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Publication Date

2002

DOI

10.1109/aps.2002.1018294

Peer reviewed

Transient Radiation by Periodic Structures: Accuracy of the (Time Domain–Floquet Wave)-FDTD Algorithm

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1. Introduction

Transient radiation by periodic structures is analyzed using a time domain (TD) transmission line network formalism based on the recently developed time-domain Floquet-waves (TD-FWs) [1],[2],[3]. The FDTD algorithm is applied to the array element cell to compute pulsed modal current generators that form the initial condition for TD-FW propagation. The transient radiated field is expressed as a sum of modal voltages and currents evaluated using previously developed propagation of TD-FWs, which form an efficient field representation for this class of problems. The efficiency and the accuracy of the method depend on the dimension of the FDTD computational domain and, on how many FW modes may be used in the signal reconstruction within the frequency range of interest. In preliminary works [1],[2],[3],[4], the authors described the basic principles and some preliminary applications to both narrow-band and wide-band arrays. Here, the accuracy problem is emphasized by numerical analysis of a simple periodic structure whose analytical solution can be easily computed, and a frequency selective surface (FSS) [5]. The analysis will help to define guidelines for the FDTD domain size and the maximum number of TD-FW modes needed.

2. Method overview

Referring to Fig. 1, we consider an infinite array with periods d_x and d_y, along x and y, respectively. We assume array elements with arbitrary geometry, simultaneously excited by short-pulse generators. The FDTD method together with appropriate boundary conditions is used to evaluate the magnetic field at a reference plane located at $z = z'$, which defines the transverse equivalent electric current distribution $\hat{\mathbf{J}}_t(\boldsymbol{\rho}, t) = \mathbf{1}_z \times \mathbf{H}(\boldsymbol{\rho}, t)$, with $\boldsymbol{\rho} = x\mathbf{1}_x + y\mathbf{1}_y$. It is convenient to define an E (TM) and H (TE) complete eigenvector set $\{\mathbf{e}_{pq}^E(\boldsymbol{\rho}), \mathbf{e}_{pq}^H(\boldsymbol{\rho}) = \mathbf{e}_{pq}^E \times \mathbf{1}_z\}$ (see [3] for a de-
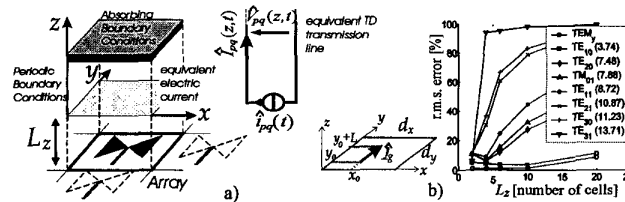

Fig. 1. Left) Geometry, array and equivalence plane where the current generators $\hat{i}_{pq}$ are placed. Right) test array of line sources $\hat{j}_g$, Problem size, (mm): $d_x = 80$, $d_y = 38$, $L = 10$, $x_0 = 22$, $y_0 = 12$. r.m.s error vs. the distance between the array and the equivalence plane at L_z. Markers at 2, 4, 6, 10, 20 FDTD cells respectively, cut-off frequencies in brackets (GHz).

tailed definition) which allows to expand the numerically-evaluated equivalent current distribution

$$\hat{\mathbf{J}}_t(\boldsymbol{\rho},t) = \hat{i}_{00}(t) + \sum_{p,q} \hat{i}^E_{pq}(t)\mathbf{e}^E_{pq}(\boldsymbol{\rho}) + \sum_{p,q} \hat{i}^H_{pq}(t)\mathbf{e}^H_{pq}(\boldsymbol{\rho}) \tag{1}$$

in terms of time dependent E and H modal current generators $\hat{i}^H_{pq}(t)$ and $\hat{i}^H_{pq}(t)$, together with the TEM generator $\hat{i}_{00}(t)$. The TD generators are computed by projecting the total current distribution, evaluated by the FDTD method, onto the vector basis set

$$\hat{i}_{pq}(t) = \int_0^{d_x} \int_0^{d_y} \hat{\mathbf{J}}_t(\boldsymbol{\rho},t) \cdot \mathbf{e}_{pq}(\boldsymbol{\rho}) dx dy; \quad \hat{i}_{00}(t) = \frac{1}{\sqrt{d_x d_y}} \int_0^{d_x} \int_0^{d_y} \hat{\mathbf{J}}_t(\boldsymbol{\rho},t) dx dy \tag{2}$$

which are numerically implemented as finite summations, with the vector modal basis sampled at the FDTD grid points. Transient radiation at $z > z'$ is evaluated using modal TD-FW voltage $\hat{V}_{pq}(z,z',t)$ and current $\hat{I}_{pq}(z,z',t)$ propagators, excited by the modal current generators $\hat{i}_{pq}(t)$ at z', as shown in Fig.1a. The magnetic field (analogous considerations hold for the electric field) is evaluated as

$$\hat{\mathbf{H}}^{tot}_t(\mathbf{r},t) = \hat{\mathbf{H}}_{00}(\mathbf{r},t) + \sum_{p,q} \hat{I}^E_{pq}(z,z',t)\mathbf{e}^E_{pq}(\boldsymbol{\rho}) + \sum_{p,q} \hat{I}^H_{pq}(z,z',t)\mathbf{e}^H_{pq}(\boldsymbol{\rho}) \tag{3}$$

in which we have also included the TEM propagator $\hat{\mathbf{H}}_{00}(\mathbf{r},t)$. In particular, the current $\hat{I}_{pq}(z,z',t)$ is evaluated by convolution between the current generator $\hat{i}_{pq}(t)$ and the impulse current response $\hat{I}^{H,imp}_{pq}{}^E(z,z',t) = \delta(t-t_0) - \tilde{\omega}_{pq}t_0\frac{J_1(\tilde{\omega}_{pq}\sqrt{t^2-t_0^2})}{\sqrt{t^2-t_0^2}}U(t-t_0)$, in which J_1 is the Bessel function of first order, $\tilde{\omega}_{pq} = c(2\pi)\sqrt{(d_x/p)^2 + (d_y/q)^2}$, and $t_0 = |z - z'|/c$ is the turn-on time of all the p,qth TD-FWs [3]. An analogous formula applies to the impulse voltage response $\hat{V}^{H,imp}_{pq}(z,z',t) = \zeta\delta(t-t_0) - \zeta\tilde{\omega}_{pq}t\frac{J_1(\tilde{\omega}_{pq}\sqrt{t^2-t_0^2})}{\sqrt{t^2-t_0^2}}U(t-t_0)$, with $\zeta = \sqrt{\mu/\epsilon}$, while $\hat{V}^{E,imp}_{pq}(z,z',t)$ is expressed in terms of more involved incomplete Lipschitz-Hankel integrals. Physical insight can be gained from frequency asymptotics applied to Fourier inversion of a FD-FW [2], which leads to the definition of the dominant local instantaneous frequencies $\omega_{pq,i}(t) = (-1)^i\tilde{\omega}_{pq}t/\sqrt{t^2-t_0^2}$, $i = 1,2$. They are real after turn-on ($t > t_0$), and determine the time-dependent dominant frequencies which govern TD-FWs. At turn-on, the pqth TD-FW has $\omega_{pq,i}(t \to t_0) \to \infty$, while $\omega_{pq,i}(t \to \infty) \to \tilde{\omega}_{pq}$, implying that at turn-on the pqth TD-FWs cannot be excited by its band-limited generator $\hat{i}_{pq}(t)$, and that after t_0 the frequency of the pqth TD-FW localizes around its cutoff frequency $\tilde{\omega}_{pq}$. Therefore, the pqth TD-FW, contributes to the total radiated field when its cutoff frequency $\tilde{\omega}_{pq}$ is lower than the maximum frequency of the spectrum of its generator $\hat{i}_{pq}(t)$. This furnishes a simple criterium to determine the total number of TD-FWs needed in (3).

Note that the algorithm does not require storage of the FDTD time-samples for each FDTD spatial location in the periodic cell. One needs to store only their projections $\hat{i}_{pq}(t)$ in (2), onto the vector basis $\mathbf{e}^{E,H}_{pq}$, since we use the scalar TD propagators $V^{imp}_{pq}(z,z',t)$ and $I^{imp}_{pq}(z,z',t)$ (network formulation).

3. Numerical analysis

Calibration. The method is calibrated against a simple canonical problem consisting of a periodic planar array of uniform $y-$polarized line sources $\hat{j}_g(x'+nd_x, y'+md_y, z' = 0,t)$, with $m,n = 0,\pm 1,\pm 2,\dots$ (see Fig.1b), located at $z' = 0$. Each source is defined
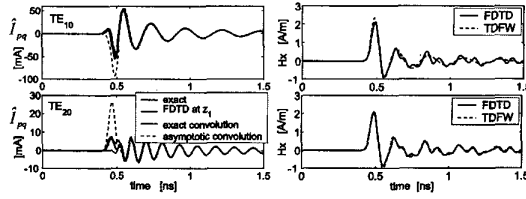
Fig. 2. Td-FW modal currents and magnetic field at a distance $z_1 = 50\Delta = 100mm$ from the array plane. Only a few modal currents $(p, q \leq 2)$ are required to construct the radiated transient magnetic field, showing that the TD-FW based representation is rapidly convergent.

by $\hat{\jmath}_g(x', y', z', t) = \hat{\imath}_g(t)\delta(x' - x_0)rect(y_0 + \frac{L}{2}, L)\delta(z')\mathbf{1_y}$ with $rect(y_0 + \frac{L}{2}, L) = 1$ if $y_0 \leq y' \leq y_0 + L$, and 0 otherwise, and with gaussian pulse $\hat{\imath}_g(t) = I_0 \exp[-(t - \tau)^2/(2T^2)]$ with parameters τ and T such to give a 20% bandwidth $[0, f_{max} = 15GHz]$ $(\hat{\imath}_g(f_{max}) = 0.2\hat{\imath}_g(0))$ and $I_0 = 1A$ for simplicity. This simple canonical problem furnishes an analytical reference solution. Indeed, the exact expression of the FW modal currents $\hat{I}_{pq}^{exact}(z, z', t)$ at any distance z from the array can be easily obtained via convolution (*) with the exact impulsive currents in Sec.II (see also [4])

$$\hat{I}_{pq}^{E,exact}(z, z', t) = \hat{I}_{pq}^{E,imp}(z, z', t) * \hat{\imath}_{g,pq}^{E}(t), \quad \text{with } \hat{\imath}_{g,pq}^{E}(t) = \hat{\imath}_g(t)\int_{y_0}^{y_0+L} e_{y,pq}^{E}(x_0, y')dy'$$

(4)

The same currents at an equivalence plane $(z = L_z)$ are evaluated using the FDTD algorithm, and their accuracy is tested against the reference solution. The FDTD algorithm is constructed using a uniform cubic mesh, with cell side $\Delta = 2mm$ (corresponding to a FDTD grid cut-off of $f^{FDTD} = \frac{c}{10\Delta} = 15$ GHz), and the computational domain is terminated (along the z axis) by 14-cell PML. Currents $\hat{I}_{pq}^{FDTD}$ have been evaluated at N_t discrete times $t_n = n\Delta t$, n=1,2,...N_t, for various choices of distances L_z between the array and the plane where the modal generators $i_{pq}(t)$ are computed. The relative root mean square (r.m.s.) error $\varepsilon_{pq}(L_z) = \frac{\|\hat{I}_{pq}^{exact}(L_z) - \hat{I}_{pq}^{FDTD}(L_z)\|_2}{\|\hat{I}_{pq}^{exact}(L_z)\|_2}100$,

where $\|g(L_z)\|_2 = \sqrt{\sum_{n=1}^{N_t} |g(L_z, t_n)|^2}$, has been computed versus the distance L_z for lower order modes. As shown in Fig.1b, the error ε_{pq} for the TEM_y and TE_{10} modes is low for every L_z, while increasingly larger errors are observed for higher order modes, for larger L_z, especially for the TE_{21}, TE_{30} and TE_{31} modes. Therefore, modal currents should be best computed at an equivalence plane (see Fig.1a) which is not too distant (1-5 cells) from the array plane, with the important benefit to keep small the FDTD computational domain. The error increase with the mode order is directly related to the sampling rate of the periodic modes, which are sinusoidal functions of the variable ρ, for the discrete approximation of the integrals in (2). This error could be reduced by a finer FDTD grid or by using a quadrature method more accurate than the finite summations. Following the discussion in[4], it has been experimentally found that the highest (p, q)-mode, whose equivalent current can still be computed with the typical FDTD accuracy is limited to $p \leq d_x/(20\Delta)$ and $q \leq d_y/(20\Delta)$. In this case $p \leq 2, q \leq 0.95$ in agreement with the results in Fig.1. The reconstruction of the magnetic field H_x using the TD-FW-FDTD algorithm in Sec.II, is analyzed in Fig.2. Equivalent current generator $\hat{\imath}_{pq}(t)$ are computed by the FDTD method at an equiv-
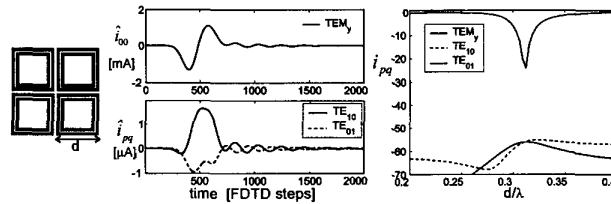
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Fig. 3. TD-FW-FDTD algorithm applied to a doubly periodic FSS with period d. Left) reflected modal current generators $\hat{i}_{pq}(t)$ v.s. time; right) spectrum i_{pq} of the modal current generators normalized by the spectrum of the incident signal.

alence plane one cell away ($L_z = 2mm$) from the array plane. Fig.2 shows currents $\hat{I}_{pq}(z_1, L_z, t)$ at a distance $z_1 = 50\Delta = 100mm$ from the array plane ($z_1 - L_z$ from the equivalence plane). Currents $\hat{I}_{pq}(z_1, L_z, t)$ are computed using convolution between the generators $\hat{i}_{pq}$, and both the exact ("exact convolution") and asymptotic ("asymptotic convolution") impulsive propagators $\hat{I}_{pq}^{imp}(z_1, L_z, t)$. These are compared with an extended FDTD estimation comprising z_1 (numerically expensive), and the "exact" reference solution $\hat{I}_{pq}^{imp}(z_1, 0, t)$ in (4). The "exact convolution" provides an excellent agreement with the "exact" reference solution, whereas, as expected, the "asymptotic convolution" provides less accurate results in the early transient. Finally, in Fig.2b the magnetic field H_x has been regenerated at point (50mm, 24mm, z_1) by summing TD-FWs. The early transient and the dominant oscillation of the signal's tail are correctly regenerated by the superposition of the sole TEM_y and TE_{10} modes, while higher order modes (those whose cut-off frequency $f_{pq}^{\text{cutoff}} \equiv \tilde{\omega}_{pq}/2\pi \leq 15$ GHz, i.e., $|p| \leq 2$, $|q| \leq 2$), have been required to more accurately reproduce the oscillating late transient. It has been experienced that a superposition of higher order modes, with cut-off outside the simulation bandwidth does not improve further the accuracy of the reconstructed signal.

Application to FSS. The method has been applied to the analysis of a doubly periodic frequency selective surface (FSS). The elementary cell with size $d \times d$ consists of two concentric square loops [5] with $(d/16) \times (d/16)$ crossection and mutual spacing $d/32$. The structure has been illuminated by a TEM_y wave with normal incidence and gaussian time waveform with $f_{\text{max}} = 4c/(3d)$. The modal current generators $\hat{i}_{pq}(t)$ for the reflected field have been computed at a an equivalent plane $d/16$ (two FDTD cells) from the FSS plane. Preliminary results in Fig.3 show lower order modal current generators (TEM, TE_{10}, and TE_{01}) in the time domain and in the frequency domain (in the FD, we have normalized by the spectrum of the incident field). As expected, mainly the TEM_y current is excited. As in the previous case, the signal reconstructed (not shown here) at $z > L_z$ by the TD-FW-FDTD algorithm is in accord with an extended FDTD computation (numerically expensive).

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