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SINGULARITIES IN THE PERTURBATIVE QCD TREATMENT OF THE PHOTON STRUCTURE FUNCTIONS

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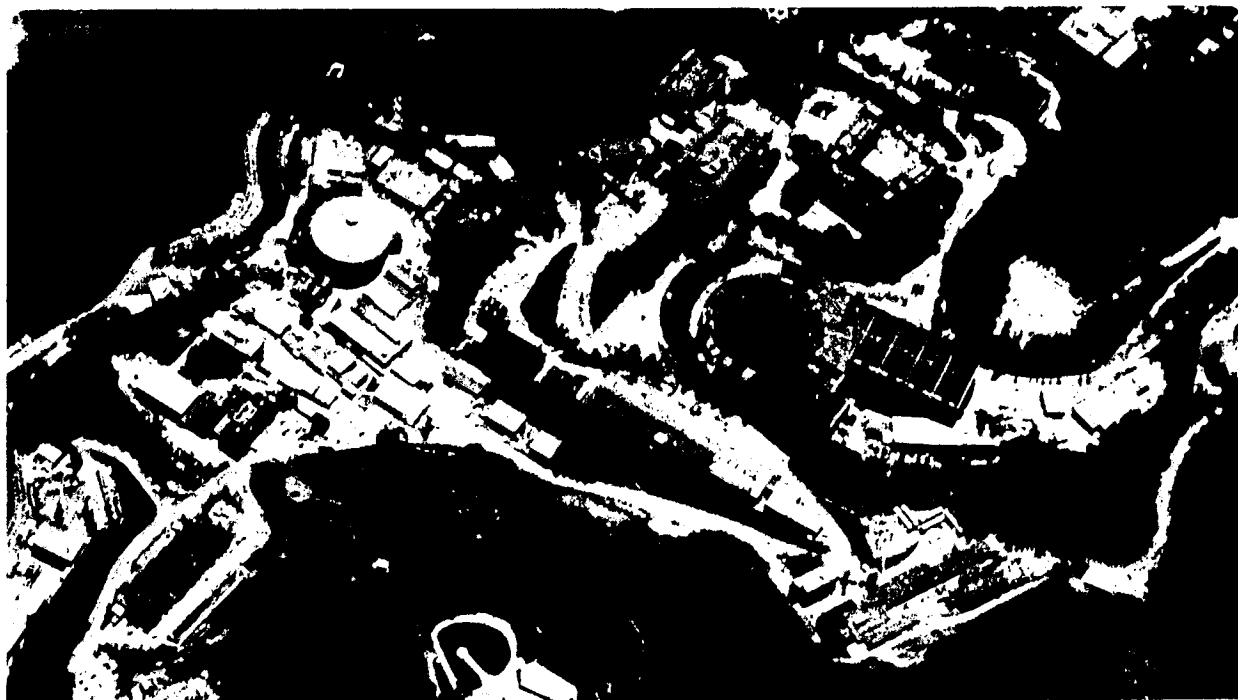
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SINGULARITIES IN THE PERTURBATIVE QCD TREATMENT  
OF THE PHOTON STRUCTURE FUNCTIONS\*

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ABSTRACT

The perturbative QCD expansion for the calculable part of the Bjorken limit photon structure functions is shown to be affected by power singularities at  $x = 0$  which spoil its convergence. Limitations on the calculability of the structure functions due to such singularities are discussed.

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In the ordinary perturbative QCD treatment [1, 2] two types of terms contribute to the photon structure functions in the Bjorken limit. Hadronlike terms exhibit the same dependence on the momentum transfer  $Q^2$  as hadronic deep inelastic scattering structure functions, whereas pointlike terms are described by a series in integer powers of the effective strong coupling constant  $\alpha(Q^2)$ . For  $F_2^Y(x, Q^2)$  this is summarized by the expression

$$F_2^Y(n, Q^2) = \int_0^1 dx x^{n-1} F_2^Y(x, Q^2) = \frac{\alpha_{EM}^n}{4\pi} \left[ \frac{4\pi}{\beta_0 \alpha} a(n) + \sum_{\ell=1}^{\infty} r_{\ell}(n) + \sum_{\ell=0}^{\infty} h_{\ell}(n) \alpha^{d_{\ell}+1} \right] \quad i = +, -, NS \quad (1)$$

here  $\alpha_{EM}$  is the fine structure constant,  $\beta_0$  is the one loop coefficient of the perturbative expansion of the  $\beta$  function:

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2} - \dots \quad (2)$$

The exponents  $d_i^n$  are given by

$$d_i^n = \lambda_i^n / 2\beta_0 \quad i = +, -, NS$$

where  $\lambda_i^n$  are the eigenvalues of the hadronic one loop anomalous dimension matrix: numerical values for  $d_i^n$  are reported in Table 1. The normalization is the same as that of Ref. [3]. The first three terms in the r.h.s. of (1) give the pointlike contribution: they are

$$a(x) \propto \left(\frac{1}{x}\right)^{n-1.596} \quad (3)$$

$$b(x) \propto \left(\frac{1}{x}\right)^{n-2} \quad (3')$$

(where  $a(n) = \int_0^1 dx x^{n-1} a(x)$ ,  $b(n) = \int_0^1 dx x^{n-1} b(x)$ ). The purpose of this letter is to point out that this behavior is general, namely that at  $x \rightarrow 0$ ,  $r_\ell(x) \propto (1/x)^{n_\ell}$  where  $n_\ell$  is the value of  $n$  at which  $d_-^n = \ell$ : since at each order the exponent  $n_\ell$  grows larger (see Table 1) one has to deal with increasingly severe singularities; these singularities are unphysical and are cancelled by corresponding singularities in the hadronlike contribution. It should be emphasized that one is dealing here with power singularities: unlike the logarithmic singularities which occur in the photon structure functions at  $x = 1$ , the effect of more and more severe power singularities at each order in  $\alpha(Q^2)$  cannot be significantly damped by the additional powers of  $\alpha(Q^2)$  at any experimentally foreseeable  $Q^2$ . As a result the perturbative expansion for the calculable part alone fails to converge in the low  $x$  range. In fact, since  $n_\ell$  increases rapidly as one goes to higher orders, at each successive order larger and larger values of  $x$  (those controlled by  $n \leq n_\ell$ ) will be affected: one should expect that for these values of  $x$  the QCD prediction for the calculable part will differ significantly from the complete result; correspondingly, hadronlike contributions which ensure the cancellation of the singularities will become more important at each successive order.

proportional to the reduced matrix element between photon states of the photon operators  $O_Y^n$  ( $\langle \gamma | O_Y^n | \gamma \rangle = 1$  in leading order in  $\alpha_{EM}$ ) and can be calculated by present methods in perturbative QCD. In particular the quantities  $a(n)$  and  $b(n)$  were first obtained by the authors of Ref. [1] and [2] respectively, while the quantities  $r_\ell(n)$  have not yet been calculated. The quantities  $h_{i,\ell}(h)$  which enter the hadronlike contribution are not calculable by present methods; each of them contains a part proportional to the reduced matrix elements between real photon states of the quark or gluon operators  $O_\psi^n$ ,  $O_G^n$ ,  $O_{NS}^n$  and a part proportional to  $\langle \gamma | O_Y^n | \gamma \rangle$ .

It has often been emphasized in the literature that the photon structure functions differ from those encountered in hadronic deep inelastic scattering in that both their  $x$  and  $Q^2$  dependence are calculable in perturbative QCD: of course, this is true only if the hadronlike contributions in (1) can be neglected in comparison to the pointlike terms or at least if their relevance is confined to a limited range of  $x$ . One can see from (1) and from the values of  $d_i^n$  given in Table 1 that even if contributions of order higher than  $\alpha$  are neglected (next to leading level) some of the hadronic terms are relevant at small  $x$ . The expressions for  $a(n)$  and  $b(n)$  given in Ref. [2] are singular (simple poles) at those values of  $n$  where  $d_-^n = -1$ ,  $d_{NS}^n = -1$  and where  $d_-^n = 0$ ,  $d_{NS}^n = 0$ ,  $d_{NS}^n = -1$  respectively; those among these poles which occur at the largest value of  $n$  determine the  $x \rightarrow 0$  behavior of the inverse Mellin transform of  $a(n)$  and  $b(n)$ :

A hint of this behavior is provided by Ref. [3]. These authors consider the photon structure functions in the kinematical region  $\Lambda^2 \ll p^2 \ll Q^2$  where  $-p^2$  is the momentum squared of the target photon:  $p^2$  can be used to fix the renormalization point and the structure functions are fully calculable in this region; they are found to be free of power singularities.

In Bjorken limit  $p^2 = 0$  one has for the nonsinglet part of the photon structure function  $F_2(n, Q^2)$

$$F_2^n(n, Q^2) = \left[ \frac{\alpha_{EM}}{4\pi} A_{NS}^n(Q_0^2) M_{NS}^n(\vec{q}_0^2, \vec{q}_0^2) + X_{NS}^n(\vec{q}_0^2, \vec{q}_0^2, \alpha_{EM}) \right] C_{NS}^n(1, \vec{q}_0^2) \quad (4)$$

here  $\vec{q}^2$  and  $\vec{q}_0^2$  are the renormalized strong coupling constant at  $Q^2$  and at the subtraction scale  $Q_0^2$  respectively ( $\alpha(Q^2) = \vec{g}^2/4\pi$ ),  $A_{NS}^n(Q_0^2)$  is the reduced matrix element of the fermion nonsinglet operator between real photon states and

$$M_{NS}^n(\vec{q}_0^2, \vec{q}_0^2) = \exp \int_{\vec{q}_0^2}^{\vec{q}^2} d\vec{q} \frac{\chi_{NS}^n(\vec{q})}{\beta(\vec{q})} \quad (5)$$

$$X_{NS}^n(\vec{q}_0^2, \vec{q}_0^2, \alpha_{EM}) = \int_{\vec{q}_0^2}^{\vec{q}^2} d\vec{q} \frac{K_{NS}^n(\vec{q}, \alpha_{EM})}{\beta(\vec{q})} M_{NS}^n(\vec{q}_0^2, \vec{q}_0^2) \quad (5')$$

The anomalous dimensions  $\gamma_{NS}^n(g)$  and  $K_{NS}^n(g, \alpha_{EM})$  and the coefficient function  $C_{NS}^n(1, \vec{g}^2)$  are calculable in perturbation theory:

$$\chi_{NS}^n(g) = \frac{g^2}{16\pi^2} \chi_{NS}^{0,n} + \frac{g^4}{(16\pi^2)^2} \chi_{NS}^{1,n} + \dots \quad (6)$$

$$K_{NS}^n(g, \alpha_{EM}) = \frac{\alpha_{EM}}{4\pi} \left( -K_{NS}^{0,n} - \frac{g^2}{16\pi^2} K_{NS}^{1,n} - \dots \right) \quad (6')$$

$$C_{NS}^n(1, \vec{g}^2) = 1 + \frac{g^2}{16\pi^2} B_{NS}^n + \dots \quad (6'')$$

Values for the coefficients in the expansions (2) and (6) can be found in Ref. [2]. One can expand the integrand in (5) to obtain

$$\frac{\chi_{NS}^n(g)}{\beta(g)} = - \frac{1}{g} \sum_{\ell=0}^{\infty} \Gamma_{\ell}(n) \left( \frac{g^2}{16\pi^2} \right)^{\ell}$$

$$(\Gamma_0(n) = 2d_{NS}^n, \Gamma_1(n) = (\chi_{NS}^{0,n} - 2\beta_1 d_{NS}^n)/\beta_0, \dots)$$
 so that

$$M_{NS}^n(\vec{q}_0^2, \vec{q}_0^2) = \left( \frac{\vec{q}^2}{\vec{q}_0^2} \right)^{d_{NS}^n} \frac{f(n, \vec{g})}{f(n, \vec{g}_0)} \quad (7)$$

with

$$f(n, g) = \exp \sum_{\ell=1}^{\infty} \frac{\Gamma_{\ell}(n)}{2\ell} \left( \frac{g^2}{16\pi^2} \right)^{\ell} = 1 + \frac{\Gamma_1(n)}{2\ell} \frac{g^2}{16\pi^2} + \dots$$

Next expanding the integrand in (5')

$$\frac{K_{NS}^n(\bar{q}, \alpha_{EM})}{\beta(\bar{q}) f(n, \bar{q})} = \frac{4n \alpha_{EM}}{\bar{q}^3} \sum_{m=0}^{\infty} \Omega_m(n) \left( \frac{\bar{q}^2}{16n^2} \right)^m$$

( $\Omega_0(n) = K_{NS}^{0n}/\beta_0, \dots$ ) one gets

$$\begin{aligned} X_{NS}^n(\bar{q}^2, \bar{q}_0, \alpha_{EM}) &= 4n \alpha_{EM} (\bar{q}^2)^{d_{NS}} f(n, \bar{q}) \sum_{m=0}^{\infty} \frac{\Omega_m(n)}{(16n^2)^m} \int_{\bar{q}}^{\bar{q}_0} d\bar{q} \bar{q}^{2m-2d_{NS}-3} \\ &= 4n \alpha_{EM} (\bar{q}^2)^{d_{NS}} f(n, \bar{q}) (I(n, \bar{q}_0) - I(n, \bar{q})) \end{aligned} \quad (8)$$

where

$$I(n, \bar{q}) = -\frac{1}{16n^2 d_{NS}^n} \sum_{m=0}^{\infty} \Pi_m(n) \left( \frac{\bar{q}^2}{16n^2} \right)^m + J(n) \quad (9)$$

Here  $J(n)$  is an arbitrary integration constant and

$$\Pi_m(n) = \frac{1}{2} \frac{\Omega_m(n)}{d_{NS}^n + 1 - m} \quad (10)$$

Inserting (7), (8) and (9) in (4) one has

$$F_2^{NS}(n, \bar{Q}^2) = \frac{\alpha_{EM} C_{NS}^n(1, \bar{q}) f(n, \bar{q})}{4n} \left[ \sum_{m=0}^{\infty} \Pi_m(n) \left( \frac{\bar{Q}^2}{16n^2} \right)^{m-1} + \left( \frac{\bar{Q}^2}{\bar{q}_0^2} \right)^{d_{NS}} \left[ \frac{A_{NS}^n(\bar{Q}_0^2)}{f(n, \bar{q}_0)} - \sum_{m=0}^{\infty} \Pi_m(n) \left( \frac{\bar{Q}_0^2}{16n^2} \right)^{m-1} \right] \right] \quad (11)$$

The first term in the r.h.s. of (11) contributes to the quantities

$a(n)$ ,  $b(n)$  and  $r_g(n)$  in (1). One can see from (10) that at each order in perturbation theory a new singularity is generated and that this singularity is absent from the complete result since

$$\lim_{A \rightarrow 0} \frac{1}{A} \left( 1 - \left( \frac{\bar{Q}^2}{\bar{Q}_0^2} \right)^A \right) = -\ln \left( \frac{\bar{Q}^2}{\bar{Q}_0^2} \right)$$

Similar conclusions can easily be drawn for the singlet sector, although the treatment of this case is somewhat more involved. The argument applies also to the longitudinal structure function  $F_L^Y(x, Q^2)$ .

Formally one can use some arbitrary subtraction procedure to eliminate from (11) the singularities and the  $Q_0^2$  dependence: for example

$$F_2^{NS}(n, \bar{Q}^2) = \frac{\alpha_{EM} C_{NS}^n(1, \bar{q}) f(n, \bar{q})}{4n} \left[ \sum_{m=0}^{\infty} \Pi_m(n) \left[ \left( \frac{\bar{Q}^2}{16n^2} \right)^{m-1} - \left( \frac{\bar{Q}^2}{16n^2} \right)^{d_{NS}} \right] + \tilde{A}_{NS}(n) \left( \frac{\bar{Q}^2}{16n^2} \right)^{d_{NS}} \right] \quad (12)$$

(a different subtraction procedure to regularize the singularity at  $d_{NS}^n = 0$  has been presented in Ref. [4]). The constant  $\tilde{A}_{NS}(n)$

appearing in (12) must be determined from experiments in the usual fashion of hadronic deep inelastic scattering: their value depends on the order at which one performs the perturbation expansion. In leading order (12) becomes

$$F_2^{NS}(n, \bar{Q}^2) = \frac{\alpha_{EM}}{4n} \left[ \Pi_0(n) \left( 16n^2 - \left( \frac{\bar{Q}^2}{16n^2} \right)^{d_{NS}} \right) + \tilde{A}_{NS}^{(0)}(n) \left( \frac{\bar{Q}^2}{16n^2} \right)^{d_{NS}} \right]$$

At this order only the region of very small  $x$  is affected: the main effect is the elimination of spikes of the form  $(1/x)^{n-1}$  and  $(1/x)^{n-1.596}$  which are present, for example, in the plots for  $h_{NS}(x)$ ,  $h_g(x)$  and  $\mathcal{L}(x)$  of Ref. [5]. In next to leading order the subtractions eliminate the unphysical negative result of the form

(3') reported in [6] for  $F_2(x, Q^2)$  in the range  $x \lesssim .25$ . One can expect that at the next order ( $O(\alpha)$ ) the results obtained for the pointlike part will be significantly modified by the hadronlike contribution in the whole range controlled by  $n \lesssim 5.3$  (i.e.  $x \lesssim .8$ ).

Vector meson dominance arguments have often been used in the literature (see for example [1, 6]) to give estimates of the hadronic terms in (1). In view of the previous discussion it is apparent that such estimates can account for only part of the hadronlike contribution. In particular the detailed singularity structure due to the perturbation expansion cannot be reproduced by the VMD model. In effect the authors of Ref. [6] fail to obtain a physical (non negative) result by adding VMD contribution to the singular pointlike next to leading order result.

In summary, the singularities examined here set considerable limitations on the calculability of the  $x$  behavior of the photon structure function in Bjorken limit: at each successive order the unphysical singular behavior of the calculable part becomes more severe and this implies that hadronlike contributions whose effect must be evaluated experimentally affect larger and larger values of  $x$ .

As mentioned before the structure functions appear to be free of these difficulties in the kinematical region  $\Lambda^2 \ll p^2 \ll Q^2$  studied by the authors of Ref. [3]. In this region the hadronlike contributions can be calculated and the additional scale  $P^2$  can be used to fix the renormalization point so that the calculability of the  $x$  behavior is not subject to the limitations discussed above: this region may therefore provide a cleaner and more significant test of the theory.

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Table 1. Values of the quantities  $d_{NS}^n$ ,  $d_-^n$ ,  $d_+^n$  (4 flavors)

n	$d_{NS}^n$	$d_-^n$	$d_+^n$
2	.427	0	.747
3	.667	.609	1.386
4	.837	.817	1.852
5	.971	.960	2.192
6	1.080	1.074	2.460

$$\begin{aligned}
 d_{NS}(n = .3099) &= -1 & d_-(n = 1.596) &= -1 \\
 d_{NS}(n = 1) &= 0 & d_-(n = 2) &= 0 \\
 d_{NS}(n = 5.250) &= 1 & d_-(n = 5.326) &= 1 & d_+(n = 2.386) &= 1 \\
 d_{NS}(n = 26.58) &= 2 & d_-(n = 26.59) &= 2 & d_+(n = 4.402) &= 2
 \end{aligned}$$

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