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Author

Pippenger, John E

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THE SOLUTION TO THE FORWARD-BIAS AND RELATED PUZZLES*

John Pippenger

Department of Economics

University of California

Santa Barbara 93106

USA

The forward-bias puzzle is probably the most important puzzle in international finance. But

there is a simple solution. Covered interest parity implies that the forward-bias puzzle is the

result of two omitted variables: (1) the future change in the forward exchange rate and (2) the

future interest rate differential. As Table 3 shows, at least for my data, the downward bias

produced by those two omitted variables completely explains the forward-bias puzzle. Covered

interest parity also solves three related puzzles.

Author: pipp@ix.netcom.com, 619-423-3618.

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The forward-bias puzzle is the most important puzzle in international finance because it suggests that there are serious informational inefficiencies in foreign exchange markets. A prodigious amount of empirical and theoretical work has produced a variety of unsatisfactory explanations and the puzzle has remained unsolved for over 20 years. But covered interest parity suggests a simple solution. The forward-bias is the result of two omitted variables.

To the best of my knowledge, all previous attempts to solve the forward-bias puzzle assume that, with risk neutrality, the forward exchange rate is the market's expectation of the future spot exchange rate. See for example Fama (1984), Goodhart, McMahon and Ngama (1992), Nikolaou and Sarno (2006), Sarno, Valente and Leon (2006), Sercu and Vinaimont (2006), Chakraboty and Haynes (2008), Chakraborty and Evans (2008) and Bacchetta and van Wincoop (forthcoming). For example, Chakraborty and Evans (2008, p. 487) say that "If agents are risk neutral then they must set today's forward rate equal to their expectation about the future spot rate,..."

As shown below, that assumption is usually inconsistent with covered interest parity. When covered interest parity holds, the expected future spot rate equals the expected future forward exchange rate plus the expected future interest rate differential. As a result, the expected future change in the spot rate equals the forward premium plus the expected future change in the forward rate minus the expected future interest rate differential. As Table 3 shows, at least for my data, the downward bias caused by omitting the future change in the forward rate and the future interest rate differential completely explains the forward-bias puzzle. If additional research supports the results in Table 3, the forward-bias puzzle is finally solved.

A convincing solution to the forward-bias puzzle should also solve three related puzzles:

(1) Why is there such strong empirical support for covered interest parity and so little support for

¹ For a discussion of some of the other puzzles see Obstfeld and Rogoff (2000).

uncovered interest parity? A risk premium does not appear to be the explanation. I call this the incompatibility puzzle. (2) Why is the coefficient for the forward exchange rate usually close to, *but less than*, one when the future spot exchange rate is regressed against the current forward exchange rate? I call this the levels puzzle. (3) Why is the variance for changes in spot exchange rates 100 to 200 times greater than the variance of the forward premium? I call this the variance puzzle. Covered interest parity also solves these three puzzles.

Section 1 briefly reviews the forward-bias puzzle. Section 2 briefly reviews the evidence regarding covered interest parity and shows what covered interest parity implies about expected future spot exchange rates. Section 3 uses Section 2 to show, I believe for the first time, that, except primarily for certain equilibrium conditions, covered and uncovered interest parity are mutually incompatible even in the absence of any risk premium. Section 4 uses Section 2 to show how covered interest parity can solve the forward-bias puzzle. Section 5 does the same for the remaining two subsidiary puzzles. Section 6 presents the evidence supporting the solutions developed in Sections 3, 4 and 5. Section 7 summarizes the article and concludes that covered interest parity solves all four puzzles.

1. The Forward-Bias Puzzle

There is a large literature claiming that the forward premium is a poor predictor of future changes in exchange rates. For a discussion of the puzzle and a review of the earlier literature see Sarno (2005). Some of the more recent articles include Nikolaou and Sarno (2006), Sarno, Valente and Leon (2006), Sercu and Vinaimont (2006), Kearns (2007), Chakraboty and Haynes (2008), Chakraborty and Evans (2008), Wang and Wang (2009), Frankel and Poonawala (2010), Hochradl and Wagner (2010) and Bacchetta and van Wincoop (forthcoming).

All that literature assumes at least implicitly that forward exchange rates are rational expectations of future spot rates. As another example of that assumption, Sercu and Vinaimont (2006, p. 2417) say the following: "The unbiased-expectations Null says that the time-t expected value for the spot currency j at time $t+\Delta$,..., equals the time-t forward rate for that horizon."

As shown below, when covered interest parity holds, in general forward rates are not expected future spot rates. As a result, the forward premium in isolation produces a biased estimate of the actual future change in the exchange rate because of two omitted variables: the future change in the forward exchange rate and the future interest rate differential.

Equation (1) describes the typical test equation in the literature. Let s_t be the logarithm of the current spot price for foreign exchange S_t . Let f_t be the logarithm of the current forward exchange rate F_t . Finally let s_{t+1} be the logarithm of the future spot rate S_{t+1} .

$$\Delta S_{t+1} = S_{t+1} - S_t = \alpha_1 + \beta_1 (f_t - S_t)$$
 (1)

Using equations like (1), estimates of β_1 are closer to zero than to one and are usually negative. For examples of such estimates using the data described later, see Table 1.

Negative estimates of β_1 seem to imply an informational inefficiency. Exchange rates fall when the forward premium seems to predict that they will rise. That apparent predictive error is the forward-bias puzzle.

Equation (1) is supposed to ask a simple question. How well does the market predict changes in exchange rates? But equation (1) does not pose that question correctly. The implicit assumption behind equation (1) is that f_t is the market's expectation of s_{t+1} . When covered interest parity holds, as appears to be the case at least between developed countries, that assumption fails under most actual conditions. Under most actual conditions, the future change

in the exchange rate depends on the forward premium, the future change in the forward exchange rate and the future interest rate differential.

2. Covered Interest Parity

Covered interest parity (CIP) is an equilibrium condition implied by effective arbitrage. Equation (2) describes covered interest parity.

$$(f_t - s_t) - (i_t - i_t^*) = \pm e_t$$
 (2)

In equation (2) i is the domestic interest rate, i* is the foreign interest rate and $\pm e_t$ captures the errors within the thresholds created by transaction costs.² The interest rates should be risk free and their maturities must match the maturity of the forward exchange rate. With effective arbitrage, covered interest parity holds whether or not there is a risk premium.

After accounting for the transaction costs, covered interest parity appears to hold on a day-to-day basis. As Akram, Rime and Sarno (2008) point out, "It seems generally accepted that financial markets do not offer risk-free arbitrage opportunities, at least when allowance is made for transaction costs." In the Conclusions to their article, Akram, Rime and Sarno explain in more detail how covered interest rate arbitrage works.

This paper provides evidence that short-lived arbitrage opportunities arise in the major FX and capital markets in the form of violations of the CIP condition. The size of CIP arbitrage opportunities can be economically significant for the three exchange rates examined and across different maturities of the instruments involved in arbitrage. The duration of arbitrage opportunities is, on average, high enough to allow agents to exploit deviations from the CIP condition. However, duration is low enough to suggest that markets exploit arbitrage opportunities rapidly. These results, coupled with the unpredictability of the arbitrage opportunities, imply that a typical researcher in international macrofinance can safely assume arbitrage-free prices in the FX markets when working with daily or lower frequency data.

See Balke and Wohar (1998) for evidence of the thresholds created by transaction costs.

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 $^{^2}$ Without logarithms, the equilibrium condition is $[F_t/S_t]/[(1+i_t)/(1+i_t^*)]=(1\pm e_t).$

For simplicity of exposition for now I ignore the thresholds created by transaction costs and assume that the forward premium, $f_t - s_t$, equals the interest rate differential, i_t .

When covered interest parity holds, the current spot exchange rate equals the forward rate minus the interest rate differential.

$$s_{t} = f_{t} - (i_{t-1}i_{t}^{*})$$
(3)

Equation (3) implies equation (4). The future spot exchange rate equals the future forward exchange rate minus the future interest rate differential.

$$S_{t+1} = f_{t+1} - (i_{t+1} - i_{t+1}^*)$$
(4)

Subtracting the current forward rate f_t from the both sides of equation (4) shows that, when covered interest parity holds, the current forward rate will not equal the future spot rate unless $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ is zero.

$$S_{t+1} - f_t = (f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$$
(5)

There are several, mostly equilibrium, conditions where $(f_{t+1} - f_t) - (i_{t+1} \cdot i_{t+1}^*)$ will be zero. But in the data set described below, $(f_{t+1} - f_t) - (i_{t+1} \cdot i_{t+1}^*)$ is occasionally small but it is never zero.

Given equation (4), equation (6) shows the rational expectation at time t of s_{t+1} .

$$\mathbf{s}_{t+1}^{E} = \mathbf{f}_{t+1}^{E} - (\mathbf{i}_{t+1}^{E} - \mathbf{i}_{t+1}^{*E})$$
 (6)

Where x_{t+1}^E is the rational expectation at time t of x_{t+1} . According to equation (6) the only way the f_t can equal s_{t+1}^E is for f_t to equal f_{t+1}^E - $(i_{t+1}^E$ - $i_{t+1}^{*E})$.

Subtracting f_t from both sides of equation (6) provides a more useful way to describe what covered interest parity implies about the relation between the current forward rate and the expected future spot rate. Equation (7) is also another way of stating the conditions necessary for uncovered interest parity to hold when covered interest parity holds. If covered interest parity holds, uncovered interest parity can hold only when s_{t+1}^E equals f_t .

$$\mathbf{s}_{t+1}^{E} - \mathbf{f}_{t} = (\mathbf{f}_{t+1}^{E} - \mathbf{f}_{t}) - (\mathbf{i}_{t+1}^{E} - \mathbf{i}_{t+1}^{*E})$$
(7)

For the forward rate to equal the expected future spot rate, $(f_{t+1}^E - f_t) - (i_{t+1}^E - i_{t+1}^{*E})$ must equal zero.

To see why s_{t+1}^E is not likely to equal f_t , consider first a situation in which they are equal. Assume a steady state in which there is neither expected nor actual inflation in either country, exchange rates are constant and real interest rates are the same in both countries. In such a steady state equilibrium s_{t+1}^E should equal f_t because $(f_{t+1}^E - f_t)$ and $(i_{t+1}^E - i_{t+1}^*)$ should both be zero. s_{t+1} should also equal f_t because $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$ also should both be zero.

But suppose all those conditions hold except that real interest rates are not the same. In that case s_{t+1} should not equal f_t and s_{t+1}^E should not equal f_t . Or suppose the real interest rates are the same but the difference in the expected rates of inflation, which determines the interest rate differential, does not equal the expected change in the forward exchange rate.

Although there are a variety of conditions under which forward rates and future spot rates might be equal, in a dynamic world subjected to many real and monetary shocks it seems highly unlikely that they would always be equal. In my data $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ is never zero.

The carry-trade, which is evaluated in Hochradl and Wagner (2010), is a simple way to try to exploit a forward bias. Under the carry-trade one borrows in the country with the low interest rate and lends in the country with the high interest rate without any forward cover. Equation (7) describes the expected return from such a strategy when covered interest parity holds.

Let the foreign country, say the U.K., be the country with the high interest rate. If $(f_{t+1}^E - f_t)$ - $(i_{t+1}^E - i_{t+1}^{*E})$ in equation (7) is zero, the expected return from the carry-trade is zero because the expected future spot rate equals the current forward rate and there is no net return from covered interest arbitrage. If $(f_{t+1}^E - f_t) - (i_{t+1}^E - i_{t+1}^{*E}) < 0$, the expected future spot price for sterling is less than the current forward rate. As a result, the expected future sterling price of dollars is more

than the forward sterling price of dollars. That price differential implies an expected loss from the carry-trade because the expected cost of repaying the dollar loan is higher than it would be using the forward market where the return is zero. If $(f_{t+1}^E - f_t) - (i_{t+1}^E - i_{t+1}^*) > 0$, the expected future dollar price for sterling is greater than the current forward rate. As a result, the expected future sterling price for dollars is less than the forward sterling price for dollars, which implies an expected gain from the carry trade because the expected cost of repaying the dollar loan is lower than it would be using the forward market where the actual return is zero.

3. CIP and UIP

Uncovered interest parity (UIP) says that the expected change in the exchange rate, s_{t+1}^E - s_t , equals the appropriate interest rate differential i_{t-1}^* . Covered interest parity implies that f_t - s_t equals the same interest rate differential. If f_t does not in general equal s_{t+1}^E because, in general $(f_{t+1}^E - f_t) - (i_{t+1}^E - i_{t+1}^*)$ is not zero, then CIP and UIP are in general mutually incompatible. Since covered interest parity appears to hold on a day-to-day basis, this incompatibility explains why there is such strong support for CIP and so little empirical support for UIP.

As pointed out above, after accounting for transaction costs, covered interest parity appears to hold day by day. But there is very little empirical support for uncovered interest parity even for monthly and quarterly data. See Bekaert, Wei and Xing (2007) for a discussion of that evidence and some new evidence regarding UIP. Perhaps the most convincing evidence of the systematic failure of uncovered interest parity is the large literature on VAR estimates of impulse response functions describing how UIP responds to monetary shocks. The seminal article in that literature is Eichenbaum and Evans (1995). Faust and Rogers (2003) critique and briefly review that literature. In their Conclusions they say that "...monetary policy shocks generate large expected root mean square UIP deviations. Even when imposing very little on the behavior of the

monetary shock, we are unable to find policy shocks that generate interest rate and exchange rate responses roughly consistent with UIP."

4. CIP and the Forward-Bias Puzzle

As far as I am aware all previous discussions, tests and explanations of the forward-bias puzzle have assumed that the forward exchange rate is the market's expectation of the future spot exchange rate. But when CIP holds the forward exchange rate is not in general the market's expectation of the future spot exchange rate. In that case equation 1 is miss-specified. When covered interest parity holds and expectations are rational, equation (8) is the appropriate way to describe the expected change in the exchange rate. Subtracting s_t from both sides of equation (6) produces equation (8).

$$\mathbf{S}_{t+1}^{E} - \mathbf{S}_{t} = (\mathbf{f}_{t+1}^{E} - \mathbf{S}_{t}) - (\dot{\mathbf{I}}_{t+1}^{E} - \dot{\mathbf{I}}_{t+1}^{*E})$$
(8)

One way to implement equation (8) is to assume rational expectations. For each expectation for t+1 at time t the current expected future value equals the actual future value plus an uncorrelated error. Ignoring those errors for the explanatory variables in equation (8) produces equation (9).

$$\mathbf{s}_{t+1}^{E} - \mathbf{s}_{t} = (\mathbf{f}_{t+1} - \mathbf{s}_{t}) - (\dot{\mathbf{i}}_{t+1} - \dot{\mathbf{i}}_{t+1}^{*})$$
(9)

Adding and subtracting f_t from the right-hand side of equation (9) produces equation (10).

$$\mathbf{s}_{t+1}^{E} - \mathbf{s}_{t} = (\mathbf{f}_{t} - \mathbf{s}_{t}) + (\mathbf{f}_{t+1} - \mathbf{f}_{t}) - (\mathbf{i}_{t+1} - \mathbf{i}_{t+1}^{*})$$
(10)

When covered interest parity holds and expectations are rational, f_t - s_t in equation (10) describes the role of the forward premium in the market's expectation of the future change in the exchange rate.

Assuming, as does equation (1), that the actual change in the exchange rate equals the expected change produces equation (11).

$$s_{t+1} - s_t = (f_t - s_t) + (f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$$
(11)

This derivation of equation (11) uses rational expectations. But equation (11) can be derived directly from covered interest parity without appealing to expectations. Subtract s_t from both sides of equation (4) and then add and subtract f_t from the right-hand side of that equation.

Whichever interpretation one prefers, covered interest parity implies that equation (1) is miss-specified because it omits the future change in the forward exchange rate, $f_{t+1} - f_t$, and the future interest rate differential, i_{t+1} - i_{t+1}^* . The forward-bias puzzle is the result of omitting those two variables.

Introducing a constant and coefficients for $(f_t - s_t)$, $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$ into equation (11) produces a correctly specified equation for testing the relationship between the current forward premium and the future change in the exchange rate.

$$\Delta S_{t+1} = \lambda_0 + \lambda_1 (f_t - S_t) + \lambda_2 (f_{t+1} - f_t) - \lambda_3 (i_{t+1} - i_{t+1}^*)$$
(12)

As a direct implication of CIP, equation (12) also provides an indirect test of CIP.

As Table 2 shows, estimates of λ_1 are positive, significant and close to 1.0. As Table 3 shows, estimates of β_1 are biased downward because equation (1) omits $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$. Indeed that downward bias completely explains the estimates of β_1 .

5. Additional Subsidiary Puzzles

Three additional subsidiary puzzles associated with the forward-bias puzzle are usually ignored in attempts to explain the forward-bias puzzle. A convincing solution to the forward-bias puzzle should solve all three. The solution to the incompatibility puzzle is described above. This section takes up the levels and variance puzzles.

5.1 Levels versus Changes

When economists first began to ask how well markets for foreign exchange predicted exchange rates, they regressed future exchange rates against current forward exchange rates.³ That regression routinely produces coefficients close to, *but less than*, 1.0. But when s_t is subtracted from both sides of that equation to obtain stationarity, the coefficient for the forward premium β_1 usually drops below zero. Section 4 explains why β_1 is usually negative. This subsection explains why the coefficient for the forward exchange rate is usually close to, but less than, one.

Covered interest parity implies that equation (13) describes the relation between forward exchange rates and future spot exchange rates. Equation (13) is simply equation (4) with appropriate coefficients B_i added.

$$S_{t+1} = B_0 + B_1 f_{t+1} - B_2 (i_{t+1} - i_{t+1}^*)$$
(13)

Equation (14) is equation (13) with f_t replacing f_{t+1} .

$$s_{t+1} = b_0 + b_1 f_t - b_2 (i_{t+1} - i_{t+1}^*)$$
(14)

Equation (15) is equation (1) in levels. Equation (15) is also equation (14) with $b_2(i_{t+1} - i_{t+1}^*)$ deleted from the right-hand side.

$$s_{t+1} = \alpha_2 + \beta_2 f_t \tag{15}$$

Covered interest parity implies that estimates of β_2 are close to, but less than, one because in equation (15) f_t is acting as a proxy for f_{t+1} . If that implication is correct, the econometric results should deteriorate as one moves from equation (13) to equation (15). In particular, b_1 should be smaller than B_1 . From the perspective of covered interest parity, estimates of β_2 are close to, *but less than*, 1.0 because using f_t as a proxy for f_{t+1} introduces measurement error.

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³ See for example Cornell (1977), Levich (1979) and Frenkel (1980).

5.2 Relative Variances

The final subsidiary puzzle is the difference between the variance for changes in spot exchange rates, Δs_t , and the variance for the forward premium, $f_t - s_t$. According to Wang and Wang (2009, p. 186), "... the variance of spot rate changes is in the range of 100-200 times the variance of the forward premium." My data produce similar results.

When one assumes that forward exchange rates equal expected future spot exchange rates, this very large difference in variances is difficult, if not impossible, to explain. But covered interest parity provides a simple explanation. From the perspective of covered interest parity, the variance of the forward premium, $f_t - s_t$, is relatively small because the variance of the interest rate differential, $i_t - i_t^*$, is relatively small. The variance of Δs_t is relatively large because the variance of Δf_t is relatively large. The variances in Table 5 support that explanation.

There are two ways to look at this explanation. One way is to use equation (11).

$$\Delta S_{t+1} = (f_t - S_t) + \Delta f_{t+1} - (i_{t+1} - i_{t+1}^*)$$
(11)

According to equation (11), the variance of Δs_{t+1} depends on the variance of Δf_{t+1} and the variances of $f_t - s_t$ and $i_{t+1} - i_{t+1}^*$ together with the relevant cross-covariances. A simpler way is to take the first difference of equation (3), which produces equation (16).

$$\Delta s_t = \Delta f_t - \Delta (i_t - i_t^*) \tag{16}$$

As shown in Table 5, the variances of $(f_t - s_t)$, $(i_{t+1} - i_{t+1}^*)$ and $\Delta(i_t - i_t^*)$ are all relatively small. It is the relatively large variance for Δf_t that explains the relatively large variance for Δs_t . I mean "explain" here strictly in the statistical sense. The issue of causation between spot and forward exchange rates is beyond the scope of this article. That issue will be taken up in future work.

6. The Evidence

In spite of small Durbin-Watson statistics, in this section I assume that the residuals from all regressions are at least globally stationary. I attribute the low Durbin-Watson statistics to the "thresholds" created by transaction costs. That assumption is based on the evidence that, after taking account of the transaction costs, covered interest parity holds day to day.⁴

6.1. The Data

The data cover two intervals between the United States and Canada and two intervals between the United States and the United Kingdom. For US-Canada, the weekly interest rates are for 13 week Treasury bills. Those interest rates are from various issues of the *Federal Reserve Bulletin* starting with the issue of October 1964. Spot and forward exchange rates are for noon and were supplied by the Bank of Canada. As the *Bulletin* makes clear, the Treasury bill rates are only approximations of the rates needed for arbitrage. The data for US-Canada run from January 1961 to June 1973. The first interval for Canada in Table 1 covers the era of pegged exchange rates that started *de facto* in December 1960 and ended in May 1970. The second interval covers a period of flexible exchange rates from June 1970 to June 1973.

For the US-UK, the data are from Balke and Wohar (1998). Their daily interest rates are one month euro rates. See Balke and Wohar (1998) for more details. Their daily data start in January 1974 and end in September 1993. To account for any possible effects of the switch to flexible rates in the early 1970s, the interval is divided into roughly two equal parts. The first

⁴ For all four intervals Δf_t is linearly stationary. For all intervals except perhaps for the flexible US-Canada interval, f_t - s_t and s_{t+1} - f_t are also linearly stationary.

⁵ For a detailed description of the interest rates, see the issue of October 1964.

⁶ The data start in January 1959 when rates were flexible. I start in January 1961 because the rates were pegged *de facto* in December of 1960. The data end in August 1973, but 13 weeks are lost due to the difference between spot and forward exchange rates.

⁷ For both US-Canada and US-UK, missing observations are replaced with the previous observation. If two observations in a row are missing, the first is replaced with the previous observation and the second with the following observation.

⁸ The data in Balke and Wohar (1998) are bid and ask. Like them, I use the geometric mean of the bid and ask.

begins in January 1974 and runs through early November 1983. The second begins the next day and ends in early September 1993.

For the Canadian data, where the interest rates are for 91 days, the future spot and future forward exchange rates are t plus 13 weeks. For the UK data, where the interest rates are for 30 days, the future spot and future forward exchange rates are t plus 22 observations.

The data are not ideal. Interest rates, future spot exchange rates and forward rates are not always matched exactly. Particularly for the US-Canadian data, the timing of the observations is not ideal. Future research should correct those shortcomings. However it seems unlikely that correcting any shortcomings in the data will change the basic message. The apparent forward-bias puzzle is the result of omitting two important variables.

In addition to reporting estimated coefficients, tables with regressions also report the adjusted R^2 or \bar{R}^2 and Durbin-Watson statistics or DW. Changes in the \bar{R}^2 can provide an indication of the effect of the specification errors. Changes in the DW statistic can indicate how the serial correlation in residuals increases as a result of specification errors.

6.2 The Forward-Bias Puzzle

Estimates of β_1 from equation (1) are often negative. But covered interest parity implies that, when the forward premium is part of an appropriate test equation as in equation (12), estimates of the coefficient for the forward premium should be about 1.0. Covered interest parity also implies that equation (1) contains two omitted variables. As Tables 1, 2 and 3 show, those omitted variables completely explain the forward-bias puzzle in my data.

Table 1 reports the estimates of equation (1) using the data described above. 9 Most of the estimates of β_1 are negative. The average $\hat{\beta}_1$ in Table 1 is -1.154. The average \bar{R}^2 is only 0.015 and the average DW is only 0.103.

Table 1 Estimates of Equation 1 $\Delta s_{t+1} = \alpha_1 + \beta_1 (f_t - s_t)$

	$\overset{\wedge}{lpha_1}$	β ₁	\bar{R}^2/DW
US-Canada	•	, -	
5 Jan 1961 to	0.251	-0.425	0.003
31 Dec 1969	(0.046)	(0.175)	0.100
US-Canada			
5 Jun 1970 to	-0.238	0.268	-0.003
29 Jun 1973	(0.093)	(0.372)	0.151
US-UK			
2 Jan 1974 to	0.657	-1.425	0.033
1 Nov 1983	(0.069)	(0.166)	0.087
US-UK			
2 Nov 1983 to	0.930	-3.034	0.025
30 Sep 1993	(0.129)	(0.406)	0.075
Averages	0.332	-1.154	0.015
-	(0.084)	(0.280)	0.103

Standard errors in parentheses.

Table 2 reports the results of estimating equation (12). In Table 2 all $\hat{\lambda}_i$ for j greater than zero have the correct sign and all are significant at well beyond the 1 percent level. More importantly, all the $\hat{\lambda}_1$, the estimated coefficient for $(f_t - s_t)$, are significantly greater than zero at well beyond the 1 percent level. In addition, the lowest \bar{R}^2 in Table 2 is 0.984. That is extraordinarily high for an equation in first differences.

⁹ Regressions in all tables use RATS with "Robusterrors".

 $\begin{array}{c} \text{Table 2} \\ \text{Estimates of Equation 12} \\ \Delta s_{t+1} = \lambda_0 + \lambda_1 (f_t - s_t) + \ \lambda_2 \Delta f_{t+1} - \ \lambda_3 (i_{t+1} \ ... \ i_{t+1}^*) \end{array}$

	$\hat{\lambda}_0$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$-\hat{\lambda}_3$	\bar{R}^2/DW
U.S-Canada	· ·	•	-	5	
5 Jan 1961 to	0.014	0.605	1.002	-0.611	0.988
31 Dec 1969	(0.006)	(0.037)	(0.004)	(0.043)	0.173
US-Canada					
5 Jun 1970 to	-0.138	0.789	0.984	-1.083	0.984
29 Jun 1973	(0.021)	(0.049)	(0.013)	(0.074)	0.266
US-UK					
2 Jan 1974 to	0.007	0.990	1.000	-1.009	0.9997
1 Nov 1983	(0.001)	(0.006)	(0.000)	(0.008)	1.468
US-UK					
2 Nov. 1983 to	0.003	0.992	0.999	-1.002	0.999
30 Sep 1993	(0.001)	(0.011)	(0.000)	(0.011)	1.776
Averages	-0.028	0.845	0.996	-0.929	0.993
-	(0.007)	(0.026)	(0.004)	(0.034)	0.842

Standard errors in parentheses.

For the data supplied by Balke and Wohar, which is much better than the data from the *Bulletin*, both $\hat{\lambda}_1$ are less than two standard errors from one. When the forward premium is part of a correctly specified equation that includes $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$, there is no forward-bias puzzle. The forward premium correctly predicts the direction of the future change in the spot exchange rate.

If the forward-bias puzzle is the result of a downward bias due to two omitted variables, then that bias should explain the estimates of β_1 in Table 1. According to equation (12), the two omitted variables are $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$. Table 3 shows the bias due to those two omitted variables. That bias equals $\hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$ where $\hat{\lambda}_2$ and $\hat{\lambda}_3$ are from Table 2, and $\hat{\Phi}_1$ and $\hat{\Theta}_1$ are obtained from equations (19) and (20).

$$(f_{t+1} - f_t) = \Phi_0 + \Phi_1(f_t - s_t)$$
(19)

and

$$(i_{t+1} - i_{t+1}^*) = \Theta_0 + \Theta_1(f_t - s_t)$$
(20)

To save space, only $\hat{\Phi}_1$ and $\hat{\Theta}_1$, and their standard errors, are reported in Table 3.

Table 3
Estimates of the Bias Due to Omitted Variables

	(f_{t+1})	$-\mathbf{f}_{t}) = \Phi_{0}$	$_0 + \Phi_1(f_t)$	- S _t),	(i _{t+1} - i	$_{t+1}^*) = \Theta_0 + \Theta_1$	$(f_t - s_t)$)
	$\hat{oldsymbol{eta}}_1$	$\hat{\Phi}_1$	$\hat{\lambda}_2 \hat{\Phi}_1$	$\hat{\Theta}_1$	$\hat{\lambda}_3\hat{\Theta}_1$	$\hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$	$\hat{\lambda}_1$	$\hat{\lambda}_1 + \hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$
US-Canada 1-5-1961 to 12-31-1969	-0.425 (0.175)	-0.845 (0.171)	-0.847	0.300 (0.036)	-0.183	-1.030	0.605	-0.425
US-Canada 6-5-1970 to 6-29-1973	0.268 (0.372)	-0.399 (0.337)	-0.393	0.118 (0.365)	-0.128	-0.521	0.789	0.268
US-UK 1-2-1974 to 11-1-1983	-1.425 (0.166)	-1.587 (0.168)	-1.587	0.820 (0.111)	-0.827	-2.414	0.990	-1.424
US-UK 11-2-1983 to 9-30-1993	-3.034 (0.406)	-3.164 (0.395)	-3.161	0.863 (0.029)	-0.865	-4.026	0.992	-3.034

Standard errors in parentheses.

Given the $\hat{\lambda}_1$ in Table 2, covered interest parity implies that the corresponding $\hat{\beta}_1$ in Table 1 should equal $\hat{\lambda}_1 + \hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$. As a comparison of the columns labeled $\hat{\beta}_1$ and $\hat{\lambda}_1 + \hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$ in

Table 3 shows, the downward bias implied by covered interest parity, namely $\hat{\lambda}_2 \hat{\Phi}_1 + \hat{\lambda}_3 \hat{\Theta}_1$, completely explains the forward-bias puzzle in Table 1.¹⁰

Tables 1, 2 and 3 include only two pairs of countries with only two intervals each. Before the forward-bias puzzle can be declared finally solved, the results in these three tables should be confirmed across various countries and intervals. However, since these are the only countries and intervals that I have analyzed, my results should hold up over space and time.

6.3 The Incompatibility Puzzle

The relevant literature assumes that the forward premium is the expected future spot exchange rate. Given that assumption, in the absence of a risk premium, if CIP holds UIP should hold. But the empirical evidence strongly supports CIP while often rejecting UIP. Since a risk premium does not seem to explain this failure of UIP, the success of CIP and the failure of UIP posses a puzzle.

As pointed out earlier, covered interest parity solves that apparent puzzle. If CIP holds, for UIP to hold $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ must be zero. Again as pointed out earlier, in my data $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ is never zero. The almost zero \bar{R}^2 s in Table 1 and the very large \bar{R}^2 s in Table 2 imply that, not only is $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ not zero, but $(f_{t+1} - f_t)$ and $(i_{t+1} - i_{t+1}^*)$ are important for explaining future changes in exchange rates.

CIP holds and UIP fails because observed $(f_{t+1} - f_t) - (i_{t+1} - i_{t+1}^*)$ are not zero. There is no need for an appeal to a risk premium. One subsidiary puzzle is solved.

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The value of $\lambda_1 + \lambda_2 \Phi_1 + \lambda_3 \Phi_1$ for the first US-UK interval for is -1.424 rather than -1.425. But that is the result of rounding error in Table 3. The computer program reports a value for $\lambda_1 + \lambda_2 \Phi_1 + \lambda_3 \Phi_1$ of -1.42475, which rounds off to -1.425.

6.4 The Levels Puzzle

Another subsidiary puzzle that any convincing solution to the forward-bias puzzle should solve is the drastic shift in coefficients from levels to changes. Estimates of β_2 from equation (15) are usually close to, but less than, one.

$$\mathbf{s}_{t+1} = \alpha_2 + \beta_2 \mathbf{f}_t \tag{15}$$

But estimates of β_1 from equation (1) are often negative.

$$\Delta \mathbf{S}_{t+1} = \alpha_1 + \beta_1 (\mathbf{f}_t - \mathbf{S}_t) \tag{1}$$

Subsection 6.2 shows that estimates of β_1 are usually negative because of two omitted variables.

As subsection 5.1 shows, covered interest parity implies that estimates of β_2 should be close to, but less than, 1.0 primarily because f_t in equation (15) is acting as a proxy for f_{t+1} . Using the same data as earlier tables, Table 4 illustrates the link from f_t to s_{t+1} through f_{t+1} . For each of the four intervals, Table 4 shows the results of estimating three equations. The first is equation (13), which is implied by covered interest parity.

$$s_{t+1} = B_0 + B_1 f_{t+1} - B_2 (i_{t+1} - i_{t+1}^*)$$
(13)

The second is equation (14). Equation (14) is the same as equation (13) except that f_t replaces f_{t+1} . Comparing the results for equations (13) and (14) illustrates how good a proxy f_t is for f_{t+1} and also illustrates the downward bias in b_1 due to the measurement error created by using f_t as a proxy for f_{t+1} .

$$S_{t+1} = b_0 + b_1 f_t - b_2 (i_{t+1} \cdot i_{t+1}^*)$$
(14)

The third equation is equation 15 which is equation (1) in levels. Comparing equations (14) and (15) illustrates how little omitting the interest rate differential affects the coefficient for the forward exchange rate.

$$\mathbf{s}_{t+1} = \alpha_2 + \beta_2 \mathbf{f}_t \tag{15}$$

Table 4
Estimates of Equations 13, 14 and 15

	$s_{t+1} = 1$	$B_0 + B_1 f_1$	$_{z+1}-B_2(i$	$i_{t+1} - i_{t+1}^*$	$s_{t+1} =$	$=b_0+b_1f$	$\hat{s}_t - b_2(i_{t+1})$	$- i_{t+1}^*)$	$s_{t+1} =$	$=\alpha_2+\beta_2$	\mathbf{f}_{t}
	$\hat{f B}_0$	$\mathbf{\hat{B}}_{1}$	$-\hat{\mathbf{B}}_2$	\bar{R}^2/DW	$\hat{\mathrm{b}}_{\mathrm{0}}$	$\hat{\mathbf{b}}_1$	$-\hat{\mathbf{b}}_2$	\bar{R}^2/DW	$\hat{\alpha}$	β̂	\bar{R}^2/DW
US-Canada											
5 Jan 1961 to	0.010	0.990	-0.883	0.994	0.319	0.703	0.090	0.792	0.319		0.792
31 Dec 1969	(0.002)	(0.002)	(0.032)	0.134	(0.042)	(0.039)	(0.424)	0.121	(0.042)	(0.039)	0.121
US-Canada											
5 Jun 1970 to	-0.057	1.055	-1.140	0.990	0.224	0.770	-2.891	0.598	0.394	0.606	0.469
29 Jun 1973	(0.008)	(0.008)	(0.063)	0.312	(0.056)	(0.055)	(0.466)	0.235	(0.040)	(0.040)	0.133
US-UK											
2 Jan 1974 to	0.000	1.000	-1.018	0.99999	0.003	0.994	-2.641	0.965	0.002	0.998	0.962
1 Nov 1983	(0.000)	(0.000)	(0.004)	1.474	(0.002)	(0.004)	(0.200)	0.090	(0.002)	(0.004)	0.084
US-UK											
2 Nov 1983 to	0.000	1.000	-1.007	0.99998	-0.018	0.933	-5.107	0.916	-0.022	0.959	0.911
30 Sep 1993	(0.000)			1.774	(0.004)			0.071		(0.008)	0.067
Averages	-0.011	1.011	-1.012	0.996	0.132	0.850	-2.637	0.818	0.173	0.816	0.784
G. 1 1	(0.002)	(0.002)	(0.026)	0.923	(0.026)	(0.027)	(0.370)	0.129	(0.022)	(0.023)	0.101

Standard errors in parentheses

As implied by covered interest parity, the results for these equations deteriorate as one moves from equation (13) to (14) and then to (15). The average \bar{R}^2 and DW statistics for equation (13) are 0.996 and 0.923 respectively. Replacing f_{t+1} with f_t reduces the average \bar{R}^2 and DW statistic to 0.818 and 0.129 respectively. Dropping the interest rate differential reduces the average \bar{R}^2 and DW statistic further to 0.784 and 0.101 respectively.

But it is the effect on the coefficients for f_{t+1} and f_t of the switch from equation (13) to (14) that is most important. For f_{t+1} in equation (13), the average coefficient is 1.011. When f_t replaces f_{t+1} in equation (13), the average coefficient for f_t falls to 0.850. Dropping the interest rate differential further reduces the average coefficient for f_t in equation (15) to 0.816. I believe Table 4 shows that estimates of B_2 are well above zero because f_t is an excellent proxy for f_{t+1} . Estimates of B_2 are less than one primarily because f_t is acting as a proxy for f_{t+1} and, as a result, f_t contains measurement error. That explanation solves the second subsidiary puzzle.

6.4 The Variance Puzzle

As long as one retains the assumption that f_t is the market's expectation of s_{t+1} , and therefore an unbiased estimate of s_{t+1} , it is almost impossible to explain why the variance of Δs_t and Δs_{t+1} is so much larger than the variance of f_t - s_t . But the large difference in the variances is fully consistent with covered interest parity. From the perspective of covered interest parity, the variance of the forward premium is small because the variance of the interest rate differential is small. The variance of changes in the spot exchange rates is very large because the variance of changes in forward exchange rates is very large. Again this "because" should not be interpreted here as causation, only correlation. I will take up the issue of causation in future research.

Using the same data as previous tables, Table 5 reports the relevant variances. In Table 5 the average variance for Δs_t and Δs_{t+1} is 6.515. But the average variances for f_t - s_t , i_{t+1} - i_{t+1}^* and $\Delta (i_t - i_t^*)$ are respectively only 0.038, 0.050 and 0.024. These estimates for Δs_t and f_t - s_t are consistent with those reported by Wang and Wang (2009). However the average variance for Δf_t is 6.518, which is essentially the same as the average variance for Δs_t and Δs_{t+1} .

Table 5
Relevant Variance

	Δs_t and Δs_{t+1}	Δf_t	$\Delta(i_t - i_t^*)$	f_t - s_t	i_{t+1} - i_{t+1}^*
U.S-Canada 5 Jan 1961 to 31 Dec 1969	0.983	1.009	0.018	0.023	0.019
US-Canada 5 Jun 1970 to 29 Jun 1973	2.201	2.119	0.042	0.050	0.026
US-UK 2 Jan 1974 to 1 Nov 1983	7.562	7.545	0.033	0.037	0.116
US-UK 2 Nov. 1983 to 30 Sep 1993	15.315	15.400	0.004	0.042	0.039
Averages	6.515	6.518	0.024	0.038	0.050

The similarity between the variances for Δs_t and Δf_t suggests that each is large because the other is large. In the purely statistical sense, the variance of Δf_t "explains" almost all the variance in Δs_t .

Equation (17) is equation (13) in differences shifted back from t+1 to t.

$$\Delta s_{t} = C_{0} + C_{1} \Delta f_{t} - C_{2} \Delta (i_{t} - i_{t}^{*})$$
(17)

Using the same data as before, Table 6 shows that, on average, equation (17) explains 98 percent of the variance in Δs_t . Dropping the interest differential produces equation (18).

$$\Delta s_t = A_0 + A_1 \Delta f_t \tag{18}$$

In Table 6, the average \hat{A}_1 from equation (18) is 1.01 and the average \bar{R}^2 is 0.977. On average the variance of Δf_t alone "explains" over 97 percent of the variance in Δs_t .

As long as the variance of the interest rate differential is small, the variance of the change in the forward exchange rate is large and covered interest parity holds, we can expect the variance of changes in spot exchange rates to be much larger than the variance of the forward premium. The third and final subsidiary puzzle is solved.

Table 6 Estimates of Equations 16 and 17

	$\Delta s_t = C_0 + C_1 \Delta f_t - C_2 (i_t - i_t^*)$				$\Delta s_t = A_0 + A_1 \Delta f_t$			
	\hat{C}_0	\hat{c}_1	-Ĉ ₂	\bar{R}^2/DW	\hat{A}_0	\hat{A}_1	\bar{R}^2/DW	
US-Canada								
5 Jan 1961 to	0.034	0.984	-0.334	0.979	0.005	0.978	0.977	
31 Dec 1969	(0.007)	(0.007) (0.067)	0.156	(0.007)	(0.010)	0.144	
US-Canada								
5 Jun 1970 to	-0.186	0.979 -	0.941	0.955	0.028	1.064	0.937	
29 Jun 1973	(0.025) (0.020) (0.112)	0.146	(0.021)	(0.017)	0.094	
US-UK								
2 Jan 1974 to	0.037	0.996 -0	0.131	0.995	0.008	0.999	0.995	
1 Nov 1983	(0.005)	(0.002)	0.016)	0.273	(0.004)	(0.002)	0.277	
US-UK								
2 Nov 1983 to	0.014	0.997 -(0.050	0.999	-0.002	0.997	0.999	
30 Sep 1993	(0.003) (0.001) (0	.009)	1.067		(0.001)	1.059	
Averages	-0.025	0.989 -(0.364	0.982	0.010	1.010	0.977	
S	(0.010) (0.051)	0.410		(0.008)	0.394	

Standard errors in parentheses.

7. Summary and Conclusion

A convincing solution to the forward-bias puzzle should solve not only that puzzle but also three subsidiary puzzles associated with the forward-bias puzzle. Unlike any previously proposed solution, the solution to the forward-bias puzzle proposed here does that.

Covered interest parity implies that the forward-bias puzzle is the result of two omitted variables: (1) future changes in forward exchange rates and (2) future interest rate differentials. As Table 3 shows, for my data the downward bias created by those two omitted variables completely explains the forward-bias puzzle.

Covered interest parity also explains the three related puzzles. One puzzle is what I call the incompatibility puzzle. That puzzle refers to the fact that there is strong empirical support for covered interest parity and very little support for uncovered interest parity. Something that a risk premium does not seem able to explain. Another is what I call the levels puzzle. That puzzle refers to the fact that, when the level of the future spot rate is regressed against the level of the current forward rate, the coefficient for the forward rate is usually close to, *but less than*, one. The third puzzle is what I call the variance puzzle. That puzzle refers to the fact that the variance of the change in the exchange rate is 100 to 200 times greater than the variance of the forward premium.

My results, which support all four solutions, are based on only two pairs of countries with two intervals each. If future empirical work confirms my results, the forward-bias puzzle and the three subsidiary puzzles are all solved.

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