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Regulation and Investment under Uncertainty
An Application to Power Grid Interconnection.*

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Abstract

Using real options, we consider a firm that must undergo a costly and time-consuming regulatory process before making an irreversible, lagged investment whose value varies randomly. We apply our model to Hydro-Québec's proposal to build a 1250 megawatts interconnection with Ontario. We find that the optimal starts of the regulatory review and of the project construction depend on the randomness of project benefits and on the duration of the regulatory authorization. A sensible limit on the latter allows the regulator to address changing circumstances at little cost to the firm. However, long and uncertain regulatory proceedings make investing less attractive.

Keywords: regulation; uncertainty; irreversibility; real options; interconnections.

JEL Classification: D81, L51.

1. INTRODUCTION

From the point of view of the firm, regulatory requirements preceding a project are a costly investment lag. Since the work of Bar-Ilan and Strange (1996), the theoretical and practical importance of investment lags in the context of irreversible investment under uncertainty has been increasingly acknowledged, as several years often elapse before some projects become operational. This lag is of course needed for project design, planning, and construction, but also to obtain the required regulatory approval, which is often linked to environmental protection.[1]

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[1] Although we focus here on environmental regulation (and more specifically on Environmental Impact Assessments or EIA), our framework also applies to safety and access regulation. We thank an anonymous referee for this suggestion.

Environmental impact assessment can take several years for energy project such as natural gas and oil pipelines, power plants and high voltage transmission lines. Distinguishing between the different causes of a project lag is important because a firm typically has little control over the cost, the duration, and even the probability of success of a regulatory review. Moreover, once secured, the regulatory green light may be valid only for a limited time. Thus far, however, these considerations appear to have received little attention in the economics literature on regulation.

The purpose of this paper is to address these issues. We propose a simple real options framework to analyze how the parameters of a regulatory review affect the decision to invest under uncertainty, which is of interest both to firms and to regulatory agencies. We consider a firm that contemplates making a lagged, irreversible investment, whose value varies randomly. The firm must incur an upfront cost to launch the regulatory process, which has an uncertain outcome. The probability of approval, however, is assumed to increase (everything else being the same) with the flow of net benefits from the project, thus reflecting the regulator's concern for social welfare. Once regulatory approval has been granted, it expires after a finite period to give the regulator some flexibility to deal with potential changes in environmental, social, or economic conditions. The firm thus faces a sequential investment problem, which entails an initial irrecoverable outlay to obtain the option to proceed with the project. To ground our numerical results, we apply our model to Hydro-Québec's recent project proposal to add a 1250 MW (megawatts) interconnection to the Ontario power grid. This interconnection will allow Hydro-Québec to perform arbitrage operations on the Ontario electricity market through the use of its large hydropower sites.

Our paper makes two contributions. First, we show that a decrease in the probability of success or an increase in the uncertainty of the regulatory process tend to delay the decision to initiate the regulatory process and to diminish the value of the project. A lengthy regulatory review can also be really costly to the firm. Regulators should thus strive to make the regulatory process reasonably predictable and short (as well as fair of course). In addition, while the duration of the regulatory green light may significantly affect the decisions to start the regulatory process and to invest, it has little impact on the value of the project we use as our case study, so the regulator may want to grant the firm an authorization to proceed for a limited time in order to keep the flexibility to deal with changing circumstances. Moreover, in spite of the presence of time lags, the firm delays starting the regulatory process and project construction (once approval has been secured) when benefits uncertainty increases. This result rests partly on the presence of a lower reflecting barrier for the flow of project benefits, i.e., when it reaches this barrier, it simply "rebounds" upwards.[2] This assumption, which is common (although typically implicit) in the investment literature, is appropriate if there is no risk that the investment opportunity disappears. It seems reasonable for our application to a power grid interconnection.

Second, our paper is one of the few applications of real options to regulation or to energy projects.[3] Following early papers by Brennan and Schwartz (1985), McDonald and Siegel (1985 and 1986), Majd and Pindyck (1987), or Paddock, Siegel, and Smith (1988), there is now a substantial literature that analyzes the impacts of uncertainty and irreversibility on optimal

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[2] An alternative could be an absorbing barrier: when the flow of benefits hits this barrier, it keeps the same value forever (see Saphores 2002).

[3] There is, however, a vast literature on regulation in the energy sector; for an analysis of environmental regulation but also technological changes on the construction of coal power plants, see for example Joskow and Rose (1985).

investment (see also Dixit and Pindyck 1994 and the references therein), but most of these papers consider unregulated firms. One exception is Teisberg (1993). Building on the characterization of rate of return regulation proposed by Joskow (1974), she analyzes the decision of electric utilities to delay investment and to build smaller plants with a shorter lead-time when regulatory policy with respect to capital cost allowance, finance cost during construction, and abandonment interacts with market uncertainty. More recently, Chaton (2001) examines the decision to invest in nuclear power plants or to decommission existing plants in France. For transmission lines, the only study dealing with investment under uncertainty that we could find is by Martzoukos and Teplitz-Sembitzky (1992). In our paper, we introduce an explicit two stages process that characterizes the role of environmental protection boards with respect to large-scale projects.

This paper is organized as follows. In the next section, we present our model. In Section 3, we apply our model to Hydro-Québec's proposed capacity addition to the interconnection of the Ontario/Quebec power grid; this project is presently under review by Quebec's environmental regulatory agency. Section 4 offers concluding comments and discusses some possible extensions.

2. THE MODEL

We consider a firm interested in developing a project such as a high voltage power line that interconnects two power grids. We assume that the flow of net project benefits, denoted by *X*, follows the geometric Brownian motion (GBM):

$$dX = \mu X dt + \sigma X dz, \qquad (1)$$

where: $\mu>0$ but $\mu<\rho$ (ρ is the relevant interest rate) in order for the present value of expected project benefits to be finite; $\sigma>0$; and dz is the increment of a standard Wiener process (Karlin and Taylor 1981). While the GBM hypothesis is not always satisfactory, it is widely used in finance and more generally in continuous-time stochastic modeling due to its tractability (e.g., see Dixit and Pindyck 1994 and the references therein).[4]

We suppose that before it can make the investment, the firm's project is subject to regulatory proceedings (such as an environmental impact assessment) that take time T_R and cost C_R, which is sunk. For simplicity, both T_R and C_R are assumed known. The firm may start the regulatory review at any time. If x is a realization of X, then $q(x)\in(0,1)$, the probability of a positive outcome, is the value at x of the cumulative distribution function (cdf) of the lognormal distribution with parameters m and ω, i.e.,

$$q(x)=\int_0^x \psi(z;m,\omega)dz=\Phi\left(\frac{\ln(x)-m}{\omega}\right), \tag{2}$$

where $\Phi(.)$ is the standard normal cdf and $\psi(z;a,b)$ is the lognormal density

$$\psi(z;a,b)\equiv\frac{1}{z\sqrt{2\pi b}}\exp\left(-\frac{1}{2}\left(\frac{\ln(z)-a}{b}\right)^2\right). \tag{3}$$

From (2), we see that $q'(x)\equiv\psi(x;m,v)>0$, so the probability of receiving the regulatory green light increases with the flow of net project benefits. Indeed, everything else being the same, higher net project benefits translate into higher social welfare and we suppose that the regulator

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[4] An interesting exception is again Teisberg (1993) who considers a regulated firm interested in a project whose value follows a mean-reverting process with a regulatory term that represents expected changes in cost allowances; she solves her model numerically.

favors projects that increase social welfare.[5] However, because of imperfect information, interest group pressures on the regulatory board and changes of board members over time, the outcome of the regulatory process is uncertain.[6]

The firm cannot proceed with its project if the regulatory outcome is negative. If, on the other hand it is successful, the firm gains the possibility of investing a known, sunk amount C_B to start its project, which begins yielding a flow of benefits after a period T_B (when construction is complete, for example).[7]

As emphasized by Dixit and Pindyck (1994), because of uncertainty and irreversibility, a standard cost-benefit approach is likely to yield incorrect decisions for the timing of the regulatory process and of the investment. Here there are two sources of uncertainty: the risk of being turned down by the regulatory body, and the randomness of project benefits. Likewise, irreversibility stems from the sunk costs associated with the regulatory process and from the implicit assumption that, once the project is built, the firm cannot recoup its investment, C_B, if it turns out that it made a mistake. There is no possibility of cost recovery as in Teisberg (1993). This assumption seems especially relevant for investments in interconnections with neighboring power grids that are used to perform arbitrage operations. The latter typically have little residual

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[5] Since X is assumed to follow a GBM, the present value of expected project benefits is proportional to the current value of X, as shown below.

[6] There is no strategic interaction between the regulatory board and the regulated firm as in the seminal work of Baron and Myerson (1982).

[7] For simplicity, C_B also includes the present value of maintenance and operation costs, which are assumed known.

value (basically the value of scrap metal).

In this context, it is fruitful to formulate this problem using concepts from the theory of real options. Thus, we see the possibility of initiating the regulatory process as a perpetual American option (we assume that the possibility to start the regulatory process does not expire); if it is exercised, it gives the firm (with probability $q(x)$) the option (also American) of building its project.[8] The firm thus needs to solve a compound optimal stopping problem. First, let us analyze the decision to start the regulatory process.

2.1 The Decision to Start the Regulatory Process

It is intuitively clear that the firm should start this process when X is high enough (recall that X is the flow of net project benefits). The firm then exchanges the option to start the regulatory process for $V_R(X)$, the expected net present value of the project itself, corrected for the risk of not getting regulatory approval and for the cost of the regulatory process.[9] Thus, for x "large enough,"

$$\Phi_R(x) = V_R(x), \tag{4}$$

where

$$V_R(x) = -C_R + q(x)e^{-\rho T_R} \int_0^{+\infty} \Phi_B(y, T_R) f(y, T_R; x)\, dy. \tag{5}$$

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[8] An American option can be exercised at any time until it expires whereas, a European option can be exercised only at the end of its life; e.g., see Wilmott, Howison, and Dewynne (1995).

[9] If the firm is turned down, it may typically resubmit after some time a modified version of its project for another regulatory review. To simplify our model, we ignore this possibility here.

In the above, $-C_R$ is the present value of the cost to the firm of the regulatory proceedings; $q(x)\in(0,1)$ is the probability that the regulatory outcome is positive; ρ is the firm's risk adjusted discount rate; T_R is the time required to complete the regulatory process; $\Phi_B(x,t)$ is the value of the option to build the project at time t and for X=x. We adjust the time clock here so that the regulatory review starts at time 0; and $f(y,T_R;x)$ is the value at y of the probability density of X at T_R time units after X=x. Since X follows a GBM, $f(y,T_R;x)\equiv\psi(y;\ln(x)+(\mu-0.5\sigma^2)T_R,\sigma^2 T_R)$, where $\psi(z;a,b)$ is defined by (3).

However, the firm is better off waiting when X is "too low". It also knows that the probability of regulatory approval increases with X (from (2)). Let x_R^* denote the frontier between the high values of X for which the regulatory process should be started and the low values of X for which waiting is optimal. For $x\in(0,x_R^*)$, the return on $\Phi_R(x)$ per unit of time should equal its expected capital gains so, after using Ito's lemma, we find that $\Phi_R(x)$ verifies the Bellman equation (a second-order linear ordinary differential equation):[10]

$$\rho F(x)=\mu x F'(x)+\frac{\sigma^2 x^2}{2}F''(x). \qquad (6)$$

It is easy to check that the general solution to (6) is a combination of two power functions (one negative and the other positive). Since the option term should be finite when x tends towards 0, we eliminate the negative power function so that

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[10] This approach is valid because we assume here that X=0 is the limit of a lower reflecting barrier (see Saphores 2002). We could also have a strictly positive reflecting barrier if HQ had a contract that guarantees an annual flow of net positive profits.

$$\Phi_R\left(x\right) = A_0 x^{\theta}, \tag{7}$$

where A_0 is an unknown constant, and

$$\theta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{8}$$

$\theta>1$ because we assume that $\rho > \mu$.

Since we solve simultaneously for x_R^* and A_0 in (7), we need another condition in addition to (4); it is the well-known "smooth-pasting" condition (Dixit and Pindyck 1994):

$$\frac{d\Phi_R}{dx}\Big|_{x=x_R^*} = e^{-\rho T_R}\frac{d}{dx}\left(q(x)\int_0^{+\infty}\Phi_B\left(y,T_R\right)f\left(y,T_R;x\right)dy\right)\Big|_{x=x_R^*}. \tag{9}$$

Equations (4), (5), (7) and (9) enable us to find Φ_R once Φ_B is known.

In order to compare the worth of the project in our case study for different values of X at the start of the regulatory process, we need to discount its value to a common reference point. We thus introduce $D_{x_R|x_0}V_R(x_R)$, the present value of expected net project benefits when $X=x_0$ if the regulatory process is started at $X=x_R>x_0>0$. The expected discount factor $D_{x_R|x_0}$ equals $E\left(\exp(-\rho T_{x_R|x_0})\right)$, where $T_{x_R|x_0}$ is the random duration between the moment $X=x_0$ and the first time X hits x_R. Since X follows a GBM, we know that (e.g., see Saphores 2002)

$$D_{x_R|x_0} = \left(\frac{x_0}{x_R}\right)^{\theta}, \tag{10}$$

where θ is given by (8).

2.2 The Decision to Start Construction

Let us now analyze the decision to build assuming that the regulatory green light has been obtained at time T_R. We suppose that the regulatory green light is valid for a limited time T_A. This constraint reflects the need to revisit regulatory approval following changes in economic, political, or environmental conditions. When the authorization to build expires, the firm looses the opportunity to invest.

For $t \in (T_R, T_R + T_A)$, the frontier that separates the values of X for which building is optimal from those for which waiting is preferable depends explicitly on time; we denote it by $x_B^*(t)$. Again, it is optimum for the firm to start building when X is high enough, and to wait otherwise. Thus, for $x \geq x_B^*(t)$ at $t \in (T_R, T_R + T_A)$, the value of the option to build equals the sum of the present value of costs plus the present value of the expected flow of project revenues. A simple integration shows that the expected value of X at time t given that $X(0)=y$ is $\int_0^{+\infty} \xi f(\xi, t; y) dy = ye^{\mu t}$, where $f(\xi, t; y)$ is the value at ξ of the probability density function of X, t time units after $X=y$. Thus, if $PV(y)$ is the present value of the flow of project benefits when $X=y$, we have $PV(y) = \int_0^{+\infty} ye^{-(\rho-\mu)t} dt = \frac{y}{\rho - \mu}$, and thus $\int_0^{+\infty} PV(y) f(y, T_B; x) dy = \frac{xe^{\mu T_B}}{\rho - \mu}$. As a result, $\forall t \in (T_R, T_R + T_A)$, $\forall x \geq x_B^*(t)$,

$$\Phi_B(x,t) = -C_B + \frac{xe^{-(\rho-\mu)T_B}}{\rho - \mu}, \tag{11}$$

and the "smooth-pasting" condition becomes,

$$\frac{\partial \Phi_B(x,t)}{\partial x}\Big|_{x=x_B^*(t)} = \frac{e^{-(\rho-\mu)T_B}}{\rho-\mu}. \tag{12}$$

As for $\Phi_R(x)$, the return per unit of time for $\Phi_B(x,t)$ should equal its expected capital gains. Time now intervenes explicitly in $\Phi_B(x,t)$ so, when we apply Ito's lemma, we obtain the Bellman equation (now a 2nd order partial differential equation)

$$\rho\Phi_B(x,t) = \frac{\partial \Phi_B(x,t)}{\partial t} + \mu(x)\frac{\partial \Phi_B(x,t)}{\partial x} + \frac{\sigma^2(x)}{2}\frac{\partial^2 \Phi_B(X,t)}{\partial x^2}, \tag{13}$$

which is valid for $t \in (T_R, T_R + T_A)$ and $x \le x_B^*(t)$. Moreover, since we assume that 0 is the limit of a reflecting barrier for *X*, we have

$$\forall t \in [T_R, T_R + T_A],\ \lim_{L \to 0} \frac{\partial \Phi_B(x,t)}{\partial x}\Big|_{x=L} = 0. \tag{14}$$

Finally, at time T_R+T_A, when the option to build expires, Φ_B is exercised on the basis of a simple cost-benefit analysis since the firm no longer has the flexibility to delay the project. Thus,

$$\Phi_B(X(T_R+T_A), T_R+T_A) = \max\left[0, -C_B + \frac{X(T_R+T_A)e^{-(\rho-\mu)T_B}}{\rho-\mu}\right]. \tag{15}$$

Equations (11) - (15) fully define $\Phi_B(X,t)$. Since there is no explicit solution for $\Phi_B(X,t)$, we solve for it numerically using finite difference methods (see Appendix).

2.3 The Deterministic Case

To better assess the impact of uncertainty on the decision to invest, let us now examine the deterministic case: now *X*, the flow of net project benefits, is known, and the regulatory review is certain to succeed so $q(x)\equiv 1$. With σ=0 in Equation (1), a simple integration shows that *X* increases at a constant rate:

$$X(t) = X(0)e^{\mu t}. \tag{16}$$

In this situation, if the project is worthwhile, the firm starts building as soon as the regulatory process is over because waiting would just reduce discounted net revenues. An upper limit in the duration of the regulatory green light is thus irrelevant here. Let T_1 be the time at which the firm triggers the regulatory proceedings. Using (16), the firm's objective function is[11]

$$\max_{0 \le T_1} e^{-\rho T_1} \left\{ -C_R + e^{-\rho T_R} \left[-C_B + e^{-\rho T_B} \frac{X(0)e^{\mu(T_1+T_R+T_B)}}{\rho-\mu} \right] \right\}. \tag{17}$$

We suppose again that $\mu<\rho$ because otherwise waiting forever would be optimal. Solving the corresponding necessary first order condition for an interior solution ($T_1>0$), we find

$$T_1 = \frac{1}{\mu}\ln\left(\frac{\rho(C_R + C_B e^{-\rho T_R})}{X(0)e^{-(\rho-\mu)(T_R+T_B)}} \right). \tag{18}$$

$\rho(C_R + C_B e^{-\rho T_R})$ represents annualized project costs and $X(0)e^{-(\rho-\mu)(T_R+T_B)}$ is the value of the flow of project benefits at the end of the construction phase if X=$X(0)$ at T_1. Both costs are expressed in \$ at the start of the regulatory process. When we combine (18) and (16) to get the value of X at which the regulatory process should start, we see that T_1 is simply the time necessary for the flow of project benefits to equal annualized project costs:

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[11] For the stochastic problem, the decision to start the regulatory process depends on the level of X and not on time because we don't know what value X will have at a given time in the future. By contrast, in the deterministic case, the firm knows perfectly how X changes over time and it can immediately make all necessary decisions.

$$x_{R0}^{*} = \frac{\rho(C_R + C_B e^{-\rho T_R})}{\mathrm{e}^{-(\rho-\mu)(T_R+T_B)}}. \tag{19}$$

Finally, inserting (18) into (17), we find that Q^*, the optimal net profit of the firm, is

$$Q^{*} = \frac{\mu}{\rho-\mu}\frac{X(0)e^{-(\rho-\mu)(T_R+T_B)}}{\rho}\left[\frac{X(0)}{x_{R0}^{*}}\right]^{\frac{\rho}{\mu}}. \tag{20}$$

A comparative statics analysis shows that:

1. T_1 is an increasing function of C_R, C_B, T_B, and ρ. Indeed, with higher regulatory or construction costs, the project is initially less attractive so the firm waits until project benefits increase. A longer time to build has the same impact because it reduces the present value of project benefits. Similarly, a higher discount rate decreases future benefits more than project costs since these are incurred upfront.

2. T_1 is a decreasing function of μ. This also makes sense: if X increases faster, the firm acts earlier as the discount rate is larger than the growth rate ($\rho>\mu$).

3. Interestingly, however, T_1 first decreases and then increases as T_R, the time to complete the regulatory review, increases. The reason is simple: for "small" values of T_R (i.e., for $T_R < \frac{1}{\rho}\ln\left(\frac{\mu}{\rho-\mu}\frac{C_B}{C_R}\right)$), a small increase in the length of the regulatory process leads to a slight discount of future project benefits but this effect is more than offset by the increase in project benefits ($\mu>0$); acting sooner is therefore optimal. The reverse is true for "large" values of T_R (i.e., for $T_R > \frac{1}{\rho}\ln\left(\frac{\mu}{\rho-\mu}\frac{C_B}{C_R}\right)$).

4. Since $x_{R0}^{*} = X(0)e^{\mu T_1}$, x_{R0}^{*} and T_1 vary similarly with C_R, C_B, T_R, T_B, and ρ. The

same holds for μ since $\frac{\partial T_1}{\partial \mu} = -\frac{1}{\mu}\left[T_1 + T_R + T_B\right] < 0$ and $\frac{\partial x_{R0}^*}{\partial \mu} = -\left[T_R + T_B\right] X(0) e^{\mu T_1} < 0$.

5. Finally, for obvious reasons, Q^* is a decreasing function of C_R, C_B, T_R, T_B, and ρ, and an increasing function of μ.

3. APPLICATION TO THE DECISION TO BUILD A POWER LINE

3.1 Data

We apply our model to one of Hydro-Quebec's (HQ, a Quebec-owned utility) recent project proposals (1998) to build a 1250 MW interconnection to the Ontario power grid in the Outaouais region. This interconnection would increase HQ's exchange capacity not only with Ontario, but also with other networks connected to Ontario's power grid (such as Western New York, Pennsylvania and Michigan).

Two reasons are given to justify the proposed investment. First, this interconnection would insure a more secure electricity supply for Quebec. HQ's domestic production and transmission capacity is concentrated in the Northern part of the province, which makes it vulnerable to catastrophic events. Access to Ontario could provide additional supply to Quebec in case of a domestic breakdown.

Second, the interconnection would provide more exchange opportunities in the Ontario wholesale electricity market, which was deregulated in May 2002. HQ's main activity would be to import electricity when outside power prices are low. Water stocked in hydroelectric reservoirs would be used to export electricity when outside prices are high.[12] For this illustration, we focus on the commercial aspects of the project because the assessment of the value of an additional supply source in the case of an emergency adds further complex issues that are beyond the scope of this paper.

According to Report 143 of the "Bureau d'Audiences Publiques sur l'Environnement" (BAPE), HQ estimates that it would take two years (T_B=2) and approximately 185 million dollars (C_B=185) to build the interconnection.[13] Before HQ can invest, however, it must go through an environmental regulatory process to obtain approval. According to the Quebec Ministry of Natural Resources (QMNR) which supervises HQ's operations, this process can take as long as two years (T_R=2) and cost approximately 2 million dollars (C_R=2). The QMNR does not want to speculate on the probability of success for this kind of proposal (note that this project is currently under review), so we assume here that when $\frac{x_R^*}{\rho - \mu}$, the expected present value of net benefits at the start of the regulatory process, equals construction costs C_B, HQ has an α=2/3 chance of getting approval.[14] The relationship between m and ω, the parameters characterizing $q(x)$ (see Equation (2)) and other project parameters is thus

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[12] Most of HQ's production capacity is hydroelectric. Because this technology is flexible, HQ can easily adjust its production; this is not the case for producers in Ontario and in the United States who rely mostly on nuclear and fossil fuel power plants

[13] All $ amounts in this paper are in Canadian dollars.

[14] In February 2001, BAPE turned down a HQ proposal to build a 315kv high voltage power line in the Outaouais region. The estimated cost of the 140 km power line was 175 million dollars. According to BAPE, the benefits for the domestic market were not deemed to be large enough.

$$m = \ln\left((\rho - \mu)C_B\right) - \omega\Phi^{-1}(\alpha) \qquad (21)$$

where Φ^{-1} is the inverse of the standard normal cdf. As ω is not readily estimable, we choose a baseline value of 1.00 and vary its value in our sensitivity analysis.

Four more parameters are needed: the discount rate ρ; the infinitesimal growth rate μ and the variance parameter σ for X; and the expiry time of the regulatory approval T_A. For ρ, the QMNR estimates that for this type of investment, HQ requires a 10% annual rate of return. In BAPE's Report 143, HQ does not detail its revenues from arbitrage activities (it is private information), so it is not possible to construct a time series for X in order to estimate μ and σ. We thus choose an arbitrary but plausible value of 2% per year for μ, and we vary σ between 0.15 and 0.8 per $\sqrt{\text{year}}$. Unfortunately, the QMNR gives no official expiry time for the regulatory approval. However, if HQ waits too long before investing, the QMNR can force HQ to undergo another regulatory review. To round up the list of parameters for our base case, we suppose that T_A equals 7 years.

Since there is substantial uncertainty concerning the value of some of our parameters, we conduct an extensive sensitivity analysis using the following values: $\alpha \in \{0.5, 0.66, 0.80\}$; $\omega \in \{0.50, 1.00, 1.50\}$ (where m changes according to (21)); $T_R \in \{1, 2, 3, 4\}$, in years; $C_R \in \{1, 2, 4\}$, in million of Canadian \$; $T_A \in \{3, 5, 7, 10, 25\}$, in years; $C_B \in \{150, 185, 220\}$, in million of Canadian \$; and $\mu \in \{0.01, 0.02, 0.044\}$, per year. In addition, we systematically vary σ between 0.15 and 0.8 per $\sqrt{\text{year}}$. Our results were generated on a PC with MatLab. Our programs are available upon request.

3.2 Results

Our main results are summarized on figures 1 to 5. We are particularly interested in how uncertainty in the value of the project (σ) combines with the regulatory parameters (α, ω, C_R, T_R, and T_A) to influence the optimal regulatory threshold x_R^*.

Figure 1 shows the variations of x_R^* with σ for three different values of α, the probability of regulatory approval when the present value of expected net benefits at the start of the regulatory process equals construction costs (i.e., when $\frac{x_R^*}{\rho - \mu} = C_B$). First, we note that x_R^* increases with σ, essentially because a higher volatility allows X to grow faster; the present value of expected project benefits and the probability of winning the regulatory green light thus increase faster so the firm waits longer (when α = 2/3, q equals 0.865 for σ=0.15 and 0.976 for σ=0.80). In addition, we observe as expected that x_R^* decreases when α increases: if the probability of regulatory approval is higher, the marginal gains from waiting for a higher q are lower so it is optimal to act earlier. These changes in α and σ are not irrelevant: when α increases from 0.5 to 0.8, the expected value of net benefits increases from \$56.3 to \$71.2 million (+26.4%) for σ=0.15, and from \$145.6 to \$153.0 million (+5.1%) when σ=0.80.

Insert figure 1 approximately here.

Figure 2A illustrates how x_R^* varies with ω, which measures the uncertainty of the regulatory process, for a wide range of values of σ. The main feature is that x_R^* increases with ω: a larger ω results in smaller marginal gains for q as x_R^* augments, so it is optimal to wait longer, but not long enough for q to increase with ω: in fact, when ω goes from 0.5 to 0.8, q decreases from 0.950 to 0.801 for σ=0.15, and from 0.996 to 0.951 for σ=0.80. An increase in ω thus also leads to a decrease in the present value of expected net project benefits (figure 2B): when ω increases from 0.5 to 0.8, they dip 16.3% for σ=0.15 and 7.8% for σ=0.80. The regulator should thus strive to reduce ω to increase welfare.

Insert figures 2A and 2B approximately here.

Figure 3A shows that x_R^* decreases when the duration of the regulatory proceedings (T_R) increases: a higher T_R makes the project less attractive as all its benefits are in the future. Since the discount rate is higher than the rate of change of net project benefits, the firm acts earlier, but it then reduces slightly the likelihood of regulatory approval. This explains that x_R^* has a simpler behavior under uncertainty than under certainty when T_R increases. Moreover, we observe that the impact on x_R^* of an increase in T_R increases with the volatility of project benefits (σ). From figure 3B we note that increasing T_R from 1 to 4 years causes the expected net present value of the project to decrease from \$157.5 to \$134.4 million when σ=0.8, so it is important for the regulator to try to minimize the duration of regulatory proceedings.

Insert figures 3A and 3B approximately here.

Figure 4 illustrates how the duration of the validity of regulatory approval, T_A, affects x_R^*. We observe that for σ fixed, x_R^* increases when T_A decreases: as the firm has less flexibility to invest, it requires a more attractive expected payoff to get started. A look at the expected present value of the project, however, indicates that decreasing T_A from 10 to 7 or even 5 years does not have a large impact on the value of the project. In our context, the regulator should thus not hesitate to reduce T_A to allow for a regulatory review in case of changing environmental or political circumstances.

Figure 5 shows the variations with uncertainty of the optimal threshold to start investment x_B^* for different values of T_A. Of course, x_B^* does not depend on T_R, C_R, α or ω. Whereas x_R^* is a mildly concave increasing function of σ, x_B^* is convex increasing. Indeed, a higher volatility increases both the incentive to wait and the opportunity cost of not investing because the risk of X taking small values is bounded from below: our assumption that X admits a lower reflecting barrier guarantees that a higher level of uncertainty will lead to potentially higher payoffs.[15] This explains that our results differ from the findings of Bar-Ilan and Strange (1996) because their model features a lower absorbing barrier where the project is abandoned. Their findings thus appear to depend more on the nature of the lower absorbing barrier than on the existence of investment lags. In addition, we note that x_B^* increases with T_A: when the regulatory green light lasts longer, the firm can invest at higher levels of the flow of benefits.

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[15] McDonald and Siegel (1986), for example, also obtain this result but they do not make clear that it depends on the assumption that X=0 is the limit of a reflecting barrier.

Insert figures 4 and 5 approximately here.

The remainder of our sensitivity analysis (results not shown) can be summarized as follows. First, for the range of parameters considered here, C_R does not have a big impact either on x_R^*, q, or the value of the project. We simply note that, as for the deterministic case, when C_R increases, so does x_R^*: undertaking the project is more costly so the firm needs to wait for a larger flow of net benefits. In addition, the impact on x_R^* of augmenting C_R increases slightly with σ: uncertainty thus exacerbates errors if C_R is known imperfectly. Finally, we find that both x_R^* and x_B^* decrease when μ increases; most importantly, changing μ from 0.01 to 0.04 per year has a large impact on the value of the project. Conversely, while both x_R^* and x_B^* decrease with C_B and increase with T_B, both C_B and T_B have a relatively modest impact on the firm's expected profits. The intuition for these results is the same as for the deterministic case.

4. CONCLUSIONS

In this paper, we present a framework based on Bar-Ilan and Strange's model (1996) to analyze the optimal decisions of a firm that needs to undergo a costly, uncertain, and time-consuming regulatory review in order to make a lagged, irreversible investment whose value varies randomly. Our results are a first step towards understanding the impact of regulatory requirements such as environmental impact assessments on the decision to make an irreversible investment in a dynamic, uncertain environment.

We apply our model to Hydro Quebec's proposal to build a 1250 megawatts interconnection to the Ontario power grid. Our numerical results show that the decisions to start the regulatory review and the project construction after regulatory approval has been secured vary differently with uncertainty. Indeed, the first one is slightly concave while the second is an increasing convex function of uncertainty; both are sensitive to the duration of the regulatory green light, although the expected value of the project for the firm is not. This suggests that the regulator has some leeway to cap the duration of the regulatory approval in order to address changing environmental, social, or political circumstances. Our results (on ω) also indicate that substantial uncertainty in the regulatory process can significantly decrease the attractiveness of the project, which suggests establishing clear rules to reduce “random” outcomes in regulatory proceedings. Finally, we find that the expected present value of the project declines substantially when the duration of the regulatory proceedings (T_R) increases from 1 to 4 years; minimizing the duration of these proceedings is thus important.

A number of extensions could be considered in future research, including: stochasticity in the duration and/or the cost of the regulatory process (i.e., T_R and/or C_R are random); alternative stochastic processes for X; uncertainty in building costs (i.e., C_B stochastic); and a general equilibrium analysis of the impact of environmental impact assessments. Finally, as more information becomes available and as deregulated electricity markets stabilize, it would be very useful to estimate actual arbitrage revenues from interconnecting power grids.

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Appendix: Numerical Solution of $\Phi_B(x,t)$

To obtain x_R^*, we first need to approximate $\Phi_B(x,t)$. Following Wilmott, Howison and Dewynne (1995) and Saphores (2000), we transform Equations (11) - (15) by making the change of variables:

$$X = C_B e^z,\ t = T_A - \frac{2\tau}{\sigma^2},\ \Phi_B(X,t) = C_B e^{\alpha z + \beta\tau} u(z,\tau), \tag{A.1}$$

where α and β solve the system:

$$\begin{cases} \beta = \alpha^2 + (k-1)\alpha - w, \\ 0 = 2\alpha + (k-1), \end{cases} \tag{A.2}$$

with $k = \frac{2\mu}{\sigma^2}$ and $w = \frac{2\rho}{\sigma^2}$.

We obtain the dimensionless heat diffusion problem:

$$\begin{cases} \frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial z^2},\ \text{for } z < z_B^*(\tau), \\ u(z,\tau) = h(z,\tau),\ \text{for } z \geq z_B^*(\tau), \end{cases} \tag{A.3}$$

where

$$h(z,\tau) = e^{\frac{1}{4}\left[(k-1)^2 + 4w\right]\tau} \max\left[0,\ -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z - (\rho-\mu)T_B}\right]. \tag{A.4}$$

The initial condition is

$$u(z,0) = h(z,0) = \max\left[0, -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z-(\rho-\mu)T_B}\right]. \tag{A.5}$$

Because of the lower reflecting barrier at L=0, we also need:

$$\lim_{z \to -\infty} \left[e^{(\alpha-1)z+\beta\tau} \left(\alpha u(z,\tau) + \frac{\partial u(z,\tau)}{\partial z} \right) \right] = 0. \tag{A.6}$$

In addition, to prevent arbitrage opportunities, the following constraint is required:

$$u(z,\tau) \geq e^{\frac{1}{4}\left[(k-1)^2+4w\right]\tau} \max\left[0, -e^{\frac{1}{2}(k-1)z} + e^{\frac{1}{2}(k+1)z-(\rho-\mu)T_B} \right]. \tag{A.7}$$

Finally, u and $\partial u/\partial z$ must be continuous at $z = z_B^*(\tau)$.

To avoid tracking the free boundary, we use the linear complementary formulation

$$\begin{cases} \left(\frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial z^2} \right) \geq 0, \quad \left(u(z,\tau) - h(z,\tau) \right) \geq 0, \\ \left(\frac{\partial u}{\partial \tau} - \frac{\partial^2 u}{\partial z^2} \right) \cdot \left(u(z,\tau) - h(z,\tau) \right) = 0, \end{cases} \tag{A.8}$$

Both expressions in the first part of (A.8) are equalities at the free boundary.

We solve the above problem using the Crank-Nicholson finite difference scheme and the projected successive over-relaxation (PSOR) algorithm (see Wilmott, Howison and Dewynne 1995) with $\delta z = 0.01$; a grid for z with $N^- = 1400$ and $N^+ = 800$; and time steps equivalent to 3 days. Details are available from the authors. To obtain the option values we reverse the change of variables.

Once we have an approximation of $\Phi_B(x,0)$, we use the continuity and smooth-pasting conditions to approximate x_R^*. Taking their ratio, x_R^* satisfies:

$$\frac{dV_R\left(x_R^*\right)}{dx} \frac{x_R^*}{V_R\left(x_R^*\right)} = \theta. \tag{A.9}$$

We use the bisection method to find the zero of (A.9).

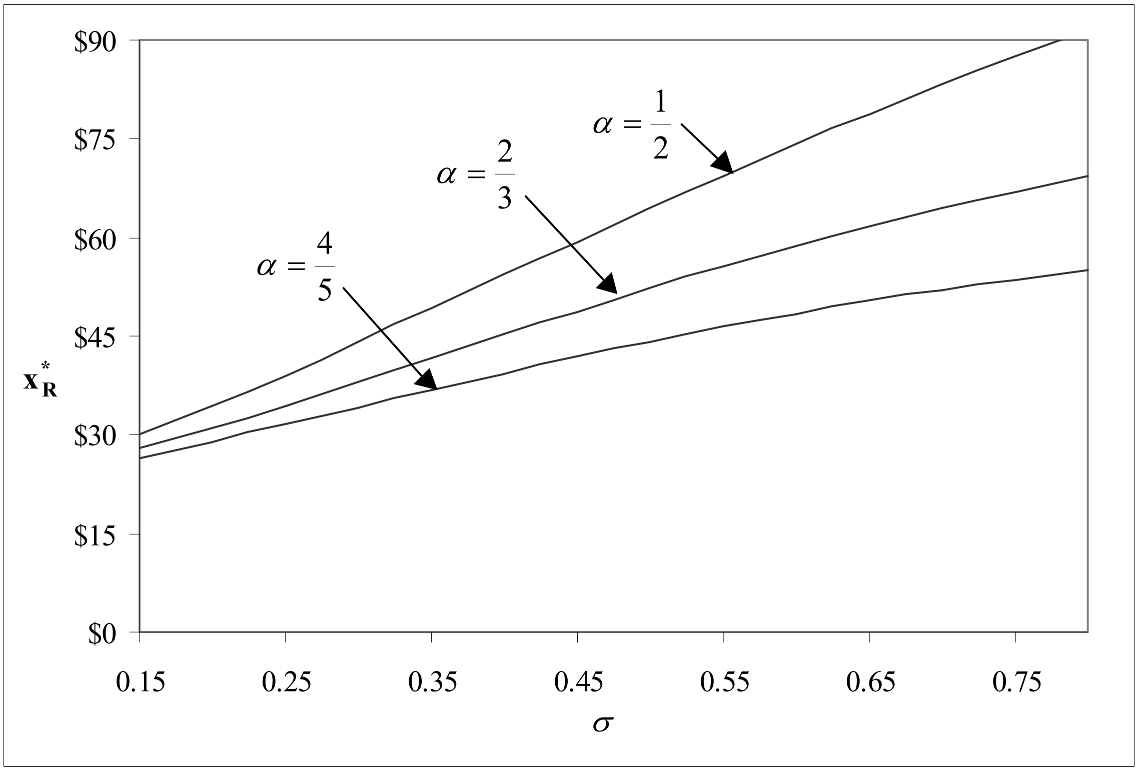

Figure 1: x_R^* versus σ for different probabilities of regulatory success.

Notes: Results above are generated with the following parameter values: ω=1.00, T_B=2 years, C_B=$185 million, T_R=2 years, C_R=$ 2 million, T_A= 7 years, μ=0.02 per year, and ρ=0.1 per year. α is the probability of regulatory approval when the expected present value of net benefits at the start of the regulatory process equals construction costs (i.e., when $\frac{x_R^*}{\rho - \mu} = C_B$). x_R^* is in million of Canadian $.

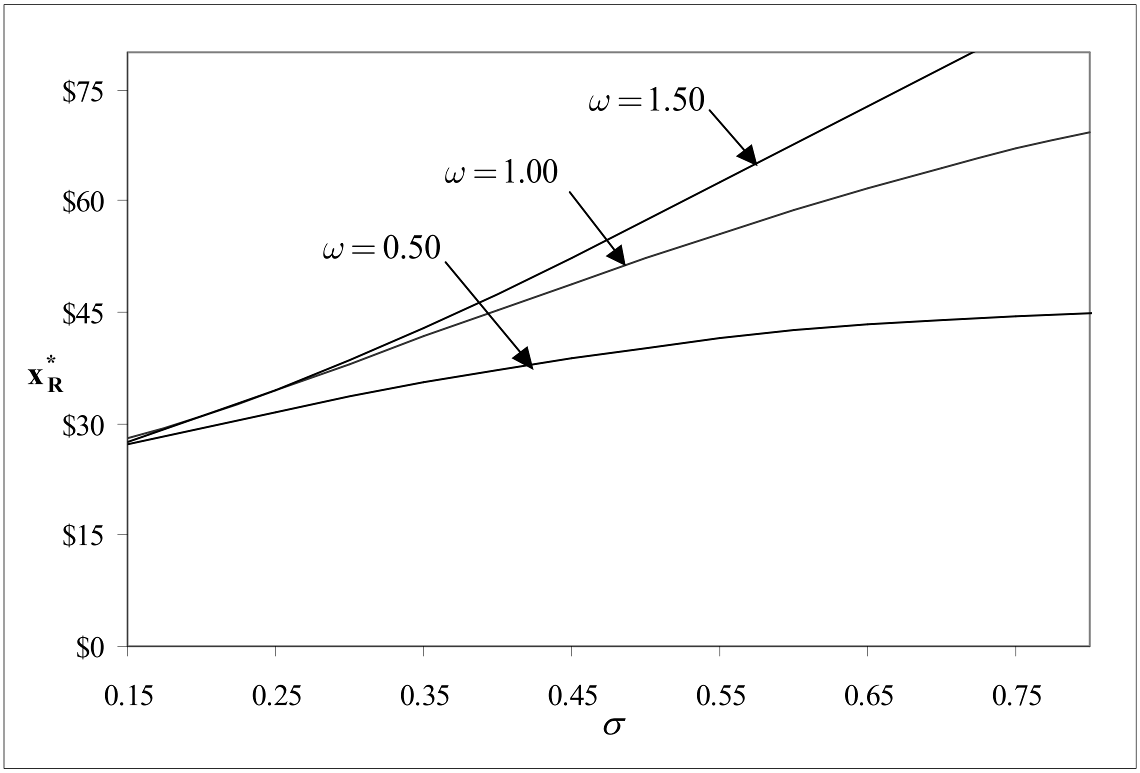

Figure 2A: x_R^* versus σ for different levels of regulatory uncertainty.

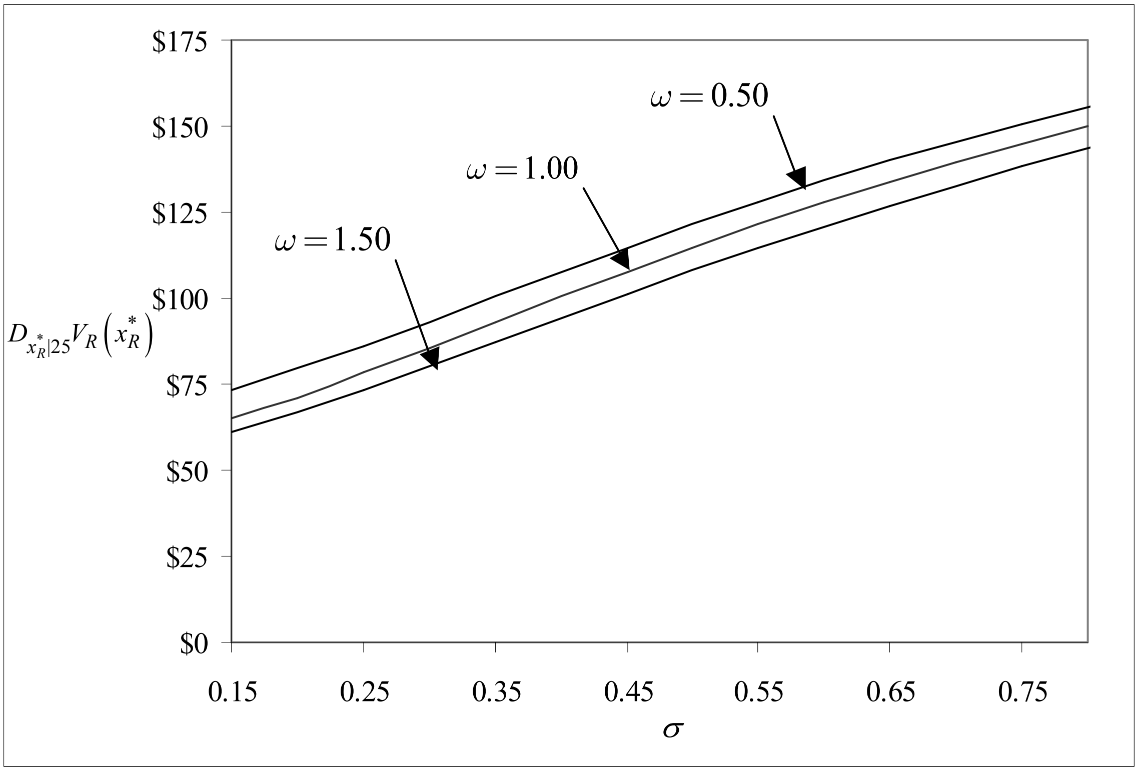

Figure 2B: Present value of the project versus σ for different levels of regulatory uncertainty.

Notes for figures 2A and 2B: Results above are generated with the following parameter values: α=2/3, T_B=2 years, C_B=$185 million, T_R=2 years, C_R=$ 2 million, T_A= 7 years, μ=0.02 per year, and ρ=0.1 per year. ω reflects the uncertainty of the regulatory process (see (2)). x_R^* and $D_{x_R^*|25}V_R(x_R^*)$ are in million of Canadian $. For $V_R(x_R^*)$, see (5). The expected discount factor $D_{x_R^*|25}$ is given by (10) with x_0=25 million of Canadian $, an arbitrary baseline.

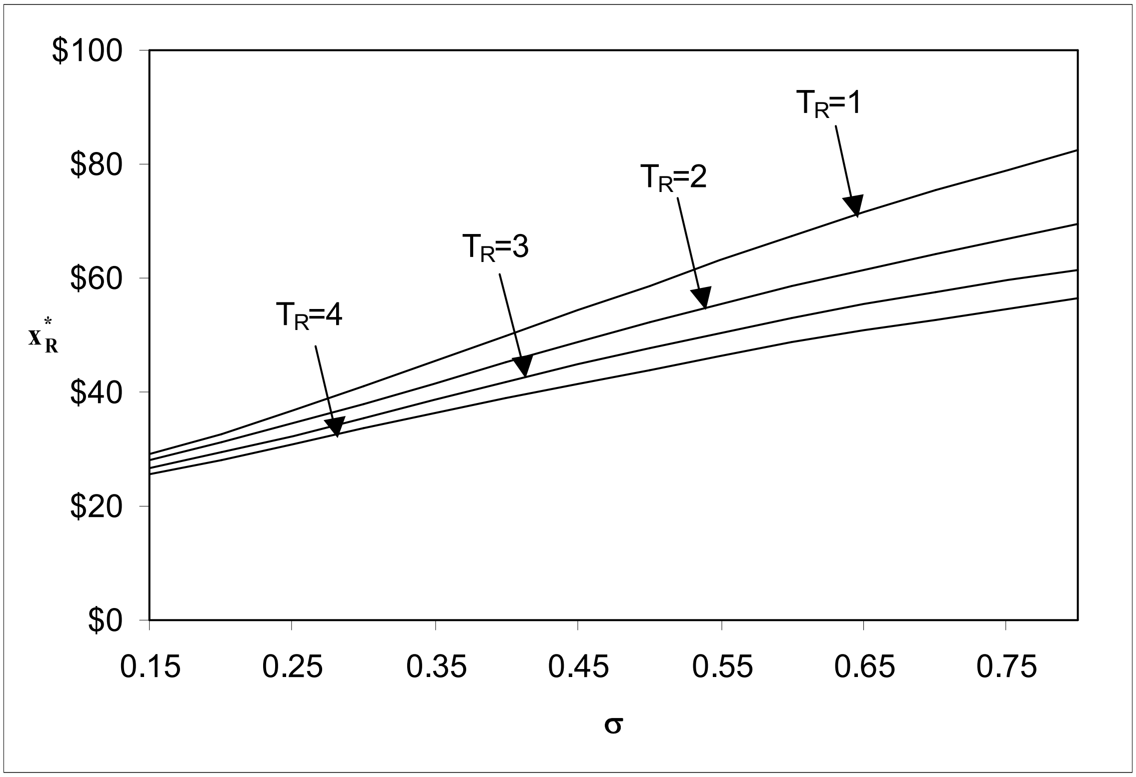

Figure 3A: x_R^* versus σ for different durations of the regulatory process.

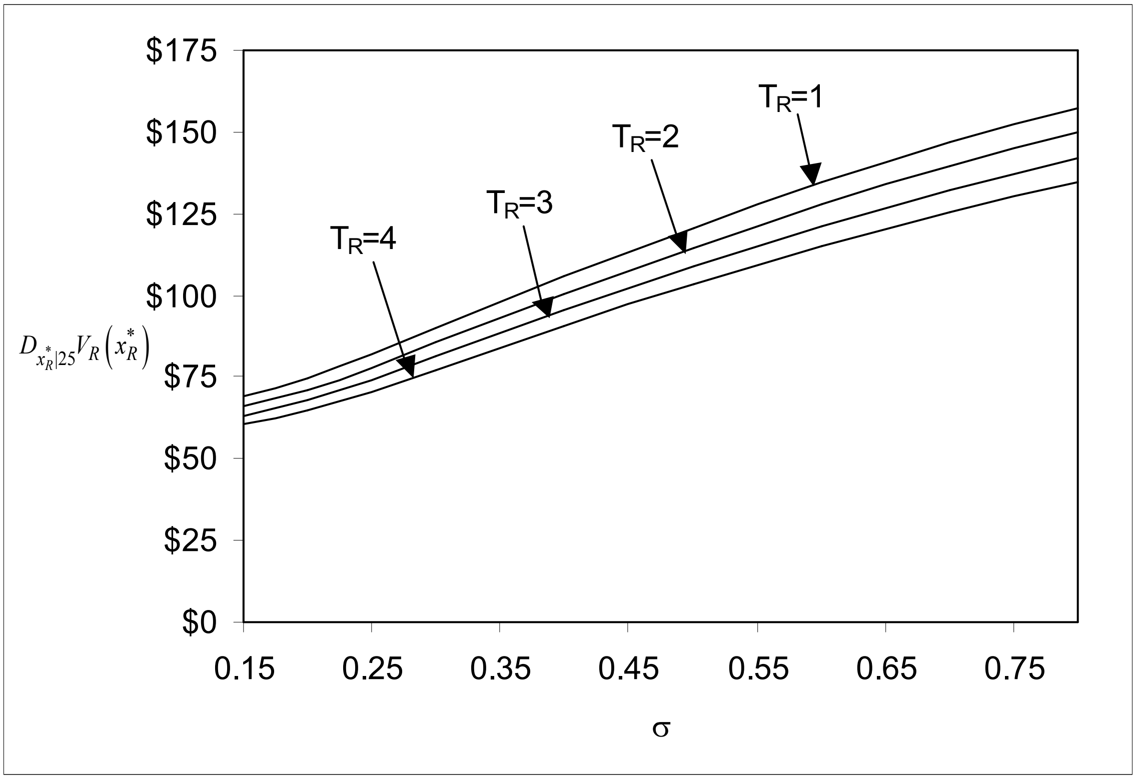

Figure 3B: Present value of the project versus σ for different durations of the regulatory process.

Notes for figures 3A and 3B: Results above are generated with the following parameter values: α=2/3, ω=1.00, T_B=2 years, C_B=$185 million, C_R=$ 2 million, T_A= 7 years, μ=0.02 per year, and ρ=0.1 per year. T_R is in years. x_R^* and $D_{x_R^*|25} V_R(x_R^*)$ are in million of Canadian $. For $V_R(x_R^*)$, see (5). The expected discount factor $D_{x_R^*|25}$ is given by (10) with x_0=25 million of Canadian $, an arbitrary baseline.

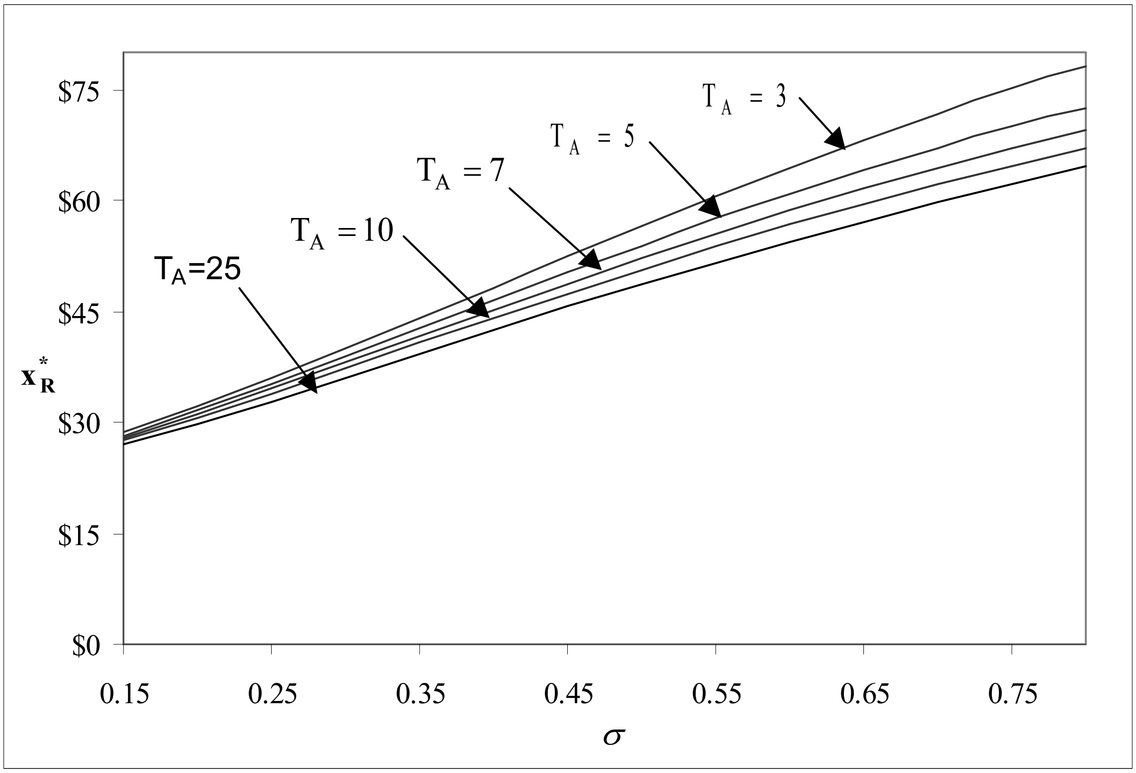

Figure 4: x_R^* versus σ for different durations of the regulatory authorization.

Notes: Results above are generated with the following parameter values: α=2/3, ω=1.00, T_B=2 years, C_B=\$185 million, T_R=2 years, C_R=\$2 million, μ=0.02 per year, and ρ=0.1 per year. T_A is the duration of the regulatory green light to proceed with the project (in years). x_R^* is in million of Canadian \$.

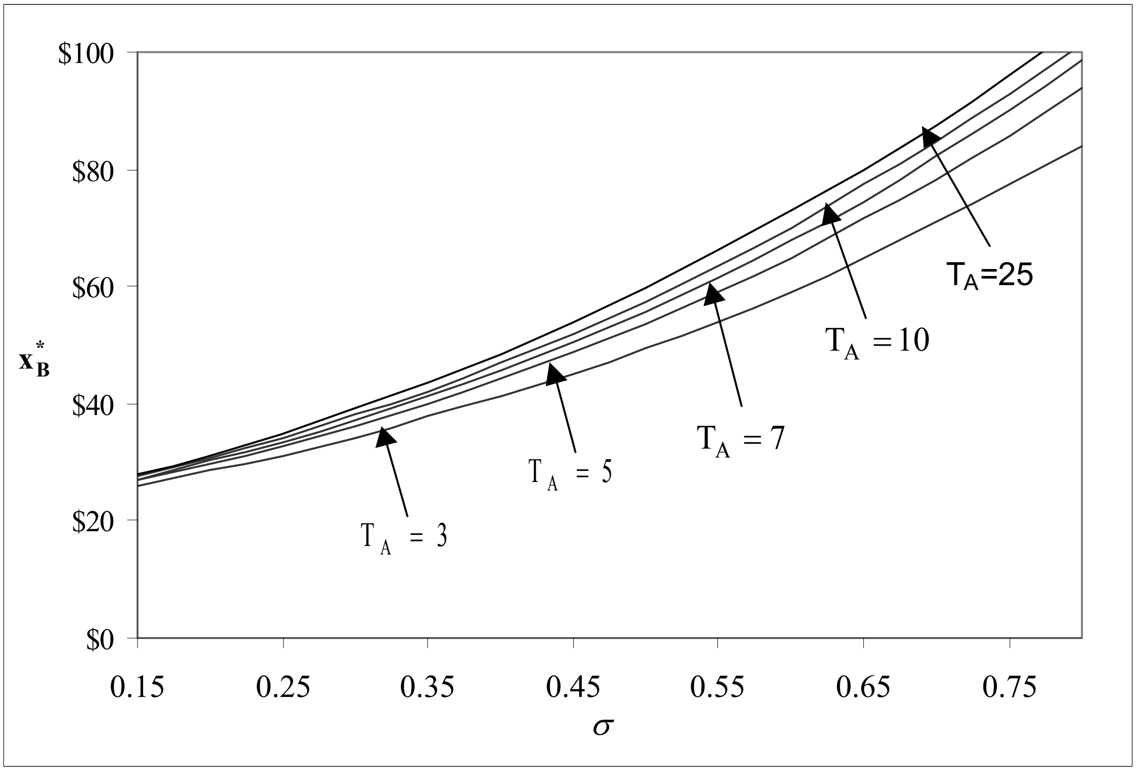

Figure 5: x_B^* versus σ for different durations of the regulatory authorization.

Notes: Results above are generated with the following parameter values: T_B=2 years, C_B=$185 million, μ=0.02 per year, and ρ=0.1 per year. T_A is the duration of the regulatory green light (in years). x_B^* is in million of Canadian $.