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Essays on Monetary Economics

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ABSTRACT OF THE DISSERTATION

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In this dissertation, I develop a monetary model where money is used in two roles: as the medium of exchange in spot transactions, and as the unit of account in credit contracts. I use this model to jointly study these two functions, comparing their properties and exploring their interactions.

In the first chapter, I present the model where money can be used as both medium of exchange and unit of account. These functions stem from limits to trade that can be partially overcome with the use of money. The unit of account role in contracts arises from the need to specify a payment, in terms of goods or money. Here, I establish the conditions for money to be chosen as the unit of account in terms of the stability of both relative prices and the general price level. I also illustrate the benefits of dollarization (writing contracts in a foreign currency) or indexation (allowing contracts to be specified in terms of an artificial unit of account).

I then analyze the stationary monetary equilibrium, where the value of money is deter-

mined from its demand as medium of exchange and a given monetary policy. As a result of this analysis, I provide several insights into these functions. For instance, the conditions that make money a good medium of exchange are different from the ones that make it a good unit of account. The unit of account role provides a rationale for a price stability goal of monetary policy, distinct from the goal of keeping a low inflation rate. Finally, the model illustrates how a currency can become a better unit of account as a result of being more widely used as a medium of exchange.

In the second chapter, I analyze two general approaches to monetary policy: inflation and price-level targeting. The former aims for a target level of inflation, while the latter attempts to keep the level of prices in an established path. In order to compare these two policies, I extend the benchmark model to incorporate long-term contracts. On one hand, price-level targeting enhances the long-term stability of the value of money, making it a better unit of account. On the other hand, inflation targeting is better for the medium of exchange role, since the inflation level is the relevant cost for the use of money in spot transactions. I argue that a combined approach resembles the recent Federal Reserve policy of "flexible average inflation targeting," and that indexation would allow monetary policy to sidestep this trade-off.

The dissertation of Diego Zúñiga is approved.

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To my parents

TABLE OF CONTENTS

LIST OF FIGURES

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CHAPTER 1

A Monetary Model of Unit of Account

1.1 Introduction

This paper develops a dynamic model in which money can have two roles: unit of account and medium of exchange. I use the model to explain when money is chosen for these roles and how changes in one of the roles affect the other. I also show how the model serves as a framework to study dollarization, indexation of contracts, and the emergence of a unit of account in the presence of multiple media of exchange. Finally, I explain how the unit-of-account role of money changes monetary policy. The monetary authority values not only a low inflation rate, as usual, but also stability of the price level. Monetary policy matters even in advanced economies where few transactions use currency because it affects the credit transactions that use money as the unit of account.

The importance of the unit-of-account role is most striking during monetary crises. In periods of high inflation and price instability, such as in some Latin American countries in the last decades of the 20th century, the prices of products and the terms of payments in contracts are not denominated in the local currency but in a foreign, stable one such as the dollar. Indeed, during a crisis, it is difficult to foresee the value of the currency in the near future and it can be better for both parties to agree on a dollar price.

The model has an infinite horizon in the style of the New Monetarist literature following Lagos and Wright (2005). Each period includes two stages: first, a decentralized market where bilateral meetings take place; second, a centralized, competitive market where the agents can trade consumption goods and money. Importantly, I add bilateral meetings as in Doepke and Schneider (2017) which create the unit-of-account role.

I model the unit-of-account role as the denomination of future payments in bilateral meetings. In particular, there is one type of bilateral meetings in which agents can sign contracts for future production of goods. The payment in the contract is to be delivered later and must consist of a specific number of goods and currency. The agents sign contracts with no default and they face risky prices and income, therefore the choice of payment denomination matters. The medium-of-exchange role emerges in the model as in standard New Monetarist models. In this second type of bilateral meetings, the transactions are anonymous and payment must be delivered immediately. Therefore, the agents must bring money in order to be able to engage in these transactions.

I start with a partial-equilibrium analysis of the choice of unit of account. The choice of denominating a contract in real or nominal terms comes down to a race between the price and income risks associated with consumption goods, on one side, and unexpected deflation risk, on the other. Unexpected deflation risk is the possibility that money will be more valuable than expected at the moment of contracting. This risk matters for payments in money because, given a promised payment, deflation increases their cost relative to the agent's income. Price and income risks matter for real contracts in different ways. With price risk, the payment could end up being much more expensive relative to its expected value when the contract was signed. This price risk affects real contracts in the same way that unexpected deflation risk affects nominal contracts. On the other hand, income risk can be partially hedged with real contracts; a benefit not present in nominal contracts.

The use of money as unit of account only depends on the unexpected deflation risk. This role is not affected by the price level or the expected rate of inflation. For instance, if the price level were lower or if a deterministic inflation rate was higher, a buyer could simply adjust the payment in the contract in order to maintain the expected value of the payment. After the adjustment, the real outcomes would be unchanged. Notice that this is consistent with the Latin American dollarization episodes. These economies experienced, at the same time, high inflation rates and high inflation volatility. In that case, unexpected deflation risk can happen as a moderate inflation rate when inflation was expected to be much higher.

The dollarization in Latin America highlights the distinction between the roles of medium of exchange and unit of account. During those times, in spite of the change in the denomination of payment, a large part of transactions was settled using the local currency. In this way, a currency can lose its unit-of-account role while maintaining its medium-of-exchange role. The framework in this paper shows that the properties of a currency that make it a good medium of exchange, low inflation rates, are different from those that make it a good unit of account, stability of the price level.

Even though the two roles tend to be played by the same instrument, it is not clear that this is ideal. For the last few decades, Chile has been using an abstract unit of account, the *Unidad de Fomento,* which has no physical counterpart and is adjusted to follow the consumer price index. Chileans participate in contracts, such as insurance, mortgages, and others, that denominate payment in *Unidad de Fomento.* Payments generally involve the local currency, the Chilean peso. This arrangement allows agents to engage in transactions without concerns about unexpected changes in inflation. One might wonder, however, about the effects of the use of this index on inflation and monetary policy. This is the type of question for which we need a model of both medium of exchange and unit of account.

I continue to a general-equilibrium analysis of the value and uses of money. In equilibrium, money can be used for the medium-of-exchange anonymous transactions and also for the unit-of-account contracts. Since the model is dynamic, an agent can choose to bring money to the contract negotiation and pay in advance to supplement his credit. In this sense, there are two ways in which money can be "used" in the unit-of-account contracts: the first one is as unit of account, in the sense that the contract specifies a payment in dollars; the second one is as collateral or down payment. The first does not generate any demand for holding money because the settlement happens in a centralized, competitive market. The second, however, does create such demand. The use of money is determined in general equilibrium. The decision to bring money to the contract negotiation will depend on the rate of inflation and on the efficiency of nominal contracts. The rate of inflation represents the cost of holding money while the efficiency of nominal contracts affects the benefit of bringing collateral to increase the size of the contract. In turn, this efficiency is related to the unexpected deflation risk which is a property of the endogenous value of money.

I use this general-equilibrium model to study the similarities and differences of the two roles. In particular, if we compare an increase in the demand for the medium-of-exchange transactions with a similar increase for unit-of-account contracts, we see different effects on real outcomes when money is used only as unit of account (and not as collateral) in contracts. When money is used as collateral, the effects are similar.

My model also provides a rationale for the goal of price stability, often stated by central banks. This goal is different from that of maintaining a low inflation rate. The former is mostly related to the unit-of-account role while the latter is related to the medium-of-exchange role. These objectives are believed to be complementary and compatible with each other.[1] In this model, I show that a monetary authority faces a trade-off between the two goals. In the presence of "money demand shocks", the monetary policy could respond and keep a stable price level or ignore the shocks and keep the expected inflation rate

[1] See, for instance, the arguments in Bernanke (2006).

low. Although this model is deliberately simple, this result shows that an argument for the complementarity between price stability and low inflation must rely on economic forces outside the current framework.

Leveraging the dual roles of money in this model, I illustrate how a currency can be a better unit of account *because* it is more widely used as medium of exchange. A widely-used currency is less exposed to idiosyncratic demand shocks. Hence, it has a more stable value and less unexpected deflation risk. This application would lie outside the scope of conventional models where money fills only one role. In this way, this framework can be used to address questions about the connection between these two roles of money.

Finally, the model provides an explanation for the relevance of monetary policy in economies that do not have many currency transactions. I study a cashless limit as a sequence of economies with fewer medium-of-exchange transactions. The equilibrium in the limit is different from the equilibrium of a non-monetary economy. Even if the output from transactions that use currency (medium-of-exchange) is very low relative to the output from contracts (unit-of-account), money can be used as the unit of account. The benefit of keeping a stable value of money comes from the unit-of-account contracts, not the medium-of-exchange transactions. In these near-cashless economies, good monetary policy pays attention to money, the medium of exchange, only because it is the unit of account.

1.1.1 Related Literature

This paper builds on Doepke and Schneider (2017) study of the unit-of-account role. Their paper studies the optimal payments in the same contracting problem that I described above. In a static setting, they analyze the optimal unit of account with random matching and payment chains. They also study the use of commodity-backed government debt as the unit of account. In contrast, I embed their contracting problem in a standard, fully-dynamic model of monetary exchange. Here, the equilibrium value of money is determined endogenously as a result of its demand as medium of exchange and unit of account. This helps me establish new results on the uses of money as unit of account, the effects of monetary instability, and the interaction of the two roles.[2]

My paper is also related to the work of Drenik, Kirpalani and Perez (forthcoming). In their paper, they extend the contracting problem of Doepke and Schneider (2017) to include an insurance motive and study the choice between denominating contracts in local or foreign currency. Using the local currency as the unit of account leads the government to pursue monetary policy that provides insurance which reinforces the benefit of this unit-of-account choice. In this context, they study dollarization and hysteresis. My environment differs in that it incorporates dynamics and the medium of exchange role. My contracting problem focuses on price and income risks, and it abstracts from the insurance motive.

My analysis embeds the unit-of-account role in a standard New Monetarist model as in Lagos and Wright (2005). Most work done in this framework is focused on the medium-of-exchange role. My paper, however, develops an economy that allows money to play both roles but does not require it. The model provides a framework for questions about the interaction between the roles and shows how price-stabilization policies can be beneficial even if money is not used in most transactions of the economy.

The present framework shares some implications with Lagos and Zhang (forthcoming). They study an environment in which monetary policy matters for real outcomes in transactions even if an arbitrarily large share of the transactions does not involve the use of currency. In their model, transactions can be done directly with cash or through interme-

[2]In order to use their contracting problem in this larger setting, I obtain explicit solutions that are easily interpretable. I also generalize their results by weakening some of their assumptions on endowments and prices.

diaries with credit. Intermediaries have market power but they are limited by the agents' option of using cash. This option affects outcomes even if it is not actually exercised in equilibrium. In my model, I obtain a similar result through a different mechanism. Monetary policy and the value of money matters for real outcomes because money is used as the unit of account in credit transactions. An erratic monetary policy makes money a worse unit of account and lowers the amount of credit that agents can take. In equilibrium, the value of money, and hence its efficiency as unit of account, is determined via its use as medium-of-exchange even if those transactions are negligible in comparison to the economy's output. The monetary authority cares about these monetary transactions because they affect the much larger market for credit transactions through the unit-of-account role. In short, in a cashless limit, monetary policy matters via the opportunity cost of holding money in Lagos and Zhang (forthcoming) and via price stability in this paper.

Freeman and Tabellini (1998) study the optimality of contingent payments in contracts denominated in money. They analyze an environment with settlement frictions in which money must be used to pay back debts. Their paper shows that contracts that specify a non-contingent payment in dollars are optimal for agents with identical CRRA preferences whose wealth is otherwise denominated in dollars. With those preferences, agents want to share risk in a way that keeps their relative wealth proportional across states and, if their wealth is already in dollars, a non-contingent nominal contract achieves this. In contrast, the contracts in my model do not involve insurance or risk-sharing motives. The unit-of-account choice is made in order to maximize the expected value of a promise while avoiding default. Their model show that non-contingent contracts can be optimal. My model starts from contracts that must be non-contingent, determines whether they will be denominated in real or nominal terms, and examines the general-equilibrium consequences in a model where money is also the medium of exchange.

My model features both money and credit. In models such as these, there is a substitutability between money and credit.[3] In this paper, credit is ruled out for medium-of-exchange transactions by assumptions on the meetings but money and credit can both be used in the unit-of-account contracts. For those contracts, the standard results apply: more credit leads to less money holdings being used for payments. Beyond this result, a key feature of this paper is that the "available credit" is determined by the properties of the value of money, specifically its unexpected deflation risk.

In this model, monetary instability harms the economy in a direct way. This is not due to sticky prices (as in New Keynesian models), prices set in advance (as in Lucas (1989)), or imperfect information (as in Lucas (1972)). Here, monetary instability makes money a worse unit of account and reduces the amount of credit in the economy.

I present a full description of the environment and the agents' problems in the next section. After that, I carry out a partial-equilibrium analysis of the problem of choosing a unit of account. Then, with those results at hand, I characterize the monetary equilibrium of the model. I establish how the roles affect each other through a series of comparative statics. Finally, I develop three extensions to the benchmark model: stochastic money demand, multiple currencies, and a cashless limit.

[3] A classic example is a standard Bewley model, see Ljungqvist and Sargent (2018) for a textbook reference. See also Gu, Mattesini and Wright (2016) for a presentation of this type of results in a more closely related model.

1.2 The Monetary Model

1.2.1 The Environment

Time is discrete and goes forever. There are three types of agents: unit-of-account (UoA) buyers, medium-of-exchange (MoE) buyers, and sellers. The measure of each type of agent is the same and normalized to one. There are two consumption goods denoted by $\{A, B\}$, two special goods, and one long-lived asset, money.

As in Lagos and Wright (2005), each period is divided into two stages. The first stage consists of a decentralized market where the buyers and sellers engage in bilateral meetings with the purpose of trading the special goods. The second stage consists of a centralized Walrasian market where all agents receive their endowments of consumption goods A, B and can trade them along with money at given prices.

Goods and Money. All agents can consume the consumption goods. Only UoA buyers can consume the UoA special good and only MoE buyers can consume the MoE special good. Sellers can produce both types of special goods. The consumption goods are received as endowments and cannot be produced or stored. Money is a long-lived asset that does not depreciate and does not pay dividends. Each period, the supply of money grows at some stochastic rate $g \in \left[g^{Min}, g^{Max}\right]$ which is distributed i.i.d. over time.

Preferences. The preferences of the agents are separable across periods. An agent's utility in a period t is given by

$$\text{Unit-of-Account Buyer: } u(x^U) + v(c_A, c_B),$$
$$\text{Medium-of-Exchange Buyer: } w(x^M) + v(c_A, c_B),$$
$$\text{Seller: } -x^U - x^M + v(c_A, c_B).$$

Here, c_A, c_B represent the consumption of goods A and B, while x^U and x^M represent the quantity of UoA and MoE special goods.

The utility over consumption goods, v, is strictly increasing, strictly concave, and homogeneous of degree one. The utility functions over special goods, u and w, are strictly increasing, strictly concave, and satisfy $u(0) = w(0) = 0$. The seller's cost of producing the special goods is linear.[4] There exist first-best levels x_U^* and x_M^* such that $u'(x_U^*) = w'(x_M^*) = 1$. All agents discount the future with a common discount factor $\beta \in (0, 1)$.

These assumptions on goods and preferences differ from the standard New Monetarist model in several respects. First, this environment has two consumption goods while the standard model has one. With two consumption goods, there can be risk on the relative price of consumption goods. This risk is key for the choice of a unit of account to be nontrivial, as will become clear shortly. Homogeneity of degree one is the generalization of the quasi-linear utility in the standard model; it eliminates wealth effects and keeps the model tractable. Second, in the standard model the agents produce the consumption good, while in this model they receive the goods as endowment. This modeling choice facilitates

[4]The linearity of the cost is without loss of generality. Given the additive separability between consumption and special goods, for any convex cost function, we can redefine the units of the special good to be units of cost to the seller.

the presentation of the model.[5]

Endowments. All agents receive endowments of consumption goods during the centralized market. The ratio of aggregate endowments of consumption goods is random. This will imply that the equilibrium relative prices for consumption goods are uncertain. Conditional on the aggregate endowments, the individual endowments are independently and identically distributed across agents.

Bilateral Meetings. In both unit-of-account (UoA) and medium-of-exchange (MoE) meetings, the buyer and seller will agree to a quantity of the special good and a payment. I restrict the bargaining process to take-it-or-leave-it offers from the buyers to the sellers. In a MoE meeting, as in Lagos and Wright (2005), the payment must be made immediately due to issues such as anonymity and non-enforceability of promises. Since the consumption goods are not storable, this payment must be made with money that the buyers have previously acquired.

In a UoA meeting, in contrast, a buyer can promise to pay the seller at the beginning of the centralized market. For a buyer, the probability of meeting a seller in the decentralized market is λ^U or λ^M. These meetings are independent across agents and periods.

Although the UoA buyer can promise to pay using consumption goods or money, he faces constraints on those promises. First, the payment must consist of a non-contingent bundle of goods and money. Second, the payment must be feasible for every state of the world, i.e., no default. Finally, the buyer can only use a fixed and commonly known fraction

[5]The standard model can be easily recast as an endowment economy. Similarly, this model can be written as a production economy. A few caveats are in order, however. First, because the unit-of-account special good is paid in the centralized market, production must be non-pledgeable, for instance, it can occur after the settlement. Second, production can affect the relative prices, which are relevant for the unit of account choice.

$\theta \in (0,1)$ of his endowment to pay.[6,7]

Formally, a contract is defined as a pair $(x, \boldsymbol{\pi}) \in R_+ \times R_+^3$ where, $\boldsymbol{\pi} = (\pi_A, \pi_B, \pi_M)$ is the promised payment bundle and x is the quantity of UoA special good. A contract $(x, \boldsymbol{\pi})$ is feasible for a buyer who holds m units of money if

$$\boldsymbol{p\pi} \leq \theta \boldsymbol{py} + \phi m$$

for all possible prices $\boldsymbol{p} = (p_A, p_B, \phi)$ and endowments $\boldsymbol{y}$.

The unit of account is the composition of the payment, $\boldsymbol{\pi}$. For example, if $\boldsymbol{\pi}$ includes only a payment in money, then the buyer is using *money as the unit of account*. This does not mean, however, that money is used as the means of payment: the settlement of the contract happens in a Walrasian market, so all agents are indifferent between giving or receiving the exact promised bundle or any other bundle with the same market value.

[6]We can imagine the agents receiving the endowment proportionally over time during the centralized market. The settlement of the contract happens before the full endowment is received.

[7]This assumption allows for two desirable results to hold at the same time. First, the buyer cannot achieve the first-best output level just from promises. Second, the buyer's equilibrium money-holding decision is interior and the distribution of money holdings, for each type of agent, is degenerate. Without this assumption, i.e., if $\theta = 1$, these two results conflict because the first one implies that there is a state of the world where the UoA buyer uses up all his income and, in that state, he cannot afford to accumulate his desired money holdings.

Timing of a period

- Decentralized Market - Bilateral Meetings
  - Unit-of-Account Buyer and Seller: contracts with future payment
  - Medium-of-Exchange Buyer and Seller: anonymous spot transaction
- Centralized Market
  - Fraction θ of endowment received
  - Settlement of UoA contracts
  - Rest of endowment and Money transfers, gM_t
  - Consumption decisions

1.2.2 The Agents' Problems

I present the agents' problems in recursive form. For each agent $i \in \{UoA, MoE, S\}$, there are two value functions W_t^i and V_t^i which represent the maximum lifetime utility of an agent in, respectively, the centralized market and the decentralized market in period t.

1.2.2.1 Centralized Market

Consider an agent of type $i \in \{UoA, MoE, S\}$ in the centralized market after the contracts have been settled and endowments have been received in full. This agent owns a vector $\boldsymbol{z} = (z_A, z_B, z_M)$ of goods and money.[8] The agent faces a Walrasian market with relative prices of goods and money $\boldsymbol{p} = (p_A, p_B, \phi)$. The wealth of that agent at that moment is $\boldsymbol{pz}$.

[8] Some entries of $\boldsymbol{z}$ might be negative for a UoA buyer who had to make a payment. Still, in equilibrium, $\boldsymbol{pz}$ will be nonnegative due to the no-default condition.

The problem of an agent at this stage consists of choosing consumption of goods A and B and money holdings for the next period m'. In particular, the agent solves the following problem.

$$W_t^i(\boldsymbol{pz}) = \max_{c_A, c_B, m'} v(c_A, c_B) + \beta V_{t+1}^i(m'),$$

subject to $p_A c_A + p_B c_B + \phi m' \leq \boldsymbol{pz}$. Since the utility is homogeneous of degree one, the optimal consumption decision will result in a utility level equal to $\boldsymbol{pz}/e(p_A, p_B)$, where $e(p_A, p_B)$ is the minimum expenditure required to obtain one unit of utility at prices p_A, p_B.

Without loss of generality, I normalize the prices (p_A, p_B, ϕ) at each centralized market to be such that $e(p_A, p_B) = 1$. With this in mind, we can write the remaining problem of choosing money holdings for next period as

$$W_t^i(\boldsymbol{pz}) = \max_{m'} \boldsymbol{pz} - \phi m' + \beta V_{t+1}^i(m'),$$

subject to $\phi m' \in [0, \boldsymbol{pz}]$.

As in Lagos and Wright (2005), I assume that the agent's wealth is large enough so that the upper-bound constraint on the money holding's decision does not bind.

Assuming differentiability of the value function, the first-order condition with respect to money holdings is

$$\phi \geq \beta \left. \frac{\partial V_{t+1}^i}{\partial m'} \right|_{m'},$$

with equality if the agent holds some money, $m' > 0$. This result, along with a quick application of the envelop theorem, implies that the marginal value of any good in the centralized market is equal to its price.[9] That is,

$$\frac{\partial W_t^i(\boldsymbol{pz})}{\partial z_j} = p_j$$

for $i \in \{UoA, MoE, S\}$ and $j \in \{A, B, M\}$.

Optimal Consumption Decision. In an agent's centralized market problem, there is a consumption decision for goods A and B. We can think of the agent's decision as choosing money holdings first and then choosing a bundle of consumption goods with his remaining income. For a given choice of future money holdings, m', the consumption decision problem is simply

$$\max_{c_A, c_B} v(c_A, c_B)$$

subject to

$$p_A c_A + p_B c_B \leq \boldsymbol{pz} - \phi m'.$$

Here, I have omitted the continuation utility of the agent which, given m', does not depend on the choice of consumption goods. This is a standard utility maximization problem. In particular, at the optimal consumption bundle, the marginal rate of substitution must equal the ratio of prices,

$$\frac{v_A(c_A, c_B)}{v_B(c_A, c_B)} = \frac{p_A}{p_B},$$

where v_i denotes the partial derivative of the utility function with respect to good $i \in \{A, B\}$. Because v is homogeneous of degree one, the left-hand side depends only on the ratio c_A/c_B. In equilibrium, the ratio of prices p_A/p_B is thus determined by the ratio of aggregate endowments Y_A/Y_B. This condition and the normalization of prices $e(p_A, p_B) =$

[9] To be more precise, the value is not necessarily linear for all wealth levels but it is linear across the wealth levels that the agents have in the centralized market.

1 pin down the prices of consumption goods in equilibrium.

The equilibrium quantities and prices in the consumption-goods market will depend only on the aggregate endowments. As we will see in the equilibrium analysis, the distribution of consumption across agents and the level of the aggregate endowments are not relevant for the market for money because of the absence of wealth effects. As in Lagos and Wright (2005), this result allows the model to be very tractable: the equilibrium value and uses of money will not depend on the specifics of the consumption-goods market. Furthermore, the value of money, which is the inverse of the price level, will not depend on the level or relative composition of the aggregate endowment. In this sense, we could say that this model fits Milton Friedman's famous quote, "Inflation is always and everywhere a monetary phenomenon."[10]

1.2.2.2 Decentralized Market

Unit-of-Account Buyers. A UoA buyer in the decentralized market who owns m units of money faces the following problem.

$$\begin{aligned} V_t^U(m) = \max_{x,\boldsymbol{\pi}} u(x) + \mathbb{E}[W_t^U(\boldsymbol{py} + \phi m - \boldsymbol{p\pi})] \\ \text{s.t. } \mathbb{E}[W_t^S(\boldsymbol{py})] \leq - x + \mathbb{E}[W_t^S(\boldsymbol{py} + \boldsymbol{p\pi})], \\ \boldsymbol{p\pi} \leq \theta \boldsymbol{py} + \phi m, \text{ for } \underline{\text{all}}\ \boldsymbol{p}, \boldsymbol{y}. \end{aligned}$$

This buyer must offer a contract $(x, \boldsymbol{\pi})$ to the seller. The buyer makes a take-it-or-leave-it offer and the seller is sequentially rational. Hence, I incorporate the seller's acceptance

[10]One might, intuitively, expect that if the aggregate endowments are low, the price level will be high, and the value of money low. This intuition is incorrect when the utility is homogeneous of degree one. The relevant objects are the marginal value of holding money and the marginal utility of consumption. Under the homogeneity assumption, the latter is independent from the level of aggregate endowments.

decision in the first constraint. Without loss of generality, I restrict attention to contracts that will be accepted, since the trivial contract ($x = 0, \boldsymbol{\pi} = \mathbf{0}$) is always acceptable. The expectation operators in the problem correspond to the centralized market prices and endowments.

Since the marginal value of a payment in the centralized market is linear, this problem can be simplified as

$$\begin{aligned} V_t^U(m) = \max_{x,\boldsymbol{\pi}} u(x) - \mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi} + \mathbb{E}[\phi]m + \mathbb{E}[W_t^U(\boldsymbol{p}\boldsymbol{y})] \\ \text{s.t. } x \leq & \mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi}, \\ \boldsymbol{p}\boldsymbol{\pi} \leq & \theta\boldsymbol{p}\boldsymbol{y} + \phi m, \text{ for } \underline{\text{all}}\ \boldsymbol{p}, \boldsymbol{y}. \end{aligned}$$

Medium-of-Exchange Buyers. A MoE buyer in the decentralized market who owns m units of money faces the following problem.

$$\begin{aligned} V_t^M(m) = \max_{x,\hat{m}} w(x) + \mathbb{E}[W_t^M(\boldsymbol{p}\boldsymbol{y} + \phi m - \phi\hat{m})] \\ \text{s.t. } \mathbb{E}[W_t^S(\boldsymbol{p}\boldsymbol{y})] \leq & -x + \mathbb{E}[W_t^S(\boldsymbol{p}\boldsymbol{y} + \phi\hat{m})], \\ \hat{m} \leq & m. \end{aligned}$$

This problem is similar to that of the UoA buyer. The MoE buyer chooses an amount of the special good and a payment. The first constraint states that the offer must be acceptable, again without loss of generality. The second constraint states that the payment must consist of money and can only be paid with the buyer's current money holdings.

As in the UoA-buyer's problem, the value function can be simplified to obtain

$$
\begin{aligned}
V_t^M(m) = \max_{x,\hat{m}} w(x) + \mathbb{E}[\phi](m-\hat{m}) + \mathbb{E}[W_t^M(\boldsymbol{py})]& \\
\text{s.t. } x \leq& \mathbb{E}[\phi]\hat{m}, \\
\hat{m} \leq& m.
\end{aligned}
$$

Sellers. A seller in the decentralized market could either meet with a buyer or not. If the seller has a meeting, the buyer could be UoA or MoE. The seller's problem consists of choosing which offers to accept. This problem is trivial because the buyers will make take-it-or-leave-it offers. In equilibrium, the seller does not capture any of the surplus from the meetings so the value function is given by

$$V_t^S(m) = \mathbb{E}[W_t^S(\boldsymbol{py} + \phi m)].$$

The interested reader can see a complete description and derivation of the seller's problem in Appendix 1.B.

1.2.3 Equilibrium Definition

Before the equilibrium analysis, I formally define an equilibrium of this economy.

Definition 1 (Equilibrium) *An equilibrium is a collection* $\{W_t^i, V_t^i, c_{A,t}^i, c_{B,t}^i, m_t^i, (x_t^U, \boldsymbol{\pi}_t), (x_t^M, \hat{m}), M_t, (p_{A,t}, p_{B,t}), \phi_t\}$ *of value functions, consumption and money holding choices, contracts, MoE transactions, money supply, and prices of consumption goods and value of money (which are functions of the history of money supply) such that (1) the value functions solve the Bellman equations, (2) consumption and money holding choices are optimal given prices and value functions, (3) UoA contracts and MoE transactions are acceptable* *for the seller and optimal for the buyers, (4) the markets for consumption goods and money clear.*

Most of my analysis focuses on the stationary monetary equilibrium. Since the supply of money is assumed to be stochastic, the notion of stationarity might not be obvious. I look for equilibria where the value of the money stock during the decentralized market is constant. This means that the value of payments from buyers to sellers is constant, in spite of the random changes to the money supply.

Definition 2 (Stationary Monetary Equilibrium) *A stationary monetary equilibrium is an equilibrium in which, for all t and histories of money supply, money is valuable, $\phi_t > 0$, and the level of aggregate real balances in the decentralized market, $E_{t-1}[\phi_t]M_t$, is constant.*

In the next section, I analyze the contracting problem by itself, taking prices as given. After that, I incorporate those results into the complete model and analyze the monetary equilibrium.

1.3 The Unit-of-Account Contracting Problem

A UoA buyer must choose a quantity of special good (or size of the contract) x and a payment $\boldsymbol{\pi}$. Specifically, the problem is

$$\begin{aligned}\max_{x,\boldsymbol{\pi}} \quad u(x) - &\mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi} \\ \text{s.t.} \quad x \leq &\mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi}, \\ \boldsymbol{p}\boldsymbol{\pi} \leq &\theta\boldsymbol{p}\boldsymbol{y} + \phi m, \text{ for } \underline{\text{all}}\ \boldsymbol{p}, \boldsymbol{y}.\end{aligned}$$

The first-best allocation would consist of x_U^* units of the special good. However, the buyer's promises are limited. In particular, the buyer is limited by the maximum expected value across feasible promises. This is the buyer's payment capacity.

Definition 3 (UoA Buyer's Payment Capacity) *The payment capacity of a buyer (who holds m units of money) is*

$$q(m) := \max_{\boldsymbol{\pi}} \mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi}$$

$$\textit{s.t. } \boldsymbol{p}\boldsymbol{\pi} \leq \theta \boldsymbol{p}\boldsymbol{y} + \phi m, \textit{ for } \underline{\textit{all}}\ \boldsymbol{p}, \boldsymbol{y}.$$

The maximized value is the maximum size of the contract that a buyer can afford. A solution to this problem defines an optimal unit of account. In fact, even if the buyer does not use his payment capacity in full, the buyer can scale down the solution bundle and offer that as payment.

Support of Prices and Endowments. The support of prices and endowments is important for the contracting problem because the payment must be affordable under all prices and endowment realizations. I assume that the supports of consumption-goods prices, value of money, and endowments are compact and unrelated to each other. Specifically, for every triple (p_A, p_B), ϕ, (y_A, y_B) of possible realizations, their joint realization is in the support.[11] I also assume that there are at least two consumption-goods price realizations and two endowment realizations; if either of these assumptions fail, the problem is trivial.

The analysis of this problem proceeds in the following steps. I start with the study of the payment capacity when the buyer does not hold any money, $m = 0$. I solve for the optimal payments using goods or money and provide conditions for money to be the unit

[11] In the full model, where prices are endogenous, this assumption is an equilibrium result under simple conditions on the idiosyncratic and aggregate endowment process. See section 1.2.2.1.

of account. Then, I consider the problem of a buyer who holds money and show that this does not change the choice of unit of account.

1.3.1 Paying with Money

Proposition 1 (Optimal Promises with Money) *The optimal payment with money is given by* $\pi_M = Y^{worst}/\phi^{Max}$ *and the expected value of that payment is*

$$\mathbb{E}[\phi]\pi_M = \frac{\mathbb{E}[\phi]}{\phi^{Max}} Y^{worst},$$

where $Y^{worst} := \min_{\boldsymbol{p},\boldsymbol{y}} \theta \boldsymbol{p}\boldsymbol{y}$ *and* ϕ^{Max} *is the maximum value of money.*

If the buyer decides to pay only with money, then his promise π_M must satisfy $\phi\pi_M \leq \theta \boldsymbol{p}\boldsymbol{y}$ for all prices and endowments. The buyer can therefore at most promise to pay $\pi_M = Y^{worst}/\phi^{Max}$ units of money. At ϕ^{Max}, the value of the payment is maximized while at Y^{worst} the income is minimized.

The value of paying with money depends on two factors: income and price risk. First, the payment capacity is proportional to the worst-case income. Second, the coefficient of proportionality, $\mathbb{E}[\phi]/\phi^{Max}$, is less or equal than one, with equality only if there is no risk in the value of money. The relevant price risk for a payment in money is the *unexpected deflation risk*.

Notice that the buyer is constrained by the worst-case outcome: minimum income and maximum cost of payment. The promise, however, is only valued according to its expected value. For a given expected value of a payment, the realizations where money has less value (inflation) are not relevant for this calculation. Importantly, only deflation relative to the expected value matters. For instance, a currency with deterministic deflation would yield a coefficient equal to 1. In turn, a currency with high but risky inflation rates would have a coefficient strictly less than one. In the second case, money would be a worse unit of account.

1.3.2 Paying with Goods

Consider now a buyer that decides to pay using only consumption goods. To facilitate the exposition, I start with a simple case: assume that for each price (p_A, p_B), the worst-case income realization is the same. That is, $\min_y \boldsymbol{py} = Y^{worst}$ for all (p_A, p_B). This case allows for a simple characterization of the optimal payment.

Proposition 2 (Optimal Promises with Goods - No Worst-Income Risk) *Suppose that the worst-case income realization is the same for all prices. The expected value of the optimal payment using goods is*

$$\chi^g \, Y^{worst},$$

where $\chi^g \leq 1$ *and it is only equal to 1 if* $\Pr(\boldsymbol{p} \textit{ is an extreme price}) = 1$.[12,13]

The problem of maximizing the value of a promise that consists only of goods is a linear programming problem. Notice that the boundaries of the set of feasible payments correspond to the extreme price realizations (and the non-negativity constraints). The slope of the expected prices must be in between the slope of the extreme price realizations so the optimal solution is at the intersection of those two constraints.

I present the problem in a two-dimensional graph with quantities of goods A and B in the axes in Figure 1.1. The value of a bundle (y_A, y_B) at prices (p_A, p_B) is given by $p_A y_A + p_B y_B$ which, due to the normalization of prices $e(p_A, p_B) = 1$, is also $\max_{c_A, c_B} v(c_A, c_B)$

[12]A price realization, $\tilde{\boldsymbol{p}}$, is an extreme price if either $\tilde{p_A} \geq p_A$ or $\tilde{p_A} \leq p_A$ for all p_A.

[13]The coefficient χ^g has an explicit solution, $\left[\left(\overline{p_B} - \underline{p_B}\right)\mathbb{E}[p_A] + \left(\overline{p_A} - \underline{p_A}\right)\mathbb{E}[p_B]\right] / \left(\overline{p_A p_B} - \underline{p_A p_B}\right)$, where $\overline{p_i} = p_i^{Max}$ and $\underline{p_i} = p_i^{Min}$ for $i \in \{A, B\}$. See appendix 1.A for a step-by-step derivation.

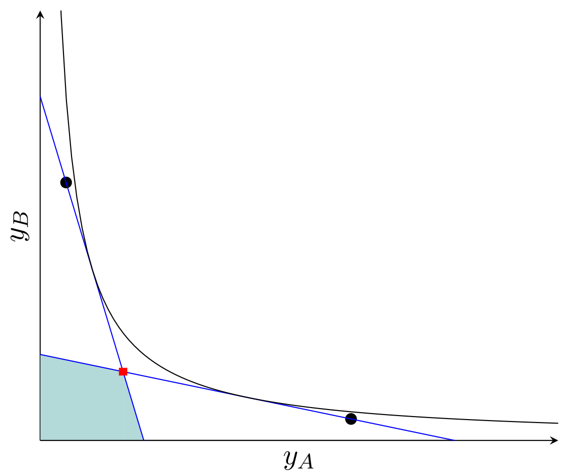

Figure 1.1. Feasible payments using only goods - No income risk.

The blue lines are the budget constraints at different relative prices. The black curve is the indifference curve of utility level Y^{worst}. The fact that the two budget constraints are tangent to the same indifference curve is equivalent to the assumption of no worst-case income risk. The red dot is the optimal payment using goods.[14]

s.t. $p_A c_A + p_B c_B \leq p_A y_A + p_B y_B$. That is, the value of a bundle at some given prices corresponds to the maximized utility of consumption that can be afforded with that bundle. Therefore, this ex-post value of a bundle at given prices corresponds to the indifference curve that is tangent to that budget constraint. In this way, the assumption that the worst-case income is the same across all prices is equivalent to the existence of an indifference curve with utility Y^{worst} to which the worst-case budget constraints at all prices are tangent.

It is easy to see that the solution corresponds to the intersection of the budget constraints in the graph. We can explicitly solve for the optimal payment using goods by finding that intersection. See appendix 1.A for the explicit solution.

Using this graph, we can see that the expected value of the optimal payment using goods equals the worst-case income when there are only extreme prices. A seller who receives the red dot bundle as payment can find herself in two situations: either good A is most expensive, or good B is. In the first case, the seller is subject to the more-vertical budget constraint in the graph, and she can attain a utility of Y^{worst}. In the second case, her budget constraint is the more-horizontal budget constraint, and she can also attain a

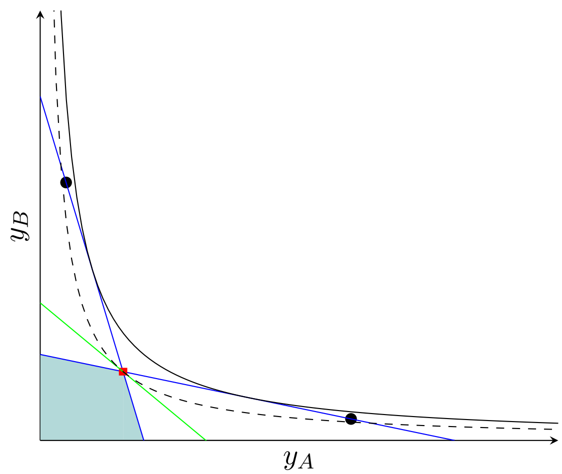

Figure 1.2. Feasible payments using only goods - No income risk - Price risk.
The green budget constraint corresponds to an intermediate relative price realization. The dashed black line is the indifference curve tangent to this new budget constraint and it is below the Y^{worst} indifference curve.

utility of Y^{worst}. Thus, if those are the only relative prices, the value of the red dot bundle is Y^{worst}.

The situation is different if there are intermediate prices. This is represented in the next graph, Figure 1.2. At the intermediate price that creates the green budget constraint, the payment bundle cannot attain the same utility level. Therefore, the value of the payment is less than Y^{worst}.

This proposition states that the expected value of the optimal payment using goods depends on two factors: income and price. First, this payment capacity is proportional to the worst-case income realization. Second, the coefficient of proportionality χ^g is less or equal than one with equality if there are only extreme consumption-goods price realizations. This coefficient of proportionality depends on the variability of relative prices of consumption goods.

Notice the similarity of this result with the previous one when paying with money. The

[14]There exist other budget constraints that do not appear in this figure. At least, for each endowment realization, there must be as many budget constraints as relative prices. Clearly, though, these budget constraints will not matter for the problem.

price-risk coefficient is simpler in the case of money because it is only one asset. In the case of goods, this coefficient is more complicated because there are two goods. Still, the underlying logic is the same, a ratio of expected to maximum value: the expectation is the value that both buyer and seller give to the bundle when they negotiate the contract, the maximum is the greatest value that the buyer can afford.

Proposition 3 (Money as Unit of Account - No Worst-Income Risk) *Suppose the worst-case income realization is the same for all prices. The optimal payment uses only money (i.e., money is the unit of account) if and only if*

$$\chi^g < \frac{\mathbb{E}[\phi]}{\phi^{Max}}.$$

With no worst-income risk, the choice of unit of account comes down to comparing consumption-goods price variability and the unexpected deflation risk.

Let us now lift the assumption of no worst-case income risk. The payment capacity using goods is not necessarily limited by the overall worst income realization anymore. The payment bundle can be chosen to hedge the worst-income risk.[15] To be more precise, the payment capacity is determined in the following way.

Proposition 4 (Optimal Promises with Goods) *The expected value of the optimal payment using goods is*

$$\chi^g\,\overline{Y^{worst}},$$

where $\overline{Y^{worst}}$ is a weighted average of the worst-case income at the extreme price realizations and χ^g is defined as in Proposition 2.

---

[15]Suppose the buyer is richer, i.e., has a greater worst-case income, when good A is most expensive. Then, the buyer can benefit from promising more of good A and less of good B relative to the promise that he would have made if the worst-case income had been the same. See Appendix 1.A for an explicit solution.

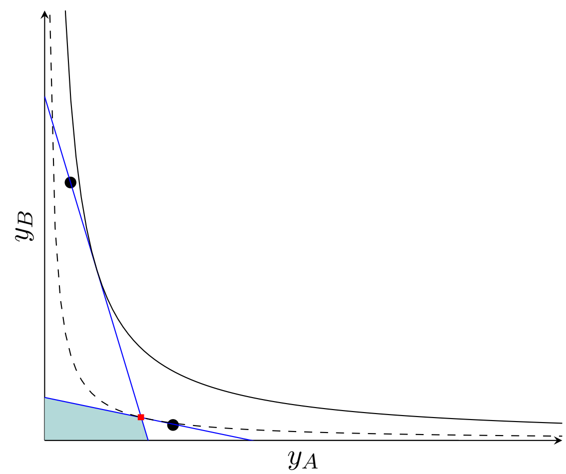

Figure 1.3. Feasible payments using goods. Income risk.
Worst-case income is greater when good A is most expensive. The red dot indicates the optimal payment. If the only price realizations are the extreme prices, the value of the payment is an average of the worst-case incomes.

We can see how the worst-case income risk changes the problem in the two-dimensional graph in Figure 1.3. In this figure, the worst-case income is greater when good A is most expensive. If only the extreme prices are possible, the the payment capacity is a simple expectation of the worst-case incomes. For the explicit solution in general, see appendix 1.A.

The payment capacity has a similar formula to the simpler case. The only difference is that the payment capacity is now proportional to an average of worst-case income at the extreme prices. Since this average will be greater than the overall worst-case income, the payment capacity using goods has an advantage over using money. In other words, suppose that $\mathbb{E}[\phi]/\phi^{Max}$ were equal to χ^g, if there is worst-case income risk, then using only goods is preferable to using money.

Proposition 5 (Money as Unit of Account) *The optimal payment uses only money (i.e.,* *money is the unit of account) if and only if*

$$\chi^g \overline{Y^{worst}} < \frac{\mathbb{E}[\phi]}{\phi^{Max}} Y^{worst}.$$

The choice of unit of account comes down to a race between, on one hand, consumption-goods price variability and income risk, and, on the other hand, the unexpected deflation risk. The advantage of using goods is due to hedging worst-case income risk, the disadvantage is relative-price variability. If the price variability is large enough and there is not too much income risk, then there is a role for money to be the unit of account. Of course, money will take on this role only if its value is relatively safe, in the sense of *unexpected deflation risk*.

1.3.3 Money holdings

I now turn to the optimal payments of a buyer who has brought money to the meeting. Holding money affects the payment capacity in a straightforward manner and, in a sense, does not change the choice of unit of account.

Proposition 6 (Optimal Payments with Money Holdings) *The optimal payment when the buyer holds m units of money consists of the optimal payment when he holds no money plus these m units.*

Notice that simply adding the m units of money to the payment leads to an increase in the payment capacity of $\mathbb{E}[\phi]m$. This increase occurs even if the buyer is otherwise paying with goods. Consider, on the other hand, using this money to pay with goods, then the buyer's worst-case income will increase by $\phi^{Min}m$ which is further affected by a factor of proportionality, $\chi^g \leq 1$. It is therefore optimal to add the money holdings to the optimal payment determined above.

Money holdings do not change the unit of account in the following sense. We can think of the m units of money as a down payment or collateral on the contract. The rest of the payment is credit which is denominated in goods or money. The unit of account, understood as the composition of only this "credit payment", is unaffected by m.[16]

1.3.4 Extensions: Indexation and Dollarization

There are two other forms of payment denomination that are relevant in a discussion of the unit-of-account role. The first one is indexation, which involves denominating payments in terms of an economic index. The second one is dollarization, in which the denomination of payments occurs in a foreign currency. I show here that both cases can be easily incorporated in this framework.

For both cases, I modify the definition of a payment bundle to allow promises in units of the index or in dollars.

1.3.4.1 Dollarization

For this section, I extend the original economy to include another currency, the dollar. This currency is traded in the centralized market along with money (the local currency) and the consumption goods. I will not attempt a full specification of the role of the dollar in the economy, at this point.[17] I assume that the dollar's value is stochastic and its support is

[16] This definition of unit of account as relating only to the credit payment is in contrast to the use of the term in Doepke and Schneider (2017) and Drenik et al. (forthcoming). In both of these papers, they consider holdings of an asset (government IOUs and foreign currency, respectively) as contributing to that asset being the unit of account. However, this is true of any asset, even risky ones, which can be held and used as payment. In this sense, I make the distinction between a payment made with holdings of an asset and a payment made purely as credit. I call the former a down payment and I reserve the label of unit of account to the composition of the latter.

[17] A full specification would require, at least, a specification of the supply of dollars and whether they are accepted as payment in MoE transactions.

unrelated to the support of the value of the local currency and the relative prices of the consumption goods.

A buyer in the possibly-dollarized economy must choose a payment bundle $\boldsymbol{\pi} = (\pi_A, \pi_B, \pi_M, \pi_D)$. Here, π_D is the quantity of dollars that the buyer owes at the beginning of the centralized market. If a buyer makes only promises in dollars, then he is limited in the same way as when making promises in local currency. The payment capacity using dollars is

$$\frac{\mathbb{E}[\phi_D]}{\phi_D^{Max}} Y^{worst}.$$

Proposition 7 (Dollarization) *The dollar is the unit of account if*

$$Y^{worst}\,\frac{\mathbb{E}[\phi_D]}{\phi_D^{Max}} > \overline{Y^{worst}}\,\chi^g \;\; \textit{and} \;\; \frac{\mathbb{E}[\phi_D]}{\phi_D^{Max}} > \frac{\mathbb{E}[\phi]}{\phi^{Max}}.$$

The first condition states that the dollar is a better unit of account than a bundle of consumption good. The second condition states that the dollar faces less unexpected deflation risk than the local currency.

This result allows me to interpret the dollarization episodes in Latin American in light of this formulation. Those economies became dollarized with high inflation rates and with high inflation volatility.[18] The high inflation rates do not exclude unexpected deflation risk. The main economic force at play here is the volatility of inflation. With volatile inflation, inflation might turn out to be moderate when we expected it to be quite high. If that were to happen, the buyer would be liable for a large payment. Hence, in anticipation, the buyer would restrict his promised payments. If the risk becomes large enough, the buyer will simply switch to promise dollar payments.

[18] See, for instance, Kehoe and Nicolini (2021).

1.3.4.2 Indexation

I now allow the agents in the economy to write contracts with payments in units of an index. This index is denominated in units of money. That is, at each centralized market,a new value for the index is reported as η units of money. A buyer who promised to pay one unit of the index is then liable to pay η units of money.

The "price" of a unit of the index is $\phi\eta$. Clearly, if the buyer uses the index to denominate the promised payment, then the expected value of that payment is at least

$$\frac{\mathbb{E}[\phi\eta]}{(\phi\eta)^{Max}}Y^{worst}.$$

The expected value will be exactly this if the support of $\phi\eta$ is unrelated to that of the consumption-goods prices and of the value of money. In general, this condition on the support will depend on the type of index under consideration, i.e., the relation of η to other prices.

Consider a price-level index. This is a simple but interesting example. At each period, the value of the index η is updated from its value in the previous period η_{-1} with the following rule,

$$\eta = \eta_{-1}\frac{\phi_{-1}}{\phi}.$$

In this way, the index incorporates changes to the price level or, equivalently, the value of money. The "price" of a unit of this index is $\phi\eta = \phi_{-1}\eta_{-1}$ for every realization of ϕ. In other words, the value of a promise made in this index is constant across all possible state realizations. Thus, a buyer who uses this index can promise an expected payment of Y^{worst}. This is just as good as if the buyer had access to riskless money.

The use of a price index for the denomination of future payments in an economy is not a mere theoretical curiosity. In Chile, the denomination of contracts, especially insurance and mortgages, is in terms of the *Unidad de Fomento*, an abstract unit that is continuously adjusted following the consumer price inflation.[19] There is no *Unidad de Fomento* currency that the agents can use to pay. The settlement of a contract is generally done with Chilean pesos. Since the use of this index is not mandated by law, the parties in the contract must find some value in its use. This model captures that economic benefit.

Indexation need not be restricted to price indices. Indices with respect to output or GDP can help agents build in contingencies in their payments that would otherwise not be allowed. This is, for instance, implicitly included in GDP-linked bonds. This type of indices has the same advantage over price indices that promises in goods have over promises in money: they allow for hedging of income risk. A borrower who does not face too much idiosyncratic risk relative to the index can benefit from such a contract.[20]

1.4 Equilibrium Analysis

Now that we have the solution to the contracting problem, we can turn to the analysis of the monetary model.

[19] See Herrera and Valdes (2005) for a report on the historical development and use of this unit.

[20] Of course, the proposals of indices and other units of account have a long history. Shiller (2009) reviews this history and proposes the creation of new indices. The model presented here captures some of the benefits of indexation that he highlights in his proposal.

1.4.1 The Demand for Money

The first-order conditions for the choice of money holdings are

$$\begin{aligned}
\phi_t \geq& \beta\mathbb{E}[\phi_{t+1}]\left(1+\lambda^U \max\{u'\left(q+\mathbb{E}[\phi_{t+1}]m_{t+1}^U\right)-1,0\}\right),\\
\phi_t \geq& \beta\mathbb{E}[\phi_{t+1}]\left(1+\lambda^M \max\{w'\left(\mathbb{E}[\phi_{t+1}]m_{t+1}^M\right)-1,0\}\right),\\
\phi_t \geq& \beta\mathbb{E}[\phi_{t+1}],
\end{aligned}$$

for UoA buyers, MoE buyers, and sellers, respectively. In each case, the condition must hold with equality if $m_{t+1}^i > 0$. In the first condition,

$$q = \max\left\{\frac{\mathbb{E}[\phi_{t+1}]}{\phi_{t+1}^{Max}} Y^{worst}, \chi^g \overline{Y^{worst}}\right\}$$

is the payment capacity of the UoA buyer if he had brought no money to the meeting.

In all of these conditions, the left-hand side is the marginal cost of increasing money holdings while the right-hand side is the marginal benefit. This marginal benefit is derived from two uses. First, the buyers can use the extra money to get larger contracts or transactions. Second, all agents can use the extra money in the next centralized market. Clearly, the sellers will not hold money in equilibrium.[21]

These three equations define the money demand at time t as a function of the current and expected future value of money. Let us denote $Z_t^U = \phi_t m_t^U$ and $Z_t^M = \phi_t m_t^M$, the real balances used by UoA and MoE buyers. In a stationary monetary equilibrium, $\mathbb{E}[Z_t^U] = \overline{Z^U}$ and $\mathbb{E}[Z_t^M] = \overline{Z^M}$ are constant across time and state realizations.

[21] I ignore the possibility of a monetary equilibrium where the MoE buyers hold no money. Although, this outcome is possible under some conditions on the utility functions u and w, it is not particularly interesting. Conditional on being in that equilibrium, the effects from and on the MoE transactions are, of course, nonexistent while the effects on the UoA contracts are the same as in the equilibrium where both buyers hold money.

The previous conditions can now be written as

$$
\begin{aligned}
Z_t^U(1+g_t) \geq & \beta \overline{Z^U}\left(1+\lambda^U \max\{u'\left(q+\overline{Z^U}\right)-1,0\}\right), \\
Z_t^M(1+g_t) = & \beta \overline{Z^M}\left(1+\lambda^M \max\{w'\left(\overline{Z^M}\right)-1,0\}\right).
\end{aligned}
$$

Here, the second condition holds with equality anticipating that money will definitely be held by the MoE buyers, since it has to be held by someone in equilibrium. The third condition can be ignored because the sellers will not hold money. We can use these conditions to solve for the real balances in a stationary monetary equilibrium.

Proposition 8 (Characterization of Stationary Monetary Equilibrium) *In a stationary monetary equilibrium, the real balances used by UoA and MoE buyers solve the following equations.*

$$
\begin{aligned}
\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} = & \beta\left(1+\lambda^M \max\{w'\left(\overline{Z^M}\right)-1,0\}\right), \\
\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} \geq & \beta\left(1+\lambda^U \max\{u'\left(q+\overline{Z^U}\right)-1,0\}\right),
\end{aligned}
$$

with equality if $\overline{Z^U} > 0$.

Furthermore, $\mathbb{E}[\phi_{t+1}]/\phi_{t+1}^{Max} = \left(1+g^{Min}\right)\mathbb{E}\left[(1+g)^{-1}\right]$ *and therefore,*

$$
q = \max\left\{\left(1+g^{Min}\right)\mathbb{E}\left[(1+g)^{-1}\right]Y^{worst}, \chi^g \overline{Y^{worst}}\right\}.
$$

The value of money in each period, ϕ_t, changes with the realized growth rate of the supply of money. Since we study a stationary equilibrium, the value of money must adjust so the aggregate real balances, i.e., the value of the total stock of money, is constant. This means that if the money supply grows by a factor $1+g$, the value of money will fall proportionally. The expected change in that value of money is then given by $\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1}$, the expected inflation rate.

This proposition states that the real balances in the stationary monetary equilibrium are determined by comparison of the expected inflation rate to the expected marginal value of holding more money in the bilateral meetings. Moreover, the ratio of expected to maximum value of money is the ratio of the minimum money-supply growth rate and the expected inflation rate. Naturally, when the money supply grows the least, the value of money is maximized.

This characterization of the stationary monetary equilibrium consists of only three equations. The first one is the real value of money holdings for MoE transactions. The second one is the total payment (credit and money holdings) for UoA transactions. The third one is the credit payment capacity for a UoA buyer, q.[22] Notice that the only effect that the consumption-goods market can have on these variables is through their potential use as unit of account.

As anticipated earlier, inflation depends only on the monetary policy.[23] More generally, the monetary outcomes, such as the value of money and where money is used, depend only on the specifics of the UoA contracts and MoE transactions. For instance, the value of money may be affected by the probability of a MoE meeting, λ^M, or the utility of the MoE special good, w. None of these outcomes depend on the particular realizations of the consumption-goods market.

[22]To be more precise, there are two more items to be pinned down: χ^g and $\overline{Y^{worst}}$. Both of them involve the consumption-goods relative prices, which can be obtained directly from the relative aggregate endowments as explained in section 1.2.2.1.

[23]See section 1.5.1 for an extension with "taste shocks" to the MoE transactions in which inflation is affected by both supply and demand shocks.

1.4.2 The Value and Uses of Money in Equilibrium

There are two types of stationary monetary equilibria in this economy. One is a *pure-UoA* equilibrium where UoA buyers do not hold money and rely only on their promised payments, $\overline{Z^U} = 0$. Another one is a *collateral* equilibrium where UoA buyers acquire money to bring to their meetings and expand their payment capacity, $\overline{Z^U} > 0$.

In the *pure-UoA* equilibrium, money is held only for the MoE transactions. The UoA contracts involve only promises which are paid in the centralized market and, while they might consist of promises of money, need not be settled with currency. In the *collateral* equilibrium, money is held both for the MoE transactions and the UoA contracts.

The following proposition characterizes the type of equilibrium as a function of the credit payment capacity, q.

Proposition 9 (Pure-UoA vs Collateral Equilibrium) *There exists a contract size $Z^* > 0$ such that the stationary monetary equilibrium*

1. *is a pure-UoA equilibrium, $\overline{Z^U} = 0$, if $q \geq Z^*$*
2. *is a collateral equilibrium, $\overline{Z^U} = Z^* - q$, if $q < Z^*$.*

The contract size Z^* is defined as the point at which the UoA-buyer is indifferent about acquiring money to bring to the meetings. That is, Z^* is implicitly defined by

$$\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} = \beta\left(1 + \lambda^U \max\{u'\left(Z^*\right) - 1, 0\}\right).$$

Proposition 10 (The Value of Money) *In a stationary monetary equilibrium, the aggregate real balances (or total value of the money stock) $\mathbb{E}[\phi_t M_t]$ is equal to $\overline{Z^M} + \max\{Z^* - q, 0\}$.*

I elaborate on the difference and similarities between these two kinds of equilibria. In the pure-unit-of-account equilibrium, money is held only by the MoE buyers. There is a key distinction between the use of money in MoE transactions and in UoA contracts. Money is held and brought to the MoE transactions whereas money is only used to denominate payments in the UoA contracts. The output in MoE transactions is determined by the expected inflation rate, $\left(\mathbb{E}\left[(1+g)^{-1}\right]\right)^{-1}$. The output in contracts is determined by the payment capacity, q. The value of money is determined purely through its role as medium-of-exchange. In that way, the medium-of-exchange role is not affected by the UoA contracts.

On the other hand, in the collateral equilibrium, money is also held by the UoA buyers. The value of money is thus determined by its demand, as money holdings, for MoE transactions and UoA contracts. Furthermore, in the collateral equilibrium, the efficiency of the unit of account does not matter for real outcomes. Since the UoA buyers choose their money holdings until the marginal cost equals the marginal benefit, the output in contracts is pinned down by the expected inflation rate. A change in the payment capacity, via less price or income risk, brings forth a change in the money brought to UoA meetings that keeps the contract output constant. This means that the value of money increases (and the price level decreases) with a decrease in the payment capacity.[24] This irrelevance result in the collateral equilibrium is to be expected because this is a model of money and credit, even if the available credit is determined by the properties of the value of money. Models of money and credit feature this type of result, sometimes phrased as how expanding inside

[24] A curious consequence of this result is that a monetary authority can temporarily reduce inflation (generate a one-time drop in the price level) by abolishing indexation or creating uncertainty about the future value of money. These actions reduce the payment capacity and increase the demand for money which increase the value of money and reduce the price level. Of course, even within the model, it is clear that these actions would reduce welfare.

money causes outside money to lose value.[25]

The fact that the use of money as unit of account does not create demand is a consequence of the assumptions on the settlement of contracts. Notice that settlement happens in a competitive market, so there is no sense in which the buyers would want to hold money to ensure that they will be able to make their payments. If, instead, it were possible for a buyer who can afford the payment to acquire the money to settle, then we would expect an extra demand for money in anticipation. I do not explore this interesting extension in this paper. Still, my model shows that the value of a currency or token designed purely for stability cannot be analyzed without looking at the difficulties in settlement of payments denominated in its terms.

1.4.3 Comparing the Roles

In this model, the difference between the medium-of-exchange and the unit-of-account role is not just semantic. There are two forms in which the roles are different. First, the economic effects of changes to the MoE transactions will generally be different from those of changes to the UoA contracts. Second, the economic effect of changes to the environment will not generally be the same for medium of exchange and unit of account.

The analysis here relies in the simple three-equations characterization of the stationary

[25]This result is present, for instance, in Bewley models. See the relevant chapter in Ljungqvist and Sargent (2018) for a textbook treatment. See also Gu et al. (2016), for a reference on models in which both money and credit can be used for payments.

monetary equilibrium. I restate those equations here for convenience.

$$\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} = \beta\left(1+\lambda^M \max\{w'\left(\overline{Z^M}\right)-1,0\}\right),$$
$$\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} \geq \beta\left(1+\lambda^U \max\{u'\left(q+\overline{Z^U}\right)-1,0\}\right) \text{ with equality if } \overline{Z^U}>0, \text{ and}$$
$$q = \max\left\{\left(1+g^{Min}\right)\mathbb{E}\left[(1+g)^{-1}\right]Y^{worst}, \chi^g \overline{Y^{worst}}\right\}.$$

1.4.3.1 Changes to the Medium-of-Exchange Role

Let us examine the effects of an increase in the demand for MoE transactions. Specifically, consider a proportional increase in the utility of MoE buyers over their special good, $\tilde{w}(x) = kw(x)$ with $k > 1$. The real balances used in the MoE transactions, $\overline{Z^M}$, will increase. Then, the aggregate real balances, $\overline{Z^M} + \max\{Z^* - q, 0\}$, will also increase. However, this does not affect the UoA output in any way in either the *pure-UoA* or the *collateral* equilibrium.[26] In order for a change in the transactions demand to affect the UoA role, it must change q which requires affecting $\mathbb{E}[\phi]/\phi^{Max}$.

Importantly, this increase in demand has no effect on the UoA contracts because it is a deterministic change. If we introduced stochastic shocks to the demand for MoE transactions, there would be an effect on $\mathbb{E}[\phi]/\phi^{Max}$ and some real effects for the unit-of-account role. See section 1.5.1 for that analysis.

1.4.3.2 Changes to the Unit-of-Account Role

Changes in demand. An increase in the demand for UoA contracts (scaling up the utility as we did for the MoE transactions above) has different effects depending on whether we

[26]The absence of a real effect on the UoA part of the economy hides a change in the *collateral equilibrium*. With the greater demand for MoE transactions, the nominal holdings of money of UoA buyers is smaller but their real value is the same due to the greater value of each unit of money.

are in the *pure-UoA* or the *collateral* regime. In the former, the second of the three equations is inactive and $\overline{Z^U} = 0$. Therefore, the aggregate real balances are simply $\overline{Z^M}$ and are thus unaffected by this change in demand. In the collateral regime, however, the aggregate real balances are $\overline{Z^M} + Z^* - q$ and Z^* will increase with the change. Therefore, the value of money increases, and it is used more in UoA contracts.[27]

If we compare the effects of the increase in demand for MoE and UoA, we can see that they are the same in the *collateral* regime and different in the *pure-UoA* regime. This is because in the *collateral* regime there is a demand for money holdings from UoA buyers just like the demand for money holdings that MoE buyers always have. In the *pure-UoA* regime, the UoA buyers do not hold money so a change in their preferences does not bring about an increased demand for holding money and the equilibrium value of money does not change.

Changes in the payment capacity. Consider a change to the environment that affects only the payment capacity, q. For instance, a change in relative-price variability, χ^g, if consumption goods are the unit of account, or a change in Y^{worst}. Any such change will have different effects in the *pure-UoA* and *collateral* equilibrium. In the first case, the UoA output changes and the value of money remains the same. In the second case, the UoA output stays the same and the value of money changes. Here, the UoA buyers will change their money holdings to exactly compensate the change in the credit payment capacity, q. This is exactly the irrelevance result in money and credit models mentioned in the previous section.

[27] Analogously to the previous exercise, $\overline{Z^M}$ remains constant as the product of two effects. The nominal holdings of money of MoE buyers are smaller but the value of each unit of money is larger.

1.4.4 Monetary Policy

We can consider many dimensions of monetary policy changes in this environment. First, consider a change in the distribution of growth rates of money such that the expected inflation rate, $\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1}$, changes but $(1+g^{Min})\mathbb{E}[(1+g)^{-1}]$ remains the same so that the efficiency of money as unit of account is unaffected. This change affects both Z^M and Z^*. Clearly, the value of money will change. In the *pure-UoA* equilibrium, the UoA output is unchanged, unlike in the *collateral* equilibrium. A version of the Friedman rule holds in this model, if $\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} = \beta$, then all buyers will accumulate money until exhausting all gains from trade in the meetings.

Now, consider a change in the distribution of growth rates of money such that the expected inflation rate remains constant but the lowest growth rate, $1+g^{Min}$, increases. Clearly, the MoE side is not affected at all. The only effect over the UoA contracts is an increase in the payment capacity, in which case the effects are those detailed in the last part of the previous section.

Importantly, in this model, monetary policy works through two channels: the (expected) inflation rate and stability in the value of money (price-level stability). In the *pure-UoA equilibrium*, the former matters for MoE transactions and the latter for UoA contracts.[28] Although price stability might sometimes be taken to mean low inflation, this model shows that these are two distinct concepts.

We can see the channels of monetary policy in the three-equations characterization of the equilibrium. I focus on the *pure-UoA* equilibrium and I simplify the expressions further

[28] In a *collateral equilibrium*, the expected inflation rate matters for the output of both transactions and contracts. The stability in the value of money affects the level of aggregate real balances but does not affect output.

to improve readability.[29]

$$\begin{aligned}\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} =& \beta\left(1-\lambda^{M}+\lambda^{M}w'\left(\overline{Z^{M}}\right)\right),\\ \left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} \geq& \beta\left(1-\lambda^{U}+\lambda^{U}u'(q)\right),\\ q =& \max\left\{\left(1+g^{Min}\right)\mathbb{E}\left[(1+g)^{-1}\right]Y^{worst}, \chi^{g}\overline{Y^{worst}}\right\}.\end{aligned}$$

Suppose the monetary authority can choose any monetary policy subject to an expected growth rate of the money supply. This is the same as requiring an expected level of seignorage revenue in a stationary equilibrium. Clearly, the best-UoA policy would consist of a deterministic growth rate equal to the requirement, since this guarantees a stable value of money. This policy is not optimal for MoE transactions. Notice that introducing randomness in the growth rate *lowers* expected inflation, $\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1}$, as can be seen from a straightforward application of Jensen's inequality. This randomness will necessarily make money a worse unit of account. In this sense, there is a trade-off in policy between favoring the medium of exchange or the unit of account role.

I am not recommending that monetary policy should be purposefully erratic. As the model shows, this would be harmful for contracts that are denominated in money. Moreover, some of the assumptions that render the model tractable eliminate other harmful effects of such policy. For instance, if information arrives during the decentralized market or if there are wealth effects, there will be other costs induced by the changes in the value of money.[30] Still, the model shows, in a stylized way, that these roles of money might benefit

[29]To be precise, I assume that in each market the output is below the first-best level. Then, the marginal gain in transactions or contracts is greater than zero, so I can drop the maximum function.

[30]One might also argue that an erratic monetary policy can help a government engage in inflationary policies while blaming the luck of the draw. The question then is if it would be optimal to have a stochastic rule that can be automated or verified.

from different policies.[31]

1.5 Some Applications

I present three variations on the benchmark environment. Throughout, I focus on the *pure-UoA* equilibrium, where money is held only by the MoE buyers. Also, I let $\lambda^U = \lambda^M = 1$ in order to keep the notation simple (except for section 1.5.3).

First, I introduce aggregate taste shocks for the MoE good, which create a stochastic money demand. These changes in demand create unexpected deflation risk, which can be addressed with changes in money supply, a price-stabilization policy. Second, I study the connection between the acceptability of a currency as MoE and its use as a unit of account. Under some conditions, wider acceptability makes money have a more stable value and a better account unit. Third, I analyze a cashless limit by studying a sequence of economies along which the probability of a medium-of-exchange meeting goes to zero. The limit is different from the non-monetary equilibrium because money can still be the unit of account.

1.5.1 Money-demand Shocks and Price Stabilization

In each period, there is a taste shock λ with support $\{\lambda^{(1)}, \ldots, \lambda^{(N)}\}$ and a probability distribution. The taste shocks are independently and identically distributed across time. The utility of any MoE-buyer at time $t+1$ is $w_{t+1}(x) = \lambda_{t+1} w(x)$. The utility of a UoA buyer is always $u(x)$. The agents learn the realization of the $t+1$ period shock in the centralized market of period t.

[31] Of course, this consideration is not only relevant for national currencies. If we were to invent a currency to be used as a medium of exchange but not as a unit of account, should its supply be stochastic or deterministic? The latter avoids the costs of bearing risk; the former, as shown above, might increase the total value of the currency stock.

Let λ and g be the realizations of the taste shock and money growth rate. A MoE buyer's decision to hold money is characterized by the following first-order condition.

$$\phi_t(\lambda, g) = \beta\mathbb{E}[\phi_{t+1}|\lambda, g]\left(\max\{\lambda w'(\mathbb{E}[\phi_{t+1}m^M_{t+1}(\lambda, g)|\lambda, g]) - 1, 0\} + 1\right).$$

Similarly, the decision for a UoA-buyer is determined by

$$\phi_t(\lambda, g) \geq \beta\mathbb{E}[\phi_{t+1}|\lambda, g]\left(\max\{u'(q + \mathbb{E}[\phi_{t+1}m^U_{t+1}(\lambda, g)|\lambda, g]) - 1, 0\} + 1\right),$$

with equality if $m^U_{t+1}(\lambda, g) > 0$.

Since we are considering a *pure-UoA* equilibrium, $m^U_{t+1} = 0$.[32] In this case, $m^M_{t+1} = M_{t+1}$ where $M_{t+1} = M_t(1+g)$. The first of the conditions can be written as

$$\phi_t(\lambda, g)M_t(1+g) = \beta\mathbb{E}[\phi_{t+1}|\lambda, g]M_{t+1}\left(\max\{\lambda w'(\mathbb{E}[\phi_{t+1}|\lambda, g]M_{t+1}) - 1, 0\} + 1\right).$$

I guess and verify that in the stationary equilibrium $\overline{Z^M} = \mathbb{E}[\phi_{t+1}|\lambda, g]M_{t+1}$ is constant for every period and every realization of λ and g. Then,

$$\frac{\phi_t(\lambda, g)}{\mathbb{E}[\phi_t]}\overline{Z^M}(1+g) = \beta\overline{Z^M}\left(\max\left\{\lambda w'(\overline{Z^M}) - 1, 0\right\} + 1\right).$$

From these equations, I can establish the following results about the stationary monetary equilibrium.

Proposition 11 (Stationary Monetary Equilibrium with Demand Shocks) *In the station-*

[32]It can be shown that this will be the case if $\lambda^{Min}/E[\lambda]$ is high enough.

ary monetary equilibrium, the aggregate real balances $\overline{Z^M}$ *are determined implicitly by*

$$1 = \beta \mathbb{E}\left[\frac{\lambda}{1+g}\right] w'(\overline{Z^M}).$$

The value of money, ϕ*, fluctuates relative to its expected value according to*

$$\frac{\phi_t(\lambda, g)}{\mathbb{E}[\phi_t]} = \frac{\lambda/(1+g)}{\mathbb{E}[\lambda/(1+g)]}.$$

Hence, the unexpected deflation risk is given by

$$\frac{\mathbb{E}[\phi_t]}{\phi_t^{Max}} = \frac{\mathbb{E}[\lambda/(1+g)]}{(\lambda/(1+g))^{Max}},$$

where $(\lambda/(1+g))^{Max}$ *is the maximum possible realization of the money-demand and money-supply shock.*

Notice that these objects do not depend on t. Under this tractable formulation, the value of money is proportional to the demand shock and inversely proportional to the growth of money supply. Greater variability of the demand shocks, in the sense of an expansion of the support of λ, makes money a worse unit of account. If monetary policy were independent from the demand shocks, the unexpected deflation risk would be given by

$$\frac{\mathbb{E}[\lambda]}{\lambda^{Max}} \cdot \frac{\mathbb{E}[1/(1+g)]}{1/(1+g^{Min})}.$$

Monetary policy in this setting involves a new consideration, how the growth rate of money depends on the demand shock. The growth of the money supply, g, affects the real aggregate balances $\overline{Z^M}$ through $\mathbb{E}[\lambda/(1+g)]$ and the UoA output through the ratio of $\mathbb{E}[\lambda/(1+g)]$ to $(\lambda/(1+g))^{Max}$.

As before, suppose the monetary authority wants to maintain a target expected growth rate of money supply, but is otherwise free to make the growth rate stochastic. We already saw that we would like g to be random in order to lower the expected inflation rate and maximize the output in MoE transactions. If the growth rate can depend on λ, the best policy for the UoA contracts aligns the growth rate of money supply with the demand shock. The best policy for the MoE transactions aims to maximize $\mathbb{E}[\lambda/(1+g)]$ which can be achieved by making the growth rate random and *negatively* related to the demand shock.[33]

Changing the supply of currency to maintain price stability is one of the goals of the Federal Reserve System. In fact, the official title of the Federal Reserve Act reads (emphasis added) "An Act To provide for the establishment of Federal reserve banks, *to furnish an elastic currency*, to afford means of rediscounting commercial paper, to establish a more effective supervision of banking in the United States, and for other purposes." As documented by Miron (1986), after the founding of the Fed, nominal interest rates became more stable and less subject to seasonal fluctuations. The benefit of an "elastic" currency in my model is that it increases credit by making money a better unit of account.

There are other interesting questions about policy which can be addressed in extensions of this model. If the length of the contract, currently set to be settled within the same period, is increased, then the UoA buyers would benefit from price stability across periods. This could be achieved with monetary policy that tries to correct for past mistakes to maintain an average inflation target. Naturally, this would create variations in inflation in each period, which would entail changes in Z^M with appropriate costs for MoE buyers. Such a policy might be optimal, if MoE transactions are less important than UoA contracts.

[33]Notice that $\mathbb{E}[\lambda/(1+g)] = \mathbb{E}[\lambda]\mathbb{E}[1/(1+g)] + \mathrm{Cov}(\lambda, 1/(1+g))$. The first term is larger if $1+g$ is random because $1/(1+g)$ is a convex function. The second term is larger if the demand shock is more negatively correlated with $1+g$ (or, positively correlated with $1/(1+g)$).

1.5.2 Unit of Account and Acceptability as Medium of Exchange

In order to capture the idea of acceptability of a currency as medium of exchange, I extend the model to include N separate regions each with an equal measure of UoA buyers, MoE buyers, and sellers. There are two currencies which are used as the medium-of-exchange in these regions. I assume that, in each region, the sellers accept only one of the currencies. In particular, let N_1 denote the number of regions in which currency 1 is used. The other $N_2 = N - N_1$ regions use currency 2. For exposition's sake, the supply of the currencies are fixed at levels M_1 and M_2.

I assume that these regions experience idiosyncratic money-demand shocks. Specifically, in the centralized market of period t in region i, a fraction μ_i of the MoE buyers are active: they learn that they will want to trade in the decentralized market in the next period. The remaining buyers learn that they will not trade, so they do not acquire any money. I make two key assumptions on the distribution of the money-demand shocks. First, the support of μ contains $M \geq N$ elements $\{\mu^{(1)}, \mu^{(2)}, \ldots, \mu^{(M)}\}$ (in ascending order) and the marginal distributions are identical across the regions. Second, the total measure of active buyers across regions is constant at some level $\bar{\mu}$. In this way, the money-demand shocks are idiosyncratic. Finally, for this part only, I specialize the utility of the MoE buyers to be $w(x) = \log(x)$.

Consider an active MoE buyer in a region i that uses currency 1. The first-order condition for that buyer is

$$\phi_t^1 = \beta \mathbb{E}[\phi_{t+1}^1] \left(\max\{w' \left(\mathbb{E}[\phi_{t+1}^1 m_{i,t+1}^1] \right) - 1, 0\} + 1 \right).$$

Let n_1 denote the measure of active buyers in regions that use currency 1. Since the buyers are identical across regions, in equilibrium, the supply of currency 1 is split evenly across all n_1 buyers. Therefore, the previous condition can be written as

$$\phi_t^1(n_1/N_1) = \beta\mathbb{E}[\phi_{t+1}^1]\left(\max\left\{w'\left(\mathbb{E}\left[\phi_{t+1}^1\frac{M_1}{n_1}\right]\right) - 1, 0\right\} + 1\right).$$

In a stationary monetary equilibrium, $\mathbb{E}[\phi_{t+1}^1]M_1 = \bar{z}_1 N_1$ is constant. Here, $\bar{z}_1$ is the aggregate real balances of currency 1 per region in which it is used. With this definition, we can write

$$\frac{\phi(n_1/N_1)}{\mathbb{E}[\phi]} = \beta\max\left\{w'\left(\frac{\bar{z}_1 N_1}{n_1}\right), 1\right\} = \beta\max\left\{\frac{n_1}{N_1}\frac{1}{\bar{z}_1}, 1\right\}.$$

Taking expectations, we obtain the following implicit expression for $\bar{z}_1$,

$$\bar{z}_1 = \beta\mathbb{E}\left[\max\left\{\frac{n_1}{N_1}, \bar{z}_1\right\}\right].$$

The unexpected deflation risk is then given by

$$\frac{\mathbb{E}[\phi]}{\phi(n_1^{Max}/N_1)} = \frac{\mathbb{E}\left[\max\left\{\frac{n_1}{N_1}, \bar{z}_1\right\}\right]}{\max\left\{\frac{n_1^{Max}}{N_1}, \bar{z}_1\right\}}.$$

I assume here that $n_1/N_1 > \bar{z}_1$ for all n_1.[34] In general, a mean preserving spread of n_1/N_1 creates an increase in the numerator, due to convexity of the maximum, and this implies that $\bar{z}_1$ must increase. Under this assumption, the numerator is linear in n_1/N_1 and a mean-preserving spread of this object has no effect on $\bar{z}_1$.

The last two equations, for $\bar{z}_1$ and the unexpected deflation risk, allow us to connect the acceptability of the currency, N_1, to its usefulness as unit of account. A currency that is

[34] A sufficient condition for this assumption is that $\mu^{(1)}/\bar{\mu}$ is high enough. In words, that the lowest demand shock are not too far from the mean.

used more widely, i.e., that has greater N_1, has less unexpected deflation risk and is a better unit of account.

First, notice that $n_1 = \sum_{i\in N_1} \mu_i$ and $\mathbb{E}[n_1] = N_1\mathbb{E}[\mu]$, by linearity of expectations. Second, $n_1^{Max} = \sum_{j=1}^{N_1} \mu^{(M-j+1)}$ and therefore n_1^{Max}/N_1 is decreasing in N_1. More generally, decreasing N_1 corresponds to a mean-preserving spread of n_1/N_1: let $\tilde{\mu}$ be this fraction for N_1 regions, then if we remove one of the regions with fraction μ, the new fraction will be $\tilde{\mu} + (\mu - \tilde{\mu})/N_1$. Since μ is chosen from the N_1 regions, the conditional expectation of the second term is zero.

Given the latest assumption, the unexpected deflation risk can be now written as

$$\frac{\mathbb{E}[\phi]}{\phi(n_1^{Max}/N_1)} = \frac{\mathbb{E}\left[\frac{n_1}{N_1}\right]}{\frac{n_1^{Max}}{N_1}} = \bar{\mu}\frac{N_1}{n_1^{Max}}.$$

The last term is increasing in N_1 as established above. Thus, a more widely accepted medium-of-exchange is a better unit of account.

This application illustrates how the use of a currency as a medium of exchange can affect its value as a unit of account. The assumptions that I imposed to get this result reveal the limits to the generality of the result. In particular, wider use will not necessarily lead to greater price stability. Imagine a currency that goes from being used in a region with fully predictable demand to being used in another region whose demand is completely unpredictable. Intuitively, the value of that currency will be less stable, and it will be a worse unit of account. However, wider use or acceptability can make a currency more stable if fluctuations in demand are largely idiosyncratic, as in this section.

1.5.3 A Cashless Limit

Since money can have more than one role in this economy, we can examine the behavior of the economy when MoE transactions are scarce. To do that, let us return to the basic environment, where the probabilities of a UoA and MoE meetings are λ^U and λ^M.

Consider a sequence of equilibria indexed by λ^M as this probability goes to zero. I focus on *pure-UoA* equilibria because that is the case where the unit-of-account role is directly relevant for output. For technical reasons, I assume that $\lim_{x\to 0} w'(x) = \infty$.[35]

The value of real balances used by the buyers is given by the following equations.

$$\begin{aligned}\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} =& \beta\left(1+\lambda^M \max\{w'\left(\overline{Z^M}\right)-1,0\}\right),\\ \left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} \geq& \beta\left(1+\lambda^U \max\{u'\left(q\right)-1,0\}\right).\end{aligned}$$

Notice that we can write the first condition as

$$\left(\mathbb{E}[(1+g)^{-1}]\right)^{-1} = \beta\left(1+\lambda^M\left(w'\left(\overline{Z^M}\right)-1\right)\right).$$

Clearly, there is a positive relation between λ^M and Z^M. Moreover, $\overline{Z^M} \to 0$ as $\lambda^M \to 0$. Still, by the same arguments of the main analysis, the value of money satisfies

$$\frac{\mathbb{E}[\phi_{t+1}]}{\phi_{t+1}^{Max}} = (1+g^{Min})\mathbb{E}[(1+g)^{-1}].$$

[35]This assumption guarantees that there will be a monetary equilibrium for all values of $\lambda^M > 0$. Without this assumption, there will exist a minimum λ^M at which the MoE buyers will choose to hold no money. This is not a problem if UoA buyers choose to hold money. However, if UoA buyers do not choose to hold money, then there will not exist a monetary equilibrium.

Therefore, for each $\lambda^M > 0$, we have that

$$q = \max\left\{(1+g^{Min})\mathbb{E}[(1+g)^{-1}]Y^{worst}, \chi^g \overline{Y^{worst}}\right\}$$

In this sense, even though the total value of MoE transactions is negligible, money is used as the unit of account, and monetary policy affects the UoA contracts. Notice that this is different from a non-monetary equilibrium. In a non-monetary equilibrium, money has no value and cannot be used to denominate contracts.

Monetary policy in these economies would be mainly concerned with the unit of account role. As such, the monetary authority would favor price stability over low inflation, as shown in the discussion of those trade-offs in section 1.4.4.

In one of these economies with few MoE transactions $\left(\text{low } \lambda^M\right)$, one might wonder why, if at all, we should care about monetary transactions since they represent a small fraction of economic output. This model answers that we should care, not because money is the medium of exchange, but because it is the unit of account.

1.6 Conclusion

I have presented a dynamic, general-equilibrium, monetary model in which money can be the medium of exchange and the unit of account. The functions of money are determined endogenously based on features of the economy. Importantly, this model combines the two roles while remaining tractable. The equilibrium of this model can be characterized in three equations. The model provides new insights about monetary policy relative to conventional models that focus on only one role. Price stability, an oft-stated goal of central banks, emerges as a key concern due to the unit-of-account role.

Besides explaining the basic economic logic in the model, I have explored questions that can benefit from this joint treatment of the roles of money. For instance, a model that explains the connection between acceptability as medium of exchange and unit of account requires both roles. An analysis of monetary policy also changes with the joint consideration of these roles. A monetary authority that is more concerned about the medium-of-exchange role (relative to the unit-of-account role) would prefer low inflation over price stability.

I believe that much more can be done in this framework. I point to a couple of directions: theoretical and quantitative. On the theoretical side, allowing for non-degenerate distributions of money holdings (as in Rocheteau, Weill and Wong (2018)) can generate interesting effects; unexpected deflation can constrain agents and lead to a credit crunch through debt deflation. Similarly, under the assumption of frictionless settlement in this paper, money being the unit of account does not create a demand for money holdings. In a model with settlement frictions, the agents might hold money in order to avoid "scrambling" for money as their payment approaches. On the quantitative side, this model has implications for the demand for money and the effects of monetary shocks in the economy. These insights can guide a quantitative exercise as in Cole and Ohanian (2002).

1.A The Contracting Problem

Proposition 2 (Optimal Promises with Goods - No Worst-Income Risk) *Suppose that the worst-case income realization is the same for all prices. The expected value of the optimal payment using goods is*

$$\chi^g \, Y^{worst},$$

where $\chi^g \leq 1$ *and it is only equal to 1 if* $\Pr(\boldsymbol{p} \text{ is an extreme price}) = 1$.

Let us denote $\overline{\boldsymbol{p}}$ and $\underline{\boldsymbol{p}}$ as the price vectors when good A is most and least expensive.

The boundaries of the set of feasible payments are given by the extreme price realizations. To see this, fix a possible endowment realization $\boldsymbol{y}$ and some intermediate price realization $\boldsymbol{p}$. There exists a convex combination of extreme prices $(1-\lambda)\overline{\boldsymbol{p}} + \lambda\underline{\boldsymbol{p}}$ that is equivalent, up to a multiplicative constant, to the intermediate price. For any given payment that is feasible for the extreme prices, we have that $\overline{\boldsymbol{p}}\boldsymbol{\pi} \leq \overline{\boldsymbol{p}}\boldsymbol{y}$, $\underline{\boldsymbol{p}}\boldsymbol{\pi} \leq \underline{\boldsymbol{p}}\boldsymbol{y}$ and $\left[(1-\lambda)\overline{\boldsymbol{p}} + \lambda\underline{\boldsymbol{p}}\right]\boldsymbol{\pi} \leq \left[(1-\lambda)\overline{\boldsymbol{p}} + \lambda\underline{\boldsymbol{p}}\right]\boldsymbol{y}$. Therefore, any payment that is feasible at the extreme prices is feasible at all prices.

With the previous result, we can write a condition for the optimal goods payment as

$$\overline{\boldsymbol{p}}\boldsymbol{\pi} = Y^{worst},$$
$$\underline{\boldsymbol{p}}\boldsymbol{\pi} = Y^{worst}.$$

It is easy to see that this problem will have a unique solution if $\overline{\boldsymbol{p}} \neq \underline{\boldsymbol{p}}$. After some algebraic manipulations, we can write the optimal payment bundle as

$$\pi_A = Y^{worst} \frac{\overline{p_B} - \underline{p_B}}{\underline{p_A}\overline{p_B} - \underline{p_A p_B}}, \; \pi_B = Y^{worst} \frac{\overline{p_A} - \underline{p_A}}{\overline{p_A p_B} - \underline{p_A p_B}},$$

where, for readability, $\overline{p_i} = p_i^{Max}$ and $\underline{p_i} = p_i^{Min}$ for $i \in \{A, B\}$. The expected value of this payment bundle is then

$$\mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi} = \mathbb{E}[p_A]\pi_A + \mathbb{E}[p_B]\pi_B = \underbrace{\frac{\left(\overline{p_B} - \underline{p_B}\right)\mathbb{E}[p_A] + \left(\overline{p_A} - \underline{p_A}\right)\mathbb{E}[p_B]}{\overline{p_A p_B} - \underline{p_A p_B}}}_{\chi^g} Y^{worst}.$$

All that remains is to show that the coefficient $\chi^g \leq 1$ and that this holds with equality if $\Pr(\boldsymbol{p}\text{ is an extreme price}) = 1$.

Suppose that $\Pr(\boldsymbol{p}\text{ is an extreme price}) = 1$. Then, consider the equations that pin down the optimal payment bundle

$$\overline{\boldsymbol{p}}\boldsymbol{\pi} = Y^{worst},$$

$$\underline{\boldsymbol{p}}\boldsymbol{\pi} = Y^{worst}.$$

The expectation of the value of the payment bundle over prices involves only these conditions. Specifically, if we multiply each condition by the probability of that price realization and add them up, we obtain $\Pr(\overline{\boldsymbol{p}})\overline{\boldsymbol{p}} + \Pr(\underline{\boldsymbol{p}})\underline{\boldsymbol{p}} = \mathbb{E}[\boldsymbol{p}]$. Therefore, $\mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi} = Y^{worst}$ so $\chi^g = 1$.

Now, suppose that $\Pr(\boldsymbol{p}\text{ is an extreme price}) < 1$. Notice that the prices of goods A and B are related so that $p_B = h(p_A)$ for a strictly decreasing, strictly convex function. Then, the value of the optimal payment bundle $\boldsymbol{\pi}$ is $\mathbb{E}[p_A\pi_A + h(p_A)\pi_B]$ where the expectation is taken over p_A. Since h is a strictly convex function and we know that the expression inside the expectation is equal to Y^{worst} at the lowest and highest price for good A, then that expression must be strictly less than Y^{worst} for any intermediate price. Thus, if $\Pr(\boldsymbol{p}\text{ is an extreme price}) < 1$, the value of the bundle is strictly less than Y^{worst} and therefore $\chi^g < 1$.

Proposition 4 (Optimal Promises with Goods) *The expected value of the optimal payment using goods is*

$$\chi^g \, \overline{Y^{worst}},$$

where $\overline{Y^{worst}}$ is a weighted average of the worst-case income at the extreme price realizations and χ^g is defined as in Proposition 2.

As before, the optimal payment bundle is given by the binding constraints at the extreme-price realizations. The solution, following the same steps as before, is

$$\pi_A = \frac{\overline{p_B} Y_A^{worst} - \underline{p_B} Y_B^{worst}}{\overline{p_A p_B} - \underline{p_A p_B}}, \pi_B = \frac{\overline{p_A} Y_B^{worst} - \underline{p_A} Y_A^{worst}}{\overline{p_A p_B} - \underline{p_A p_B}},$$

where Y_i^{worst} is the worst-case income realization when the price of good i is the largest. The calculation of the expected value of this payment bundle is straightforward. The expected value $\mathbb{E}[p_A]\pi_A + \mathbb{E}[p_B]\pi_B$ equals

$$\chi^g \underbrace{\left(\frac{\overline{p_A}\mathbb{E}[p_B] - \underline{p_B}\mathbb{E}[p_A]}{\left(\overline{p_B} - \underline{p_B}\right)\mathbb{E}[p_A] + \left(\overline{p_A} - \underline{p_A}\right)\mathbb{E}[p_B]} Y_B^{worst} + \frac{\overline{p_B}\mathbb{E}[p_A] - \underline{p_A}\mathbb{E}[p_B]}{\left(\overline{p_B} - \underline{p_B}\right)\mathbb{E}[p_A] + \left(\overline{p_A} - \underline{p_A}\right)\mathbb{E}[p_B]} Y_A^{worst} \right)}_{\overline{Y^{worst}}}.$$

1.B Seller's Value Function in the Decentralized Market

Consider a seller who meets a UoA buyer and is offered a contract $(x, \boldsymbol{\pi})$. This seller must decide between accepting and rejecting. The lifetime utility of accepting and rejecting are

$$V_t^{S,\text{accept}}(m, (x, \boldsymbol{\pi})) = -x + \mathbb{E}[W_t^S(\boldsymbol{py} + \boldsymbol{p\pi})],$$

$$V_t^{S,\text{reject}}(m) = \mathbb{E}[W_t^S(\boldsymbol{py})].$$

Then, the value function for the seller is

$$V_t^S(m,(x,\boldsymbol{\pi})) = \max\{V_t^{S,\text{accept}}(m,(x,\boldsymbol{\pi})), V_t^{S,\text{reject}}(m)\}.$$

The seller's optimal decision is to accept if $x \leq \mathbb{E}[\boldsymbol{p}]\boldsymbol{\pi}$.

Now, consider a seller who meets a MoE buyer and is offered a transaction $(x,\hat{m})$. As before, the value of accepting and rejecting are, respectively,

$$V_t^{S,\text{accept}}(m,(x,\hat{m})) = -x + \mathbb{E}[W_t^S(\boldsymbol{p}\boldsymbol{y} + \phi m + \phi\hat{m})],$$

$$V_t^{S,\text{reject}}(m) = \mathbb{E}[W_t^S(\boldsymbol{p}\boldsymbol{y} + \phi m)].$$

Then, the value function for the seller is

$$V_t^S(m,(x,\boldsymbol{\pi})) = \max\{V_t^{S,\text{accept}}(m,(x,\hat{m})), V_t^{S,\text{reject}}(m)\}.$$

The seller's optimal decision, again, is to accept if $x \leq \mathbb{E}[\phi]\hat{m}$.

In equilibrium, since buyers will make the seller indifferent between accepting and rejecting, the value function for the seller will satisfy

$$V_t^S(m) = \mathbb{E}[W_t^S(\boldsymbol{p}\boldsymbol{y} + \phi m)].$$

CHAPTER 2

Monetary Policy: Inflation and Price-Level Targeting

2.1 Introduction

I compare two approaches to monetary policy that are based on monetary stability. The key distinction between these approaches is due to the dynamic nature of stability. The first approach, inflation targeting, keeps expected inflation stable in each period. The second approach, price-level targeting, attempts to keep the level of prices close to a predetermined path. I show that inflation targeting favors the medium-of-exchange role, while price-level targeting favors the unit-of-account role in long-term contracts.

The key difference between these approaches is in their response to past deviations of inflation from the target. For example, suppose that the inflation target is 2% a year and the last year inflation rate was 7%. Under inflation targeting, the monetary authority still targets an inflation of 2% for the current year. Under price-level targeting, the monetary authority aims for deflation so that the price-level is up 4% by the end of the year. In this way, price-level targeting corrects past deviations, while inflation targeting only takes them into account to the extent that they might affect future inflation.

These policies can have different effects on the unit-of-account and medium-of-exchange roles. The stability benefits of price-level targeting are not apparent with the short-term contracts of the benchmark model. Therefore, I introduce a simple variation with long-term contracts. When some unit-of-account contracts last for more than one period, there is a benefit to target the price level. This benefit is due to the reduced long-term risk in the value of money.

Importantly, this does not mean that price-level targeting is optimal. One downside of this policy is that it must allow for variations in the inflation level across periods. This is inefficient for the medium-of-exchange transactions, relative to a constant inflation rate. I conjecture that an optimal monetary policy in this environment would be a compromise between these approaches. Such a compromise would take the form of "flexible average inflation targeting" (as the Federal Reserve announced in August 2020).

The notion that there are many ways in which monetary stability can be pursued has long been recognized. For instance, Woodford (2003, p.382) wrote

> Should greater priority perhaps be given to reducing the variability of unforecastable inflation, on the grounds that this is what causes unexpected modifications of the real consequences of preexisting nominal contracts, whereas forecastable variations in inflation can simply be incorporated into contracts? Or should greater priority be given to stabilization of forecastable inflation, on the grounds that expected inflation distorts incentives (like an anticipated tax), whereas unforecastable inflation has no incentive effects (like an unanticipated wealth levy)?

In this model, the key trade-off is between unexpected and expected inflation. Changes in expected inflation distort incentives for medium of exchange. While changes in unexpected inflation reduce the trade in *new* nominal contracts. In this models, agents foresee the variations in inflation and anticipate their possibility of default when entering a new contract. Unless the agents were able to sign contingent contracts (and in this framework they cannot), only the expected inflation can be incorporated into the contract. Thus only the variability of inflation relative to its expected value matters for the unit of account role.

2.1.1 Related Literature

There is a large literature on the analysis of monetary policy rules in the context of New Keynesian models. In those models, the main mechanism for the effects of monetary policy are through the distribution of prices which are assumed to be sticky. In contrast, in my model, monetary policy matters due to its effects on the credit capacity in money-denominated contracts.

The costs of instability of the price level are therefore different in these models. A commitment to price stability allows firms to anticipate a return to the established price path and, therefore, reduces the incentive to adjust prices. Hence, price dispersion and misallocation is lower under price-level targeting. The model presented here does not rely on sticky prices; instead, instability of the price level reduces the credit capacity of buyers in contracts denominated in money.

Regarding the benefits of price-level targeting, Coibion, Gorodnichenko and Wieland (2012) find that this type of policy can greatly enhance welfare because it stabilizes expectations and dampens the volatility of inflation. Consequently, the zero lower bound is hit less frequently. The results in this chapter are not related to zero lower bound episodes.

The standard approach to evaluate targeting rules in the New Keynesian literature consists of solving for optimal linear policies that minimize a quadratic approximation of a loss function which includes a penalty for deviations of output and another one for the desired target.[1] In this chapter, I take each monetary policy rule as a commitment to a set of decisions, not as a preference over a target. In other words, the monetary authority in my model follows inflation or price-level targeting without consideration of output gaps. The analysis here does not solve for the optimal monetary policy. Instead, I first establish

[1] Woodford (2003), Galí (2015)

results for inflation and price-level targeting separately and then I show that those results imply relative benefits of one policy over the other.

In the New Monetarist literature, Berentsen and Waller (2011) study price-level targeting and the stabilization of demand shocks. They establish results on the effects of monetary expansions under price-level targeting which are also present in the analysis of medium-of-exchange buyers in this chapter. The main difference between their paper and this chapter is that I focus on the comparison of inflation and price-level targeting, as opposed to deriving optimal stabilization policy. Since the main difference between inflation and price-level targeting is in their response to the history of inflation and policy, I analyze these policies in an environment with policy errors and without demand shocks.

2.2 A Model with Long-term Contracts

2.2.1 The Environment

Consider a variation of the benchmark model with long-term contracts. The main difference is that the unit-of-account buyers are divided into two types: short-term and long-term buyers. Less importantly, in order to simplify notation, I take the probability of a meeting for a medium-of-exchange buyer to be $\lambda^M = 1$.

Long-term Buyers. The short-term buyers are identical to the UoA buyers in the benchmark model. The long-term buyers are active every two periods, in the following way. A long-term buyer participates in the centralized market at time $t-1$, the decentralized market at time t, and the centralized market at time $t+1$. In this way, the long-term buyer skips the centralized market at time t and the decentralized market at time $t+1$.[2] Half of the

[2] The purpose of this timing assumption is to keep the budget constraint of the long-term buyers relevant.

long-term buyers participate in the decentralized market in the even periods, and the other half in the odd periods.

Timing and Information. Since the contracting problem involves specifying a payment further into the future, it is helpful to clarify the information of the agents at each point. In particular, all (monetary policy) shocks are realized during the centralized market of each period. No information is revealed during the decentralized market. For this reason, the information of an agent at the centralized market at time t and at the decentralized market at time $t+1$ is the same. I therefore write the expectations of future prices or value of money as $\mathbb{E}_t[\boldsymbol{p}_{t+1}]$ or $\mathbb{E}_t[\phi_{t+1}]$.

Equilibrium. As before, I focus on stationary monetary equilibria. The only difference in the equilibrium definition of this variation of the model is that it includes contract terms for long-term buyers, which are determined optimally given prices, as shown next.

Furthermore, in order to highlight the unit-of-account role, I focus on the "pure unit-of-account" equilibria. As mentioned before, in this type of equilibrium, the stock of money is held only by the medium-of-exchange buyers.

2.2.2 Long-Term Contracts

A time-t UoA meeting between a long-term buyer and a seller works in the same way as that of a short-term buyer, except that the contract is planned for the buyer's next visit to the centralized market. That is, the buyer and seller agree to a contract $(x, \boldsymbol{\pi})$ to be fulfilled at time $t+1$. Specifically, the contracting problem of a long-term buyer is

$$\max_{x,\boldsymbol{\pi}} \quad u(x) - \mathbb{E}_{t-1}[\boldsymbol{p}_{t+1}]\boldsymbol{\pi}$$

$$\text{s.t. } \; x \leq \mathbb{E}_{t-1}[\boldsymbol{p}_{t+1}]\boldsymbol{\pi},$$

$$\boldsymbol{p}_{t+1}\boldsymbol{\pi} \leq \theta \boldsymbol{p}_{t+1}\boldsymbol{y} + \phi m, \text{ for } \underline{\text{all}}\ \boldsymbol{p}_{t+1}, \boldsymbol{y}.$$

Notice that the relevant prices are those at time $t+1$, since that is the date when the contract is fulfilled. The expectation is taken with respect to the information at the decentralized market at time t, which is the same as that at the centralized market at $t-1$.

Since this problem is identical to the original contracting problem, I invoke those results here without proof. The payment capacity of a long-term buyer is given by the now familiar expression

$$q_t^{LT} = \max\left\{ \frac{\mathbb{E}_{t-1}[\phi_{t+1}]}{\phi_{t+1}^{Max,t-1}} Y^{worst}, \chi^g \overline{Y^{worst}} \right\}.$$

Here, the value of money $\phi_{t+1}^{Max,t-1}$ is the maximum value of money that could be realized at time $t+1$, given the information at $t-1$. It is important to see that this value is potentially different from $\phi_{t+1}^{Max,t}$ because the information at t could impose further limits on this maximum value.

The time-t long-term buyer's payment capacity using money takes into account the *unexpected deflation risk* based on the information available at the time-t decentralized market. In contrast, a short-term buyer who writes a contract for goods also delivered at time $t+1$ has the following payment capacity.

$$q_{t+1}^{ST} = \max\left\{ \frac{\mathbb{E}_t[\phi_{t+1}]}{\phi_{t+1}^{Max,t}} Y^{worst}, \chi^g \overline{Y^{worst}} \right\}.$$

Intuitively, money will be less useful as a unit of account for long-term contracts, relative to short-term ones, because there is more uncertainty over the value of money at a longer horizon. However, the uncertainty over the value of money depends on the choice of monetary policy, as I explain in the next section.

2.3 Monetary Policy

The goal of this section is to showcase the trade-offs involved in different approaches to monetary policy: inflation targeting and price-level targeting. In order to do that, I abstract away from demand shocks and consider monetary policy as a possibly dynamic choice of the growth rate of the supply of money, subject to to stochastic errors.

The growth rate of money at time t is given by $\epsilon_t(1+g_t)$, where g_t is the choice of the monetary authority and ϵ_t is an error term. The error terms are independent and identically distributed over time with $\mathbb{E}\left[\epsilon_t^{-1}\right]=1$ and with a support bounded from below at $\epsilon_{Min}>0$. The monetary authority chooses g_t in between the decentralized and centralized markets of time t, before ϵ_t is realized. In this sense, the choice of the monetary authority cannot prevent the errors, but, importantly, can correct them in future periods. Without loss of generality, I normalize the initial stock of money M_0 to be one.

This timing assumption allows for persistent errors in the path of money supply, which is a key component of the comparison between the policies under study. The trade-off between providing a stable price-level path and stable inflation rates will depend on the response of monetary policy to these past errors. The former will require correcting them, while the latter will require ignoring them.

The distributional assumptions on the error terms simplify the analysis without obscuring the main point. Adding serial correlation on the error terms would change the implementation of the policies, but it would not affect the properties of the stationary monetary equilibria.

In the following two sections, I provide explicit definitions of the policies and establish the properties of the equilibrium value of money. After that, I compare the policies with an emphasis on the welfare trade-offs between medium-of-exchange and unit-of-account buyers.

2.3.1 Inflation Targeting

I first consider a policy with the goal of maintaining a stable level of (expected) inflation. At each time t, the growth rate of money is chosen so that the expected one-period-ahead inflation is equal to a target. This is the goal declared by many central banks, with the inflation target commonly set to be 2%.

Definition 4 (Inflation targeting) *A monetary policy is called inflation targeting if*

$$\frac{\phi_t}{\mathbb{E}_t[\phi_{t+1}]} = \Pi$$

across all periods and histories, for some constant Π.

Given the set-up of the monetary policy shocks, the monetary authority can pursue inflation targeting with a simple rule: choosing a constant growth rate of money at all times and across all histories.

Proposition 5 (Constant money growth achieves inflation targeting) *Under a constant money growth choice, $g_t = g$, the expected inflation rate is constant and equal to*

$$\frac{\phi_t}{\mathbb{E}_t[\phi_{t+1}]} = \mathbb{E}\left[\epsilon_t^{-1}(1+g)^{-1}\right]^{-1} = (1+g)\mathbb{E}\left[\epsilon_t^{-1}\right]^{-1}.$$

The result comes from the first-order condition of the medium-of-exchange buyers and the assumption of being in the *pure-UoA* equilibrium. Since a similar version of this condition was derived earlier, I write the expression directly. At any time t, the value of money can be written as

$$\phi_t = \beta \mathbb{E}_t[\phi_{t+1}] w' \left(\mathbb{E}_t[\phi_{t+1}] M_t \right).$$

Multiplying by the money supply at the end of period t, M_t, we get

$$\phi_t M_{t-1} \epsilon_t (1+g) = \beta \mathbb{E}_t[\phi_{t+1}] M_t w' \left(\mathbb{E}_t[\phi_{t+1}] M_t \right),$$

since $M_t = \epsilon_t (1+g) M_{t-1}$. Consider a stationary monetary equilibrium with constant aggregate real balances, $\mathbb{E}_t[\phi_{t+1}] M_t = Z^M$ for all t and across all histories. Then,

$$\frac{\phi_t}{\mathbb{E}_{t-1}[\phi_t]} Z^M \epsilon_t (1+g) = \beta Z^M w'(Z^M),$$

Thus, rearranging ϵ_t and taking expectations, the aggregate real balances Z^M are implicitly defined by

$$1 = \beta \mathbb{E}\left[\frac{1}{\epsilon_t (1+g)} \right] w'(Z^M) = \frac{\beta}{1+g} \mathbb{E}\left[\epsilon_t^{-1} \right] w'(Z^M).$$

Therefore, the expected inflation rate is

$$\frac{\phi_t}{\mathbb{E}_t[\phi_{t+1}]} = \beta w'(Z^M) = (1+g) \mathbb{E}\left[\epsilon_t^{-1} \right]^{-1}.$$

The fact that inflation targeting can be achieved with such a simple rule is due to the assumption of i.i.d. errors. In a more general case, the growth rate of money would have to be adjusted over time to compensate for the expected future error terms. In either case, however, inflation targeting does not respond to previous error terms. Even if inflation turns out to be high in one period, the monetary policy does not attempt to compensate with lower inflation in the future. Doing so would imply falling short of the target.

This type of policy is particularly suited for the medium-of-exchange role. An agent that considers using money as the medium of exchange takes into account the expected inflation rate as an extra marginal cost for spot transactions. Under inflation targeting, the aggregate real balances are constant across all histories. On the other hand, if the monetary authority followed a different rule, then the aggregate real balances would be changing over time. I will show later that these fluctuations generate a welfare loss for those buyers.

Unfortunately, inflation targeting is not as well suited for the unit of account role. In particular, inflation targeting will result in a less stable value of money and worse prospects for the use of money as unit of account in long-term contracts. The problem is that, under inflation targeting, errors accumulate without correction, so the price level can wildly deviate from its expected path.

Proposition 6 (Unexpected Deflation Risk with Inflation Targeting) *The unexpected deflation risk for long-term contracts is given by*

$$\frac{\mathbb{E}_{t-1}[\phi_{t+1}]}{\phi_{t+1}^{Max,\,t-1}} = \epsilon_{Min}^2,$$

where ϵ_{Min} is the lowest possible value of ϵ.

Since aggregate real balances are constant, we have that

$$Z^M = \mathbb{E}_t[\phi_{t+1}]M_t = \mathbb{E}_t[\phi_{t+1}]M_{t-1}\epsilon_t(1+g),$$

where the last equality follows from the growth policy. In the previous proposition, I established that

$$\phi_{t+1} = \mathbb{E}_t[\phi_{t+1}]\mathbb{E}_t[\epsilon_{t+1}^{-1}]^{-1}\frac{1}{\epsilon_{t+1}}.$$

Thus, we can write

$$\phi_{t+1} = \frac{Z^M}{M_{t-1}(1+g)} \frac{1}{\epsilon_t} \cdot \mathbb{E}_t[\epsilon_{t+1}^{-1}]^{-1} \cdot \frac{1}{\epsilon_{t+1}}.$$

Taking expectations at the centralized market of period $t-1$,

$$\mathbb{E}_{t-1}[\phi_{t+1}] = \frac{Z^M}{M_{t-1}(1+g)} \mathbb{E}_{t-1}\left[\frac{1}{\epsilon_t} \cdot \frac{1}{\epsilon_{t+1}}\right] \mathbb{E}_{t-1}[\epsilon_{t+1}^{-1}]^{-1} = \frac{Z^M}{M_{t-1}(1+g)} \mathbb{E}_{t-1}[\epsilon_t^{-1}].$$

Therefore, the realized value of money at $t+1$, relative to its $t-1$ expectation is

$$\frac{\mathbb{E}_{t-1}[\phi_{t+1}]}{\phi_{t+1}} = \mathbb{E}[\epsilon^{-1}]^2 \cdot \epsilon_t \cdot \epsilon_{t+1}.$$

This result highlights the weakness of inflation targeting in providing a stable unit of account. At time t, a long-term buyer commits to a contract, promising money at time $t+1$ without knowing the actions of the monetary authority at times t and $t+1$. If at time t, the economy goes through relative deflation due to a low ϵ, the monetary authority will not compensate with higher inflation at time $t+1$. Therefore, it is possible that the economy goes through further deflation. Anticipating this, the long-term buyer will restrict the size of his contracts.

2.3.2 Price-Level Targeting

Having established the properties of money under inflation targeting, I turn to the analysis of price-level targeting. This is a policy that aims to keep the value of money stable over time, not only in the short-run.

In spite of inflation targeting being the most common policy objective used by central banks, many of them were created with the goal of achieving price stability. Whereas inflation targeting leads to a stable rate of inflation, price-level targeting focuses on stability of the level of prices, i.e., the value of money.

Definition 5 (Price-Level Targeting) *A monetary policy follows price level targeting if*

$$\mathbb{E}_t[\phi_{t+1}] = \frac{P_1}{\Pi^t}$$

across all histories, for some constants P_1 and Π.

I define price-level targeting here as a policy that makes the price-level follow a given expected path, determined by some growth rate Π. This definition makes the expected future value of money independent of both the current shock and the history of past shocks. Therefore, price-level targeting requires that past errors in monetary policy be corrected once they occur.

Price-level targeting, as defined here, can be achieved by setting the growth rate of money to adjust for the previous error. In particular, for a given target growth rate of the money supply, the policy should be $1 + g_t = (1 + g)/\epsilon_{t-1}$. With this policy, the money supply at time t will be $M_t = (1 + g)^t \epsilon_t$. The following proposition proves this claim.

Proposition 7 (Price-level targeting requires changes in money supply growth) *Suppose the monetary authority chooses the growth rate of money as $1 + g_t = (1 + g)/\epsilon_{t-1}$. Then, the expected value of money is*

$$\mathbb{E}_t[\phi_{t+1}] = P_1(1 + g)^{-t},$$

where P_1 is determined implicitly in

$$1 = \frac{\beta}{1 + g}\mathbb{E}\left[w'\left(P_1\epsilon\right)\right].$$

The unexpected deflation risk is given by

$$\frac{\mathbb{E}_{t-1}[\phi_{t+1}]}{\phi_{t+1}^{Max,\,t-1}} = \frac{\mathbb{E}[w'\left(P_1\epsilon\right)]}{w'\left(P_1\epsilon_{Min}\right)}.$$

In particular, if $w(q) = \log(q)$*, the unexpected deflation risk is simply* $\mathbb{E}\left[\epsilon^{-1}\right]\epsilon_{Min}$.

Given the policy rule, I conjecture that the equilibrium aggregate real balances will be a function of only the current error ϵ_t. This means that $\mathbb{E}_t[\phi_{t+1}]M_t = Z(\epsilon_t)$. Since we know the path for the money supply, we can see that $\mathbb{E}_t[\phi_{t+1}] = P_1(1+g)^{-t}$. Moreover, the value of money at time t must be

$$\phi_t = \beta P_1(1+g)^{-t}w'\left(P_1(1+g)^{-t}M_t\right) = \beta P_1(1+g)^{-t}w'\left(P_1\epsilon_t\right).$$

Multiplying each side by M_t, we can obtain the following expression

$$\frac{\phi_t}{\mathbb{E}_{t-1}[\phi_t]} = \frac{\beta}{1+g}w'\left(P_1\epsilon_t\right).$$

Taking expectations at time t, we obtain an implicit equation for P_1. The term P_1 in the value of money is determined via

$$1 = \frac{\beta}{1+g}\mathbb{E}\left[w'\left(P_1\epsilon\right)\right].$$

Combining the last two equations, we get that the short-term unexpected deflation risk under price-level targeting is

$$\frac{\mathbb{E}_{t-1}[\phi_t]}{\phi_t^{Max,t-1}} = \frac{w'\left(P_1\epsilon_{Min}\right)}{\mathbb{E}\left[w'\left(P_1\epsilon\right)\right]}.$$

Crucially, this expression only depends on the last error term. This implies that the long-term unexpected deflation risk is the same as the short-term one.

Another way to see this result is to note that $\mathbb{E}_{t-1}[\phi_{t+1}] = \mathbb{E}_{t-1}[\mathbb{E}_t[\phi_{t+1}]] = \mathbb{E}_{t-1}[P_1(1+g)^{-t}] = P_1(1+g)^{-t}$. Since $\phi_{t+1} = \mathbb{E}_t[\phi_{t+1}]]\frac{\beta}{1+g}w'(P_1\epsilon_{t+1})$, the deviation of ϕ_{t+1} from its expected value can only come from the realization of ϵ_{t+1}.

The main benefit of this form of price-level targeting is that it makes the value of money stable over the long run. The policy compensates for previous errors so all fluctuations are only due to the latest error. Thus, long-term contracts face the same risk as short-term ones.

The drawback of this policy is that it creates variation in the level of aggregate real balances, $Z(\epsilon) = P_1\epsilon$. The reason for this variation is simply that the expected future value of money is constant, because of the anticipated adjustments to future monetary policy. Then, a greater than expected monetary expansion necessarily increases the aggregate level of real balances. Since the utility is concave, this policy creates a second-order loss in medium-of-exchange transactions. In the next section, I show that price-level targeting lowers the welfare of medium-of-exchange buyers when their utility function is CRRA.

2.4 Comparison of the Policies

The sources of unexpected deflation risk under these two policies are different. In general, the value of money depends on both the marginal value of real balances and the growth rate of the money supply. Thus, the value of money can be decomposed into the level of aggregate real balances and the total supply of money. Under inflation targeting, aggregate real balances are constant, while the supply of money is affected by the random errors. Thus, the value of money fluctuates directly in proportion to the error, and the unexpected deflation risk is just $\mathbb{E}[\epsilon^{-1}] \cdot \epsilon_{Min}$.

Under price-level targeting, aggregate real balances fluctuate due to the random errors.

This is because aggregate real balances are the product of the expected value of money and the total supply of money. Since the policy holds the expected value of money constant, an error that increases the supply of money also increases the aggregate real balances. Furthermore, the policy eliminates the effect of random errors on the long-run money supply. Thus, the only source of variation in the value of money is the variation in the marginal value of real balances.

2.4.1 Welfare Implications for Medium of Exchange

These fluctuations in aggregate real balances have two effects for the welfare of medium-of-exchange buyers. First, since utility is concave, the welfare will be lower, relative to having a constant level of real balances equal to their expected value. Second, the expected level of real balances will itself increase due to a precautionary savings motive: the expected marginal value of real balances is greater than the marginal value of the expected level. I conjecture that the first effect will dominate in general, and I show that this is so when the buyer's utility features constant relative risk aversion.

Since the errors in monetary policy are stochastic, the expected seignorage levels might differ between the two policies even for a given growth rate decision. The appropriate welfare comparison of the two policies in this chapter should hold the seignorage levels constant. The following proposition shows the seignorage levels for each policy as a function of the growth rate of money chosen.

Proposition 8 (Seignorage) *Suppose that* $w(q) = (1-\gamma)^{-1}q^{1-\gamma}$*. Under inflation target-*

ing, the expected seignorage level is

$$\begin{aligned} s^{IT} &= \beta Z^M w'(Z^M) - Z^M \\ &= \left(\frac{\beta}{1+g}\right)^{1/\gamma} \mathbb{E}[\epsilon^{-1}]^{1/\gamma} \left((1+g)\mathbb{E}[\epsilon^{-1}]^{-1} - 1\right). \end{aligned}$$

Under price-level targeting, the expected seignorage level is

$$\begin{aligned} s^{PLT} &= \mathbb{E}[\beta(P_1\epsilon)w'(P_1\epsilon) - w(P_1\epsilon)] \\ &= \left(\frac{\beta}{1+g}\right)^{1/\gamma} \mathbb{E}[\epsilon^{-\gamma}]^{1/\gamma} \left((1+g)\mathbb{E}[\epsilon^{-\gamma}]^{-1}\mathbb{E}[\epsilon^{1-\gamma}] - \mathbb{E}[\epsilon]\right). \end{aligned}$$

The seignorage generated from these policies is different because the policies induce different expected levels of aggregate real balances and price-level targeting imposes fluctuations on that expected level. It is easy to verify that in the case of no error terms, i.e., when ϵ is a degenerate random variable, the policies are identical provided that $(1+g^{IT}) = \epsilon(1+g^{PLT})$.

The properties of the seignorage levels in general will depend on the shape of the utility function. For the present discussion, however, the only important property is that there is a region, for small enough growth rate of the money supply g, for which the seignorage levels are increasing in g.

I will show that price-level targeting is worse for the welfare of medium-of-exchange buyers, relative to price-level targeting, at the same seignorage levels, using the following argument. First, I fix a level of welfare for those buyers under both policies, given as a deterministic consumption equivalent (because the consumption level under price-level targeting is stochastic). Then, I rewrite seignorage levels as a function of this consumption equivalent. Finally, I show that seignorage is lower under price-level targeting. Indeed, since seignorage and consumption follow an inverse realtionship in the parameter region under study, this result implies my initial claim.

Lemma 1 (Consumption equivalent) *Let q be a (fixed) level of consumption of the medium-of-exchange good. Then, q is the certainty equivalent under price-level targeting if*

$$q = \left(\frac{\beta}{1+g}\right)^{1/\gamma} \mathbb{E}[\epsilon^{-\gamma}]^{1/\gamma}\mathbb{E}[\epsilon^{1-\gamma}]^{1/(1-\gamma)}.$$

Similarly, the certainty equivalent under inflation targeting is

$$q = \left(\frac{\beta}{1+g}\right)^{1/\gamma} \mathbb{E}[\epsilon^{-1}]^{1/\gamma}.$$

Notice that for q to be the certainty-equivalent consumption level, it must be that

$$\begin{aligned}
w(q) &= \mathbb{E}[w(P_1\epsilon)], \\
&= P_1^{1-\gamma}\mathbb{E}[w(\epsilon)], \\
&= P_1^{1-\gamma} w\left(\left(\mathbb{E}[\epsilon^{1-\gamma}]\right)^{1/(1-\gamma)}\right) = w\left(P_1 \left(\mathbb{E}[\epsilon^{1-\gamma}]\right)^{1/(1-\gamma)}\right).
\end{aligned}$$

Then, the result follows from substituting P_1 in the last expression. This term can be found from the previously found equation:

$$1 = \frac{\beta}{1+g}\mathbb{E}\left[w'\left(P_1\epsilon\right)\right] = \frac{\beta}{1+g}P_1^{-\gamma}\mathbb{E}\left[\epsilon^{-\gamma}\right],$$

which can be rearranged to show

$$P_1 = \left(\frac{\beta}{1+g}\right)^{1/\gamma} \mathbb{E}[\epsilon^{-\gamma}]^{1/\gamma}.$$

Since inflation-targeting features constant aggregate real balances, the second result simply states that $q = Z^M$, where Z^M can be found in the same way:

$$1 = \frac{\beta}{1+g}\mathbb{E}[\epsilon^{-1}]\left(Z^M\right)^{-\gamma},$$

which can be rearranged to show

$$Z^M = \left(\frac{\beta}{1+g}\right)^{1/\gamma}\mathbb{E}[\epsilon^{-1}]^{1/\gamma}.$$

In both cases, the consumption equivalent level is strictly decreasing in the growth rate of the money supply. Since the analysis focuses on the part where the seignorage is a strictly increasing function of the growth rate, we can write this seignorage as a function of the consumption equivalent. The resulting expressions are shown in the next proposition.

Proposition 9 (Seignorage as a function of consumption equivalent) *Let q be a (fixed) level of consumption of the medium-of-exchange good. The seignorage level consistent with q is*

$$s^{IT} = \beta q^{1-\gamma} - q,$$

for inflation targeting and

$$s^{PLT} = \beta q^{1-\gamma} - q\left(\mathbb{E}[\epsilon]\mathbb{E}[\epsilon^{1-\gamma}]^{\frac{-1}{1-\gamma}}\right),$$

for price-level targeting.

Importantly, attaining a level of welfare (or consumption-equivalent q) implies different choices for the growth rate of the money supply under these different policies.

The main result of this section can now be easily shown. The seignorage level under price level-targeting is lower if $\mathbb{E}[\epsilon]\mathbb{E}[\epsilon^{1-\gamma}]^{\frac{-1}{1-\gamma}} \geq 1$. The following argument shows this

$$\begin{aligned}\mathbb{E}[\epsilon^{1-\gamma}]^{\frac{1}{1-\gamma}} &= w^{(-1)}\left(\mathbb{E}[w(\epsilon)]\right), \\ &\leq w^{(-1)}\left(w\left(\mathbb{E}[\epsilon]\right)\right), \\ &= E[\epsilon].\end{aligned}$$

The first line is a convenient rewriting of the expression on the left-hand side. The second line is an application of Jensen's inequality, using the fact that w is a concave function. Therefore, multiplying each side to complete the desired expression, we obtain

$$1 = \mathbb{E}[\epsilon^{1-\gamma}]^{\frac{-1}{1-\gamma}}\mathbb{E}[\epsilon^{1-\gamma}]^{\frac{1}{1-\gamma}} \leq \mathbb{E}[\epsilon^{1-\gamma}]^{\frac{-1}{1-\gamma}}E[\epsilon]$$

This result shows that the fluctuations in aggregate real balances reduce the welfare of medium-of-exchange buyers. Although this result might appear trivial, the expected aggregate real balances are indeed higher under price-level targeting. This is simply a consequence of a precautionary-savings motive (a convex marginal utility). However, this increase in the expected aggregate real balances is not enough to compensate for the cost that the fluctuations impose on a buyer with concave utility.

2.4.2 Welfare Implications for Unit of Account

I have already shown that price-level targeting eliminates the increase in risk of the value of money in the long run. However, this does not men that price-level targeting will make money a better unit of account in the short tem. In other words, although price-level targeting enhances the stability of the value of money over long periods of time, it is possible for it to make the value of money *less* stable in the short-term, relative to inflation targeting.

This possibility arises because the fluctuations in the value of money under price-level targeting come from changes in the marginal utility of real balances, as opposed to changes in the supply of money.

Thus, this result will depend on the shape of the utility function. In particular, when the utility function is CRRA, the result can be shown to depend on whethter the coefficient of relative risk aversion is less or greater than one.

Proposition 10 (Price-level targeting can lead to less stability) *Suppose that $w(q) = (1-\gamma)^{-1}q^{1-\gamma}$. The short-term (i.e., one-period-ahead) unexpected deflation risk is greater under price-level targeting, relative to inflation targeting, if $\gamma \geq 1$.*

Under price-level targeting, the unexpected deflation risk is given by

$$\frac{\mathbb{E}[\phi]}{\phi} = \frac{\mathbb{E}[w'(P_1\epsilon)]}{w'(P_1\epsilon)} = \mathbb{E}[\epsilon^{-\gamma}]\epsilon^{\gamma}.$$

On the other hand, under inflation targeting, the unexpected deflation risk is

$$\frac{\mathbb{E}[\phi]}{\phi} = \mathbb{E}[\epsilon^{-1}]\epsilon.$$

Thus, we can see that the short-term unexpected deflation risk is equal under both policies when the utility is logarithmic. In order to compare them, notice that we can rewrite the former as

$$\mathbb{E}[\epsilon^{-\gamma}]\epsilon^{\gamma} = \mathbb{E}\left[\left(\frac{\epsilon_{Min}}{\epsilon}\right)^{\gamma}\right].$$

If $\gamma > 1$, then

$$\mathbb{E}\left[\left(\frac{\epsilon_{Min}}{\epsilon}\right)^{\gamma}\right] \leq \mathbb{E}\left[\frac{\epsilon_{Min}}{\epsilon}\right];$$

with the reverse inequality if $\gamma < 1$. Therefore, price-level targeting makes the value of money less stable in the short-term when $\gamma > 1$ because, in that case, marginal utility is more sensitive to changes in aggregate real balances.

The preceding section has described the benefits and drawbacks of both policies. In summary, price-level targeting enhances the long-term stability of the value of money, with ambiguous effects on the short-term stability. Price-level targeting also imposes a welfare loss on medium-of-exchange transactions due to its necessary fluctuations on expected inflation rates. On the other hand, inflation targeting keeps expected inflation rates constant, to the benefit of medium-of-exchange transactions; yet, it introduces long-term instability in the value of money, as past policy errors are not taken into account in current policy decisions.

2.5 Discussion

The results of this chapter underscore an important lesson for monetary policy. To the extent that money plays two distinct roles in an economy, monetary policy will generally involve trade-offs between these roles. In this case, the monetary authority must choose between providing a good medium of exchange or a good unit of account.

One simple solution to this problem is to unbundle the functions of money. Indeed, in an economy with widespread indexation of contracts, the credit capacity depends on the quality of the index and real variables, not on the stability of the currency. In that economy, monetary policy can focus on maximizing the usefulness of money as medium of exchange. Although this solution eliminates the problem in a neat way, it also generates additional questions beyond the scope of the current framework. For instance, what is the optimal index or indices to provide as the unit of account? What are the implications of widespread indexation for financial stability in a world with settlement frictions? These questions require a deeper understanding of the implications of the role of unit of account.

If money is to remain both medium of exchange and unit of account, then it seems reasonable that an optimal policy would be a compromise between inflation and price-level targeting. For instance, the fluctuations in the level of real balances can be reduced with a less strict commitment to price-level targeting. Under this intermediate policy, the monetary authority avoids large adjustments to the inflation rate, but it eventually corrects the deviations from the price path. This is the core of the approach called "flexible average inflation targeting" which the Federal Reserve announced in August 2020. In that announcement, the Federal Reserve specified that it would aim for average yearly inflation to be 2%, over an unspecified period of time.[3] The policy is called flexible because it does not imply that a large inflation should be immediately countered with a large deflation.

It is important to note that a policy that makes money a worse unit of account does not necessarily make it a better medium of exchange. There are policies that can be uniformly worse for both roles. For instance, in a variation of the model with an additional foreseeable shock, a policy of constant monetary growth will be worse for both roles. It will introduce unnecessary fluctuations to the level of real balances (a worse medium of exchange) and allow for greater fluctuations on the value of money (a worse unit of account).

In the context of privately-issued independent currencies, the seignorage results in this chapter suggest that the incentives of an issuer are oriented towards providing better media of exchange, rather than units of account. Indeed, this is true in this model, due to the fact

[3]The transcript of the speech is available at https://www.federalreserve.gov/newsevents/speech/powell20200827a.htm

that agents demand a currency to use it as a medium of exchange, whereas using it as a unit of account creates no demand. It remains an open question whether this result holds in a unit-of-account model with settlement frictions.

2.6 Conclusions

In this chapter, I examined two approaches to monetary policy: inflation and price-level targeting. These approaches differ in their goal, the former aims to keep inflation rates stable, while the latter aims for stable prices over time.

These goals are not compatible in the long run, even though they might appear so in a short-term, one-period-ahead analysis. Price-level targeting requires fluctuations in the inflation rate; inflation targeting allows the price level to diverge from its long-run path. Relative to inflation targeting, price-level targeting favors the role of money as unit of account at the expense of its value as medium of exchange.

In an economy where money is the medium of exchange and the unit of account, monetary policy is torn between providing stable inflation rates and stable prices. It is not surprising that this challenge arises. As shown in the previous chapter, these roles of money are distinct and the properties that make money a good unit of account are different from those that make it a good medium of exchange.

2.A Serially-correlated Policy Shocks

The results in the main text do not rely on the assumption of independent and identically distributed errors. I show that the results hold in a more general setting where the error terms are assumed to follow a first-order Markov process.

In particular, I assume that, given ϵ_{t-1}, ϵ_t is distributed according to a cumulative distribution function $G(\epsilon_t; \epsilon_{t-1})$. This is a function $G : [\epsilon_{Min}, \epsilon_{Max}]^2 \to [0, 1]$, which is weakly increasing and right-continuous on its first argument.

2.A.1 Inflation Targeting

In order to keep the expected inflation rate constant, inflation targeting requires that the growth rate of money be chosen as $1 + g_t = \mathbb{E}_{t-1}\left[\epsilon_t^{-1}|\epsilon_{t-1}\right](1 + g)$. This is a similar expression to the one in the main text, except that this one corrects for the expected error in the current period.

Proposition 11 (Inflation targeting with serial correlation) *A monetary policy that sets the growth rate of money as*

$$1 + g_t = \mathbb{E}_{t-1}\left[\epsilon_t^{-1}|\epsilon_{t-1}\right](1 + g)$$

attains a constant expected inflation rate equal to

$$\frac{\phi_t}{\mathbb{E}_t[\phi_{t+1}]} = 1 + g.$$

I start by conjecturing a constant level of real balances, Z^M. Then, notice that

$$Z^M = \mathbb{E}_t[\phi_{t+1}]M_t = \mathbb{E}_t[\phi_{t+1}](1 + g_t)\epsilon_t M_{t-1}.$$

Since the level of real balances is constant, we can write

$$\begin{aligned}\mathbb{E}_t[\phi_{t+1}](1+g_t)\epsilon_t M_{t-1} &= \mathbb{E}_{t-1}[\phi_t]M_{t-1},\\ \mathbb{E}_t[\phi_{t+1}] &= \mathbb{E}_{t-1}[\phi_t]\cdot\frac{1}{1+g_t}\cdot\frac{1}{\epsilon_t},\\ \mathbb{E}_{t-1}[\phi_{t+1}] &= \mathbb{E}_{t-1}[\phi_t]\cdot\frac{1}{1+g_t}\cdot\mathbb{E}\left[\frac{1}{\epsilon_t}\right],\end{aligned}$$

where the second line is a rearrangement of the first one and the third line follows from applying expectations at time $t-1$. Since we are under inflation targeting, it must be that

$$\phi_t = \mathbb{E}_t[\phi_{t+1}]\Pi,$$

$$\rightarrow \mathbb{E}_{t-1}[\phi_t] = \mathbb{E}_{t-1}[\phi_{t+1}]\Pi.$$

Thus, we can solve for the choice of money supply growth as

$$1+g_t = \Pi\cdot\mathbb{E}_{t-1}\left[\frac{1}{\epsilon_t}\right].$$

Setting $\Pi = (1+g)$, the result follows.

For the sake of completeness, I will mention a few more properties of the equilibrium under inflation targeting, and I will show that they coincide with the results in the main text. First, the aggregate real balances under inflation targeting Z^M are implicitly defined by

$$1 = \frac{\beta}{1+g}w'(Z^M).$$

Second, the unexpected deflation risk is also given by a familiar expression. Starting from the equation for the value of money

$$\begin{aligned}
\phi_t &= \beta \mathbb{E}_t\left[\phi_{t+1}\right] w'(Z^M), \\
\frac{\phi_t}{\mathbb{E}_{t-1}\left[\phi_t\right]}\mathbb{E}_{t-1}\left[\phi_t\right] M_t &= \beta \mathbb{E}_t\left[\phi_{t+1}\right] M_t w'(Z^M), \\
\frac{\phi_t}{\mathbb{E}_{t-1}\left[\phi_t\right]} Z^M \epsilon_t (1+g_t) &= \beta Z^M w'(Z^M).
\end{aligned}$$

Now, I rearrange this last expression and use the implicit equation for aggregate real balances. I also iterate this expression forward, to present it in the familiar form.

$$\frac{\phi_{t+1}}{\mathbb{E}_t\left[\phi_{t+1}\right]} = \mathbb{E}_t\left[\epsilon_{t+1}^{-1}\right]^{-1} \epsilon_{t+1}^{-1}.$$

The only difference between this expression and the one in the case of i.i.d. errors is due to the conditional expectation. The main logic remains the same. The long-term unexpected deflation risk can be obtained in the following way.

$$\begin{aligned}
\phi_{t+1} &= \mathbb{E}_t\left[\phi_{t+1}\right] \cdot \mathbb{E}_t\left[\epsilon_{t+1}^{-1}\right]^{-1} \epsilon_{t+1}^{-1}, \\
&= \frac{Z^M}{M_{t-1}(1+g_t)\epsilon_t} \cdot \mathbb{E}_t\left[\epsilon_{t+1}^{-1}\right]^{-1} \epsilon_{t+1}^{-1}, \\
&= \frac{Z^M}{M_{t-1}} \frac{1}{1+g} \mathbb{E}_{t-1}[\epsilon_t^{-1}]^{-1} \epsilon_t^{-1} \cdot \mathbb{E}_t\left[\epsilon_{t+1}^{-1}\right]^{-1} \epsilon_{t+1}^{-1}.
\end{aligned}$$

Taking expectations at time $t-1$, we obtain

$$\mathbb{E}_{t-1}\left[\phi_{t+1}\right] = \frac{Z^M}{M_{t-1}} \frac{1}{1+g}.$$

Therefore, the unexpected deflation risk is

$$\frac{\mathbb{E}_{t-1}\left[\phi_{t+1}\right]}{\phi_{t+1}^{Max,t-1}} = \left(\mathbb{E}_{t-1}[\epsilon_t^{-1}]\epsilon_t \cdot \mathbb{E}_t\left[\epsilon_{t+1}^{-1}\right]^{-1} \epsilon_{t+1}\right)^{Min},$$

where in general the expression in brackets might not correspond to getting $\epsilon_t = \epsilon_{t+1} = \epsilon_{Min}$. Still, this long-run unexpected deflation risk is certainly more severe than the short-run one.

2.A.2 Price-Level Targeting

In a similar manner, the presence of serial correlation in policy shocks does not qualitatively affect the results for price-level targeting. Although, in the presence of serial correlation, it is in general not possible to achieve price-stability in the sense of

$$\mathbb{E}_t[\phi_{t+1}] = P_1 \Pi^{-t},$$

the monetary policy can be designed so that the value of money at time t depends only on the most recent error term and the desired long-run growth of money supply. That is, we can obtain

$$\mathbb{E}[\phi_{t+1}] = P_1 \Pi^{-t}.$$

My claim of the impossibility result can be verified with the equation for the value of money. Suppose that the result is true, then we can write

$$\phi_t = \beta P_1 \Pi^{-t} w'(P_1 \epsilon_t).$$

Taking expectations at time $t-1$, the resulting expression is

$$P_1 \Pi^{-(t-1)} = \mathbb{E}_{t-1}[\phi_t] = \beta P_1 \Pi^{-t} \mathbb{E}_{t-1}[w'(P_1 \epsilon_t)].$$

Consider two possible histories up to time $t-1$, with different shocks $\tilde{\epsilon}$ and $\hat{\epsilon}$. Then, the previous result implies that

$$\mathbb{E}_{t-1}[w'(P_1\epsilon_t)|\epsilon_{t-1} = \hat{\epsilon}] = \mathbb{E}_{t-1}[w'(P_1\epsilon_t)|\epsilon_{t-1} = \tilde{\epsilon}].$$

This equality will in general not hold if there is serial correlation.

Nevertheless, a policy of "correcting previous errors" delivers price stability in the second sense.

Proposition 12 (Price-Level targeting with serial correlation) *A monetary policy that sets the growth rate of money as* $(1+g_t) = (1+g)/\epsilon_{t-1}$ *achieves price stability in the sense of*

$$\mathbb{E}[\phi_{t+1}] = P_1\Pi^{-t}.$$

The value of money will satisfy the following equation

$$\phi_t(\epsilon_t) = \beta\mathbb{E}_t[\phi_{t+1}|\epsilon_t]w'\left(\mathbb{E}_t[\phi_{t+1}|\epsilon_t]M_t\right).$$

This policy rule results in the following money supply, $M_t = (1+g)^t\epsilon_t$. Natirally, a stationary monetary equilibrium requires that $\Pi = 1+g$, and therefore we can write

$$\phi_t(\epsilon_t) = \phi(\epsilon_t)(1+g)^{-t}.$$

The function $\phi(\epsilon)$ must then satisfy

$$\phi(\epsilon) = \frac{\beta}{1+g}\mathbb{E}[\phi(\epsilon_{+1})|\epsilon]w'(\phi(\epsilon)\epsilon).$$

There is no general solution for this expression, except in some simple cases. For instance, if $w(q) = \log(q)$, then $\phi(\epsilon) = \epsilon^{-1}$, just as in the i.i.d. case.

Still, the fact that this policy makes the value of money depend only on the realization of the most recent error means that the risk in the value of money is the same for both short-term and long-term contracts. In the former case, the unexpected deflation risk is

$$\frac{\mathbb{E}_t[\phi_{t+1}(\epsilon_{t+1})]}{\phi_{t+1}(\epsilon_{t+1})} = \frac{\mathbb{E}_t[\phi(\epsilon_{t+1})]}{\phi(\epsilon_{Min})}.$$

In the latter, the risk is

$$\frac{\mathbb{E}_{t-1}[\phi_{t+1}(\epsilon_{t+1})]}{\phi_{t+1}(\epsilon_{t+1})} = \frac{\mathbb{E}_{t-1}[\phi(\epsilon_{t+1})]}{\phi(\epsilon_{Min})}.$$

Again, price-level targeting reduces the long-run instability in the value of money, relative to inflation targeting.

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