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One piece at a time: Learning complex rules through self-directed sampling

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Abstract

Self-directed information sampling—the ability to collect information that one expects to be useful—has been shown to improve the efficiency of concept acquisition for both human and machine learners. However, little is known about how people decide which information is worth learning about. In this study, we examine self-directed learning in a relatively complex rule learning task that gave participants the ability to "design and test" stimuli they wanted to learn about. On a subset of trials we recorded participants' uncertainty about how to classify the item they had just designed. Analyses of these uncertainty judgments show that people prefer gathering information about items that help refine one rule at a time (i.e., those that fall close to a pairwise category "margin") rather than items that have the highest overall uncertainty across all relevant hypotheses or rules. Our results give new insight into how people gather information to test currently entertained hypotheses in complex problem solving tasks.
Keywords: self-directed learning, categorization, active learning, information search, rule learning

Introduction

A cornerstone of many educational philosophies is that people learn more effectively when they direct or control their own learning experiences (Bruner, 1961). While there are many ways that control might influence learning, an important factor is the ability to actively gather information that one considers potentially useful while avoiding data that is potentially redundant, a behavior referred to as *self-directed sampling* (Gureckis & Markant, in revision).

One recent study directly examined the interaction of self-directed information sampling and learning (Markant & Gureckis, 2010, in revision). In this study, people attempted to learn simple dichotomous categories of objects that varied along two perceptual dimensions (circles that differed in size and the orientation of a central line segment, see Figure 1). In contrast to traditional categorization training procedures, we allowed participants to "design" stimuli that they wanted to learn more about on each trial. Like a child asking their parent to label an unfamiliar object, self-directed "designing" or "sampling" allows the learner to focus on information they want rather than be limited by the flow of passive experience.

The major finding from this study was that for simple unidimensional rules, self-directed learners acquired the correct category rule faster than "passive" participants who were provided samples from an experimenter-defined distribution. In addition, self-directed learners out-performed a set of "yoked" learners who viewed the same examples but did not get to make information sampling decisions themselves (consistent with studies of causal learning with similar yoked comparisons, Lagnado and Sloman, 2004; Sobel and Kushnir, 2006).

How do people make information sampling decisions?

In light of evidence that self-directed sampling can speed learning, it is important to understand *how* people decide what data to collect. Given a potential observation, what information do people rely on to decide if it will be useful?

One aspect that may help explain a person's decision to sample an item is their uncertainty in how to classify it (or more generally, their uncertainty about the outcome of any test performed on the item). Intuitively, a self-directed learner should direct their attention to items that are high in uncertainty while ignoring items that can already be confidently classified or predicted. Consistent with this strategy, the pattern of stimuli sampled by self-directed learners in our previous study (see Figure 1) revealed that participants systematically directed their samples toward the category boundary as the task progressed. Intuitively, the learner is mostly likely to be uncertain about these items (e.g., most of the errors in classification occur near the category boundary).

In the current study, we examine how subjective uncertainty in how to classify an item can be used to predict whether or not it is sampled. We begin by presenting three psychologically motivated proposals for how sampling decisions relate to judgments of uncertainty, and then test these models in a new experiment that extends the "self-directed" classification learning paradigm used in Markant and Gureckis (2010). Our results highlight the need for models of sampling behavior that go beyond monolithic measures of information value to consider how people collect and use data during the sequential learning of concepts.

Three models for relating uncertainty and information sampling decisions

The following sections lay out three possible ways in which uncertainty might guide information sampling decisions.

Model 1: Sampling to reduce global uncertainty

Prior work on how people gather information has often focused on diagnostic reasoning problems in which the learner is given a set of alternatives (e.g., different diseases) and asked to sample observable features (e.g., symptoms) in order to identify the true diagnosis (Nelson, McKenzie, Cottrell, & Sejnowski, 2010; Skov & Sherman, 1986; Trope & Bassok, 1982). From a computational perspective, various authors have proposed *sampling norms* that attempt to predict information sampling decisions based on a learner's representation of the task (Nelson, 2005; Nelson et al., 2010; Oaksford & Chater,

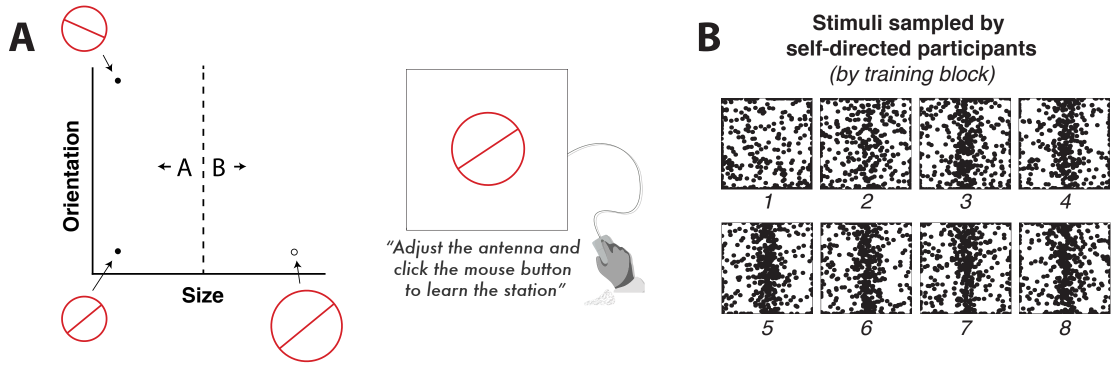

Figure 1: **A:** Abstract stimulus space used in Markant and Gureckis (2010, in review) and which is adapted for the current study. Stimuli were circles which varied in size and orientation of a central diagonal. In Markant and Gureckis, these objects were assigned to one of two categories ("A" or "B"). Participants "designed" a stimulus they wanted to learn about using the mouse. Clicking the mouse button reveals the category membership of the item. **B:** The pattern of sampling behavior observed by self-directed learners in Markant and Gureckis (2010) across eight training blocks. Each dot represents a single stimulus which was selected by a participant. In the first block, participants distributed their samples widely over the entire stimulus space but then gradually focused their choices on the region surrounding the category boundary.

1994). Much of this work has focused on what we will call "prospective" models (e.g., *probability gain*, *information gain*, etc.) that estimate the expected drop in uncertainty that will result from making an observation (taking into account all possible outcomes). While in many contexts these models make similar predictions, Nelson et al. (2010) designed a diagnostic reasoning task which found that participants' choices were best fit by *probability gain*, which values a potential observation according to how much it increases the chance of classifying an item correctly.

In the context of learning a classification rule, this approach is consistent with a preference for sampling items that the learner is least certain about how to classify. Assume that for a given stimulus $x = (f_1...f_d)$ with d observable features the learner represents the probability that x is a member of each possible category y in the distribution $P(y|x)$. We can then define the *least certain* measure as:

$$LC(x) = 1 - max(P(y|x)) \tag{1}$$

for all stimuli x. Note that there are alternative norms that make similar predictions to *least certain*, such as using the Shannon entropy of the marginal distribution to calculate uncertainty (see Settles, 2009 for a review). Regardless of the particular form, the important property of this approach is that the most valuable observation is always an item that is considered equally likely to belong to all possible categories. In general, choosing items which maximize $LC(x)$ should convey the greatest amount of information to the learner.

Model 2: Isolating individual rule components through margin sampling

While focusing on items that are the most "globally" uncertain or unpredictable seems intuitively useful, there is reason to expect that it may not be the sampling strategy humans use, particularly when learning in complex, multivariate environments. One natural strategy, not captured by *least certain*, might be to decompose a complex task into a series of simpler problems. For example, when multiple features may be related to an outcome, a learner might choose to hold one feature constant while varying the other across multiple samples (Rottman & Keil, 2012). Such a strategy is related to the "control of variables" strategy which is essential to scientific reasoning. Isolating variables often helps people to more efficiently search the space of potential hypotheses (Klahr & Dunbar, 1988) and is a key part of "learning to learn" about complex concepts (Kuhn & Dean, 2005). In an experiment similar to that of Markant and Gureckis (2010), Avrahami et al. (1997) had participants choose samples to teach a partner about a single-dimensional rule and found that the "teachers" frequently used a strategy of isolating individual features. Moreover, their students learned better from this strategy than when given items that were closest to the category boundary.

We can formalize the strategy of focusing on separate components in a sampling model that values uncertainty about any individual boundary between only two categories. *Label margin* predicts that the learner will prefer instances for which the likelihood of any two categories is similar, independent of any other categories. For example, when there are three categories and the marginal distribution is defined as $P(y|x) = (p_1, p_2, p_3)$:

$$LM(x) = 1 - min[|p_1 - p_2|, |p_1 - p_3|, |p_2 - p_3|] \tag{2}$$

Critically, label margin does *not* preferentially select items for which the learner is globally uncertain. Instead, by this approach, a learning problem is decomposed into simpler problems and items are selected which are expected to resolve uncertainty about the sub-components.

Model 3: Seeking confirmation

While the previous two models propose that people search for uncertain items, previous work on hypothesis testing suggests that people may prefer items that they already know how to

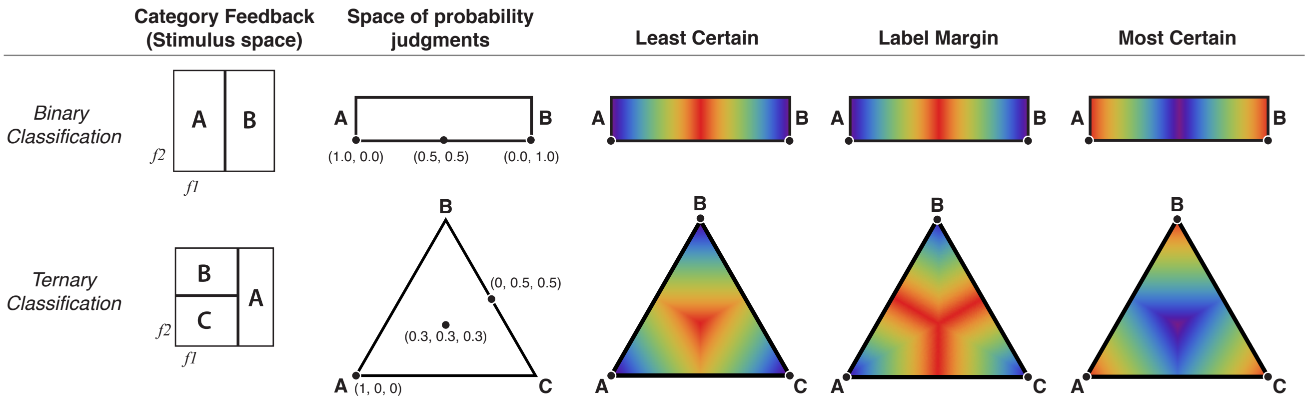

Figure 2: Comparing predictions of sampling norms (red = more highly valued choices, blue = less valued choices) **Top:** For a binary classification problem, a new observation in stimulus space will correspond to a location on the probability judgment scale, where the leftmost point reflects confidence that the observation will be classified "A" and the rightmost point reflects confidence it will be classified "B". For the binary problem, the predictions of *least certain* and *label margin* are identical. **Bottom:** In a ternary classification problem, an item in stimulus space will correspond to a location in the 3-category simplex depending on the learner's uncertainty. Here the predictions of *least certain* and *label margin* diverge, allowing us to test which of the two models better account for sampling behavior.

classify. For example, people have a well-documented bias toward seeking positive evidence of the hypothesis they are considering (Klayman & Ha, 1989; Wason, 1960), a strategy that in certain conditions is aligned with the goals of maximizing uncertainty reduction (Austerweil & Griffiths, 2011; Navarro & Perfors, 2011; Nelson & Movellan, 2001). To quantify this strategy, we define the *most certain* measure as:

$$MC(x) = max(P(y|x)) \tag{3}$$

The predictions of this model directly contrast those of *least certain*, with the highest value assigned to items that can already be classified with confidence. One may also think of the *most certain* measure as instantiating confirmation bias—it shows a preference for items for which the learner has a strong prediction about the category label.

Empirically distinguishing these alternatives

Given these various approaches, a key question is if they are distinguishable based on empirical data. The predictions of each model are shown for a binary classification problem (like the task used in Markant and Gureckis, 2010) in the top row of Figure 2. Each heatmap describes the value assigned to a potential observation depending on the learner's uncertainty in how to classify it. For example, an item that can be confidently classified (e.g., $p(y|x) = (1, 0)$) would be assigned a high value by *most certain* and a low value by *least certain*. Note that for the binary classification problem, *least certain* and *label margin* make identical predictions about how items will be valued (i.e., items close to the center of the space are preferred), making it impossible to separately test these models. However, an interesting observation made by Settles (2009) is that the predictions of these models strongly diverge when considering more complex categorization tasks. For example, in a ternary classification task (see bottom row of Figure 2), *label margin* assigns the maximum value to any items for which one category is highly unlikely but the learner is uncertain about the other two (shown in Figure 2 by the high predicted value along the radial axes of the simplex, including the midpoints of each edge). In short, this model predicts that samples are likely to be allocated close to any boundary (i.e., "margin") between two categories. In contrast, *least certain* predicts sampling close to the *junction* of the category boundaries, where all three classes are likely.

Overview of the current study

The design of our experiment capitalizes on the distinction described in the previous section by extending the paradigm used in Markant and Gureckis (2010) to a ternary classification problem. In the experiment participants collect data by sampling new instances and receiving feedback about their category membership. As shown above, using the ternary classification problem allows us to separate the predictions of the three sampling models, two of which were confounded in our previous design. In order to obtain an estimate of the learner's uncertainty at any point in time, on a subset of sampling trials participants report how likely they believe the instance they created will belong to each of the three categories (before receiving feedback). The goal of our analysis is to use these subjective judgments to test which model provides the best account of their sampling decisions.

Experiment

Participants Fifty-seven undergraduates at New York University participated in the study for course credit. The experiment was run on standard Macintosh computers in a single 1-hour session.

Stimuli Stimuli were defined by a two-dimensional continuous-valued feature space corresponding to the size (radius) of a circle and the angle of a central diameter. These feature values were mapped to a limited range of orientations and sizes on the display. Orientation could vary over only 150 degrees, ensuring that a full rotation of the stimulus was not possible. The two halves of the central diameter were given different colors, further reducing the perceptual similarity of stimuli at the two extremes of the orientation dimension.

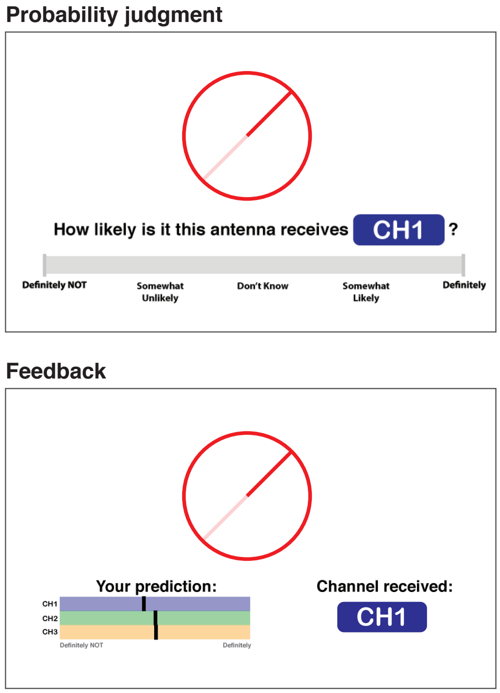

Figure 3: **Top:** Probability judgments were entered by clicking on a scale for each of the three categories (CH1, CH2, and CH3). **Bottom:** Probability judgments were displayed alongside the category label during feedback.

The minimum radius and orientation were randomized so that the boundary between the categories corresponded to a unique boundary in perceptual space for each participant. A total of 256 stimuli were sampled from a uniform grid over the feature space and used as test items for all participants, presented in random order.

The category label associated with each stimulus was deterministically defined by a conjunctive, ternary classification rule of the form shown in Figure 2. In addition to the structure that is shown, three more rules were created through different rotations (90, 180, and 270 degrees) of the same boundaries in stimulus space. Each participant was randomly assigned to one of the four rules.

Procedure Participants were instructed that the stimuli in the experiment were television "loop antennas" and that each unique antenna received one of three channels (CH1, CH2, or CH3). Their goal was to learn the difference between the three types of antennas so that they could correctly classify new antennas during the test blocks. Participants were told that the experiment would end when they correctly classified 20 consecutive test items. If a participant failed to reach that goal the experiment ended after 32 blocks or at the end of an hour (whichever occurred first).

Training Trials. Participants "designed" antennas by adjusting the size and orientation and receiving feedback about which channel was received. They were instructed that they should design antennas they thought were useful and that would help them to predict the TV channel for other designs they had not yet tested.

Each trial began with the presentation of a randomly generated antenna. Participants then adjusted the size and orientation by moving the mouse from left to right while holding either the 'Z' or 'X' key, respectively. Only one dimension could be changed at a time, but participants could make any number of changes and were self-paced. When the stimulus was the desired size and orientation, they pressed the mouse button to reveal the channel received, displayed above the stimulus for 4 seconds.

Probability judgments. Half of the training trials in each block were randomly selected to include probability judgments. On these trials, after participants had designed an antenna but before the category label was shown, they judged the likelihood that the antenna would receive each of the three channels using a sequence of rating scales (shown in Figure 3). The three scales were presented independently such that only one was visible at a time. When each scale appeared, the participant clicked on a location in the scale according to their belief that the antenna they had designed would receive that channel. A response was required for each scale, and there was no time limit for entering the response. The initial position of the mouse cursor within each scale was randomized, allowing us to record whether responses were influenced by the starting position. After probability judgments were recorded, they were displayed alongside the category label for the same duration as in regular training trials. This allowed the participant to evaluate the accuracy of their prediction given the true category label.

Test Trials. Each block of 6 training trials was followed by 8 test trials. On each test trial, a single item was presented in the center of the display and the participant classified the item according to the channel they believed it was most likely to receive. A response was required to complete the trial, and participants responded at their own pace. No feedback was provided on individual test trials. At the end of each block participants were told their accuracy during the block they just completed, as well as the number of consecutive correct responses.

Results

Three participants were excluded from analysis for failing to complete the task, leaving $N = 54$. Thirty-one people reached the goal of correctly classifying 20 items in a row. However, there were a number of additional people who achieved similarly high rates of accuracy. For each subject we computed a moving average of their classification accuracy with a window of 3 blocks, and found 43 people for whom this average exceeded 83% at any point in the experiment (i.e., they correctly classified 20 of 24 items within any three consecutive blocks).

Probability judgments. On half of participants' sampling trials they judged how likely the stimulus they selected belonged to each of the three categories, resulting in three values between 0 and 1. In order to verify that participants were not simply responding based on the position of the cursor, for each rating we measured the difference between the initial (random) position and the participant's response. One participant was excluded from further analysis because the majority of their ratings (82%) did not change by more than 0.01% from the initial values (for the remaining subjects, the average proportion of samples that met the same condition was $M = 0.04$, $SD = 0.05$).

Fitting the alternative sampling models. Our first goal was to assess the overall fit of the three sampling models to each participant's full set of probability judgments. For each model we computed the normalizing constant necessary to define the probability density function. Each triplet of ratings was normalized so that they summed to one. We then calculated the log-likelihood of each judgment made by a participant and summed across all trials to get an overall score for each model.

Classifying participants according to the model with the highest log-likelihood, we found that 3 people were best-fit by *least certain*, 25 people were best-fit by *label margin*, and the

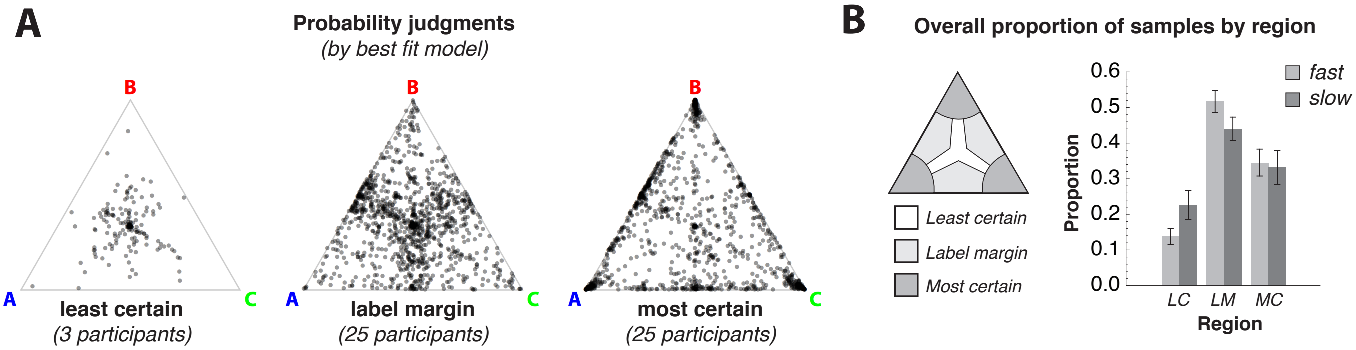

Figure 4: **A:** Probability judgments are plotted using the 3-category simplex for participants best-fit by each of the three models (see Figure 2 for reference). Each point represents a single judgment after normalization. **B:** Each judgment was classified according to the model assigning it the highest likelihood, effectively dividing the probability space into three regions. Participants were divided into two groups based on the number blocks they required to complete the task ("fast" and "slow"), and the relative frequency of sampling in each region is shown at right.

remaining 25 people were best-fit by *most certain*. Judgments made by participants, separated by the best-fitting model, are plotted using the 3-category simplex in Figure 4A, with each point a single sample chosen by a participant. A higher density of points reflects an increased tendency (as a group) to sample stimuli in a given region of probability space. Upon visual inspection, the overall pattern for each group corresponds to the predictions of the best-fitting model (Figure 2).

Relating sampling decisions to learning efficiency. We next tested whether a participant's overall success at learning the target concept was related to the sampling behavior reflected in their probability judgments. Our approach was to divide participants into two groups based on the number of blocks they required to complete the task. We performed a median split on the number of blocks (median = 16) to create a group of "fast" ($N = 26$) and "slow" ($N = 27$) learners. With respect to overall model fits, however, there was no difference in the proportion of participants best fit by each model between groups (fast learners: $N_{LC} = 1$, $N_{LM} = 12$, $N_{MC} = 13$; slow learners: $N_{LC} = 2$, $N_{LM} = 13$, $N_{MC} = 12$).

While overall model fits provide a measure of each participant's sampling behavior in general, inspection of the data showed that most subjects had relatively mixed strategies. For example, a participant best-fit by *most certain* may have made a number of judgments consistent with *label margin*. Given this heterogeneity, we classified individual probability judgments according to the model that assigned it the highest likelihood, effectively dividing the probability space into three regions corresponding to each model (Figure 4B). Multinomial logistic regression was used to test for differences between the relative frequency with which fast and slow learners sampled in each of the three regions. Overall frequency differed significantly between the two groups ($\chi^2 = 24.7, df = 2, p < .001$). Post-hoc tests showed that fast learners sampled somewhat more frequently in the *label margin* region ($t(51) = 1.68$, $p = .09$) and less frequently in the *least certain* region overall ($t(51) = -1.91$, $p = 0.06$), suggesting that this pattern of sampling was related to success in the task.

Discussion

Theories of rational information acquisition propose that the decision to make an observation is related to the amount of information it conveys (Nelson, 2005; Oaksford & Chater, 1994). Sampling norms such as *probability gain* prospectively evaluate the change in uncertainty that is expected to occur following an observation, and a rational learner should choose the data that maximizes that measure. Our results illustrate the relative inadequacy of these models when applied to even a basic rule learning task. Very few of our participants were best fit assuming they preferentially selected observations they were least certain about. In addition, the heterogeneity of participants' sampling strategies is a noteworthy finding. For example, about 20% of samples in the first 4 blocks were "confirmatory" (i.e., data that the learner could already classify with relative confidence), and overall there was no difference in the frequency of this kind of sampling between fast and slow learners. Confirmatory sampling could serve a number of purposes, including helping to organize the representation of a rule in mind (Mathy & Feldman, 2009) or to facilitate comparisons between successive observations, but further work is required to understand its exact role in this task.

Margin sampling vs. information maximization A second way in which participants' behavior diverged from the "rational" prediction was their preference for samples that fell along the category margins over items that offered information about all three categories (i.e., those located at the junction of the boundaries). From the perspective of an ideal observer (i.e., a model that can represent the full set of possible hypotheses and use Bayesian inference to update its beliefs), the most efficient strategy is to maximize the amount of information contained in each observation. Sampling at the category margins should only decrease the efficiency of learning since it will tend to rule out a smaller number of hypotheses, which raises the question of why this kind of behavior was so common in our task.

In our discussion we motivated the margin sampling model by noting that people might decompose a complex problem into simpler pieces. The use of such a strategy may reflect a participant's limited ability to simultaneously represent all possible alternatives and to remember prior observations. Thus, margin sampling may reflect an adaptation whereby people isolate individual components to learn in succession. Separately testing the role of different features is an important part of scientific thinking in general (Klahr & Dunbar, 1988; Kuhn & Dean, 2005), particularly when intervention is necessary to remove the effect of confounding variables. Importantly, our results do not reveal the particular cognitive processes underlying participants' decisions, but merely provide a descriptive account of their overall behavior. Nonetheless, they provide a useful constraint for theories of information sampling, particularly when applied to more complex tasks that involve sequential learning and memory demands.

Measuring subjective uncertainty. It is important to consider that the probability judgments we collected provide an incomplete picture of participants' uncertainty over the course of the task. Although we found some evidence that fast and slow learners differed in the kinds of samples they collected, because uncertainty judgments were assessed on only half of sampling trials it is difficult to draw strong conclusions about the impact of those samples on classification accuracy. Moreover, we cannot be sure that the judgments reported by participants accurately reflected their subjective belief since there were no costs for failing to report accurately[1]. These issues arise whenever considering sampling models based on a learner's subjective uncertainty rather than objective measures of value such as *information gain*, and as such present an important challenge to be addressed in future work.

Conclusion

Past accounts of information sampling have suggested that a single normative model might account for people's decisions across many learning problems, and that people tend to seek out data that lead to the greatest reduction in uncertainty. In contrast, we found little evidence of a single sampling norm that was consistently applied across individuals. Instead, participants' sampling choices seem to reflect ongoing aspects of constructive problem solving. Our approach highlights the need for theories of self-directed learning to move beyond individual measures of information value to capture interactions with task demands and cognitive constraints.

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[1]Two aspects of our procedure may lessen the impact of this concern. First, we were able to measure a failure to respond by randomly initializing the cursor position before each rating, and found that people changed the position in about 95% of ratings. Second, because judgments were displayed alongside the category label feedback, participants may have been encouraged to respond accurately in order to facilitate processing of that feedback

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