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Waveguide and Radiation Applications of Modes in Linear Chains of Plasmonic Nanospheres

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Abstract—**We apply an analytical procedure that uses a periodic Green's function based on the Ewald representation to compute the modes in a linear chain of plasmonic nanospheres, including the case of complex modal wave numbers. We analyze both transverse and longitudinal polarizations (with respect to the array axis) of the nanospheres' electric dipole moments. Among all the modes, we discuss those excitable and that can be used for wave guidance through bound modes or for radiation through leaky modes.**

I. INTRODUCTION

Linear chains of nanoparticles are one dimensional periodic arrays that have been studied to miniaturize photonic devices (e.g., biosensors [1]) and to efficiently transport electromagnetic energy (e.g., directive radiators [2], [3] and wave guiding structures [4]-[6]) required for the design of integrated photonic circuits. Linear chains reside their properties onto the excitation of *collective* resonances in periodic arrays with subwavelength inter-particle distance stemming from the resonance of the individual nanoparticles [7].

Over the years, researchers have tried to address the problem of mode propagation in linear chains of nanoparticles [8]-[33]. In particular, full characterization of the modes with real and complex wavenumber in the linear chain of plasmonic nanospheres has been provided in [23] and here briefly summarized. The inherent difficulty to handle the complex wave vector space in open cylindrical coordinates has also been discussed in [23], [34]. The modal classification here discussed provides with the knowledge of the physical bound (non-radiating) and leaky (radiating) modes excitable in the linear chain of plasmonic nanospheres. We then show in this paper how these modes can be used to obtain nano-waveguides and nano-antennas.
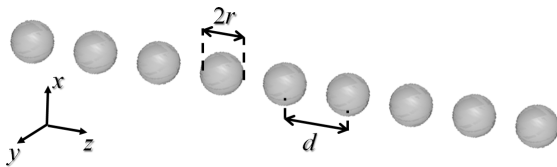

Fig. 1. Linear chain of plasmonic nanospheres (radius r, period d) embedded in a homogeneous host environment with relative permittivity ε_h. The linear chain is supposed to extend to infinity along the z direction.

II. ELECTRODYNAMIC MODEL FOR THE COMPUTATION OF THE PHYSICAL MODES EXCITABLE IN LINEAR CHAINS OF PLASMONIC NANOSPHERES

Consider the linear chain of plasmonic nanospheres embedded in a homogeneous background with relative permittivity ε_h in Fig. 1. Each nanosphere is described by a dipole-like electric polarizability (good approximation when the dipolar term dominates the scattered-field multipole expansion) and is placed at positions $\mathbf{r}_n = z_n \hat{\mathbf{z}}$, with $z_n = nd$, $n = 0, \pm 1, \pm 2, \dots$, and d is the period of the chain.

Suppose that the array supports a mode with dielectric dipole moments equal to $\mathbf{p}_n = \mathbf{p}_0 \exp(ik_z z_n)$ [the monochromatic time harmonic convention $\exp(-i\omega t)$ is assumed], hence the field is periodic except for a phase shift described by the Bloch wavenumber k_z aligned with the chain axis, which also accounts for decay when k_z is complex. The induced electric dipole moment $\mathbf{p}_0$ of the nanosphere at position $\mathbf{r}_0$ is given by

$$\mathbf{p}_0 = \alpha_e \mathbf{E}^{\text{loc}}(\mathbf{r}_0), \qquad (1)$$

where α_e is its isotropic electric polarizability according to Mie theory [7]. The term $\mathbf{E}^{\text{loc}}(\mathbf{r}_0)$ in (1) is the local electric field acting at $\mathbf{r}_0$ produced by all the nanospheres of the array except the one at $\mathbf{r}_0$, and is given by

$$\mathbf{E}^{\text{loc}}(\mathbf{r}_0) = \underline{\breve{\mathbf{G}}}^{\infty}(\mathbf{r}_0, \mathbf{r}_0, k_z) \cdot \mathbf{p}_0, \qquad (2)$$

where $\underline{\breve{\mathbf{G}}}^{\infty}(\mathbf{r}_0, \mathbf{r}_0, k_z)$ is the electric field periodic dyadic Green's function (GF) for the phased periodic array of electric dipoles, without considering the self coupling (the reader is addressed to [23] for more details). We employ the Ewald method for fast computation of the regularized GF in (2) because it provides with the analytic continuation of k_z into the complex wavenumber space, and the expressions for the required spatial and spectral terms have been reported elsewhere [23]. Furthermore, the Ewald representation has the spatial singularity explicitly shown, which makes it simple to regularize by subtracting the self term from the periodic GF.

By combining (2) with (1), one obtains a linear system from which one could compute the modal k_z wavenumber by solving

$$\det\left[\underline{\mathbf{I}} - \alpha_e \underline{\breve{\mathbf{G}}}^\infty (\mathbf{r}_0, \mathbf{r}_0, k_z)\right] = 0 \qquad (3)$$

for complex k_z.

The field relative to a mode in the linear chain is expressed in terms of spatial Floquet waves as

$$\mathbf{E}^{\text{mode}}(\mathbf{r}, k_z) = \sum_{p=-\infty}^{+\infty} \mathbf{e}_p^{\text{mode}}(x, y, k_z) e^{ik_{z,p}z}, \qquad (4)$$

where $k_{z,p} = k_z + 2\pi p / d = \beta_{z,p} + i\alpha_z$ are the Floquet wavenumbers, p is the order of the Floquet harmonic, and $\mathbf{e}_p^{\text{mode}}$ is the transverse eigenvector of the p-th Floquet harmonic. Moreover, each Floquet wavenumber has the same attenuation constant α_z and periodic (in the complex wavenumber domain) propagation constant $\beta_{z,p}$, with period $2\pi p / d$. Due to this periodicity, a mode will be described by the wavenumber of its Floquet wave in the fundamental Brillouin zone (BZ), defined as $-\pi / d < \beta_{z,0} < \pi / d$. Here we assume here that the wavenumber k_z is the one that lies in the fundamental BZ, i.e., $k_z = k_{z,0}$. Furthermore, the radial wavenumber is $k_{\rho,p} = \sqrt{k^2 - k_{z,p}^2} = \beta_{\rho,p} + i\alpha_{\rho,p}$, where $k = \omega\sqrt{\varepsilon_0 \varepsilon_h \mu_0}$ is the wavenumber in the host material. Proper waves, i.e., those that vanish for $\rho = \sqrt{x^2 + y^2} \to \infty$, are defined as those with $\alpha_{\rho,p} > 0$. Improper waves, i.e., those that grow for $\rho \to \infty$, are those with $\alpha_{\rho,p} < 0$.

TABLE I
CLASSIFICATION OF PHYSICAL MODES WITH COMPLEX WAVENUMBER OF THE FUNDAMENTAL FLOQUET HARMONIC

	Forward Wave $\beta_{z,0}\alpha_z > 0$	Backward Wave $\beta_{z,0}\alpha_z < 0$
Slow Wave	$\|\beta_{z,0}\| > k$ $\alpha_{\rho,0} > 0$ (proper, bound)	$\|\beta_{z,0}\| > k$ $\alpha_{\rho,0} > 0$ (proper, bound)
Fast Wave	$\|\beta_{z,0}\| < k$ $\alpha_{\rho,0} < 0$ (improper, leaky)	$\|\beta_{z,0}\| < k$ $\alpha_{\rho,0} > 0$ (proper, leaky)

Among all the mathematical solutions of (4), only a subset represents physical waves, i.e., those that can be excited by a localized source, a defect, or array truncation. Note that modes that are not classified as physical in this paper may however be excited by much more complicated source distributions, though this study is not within the scope of this paper. The physical waves are summarized in Table I based on the complex wavenumber of the fundamental Floquet harmonic in the first BZ, i.e., the one with $p = 0$. Modes are classified as backward when $\beta_z\alpha_z < 0$ and forward when $\beta_z\alpha_z > 0$; bound when $|\beta_z| > k$ (slow wave) and leaky when $|\beta_z| < k$ (fast wave). The periodic condition $k_{z,p} = k_z + 2\pi p / d$ would determine the behavior of Floquet harmonics with wavenumbers in other BZs. More details about classification of physical modes can be found in [23].

III. DISPERSION DIAGRAMS

We show the dispersion diagrams $(\beta_z, \alpha_z) - k$, for both transverse (T-pol, Fig. 2) and longitudinal (L-pol, Fig. 3) polarization states relative to the physical modes excitable in a linear chain of silver nanospheres embedded in free space with $\varepsilon_h = 1$. The reader is addressed to [23] for a detailed modal description, including nonphysical modes and modal wavenumber frequency evolution in the complex k_z plane. The radius of the nanospheres is $r = 35$ nm, the relative permittivity of silver is described by a tabulated measured dielectric function [35], and the period is $d = 75$ nm.
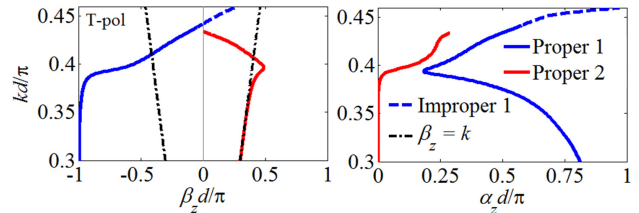

Fig. 2. Dispersion diagram showing the physical modes for T-pol. (a) Real part and (b) imaginary part of the wavenumber of the fundamental Floquet wave $k_z = \beta_z + i\alpha_z$. Mode 'Proper 2' is physical only when $\beta_z > k$, i.e., under the light line.
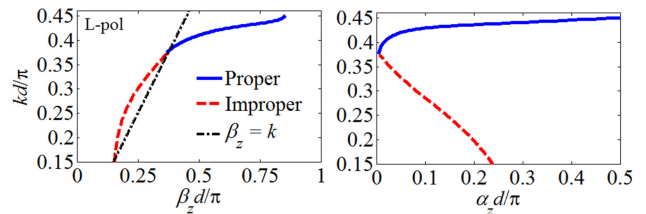

Fig. 3. As in Fig. 2, for L-pol. Mode 'Improper' is physical only when $\beta_z < k$, i.e., above the light line.

Modes 'Proper 2' (T-pol) and 'Proper' (L-pol) are good candidates for the development of nano-waveguides because they exhibit a small attenuation constant α_z for some frequency ranges (refer to Fig. 2 and Fig. 3); analogously, mode 'Improper' (L-pol) may be used for the design of directive nano-antennas. Indeed, it is well-known that a small attenuation constant α_z is a required condition for long mode propagation or directive radiation. The attenuation constant α_z can be further lowered by optimizing the chain dimensions [23], [28] or by employing active gain materials

[36]-[43]. For example, the latter method may be used to lower the attenuation constant of mode 'Proper 1' (T-pol) whose minimum value is now limited to $\alpha_z d / \pi \approx 0.18$.

IV. PROPAGATION LENGTH FOR WAVEGUIDE APPLICATIONS

Considering exciting a wave travelling as $\exp(ik_z z)$ along the linear chain with wavenumber k_z, we estimate its propagation length as $L = 1/\alpha_z$ ($1/e$ criterion), reported in Fig. 4 normalized to the free space wavelength $\lambda_0 = 2\pi\sqrt{\varepsilon_h}/k$ for both T- and L-pol states.

We note that mode 'Proper 2' (T-pol) is able to propagate very large distances for a large frequency range. A fairly large propagation distance is travelled also by both modes for L-pol around 750 THz. Mode 'Proper 1' (T-pol) can propagate, in the best case, a length of $L \approx 0.34\lambda_0$ at about 788 THz in the analysed linear chain.
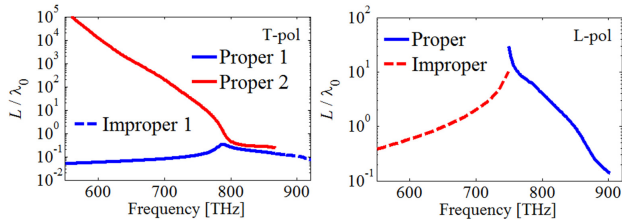

Fig. 4. Propagation length $L = 1/\alpha_z$ normalized to the free space wavelength λ_0 relative to the linear chain used in Fig. 2 and Fig. 3.

V. LEAKY MODES FOR DIRECTIVE RADIATION

We apply array factor theory [44], [45] to determine the radiation pattern of $|E_\theta|$ in Fig. 5 of the 'Improper' (L-pol) forward leaky mode excited in a finite chain of 20 elements, along positive z, to achieve radiation at a certain angle θ (the angle from the z axis in Fig. 1) from the end-fire direction.
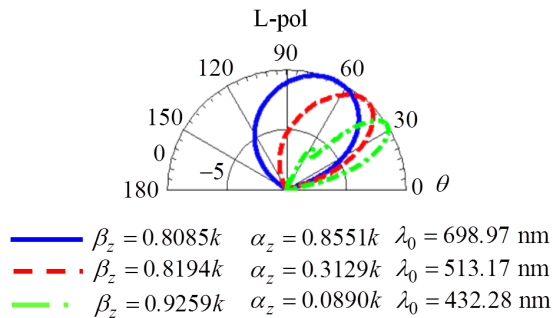

Fig. 5. Radiation pattern of $|E_\theta|$ relative to the excitation of the 'Improper' (L-pol) forward leaky mode estimated using array factor theory at the three frequencies in the legend. The leaky wave parameters β_z and α_z are also provided in the legend.

We have chosen three frequencies pertaining to the red dashed dispersion modal curve in Fig. 3. From the dispersion curve we note that an increase in frequency determines an increase of the propagation constant β_z and a decrease of the

attenuation constant α_z. This will in turn determine the beam to approach a pencil beam shape as well as to point at a smaller angle, as can be clearly seen by looking at the solid blue, dashed red and dashed-dotted green curves in Fig. 5.

VI. CONCLUSION

The strength of the shown analytical procedure based on a periodic Green's function is that it allows for the determination of the physical modes in linear chains of plasmonic nanospheres excitable by a localized source, a defect, or array truncation. The Ewald representation of the periodic GF is very suitable for linear arrays because a fully spectral representation cannot be used when the observer is on the array axis, and representations based on poly-logarithmic functions [8], [9], [13] render the classification of complex modes in their Riemann sheets more difficult. Classification is important for determining if a mode can be excited (physical), and if it is proper/improper, bound/leaky, forward/backward. Complex mode analysis enables the use of the described method for designing nano-waveguides and nano-antennas. Future developments will consider the excitation of a linear chain with finite extent through a dipole source.

REFERENCES

[1] S. M. H. Rafsanjani, T. Cheng, S. Mittler, and C. Rangan, "Theoretical proposal for a biosensing approach based on a linear array of immobilized gold nanoparticles," *J. Appl. Phys.,* vol. 107, pp. 094303-6, 2010.

[2] A. F. Koenderink, "Plasmon Nanoparticle Array Waveguides for Single Photon and Single Plasmon Sources," *Nano Lett.,* vol. 9, pp. 4228-4233, Dec 2009.

[3] X. Liu and A. Alu, "Subwavelength leaky-wave optical nanoantennas: Directive radiation from linear arrays of plasmonic nanoparticles," *Phys. Rev. B,* vol. 82, p. 144305, 2010.

[4] M. Quinten, A. Leitner, J. R. Krenn, and F. R. Aussenegg, "Electromagnetic energy transport via linear chains of silver nanoparticles," *Opt. Lett.,* vol. 23, pp. 1331-1333, 1998.

[5] S. Lal, S. Link, and N. J. Halas, "Nano-optics from sensing to waveguiding," *Nat. Photon.,* vol. 1, pp. 641-648, 2007.

[6] J. Du, S. Liu, Z. Lin, J. Zi, and S. T. Chui, "Guiding electromagnetic energy below the diffraction limit with dielectric particle arrays," *Phys. Rev. A,* vol. 79, p. 051801, 2009.

[7] C. F. Bohren and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*. New York: Wiley, 1983.

[8] R. A. Shore and A. D. Yaghjian, "Complex Waves on 1D, 2D, and 3D Periodic Arrays of Lossy and Lossless Magnetodielectric Spheres," Air Force Research Laboratory, 2010.

[9] R. Shore and A. D. Yaghjian, "Complex waves on periodic arrays of lossy and lossless permeable spheres. Part 1: Theory," *Radio Sci.,* vol. 47, p. RS2014, 2012.

[10] R. A. Shore and A. D. Yaghjian, "Complex waves on periodic arrays of lossy and lossless permeable spheres: 2. Numerical results," *Radio Sci.,* vol. 47, p. RS2015, 2012.

[11] A. F. Koenderink and A. Polman, "Complex response and polariton-like dispersion splitting in periodic metal nanoparticle chains," *Phys. Rev. B,* vol. 74, p. 4, Jul 2006.

[12] A. F. Koenderink, R. de Waele, J. C. Prangsma, and A. Polman, "Experimental evidence for large dynamic effects on the plasmon dispersion of subwavelength metal nanoparticle waveguides," *Phys. Rev. B,* vol. 76, p. 4, Nov 2007.

[13] A. Alu and N. Engheta, "Theory of linear chains of metamaterial/plasmonic particles as subdiffraction optical nanotransmission lines," *Phys. Rev. B,* vol. 74, p. 18, Nov 2006.

[14] W. H. Weber and G. W. Ford, "Propagation of optical excitations by dipolar interactions in metal nanoparticle chains," *Phys. Rev. B,* vol. 70, p. 125429, 2004.

[15] S. Y. Park and D. Stroud, "Surface-plasmon dispersion relations in chains of metallic nanoparticles: An exact quasistatic calculation," *Phys. Rev. B,* vol. 69, p. 7, Mar 2004.

[16] C. R. Simovski, A. J. Viitanen, and S. A. Tretyakov, "Resonator mode in chains of silver spheres and its possible application," *Phys. Rev. E,* vol. 72, p. 066606, 2005.

[17] D. S. Citrin, "Plasmon-polariton transport in metal-nanoparticle chains embedded in a gain medium," *Opt. Lett.,* vol. 31, pp. 98-100, Jan 2006.

[18] S. A. Maier, P. G. Kik, and H. A. Atwater, "Optical pulse propagation in metal nanoparticle chain waveguides," *Phys. Rev. B,* vol. 67, p. 205402, May 2003.

[19] D. Van Orden, Y. Fainman, and V. Lomakin, "Optical waves on nanoparticle chains coupled with surfaces," *Opt. Lett.,* vol. 34, pp. 422-424, Feb 2009.

[20] R. A. Shore and A. D. Yaghjian, "Travelling electromagnetic waves on linear periodic arrays of lossless spheres," *Electron. Lett.,* vol. 41, pp. 578-580, May 2005.

[21] K. H. Fung and C. T. Chan, "Plasmonic modes in periodic metal nanoparticle chains: a direct dynamic eigenmode analysis," *Opt. Lett.,* vol. 32, pp. 973-975, 2007.

[22] S. Steshenko and F. Capolino, "Single Dipole Approximation for Modeling Collections of Nanoscatterers," in *Theory and Phenomena of Metamaterials*, F. Capolino, Ed., ed Boca Raton, FL: CRC Press, 2009, p. 8.1.

[23] S. Campione, S. Steshenko, and F. Capolino, "Complex bound and leaky modes in chains of plasmonic nanospheres," *Opt. Express,* vol. 19, pp. 18345-18363, 2011.

[24] M. Conforti and M. Guasoni, "Dispersive properties of linear chains of lossy metal nanoparticles," *J. Opt. Soc. Am. B,* vol. 27, pp. 1576-1582, 2010.

[25] C. Tserkezis and N. Stefanou, "Calculation of waveguide modes in linear chains of metallic nanorods," *J. Opt. Soc. Am. B,* vol. 29, pp. 827-832, 2012.

[26] Y. Hadad and B. Z. Steinberg, "Green's function theory for infinite and semi-infinite particle chains," *Phys. Rev. B,* vol. 84, p. 125402, 2011.

[27] B. Rolly, N. Bonod, and B. Stout, "Dispersion relations in metal nanoparticle chains: necessity of the multipole approach," *J. Opt. Soc. Am. B,* vol. 29, pp. 1012-1019, 2012.

[28] I. B. Udagedara, I. D. Rukhlenko, and M. Premaratne, "Surface plasmon-polariton propagation in piecewise linear chains of composite nanospheres: The role of optical gain and chain layout," *Opt. Express,* vol. 19, pp. 19973-19986, 2011.

[29] S. M. Raeis Zadeh Bajestani, M. Shahabadi, and N. Talebi, "Analysis of plasmon propagation along a chain of metal nanospheres using the generalized multipole technique," *J. Opt. Soc. Am. B,* vol. 28, pp. 937-943, 2011.

[30] A. Hochman and Y. Leviatan, "Rigorous modal analysis of metallic nanowire chains," *Opt. Express,* vol. 17, pp. 13561-13575, 2009.

[31] A. A. Govyadinov and V. A. Markel, "From slow to superluminal propagation: Dispersive properties of surface plasmon polaritons in linear chains of metallic nanospheroids," *Phys. Rev. B,* vol. 78, p. 035403, 2008.

[32] G. S. Blaustein, M. I. Gozman, O. Samoylova, I. Y. Polishchuk, and A. L. Burin, "Guiding optical modes in chains of dielectric particles," *Opt. Express,* vol. 15, pp. 17380-17391, 2007.

[33] D. Szafranek and Y. Leviatan, "A Source-Model Technique for analysis of wave guiding along chains of metallic nanowires in layered media," *Opt. Express,* vol. 19, pp. 25397-25411, 2011.

[34] A. I. Nosich, "Radiation conditions, limiting absorption principle, and general relations in open wave-guide scattering," *J. Electromagnet. Wave,* vol. 8, pp. 329-353, 1994.

[35] P. B. Johnson and R. W. Christy, "Optical Constants of the Noble Metals," *Phys. Rev. B,* vol. 6, pp. 4370-4379, 1972.

[36] J. A. Gordon and R. W. Ziolkowski, "CNP optical metamaterials," *Opt. Express,* vol. 16, pp. 6692-6716, 2008.

[37] P. Holmstrom, L. Thylen, and A. Bratkovsky, "Composite metal/quantum-dot nanoparticle-array waveguides with compensated loss," *Appl. Phys. Lett.,* vol. 97, pp. 073110-3, 2010.

[38] S. Campione, M. Albani, and F. Capolino, "Complex modes and near-zero permittivity in 3D arrays of plasmonic nanoshells: loss compensation using gain [Invited]," *Opt. Mater. Express,* vol. 1, pp. 1077-1089, 2011.

[39] S. Campione and F. Capolino, "Composite material made of plasmonic nanoshells with quantum dot cores: loss-compensation and ε-near-zero physical properties," *Nanotechnology,* vol. 23, p. 235703, 2012.

[40] S. D. Campbell and R. W. Ziolkowski, "The Performance of Active Coated Nanoparticles Based on Quantum-Dot Gain Media," *Adv. Optoelectron.,* vol. 2012, p. 6, 2012.

[41] A. Ciattoni, R. Marinelli, C. Rizza, and E. Palange, "|\epsilon|-Near-zero materials in the near-infrared," *Appl. Phys. B: Lasers Opt.,* pp. 1-4, 2012.

[42] S. Xiao, V. P. Drachev, A. V. Kildishev, X. Ni, U. K. Chettiar, H.-K. Yuan, and V. M. Shalaev, "Loss-free and active optical negative-index metamaterials," *Nature,* vol. 466, pp. 735-738, 2010.

[43] G. Strangi, A. De Luca, S. Ravaine, M. Ferrie, and R. Bartolino, "Gain induced optical transparency in metamaterials," *Appl. Phys. Lett.,* vol. 98, pp. 251912-3, 2011.

[44] D. R. Jackson, J. Chen, R. Qiang, F. Capolino, and A. A. Oliner, "The role of leaky plasmon waves in thedirective beaming of light through asubwavelength aperture," *Opt. Express,* vol. 16, pp. 21271-21281, 2008.

[45] Q. Song, S. Campione, O. Boyraz, and F. Capolino, "Silicon-based optical leaky wave antenna with narrow beam radiation," *Opt. Express,* vol. 19, pp. 8735-8749, 2011.