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Essays on Open-Economy Macroeconomics

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ABSTRACT OF THE DISSERTATION

Essays on Open-Economy Macroeconomics

by

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This dissertation covers several topics in macroeconomics. Chapter one provides an overview for this dissertation. Chapter two explores the role of demand shocks, as an alternative to productivity shocks, in driving both domestic and international business cycles. In addition to those well-documented domestic and international business cycle properties, this paper focuses on two additional stylized facts in the industrialized countries: procyclical trade openness (GDP fraction of trade volume) and countercyclical government size (GDP fraction of government spending). Using a parsimonious dynamic stochastic general equilibrium model, I show that the model's predictions under productivity shocks are not consistent with these facts. Instead, a demand-shock-driven model replicates the above facts while matching other business cycle properties.

Chapter three examines the long-run relationship between trade openness and government size in a two-country dynamic general equilibrium model. I analytically

show that, if the non-tradable sector is more capital intensive, higher government expenditures drive up the relative capital stock in the tradable sector in steady state. This gives rise to a relatively higher output and a relatively lower price in the tradable sector. As a result, when trade openness increases, a benevolent government would expand public expenditures to push up (down) the relative output (price) of tradables so as to achieve agents' *desired* consumption plan with more consumption of tradables. Therefore, a positive relationship between trade openness and government size is observed. On the contrary, if the tradable sector is more capital intensive, a negative correlation follows.

In chapter four, I present new estimates of the factor substitution elasticity and biased factor-augmenting technical progress using the supply-side system for the aggregate U.S. economy during the period 1948-2012. On the basis of recursive scheme estimations, I first show that significant variations of estimated model parameters arise from different sample periods. I next incorporate labor market friction into the supply-side system and show that the augmented model fits the data better. With labor market friction, the estimated elasticity of substitution between capital and labor does not statistically significantly differ from unity. The long-run technical progress tends to be purely labor-augmenting although non-negligible variations arise during some sample periods.

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Chapter 1

Introduction

This dissertation covers several topics in macroeconomics. Chapter two focuses on domestic and international business cycles and explores its driving forces. Chapter three examines, in the long run, whether a country needs to have a large or small government when its economy becomes more open. Chapter four investigates the interaction of labor market friction and economic growth.

In the second chapter, I explores the role of demand shocks, as an alternative to productivity shocks, in driving both domestic and international business cycles within an international real business cycle (IRBC) framework. Data of the industrialized countries show that (i) government consumption and government size (GDP fraction of government consumption) are countercyclical and (ii) trade openness (GDP fraction of trade volume) is procyclical and net export is countercyclical. In this chapter, I study a parsimonious two-country dynamic stochastic general equilibrium (DSGE) model driven by demand shocks that is able to generate all above characteristics in addition to matching other business cycle properties observed in the data.

With demand shocks, the substitution between consumption of private and public goods generates a countercyclical government spending and government size. This feature continues to hold when the government provides both public goods and public capital. Second, according to my model, international trade decreases more than output during economic downturns in the presence of demand shocks. The predicted procyclicality of trade openness is consistent with the data and my model implies that the recent Great Trade Collapse may be largely demand driven. Third, the current model exhibits the possibility for demand shocks to serve as a common driving force behind the domestic and international business cycles and explain a large set of regularities observed in the industrialized countries, including the well-documented comovement puzzle and the Backus-Smith puzzle.

The third chapter focuses on the long-run relationship between a country's trade openness and government size. Globalization increasingly facilitates the bilateral and multilateral connections across countries and integrates world market closely. In the context of globalization, the debate on the role of government is attracting more attention. However, empirical studies so far cannot provide a consensus on the long-run relationship between government size and trade openness. Both positive and negative correlation has been found in the existing empirical literature. Given the mixed evidence on their long-run relationship, using a two-country dynamic general equilibrium model, I analytically show that the way a benevolent government responds to trade openness may be impacted by the differentiated factor intensities in the production of the tradable and non-tradable sectors. If the non-tradable (tradable) sector is more capital intensive, expansionary (contractionary) government spending drives down the relative prices of tradable vs. non-tradable goods. When trade openness increases and households would like to consume more tradables compared to non-tradables, a benevolent government responds by expanding (contracting) its spending to lower the relative prices of tradable goods. Therefore a positive (negative) relationship of government size and openness follows.

The capability of our model to generate a positive/negative steady state relationship between trade openness and government size reconciles the empirical ambiguity observed in the existing literature. When countries under concern exhibit different relative factor intensities in their tradable and non-tradable sectors, our model predicts that both positive and negative correlations between openness and government size are possible.

In chapter four, I switch to economic growth and labor market friction. Estimating the elasticity of substitution between capital and labor has been the central theme of numerous studies ever since the constant elasticity of substitution (CES) production function is introduced into economics. In this chapter, I present new estimates of the factor substitution elasticity and biased factor-augmenting technical progress using the supply-side system (Klump et al., 2007) for the aggregate U.S.

economy during the period 1948-2012. On the basis of recursive scheme estimations, I first show that significant variations of estimated model parameters arise from different sample periods, calling for attention on the impacts of sample periods on parameter estimations. I next incorporate labor market friction (Farmer, 2012) into the supply-side system and show that the augmented model fits the data better. In the presence of labor market friction, the estimated elasticity of substitution between capital and labor does not statistically significantly differ from unity in most of the sample periods. The long-run technical progress tends to be purely labor-augmenting although non-negligible variations arise during some sample periods.

Chapter 2

By Force of Demand: Explaining Cyclical Fluctuations of International Trade and Government Spending

2.1 Introduction

The purpose of this chapter is to explore the role of demand shocks for both domestic and international business cycles. As in this chapter, some authors have shown that, as an alternative to productivity shocks, demand shocks have different yet important implications for international comovements.[1] However, the cyclical fluctuations of international trade and government spending and the comovement between these two variables have not been fully explored. In this chapter, I investigate the potential of an international real business cycle (IRBC) model driven by demand shocks and show its ability to explain a large set of regularities observed in the data of industrialized countries, including the cyclical fluctuations of international trade and government spending.

Pioneered by Backus, Kehoe, and Kydland (1992; 1994) (BKK henceforth), a well-known stylized fact in the IRBC literature is the countercyclical net export, customarily calculated as the ratio of trade balance to output, which implies that domestic absorption is more volatile than output. Recently, Raffo (2008) finds that the countercyclical net export observed in the data is mainly driven by the consumption smoothing and imports, not because of the dynamics of capital; the countercyclical net export are driven by the countercyclicality of the net quantity of goods traded across countries and international price plays a minor role. Consistent with these facts, I show that when the model fluctuations are driven by demand shocks, these is less capital mobility across countries. The countercyclical net export is mainly driven by the fluctuations of net quantity of international trade, not the price variations. In other words I identify an alternative source of shocks that generates the business cycles of net export consistent with the stylized fact.

[1] Important contributions on demand shocks include Baxter and King (1991); Stockman and Tesar (1995); Hall (1997); and Wen (2007), among others.

On top of the countercyclical net export, a defining feature of this paper is to explore another important but largely ignored cyclical property of international trade: the procyclical trade openness, customarily calculated as the ratio of trade volume (imports plus exports) to output. Data from the industrialized countries show that trade openness decreases (increases) during economic recessions (expansions). This is interesting because this finding implies that trade volume responds more than output to exogenous shocks over business cycles. For example, during the period of the Great Recession, the nominal GDP in the United States drops approximately by 2% from 2008 to 2009; in contrast, trade volume drops by approximately 20% during the same period, about 10 times the size of that change in the nominal GDP. The drop of trade volume is so prominent during this crisis that this phenomenon has been termed as the "Great Trade Collapse". It turns out that this pattern of trade volume fluctuations is not specific to this period. Figure 2.1 plots the cyclical fluctuations of real GDP and trade openness for the United States during the period 1954-2010, with both series detrended with the Hodrick–Prescott (1997) filter. I observe a significantly positive comovement relation between these two variables. This *procyclicality* of trade openness is not unique to the United States. When I calculate the correlation of trade openness and real output over business cycles for other G7 countries, I am informed of the average correlation coefficient of 0.51.[2] Looking at this pattern from the eye of an IRBC model, a conventional productivity-shock-driven model cannot generate the

[2]The data come from PWT 7.1. All series are from 1950 to 2010 but the time span of different countries varies due to its availability. All calculations are based on the H-P filtered series.

procyclical trade openness. This is because a positive (negative) productivity shock will unambiguously increase (decrease) output more than trade volume. On the contrary, a demand-shock-driven model predicts a larger demand-induced response of trade volume than output in the presence of exogenous demand shocks. The predicted procyclicality of trade openness is consistent with the data and my model implies that the recent Great Trade Collapse may be largely demand-driven.

Another feature of my demand-shock-driven model is its ability to explain the cyclical properties of government consumption.[3] Data from the industrialized countries show that government consumption is either acyclical or countercyclical while government size (the GDP fraction of the government consumption) is unambiguously countercyclical, consistent with the notion of countercyclical fiscal policy.[4] Take the United States as an example. The contemporaneous correlation between real government consumption (government size) and real GDP is –0.23 (-0.74). Figure 2.2 plots the real GDP, government size, and government consumption over business cycles for the United States during the period 1954-2010. All series are H–P filtered. From Figure 2.2, I observe a negative comovement between output and government size. During all recessions (the shaded areas), there is a significant and almost instantaneous jump in government size. Figure 2.2 also shows that this negative

[3]Ravn (1997) and Roche (1996) explore the cyclical performance of government spending in an open economy. However, government spending in both studies is specified as exogenous fiscal shocks, not endogenously determined. Roche (1996) shows that government spending could potentially facilitate a standard IRBC model to match the key stylized facts in the data.

[4]See, for example, the discussions in Kaminsky et al. (2005).

relationship is not driven by a flat government consumption; instead, a moderate countercyclical government consumption contributes to the significant countercyclicality of government size. This phenomenon does not uniquely apply to the United States. A calculation on the correlation of government size (consumption) and output for G7 countries reveals that the average correlation coefficient is -0.77 (-0.16). When I evaluate the performance of a productivity-shock-driven IRBC model on government spending, its prediction is counterfactual. In the presence of a positive productivity shock, the large positive wealth effects induce more private and public consumption. Therefore government consumption under productivity shocks is predicted to be procyclical, which is counterfactual. On the contrary, when the model is driven by demand shocks, a positive demand shock to the private consumption induces agents to substitute out the consumption of public goods for private goods. Therefore government consumption decreases when output increases, which generates both countercyclical government consumption and government size.

More broadly, this paper falls into the literature exploring the relation between trade openness and government size. When the economy becomes more open, on the one hand, a large public sector may serve as a safe sector to cushion the economy from the uncertainty imposed by global markets; on the other hand, a large public sector may cause the loss of international competitiveness. More recently, Epifani and Gancia (2009) and Jiang (2013), among others, discuss the response of government spending to the (exogenous) variations of trade openness. Provided that both trade openness and government size are simultaneously determined in a general equilibrium model, it is interesting to investigate the spontaneous comovement of these two variables at the business cycle frequency. Due to the countercyclicality of government size and the procyclicality of trade openness in the data, these two variables should move in the opposite directions over business cycles. Figure 2.3 confirms this and reports the correlation coefficient between these two variables to be -0.50 and -0.51 for the United States and G7 countries, respectively. A successful business cycle model therefore should be able to generate this stylized fact. I show that this is the prediction of a demand-shock-driven model, but not a productivity-shock-driven model.

Despite my focus on the cyclical properties of international trade and government spending, this paper also accounts for other well-documented business cycle properties. For the closed-economy variables, the demand-shock-driven model generates an output which is more volatile than consumption but less volatile than investment; consumption, investment, labor hours, and labor productivity are all positively related to output. As for the international business cycles, the demand-shock-driven model generates a higher cross-country correlation of output than consumption, solving the comovement puzzle (BKK, 1992). The model also generates the negative correlation between the real exchange rate (the relative consumption price index) and the relative consumption, hence resolving the Backus-Smith (1993) puzzle documented in the literature. These findings generally manifest the potential of an IRBC model to explain a large set of both domestic and international business cycle characteristics under demand shocks.

Finally, this paper exhibits the potential of a simple one-sector IRBC model, with complete asset markets and isoelastic (additively separable) preferences, to generate the empirically plausible cyclical fluctuations. More recently, several authors (Canova and Ubide, 1998; Karabarbounis, 2014; Nguyen, 2010; Raffo, 2008 and 2010) explored the significant impacts of having an independent home production sector or adopting its reduced form, i.e., the GHH preferences, on the international business cycles.[5] Corsetti et al. (2008) explored the significance of the incomplete asset market and trade elasticity of tradables on international risk sharing. With an one sector IR-BC model driven by demand shocks, this paper shows that complete asset markets and isoelastic preferences could coexist with the low international risk sharing while accounting for a large set of domestic and international business cycle fluctuations.

The remainder of this paper is organized as follows. Section 2 describes the stylized facts of G7 countries on domestic and international business cycles, with a focus on the international trade and government spending. Section 3 introduces a simple model to gain intuitions. Section 4 examines the performance of a more general model. The robustness of the model with public capital and alternative parameters values

[5]Benhabib et al. (1991) shows that GHH utility function can be obtained analytically as a reduced form case from a model that includes home activities, provided that home and market consumptions are close substitutes.

is examined in section 5. Section 6 concludes the paper with remarks for further research.

2.2 Data Regularities

In this section, I briefly report the stylized facts observed in the G7 countries. These facts are used to gauge the success of the model driven by demand shocks relative to productivity shocks. All data come from the Penn World Table 7.1 (Heston et al., 2012). All statistics refer to the cyclical components obtained after applying the H-P filter to the natural log of each series. No logarithm transformation is made for net export. All series are in real terms and, consistent with the PWT 7.1, the sample period is 1950 to 2010.[6]

Table 2.1 reports the standard deviations of output and the standard deviations of other variables relative to output in G7 countries. As documented in the literature, output is more volatile than consumption but less volatile than investment and is approximately as volatile as labor hours. Labor productivity (defined as the ratio of output to labor hours) is less volatile than output.

Table 2.2 reports the correlations of different variables. As expected, consumption, investment, labor hours, and labor productivity are all procyclical. Government consumption, however, is countercyclical for the majority of these countries, with an

[6]All series are from 1950 to 2010 but the time span of different countries varies due to its availability. All series of the United States start at 1954, corresponding to the after-war period.

average correlation coefficient -0.16, leading to a countercyclical government size. As for international trade, the net export is countercyclical, indicating a more volatile domestic absorption than output. Trade openness is procyclical, implying that trade volumes fluctuate more than output over business cycles. The countercyclical government size and the procyclical trade openness simultaneously determine a negative correlation between these two variables. In addition, the correlation between the relative (home to foreign) consumption and the real exchange rate is negative, a fact documented in Backus and Smith (1993).

Table 2.3 reports the international comovements of output, consumption, investment, and labor hours between the United States the other six countries. All variables tend to be positively correlated across countries and output is more correlated than consumption, indicating a relatively low international consumption risk sharing, a fact investigated in BKK (1992).

2.3 The Basic Model

2.3.1 A World Economy

In this section I introduce the basic model, a standard two-country two-good IRBC model (BKK, 1992; 1994) adapted to allow for the country-specific public goods. There exist two countries in the model economy, *home* and *foreign*, labelled as h and f, respectively. Each country is represented by a large number of identical agents and a production technology. Agents have symmetric preferences and production technology across countries. Each country specializes in the production of a single good: the home country specializes in the production of good 1 and the foreign country specializes in the production of good 2. Labor is internationally immobile but capital is free to flow across borders. Since the model has two countries (h and f) and two goods (1 and 2), I use X_{sub}^{sup} to denote the final use of good sub in country sup (X denotes consumption, investment, etc.). The superscript $sup = h, f$ represents the country of uses and the subscript $sub = 1, 2$ represents the index of goods. In what follows, I focus on the analysis of home country. All specifications in the foreign country are symmetric.

Preferences of the representative agent in the home country (h) are characterized by utility functions of the form

$$u(C_t^h, N_t^h, G_t^h) = \Delta_t^h \ln C_t^h - \eta N_t^h + A \ln G_t^h \quad (2.1)$$

$$C_t^h = \left[\theta \left(c_{1t}^h\right)^\phi + (1-\theta)\left(c_{2t}^h\right)^\phi\right]^{\frac{1}{\phi}} \quad (2.2)$$

where C_t^h is the consumption bundle in the home country, including consumption of good 1, c_{1t}^h, and consumption of (imported) good 2, c_{2t}^h; N_t^h is the labor hours worked; G_t^h is the consumption of a country-specific public good provided by the home government; Δ_t^h is a country-specific random shock to the marginal utility of private consumption.

Due to my interest in the cyclical properties of government spending and government size, the agent's preferences are augmented with the utility-generating government spending (public goods and services). Since public expenditure is significantly home biased[7], to carefully investigate the cyclical properties of government spending, I do not assume that the provision of public goods requires both domestic and foreign content as in the case of private consumption (equation 2.2); instead, I assume that government uses only domestic good to produce the public good. The additive separability in G_t^h implies that the consumption of public goods does not affect the consumption of private goods, a specification supported by empirical estimations.[8] This specification simplifies the computations as the G_t^h term can be ignored when the household optimization conditions are derived.[9]

Following the literature (*e.g.* Hansen, 1985; Rogerson, 1988), I adopt the specification of indivisible labor where the utility function becomes linear in leisure. As discussed in Wen (2007), since the marginal utility of leisure is not affected by the preference shock Δ_t^h, the equilibrium employment can only be affected by this shock

[7]Epifani and Gancia (2009) shows that, in a sample of 14 countries, the average import share in government consumption is only about 1%.

[8]Karras (1994) and Amano and Wirjanto (1998) indicate that one cannot reject the hypothesis of additive separability in private and public consumption. Also see the analysis in McGrattan, Rogerson, and Wright (1997).

[9]Karabarbounis (2014) explores the role of non-separable preferences between market consumption and (non-market) home consumption for labor wedge and international business cycles. S-tockman and Tesar (1995) explore the impacts of non-separable preferences between tradable and non-tradable goods for international business cycles. In this paper, the (additively separable) specification of public goods, supported by empirical studies, differs from the specification of (non-separable) non-traded goods or goods produced in the (non-market) home sector. See also Guo and Lansing (1999) and Lansing (1998) for other studies adopting similar specifications.

through its impact on the changing shadow prices of consumption goods. So the preference shock (Δ_t^h), under indivisible labor, is effectively a "demand" shock.

Production of good 1 in the home country takes place according to the constant-return-to-scale technology

$$Y_{1t} = z_t^h \left(K_t^h\right)^{\alpha} \left(N_t^h\right)^{1-\alpha} \tag{2.3}$$

$$K_t^h = \left[\rho \left(k_{1t}^h\right)^q + (1-\rho)\left(k_{2t}^h\right)^q\right]^{\frac{1}{q}} \tag{2.4}$$

where z_t^h represents the home total factor productivity (TFP) shocks; N_t^h represents the home labor hours worked. K_t^h denotes the capital used in the home production, which includes not only the capital originated from good 1 (k_{1t}^h) but also the capital originated from good 2 (k_{2t}^h). The resource constraint in the home country is given by

$$Y_{1t} = c_{1t}^h + c_{1t}^f + k_{1t+1} - (1-\delta)k_{1t} + G_t^h \tag{2.5}$$

$$k_{1t} = k_{1t}^h + k_{1t}^f \tag{2.6}$$

where k_1, named as capital 1, denotes the total existing capital stock originated from good 1. Capital 1 is decomposed into two parts: k_{1t}^h, used in the home production, and k_{1t}^f, used in the foreign production. Resource constraint (2.5) implies that, out of all good 1 produced in the home country, c_{1t}^h is consumed domestically; c_{1t}^f is *exported* to the foreign country for consumption; G_t^h is used by the home government to produce public goods; and the remaining part, $k_{1t+1} - (1-\delta)k_{1t}$, *i.e.*, total investment of good

1, is either used to accumulate capital 1 in the home economy (k_{1t}^h) or *exported* to accumulate capital 1 in the foreign economy (k_{1t}^f).

Due to its international transfer, capital used in the home country may not be owned by the home residents. I use $f_{it}^h(\equiv k_{it}^h/k_{it})$ and $f_{it}^f(\equiv k_{it}^f/k_{it})$ to denote the fraction of capital i ($= 1, 2$) used in the production of home and foreign countries, respectively. π^h and π^f represent the fraction of the world population residing in the home and foreign countries, with $\pi^h + \pi^f = 1$.[10] Therefore investment of good 1 in the home and foreign countries can be defined as

$$I_{1t}^h = \pi^h \left[k_{1t+1} - (1-\delta)k_{1t}\right] + \left(f_{1t}^h - \pi^h\right) k_{1t} \tag{2.7}$$

$$I_{1t}^f = \pi^f \left[k_{1t+1} - (1-\delta)k_{1t}\right] + (f_{1t}^f - \pi^f)k_{1t} \tag{2.8}$$

where δ is the exogenous capital depreciation rate; I_{1t}^h (I_{1t}^f) represents the home (foreign) investment expenditure on good 1. $k_{1t+1} - (1-\delta)k_{1t}$ is the aggregate world investment of good 1; the second term in equation (2.7) and (2.8) indicates the amount of other country's capital operating in domestic economy at period t, which is called the foreign direct investment. If I sum up equation (2.7) and (2.8), I recover the total investment, $k_{1t+1} - (1-\delta)k_{1t}$ in equation (2.5).[11]

The foreign country has symmetric specifications on technology. The total investment originated from good 2, $k_{2t+1} - (1-\delta)k_{2t}$, is partly used to accumulate capital 2 in the foreign country (k_{2t}^f) and partly *imported* into the home country to accumulate

[10] Due to symmetry, $\pi^h = \pi^f = 1/2$.

[11] Note that $\pi^h + \pi^f = 1$ and $f_{it}^h + f_{it}^f = 1, i = 1, 2$.

capital 2 at home (k^h_{2t})

$$I^h_{2t} = \pi^h \left[k_{2t+1} - (1-\delta)k_{2t}\right] + \left(f^h_{2t} - \pi^h\right) k_{2t} \quad (2.9)$$

$$I^f_{2t} = \pi^f \left[k_{2t+1} - (1-\delta)k_{2t}\right] + \left(f^f_{2t} - \pi^f\right) k_{2t} \quad (2.10)$$

where I^f_{2t} (I^h_{2t}) represents the foreign (home) investment expenditure of good 2.

The home country exports c^f_{1t} and I^f_{1t} amount of good 1 to the foreign country for consumption and investment, respectively, and imports c^h_{2t} and I^h_{2t} amount of good 2 from the foreign country accordingly. If I use good 1 as the *numeraire* and denote the relative price of good 2 as p_t, then p_t also stands for the (home) terms of trade. The total value of consumption expenditures (TC), total value of investment expenditures (TI), and total value of output (TY) in the home and foreign countries can be computed in the unit of good 1

$$TC^h_t = c^h_{1t} + p_t c^h_{2t} \quad (2.11)$$

$$TC^f_t = p_t c^f_{2t} + c^f_{1t} \quad (2.12)$$

$$TI^h_t = I^h_{1t} + p_t I^h_{2t} \quad (2.13)$$

$$TI^f_t = p_t I^f_{2t} + I^f_{1t} \quad (2.14)$$

$$TY^h_t = Y_{1t} \quad (2.15)$$

$$TY^f_t = p_t Y_{2t} \quad (2.16)$$

Given the above structure of international trade, the trade balance (TB) and trade volume (TV) in the home country are computed as[12]

$$
\begin{aligned}
TB_t^h &= EX_t^h - IM_t^h = c_{1t}^f + I_{1t}^f - p_t \left(c_{2t}^h + I_{2t}^h \right) && (2.17) \\
TV_t^h &= EX_t^h + IM_t^h = c_{1t}^f + I_{1t}^f + p_t \left(c_{2t}^h + I_{2t}^h \right) && (2.18)
\end{aligned}
$$

When the relative price p_t is evaluated at its steady state level, equation (2.17) and (2.18) reflect the constant-price net trade and trade volume. This allows us to examine the impacts of variations of relative prices on business cycles (Raffo, 2008). I define net export to GDP ratio (nx) and trade openness ($open$) in the home country as

$$
\begin{aligned}
nx_t^h &= \frac{TB_t^h}{TY_t^h} && (2.19) \\
open_t^h &= \frac{TV_t^h}{TY_t^h} && (2.20)
\end{aligned}
$$

Government size in the home country is defined as the GDP fraction of the government consumption

$$size_t^h = \frac{G_t^h}{TY_t^h} \qquad (2.21)$$

Given equation (2.2) and the relative price p_t, the price index for the utility-based consumption bundle C_t^h is given by

$$P_t^h = \left[\theta^{\frac{1}{1-\phi}} + (1-\theta)^{\frac{1}{1-\phi}} p_t^{\frac{\phi}{\phi-1}} \right]^{\frac{\phi-1}{\phi}} \qquad (2.22)$$

[12]Note that the trade balance can also be computed from the difference between total output (Y_{1t}) and total absorption ($TC_t^h + TI_t^h + G_t^h$) in the home country.

Similarly, the price index for the utility-based consumption bundle C_t^h in the foreign country is

$$P_t^f = \left[(1-\theta)^{\frac{1}{1-\phi}} + \theta^{\frac{1}{1-\phi}} p_t^{\frac{\phi}{\phi-1}} \right]^{\frac{\phi-1}{\phi}} \tag{2.23}$$

The real exchange rate in the home country can be defined as

$$RER_t = \frac{P_t^f}{P_t^h} \tag{2.24}$$

Fluctuations in the model are driven by stochastic TFP or demand shocks. These underlying shocks to the economy are governed by the independent bivariate autoregressions.[13] The TFP shocks follow

$$\mathbf{z}_t = \lambda \mathbf{z}_{t-1} + \varepsilon_t \tag{2.25}$$

where $\mathbf{z}_t = \left(\ln z_t^h \;\; \ln z_t^f \right)$ and ε_t is distributed normally and independently over time with variance $\mathbf{V}_\varepsilon$. The correlation between z_t^h and z_t^f is determined by the off-diagonal elements of λ and $\mathbf{V}_\varepsilon$. Similarly, demand shocks are governed by

$$\mathbf{\Delta}_t = \mathbf{d}\mathbf{\Delta}_{t-1} + \xi_t \tag{2.26}$$

where $\mathbf{\Delta}_t = \left(\ln \Delta_t^h \;\; \ln \Delta_t^f \right)$ and ξ_t is distributed normally and independently over time with variance $\mathbf{V}_\xi$.

I solve the equilibrium allocations by studying the problem facing a social planner who maximizes the life-time utility of the agents in both countries. An equilibrium of

[13] I do not focus my attention on the interaction of these shocks in driving the model economy. Instead, as shown later, I am interested in comparing the performance of the model under different shocks.

this economy could be computed as the solution to the following planning problem:

$$\max mE_0 \sum_{t=0}^{\infty} \beta^t u(C_t^h, N_t^h, G_t^h) + (1-m)E_0 \sum_{t=0}^{\infty} \beta^t u(C_t^f, N_t^f, G_t^f) \tag{2.27}$$

subject to the resource constraints in both countries. m is the planner's social welfare weight for the home country. Following BKK (1992, 1994) and Wen (2007), I compute the equilibrium associated with $m = 1/2$. The equilibrium is characterized by the following first order conditions (FOCs)[14]

$$\lambda_t^h = \frac{\theta \Delta_t^h \left(c_{1t}^h\right)^{\phi-1}}{\theta \left(c_{1t}^h\right)^{\phi} + (1-\theta)\left(c_{2t}^h\right)^{\phi}} \tag{2.28}$$

$$\lambda_t^f = \frac{(1-\theta)\,\Delta_t^h \left(c_{2t}^h\right)^{\phi-1}}{\theta \left(c_{1t}^h\right)^{\phi} + (1-\theta)\left(c_{2t}^h\right)^{\phi}} \tag{2.29}$$

$$\chi N_t^h = (1-\alpha)\lambda_t^h Y_{1t} \tag{2.30}$$

$$\lambda_t^h = \frac{A}{G_t^h} \tag{2.31}$$

$$\begin{aligned} \lambda_t^h = {} & \beta E_t \left\{ \lambda_{t+1}^h \left[\alpha\rho Y_{1t+1} \left(K_{t+1}^h\right)^{-q} \left(f_{1t+1}^h\right)^q k_{1t+1}^{q-1} + 1 - \delta \right] \right\} \\ & + \beta E_t \left[\alpha(1-\rho)\lambda_{t+1}^f Y_{2t+1} \left(K_{t+1}^f\right)^{-q} (f_{1t+1}^f)^q k_{1t+1}^{q-1} \right] \end{aligned} \tag{2.32}$$

where λ_t^h and λ_t^f are the Lagrangian multipliers associated with the home and foreign resource constraints, respectively. I solve for the unique steady state of the model and approximate the dynamics of the model in response to random shocks by log-linearizing the FOCs around the steady state. I then solve for the equilibrium sequences of allocations in the basic model. Since λ_t^h and λ_t^f also indicate the shadow

[14] I only show the first order conditions associated with the home country variables. Similarly there are corresponding FOCs associated with the foreign variables.

prices (in utility measure) of one unit of good 1 and good 2, respectively, the relative price of good 2, p_t, is then computed as the ratio of these shadow prices

$$p_t = \frac{\lambda_t^f}{\lambda_t^h} = \frac{\theta}{1-\theta}\left(\frac{c_{1t}^h}{c_{2t}^h}\right)^{\phi-1} \tag{2.33}$$

Through equation (2.22) to (2.24), the real exchange rate is also determined, which allows us to relate the fluctuations of the relative consumption sequence to the relative prices (the terms of trade and the real exchange rate) to address the Backus-Smith puzzle. A limitation of this approach is that the relative consumption sequence (c_{1t}^h/c_{2t}^h) is rather stable in equilibrium (regardless of demand or productivity shocks) and as a result this model does not generate enough fluctuations of the terms of trade or real exchange rate.[15]

2.3.2 Steady State and Parameter Calibrations

To check how the basic model's predictions match the data, I first define the steady state of the basic model by exploring the rest point of the first order conditions that characterize the equilibrium.[16] In the calibration process, values of all parameters are either taken from the existing literature or calibrated such that the steady state values of N^h, $size^h$, and $open^h$ match the empirical counterparts and are therefore set

[15] See Chari et al. (2002) and Corsetti et al. (2008) for discussions on volatility of international prices.

[16] I assume that $\rho = \theta$ and $q = \phi$. Given the fact that only one good is produced in each country, which is used for consumption and investment, this assumption implies that the final uses of goods, both consumption and investment, have the domestic and foreign content and in the same proportions, the spirit of the Armington aggregator in the general equilibrium trade models.

without regard for their business cycle performance. The time period in this paper is one year. The annual discount factor β is set to 0.96, which implies an annual interest rate of approximately 5%. The capital income share, α, is set to 0.36 (BKK 1992, Hansen 1985). The parameter ϕ is calibrated such that the elasticity of substitution between domestic and foreign goods, $1/(1-\phi)$, equals 1.5 (BKK, 1994). The following three free parameters, η, A, and θ, are calibrated such that three targets observed in the data are matched: (1) the steady state working hours $N = 1/3$, *i.e.*, one third of the total time is devoted to working (Hansen, 1985); (2) the steady state government size $size = 7.25\%$, which is the average government size in the G7 countries; (3) the steady state trade openness $open = 31\%$, the average level of trade openness for the G7 countries.[17]

As for the shock parameters, I follow BKK's (1992; 1994) estimations on Solow residual and take their estimates of TFP shock parameters. There is no immediate empirical estimations on demand shock parameters. I instead construct the series of demand shocks Δ_t^h based on the first order conditions (4.8) and (2.30) and estimate equation (2.26) to obtain the transition matrix $\mathbf{d}$, the cross-country correlation ρ_ξ, and the standard deviation σ_ξ. See the appendix for detailed estimation process. The estimated parameters are $d_{11} = d_{22} = 0.8960$ and $d_{12} = d_{21} = 0.1038$, $\sigma_\xi = 0.015$ and $\rho_\xi = 0.3647$.

[17]BKK (1994) set the import ratio to 15%, equivalent to a 30% trade openness in my model. The average government size for the United States is about 10%, also very close to the target in this paper.

2.3.3 Findings

I now turn to the performance of the basic model. In Table 2.5, column 1 reports the empirical moments for the United States. Column 2 reports the range of the same moments observed in the G7 countries. Column 3 reports the predicted moments of the basic model driven solely by TFP shocks while column 4 reports the predicted moments under demand shocks only. I first discuss the case of TFP shocks and then compare it with the demand shocks.

When the model is purely driven by TFP shocks, first, I find that public consumption G is strongly procyclical. This counterfactual prediction is due to the strong wealth effects in the presence of TFP shocks. When home country's productivity increases, home agents become wealthier and consume more private and public goods. Therefore government consumption increases with output. Since the productivity drive up the output more than public consumption, government size decreases when output increases.

Second, trade openness is countercyclical under TFP shocks. This counterfactual prediction is due to the excessive cross-country capital flows, revealed by the significant volatility of investment. When home country is hit by a positive TFP shock, capital flows into the home country but out of the foreign country. As a result, trade flows are dominated by trade on investment goods. To take advantage of a high productivity at home, home agents imports more good 2 for investment purpose but export less good 1 to the foreign country. Although the increasing imports and decreasing exports generate the countercyclical net export, the trade volume barely changes when output increases significantly. Therefore trade openness is countercyclical, inconsistent with the data. This is shown in the impulse response in Figure 2.4.

Third, consistent with the existing literature, under TFP shocks, the basic model reports a positive correlation between the relative consumption and the real exchange rate (the Backus-Smith puzzle) and a lower (actually negative) cross-country correlation of output than consumption (the comovement puzzle); both predictions are counterfactual. On the one hand, with a positive TFP shock, the relative supply of good 1 increases and makes good 2 more expensive (a higher p_t). The deterioration of the home terms of trade induces a higher real exchange rate for the home country (a depreciation). Due to the home bias in private consumption ($\theta > 1/2$), the relative consumption (TC^h/TC^f) increases for the home agents, leading to its positive comovement with the real exchange rate. On the other hand, a positive shock at home induces cross-border capital flows from the foreign to the home country. Output, investment, and labor increase (decrease) in the home (foreign) countries accordingly. Consumption correlation, however, due to the agents' risk-sharing across countries, is perfect. This is not surprising considering the additively separable preferences.

When the model fluctuations are purely driven by demand shocks, the predictions are different. First, the public consumption becomes countercyclical. With a positive demand shock to the marginal utility of private consumption, the consumption of private goods substitutes out the consumption of public goods. Therefore government consumption decreases when demand-induced output increases, consistent with the data. The countercyclical government consumption leads to the countercyclical government size. (Figure 2.5)

Second, trade openness become procyclical. With positive demand shocks, agents consume more good 1 as well as good 2, i.e., the *world* demand for both commodities is higher. With the higher world demand, productions in both countries tend to increase. Therefore less capital moves across countries and investment volatility falls. Since foreign agents need to buy more good 1 for investment purpose to satisfy the higher demand, home country's exports tend to increase.[18] Imports, on the other hand, also increase as home agents would like to buy more goods for consumption and investment purpose in response to a positive demand shock. With both higher exports and imports, the impulse response in Figure 2.5 shows that trade volume increases more than output; hence trade openness is procyclical. Intuitively, demand shocks on private consumption induces larger variations on international trade (for both consumption and investment purpose) than on output. As imports are more correlated with output than exports, net export is still countercyclical. Moreover, I also verify the cyclical fluctuations of net trade, i.e., evaluating the trade balance

[18]In Figure 2.5, the initial response of exports drops a little bit. That is because of the one-time shock. When the demand shocks are persistent, as shows in the simulations, foreign agents have stronger expectations on the higher demand for good 2 and purchase more good 1 for investment, giving rise to the procyclical exports of good 1.

in equation (2.17) at the steady state price level. The resulting net trade is still countercyclical. Therefore the countercyclicality of net export ratio in this paper is not driven by the price variations, consistent with the findings in Raffo (2008).

Third, the correlation between the relative consumption and the real exchange rate becomes negative and the cross-country correlation of output is higher than consumption. Different from the TFP shocks, a positive demand shock increases the demand for both commodities. Due to the home bias on the consumption of domestically produced goods, the relative demand for good 1 increases when the home country is hit by a positive demand shock. Therefore good 2 becomes cheaper and the terms of trade improves in the home country. What follows is the lower home real exchange rate (an appreciation) and its negative relation with home agents' higher relative consumption (TC^h/TC^f). Therefore the Backus-Smith puzzle is solved under demand shocks. In addition, a positive demand shock tends to increase output in both countries and therefore output correlates positively, which partly explains the comovement puzzle.

The predictions of the basic model are not perfect and miss several important properties of domestic and international business cycles (to be addressed in the next section). But, as an alternative to the TFP shocks, demand shocks deliver a different yet promising mechanism that drives business cycles. With the estimated shock parameters presented above, demand shocks alone account for 36% to 56% of the observed output volatility. In a world where demand shocks coexist with many other shocks, the contribution of demand shocks is certainly non-negligible.

2.4 The General Model

This section is an extension of the previous analysis. The general model presented here inherits the insights of the basic model and attempts to improve upon the limitations of the basic model. Column 4 in Table 2.5 shows that consumption is too volatile under demand shocks. The volatile consumption is channelled to the cyclical fluctuations of labor hours due to the complementarity between consumption and leisure. Figure 2.5 (the upper left panel) shows that labor hours respond more than output in response to demand shocks, leading to the countercyclical labor productivity. In addition, the excessive volatility of consumption worsens its cross-country correlation and makes it negative.

To improve upon these limitations, I first incorporate consumption habit into the agents' preferences, which, as shown in Wen (2004; 2007), contributes to consumption smoothing in the case of demand shocks. Second, I introduce factor hoarding (the variable labor efforts and capital utilization) *a la* Burnside et al. (1993) and Burnside and Eichenbaum (1996), which helps lower the consumption volatility, and more importantly, generate the procyclicality of labor productivity in the presence of demand shocks.

2.4.1 A Revised World Economy

With consumption habit and factor hoarding, preferences of the representative agent in the home country become

$$u(C_t^h, N_t^h, G_t^h) = \Delta_t^h \ln\left(C_t^h - aC_{t-1}^h\right) + N_t^h \ln(T - B - be_t^h) + (1 - N_t^h)\ln T + A\ln G_t^h \tag{2.34}$$

where e_t^h represents the level of labor effort while the parameters T, B, and b stand for the total time endowment, the cost of time from going to work and the length of working hours per shift, respectively. On the production side, a similar constant-return-to-scale technology is used

$$Y_{1t} = z_t^h \left(u_t^h K_t^h\right)^\alpha \left(e_t^h N_t^h\right)^{1-\alpha} \tag{2.35}$$

where u_t^h is the variable capital utilization rate. Due to this variable capital utilization, capital depreciation are *endogeneized*

$$\delta_t^h = \frac{1}{\chi}\left(u_t^h\right)^\chi \tag{2.36}$$

where $\chi > 1$ indicates the convexity of capital depreciations. All investment process in (2.7) to (2.10) is adapted accordingly. The resource constraints in the home and foreign countries are given by

$$Y_{1t} = c_{1t}^h + c_{1t}^f + k_{1t+1} - (1 - \overline{\delta}_{1t})k_{1t} + \frac{\omega}{2}\left(N_t^h - N_{t-1}^h\right)^2 k_{1t} + G_t^h \tag{2.37}$$

where $\overline{\delta}_{it}$ is the overall depreciation rate of capital i[19]; the parameter ω measures the size of dynamic adjustment cost of labor employment compared to its previous level. The introduction of the adjustment cost provides the incentive for hoarding labor (Wen, 2004; 2007). k_{it}, $i = 1, 2$, in the adjustment cost term is a way to normalize the adjustment cost in the steady state which facilitates the calibration of the parameter ω. The introduction of this terms does not affect the dynamics of the model as this terms will drop out with the first-order Taylor expansion.

2.4.2 Parameter Calibrations

Calibrations of all parameter values in the general model are similar to the basic model. Following Wen (2007) I set the time endowment $T = 5476$ since the time period considered here is one year. The following five parameters, χ, A, θ, B,and b, are calibrated such that five targets are matched: (1) the steady state annual depreciation rate $\delta = 10\%$; (2) the steady state government size $size = 7.25\%$; (3) the steady state trade openness $open = 31\%$; (4) the steady state employment rate $N = 0.94$; and (5) the steady state labor effort $e = 1$. The empirical estimates of the consumption habit parameter vary between $(0.64, 0.97)$ (Ferson and Constantinides, 1991; Braun et al., 1993). In the baseline calibration, I choose the consumption habit

[19] As for capital i, since f_{it}^h fraction is used in the home country and f_{it}^f fraction is used in the foreign country, the overall depreciation rate $\overline{\delta}_{it}$ is given by $\overline{\delta}_{it} = f_{it}^h \delta_t^h + f_{it}^f \delta_t^f$, $i = 1, 2$.

parameter $a = 0.8$ and the labor adjustment cost parameter $\omega = 0.5$ as in Wen (2007) and experiment with high and low values.

2.4.3 Findings

The predicted moments of the general model are reported in column 5 and 6 of Table 2.5. Compared with the basic model (column 3 and 4), the general model does not improve upon the moments of interest under the TFP shocks. Under the demand shocks, several improvement arises. First, consumption becomes less volatile than output. This improvement relative to the basic model is (i) due to the consumption habit, which stabilizes consumption expenditures over time, and (ii) due to the variable capital utilization, which allows agents to utilize the existing capital stock with different intensity, reduces the cross-border capital flows, and stabilizes the economy.

Second, labor productivity becomes procyclical. Due to the costly adjustment of labor, when a positive demand shock drives up the demand for labor hours, instead of only adjusting the labor hours, agents can increase the labor effort. Therefore the total output increases more than labor hours, leading to a procyclical labor productivity. The introduction of labor adjustment cost justifies the behavior of hoarding labor and is indispensable for the model to have a procyclical labor productivity.

Third, as for the international comovements, all variables including investment and consumption, are positively correlated across countries and, more important, the general model predicts a much larger cross-country correlation of output than consumption. Due to the persistence of demand shocks, agents in both countries would like to accumulate more capital for next period when positive shocks occur. This can only be achieved by investing more because there is barely movement of the existing capital under the variable capital utilization and demand shocks. Then a highly positive investment correlation follows. As for the consumption comovement, consumption habit permits smooth consumption series and contributes to the positive cross-country comovement of consumption with the help of positively correlated demand shock innovations across countries. The resulting consumption correlation is much lower compared with output.

With the general model, the predictions on international trade and government spending under demand shocks are qualitatively the same. Government consumption and government size are both countercyclical; trade openness is procyclical while net export is countercyclical. All previously discussed insights still apply. It seems that the general model now only accounts for 16% to 25% of the observed output volatility. This perception is misleading as the general model, in the simulation exercise, is fed with the demand shocks estimated from the basic model. With the consumption habit and factor hoarding, one can no longer obtain a clean expression of demand shock as I did in the case of the basic model. Given my focus on the cyclical properties of international trade and government spending, the estimation of demand shock process in the general model is left for future work.

2.5 Robustness

Government certainly provides more than just public goods and services in the model economy. The question one may ask is how the model's predictions differ when the government, in addition to providing public goods and services, also provides public capital? With this question in mind, I incorporate public capital into the model and analyze its impacts on the model's predictions. I also examine whether the performance of the general model, driven by demand shocks, is sensitive to the modest changes of parameter values.

2.5.1 Public Capital

With public capital, the production technology in equation (4.27) becomes

$$Y_{1t} = z_t^h \left(u_t^h K_t^h\right)^{\alpha_1} \left(e_t^h N_t^h\right)^{\alpha_2} \left(u_{gt}^h K_{gt}^h\right)^{\alpha_3} \tag{2.38}$$

where $\alpha_1 + \alpha_2 + \alpha_3 = 1$; K_{gt}^h and u_{gt}^h represent the public capital and its utilization, respectively. The resource constraint takes the same form as in equation (2.37), but part of the investment of good 1 is now used to accumulate the public capital K_{gt}^h at home. The government spending includes the government consumption G_t and government investment

$$I_{gt}^h = K_{gt+1}^h - \left(1 - \overline{\delta}_t\right) K_{gt}^h \tag{2.39}$$

Government size is re-defined as

$$size_t^h = \frac{G_t^h + I_{gt}^h}{TY_t^h} \tag{2.40}$$

The calibration process is the same and α_3 is calibrated to match the US economy in which the steady state K_{gt}^h/TY_t^h ratio is about 0.61 (Guo and Lansing, 1997). The predictions of the model are reported in column 7 and 8 of Table 2.5. Comparing column 8 with column 6, the qualitative predictions of the model does not change. Government size and government consumption are still countercyclical; trade openness are procyclical and net export is countercyclical. Since public investment I_g is procyclical, government size becomes less procyclical and less volatile. With both public consumption and investment, government substitutes public consumption for investment in response to a positive demand shock to the marginal utility of private consumption. The abundant public capital encourages agents to export more good 1 abroad for investment purpose. Therefore exports are more procyclical and net export is less countercyclical.

2.5.2 Alternative Parameters

In this section, I examine the robustness of the general model with different parameter values. In Table 2.6, column 4 and 5 examine the impacts of a low and high adjustment cost. The labor adjustment cost helps rationalize the adoption of labor hoarding. When the adjustment cost is higher, the labor variations become smaller and consequently, labor productivity becomes more volatile and more procyclical. Besides, consumption responses more to the shocks as the adjustment of leisure (labor) is more costly. Column 6 and 7 check the robustness of the model with a weak and strong consumption habit. The introduction of a stronger consumption habit stabilizes consumption expenditure. As consumption barely changes, demand shocks induce more variations of investment and labor hours. Labor productivity becomes less procyclical.

In all cases, government consumption and government size are countercyclical and trade openness is procyclical. These moments do not vary with the parameter values, which exhibits the ability of the model to explain the stylized facts.

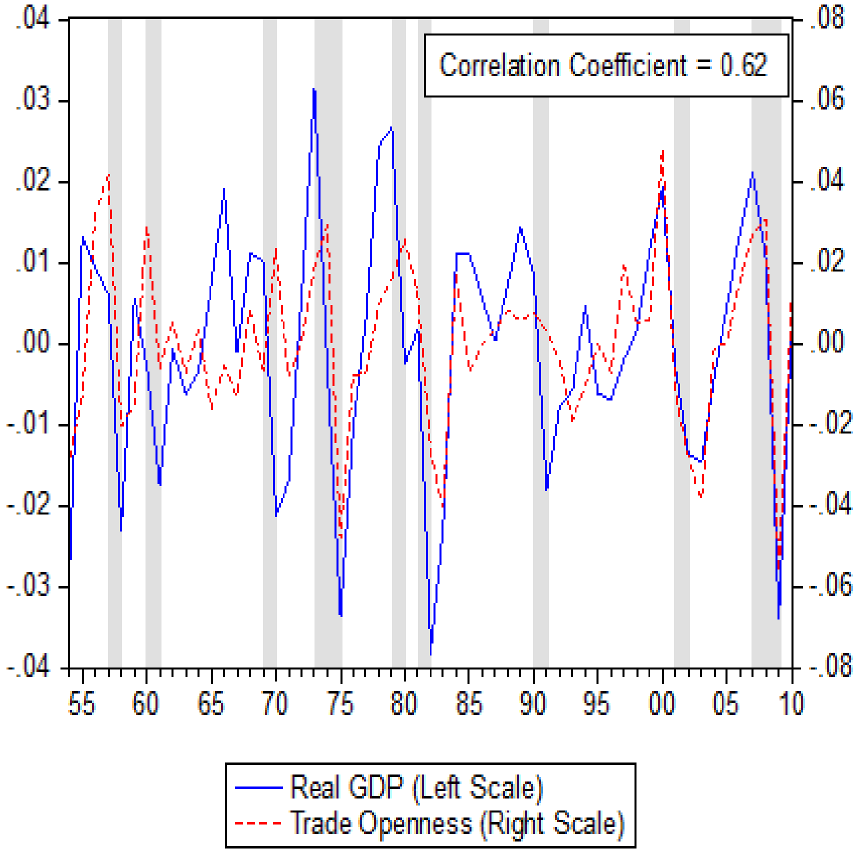
2.6 Conclusions

The following stylized facts are observed in the data of the industrialized countries: (i) government consumption and government size are countercyclical; (ii) trade openness is procyclical and net export is countercyclical. In this paper, I study a parsimonious two-country dynamic stochastic general equilibrium model driven by demand shocks. The demand-shock-driven model is able to generate all of the above characteristics in addition to matching other business cycle properties observed in the data.

With demand shocks, the substitution between consumption of private and public goods generates a countercyclical government spending and government size. This feature continues to hold when the government provides both public goods and public capital. Second, according to the model, international trade decreases more than output during economic downturns in the presence of demand shocks. The predicted procyclicality of trade openness is consistent with the data and the model implies that the recent Great Trade Collapse may be largely demand driven (Warner, 1994; Cheung and Guichard, 2009). Third, the model exhibits the possibility for demand shocks to serve as a common driving force behind the domestic and international business cycles and explain a large set of regularities observed in the industrialized countries, including the well-documented comovement puzzle and the Backus-Smith puzzle.

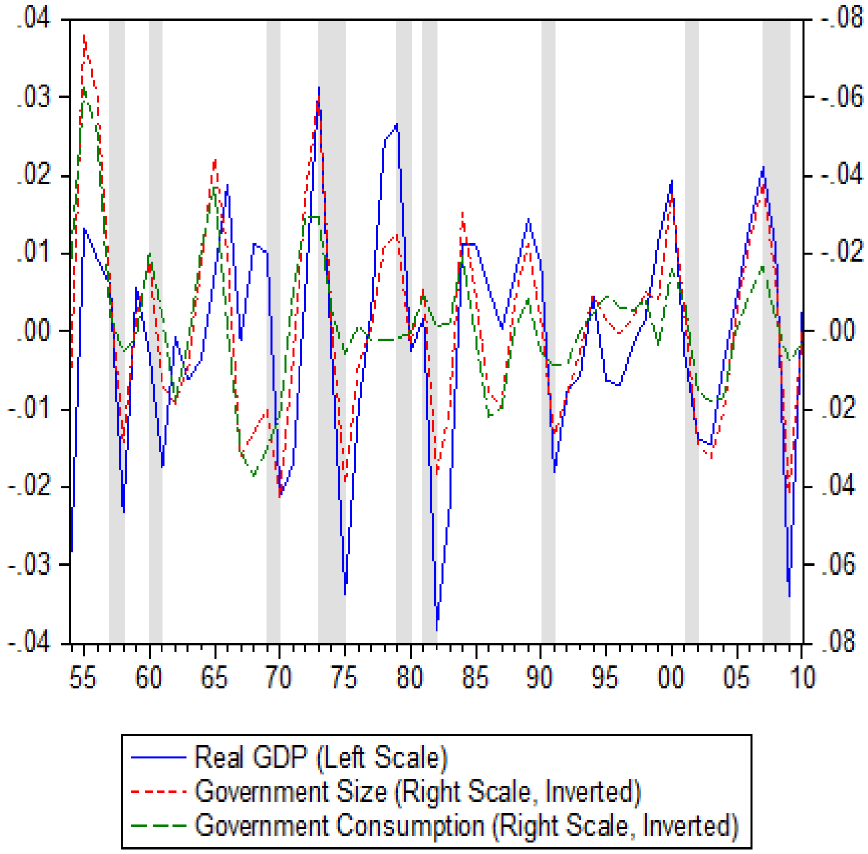
Some important factors are left undiscussed in this paper. First, the volatility of the relative price (the terms of trade and real exchange rate) is much higher than output in the data. In the model, however, these two variables are directly related to consumption expenditures, whose volatility is low due to consumption smoothing. Second, government spending in the current model is financed by a lump-sum tax. Allowing the distortionary taxes may bring in more dynamics of government spending and facilitate the discussions of the government's fiscal policy from the financing side. These interesting topics are not covered in the current paper and left for future research.

Figure 2.1: Real GDP and Trade Openness over Business Cycles in the U.S.



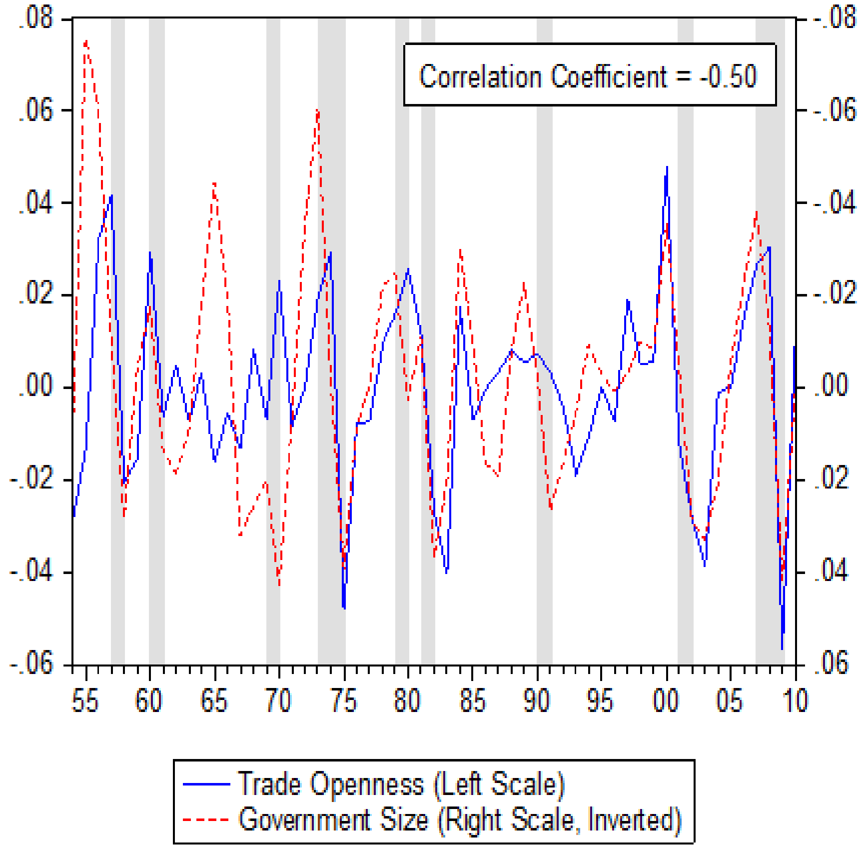
Note: This figure plots the H-P filtered components of real GDP and trade openness (the ratio of trade volume to output) for the United States during the period 1954 to 2010. The shaded areas represent recessions dated by NBER. Data Source: PWT 7.1.

Figure 2.2: Real GDP and Government Size over Business Cycles in the U.S.



Note: This figure plots the H-P filtered components of real GDP, government size (the ratio of government consumption to output), and government consumption for the United States during the period 1954 to 2010. The shaded areas represent recessions dated by NBER. Data source: PWT 7.1.

Figure 2.3: Trade Openness and Government Size over Business Cycles in the U.S



Note: This figure plots the H-P filtered components of trade openness (the ratio of trade volume to GDP) and government size (the ratio of government consumption to output) for the United States during the period 1954 to 2010. The shaded areas represent recessions dated by NBER. Data source: PWT 7.1.

Figure 2.4: Impulse Response to a 1 Percent TFP Shock from the Home Country (Basic Model)

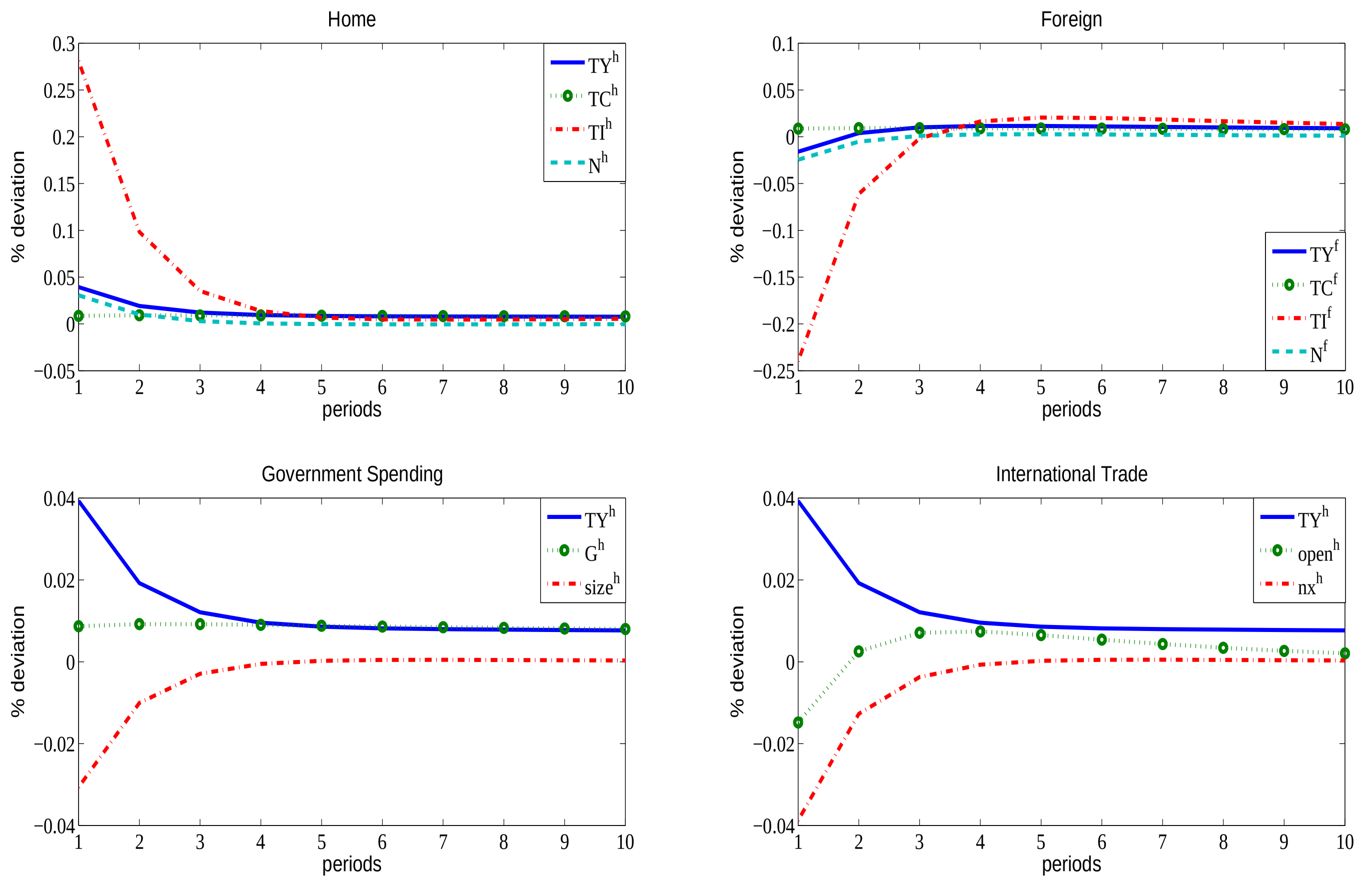


Figure 2.5: Impulse Response to a 1 Percent Demand Shock from the Home Country (Basic Model)

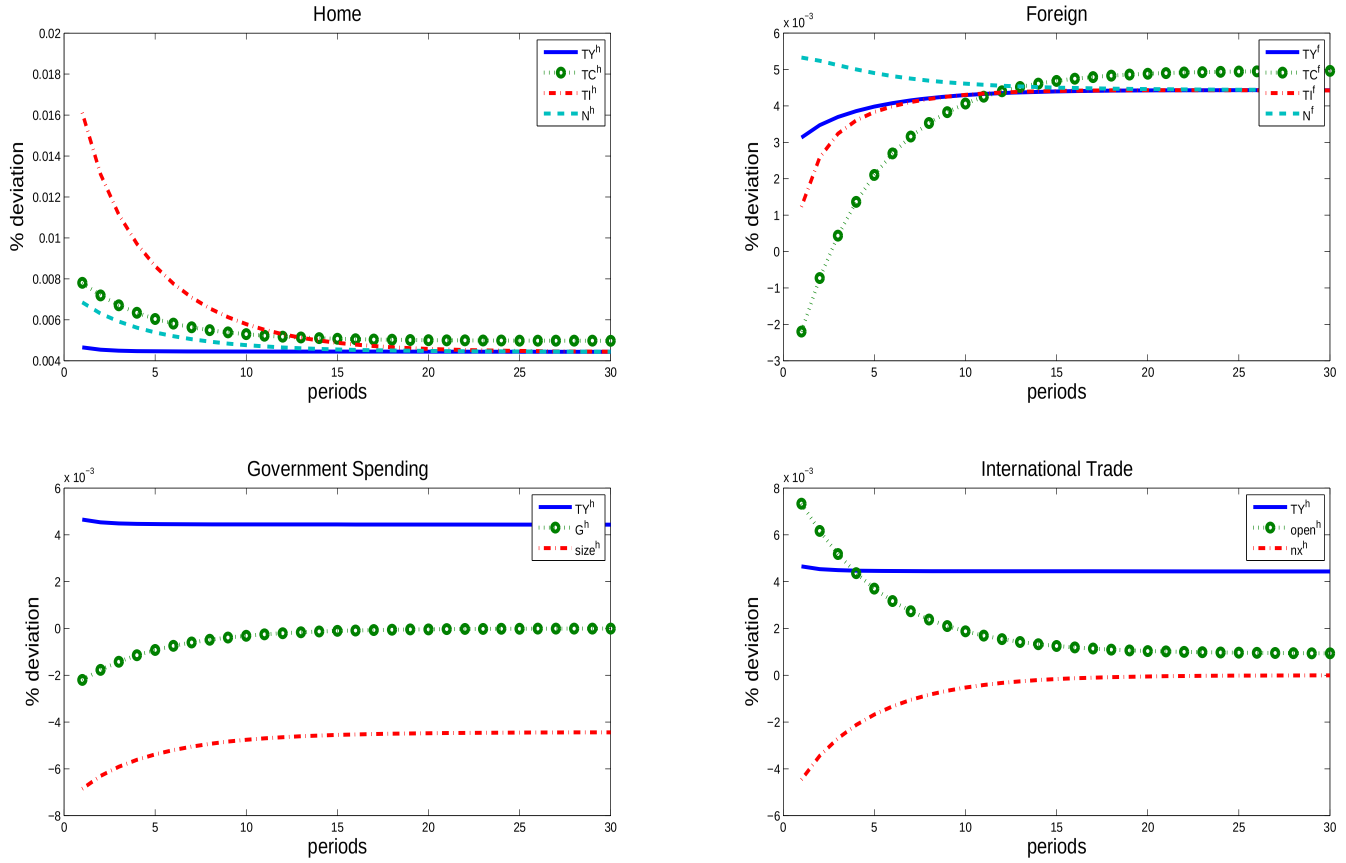


Table 2.1: Standard Deviations

Country	Std Dev of output (%)	C	I	N	Y/N	G	$size$	nx	$open$
		Standard Deviation relative to output							
Canada	1.52	0.68	3.73	0.94	0.80	1.76	1.88	0.40	1.50
France	1.05	0.80	3.71	0.78	0.73	1.05	1.42	0.31	1.95
Germany	1.37	0.61	3.18	0.70	0.57	0.89	1.48	0.41	1.61
Italy	1.45	0.68	3.24	0.76	0.83	0.68	1.25	0.41	1.67
Japan	1./61	0.73	3.39	0.60	0.69	1.26	1.60	0.28	2.32
United Kingdom	1.32	0.91	4.30	0.94	0.73	1.48	1.88	0.28	1.36
United States	1.52	0.70	3.77	0.94	0.37	1.18	1.72	0.21	1.39
average	1.41	0.73	3.62	0.81	0.67	1.19	1.60	0.33	1.69
min	1.05	0.61	3.18	0.60	0.37	0.68	1.25	0.21	1.36
max	1.61	0.91	4.30	0.94	0.83	1.76	1.88	0.41	2.32

Notes: This table reports the standard deviations of output and other variables relative to output, including consumption (C), investment (I), labor hours (N), labor productivity (Y/N), government consumption (G), government size ($size$), net export ratio (nx), and trade openness ($open$). The data for G7 countries are from the Penn World Table 7.1. All statistics refer to the cyclical components obtained after applying the H-P filter to the natural log of each series. No logarithm transformation is made for net export. All series are in real terms and, consistent with the PWT 7.1, the sample period is generally 1950 to 2010. The time span of different countries varies due to its availability. All series of the United States start at 1954, corresponding to the after-war period.

Table 2.2: Domestic Correlations

Country	Correlation with output									
	C	I	N	Y/N	G	$size$	nx	$open$	$\rho(open, size)$	$\rho(C^h/C_f, RER)$
Canada	0.88	0.86	0.68	0.36	-0.31	-0.81	-0.21	0.54	-0.50	-0.13
France	0.79	0.88	0.65	0.63	0.04	-0.67	-0.50	0.61	-0.53	-0.20
Germany	0.64	0.91	0.82	0.72	-0.23	-0.81	0.19	0.56	-0.51	-0.23
Italy	0.70	0.96	0.62	0.68	-0.24	-0.92	-0.31	0.40	-0.45	-0.09
Japan	0.82	0.91	0.77	0.80	0.00	-0.80	-0.12	0.43	-0.57	-0.15
United Kingdom	0.85	0.88	0.74	0.46	-0.14	-0.66	-0.52	0.40	-0.51	-0.35
United States	0.93	0.94	0.93	0.34	-0.23	-0.74	-0.51	0.62	-0.50	-
average	0.80	0.91	0.74	0.57	-0.16	-0.77	-0.29	0.51	-0.51	-0.19
min	0.64	0.86	0.62	0.34	-0.31	-0.92	-0.52	0.40	-0.57	-0.35
max	0.93	0.96	0.93	0.80	0.04	-0.66	0.19	0.62	-0.45	-0.09

Notes: This table reports the domestic correlations of variables discussed in the paper, including consumption (C), investment (I), labor hours (N), labor productivity (Y/N), government consumption (G), government size ($size$), imports (IM), exports (EX), net export ratio (nx), and trade openness ($open$). The United States is regarded as the home country when the correlation between relative consumption and real exchange rate (RER) in the last column is calculated. The data for G7 countries are from the Penn World Table 7.1. All statistics refer to the cyclical components obtained after applying the H-P filter to the natural log of each series. No logarithm transformation is made for net export. All series are in real terms and, consistent with the PWT 7.1, the sample period is generally 1950 to 2010. The time span of different countries varies due to its availability. All series of the United States start at 1954, corresponding to the after-war period.

Table 2.3: Cross Country Correlations

Country	Cross-country correlations			
	Y	C	I	N
Canada	0.82	0.63	0.64	0.65
France	0.48	0.33	0.37	0.49
Germany	0.59	0.38	0.54	0.54
Italy	0.39	-0.03	0.31	0.17
Japan	0.49	0.44	0.42	0.57
United Kingdom	0.73	0.52	0.65	0.70
average	0.58	0.38	0.49	0.52
min	0.39	-0.03	0.31	0.17
max	0.82	0.63	0.65	0.70

Notes: This table reports the cross-country correlations of output (Y), consumption (C), investment (I), and labor hours (N) between the United States and other countries. The data for G7 countries are from the Penn World Table 7.1. All statistics refer to the cyclical components obtained after applying the H-P filter to the natural log of each series. No logarithm transformation is made for net export. All series are in real terms and, consistent with the PWT 7.1, the sample period is generally 1950 to 2010. The time span of different countries varies due to its availability. All series of the United States start at 1954, corresponding to the after-war period.

Table 2.4: Calibration of the Basic Model

Parameter	Interpretation	Value
m	Social welfare weight	0.5
β	Subjective discount factor	0.96
α	Capital income share	0.36
ϕ	Elasticity of substitution parameter	0.3333
η	Indivisible labor parameter	2.8834
A	Useful Government spending parameter	0.1201
θ	Distribution parameter	0.5638
$\sigma_\epsilon(\sigma_\xi)$	s.d. of TFP (demand) shocks	0.00852 (0.015)
$\lambda_{11}, \lambda_{22}$ (d_{11}, d_{22})	Diagonal element in $\boldsymbol{\lambda})(\boldsymbol{d})$	0.906 (0.8960)
$\lambda_{12}, \lambda_{21}$ (d_{12}, d_{21})	Off-diagonal element in $\boldsymbol{\lambda})(\boldsymbol{d})$	0.088 (0.1038)
ρ_ϵ (ρ_ξ)	Corr. of TFP (demand) innovations	0.258 (0.3647)

Table 2.5: Predicted Moments with TFP or Demand Shocks

	US	G7	Basic Model		General Model		Public Capital	
	Data	Range	TFP	demand	TFP	Demand	TFP	Demand
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Volatility								
$\sigma_{TY}(percent)$	1.52	(1.05 1.61)	2.58	0.59	0.98	0.26	1.09	0.26
σ_{TC}/σ_{TY}	0.70	(0.61 0.91)	0.12	1.18	0.21	0.69	0.18	0.67
σ_{TI}/σ_{TY}	3.77	(3.18 4.30)	8.22	2.73	4.93	2.91	5.32	2.58
σ_{N}/σ_{TY}	0.94	(0.60 0.94)	0.88	1.60	0.21	0.91	0.19	0.88
$\sigma_{prod}/\sigma_{TY}$	0.37	(0.37 0.83)	0.17	0.61	0.85	0.27	0.87	0.34
σ_{G}/σ_{TY}	1.18	(0.68 1.76)	0.17	0.61	0.57	0.63	0.52	0.82
σ_{nx}/σ_{TY}	0.21	(0.21 0.41)	1.19	0.85	0.56	0.59	0.74	0.34
$\sigma_{open}/\sigma_{TY}$	1.39	(1.36 2.32)	0.97	2.24	0.69	0.70	1.12	1.83
$\sigma_{size}/\sigma_{TY}$	1.72	(1.25 1.88)	0.88	1.60	0.64	1.56	0.60	0.40
Domestic correlations								
$\rho(TC,TY)$	0.93	(0.64 0.93)	0.75	0.69	0.85	0.74	0.84	0.70
$\rho(TI,TY)$	0.94	(0.86 0.96)	0.99	0.91	0.97	0.96	0.94	0.97
$\rho(N,TY)$	0.93	(0.62 0.93)	0.99	0.99	0.76	0.97	0.76	0.94
$\rho(prod,TY)$	0.34	(0.34 0.80)	0.75	-0.96	0.98	0.47	0.99	0.51
$\rho(G,TY)$	-0.23	(-0.31 0.04)	0.75	-0.96	0.80	-0.83	0.80	-0.87
$\rho(size,TY)$	-0.74	(-0.92 -0.66)	-0.99	-0.99	-0.85	-0.97	0.98	-0.84
$\rho(nx,TY)$	-0.51	(-0.52 0.19)	-0.97	-0.15	-0.99	-0.81	-0.80	-0.06
$\rho(open,TY)$	0.62	(0.40 0.62)	-0.77	0.96	-0.60	0.98	0.03	0.98
$\rho(size,open)$	-0.50	(-0.57 -0.45)	0.83	-0.98	0.91	-0.97	0.09	-0.81
$\rho(RER,TC^h/TC^f)$	-0.19	(-0.35 0.09)	1.00	-0.92	0.74	-0.83	0.74	-0.80
International correlations								
$\rho(TY^h,TY^f)$	0.58	(0.39 0.82)	-0.87	0.96	-0.03	0.99	-0.05	0.99
$\rho(TC^h,TC^f)$	0.38	(-0.03 0.63)	1.00	-0.32	0.99	0.16	0.99	0.08
$\rho(TI^h,TI^f)$	0.49	(0.31 0.65)	-0.99	0.43	-0.66	0.92	-0.80	0.91
$\rho(N^h,N^f)$	0.52	(0.17 0.70)	-0.97	0.98	-0.98	0.99	-0.89	0.99

Notes: This table reports the predicted moments of variables in the basic and the general models with TFP or demand shocks. The predicted statistics are based on 500 simulations with the sample length of 50. All series are H-P filtered.

Table 2.6: Robustness of General Model with Alternative Parameter Values

	US Data	G7 Range	Baseline	Low Adj. Cost $\omega = 0.35$	High Adj. Cost $\omega = 1.00$	Weak Con. Habit $a = 0.7$	Strong Con. Habit $a = 0.9$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Volatility							
$\sigma_{TY}(percent)$	1.52	(1.05 1.61)	0.26	0.30	0.23	0.31	0.17
σ_{TC}/σ_{TY}	0.70	(0.61 0.91)	0.69	0.62	0.76	0.84	0.55
σ_{TI}/σ_{TY}	3.77	(3.18 4.30)	2.91	2.99	2.88	2.66	3.14
σ_{N}/σ_{TY}	0.94	(0.60 0.94)	0.91	0.99	0.79	0.88	0.95
$\sigma_{prod}/\sigma_{TY}$	0.37	(0.37 0.83)	0.27	0.18	0.39	0.29	0.23
σ_{G}/σ_{TY}	1.18	(0.68 1.76)	0.63	0.49	0.79	0.63	0.64
σ_{nx}/σ_{TY}	0.21	(0.21 0.41)	0.59	0.59	0.61	0.61	0.59
$\sigma_{open}/\sigma_{TY}$	1.39	(1.36 2.32)	0.70	0.64	0.74	0.80	0.61
$\sigma_{size}/\sigma_{TY}$	1.72	(1.25 1.88)	1.56	1.43	1.72	1.57	1.56
Domestic correlations							
$\rho(TC, TY)$	0.93	(0.64 0.93)	0.74	0.74	0.72	0.74	0.74
$\rho(TI, TY)$	0.94	(0.86 0.96)	0.96	0.97	0.96	0.96	0.98
$\rho(N, TY)$	0.93	(0.62 0.93)	0.97	0.98	0.93	0.96	0.97
$\rho(prod, TY)$	0.34	(0.34 0.80)	0.47	0.14	0.67	0.53	0.32
$\rho(G, TY)$	-0.23	(-0.31 0.04)	-0.83	-0.82	-0.85	-0.85	-0.80
$\rho(size, TY)$	-0.74	(-0.92 -0.66)	-0.97	-0.98	-0.97	-0.98	-0.97
$\rho(nx, TY)$	-0.51	(-0.52 0.19)	-0.81	-0.85	-0.77	-0.73	-0.88
$\rho(open, TY)$	0.62	(0.40 0.62)	0.98	0.98	0.99	0.99	0.98
$\rho(size, open)$	-0.50	(-0.57 -0.45)	-0.97	-0.98	-0.97	-0.97	-0.98
$\rho(RER, TC^h/TC^f)$	-0.19	(-0.35 0.09)	-0.83	-0.79	-0.85	-0.88	-0.76
International correlations							
$\rho(TY^h, TY^f)$	0.58	(0.39 0.82)	0.99	0.99	0.99	0.99	0.99
$\rho(TC^h, TC^f)$	0.38	(-0.03 0.63)	0.16	0.18	0.13	0.09	0.25
$\rho(TI^h, TI^f)$	0.49	(0.31 0.65)	0.92	0.94	0.91	0.88	0.96
$\rho(N^h, N^f)$	0.52	(0.17 0.70)	0.99	0.99	0.99	0.99	1.00

Notes: This table reports the predicted moments of variables in the general model, driven by demand shocks, with alternative parameter values. The predicted statistics are based on 500 simulations with the sample length of 50. All series are H-P filtered.

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Chapter 3

Trade Openness, Government Size and Factor Intensities

3.1 Introduction

During the process of economic globalization and trade integration, there are many debates in the academic literature on the government's response to the increasing trade openness in the long run. However, the empirical evidence on the relationship between trade openness (the GDP fraction of trade volume) and government size (the GDP fraction of government expenditure or consumption) is mixed. In an earlier study, Cameron (1978) documents a *positive* association between these two variables. The positive relationship is also found by recent studies including Adsera and Boix (2002), Alesina and Wzcziarg (1998), Balle and Vaidya (2002), Bretschger and Hettich (2002), Epifani and Gancia (2009), Ram (2009) and Rodrik (1998). In the meanwhile, Garrett and Mitchell (2001) obtain a *negative* correlation between trade openness and government size, casting doubt on the reliability and robustness of the afore-mentioned positive association. Similar negative correlations are also found in Abezadeh (2005), Busemeyer (2009), Garen and Trask (2005), Garrett (2001), Kaufman and Segura-Ubiergo (2001), and Liberati (2007). Islam (2004) actually finds mixed relationship between these two variables for different countries. Due to the fact that empirical estimations are sensitive to the sample countries and time spans, there is still no consensus on the empirical long-run relationship between trade openness and government size.

Compared to numerous empirical studies, little theoretical work has been done to uncover the economic rationale behind trade openness and government size. To the best of my knowledge, there is no existing theoretical work that reconciles the mixed empirical regularities, as observed in the empirical literature, between trade openness and government size. The current paper attempts to fill the gap in the literature and provides a theory that explains the mixed long-run relationship between these two variables.

This work is closely related to a recent study on this topic by Epifani and Gancia (2009), who build up a static general equilibrium model and put forward a promising argument, *i.e.*, the terms of trade externality, to account for the (positive) correlation between trade openness and government size. According to this mechanism, when government expands its expenditure, provides more public goods and hires more workers, the contraction of the employment (hence output) in the private firms leads to a terms of trade improvement in the tradable sector. Therefore as openness increases, the non-cooperatively behaving government tends to take advantage of this terms of trade externality by expanding public expenditure. The current paper differs from Epifani and Gancia (2009) in several ways. First, I build a *dynamic* model and emphasize the dynamics of capital accumulations in both the tradable and non-tradable sectors, while the model setup in Epifani and Gancia (2009) is *static* and capital does not play a role. As shown later, a change of the tax rate (government size) in the model induces *different* capital accumulations in the tradable and non-tradable sectors. This change of relative capital stocks eventually feeds back to the government's decision on its size. Second, my model relaxes the assumption that domestically produced goods contribute to a negligible fraction of domestic consumers' consumption basket. Although this is a tempting assumption considering the numerous items traded across countries, there exist some categories of tradable goods (*e.g.*, automobile, gasoline, etc.) in which the consumption fraction of domestic output is not negligible.[1] Once this assumption is relaxed, when government expands its expenditure and contracts the output of domestic private firms, the resulting lower consumption of domestically produced goods counteracts households' benefits from

[1]For many countries like the US, only a small fraction of GDP is imported (Baumol and Blinder 2011, p. 23) and most consumption goods are still manufactured domestically. So the current framework should be regarded as an alternative to the one studied by Epifani and Gancia (2009).

the terms of trade improvement in the tradable sector. In this case government has no incentives to expand its expenditure and the terms of trade externality mechanism fails to generate any relationship between trade openness and government size.

In this paper, I introduce an alternative mechanism, *i.e.*, the differentiated factor intensities in the production of the tradable and non-tradable sectors, to link the steady state government size to trade openness. I base my analysis on Epifani and Gancia (2009) and relax the assumption of negligible consumption of domestically produced goods. Specifically I build up a two-country dynamic general equilibrium model with differentiated factor intensities in the tradable and non-tradable sectors. I analyze the long-run relationship between trade openness and government size, with the latter determined by a benevolent government. I show that, without the terms of trade mechanism, different factor intensities in the tradable and non-tradable sectors is an alternative driving force behind the observed mixed association of trade openness and government size. When the non-tradable (tradable) sector is more capital intensive than the tradable (non-tradable) sector, an expansionary (contractionary) government expenditure increases the steady state capital stock and output in the tradable sector relative to the non-tradable sector. It further gives rise to lower steady state prices of tradable goods relative to non-tradable goods. This transmission mechanism is essential because the households love varieties and *would like to* consume more tradables relative to non-tradables as a country's openness increases (the tradable sector expands). The households' desired consumption plan is achieved with the help of a benevolent government, who expands (contracts) its expenditure to push down the relative prices of tradable goods. So a positive (negative) long-run relationship between trade openness and government size is observed. Without the terms of trade externality, the current model is able to relate government size to openness through the government's desire to lower the steady state relative prices of tradable goods. Different factor intensities in the tradable and non-tradable sectors are the foundations of the proper functioning of this mechanism. Moreover, the capability of the current model to generate a positive/negative steady state relationship between trade openness and government size reconciles the empirical ambiguity observed in the existing literature. When countries under concern exhibit different relative factor intensities in their tradable and non-tradable sectors, the model predicts that both positive and negative correlations between openness and government size are possible.

I proceed as follows. In section 2, I introduce the model and define the competitive equilibrium. Section 3 deals with the analysis of the steady state relationship of trade openness and government size. The case with productive government spending is discussed in section 4. Section 5 makes concluding remarks.

3.2 Model

In the model setup, there exist two symmetric countries, home and foreign. Due to the symmetry across countries, I focus the analysis on the home country. In what follows, I describe the problems of firms, households and government, sequentially.

3.2.1 Firms' Problem

There is a continuum of private industries $s \in [0, 1]$ in each country. Among all industries $s \in [0, 1]$, index $i \in [0, \theta]$ denotes an industry in the tradable sector while index $j \in [\theta, 1]$ denotes an industry in the non-tradable sector. θ is a threshold value that measures trade openness. A higher θ represents a larger tradable sector and a smaller non-tradable sector, hence a higher degree of trade openness.[2] Each industry s specializes in the production of a country-specific variety. I assume that all industries in both the tradable and non-tradable sectors use labor (L) and capital (K) to produce output (y), but the capital elasticity parameters are different in the production functions of the tradable and non-tradable sectors. Labor is allowed to move freely across industries within a country. Capital, however, is industry-specific and can not be moved across industries.[3]

[2]In a static model, e.g., Epifani and Gancia (2009), the openness measure defined in this way also equals the GDP fraction of trade volume. Due to the dynamic feature introduced in this paper, the openness measure, θ, does not exactly equal the GDP fraction of trade volume.

[3]See, for example, Williamson (1985) and McGuinness (1994) for discussions on capital specificity. On top of that, the assumption of industry-specific capital also prevents the occurrence of a corner solution in the equilibrium of the model.

Within the tradable sector, I assume a Cobb-Douglas production function for industry $i \in [0, \theta]$:

$$y_{it}^{T} = \left(K_{it}^{T}\right)^{\alpha} \left(L_{it}^{T}\right)^{1-\alpha} \qquad \alpha \in (0,1) \tag{3.1}$$

The subscripts i and t denote industry i and period t; the superscript T denotes the "Tradable" sector. Profit maximizing firms and competitive factor markets imply that factors of production are paid at their values of marginal product:

$$\frac{W_t}{p_{it}^{T}} = (1-\alpha)\left(K_{it}^{T}\right)^{\alpha} \left(L_{it}^{T}\right)^{-\alpha} \tag{3.2}$$

$$\frac{R_{it}^{T}}{p_{it}^{T}} = \alpha\left(K_{it}^{T}\right)^{\alpha-1} \left(L_{it}^{T}\right)^{1-\alpha} \tag{3.3}$$

where W_t, R_{it}^{T} and p_{it}^{T} are the nominal wage rate, nominal rental rate, and nominal price of goods produced in the tradable sector, respectively. Note that the capital rental rate is also industry-specific due to the immobility of capital across industries. Within the non-tradable sector, I assume a similar Cobb-Douglas production function in industry $j \in [\theta, 1]$ with *different* factor intensities:

$$y_{jt}^{N} = \left(K_{jt}^{N}\right)^{\gamma} \left(L_{jt}^{N}\right)^{1-\gamma} \qquad \gamma \in (0,1) \tag{3.4}$$

The subscripts j and t denote industry j and period t; the superscript N denotes the "Non-tradable" sector. Competitive factor markets imply that

$$\frac{W_t}{p_{jt}^{N}} = (1-\gamma)\left(K_{jt}^{N}\right)^{\gamma} \left(L_{jt}^{N}\right)^{-\gamma} \tag{3.5}$$

$$\frac{R_{jt}^N}{p_{jt}^N} = \gamma \left(K_{jt}^N\right)^{\gamma-1} \left(L_{jt}^N\right)^{1-\gamma} \tag{3.6}$$

where W_t, R_{jt}^N and p_{jt}^N are the nominal wage rate, nominal rental rate, and nominal price of goods produced in the non-tradable sector, respectively. Due to the free labor mobility across industries, the nominal wage rate is equalized across all industries in both tradable and non-tradable sectors.

3.2.2 Households' Problem

There are L identical agents in each country and each agent is endowed with one unit of time. All agents work, accumulate capital and rent it to firms. Subject to the constraint of (after-tax) labor and capital income, each agent derives utility from the consumption of private goods, produced in all domestic sectors and in the foreign tradable sector, as well as a country-specific public good provided by the government. In particular, the private consumption bundle (C_{it}) in industry i of the tradable sector and the bundle (C_{jt}) in industry j of the non-tradable sector are defined as:

$$C_{it} = \left(\xi \left(c_{it}^T\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)\left(c_{it}^{T^*}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \qquad \sigma > 0, \xi \in (0,1), i \in [0,\theta] \tag{3.7}$$

$$C_{jt} = c_{jt}^N \qquad j \in [\theta, 1] \tag{3.8}$$

where c_{it}^T and $c_{it}^{T^*}$ represent the consumption of domestically and foreign produced tradable goods in industry $i \in [0, \theta]$, respectively; c_{jt}^N represents consumption of domestically produced non-tradable goods in industry $j \in [\theta, 1]$; σ is the elasticity of substitution across domestic and foreign varieties in industry $i \in [0, \theta]$ from the tradable sector; ξ is a distribution parameter. Consumption bundle (C_{it}) in the tradable sector includes both domestic and imported varieties while consumption bundle (C_{jt}) in the non-tradable sector includes only domestic variety.[4]

With the above definitions in (3.7) and (3.8), the agents' preference is specified by the following CRRA utility function:

$$U_t = \frac{\left[\left(\exp \int_0^1 \log C_{st} ds\right)^{\eta} G_t^{1-\eta}\right]^{1-\rho}}{1-\rho}, 0 < \eta < 1, \rho > 0, \rho \neq 1 \tag{3.9}$$

where C_{st} denotes the consumption bundle of private goods in industry $s \in [0, 1]$; G_t denotes the consumption of public good. ρ is the inverse of the intertemporal elasticity of substitution.

Given the above preference, agents maximize life-time utility by making decisions over the consumption of tradable goods (both domestic c_{it}^T and imported $c_{it}^{T^*}$) and non-tradable goods c_{jt}^N as well as the accumulation of industry-specific capital (both in the tradable sector K_{it+1}^T and non-tradable sector K_{jt+1}^N) at each period t:

$$\max_{\left\{c_{it}^T, c_{it}^{T*}, c_{jt}^N, K_{it+1}^T, K_{jt+1}^N, i \in [0,\theta], j \in [\theta, 1]\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left[\left(\exp \int_0^1 \log C_{st} ds\right)^{\eta} G_t^{1-\eta}\right]^{1-\rho}}{1-\rho} \right\}$$

subject to the period budget constraint:

[4] Similar specifications are used in Obstfeld and Rogoff (2001), Hanslin (2008), and Epifani and Gancia (2009).

$$\int_0^\theta c_{it}^T p_{it}^T di + \int_0^\theta c_{it}^{T^*} p_{it}^{T^*} di + \int_\theta^1 c_{jt}^N p_{jt}^N dj + \int_0^\theta \left(K_{it+1}^T - K_{it}^T\right) p_{it}^T di$$

$$+ \int_\theta^1 \left(K_{jt+1}^N - K_{jt}^N\right) p_{jt}^N dj \leq \left(W_t L + \int_0^\theta R_{it}^T K_{it}^T di + \int_\theta^1 R_{jt}^N K_{jt}^N dj\right)(1-\tau_t) \quad (3.10)$$

where $p_i^{T^*}$ denotes the price of foreign tradable goods. The LHS of (3.10) describes the agent's total expenditure on consumption and investment while the RHS gives the agent's after-tax labor and capital income, with $\tau_t \in (0,1)$ as the income tax rate. Without losing any insight into the model, I simplify the algebra by assuming that capital does not depreciate over time.

From the FOCs I have the following intratemporal relations

$$c_{it}^{T^*} = \left(\frac{1-\xi}{\xi}\right)^\sigma \left(\frac{p_{it}^T}{p_{it}^{T^*}}\right)^\sigma c_{it}^T \quad (3.11)$$

$$c_{jt}^N = \left(\frac{p_{it}^T}{p_{jt}^N}\right) \left[1 + \left(\frac{1-\xi}{\xi}\right)^\sigma \left(\frac{p_{it}^T}{p_{it}^{T^*}}\right)^{\sigma-1}\right] c_{it}^T \quad (3.12)$$

and two dynamic consumption Euler equations

$$\lambda_t p_{it}^T = \beta \lambda_{t+1} \left[R_{it+1}^T (1-\tau_t) + p_{it+1}^T\right] \quad (3.13)$$

$$\lambda_t p_{jt}^N = \beta \lambda_{t+1} \left[R_{jt+1}^N (1-\tau_t) + p_{jt+1}^N\right] \quad (3.14)$$

where λ_t is the Lagrangian multiplier associated with the budget constraint (3.10).

3.2.3 Government's Problem

The government's role in the model is twofold. First, the government provides a country-specific public good (G_t) according to a linear production function:

$$G_t = L_{gt} \tag{3.15}$$

where L_{gt} is the public employment. Second, government raises income tax at the rate of τ_t to finance its expenditure (wage bill of the public employment):

$$W_t L_{gt} = \tau_t \left(W_t L + \int_0^\theta R_{it}^T K_{it}^T di + \int_\theta^1 R_{jt}^N K_{jt}^N dj \right) \tag{3.16}$$

Government size is defined as the fraction of government expenditure out of total GDP, which happens to be the tax rate τ_t according to equation (3.16).

3.2.4 Competitive Equilibrium

With the above analysis of firms, households, and government, market clearing conditions need to be satisfied before I define a competitive equilibrium. The market clearing conditions include

(1) the labor market:

$$\int_0^\theta L_{it}^T di + \int_\theta^1 L_{jt}^N dj + L_{gt} = L \tag{3.17}$$

(2) the tradable goods market:

$$y_{it}^{T} = c_{it}^{T} + c_{it}^{T\prime} + K_{it+1}^{T} - K_{it}^{T}, \quad i \in [0, \theta] \tag{3.18}$$

where $c_{it}^{T\prime}$ denotes the amount of tradable goods in industry i that is exported to the foreign country; and

(3) the non-tradable goods market:

$$y_{jt}^{N} = c_{jt}^{N} + K_{jt+1}^{N} - K_{jt}^{N}, \quad j \in [\theta, 1] \tag{3.19}$$

Given the above model structure, a competitive equilibrium consists of sequences of allocations $\left\{c_{it}^{T}, c_{it}^{T^*}, c_{it}^{T\prime}, c_{jt}^{N}, L_{it}^{T}, L_{jt}^{N}, L_{gt}, K_{it+1}^{T}, K_{jt+1}^{N}, y_{it}^{T}, y_{jt}^{N}, G_t, \lambda_t\right\}_{t=0}^{\infty}$ and sequences of prices $\left\{W_t/p_{it}^{T}, W_t/p_{jt}^{N}, R_{it}^{T}/p_{it}^{T}, R_{jt}^{N}/p_{jt}^{N}\right\}_{t=0}^{\infty}$ for all industries $i \in [0, \theta]$ in the tradable sector and all industries $j \in [\theta, 1]$ in the non-tradable sector, such that

(1) Given prices $\left\{W_t/p_{it}^{T}, W_t/p_{jt}^{N}, R_{it}^{T}/p_{it}^{T}, R_{jt}^{N}/p_{jt}^{N}\right\}_{t=0}^{\infty}$, tax rate $\{\tau_t\}_{t=0}^{\infty}$ and government expenditure $\{G_t\}_{t=0}^{\infty}$, households maximize life-time utility;

(2) Given prices $\left\{W_t/p_{it}^{T}, W_t/p_{jt}^{N}, R_{it}^{T}/p_{it}^{T}, R_{jt}^{N}/p_{jt}^{N}\right\}_{t=0}^{\infty}$, tax rate $\{\tau_t\}_{t=0}^{\infty}$, firms in all industries maximize profits;

(3) Government hires labor to produce public goods and balances its budget in each period;

(4) All markets clear;

(5) Balanced trade holds due to the symmetry across countries:

$$\int_0^{\theta} p_{it}^{T} c_{it}^{T\prime} di = \int_0^{\theta} p_{it}^{T*} c_{it}^{T*} di; \text{ and} \tag{3.20}$$

(6) Prices of tradable goods are equalized across countries due to symmetry:

$$p_{it}^{T} = p_{it}^{T*}, i \in [0, \theta]. \tag{3.21}$$

The competitive equilibrium is characterize by equation (3.1) to (3.6) and equation (3.11) to (3.21).

3.3 Steady State Analysis

Based on the competitive equilibrium defined in last section, I now solve the *unique* steady state of the model. All variables without time subscript denote steady state variables. Below is part of the steady state expressions.

$$\begin{aligned}
L_i^T &= \frac{(1-\alpha)(1-\tau)L}{\theta(1-\alpha+\alpha\tau)+(1-\theta)(1-\gamma+\gamma\tau)} \\
L_j^N &= \frac{(1-\gamma)(1-\tau)L}{\theta(1-\alpha+\alpha\tau)+(1-\theta)(1-\gamma+\gamma\tau)} \\
L_g &= \frac{\tau L}{\theta(1-\alpha+\alpha\tau)+(1-\theta)(1-\gamma+\gamma\tau)} \\
\frac{W}{p_i^T} &= (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{\beta}-1\right)^{-\frac{\alpha}{1-\alpha}}(1-\tau)^{\frac{\alpha}{1-\alpha}} \\
c_i^T &= \left[1+\left(\frac{1-\xi}{\xi}\right)^{\sigma}\right]^{-1}\frac{W}{p_i^T}\frac{L_i^T}{1-\alpha} \\
c_j^N &= \frac{1-\gamma}{1-\alpha}\gamma^{\frac{\gamma}{1-\gamma}}\left(\frac{1}{\beta}-1\right)^{-\frac{\gamma}{1-\gamma}}(1-\tau)^{\frac{\gamma}{1-\gamma}}L_i^T
\end{aligned} \tag{3.22}$$

$$\begin{aligned} G &= L_g \\ C_i &= \left(1+\left(\frac{1-\xi}{\xi}\right)^{\sigma}\right)^{\frac{\sigma}{\sigma-1}} \xi^{\frac{\sigma}{\sigma-1}} c_i^T \\ C_j &= c_j^N \end{aligned}$$

The derivation of the model's steady state assumes the symmetry across all industries in the tradable sector (and similar conditions across all industries in the non-tradable sector). Due to these symmetry conditions, the utility function (3.9), in steady state, reduces to

$$\begin{aligned} U &= \frac{\left[\left(\exp \int_0^1 \log C_s ds\right)^{\eta} G^{1-\eta}\right]^{1-\rho}}{1-\rho} \\ &= \frac{\left[\left(C_i^{\theta} C_j^{1-\theta}\right)^{\eta} G^{1-\eta}\right]^{1-\rho}}{1-\rho} \end{aligned} \tag{3.23}$$

For a given level of openness, θ, with the steady state expressions of G, C_i, and C_j in equation (3.22), a benevolent government maximizes (3.23) by choosing a utility-maximizing long-run government size τ, which is also the steady state value of the income tax rate. The first order condition of the above maximization problem characterizes the relationship between openness and government size:[5]

$$\begin{aligned} F(\theta,\tau) &\equiv \frac{\partial \log U}{\partial \tau} = \frac{1}{U}\frac{\partial U}{\partial \tau} \\ &= \frac{\theta(\alpha-\gamma)+\gamma-\alpha\gamma}{(1-\alpha)(1-\gamma)}\frac{-\eta}{1-\tau} - \frac{\eta}{1-\tau} + \frac{1-\eta}{\tau} \end{aligned}$$

[5]The second order condition is also examined to guarantee that the maximum of the utility function is achieved. Please refer to the appendix for corresponding discussions.

$$
\begin{aligned}
& -\frac{\gamma - \theta(\gamma - \alpha)}{1 - \gamma + \gamma t + \theta(\gamma - \alpha)(1 - \tau)} \\
= \; & 0
\end{aligned}
$$

The following proposition describes the steady state relationship of openness and government size.

Proposition

Given the model described in previous sections with $0 < \alpha, \gamma, \eta < 1$, I have

$$
sign\left(\frac{\partial \tau}{\partial \theta}\right) = sign\,(\gamma - \alpha)
$$

for any $\tau \in (0, 1)$ and any $\theta \in (0, 1)$ if $max\,\{\alpha, \gamma\} < \sqrt{1 - \eta}$.

Proof: See the appendix.

Note that the above proposition introduces a sufficient condition ($i.e.$, $max\,\{\alpha, \gamma\} < \sqrt{1 - \eta}$) for the correlation of openness and government size to be determined by the relative factor intensities in the tradable and non-tradable sectors. When the non-tradable (tradable) sector is more capital intensive, government size moves in the same (opposite) direction with trade openness. The validity of the condition, $max\,\{\alpha, \gamma\} < \sqrt{1 - \eta}$, depends on the relative size of α, γ and η. Given the empirically plausible range of $\eta \in (0.64, 0.75)$ (Ni 1995), this condition is easily satisfied. Therefore the above condition does not seem to be very restrictive.

How do we understand the economic rationale behind the above proposition? First of all, government could impact the relative steady state prices of tradables to non-tradables, p_i^T/p_j^N, by changing the tax rate τ. To see how this works, let us consider a country with a relatively capital intensive non-tradable sector (*i.e.*, $\alpha < \gamma$). When the income tax rate increases, the effective rate of return on capital becomes lower and capital accumulation drops in both the tradable and non-tradable sectors. Since capital is industry-specific and the non-tradable sector has a larger capital income share, the drop of capital stock will be larger in the non-tradable sector than that in the tradable sector. Hence a higher income tax rate leads to a *relative* increase in K_i^T/K_j^N.[6] Meanwhile, because of the complementarity between labor and capital, lower capital accumulation discourages employment in both the tradable and non-tradable sectors. The steady state distribution of private employment across industries L_i^T/L_j^N, however, is constant due to the free labor mobility and the resulting nominal wage equalization across all industries.[7] A constant labor ratio (L_i^T/L_j^N) and a higher capital ratio (K_i^T/K_j^N) give rise to higher relative supplies (y_i^T/y_j^N) and hence lower relative prices (p_i^T/p_j^N) of tradable goods against non-tradable goods.[8] To the contrary, if the tradable sector is relatively capital intensive (*i.e.*, $\alpha > \gamma$), a higher tax

[6]In steady state, K_i^T/K_j^T is proportional to $(1-\tau)^{\frac{(\alpha-\gamma)}{(1-\alpha)(1-\gamma)}}$. A higher τ leads to a higher K_i^T/K_j^T when $\alpha < \gamma$.

[7]With the free labor mobility across industries, the labor distribution across industries is determined by $L_i^T/L_j^N = (1-\alpha)/(1-\gamma)$ in steady state. The employment distribution across the tradable and non-tradable sectors is given by $\theta(1-\alpha)/(1-\theta)/(1-\gamma)$.

[8]In fact, due to the lower capital accumulation, labor become less productive in all sectors. However, a larger drop of K_j^N than K_i^T leads to a relatively higher K_i^T/K_j^N and a relatively higher MPL_i^T/MPL_j^N, *i.e.*, the relative labor productivity is higher in the tradable sector than in the non-tradable sector. Therefore the relative output in the tradable sector increases and drives down its relative price. The higher MPL_i^T/MPL_j^N and lower p_t^T/p_j^N together generate the equalized nominal values of marginal product of labor across sectors, giving rise to the constancy of L_i^T/L_j^N in steady state equilibrium.

rate will lead to a relative lower K_i^T/K_j^N. The steady state output ratio y_i^T/y_j^N will be lower, driving up the relative prices. To summarize, a higher tax rate (government size) gives rise to lower (higher) steady state relative prices of tradables vs. non-tradables when the non-tradable (tradable) sector is relatively capital intensive.

When the openness measure θ increases, more industries become tradable. Due to the households' "love of variety" preference in (3.7), the marginal utility of tradable goods relative to non-tradable goods increases.[9] Therefore households *would like to* consume more tradable goods relative to non-tradable goods, which *could be* realized through the lower relative prices of tradable goods. According to the mechanism above, when the non-tradable (tradable) sector is more capital intensive, a benevolent government will increase (decrease) tax rate τ to push down the relative price of tradable goods, achieving the households' desired consumption plan. The government's choice of the tax rate eventually leads to a positive (negative) association between trade openness and government size in the steady state in the presence of a relatively capital-intensive non-tradable (tradable) sector.

What would happen when $\alpha = \gamma$, *i.e.*, the tradable and non-tradable sectors have the same factor intensities? It turns out that government size in this case does not depend on the openness measure. The intuition is as follows. When the tradable and non-tradable sectors share the same factor intensities, changing tax rates could not change the distribution of private employment or capital stocks, where

[9] This is clear according to equation (3.23).

$L_i^T/L_j^N = K_i^T/K_j^N = 1$ in the steady state. Tax rate has no impact on the relative prices of tradables against non-tradables. Therefore government has no incentives to change tax rate as openness changes.

A special case occurs when $\alpha = \gamma = 0$, *i.e.*, capital is not accumulated in the tradable or non-tradable sector. Based on the analysis above, one may conjecture that government size is independent of the openness measure. This turns out to be true. Moreover, since capital is not accumulated, the model reduces to the static case discussed in Epifani and Gancia (2009), with the difference that domestically produced goods now contribute to a non-negligible fraction of the households' consumption bundle. In this case, the tax rate chosen by a benevolent government is not affected by openness. The terms of trade mechanism in Epifani and Gancia (2009) still works but, when a higher tax rate reduces domestic supply of tradable goods and improves the terms of trade in the tradable sector, domestic households get hurt simultaneously because of their lower consumption of these domestically produced goods. The benefits of the terms of trade improvement in the tradable sector is counteracted by the lower consumption of domestic output. Therefore when openness changes, government has no incentives to change the tax rate. Hence the terms of trade externality mechanism does not generate any correlation between openness and government size when both the tradable and non-tradable sectors share the same factor intensities.

Several points need to be further analyzed. First, as shown in the proposition (as well as equation (B.2), (B.3), and (B.5) in the appendix), the relationship between openness and government size $\left(\frac{\partial \tau}{\partial \theta}\right)$ is completely determined by α, γ, and η, independent of σ, the elasticity of substitution across varieties. This is different from Epifani and Gancia (2009) in which the magnitude of σ matters. The difference results from the removal of the assumption on the households' negligible consumption of domestically produced goods. In other words, lower consumption of domestic goods offsets the impact of terms of trade improvement in the current model. I instead introduce an alternative channel, the differentiated factor intensities, to relate government expenditure to openness, which does not depend on the elasticity parameter σ. Second, according to this model, an increasing (decreasing) government size should be welcome in the presence of increasing openness if the non-tradable (tradable) sector is relatively capital-intensive. A benevolent government chooses government size to maximize the households' utility without considering any externality. This is also different from the terms of trade externality argument, in which government expands its expenditure in order to take advantage of the externality, leading to an over-sized government in the process of globalization.

3.4 Productive Government Expenditure

When I introduced the public sector in section 2, I focused on the utility-generating public expenditure. It turns out that the main results reported in section 3 do not change qualitatively when the utility-generating government expenditure is replaced by a productive government expenditure:

$$U_t = \frac{\left(\exp \int_0^1 \log C_{st} ds\right)^{1-\rho}}{1-\rho}, \rho > 0, \rho \neq 1 \quad (3.24)$$

$$y_{it}^T = \left(K_{it}^T\right)^{\alpha} \left(L_{it}^T\right)^{1-\alpha} G_t^{\chi} \qquad \alpha, \chi \in (0,1), \alpha + \chi < 1 \quad (3.25)$$

$$y_{jt}^N = \left(K_{jt}^N\right)^{\gamma} \left(L_{jt}^N\right)^{1-\gamma} G_t^{\chi} \qquad \gamma, \chi \in (0,1), \gamma + \chi < 1 \quad (3.26)$$

In equation (3.24) preferences still take the CRRA formulation but the government expenditure G_t, taken as given by the firms, enters into the production functions (3.25) and (3.26) in the tradable and non-tradable sectors. Based on these specifications, I define the competitive equilibrium and solve the corresponding steady state of the model. The steady state relationship between openness and government size turns out to be affected by the relative factor intensities in a similar fashion to the case of utility-generating government expenditure. The results are qualitatively similar even though the sufficient condition needed for $sign\left(\frac{\partial\tau}{\partial\theta}\right) = sign\left(\gamma - \alpha\right)$ is much more involved.[10] In particular, given the specifications in equation (3.24) to (3.26) with

[10] The proof for this sufficient condition is available from the author upon request.

$0 < \alpha, \gamma, \chi < 1$, I have

$$sign\left(\frac{\partial \tau}{\partial \theta}\right) = sign\left(\gamma - \alpha\right)$$

for any $\tau \in (\underline{\tau}, 1)$ and any $\theta \in (0, 1)$ if $\chi - M\sqrt{\chi} + N < 0$, where

$$\begin{aligned} M &= \min\left\{\frac{4(1-\alpha)^2}{\alpha(1-\gamma)}, \frac{4(1-\gamma)^2}{\gamma(1-\alpha)}\right\} \\ N &= \min\left\{1-\alpha, 1-\gamma\right\} \end{aligned}$$

$$\begin{aligned} \underline{\tau} &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \text{ if } A \neq 0 \text{ and, otherwise} \\ \underline{\tau} &= \max\left\{\begin{array}{c} \frac{\chi(1-\gamma)}{(1-\gamma)+(1-\alpha)(1-\gamma)+\chi(2-\alpha-\gamma)}, \\ \frac{\chi(1-\alpha)}{(1-\alpha)+(1-\alpha)(1-\gamma)+\chi(2-\alpha-\gamma)} \end{array}\right\} \end{aligned}$$

where

$$\begin{aligned} A &= \min\left\{\begin{array}{c} \gamma + \chi\gamma - (1-\alpha)(1-\gamma) - \chi(1-\alpha+\gamma), \\ \alpha + \chi\alpha - (1-\alpha)(1-\gamma) - \chi(1-\gamma+\alpha) \end{array}\right\} \\ B &= \max\left\{\begin{array}{c} 1-\gamma + \chi(2-\alpha-\gamma) + (1-\alpha)(1-\gamma), \\ 1-\alpha + \chi(2-\alpha-\gamma) + (1-\alpha)(1-\gamma) \end{array}\right\} \\ C &= \min\left\{-\chi(1-\gamma), -\chi(1-\alpha)\right\} \end{aligned}$$

The fact that the steady state relationship of openness and government size in the presence of the productive government spending is also affected by the relative factor intensities in the tradable and non-tradable sectors is not surprising if we recall how government size is linked to openness. Regardless of the utility-generating or productive government expenditure, the government responds to a higher openness by changing tax rates to lower the relative price of tradable vs. non-tradable goods, with the hope that this will increase the households' relative consumption of tradables. This mechanism works through the reallocation of employment between the public and private sectors and through the adjustment of the relative steady state capital stocks in the tradable and non-tradable sectors. These changes are then channelled to the relative supplies and the relative prices of tradable vs. non-tradable goods. The proper functioning of this mechanism does not depend on how the government expenditure is spent, which explains why I observe similar results with different specifications of useful government expenditure.

3.5 Concluding Remarks

Globalization increasingly facilitates the bilateral and multilateral connections across countries and integrates world market closely. In the context of globalization, the debate on the role of government is attracting more attention. Given the mixed long-run relationship of trade openness and government size found in the empirical literature, this paper argues that the way a benevolent government responds to trade openness may be impacted by the differentiated factor intensities in the production of the tradable and non-tradable sectors.

In particular, if the non-tradable (tradable) sector is more capital intensive, expansionary (contractionary) government spending drives down the relative prices of tradable vs. non-tradable goods. When trade openness increases and households *would like to* consume more tradables compared to non-tradables, a benevolent government responds by expanding (contracting) its spending to lower the relative prices of tradable goods. Therefore a positive (negative) relationship of government size and openness follows.

The mechanism proposed in this paper is an alternative to the existing theory (*e.g.*, the terms of trade externality), which accounts for the observed mixed long-run relationship between openness and government size. A direct measure of factor intensities in the tradable and non-tradable sectors will be informative in evaluating the validity and relevance of the theory in this paper. Given the framework of this paper, many factors have been left undiscussed. One important extension is to incorporate the financial openness. Globalization brings about not only the goods market integration but also the financial interdependence. It is promising to bring the financial openness into the current framework and explore the behavior of government. A second extension may come from the provision of public goods. In the current framework, following the literature, I assume a linear production function in the public sector. However, the accumulation and utilization of the public capital may potentially change the propagation mechanism. The government's response to the globalization in the presence of public capital offers another interesting research direction.

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Chapter 4

Factor Substitution and Labor Market Friction in the U.S.: 1948-2012

4.1 Introduction

Estimating the elasticity of substitution between capital and labor has been the central theme of numerous studies ever since the constant elasticity of substitution (CES) production function is introduced into economics by the seminal work of Solow (1956) and formally derived in Arrow et al. (1961). The long-lasting research interest in this factor substitution elasticity arises from its critical role in various areas. For instance, in the neoclassical growth theory, the magnitude of elasticity of substitution is essential to perpetual economic growth (Antras, 2004; Barro and Sala-i-Martin, 1995) and direction of biased technical progress (Acemoglu, 2002); it affects the economic production level as well as the growth rate (de la Grandville, 1989; Klump and Preissler, 2000). In business cycle literature, the elasticity parameter affects the generation of dynamic indeterminacy and business cycle propagation mechanism (Guo and Lansing, 2009) and the occurrence of steady state indeterminacy and the monotonic relationship between asset prices and unemployment rate (Farmer, 2013). In public finance, factor substitutability constitutes an important determinant of the response of investment behavior to tax policy (Chirinko, 2002). Even for government policy, elasticity of substitution generates direct influence on policy forecasting and policy analysis (Miller, 2008).

Despite the consensus on the importance of the elasticity of substitution between capital and labor, there is much less consensus on its value. Moreover, recent studies call for attention on the interplay between the factor-augmenting technical progress and factor substitutability. Antras (2004) shows that the assumption of Hicks neutrality, despite its popularity in the existing literature, biases the estimation of the elasticity parameter upwards. Acemoglu (2002; 2003) and Jones (2003) analyze the impacts of the directions of technical progress on economic growth in the short and long run. As pointed out by Chirinko (2008), there tends to be the tension between the long-run model and the short run data in most of studies. Based on these studies, the first goal of this paper amounts to the *joint* estimation of the factor substitutability and the *biased* factor-augmenting technical progress. In particular, given that most of the existing studies focus on a specific sample and deliver estimates specific to the sample period chosen[1], this paper moves one step further by examining the possible variations of model parameters across different sample periods. On the one hand, this approach provides an analysis on the tension between the long-run model and the short run data; on the other hand, the revealed sensitivity of model parameters to different sample periods may constitute an important source giving rise to the empirical ambiguity on parameter estimations.[2]

Another salient feature of this paper is its joint modelling and estimation of the aggregate production and labor market friction (search and matching). In a series of articles, Farmer (2012, 2013a, 2013b, 2013c) and Plotnikov (2013) study the existence of multiple steady state equilibria in an otherwise standard dynamic stochastic general equilibrium (DSGE) model augmented with labor search and matching. They argue that the induced occurrence of multiple steady state equilibria explains recent Great Recession, stock market crash, and persistent unemployment and attribute these drastic cyclical fluctuations of aggregate economy to a collective change of the self-fulfilling belief. Since unemployment is an equilibrium phenomenon and a change of

[1]One exception is Oberfield and Raval (2012) who discuss the time variations of estimations but focus on cross-sectional estimation at each time point; this paper resort to a recursive scheme so that I could explore how sensitive the estimations are to sample periods.

[2]In this paper, I focus my attention on the *constant* elasticity of substitution production function and explore the sensitivity of model parameters to different sample periods. This approach is different from the time-varying coefficient model. Please refer to Sato (1970), Barro and Sala-i-Martin (2003), and Pereira (2003) for an example of time-varying elasticity of substitution.

unemployment has long-lasting impacts on the economy, labor market friction may affect economic growth in the medium and long run as well. In particular, to the best of my knowledge, how labor market fluctuations affects factor substitutability and technical progress has not been subject to any empirical examination. This paper incorporates labor search and matching into a neoclassical growth model to jointly estimate and evaluate the interactions of aggregate production and labor market friction.

Methodologically, this paper extends the *system* approach proposed by Klmup et al. (2007) to incorporate labor market friction. The current supply-side system includes the production function as well as two first order conditions (FOCs) with respect to labor and capital and estimates factor substitution, biased technical progress, and labor market friction simultaneously. Klmup et al. (2007) and Leon-Ledesma et al. (2010) show that (1) the system approach has significant superiority compared with the conventional one-equation or two-equation approach in terms of parameter identifications; (2) this superiority is reinforced in the presence of the "normalization" process that initially proposed by de la Grandville (1989) and then adopted in Klump and de la Grandville (2000) and Klump and Preissler (2000).[3] With normalization, model parameters have clear theoretical and empirical interpretations, which facilitates parameter identification and estimation. In addition, a two-step

[3]The normalization process arises from the observation that the same family of CES production functions differentiated solely by the elasticity of substitution should have the same baseline values of input, output, and factor income shares. See section 2 for a detailed discussion on normalization.

semi-parametric estimation strategy is adopted to handle the additional non-linearity and complex brought by the labor market friction.

Using the data of aggregate U.S. economy (1948-2012) on a recursive scheme, this paper shows that the estimates of factor substitutability and factor-augmenting technical progress change with sample periods. Within the same theoretical framework, resorting to different sample periods delivers different estimation results. The tension between the long-run model and the short-run data gains some support. Adoption of different sample periods can be one reason for the empirical ambiguity of parameter estimates.

More importantly, it is shown that the labor market friction indeed has non-negligible impacts on the production system. Augmented with labor search and matching, the system fits the data better in the statistical sense and the impact of labor market friction is statistically significant in most of the sample periods. With labor market friction, the elasticity parameter does not statistically significantly differ from unity for most of the sample periods. In the long run, the technical progress tends to be purely labor-augmenting, although capital-augmenting technical progress may arise in the short run. These findings provide support for the specification of the Cobb-Douglas production function and shed light on the role of labor market friction on economic growth.

The remaining part of this paper is structured as follows. Section 2 introduces the supply-side system and incorporates labor market friction into the system. Section 3 discusses the estimation strategy and section 4 reports the estimation results. Section 5 concludes.

4.2 Theory

4.2.1 A Supply-Side System

Recently Klump et al. (2007) and Leon-Ledesma et al. (2010) have shown how an equation *system* facilitates the identification and estimation of structural model parameters. Using the production function together with two first order conditions with respect to capital and labor, these studies construct a supply-side system to estimate biased technical progress and elasticity of substitution between capital and labor simultaneously. In this section, I briefly introduce the construction of this equation system.

Assume that the aggregate U.S. economy is described by a CES production function of the form

$$Y_t = \left[\left(E_t^N N_t\right)^{-\rho} + \left(E_t^K K_t\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{4.1}$$

where aggregate output Y_t is produced by labor service N_t and capital service K_t; these two factors of production are augmented by technology E_t^N and E_t^K, respectively. $\rho > -1$ is a parameter such that the elasticity of substitution between capital and labor is $\sigma = \frac{1}{1+\rho}$. When $\rho = 0$, the elasticity equals unity and the CES production function reduces to the Cobb-Douglas form. The factor-augmenting technical progress is described by the following equations

$$E_t^N = E_{t_0}^N e^{g_N(t_0,t)} \tag{4.2}$$

$$E_t^K = E_{t_0}^K e^{g_K(t_0,t)} \tag{4.3}$$

$$g_N(t_0,t) = \frac{t_0 \gamma_N}{\lambda_N} \left(\left(\frac{t}{t_0} \right)^{\lambda_N} - 1 \right) \tag{4.4}$$

$$g_K(t_0,t) = \frac{t_0 \gamma_K}{\lambda_K} \left(\left(\frac{t}{t_0} \right)^{\lambda_K} - 1 \right) \tag{4.5}$$

where $E_{t_0}^N$ and $E_{t_0}^K$ are the levels of factor-augmenting technical efficiency at the reference time t_0; $g_N(t_0,t)$ and $g_K(t_0,t)$ characterize the growth of *biased* factor-augmenting technology at time t. Both labor- and capital-augmenting technical progress are characterized by the flexible Box-Cox (1964) transformations: γ_N and γ_K represent the growth rates of labor- and capital- augmenting technical progress at time t_0; λ_N and λ_K control the shape of the time-evolution of the technical progress. For example, if λ_N (λ_K) is equal to unity, the growth rate of labor- (capital-) augmenting technology is linear in time; if λ_N (λ_K) equals zero, the growth rate is logarithmic in time; the growth rate is hyperbolic (exponential) if λ_N and λ_K are negative (positive).

In light of de la Grandville (1989), Klump and de la Grandville (2000), and K-lump and Preissler (2000), a normalization process is introduced into the production function. The normalization is based on the observation that a family of CES production functions, whose members are distinguished only by the different elasticities of substitution, should have a common baseline point. This implies that a family of CES production functions should share the same baseline values for factor income shares. Specifically, normalization of the CES production function in equation (4.1) requires the constant factor income shares at the reference time point t_0, regardless of the value of elasticity of substitution

$$1-\pi_0 = \frac{w_{t_0}N_{t_0}}{w_{t_0}N_{t_0}+q_{t_0}K_{t_0}} \quad (4.6)$$

$$\pi_0 = \frac{q_{t_0}K_{t_0}}{w_{t_0}N_{t_0}+q_{t_0}K_{t_0}} \quad (4.7)$$

where π_0 and $1-\pi_0$ are the capital and labor income share at t_0; w and q represent the nominal wage rate and capital rental rate, respectively. On the one hand, normalization allows the elasticity parameter to be independent of the distribution parameters and facilitates the parameter identification from the data. On the other hand, normalization brings clear theoretical interpretations and well-defined range to each parameter.[4] Using the firm's profit maximization conditions

$$\frac{w_t}{p_t} = MPN_t \quad (4.8)$$

$$\frac{q_t}{p_t} = MPK_t \quad (4.9)$$

and the production function (4.1), the following conditions are derived from (4.6) and (4.7)

$$E^N_{t_0} = \left(\frac{1}{1-\pi_0}\right)^{\frac{1}{\rho}}\frac{Y_{t_0}}{N_{t_0}} \quad (4.10)$$

$$E^K_{t_0} = \left(\frac{1}{\pi_0}\right)^{\frac{1}{\rho}}\frac{Y_{t_0}}{K_{t_0}} \quad (4.11)$$

[4]See Klump et al. (2007) for more discussions on normalization in the supply-side system.

Klump et al. (2007) suggest that the baseline values (hence the choice of t_0) should be the sample geometric averages since cyclical variations have netted out over time. However, the choice of geometric averages introduces a scale problem as those geometric averages of times series are calculated separately and may not satisfy the original production function. So a scale parameter A is introduced into the system such that $Y_{t_0} = A\overline{Y}, N_{t_0} = \overline{N}$, $K_{t_0} = \overline{K}$, where all bar variables denote the geometric averages except $t_0 = \overline{t}$ is the simple average of time periods. With those expressions in equation (4.2) to (4.7), I rewrite the FOCs (4.8) and (4.9) as well as the production function (4.1) to obtain the following normalized supply-side system (*system one*)[5]

$$\log \frac{w_t N_t}{p_t Y_t} = \log(1-\pi) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{N_t/\overline{N}}\right) - \log A - g_N(t_0,t)\right] \quad (4.12)$$

$$\log \frac{q_t K_t}{p_t Y_t} = \log(\pi) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{K_t/\overline{K}}\right) - \log A - g_K(t_0,t)\right] \quad (4.13)$$

$$\begin{aligned} \log \frac{Y_t}{N_t} = {} & \log\left(\frac{A\overline{Y}}{\overline{N}}\right) + g_N(t_0,t) \\ & - \frac{\sigma}{1-\sigma}\log\left[\overline{\pi} e^{\frac{1-\sigma}{\sigma}[g_N(t_0,t)-g_K(t_0,t)]}\left(\frac{K_t/\overline{K}}{N_t/\overline{N}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\overline{\pi})\right] \end{aligned} \quad (4.14)$$

4.2.2 Labor Market Friction

Labor market friction is introduced in this subsection a la the recent work of Farmer (2012, 2013a, 2013b, 2013c) and Plotnikov (2013) to explore its impacts on

[5]This is the system derived in Klump et al. (2007) except that I assume away the markup term. In the aggregate economy, total income consists of only labor income and capital income. Business profits are included in the capital income.

factor-augmenting technical progress and factors substitution elasticity. The aggregate economy is characterized by the following CES production function

$$Y_t = \left[\left(E_t^N X_t\right)^{-\rho} + \left(E_t^K K_t\right)^{-\rho}\right]^{-\frac{1}{\rho}} \quad \rho \geq -1 \tag{4.15}$$

$$X_t = N_t - V_t \tag{4.16}$$

where output Y_t is produced by capital K_t and labor X_t. Total workers N_t are divided and assigned into two departments: X_t represents the amount of workers assigned to the producing department and V_t represents the amount of workers assigned to the recruiting department. In the simplest case, the firm's recruiting process is described by

$$N_t = \lambda_t V_t \qquad \lambda_t > 0 \tag{4.17}$$

where λ_t is the recruiting efficiency of the workers in the personnel department and $1/\lambda_t$ measures the recruiting cost. During the recruiting process, firms attract as many applications in the beginning as possible and then set up a screening process to choose those workers that fit better. Within each time period, the firms decide the total labor demand N_t by choosing the size of the recruiting department V_t and taking the recruiting efficiency λ_t as given. According to equation (4.16) and (4.17)

$$X_t = N_t - V_t = \left(1 - \frac{1}{\lambda_t}\right) N_t \equiv \Theta_t N_t \tag{4.18}$$

$$\Theta_t \equiv 1 - \frac{1}{\lambda_t} \tag{4.19}$$

Θ_t is defined as a recruiting externality that is taken as given by the firms. Insert equation (4.18) into (4.15) and the production function augmented with recruiting externality Θ_t becomes

$$Y_t = \left[\left(E_t^N \Theta_t N_t\right)^{-\rho} + \left(E_t^K K_t\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{4.20}$$

Following Farmer (2012), I assume that the number of employed workers is determined by the following matching technology in a symmetric equilibrium[6]

$$N_t = H_t^{1-\theta}(V_t \Gamma)^{\theta} \tag{4.21}$$

where H_t is the labor force. θ is the search elasticity associated with the recruiting department while Γ is a scale parameter. The recruiting externality Θ_t, by combining (4.17), (4.19) and (4.21), is derived as follows

$$V_t = \frac{1}{\Gamma} N_t^{\frac{1}{\theta}} H_t^{1-\frac{1}{\theta}} \tag{4.22}$$

$$\lambda_t = \Gamma \left(\frac{N_t}{H_t}\right)^{1-\frac{1}{\theta}} \tag{4.23}$$

$$\Theta_t = 1 - \frac{1}{\Gamma}\left(\frac{N_t}{H_t}\right)^{\frac{1}{\theta}-1} \tag{4.24}$$

With the same specifications of factor-augmenting technical progress in equation (4.2) to (4.5), the normalized supply-side system with labor market friction is derived in a similar fashion (*system two*)

$$\log \frac{w_t N_t}{p_t Y_t} = \log(1-\overline{\pi}) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{N_t/\overline{N}}\right) - \log A - g_N(t_0, t)\right]$$

[6]Following Farmer (2012), I assume that each period the firms fire and rehire all workers. Hence the whole labor force H_t is searching for jobs in each period and V_t represent the size of recruiting department. This assumption simplifies significantly the algebra; otherwise equation (4.17) becomes a dynamic equation because labor N_t has to be treated as a state variable as in a standard search model.

$$+\frac{\sigma-1}{\sigma}\log\left(\frac{\Theta_t}{\Theta_{t_0}}\right) \tag{4.25}$$

$$\log\frac{q_tK_t}{p_tY_t} = \log(\overline{\pi}) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{K_t/\overline{K}}\right) - \log A - g_K(t_0,t)\right] \tag{4.26}$$

$$\begin{aligned}\log\frac{Y_t}{N_t} &= \log\left(\frac{A\overline{Y}}{\overline{N}}\right) + g_N(t_0,t) \\ &\quad -\frac{\sigma}{1-\sigma}\log\left[\begin{array}{c}\overline{\pi}e^{\frac{1-\sigma}{\sigma}[g_N(t_0,t)-g_K(t_0,t)]}\left(\frac{K_t/\overline{K}}{N_t/\overline{N}}\right)^{\frac{\sigma-1}{\sigma}} \\ +(1-\overline{\pi})\exp\left[\frac{\sigma-1}{\sigma}\log\left(\frac{\Theta_t}{\Theta_{t_0}}\right)\right]\end{array}\right]\end{aligned} \tag{4.27}$$

Comparing system two (4.25) to (4.27) with the original system one (4.12) to (4.14), I show that the recruiting externality term enters the first and the third equations. The labor market friction has a direct impact on the labor income share and an indirect impact on the capital income share. In addition, the term $\frac{\sigma-1}{\sigma}\log(\Theta_t/\Theta_{t_0})$, left out in system one, enters equation (4.25) in an additive fashion, which facilitates the estimation strategy discussed in the next section.

4.3 Estimation Strategy

In the estimation process, both systems are estimated by the Full-Information Maximum Likelihood (FIML) method. Due to the high non-linearity of both systems, I experiment carefully with different initial values through grid search to numerically reach the global maximum of the log likelihood. What remains to do is estimating system one (4.12) to (4.14) and system two (4.25) to (4.27) separately and then make a careful comparison. A direct estimation of system one is immediate. A direct estimation of system two, however, is not immediate due to the additional nonlinearity in labor market friction Θ_t/Θ_{t_0}. In particular, the parameters Γ and θ could not be identified unless some strong assumptions are imposed.[7] To circumvent this identification problem, I resort to the following two-step semi-parametric procedure. In the first step, I attempt to obtain a proxy for the whole externality term $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ without estimating Γ or θ explicitly. Then in the second step, I estimate system two with labor market friction by replacing the externality term $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ with the proxy obtained in the first step.[8]

How to obtain a proxy for the externality term $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ in the first step? As mentioned in last section, the externality term $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ is left out in equation (4.12) of system one and therefore becomes part of the random error in that equation. Given this observation, the strategy amounts to estimating the system one (4.12) to (4.14) first and retrieving the residual $\widehat{u}_t$ from equation (4.12). The residual $\widehat{u}_t$, in principle, reveals the variations of the labor market friction term $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ and other possible noise. Since $\frac{\sigma-1}{\sigma}\log\left(\Theta_t/\Theta_{t_0}\right)$ is a nonlinear func-

[7]This identification problem is theoretically foreseen in Farmer (2013a), in which most variables, including the recruiting and externality variables are linear in employment in a continuum of steady state equilibrium within the empirically plausible range of unemployment rate in the U.S. history.

[8]Since Θ_{t_0} is a constant and the logarithmic transformation is monotonic, Θ_t/Θ_{t_0} and $\log(\Theta_t/\Theta_{t_0})$ describe the same time patter of the externality as Θ_t does. In the theoretical derivation, I define the recruiting externality as Θ_t. In the following estimations, externality term refers to monotonic transformations of Θ_t.

tion of employment rate N_t/H_t according to equation (4.24), what follows next is a non-parametric regression: $\widehat{u}_t = m(N_t/H_t) + v_t$, where v_t is a well-defined random error. The fitted value $\widehat{m}(N_t/H_t)$ is a natural proxy for the labor market friction term $\frac{\sigma-1}{\sigma}\log(\Theta_t/\Theta_{t_0})$, which is left in the residual of equation (4.12) and only accounted for by the variations of employment rate (N_t/H_t). Using the non-parametric estimation has two additional advantages. First, I do not worry about the joint identification of θ and Γ in the current framework; the only thing that matters is the fitted value of the externality term $\widehat{m}(N_t/H_t)$. Second, the non-parametric estimation in principle allows for the potential time variations of θ and Γ, on top of the time-varying employment rate N_t/H_t. Even when θ and Γ are not time-varying, the non-parametric estimation will not deliver a worse estimation compared to its parametric counterpart.[9]

With the proxy $\widehat{m}(N_t/H_t)$ replacing the externality term $\frac{\sigma-1}{\sigma}\log(\Theta_t/\Theta_{t_0})$, system two becomes:

$$\begin{aligned}
\log\frac{w_tN_t}{p_tY_t} &= \log(1-\overline{\pi}) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{N_t/\overline{N}}\right) - \log A - g_N(t_0,t)\right] \\
&\quad + c\cdot\widehat{m}(N_t/H_t) \qquad (4.28) \\
\log\frac{q_tK_t}{p_tY_t} &= \log(\overline{\pi}) + \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\overline{Y}}{K_t/\overline{K}}\right) - \log A - g_K(t_0,t)\right] \qquad (4.29) \\
\log\frac{Y_t}{N_t} &= \log\left(\frac{A\overline{Y}}{\overline{N}}\right) + g_N(t_0,t) \\
&\quad - \frac{\sigma}{1-\sigma}\log\left[\begin{array}{c} \overline{\pi}e^{\frac{1-\sigma}{\sigma}[g_N(t_0,t)-g_K(t_0,t)]}\left(\frac{K_t/\overline{K}}{N_t/\overline{N}}\right)^{\frac{\sigma-1}{\sigma}} \\ +(1-\overline{\pi})\exp\left[c\cdot\widehat{m}(N_t/H_t)\right] \end{array}\right]
\end{aligned}$$

[9]See Pagan and Ullah (1999) for an introduction on non-parametric estimations.

$$(4.30)$$

A free parameter c is intentionally added in front of $\widehat{m}(N_t/H_t)$ in the above system. First, this allows us to connect system two (augmented with labor market friction) to system one (the Klump et al. (2007) system). When $c = 0$, the labor market friction term drops out and system two reduces to system one. Therefore, the statistical significance of the parameter c serves as an indicator of the significance of labor market friction. Second, the existence of the parameter c enhances the capability and flexibility of the model to fit the data. The labor market friction modelled above takes a simplified theoretical specification.[10] Introducing this free parameter improves upon the possible model mis-specification due to the simplification assumption. In particular, given my focus on the time sensitivity of model parameters (by estimating the same model using different sample periods) and the fact that labor market tightness may well change over time, it is important to have this flexible free parameter to respond to the additional labor market dynamics on top of those captured by the fluctuating employment rate N_t/H_t.

[10]See, for example, Farmer (2013a) for a model with a more general specification of labor recruiting dynamics where labor is modelled as a predetermined state variable.

4.4 Empirical Estimations

4.4.1 Data

Before proceeding to estimation results, I briefly introduce the data used in my estimations. Several authors (e.g., Berndt, 1974; Jorgenson and Ho, 2000; Antras, 2004; Klump et al., 2007; among others) have pointed out the importance of high-quality data to the estimation results. With different research questions and significant effort on data refinement (capital user cost, quality adjusted labor services, capital stock, etc.), these studies mostly concentrate on a subset of output, capital and labor and try hard to maintain a consistent measure across all variables. This paper emphasizes the impact of the overall labor market friction, i.e., the (un)employment status of the whole labor force, on the aggregate economic growth. Therefore I adopt the complete set of all variables: output (Y), labor input (N), capital input (K), and labor force (H) are represented by series of gross domestic product, employment level, net stock of fixed assets, and civilian labor force, respectively. These series make the estimations not directly comparable with some existing studies but are internally consistent and necessary to address my concerns.

To be specific, the full sample for the aggregate U.S. economy spans from 1948 to 2012. The major data source is the National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA); employment and labor force series come from the Bureau of Labor Statistics (BLS). Labor income (wN) is calculated as compensation of employee plus nonfarm proprietors' income while the capital income (qK) is defined as the difference between nominal GDP and labor income. The series of nominal capital stock is defined as the current-cost net stock of fixed assets from BEA, which is deflated by the price index to generate real capital stock (K).

4.4.2 Recursive Estimations

Given my interest in the possible variations of the model parameters across sample periods, I estimate both systems on the recursive scheme. The first sample spans from 1948 to 1983. The remaining 29 samples cover the period 1948 to 1983+i, $i = 1, 2, \ldots 29$, respectively. With the recursive scheme, each following sample includes one more year than the previous sample and the last sample coincides with the full sample 1948-2012. The first sample starts at 1983 so that (1) estimations in the first sample period have enough observations and (2) I can trace the evolution of estimated parameters for a relatively long period (1983-2013). The estimation results are reported in Figure 4.1 and 4.2 for system one (no labor market friction) and in Figure 4.3 and 4.4 for system two (with labor market friction). In each panel of these figures, the horizontal axis represents the last year included in that specific sample period. The solid line denotes the point estimate while the dashed lines depict 95% confidence intervals. Figure 4.5 compares the performance of two systems using different indicators.

Several observations arise. First, from Figure 4.1 to Figure 4.4, there exist variations for all parameters in both systems across different sample periods. The variation is more significant in factor-augmenting technical progress (γ_N and γ_K) and the factor substitution elasticity parameter (σ). In Figure 4.1, the elasticity parameter is estimated below unity during two sets of sample periods but around unity during other sample periods. Fluctuations are also observed from the estimated growth rates of factor-augmenting technical progress. Given the long-run growth model, the resulting estimates indeed change with sample periods. The tension between the long-run model and short-run sample periods calls for attention on the sample sensitivity of the estimations. In some sample periods in the short run, cyclical fluctuations may still play a role and affect the estimation of growth parameters.

Second, panel 1 and 2 in Figure 4.5 report the non-negligible impacts of labor market friction on the supply-side system. The first panel shows that, in terms of the log likelihood, the fitness of the model is significantly improved with the labor market friction as the log likelihood in system two is systematically higher than in system one. The inclusion of labor market friction improves the model's capacity to explain the data. The second panel in Figure 4.5 plots the estimated parameter c over all sample periods, which significantly differs from zero for most of the sample periods. This finding confirms the impact of labor market friction on the system and suggests that the strength of this impact changes over time. Due to the non-negligible role of labor market friction in the supply-side system, the estimations from system one

(Figure 4.1 and 4.2) are subject to the omitted variables and are possibly inconsistent and biased.

Third, labor market friction not only enhances the fitness of the model but also affects factor substitution elasticity. In the presence of labor market friction, panel three in Figure 4.5 shows that the estimated elasticity of substitution σ in system two tends to be larger than the original estimates in system one. Panel 1 in Figure 4.3 actually shows that the 95% confidence intervals of the elasticity parameter include unity for most of the sample periods, making it hard to reject the Cobb-Douglas specification of the production function. This finding, different from some existing studies, reveals the impact of labor market friction on factor substitution. On the one hand, with labor search and matching captured explicitly in the system, the decisions that workers make on the extensive margin of the labor market is also taken into account. Workers' status on being employed or unemployed and staying in or out of the labor force affect the factor substitution. The explicit modelling of unemployed workers and discouraged workers (out of the labor force) increase the estimated substitutability between capital and labor. On the other hand, with the current labor search framework, there exists internal re-allocations of employed workers between the recruiting department and the producing department. From equation (4.15), the elasticity describes the substitutability between factors of productions, i.e., between capital and worker assigned to the producing department. If workers are able to move across different departments, the substitutability between capital and the workers assigned to producing department will be higher than in system one when this option is not taken into account.

Fourth, for most of the sample periods in panel 2 and 3 of Figure 4.3, labor-augmenting technical progress (γ_N) dominates capital-augmenting technical progress (γ_K). The exception occurs in the set of sample periods ending between 2002 and 2009. These findings imply that the long-run technical progress tends to be purely labor-augmenting but capital-augmenting technical progress may also arise in the short run.

Figure 4.3 and 4.4 also show some variations of other parameters across different periods. The most drastic variations again arise during the set of sample periods ending between 2002 and 2009; the labor-augmenting technical progress is significantly lower but capital-augmenting technical progress is significantly higher during this set of sample periods. According to panel 1 in Figure 4.3, the elasticity parameter is obviously not significantly below unity, suggesting that capital and labor services are gross substitutes rather than complements. As shown in detail in the Appendix, with a CES production function and two factors of production as gross substitutes, the labor-augmenting technical progress is also labor-biased in the sense that a positive labor-augmenting technical progress will increase the marginal product of labor more than the marginal product of capital. As a result, labor income share moves in the same direction as the labor-augmenting technical progress. Similarly a capital-augmenting technical progress is capital biased and the capital income share co-moves with capital-augmenting technical progress. Estimations in panel 1 of Figure 4.3 show that the technical progress is more capital-augmenting and less labor-augmenting during the sample periods ending between 2002 and 2009. Therefore capital income share should arise while labor income share should drop according to the estimations. This prediction is confirmed by the data. The bottom panel in Figure 4.5 shows that, during the period 2002 to 2009, there is indeed a downward trend for the labor income share. This trend is captured by the recursive estimation and leads to variations of model parameters across different sample periods. The above analysis, to some extent, exhibits and justifies the compatibility of system two with the data. It again points out how sensitive an estimation could be to the choice of different sample periods; the nature of the system may be blurred by an inappropriate choice of sample period.

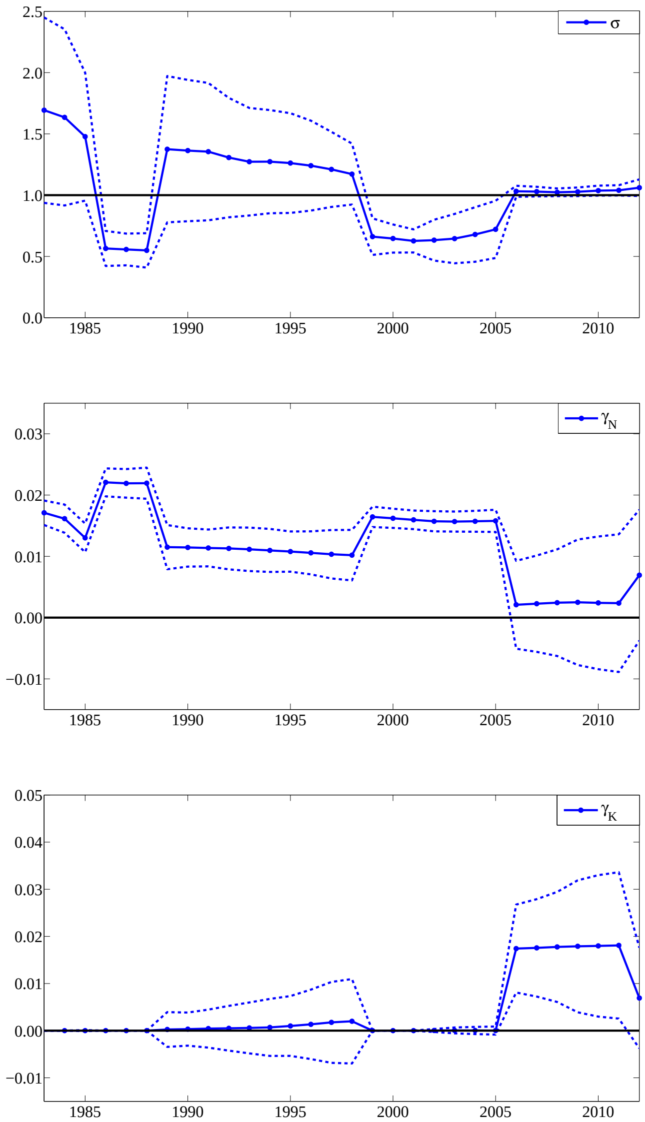
4.5 Concluding Remarks

The contribution of this paper is two-fold. First, with simultaneous estimations of factor substitutability and biased factor-augmenting technology, I take the possible variations of the model parameters with sample periods seriously. Despite the theoretical importance of elasticity of substitution between capital and labor, there is much less consensus in the empirical studies on its value. Obtaining the value of this elasticity parameter becomes even more subtle in the presence of biased factor-augmenting technical progress. Given that most of the existing studies focus on a specific sample, this paper investigates the possible variations of model parameters (factor substitutability and biased factor-augmenting technical progress) across different sample periods. There is evidence for the tension between the long-run model and the short run data as the parameters do vary across different sample periods. The sensitivity of model parameters to different sample periods constitutes an important and plausible source of empirical disagreement on parameter values of factor substitutability.

Second, this paper sheds light on the impacts of labor market friction on the growth parameters. With the joint modelling and estimation of the aggregate production and labor market friction, this paper shows that labor market friction plays a non-negligible role in the production system. In the presence of labor market friction, the system fits the data better and the elasticity parameter does not differ from unity significantly for most of the sample periods. These findings support the Cobb-Douglas production function specification. In the long run, the technical progress tends to be purely labor-augmenting, but capital-augmenting technical progress may arise in the short run as well.

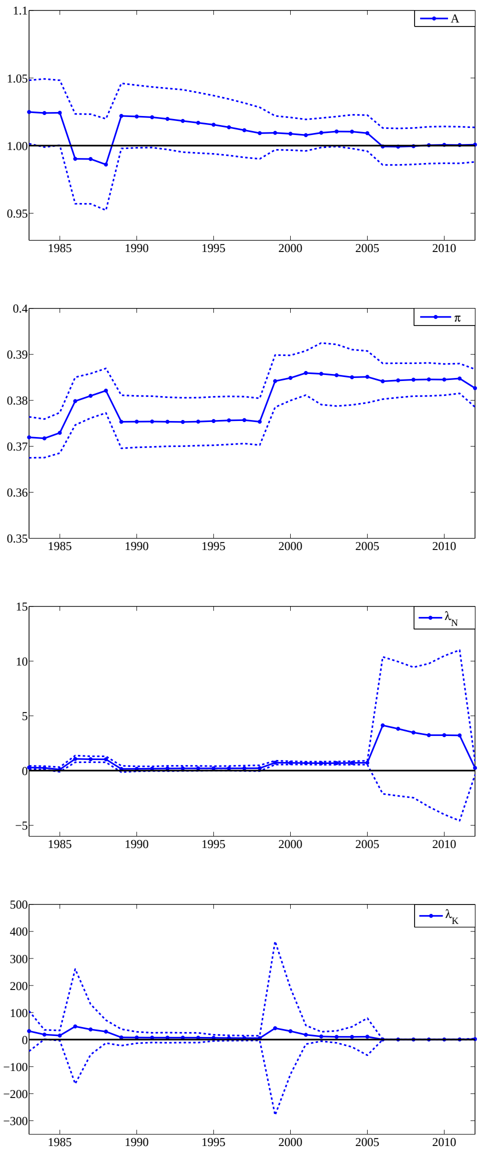
Two issues are worth more attention but left undiscussed in the current paper. First, the above analysis is drawn on the data for the aggregate U.S. economy during the period 1948-2012 on a recursive scheme. From the perspective of economic growth, even the full sample period 1948-2012 is still a relatively short period. Whether the above analysis will change qualitatively as the longer sample period becomes available is subject to further empirical examination. Second, the current paper empirically examines the impacts of labor market friction on economic growth and production. The transmission mechanism from labor market friction to economic growth (maybe the inverse direction as well) has not been fully explored and also left for future research.

Figure 4.1: Estimating System One across Different Sample Periods (I)



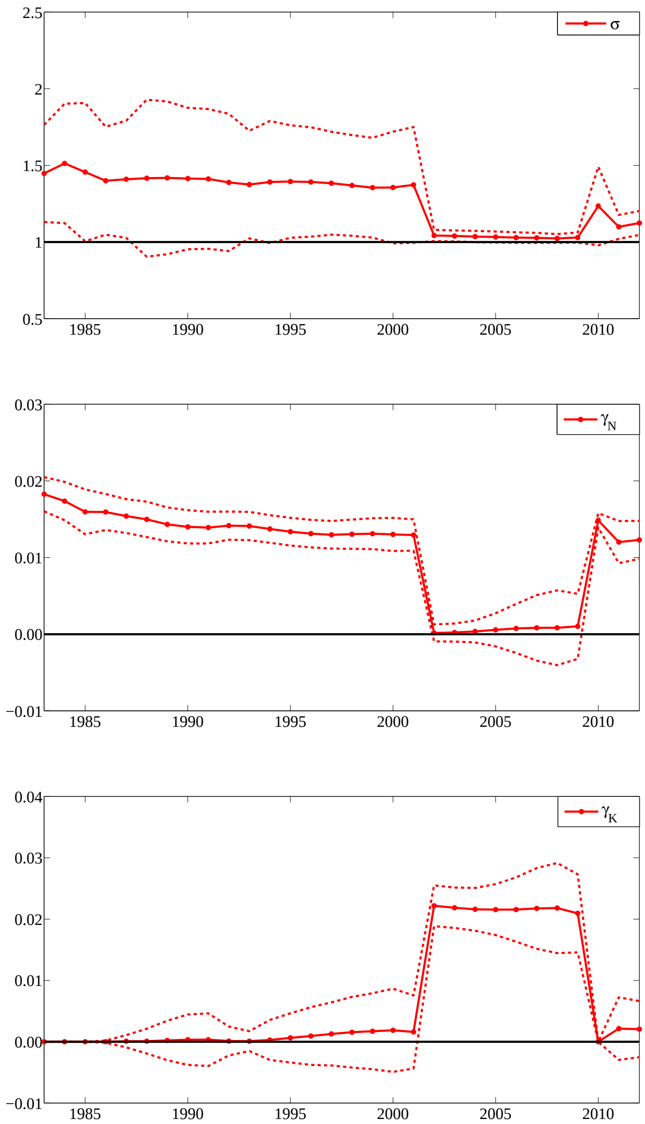
Note: This figure plots the estimation results on a recursive scheme for the Klump et al. (2007) system. All sample periods start in 1948 and the horizontal axis indicates the last year included in each sample. The solid line reports the point estimate for each parameter and the dashed lines report the corresponding 95% confidence intervals. Parameters reported in this figure include: elasticity of substitution σ, average growth rate of labor-augmenting technical progress γ_N, and average growth rate of capital-augmenting technical progress γ_K.

Figure 4.2: Estimating System One across Different Sample Periods (II)



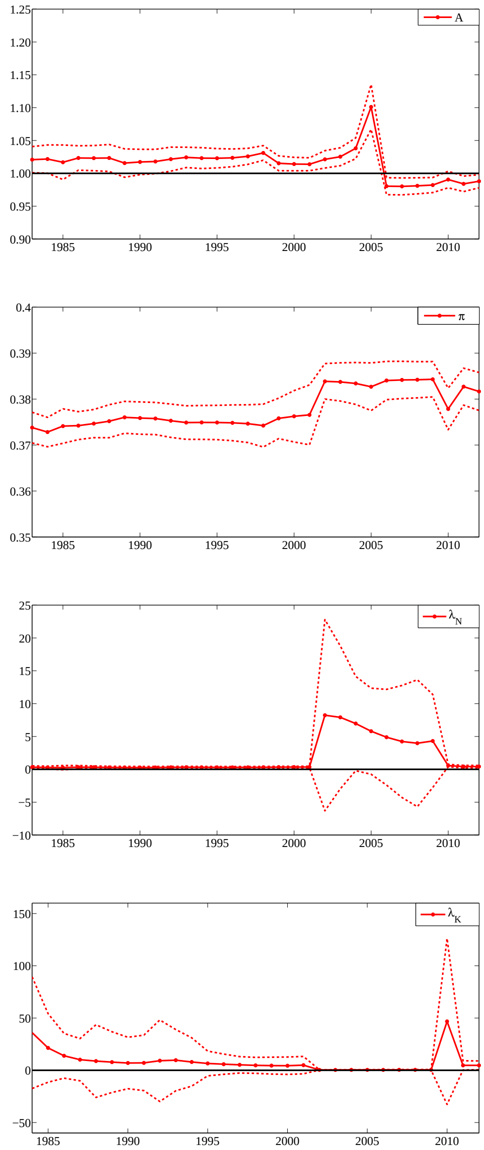
Note: This figure plots the estimation results on a recursive scheme for the Klump et al. (2007) system. All sample periods start in 1948 and the horizontal axis indicates the last year included in each sample. The solid line reports the point estimate for each parameter and the dashed lines report the corresponding 95% confidence intervals. Parameters reported in this figure include: normalization scale parameter A, average capital income share parameter π, curvature parameter of labor-augmenting technical progress λ_N, and curvature parameter of capital-augmenting technical progress λ_K.

Figure 4.3: Estimating System Two across Different Sample Periods (I)



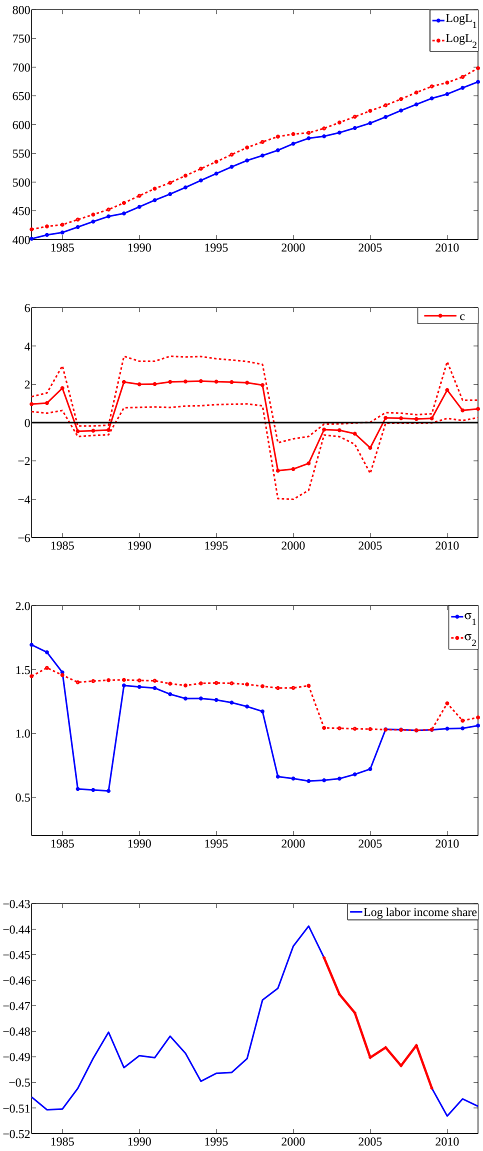
Note: This figure plots the estimation results on a recursive scheme for the supply-side system augmented with the labor market friction. All sample periods start in 1948 and the horizontal axis indicates the last year included in each sample. The solid line reports the point estimate for each parameter and the dashed lines report the corresponding 95% confidence intervals. Parameters reported in this figure include: elasticity of substitution σ, average growth rate of labor-augmenting technical progress γ_N, and average growth rate of capital-augmenting technical progress γ_K.

Figure 4.4: Estimating System Two across Different Sample Periods (II)



Note: This figure plots the estimation results on a recursive scheme for the supply-side system augmented with the labor market friction. All sample periods start in 1948 and the horizontal axis indicates the last year included in each sample. The solid line reports the point estimate for each parameter and the dashed lines report the corresponding 95% confidence intervals. Parameters reported in this figure include: normalization scale parameter A, average capital income share parameter π, curvature parameter of labor-augmenting technical progress λ_N, and curvature parameter of capital-augmenting technical progress λ_K.

Figure 4.5: Comparing Estimations of Two Systems



Note: This figure compares the estimations of the supply-side system with and without labor market friction. All sample periods start in 1948 and the horizontal axis indicates the last year included in each sample. The first panel compares log likelihood; the second panel reports the estimated free parameter c; the third panel compares elasticity and the last panel reports the log labor income share observed in the data.

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Appendix A

Appendix for Chapter 1

A.1 Estimate the Demand Shock Parameters

To estimate the demand shock parameters, i.e., the transition matrix **d**, the standard deviation of demand shock innovation σ_ξ, and the correlation coefficient ρ_ξ, I need to first construct the demand shocks Δ_t. From the FOCs with respect to c^h_{1t} and N^h_t in the home country, I have

$$\lambda^h_t = \frac{\theta \Delta^h_t \left(c^h_{1t}\right)^{\phi-1}}{\theta \left(c^h_{1t}\right)^{\phi} + (1-\theta)\left(c^h_{2t}\right)^{\phi}} \tag{A.1}$$

$$\chi N^h_t = (1-\alpha)\lambda^h_t Y_{1t} \tag{A.2}$$

Eliminating the shadow price λ^h_t and rewriting the equations give the following expression for demand shocks in the home country

$$\Delta^h_t = \frac{\chi N^h_t}{(1-\alpha)Y_{1t}} \frac{\theta \left(c^h_{1t}\right)^{\phi} + (1-\theta)\left(c^h_{2t}\right)^{\phi}}{\theta \left(c^h_{1t}\right)^{\phi-1}} \tag{A.3}$$

The main data set (the PWT 7.1) does not have the separate series of consumption of domestic goods (c^h_{1t}) and consumption of imported goods (c^h_{2t}); instead I turn to the detailed import flow data from the International Trade by Commodity Statistics

(ITCS) classified according to the Harmonized System 1988 (HS 1988, available at the OECD StatExtracts). Among all 99 groups of commodities in HS 1988, I identify 58 commodities as consumption goods and construct the imported consumption series (c_{2t}^h) for all G7 countries. The consumption of domestic goods (c_{1t}^h) is then obtained as the difference between the total consumption and the imported consumption. With the total consumption and output series obtained from the same data set and labor hours obtained from PWT 7.1, the demand shock series are constructed according to equation (A.3) for the home country (the US) and the foreign country (the aggregation of the other six countries). Values of χ, α, θ and ϕ are calibrated according to the same procedure as described in the text.

A bivariate VAR process is then estimated

$$\begin{bmatrix} \ln \Delta_t^h \\ \ln \Delta_t^f \end{bmatrix} = \begin{bmatrix} \widetilde{d}_{11} & \widetilde{d}_{12} \\ \widetilde{d}_{21} & \widetilde{d}_{22} \end{bmatrix} \begin{bmatrix} \ln \Delta_{t-1}^h \\ \ln \Delta_{t-1}^f \end{bmatrix} + \begin{bmatrix} \xi_t^h \\ \xi_t^f \end{bmatrix} \tag{A.4}$$

The estimation results of the transition matrix is reported below with the standard errors reported in the parenthesis

$$\begin{bmatrix} \widetilde{d}_{11} & \widetilde{d}_{12} \\ \widetilde{d}_{21} & \widetilde{d}_{22} \end{bmatrix} = \begin{bmatrix} 0.8854 & 0.1117 \\ (0.1333) & (0.0401) \\ 0.0954 & 0.9067 \\ (0.1111) & (0.0334) \end{bmatrix} \tag{A.5}$$

The estimated standard errors of demand shock innovations and the correlation coefficient are

$$\begin{bmatrix} \widetilde{\sigma}_\xi^h \\ \widetilde{\sigma}_\xi^f \end{bmatrix} = \begin{bmatrix} 0.0201 \\ 0.0060 \end{bmatrix}, \qquad \rho_\xi = 0.3647 \tag{A.6}$$

Since the two countries are symmetric in the model, I follow BKK (1992) and retrieve the unique symmetric version of the above estimated transition matrix such that (1) the new transition matrix has the same eigenvalues as the original one and (2) it features the same diagonal elements and, up to a sign difference, the same off-diagonal elements. The unique symmetric matrix turns out to be

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} 0.8960 & 0.1038 \\ 0.1038 & 0.8960 \end{bmatrix} \tag{A.7}$$

So I have $d_{11} = d_{22} = 0.8960$ and $d_{12} = d_{21} = 0.1038$ in the baseline calibration. I also set the symmetric standard deviations of demand innovations as $\sigma_\xi^h = \sigma_\xi^f = 0.015$ and $\rho_\xi = 0.3647$ according to the above estimations.

Appendix B

Appendix for Chapter 2

B.1 Proof of Proposition

The proof of proposition in section 3 includes three steps.

Step 1: Government size τ is determined by the first order necessary condition from the benevolent government's maximization problem (3.23):

$$\begin{aligned} F(\theta, \tau) &= \frac{-\eta\left[\theta(\alpha-\gamma)+\gamma-\alpha\gamma\right]}{(1-\alpha)(1-\gamma)\left(1-\tau\right)} - \frac{\eta}{1-\tau} + \frac{1-\eta}{\tau} \\ &\quad - \frac{\gamma-\theta(\gamma-\alpha)}{1-\gamma\left(1-\tau\right)-\theta(\alpha-\gamma)(1-\tau)} \\ &= 0 \end{aligned} \tag{B.1}$$

Equation (B.1) is an implicit function of openness θ and government size τ. I resort to the implicit function theorem to uncover the steady state relationship between these two variables:

$$\frac{\partial \tau}{\partial \theta} = -\frac{\partial F(\theta, \tau)/\partial \theta}{\partial F(\theta, \tau)/\partial \tau} \tag{B.2}$$

From (B.1), I have

$$\begin{aligned}\frac{\partial F(\theta,\tau)}{\partial\theta} &= \frac{-\eta\,(\alpha-\gamma)}{(1-\alpha)(1-\gamma)\,(1-\tau)} - \frac{\alpha-\gamma}{[1-\gamma(1-\tau)-\theta(\alpha-\gamma)(1-\tau)]^2} \\ &= (\gamma-\alpha)\left[\frac{\eta}{(1-\alpha)(1-\gamma)\,(1-\tau)} + \frac{1}{[1-\gamma(1-\tau)-\theta(\alpha-\gamma)(1-\tau)]^2}\right]\end{aligned} \tag{B.3}$$

Given $\alpha,\gamma,\eta\in(0,1)$, for any $\theta,\tau\in(0,1)$, equation (B.3) implies that

$$sign\left(\frac{\partial F(\theta,\tau)}{\partial\theta}\right) = sign(\gamma-\alpha) \tag{B.4}$$

Step 2: Given $\alpha,\gamma,\eta\in(0,1)$ and $\theta,\tau\in(0,1)$, if I could show $\frac{\partial F(\theta,\tau)}{\partial\tau}<0$ when $max\{\alpha,\gamma\}<\sqrt{1-\eta}$, then equation (B.2) and (B.4) imply that $sign\left(\frac{\partial\tau}{\partial\theta}\right)=sign(\gamma-\alpha)$.

To start, from (B.1), I have

$$\begin{aligned}\frac{\partial F(\theta,\tau)}{\partial\tau} &= \frac{-\eta\,[\theta\,(\alpha-\gamma)+\gamma-\alpha\gamma]}{(1-\alpha)(1-\gamma)\,(1-\tau)^2} - \frac{\eta}{(1-\tau)^2} \\ &\quad -\left\{\frac{1-\eta}{\tau^2} - \frac{[\gamma-\theta\,(\gamma-\alpha)]^2}{[1-\gamma(1-\tau)-\theta(\alpha-\gamma)(1-\tau)]^2}\right\}\end{aligned} \tag{B.5}$$

It is obvious to show that the first two terms in equation (B.5) are both negative. I need to show that the third term is also negative when $max\{\alpha,\gamma\}<\sqrt{1-\eta}$.

Let $M=\gamma-\theta\,(\gamma-\alpha)\in(0,1)$. In order for the third term to be negative, I need

$$\frac{[\gamma-\theta\,(\gamma-\alpha)]^2}{[1-\gamma(1-\tau)-\theta(\alpha-\gamma)(1-\tau)]^2} < \frac{1-\eta}{\tau^2}$$

or

$$\frac{M^2}{[1-(1-\tau)M]^2} < \frac{1-\eta}{\tau^2}$$

It implies

$$\frac{M}{1-(1-\tau)M} < \frac{\sqrt{1-\eta}}{\tau}$$

since $M \in (0,1)$. Rewriting this condition gives

$$\tau < \frac{\sqrt{1-\eta}\,(1-M)}{M\left(1-\sqrt{1-\eta}\right)} \tag{B.6}$$

If the RHS of (B.6) is bigger than one

$$\frac{\sqrt{1-\eta}\,(1-M)}{M\left(1-\sqrt{1-\eta}\right)} > 1$$

then inequality (B.6) holds for all $\tau \in (0,1)$. This implies

$$\theta(\alpha-\gamma) < \sqrt{1-\eta} - \gamma \tag{B.7}$$

In order for (B.7) to hold for all $\theta \in (0,1)$, the following two conditions need to be satisfied, depending on the relative size of α and γ:

$$\begin{aligned} \alpha &< \sqrt{1-\eta} \text{ when } \alpha > \gamma \\ \gamma &< \sqrt{1-\eta} \text{ when } \alpha < \gamma \end{aligned}$$

These two conditions could be summarized as

$$max\,\{\alpha, \gamma\} < \sqrt{1-\eta}$$

Step 3: The last step is to check that, under all conditions in the proposition, the second order condition of the government's maximization problem (3.23) is satisfied, *i.e.*, indeed I reach the maximum. This requires the following condition:

$$\frac{\partial^2 \log U}{\partial \tau^2} = \frac{\partial F(\theta, \tau)}{\partial \tau} < 0 \tag{B.8}$$

Notice that this condition (B.8) coincides with (B.5), which has already been shown to hold in step 2. This completes the proof of the proposition.

Appendix C

Appendix for Chapter 3

C.1 Technical Progress and Factor Substitution

In this section, we briefly review the biased factor-augmenting technical progress, factor substitution elasticity, and their relation to factor income shares. Consider the CES production function $F(K, N)$ adopted in the text

$$F(K,N) = \left[\left(E^N N\right)^{-\rho} + \left(E^K K\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{C.1}$$

where N and K represent the labor service and capital service in the production process while E^N and E^K denote the factor-augmenting technical efficiency. The elasticity of substitution between capital and labor is $\sigma = \frac{1}{1+\rho} > 0$. When $\sigma > 1$, these two factors are gross substitutes; when $0 < \sigma < 1$, these two factors are gross complements; when $\sigma = 1$, the CES production reduces to the Cobb-Douglas production form. To illustrate the relation between factor-augmenting and factor-biased technical progress, consider the ratio of marginal product of labor to capital

$$\frac{MP_N}{MP_K} = \left(\frac{E^K}{E^N}\right)^{\rho} \left(\frac{K}{N}\right)^{1+\rho} = \left(\frac{E^N}{E^K}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K}{N}\right)^{\frac{1}{\sigma}} \tag{C.2}$$

It is clear from equation (C.2) that, when capital and labor are gross substitutes, i.e., $\sigma > 1$, labor- (capital-) augmenting technical progress is also labor- (capital-) biased since $\frac{\partial \frac{MP_N}{MP_K}}{\partial E^N} > 0$.[1] When capital and labor are gross complements, i.e., $0 < \sigma < 1$, labor- (capital-) augmenting technical progress becomes capital- (labor-) biased since $\frac{\partial \frac{MP_N}{MP_K}}{\partial E^N} < 0$ because a labor-augmenting technical progress will increases the demand for capital more than for labor, therefore MP_K increases more than the MP_N.

With these notations, the labor and capital income shares are derived as

$$N - share = \frac{MP_N \cdot N}{MP_K \cdot K + MP_N \cdot N} = \frac{1}{1 + \left(\frac{E^N N}{E^K K}\right)^{\frac{1-\sigma}{\sigma}}} \tag{C.3}$$

$$K - share = \frac{MP_K \cdot K}{MP_K \cdot K + MP_N \cdot N} = \frac{1}{1 + \left(\frac{E^N N}{E^K K}\right)^{\frac{\sigma-1}{\sigma}}} \tag{C.4}$$

The above expressions imply that, when capital and labor are gross substitutes, i.e., $\sigma > 1$, the labor income share will *increase* in the presence of a higher labor-augmenting technical efficiency; when capital and labor are gross complements, i.e., $0 < \sigma < 1$, the labor income share will *decrease* in the presence of the same higher labor-augmenting technical efficiency. Capital income share, on the other hand, moves in the opposite direction to labor income share.

[1] For a general production function $F(K, N, A)$, where A denotes the technical efficiency, the technical progress is N-biased if $\frac{\partial \frac{\partial F/\partial N}{\partial F/\partial K}}{\partial A} > 0$, i.e., if the technical change A increases the marginal product of N more than that of K. See Acemoglu (2002) for more discussions.