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FREEWAY TRAFFIC OSCILLATIONS: OBSERVATIONS AND PREDICTIONS

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ABSTRACT

Freeway traffic was observed over multiple days and was found to display certain regular features. Oscillations arose only in queues; they had periods of several minutes; and their amplitudes stabilized as they propagated upstream. They propagated at a nearly constant speed of about 22 to 24 kilometers per hour, independent of the location within the queues and the flow measured there; this was observed for a number of locations and for queued flows ranging from about 2,000 to 850 vehicles per hour per lane. The effects of the oscillations were not felt downstream of the bottleneck. Thus, the only effect on upstream traffic was that a queue's tail meandered over time by small amounts. (For the long queues studied here, the tails deviated by no more than about 16 vehicle spacings, as compared with predictions that ignored the oscillations). Notably, the character of queued traffic at fixed locations did not change with time, despite the oscillations; i.e., traffic did not decay.

There were changes over space, however. New oscillations formed in moderately dense queues near ramp interchanges and then grew to their full amplitudes while propagating upstream, even though the range of wave speeds was narrow. The formations of these new oscillations are strongly correlated with vehicle lane changing. But this pattern of formation and growth was less evident in a very dense queue (caused by an incident), although frequent lane changing occurred near the interchanges. It thus appears that the oscillations were triggered by random lane changing in moderately dense queues more than by car-following effects.

Finally, kinematic wave theory was found to describe the propagation of the oscillations to within small errors. For distances approaching one kilometer, and for 2-hour periods, the theory predicted the locations of vehicles to within about 5 vehicle spacings. Further analysis showed that some of these small discrepancies are explained by differences in car-following behavior across drivers.

1. INTRODUCTION

This manuscript reports on a study of freeway traffic in long queues. Its findings indicate that oscillations apparently arose due to vehicle lane changing near interchanges; that the oscillations displayed some very regular features that did not cause the character of traffic to change over time; and that their propagation is described by simple theories.

The freeway site used for this study, and the many hours of traffic data collected there, are described in the following section. In section 3, some of the notable features observed in the oscillations are presented. Evidence that the oscillations were triggered by vehicle lane changing is provided in section 4. Section 5 shows that the oscillations propagated through queued traffic in ways that closely matched the descriptions of simple theories. Some of the implications of these findings are briefly discussed in the manuscript's sixth and final section.

2. THE SITE AND ITS DATA

Data came from the 10-kilometer stretch of the Queen Elizabeth Way shown in Figure 1(a). The site's 15 loop detector stations are labeled in the figure as per the numbers assigned them by the regional transport authority. They recorded vehicle counts, occupancies (a dimensionless measure of density) and average vehicle speeds over 20-second intervals.

The detector data were examined over six weekday mornings spanning the rush. On each of these days, the segment between detectors 51 and 52 contained an active bottleneck; i.e., a bottleneck characterized by queues upstream and freely flowing traffic downstream (Daganzo, 1997).

Figure 1(b) verifies this for part of a rush (and it serves to illustrate the methods used for making this determination). Shown in this figure are specially transformed curves of cumulative vehicle count, N, versus time, t, measured across all travel lanes at detectors 50 - 53. The curves were constructed such that the vertical separations between any two of them are the excess vehicle accumulations between their respective detectors due to vehicular delays.

An oblique coordinate system was used in the figure to plot $N - q_0 \times (t - t_0)$ versus *t* for each curve's starting time, t_o , and some choice of a background flow, q_o ; the latter was selected so that the range of $N - q_0 \times (t - t_0)$ was small as compared with the *N* itself. This coordinate system magnified the figure's vertical axis which, in turn, amplified not only the curves' vertical separations, but also the changing slopes of the curves themselves. Since each curve was drawn



Figure 1 (a) Queen Elizabeth Way, Ontario, Canada (b) Oblique N-curves at the Active Bottleneck

using piece-wise linear interpolations through the detectors' 20-sec counts, flow changes were made more visible by these amplifications in the slopes.¹

It is clear from Figure 1(b) that traffic conditions between detectors 52 and 53 remained freely flowing over the observation period shown; i.e., the two oblique N-curves at these locations remained approximately superimposed. But curves 51 and 50 eventually diverged from their two downstream counterparts, indicating the presence of excess vehicle accumulations. Segment 51-52 was thus identified as the location of an active bottleneck. We suspect this activation was due to merging; apparently some vehicles from the Cawthra Road on-ramp merged after traveling a good distance on the freeway's shoulder.²

Oblique N-curves constructed for longer periods confirmed that this bottleneck remained active for more than two hours each rush Similar curves also showed that the resulting queues grew to fill all or most of the freeway stretch upstream. And finally, curves of this type revealed that flows diminished at queued locations further upstream from the active bottleneck.

The latter phenomena occurred because the inflows from each of the site's on-ramps exceeded the exit flows to each off-ramp. The net inflows restricted freeway traffic arriving at the interchanges such that each became a bottleneck, although not an active one (see Cassidy and Mauch, 2001). This is noteworthy in that it provided for a range of observed flows (and densities) within each long queue.

The range of observed traffic conditions was further extended, thanks to an incident that occurred a short distance downstream of detector 50 and persisted for about 40 minutes. It was confirmed from detailed incident records maintained by the regional transport authority. And it created a long queue that was more dense than those typical of a rush; i.e., the incident was a severe one. (Its average discharge flow was less than 4,600 vph while the recurrent bottleneck's discharge rate was about 6,200 vph).

3. Some observed features of the oscillations

Curves like those in Figure 2(a) reveal a number of details about oscillations. Each curve shown here was transformed such that its slopes are deviations from 15-min moving average flows; i.e.,

¹ Extended discussion on the construction and interpretation of oblique N-curves is available from a number of sources, including Cassidy and Bertini (1999b).

 $^{^{2}}$ We know for certain that the activation occurred each day at approximately the same time and location and that following this, the bottleneck exhibited a reproducible average vehicle discharge rate (see Cassidy and Bertini, 1999a). These point to exogenous cause(s) for the bottleneck's activation.



Figure 2(a) Deviation Curves at Each Detector

from the cumulative count to time t, N(t), the N(t) - [N(t + 7.5 min) - N(t - 7.5 min)]/2 was computed for each 20-sec interval and plotted over time. The vertical displacements between one of these deviation curves and its horizontal trend line are denoted $N - \overline{N}_{15}$.

Each deviation curve in Figure 2(a) is vertically displaced from its neighbor in proportion to the distance actually separating their detectors. (And most of the freeway stretch is reproduced along the left edge of the figure as a convenience to the reader). The counts for these curves were measured across all lanes. These were taken when the queue from the active bottleneck had filled the entire upstream portion of the freeway stretch. Some of the average flows measured during the one-hour period shown in the figure are annotated to illustrate that flows diminished upstream of the junctions.

The wiggles made prominent on some of the deviation curves are the oscillations themselves. They are characterized by sequences of high and low flows with periods of several minutes each.³ The amplitude of each oscillation eventually stabilized; the $N - \overline{N}_{15}$ are not more than 50 vehicles (or about 16 vehicles per lane) and a scale is provided in Figure 2(a) to verify this.

Also included in this figure are dashed lines tracing the motion of some oscillations. (These lines are shown connecting the peaks of wiggles, as this made for an uncluttered presentation). The lines show the oscillations propagated upstream, against the flow of traffic. That the lines are parallel indicates that the wave speeds were nearly constant, despite the presence of ramp interchanges and the reduced flows that prevailed upstream of each of them.⁴

The oscillations did not affect freely flowing traffic upstream of a queue's tail. As evidence of this, Figure 2(b) presents deviation curves from a second observation day. The features of these new curves are much the same as those of the previous figure. But on this second day, the tail of the bottleneck's queue did not quite fill the entire upstream freeway stretch; i.e., it propagated beyond detector 41 (as before), but it did not reach detector 40 further upstream Figure 2(b) shows that the oscillations, in turn, moved through the queue but that they did not continue beyond the queue's tail; i.e., the wiggles displayed on curve 41 and most of its downstream counterparts are not evident on curve 40.

Moreover, the oscillations originated within the queue and their effects did not propagate

³ These periods are far greater than what is described by the well-known car-following theories that generate oscillatory periods comparable to driver reaction times (Chandler, et.al., 1958; Kometani and Sasaki, 1958; 1961; Herman, et.al., 1959; Helly, 1961).

⁴ Further analyses showed that average wave speeds on each segment ranged from about 22 to 24 kilometers per hour and that flow had no systematic effects on these (Mauch, 2001). Small reductions in average wave speeds did occur near interchanges, but we suspect this was linked to vehicle lane changing. More is said about lane-changing effects later in the manuscript.



Figure 2(b) Deviation Curves at Each Detector, a Second Observation Day

downstream beyond the head of the queue. In both Figures 2(a) and (b), the deviation curves at detector 52 describe the bottleneck's discharge and these remained smooth, despite the oscillations upstream. It follows that the oscillations had relatively little effect on the growth of queues; i.e., they caused a queue's tail to meander over time by small amounts. The extent of this meandering would not exceed about 16 vehicle spacings.

Of further note, the effects of these oscillations did not change systematically over time. Figure 3 presents evidence of this for several queued locations. These plotted lines display root mean squared errors taken over 20-min periods; i.e., shown for each detector interval ending at time t are

the
$$\left[\sum_{k=t-10\min}^{t+10\min} (N(k) - \overline{N}_{15}(k))^2 / 20\min\right]^{1/2}$$
. The values in Figure 3 are actually averages taken over

four days. These show no systematic trends over time. The oscillations did not steadily grow in amplitude and cause traffic to decay.



But the oscillations and their effects did change with location. For example, traffic did not oscillate between the same two flows at all points within the queue. Rather, both the high and low flows diminished with distance from the active bottleneck in much the same fashion as did the longer-run average flows. Evidence of this is provided in Table 1, which lists some of the average flows (and vehicle speeds) measured during the queued portion of a rush.

Figure 3 also shows the oscillations' effects (i.e., the flow variations they caused) increased in the upstream direction. This occurred because new oscillations arose at different locations in the queue and grew to their full amplitudes while propagating upstream. Numerous examples of this are apparent in Figures 2(a) and (b).

	Long-Run	Average	Average	Average	Average
Detector	Average Flows	High Flows	Low Flows	High Speeds	Low Speeds
	(vph)	(vph)	(vph)	(kph)	(kph)
40	3,990	4,650	2,640	41	15
42	3,990	4,650	2,640	41	15
44	4,380	5,250	3,030	57	18
46	4,920	5,550	3,660	68	25
48	5,280	5,790	3,960	78	29

Table 1Traffic Conditions in a Long Queue

Still more evidence is shown in Figure 4(a). It displays, for each detector, the root mean squared error (RMSE) of $N - \overline{N}_{15}$; these are the averages measured for 2 hours on each of four days. The figure reveals certain telling trends. Namely, the oscillations' effects grew as they propagated over freeway segments near interchanges. But this growth did not continue on segments located further upstream where no interchanges were present.

This growth might seem puzzling in light of the very regular ways the oscillations propagated through queued traffic. Since each traveled at a nearly constant speed, the occurrence of new oscillations cannot be caused by diverging waves; i.e., these new formations cannot be explained by theories of traffic instability (e.g., Newell, 1962).

The answer instead appears to lie with the freeway's geometric features and the lane changing these induced. Evidence of this is provided in the following section.

4. FREEWAY GEOMETRY AND LANE CHANGING

Strong correlation between vehicle lane changing and the formation and growth of new oscillations is evident in Figures 4(a) and (b). The latter of these figures shows long-run average flows measured by the detectors in individual lanes; i.e., these flows were measured over 2-hr periods and averaged over 4 days. Each lane's flow remained nearly fixed across detectors 40-42, indicating an absence of systematic lane changing on the two upstream-most freeway segments. The flows tell a different

story for the downstream segments, however. There they changed over space in ways that reveal lane changing. As an example, vehicles entered the shoulder lane while traveling between detectors 45 and 46; the latter measured higher flow in the shoulder lane and corresponding flow reductions in the center and median lanes.

But lane changing is described in Figure 4(b) with very coarse time scales and certain patterns of interest might not be revealed at this scale. To verify that higher systematic lane-changing persisted near interchanges, we turn to Figures 5(a) and (b). Shown here are oblique N-curves measured while an oscillation propagated past detectors 46 and 45. The curves from downstream detector 46 were shifted horizontally (by the oscillation's trip time on the segment) and vertically to superimpose the initial portion of each pair of curves.



(a) Average Root Mean Squared Errors Vs. Location(b) Average Long-Run Flows in Each Lane

Figure 5(a) displays N-curves measured in the shoulder lane. The one at detector 46 rises above the one at 45; i.e., a higher flow was measured by the downstream detector. As in the long-run observations in Figure 4(b), lane-changers entered the shoulder lane. Figure 5(a) shows this occurred during a (short) time comparable to this oscillation's period.

The curves in Figure 5(b) describe the oscillation as measured in the center and median lanes together. By grouping these lanes, their oblique N-curves exhibit effects opposite to those in the shoulder lane; i.e., the curves in Figure 5(b) reveal the net defection of vehicles.



(c) Shown for a Longer Period

The curves in Figure 5(c) were constructed for a longer time period so that short-run has changing could be studied over more of the rush. These curve pairs were translated much like

before; the downstream curves at 46 were shifted horizontally by the average trip time for oscillations traversing this segment, T (measured here in mins). The rate of change in the vertical separation between each pair of curves was computed every 20 secs using N-curves like those in Figure 5(c), i.e.,

$$r(t)^{l} = [N(t+T) - N(t-20\sec+T)]_{u}^{l} - [N(t) - N(t-20\sec)]_{d}^{l} ,$$

where

 $r(t)^{l}$ = the rate of change for the 20-sec interval ending at *t* for lane group *l*, l = (sh) for the shoulder lane, or l = (cm) for the center and median lanes; and

u and *d* denote counts from the upstream and downstream detectors, respectively.

The cross-correlation term is the product of $r(t)^{sh}$ and $r(t)^{cm}$. A 10-min moving average was used to smooth fluctuations in these cross correlations. Values were less than zero when lane-changing occurred (and small positive values greater than zero were interpreted as noise).

Figures 6(a) and (b) provide typical examples of cross-correlations measured in queued traffic. They indicate not only that lane-changing rates fluctuated sharply, but also that these rates were consistently higher on segments near interchanges. Not so on segments further upstream, away from interchanges.



(b) Downstream Freeway Segments

Having shown the correlation between the oscillations' spatial growth and short-run lane changing, we conclude this section with a puzzling observation. The correlation was less evident in the very dense queue caused by the incident (just downstream of detector 50), as evident in Figures 7(a) and (b). These figures were constructed in the identical manner as Figures 4(a) and (b), but with data from the incident-induced queue. Detector 42 was not functioning during this time. Yet Figures 7(a) and (b) still show that the spatial growth in the oscillations' effects was small. This despite frequent lane changing on some segments, as evident in Figure 7(c). Apparently, lane changing had less of an effect in queues of very slow-moving vehicles.

5. TESTING SIMPLE THEORIES

The oscillations propagated upstream in ways consistent with the simplified version of kinematic wave theory proposed by Newell (1993).⁵ This was verified for freeway segments without intervening interchanges using N-curves from the segment's upstream and downstream detectors. The curve from the latter was shifted horizontally by the oscillations' average trip time on the segment and vertically in an effort to superimpose this curve on its upstream neighbor; (the vertical translation is the average number of vehicles through which the waves passed while traversing the segment).

This recipe was carried out for continuous 2-hour periods on each of 4 days. The resulting root mean squared errors (RMSE) are provided in column 2 of Table 2; these were computed from the deviations measured each 20-sec interval and the values shown are the 4-day averages. For each segment studied, the RMSE's were less than 15 vehicles (in all three lanes). Thus, for distances approaching one kilometer, kinematic wave theory predicted vehicle locations (in each lane, on average) to within 5 vehicle spacings or less.

Column 3 of the same table provides the errors that resulted from a more naive approach to predicting queued flows. Here the long-run (2-hr) average flow measured by a downstream detector served as the estimate of the neighboring N-curve upstream (This represents how one might estimate queued traffic in the absence of any theory). Column 4 lists the correlation coefficients from the two recipes. These indicate that kinematic wave theory explained 99 percent of the additional variation (i.e., the wiggles and other time dependent changes in flow) measured on each

⁵ This simplified version describes the occurrence of kinematic waves as per Lighthill and Whitham (1955), with wave speed presumed independent of flow.



Figure 7 (a) Average Root Mean Squared Errors During Incident (b) Average Long-Run Flows During Incident (c) Cross Correlation Vs Time; Downstream Segments During Incident

Freeway		RMSE	Correlation
Section	RMSE (veh)	Naive Approach (veh)	Coefficient
(1)	(2)	(3)	(4)
40-41	14.22	94.58	0.99
41-42	13.54	96.70	0.99
45-46	14.93	105.33	0.99
48-49	12.93	97.13	0.99

upstream N-curve. These findings show that queued flows are constrained from downstream much as described by this simple theory.

Table 2Test of Kinematic Wave Theory

It turns out that some of these small discrepancies between kinematic wave theory and measurement can be explained by driver differences; i.e., by variations in the ways that different drivers respond to traffic downstream. This finding is a logical one. After all, the drivers passing through a wave at some downstream location are different from those who pass through the wave further upstream. And different drivers choose different spacings for following the vehicle ahead of them (Cassidy and Windover, 1998).

A model of driver differences was proposed by Newell (1999). According to this simple theory, the trajectory of some j^{th} vehicle is obtained by translating the $(j-1)^{\text{th}}$ trajectory horizontally by duration \mathbf{t}_j and vertically by distance d_j in the manner shown in Figure 8. The \mathbf{t}_j and d_j vary with

each j^{th} driver such that the kinematic waves propagate from one vehicle to the next as a random walk. The wave's average speed is equal to the ratio of d to t, the expected values of the vertical and horizontal translations required to superimpose two neighboring trajectories.

Given the above, the total time, T(n), and distance, D(n), covered by a wave propagating through *n* vehicles is a bivariate process with independent increments, and therefore described by a bivariate normal distribution with mean and covariance matrix proportional to *n*; i.e.,

$$[T(n), D(n)] \sim \text{BVN}\left([\boldsymbol{t} \times n, d \times n]; \begin{bmatrix} \boldsymbol{s}_{t}^{2} & \boldsymbol{s}_{dt} \\ \boldsymbol{s}_{td} & \boldsymbol{s}_{d}^{2} \end{bmatrix} \times n \right)$$
Eq. 1

The present data were obtained from detectors at fixed locations. Therefore, the wave's trip time on segment *i*, T_i , and the number of vehicles through which it propagated, N_i , (see Figure 8) were



Figure 8 Hypothetical Trajectories Traveling Between Two Detectors in a Single Freeway Lane

measured in each lane using oblique N-curves.⁶

The $[T_i, N_i]$ are described by a bivariate normal distribution with mean and covariance proportional to the segment's length, L_i ; i.e.,

$$[T_i, N_i] \sim \text{BVN}\left(\left[t \frac{L_i}{d}, \frac{L_i}{d}\right]; \left[\begin{array}{c} \mathbf{s}_t^2/d & \mathbf{s}_{dt}/d \\ \mathbf{s}_{td}/d & \mathbf{s}_d^2/d \end{array}\right] \times L_i\right)$$
Eq. 2

The joint distribution of the $[T_i, N_i]$ would thus be independent of exogenous factors, including the flow and the freeway segment from which measurements came.

This was tested by plotting joint observations from queued freeway segments with different flows. For example, measurements from segments with moderately dense queues typical of a rush were compared with those from the very dense queue upstream of the incident. These observations were transformed such that, if the theory holds, samples drawn from different L_i would still come

⁶ The N_i in each lane were estimated using cumulative curves that were constructed from counts simultaneously measured across all travel lanes. The T_i came from curves separately constructed for each lane.

from a common distribution.⁷ To this end, we note that

$$\left\lfloor \frac{T_i - \mathrm{E}\{T_i\}}{\sqrt{\mathrm{E}\{N_i\}}}, \frac{N_i - \mathrm{E}\{N_i\}}{\sqrt{\mathrm{E}\{N_i\}}} \right\rfloor \sim \mathrm{BVN}\left([0, 0]; \ \Sigma L_i \frac{1}{\mathrm{E}\{N_i\}}\right) \equiv \mathrm{BVN}\left([0, 0]; \ \Sigma d\right), \qquad \text{Eq. 3}$$

where Σ is the covariance matrix in Eq 2. And since

$$\left[\frac{T_i - \mathbf{t}(L_i/d)}{(L_i/d)^{1/2}}, \frac{N_i - L_i/d}{(L_i/d)^{1/2}}\right] = \left[\mathbf{t}\left\{\frac{T_i/\mathbf{t} - L_i/d}{(L_i/d)^{1/2}}\right\}, \left\{\frac{N_i - L_i/d}{(L_i/d)^{1/2}}\right\}\right], \quad \text{Eq. 4}$$

we plotted only the braced terms on the right-hand-side of Eq. 4, denoted as $[T^*, N^*]$. They are dimensionless and the collection of all such points are normally distributed about mean [0, 0].

Figure 9(a) presents observations of $[T^*, N^*]$ measured in moderately dense queues on segments 40-41 and 41-42 (shown with unshaded data points) and another set of observations taken from the very dense incident-induced queue on segment 40-41 (shaded points). These upstreammost segments were selected because lane changing was infrequent there; observations from segment 41-42 were not available during the incident because detector 42 malfunctioned.

Despite the different flows, the similar scatter in both sets of data indicate that all were drawn from a common distribution⁸ Newell's simple theory thus provided a reasonable description of the driver differences in these traffic streams. These differences explain some of the additional effects not captured by kinematic wave theory.

But Newell's theory does not describe the effects of lane changing. Figure 9(b) presents observations of $[T^*, N^*]$ from downstream segments in moderately dense queues; segments marked by frequent lane changing. The wide scatter in these data indicate the variance of the t_j and the d_j increased along the wave paths. The insertion and defection of (lane-changing) vehicles evidently disrupted traffic streams.

Yet these effects were not visible in the incident's very dense queue. Figure 9(c) displays measurements of $[T^*, N^*]$ during the incident. One of the two freeway segments used for this figure was marked by frequent lane changing and one was not; the reader can refer back to Figure 7(c) to verify this. But the scatter of the data from both segments is small.

⁷ Without the transformations, different segment lengths would affect comparisons, since the mean values of N_i and T_i increase linearly with L_i while their standard deviations increase with the square root of L_i .

⁸ Statistical tests for the differences of parameter estimates between these samples did not hold at the 95 percent significance level. We suspect this was due to the small sample sizes and because the data were only available in 20-sec increments. Notably, two random samples (of sizes identical to the samples in Figure 9(a)) drawn from a common BVN distribution and aggregated to 20-sec intervals likewise failed at the 95 percent significance level.





- (b) Downstream Segments in Non-incident Queues
- (c) Downstream Segments in Incident Queues

6. CONCLUSIONS

The observed oscillations displayed regular features that did not cause the characteristics of queued traffic to change over time. The propagation of these oscillations was described well by kinematic wave theory. Driver differences added small effects, but marked improvements in traffic flow theories will likely come by incorporating lane-changing effects. After all, it appears that in moderately dense queues, oscillations were due more to lane changing than to (endogenous) carfollowing effects. Any new theories aimed at improving traffic prediction can be tested against the present findings. (The predictions of these new theories need to be at least as good as those of the simple models examined here).

That lane-changing effects were less evident in the incident's very dense queue is puzzling. But this finding is consistent with previous reports that speeds, flows, densities, etc. exhibit greater variation when measured in moderately dense queues, as compared with measurements in queues of higher density (Mika, et.al., 1969; Koshi, et.al., 1983; Kerner and Rehborn, 1997).

But this finding does not necessarily imply that queued traffic is composed of two distinct classes; i.e., synchronized and jammed. In the present study, data were drawn from moderately dense queues that formed daily and from one very dense queue created by an incident. There are ranges of density for which we had no observations. It may be that data scatter (i.e., correlation) changes continuously with density.

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