Title
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The consistency of children's responses to logical statements: Coordinating components of formal reasoning

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Abstract

Processing of formal statements has three distinct phases: assessing truth-values a priori, requesting evidence, and if requested, evaluating this evidence. Previous investigations of children's ability to process formal assertions have studied each of these phases in isolation, but have not asked whether responses to each processing phase are coordinated into consistent response patterns. This study examined the consistency of 28 third (*M*= 8.9) and 33 sixth graders (*M*= 12.0) responses to four types of logical statements: tautologies, contradictions, conjunctions, and disjunctions. The results indicated that third graders' between-item responses are significantly less consistent than sixth graders, that sixth graders are more likely to give correct response on each question phase than third graders, and that development on each question phase may occur independently.

An important milestone in the development of scientific reasoning is the point at which children recognize that some statements are true or false simply because of the their formal structure while others require the gathering and evaluation of empirical evidence. We propose that there are three phases in processing theoretical assertions: 1) evaluating a priori truth values, 2) requesting evidence only if a statement's truth-value is unable to be determined a priori, and 3) correctly mapping evidence to formal states, for statements requiring evidence, so that correct conclusions can be drawn. Thus, a complete account of how children process formal properties requires an examination not only of the development of all three phases in isolation but also of their coordination.

Most previous research in young children's logical reasoning has focused only on one component at a time. That is, research has focused separately on a priori truth-values of statements (Braine & O' Brien, 1998), evidence requests (Osherson & Markman, 1975), or evidence evaluation (Suppes & Feldman, 1971).

This piecemeal focus has led to seemingly contradictory results. For example, preschool children correctly evaluate the a priori truth-values of contradictions as false but incorrectly request evidence for these statements (Osherson & Markman, 1975; Braine & Rumain, 1981). The goal of the present study is to examine children's coordination of all three components involved in processing formal properties.

Logical Statements and Formal Properties

Formal properties describe the relationship between form, evidence and truth-values. A logical statement's formal properties are defined by values on three phases: 1) truth-values before evidence (true, not true, can't tell), 2) whether evidence is necessary (yes, no), and 3) if evidence is necessary, correctly evaluating evidence (true, not true, can't tell). We will focus on the following statement forms: conjunctions, disjunctions, tautologies and contradictions. Each statement type represents different formal properties. The values on each phase distributes the statements into two classes: *logically indeterminate* statements that require evidence to determine their truth-values (hereafter called indeterminate) and *logically determinate* statements that do not require evidence to determine their truth-values (hereafter called determinate) (Suppes, 1957). The statements and components are described in Table1.

Research into young children's processing of formal properties shows mixed evidence for children's competence. While preschool and elementary children show some sensitivity to statements such as contradictions (often evaluating them as false) (Braine & Rumain, 1981) children of the same age tend to request evidence for most problems even when unnecessary and rarely coordinate evaluation and evidence requests correctly (Osherson & Markman, 1975; Morris & Sloutsky, 1999; Ruffman, 1999).

To address this limitation, we examined children's response patterns to determine the degree of between-phase dependence as an index of processing competence. That is, if children correctly process formal properties, then we would expect correct responses in each of the necessary phases. For example, correctly responding to a contradiction requires processing two phases: an a priori evaluation of "not true" and denying a request for evidence. However, a child who lacks such an understanding may not only err on individual question phases (e.g., "true" a priori evaluation) but may fail to link response phases (e.g.,

Table 1- Comparison of Logical Statements by Dimension

	Statement	Example	A Priori?	Evidence Necessary	Evidence/Form Mapping
Determinate	Tautology	The shape is a circle or the shape is not a circle	True	No	All True states
	Contradiction	The shape is a triangle and the shape is not a triangle	False	No	No True States
Indeterminate	Conjunction	The shape is a square and the shape is not red	Can't Tell	Yes	1 True State (square, not red)
	Disjunction	The shape is a square or the shape is not blue	Can't Tell	Yes	2 True States (square) (not blue)

denying request for evidence). In examining this question, we implicitly examined a related question: Are children's responses from a single dimension of formal properties (e.g., a priori evaluations) diagnostic of their overall processing competence?

To examine these research questions, we presented a 'game' to a group of third and sixth grade children in which they were asked to evaluate 16 logical statements. For each statement, each child was asked up to three questions ("phases") per statement: a priori evaluation, a request for evidence, and, if evidence was requested, evidence evaluation. Each child's response to each question phase was coded as correct or incorrect then these question phases were compiled into a response pattern for each of the 16 statements. Response patterns were aggregated and compared to levels predicted by 'chance' decisions at each question phase.

Method

Participants

A total of 28 third graders (8.5-9.5 years, *M*= 8.9) and 33 sixth graders (11.3-13.6 years, *M*= 12.0) participated. The children were recruited from three public schools in Pittsburgh, PA.

Materials

There were four examples of each of four statement types: tautology, contradiction, disjunction, and conjunction. 'Evidence' was provided in the form of a picture (approximately 5"x7") that displayed information related to each statement card. For example, given the statement "The circle is red AND the circle is not red" the picture displays a red circle. Four additional cards and pictures were used for the warm-up tasks. The statements and evidence were designed to allow us to differentiate among response patterns.

Procedure

The experiment was conducted in a single 15-20 minute session that included two segments: a warm-up and an experimental segment. Each participant was tested individually. In the warm-up segment, the interviewer read a set of instructions explaining the game's purpose. The warm-up period lasted approximately five minutes and consisted of four trials. All instructions and statements for the experimental segment were read to each participant and repeated if requested. Two cards were placed in front of each child: a statement card (face up) and a picture card (face down). The order of presentation was counterbalanced across participants. Each child was given 16 trials, with 1 statement per trial.

For each trial, the following procedure was used. The statement card and picture were placed in front of the child. The child was read the statement card. (Q1) The child was then asked the first question phase: An evaluation of truth status of the statement before evidence (*a priori* evaluation): "Is this statement True, Not True, or Can't Tell. (Q2) After the child responded to the statement, they were asked a second question: Do you need to see the picture to help figure out the sentence? Yes or No. (Q3) If the child requested to see the picture, then they were shown the picture and asked the following: "Now that you have seen the picture, is the statement True, Not True, or Can't Tell."

Coding

Compiled Phase Analysis (CPA) Traditionally, children's responses to each phase of questioning have been analyzed independent of their responses to the other phases. We suggest an analysis of children's response patterns, that is, the frequency of the different types of sequential responses that children generated as they moved through each of the three processing phases for each statement. Because each pattern represents the compilation of responses across the three phases, we call this the Compiled Phase Analysis (CPA).

The set of all possible response patterns – both correct and incorrect – is listed in Table 2. For example, as shown in the top row of Table 2, if a child's responses to a Contradiction were A priori- "False" and Evidence Request- "No", we would code that pattern as C-C because each phase was answered correctly. But many other patterns could – and did - occur. For example, one erroneous response pattern for a contradiction is: A priori- "False", Evidence Request- "Yes" and, Evidence Evaluation- "True," which would be coded as C-I-I, (Table 2 last row of upper section) because the answer to the first phase is correct, but the other two are incorrect. This pattern demonstrates correct a priori evaluation, but incorrect evidence request and evaluation. Another erroneous response pattern for a contradiction is A priori- "Can't Tell", Evidence Request- "Yes", Evidence Evaluation- "False". (Coded as I-I-C). This pattern reveals a different type of misunderstanding: one in which, because the contradiction is not recognized as such, evidence is (incorrectly) requested, but then correctly evaluated. The other patterns listed in Table 2 imply other types of errors. For the remainder of this section, we will look more closely at the distribution of these patterns, without further discussing their underlying implications.

These distributions are informative because a child responding at random to each phase could have generated any particular response pattern. By analyzing the relative frequency of different response patterns – in particular the extent to which they deviate from what would be expected from a random responder - we can begin to understand whether children are responding consistently, albeit erroneously, to these statements or whether their responses to each phase are random, and based only on guessing.

Examining Response Patterns In this section we investigate the extent to which children's response patterns deviate from a randomly generated set. Chance values are based on the assumption that, for each phase, each alternative response is equally likely and is independent of all other phases. Response patterns consisted of either three questions (if evidence was requested) or two questions (if evidence was not requested). The first, a priori evaluation, has three possible responses: "true," "not true" or "can't tell," only one of which is correct, with probability 1/3. The second phase, request for evidence, had two possible responses: "yes or no". Thus the probability of correctly answering this question was 1/2. The third phase, evidence evaluation, occurred only if evidence had been requested (and obtained). Like the first question, this question had three possible responses, with the probability of the one correct response being 1/3.

These probabilities were used to generate the expected chance distribution of the different response patterns for determinate and indeterminate statements. The expected and observed results for each type of pattern are displayed in Table 2 as counts. Also displayed in Table 2 are the observed proportions of each type of pattern for each grade level.

For determinate and indeterminate statements, there were 6 possible response patterns, one correct and five incorrect. Chance values were calculated by computing the probability of choosing a correct or incorrect response on each question phase. For example, for the determinate pattern (I-I-I), the probability of an incorrect a priori response is 2/3, an incorrect evidence request is 1/2, and an incorrect evidence evaluation is 2/3. The conditional probability of selecting this pattern of responses is 2/3*1/2*2/3=2/9. Once the conditional probability was calculated for each possible response pattern, we calculated the total number of patterns we would expect by chance given the number of responses in the data set (229, 3rd graders). So the numbers in Table 2 reflect the number of times we would expect to see each response pattern if a child simply 'flipped a coin' at each decision point.

Finally, the response pattern analysis also indicates the degree to which children correctly process individual response phases. That is, if children err in processing one question phase (e.g., a priori evaluations only) then errors should focus on those patterns that indicate correct response on two phases and errors on one phase. Further, the overall distribution of responses should deviate from chance on patterns that reflect errors on this particular response phase. Such evidence would support the notion of independent developmental trajectories for each question phase.

Results

Configurational Frequency Analysis

We conducted a configurational frequency analysis to compare the observed and expected values from the CPA analysis (von Eye, 1990). This analysis uses assumptions similar to a chi-square analysis to compare the distribution of expected and observed response frequencies. This analysis controls the overall significance level by using the Bonferroni adjustment. The results provide a significance level for the difference between the observed and expected values. When these differences are significant the CFA indicates two classifications: types (in which the observed value is significantly higher then the predicted value) and antitypes (in which the observed value is significantly lower then the predicted value). The analysis was conducted using the 'CFA program for 32 bit operation systems' (von Eye, 1998). The results of the analysis determinate the significance levels indicated in Table 2. Recall that for each of the possible CPA patterns, there is one correct and five incorrect patterns for both determinate and indeterminate statements.

For indeterminate statements, sixth grader's correct CPA responses deviate significantly from chance, while third graders CPA responses do not differ significantly from chance. Sixth graders are below chance on one incorrect pattern. Conversely, third grader's response patterns were above chance on one pattern- I-C-C- and below chance on two patterns- C-I and I-I. This suggests that children requested evidence for most problems failing to distinguish when evidence was unnecessary and often failed to assign correct truth-values before evidence. For determinate statements, neither third nor sixth graders' correct patterns were above chance. Of note however, is that both sixth and third grader's response patterns were significantly above chance for one incorrect pattern- I-C- suggesting that they were not processing a priori and evidence request responses dependently. Third graders were above chance for one incorrect pattern- I-I-I indicating that they erred on all question phases.

Finally, an examination of the types (observed levels are significantly above those predicted by chance) and anti-types (observed levels are significantly below those predicted by chance) supports the notion of independent developmental trajectories for each question phase. Third graders response patterns demonstrated a lack of understanding of the necessity of evidence often erroneously requesting evidence for determinate statements. Few third (or sixth) graders failed to request evidence for indeterminate statements (I-I and C-I patterns) suggesting that in general, children erred on the side of evidence requests.

For determinate statements, third and sixth graders also made fewer I-C patterns than expected by chance. Thus, children rarely made an error on a priori evaluations and were correct on evidence requests suggesting that the former is better established in third graders than the latter. Overall, children's patterns suggested that errors on evidence evaluation were the least frequent while errors on evidence requests were the most frequent.

Correct Response Patterns

In order to compare the number of correct responses by age and type a 2 (grade) X 2 (statement type: determinate v. indeterminate) ANOVA was performed with grade as a between-subjects factor and statement type as a within-subjects factor. Each correct pattern was scored as 1 while each incorrect pattern was scored as 0. Results indicated that sixth graders gave significantly more correct responses on both determinate $\underline{F}$ (1, 114) = 39, p <.001 and determinate statements $\underline{F}$ (1, 114) = 42, p <.001 than third graders.

CPA: Summary

The Compiled Phase Analysis examined the degree to which children are correctly processing each question phase. The formal properties of each statement type require a particular response pattern in which each question phase is dependent on the previous response. The CPA compared the number of responses due to chance responding on each phase to the number of observed responses. The results suggest 1) a high amount of variability in all children's between phase processing, 2) sixth graders were more likely than third graders to correctly process all phases, and 3) each processing phase develops independently. However, while the CPA indicated phase to phase dependencies, it does not reveal the specific strategies that could produce these patterns (see Morris & Klahr, in review for such as analysis).

Table 2- Predicted and Observed Compiled Response Patterns

Type of response patterns[a]					*Number of responses of each type*[b]			
Statement Type	*A Priori*	*Evidence Request*	*Evidence Evaluation*	*Pattern Code*	*Third Grade*		*Sixth Grade*	
Determinate	**Correct**	**Correct**	**N/A**	**C-C**	15	(38)	85	(44)
	Incorrect	Correct	N/A	I-C	12	(74)+	21	(87)+
	Incorrect	Incorrect	Incorrect	I-I-I	96	(50)*	86	(58)
	Incorrect	Incorrect	Correct	I-I-C	51	(25)	40	(29)
	Correct	Incorrect	Correct	C-I-C	27	(14)	17	(16)
	Correct	Incorrect	Incorrect	C-I-I	23	(25)	14	(29)
					224		263	
Indeterminate	**Correct**	**Correct**	**Correct**	**C-C-C**	35	(13)	110	(16)*
	Correct	Correct	Incorrect	C-C-I	25	(25)	40	(29)
	Correct	Incorrect	N/A	C-I	1	(38)+	3	(44)+
	Incorrect	Correct	Incorrect	I-C-I	57	(48)	22	(58)
	Incorrect	Correct	Correct	I-C-C	78	(25)*	40	(29)
	Incorrect	Incorrect	N/A	I-I	27	(74)+	47	(86)
					223		262	

a. Responses are coded as being ‘correct’ or ‘incorrect’ compared to optimal response patterns. Correct response patterns are in bold.

b. Entries indicate the total number of individual response patterns summed over all children’s responses in each grade. A total of 28 3rd graders and 33 6th graders were given 8 determinate and 8 indeterminate statements. Numbers in parentheses indicate the number of responses expected from a random response pattern.

* Indicates a type (significantly above chance at the p< .05 level) while + indicates and anti-type (significantly below chance at the p< .05 level).

Discussion

We have argued that one previously overlooked component of logical reasoning is the coordination of individual responses into consistent response patterns that reflect an understanding of formal properties. The present study attempted to examine simultaneously children’s responses to each component of formal properties (operationalized as component question phases) and to compare the distribution of children’s responses to what would be expected if children answered each question phase as if they were independent of each other.

The results indicated that sixth graders’ compiled response patterns demonstrated a greater degree of question phase dependence than the response patterns of third graders. When the aggregated patterns were compared to chance, sixth graders’ responses were significantly above chance for ‘correct’ patterns on indeterminate statements while third graders patterns were above chance only for a specific error pattern for indeterminate problems (I-C-C). Overall, third graders response patterns deviated from levels predicted by ‘chance’ responding only on incorrect response patterns.

The results suggest that each of the component properties of formal properties has an individual developmental course and that once each component property is established, the individual components must then be coordinated. The distributions of response patterns suggest the following: 1) children are likely to err when requesting evidence demonstrated by high levels of incorrect evidence requests for determinate statements and low levels of evidence refusal for indeterminate statements, 2) sixth graders often evaluated evidence correctly, even when such evidence is unnecessary. Taken together, the evidence suggests that correct processing on all phases is not present early in development and that correct processing may occur at different times for each phase.

While sixth grader's performance was significantly better than third graders, they still erred on a large number of statements suggesting that they do not correctly process formal properties.

Finally, the large amount of variance in children's response patterns suggests that a focus on one component of formal properties is not diagnostic of children's overall understanding of formal properties. That is, children's responses on one component may erroneously suggest competence (or lack of competence) when only partial competence exists.

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