing this concern fully is beyond the scope of the present paper, we follow a suggestion in Acemoglu (2002) [1] and re-calibrate σ and ρ while allowing for trend growth in the productivity of the composite of skilled labor and capital equipment relative to unskilled labor. Specifically we generalize equation (2.5) by replacing the term $(1 - \mu_i)^{1/\sigma}$ with $T(t) (1 - \mu_i)^{1/\sigma}$, where $T(t) = \exp(\vartheta t)$ and ϑ denotes the annual trend.

In this extended version of the model, we obtain a version of Proposition 1 generalized as follows. In any equilibrium in period t , the skill premium in country i is given by a generalized version of equation (2.8) in which $T(t)$ multiplies the right-hand side. The change in the skill premium in country i across two steady-states in years t and $t' > t$ is, to a first-order approximation, given by a generalized version of equation (2.9) in which

$\left(1 + \Theta_i - \frac{\xi_i^L + \xi_i^H}{\rho \xi_i^L + \sigma \xi_i^H}\right) \frac{\sigma}{\sigma - 1} \vartheta (t' - t)$ is added to the right-hand side. Note that in the special case in which $\sigma = \rho$, this final term simplifies to $\vartheta (t' - t)$.

Given this extension, we re-calibrate ρ and σ under two alternative values for the annual trend growth, $\vartheta = 0.01$ and $\vartheta = 0.02$, using equation (2.14) and an adjusted version of equation (2.15). The strength of capital-skill complementarity, as represented by Θ , falls from 0.39 to 0.30 and 0.22 if $\vartheta = 0.01$ and $\vartheta = 0.02$, respectively. Our approximation implies that this reduces the elasticity of the skill premium with respect to $\pi_{ii}(j)$ to slightly more than 3/4 and 1/2 of its baseline level, respectively; this is confirmed when we re-run our counterfactual exercises using these parameters. We continue to infer capital-skill complementarity, $\sigma > \rho$, as long as annual trend growth is less than 0.052.

Another concern in the literature is that although there is cross-country evidence supportive of capital-skill complementarity, the evidence is not strong; see e.g. Duffy, Papageorgiou and Perez-Sebastian (2004) [17]. Again, whereas we do not aim to fully address this concern, we do assess the degree of capital-skill complementarity in a developing country that is a net importer of capital equipment, Chile, over the years 1983-2000; we provide details in the Appendix 2.6.3.²⁴ Together with the factor shares, the resulting elasticities imply stronger capital-skill complementarity, as represented by Θ , in Chile than in the US: $\Theta = 0.63$. Table 2.4 reports correspondingly larger effects in the median country of both of our counterfactual exercises.

Alternative values for productivity dispersion: There is a similar debate regarding the correct value of the aggregate trade elasticity; see e.g. Anderson and Van Wincoop (2004) [5], Donaldson (2010) [16], Simonovska and Waugh (2011) [39], Eaton, Kortum and Kramarz (2011) [20], and Costinot, Donaldson, and Komunjer (2012) [15]. To understand the sensitivity of our results to our choice of sector-level trade elasticities, determined by $\theta(j)$, we choose two alternative values of $\theta(j)$: $\theta(j) = 0.15$ and $\theta(j) = 0.25$ for $j = E, M$. A higher value of $\theta(j)$ raises the elasticity of the skill premium (and of the real wage of skilled and unskilled workers) to changes in domestic sectoral expenditure shares, as shown in equation (2.9) (and in equations (2.12) and (2.13), respectively) and as described in section 2.3. Table 2.4 reports correspondingly smaller (larger) effects in the median

²⁴We choose 1983 as an initial year to focus on the period in the aftermath of the Chilean debt crisis. We obtain a similar value for Θ if we choose 1974 as a starting year.

country of both of our counterfactual exercises under the assumption that $\theta(j) = 0.15$ ($\theta(j) = 0.25$, respectively) for all j .

Alternative values for the share of structures: Because we do not directly observe separately the share of payments to structures and equipment capital in total factor payments, we disaggregate total capital payments using data on the value of capital stocks and the steady-state Euler equations of our model, as discussed above in section 2.1 and in more depth in Appendix 2.6.3. The implied split of payments to capital is 2/3 to equipment and 1/3 to structures (implying $\alpha = 0.10$) in our baseline parameterization. Given the complication of measuring this share, here we perform sensitivity by considering a lower share of 1/2 (corresponding to $\alpha = 0.15$) and a higher share of 3/4 (corresponding to $\alpha = 0.076$) of capital payments accruing to equipment.

A higher share of capital payments accruing to equipment (a lower α) mechanically lowers $\xi_i^H = s_i H_i / (r_i K_i(E))$ and $\xi_i^L = w_i L_i / (w_i L_i + s_i H_i + r_i K_i(E))$. Lower values of ξ_i^H and ξ_i^L are associated with a higher value of $\Theta_i = (\sigma - \rho) / (\rho \xi_i^L + \sigma \xi_i^H)$. Hence, a higher share of capital payments accruing to equipment is associated with stronger capital-skill complementarity and, therefore, a larger impact of changes in domestic sectoral expenditure shares on the skill premium (and real wages). This intuition is confirmed in Table 2.4.

2.5 Conclusions

Given the difficulty of empirically measuring the impact of international trade on the aggregate stock of capital equipment and, through capital-skill complementarity, the skill premium, we use a model to do so in this paper. Our framework combines a standard quantitative trade model with a basic component of macroeconomic models of inequality, an aggregate production function that exhibits capital-skill complementarity. We provide simple analytic expressions relating steady-state changes in the skill premium and the real wages of skilled and unskilled workers to changes in domestic expenditure shares, domestic productivities, and domestic labor endowments. Changes in domestic expenditure shares by sector fully summarize the effects of international trade, whether generated by changes in foreign or domestic technologies, foreign or domestic labor endowments, or

trade costs. Using these results, we perform a range of simple counterfactual exercises to assess the importance of international trade on real wages and, through capital-skill complementarity, on the skill premium. We find that international trade can have a substantial impact on the skill premium, especially in countries that import a large fraction of their equipment. While our quantitative analysis is only suggestive—as there is an active debate on the role of capital-skill complementarity in accounting for changes in the skill premium—we view our main contribution as providing a simple set of analytic equations linking observable changes in domestic sectoral expenditure shares to changes in the real wages of skilled and unskilled workers for any given parameter values.

In our quantitative analysis, we make three choices in the pursuit of tractability that deserve further discussion. First, we focus on steady-state equilibria, abstracting from transition dynamics as countries open up to trade and gradually accumulate capital; see e.g. Stokey (1996) [40]. Second, we parameterize the degree of capital-skill complementarity to match observed changes in aggregate factor shares and the skill premium in the US and in Chile. An alternative approach would be to make use of micro-level evidence on the relationship between skill intensity and capital intensity at the producer level. This would require extending the model to allow for heterogeneity in factor intensities across producers within a country and sector.²⁵ Third, we assume that the degree of capital-skill complementarity is common across each type of capital equipment. If, however, different types of equipment exhibit different degrees of capital-skill complementarity, then countries might choose to invest in and import different mixes of equipment depending on their relative endowment of skilled to unskilled labor; see e.g. Caselli and Wilson (2004) [12].²⁶

While we focus on the implications of changes in trade patterns for real wages and the skill premium, our framework can be applied to study the importance of skill-biased technical change as well. In particular, by incorporating factor-specific technical change into our production function, as we do in the sensitivity analysis in Section 2.4.4, we

²⁵Burstein and Vogel (2012) [9] provide a related model in which producer productivity is positively correlated with skill intensity. With this heterogeneity, one loses the tractable gravity structure of the model, even at the sectoral level, and the model must be solved numerically.

²⁶Such an extension would have to be consistent with our finding that the extent of capital-skill complementarity is similar in the US and Chile. Moreover, if imported capital exhibits a greater degree of capital-skill complementarity than domestically produced capital, then trade would produce a larger rise in the skill premium.

obtain an equation that extends Tinbergen’s (1974, 1975) [42][43] pioneering work—what Acemoglu and Autor (2010) [3] call the *canonical model*—to include the effects on the skill premium not only of labor endowment and skill-biased technical changes, but also of changes in the pattern of international trade.

Finally, in this paper we model the international transfer of skill-biased technology through trade in capital goods. We abstract from other potentially important channels by which technologies diffuse across countries, such as multinational production, see, e.g., Burstein and Monge-Naranjo (2009) [7] and Ramondo and Rodriguez-Clare (2009) [37]; migration, see, e.g., Gandal, Hanson, and Slaughter (2004) [24]; or spillovers, see, e.g., Coe and Helpman (1995) [14] and Gancia, Müller, and Zilibotti (2011) [23]. We also abstract from endogenous skill-biased technical change through innovation, see, e.g., Acemoglu (2003) [2]. Understanding the quantitative link between globalization and inequality through these alternative channels remains an important area for future research.

2.6 Appendix

2.6.1 Equilibrium

In this section, we characterize a steady-state equilibrium. We show how to solve the key steady-state variables of interest as a function of domestic expenditure shares, $\pi_{ii}(j)$ s. In addition, we provide a system of six equations with which we can solve for changes in country i ’s skill premium and both of its real wages as functions of changes in its domestic expenditure shares, $\pi_{ii}(j)$ s; its domestic technologies, $A_i(j)$ s; and its domestic labor endowments, H_i and L_i .

2.6.1.1 Steady-State Equilibrium

We now define and characterize the steady state equilibrium for the world economy. In doing so, we show how aggregate quantities and prices can be determined before solving for product level variables. A steady-state equilibrium for the aggregate variables in the world economy consists of a set of prices $\{v_i, w_i, r_i, s_i\}_{i \in I}$, $\{p_{b1,i}, p_{b2,i}, p_{b3,i}, p_{b4,i}, c_i\}_{i \in I}$, $\{P_i(S), P_i(M), P_i(E)\}_{i \in I}$, aggregate quantities $\{K_i(S), K_i(E), X_i(M), X_i(S)\}_{i \in I}$ and

$\{C_i(M), C_i(S)\}_{i \in I}$, $\{Y_i(M), Y_i(S), Y_i(E)\}_{i \in I}$, and trade shares $\{\pi_{in}(j)\}_{i,n \in I, j \in \mathcal{J}}$, such that, given factor supplies, $\{H_i, L_i\}_{i \in I}$, technologies, $\{A_i(S), A_i(M), A_i(E)\}_{i \in I}$, and net exports, $\{nx_i\}_{i \in I}$, in each country, the following are satisfied:

1. Household's maximize utility subject to their budget constraints: The household's optimality conditions in steady state are given by the Euler equations,

$$1/\beta = r_i/P_i(E) + 1 - \delta_i(E), \quad (2.16)$$

$$1/\beta = v_i/P_i(S) + 1 - \delta_i(S), \quad (2.17)$$

the intra-temporal consumption equation,

$$P_i(M) C_i(M) = \frac{\phi}{1-\phi} P_i(S) C_i(S), \quad (2.18)$$

and the budget constraint,

$$(w_i L_i + s_i H_i + v_i K_i(S) + r_i K_i(E)) (1 + nx_i) = P_i(E) \delta_i(E) K_i(E) + P_i(M) C_i(M) + P_i(S) [C_i(S) + \delta_i(S) K_i(S)] \quad (2.19)$$

where nx_i denotes net exports as a share of GDP, which we take as a parameter.

2. Cost minimization by producers of intermediate goods: We first define the following input bundles to simplify notation,

$$\begin{aligned} b_{1,i} &= \left[\lambda_i^{1/\rho} k_E^{(\rho-1)/\rho} + (1 - \lambda_i)^{1/\rho} h^{(\rho-1)/\rho} \right]^{\rho/(\rho-1)} \\ b_{2,i} &= \left[\mu_i^{1/\sigma} l^{(\sigma-1)/\sigma} + (1 - \mu_i)^{1/\sigma} b_1^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \\ b_{3,i} &= k_S^{\alpha_i} b_2^{1-\alpha_i} \\ b_{4,i} &= x_S^{\varepsilon_i} x_M^{1-\varepsilon_i} \end{aligned}$$

so that the production function of (ω, j) intermediate good producers can be written as:

$$y_i(\omega, j) = A_i(j) z_i(\omega, j) b_{3,i}^{\zeta_i} b_{4,i}^{1-\zeta_i}.$$

Cost minimization implies that the unit cost of production for the domestic market of a

producer with productivity $A_i(j) z_i(\omega, j) = 1$, c_i , is given by

$$c_i = p_{b_{3,i}}^{\zeta_i} p_{b_{4,i}}^{1-\zeta_i} / \left[\zeta_i^{\zeta_i} (1 - \zeta_i)^{1-\zeta_i} \right], \quad (2.20)$$

where

$$p_{b_{1,i}} = [\lambda_i r_i^{1-\rho} + (1 - \lambda_i) s_i^{1-\rho}]^{1/(1-\rho)} \quad (2.21)$$

$$p_{b_{2,i}} = [\mu_i w_i^{1-\sigma} + (1 - \mu_i) p_{b_{1,i}}^{1-\sigma}]^{1/(1-\sigma)} \quad (2.22)$$

$$p_{b_{3,i}} = v_i^{\alpha_i} p_{b_{2,i}}^{1-\alpha_i} \alpha_i^{-\alpha_i} (1 - \alpha_i)^{\alpha_i-1} \quad (2.23)$$

$$p_{b_{4,i}} = P_i(S)^{\varepsilon_i} P_i(M)^{1-\varepsilon_i} \varepsilon_i^{-\varepsilon_i} (1 - \varepsilon_i)^{\varepsilon_i-1}. \quad (2.24)$$

Here, $p_{b_{1,i}}$, $p_{b_{2,i}}$, $p_{b_{3,i}}$, and $p_{b_{4,i}}$ denote the unit costs of the input bundles $b_{1,i}$, $b_{2,i}$, $b_{3,i}$, and $b_{4,i}$ in country i . Given these prices, factors demanded in the production of intermediate good (ω, j) in country i for goods sold in country n are given by

$$\begin{aligned} l_{in}(\omega, j) &= \mu_i (p_{b_{2,i}}/w_i)^\sigma b_{2,in}(\omega, j) \\ h_{in}(\omega, j) &= (1 - \lambda_i) (p_{b_{1,i}}/s_i)^\rho b_{1,in}(\omega, j) \\ k_{S,in}(\omega, j) &= \alpha_i p_{b_{3,i}} b_{3,in}(\omega, j) / v_i \\ k_{E,in}(\omega, j) &= \lambda_i (p_{b_{1,i}}/r_i)^\rho b_{1,in}(\omega, j) \\ x_{S,in}(\omega, j) &= \varepsilon_i p_{b_{4,i}} b_{4,in}(\omega, j) / P_i(S) \\ x_{M,in}(\omega, j) &= (1 - \varepsilon_i) p_{b_{4,i}} b_{4,in}(\omega, j) / P_i(M) \end{aligned}$$

where

$$\begin{aligned} b_{1,in}(\omega, j) &= (1 - \mu_i) (p_{b_{2,i}}/p_{b_{1,i}})^\sigma b_{2,in}(\omega, j) \\ b_{2,in}(\omega, j) &= (1 - \alpha_i) p_{b_{3,i}} b_{3,in}(\omega, j) / p_{b_{2,i}} \\ b_{3,in}(\omega, j) &= \zeta_i p_n(\omega, j) q_n(\omega, j) \mathbb{I}_{in}(\omega, j) / p_{b_{3,i}} \\ b_{4,in}(\omega, j) &= (1 - \zeta_i) p_n(\omega, j) q_n(\omega, j) \mathbb{I}_{in}(\omega, j) / p_{b_{4,i}} \end{aligned}$$

Here, $\mathbb{I}_{in}(\omega, j)$ is an indicator function that takes the value of one when country i supplies country n with intermediate good (ω, j) and is zero otherwise.

3. Cost minimization by producers of final goods: Cost minimization by final good producers implies that demand for variety (ω, j) in country i is given by

$$q_i(\omega, j) = \left(\frac{p_i(\omega, j)}{P_i(j)} \right)^{-\eta_i(j)} Y_i(j).$$

As shown in EK under our same distribution assumptions, price indices for final goods in any time period (even out of steady-state) are given by

$$P_i(j) = \gamma_i(j) \left\{ \sum_{k=1}^I \left[\tau_{kn}(j) \frac{c_k}{A_k(j)} \right]^{-1/\theta(j)} \right\}^{-\theta(j)} \quad (2.25)$$

where $\gamma_i(j) = \{\Gamma(1 + \theta(j)[1 - \eta_i(j)])\}^{1/[1 - \eta_i(j)]}$ and Γ is the Gamma function in country i . Trade shares between any pair of countries are given by equation (2.7).

4. Aggregate factor market clearing: Integrating factor demands across producers, adding across all destination countries n and sectors j , substituting for the demand each for variety $q_i(\omega, j)$ and using equation (2.6), we can write the aggregate factor market clearing conditions as,

$$v_i K_i(S) = \zeta_i \alpha_i \Phi_i, \quad (2.26)$$

$$w_i L_i = \zeta_i \mu_i (1 - \alpha_i) (p_{b_{2,i}}/w_i)^{\sigma-1} \Phi_i, \quad (2.27)$$

$$r_i K_i(E) = \zeta_i \lambda_i (1 - \alpha_i) (1 - \mu_i) \left(\frac{p_{b_{1,i}}}{r_i} \right)^{\rho-1} \left(\frac{p_{b_{2,i}}}{p_{b_{1,i}}} \right)^{\sigma-1} \Phi_i, \quad (2.28)$$

$$s_i H_i = \zeta_i (1 - \alpha_i) (1 - \mu_i) (1 - \lambda_i) \left(\frac{p_{b_{1,i}}}{s_i} \right)^{\rho-1} \left(\frac{p_{b_{2,i}}}{p_{b_{1,i}}} \right)^{\sigma-1} \Phi_i, \quad (2.29)$$

and,

$$P_i(S) X_i(S) = \varepsilon_i (1 - \zeta_i) \Phi_i, \quad (2.30)$$

$$P_i(M) X_i(M) = (1 - \varepsilon_i) (1 - \zeta_i) \Phi_i. \quad (2.31)$$

where:

$$\Phi_i \equiv \sum_n \sum_j \pi_{in}(j) P_n(j) Y_n(j) \quad (2.32)$$

denotes total revenue accruing to all country i producers across all sectors.

5. Aggregate goods markets clear in each country:

$$Y_i(M) = C_i(M) + X_i(M), \quad (2.33)$$

$$Y_i(S) = C_i(S) + X_i(S) + \delta_i(S) K_i(S), \quad (2.34)$$

$$Y_i(E) = \delta_i(E) K_i(E), \quad (2.35)$$

Note that, after choosing a numeraire, $(21 \times I + I \times I - 1)$ aggregate variables must be determined in equilibrium. Equations (2.7) and (2.16) – (2.35) give a system of $(21 \times I + I \times I - 1)$ independent equations, since the market clearing conditions together with the budget constraints and the definition of revenues make one budget constraint redundant.

2.6.1.2 Solving in Terms of Domestic Expenditure Shares

In this section we show how to solve for all domestic variables as functions of domestic expenditure shares, $\pi_{ii}(j)$. The problem can be split into two parts. First, we use a subset of equations to solve for all domestic prices. Second, we use these prices and the remaining equations to solve for quantities.

From equations (2.7) and (2.25), we can write aggregate price indices as functions of domestic expenditure shares

$$P_i(j) = \gamma_i(j) c_i \pi_{ii}(j)^{\theta(j)} / A_i(j), \quad (2.36)$$

and from equations (2.27) and (2.29) we obtain

$$\frac{s_i^\rho}{w_i^\sigma} \frac{H_i}{L_i} = (1 - \lambda_i) \frac{1 - \mu_i}{\mu_i} p_{b,i}^{\rho-\sigma}. \quad (2.37)$$

The 3 price index equations (2.36), together with equation (2.37) the Euler equations (2.16) – (2.17) and the cost minimization equations (2.20)–(2.24) make a system of 11 equations. Together with a choice of numeraire these equations can be used to solve for the 12 domestic prices.

Given prices, we can solve for quantities as follows. First, solve for $K_i(E)$ and $K_i(S)$

using (2.26), (2.28), and (2.29). Second, adding equations (2.26) – (2.29), we solve for Φ_i as

$$\zeta_i \Phi_i = v_i K_i(S) + w_i L_i + r_i K_i(E) + s_i H_i.$$

Third, using equations (2.30) and (2.31), we obtain intermediate inputs $X_i(M)$ and $X_i(S)$. Fourth, from equations (2.18) and (2.19) we can solve for the consumption levels $C_i(S)$ and $C_i(M)$. Finally, from the market clearing equations (2.33) – (2.35) we obtain total production in each sector.

2.6.1.3 Solving for Price Changes Across Steady States

In this section we derive steady-state changes in the skill premium and real wages in country i using the following system of six equations:

$$\tilde{r}_i = \left[\tilde{A}_i(S) / \tilde{A}_i(E) \right] \tilde{\pi}_{ii}(E)^{\theta(E)} \quad (2.38)$$

$$\tilde{s}_i^\rho / \tilde{w}_i^\sigma = \tilde{p}_{b_{1,i}}^{\rho-\sigma} \left(\tilde{L}_i / \tilde{H}_i \right) \quad (2.39)$$

$$\tilde{p}_{b_{1,i}} = \left[\frac{1}{1 + \xi_i^H} \tilde{r}_i^{1-\rho} + \frac{\xi_i^H}{1 + \xi_i^H} \tilde{s}_i^{1-\rho} \right]^{1/(1-\rho)} \quad (2.40)$$

$$\tilde{p}_{b_{3,i}}^{1/1-\alpha_i} = \left[\xi_i^L \tilde{w}_i^{1-\sigma} + (1 - \xi_i^L) \tilde{p}_{b_{1,i}}^{1-\sigma} \right]^{1/(1-\sigma)} \quad (2.41)$$

$$\tilde{p}_{b_{3,i}} = \tilde{A}_i(S)^{(\varepsilon_i + \zeta_i - \varepsilon_i \zeta_i)/\zeta_i} \left[\tilde{A}_i(M) / \tilde{\pi}_{ii}(M)^{\theta(M)} \right]^{(1-\varepsilon_i)(1-\zeta_i)/\zeta_i} \quad (2.42)$$

$$\tilde{P}_i(C) = \left(\tilde{\pi}_{ii}(M)^{\theta(M)} \tilde{A}_i(M) / \tilde{A}_i(S) \right)^\phi \quad (2.43)$$

where, $\tilde{x} \equiv x'/x$ denotes the ratio of a variable between the new and initial equilibrium, and where $\xi_i^H = \frac{s_i H_i}{r_i K_i(E)}$ and $\xi_i^L = \frac{w_i L_i}{w_i L_i + s_i H_i + r_i K_i(E)}$ denote relative factor shares in the initial equilibrium.

We proceed in order. Taking changes between the new and initial equilibrium using equation (2.36) gives

$$\tilde{P}_i(j) = \tilde{c}_i \tilde{\pi}_{ii}(j)^{\theta(j)} / \tilde{A}_i(j). \quad (2.44)$$

Similarly, by equation (2.18), we have

$$\tilde{r}_i = \tilde{P}_i(E). \quad (2.45)$$

Equations (2.44) and (2.45) imply equation (2.38). Equations (2.39) and (2.40) follow directly from expressing equations (2.37) and the definition of $p_{b_{1,i}}$ in changes, respectively.

To obtain equations, (2.41) and (2.42), we express the remaining marginal cost equations in changes,

$$\tilde{c}_i = \tilde{p}_{b_{3,i}}^{\zeta_i} \tilde{p}_{b_{4,i}}^{1-\zeta_i} \quad (2.46)$$

$$\tilde{p}_{b_{4,i}} = \tilde{P}_i(S)^{\varepsilon_i} \tilde{P}_i(M)^{1-\varepsilon_i} \quad (2.47)$$

$$\tilde{p}_{b_{3,i}} = \tilde{v}_i^{\alpha_i} \tilde{p}_{b_{2,i}}^{1-\alpha_i} \quad (2.48)$$

$$\tilde{p}_{b_{2,i}} = \left[\xi_i^L \tilde{w}_i^{1-\sigma} + (1 - \xi_i^L) \tilde{p}_{b_{1,i}}^{1-\sigma} \right]^{1/1-\sigma}. \quad (2.49)$$

Letting $P_i(S) = 1$ be the numeraire, equation (2.17) implies $\tilde{v}_i = \tilde{P}_i(S) = 1$. Hence, equations (2.48) and (2.49) imply equation (2.41). By equation (2.44) and $\pi_{ii}(S) = 1$, we have

$$\tilde{c}_i = \tilde{A}_i(S). \quad (2.50)$$

By equations (2.46), (2.47), and (2.50), we have

$$\tilde{A}_i(S) = \tilde{p}_{b_{3,i}}^{\zeta_i} \tilde{p}_{b_{4,i}}^{1-\zeta_i}$$

and

$$\tilde{p}_{b_{4,i}} = \tilde{P}_i(M)^{1-\varepsilon_i} = \left\{ \tilde{A}_i(S) \tilde{\pi}_{ii}(M)^{\theta(M)} / \tilde{A}_i(M) \right\}^{1-\varepsilon_i}.$$

The two previous equations imply equation (2.42).

Finally, we obtain equation (2.43) using equation (2.44), recalling that $P_i(S)$ is the numeraire.

2.6.2 Proofs

In this section, we prove Propositions 1 and 2.

Derivation of Equation (2.8). By equations (2.27) and (2.29), we have

$$\left(\frac{r_i}{s_i} \right)^{1-\rho} = \left[\frac{1 - \lambda_i}{\lambda_i} \frac{K_i(E)}{H_i} \right]^{\frac{\rho-1}{\rho}}. \quad (2.51)$$

From the definition of $p_{b_{1,i}}$ and equation (2.51), we have

$$\frac{p_{b_{1,i}}}{s_i} = (1 - \lambda_i)^{-\frac{1}{\rho}} \left\{ \lambda_i^{\frac{1}{\rho}} \left[\frac{K_i(E)}{H_i} \right]^{\frac{\rho-1}{\rho}} + (1 - \lambda_i)^{\frac{1}{\rho}} \right\}^{\frac{1}{1-\rho}}. \quad (2.52)$$

In addition, equations (2.27) and (2.29) imply

$$\frac{s_i}{w_i} = (1 - \lambda_i)^{\frac{1}{\sigma}} \left(\frac{1 - \mu_i}{\mu_i} \right)^{\frac{1}{\sigma}} \left(\frac{p_{b,i}}{s_i} \right)^{\frac{\rho-\sigma}{\sigma}} \left(\frac{L_i}{H_i} \right)^{\frac{1}{\sigma}}. \quad (2.53)$$

From equations (2.52) and (2.53), we obtain equation (2.8).

Derivation of Equation (2.9). Let $\hat{x} \equiv \log(\tilde{x})$. Using this notation, we express equations (2.38), (2.39), and (2.42) as

$$\hat{r}_i = \hat{A}_i(S) - \hat{A}_i(E) + \theta(E) \hat{\pi}_{ii}(E) \quad (2.54)$$

$$\rho \hat{s}_i - \sigma \hat{w}_i = (\rho - \sigma) \hat{p}_{b_{1,i}} - (\hat{H}_i - \hat{L}_i) \quad (2.55)$$

$$\hat{p}_{b_{3,i}} = \frac{\zeta_i + \varepsilon_i - \zeta_i \varepsilon_i}{\zeta_i} \hat{A}_i(S) + \frac{(1 - \zeta_i)(1 - \varepsilon_i)}{\zeta_i} \left[\hat{A}_i(M) - \theta(M) \hat{\pi}_{ii}(M) \right]. \quad (2.56)$$

Using the first-order approximation, $\exp(\hat{x}) \approx 1 + \hat{x}$, we express equations (2.41) and (2.40) as

$$\hat{p}_{b_{1,i}} = \frac{1}{(1 - \xi_i^L)(1 - \alpha_i)} \hat{p}_{b_{3,i}} - \frac{\xi_i^L}{1 - \xi_i^L} \hat{w}_i \quad (2.57)$$

$$\hat{p}_{b_{1,i}} = \frac{1}{1 + \xi_i^H} \hat{r}_i + \frac{\xi_i^H}{1 + \xi_i^H} \hat{s}_i \quad (2.58)$$

We now solve equations (2.54) – (2.58) for $\hat{s}_i - \hat{w}_i$. By equations (2.57) and (2.55), we have

$$\hat{s}_i - \hat{w}_i = \frac{\sigma - \rho}{\rho} \frac{1}{1 - \xi_i^L} \left[\hat{w}_i - \frac{\hat{p}_{b_{3,i}}}{(1 - \alpha_i)} \right] - \frac{1}{\rho} (\hat{H}_i - \hat{L}_i), \quad (2.59)$$

whereas by equations (2.57) and (2.58), we have

$$\hat{s}_i = \frac{1 + \xi_i^H}{\xi_i^H (1 - \xi_i^L)} \frac{\hat{p}_{b_{3,i}}}{(1 - \alpha_i)} - \frac{\xi_i^L (1 + \xi_i^H)}{(1 - \xi_i^L) \xi_i^H} \hat{w}_i - \frac{1}{\xi_i^H} \hat{r}_i.$$

Using the two previous expressions to solve for $\widehat{w}_i - \frac{\widehat{p}_{b_{3,i}}}{1-\alpha_i}$ we obtain

$$\widehat{w}_i - \frac{\widehat{p}_{b_{3,i}}}{1-\alpha_i} = \frac{\rho(1-\xi_i^L)}{\rho\xi_i^L + \sigma\xi_i^H} \left[\frac{\widehat{p}_{b_{3,i}}}{(1-\alpha_i)} - \widehat{r}_i + \xi_i^H \frac{1}{\rho} (\widehat{H}_i - \widehat{L}_i) \right]. \quad (2.60)$$

By equations (2.48), (2.59), and (2.60) we have

$$\widehat{s}_i - \widehat{w}_i = -\frac{\xi_i^H + \xi_i^L}{\sigma\xi_i^H + \rho\xi_i^L} (\widehat{H}_i - \widehat{L}_i) + \Theta_i (\widehat{p}_{b_{2,i}} - \widehat{r}_i). \quad (2.61)$$

Then, given factor supplies in the domestic country, changes in the skill premium are determined by changes in the price of the composite bundle of equipment and both types of labor, $\widehat{p}_{b_{2,i}}$, relative to changes in the price of equipment, \widehat{r}_i . Note that Θ_i is the elasticity of the skill premium with respect to this relative price.

Finally, by equations (2.54), (2.56), (2.48) and (2.61), we have equation (2.9).

Derivation of Equations (2.12) and (2.13). We can write the log change in the consumption price index as

$$\widehat{P}_i(C) = \phi \widehat{A}_i(S) - \phi \left[\widehat{A}_i(M) - \theta(M) \widehat{\pi}_i(M) \right] \quad (2.62)$$

Using equation (2.60) to substitute for \widehat{w}_i in (2.59) and solving for \widehat{s}_i we obtain

$$\widehat{s}_i = \frac{\sigma(1+\xi_i^H)}{\rho\xi_i^L + \sigma\xi_i^H} \frac{\widehat{p}_{b_{3,i}}}{(1-\alpha_i)} - \frac{\sigma - \rho\xi_i^L}{\rho\xi_i^L + \sigma\xi_i^H} \widehat{r}_i - \xi_i^L \frac{1+\xi_i^H}{\rho\xi_i^L + \sigma\xi_i^H} (\widehat{H}_i - \widehat{L}_i),$$

Together with equations (2.54), (2.56) and (2.62), the previous expression gives

$$\begin{aligned} \widehat{s}_i - \widehat{P}_i(C) &= -\xi_i^L \frac{1+\xi_i^H}{\rho\xi_i^L + \sigma\xi_i^H} (\widehat{H}_i - \widehat{L}_i) + \left[\frac{\sigma(1+\xi_i^H)}{\rho\xi_i^L + \sigma\xi_i^H} \frac{\zeta_i + \varepsilon_i - \zeta_i \varepsilon_i}{\zeta_i(1-\alpha_i)} - \frac{\sigma - \rho\xi_i^L}{\rho\xi_i^L + \sigma\xi_i^H} - \phi \right] \widehat{A}_i(S) \\ &\quad + \left[\frac{\sigma(1+\xi_i^H)}{\rho\xi_i^L + \sigma\xi_i^H} \frac{(1-\zeta_i)(1-\varepsilon_i)}{\zeta_i(1-\alpha_i)} + \phi \right] \left[\widehat{A}_i(M) - \theta(M) \widehat{\pi}_{ii}(M) \right] \\ &\quad + \frac{\sigma - \rho\xi_i^L}{\rho\xi_i^L + \sigma\xi_i^H} \left[\widehat{A}_i(E) - \theta(E) \widehat{\pi}_{ii}(E) \right]. \end{aligned}$$

The previous expression and the definitions of $\nu_i(j)$, Θ_i , and $\varkappa_i(j)$ yield equation (2.12).

Solving for \widehat{w}_i in equation (2.60), subtracting (2.62), and substituting $\widehat{p}_{b_{3,i}}$ and \widehat{r}_i using

(2.56) and (2.54) gives

$$\begin{aligned}\widehat{w}_i - \widehat{P}_i(C) &= \frac{(1 - \xi_i^L) \xi_i^H}{\rho \xi_i^L + \sigma \xi_i^H} (\widehat{H}_i - \widehat{L}_i) + \left[\frac{\rho + \sigma \xi_i^H}{\rho \xi_i^L + \sigma \xi_i^H} \frac{\zeta_i + \varepsilon_i - \zeta_i \varepsilon_i}{\zeta_i (1 - \alpha_i)} - \frac{\rho (1 - \xi_i^L)}{\rho \xi_i^L + \sigma \xi_i^H} - \phi \right] \widehat{A}_i(S) \\ &+ \left[\frac{\rho + \sigma \xi_i^H}{\rho \xi_i^L + \sigma \xi_i^H} \frac{(1 - \zeta_i) (1 - \varepsilon_i)}{\zeta_i (1 - \alpha_i)} + \phi \right] [\widehat{A}_i(M) - \theta(M) \widehat{\pi}_{ii}(M)] \\ &+ \frac{\rho (1 - \xi_i^L)}{\rho \xi_i^L + \sigma \xi_i^H} [\widehat{A}_i(E) - \theta(E) \widehat{\pi}_{ii}(E)].\end{aligned}$$

The previous expression and the definitions of $\nu_i(j)$, Θ_i , and $\varkappa_i(j)$ yield equation (2.13).

2.6.3 Data and Parameterization

Domestic Expenditure Shares: For trade data, we define equipment trade as the sum of BEA industry codes 20-27 and 33. These codes are: Farm and Garden Machinery; Construction, Mining, etc.; Computer and Office Equipment; Other Nonelectric Machinery; Household Appliances; Household Audio and Video, etc.; Electronic Components; Other Electrical Machinery; and Instruments and Apparatus.

For gross output data, we define capital equipment goods as the sum of ISIC Rev. 3 codes 29-33. These codes are: Manufacture of machinery and equipment n.e.c.; Manufacture of office, accounting and computing machinery; Manufacture of electrical machinery and apparatus n.e.c.; Manufacture of radio, television and communication equipment and apparatus; and Manufacture of medical, precision and optical instruments, watches and clocks.

Disaggregating capital payments into structures and equipment: For a given share of payments to capital in value added, i.e.

$$\frac{v_i K_i(S) + r_i K_i(E)}{s_i H_i + w_i L_i + v_i K_i(S) + r_i K_i(E)},$$

the parameter α_i determines the ratio of payments to capital structures relative to the payments to equipment capital, i.e. $v_i K_i(S) / [r_i K_i(E)]$. Given the difficulty of measuring capital rental rates, we construct them using the steady-state Euler equations for the

accumulation of each type of capital,

$$\begin{aligned} 1 + R_i &= \frac{P_{i,t+1}(S)/P_{i,t+1}(C)}{P_{i,t}(S)/P_{i,t}(C)} \left\{ 1 - \delta_i(S) + \frac{v_{i,t+1}}{P_{i,t+1}(S)} \right\} \\ &= \frac{P_{i,t+1}(E)/P_{i,t+1}(C)}{P_{i,t}(E)/P_{i,t}(C)} \left\{ 1 - \delta_i(E) + \frac{r_{i,t+1}}{P_{i,t+1}(E)} \right\} \end{aligned}$$

where R_i denotes the consumption-based real-interest rate and $P_{i,t}(C)$ denotes the price of the final consumption good in year t . Note that, in this calculation we allow for trends in relative prices (as above, introducing growth into our model does not change our results on the impact of trade on the skill premium).

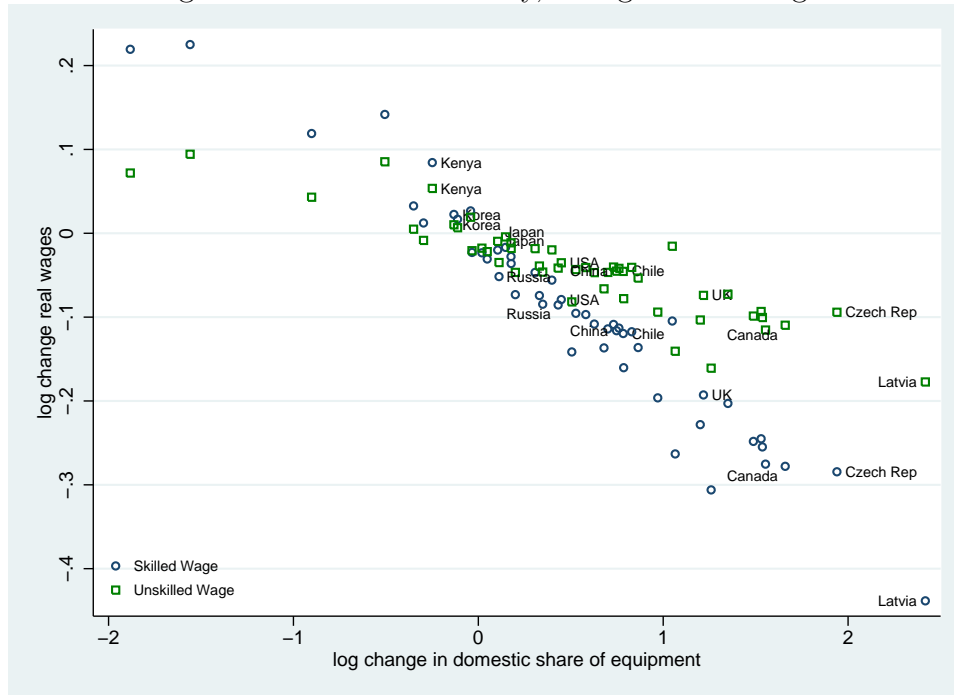
To solve for the rental rates, we use data from NIPA for the 1963-2000 period. We define non-residential equipment and software as the equipment sector E , and non-residential structures as the structure sector, S . We take $P_{i,t+1}(E)/P_{i,t}(E)$ and $P_{i,t+1}(S)/P_{i,t}(S)$ from NIPA's price indices for private investment (NIPA table 5.3.4). We use the GDP deflator from NIPA for $P_{i,t+1}(C)/P_{i,t}(C)$. We construct the annual depreciation rates of equipment and structures, $\delta_i(E)$ and $\delta_i(S)$, as the ratio of the current-cost depreciation (NIPA fixed assets table 4.4) to the current cost capital stock (NIPA fixed assets table 4.1) in these two sectors. We set the real interest rate R_i to 4%.

We use the 1963-2000 average of these variables and the Euler equations to obtain the relative return for equipment and structures $v_i/P_i(S)/[r_i/P_i(E)]$. We multiply this by the relative value of the capital stocks $[P_i(S)K_i(S)/P_i(E)K_i(E)]$ to obtain $v_iK_i(S)/[r_iK_i(E)]$. We use the 1963-2000 average current cost capital stock of non-residential equipment and non-residential structures (NIPA fixed assets table 4.1) for $P_i(E)K_i(E)$ and $P_i(S)K_i(S)$. Finally, to compute the share of payments to structures capital in value added, α , we use the relative payments to structures and equipment and the share of payments to capital in value added (equal to one minus the average labor share, as defined in the body of the paper). We obtain a very similar value for α if we first calculate, year by year, the relative payments to equipment and structures and the share of capital, and then average these over time.

Chilean data and calibration: We use data on changes in the skill premium and on the stocks of capital equipment (not adjusted for quality), skilled labor and unskilled

labor for the time period 1983-2000 from Gallego (2012) [22]. We adjust the stock of capital equipment using the same adjustment factor as in the US, obtained from Polgreen and Silos (2008) [36]. We calculate the labor share in value added as the ratio of the sum of compensation for employees and the surplus of enterprises owned by households to the sum of compensation for employees and all operating surplus.²⁷ Due to a lack of data on prices and on depreciation rates of capital equipment and structures, we assume that the share of structures in value added is the same in Chile as in the US, $\alpha = 0.1$. Finally, the share of value added in gross output ζ , and the share of services in intermediate inputs, ε , used to compute $\varkappa_i(M)$ for Chile are set at their average shares in 1996 and 2003 from the OECD Input-Output database [34].

Figure 2.1: Move to autarky, change in real wages



²⁷We only have data on surplus of enterprises owned by households (Mixed Income) between 1996-2002. We assume that in the years 1983-2000, the ratio of Mixed Income to Operating Surplus equals 0.196, which is the average for the 1996-2002 period. The source of this data is the National Accounts Official Country Data from the United Nations Statistics Division.

Figure 2.2: Move to autarky, change in the skill premium

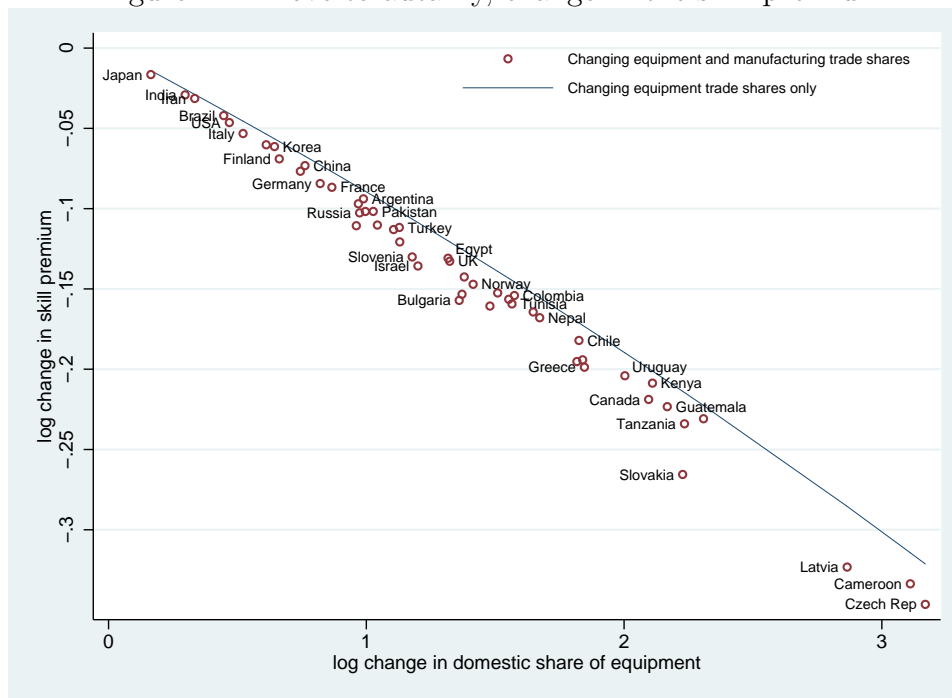


Figure 2.3: Observed changes in trade shares between 1963 and 2000

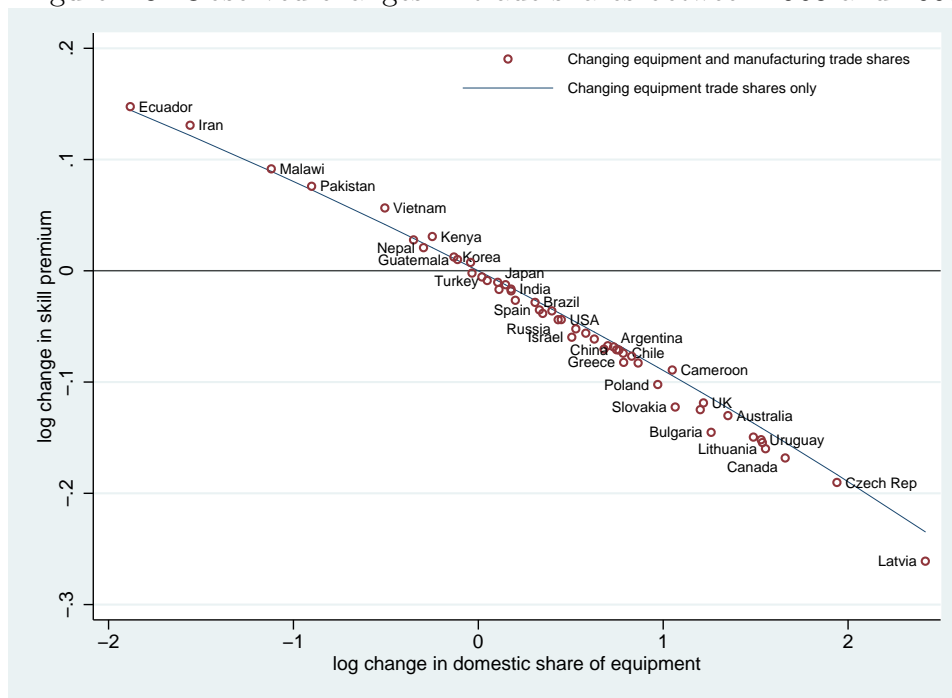


Table 2.1: Baseline parameter values

	US	Chile
σ	1.56	1.54
ρ	0.63	0.38
ζ	0.54	0.49
ε	0.62	0.6
ξ_i^H	1.37	1.12
ξ_i^L	0.44	0.31
ϕ	0.2	Not used for skill premium
α	0.1	0.1
$\theta(E)$	0.22	0.22
$\theta(M)$	0.19	0.19

We impose that α and $\theta(j)$ are equal in Chile and the US

Table 2.2: Domestic expenditure shares

Country	Initial eqm yr	Counterf. eqm yr	$\pi_{ii}(E)$	$\pi'_{ii}(E)$	$\pi_{ii}(M)$	$\pi'_{ii}(M)$
Argentina	2000	1984	0.37	0.77	0.83	0.95
Australia	2000	1963	0.19	0.74	0.70	0.87
Austria	2000	1963	0.16	0.54	0.47	0.79
Bangladesh	1998	1972	0.37	0.43	0.65	0.65
Brazil	2000	1990	0.64	0.87	0.89	0.95
Bulgaria	2000	1980	0.26	0.90	0.35	0.95
Cameroon	2000	1970	0.04	0.13	0.60	0.50
Canada	2000	1963	0.12	0.65	0.56	0.87
Chile	2000	1963	0.16	0.35	0.71	0.84
China	2000	1977	0.47	0.99	0.81	0.97
Colombia	2000	1963	0.21	0.44	0.76	0.88
Czech Rep	2000	1995	0.04	0.29	0.51	0.64
Denmark	2000	1963	0.23	0.54	0.46	0.56
Ecuador	2000	1963	0.10	0.02	0.78	0.68
Egypt	1998	1964	0.27	0.27	0.70	0.81
Finland	2000	1963	0.52	0.50	0.68	0.83
France	2000	1963	0.42	0.79	0.72	0.90
Germany	2000	1991	0.44	0.65	0.67	0.71
Greece	1998	1963	0.16	0.35	0.46	0.71
Guatemala	1998	1968	0.11	0.10	0.62	0.61
India	1999	1963	0.74	0.89	0.88	0.92
Iran	2000	1963	0.72	0.15	0.91	0.60
Israel	2000	1963	0.30	0.50	0.41	0.72
Italy	2000	1967	0.59	0.71	0.76	0.84
Japan	2000	1963	0.85	0.94	0.91	0.96
Kenya	2000	1963	0.12	0.09	0.80	0.54
Korea	2000	1963	0.53	0.46	0.84	0.80
Kyrgyzstan	2000	1992	0.21	0.98	0.66	0.98
Latvia	2000	1992	0.06	0.64	0.36	0.76
Lithuania	2000	1992	0.16	0.75	0.52	0.87
Malawi	2000	1965	0.02	0.01	0.59	0.54
Nepal	1996	1986	0.19	0.14	0.68	0.79
Norway	2000	1963	0.24	0.43	0.57	0.68
Pakistan	2000	1963	0.36	0.15	0.72	0.63
Poland	2000	1982	0.35	0.93	0.57	0.97
Portugal	2000	1963	0.25	0.28	0.59	0.77
Romania	2000	1985	0.22	0.98	0.65	0.97
Russia	2000	1996	0.38	0.58	0.59	0.75
Slovakia	2000	1993	0.11	0.31	0.22	0.54
Slovenia	2000	1992	0.31	0.44	0.46	0.62
Spain	2000	1963	0.38	0.53	0.70	0.90
Sweden	2000	1963	0.33	0.67	0.64	0.78
Switzerland	2000	1986	0.25	0.58	0.41	0.45
Macedna	2000	1993	0.38	0.47	0.43	0.61
Tanzania	1999	1965	0.11	0.08	0.56	0.59
Tunisia	2000	1963	0.21	0.20	0.63	0.54
Turkey	2000	1963	0.32	0.34	0.72	0.85
UK	2000	1963	0.27	0.90	0.67	0.89
USA	2000	1963	0.63	0.98	0.82	0.97
Ukraine	2000	1992	0.48	0.94	0.68	0.99
Uruguay	2000	1968	0.13	0.62	0.65	0.91
VietNam	2000	1998	0.32	0.19	0.53	0.29
Zimbabwe	1996	1964	0.54	0.92	0.79	0.99

Note: "Initial eqm yr." — the year used to obtain trade shares for the initial steady state of our counterfactuals.

"Counterf. eqm. yr." — the year used to obtain trade shares in the new steady state of counterfactual 2.

Table 2.3: Counterfactual changes in real wages and the skill premium

Country	Cf 1: Autarky			Cf 2: Observed trade		
	S/P	W/P	S/W	S/P	W/P	S/W
Argentina	-0.15	-0.05	-0.09	-0.11	-0.04	-0.07
Australia	-0.26	-0.10	-0.16	-0.20	-0.07	-0.13
Austria	-0.35	-0.15	-0.20	-0.23	-0.10	-0.12
Bangladesh	-0.19	-0.08	-0.10	-0.02	0.00	-0.01
Brazil	-0.07	-0.03	-0.04	-0.05	-0.02	-0.03
Bulgaria	-0.33	-0.17	-0.16	-0.31	-0.16	-0.15
Cameroon	-0.51	-0.18	-0.33	-0.10	-0.02	-0.09
Canada	-0.36	-0.14	-0.22	-0.28	-0.11	-0.17
Chile	-0.29	-0.10	-0.18	-0.12	-0.05	-0.07
China	-0.12	-0.05	-0.07	-0.12	-0.05	-0.07
Colombia	-0.24	-0.09	-0.15	-0.11	-0.04	-0.07
Czech Rep	-0.54	-0.20	-0.35	-0.28	-0.09	-0.19
Denmark	-0.30	-0.14	-0.16	-0.14	-0.05	-0.08
Ecuador	-0.34	-0.11	-0.23	0.22	0.07	0.15
Egypt	-0.22	-0.09	-0.13	-0.02	-0.02	-0.01
Finland	-0.14	-0.07	-0.07	-0.02	-0.02	0.00
France	-0.15	-0.07	-0.09	-0.11	-0.05	-0.06
Germany	-0.16	-0.07	-0.08	-0.06	-0.02	-0.04
Greece	-0.36	-0.16	-0.20	-0.16	-0.08	-0.08
Guatemala	-0.36	-0.13	-0.22	0.02	0.01	0.01
India	-0.05	-0.03	-0.03	-0.03	-0.01	-0.02
Iran	-0.05	-0.02	-0.03	0.23	0.09	0.13
Israel	-0.28	-0.15	-0.14	-0.14	-0.08	-0.06
Italy	-0.10	-0.05	-0.05	-0.04	-0.02	-0.02
Japan	-0.03	-0.02	-0.02	-0.02	-0.01	-0.01
Kenya	-0.31	-0.10	-0.21	0.08	0.05	0.03
Korea	-0.10	-0.04	-0.06	0.02	0.01	0.01
Kyrgyzstan	-0.26	-0.10	-0.16	-0.25	-0.10	-0.15
Latvia	-0.55	-0.23	-0.32	-0.44	-0.18	-0.26
Lithuania	-0.34	-0.14	-0.19	-0.28	-0.12	-0.16
Malawi	-0.66	-0.22	-0.44	0.14	0.05	0.09
Nepal	-0.27	-0.10	-0.17	0.01	-0.01	0.02
Norway	-0.26	-0.12	-0.15	-0.10	-0.04	-0.06
Pakistan	-0.17	-0.07	-0.10	0.12	0.04	0.08
Poland	-0.21	-0.10	-0.11	-0.20	-0.09	-0.10
Portugal	-0.25	-0.11	-0.14	-0.05	-0.03	-0.02
Romania	-0.26	-0.10	-0.15	-0.25	-0.10	-0.15
Russia	-0.20	-0.09	-0.10	-0.09	-0.04	-0.04
Slovakia	-0.52	-0.26	-0.27	-0.26	-0.14	-0.12
Slovenia	-0.26	-0.13	-0.13	-0.08	-0.05	-0.04
Spain	-0.17	-0.07	-0.10	-0.07	-0.04	-0.04
Sweden	-0.20	-0.09	-0.11	-0.11	-0.05	-0.07
Switzerland	-0.31	-0.15	-0.15	-0.12	-0.04	-0.08
Macedonia	-0.24	-0.13	-0.11	-0.07	-0.05	-0.03
Tanzania	-0.38	-0.15	-0.23	0.03	0.00	0.03
Tunisia	-0.27	-0.11	-0.16	0.03	0.02	0.01
Turkey	-0.19	-0.08	-0.11	-0.03	-0.02	-0.01
UK	-0.22	-0.09	-0.13	-0.19	-0.07	-0.12
USA	-0.09	-0.04	-0.05	-0.08	-0.04	-0.04
Ukraine	-0.15	-0.07	-0.08	-0.14	-0.07	-0.07
Uruguay	-0.33	-0.12	-0.20	-0.25	-0.09	-0.15
Viet Nam	-0.23	-0.11	-0.12	0.14	0.09	0.06
Zimbabwe	-0.11	-0.05	-0.06	-0.10	-0.04	-0.05
Median	-0.25	-0.10	-0.14	-0.10	-0.04	-0.05

“Cf 1: Autarky” — counterfactual moving from 2000 trade to autarky. “Cf 2: Observed trade” — counterfactual moving from 2000 to start of sample trade. S/P : real wage of skilled workers, W/P : real wage of unskilled workers, S/W : skill premium. S/W may not equal $S/P - W/P$ in the table due to rounding error.

Table 2.4: Alternative parameterizations

Parameterization	Implied elasticity of the skill premium to sectoral import shares:		Counterfactual change in the skill premium: median country	
	Equipment: $\Theta\theta(E)$	Manufacturing: $\Theta\theta_i(M)\theta(M)$	Cf1: Autarky	Cf 2: Observed trade
Baseline with US data	0.085	0.026	-0.14	-0.05
Calibration with Chilean data	0.138	0.055	-0.26	-0.09
Skill-biased trend $\vartheta = 0.01$	0.066	0.020	-0.11	-0.04
Skill-biased trend $\vartheta = 0.02$	0.048	0.015	-0.08	-0.03
$\theta(E) = \theta(M) = 0.15$	0.058	0.021	-0.09	-0.04
$\theta(E) = \theta(M) = 0.25$	0.096	0.035	-0.16	-0.06
$r_i K_i(E)/v_i K_i(S) = 1$; i.e. $\alpha = 0.15$	0.071	0.023	-0.11	-0.04
$r_i K_i(E)/v_i K_i(S) = 3$; i.e. $\alpha = 0.076$	0.091	0.028	-0.15	-0.06
Note: Elasticities are derived to a first-order approximation. Counterfactuals are computed using the exact solution. "Cf 1: Autarky" refers to the counterfactual moving from 2000 trade levels to autarky. "Cf 2: Observed trade" refers to the counterfactual moving from 2000 to start of sample trade levels.				

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CHAPTER 3

Exchange Rates, Aggregate Productivity and the Currency of Invoicing of International Trade

3.1 Introduction

Nominal exchange rates often experience dramatic fluctuations.¹ How do these movements affect the real economy? A large literature in international economics has emphasized two mechanisms through which nominal exchange rate movements can impact real output and productivity. First, exchange rate movements can stimulate (depress) output by inducing a switch in expenditures between domestic and foreign goods.² Second, exchange rates can affect efficiency in the allocation of factors across firms by inducing price movements that are not related to changes in marginal costs.³ Understanding the quantitative importance of these two mechanisms is key for the design of exchange rate policy.

This paper sheds light on the strength of these forces guided by a novel dataset from Chilean customs that contains information on the currency used to invoice transactions. The first contribution of the paper is to use these data to identify the expenditure switching effects of exchange rates. The second contribution is to measure how exchange rate movements impact aggregate productivity using a quantitative model of international prices and nominal rigidities guided by the data.

A major challenge in identifying the expenditure switching effects is that exchange rates respond to shocks that simultaneously affect supply and demand conditions in the domestic and foreign economies. I am able to identify these effects in the customs data by

¹For instance, the euro/dollar exchange rate fell 35 percent between 2002 and 2004, only to increase 16 percent in 2005. The pound/ dollar exchange rate increased 40 percent during 2008.

²This topic has motivated a vast literature on how exchange rate movements affect international relative prices. See Burstein and Gopinath (2012) [6] for a summary.

³See Corsetti et al. (2010) [8].

exploiting the fact that Chilean exporters use different currencies to invoice transactions. In particular, I compare how exporters selling the same product into the same destination but invoicing in different currencies respond to changes in the exchange rate. Consistent with previous findings in the literature (Gopinath et al. (2010) [28]), I show that export prices are rigid in the currency in which they are invoiced, so the relative price of two products invoiced in different currencies fluctuates almost one-to-one with the exchange rate. The response in relative quantities to this change in relative prices can be used to identify the elasticity.⁴ I estimate an elasticity of quantities in response to the exchange rate that is in the range of -1 and -2. These values are in line with those used by the international business cycle literature to match the observed comovements between the terms of trade and the trade balance. In contrast, my estimates are obtained directly from microdata on the response of prices and quantities to changes in the exchange rate. Such low elasticities indicate that the expenditure switching effects are limited, even when exchange rate movements change relative prices.

I then measure how movements in exchange rates impact aggregate productivity using a quantitative model of international prices with Calvo sticky prices. An extensive literature studies how exchange rates affect efficiency by inducing price movements across firms that are not driven by changes in marginal costs. This literature has been mainly theoretical in nature, and typically assumes constant desired markups and no heterogeneity in invoicing.⁵ In contrast, I design a quantitative model of international relative prices that is consistent with the following three features of the Chilean data. First, there is substantial heterogeneity in the currency in which exporters invoice transactions. Second, the relative price of exports invoiced in different currencies displays persistent changes that comove with the exchange rate. Third, firms that invoice in the destination market's currency have a higher markup elasticity with respect to changes in the exchange rate than firms invoicing in U.S. dollars.⁶ I show that by incorporating these assumptions

⁴This assumes that the currency of invoicing is set before the exchange rate changes, and that relative demand shocks affecting both firms are uncorrelated with nominal exchange rates. As I show in the empirical section, these assumptions are likely to hold in these data.

⁵See for example Engel (2002, 2011) [19], [17] and Gali and Monacelli (2005) [22]. Dotsey and Duarte (2010) [15] and Gust et al. (2009) [29] are examples of richer quantitative models that evaluate the role of exchange rate pass-through on aggregate variables such as the trade balance. None of these papers focus on how exchange rates affect production efficiency.

⁶Gopinath et al. (2010) [28] provide related evidence of this fact by documenting substantial differences in pass-through into the United States of the average good priced in dollars versus non-dollars

into the model, I obtain very different measures of efficiency losses due to exchange rate movements than those obtained under the standard assumptions made in the literature.

I parameterize a three-country version of the model, taking the countries to represent Chile, the U.S. and Europe. I simulate an appreciation of the euro against all currencies and evaluate its effect on Chilean output per worker, where output per worker is calculated following the procedures used by statistical agencies. In my baseline calibration, a 10 percent appreciation of the euro reduces Chilean productivity in the tradable sector by 0.5 percent. These effects are persistent: productivity is still 0.25 percent below the initial steady state a year after the shock. To evaluate the role of heterogeneity in invoicing, I reparameterize the model by assuming that all Chilean firms selling in each destination invoice in the same currency. I also conduct counterfactual parameterizations that assume that invoicing is uncorrelated to the elasticity of desired markups. These alternative parameterizations predict changes in productivity that are at least five times smaller than my baseline results. This indicates that taking heterogeneity in invoicing into account is crucial for understanding how exchange rate fluctuations affect productive efficiency.

The response of productivity to an exchange rate shock depends on whether the shock magnifies or reduces the initial dispersion in markups. In the model, larger firms have higher desired markups. I show in the data that the invoicing is strongly correlated with firm's size and use this correlation to put discipline on the relation between desired markups and invoicing in the model. Other key parameters are the share of firms invoicing in each currency and the elasticity of substitution across products. The first of these parameters is directly observable in the data, while the elasticity is estimated in the first section of the paper.

Some final considerations are in order. First, firms may respond differently to changes in the exchange rate if they use imported intermediate inputs from different source countries.⁷ In such instances, changes in the relative price of these firms would be an efficient response to the changes in input costs generated by the exchange rate. I show, however, that the changes in relative prices that I document arise from movements in relative

after conditioning on a price change.

⁷Using data from Belgium, Amiti et al. (2012) [1] argue that about half of the lack of exchange rate pass-through into prices comes from this channel.

markups rather than from relative costs. In particular, in estimating how prices respond to the exchange rate, I exploit the fact that Chilean firms sell into more than one destination and use a fixed-effect strategy to control for changes in firm level marginal costs that are common across destinations.⁸

Second, the currency in which exporters invoice their exports is exogenously determined in the model. From the observed correlation between firm size and invoicing, it seems clear that firms select into invoicing currencies. The observed correlation between size and pass-through suggests that firms with a low desired pass-through choose to invoice using the destination market's currency. This correlation between desired pass-through and invoicing is taken into account for the calibration of the model. While these invoicing decisions can be endogenized, I do not expect this modification to significantly affect the quantitative results of the paper.

Finally, it is well known that the efficiency losses from inflation are smaller when price rigidities are state-dependent rather than time-dependent, as is assumed in my model.⁹ In this sense, one could interpret the results from my quantitative exercises as evidence that heterogeneity in invoicing greatly amplifies the effects of exchange rates on productivity rather than focusing on the absolute numbers of the counterfactuals.

Relation to existing literature: This paper is related to various strands of literature. First, there is a growing literature that uses firm or product level data to document how international prices respond to changes in the nominal exchange rate.¹⁰ From this literature, the paper that is the closest to mine is Gopinath et al. (2010) [28], who document differences in pricing practices by firms importing into the U.S. in dollars vs. non-dollars. My contribution to this literature is to document how these differences in prices are reflected in quantities. This is essential for establishing how exchange rate movements affect actual allocations and for measuring the expenditure switching effects of exchange rates.

⁸Fitzgerald and Haller (2012) [20] use a similar fixed effects strategy to document pricing-to-market by Irish firms selling in Ireland and the UK, for firms invoicing their exports in pounds.

⁹See Golosov and Lucas (2007) [26] and Burstein and Hellwig (2008) [5]. Time dependent (Calvo) pricing is the common assumption in open economy literature with price rigidities.

¹⁰Some of these papers are Berman et al. (2012) [3], Amiti et al. (2012) [1], Fitzgerald and Haller (2012) [20], Burstein and Jaimovich (2012) [7], and Burstein and Gopinath (2012) [6]. See Goldberg and Knetter (1997) [25] for a summary of an older literature measuring exchange rate pass-through using sector level data.

Second, the paper relates to the literature on the international elasticity puzzle (see Ruhl (2008) [31] and Fitzgerald and Haller (2012) [21]). This literature documents that trade flows are very responsive to changes in tariffs, but not to changes in the exchange rate. I contribute to this discussion by providing a new micro-estimate of the short run trade elasticity by exploiting special features of the Chilean data.

Finally, as discussed above, there is an extensive literature that uses open economy models with sticky prices to study the transmission of monetary shocks across countries.¹¹ In contrast, I use a quantitative model calibrated to microdata to measure the effects of exchange rate movements on productivity. In doing so, I provide evidence linking invoicing to firm characteristics. This evidence can shed light on the determinants of invoicing practices.¹²

The rest of the paper is organized as follows. The empirical evidence is presented in the next section. Section 3.3 introduces the model. Section 3.4 describes the parameterization and the quantitative exercises, and the last section concludes.

3.2 Empirical Evidence

3.2.1 Data

I use two different datasets from Chilean customs. The first dataset contains all export shipments between the years 2009 and 2011. The second dataset only covers wine export shipments, but spans more years, from 2003 to 2011. I use both datasets in the empirical section below.

The data contain information on each export shipment originating in Chile during these periods. Before shipping their products abroad, Chilean exporters, to be authorized by customs, must file an export authorization form.¹³ This form records, among other information, the date, the value and quantity of the shipment, the exporter tax id, the destination port and country, the HS8 category of the product, and the product brand

¹¹A non-exhaustive list of these papers is Obstfeld and Rogoff (1995) [30], Corsetti-Pesenti (2005) [10], Devereux and Engel (2006) [12], Gali and Monacelli (2005) [22] and Engel (2002, 2011) [19] [17].

¹²See for example Goldberg and Tille (2008, 2009) [23] [24], Bacchetta and van Wincoop (2005) [2], Engel (2006) [18].

¹³More precisely, exporters need to get a "Documento Unico de Salida" or "DUS" authorized by customs to be able to get their products out of the country.

and description.¹⁴ The form records the currency in which the transaction was settled, for which the exporter must provide the receipt. I refer to this as the currency of invoicing.

As it is typically the case with customs data, I use firm-product-destination level unit values as proxy for prices. A disadvantage of using unit values is that I cannot measure price stickiness directly, because I do not observe the frequency at which firms adjust prices. On the other hand, an important advantage of the data relative to survey data on prices is that it records values and quantities of actual transactions.

Finally, I take the period average nominal exchange rate from the IMF International Financial Statistics. Data on nominal GDP and domestic inflation are taken from the same source.

3.2.1.1 Summary Statistics

Table 3.1 provides summary statistics for the manufacturing and wine datasets. There were over 3 million manufacturing shipments between 2009 and 2011, and over a million wine shipments between 2003 and 2011. These were made by 11,596 and 816 exporters respectively. Finally, note that Chilean exporters sold a wide variety of products (almost 6000 HS8 products) to over 170 destinations during this period.

The second and third panels of the table show the distribution of price and nominal exchange rate changes used in the estimations (both computed as log differences). The distributions are plotted in Figure 3.1. First, note that there is significant heterogeneity in how firms change prices. Second, note that the changes in exchange rates during this period are significant relative to this variation in prices. The median change in the exchange rate in each sample is -0.04, indicating that this was a period during which currencies appreciated relative to the dollar.

Invoicing: I summarize some important features of the data before proceeding with the econometric analysis. First, most of the invoicing by Chilean exporters is done using the U.S. dollar, while the Chilean peso is seldom used. This is in line with what previous studies have found in developing countries.¹⁵ Figure 3.2 shows the predominance of the

¹⁴HS8 is a very detailed classification system that Chilean customs uses to impose tariffs. This classification contains over 6000 products and has a level of disaggregation that is equivalent to the HS10 classification used in the US or the CN8 classification used in Europe.

¹⁵See Goldberg and Tille (2008) [23] for some aggregate facts on how countries invoice their exports.

dollar in selected destination countries. We can see that in some markets a significant fraction of the invoicing is done in the currency of the destination country. For instance, about 50 percent of the exports to the UK and Europe are invoiced in pounds and euros respectively. In a given destination, Chilean exporters typically use either the dollar or the destination's currency, while exports in a third currency are extremely infrequent. In addition, over 85 percent of the exporters in my sample use only one currency in a given destination during the period. This suggests that i) exporters play a major role in determining the currency in which international transactions are invoiced, and ii) exporters rarely switch currencies over time in a particular destination. Finally, the volume of exports of firms invoicing in the destination market's currency is 82 percent larger on average than that of firms invoicing in U.S. dollars.

3.2.2 Empirical Strategy

In this section I describe the empirical strategy for estimating the expenditure switching effects of exchange rates. I compare how exporters selling the same product into the same destination but invoicing in different currencies respond to changes in the exchange rate. Assuming that: (i) the invoicing currency is set before the exchange rate changes, and (ii) differences in the shocks to exporters invoicing in different currencies are not correlated with the bilateral exchange rate, the response in relative quantities to changes in relative prices generated by the exchange rate can be used to identify the elasticity. Both assumptions are likely to hold in this setting. The first assumption holds since exporters do not change their invoicing currency during the period. The second assumption is also likely to hold, since exporters from the same country who sell the same product into the same destination are likely to be affected by the same set of aggregate shocks.

I proceed by estimating the following equation at the firm-product-destination level:

$$\begin{aligned} \Delta \log Y_{f,p,d,t} = & \beta_{dc} \times D_{f,p,d} \times \Delta \log NER_{d,t} + \beta_{\$} \times [1 - D_{f,p,d}] \times \Delta \log NER_{d,t} \\ & + \gamma Z'_{d,t} + v_{f,p,t} + \gamma_d + \varepsilon_{f,d,t}. \end{aligned} \quad (3.1)$$

Here, $\Delta \log Y_{f,p,d,t}$ is the dependent variable, which can be either the log change in the price (expressed in the destination market's currency) or the quantity sold by firm f into

destination d in year t . $\Delta \log NER_{d,t}$ is the log change in the destination market's nominal exchange rate, expressed in units of the destination market's currency per U.S. dollar. D_{fpd} is a dummy that takes the value of 1 if the good is priced in the destination market's currency and zero if it is priced in dollars. $Z'_{d,t}$ includes controls for the change in the destination's price level and nominal GDP. γ_d is a set of destination fixed effects. $\nu_{fp,t}$ are firm-product-year fixed effects that control for changes in firm-product level marginal costs or demand that are common across destinations. The coefficients of interest are β_{dc} and $\beta_{\$}$, and capture the elasticity of prices or quantities to changes in nominal exchange rates for firms invoicing in the destination's currency and dollars, respectively.

Since I am interested in the differential response of firms that invoice in different currencies, I exclude from the sample those firms that use multiple currencies in the same destination. In addition, I follow Gopinath et al. (2010) [28] by focusing on product-destination pairs where multiple currencies are used. Finally, I aggregate shipments by year to obtain a more accurate interpretation of quantities and avoid seasonality issues.

Below I present my benchmark results using both the manufacturing and the wine datasets. To mitigate concerns about selection, the baseline regressions only include exporters that are active in a destination during the entire period. Subsection 3.2.3.3 presents robustness checks using different samples and different fixed effects estimators.

3.2.3 Results

3.2.3.1 Exchange Rates and Prices

The results from estimating equation (3.1) using the change in price as the dependent variable, $\Delta \log Y_{fpd,t} = \Delta \log P_{fpd,t}$, are presented in Table 3.2. Columns 1 and 5 show my benchmark results using the manufacturing and the wine datasets respectively. Note first that the coefficient β_{dc} is not statistically different from zero in either sample. This coefficient captures the price elasticity with respect to the exchange rate for exporters that invoice using the destination's currency. Since prices are denominated in the destination's currency, a zero coefficient indicates that these firms do not change nominal prices in response to changes in the destinations' nominal exchange rate.

In contrast, the elasticity for firms invoicing in U.S. dollars, $\beta_{\$}$, is close to one in both

the manufacturing and wine samples. In fact, we cannot reject the null hypothesis that $\beta_{\$} = 1$ in either sample. This implies that nominal prices for these firms are rigid in U.S. dollars, so that these prices move one-to-one with the destination's exchange rate once they are denominated in the destination market's currency. This evidence suggests that prices are very rigid in the currency in which they are invoiced. This rigidity implies that relative prices move one-to-one with the nominal exchange rate. This result is in line with Gopinath et al. (2010) [28], who document a similar finding for firms importing into the U.S. using dollars vs. non-dollars.¹⁶

A possible interpretation for such an extreme difference in relative prices may be that firms invoicing in different currencies use intermediate inputs sourced from different countries, so that exchange rate changes affect relative marginal costs across firms.¹⁷ An important characteristic of my data is that I can include fixed effects at the firm-product-year level to control for changes in marginal costs that are common across destinations. Assuming that each firm uses the same set of inputs to source every destination, the difference in the coefficients can be attributed to changes in relative markups rather than to changes in marginal costs brought forth by the exchange rate.¹⁸

Note also that this rigidity seems to be beyond what can be explained by price stickiness. Table 3.3 repeats the regressions aggregating the wine dataset over periods of two years.¹⁹ Although the coefficient β_{dc} in these estimations turns out to be positive, there continues to be a significant difference between the response of firms invoicing in U.S. dollars relative to those using the destination's currency. The literature on nominal rigidities documents a median price duration of a year. In contrast, I find a difference in markups that moves one-to-one with the exchange rate over a period of a year, and that is still significant over a period of two years. Such stark responses are in line with those reported by Gopinath et al. (2010) [28] and Fitzgerald and Haller (2012) [20].

¹⁶Fitzgerald and Haller (2012) [20] also document extreme pricing to market (i.e., an elasticity close to zero) for Irish firms selling in pounds into the UK.

¹⁷Using data from Belgium, Amiti et al. (2012) [1] argue that about half of the lack of exchange rate pass-through into prices comes from this channel.

¹⁸Fitzgerald and Haller (2012) [20] use a similar fixed effects strategy to document pricing to market by Irish firms.

¹⁹Unfortunately, the manufacturing dataset does not span enough years to do this exercise.

3.2.3.2 Exchange Rates and Quantities

I now present the results for quantities. Table 3.4 displays the results from estimating equation (3.1) using the change in quantities as the dependent variable, $\Delta \log Y_{fpd,t} = \Delta \log Q_{fpd,t}$. My benchmark results are presented in columns 1 and 5 for the manufacturing and wine samples, respectively. There is no significant response in quantities for firms invoicing with the destination market's currency, and I cannot reject that the null that the elasticity β_{dc} equals zero in either sample. These are the firms whose price in the destination market did not change in response to the exchange rate. On the other hand, the coefficient for the firms that invoice in dollars, $\beta_{\$}$, comes out negative and significant as expected. These are the firms whose price was rigid in dollars and increased in the destination market's currency when the destination's currency depreciated, as shown in Table 2. The difference in the coefficients is statistically significant and equals 1.35 in our benchmark specification. Note that although relative quantities move in the expected direction, the implied elasticity is low. As mentioned above, such low elasticities are in line with those used by the international business cycle literature to match the observed comovements between the terms of trade and the trade balance. Here, the elasticity is identified from the variation in prices across firms invoicing in different currencies in response to a change in the exchange rate.

3.2.3.3 Robustness

I now conduct several robustness checks for the results established in the previous two sections. In particular, I conduct the following exercises: First, I repeat the regressions including the entire sample of firms, instead of using only continuing firms. These results are displayed in columns 2 and 6 of Tables 3.2 and 3.4 for the manufacturing and wine datasets, respectively. The results are robust to these alternative samples, although the difference in coefficients for the quantity regression is somewhat smaller. Second, I run the regressions controlling for different types of fixed effects. The results of these estimates are presented in columns 3 and 4 of the tables. None of these changes modify the conclusions of the previous subsections.

3.2.3.4 Discussion

The data presented in this section establish that: i) relative prices across firms that invoice using different currencies fluctuate one-to-one with the exchange rate, ii) these price fluctuations can be attributed to variations in destination specific markups, as opposed to changes in firm level marginal costs that are common across destinations, and iii) relative quantities respond to the exchange rate in the expected direction, with an implied elasticity that is between -1 and -2. Such low elasticities are indicative of limited expenditure switching effects of changes in the exchange rates. The elasticities estimated in this section are for goods in the same product category that were sourced from the same country (Chile). To the extent that goods from different source countries are less substitutable than goods from the same country, the expenditure switching effects would be even weaker than those implied by these estimates.

In addition, the evidence provided so far shows that exchange rate changes affect relative markups and the allocation of production across firms invoicing in different currencies. The model developed in the next section provides a framework for evaluating how these changes in nominal exchange rates translate to aggregate output per worker.

3.3 Model

This section introduces a quantitative open economy model of international relative prices to measure how exchange rates affect aggregate productivity.

Preliminaries: The structure of the model is relatively standard. There are three countries indexed by $i = c, s, e$. Each country is inhabited by n_i agents and produces n_i goods. I normalize the world population and the number of available goods in the world to 1. Identical households in each country consume a final good and supply labor. In addition, in each country there is a continuum of n_i monopolistically competitive intermediate producers, each producing a differentiated good. These producers use labor as their sole input of production and differ in their productivities. The output of these intermediate producers is aggregated by the consumers into a final good with a Dixit-Stiglitz CES aggregator. I introduce endogenous variable markups in a tractable way

by assuming that intermediate goods must be combined with nontradable distribution services in fixed proportions to be delivered to consumers.²⁰ How distribution costs affect desired markups is explained below. Finally, money is introduced in the model assuming a cash-in-advance constraint.

Households: The utility function of a household in country i is given by

$$U_{i,t} = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{i,t}^{1-\sigma}}{1-\sigma} - \frac{N_{i,t}^{1+\phi}}{1+\phi} \right],$$

where $C_{i,t}$ is an aggregate bundle of tradable and nontradable consumption goods and $N_{i,t}$ denotes labor effort. The parameters σ and ϕ control the intertemporal elasticity of substitution for consumption and the Frisch elasticity of the labor supply, respectively. I assume that households in each country can trade a full set of state contingent nominal bonds. This gives rise to two familiar optimality conditions: the intratemporal consumption-leisure condition,

$$C_{i,t}^{\sigma} N_{i,t}^{\phi} = W_{i,t} / P_{i,t},$$

and the risk sharing condition,

$$\left(\frac{C_{i,t}}{C_{j,t}} \right)^{\sigma} = \frac{E_{ij,t} P_{j,t}}{P_{i,t}} \equiv Q_{ij,t}.$$

Here, $W_{i,t}$ and $P_{i,t}$ denote the nominal wage and the consumption price index in country i , respectively. $E_{ij,t}$ denotes the bilateral nominal exchange rate, expressed as units of country i 's currency per currency unit of country j 's currency. The first condition states that households equalize the ratio of the marginal utilities between consumption and leisure to the real wage. The risk sharing condition states that the marginal utility of a dollar is equalized across countries. This means that the ratio of marginal utilities between countries i and j must equal the real exchange rate between country i and

²⁰One interpretation is that all goods use the same "shelf space", regardless the technology used for production. This way of generating endogenous variables markups was first introduced by Corsetti and Dedola (2005) [10].

country j , denoted by $Q_{ij,t}$. Finally, the cash-in-advance constraint implies

$$P_{i,t}C_{i,t} \leq M_{i,t}.$$

Preferences and demands: Aggregate consumption in each country is a composite of nontradable and tradable goods. We break down this bundle in steps. First, aggregate consumption is given by $C_{i,t} = C_{iT,t}^\alpha C_{iN,t}^{1-\alpha}$, where $C_{iT,t}$ and $C_{iN,t}$ are bundles of tradable and nontradable goods, respectively. The tradable good bundle is a composite of intermediate tradable goods produced in each country:

$$C_{iT,t} = \sum_j \left[\nu_{ji}^{\frac{1}{\xi}} C_{jiT,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}.$$

Here, $C_{jiT,t}$ is a composite of consumption goods sold from country j into country i , and ξ is the elasticity of substitution across tradable varieties. The weights that composites from different source countries receive in the aggregate bundle are given by $\nu_{ji} \equiv n_j \lambda$ and $\nu_{ii} \equiv 1 - (1 - n_i) \lambda$. This specification allows weights ν_{ji} to depend on a parameter λ that determines home bias, and on the number of goods n_j produced in the source country j . Hence, consumption shares depend directly on country size n_j . This is a tractable way of making the size of the Chilean economy arbitrarily small in the quantitative exercises below.²¹ In addition, the consumption composites $C_{jiT,t}$ are aggregate bundles of the intermediate goods produced in each country. These bundles are given by

$$C_{jiT,t} = \left[\left(\frac{1}{n_j} \right)^{\frac{1}{\xi}} \int_0^{n_j} C_{jiT,t}(f)^{\frac{\xi-1}{\xi}} df \right]^{\frac{\xi}{\xi-1}},$$

where, $C_{jiT,t}(f)$ denotes consumption of good f . Finally, the nontradable bundle in each country is a composite of domestically produced intermediate goods, given by:

$$C_{iN,t} = \left[\left(\frac{1}{n_i} \right)^{\frac{1}{\rho}} \int_0^{n_i} C_{iN,t}(f)^{\frac{\rho-1}{\rho}} df \right]^{\frac{\rho}{\rho-1}},$$

where ρ is the elasticity of substitution across nontradable intermediate goods.

Cost minimization implies that demands for tradable and non-tradable goods are

²¹This specification has previously been used in Sutherland (2005) [32] and Di Paoli (2009) [14].

given by:

$$C_{iT,t} = \frac{\alpha P_{i,t} C_{i,t}}{P_{iT,t}}; \quad C_{iN,t} = \frac{(1-\alpha) P_{i,t} C_{i,t}}{P_{iN,t}};$$

where $P_{iT,t}$ and $P_{iN,t}$ are the price indexes for tradable and nontradable consumption. Demand for goods originating in country j is:

$$C_{jiT,t} = \nu_{ji} \left[\frac{P_{jiT,t}}{P_{iT,t}} \right]^{-\xi} C_{iT,t}.$$

while the demands for individual varieties are:

$$C_{jiT,t}(f) = \frac{1}{n_j} \left[\frac{P_{jiT,t}(f)}{P_{jiT,t}} \right]^{-\xi} C_{jiT,t}; \quad C_{iN,t}(f) = \frac{1}{n_i} \left[\frac{P_{iN,t}(f)}{P_{iN,t}} \right]^{-\rho} C_{iN,t};$$

where $P_{jiT,t}(f)$ is the consumer price in country i of good f produced in country j , and $P_{jiT,t}$ is the ideal consumer price index for goods sold in from country i into country j . $P_{iN,t}(f)$ is the price of nontradable intermediate f in country i . The relative consumption between goods being sourced from different countries is given by:

$$C_{ijT,t}/C_{jjT,t} = (P_{ijT,t}/P_{jjT,t})^{-\xi}.$$

I refer to changes in this ratio following a change in the nominal exchange rate $E_{ij,t}$ as expenditure switching effects. These are determined by changes in retail prices and the elasticity of substitution ξ .

Pricing: There are two sources of price rigidities in the model. First, producer prices are sticky in the currency in which they are invoiced. Second, there are endogenous variable markups. To introduce endogenous variable markups in a tractable way, I follow Corsetti and Dedola (2005) [10] and assume that competitive retailers combine the intermediate goods with local nontradable goods in fixed proportions to deliver these goods to consumers. This implies that the consumer price in country j for tradable good f produced in country i is given by:

$$P_{ijT,t}(f) = P_{ijT,t}^p(f) + \eta P_{jN,t}.$$

Here, $P_{ijT,t}(f)$ is the consumer price of good f in country j , $P_{ijT,t}^p(f)$ is the producer price of good f denoted in j 's currency, and η controls the share of distribution costs in

the consumer's price. Note that distribution costs use nontradables from the destination country. Below I describe how the introduction of distribution costs affects the pricing problem of the firm.

Flexible prices: I start by solving for the optimal price under flexible prices. The problem of producer f from country i selling into country j is given by:

$$\max_{P_{ijT,t}^p(f)} \left[P_{ijT,t}^p(f) E_{ij,t} - \frac{W_{i,t}}{z(f)} \right] \left[\frac{P_{ijT,t}(f)}{P_{ijT,t}} \right]^{-\xi} C_{ijT,t},$$

where $z(f)$ denotes the productivity of firm f . The optimal flexible price expressed in the producer's currency is a markup over the marginal costs

$$P_{ijT,t}^p(f) E_{ij,t} = \mu_{ij,t}(f) \frac{W_{i,t}}{z(f)},$$

where the markup is given by:

$$\mu_{ij,t}(f) \equiv \frac{\xi}{\xi - 1} \left[1 + \frac{\eta z(f) P_{jN,t} E_{ij,t}}{W_{i,t}} \right].$$

To provide intuition on why markups $\mu_{ij,t}(f)$ vary by source country, destination and firm, note that the elasticity of demand faced by the producer is:

$$\varepsilon_{ij,t}(f) \equiv -\frac{d \log C_{ijT,t}(f)}{d \log P_{ijT,t}^p(f)} = \xi (1 - s_{ij,t}(f)),$$

where $s_{ij,t}(f) \equiv \frac{\eta P_{jN,t}}{P_{ijT,t}^p(f) + \eta P_{jN,t}}$ is the share of distribution services in the consumer price.

Note also that the elasticity of markups with respect to the producer price is given by:

$$\Gamma_{ij,t}(f) \equiv -\frac{d \log \mu_{ij,t}(f)}{d \log P_{ijT,t}^p(f)} = \left[(\xi - 1) \frac{1 - s_{ij,t}(f)}{s_{ij,t}(f)} + 1 \right]^{-1}.$$

Markups depend on the demand elasticity with respect to the producer price, $\varepsilon_{ij,t}(f)$, which depends on the share that the producer price has in the price paid by consumers $s_{ij,t}(f)$. This share is determined by the firm's productivity $z(f)$. More productive firms have lower marginal costs, higher $s_{ij,t}(f)$, and higher desired markups $\mu_{ij,t}(f)$. In addition, these firms have a higher markup sensitivity to changes in the price, $\Gamma_{ij,t}(f)$.

Finally, with flexible prices, firms set the same price regardless of the currency that is used for invoicing. That means that the flexible price for a producer that invoices in currency l , $P_{ijT,t}^{pl}(f)$, is given by:

$$P_{ijT,t}^{pl}(f) E_{lj,t} = \mu_{ij,t}(f) \frac{W_{i,t}}{z(f)}.$$

Nominal rigidities: I now introduce Calvo-style nominal rigidities. In particular, intermediate producers in the tradable sector can reset their price with probability $1 - \theta_T$, and producers in the nontradable sector reset their prices with probability $1 - \theta_N$. I assume that producer prices are rigid in the currency in which they are invoiced. This gives rise to the familiar pricing equations in logs:

$$\bar{p}_{ijT,t}^{pl}(f) = (1 - \beta\theta_T) \sum_{k=0}^{\infty} (\beta\theta_T)^k E_t \left[\tilde{p}_{ijT,t+k}^{pl}(f) \right].$$

Here $\bar{p}_{ijT,t}^{pl}(f)$ is the log of the reset producer price of firm f selling from country i to country j invoicing in currency l . $\tilde{p}_{ijT,t}^{pl}(f)$ is the log of the price that the firm would set if prices were flexible, which to a first order approximation is given by:

$$\tilde{p}_{ijT,t}^{pl} = \frac{1}{1 + \Gamma(f)} [\hat{w}_{i,t} - e_{il,t} + \Gamma(f) [\tilde{p}_{jN,t} + e_{lj,t}]].$$

Finally, retail prices are flexible.

Money supply: The law of motion for the money supply follows: $\Delta \log M_{i,t} = v_{it}$, where $v_{it} \sim N(0, \sigma_m)$.

Market clearing: Goods market clearing in the tradable and nontradable sector implies:

$$Y_{iT,t} = \frac{1}{n_i} \sum_j n_j C_{ijT,t}, \quad (3.2)$$

and:

$$Y_{iN,t} = C_{iN,t} + D_{iN,t}.$$

Here $D_{iN,t} \equiv \frac{1}{\eta} \sum_j \int_0^{n_j} c_{ji,t}(f) df$ denotes the amount of nontradable goods used for distribution services.

The amount of labor used in the tradable sector is given by:

$$n_i N_{iT,t} = \int_0^{n_i} N_{iT,t}(f) df,$$

we can write this condition as:

$$N_{iT,t} = \frac{1}{n_i} \sum_j n_j C_{ijT,t} V_{ijT,t}, \quad (3.3)$$

where $V_{ijT,t} \equiv \left[\frac{1}{n_i} \int_0^{n_i} \left[\frac{p_{ijT,t}(f)}{P_{ijT,t}} \right]^{-\xi} \frac{1}{z(f)} df \right]$ is a term capturing the dispersion in tradable prices. The amount of labor used in the non-tradable sector is:

$$N_{iN,t} = [C_{iN,t} + D_{iN,t}] V_{iN,t},$$

with $V_{iN,t} \equiv \left[\frac{1}{n_i} \int_0^{n_i} \left[\frac{p_{ijN,t}(f)}{P_{ijN,t}} \right]^{-\rho} df \right]$. Labor market clearing implies:

$$N_{i,t} = N_{iT,t} + N_{iN,t}.$$

I solve the model by log-linearizing the equilibrium conditions around the steady-state and solving the resulting system of linear difference equations.

Measuring aggregate output per worker: I define the change in define output per worker in the tradable sector as:

$$TFP_{iT,t}/TFP_{iT,0} = [RGDP_{iT,t}/N_{iT,t}] / [RGDP_{iT,0}/N_{iT,0}],$$

where $RGDP_{iT,t}$ is real GDP in the tradable sector. I will compute $RGDP_{iT,t}$ following as closely as possible the procedures used in the United States' National Income and Product Accounts (NIPA) by the Bureau of Economic Analysis to compute real GDP.²² In particular, I use a Fisher formula, which is a geometric average of a Laspeyres and a Paasche quantity index. For example, real GDP in period t relative to period $t - 1$ is

²²See, e.g. Concepts and Methods of the U.S. National Income and Product Accounts (2009). The procedures that we consider are broadly consistent with the recommendations by the United Nations in their System of National Accounts.

given by

$$\frac{RGDP_{iT,t}}{RGDP_{iT,t-1}} = \left[\frac{\sum_j n_j \int_0^{n_i} p_{ijT,t-1}(f) c_{ijT,t}(f) df}{\sum_j n_j \int_0^{n_i} p_{ijT,t-1}(f) c_{ijT,t-1}(f) df} \times \frac{\sum_j n_j \int_0^{n_i} p_{ijT,t}(f) c_{ijT,t}(f) df}{\sum_j n_j \int_0^{n_i} p_{ijT,t}(f) c_{ijT,t-1}(f) df} \right]^{0.5}, \quad (3.4)$$

where $p_{ijT,t-1}(f)$ and $c_{ijT,t}(f)$ denote prices and quantities in period t of the detailed components of GDP.²³ The first term in expression (3.4) is a Laspeyres quantity index (based on $t-1$ prices), while the second term is a Paasche quantity index (based on t prices).²⁴ Real GDP in period L relative to period 0 is given by:

$$\frac{RGDP_{iT,L}}{RGDP_{iT,0}} = \prod_{t=1}^L \frac{RGDP_{iT,t}}{RGDP_{iT,t-1}}. \quad (3.5)$$

I assume there are two types of firms; z_H and z_L . Using the equilibrium conditions, I show in the appendix that the log-linearized versions of (3.3) and (3.4) can be combined as:

$$\hat{v}_{iT,t} = \hat{y}_{iT,t} - \hat{n}_{iT,t},$$

where a \hat{x} denotes log deviations from the non-stochastic steady state. Here $\hat{v}_{iT,t}$ denotes the log change in change in tradable productivity. In the Appendix, I show that $\hat{v}_{iT,t}$ can be written as:

$$\hat{v}_{iT,t} \equiv -\xi (\omega^v - \omega) \sum_j \nu_{ij} [\hat{p}_{ijT,t}^H - \hat{p}_{ijT,t}^L], \quad (3.6)$$

where $\omega \equiv \left[1 + \left(\frac{1-\kappa}{\kappa} \right) \left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)} \right]^{1-\xi} \right]^{-1}$ and $\omega^v \equiv \left[1 + \left(\frac{1-\kappa}{\kappa} \right) \left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)} \right]^{-\xi} \frac{z(H)}{z(L)} \right]^{-1}$. In the following sections we will focus on how changes in nominal exchange rates affect productivity in the tradable sector, $\hat{v}_{iT,t}$, in Chile.

3.3.1 Exchange Rates, Markups and Productivity

This section describes how exchange rates affect markup dispersion and productivity in the model. Exchange rate fluctuations affect markup dispersion through three different

²³See Burstein and Cravino (2012) [4] for a more detailed discussion of these measures.

²⁴The implicit GDP deflator is calculated as the ratio of current-dollar GDP to real GDP, $(\sum p_t q_t / \sum p_{t-1} q_{t-1}) / (RGDP_t / RGDP_{t-1})$, which is equal to a geometric average of a Laspeyres and a Paasche price index.

channels. First, since producer prices are sticky, relative markups across firms invoicing in different currencies fluctuate with the exchange rate. Second, for the firms that are able to reset prices each period, there is dispersion across exporters with different productivities and different desired markups. Finally, there is dispersion originating from the staggered price adjustment caused by the Calvo price stickiness.

Consider an appreciation of the euro against all currencies. Distribution costs in Europe increase relative to production costs in Chile following the appreciation, so all Chilean firms exporting to Europe increase markups. The effects are larger for more productive firms, since they have a higher markup elasticity. Second, since prices are sticky and firms invoice in different currencies, an appreciation of the euro increases relative markups of firms invoicing in euros relative to those invoicing in U.S. dollars. Markup dispersion affects productivity in much the same way that inflation affects efficiency in closed economy models with staggered price adjustment. This is captured by the term $V_{ij,T}$ in equation (3.3). How exchange rate movements affect productivity depends on how invoicing and desired markups correlate with the initial markup dispersion. This implies that productivity can move in either direction in response to an exchange rate shock depending on whether the shock magnifies or reduces the initial markup dispersion. Finally, the shock generates markup dispersion across identical firms that reset prices at different times, as is usual with Calvo pricing. In the next section, I calibrate the model using the Chilean data and evaluate the strength of these mechanisms.

3.4 Quantitative Results

In this section, I parameterize the model using the Chilean data and evaluate the impact of exchange rate movements in aggregate output per worker. In what follows, I describe what aspects of the data identify the key parameters in my model. I next present my baseline quantitative results. Finally, I conduct alternative parameterizations and sensitivity analyses to show the importance of different assumptions regarding invoicing for the effects of exchange rates in productivity.

3.4.1 Parameterization

I parameterize the model assuming that there are two types of firms, z_H and z_L . Then, the parameters that I must choose are the elasticity of substitution across varieties, ξ , the share of firms invoicing in each currency in each country, the ratio of productivities across firms, $z_r = z_H/z_L$, the steady state share of distribution costs in the retail price, $\frac{\eta P_N}{P_T}$, the degree of price stickiness in the tradable and nontradable sectors, θ_T and θ_N , the share of goods that are exported, λ , and the relative country sizes, n_i . I also need to assign values for the parameters in the utility function σ and ϕ , the share of nontradables in consumption α , and the discount factor β . I now provide an overview of my baseline parameterization procedure, the results of which are summarized in Table 3.5.

The calibration of most of these parameters is standard. I take the consumption, output, and trade shares in manufacturing for Chile from the OECD-STAN Input-Output Database. This results in setting $\alpha = 0.37$ and $\lambda = 0.4$. I set the country sizes to $n_c \rightarrow 0$, and $n_s = 0.52$, $n_e = 0.48$, so that the size of Chile in the world economy is negligible and to match the share of Chilean manufacturing exports to the US and Europe respectively. Since I use unit values as proxy for prices, I cannot observe the frequency of price changes in my data. Hence, I take the price stickiness parameters θ_N and θ_T from the literature on nominal rigidities and set both of these parameters to equal 0.75, which implies a median price duration of a year. Finally, I set the parameters in the utility function to be $\sigma = 1$ and $\phi = 0$. These choices are made purely for convenience, to ensure that a monetary shock does not generate an overshooting of the nominal exchange rate, and that the change in the exchange rate following the shock is permanent. The values of these parameters do not affect the response of productivity for a given path of the nominal exchange rate.

The remaining parameters are calibrated to the microdata. I first need to establish the currency in which the invoicing is done. I set the share of Chilean firms using the dollar to sell into the U.S. equal to one. I set the share of firms using euros when exporting to Europe equal to 0.38. Both of these shares are directly observable in the Chilean customs data. I assume that the H firms invoice in euros, while the L firms invoice in U.S. dollars in Europe to match the correlation between invoicing and size in the data. Although I

do not observe the currency used to sell into the domestic country, I assume that Chilean firms use the Chilean peso when selling into Chile. Finally, I assume that all U.S. firms invoice in dollars and that all European firms invoice in euros in every destination. I will not be focusing on how invoicing affects productivity in these countries.

Finally, there are 3 key parameters that need to be jointly calibrated. These are the elasticity of substitution ξ , the relative productivities between firms, z_r , and the share of distribution costs in the final price $\frac{\eta P_N}{P_T}$. I choose these parameters to target the following three moments: i) the response of relative quantities to the exchange rate, ii) the relative size of firms invoicing in different currencies, and iii) the average share of distribution costs in the retail price. I take the first two moments from the data, while I take the last moment from the literature and set it equal to 0.5.

Note that the low level of the elasticity implies an extremely high level of markups. However, the level of markups does not enter the log-linear system of equations that characterize the solution of the model.

3.4.2 Baseline Results

I simulate a change in European money supply that generates a permanent appreciation of the euro against all other currencies. The first panel of Figure 3.3 depicts the nominal exchange rate shock. The responses of relative prices and quantities of Chilean firms invoicing in different currencies are displayed in the second panel. The dashed red line displays the change in the relative price of firms invoicing in euros relative to those invoicing in U.S. dollars. The dotted blue line shows the corresponding relative quantities. The shock increases the relative price of firms invoicing in euroz relative to those invoicing in U.S. dollars, and decreases the relative quantities between these two types of firms. Since the elasticity is low in the baseline calibration, the resulting change in quantities is small. This corresponds to the limited expenditure switching effect described in first part of the paper.

Notice that the persistence of the change in prices is lower than in the data. This is a common feature of models with sticky prices. One way to generate higher persistence would be to introduce a larger degree of price stickiness. In that case, the resulting

change in productivity would be even larger and more persistent than those in the results reported below.

Exchange rates and output per worker: Figure 3.4 shows the response of Chilean output per worker in the tradable sector, given by equation 3.6, to the change in the European exchange rate. The y-axis shows the deviation in output per worker from the initial steady state. Output per worker falls by 0.5 percent on impact and is still 0.25 percent below the initial steady state four quarters after the shock. Following an appreciation of the euro there are two effects. First, most firms cannot change their nominal price, so the markups of firms invoicing in U.S. dollars decline relative to the markups of the firms invoicing in euros. Second, as the appreciation of the euro increases the share of distribution costs in Europe, all Chilean firms increase markups, in particular large firms. Since large firms are precisely those invoicing in euros, these two effects reinforce each other in increasing the relative markups of the larger firms. Since large firms had higher markups before the exchange rate shock, the shock increases the initial dispersion in markups, generating the drop in productivity. In contrast, a depreciation of the euro would close the initial dispersion in markups and have the opposite effect on productivity.

3.4.3 Alternative Parameterizations

I now evaluate the role of variable markups and heterogeneity in invoicing in driving these results. For each of the following exercises, I recalibrate the entire model to be consistent with the corresponding assumptions.

Endogenous variable markups: I first solve a version of the model with multiplicative distribution costs to analyze the importance of variable markups.²⁵ Under this assumption, markups are constant, with the only effects of the monetary shock on productivity being those arising from the staggered price setting. I repeat the counterfactual exercise and show the results in Figure 3.5. This is the case depicted by the dashed light blue line labeled "no variable markups." Note that the productivity losses in this case are minus-

²⁵In particular, I assume that the consumer price is given by $P_{ijT,t}(f) = \left[P_{ijT,t}^p(f) \right]^{1-\eta} [P_{jNt}]^\eta$. In this case, the share of distribution costs in the retail price is constant and all firms set the same constant markup $\xi/(\xi - 1)$.

cule. It is worth emphasizing that these are the losses typically studied in the literature. By starting from an inefficient allocation, heterogeneity in markups makes the effects of exchange rates in productivity an order of magnitude larger.

Flexible tradable prices: I now analyze the role of sticky prices in the tradable sector. Even without sticky prices in the tradable sector, changes in the nominal exchange rate generate a change in productivity because of the markup dispersion induced by the endogenous markup channel. This is depicted by the dotted blue line in Figure 3.5. The figure shows, however, that price stickiness in the currency of invoicing is important: the change in productivity when prices are sticky is only a third of those in the baseline calibration.

Heterogeneity in invoicing: Figure 3.5 shows the case in which there is no heterogeneity in invoicing. I consider the cases in which all Chilean firms export using either euros or dollars. These correspond to the circled and crossed lines in Figure 3.5. The figure shows that the response in either case is about five times smaller than in the benchmark scenario. As in the case of flexible tradable prices, exchange rates still affect markup dispersion through the endogenous markup channel despite their lack of heterogeneity in invoicing. However, the effects are even smaller than with flexible tradable prices, since only a small fraction of the firms have the opportunity to reset markups in response to the exchange rate under this specification.

Random invoicing: Finally, I repeat the counterfactual in an environment of multiple invoicing currencies, but where invoicing is uncorrelated to desired markups. That is, the calibration ignores the correlation between invoicing and firm size in the data. This case is shown by the dashed red line in Figure 3.5. The response of productivity under this scenario is significantly smaller, as firms that increase markups are not necessarily those that had higher markups before the change in the exchange rate.

Role of elasticity: Finally, I recalibrate the model assuming an elasticity of quantities to the exchange rate equal to 4. Such high elasticities are common in the international trade literature (typically using trade elasticities in the range of 5 and 10, see Eaton and Kortum (2002) [16]). Figure 3.6 shows the results. Not surprisingly, the first panel shows that the expenditure switching effects of exchange rates would be much higher under a higher elasticity. The second panel shows the response of aggregate output per worker.

A higher trade elasticity implies a higher elasticity of substitution ξ . Then, for a given change in relative quantities, the response in productivity would be smaller.

3.4.4 Discussion

The counterfactual parameterizations in this section show that models that ignore heterogeneity in invoicing, endogenous variable markups, or the correlation between markups and invoicing, greatly understate how changes in the exchange rate affect productivity. The intuition is that if firms invoice in different currencies and prices are sticky, changes in the exchange rate have a dramatic impact on relative markups. This effect is reinforced when the invoicing currency is correlated with the initial dispersion in markups. As mentioned above, these features are typically absent in models used to evaluate optimal exchange rate policy.

Some final considerations on how to interpret these results are in order: First, in the model, all changes in relative prices arise from changes in markups rather than from changes in marginal costs. This is consistent with the fixed effects method for estimating changes in relative prices in the empirical section of the paper. Second, although the currency in which exporters invoice their exports is exogenously determined in the model, the correlation between invoicing and desired markups in the baseline parameterization is in line with the predictions of models of endogenous currency choice (see Engel 2006 [18]). While I expect to endogenize these decisions in future versions of the paper, I do not expect this modification to significantly affect my quantitative results. Finally, the nominal rigidities in the model arise from Calvo pricing. A more realistic assumption is that firms must incur in menu costs to be able to reset prices. The nominal rigidities literature indicates that losses from inflation are smaller when price rigidities are state-dependent rather than time-dependent. In light of this concern, my results can be interpreted as evidence that heterogeneity in invoicing greatly amplifies the effects of exchange rates on productivity.

3.5 Conclusions

A large literature in international economics has emphasized expenditure switching and misallocation effects as mechanisms through which nominal exchange rates can affect real output and productivity. This paper provides a quantitative exploration of these mechanisms guided by a novel dataset from Chilean customs and a quantitative model of international prices with nominal rigidities. I exploited differences in the response of Chilean firms invoicing exports in different currencies to identify an elasticity of export quantities in response to the exchange rate that is in the range of -1 and -2. Such a low elasticity indicates that the expenditure switching effects of exchange rates are limited not only because price rigidities limit exchange rate pass-through into prices, but also because quantities are not very responsive to these changes in prices.

I then designed a quantitative model of international relative prices that is consistent with the salient features of the Chilean data to measure how exchange rates affect aggregate productivity. I have shown that by incorporating these features, I obtain very different measures of efficiency losses due to exchange rate movements than those obtained under the standard assumptions made in the literature. As noted above, both the currency of invoicing and the timing of price changes are exogenous in the model. I intend to endogeneize both of these decisions in future versions of the paper. The results presented here show that taking heterogeneity in invoicing and endogenous variable markups into account is key for the discussion in optimal exchange rate policy.

Finally, in light of my results, a natural question is how developing countries should design exchange rate policy. This question has received surprisingly little attention in the literature of optimal exchange rate policy, which typically focuses on cases where all the invoicing is done either in the producer's currency (PCP) or the destination's currency (LCP) (Corsetti et al. 2010 [8]). The available evidence suggests that, as in Chile, developing countries use the dollar to invoice a large fraction of their exports. The results in this paper suggest that this is a fruitful area for future research.

3.6 Appendix

3.6.1 Measuring output per worker

In this appendix I derive equation (3.6) in the paper. I start by approximating $RGDP_{iT,L}/RGDP_{iT,0}$ around the non stochastic steady state. A first order approximation to equation (3.4) gives:

$$\begin{aligned} y_{iT,t} - y_{iT,t-1} &= \int_0^{n_i} \frac{\sum_j n_j \bar{P}_{ijT}(f) \bar{C}_{ijT}(f)}{\int_0^{n_i} \sum_j n_j \bar{P}_{ijT}(f) \bar{C}_{ijT}(f)} (c_{ijT,t}(f) - c_{ijT,t-1}(f)) df \\ &= \sum_j \frac{n_j \bar{P}_{ijT} \bar{C}_{ijT}}{\sum_j n_j \bar{P}_{ijT} \bar{C}_{ijT}} \int_0^{n_i} \frac{\bar{P}_{ijT}(f) \bar{C}_{ijT}(f)}{\bar{P}_{ijT} \bar{C}_{ijT}} (c_{ijT,t}(f) - c_{ijT,t-1}(f)) \bar{d}f \\ &= \sum_j \nu_{ij} (c_{ijT,t} - c_{ijT,t-1}), \end{aligned}$$

where $y_{iT,t}$ denotes the log of real GDP, and we used $c_{ijT,t} = \int_0^{n_i} \frac{\bar{P}_{ijT}(f) \bar{C}_{ijT}(f)}{\bar{P}_{ijT} \bar{C}_{ijT}} c_{ijT,t}(f)$ in the derivation. We can then write the log-linear version of (3.5) as:

$$\hat{y}_{iT,t} = \sum_j \nu_{ij} \hat{c}_{ijT,t}.$$

In addition, equation (3.3) can be approximated as:

$$n_{iT,t} = \sum_j \frac{n_j \bar{C}_{ijT} \bar{V}_{ijT}}{\sum_j n_j \bar{C}_{ijT} \bar{V}_{ijT}} (c_{ijT,t} + v_{ijT,t}).$$

In a symmetric SS, $\bar{V}_{ij,T} = \bar{V}$ and $\bar{P}_{ijT} = \bar{P}_T$. Then,

$$\begin{aligned} \hat{n}_{iT,t} &= \sum_j \nu_{ij} (\hat{c}_{ijT,t} + \hat{v}_{ijT,t}) \\ &= \hat{y}_{iT,t} - \hat{v}_{iT,t}, \end{aligned}$$

where: $v_{iT} \equiv -\sum_j \nu_{ij} v_{ijT,t}$. Finally, approximating $V_{ij,T}$ we obtain:

$$\begin{aligned}
V_{ij,T} &\equiv \left[\frac{1}{n_i} \int_0^{n_i} \left[\frac{p_{ijT,t}(f)}{P_{ijT,t}} \right]^{-\xi} \frac{1}{z(f)} df \right] \\
v_{ij,t} &= \xi \left[\int_0^{n_i} \frac{[\bar{p}_{ijT}(f)]^{-\xi} \frac{1}{z(f)}}{\int_0^{n_i} [\bar{p}_{ijT}(f)]^{-\xi} \frac{1}{z(f)} df} [p_{ijT,t}(f) - P_{ijT,t}] df \right] \\
&= \xi \int_0^{n_i} \left[\frac{[\bar{p}_{ijT}(f)]^{-\xi} \frac{1}{z(f)}}{\int_0^{n_i} [\bar{p}_{ijT}(f)]^{-\xi} \frac{1}{z(f)} df} - \int_0^{n_i} \frac{[\bar{p}_{ijT}(f)]^{1-\xi}}{\int_0^{n_i} [\bar{p}_{ijT}(f)]^{1-\xi} df} \right] p_{ijT,t}(f) \\
&= \xi \left[\frac{\frac{\kappa [\bar{p}_{ijT}(H)]^{-\xi} \frac{1}{z(H)}}{\kappa [\bar{p}_{ijT}(H)]^{-\xi} \frac{1}{z(H)} + (1-\kappa) [\bar{p}_{ijT}(L)]^{-\xi} \frac{1}{z(L)}} - \frac{\kappa [\bar{p}_{ijT}(H)]^{1-\xi}}{\kappa [\bar{p}_{ijT}(H)]^{-\xi} \frac{1}{z(H)} + (1-\kappa) [\bar{p}_{ijT}(L)]^{-\xi} \frac{1}{z(L)}} \right] p_{ijT,t}(H) \\
&\quad + \left[\frac{\frac{(1-\kappa) [\bar{p}_{ijT}(L)]^{-\xi} \frac{1}{z(L)}}{\kappa [\bar{p}_{ijT}(H)]^{-\xi} \frac{1}{z(H)} + (1-\kappa) [\bar{p}_{ijT}(L)]^{-\xi} \frac{1}{z(L)}} - \frac{(1-\kappa) [\bar{p}_{ijT}(L)]^{1-\xi}}{\kappa [\bar{p}_{ijT}(H)]^{-\xi} \frac{1}{z(H)} + (1-\kappa) [\bar{p}_{ijT}(L)]^{-\xi} \frac{1}{z(L)}} \right] p_{ijT,t}(L) \\
&= \xi \left[\begin{aligned} &\left[1 + \left(\frac{1-\kappa}{\kappa} \right) \left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)} \right]^{-\xi} \frac{z(H)}{z(L)} \right]^{-1} \\ &- \left[1 + \left(\frac{1-\kappa}{\kappa} \right) \left[\frac{\bar{p}_{ijT}(L)}{\bar{p}_{ijT}(H)} \right]^{1-\xi} \right]^{-1} \end{aligned} \right] [p_{ijT,t}(H) - p_{ijT,t}(L)]
\end{aligned}$$

then, the change in productivity is:

$$\hat{y}_{iT,t} - \hat{n}_{iT,t} = \hat{v}_{iT,t} = -\xi (\omega^v - \omega) \sum_j \nu_{ij} [\hat{p}_{ijT,t}^H - \hat{p}_{ijT,t}^L],$$

where ω and ω^v are defined in the text.

Table 3.1: Summary statistics

	Manufacturing	Wines
Number of Shipments	3,142,211	1,388,131
Number of exporters	11,596	816
Number of HS8 products	5,746	26
Number of destinations	171	140
Number of currencies used	27	16
Distribution of price changes, expressed in destination's currency*		
10%	-0.19	-0.21
25%	-0.06	-0.09
50%	0.02	0.00
75%	0.12	0.07
90%	0.29	0.18
Distribution of NER changes, destination's currency per U.S. dollar*		
10%	-0.05	-0.10
25%	-0.05	-0.09
50%	-0.04	-0.04
75%	0.05	0.01
90%	0.05	0.05

*Computed for the observations used in the benchmark estimation

Table 3.2: Exchange rates and prices

	Manufacturing				Wines	
	(1)	(2)	(3)	(4)	(5)	(6)
β_{dc}	-0.213 (0.189)	-0.215 (0.188)	-0.061 (0.117)	-0.089 (0.115)	-0.045 (0.087)	-0.043 (0.066)
$\beta_{\$}$	1.249*** (0.21)	1.128*** (0.21)	1.287*** (0.116)	1.280*** (0.111)	0.889*** (0.10)	0.911*** (0.061)
Cty FE	Yes	Yes	No	No	Yes	Yes
Continuing	Yes	No	Yes	No	Yes	No
Observations	9,113	9,891	9,113	9,891	9,637	21,282
Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1						

Table 3.3: Wines sample

	(1)	(2)
β_{dc}	0.134 (0.096)	0.167** (0.078)
$\beta_{\$}$	0.929*** (0.097)	0.961*** (0.069)
Cty FE	Yes	Yes
Continuing	Yes	No
Observations	6,172	11,067
Robust standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.1		

Table 3.4: Exchange rates and quantities

	Manufacturing				Wines	
	(1)	(2)	(3)	(4)	(5)	(6)
β_{dc}	-0.519 (0.746)	-0.381 (0.735)	0.335 (0.463)	0.482 (0.455)	-0.408 (0.348)	-0.366 (0.287)
$\beta_{\$}$	-1.873** (0.823)	-1.320* (0.795)	-1.369*** (0.436)	-1.395*** (0.417)	-1.784*** (0.464)	-0.996*** (0.290)
$\beta_{\$} - \beta_{dc}$	1.35**	0.95	1.704***	1.877***	1.37**	0.63
Cty FE	Yes	Yes	No	No	Yes	Yes
Continuing	Yes	No	Yes	No	Yes	No
Observations	9,113	9,891	9,113	9,891	9,637	21,282
Robust standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1						

Table 3.5: Baseline calibration

Parameter	Value	Parameter	Value
n_c, n_s, n_e	0, 0.52, 0.48	$\kappa_{cs}, \kappa_{ce}, \kappa_{cc}$	1, 0.38, 1
σ	1	ξ	2.53
ϕ	0	z_r	2.18
β	0.99	λ	0.4
θ	0.75	α	0.37
θ_N	0.75	$\frac{\eta \bar{P}_N}{P_T(H)}$	0.55
		$\frac{\eta \bar{P}_N}{P_T(L)}$	0.44

Figure 3.1: Distribution of price changes

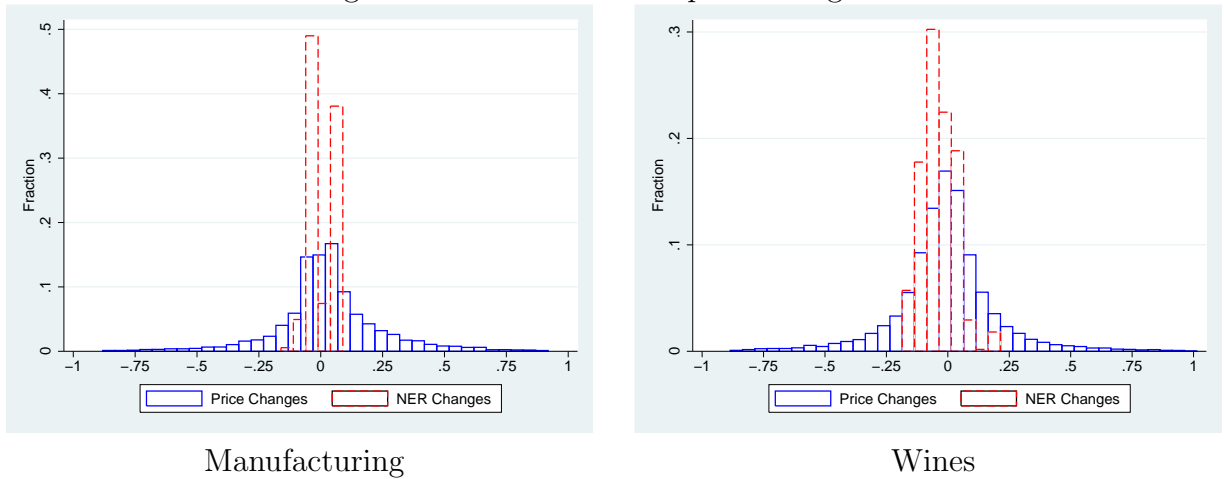


Figure 3.2: Share of sales invoiced in U.S. dollars

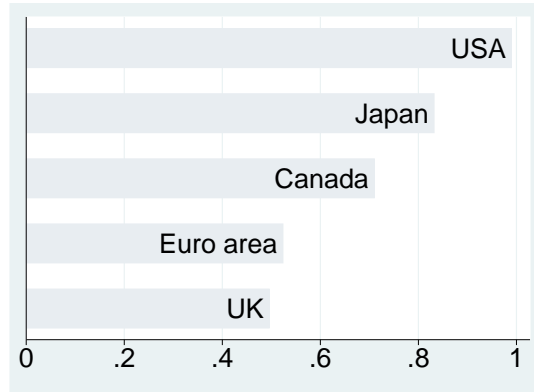


Figure 3.3: Exchange rate shock, baseline calibration

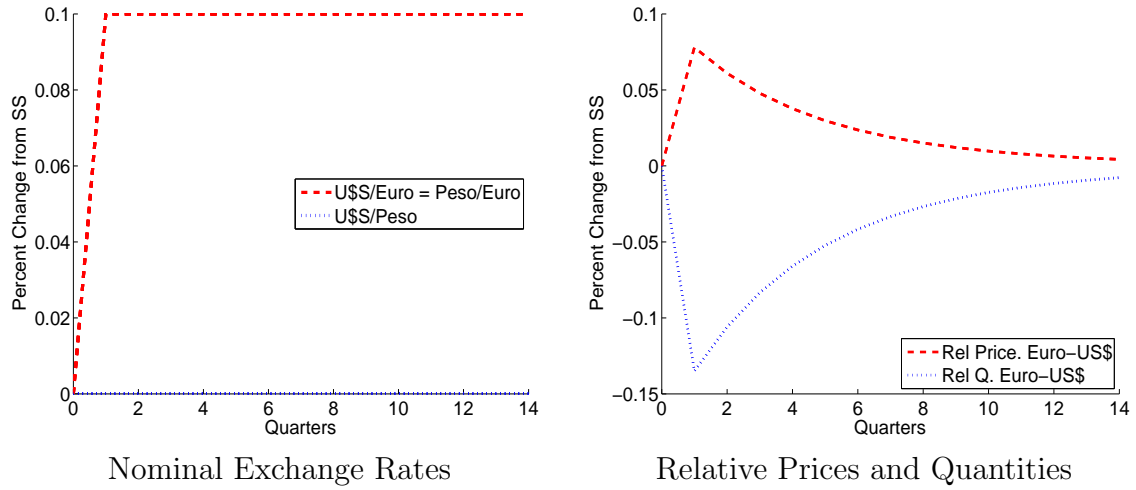


Figure 3.4: Exchange rates and productivity

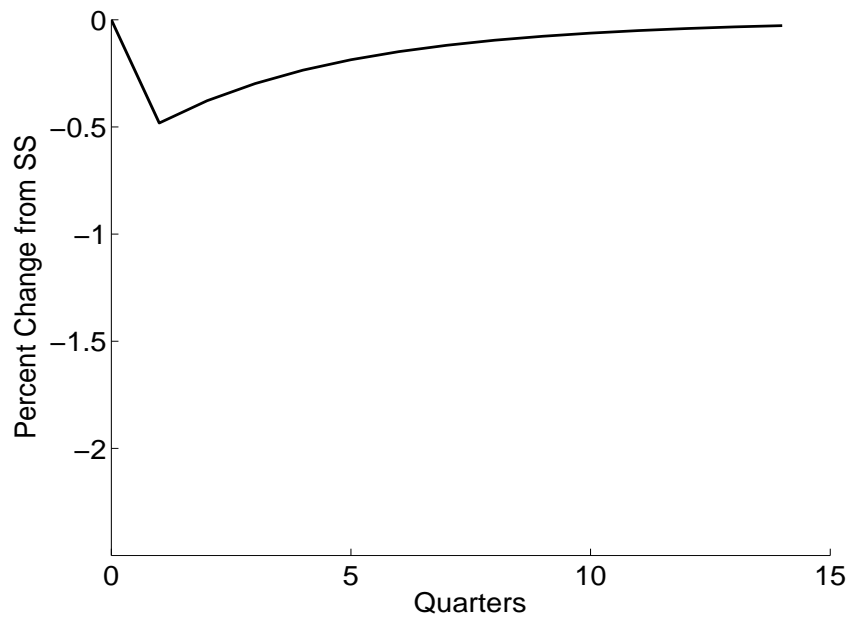


Figure 3.5: Alternative parameterizations

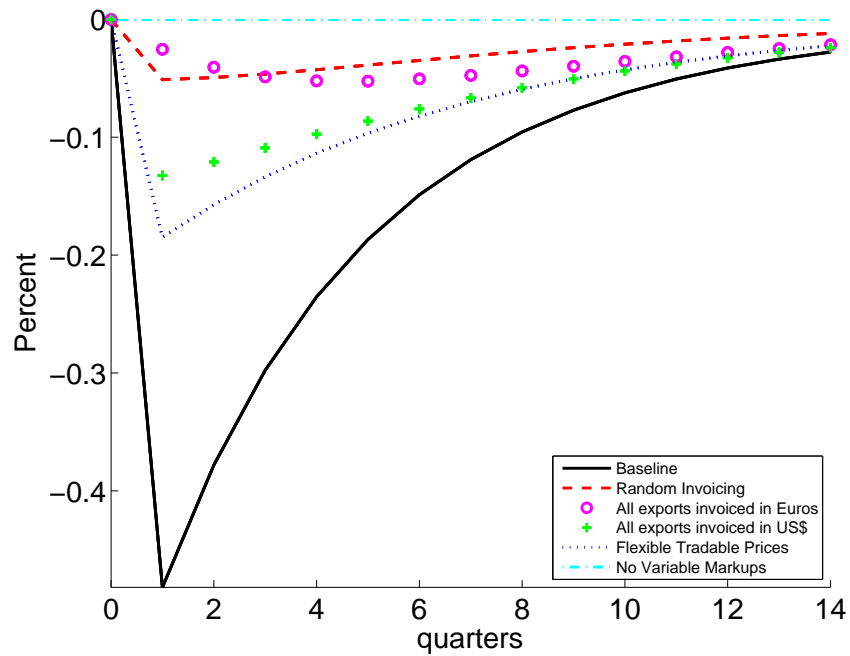
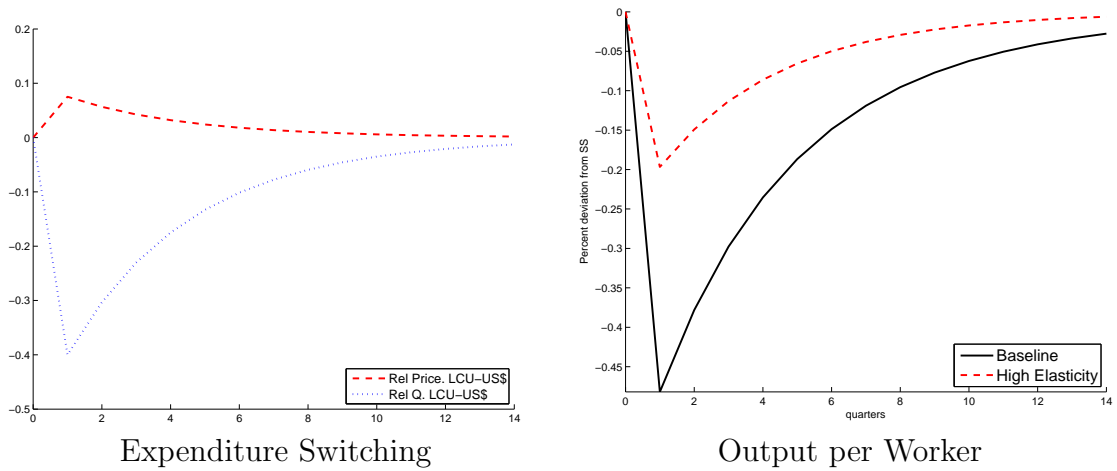


Figure 3.6: Exchange rate shock, high elasticity



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