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An Approach to Reducing Bus Bunching

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An Approach to Reducing Bus Bunching

by

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University of California, Berkeley

An Approach to Reducing Bus Bunching

Abstract

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by

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Doctor of Philosophy in Engineering

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The tendency of buses to bunch is a problem that was defined almost 50 years ago. Since then, there has been a significant amount of work done on the problem; however, the tendency of the current literature is either to only focus on the surface causes or to rely on simulation to create results instead of model formulation. With GPS installed on many buses throughout the world, the data is only being used for monitoring and informing the user. This research proposes a new approach to solving the problem that uses the GPS data to directly counteract the cause of the bunching by allowing the buses to cooperate with each other and determine their speed based on relative position. A continuum approximation model is presented as a tool to systematically analyze the behavior of the system and test the proposed control. In order to validate the model and the control, a simulation tool is used to model the system in a more realistic, discrete way. The control is shown to produce bounded deviations in spacing consistent with those predicted by the model. The resulting bus system will not bunch with only a small reduction in commercial speed.

To all my parents,

for your constant love and support through every joy and hardship

you made me who I am today

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Chapter 1

Introduction

Modern transportation provides many modes as options for people's daily travel needs. Private modes (personal car, bicycle, walking) share similar features in that the user defines most aspects of a trip and is responsible for the full operational and maintenance cost of the mode. Public transportation is a unique mode in that the operational and maintenance cost is shared by all users as well as subsidized in most cases by the government. This shared cost creates a situation where the stakeholders have opposing objectives. The question is how to provide reasonable transportation service for a diverse set of users and desired trips without the cost of the service being too expensive. In order to do this, users' trips are consolidated, both spatially along predefined bus routes and temporally at some frequency of vehicle dispatch.

The time it takes a transit vehicle to travel a route can be broken into two parts: time spent overcoming distance and time spent at stops. The time spent overcoming distance depends on the cruising speed of the vehicle, the speed at which it can move between stops. This depends both on the physical capabilities of the vehicle as well as any externalities that can impede its progress, such as traffic congestion and traffic signals. The time spent at stops can be broken into several parts. There is a fixed amount of time at each stop that must be spent decelerating to a stop, opening and closing the doors, and then accelerating back to cruising speed. There is also an amount of time spent allowing passengers to board and alight. The long-term average speed at which the vehicle can move, including both the time spent traveling and stopped, is called the commercial speed. This is the speed which transit schedules are based on and which a user experiences when traveling to a destination. Because there is an amount of time lost with each stop, the more densely the stops are placed, the slower the commercial speed of the mode will be. However, the total time users spend on their trip also depends on how long it takes them to access a transit stop from their origin, and how long it takes them to progress to their destination from a transit stop. If stops are located too far apart, this time can outweigh the time savings from having fewer stops. This is part of the trade-off that must be taken into account when designing a transit network. The other part of the trade-off is determining frequency of service. The headway, defined as the time between successive bus arrivals to a point in space, determines the number of users served by each vehicle and how long users must wait for a vehicle to arrive to a transit stop. Vehicles must be dispatched often enough that they do not become over crowded and users do not have to wait too long for one to arrive. However, more vehicles are required to provide more frequent service and each additional vehicle carries with it the cost of an operator as well as capital and maintenance costs.

All of these components are taken into account during the design process to provide a certain level of service to the user. However, the actual level of service users experience depends on the reliability of the system to operate as designed. Each component carries with it some level of reliability, and when taken as a whole they determine how well the system behaves. For example, having a dedicated right-of-way allows a vehicle to travel at a constant speed uninterrupted by externalities between stops, and having a fixed dwell time removes any randomness caused by random passenger arrivals at transit stops. Most heavy rail systems have both of these features and so under ordinary circumstances have very high reliability and schedule adherence. Most bus lines operate within general traffic and have dwell times based on the number of users at each bus stop (even skipping stops when there is no reason to stop). Because of this, bus transit can can have very low reliability.

1.1 The Bunching Problem

1.1.1 Why do buses bunch

In addition to being more susceptible to external disturbances, as the headways between buses change from the designed value these disturbances are magnified over time until buses can travel in pairs instead of evenly spaced. This effect is referred to as bunching. The main cause for bunching comes from the fact that the time a bus spends at a bus stop increases with the number of users that need to board and alight the bus. On most buses the number of users boarding tends to control the time spent at a stop as there is only one point of entry and users must interact with the driver to pay their fare. Additionally, the expected number of users waiting to board at any bus stop generally increases with the time between successive bus arrivals. These two properties in tandem cause a positive feedback effect. If a bus is delayed or slowed a small amount so that the number of passengers waiting at each bus stop is larger than expected, the bus will have to dwell longer, slowing it down further. Similarly, a bus that is momentarily sped up will arrive to a bus stop sooner, and have fewer passengers to board, speeding it up further. This effect grows exponentially over time and if the space between two buses starts decreasing, they will eventually have little or no space between them and start moving as one unit since they are serving the same demand. If the number of buses on a route is fixed for most of the day and the length of a route is constant, for every group of bunched buses, there will be locations along the route not served by buses for long periods of time.

1.1.2 What effect does it have

Effect on transit users

When determining door-to-door travel time there are several parts of a trip to consider. When using a private mode, individuals must access their vehicles, travel in their vehicles to a parking area close to their destination, and then progress to their destination.

In most cases, the access, travel, and egress times are able to be estimated with a reasonable margin of error and individuals can arrive at their destination at their desired time. When using public transit however there is an added component. Users are restricted to a predetermined set of arrival times based on the schedule and/or the frequency of service offered. Transit users must plan their trip taking this into account, and even if public transit vehicles run perfectly on schedule, the earliness time at their destination before the desired arrival time would be added to the time of the trip. In the case of constant headway service without a schedule when the time of arrival is not known, users must budget a full headway of waiting time to their expected trip time to be split between waiting for a vehicle and arriving early at their destination.

Some transit modes, like heavy rail, operate on a designated right-of-way and require strict centralized control for safety reasons. This allows deviations from the schedule during everyday operation to be very small. If users know they can rely on a schedule to accurately predict vehicle arrival they can plan their trip with very little wasted time. However, with buses and other modes that operate within general traffic there are very few if any controls to ensure schedule adherence. As a result, users must build in much more buffer time to their trip to guarantee arrival to their destination by the desired time.

In addition to the increased waiting time experienced by users of a bus system, there are other negative effects of bunching as well. Because demand served by a bus is proportional to the time between bus arrivals to each stop, many more passengers are served by late buses than by early buses. This compounds the frustration of having to wait longer for a bus with having to then travel on a crowded bus. These crowded buses running behind schedule also travel at a slower average speed because of the time spent boarding and alighting an increased number of passengers, causing slower travel times.

Effect on the provider and society

As more and more people move toward urban areas, taxing the existing infrastructure, the role of public transportation as an efficient means of mass transportation continues to increase. Additionally, with political pressure towards promoting more environmentally friendly transportation options there is a growing awareness of public transportation as a means to travel. There is a portion of the population that is captive and relies on public transportation, regardless of the level of service. However, beyond this, agencies that provide public transportation are not guaranteed a mode share. In order to get people to ride, transit must be fast, convenient, and reliable. Bus bunching negatively affects all three of these qualities. If a bus line has a reputation for being unreliable, that is a disincentive for users to ride the bus. Additionally, the commercial speed of a bus will decrease as it falls behind schedule and must bear more of the passenger load.

If not enough people ride buses, two things can happen. Either the buses operate with lower occupancy and higher fares, or service can be reduced. Since buses output more emissions than private vehicles, they will be more polluting per user than private vehicles if they are run at a low enough occupancy. This could nullify the environmental benefits of transit. If fares are raised or service is reduced, those with no other option than to ride the bus are negatively impacted and those who have a choice are more likely to choose a different mode. Because transit is funded in part by the government, there is also the issue of wasting resources to provide a poorly performing and largely unused service.

Finally, there is an effect on the bus drivers. Depending on how far behind schedule a bus is running, a driver may find their break time cut short or missed entirely. In addition, there is often hostility towards drivers generated when a bus arrives to a bus stop late. These can lead to dissatisfaction among the drivers and a hostile work environment.

1.2 Dissertation Overview

1.2.1 Main Contributions

The three main contributions of this research are: 1. To propose a control that will mitigate the cause of bus bunching; 2. To demonstrate the feasibility of the control; 3. To evaluate its performance.

1.2.2 Organization

The dissertation is organized as follows. Chapter 2 will discuss examples of the current state of practice for avoiding bus bunching as well as review research pertaining to bus bunching. Chapter 3 will present a framework for modeling the physics of bus behavior. This framework will be used to explain the phenomenon of bus bunching. Chapter 4 will propose a control for mitigating the cause of bus bunching. The framework presented in Chapter 3 will be used to model the control and calculate expected results. Validation will be provided through simulation. Chapter 5 will summarize the findings and discuss possible future work.

Chapter 2

Literature Review and Current Practices

After a transit route has been designed and implemented, such that stop locations and service frequencies are fixed, the only remaining objective for the transit agency should be to provide their users door-to-door service as reliably as possible. The relevant components of the trip are waiting time, travel time, and earliness. Lisco (1967) found that users value their time spent waiting at two to three times more than their time spent traveling. Mohring et al. (1987) expands this analysis by using ridership survey data from routes offering their users a wide range of cost and frequency trade-offs. They find that peak period users value their time spent waiting between 75%–130% of their household wage rate and during the off peak value their time spent waiting between 60%–120% of their household wage rate. They assume that nonwage earners make up most of the off peak demand concluding that even though nonwage earners don't value their waiting time as highly as wage earners, it still carries a high value. This reinforces the idea that minimizing user's waiting time should be a priority for transit providers.

2.1 Reducing Trip Time

When users plan a trip they must take into account the amount of time spent waiting, the amount of time spent traveling, and how early they will arrive at their destination. Users can minimize their total trip time by having accurate knowledge of when a bus will arrive at their origin and destination bus stops. Transit agencies are responsible for providing this information to users in some form.

2.1.1 Scheduled Bus Arrivals and Scheduled Headways

Most bus routes operate with predetermined time tables for each bus, available publicly online and at most bus stops as a matrix of locations and scheduled times of arrival at each location (e.g. actransit.org). From this a user can determine the expected time between successive buses as well as an expected travel time between two locations along a route. Many agencies provide online tools to aid users in planning trips (e.g. 511.org for the

SF Bay Area) and Google has developed a service (Google transit) that references available data from multiple transit agencies to aid users in planning trips across multiple modes.

Some rapid transit lines operate at constant headways with no published schedule but rather a planned time between successive bus arrivals. Users must then rely on the schedule of another parallel line or personal experience to determine travel time, though there are examples of published expected travel time for headway based routes (Transport for London). In order to avoid excessive waiting times, this strategy of running with constant headways instead of scheduled arrival times is only useful for high frequency routes when passengers arrive independently of expected bus arrival times. Arrivals have been empirically shown to be uniform on high frequency routes. Jolliffe and Hutchinson (1975) and Marguier and Ceder (1984) both determine this to be a frequency of at least a bus every 12-13 minutes. Bowman and Turnquist (1981) also find that with smaller headways, passenger arrival tends to be uniform.

2.1.2 Real Time Bus Location Information

Automatic Vehicle Location (AVL) devices have been used by transit agencies for decades with varying levels of technological complexity. Older systems relied on fixed location sensors that could track when a bus passed or would triangulate the location of the vehicle using radio signals. Recently, Global Positioning Systems (GPS) have been installed by many transit agencies around the world allowing them to continuously monitor the location of their buses. Depending on the agency, this information is publicly available both at bus stops and online for users. There have been several studies on predicting expected arrival times based on GPS information. Jula et al. (2008) state that under mild conditions, the errors in predicting travel times from GPS data are bounded. Jariasunant et al. (forthcoming) offer a trip planner that finds an optimal path for a user using predicted vehicle arrivals instead of scheduled arrivals and conclude that the accuracy of the travel time of the resulting trip is marginally improved. The effect on users' behavior of providing real time bus arrival information has also been examined. Hickman and Wilson (1995) simulate users who are given real time information of vehicle arrivals and travel times to determine their route choice across modes. Their findings show that with route choice the effect of real time information on door-to-door travel times and variability of trip times is small. Ridho and Sumi (2009) show however that without route choice, communicating real time bus arrival estimates to users via cellphones can reduce average waiting time by 44% and reduce total trip times by up to 19%.

2.2 Increasing Reliability

Providing reliable service is much more important than providing users with system information, though solutions are more complex. Bus bunching has been a well known problem as long as there has been high enough transit demand to cause it. The cause was first introduced by Newell and Potts (1964). As small random effects create deviations from the desired headways between buses, the errors will grow over time. A bus with a larger headway will on average have more passengers to board than a bus with a smaller headway.

Therefore a bus with a large headway will be delayed longer at each bus stop increasing the headway, and a bus with a small headway will be delayed less at each stop, decreasing the headway. This also means that there is a greater chance that a random user will have to wait for a bus with a larger headway than for one with a small headway. The result is that the expected waiting time for users increases proportionally with variance in headways, meaning that the more headways vary between buses, the longer the expected waiting time is for passengers.

2.2.1 Metrics of Reliability

Before the general goal of increasing reliability can be approached, a metric for reliability must be clearly defined. The two main indicators for bus performance are widely accepted to be on-time performance and service regularity. Nakanishi (1997) examines New York City's performance indicator program and what each indicator actually measures. The program defines on-time performance as the percentage of trips that depart from a time point within 5 minutes of the scheduled departure time. Service regularity is defined as the percentage of headways within 50% of the scheduled headway for headways of 10 minutes or less. Nakanishi concluded that these indicators accurately portray the user experience with transit. Senevirante (1990) proposes simulation as a method to compare the effect on reliability of different operating strategies, and also uses the same indicators as a metric of comparison. Bullock et al. (2005) show how GPS can be a cost effective method for measuring the reliability and performance of bus systems. Strathman and Hopper (1993) offer a review of many empirical studies on on-time performance and examine data from Portland, Oregon's bus system in order to identify factors which contribute to a decrease in reliability. They find that reliability decreases during the PM peak period, with larger headway variance, higher demand, and as buses travel along their routes. The amount of experience a driver has is also shown to affect reliability.

2.2.2 Basic Slack and Holding Strategies

One of the first, and still widely used, strategies for increasing reliability is to insert extra time, or *slack*, into a schedule as a buffer to prevent the propagation of disturbances. Osuna and Newell (1972) and Newell (1974) both suggest and provide a mathematical analysis of holding buses at predefined stops when they arrive ahead of schedule. However, after admitting the complexity of the problem only a simple system with two vehicles is modeled. Adamski (1996) presents a flexible dispatching and holding method and supports computerized tools over more subjective human agents. Rossetti and Turitto (1998) examine the difference between static and dynamic headway thresholds for whether or not to hold a bus at a control point and conclude that setting the threshold at the scheduled headway results in the lowest variance in headways, however this results in extra delay to passengers. Dessouky et al. (2003) present a method for better predicting bus arrivals for use with holding strategies; however this is a centralized control most useful for systems with large headways and small slack times. Abkowitz and Tozzi (1987) offers a review of multiple control strategies, including where and how to place control points and when to implement a control.

While holding strategies and slack time allow buses ahead of schedule to slow down, it is also necessary to allow buses behind schedule a way to regain lost time. Ling and Shalaby (2003) offer a method of using adaptive signal priority as a way to do this. Ling and Shalaby (2005) then advance this idea by using computerized agents to not only speed up buses behind schedule, but as a way to break up paired buses by impeding the following bus. Sun and Hickman (2005) offer stop-skipping as a way to recover lost time and find that the strategy is most productive in areas with a high density of stops.

2.2.3 Manipulation Strategies

Finnamore and Jackson (1978) provide a comprehensive summary of ways a bus route can be adjusted and the effects of each action. To break up bunches, buses can be be told to skip stops or depart from a control point early. In order to fill gaps in service, a bus can travel out of service to an existing gap or an additional bus can be added to a route.

2.2.4 Strategies with Real Time Information

Eberlein et al. (2001) comment on the lack of research using real-time data and present an algorithm for finding the optimal holding strategy when real-time information on headways is available, concluding that the solution is very dependent on bus headways and is most effective with one control point at the dispatching station. Daganzo (2009) proposes a holding strategy based on real time headways at multiple locations along a route. He shows that by using his strategy, as long as the random noise inserted into the system is bounded, headways will be bounded as well. Chandrasekar and Chin (2002) offer the idea of speed control without schedule constraints as a viable control method. They propose a binary speed control such that a bus will run at maximum speed when far behind the bus ahead and at a slower speed when closer to the bus ahead. They also propose using transit signal priority to advance a bus behind schedule. This control is analyzed by simulation using PARAMICS software but without an analytical model. The results suggest that decentralized speed control is a workable solution.

2.3 Current Practices

2.3.1 Practices in US Metropolitan Areas

Peng et al. (2008) gives a comprehensive review of strategies in use in major US cities. The information was gathered by survey. Seattle, Los Angeles, Boston, Portland, San Antonio, and Washington D.C. all have AVL on 99%-100% of their buses. The AVL ranges from fixed location sensors to onboard GPS depending on when AVL systems were installed. The refresh rate of the AVL data also varies up to 5 minutes between data points. St. Louis and New York City both have AVL installed on only a small portion of their bus fleet as of the date of the survey, however New York has plans to increase their AVL coverage.

The most common practice is to notify operators when they are running early or late, or if they are in danger of bunching. Operators can be to told to skip parts of their route or suspend service if they get too far behind schedule. In addition, buses can be inserted into a route where large gaps are located. Buses ahead of schedule are then held at control points.

2.3.2 Bus Rapid Transit

Another approach to improving reliability of a bus system is to reduce the severity of the perturbations that can affect the components of a bus' travel time. There are several ways to do this, all of which speed up bus trips, that fall under the general term of Bus Rapid Transit (BRT). Giving a bus a dedicated lane allows it to travel without being delayed by general traffic. Installing Transit Signal Priority (TSP) systems allow buses to avoid stopping at many traffic signals. Preboarding ticketing systems and aligned platforms reduce the amount of time each passenger takes to board a bus. All of these components reduce the amount of randomness that can be added to a bus' trip which results in less control needed to provide on-time performance and service regularity. Well known examples of BRT are in Curitiba, Bogata, and Seoul. While many cities in the US make use of different aspects of BRT, full implementation is rare.

2.4 Discussion

2.4.1 Gaps in Current Research

The majority of studies examining transit performance and offering possible strategies for mitigation are entirely empirically based. Without a physical model to explain how the system responds to a control it is difficult to calibrate or predict how a given system will respond. The main focus of controls is holding strategies. This allows for an easier implementation since the control is restricted to predefined locations along a route, however this allows more time for errors to propagate through the system. Because errors from desired headways grow exponentially over time, a longer time between control points allows more damage to the system overall. While many proposed strategies make use of real time data, only Chandrasekar and Chin (2002) study a continuous control, but only with an analysis of simulated data. There is no systematic analysis of real time control until Daganzo (2009) and the proposed control is headway based and limited in response by the frequency of control points.

2.4.2 Deficiencies in Current Practice

While AVL units are widespread allowing real time monitoring of buses, there seems to be very little methodical use of the data for improving reliability. The two main uses of the data are informing users of estimated bus arrivals and allowing agencies to know the on-time performance of their buses. The actual controls tend to only be implemented when bunching has already occurred instead of using preventive measures. As a result, methods to restore service can have a negative affect on users. Dwelling for unknown periods of time at transit stops or skipping stops or potions of the route can cause confusion or frustration. Heavy rail systems use real time data for continuous control and as a result have very high reliability. This however, to this author's knowledge, has not been implemented for bus systems because of the complexity added by traveling in mixed traffic.

Additionally, most agencies control their system from a central location. A decentralized control allowing for buses to act independently would produce an easier implementation and quicker response.

Chapter 3

Model and Analysis of Bus Travel

This chapter will present a framework for modeling bus travel. The framework will assume certain aspects of the system are continuous over time and space to allow for a systematic analysis. The model will then be used to analyze the behavior of an uncontrolled bus system. The continuum assumptions will then be relaxed and a simulation tool used to validate the results in a more realistic setting.

3.1 Definitions and Assumptions

3.1.1 Route Definition and Assumptions

In order to model the operation of a system of buses along a route, the physical traits that describe the route and the designed level of service set by the agency must be defined. These are declared as constants and will be denoted by capital letters throughout the formulation.

Physical Characteristics

The route is considered to be a loop with length, L. Deployment and retraction of buses are be allowed to happen anywhere along the route. For simplicity the route is homogeneous, with uniform demand along the route defined as a demand rate density, Λ, measured in passengers generated per unit of time per unit of distance along the route, and uniform stop density along the route, K, measured in bus stops per unit of distance. The number of stops along a route KL is an integer.

Service Characteristics

There are N buses deployed on the route, indexed $n = 1, 2, ..., N$, resulting in a desired equilibrium spacing, $S = \frac{L}{N}$. N is always an integer. While N (and therefore S) may change over a long period of time, it is constant on the short term and is considered as such for the formulation. The average cruising speed of a bus in traffic, affected by traffic signals and congestion, is defined as V. The equilibrium commercial speed, E, is the average speed at which a bus travels, including stops, when buses are spaced evenly along the route.

This speed determines the travel time passengers experience while on the bus, assuming equilibrium spacing. The frequency of service is defined by an equilibrium headway, H, between bus arrivals at any given stop. The headway between successive bus arrivals is determined by the spacing between two consecutive buses divided by the commercial speed of a bus. When buses are evenly spaced, the resulting headways are equal to the equilibrium headway.

$$H = \frac{S}{E} \tag{3.1}$$

The headway is considered to be short enough so that users arrive uniformly to bus stops and there is no need for a scheduled timetable of arrivals.

At equilibrium, the amount of dwell time per boarding passenger added to a bus' travel time, B, is calculated as the sum of the time required for a passenger to board a bus, b, and the loss time per passenger generated at each stop, τ, in order to decelerate to a stop, open and close the doors, and accelerate back to cruising speed. The dwell time is calculated as:

$$B = b + \frac{\tau K}{H\Lambda} \approx b + \frac{\tau K E}{S\Lambda} \tag{3.2}$$

This assumes that buses stop at each bus stop regardless of demand, a valid assumption for routes with high frequency and demand with evenly spaced buses.

3.1.2 Bus Definitions and Assumptions

Relationships Between Trajectories

The position of bus n at time t is defined as $x_{n,t}$ where x is the distance measured along the route in the direction of travel from a predefined point (a terminal location or a control point). The position can be determined at near real time with GPS devices installed in the buses. The process of doing this is discussed in Greenfeld (2002). The trajectory of bus n is then the set of $x_{n,t}$ over all t. Because the route is a loop, any arithmetic pertaining to position or bus index is modular and is denoted with $\oplus$ for addition and $\ominus$ for subtraction. Bus $n \ominus 1$ refers to the bus in front of bus n and bus $n \oplus 1$ refers to the bus behind bus n. Position is restricted to the range $x_{n,t} \in [0, L)$. The spacing $s_{n,t}$ of bus n at time t is defined as the distance measured along the route between bus n and the bus in front of it $n \ominus 1$:

$$s_{n,t} = x_{n\ominus 1,t} \ominus x_{n,t} \tag{3.3}$$

When calculating spacing, $\ominus$ refers to subtraction modulo L. Initial conditions are set such that:

$$s_{n,0} = S \tag{3.4}$$

It is assumed that buses will not pass one another on a route so that $s_{n,t}$ is always positive. Spacing is shown graphically in Figure 3.1 as the distance between two consecutive trajectories at a point in time. Because the length of the route is fixed, the sum of spacings over all buses is constant:

$$\sum_{n=1}^{N} s_{n,t} = L \forall t \tag{3.5}$$

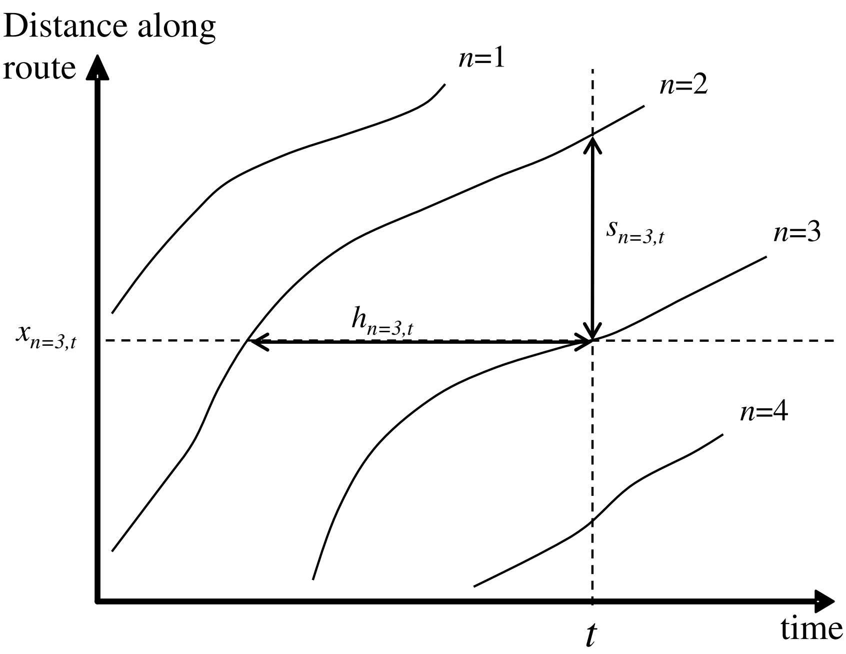

Figure 3.1: Trajectory Relationship

The headway $h_{n,t}$ of bus n at time t at some location is defined as the time that has passed since bus $n \ominus 1$ was at the location in question. Headway is shown graphically in Figure 3.1 as the time between two consecutive trajectories at a point in space. There is no corresponding conservation for the sum of headways.

Parts of a Trajectory

As explained in Chapter 1, the path of a bus can be broken into two main parts: time spent traveling between stops and time dwelling at stops. The behavior of a bus at a stop is broken down into its component parts in Figure 3.2. (1) The bus approaches a stop at speed V; (2) decelerates to a stop; (3) the doors open; (4) passengers board; (5) the doors close; (6) the bus accelerates back to speed V; (7) and leaves at speed V. The time spent decelerating and accelerating can be approximated with a time spent traveling at speed V and loss time spent dwelling at the stop as illustrated by the dashed line in Figure 3.2. This allows bus trajectories to be accurately represented as piecewise linear with only two speeds: V when moving, and 0 when at a stop. The loss time spent boarding, (4), is equal to the number of passengers who need to board multiplied by b. The loss time associated with a stop, τ, presented earlier is the sum of all other loss times. The trajectory of a bus will alternate between these two speeds every time it arrives at a stop, many times over the course of a route.

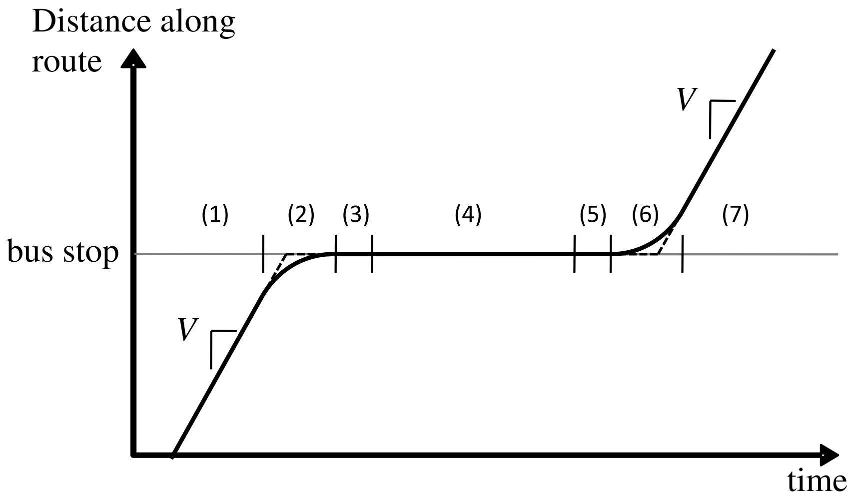

Figure 3.2: Parts of a Trajectory

3.2 Continuous Approximation Model

With perfect information (bus stop locations, arrival of passengers to bus stops, traffic conditions, driver behavior, etc.) it would be possible to accurately model the trajectory of a bus. However, most of that information is unobtainable in real time and the resulting model would be very complicated and wouldn't provide many insights. Instead, trajectories can be smoothed out by approximating all the components of the route in a continuous manner. The loss times of passenger loading and bus stops would continually act on the speed of the bus instead of being concentrated at discrete bus stops. The resulting trajectory would model the commercial speed of a bus. The commercial speed of the bus is defined as the average speed of a bus over a distance D, where D is long enough to include the effect of several stops (Figure 3.3). The commercial speed determines travel time for passengers and the amount of time a bus needs to travel the length of the route.

However, the commercial speed of a bus is not necessarily constant over the length of the route. It was discussed earlier that a bus with a larger spacing will travel slower because it must serve more passengers and that a bus with a smaller spacing will travel faster because it has fewer passengers to serve. Therefore the commercial speed of a bus is dependent on the spacing, which is continuously changing. By calculating the average speed of a bus with the assumption that the passenger demand generated by the spacing during a single time step was held constant over a longer time, the result could be thought of as the instantaneous commercial speed of that bus for that time step.

3.2.1 Instantaneous Commercial Speed

The instantaneous commercial speed, $v_{n,t}$, of bus n at time t is defined using a continuous approximation (CA) model that will accurately model the behavior of the system on a long scale. Whereas the actual trajectory of a bus will have periods of dwelling when

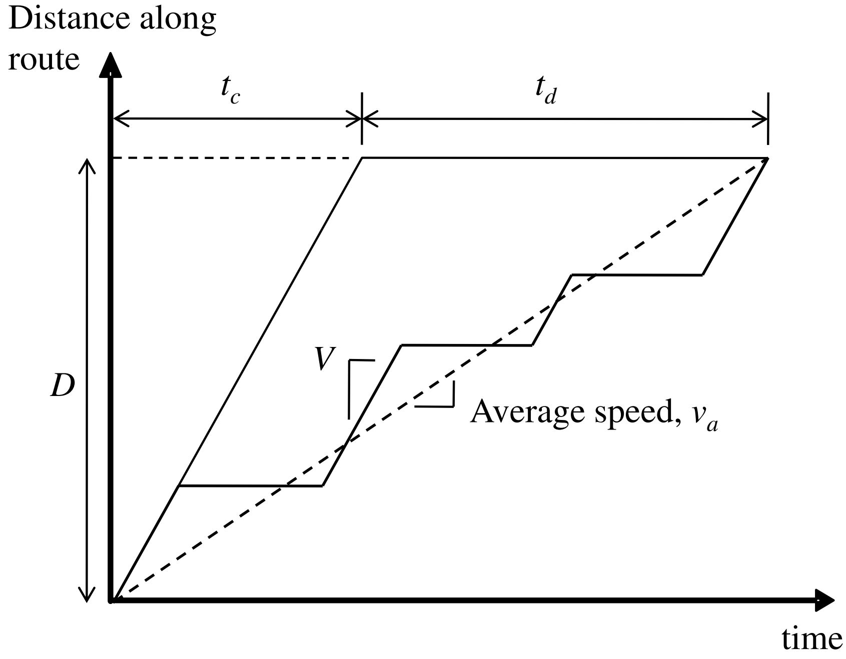

Figure 3.3: Calculating Average Speed

the bus is at a stop or cruising between stops, the CA trajectory of a bus will continually move during each time step at the instantaneous commercial speed. The CA trajectory is defined as the set of $y_{n,t}$ over all t where $y_{n,t}$ is the CA location of bus n at time t determined by the rule:

$$y_{n,t+\Delta t} = y_{n,t} + v_{n,t}\Delta t \tag{3.6}$$

Because the CA trajectory of a bus is modeled to approximate the behavior of a real trajectory, the behavior of the spacing between two CA trajectories should be a good approximation for the behavior of the spacing between two actual trajectories. With this assumption we can model spacing such that:

$$s_{n,t} \approx y_{n\ominus 1,t} \ominus y_{n,t} \tag{3.7}$$

It is also assumed that $v_{n,t}$ is slow changing over time and that nearby buses have similar commercial speeds. This allows the headway of a bus to be approximated with knowledge of spacing.

$$h_{n,t} \approx \frac{s_{n,t}}{v_{n,t}} \tag{3.8}$$

This is advantageous because spacing can be easily known in real time from GPS data whereas headway cannot.

As described earlier, the trajectory of a bus can be approximated as piecewise linear with two possible speeds. As such, the average speed of the bus, v_a, over a distance, D, can be calculated as the ratio between the distance traveled and the sum of the expected

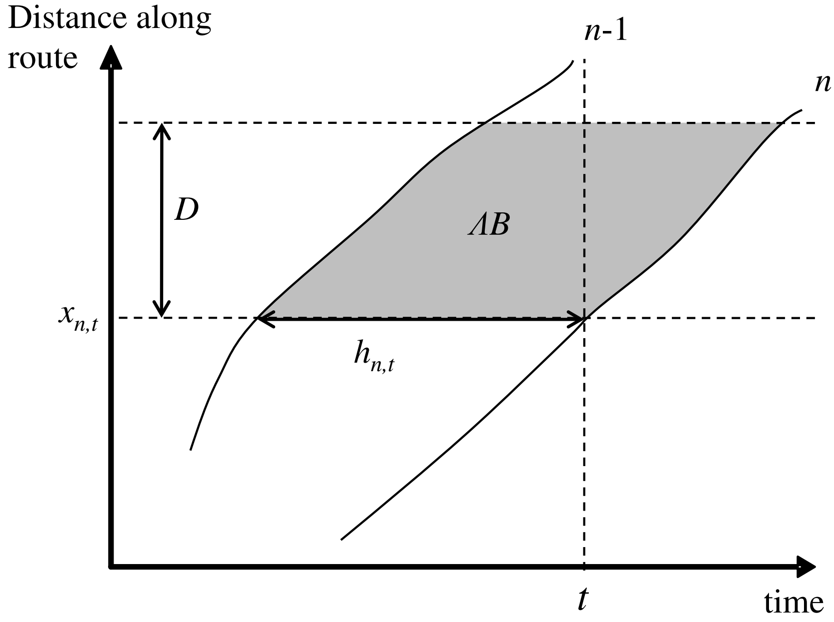

Figure 3.4: Area of Demand

time spent cruising, t_c, and the expected time spent dwelling at stops, t_d:

$$v_a = \frac{D}{t_c + t_d} \tag{3.9}$$

Because trajectories are assumed to be piecewise linear, the trajectory can be redrawn by grouping the periods with the same slope without changing the average speed as shown in Figure 3.3. The expected time spent cruising is equal to the distance traveled divided by the cruising speed of the bus:

$$t_c = \frac{D}{V} \tag{3.10}$$

The time spent dwelling depends on the expected number of passengers produced over D since the last bus passed and can be visualized as the area between two consecutive trajectories over D times the demand rate density and the loss time per passenger (Figure 3.4). With the assumption that commercial speeds are similar between buses, the expected time spent dwelling is $t_d = D\Lambda B h_{n,t}$. With Equation 3.8 this can be approximated as:

$$t_d \approx \frac{D\Lambda B s_{n,t}}{v_{n,t}} \tag{3.11}$$

By substituting Equations 3.10 and 3.11 into Equation 3.9 we can define the result as the instantaneous commercial speed, which is independent of D.

$$v_a \approx \left(\frac{1}{V} + \frac{\Lambda B s_{n,t}}{v_{n,t}}\right)^{-1} \approx v_{n,t} \tag{3.12}$$

This can be simplified to:

$$v_{n,t} \approx V(1 - \Lambda B s_{n,t}) \tag{3.13}$$

which describes the commercial speed of the bus as a proportion of the cruising speed decreasing linearly as spacing increases. This is a formulation of the behavior described earlier: as spacing (and proportionally headway) increases, more passengers arrive, causing the bus to dwell longer and commercial speed to decrease.

By substituting the desired spacing S for $s_{n,t}$ in Equation 3.13, the equilibrium commercial speed, E, the commercial speed at which buses travel when evenly spaced, is found to be:

$$E = V(1 - \Lambda BS) \tag{3.14}$$

By substituting Equation 3.14 into Equation 3.1, the equilibrium headway, H, is found to be:

$$H = \frac{S}{V(1 - \Lambda BS)} \tag{3.15}$$

This result can be substituted into Equation 3.2 in order to define the average dwell time per passenger, B, in terms of system constants, assuming that the bus stops at every stop:

$$B = \frac{b\Lambda S + \tau KV}{\Lambda S + \tau KV\Lambda S} \tag{3.16}$$

When Λ and/or H is large enough that the chance of a bus arriving to a stop with no passengers desiring to board or alight is very small, then buses will stop at every stop and this is a good approximation for loss time per passenger. However, as the probability of a bus skipping a stop grows larger, the effect of τ can overwhelm b and the resulting B can be unreasonably large. The expected number of people, p, waiting to board a bus at a stop is given by:

$$\mathrm{E}(p) = \frac{H\Lambda}{K} \tag{3.17}$$

Given uniform demand, the expected number of people waiting to alight at a stop is the same and assumed to be independent. Assuming Poisson arrivals, the probability of $p = 0$ is equal to:

$$\mathrm{P}(p = 0) = e^{-\frac{H\Lambda}{K}} \tag{3.18}$$

Therefore the probability of a bus stopping at a bus stop and incurring loss time τ is given by:

$$\mathrm{P}(\text{stopping}) = 1 - e^{-\frac{2H\Lambda}{K}} \tag{3.19}$$

Substituting Equation 3.15 for H gives:

$$\mathrm{P}(\text{stopping}) = 1 - e^{-\frac{2S\Lambda}{KV(1-\Lambda BS)}} \tag{3.20}$$

Because the time penalty for a bus stop is not experienced if it is skipped, the values τ in Equation 3.16 will be multiplied by Equation 3.20, giving an equation for the average dwell time per passenger without assuming that a bus will stop at every stop:

$$B = \frac{b\Lambda S + \tau KV\left(1 - e^{-\frac{2S\Lambda}{KV(1-\Lambda BS)}}\right)}{\Lambda S + \tau KV\Lambda S\left(1 - e^{-\frac{2S\Lambda}{KV(1-\Lambda BS)}}\right)} \tag{3.21}$$

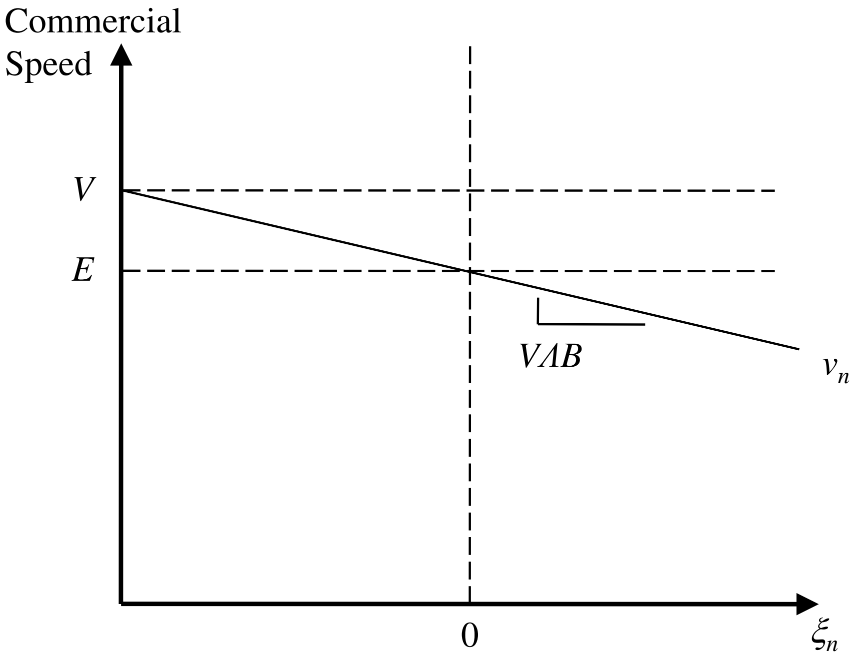

Figure 3.5: Uncontrolled Commercial Speed

An explicit formula for B cannot be found, however B can be determined numerically. It is noted that when $\tau = 0$, $B = b$.

Because there is a desired spacing S, it is useful to define a state variable, $\xi_{n,t}$ that is the deviation of the spacing of bus n at time t from the desired spacing:

$$\xi_{n,t} = s_{n,t} - S \tag{3.22}$$

Following from Equation 3.5, the sum of deviations over all buses is equal to zero:

$$\sum_{n=1}^{N} \xi_{n,t} = 0 \forall t \tag{3.23}$$

By replacing $s_{n,t}$ with $(S + \xi_{n,t})$ in Equation 3.13 and combining the result with Equation 3.14, commercial speed can be expressed as a difference from equilibrium commercial speed, changing linearly with the deviation from desired spacing.

$$v_{n,t} \approx E - V\Lambda B\xi_{n,t} \tag{3.24}$$

The slope $V\Lambda B$ is equal to the approximate rate at which the instantaneous commercial speed decreases from an increase in spacing. This is shown in Figure 3.5.

3.2.2 Trajectories

During a time step, Δt, a bus can be expected to advance a distance of $v_{n,t}\Delta t$. There is also expected to be some random noise effect on the distance traveled caused by fluctuations in passenger arrivals and traffic effects. The noise, $\nu_{n,t}$, experienced by bus n during the time step starting at time t is assumed to be normally distributed with variance σ_0^2 for a time step t_0. For a time step, Δt, the variance would be:

$$\sigma_{\Delta t}^2 = \sigma_0^2 \frac{\Delta t}{t_0} \tag{3.25}$$

The resulting CA position of bus n at time $t + \Delta t$ is equal to:

$$y_{n,t+\Delta t} = y_{n,t} + v_{n,t}\Delta t + \nu_{n,t} = y_{n,t} + E\Delta t - V\Lambda B\Delta t \xi_{n,t} + \nu_{n,t} \tag{3.26}$$

When the deviation from the desired spacing, $\xi_{n,t}$,is zero and there is no random noise, $\nu_{n,t} = 0$, it is easy to see that a bus will travel at the equilibrium commercial speed.

3.2.3 Deviation from Desired Spacing

Substituting Equation 3.26 for buses n and $n - 1$ into Equation 3.7 shows how spacing changes over time with deviations from the desired spacing.

$$s_{n,t+\Delta t} \approx s_{n,t} - V\Lambda B\Delta t \xi_{n\ominus 1,t} + V\Lambda B\Delta t \xi_{n,t} + \nu_{n\ominus 1,t} - \nu_{n,t} \tag{3.27}$$

Subtracting the desired spacing, S, from each side and combining terms results in the state equation for the system in terms of the state variable, $\xi_{n,t}$.

$$\xi_{n,t+\Delta t} \approx -V\Lambda B\Delta t \xi_{n\ominus 1,t} + (1 + V\Lambda B\Delta t)\xi_{n,t} + \nu_{n\ominus 1,t} - \nu_{n,t} \tag{3.28}$$

By defining $\beta = V\Lambda B\Delta t$ and combining the noise terms that affect the spacing of bus n as $\varphi_{n,t} = \nu_{n\ominus 1,t} - \nu_{n,t}$, Equation 3.28 can be rewritten in the simple form:

$$\xi_{n,t+\Delta t} \approx -\beta \xi_{n\ominus 1,t} + (1 + \beta)\xi_{n,t} + \varphi_{n,t} \tag{3.29}$$

3.3 Analysis of Continuum Model

Vector Notation

The state equation 3.29 is of the same form as the one studied in Daganzo (2009) and so the same formulation can be used to examine the stability of the system. By introducing constants $f_0 = (1 + \beta)$, $f_1 = -\beta$, and $f_j = 0$ for all other integers, Equation 3.29 can be rewritten as:

$$\xi_{n,m\Delta t} \approx \sum_{j=1}^{N} f_{n\ominus j} \xi_{j,(m-1)\Delta t} + \varphi_{n,(m-1)\Delta t} \tag{3.30}$$

the stochastic part of which is a convolution. Using boldface for vectors and $*$ as the convolution operator, Equation 3.30 can be rewritten as:

$$\boldsymbol{\xi}_{t+\Delta t} \approx \boldsymbol{f} * \boldsymbol{\xi}_{(m-1)\Delta t} + \boldsymbol{\varphi}_{(m-1)\Delta t} \tag{3.31}$$

where $\boldsymbol{f}$ is the kernel of the convolution. By substituting $\boldsymbol{\xi}_t$ in the RHS of Equation 3.31 with its expression according to Equation 3.31, the result is:

$$\boldsymbol{\xi}_{m\Delta t} \approx \boldsymbol{f} * (\boldsymbol{f} * \boldsymbol{\xi}_{(m-2)\Delta t} + \boldsymbol{\varphi}_{(m-2)\Delta t}) + \boldsymbol{\varphi}_{(m-1)\Delta t} \tag{3.32}$$

Continuing this process with the logic in Daganzo (2009) and using $\boldsymbol{f}_{|j}$ as notation for the kernel created by convolving $\boldsymbol{f}$ with itself j times, the resulting equation in terms of ξ_0 is:

$$\boldsymbol{\xi}_{m\Delta t} \approx \boldsymbol{f}_{|m}\boldsymbol{\xi}_0 + \sum_{j=0}^{m-1} \boldsymbol{f}_{|j} * \boldsymbol{\varphi}_{(m-j)\Delta t} \tag{3.33}$$

Equations 3.4 and 3.22 set $\boldsymbol{\xi}_0 = 0$ which removes the first term, however Equation 3.33 can be used to examine the behavior of a system with any initial conditions. Using $f_{i|j}$ as the ith term of $\boldsymbol{f}_{|j}$, Equation 3.33 can be rewritten in scalar form as:

$$\xi_{n,m\Delta t} \approx \sum_i f_{i|m}\xi_{i,0} + \sum_{j=0}^{m-1}\sum_i f_{i|j}\varphi_{n\ominus i,(m-j)\Delta t} \tag{3.34}$$

By decomposing the noise term, $\varphi_{n\ominus 1,(m-j)\Delta t}$ back into the individual noise components $(\nu_{n\ominus 2,(m-j)\Delta t} - \nu_{n\ominus 1,(m-j)\Delta t})$ and combining terms with the same $\nu_{n,t}$ the result is:

$$\xi_{n,m\Delta t} \approx \sum_i f_{i|m}\xi_{i,0} + \sum_{j=0}^{m-1}\sum_i (f_{i|j} - f_{i\ominus 1|j})\nu_{n\ominus i,(m-j)\Delta t} \tag{3.35}$$

Because $(1+\beta)$ is greater than 1, as $m \to \infty$, $f_{i|m}$ will go to infinity or negative infinity as will $f_{i|m} - f_{i-1|m}$. This means that deviations from desired spacing will go to infinity or negative infinity depending on how they are perturbed, however realistically ξ is bounded below by $-S$ and above by $L-S$ where $\xi = -S$ means a bus has bunched with the bus in front of it and $\xi = L-S$ means all buses are bunched behind that bus. There is no easy way to solve for the distribution of the time to bunching, however it is the solution of setting Equation 3.34 equal to $-S$. It can however be modeled by simulation.

Matrix Notation

Because the model has a fixed number of buses operating on a loop, it is possible to rewrite Equations 3.31–3.33 in matrix notation, removing the need for the modular arithmetic used in Equation 3.30.

By defining $\mathbf{F}$ as an $N \times N$ matrix with terms $\mathbf{F}(i,j) = f_{i\ominus j}$ and $\mathbf{\Phi}$ as an $N \times N$ matrix $\mathbf{\Phi}(i,j) = 1$ for $j = i \ominus 1$; $\mathbf{\Phi}(i,j) = -1$ for $j = i$ and $\mathbf{\Phi}(i,j) = 0$ for all others the equations can be rewritten as:

$$\boldsymbol{\xi}_{m\Delta t} \approx \mathbf{F}\boldsymbol{\xi}_{(m-1)\Delta t} + \mathbf{\Phi}\nu_{(m-1)\Delta t} \tag{3.36}$$

$$\boldsymbol{\xi}_{m\Delta t} \approx \mathbf{F}(\mathbf{F}\boldsymbol{\xi}_{(m-2)\Delta t} + \mathbf{\Phi}\nu_{(m-2)\Delta t}) + \mathbf{\Phi}\nu_{(m-1)\Delta t} \tag{3.37}$$

$$\boldsymbol{\xi}_{m\Delta t} \approx \mathbf{F}^m\boldsymbol{\xi}_0 + \sum_{j=0}^{m-1} \mathbf{F}^j\mathbf{\Phi}\boldsymbol{\nu}_{(m-1)\Delta t} \tag{3.38}$$

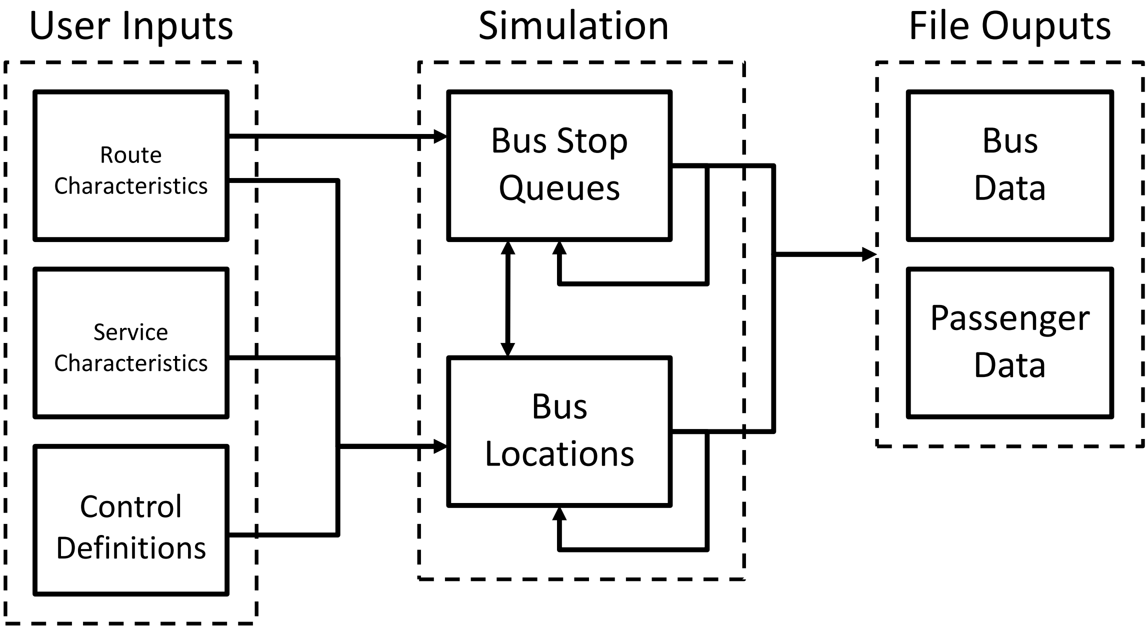

Figure 3.6: Simulation Tool Flow Chart

3.4 Microscopic Simulation Tool

In order to test the continuous model in a more realistic discrete setting it is necessary to use simulation to model the more complicated trajectories that actual buses travel. The main difference between the CA model and the simulation is the discretization of the effect of bus stops on buses. By generating passenger arrivals at discrete locations and only allowing a bus to be delayed by passengers at these locations, the bunching effect is reduced. Therefore the CA model is more conservative, and any result that holds for it, should hold in a simulated environment. The effect of traffic and other external disturbances remains constant between the CA model and the simulation.

The microscopic simulation tool was programmed in Microsoft Visual Studio 2005 using Microsoft Visual Basic .NET Framework 2.0. The code for the simulation tool can be found in the Appendix. The flow of information in the simulation is shown in Figure 3.6. The user inputs are described in Section 3.4.1, the file outputs are described in Section 3.4.2, and the simulation itself is described in Section 3.4.3.

3.4.1 Simulation Inputs

The inputs for the simulation are divided into three categories.

Route Characteristics

The route is defined to have length, L. Passenger arrivals are defined by either a uniform stop density, K, and arrival demand rate density, Λ, or an input file with bus stop locations and an origin/destination table. The effect of traffic and other external disturbances on the speed of the bus are defined by a normally distributed random variable, ν with a mean of 0, an standard deviation σ_0, and a time of effect t_0.

Service Characteristics

The number of buses, N, serving the route is defined. The buses are defined by a cruising speed, V, and passenger capacity. The dwell times are defined by a fixed loss time for each bus stop, τ, and the time to board a passenger, b.

Control Definition

The cruising speed of the buses can be set to be dynamic or static. This will be defined in further detail in the next Chapter.

3.4.2 Simulation Outputs

The outputs for the simulation are divided into two categories.

Bus data

The location and the cruising speed of each bus is recorded for every bus for every time step. From this data spacing, headway, and commercial speed can be calculated.

Passenger Data

The origin and destination for every passenger is recorded. Each passenger's trip time is recorded, including time of generation, time of boarding, and time of alighting. From this data waiting time, travel time, and door to door time can be calculated.

3.4.3 Simulation Logic

Initial Conditions

After the user tells the simulation to start, empty buses are given initial positions along the route and bus stops are initialized with no queues. By default the initial bus positions are evenly spaced along the route. Bus Stop locations are defined by the user.

Passenger Generation

In order to approximate Poisson passenger arrivals, each arrival event is modeled as a Bernoulli random variable. Because passenger generation is modeled as a demand rate density (passenger arrivals per unit of time per unit of distance), each Bernoulli trial represents the probability of a passenger arriving at a bus stop during a time step and generated from a range of positions along the route (Figure 3.7). The simulation uses a time step of one second and a range of $\frac{1}{10}$km. This is repeated for each destination according to the demand values in the Origin/Destination table. Whenever an arrival is generated it is added to the back of the passenger queue of the closest bus stop.

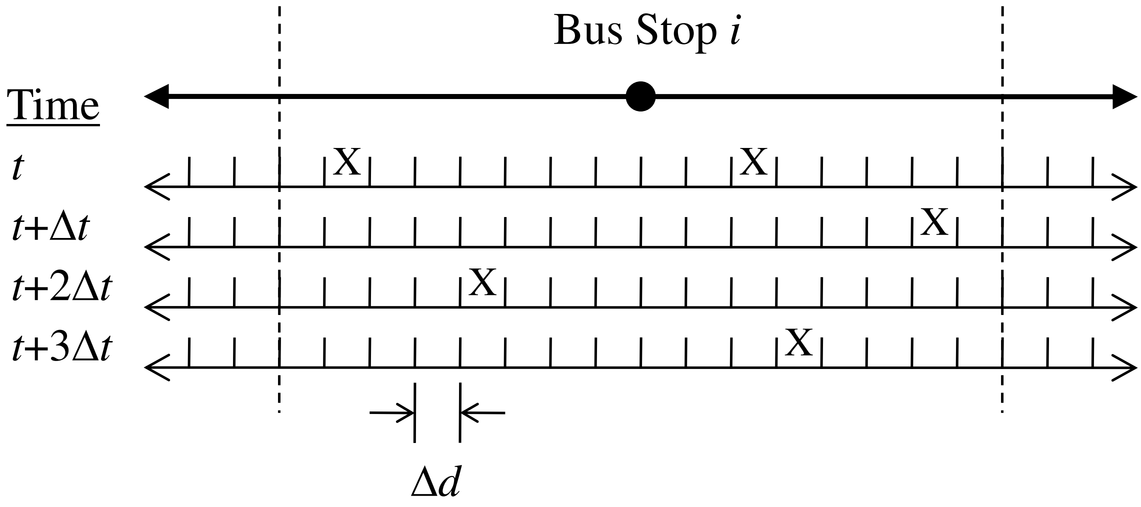

Figure 3.7: Passenger Generation

Bus Movement

During a time step each bus is determined to be either moving or dwelling. A bus is determined to be dwelling if its movement during a time step passes a bus stop and there are passengers waiting at a that bus stop or passengers on the bus with a destination of that bus stop. Once it is determined that a bus needs to stop it dwells for the loss time associated with a stop. All passengers waiting at the bus stop then board while all passengers at their destination alight. The position of a bus while dwelling remains constant. Once all passengers have been served, the bus starts moving again.

The position of a moving bus is updated based on two parameters. First the position is increased by the distance covered by traveling at the defined cruising speed for a time step. The position is then modified by the random noise variable representing the effect of traffic and other external disturbances.

3.5 Simulation Analysis

In order to simulate the time it takes for the uncontrolled system to bunch, the following inputs were used: $L = 24$ km; $K = 1$ stop/km; $\Lambda = 50$ pax/hr·km; $\sigma_0 = 0.086$ km; $t_0 = 1$ min; $N = 8$ buses; $V = 30$ km/hr; $\tau = 30$ sec/stop; $b = 4$ sec/pax. This results in an equilibrium commercial speed of 25 km/hr.

The Simulation Tool was run 10 times with the above inputs. In order to compare, the Monte Carlo method was used to simulate 10 runs using the CA model given in Equation 3.29 with the same inputs and time step as the simulation. For both simulations, the minimum spacing is graphed over time in Figure 3.8 with a spacing of zero representing a bunching event. In every case bunching occurred and deviations increased. However, the simulated CA model bunched sooner than the discrete simulation in every case. This suggests that the CA model is more difficult to control than the discrete simulation. There-

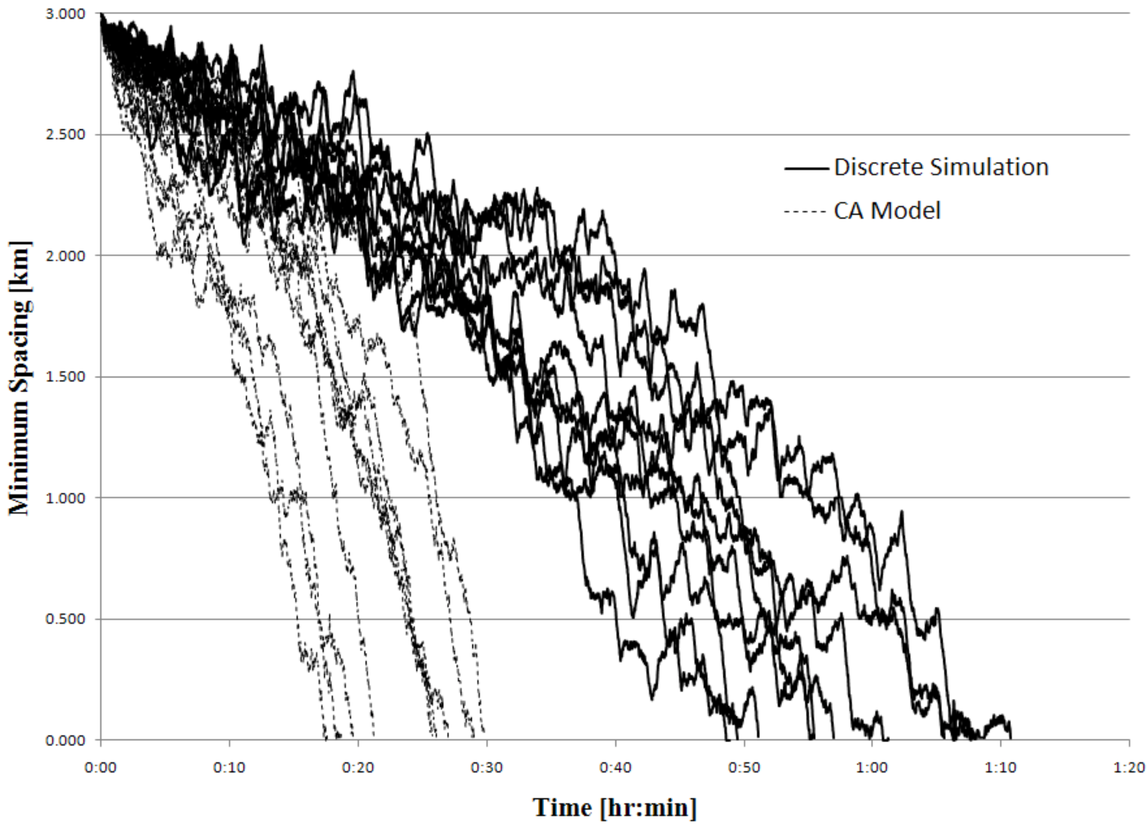

Figure 3.8: Time Until Bunching for an Uncontrolled System

fore, a control method that would work with the continuous model should also work for the simulation.

The advantage of the CA model formulated in this chapter is that it allows systematic analysis of a complex system so that a control can be designed. It however makes several continuum assumptions of how demand affects the system. The simulation tool allows these assumptions to be lifted and the system examined in a more realistic environment.

The methodology and simulation tool presented in this chapter will be used to examine the effect of a proposed control in Chapter 4.

Chapter 4

Determination of a Control

The main cause of bunching is simply that buses with larger spacings travel slower than buses with smaller spacings (Figure 3.5.) Regardless of any external effects or disturbances, as the spacing in front of a bus increases, the commercial speed of the bus decreases, and a decrease in spacing causes an increased commercial speed. In general, spacings between buses tend to move away from the desired spacing instead of towards it. The question posed is, by changing the relationship between spacing and commercial speed, can the tendency of buses to bunch be alleviated?

4.1 Intuition of Control

To achieve equal spacing, buses should tend towards the desired spacing instead of away from it. In order to do this, the current relationship between spacing and commercial speed must be changed so that an increase in spacing corresponds to an increase in commercial speed. The commercial speed shown in Figure 3.5 corresponds to the highest cruising speed of a bus, and so represents an upper bound on the commercial speed that a bus can achieve for a given deviation from the desired spacing. Therefore, in order to achieve faster commercial speeds with larger spacings, the cruising speed of a bus must be decreased as spacing decreases. There is however a trade-off. Decreasing the cruising speed may increase reliability of service, but it will also decrease the average commercial speed of the system. This will cause longer travel times for passengers and possibly require more buses to provide the same frequency of service. The concept of slowing down a bus to increase reliability is not new though. Described in Section 2.2.2, the strategy of adding extra time to a bus' expected run time is commonly used. This dissertation however proposes a more efficient way of achieving this with the goal of providing a certain level of reliability while providing the fastest commercial speed possible.

4.2 Control Formulation in the CA Model

The control formulation is based on a commercial speed definition of the form given in Equation 3.24. In order to have buses tend toward the desired spacing, the commercial

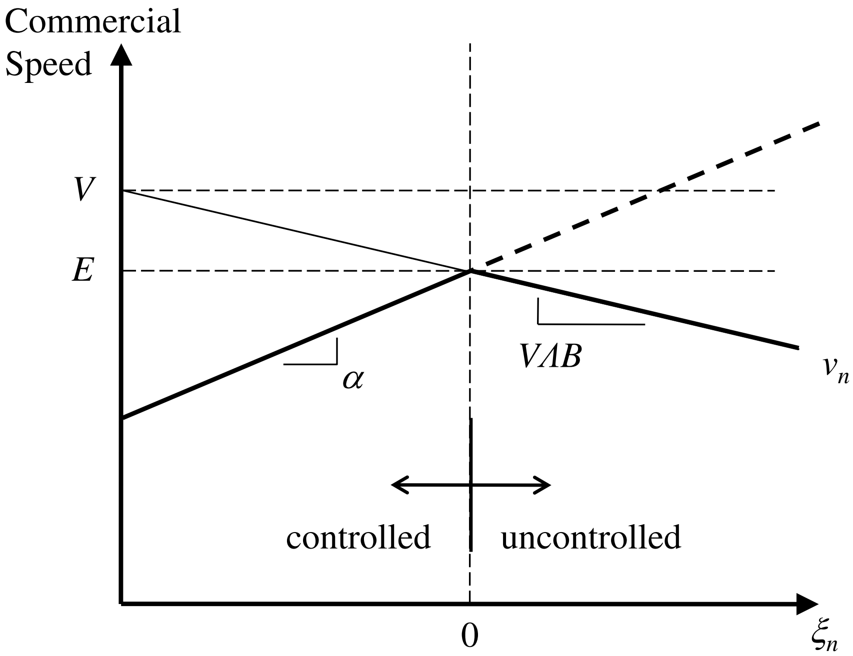

Figure 4.1: Simple Control Rule

speed should increase with deviation from desired spacing. Using a prime to denote variables under control, a simple commercial speed control rule would be:

$$v'_{n,t} = E + \alpha\xi_{n,t} \tag{4.1}$$

where α is the desired rate at which commercial speed would increase with an increase in spacing. The commercial speed however is constrained above by Equation 3.24, the commercial speed achieved by cruising at the maximum speed. Thus the control rule is in effect:

$$v'_{n,t} = E + \min\{\alpha\xi_{n,t}, -V\Lambda B\xi_{n,t}\} \tag{4.2}$$

as shown by the thick line of Figure 4.1. The point of intersection of the two equations is $\xi_{n,t} = 0$, meaning that a bus with a large spacing will travel uncontrolled, and therefore with a spacing that continues to grow, and a bus with a small spacing will travel slower than it can. This will only remove the bunching effect for buses with smaller spacings and it will result in an overall average speed lower than B.

4.2.1 Slowing the Buses

In order to control buses with a spacing larger than the desired spacing, the commercial speed rule must be decreased so that the point of intersection occurs to the right of the equilibrium spacing (Figure 4.2). The reduced commercial speed control rule would be:

$$v'_{n,t} = E - \delta + \alpha\xi_{n,t} \tag{4.3}$$

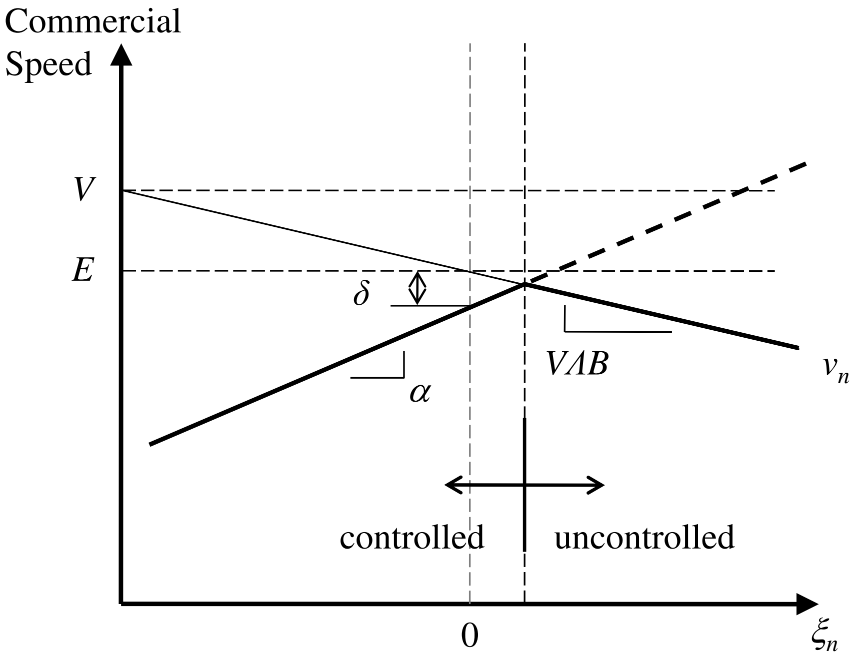

Figure 4.2: Reduced Control Rule

where δ is defined to be the reduction in commercial speed at equilibrium spacing. When all buses are equally spaced, the controlled equilibrium commercial speed will be $E' = E - \delta$.

The constraint on commercial speed results in a range where the linear control rule is valid:

$$\xi_{n,t} \leq \frac{\delta}{\alpha + V\Lambda B} \tag{4.4}$$

Using this reduced commercial speed control rule, while the condition in Equation 4.4 holds, a bus with a large spacing can still tend back toward equilibrium. However, if spacing becomes too large, the commercial speed will become constrained, the bus will travel uncontrolled and will not be able to catch up. This means the system is not robust to large disruptions.

4.2.2 Two-Way Cooperation

This problem can be avoided by recognizing that the spacing between two buses is affected by the speed of both buses. Therefore, if a bus' spacing is too large, it can be reduced by speeding up the bus and/or by slowing down the bus in front. In the same way, if a bus' spacing is too small, it can be increased by slowing down the bus and/or speeding up the bus in front. By enabling such two-way cooperation, the control takes advantage of the fact that a bus can affect the spacing in front and behind it. The proposed two-way commercial speed control rule is:

$$v'_{n,t} = E - \delta + \alpha\xi_{n,t} - \alpha\xi_{n\oplus1,t} \tag{4.5}$$

The constraint for where this linear control rule is valid becomes:

$$\xi_{n,t} \leq \frac{\delta - \alpha(\xi_{n,t} - \xi_{n\oplus 1,t})}{V\Lambda B} \tag{4.6}$$

which depends on the difference between the spacing in front of a bus and the spacing behind it. When all buses are equally spaced, the controlled equilibrium commercial speed will be $E' = E - \delta$. This will also be the commercial speed of a bus with equal spacing in front and behind. In effect, each bus will tend to center itself between the bus in front of it and the bus behind it. At equilibrium, this condition will be true for all buses. This is the control rule upon which the rest of the analysis will be based.

4.2.3 Defining the Cruising Speed

The commercial speed given by the control rule can be obtained by defining a cruising speed for the driver. Equation 3.13 gives the relationship between the cruising speed and the resulting instantaneous commercial speed for any given spacing. By substituting the control rule in Equation 4.5 for the commercial speed in Equation 3.13, the cruising speed, $c'_{n,t}$, necessary to achieve the control rule can be determined:

$$c'_{n,t} \approx \frac{v'_{n,t}}{1 - \Lambda B s_{n,t}} = \frac{E - \delta + \alpha\xi_{n,t} - \alpha\xi_{n\oplus 1,t}}{1 - \Lambda B s_{n,t}} \tag{4.7}$$

By substituting Equation 3.14 for E, Equation 3.22 for $s_{n,t}$, and with some manipulation, an equation for the cruising speed to achieve the control rule as a difference from the maximum cruising speed, V, in terms of the state variable is found to be:

$$c'_{n,t} \approx V + \frac{-\delta + (\alpha + V\Lambda B)\xi_{n,t} - \alpha\xi_{n\oplus 1,t}}{(1 - \Lambda B S) - \Lambda B \xi_{n,t}} \tag{4.8}$$

Notice that the constraint given in Equation 4.6 is consistent with constraining $c'_{n,t} \leq V$.

4.2.4 State Equation under Control

The following formulation assumes the constraint on cruising speed holds and that the desired cruising speed given by the control for a bus can be achieved. The conditions for this assumption and the resulting behavior when it is not met will be discussed in Section 4.5.

Using the CA trajectory definition given in Equation 3.26, the CA position of bus n at time t under control is equal to:

$$y'_{n,t+\Delta t} = y'_{n,t} + v'_{n,t}\Delta t + \nu_{n,t} = y'_{n,t} + E'\Delta t + \alpha\xi_{n,t} - \alpha\xi_{n\oplus 1,t} + \nu_{n,t} \tag{4.9}$$

Substituting Equation 4.9 for buses n and $n-1$ into Equation 3.7 shows how spacing changes over time while under the control.

$$s'_{n,t+\Delta t} \approx s_{n,t} + \alpha\Delta t\xi_{n\ominus 1,t} - 2\alpha\Delta t\xi_{n,t} + \alpha\Delta t\xi_{n\oplus 1,t} + \nu_{n\ominus 1,t} - \nu_{n,t} \tag{4.10}$$

Subtracting the desired spacing, S, from each side and combining terms results in the state equation for the controlled system in terms of the state variable.

$$\xi'_{n,t+\Delta t} \approx \alpha\Delta t\xi_{n\ominus 1,t} + (1 - 2\alpha\Delta t)\xi_{n,t} + \alpha\Delta t\xi_{n\oplus 1,t} + \nu_{n\ominus 1,t} - \nu_{n,t} \tag{4.11}$$

4.3 Analysis of Controlled System

Recall the convolution kernel in Section 3.3. By introducing constants $f'_{-1} = \alpha\Delta t$, $f'_0 = (1 - 2\alpha\Delta t)$, $f'_1 = \alpha\Delta t$, and $f'_j = 0$ for all other integers and using the formulation in Section 3.3 the deviation for bus n at time $m\Delta t$ can be written as:

$$\boldsymbol{\xi}_{m\Delta t} \approx \boldsymbol{f}'_{|m}\boldsymbol{\xi}_0 + \sum_{j=0}^{m-1} \boldsymbol{f}'_{|j} * \boldsymbol{\varphi}_{(m-j)\Delta t} \tag{4.12}$$

or in scalar notation as:

$$\xi_{n,m\Delta t} \approx \sum_i f'_{i|m}\xi_{i,0} + \sum_{j=0}^{m-1}\sum_i (f'_{i|j} - f'_{i\ominus 1|j})\nu_{n\ominus i,(m-j)\Delta t} \tag{4.13}$$

By defining $\mathbf{F}'$ as an $N \times N$ matrix with terms $\mathbf{F}'(i,j) = f_{i\ominus j}$ and $\mathbf{F}'_{\ominus 1}$ as an $N \times N$ matrix with terms $\mathbf{F}'_{\ominus 1}(i,j) = \mathbf{F}'(i \ominus 1, j)$, Equation 4.12 can be rewritten in matrix notation as:

$$\boldsymbol{\xi}_{m\Delta t} \approx \mathbf{F}'^m\boldsymbol{\xi}_0 + \sum_{j=0}^{m-1}(\mathbf{F}'^j - \mathbf{F}'^j_{\ominus 1})\boldsymbol{\nu}_{(m-1)\Delta t} \tag{4.14}$$

Recognizing that $\boldsymbol{f}'$ is a p.m.f., the repeated convolution of $\boldsymbol{f}'$ will also be a p.m.f. If there were an infinite number of buses along the loop, the coefficients of $\boldsymbol{f}'$ would approach the normal distribution as $m\Delta t$ increased. However, since the number of buses is finite and they are located along a loop, the coefficients of $\boldsymbol{f}'$ should approach a uniform distribution with probability of N^{-1} as $m\Delta t$ increases.

Because $f'_{i|m}$ tends toward the constant N^{-1} as m increases, for large enough m, the first term of Equation 4.13, representing the effect of the initial conditions of the system, can be rewritten as:

$$N^{-1}\sum_i \xi_{i,0} \tag{4.15}$$

Equation 3.23 states that the sum of deviations over all buses is zero, therefore as m increases, the effect of the initial conditions on the current state of the system goes to zero as long as the cruising speed constraint holds.

Setting the initial condition term to zero results in an equation for deviation from desired spacing of the same form as the equation for deviation from ideal headway analyzed in Daganzo (2009). Taking the variance of Equation 4.13 results in a variance amplification of the noise term of:

$$k^2_{\xi,m\Delta t} = \sum_{j=0}^{m-1} Q_j \text{ where } Q_j = \sum_i (f'_{i|j} - f'_{i\ominus 1|j})^2 \tag{4.16}$$

For the case of infinite buses along an infinite loop, the results given in Daganzo should hold and the variance of $\xi_{n,t}$ should be bounded. For the case with a discrete number of buses, Equation 4.16 was calculated over a large number of time steps and fit to the reciprocal of the variance of the kernel: $\mathrm{Var}(\boldsymbol{f}') = 2\alpha\Delta t$. The result shows the deviation from desired spacing to be approximately Gaussian wither bound on the variance amplification of:

$$k_\xi^2 \approx (2\alpha\Delta t)^{-1} \tag{4.17}$$

Given a variance of $\nu_{n,t}$ from Equation 3.25, the variance of ξ, σ_ξ^2, is expected to be:

$$\sigma_\xi^2 \approx \frac{\sigma_0^2}{2\alpha t_0} \tag{4.18}$$

4.4 Determination of Control Variables

In order for the analysis to be valid, the constraint given in Equation 4.6 must hold. By solving for δ, the minimum reduction in commercial speed, δ' for the constraint to hold can be found for given deviations, $\xi_{n,t}$, and $\xi_{n\oplus 1,t}$.

$$\delta' \approx (\alpha + V\Lambda B)\xi_{n,t} - \alpha\xi_{n\oplus 1,t} \tag{4.19}$$

While all $\xi_{n,t}$ are identically distributed with a mean of 0 and standard deviation given in Equation 4.18, there is expected to be some negative covariance in consecutive bus' deviation, since the action of a bus will have opposite effects on the spacing in front and the spacing behind. Taking the variance of both sides of Equation 4.19 results in:

$$\mathrm{Var}(\delta') \approx (\alpha + V\Lambda B)^2\mathrm{Var}(\xi) - \alpha^2\mathrm{Var}(\xi) + 2(\alpha^2 + \alpha V\Lambda B)\mathrm{Cov}(\xi_n, \xi_{n\oplus 1}) \tag{4.20}$$

Because ξ_n and $\xi_{n\oplus 1}$ are identically distributed, the covariance between the two can be approximated as $\mathrm{Cov}(\xi_n, \xi_{n\oplus 1}) \approx \rho\mathrm{Var}(\xi)$ where ρ is the correlation between consecutive deviations. For systems with more than 2 buses, the correlation coefficient, ρ, is expected to be in the range $-0.50 < \rho < 0.15$ which will be verified from simulation results. This is because in a system with at least three buses, a single bus is only one of two buses to determine spacing. Using this relationship and substituting Equation 4.18 results in:

$$\mathrm{var}(\delta') \approx \left((2-2\rho)\alpha + (2-2\rho)V\Lambda B + (V\Lambda B)^2\alpha^{-1}\right)\frac{\sigma_0^2}{2t_0} \tag{4.21}$$

To ensure that the linear control rule is valid 99% of the time, 3 standard deviations of δ' are used resulting in a safe commercial speed reduction of:

$$\delta \approx 3\left((2-2\rho)\alpha + (2-2\rho)V\Lambda B + (V\Lambda B)^2\alpha^{-1}\right)^{\frac{1}{2}}\frac{\sigma_0}{\sqrt{2t_0}} \tag{4.22}$$

Equations 4.18 and 4.22 provide a relationship between the control inputs, α and δ, and the resulting variance in deviation from equilibrium, σ_ξ^2. This relationship can be used by a transit agency in order to calibrate the control to their priorities. If the priority is to obtain the fastest commercial speed while avoiding bunching, then the value of α that minimizes Equation 4.21 is:

$$\alpha^* \approx \frac{V\Lambda B}{\sqrt{2-2\rho}} \tag{4.23}$$

This gives an minimum reduction in commercial speed of:

$$\delta^* \approx 3\sigma_0 \sqrt{\left(2\sqrt{2-2\rho} + (2-2\rho)\right) \frac{V\Lambda B}{2t_0}} \tag{4.24}$$

For $\rho \in (-0.50, 0.15)$, this will range from $4.40\sigma_0\sqrt{\frac{V\Lambda B}{t_0}} < \delta^* < 5.39\sigma_0\sqrt{\frac{V\Lambda B}{t_0}}$

If instead, a bound on variance is desired, Equation 4.18 will provide the necessary α and Equation 4.22 will give the needed reduction in commercial speed, δ.

4.5 Non-Linear Behavior of System

If the differential between the spacing of two buses becomes too large, the constraint in Equation 4.6 will no longer hold and the behavior of the system becomes non-linear. While it is too complex to analytically model the non-linear system as a whole, the behavior of a single bus under the non-linear conditions can be examined and insights derived.

The non-linear conditions occur when bus n has a large enough spacing in front and a small enough spacing behind to cause the control cruising speed to be larger than V. In this case, the bus will no longer be able to speed up enough to overcome the bunching effect. However, these conditions will cause the bus in front, $n \ominus 1$, under the control rule to slow down. The resulting commercial speeds are:

$$v'_{n,t} \approx E - V\Lambda B\xi_{n,t} \tag{4.25}$$

$$v'_{n\ominus 1,t} \approx E - \delta + \alpha\xi_{n\ominus 1,t} - \alpha\xi_{n,t} \tag{4.26}$$

This results in a state equation for bus n of:

$$\xi_{n,t+\Delta t} \approx \alpha\Delta t\xi_{n\ominus 1,t} + (1 - \alpha\Delta t + V\Lambda B\Delta t)\xi_{n,t} - \delta\Delta t + \nu_{n\ominus 1,t} - \nu_{n,t} \tag{4.27}$$

In order for the bus to recover, the deviation must decrease over time such that $\xi_{n,t+\Delta t} - \xi_{n,t} < 0$. Rearranging Equation 4.27 to express the decrease in deviation over time and grouping positive and negative terms gives:

$$\xi_{n,t+\Delta t} - \xi_{n,t} \approx (\alpha\Delta t\xi_{n\ominus 1,t} + V\Lambda B\Delta t\xi_{n,t} + \nu_{n\ominus 1,t}) - (\alpha\Delta t\xi_{n,t} + \delta\Delta t - \nu_{n,t}) \tag{4.28}$$

In order for the deviation to decrease over time, the following condition must hold:

$$\alpha\Delta t\xi_{n,t} + \delta\Delta t - \nu_{n,t} > \alpha\Delta t\xi_{n\ominus 1,t} + V\Lambda B\Delta t\xi_{n,t} + \nu_{n\ominus 1,t} \tag{4.29}$$

In order for this to hold, α should be larger than $V\Lambda B$ and the deviation needs to decrease to within the linear region before bus $n \ominus 1$ slowing down causes $\alpha\xi_{n\ominus 1,t} > \delta$. If, however, the spacing of bus $n \ominus 1$ is small, bus n should be able to recover.

parameter	range
N	[3,20]
V [km/hr]	[25,60]
Λ [pax/km·hr]	[10,100]
τ [sec]	0 or 30
b [sec/pax]	2 or 4
S [km/bus]	[2,6]
K/S [stops/bus]	2, 4 or 8
$\frac{\sigma_0^2}{t_0}$ [km^2/hr]	0.1 or 0.4
$\alpha/V\Lambda b$	0.5, 1 or 2
ρ	-0.25
δ [km/hr]	Equation 4.22
Δt [sec]	5 or 20

Table 4.1: Simulation Parameters

4.6 Simulation Results

The simulation tool presented in Section 3.4 was used to run 200 simulations with parameters chosen at random from Table 4.1. Each simulation was run for 8 hours of simulated time. Control parameter α is chosen proportional to the constants given in Equation 4.23 and δ is calculated from Equation 4.22. For each simulation, the expected variance in ξ, equivalent to the variance in spacing, is calculated according to Equation 4.18. The calculated variance is plotted versus the actual variance from the simulation output in Figure 4.3 for simulations where $\tau = 0$. From this figure, the variance given in Equation 4.18 is shown to be a good predictor for the more realistic simulated case, and an upper-bound for most of the simulations. Given the conservative assumptions this is a very encouraging result. In all cases, the simulation results show clearly that the proposed control is successful in preventing bunching. A representative sample from a simulation is shown in Figure 4.4. The minimum and maximum spacing over all buses is plotted for each time step. The calculated mean is plotted as the straight solid line and the calculated 99% bounds on spacing, $S \pm 3\sigma_\xi$ are plotted as the straight dashed lines. The spacings are often within 10% of the mean and are never outside the 99% bounds. The values of ρ from the simulations are shown in Figure 4.5 and shown to be mainly negative and in the expected range given in Section 4.4.

For the simulations where $\tau > 0$ sec, the effect of discrete stops on the behavior of the simulated system will be more pronounced and the CA model is expected to less accurately model the behavior of the system. Averaging the loss time per stop over all boarding passengers can result in $B >> b$. Using this value to determine the optimal control parameters may over control the system under these conditions. Figure 4.6 gives the calculated variance plotted versus the actual variance from the simulation output for simulations where $\tau = 30$ sec. While the variance calculated from the CA model remains a good predictor and an upper-bound for many of the simulated runs, there are many runs with a much smaller variance than expected. This supports the expectation of the calculated

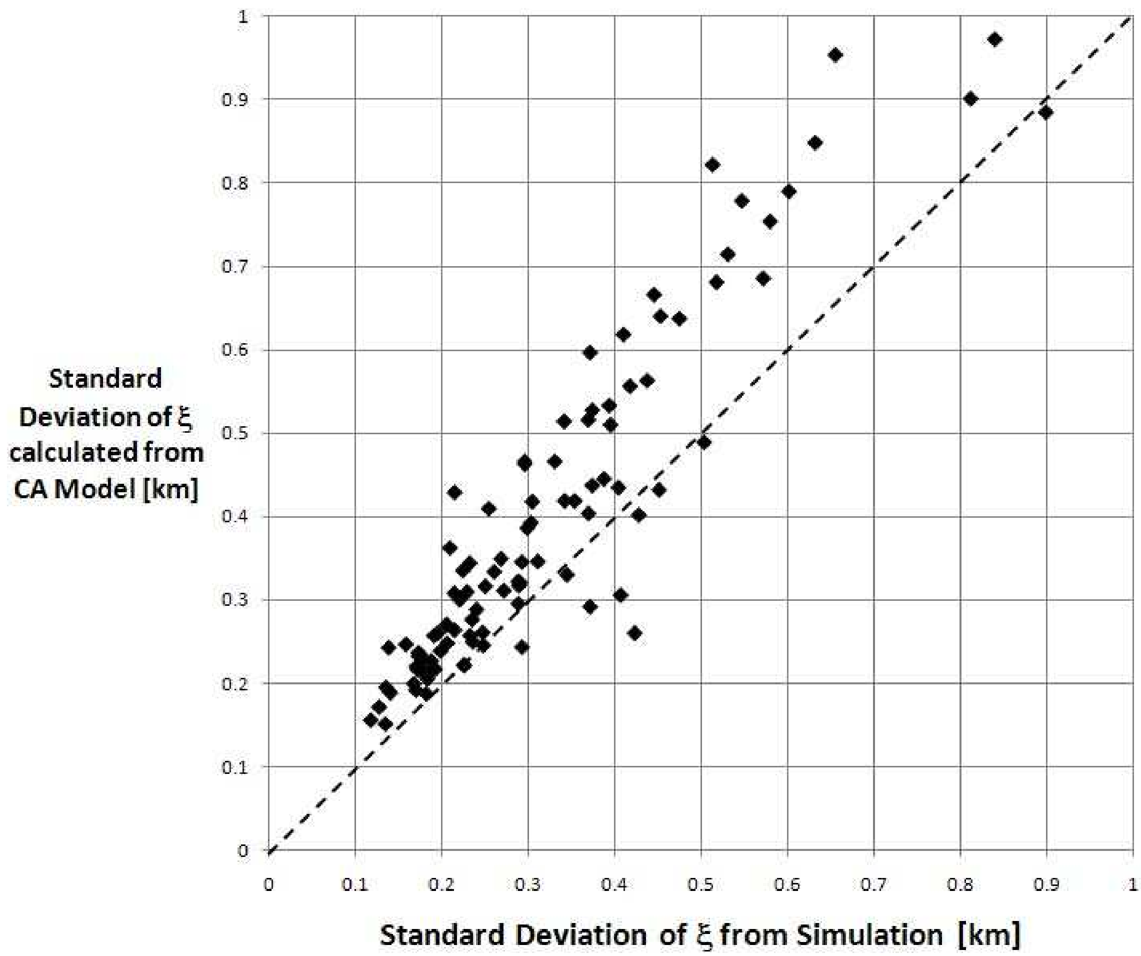

Figure 4.3: Comparison of Calculated and Simulated Standard Deviation of ξ, $\tau = 0$s

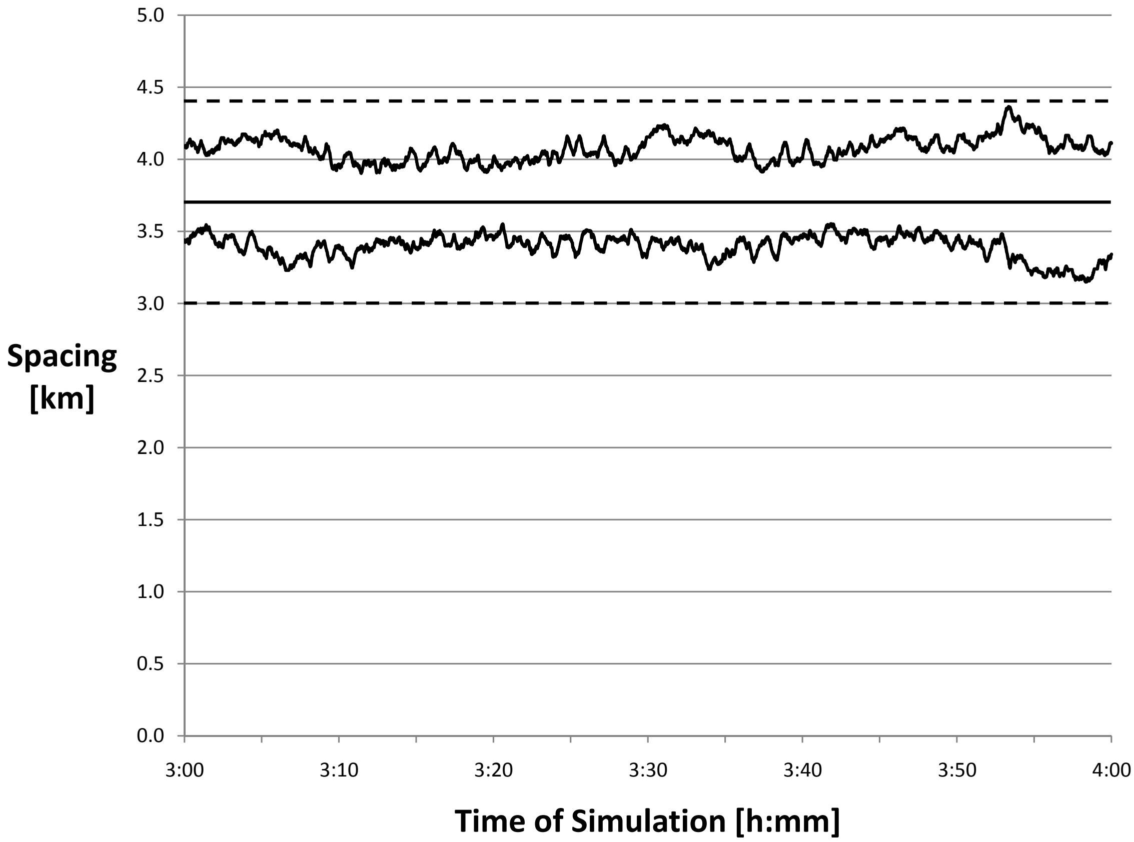

Figure 4.4: Minimum and Maximum Spacing with Calculated Bounds

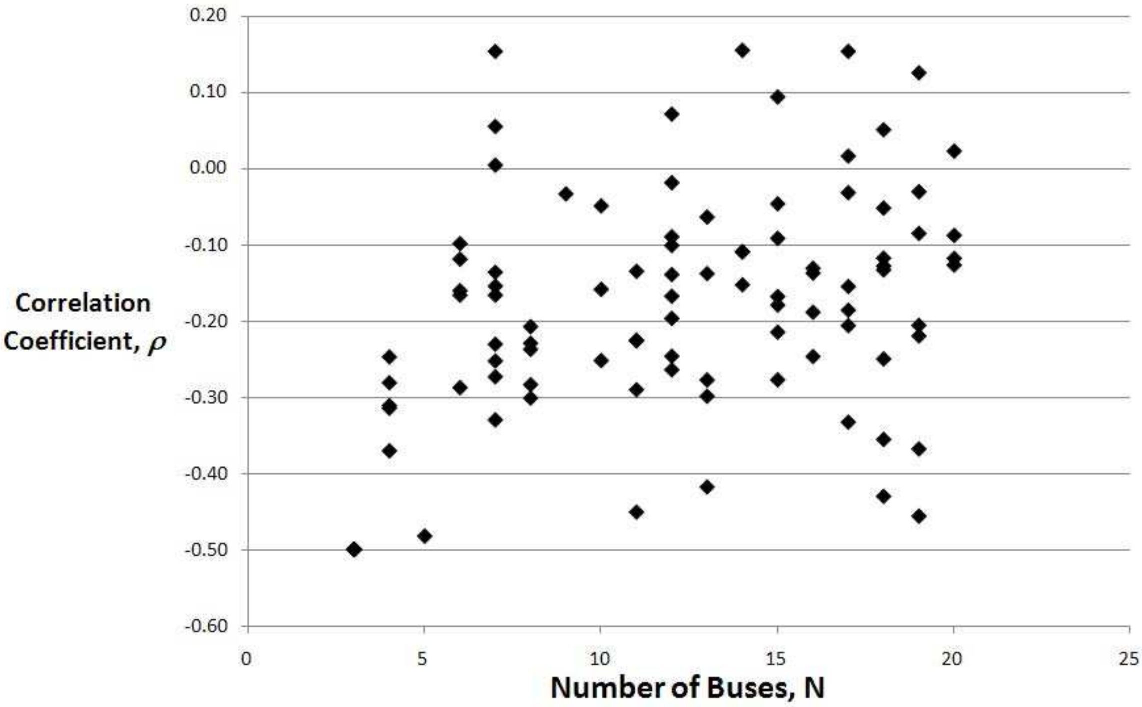

Figure 4.5: Simulated Range of Covariance Coefficient

optimal control parameters over controlling the system. However bunching is still avoided in all cases.

4.7 Final Remarks

The control presented in this chapter overcomes the effect of bunching by allowing buses that have a larger spacing to move faster than buses with a smaller spacing. This can be done by determining a desired cruising speed for a driver and allowing buses to cooperate with neighboring buses to achieve even spacing. To do this requires a reduction in commercial speed, however the trade-off between commercial speed and variance in spacing is defined so that a transit agency can determine a level of control based on their priorities. The control is defined using assumptions to simplify the system, however by using a simulation tool to lift some of the assumptions, the relationships presented in the model are shown to be good estimates.

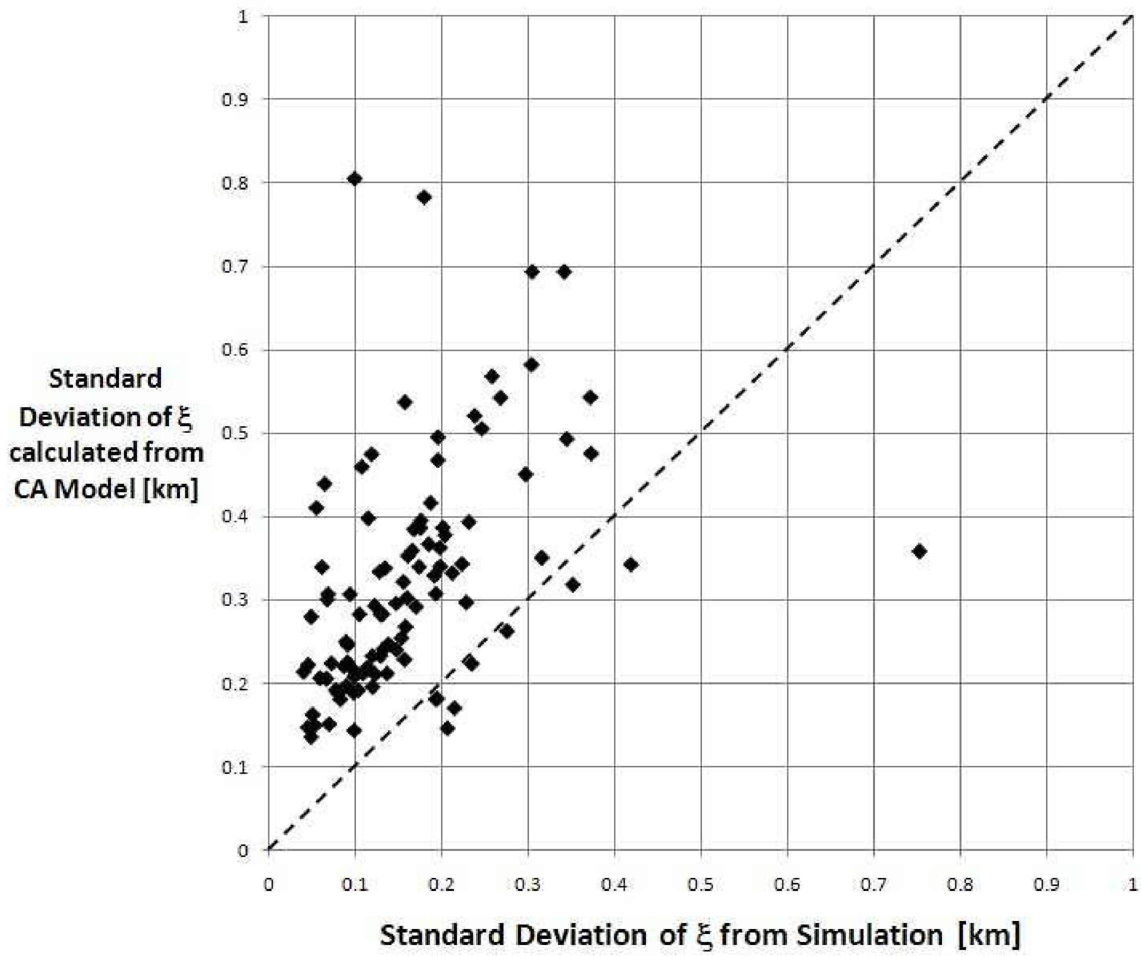

Figure 4.6: Comparison of Calculated and Simulated Standard Deviation of ξ, $\tau = 30$s

Chapter 5

Conclusions

This final chapter summarizes the findings from this dissertation and proposes future work based on the ideas presented.

5.1 Summary of Findings

In order to provide reliable service to users, buses should be evenly spaced along a route. However, it is well known that when a bus system is uncontrolled, fluctuations in passenger arrivals and external disturbances can trigger a bunching effect, causing buses to pair and the spacings between buses to be very uneven.

In this dissertation a continuum approximation model was presented as a systematic tool to examine the behavior of a bus system. By assuming the effect of passenger generation on the commercial speed of a bus is continuous, the speed and position of buses over time can be approximated. Using this model a control is proposed to overcome the bunching effect and allow buses to maintain equal spacing by determining the speed of each bus depending on its relative location to neighboring buses. The control is shown to produce bounded variance in spacing such that deviations from equal spacings will not grow unbounded and buses will not bunch. Additionally, a relationship between the control parameters and the resulting variance in spacing was determined.

A simulation tool was created in order to test the system with with discrete bus stops. The continuous model was expected to provide an upper bound because of the conservative assumptions made. A large number of simulations were run with random system constants and control parameters and the variances predicted by the model were shown to be good estimates for the more realistic simulation.

5.2 Future Work

There are several directions of research in which the ideas presented in this dissertation can be continued. They can be generally grouped into two categories: further refinement of the model and proposed control presented in this dissertation, and development of the proposed control towards implementation.

5.2.1 Refinement of Theory

Relaxation of Homogeneity Assumptions

The model and simulation presented in this dissertation assume uniform service and demand in space and time. While this is an idealized condition, it is not realistic. A localized spike in demand or an area of traffic congestion can disrupt the operation of a bus line even under control. If the disruption is large enough, there is a chance that equilibrium operation cannot be regained just through the control. By modeling demand as a function over time and space and allowing the control parameters to be dynamic, the effect of localized disruptions can better mitigated.

Better Understanding of the Non-Linear Control

In this dissertation the parameter for reducing the commercial speed of the bus, δ, is determined such that the system operates for the most part where the linear control rule is valid. However, because of the two-way cooperation, the system is still expected to be stable for some conditions when the linear control rule is not valid. By reducing δ towards 0, the commercial speed of the system can be increased, though at the cost of higher variance of spacing. Though difficult to analyze mathematically, the behavior of the system in this state can be easily analyzed through simulation.

Developing Better Controls

This dissertation presents a feasible, simple control that prevents the bunching of buses and requires only knowledge of the positions of the buses at any time. This is done through the use of conservative assumptions that may not accurately portray the discreteness of the system. Since the control can only affect a bus while it is moving, routes where a bus spends a large amount of time dwelling (because of large demand or high stop frequency) can not take full advantage of the control. Because the CA model assumes passenger demand as continuously affecting the system, routes with a lower density of stops may be over-controlled and travel slower than necessary.

By developing more complicated controls that recognize the discreteness of the system and require more information, a better understanding of the system and therefore a better control over the system can be achieved.

5.2.2 Implementation of Control

Necessary Infrastructure

In order to implement the proposed control on a bus route, each bus serving the route requires a certain amount of hardware. A GPS unit is required to determine the location of the bus as well as communication equipment capable of transmitting the information to neighboring buses. This can be done directly between buses or by transmitting the information to and from a central server. Each bus will then need a computer able to input the GPS data and determine the cruising speed for the driver according to the control algorithm. It is essential to have this equipment on all buses serving the route.

Drivers

It will be necessary to develop an interface to communicate the desired cruising speed to the driver in a safe, non-distracting way. Additionally, drivers must be trained and encouraged to use the control.

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Appendix A

Glossary of Symbols

Physical Characteristics

L – Route length [km]
Λ – Demand Rate Density [pax/hr·km]
K – Stop Density [stop/km]

Service Characteristics

N – Number of Buses [bus]
S – Desired Equilibrium Spacing [km]
V – Average Cruising Speed in Traffic [km/hr]
E – Equilibrium Commercial Speed [km/hr]
H – Equilibrium Headway [hr]
b – Loss Time per Passenger (boarding) [hr/pax]
τ – Loss Time per Stop [hr/stop]
B – Loss Time per Passenger (boarding + stop) [hr/pax]

State Variables and Indices

n – Bus Index [bus]
t – Time Index [hr]
$x_{n,t}$ – Position of Bus [km]
$s_{n,t}$ – Spacing of Bus [km]
$y_{n,t}$ – CA Position of Bus [km]
$h_{n,t}$ – Headway of Bus [hr]
$v_{n,t}$ – Instantaneous Commercial Speed of Bus [km/hr]
$\xi_{n,t}$ – Deviation from Equilibrium Spacing [km]
$\nu_{n,t}$ – Traffic Noise Random Variable [km]
t_0 – Time of Effect for $\nu_{n,t}$ [hr]
σ_0^2 – Variance of $\nu_{n,t}$ for time t_0 [km^2]
$\varphi_{n,t}$ – Sum of Noise Terms Affecting a Spacing [km]

Analysis Variables

D – Distance Traveled [km]
v_a – Average Speed for a distance D [km/hr]
t_c – Time Spent Cruising for a Distance D [hr]
t_d – Time Spent Dwelling for a Distance D [hr]
i – Bus Index [bus]
Δt – Time Step [hr]
m – Time Step Index
j – Time Step Index
β – Bunching Coefficient [hr^{-1}]
f_i – Convolution Coefficient
$\mathbf{F}$ – f Matrix
$\boldsymbol{\Phi}$ – ν Matrix
ρ – Correlation Coefficient for Consecutive ξ
k_ξ^2 – Variance Amplification
σ_ξ^2 – Variance of Deviation (and Spacing) [km^2]
Δd – Passenger Generation Distance Step [km]

Control Variables

α – Slope of Control [hr^{-1}]
δ – Reduction in Commercial Speed of Control [km/hr]
$E\prime$ – Controlled Equilibrium Commercial Speed [km/hr]
$v'_{n,t}$ – Controlled Instantaneous Commercial Speed of Bus [km/hr]
$c'_{n,t}$ – Controlled Cruising Speed of Bus [km/hr]
$y'_{n,t}$ – Controlled CA Position of Bus [km]
$s'_{n,t}$ – Controlled Spacing of Bus [km]
$\xi'_{n,t}$ – Controlled Deviation from Equilibrium Spacing [km]
δ' – Minimum δ to Maintain Linear Control (99% CI) [km/hr]
α^* – α to Minimize δ [hr^{-1}]
δ^* – Minimum δ [km/hr]
f'_i – Controlled Convolution Coefficient
$\mathbf{F}$ – f' Matrix