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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 33(33)

ISSN
1069-7977

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Publication Date
2011

Peer reviewed

Do children and adults learn forward and inverse conditional probabilities together?

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Abstract

Learning p(A|B) often provides information about p(B|A). Do learners attend to this information? In Experiment 1, preschool-aged children learned to predict the sound of an alien from its color. The predictability of color from sound did not have a large effect on learning rate. During testing children seemed to use the learned probabilities, p(sound|color) to make judgments of the inverses, p(color|sound) rather than the actual encountered frequency distribution. In Experiment 2 adults showed a similar pattern. Adults used the probabilities they were trained on, either p(sound|color) or p(color|sound), to make judgments of the inverses. These results support previous demonstration of an "inverse fallacy" and suggest that both young children and adults show very task-specific learning.

Keywords: probability learning; development; inverse fallacy.

Many learning problems can be understood in terms of conditional probabilities. In particular, categorization involves the probability of class membership given (conditional upon) possession of some properties. The inverse of categorization is property projection. In this case the task is to estimate the probability of some property given class membership. A categorization problem is, "If something barks is it likely to be a dog?" A projection problem is, "If something is a dog is it likely to bark?" The focus of the current study is the relation between these two problems, the relation between forward and inverse conditional probabilities.

In theory there is no particular relation between a forward conditional probability and its inverse. By itself, that p(A|B) is high implies almost nothing about p(B|A). Psychologically the expected relation between these two quantities is less clear. People often confuse the two quantities, the so-called "inverse fallacy" (Dawes, Mirels, Gold, & Donahue, 1993; Villejoubert & Mandel, 2002). Young children seem to interpret conditional statements ("if A then B") as biconditional ("if A then B, and if B then A" Barrouillet & Lecas, 2002). At the same time, the categorization literature suggests that people fail to learn about p(B|A) when trained on p(A|B), even though both quantities may be estimated from the same experience. The hypothesis is that learners focus on one predictive problem to the exclusion of others.

There are many ways to learn conditional probabilities, but learning from examples seems the most basic. Consider an environment consisting of objects composed of two binary features (e.g., red and blue, loud and quiet). Assume the learner is able to keep track of frequencies of encounters with the different kinds of objects, to fill in something like a contingency table (see Figure 1). Those frequencies are informative about both p(red|loud) and p(loud|red). In most experimental conditions, and perhaps in the real world as well, experience of the features is sequential. The learner first observes one feature ("this is red") and then learns another ("it is loud"). If color is always observed first, the learner gets experience making judgments of p(loud|red) and p(loud|blue). We know from a long history of experimental psychology that people will soon get good at estimating these probabilities: They will learn what red and what blue predict. Will they also learn anything about the inverse probabilities, p(red|loud) and p(red|quiet)?

Research on category learning suggests that training on p(A|B) may not produce learning of p(B|A). For example, in a supervised categorization task, people learn which cues predict membership in which category, p(category|feature). However they seem not to learn about the characteristic properties of the categories, p(feature|category) (see, Taylor & Ross, 2009). They learn "what predicts category A" but not "what category A predicts." Similar results come from the literature on perceptual learning: People often learn the discrimination they are trained on, but little else (Ahissar, 1999). In contrast, there is considerable evidence that learned associations are symmetric (Kahana, 2002). For example, in paired associates, training A from B also trains B from A. The literature on the inverse fallacy, p(disease|symptom) = p(symptom|disease), supports the hypothesis of symmetry. Although somewhat inconsistent, these lines of work are not exactly contradictory. If people confuse (or assume symmetry) of forward and inverse conditional probabilities then they are not really learning both at the same time. They learn one, and then infer the other based on the first. It is one thing to "learn" that p(A|B) = p(B|A). The more interesting cases involve learning that the two probabilities are NOT equal.

The focus of the current report is young children's learning. One hypothesis is that young children learn general associations between features, rather than specific conditional probabilities. For example from experience with the environments represented in Figure 1, young children might learn that red "goes with" loud, and blue "goes with" quiet. They form an overall, gist, impression of the relation (Reyna & Brainerd, 1994). The key distinction between an association and a set of conditional probabilities is that associations are symmetric. The correlation between A and B is the same as the correlation between B and A. Thus, if

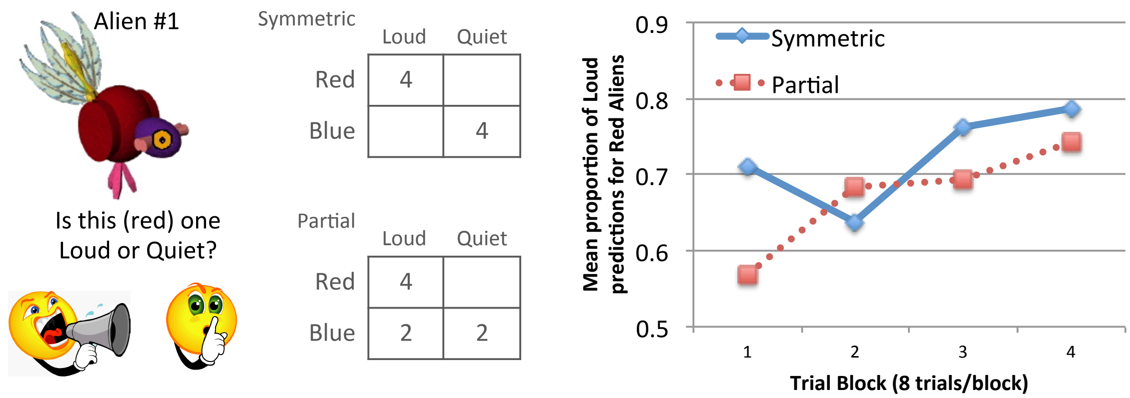

Figure 1: Distributions of Exemplars and Learning Rate in Experiment 1
Graph shows mean proportions of predictions that Red aliens will be Loud. In both Symmetric and Partial conditions p(Loud|Red) = 1

children learn a forward relation between two features (e.g., loud things are red) as an association they should expect the same relation to hold in the inverse (e.g., red things are loud). In contrast, conditional probabilities need not be symmetric: p(Loud|Red) ≠ p(Red|Loud). Thus the specific focus of the current study is the distinction between forward and inverse conditional probabilities. Are they learned separately or together? Does learning one automatically inform people about the other?

Kalish (2010) found that preschool-aged children are better able to make conditional predictions (e.g., "Is this red thing loud or quiet?") given a perfect correlation between features than given a perfect conditional probability. That is, children learned "All red things are loud" more easily when all loud things were also red. Similarly, preschool-aged children did not distinguish the implications of counter examples. Encounters with loud blue things led to chance-level estimates of both p(red|loud) and p(loud|red) (Kalish, Kim, & Young, 2011). In these tasks, children encountered both features simultaneously (did not predict one feature from the other). The experience encouraged learning both forward and inverse probabilities together. Perhaps when learning one probability was difficult (because of inconsistent examples) children had difficulty learning the other as well. This raises the question of what would happen if children were trained in one direction only. Would they learn the inverse as well? And would the reliability of the inverse affect learning of the forward probability? That is, is it difficult to learn that p(loud|red) = 1 when p(red|loud) ≠ 1?

Experiment 1

Methods Forty-two 4- to 5-year-old children (Mean= 4:9, range 4:0-5:6) participated. Twenty-seven children participated in the Partial condition, 15 in the Symmetric condition. More children were included in the Partial condition because of the expectation that this condition might prove difficulty. This condition was over-sampled so non-learners could be excluded if necessary. Children were recruited from preschools serving a largely middle-class population in Madison, WI.

Children were invited to play a computer game about some space explorers who have discovered aliens on a new planet. The child's job was to help the explorers learn about the aliens. An experimenter led the participant through the task, reading all text, explaining all pictures, and (usually) making all responses based on the child's verbal instructions. Aliens varied on two binary dimensions: Red or Blue, and Loud or Quiet. In the Training phase of the experiment children guessed the sound made by 32 aliens, 16 red and 16 blue. Each trial began with presentation of an alien paired with two choices (Loud or Quiet). When a child selected an option they received immediate corrective feedback: "Right this one is X" or "Wrong, this one is actually Y". All red aliens were visually identical, as were all blue. Aliens were distinguished by number (e.g., "Alien number 1"). In the Symmetric condition there was a perfect correlation between color and sound: All red aliens were loud, and all blue were quiet. In the Partial condition all red aliens were loud, but half the blue were loud and half were quiet. Thus there was a perfectly predictive relation for two features (p(loud|red) and p(blue|quiet) both = 1) but not for the other two (p(red|loud) = .67 and p(quiet|blue) = .5, see Figure 1). Order of presentation was randomized within blocks of 8 trials: Every block of 8 trials had the specified frequency distribution.

Following the Training phase, children made (and evaluated) a series of conditional predictions in the Testing phase. Each trial began with a cartoon image of a child. The experimenter explained that it was this child's turn to guess about the aliens and that the participant could help him/her. An image representing an alien known to have one feature (red, blue, loud, quiet) appeared. Participants indicated what the new child should guess (e.g., loud or quiet for a red alien). The cartoon child then indicated his/her guess. The participant then rated the cartoon child's guess on a 20-point scale by moving a scroll bar with endpoints labeled "definitely wrong" to "definitely right". There were 12 conditional predictions: two each of red and blue (the "forward" predictions), and four each of loud and quiet (the "inverse" predictions). Cartoon children guessed each feature half the time (e.g., one child guessed loud given red, the other blue). These guesses were not contingent on the participant's responses. Prediction requests appeared in random order.

Results & Discussion In the Training phase, one relation was perfectly predictable in both the Symmetric and the Partial conditions, p(loud|red). Figure 1 presents the mean proportion of correct predictions that red aliens would be loud in the training phases, by 8-trial block. Although children seemed to learn faster in the Symmetric condition, there were no statistically reliable differences during any block, or when looking at overall performance. In part, this null result is due to high variability in children's learning. Eleven of the 27 participants averaged below 60% correct in the Partial condition. In the Symmetric condition, the rate was six of 15. Some children failed to learn anything. If the analysis is restricted to learners (>60% correct) then children in the Symmetric condition outperformed those in the Partial condition, $t(24) = 2.4$, $p<.05$, but only in the initial block. Overall, the structure of the Partial condition may have made it slightly more difficult to learn the predictive relation, but not by much. Children learned somewhat faster in the Symmetric condition, but there were no overall differences, and about the same proportion failed to learn in either condition. These results suggest that children may be learning p(Loud|Red) independently of p(Red|Loud). Children learned that p(Loud|Red) was high despite variation in p(Red|Loud). Note that this effect was not simply due to children making the inverse fallacy. Children in the Partial condition guessed randomly for blue aliens (M = .51 loud predictions). That is, they recognized that blue aliens could be either loud or quiet. Thus they did not seem to treat p(Loud|Red) as equal to p(Red|Loud). They did recognize that there were loud blue aliens. At least from these data, the reliability of the inverse did not seem to affect learning of the forward probability.

In the interests of space, we just consider participants' evaluations of others' predictions (and just those participants who did learn in the Training phase). The patterns in participants' own predictions in the Testing phase were roughly similar. Panel 1 of Figure 2 shows ratings of "forward" predictions. Children used the relations in the Training phase to evaluate character's predictions. In the Symmetric condition, children thought it was better to guess Loud than Quiet given Red, and better to guess Quiet than Loud given Blue. In the Partial condition children also preferred predictions of Loud given Red, but showed no preference given Blue.

In the Symmetric condition, children's ratings of "inverse" predictions were also consistent with the relations in the Training phase. They thought a Loud object should be predicted to be Red, and a Quiet object predicted to be Blue. In the Partial condition, children thought that Red was a better guess for a Loud alien than Blue. This preference is consistent with the conditional probability in the Training phase, p(red|loud)= .67. However, children showed no reliable preference for predictions about Quiet aliens, even though all the Quiet aliens previously encountered had been blue. One interpretation of this pattern of results is that children were not actually learning the inverse conditional probabilities from experience in the Training phase. Rather, they only learned the forward conditional probabilities, and then used those to make inverse predictions. That is, children learned "red things are loud", and on that basis, predicted "loud things would be red." In the Symmetric condition they also learned "blue things are quiet" and so predicted, "quiet things are blue." In the Partial condition, children did not learn "blue things are quiet", so they did not predict, "quiet things are blue." Although children made reliable inverse predictions, the basis for those predictions

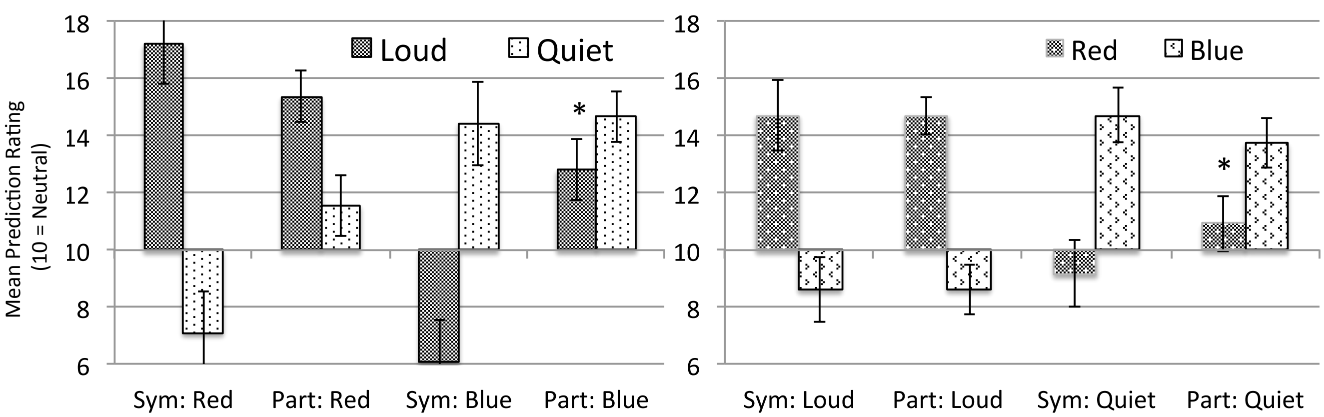

Figure 2: Mean Evaluations of Others' Guesses, Experiment 1
X axis represents the condition and given feature. For example, "Sym:Red" is ratings of Loud and Quiet predictions for a Red alien in the Symmetric condition. Bars represent mean ratings of predictions of each feature . *indicates that mean ratings of the two predicted features in a pair were not significantly different ($p<.05$, paired t-tests). Error bars represent 1 standard error.

seemed to be the forward probabilities they had learned. An illustration of this result is that the left and right panels of Figure 2 are very similar. Thus the conclusion is that children did not learn inverse probabilities from experience making forward probability judgments.

Experiment 2

Young children seemed to learn the conditional probabilities they were trained on but not the inverse probabilities. Would adults learn both? We expected ceiling performance from adults in the Symmetric condition, thus Experiment 2 presented two conditions involving the Partial distribution. The "Color-to-Sound" condition was the same as Experiment 1. The "Sound-to-Color" presented the same distribution of exemplars, but during the Training phase people predicted alien color (red or blue) from sound (loud or quiet).

Methods Thirty college students at UW-Madison participated for course credit. Fifteen participated in the "Color-to-Sound" (CtS) condition, which was identical to the Partial condition from Experiment 1. Fifteen participated in the "Sound-to-Color" (StC), which varied only in the Training phase. StC participants were told the sound an alien made (loud or quiet) and asked to predict its color. Upon making a guess, the picture of the alien appeared and participants received corrective feedback. Critically, participants encountered the same exemplars in both conditions. The only difference was which probabilities were forward and which were inverse. In the CtS condition, p(sound|color) was forward and p(color|sound) was inverse. Forward conditional probabilities are those the participants made during the training phase (e.g., predicting sound from color). Inverse probabilities are those the participants were not trained to make (e.g., they did not predict color from sound in the training phase). In the StC condition, p(color|sound) sound was forward and p(sound|color) inverse. Adults participated in the same alien explorer computer task as did young children in Experiment 1. The only procedural change from Experiment 1 was that adults were tested in groups working at individual computers.

Results & Discussion Figure 3 presents the mean ratings of others' guesses. Of interest is whether the direction of training made a difference. First, participants did learn the trained forward predictions. In the CtS condition participants judged predictions of Loud better than Quiet given Red, but not Blue. In IP predictions of Blue were better than Red given Quiet, and predictions of Red were better given Loud. These ratings are consistent with the actual distributions of exemplars. The same predictions received significantly different ratings when they appeared as inverse. While participants in CtS (forward) recognized that Loud or Quiet were equally good predictions about a Blue alien, participants in StC (inverse) rated Quiet as a significantly better prediction. The rating of predictions for Blue aliens differed significantly between conditions, $t(29) = 4.89$, $p<.001$. Similarly, in StC (forward) participants preferred a guess of Red given Loud, but recognized that Blue was plausible. In contrast, CtS participants (inverse) exaggerated the difference and rated Blue as a poor guess. Ratings of predictions for Loud aliens differed in the two conditions, $t(29) = 6.98$, $p<.001$.

The adults' pattern is roughly consistent with children's from Experiment 1. In both cases participants seemed to learn the forward conditional probabilities and use those to estimate inverse conditional probabilities. Adults in CtS learned that red aliens were loud (forward). They then judged that loud things would be red (inverse). Adults in StC learned that quiet things were blue (forward). They then judged that blue things would be quiet (inverse).

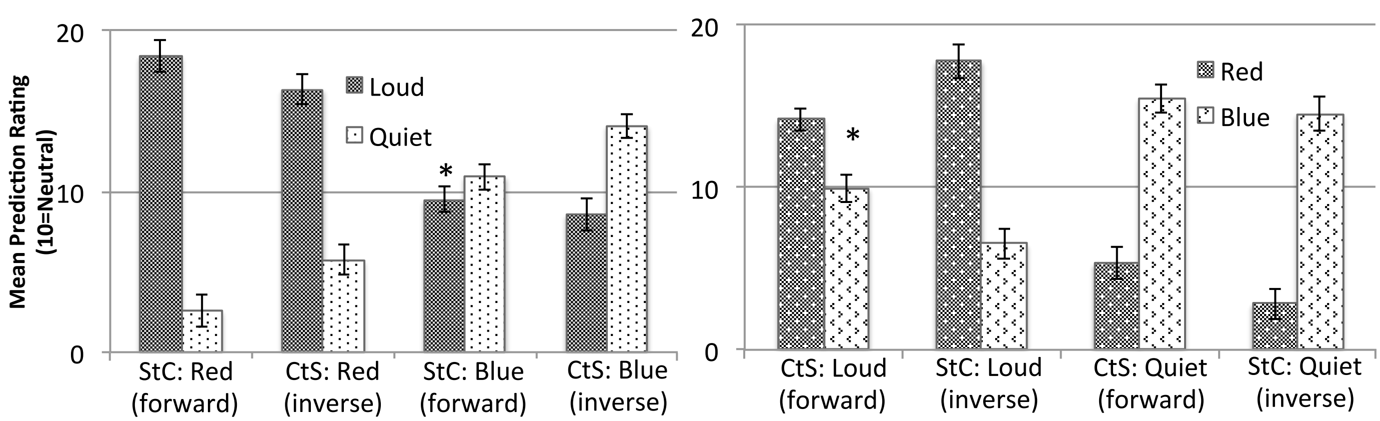

Figure 3: Mean Evaluations of Others' Predictions, Experiment 2
X axis represents the condition and given feature. For example, "StC: Red" is ratings of Loud and Quiet predictions for a Red alien in the Sound-to-Color training condition. Bars represent mean ratings of the predicted features. *indicates that mean ratings of the two predicted features in a pair were not significantly different ($p<.05$, paired t-tests). Error bars represent 1 standard error.

Conclusions

The results of the two experiments are generally consistent with findings from the categorization and perceptual learning literature: Participants learn the distinctions they are trained on (forward). Taught to predict sound from color participants formed accurate representations of p(loud|red) and p(loud|blue). During the course of learning, participants were exposed to exemplars that also provided information about the inverse conditional probabilities, p(red|loud) and p(red|quiet). However, people did not seem to use this information, or did not use it as effectively as the information about the trained discriminations. When called upon to use the conditional probabilities not directly trained, people seemed to "work backwards" from the ones they were trained on. In effect, both adults and young children showed the inverse fallacy (Villejoubert & Mandel, 2002), they used p(A|B) to estimate p(B|A). They relied on this fallacy rather than actually learning p(B|A) as they were learning p(A|B).

The current study has some important limitations. First the sample sizes were small, and there were a large number of non-learners among children. Second, the measures of participants' probability judgments were very imprecise. This is likely an unavoidable limit with children, but probability estimates could be measured more directly with adults. In any case it would be useful to ask for frequency estimates. Previous research has shown that young children may have accurate memories for relative frequencies of exemplars, even as their predictions/probability estimates seem at odds with these frequencies (Kalish, 2010). Finally, the differences between forward and inverse conditional probabilities were rather subtle. The relations between learning forward and inverses might be different when probabilities reverse (e.g., p(A|B)>.5, p(B|A)<.5). The working assumption has been that young children are sensitive to only very large qualitative differences in probabilities (e.g., all, none, unpredictable), but this remains to be explored.

One of the big remaining questions is whether forward and inverse probabilities are ever learned together. Are there some learning tasks that promote both? One way to characterize the pattern observed is that people get very good at solving the problem posed to them, but at a cost of not forming more general representations that could be useful for solving other problems. Perhaps less "goal-directed" training would support generative as well as discriminative learning. The previous studies, in which children simply encountered exemplars (e.g., saw red and loud at the same time), suggest that forward and inverse conditional probabilities do affect each other. Of course, the flip side to this question is the fact that experience is not always informative about all probabilities. For example, depending on how the training exemplars have been selected it may be only possible to generalize forward probabilities to a larger population and not inverses. In general, it seems most useful to learn both forward and inverse probabilities. We want to learn not only "what tells me something is a dog?" but also "what does knowing something is a dog tell me?" However, we know little about the conditions under which children and adults learn these relations together or separately.

Acknowledgments

Thanks to Andrew Young, Annie Riggs, and Jordan Thevenow-Harrison for help with design and data collection. This research was supported by a grant from the NSF (DRM/DLS) 0745423.

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