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How much water will we have by the end of summer? Progress and pitfalls along the path to prediction.

by

David Dralle

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy

 in

Engineering - Civil and Environmental Engineering

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Assistant Professor Sally E. Thompson, Chair Professor Mark Stacey Assistant Professor Laurel Larsen

Spring 2016

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Abstract

How much water will we have by the end of summer? Progress and pitfalls along the path to prediction.

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Doctor of Philosophy in Engineering - Civil and Environmental Engineering

University of California, Berkeley

Assistant Professor Sally E. Thompson, Chair

Seasonally dry climates exhibit annual swings in precipitation that strongly influence surface water availability. Reliable prediction and characterization of these resources are important for ecosystems and society, yet few hydrologic models and methods have been tailored to cope with the complexities of a non-stationary climate.

This dissertation focuses on techniques to improve prediction and analysis of soil moisture, hillslope flow generation, and catchment scale flows in seasonally dry climates. It evaluates the performance of three new models for these hydrologic variables, which span the relevant range of scales for conservation and management purposes. It examines in fine detail a common mathematical model for the streamflow recession, the power law differential equation $dq/dt = -aq^b$, which is one of the simplest and most broadly used functions to describe the draining of a catchment. For seasonally dry watersheds that spend much of the year in a state of recession, the power law model is an important tool for streamflow modeling and analysis. The work here assesses some of the mathematical challenges associated with application of the power law recession model, and quantifies uncertainty associated with determination of the power law parameters, a and b.

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Acknowledgments

Chapter 1

Introduction

1.1 Seasonally dry ecosystems

Seasonally dry ecosystems (SDEs), which include Mediterranean, tropical monsoonal and tropical savanna climates, cover approximately 30% of the planet's land area [Peel and Finlayson, 2007] and contain numerous biodiversity hot spots [Klausmeyer and Shaw, 2009, Miles et al., 2006]. SDEs exhibit a distinct rainy season, followed by a pronounced dry season during which rainfall makes a minimal contribution to the water balance. Many SDEs are considered to be highly vulnerable to environmental degradation under projected climate scenarios [Dominguez et al., 2012, Gao and Giorgi, 2008, García-Ruiz et al., 2011b], a vulnerability exacerbated by anthropogenic pressures and increased water use for agriculture and domestic water supplies [Miles et al., 2006, Underwood et al., 2009]. Forecasting the specific risks to these ecosystems and the services they provide requires being able to predict and characterize hydrologic variability as a function of climate and land use variables [Müller et al., 2014].

1.2 Dry season hydrology in Mediterranean SDEs

In some seasonally dry areas, such as the Mediterranean climatic zones found in northern California and southern Oregon, the annual dry period coincides with a warm summer, when ecosystems peak in productivity. Summer ecosystems are thus dependent on the carryover storage of wet season water (in the form of soil moisture or deeper saturated stores), the availability of which is controlled by a suite of physical processes related to the drainage and drying of the catchment [Andermann et al., 2012, Brahmananda Rao et al., 1993, Power et al., 2008, Rodríguez-Iturbe et al., 2001, Samuel et al., 2008]. To properly describe dry season ecosystem functioning, the relationship between climatic variability and the hydrologic dynamics of these seasonal dry down periods must be characterized and ultimately predicted [Bonada and Resh, 2013, Dallas, 2013, Power et al., 2013, Rodríguez-Iturbe et al., 2008]. The dependence of dry season water availability on

wet season storage dynamics makes this task challenging, and requires hydrological theory and models that describe the tremendous range of moisture states that characterize SDEs.

Even at the scale of a one-dimensional soil column, many soil moisture regimes can be generated based on the interplay of the timing and magnitude of rainfall events and soil drying through evapotranspiration [Laio et al., 2001a, Miller et al., 2007, Porporato et al., 2004, Rodriguez-Iturbe et al., 1999]. These one-dimensional processes affect recharge from the unsaturated soils into hillslope aquifers, which themselves exhibit rich diversity in discharge dynamics induced by spatial patterns of recharge, hillslope plan shape, and hillslope profile curvature [Dralle et al., 2014, Troch et al., 2003, Woods et al., 1997]. Catchment and basin scale dynamics are further influenced by emergent spatial variation in geology, geomorphology and the channel network [Biswal and Marani, 2010, Clark et al., 2009, Harman and Sivapalan, 2009, Harman et al., 2009].

Explicitly resolving the effects of these sources of heterogeneity in natural watersheds presents a significant challenge [Beven, 2001], which can be circumvented at the catchment scale, for example, by using simple hydrologic models of basin-scale response [Sivapalan, 2003]. Such 'lumped' models encode the collective behavior of sub-resolution processes using emergent hydrologic relationships [e.g. Xu et al., 2013], scaling techniques [e.g. Thompson et al., 2011], or closure models [e.g. Kirchner, 2009]. In the case of lumped basin-scale models, the catchment is generally treated as an individual control volume, and drainage or recession processes are governed by phenomenological 'storage-discharge' functions, which relate the catchment storage state to catchment discharge [Wittenberg and Sivapalan, 1999, Wittenberg, 1999b]. This and analogous models at the patch and hillslope scales have the potential to capture the essential dynamics of hydrologic systems using a relatively sparse set of physically meaningful parameters. The typically minimal structure of lumped frameworks simplifies parameter calibration, facilitates iterative model refinement [e.g. Clark et al., 2009], and aids in the identification of intuitive metrics for catchment classification [e.g Botter et al., 2013].

Despite the advantages of these minimal approaches, only a few such models have been formulated specifically for SDEs, and with few exceptions [Müller et al., 2014], most of these focus primarily on the soil water balance [Feng et al., 2012, 2015, Viola et al., 2008]. While a number of statistical and deterministic modeling methods have been used to characterize larger scale hydrologic variables, such as low flow conditions in both gauged and ungauged basins [Castellarin et al., 2007, Ganora et al., 2009, Laaha and Blöschl, 2007, Müller and Thompson, 2015, Nathan and McMahon, 1992, Skøien et al., 2006, among others], many such approaches struggle to account for climatic non-stationarities [Müller and Thompson, 2015], or are heavily parameterized [e.g. Arnold et al., 1998] and thus difficult to apply in sparsely instrumented regions.

This dissertation expands minimal, process-driven modeling frameworks and methods to predict and analyze dry season surface water availability in seasonally dry climates. The work spans the relevant spatial scales of ecological and human water uses, describing dynamics at patch, hillslope and small-watershed scales. The focus is on Mediterranean climates, using watersheds from northern California and Southern Oregon as case studies. The dissertation is divided into two major thematic components:

- Part I focuses on developing and testing models of hydrologic dynamics in SDEs. Specifically, the models target the carry-over of soil moisture from wet seasons into dry seasons, the effects of hillslope heterogeneity on discharge generation and persistence during dry periods, and the risk of catchment-scale flows crossing ecologically significant thresholds during the dry season.
- Part II addresses the use of a common basin-scale closure relationship the power law storage-discharge function – as a means for hydrograph recession analysis and catchment inference. The work identifies a ubiquitous mathematical artifact that hinders robust inference from such analyses, and presents methods to correct that artifact. Methodological issues arising from numerical procedures and selection of analysis tools are addressed in the final chapter of the dissertation.

1.3 Part I: Predicting dry season water availability

Part I of this dissertation develops three simple models for dry season water availability, at the point, hillslope, and catchment scales. At the point scale, a minimal stochastic soil moisture model is extended to include non-stationary, seasonal climatic forcing. At the hillslope scale, a minimal model for the dynamics of unconfined, hillslope water tables is solved to explicitly examine the potential impacts of spatial heterogeneity in recharge of the water table (as might be driven by spatially heterogenous vegetation cover often present in seasonally dry ecosystems) to the rate of streamflow recession. Finally, a stochastically forced model for catchment response in seasonal climates is extended to examine the persistence of dry season high flow conditions.

Chapter 1 overview: Soil moisture in seasonal climates

Using stochastic analytical models of soil moisture to capture the effects of rainfall seasonality has been hindered by the need to mathematically connect dry season soil moisture dynamics to the initial condition introduced by the wet season. This chapter presents a simple process based stochastic model for soil moisture, which explicitly models inter-seasonal transient effects by accounting for carry-over soil moisture storage between the wet and dry seasons, and allows a derivation of an analytical expression for mean duration from the start of the dry season before dry season soil moisture drops below a specified threshold value. Such 'first-passage' or 'crossing' times pose controls on both vegetation productivity and water stress during dry summers. The new model, along with an existing model that incorporates non-zero dry season rainfall but not variability in the soil moisture condition at the start of the dry season, are tested against data from the Tonzi Ranch Ameriflux site. Both models predict first passage times well for high soil moisture thresholds, but the new model improves prediction at lower thresholds. The annual soil moisture probability distribution function (PDF) from the new model also compares well with observations. The work presented in this chapter is published in Water Resources Research [Dralle and Thompson, 2016]

Chapter 2 overview: Dry season hillslope recession

The linearized hillslope Boussinesq equation, introduced by Brutsaert [1994], describes the dynamics of saturated, subsurface flow from hillslopes with shallow, unconfined aquifers. In this chapter, a new analytical technique is used to solve the linearized hillslope Boussinesq equation to predict water table dynamics and hillslope discharge to channels. The new solutions extend previous analytical treatments of the linearized hillslope Boussinsg equation to account for the impact of spatiotemporal heterogeneity in water table recharge, while also providing a simpler mathematical procedure to an analytical solution. The results indicate that the spatial character of recharge may significantly alter both steady-state subsurface storage characteristics and the transient hillslope hydrologic response, depending strongly on similarity measures of controls on the subsurface flow dynamics. Additionally, the work presents new analytical solutions for the linearized hillslope-storage Boussinesq equation and explores the interaction effects of recharge structure and hillslope morphology on water storage and baseflow recession characteristics. A theoretical recession analysis, for example, demonstrates that decreasing the relative amount of downslope recharge has a similar effect as increasing hillslope convergence. In general, the theory suggests that recharge heterogeneity can serve to diminish or enhance the hydrologic impacts of hillslope morphology. The work presented in this chapter is published in Water Resources Research [Dralle et al., 2014]

Chapter 3 overview: Dry season streamflow persistence

Seasonally dry ecosystems exhibit periods of high water availability followed by extended intervals during which rainfall is negligible and streamflows decline. Eventually, such declining flows will fall below the minimum levels required to support ecosystem functions or services. The time at which dry season flows drop below these minimum levels (Q_*) , relative to the start of the dry season, is termed the 'persistence time' (T_{Q_*}) . The persistence time determines how long seasonal streams can support various human or ecological functions during the dry season. This chapter extends recent work on the stochastic hydrology of seasonally dry climates to develop an analytical model for the probability distribution function of the persistence time. The proposed model accurately captures the mean of the persistence time distribution, but underestimates its variance. It is demonstrated that this underestimation arises in part due to correlation between the parameters used to describe the dry season recession, but that this correlation can be removed by re-scaling the flow variables. The work presented in this chapter is published in Water Resources Research [Dralle et al., 2016b]

1.4 Part II: Power law recession analysis

As applied in Chapter 3, power law functions, $dq/dt = -aq^b$, are commonly used to model the falling limb of the hydrograph – the recession. This power law form is empirically and theoretically motivated: it accommodates observed non-linearity in the hydrograph recession, and is a predicted outcome of well known solutions from the hydraulic groundwater equations, which have been used as a basis for streamflow characterization for over 100 years [Boussinesq, 1904]. In these solutions, the power-law parameters represent multiplicative combinations of parameters describing aquifer geometry and conductivity. These results have been leveraged for inverse modeling and the development of flow separation algorithms [Huyck et al., 2005a, Mendoza et al., 2003, Rupp and Selker, 2006b, Rupp et al., 2004, Szilagyi et al., 1998b,b, Vogel and Kroll, 1992].

While traditional recession models and methods of recession analysis stem from the results of hydraulic groundwater theory, recent research has spurred a 'revival' in power law recession analysis, owing primarily to new theories concerning catchment function [Biswal and Marani, 2010, Clark et al., 2009, Harman et al., 2009]. Each of these theories poses its own radically different understanding of how the catchment state relates to the character of the streamflow recession. Yet, the convergent predictions of the models – that the streamflow recession should take a power law form – pose two related questions for recession analysis and modeling: 1) Which, if any, of the proposed recession mechanisms dominates the dynamics in a particular catchment, and 2) Is there a procedure to infer the dominant process solely from hydrograph analysis? One essential step towards addressing these questions is to establish unambiguous methods to characterize recession events. While a multitude of recession analysis techniques exist, it is unclear which of these is most appropriate for any particular application, and to what extent the results of any one analysis will remain unchanged under different combinations of methodological choices.

Part II of this dissertation aims establish less ambiguous methods of recession analysis by identifying sources of error and uncertainty that could confound attempts to extract physically meaningful information from power law recession models. Chapter 4 introduces a novel method to interpret power law recession parameter fits and Chapter 5 attempts to quantify uncertainty stemming from subjective choices in procedures for recession extraction and fitting. These results contribute to a methodological foundation that can be relied upon for more consistent recession analysis. This will foster the development of theoretical frameworks that can use solely hydrograph data to identify the dominant processes governing catchment drainage, and will ultimately improve our ability to forecast dry season stream water availability under changing conditions.

Chapter 4 overview: a, b careful!

Recessions are often characterized in terms of their power law parameters (a, b). The empirical determination and interpretation of the parameter a is typically biased by the presence of a ubiquitous mathematical artifact resulting from the scale-free properties of the power

law function. This reduces the information available from recession parameter analysis, and creates several heretofore-unaddressed methodological 'pitfalls'. This chapter outlines the artifact, demonstrates its genesis, and presents an empirical re-scaling method to remove artifact effects from fitted recession parameters. The re-scaling process reveals underlying climatic patterns obscured in the original data, and could maximize the information content of fitted power laws. The contents of this chapter are published in the journal Geophysical Research Letters [Dralle et al., 2015].

Chapter 5 overview: Streamflow recession variability

There is no universally agreed up procedure for performing power law recession analysis, however, most approaches are comprised of two key steps: 1) Extract streamflow recessions; 2) Fit a power law model to the extracted streamflow recessions.

In classical recession analysis, this procedure is as simple as performing some type of linear fit to a scatter plot of $(q, \log -dq/dt)$ point pairs, with dq/dt < 0, for the set of recessions extracted from the hydrograph record [e.g. Brutsaert and Nieber, 1977, Stoelzle et al., 2013, Tague and Grant, 2004]. However, more complex methods exist, and the choice of analysis generally depends on the type of information sought from the recessions.

Recent work has attempted to attribute physical meaning to variability <u>across</u> recessions within a single catchment [Bart and Hope, 2014, Biswal and Marani, 2010, 2014, Biswal and Nagesh Kumar, 2012, Clark et al., 2009, Harman et al., 2009, Mutzner et al., 2013]. Whereas classical recession analysis seeks a single recession parameter pair that best fits all hydrograph recessions, these new theories have triggered a rapid increase in the number of studies that perform 'event-scale recession analysis', where the power law model is fit separately to each extracted recession limb.

Before broadly applying these event scale recession analyses to flow data sets in order to validate new theories, uncertainty deriving simply from the particular choice of recession extraction and fitting methodology should first be quantified. Chapter 5 of this dissertation examines this issue by performing recession analysis for a number of distinct extraction and fitting methodologies. It then compares the fitted results across these procedures to estimate the amount of variability that can be attributed purely to methodological choices.

Part I Prediction

Chapter 2

A minimal probabilistic model for soil moisture in seasonally dry climates

2.1 Introduction

Seasonally dry ecosystems (SDEs), which include Mediterranean, tropical monsoonal and tropical savannah climates, cover approximately 30% of the Earth's land area [Peel and Finlayson, 2007] and contain several biodiversity hot spots [Klausmeyer and Shaw, 2009, Miles et al., 2006]. Pronounced climatic variability is a common feature of these regions [Fatichi et al., 2012] and is projected to intensify in future climate scenarios [Dominguez et al., 2012, Gao and Giorgi, 2008, García-Ruiz et al., 2011b]. Consequently, a number of studies classify SDEs and their water resources as climatically vulnerable [Gao and Giorgi, 2008, García-Ruiz et al., 2009, Nohara et al., 2006, Parry, 2007]. Projecting the variability of water availability and the risks of water shortfalls in these regions could therefore provide useful insights into vegetation and ecosystem risk [Müller et al., 2014, Vico et al., 2014].

Process based stochastic methods provide a minimal modeling framework to obtain the probability distributions of soil moisture and streamflow [Botter et al., 2007a, Laio, 2002, Milly, 1993, Rodriguez-Iturbe et al., 1999]. Since hydroclimatic variation strongly impacts plants through soil moisture [Taiz and Zeiger, 2010, Thompson and Katul, 2012], these models have also been used to predict ecological response and to assess the vulnerability of ecosystems [Porporato et al., 2004, Thompson et al., 2013, 2014, Viola et al., 2008]. To date, the majority of methods have been developed under conditions where either the climatic forcing can be considered stationary in time [Botter et al., 2007a, Porporato et al., 2004, Rodriguez-Iturbe et al., 1999], or where transient dynamics between seasons are not considered [Kumagai et al., 2009, Miller et al., 2007]. Studies that considered the effects on soil moisture of seasonality in rainfall or evaporative demand, or transient dynamics between seasons, have typically focused on the mean soil moisture dynamics [D'odorico et al., 2000, Feng et al., 2012, Laio et al., 2001b].

Viola et al. [2008] first investigated the impacts of transient soil moisture dynamics on plant water stress during the growing season in Mediterranean ecosystems. In that study, the steady state PDF of soil moisture during the wet season represents the end of wet season conditions, after which the dry season proceeds following a step change in rainfall statistics and potential evapotranspiration. For a site with shallow soil or a small mean rainfall depth (relative to the total possible amount of soil water storage), this approach is appropriate because the variance of wet season conditions will be small. However, to accurately quantify soil moisture variability at sites with large average rainfall depths or soil storage, a more precise description of the conditions at the start of the dry season may be required.

This work developed a simplified but fully analytic stochastic theory for soil moisture probability distribution functions for seasonally dry regions. The model yields a single, analytical expression for the annually integrated soil moisture PDF under seasonal climates, and a similarly minimal analytical expression for the mean first passage time of dry season soil moisture below a given threshold. Such probabilistic descriptions of first passage times are used to link soil moisture dynamics to plant water stress [Rodríguez-Iturbe et al., 2001, Viola et al., 2008].

The model of Porporato et al. [2004] was used to represent the probabilistic dynamics of soil moisture during the wet season. In this framework, the wet season is characterized by stationary hydroclimatic and rainfall properties. At the onset of the dry season, the soil moisture initial condition is a random variable, described by the soil moisture PDF following the final storm of the wet season, which is an improved representation of conditions at the beginning of the dry season, compared to the steady state wet season PDF. The dry season has zero rainfall and a deterministic soil moisture dry down, which proceeds uninterrupted until the following wet season. Consequently, the approach assumes rainfall seasonality is perfectly binary; i.e. a wet season, characterized by statistically stationary rainfall, is followed by a dry season with no significant rainfall at all. This is most appropriate in locations where a pronounced wet season is followed by a dry season during which rainfall is negligible (or where rainfall inputs are so low that the majority of rainfall is intercepted and does not contribute to soil moisture variations over the rooting depth of local vegetation) [Savenije, 2004]. If dry season rainfall is negligible, the time-integrated dry season soil moisture PDF can be computed, making it possible to analytically compute the full, annual soil moisture PDF for seasonally dry regions. We note that one of the advantages of the approach here is that it facilitates other stochastic derivations of ecological relevance, for instance the dry season soil moisture crossing properties.

The model is analogous to the streamflow model of Müller et al. [2014], where an existing stochastic streamflow model [Botter et al., 2007a] was modified by assuming that the dry season streamflow is a function of the water storage in the catchment at the end of the wet season (a stochastic variable), followed by a deterministic, seasonal recession. Here, rather than assuming a deterministic form of streamflow recession during the dry season, we prescribe a deterministic loss rate of water to evapotranspiration, allowing stochasticity to arise in the soil water storage at the end of the last wet season storm.

We tested the model at a seasonally-dry site monitored within the AmeriFlux network.

AmeriFlux sites record micrometeorological and soil moisture data at high temporal resolutions, making them good candidates for comparisons to theory [Miller et al., 2007]. We first compare the empirical annual soil moisture PDF to the modeled annual soil moisture PDF. The annual time-scale PDF provides a parsimonious model test: it encodes all the features of a seasonal model, and allows all available soil moisture data to be brought to bear on model testing. Following this, the model's prediction of the dry season mean first passage time below a soil moisture threshold is compared with a previously developed expression for this crossing time [Viola et al., 2008] and with data. The model developed by Viola et al. [2008] provides an interesting contrast in simplifying assumptions: the current model accounts for end of wet season variability and not dry season rainfall, and the model of Viola et al. [2008] are able to incorporates dry season rainfall but neglects variability in the dry season soil moisture initial condition.

2.2 Methods

Symbols Used

Throughout this section, $\Gamma(*)$ refers to the gamma function, $\Gamma(*, *)$ to the generalized incomplete gamma function, and $\delta(*)$ to the Dirac delta function [Abramowitz and Stegun, 1964]. The probability density function is represented by p and the cumulative density function (CDF) by P. Subscripts, in upper case, denote the random variable being described by the PDF or CDF, and the the corresponding lower case characters denote the observed value of the random variable. For example, the PDF and the CDF of the soil moisture S at value s are denoted as $p_S(s)$ and $P_S(s)$ respectively.

Seasonally dry stochastic soil moisture: Wet season soil moisture and the dry season initial condition

The mass balance for water with constant density within a one dimensional control volume spanning the active rooting zone depth, Z_r [L] is given by:

$$nZ_r \frac{ds}{dt} = R(t) - \operatorname{ET}[s(t)] - \operatorname{LQ}[s(t), t], \qquad (2.1)$$

where n [] is the porosity, LQ [L T⁻¹] is the flux of soil moisture leaving the control volume as runoff or deep drainage and ET [L T⁻¹] is the flux of soil moisture lost due to evapotranspiration. Following Porporato et al. [2004], this model simplifies the dependence of evapotranspiration and drainage dynamics on soil water content by assuming that any water storage in excess of field capacity (s_1) is instantly drained. While evapotranspiration is assumed to occur at a prescribed maximum rate ET_{max} at s_1 , this rate of loss declines linearly until it goes to zero evapotranspiration at the wilting point (s_w) :

$$ET(s) = \begin{cases} 0 & : s \in (0, s_w) \\ ET_{\max} \cdot \frac{s - s_w}{s_1 - s_w} & : s \in [s_w, s_1] \end{cases}$$
(2.2)

Rainfall, R(t) is modeled on daily timescales as a Poisson process with exponentially distributed depths, making Equation 2.1 a stochastic differential equation.

Under stationary climate conditions, assumed to prevail during the wet season in a seasonally dry climate, the steady state PDF of the non-dimensional, wet-season, relative soil moisture X_w (scaled to assume a value of zero at the wilting point and a value of one at field capacity, $x = \frac{s-s_w}{s_1-s_w}$) can be obtained [Porporato et al., 2004]:

$$p_{X_w}(x_w) = \frac{N}{\eta} x_w^{\lambda/\eta - 1} e^{-\gamma x_w} \quad \text{for} \quad x_w \in [0, 1],$$
 (2.3)

The model is characterized by the two non-dimensional parameters:

$$\gamma = \frac{w_0}{\alpha} \quad \text{and} \quad \frac{\lambda}{\eta} = \frac{\lambda w_0}{\text{ET}_{\text{max}}},$$
(2.4)

where $w_0[L] = (s_1 - s_w)nZ_r$ is the total available water storage, λ [T⁻¹] is the reciprocal of the mean waiting time between rainfall events, α [L] is the mean depth of the rainfall events, ET_{max} [L T⁻¹] is the maximum rate of evapotranspiration from the soil, and N is a normalization constant. Due to seasonal changes in temperature, insolation and relative humidity, ET_{max} is likely to vary between the wet and dry seasons, leading also to different values of η . We therefore use η_w to describe the wet season dynamics and η_d to describe the dry season dynamics.

Moisture dynamics during the dry season are assumed to consist of a deterministic dry down due to ongoing evapotranspiration from the soil. The dry season soil moisture initial condition, X_0 , is treated as a stochastic variable generated by the last significant storm of the wet season. This soil moisture state at the onset of the dry season is the sum of the soil moisture condition that prevailed before the final wet season storm and the soil moisture increment introduced by this storm (this biases X_0 towards a more saturated state than prevails during the wet season as a whole). To determine the distribution of this initial condition, we first nondimensionalize the depth of each incoming rainfall event (R) by the transformation:

$$R' = \frac{R}{nZ_r},\tag{2.5}$$

and then perform the rescaling:

$$H = \frac{R'}{s_1 - s_w} = \frac{R}{w_0},$$
(2.6)

which ensures that the new increment, H, has the same non-dimensional scale as X_w . Based on the assumption that rainfall depths are exponentially distributed, the CDF and PDF of H are given by:

$$P_H(h) = 1 - e^{-\gamma h} \quad \text{and} \quad p_H(h) = \gamma e^{-\gamma h}. \tag{2.7}$$

Since the Poisson rainfall process is memoryless, the soil moisture conditions prior to the final wet season storm are described simply by the steady-state wet season soil moisture PDF, p_{X_w} . The dry season soil moisture initial condition (X_0) is the sum of the wet season soil moisture (X_w) and the rainfall depth (H) random variables:

$$X_0 = X_w + H, \tag{2.8}$$

assuming $X_w + H < 1$. There is a finite probability that the final rainfall increment of the wet season will lead to saturated soil conditions, so the initial condition distribution also contains an atom of probability at $X_0 = 1$. For a final rainfall increment that causes $X_0 < 1$, the PDF of X_0 conditioned on $X_0 < 1$ is:

$$p_{X_0|X_0<1}(x_0) = \int_0^{x_0} p_{X_w}(x) p_H(x_0 - x) dx = \frac{N\gamma}{\lambda} e^{-\gamma x_0} x_0^{\frac{\lambda}{\eta_w}} \quad \text{for} \quad x_0 \in (0, 1).$$
(2.9)

The atom of probability at $X_0 = 1$ is given by:

$$\Pr(X_0 = 1) = \int_0^1 p_{X_w}(x) \left(1 - P_H(1 - x)\right) dx = \frac{N}{\lambda} e^{-\gamma}, \qquad (2.10)$$

where $1 - P_H(1 - x)$ is the probability that the final rainfall increment is greater than 1 - x, leading to saturated conditions at the end of the wet season. The PDF for the soil moisture conditions at the start of the dry season is:

$$p_{X_0}(x_0) = \frac{N\gamma}{\lambda} e^{-\gamma x_0} x_0^{\frac{\lambda}{\eta w}} + \frac{N}{\lambda} e^{-\gamma} \cdot \delta(x_0 - 1) \quad \text{for} \quad x_0 \in (0, 1].$$
(2.11)

Dry season soil moisture PDF

In addition to previous studies, which derive the probabilistic dynamics of dry season soil moisture at the daily (X_d^t) timescale, we derive the PDF for dry season soil moisture at the annual (X_d) timescale. X_d^t and X_d represent slightly different ways to quantify soil moisture values. The former is a soil moisture PDF that changes as a function of the number of days (t) from the start of the dry season. On the first day (t = 0), X_d^t is necessarily distributed as X_0 . In previous studies [Viola et al., 2008], X_d^t also varies depending on the likelihood of rainfall occurring. In this study, assuming no significant dry season rainfall, evapotranspirative processes deplete soil water stores as the dry season progresses, biasing X_d^t towards more dry conditions. In contrast, the time-independent dry season soil moisture

distribution, X_d , represents the time-integrated probability distribution of soil moisture over the course of the dry season. It is therefore independent of time. This distribution combines all dry season soil moisture data into a single, lumped distribution, encapsulating variability from the antecedent wet season and from the temporal evolution of the daily dry season soil moisture.

The distribution of X_d^t

In the absence of dry season rainfall, the dry season soil moisture dynamics given by the soil water balance (Equation 2.1) are described by a deterministic, exponential dry down:

$$x_d^t = x_0 e^{-\eta_d t} \implies x_0 = x_d^t e^{\eta_d t}, \tag{2.12}$$

where η_d is the dry season equivalent of η_w . Equation 2.12 specifies a unique dry season soil moisture value for a given initial condition (x_0) and a given number of days into the dry season (t). This implies that the time-dependent, dry season soil moisture is a derived random variable (X_d^t) of the dry season initial condition:

$$p_{X_d^t}\left(x_d^t\right) = p_{X_0}\left(x_0\left(x_d^t\right)\right)\frac{dx_0}{dx_d^t}$$
$$= e^{\eta_d t} \cdot \left[\frac{N\gamma}{\lambda}\exp\left(-\gamma x_d^t e^{\eta_d t}\right)\left(x_d^t e^{\eta_d t}\right)^{\lambda/\eta_w} + \frac{N}{\lambda}e^{-\gamma}\delta\left(x_d^t - e^{-\eta_d t}\right)\right] \text{ for } x_d^t \in \left(0, e^{-\eta_d t}\right]. (2.13)$$

The time-dependent moments of X_d^t can be obtained from its moment generating function M(c) [Ross, 2009], which is defined as:

$$M(c) = \mathbb{E}[\exp(c \cdot x_0 e^{-\eta_d t})] = \frac{N}{\lambda} e^{-\gamma} \cdot \exp\left(c \cdot e^{-\eta_d t}\right) + \int_0^1 p_{X_0}(x_0) \cdot \exp(c \cdot x_0 e^{-\eta_d t}) dx_0$$
$$= \frac{N\left[\gamma \left(\gamma - c e^{-\eta_d t}\right)^{-\frac{\eta_w + \lambda}{\eta_w}} \left[\Gamma\left(\frac{\eta_w + \lambda}{\eta_w}\right) - \Gamma\left(\frac{\eta_w + \lambda}{\eta_w}, \gamma - e^{-\eta_d t}c\right)\right] + e^{c e^{-\eta_d t} - \gamma}\right]}{\lambda}.$$
 (2.14)

For example, the mean of X_d^t is then calculated from M(c) as:

$$\langle X_d^t \rangle = \frac{dM}{dc} \Big|_{c=0}$$

$$= \frac{N(\eta_w + \lambda)\gamma^{-\frac{\eta_w + \lambda}{\eta_w}} \left[\Gamma\left(\frac{\eta_w + \lambda}{\eta_w}\right) - \Gamma\left(\frac{\eta_w + \lambda}{\eta_w}, \gamma\right)\right]}{\eta_w \lambda} \cdot e^{-\eta_d t} = \langle X_0 \rangle e^{-\eta_d t},$$

$$(2.15)$$

which is exactly the form of the exponential dry down with $x_0 = \langle X_0 \rangle$. This approach could be extended to calculate higher order moments of X_d^t , such as its variance, or to incorporate temporal variability in the climate parameters. For example, $\eta_d = \eta_d(t)$ can be substituted into Equation 2.14 without affecting the analytical tractability of the moment calculations.

The distribution of X_d

For simplicity, we assume that the duration of the dry season (t_d) is constant from year to year. Provided that the available soil storage does not grossly exceed the mean soil moisture increment and that the wet season duration is long compared to the average rainfall interarrival time, the duration of the transient wet-up period at the start of the wet season can be assumed insignificant compared to the duration of the entire wet season, and the distribution of soil moisture during the wet season will be independent of the wet season length (note that both of these conditions would likely be violated in very arid regions, meaning that these models are inappropriate for representing soil moisture dynamics in true deserts). If the assumption of independence is valid, then the annual soil moisture PDF can be calculated as a weighted sum of the dry season and wet season soil moisture PDFs:

$$p_X(x) = \left(1 - \frac{t_d}{365}\right) p_{X_w}(x) + \frac{t_d}{365} p_{X_d}(x).$$
(2.16)

To calculate the time-integrated dry season PDF, we first note that the CDF of the dry season soil moisture (X_d) conditioned on the value of X_0 is:

$$P_{X_d|X_0}(x_d, x_0) = P\{X_d \le x_d | X_0 = x_0\}$$

$$= \begin{cases} 0 & : x_d \in (0, x_0 e^{-\eta_d t_d}) \\ \frac{\ln\left(\frac{x_0}{x_d}\right)}{\eta_d t_d} & : x_d \in [x_0 e^{-\eta_d t_d}, x_0) \\ 1 & : x_d \in [x_0, 1). \end{cases}$$
(2.17)

The logic behind the second line of (2.17) is the (admittedly obvious) observation that time itself is uniformly distributed over the course of the dry season; that is, each day from the dry season has an equal probability of being selected in a random sample of days from the dry season. This implies that the CDF of time over the dry season is a simple linear function: $P_T(t) = t/t_d$: $t \in [0, t_d]$. Since, for a given initial condition, the dry season soil moisture is only a function of time, the distribution of the integrated dry season soil moisture random variable is a derived distribution of the uniform distribution of time. The expression $x_d = x_0 e^{-\eta_d t}$ therefore can be solved for t and then substituted into $P_T(t) = t/t_d$ to obtain the CDF of X_d (with an appropriate transformation of the domain) which is the time-integrated form of X_d^t . The time-integrated dry season PDF of X_d given $X_0 = x_0$ is then:

$$p_{X_d|X_0}(x_d, x_0) = \frac{dP_{X_d|X_0=x_0}}{dx_d}$$

= $\frac{1}{t_d x_d \eta_d}$ for $x_0 e^{-\eta_d t_d} \le x_d \le x_0.$ (2.18)

The unconditional, time-integrated dry season PDF can be found from the conditional distribution of Equation 2.18 by integrating over the distribution of X_0 :

$$p_{X_{d}}(x_{d}) = \int_{X_{0}} p_{X_{d}|X_{0}}(x_{d}, x_{0}) p_{X_{0}}(x_{0}) dx_{0}$$

$$= \begin{cases} \int_{x_{d}}^{x_{d}e^{\eta_{d}t_{d}}} p_{X_{d}|X_{0}}(x_{d}, x_{0}) p_{X_{0}}(x_{0}) dx_{0} & : x_{d} \in (0, e^{-\eta_{d}t_{d}}) \\ \int_{x_{d}}^{1} p_{X_{d}|X_{0}}(x_{d}, x_{0}) p_{X_{0}}(x_{0}) dx_{0} & : x_{d} \in [e^{-\eta_{d}t_{d}}, 1] \end{cases}$$

$$= \begin{cases} \frac{N\gamma^{-\frac{\lambda}{\eta_{w}}} \left[\Gamma(\frac{\eta_{w}+\lambda}{\eta_{w}}, x_{d}\gamma) - \Gamma(\frac{\eta_{w}+\lambda}{\eta_{w}}, x_{d}\gamma e^{t_{d}\eta_{d}}}) \right]}{t_{d}\eta_{d}\lambda x_{d}} & : x_{d} \in (0, e^{-\eta_{d}t_{d}}) \\ \frac{N\gamma^{-\frac{\lambda}{\eta_{w}}} \left[\Gamma(\frac{\eta_{w}+\lambda}{\eta_{w}}, x_{d}\gamma) - \Gamma(\frac{\eta_{w}+\lambda}{\eta_{w}}, \gamma) \right]}{t_{d}\eta_{d}\lambda x_{d}}} + \frac{Ne^{-\gamma}}{\lambda t_{d}x_{d}\eta_{d}} & : x_{d} \in [e^{-\eta_{d}t_{d}}, 1]. \end{cases}$$

$$(2.19)$$

There are two distinct domains for the distribution p_{X_d} : 1) $x_d \in (0, e^{-\eta_d t_d})$, and 2) $x_d \in [e^{-\eta_d t_d}, 1)$. The only way for the dry season soil moisture to take on a value in the first domain (that is, the only source of probability density in that domain) comes from a dry season soil moisture initial condition that is greater than the value of x_d itself (hence the lower bound), but less than the initial condition value which leads to a dry season soil moisture value of x_d on the very last day of the dry season (hence the upper bound at $x_d e^{\eta_d t_d}$). In the second domain, however, there exist values for x_0 that are greater than 1 (which is outside the domain of p_{X_0}) which could lead to a dry season soil moisture value $x_d \in [e^{-\eta_d t_d}, 1]$. Therefore, the upper bound on x_0 should be fixed to 1 in this domain, while the lower bound remains the same.

The annual soil moisture PDF is then calculated as the weighted sum of the dry season and wet season soil moisture PDFs, according to Equation 2.16.

Dry season mean first passage time

As a simplified measure of dry season plant water stress in regions where the growing season and the dry season coincide, Rodríguez-Iturbe et al. [2001] and Viola et al. [2008] consider

the mean fraction of the dry season that soil moisture is less than some threshold (s^*) , below which plants become water stressed:

Plant water stress
$$\propto \frac{t_d - \bar{T}_{s^*}}{t_d}$$
. (2.20)

Here, \bar{T}_{s^*} is the mean time from the start of the growing (dry) season to reach the water stress threshold s^* . Using a number of simplifying assumptions, Viola et al. [2008] derive the following approximate expression for \bar{T}_{s^*} :

$$\bar{T}_{s^*} = \frac{\min\left[(s_1 - s^*) \, nZ_r, t_w \left(\alpha_w \lambda_w - ET_{max,w}\right) - (s^* - s_w) \, nZ_r\right]}{ET_{max,d} - \alpha_d \lambda_d},\tag{2.21}$$

where α_w, λ_w are wet season rainfall statistics (mean depth and frequency), t_w is the duration of the wet season, α_d, λ_d are dry season rainfall statistics, and $ET_{max,w}, ET_{max,d}$ are the wet and dry season maximum rates of evapotranspiration. This expression assumes that the soil moisture conditions at the beginning of the growing season are well approximated by the mean wet season soil moisture conditions and that the threshold crossing time is a linear function of this initial condition.

The model derived by Viola et al. [2008] accounts for stochasticity in T_{s^*} that results from dry season rainfall processes. The theory presented here yields an analogous expression for T_{s^*} that instead accounts for variability in the dry season initial soil moisture condition. First, we use Equation 2.12 to obtain an expression for T_{x^*} (where x^* is the normalized equivalent of the relative soil moisture threshold, $x^* = \frac{s^* - s_w}{s_1 - s_w}$) as a function of the dry season initial condition:

$$x^* = x_0 e^{-\eta_d T_{x^*}} \implies T_{x^*}(x_0) = \frac{\ln \frac{x_0}{x^*}}{\eta_d}.$$
 (2.22)

Making the simplifying assumption that the dry season initial condition is greater than the soil moisture stress threshold with high probability $(\operatorname{Prob}[X_0 > x^*] \gg \operatorname{Prob}[X_0 < x^*])$, the mean of T_{x^*} is easily obtained by integrating over the PDF for X_0 :

$$T_{x^*} = \int_{x^*}^{1} p_{X_0}(x_0) T_{x^*}(x_0) x_0$$

= $\frac{N}{\eta_d \lambda} \left\{ \frac{\gamma \eta_w^2 x^* \frac{\eta_w + \lambda}{\eta_w} A\left(\frac{\lambda}{\eta_w} + 1, \frac{\lambda}{\eta_w} + 1; \frac{\lambda}{\eta_w} + 2, \frac{\lambda}{\eta_w} + 2; -x^*\gamma\right)}{(\eta_w + \lambda)^2} - \frac{\gamma \eta_w^2 A\left(\frac{\lambda}{\eta_w} + 1, \frac{\lambda}{\eta_w} + 1; \frac{\lambda}{\eta_w} + 2, \frac{\lambda}{\eta_w} + 2; -\gamma\right)}{(\eta_w + \lambda)^2} + \ln x^* \left[\gamma \operatorname{E}\left(-\frac{\lambda}{\eta_w}, \gamma\right) - \gamma^{-\frac{\lambda}{\eta_w}} \Gamma\left(\frac{\eta_w + \lambda}{\eta_w}\right) - e^{-\gamma}\right] \right\},$ (2.23)

where E(n, z) is the exponential integral function $E_n(z) = \int_1^\infty \frac{e^{-zt}}{t^n} dt$ and A is a version of the generalized hypergeometric function [Abramowitz and Stegun, 1964]:

$$A(a_1, a_2; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!}, \quad \text{with}$$
(2.24)
$$(a)_0 = 1 \quad \text{and} \quad (a)_n = a(a+1)(a+2)...(a+n-1), \quad n \ge 1$$

Case study

Site

A case study from the AmeriFlux station at Tonzi Ranch is presented to empirically test the annual soil moisture distribution and mean first crossing time models. Tonzi Ranch is an oak savanna woodland located in the foothills of the Sierra Nevada near Ione, California. The climate is Mediterranean, characterized by wet, cool winters and hot, dry summers [Baldocchi et al., 2010, Ma et al., 2007]. Twelve years of soil moisture data (2001 - 2013), collected at depths of 5cm, 20cm, and 50cm using Theta Probe model ML2-X impedance sensors [Delta-T Devices Miller et al., 2007], are analyzed for this study.

Model Parameterization

The eight model parameters $(t_d, \text{ET}_{\max}, \alpha, \lambda, s_w, s_1, Z_r, n)$ were computed from the Ameri-Flux datasets. One of the challenges in applying our model is that the assumption of binary rainfall seasonality is an approximation at best, and the model user must make decisions about how to pragmatically separate the wet and dry periods. Numerous rubrics could feasibly be used to differentiate these seasons, leading to variable results. Here we present results based on two distinct partitioning rubrics, to explore the sensitivity of the results to reasonable choices.

For Rubric 1, we simply examine the spring months from March through the end of May and choose the latest well-defined (change in sign of the first derivative) soil moisture peak that is greater than a threshold, chosen here to be x = 0.6. For Rubric 2, we first extract each well-defined soil moisture peak during the spring period as potential start days for the dry season. We then calculate the mean of all the soil moisture local minima from January 1 to each potential dry season starting peak. The chosen peak is the final peak of the wet season greater than the mean of the preceding soil moisture minima. This ensures that the selection of the initial condition is strongly correlated with wet season conditions. In both cases, the end of the dry season is chosen as the minimum soil moisture value between the start of the dry season and the start of the next calendar year. Figure 2.1 presents a plot of the seasonally partitioned soil moisture time series corresponding to each rubric. The dry season length (t_d) is calculated by subtracting the median wet season length from 365. Each extracted dry season is used to calculate the first passage time below soil moisture thresholds ranging from 1% to 50% of the soil moisture wilting point.



Figure 2.1: Relative soil moisture time series data from Tonzi Ranch. Extracted dry seasons are denoted with shaded rectangles. The top row uses seasonality partitioning Rubric 1, and the bottom row uses Rubric 2.

We estimated ET_{max} using the Priestly-Taylor equation:

$$ET_{max} = 1.26 \frac{e'_s}{(e'_x + g) L} (R_n - G)$$
(2.25)

where g is the psychrometric constant, L is the latent heat of vaporization of water, G the ground heat flux, R_n the net radiation, and e'_s is the derivative of the saturation vapor pressure (calculated using the Clausius-Clapeyron equation) with respect to temperature. The value of ET_{max} was computed separately for each season based on the seasonal daily mean value of ET_{max} . The mean rainfall depth and mean waiting time between rainfall events (for wet and dry seasons), α and λ , were derived by aggregating 30-minute AmeriFlux rain gauge data to the daily time scale. Due to the fact that the duration of Pacific coastal storm systems is typically longer than one day, we treated multi-day storm events as single events. Similarly to previous sensitivity analyses using stochastic streamflow models [Müller et al., 2014], however, we find that this deviation from the exact specification of rainfall as a Poisson process does not significantly degrade model performance.

The soil depth (Z_r) and porosity (n) were obtained from the Ameriflux biological data. Although soil textural data are available for the site, using a pedotransfer function [e.g.

Parameter	Rubric 1	Rubric 2
t_d [days]	202	212
ET_{max} wet season (dry season) [mm/day]	2.01(5.54)	1.88(5.50)
α wet season (dry season) [mm]	24.45(5.52)	24.63(6.67)
λ wet season (dry season) [day ⁻¹]	0.14(0.04)	0.14(0.04)
s_w []	0.26	0.26
s_1 []	0.82	0.82
$Z_r \; [\mathrm{mm}]$	600	600
n []	0.45	0.45

Tonzi Ranch - Site characteristics calculated from AmeriFlux data.

Table 2.1:

Saxton and Rawls, 2006] to estimate the model parameters is problematic for the Porporato et al. [2004] model, since the simplifications made to the drainage and evaporation processes mean that s_1 and s_w in the model do not map precisely to conventional definitions of field capacity and wilting point. We therefore calibrate these parameters, recognizing that these values primarily affect the validity of the wet season PDF (i.e. the underlying Porporato et al. [2004] model).

Bulk volumetric soil water content is calculated by zonally averaging the soil moisture measurements over the depth of the soil column. For this type of averaging, each depth is assigned the soil moisture value of the nearest measured value, then the standard integrated average of the resulting profile is computed. Table 2.1 summarizes model parameters.

Model Evaluation

To evaluate the performance of our model, we used the Nash-Sutcliffe efficiency (NSE) applied to the soil moisture quantiles:

$$NSE = 1 - \frac{\sum_{i=1}^{364} (\hat{q}_i - q_i)^2}{\sum_{j=1}^{364} (q_j - \frac{1}{364} \sum_{k=1}^{364} q_k)^2},$$
(2.26)

where \hat{q}_i and q_i are the empirical and theoretical relative soil moisture values associated with quantile *i* of the annual soil moisture PDF. The *NSE* has been used extensively for the assessment of hydrologic models [Castellarin et al., 2004, Müller et al., 2014, Nash and Sutcliffe, 1970].

2.3 Results and Discussion

The case study shown in Figure 2.2 supports the applicability of the model for capturing the annual PDF of soil moisture in seasonally dry regions. Although the domain of the analytical PDF is supported on $x \in [0, 1]$, the raw data take on values outside of this range.



Figure 2.2: Integrated annual soil moisture PDFs compared to AmeriFlux soil moisture data, partitioned using Rubric 1 (a) and Rubric 2 (b).

Figure 2.2 therefore shows the raw empirical histogram and a truncated histogram, the latter computed by setting all soil moisture values above field capacity (x = 1) to field capacity and all values below the wilting point (x = 0) to the wilting point. The two seasonal partitioning rubrics (Figure 2.2 a,b) yield similar results at the Tonzi Ranch site. Using Rubric 1 and un-truncated soil moisture data, the model NSE is 0.85, indicating that even the simple bounded model yields a good fit to field measurements. For the truncated quantiles, the NSE for the analytical model is 0.86. Using Rubric 2, the model NSE is 0.89 against the un-truncated soil moisture data and 0.90 against the truncated soil moisture data.

The computed mean first passage times for each partitioning rubric are plotted in Figure 2.3. There is strong agreement between the mean first passage time for the current model using both rubrics, suggesting that dry season rainfall plays an insignificant role at Tonzi Ranch in determining the time for soil moisture to drop below the presented range of soil moisture thresholds. We also found that even for the lowest threshold, set to 1% of the soil moisture wilting point, all dry seasons were long enough for soil moisture levels to drop below the threshold. This implies that variability in the dry season length at this site is not an issue when considering first passage times, as truncation (due to the beginning of the next wet season) of the seasonal dry down (above ecological stress thresholds, for instance) is unlikely. The model of Viola et al. [2008] works well for higher soil moisture thresholds, but underestimates the mean crossing time for lower thresholds. These differences likely arise from the fact that, in order to obtain an approximate analytical expression for \bar{T}_{s^*} , Viola et al. [2008] both linearize the dry season soil moisture dynamics and use a simplified expression for the soil moisture conditions at the start of the dry season. Consequently, the new model, which includes a more accurate description of the dry season initial conditions



Figure 2.3: Mean time from the beginning of the dry season to reach a range of soil moisture thresholds (x^*) using Ameriflux data (blue dashed line), the current model (solid line), and the model developed by Viola et al. [2008] (finely dashed line).

and does not linearize the functional form of the soil moisture dry down, outperforms the model presented Viola et al. [2008]. We suspect, however, that at sites with more significant dry season rainfall contributions, the model of Viola et al. [2008] would become increasingly more appropriate.

Clearly the model presented here could be further elaborated, for example through use of a more complete soil moisture model, such as Laio [2002]. More realistic loss functions can be incorporated into the soil moisture model without significantly altering the logic of the approaches illustrated here. However, the use of a more complex loss function leads to considerably more complex algebra to manage the conditionality associated with multipart piecewise functions, variable initial conditions, and a finite dry season length. Here we elected to base the analysis on the simpler Porporato et al. [2004] model to ensure that the logic of the approach was not obscured.

The model also makes the seemingly inconsistent assumption that the dry season can have a variable start date, but that season lengths are fixed. To resolve this, the stochastic model should not be treated as a one-to-one mapping of the soil moisture time series into a probabilistic domain, but rather as a model developed in the probabilistic domain. The parameterization then represents the deliberate decision to capture the important source of variation imposed by conditions at the end of the wet season, while retaining analytical tractability. The model then assumes that this form of variability is more important than creating a perfect mapping between the time series and probability domains.

Other sources of stochasticity, such as the dry season maximum evapotranspiration rates, could also be incorporated. Inter-annual variations in ET_{max} , however, are subordinate to inter-annual variations in rainfall when driving hydrological processes [see for example Milly

and Dunne, 2002], while variability in the length of the dry season could only be expected to influence the soil moisture PDF when the timescales of dry down approach the mean dry season length. Thus, the level of complexity used here sufficiently captures key stochastic drivers of soil moisture dynamics in seasonally dry systems.

2.4 Conclusion

This work presents an analytical model to compute the PDF of a bounded random variable, soil moisture, in climates with two distinct seasons. The formulation is used to derive a simple analytical expression for the dry season mean time to reach a threshold of water stress s^* . The presented model and an existing model are tested and compared using soil moisture data from the Tonzi Ranch Ameriflux site. The case study demonstrates that the current model performs well, despite simplifications in the underlying evapotranspiration and drainage dynamics, and may be particularly valuable in regions such as California characterized by pronounced seasonality in rainfall, and large fluctuations around mean wet season soil moisture.

Chapter 3

Spatially variable water table recharge and the hillslope hydrologic response: Analytical solutions to the linearized hillslope Boussinesq equation

3.1 Introduction

The hillslope Boussinesq equation (HB) forms a key component of hydraulic groundwater theory. It describes the spatiotemporal evolution of a slowly-draining water table along a one-dimensional hillslope, allowing storage and discharge from the hillslope to be estimated at any point in time. The HB is derived under the assumption that saturated subsurface flow occurs purely parallel to an impermeable bottom boundary (the hydrostatic assumption), that the effects of capillarity can be lumped into the water table evolution (an assumption reflected in the use of the 'drainable porosity parameter', n_e , rather than the true porosity, n, when computing mass balances in the subsurface [Hilberts et al., 2007]), and that a net recharge parameter, R(x,t), can be specified to account for the effects of both vertical infiltration from the surface to the water table and losses of water due to evaporation and root uptake in the unsaturated zone [Brutsaert, 2005]. These conditions and their outcomes are illustrated in Figure 3.1.

With these assumptions, Darcy's Law can be combined with the mass continuity equation to obtain HB as the governing partial differential equation for a homogeneous, unconfined aquifer set above a sloping impermeable layer:

$$\frac{\partial \eta}{\partial t} = \frac{k_0 \cos \theta}{n_e} \frac{\partial}{\partial x} \left(\eta \frac{\partial \eta}{\partial x} \right) + \frac{k_0 \sin \theta}{n_e} \frac{\partial \eta}{\partial x} + \frac{R(x, t)}{n_e}.$$
(3.1)

Here η [L] is the height of the water table above the impermeable layer in the sloping reference frame, k_0 [L/T] is the hydraulic conductivity, n_e is the drainable porosity, θ is



Figure 3.1: One-dimensional profile of a hillslope at angle θ with a water table height profile (η) and initial water table depth (D). Hillslope length (B_x) is defined along the hillslope axis (x) and the water table recharge (R) can vary in both space (x) and time (t). After Brutsaert [2005].

the slope angle of the underlying impermeable layer, and R(x,t) [L/T] is a source term representing any recharge to the water table.

No general analytical solutions are known for the full HB, although a limited set of solutions are available through approximation methods [Boussinesq, 1877, Polubarinova-Kochina, 1962]. To progress towards a more generalizable solution method, Brutsaert [1994] linearized (3.1) by expanding the diffusive term:

$$\frac{\partial}{\partial x} \left(\eta \frac{\partial \eta}{\partial x} \right) = \left(\frac{\partial \eta}{\partial x} \right)^2 + \eta \frac{\partial^2 \eta}{\partial x^2}, \tag{3.2}$$

and noting that if η fluctuates locally around an average water table height (η_0) the term $\left(\frac{\partial \eta}{\partial x}\right)^2$ is negligibly small and η in the second term of (3.2) can be replaced with η_0 . The linearized HB (LHB) is then given by:

$$\frac{\partial \eta}{\partial t} = \frac{k_0 \eta_0 \cos \theta}{n_e} \frac{\partial^2 \eta}{\partial x^2} + \frac{k_0 \sin \theta}{n_e} \frac{\partial \eta}{\partial x} + \frac{R(x, t)}{n_e}.$$
(3.3)

Equation (3.3) has been successfully used to develop minimalistic, physically based hydrologic models and baseflow separation algorithms [Huyck et al., 2005b, Pauwels et al., 2002, Rupp et al., 2009]. Here, we extend the analytical treatment of Equation (3.3). While existing solution methodologies [Brutsaert, 1994, Pauwels et al., 2002, Troch et al., 2004, 2013, Verhoest and Troch, 2000] assume spatially homogeneous recharge, the new solution method admits analytical solutions for spatiotemporally-variable forcing. We are motivated by high quality, remotely sensed data that highlights spatial heterogeneity in the distribution
of vegetation, which we hypothesize could introduce significant spatial variability in water balance partitioning. With this in mind, we set two aims for the study:

- 1. To apply a recently developed solution technique for the analytical solution of spatially forced advection-diffusion equations, such as the linearized hillslope Boussinesq equation;
- 2. To explore the potential effects of spatially variable water table recharge and hillslope morphology on the hillslope hydrologic response and to determine when the new solution can be used in topographically complex situations.

In pursuit of these aims, we first develop a novel solution for the linearized HB based on recently developed methods for solving advection-diffusion equations [Guerrero et al., 2009]. In addition to a spatiotemporally varying recharge term, the new methods do not require the use of Laplace transforms for solving the partial differential equations, leading to a simpler analytical formulation for the problem. The solution methodology is extended to also address the linearized hillslope-storage Boussinesq (LHSB) equation, which accounts for the effects of convergence or divergence in hillslope morphology on water table dynamics Troch et al. [2003].

Following the development of new solutions to the LHB and the LHSB, we examine the impact of spatially heterogeneous water table recharge on: (i) instantaneous water table and baseflow responses to pulse inputs of water; (ii) seasonal hillslope storage volumes and seasonal baseflow recession behavior; and (iii) hillslope flow recession characteristics in topographically complex situations. Topographic complexity is an important control on patterns of hillslope discharge and should be considered, but the linearization of the HSB requires more approximations in comparison to the HB [Troch et al., 2003, Woods et al., 1997]. Accordingly, we provide a detailed error analysis of LHSB performance across a variety of hillslopes and recharge patterns.

All cases are explored in the context of seasonally dry climates (where the volume, persistence and profile of baseflow is of critical ecological and anthropic importance), on hillslopes where discharge to the channel is dominated by saturated subsurface flow. Throughout the study we employ simple (smoothed and homogeneous) representations of hillslope geometry, hydraulic properties, seasonal climate dynamics and spatial changes in hillslope water table recharge. While we recognize that deviations from these idealized conditions could alter the details of the results presented, the simplified cases capture the critical features of seasonality, hillslope topography, and the possible effects of spatially variable vegetation on recharge fluxes.

3.2 Methods

Equation (3.3) is a linear, constant-coefficient, non-homogeneous advection-diffusion equation (non-homogeneous in this case means there exists a term in the PDE which is not proportional to the dependent variable or its derivatives). It has been solved explicitly for a variety of cases, including a no-recharge case [Brutsaert, 1994], with constant recharge [Verhoest and Troch, 2000], and with temporally variable recharge conditions (R(t)) [Pauwels et al., 2002]. These solutions and the broad utility of the LHB model were recently reviewed [Troch et al., 2013]. To date, all solutions presented to the LHB rely on the use of Laplace Transforms. While useful, Laplace Transform based methods are complex to implement, and constrain the range of solutions that can be obtained.

Here we outline an alternative, simpler solution methodology which allows the development of a more general analytical solution to the LHB. The solution methodology is based on a change of variable followed by the use of the Generalized Integral Transform Technique [Cotta, 1993, Guerrero et al., 2009].

General solution

We seek a general solution to (3.3) using the boundary conditions established by Brutsaert [1994]: (i) a no-flux boundary condition at the upslope boundary of the hillslope, corresponding to the presence of a groundwater divide, (ii) a water-table height of zero at the stream, approximating the conditions that prevail at the boundary where a water table meets a shallow, upland stream [Brutsaert, 2005], and (iii) an initial condition that assumes a uniform water table depth above the bottom boundary along the entire hillslope. Mathematically, these conditions are:

stream boundary condition: $\eta = 0$ x = 0 t > 0, divide boundary condition: $\eta_0 \cos \theta \frac{\partial \eta}{\partial x} + \eta \sin \theta = 0$ $x = B_x$ t > 0, (3.4) initial condition: $\eta = D$ $0 \le x \le B_x$ t = 0.

Following Brutsaert [2005], we non-dimensionalize the variables with the following scalings:

$$x_{+} = \frac{x}{B_{x}}; \quad t_{+} = \frac{k_{0}\eta_{0}\cos\theta}{n_{e}B_{x}^{2}}t; \quad \eta_{+} = \frac{\eta}{D}; \quad R_{+} = \frac{R}{R_{\max}},$$
(3.5)

Many authors (e.g. Verhoest and Troch [2000]) non-dimensionalize the water table height by the depth of the aquifer; here we scale the water table height by its initial value and assume that the aquifer depth is great enough to contain all of the simulated water table heights. For the remainder of this derivation, we drop the subscripts for clarity and all variables will be dimensionless. The non-dimensionalization transforms (3.3) into:

$$\frac{\partial \eta}{\partial t} = \frac{\partial^2 \eta}{\partial x^2} + H_i \frac{\partial \eta}{\partial x} + H_r R(x, t), \qquad (3.6)$$

with:

$$H_i = \frac{B_x \tan \theta}{\eta_0}, \quad H_r = \frac{B_x^2 R_{\max}}{Dk_0 \eta_0 \cos \theta}.$$
(3.7)

The initial and boundary conditions must also be recast in dimensionless form:

dimensionless stream boundary condition: $\eta = 0$ x = 0 $t \ge 0$, dimensionless divide boundary condition: $\frac{\partial \eta}{\partial x} + H_i \eta = 0$ x = 1 $t \ge 0$, (3.8) dimensionless initial condition: $\eta = 1$ $0 \le x \le 1$ t = 0.

Using the technique presented by Guerrero et al. [2009], we make the following substitution:

$$\eta(x,t) = u(x,t) \exp\left(at + bx\right), \qquad (3.9)$$

$$a = -\frac{H_i^2}{4}, \quad b = -\frac{H_i}{2}.$$
 (3.10)

With this substitution, (3.6) is transformed into a purely diffusive problem with a modified source term and boundary conditions:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{H_r R(x,t)}{\exp\left(at + bx\right)},\tag{3.11}$$

$$u = 0 \quad x = 0 \quad t \ge 0,$$

$$\frac{\partial u}{\partial x} + \frac{H_i}{2}u = 0 \quad x = 1 \quad t \ge 0,$$

$$u = \exp(-bx) \quad 0 \le x \le 1 \quad t = 0.$$

(3.12)

The Generalized Integral Transform Technique, which is related to the method of eigenfunction expansion, can be used to solve (3.11) [Cotta, 1993, Mikhailov and Ozisik, 1987]. The method of eigenfunction expansion is a standard solution method for partial differential equations, which we do not outline completely here. The reader is referred to Haberman [2012] for the details of this solution method.

We assume that any solution to (3.11) is separable and of the form:

$$u(x,t) = \sum_{m=1}^{\infty} c_m(t)\phi_m(x).$$
 (3.13)

This separable solution can be obtained by first solving the eigen-problem:

$$\frac{d^2\phi}{dx^2} + \lambda^2\phi = 0. \tag{3.14}$$

Here ϕ and λ are the eigenfunction and eigenvalue of the boundary value problem associated with Equation (3.11). For the fixed-interval boundary conditions listed in (3.12), Equation (3.14) has eigenfunction solutions:

$$\phi_m = \sin(\lambda_m x) \quad m = 1, 2, 3, ...,$$
(3.15)

where the eigenvalues, λ_m , are the (infinitely many) transcendental roots of:

$$\lambda_m = -\frac{H_i \tan \lambda_m}{2} \quad \lambda_m > 0 \quad m = 1, 2, 3, \dots$$
(3.16)

The subscript m indicates that the eigen-problem has a countably infinite number of solutions for the chosen boundary conditions, allowing one to find an infinite series solution for (3.11) in the form of Equation (3.13). Substituting (3.13) into (3.11) yields:

$$\sum_{m=1}^{\infty} \frac{dc_m}{dt} \phi_m = \sum_{m=1}^{\infty} c_m \frac{d^2 \phi_m}{dx^2} + \frac{H_r R}{\exp(at + bx)}.$$
(3.17)

With Equation (3.14), Equation (3.17) can be rewritten as:

$$\sum_{m=1}^{\infty} \left[\left(\frac{dc_m}{dt} + \lambda_m^2 c_m \right) \phi_m \right] = \frac{H_r R}{\exp\left(at + bx\right)}.$$
(3.18)

Multiplying (3.18) by ϕ_n and integrating from 0 to 1 causes all terms except for that with m = n to go to zero (the orthogonality property of the eigenfunctions), yielding:

$$\frac{dc_n}{dt} + \lambda_n^2 c_n = \frac{\int_0^1 \frac{H_r R}{\exp(at+bx)} \phi_n dx}{\int_0^1 \phi_n^2 dx} = G(t).$$
(3.19)

Equation (3.19) requires an initial condition, which is obtained from the initial conditions of (3.11):

$$\sum_{n} c_n(0)\phi_n(x) = \exp(-bx).$$
(3.20)

Once again, the orthogonality property allows the series expansion to be evaluated:

$$C_n = c_n(0) = \frac{\int_0^1 \exp\left(-bx\right) \phi_n dx}{\int_0^1 \phi_n^2 dx},$$

$$4e^{-b}\lambda_n \left(b\sin\left(\lambda_n\right) + \lambda_n \left(\cos\left(\lambda_n\right) - e^b\right)\right)$$
(3.21)

$$=\frac{4e^{-b}\lambda_n \left(b\sin\left(\lambda_n\right)+\lambda_n \left(\cos\left(\lambda_n\right)-e^b\right)\right)}{\left(b^2+\lambda_n^2\right) \left(\sin\left(2\lambda_n\right)-2\lambda_n\right)}.$$

The solution to (3.19) is:

$$c_n(t) = \exp\left(-\lambda_n^2 t\right) \left[C_n + \int_0^t G(\tau) \exp\left(\lambda_n^2 \tau\right) d\tau \right], \qquad (3.22)$$

where τ is a dummy integration variable. Substituting (3.22) into (3.13) completes the solution for the transformed variable (u(x,t)). Using the variable transformation, Equation (3.9), and the solution for u(x,t), we can write the final, dimensionless solution for the water table height:

$$\eta(x,t) = \exp(at + bx) \sum_{n=1}^{\infty} c_n(t)\phi_n(x).$$
(3.23)

The volumetric outflow per length of channel $q [L^2/T]$ can be computed from the water table profile using the linearized version the water table flux at any point along the hillslope [Brutsaert, 2005]; in dimensionless form (with the outflow scaled by $k_0\eta_0 D \cos\theta/B_x$), this is:

$$q = -\left(\frac{\partial\eta}{\partial x} + H_i\eta\right). \tag{3.24}$$

At the stream, the expression for dimensionless q simplifies to:

$$q_{stream} = -\frac{\partial \eta}{\partial x}.$$
(3.25)

Equation (3.25) represents the volume of water that is delivered to the adjacent stream per length of channel.

Incorporation of spatio-temporally variable recharge

The formulation of particular solutions for Equation (3.22) is possible for a wide range of functional descriptions of the recharge dynamics in space and time (R(x,t)). Here we explore solutions that account for spatial variation of recharge as well as for variation of the total recharge volume in time due to the seasonality of rainfall or evaporation. To achieve this in general terms, we treat the recharge as a product of two functions, one of which varies in space and the other of which varies in time:

$$R(x,t) = M(t) N(x).$$

$$(3.26)$$

R is the total recharge normalized by its maximum rate, so M(t) and N(x) must be chosen to lie in the closed interval [0, 1]. Substituting Equation (3.26) into Equation (3.19), the right hand side of (3.19) becomes:

$$\frac{\int_{0}^{1} \frac{H_{r}M(t)N(x)}{\exp(at+bx)} \phi_{n} dx}{\int_{0}^{1} \phi_{n}^{2} dx} = \Phi_{n} M(t) e^{-at},$$
(3.27)

with

$$\Phi_n = \frac{\int_0^1 H_r N(x) \exp(-bx) \phi_n dx}{\int_0^1 \phi_n^2 dx}.$$
(3.28)

The solution to Equation (3.19) is then:

$$c_n(t) = \exp\left(-\lambda_n^2 t\right) \left[C_n + \Phi_n \int_0^t M(\tau) \exp\left(\lambda_n^2 \tau - a\tau\right) d\tau \right], \qquad (3.29)$$

where τ is again a dummy integration variable. Using Equations (3.25), (3.23), and (3.15), the dimensionless expression for outflow at the stream is found as:

$$q_{stream} = -e^{at} \sum_{n} \lambda_n c_n \left(t \right) \tag{3.30}$$

Operationally, the solution can be implemented as follows: First, specify M(t) and N(x)(defined within the unit interval for dimensionless space and time variables) for the desired form of water table recharge. Next, integrate Equation (3.28) using N(x) to find Φ_n . Now, substitute Φ_n and M(t) into Equation (3.29) to find the time dependent series coefficients $(c_n(t))$. With the eigenfunctions (ϕ_n) and the eigenvalues (λ_n) from Equation (3.16), substitute $c_n(t)$ into Equation (3.23) to obtain the final (dimensionless) solution for the spatiotemporal evolution of the water table surface.

The hillslope-storage Boussinesq equation

The solution methodology applied to Equation (3.3) can also be used to find solutions to the more general linearized hillslope-storage Boussinesq equation [Troch et al., 2003] which extends the LHB to account for topographic variation associated with hillslope divergence or convergence. As with the LHB, the LHSB equation is more tractable than the nonlinear form and has been solved for constant recharge [Troch et al., 2004]. Here we briefly outline the solution method and solution obtained for the LHSB with spatio-temporally varying recharge. The LHSB takes the form:

$$\frac{\partial S}{\partial t} = K \frac{\partial^2 S}{\partial x^2} + U \frac{\partial S}{\partial x} + Rw, \qquad (3.31)$$

where S [L²] is soil moisture storage per longitudinal length of hillslope, R [L/T] is some spatiotemporally varying recharge term , and w [L], K [L²/T], and U [L/T] are defined as:

$$w = g e^{hx}, \tag{3.32}$$

$$K = \frac{k_0 \eta_0 \cos \theta}{n_c},\tag{3.33}$$

$$U = \frac{k_0 \sin \theta - h k_0 \eta_0 \cos \theta}{n_e}.$$
(3.34)

Hillslope storage is directly related to water table height via the relation:

$$S(x,t) = n_e w(x)\eta(x,t).$$
(3.35)

The critical component of this reformulation is the hillslope width function (w), which prescribes the geometric width of the hillslope as a function of distance upslope (x). The parameters of the hillslope width function set the hillslope width at the stream boundary (the parameter g [L]) and the rate of increase or decrease of width moving in the upslope direction (the parameter h [1/L]). This allows for variable hillslope geometries (such as a divergent hillslope, where hillslope width (w) decreases with distance (x) from the stream, or a convergent hillslope where w increases with x) to be represented in a simple, one-dimensional Boussinesq framework. For constant hillslope width (w = constant), the HSB simplifies to the HB.

We continue to adopt a zero head boundary at the stream, a no flux boundary at the upslope water table divide and a constant initial water table height above the bedrock as boundary conditions for the LHSB. Mathematically these conditions are expressed for the LHSB as:

stream boundary condition:
$$S = 0$$
 $x = 0$ $t > 0$,
divide boundary condition: $K \frac{\partial S}{\partial x} + US = 0$ $x = B_x$ $t > 0$, (3.36)
initial condition: $S = Dn_e g e^{hx}$ $0 \le x \le B_x$ $t = 0$.

Again we consider spatiotemporally variable recharge of the form $R(x, t) = R_{max}M(t)N(x)$, and repeat the methods used to solve Equation (3.3). This allows the following solutions to the LHSB to be found:

$$S(x,t) = \exp\left(at + bx\right) \sum_{n=1}^{\infty} f_n\left(t\right) \psi_n\left(x\right),$$

$$a = -\frac{U^2}{4K},$$

$$b = -\frac{U}{2K},$$
(3.37)

with

$$\psi_n = \sin\left(\mu_n x\right),\tag{3.38}$$

$$\frac{2K\mu_n}{U} = \tan\left(\mu_n B_x\right),\tag{3.39}$$

and

$$f_n(t) = \exp\left(-K\mu_n^2 t\right) \left[F_n + \Psi_n \int_0^t M(\tau) \exp\left(K\mu_n^2 \tau - a\tau\right) d\tau\right], \qquad (3.40)$$

with

$$\Psi_n = \frac{\int_0^{B_x} R_{\max} N w \psi_n e^{-bx} dx}{\int_0^{B_x} \psi_n^2 dx},$$
(3.41)

and

$$F_n = \frac{4Dgn_e\mu_n \left(e^{(h-b)B_x} \left((h-b)\sin\left(B_x\mu_n\right) - \mu_n\cos\left(B_x\mu_n\right)\right) + \mu_n\right)}{\left((b-h)^2 + \mu_n^2\right) \left(2B_x\mu_n - \sin\left(2B_x\mu_n\right)\right)}.$$
 (3.42)

3.3 Validation of analytical solutions

In Appendix A, we test the solutions in three ways to ensure their applicability and validity when representing spatially variable recharge for both the HB and the HSB:

- 1. We compare our analytical solutions to previous analytical results obtained using the Laplace Transform method for spatially uniform recharge [Troch et al., 2003, 2004, Verhoest and Troch, 2000] and confirm that our solution exactly reproduces these previous results.
- 2. We compare analytical solutions of the LHB and LHSB with spatially variable recharge to numerical solutions of the LHB and LHSB equations obtained using a finite difference scheme [Brutsaert, 1994, Troch et al., 2003, Verhoest and Troch, 2000]. Our analytical solutions perfectly match the numerical solutions, confirming the correctness of our solutions for the previously unsolved case of spatially variable recharge. This item also provides a clear demonstration of the advantages of the analytical LHB solution: The numerical solution to the LHB equation took $\mathcal{O}(100)$ times longer to compute than the analytical solution, depending on the number of eigenvalues computed (for the analytical solution) and the grid size (for the numerical solution).
- 3. We compare the analytical solutions from the linearized equations to numerical solutions of the full nonlinear equations for equivalent boundary and initial conditions. This final test is intended to explore the validity of the linearization assumption in the context of abrupt changes in recharge fluxes through space, and in particular to determine whether or not large spatial gradients in recharge might induce large spatial gradients in the water table and violate the linearization assumption, $\left(\frac{\partial \eta}{\partial x}\right)^2 \approx 0$. We conclude that any additional error induced by the recharge pattern is negligible.

We find additional support for the third item in Section 3.5, where we perform an in depth error analysis of the LHSB. The results show no significant error trend between the



Figure 3.2: Illustration of the step function model for spatially variable water table recharge.

linear and nonlinear equations as the spatial pattern of recharge is varied, suggesting that the linearized equations appropriately capture hillslope sensitivity to recharge structure. Nevertheless, in agreement with Troch et al. [2003], we find that the absolute linearization error can be significant, especially on shallow, divergent slopes. Troch et al. [2003] also report that the choice of the mean water table height, η_0 , can significantly alter the magnitude of the error. However, in this paper we only report linearization error sensitivity to hillslope geometry and recharge heterogeneity, leaving an analysis of the effects of the choice of the mean water table height for future work.

3.4 Modeling scenarios

The solutions to the LHB and the LHSB outlined above were used to model three different scenarios of water table and baseflow dynamics. In Scenarios 1 and 2, we explore two 'end member' cases, each of which divides a hillslope into two regions of homogenous recharge cover, with the extent of each region varying along the main hillslope axis (the 'x' axis in Figure 3.1). We assume that one of these recharge types occurs at a maximum dimensionless rate of R = 1 while the other is set to a fraction, α , of the maximum rate (for the purposes of our simulation, we set the low recharge to zero, $\alpha = 0$). An example of this type of recharge configuration is illustrated in Figure 3.2. We present analytical results for the case of an upslope step decrease in recharge and for an upslope step increase in recharge. These solutions are shown in Table 3.1. A third solution for a case in which recharge varies linearly with distance upslope is also presented in Table 3.1, but is not analyzed further. Scenario 1 compares the hillslope response to a pulse input of rainfall under different spatial distributions of recharge on linear hillslopes. Scenario 2 uses the same spatial recharge distributions, but explores their effects on water table and baseflow variations on seasonal timescales, specifically during the transition from a wet season steady-state condition to a dry season. A third scenario examines hillslope recession behavior, but in the context of the HSB model, allowing convergent and divergent hillslopes as well as linear hillslopes to be modeled.

	Hillslope A	Hillslope B
Slope length (B_x) [m]	100	100
Slope angle (θ) [degrees]	0.1	6
Hydraulic conductivity (k_0) [m/day]	1000	86.4
Drainable porosity (n_e)	0.08	0.34
Mean water table height (η_0) [m]	0.67	0.67
Response timescale (t^*) [days]	4.58	3.76
Hillslope number (H_i)	0.26	15.78

Table 3.2: Hillslope parameterizations for Scenario 1

Scenario 1: Effects of spatially heterogeneous recharge on storage-discharge responses to an input pulse of water

The first model scenario addresses a recharge pulse lasting for two days entering dry hillslopes where the initial water table level is at the bedrock boundary ($\eta(x, 0) = D = 0$). Two cases are simulated: (i) upslope 50% high recharge and downslope 50% zero recharge, and (ii) upslope 50% zero recharge and downslope 50% high recharge. In both cases, the total hillslope integrated recharge per length of channel ($\int_0^{B_x} R(x, t) dx$) was fixed at 3.6 m²/day during the two day pulse (this corresponds to a maximum recharge rate of 7.2 cm/day for the 100m long heterogeneously recharged hillslopes).

For each case, we examine two hillslopes (Hillslopes A and B in Table 3.2) with contrasting hydrogeologic characteristics. The specific parameterizations for this scenario are listed in Table 3.2, and the variety of observed dynamics can be interpreted using the scaling framework introduced in the derivation of Equation (3.6). This non-dimensionalization of the LHB yielded the dimensionless number, $H_i = \frac{B_x \tan \theta}{\eta_0}$, the so called 'Hillslope number' [Brutsaert, 2005]. The Hillslope number is a type of Péclet number and therefore represents the relative strength of advective and diffusive behaviors. Additionally, we consider the absolute advective and diffusive response timescales for linear hillslopes. Following Harman and Sivapalan [2009], we examine $t^* = \frac{B_x n_e}{k_0 \sin \theta}$, which scales proportionally with both the advective $(\frac{t^*}{2})$ and diffusive $(\frac{t^*}{2\eta_0})$ response timescales for linear hillslopes. We use these similarity relationships to discuss the observed behaviors of Hillslopes A and B and to ensure that this study fully explores the variety of possible hillslope responses to spatially heterogeneous recharge.

Scenario 2: Effects of recharge spatial distribution on seasonal storage - discharge responses in a seasonally dry climate

The second model scenario addresses the effect of variable recharge on seasonal storagedischarge responses using a zeroth-order representation of recharge seasonality which mimics the canonical behavior of a seasonally-dry climate. In such climates, rainfall events (and thus recharge) are concentrated over several months, which are followed by a prolonged dry season. The persistence of baseflow supplied by discharge from groundwater and hillslope aquifers in the dry season has important implications for water resources and ecosystem health in seasonally dry climates [Vico et al., 2014].

In this scenario, we explore the persistence of baseflow from a hillslope aquifer as a function of recharge distribution for a step-decrease in recharge through time; represented by M(t) = 1 during the wet season (mathematically, we impose wet season conditions for $t < t_d$, where t_d is the seasonal transition time from wet to dry season) and M(t) = 0 during the dry season. Here, M(t) = 1 represents a scenario where recharge occurs at the maximum rate specified by the spatial function N(x). Again, analytical results for the water table dynamics are presented in Table 3.1 for this solution, as well as for temporally uniform recharge M(t) = 1 and for a step-increase in recharge in time from M(t) = 0 to M(t) = 1, providing a zeroth-order representation of the onset of the wet season. Equation (3.3) may also be solved for more complex forms of M(t) (e.g. sinusoids), but the step functions used here capture the key features of a seasonal transition while maintaining as much analytical simplicity as possible.

To simulate hydrologic conditions at the end of a long wet season, we formulate the step function so that the maximum recharge rate is maintained until the water table and discharge approximate steady-state conditions. We compute this timescale by noting that in the series solution for $\eta(x,t)$ it is the eigenvalues (λ_n) that control the rate at which the water table relaxes to its steady-state condition. The dominant (smallest) eigenvalue (λ_1) characterizes the slowest of these transient dynamics. It provides a timescale $\tau = \frac{1}{\lambda_1^2}$ for this relaxation. By choosing the transition timescale in the step function (t_d) such that $t_d \gg \tau$, we ensure that the hillslope has reached a close approximation to its steady-state wet season condition, and that the dynamics following the cessation of recharge reflect a realistic initial condition. This approach ensures that the initial water table profile during the seasonal transition is realistic [Verhoest and Troch, 2000], without having to re-derive the analytical solution to Equation (3.3) for a new suite of initial conditions. For Scenario 2, we choose D = 1.5m, and, to ensure a steady-state is reached, take $t_d = 1000$ days and set M(t) = 0 for $t > t_d$. For all cases modeled here, the pseudo-steady water table approximated the true steadystate condition to within 5% after 120 days, comparable to the response timescale for these hillslopes, $t^* \approx 112$ days. This indicates that these analyses are relevant in climates with long (> 4 months) wet seasons. We impose the same recharge cases as those used in Scenario 1, where the recharge transition occurs at the hillslope midpoint, and present analyses for two hillslopes: a diffusion dominated hillslope (Hillslope 1) and an advection dominated hillslope (Hillslope 2). The specific parameterizations of Hillslopes 1 and 2 are listed in Table 3.3. The total hillslope integrated recharge rate is set to 1.5 m^2/day , corresponding to a maximum recharge rate of 1 cm/day on the 300m long heterogeneously recharged hillslopes.

	Hillslope 1	Hillslope 2
Slope length (B_x) [m]	300	300
Slope angle (θ) [degrees]	0.1	6
Hydraulic conductivity (k_0) [m/day]	230	8.64
Drainable porosity (n_e)	0.15	0.34
Mean water table height (η_0) [m]	0.67	0.67
Response timescale (t^*) [days]	112.10	112.94
Hillslope number (H_i)	0.78	47.29

Table 3.3: Hillslope parameterizations for Scenario 2

Scenario 3: Topographically variable hillslopes

In this scenario we explore the interplay between recharge and hillslope morphology by varying the parameter h in the hillslope width function from h = -0.03 to h = 0.03 (very divergent to very convergent hillslopes) and varying the extent of the downslope fraction of the hillslope which receives zero recharge (from 0 to 75%). The analytical results for the LHSB for the case of high upslope recharge are shown in Table 3.4. We again compute the water table and discharge dynamics over a step-decrease in recharge in time at t = 1000 days (to ensure that steady-state conditions are achieved before the onset of recession).

Table 3.4: Hillslope-storage Bouss	inesq forcing function for complex hillslopes with recharge $R(x,t) = R_{\max}N(x)M(t)$.
Ν	Ψ_n
$N(x) = \begin{cases} \alpha, & \text{if } 0 \le x \le x_r \\ 1, & \text{if } x_r < x \le B_x \end{cases}$	$\begin{cases} 4gR_{\max}\mu_n\left\{(b-h)\left(e^{(h-b)B_x}\sin\left(B_x\mu_n\right) + (\alpha-1)e^{(h-b)x_r}\sin\left(\mu_nx_r\right)\right) \\ + \mu_n\left(-\alpha + e^{(h-b)B_x}\cos\left(B_x\mu_n\right) + (\alpha-1)e^{(h-b)x_r}\cos\left(\mu_nx_r\right)\right) \\ /\left\{\left((b-h)^2 + \mu_n^2\right)\left(\sin\left(2B_x\mu_n\right) - 2B_x\mu_n\right)\right\} \end{cases}$
M	$f_n(t)$
$M(t) = \begin{cases} 1, & \text{if } 0 \le t \le t_d \\ \gamma, & \text{if } t > t_d \end{cases}$	$e^{at} \left(F_n e^{at - Kt\mu_n^2} + \Psi_n \frac{\gamma e^{at - Kt\mu_n^2} - e^{Kt_d\mu_n^2 - at_d + at - Kt\mu_n^2} + e^{at - Kt\mu_n^2 - \gamma}}{a - K\mu_n^2} \right) \text{valid for } t \ge t_d$
The form of $f_n(t)$ displayed is only valid	l after the temporal transition at $t = t_{\beta}$; $x_{\gamma} \in [0, B_{\gamma}]$ and $\alpha, \gamma \in [0, 1]$.

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We control for the total input volume of recharge by firstly controlling for the total hillslope area as the hillslope width function $(w = ge^{hx})$ changes (we fix this area as A = 2000 m²), and then adjusting the maximum recharge rate to fix the total recharge volume entering the hillslope at $20m^3/day$. Hillslope convergence is varied with the exponential parameter (h) in the hillslope width function (w), constraining the other hillslope width parameter (g):

$$A = \int_{0}^{B_{x}} w(x) \, dx = \int_{0}^{B_{x}} g e^{hx} dx \quad \to \quad g = \frac{Ah}{e^{B_{x}h} - 1} \tag{3.43}$$

We parameterize the LHSB model as follows: hillslope length $B_x = 100$ m; hydraulic conductivity $k_0 = 1$ m/hr; drainable porosity $n_e = 0.30$; initial water table height D = 0.4m and mean water table height $\eta_0 = 0.9$ m. We run the simulations for two hillslope angles: 5° and 15°. For complex hillslopes such as these, Troch et al. [2004] follow Brutsaert [1994] and suggest a type of Péclet number:

$$H_{\text{topo}} = \frac{B_x \tan \theta}{2\eta_0} - \frac{hB_x}{2}, \qquad (3.44)$$

which is analogous to the Hillslope number (H_i) for topographically complex hillslopes with exponential width functions (*h* is the exponential rate of convergence or divergence of the hillslope width function). For $h \in [-0.03, 0.03] \text{ m}^{-1}$ and $\theta \in [5^{\circ}, 15^{\circ}]$, H_{topo} ranges from 3.36 (for shallow, highly convergent hillslopes) to 16.38 (for steep, highly divergent hillslopes), indicating that these hillslopes are advection dominated.

In this scenario, we allow the water table to approach a steady-state condition and then 'turn off' recharge completely and compute the *e*-folding time (the recession constant, or the time required for outflow to decline to $\frac{1}{e}$ of its original value) for the flow recession curve. We propose this *e*-folding recession time as representative of the storage and discharge variability between different topography - recharge combinations. The linearization of HSB, however, requires more simplifying assumptions (as compared to the linearized HB) and therefore introduces errors that can significantly affect single-valued flow recession metrics, such as the *e*-folding time. To assess the validity of the LHSB for measuring flow recession characteristics, we also calculate the *e*-folding recession time for select hillslopes using the full, nonlinear HSB equation. With this, we compute linearization error surfaces for the *e*-folding time across a variety of hillslopes.

3.5 Results

Scenario 1: Effects of recharge spatial distribution on storage discharge responses to an input pulse of water

The hillslope discharge per length of channel induced by a two day water pulse recharging a dry water table is shown as a function of time for high upslope and downslope recharge on Hillslopes A and B in Figure 3.3 (a and b respectively). Although both hillslopes receive



Figure 3.3: Outflow response to a two day pulse of recharge $(M = M_2, \gamma = 0, t_d = 2 \text{ days})$ for Hillslopes A and B ((a) and (b) in the figure). Dashed lines illustrate results for high upslope recharge $(N = N_1, x_r = 50\text{m}, \alpha = 0, R_{\text{max}} = 7.2 \text{ cm/day})$ and solid lines for high downslope recharge $(N = N_2, x_r = 50\text{m}, \alpha = 0, R_{\text{max}} = 7.2 \text{ cm/day})$.

and eventually discharge the same total volume of water, the spatial location of recharge and the hydrogeologic character of the hillslopes clearly alter the peak flow and timing of this discharge. For Hillslope B (advection dominated), high upslope recharge exhibits a large lag between the cessation of recharge and the timing of peak flow (as compared to high downslope recharge). In contrast, Hillslope A (diffusion dominated) exhibits an insignificant lag in peak flow after the cessation of recharge under both high upslope and high downslope recharge. High downslope recharge produces a more rapid response on both Hillslopes A and B, although the peak size for high downslope recharge relative to the peak size for high upslope recharge strongly depends on the advective/diffusive characteristics of the hillslope. Peak discharge values are primarily driven by response time, and vary from $3.58 \text{ m}^2/\text{day}$ for high downslope recharge on Hillslope A to $2.29 \text{ m}^2/\text{day}$ for high upslope recharge on Hillslope B.

Scenario 2: Effects of recharge spatial distribution on seasonal storage - discharge responses in a seasonally dry climate

Steady-state water table profiles for Hillslopes 1 and 2 are illustrated in solid lines in panels (a-b) and (d-e) of Figure 3.4, respectively. For each hillslope, snapshots of the water table



Figure 3.4: Recessions from steady-state water table profiles for Hillslopes 1 and 2 for high upslope recharge (a,d) and for high downslope recharge (b,e). Forcing functions are obtained from Table 3.1 letting $M = M_2$, $\gamma = 0$, $t_d = 1000$ days with $N = N_1$, $x_r = 150$ m, $\alpha = 0$ for high upslope recharge (a,d) and $N = N_2$, $x_r = 150$ m, $\alpha = 0$ for high downslope recharge (b,e). Water table profiles are plotted at 0, 20, 50, and 100 days after the cessation of recharge. The outflows corresponding to the draining water table profiles are plotted in (c,f), where time is in days after the dry-down transition ($t = t_d$) at 1,000 days.

draw-down are shown for times t = 20, 50, and 100 days following both a seasonal high upslope recharge event (a,d) and a seasonal high downslope recharge event (b,e). It is clear that the steady-state storage profiles are strongly influenced by the recharge distribution, with generally higher steady-state storage for the cases of high upslope recharge. Following the cessation of recharge, each of these water tables drains to the channel, generating the outflow recession curves plotted in Figure 3.4(c,f). As a consequence of the enforced steadystate condition and equivalent hillslope total recharge volume in the model runs, the initial outflows from all four simulations are the same ($q = 1.5 \text{ m}^2/\text{day}$). As the hillslopes begin to drain, however, the outflow curves diverge from each other. For the cases of high upslope recharge, steady-state storages are larger and thus outflows tend to recess more slowly than for the high-downslope recharge profiles.



Figure 3.5: Contour plots of recession constants for the linearized HSB simulations (a,d) and the nonlinear HSB (c,f) for varying recharge distributions and hillslope geometries (calculated using Table 3.4 with $t_d = 1000$ days, $\gamma = 0$, $\alpha = 0$). Circles on the contour plot indicate the parameter combinations for which the nonlinear HSB was numerically solved. The vertical axis in plots (a,c,d,f) indicates the downslope proportion (by area) of the hillslope that receives no recharge. The horizontal axis indicates the strength of hillslope convergence or divergence, as measured by the parameter h in the hillslope width function (w). The illustrated hillslopes on the horizontal axis correspond in scale to approximately h = 0.02(convergent) and h = -0.02 (divergent). The recharge rate on homogeneously recharged hillslopes (dry proportion = 0) is 10 mm/day and the slope length $B_x = 100$ m. Plots (b,e) contour the percent error of the recession constant computed by the linearized HSB against the recession constant computed by the nonlinear HSB.

Scenario 3: Topographically variable hillslopes

Figure 3.5(a,d) presents contours of the recession constant (*e*-folding time) for different hillslope geometries and recharge extents using the LHSB. Plots (a,d) indicate that the influence of recharge variability on discharge is maximized in divergent hillslopes. At larger hillslope angles, however, the contours are generally parallel, meaning that steep divergent and convergent slopes are comparably sensitive to recharge heterogeneity.

Figure 5 (c,f) replicates the recession contours presented in Figure 5 (a,d) except that the contours are calculated using a numerical solution of the full nonlinear HSB. Qualitatively,

trends in the *e*-folding recession time using the nonlinear HSB match the plots created using the linearized equation. Error surfaces (b,e) demonstrate that the accuracy of the LHSB depends only somewhat on the spatial character of recharge and strongly on the hillslope morphology. Relative error is largest in shallow divergent hillslopes, in agreement with error trends found by Troch et al. [2003] for hillslopes with spatially homogeneous recharge.

3.6 Discussion

Spatially heterogeneous recharge effects on water storage and runoff generation

All three scenarios demonstrate that the spatial distribution of recharge could significantly alter the hillslope water table profile as well as the temporal character of hillslope discharge. It is apparent that topography and the hydrogeologic character of a given hillslope mediate the impact of spatially heterogeneous recharge, resulting in diverse subsurface storage and discharge behaviors.

Scenario 1

The results of Scenario 1, the case of a pulse input of water to the subsurface, suggest that (all other factors being equal) hillslopes with lower downslope recharge (relative to upslope recharge) are likely to exhibit longer lags between a rainfall input and the peak of baseflow response. We find that these slopes exhibit lower peak flow rates and longer baseflow persistence compared to hillslopes where recharge is concentrated downslope. Additionally, increased travel time for water entering the groundwater store upslope increases the amount of time available for diffusion, resulting in enhanced dispersion of the discharge response.

The scaling framework of Harman and Sivapalan [2009] provides a more thorough interpretation of the variety of behaviors exhibited in the plots of Figure 3.3. For example, the highly diffusive (low Hillslope number) nature of Hillslope A leads to a discharge response that appears to have little sensitivity to the spatial structure of the recharge. Hillslope A also has a rapid response timescale (low t^*) and drains nearly all of its stored water within four days of the recession onset. The result is a discharge signal that responds rapidly, exhibiting only minor differences between the cases of high upslope recharge and high downslope recharge. In comparison, Hillslope B responds on a timescale comparable to Hillslope A, but the advective nature of the hillslope (higher Hillslope number) leads to greater preservation of the spatial structure of the recharge signal, leading to a lagged response for uphill recharge. These results are characteristic of most advective-diffusive systems; diffusion tends to dampen spatial heterogeneity while advection serves to preserve it. Given that the advective nature of a hillslope is proportional to hillslope length and the tangent of the hillslope angle $(H_i = \frac{B_x \tan \theta}{\eta_0})$, we conclude that recharge heterogeneity at event timescales is the most hydrologically relevant on long, steep hillslopes.

Scenario 2

Under steady-state recharge conditions (Scenario 2), it is evident that the spatial pattern of recharge places a strong control on the volume and profile of storage in the water table (Figure 3.4). Where recharge primarily occurs upslope, the whole hillslope is available for water storage, increasing total storage at steady-state. Recharge that occurs primarily downslope is effectively stored in a smaller volume of soil, since groundwater movement in the upslope direction is limited, and the connection to the channel over a relatively short distance reduces the volumetric storage needed to achieve the hydraulic gradients for steady-state outflow conditions.

Although recharge heterogeneity significantly alters the spatial structure of steady-state storage, the relative strengths of hillslope advective and diffusive behaviors mediate the strength of its impact. For example, in Hillslope 1 (a diffusion dominated hillslope) storage is more evenly dispersed in the subsurface at steady-state due to the hillslope's low topographic gradient. The steep topographic gradients in Hillslope 2, however, restrict the spatial redistribution of storage, leading to groundwater piling near the outlet. This is characteristic of strongly advective hillslopes. Thus, storage is more evenly distributed throughout Hillslope 1 for both upslope and downslope high recharge, while in Hillslope 2, storage is limited to the lower half of the hillslope for the case of high downslope recharge.

Figure 3.4(c,f) indicates that these storage effects are propagated into the recession hydrographs. The results indicate that baseflow is more likely to persist from hillslopes with higher upslope recharge than hillslopes with higher downslope recharge under equivalent climatic forcing. This effect is weaker in Hillslope 1 due to greater storage redistribution. In Hillslope 2, there is a greater disparity in total storage between the cases of high upslope recharge and high downslope recharge. The steady-state storage profiles of Hillslope 2 are preserved by its advective nature, resulting in a large lag before any significant recession of flow for the case of high upslope recharge.

Scenario 3

In order to explore the potential effects of heterogeneous recharge on the hydrology of topographically complex hillslopes, we compared the hillslope outflow recession constant over a range of recharge patterns and hillslope geometries. Figure 3.5 suggests that specific changes in recharge spatial structure may lead to near-equivalent recession behavior between hillslopes with different topographic characteristics. For example, the recession constants are equivalent between a highly convergent ($h \approx 0.02$), 15° angled hillslope with homogeneous recharge and a linear, 15° angled hillslope with the upper half of its surface receiving high recharge. The steady-state volume of hillslope water storage (not shown) was found to follow nearly the same pattern as the recession constant, with larger storage volumes corresponding to longer recessions. The spatial distribution of recharge was also shown to amplify or dampen the effects of hillslope morphology. Increasing convergence increases hillslope storage at steady-state, which leads to longer recessions. The effect is similar for an increase in the downslope portion of a hillslope that receives low recharge. In other words, reducing the amount of recharge downslope appears to diminish the divergent character or enhance the convergent character of a hillslope. The relative effects, however, of spatially heterogeneous recharge are less important on more diffusive hillslopes, as evidenced by the steeper contour slopes in Figure 3.5(a). Although, increasing divergence can counteract the effects of diffusion by increasing the advective nature of the hillslope (according to Equation (3.44)), preserving more of the recharge structure in the discharge signal. We conclude that steep, divergent hillslopes are most affected (in the relative sense) by spatially heterogeneous recharge.

To evaluate the effects of the linearization assumptions on the computed *e*-folding times, we repeated the analysis using the full nonlinear HSB. The general trend of increasing recession time for increasing hillslope convergence remains the same using the full HSB. Error surfaces demonstrate that the LHSB successfully captures the strength of this trend on the 15° hillslope. On the 5° hillslope, the trend is captured on all slopes but absolute accuracy erodes for increasingly divergent hillslopes. The error is increasingly negative for decreasing h, showing that the LHSB underestimates the actual recession timescale.

The effects of increasing recharge heterogeneity are appropriately captured on both the steep and shallow hillslopes. Although the absolute error is significant for the 5° hillslope, there is essentially no trend in the error with changing the extent of the hillslope that receives zero recharge. Error on the 15° hillslope exhibits no strong trend and appears to be quite noisy, presumably due to numerical error in the solution of the nonlinear HSB. This suggests that for these hillslopes, linearization and numerical approximations may contribute comparably significant errors.

Although it is clear from Figure 3.5 that topography remains the dominant driver of steady-state storage and recession timing, the effects of recharge spatial distribution are not negligible and could be very significant. Recession constant trends calculated using both the nonlinear and linearized HSB indicate that hillslopes with more recharge heterogeneity generally exhibit longer recession timescales with a stronger relative effect on steep, divergent hillslopes. For the purposes of modeling, however, these simulations also suggest that the LHSB should be used with caution.

Model limitations

Clearly this theoretical study has numerous limitations in terms of its realistic representation of the spatial character of recharge to the water table. Variations in net recharge to unconfined hillslope aquifers are not empirically quantified and may be offset by heterogeneity in other hydrogeological characteristics (e.g. soil depth, bedrock slope, bedrock topography, porosity, or hydraulic conductivity) that are not considered.

The analysis is also limited by the assumptions and linearization of the hillslope Boussinesq equation, which introduce varying degrees of error for water table and outflow predictions. Troch et al. [2013] provide review of the problems and benefits associated with the use of the Boussinesq approach. Despite these limitations, the study goals — to explore the potential for spatially organized recharge patterns to alter hydrology — are reasonably met by the simplified spatiotemporal dynamics, which capture the gross features of spatial variation in recharge patterns and temporal variation in recharge due to seasonality that influences many real systems. With this in mind, we propose avenues for future work.

3.7 Conclusion

This study developed a novel and (comparatively) straightforward analytical solution approach to the linearized hillslope Boussinesq and hillslope-storage Boussinesq equations. Our simple simulations demonstrate that, in conjunction with hillslope topography, the spatial distribution of water table recharge may cause a non-trivial modification of catchment storage and discharge responses on multiple timescales. We hypothesize that vegetation has the potential to alter the spatial character of recharge and suggest the Boussinesq framework as a promising avenue for the development of new hillslope-scale ecohydrologic theory.

Chapter 4

Streamflow persistence in seasonal climates

4.1 Introduction

Pronounced variability in precipitation is the defining characteristic of seasonally dry ecosystems (SDE) [Fatichi et al., 2012, Vico et al., 2014], which cover nearly 30% of the planet and contain about 30% of the Earth's population [CIESIN, 2012, Peel and Finlayson, 2007]. In these regions, a distinct rainy season is followed by a pronounced dry season during which rainfall makes a small or negligible contribution to the water balance. As a consequence, the availability of dry season surface water resources depends strongly on streamflow, which is generated primarily from the storage and subsequent discharge of antecedent wet season rainfall in the subsurface [Andermann et al., 2012, Brahmananda Rao et al., 1993, Samuel et al., 2008]. Because these transient stores are strongly influenced by the characteristics of the wet season climate, dry-season water availability can be highly variable from year to year in many SDE's [Andermann et al., 2012, Samuel et al., 2008]. This hydroclimatic variability leaves SDE's, considered important 'hot spots' of biodiversity [Klausmeyer and Shaw, 2009, Miles et al., 2006, and the human populations that depend upon them susceptible to threats, such as soil erosion, deforestation, and water diversions [Miles et al., 2006, Underwood et al., 2009]. Future climate scenarios are projected to further intensify wet season rainfall variability in many SDEs [e.g. Dominguez et al., 2012, Gao and Giorgi, 2008, García-Ruiz et al., 2011a, necessitating models which can predict the response of water resources to climatic change in order to measure the corresponding risk to local ecosystems and human populations [Müller et al., 2014, Vico et al., 2014].

Stochastic methods have a 30-year history of use in deriving simple, process-based models for the probability distributions of hydrologic variables, such as soil moisture, streamflow, and associated ecological responses [Botter et al., 2007b, Laio, 2002, Milly, 1993, Rodriguez-Iturbe et al., 1999, Szilagyi et al., 1998a, Thompson et al., 2013, 2014]. To date, the majority of stochastic analytical models for hydrology have been developed under conditions where the climatic forcing does not exhibit strong seasonality [Botter et al., 2007b, Porporato et al., 2004, Rodriguez-Iturbe et al., 1999]. Those studies that have considered the effects of seasonality in rainfall or evaporative demand have either focused on the mean dynamics of the variable of interest [Feng et al., 2012, 2015, Laio, 2002] or excluded a treatment of the transient dynamics between the wet and dry seasons [D'odorico et al., 2000, Kumagai et al., 2009, Miller et al., 2007]. This is problematic in situations when such transient hydrologic dynamics have a large impact on the availability of water - i.e. in SDEs [Feng et al., 2015, Müller et al., 2014, Viola et al., 2008]. Recently, stochastic analytical models for streamflow and soil moisture have been extended to include seasonal transitions in SDEs [Feng et al., 2012, 2015, Müller et al., 2014, Viola et al., 2008]. In the case of streamflow models, this is accomplished by explicitly accounting for the dynamics of the seasonal streamflow recession during the dry season [Müller et al., 2014].

In SDE's, the question, "how long do dry season streamflows persist above a given level?" is highly pertinent to the habitat quality and ecological functions sustained by streams, and to the legal and management frameworks applied to support those functions. For example, ecosystem managers often assume a correspondence between habitat availability and the stream wetted perimeter in order to determine critical minimum flow values [below which instream conditions become sub-optimal for habitat protection, e.g Annear and Conder, 1984, Nelson, 1980, Parker and Armstrong, 2001]. These minimum flows are frequently adopted as regulatory measures, and thus set conditions beyond which in-stream water abstractions are prohibited. Other ecological transitions are also flow-dependent; for instance, lower bed-shear stresses associated with low flows are suspected to promote blooms of toxic cyanobacteria in some northern California watersheds [Power et al., 2015, *Keith Bouma-Gregson*, pers. comm.]. The flow regime also appears to be the primary determinant for temperature-driven stratification of deep river pools, with implications for organisms that depend on these pools for summer survival [Nielsen et al., 1994, Turner and Erskine, 2005].

A number of statistical and deterministic modeling methods have been developed to characterize low flow conditions in both gauged and ungauged basins [Arnold et al., 1998, Castellarin et al., 2007, Ganora et al., 2009, Laaha and Blöschl, 2007, Müller and Thompson, 2015, Nathan and McMahon, 1990, Skøien et al., 2006, among others]. Regression-based and geostatistical techniques employ the concept of hydrologic similarity – the idea that catchments with similar geomorphologic and hydroclimatic features should also exhibit similar low flow features [Blöschl et al., 2013]. Deterministic rainfall-runoff models [e.g. Arnold et al., 1998] can be used for low flow estimation, though such models cannot provide a probabilistic characterization without computationally intensive Monte Carlo techniques.

Compared to their statistical and deterministic counterparts, process oriented stochastic methods possess a number of advantages. Their simplicity facilitates applications even when data are sparse, and their mechanistic underpinnings overcome the limitations of statistical models which cannot, for example, distinguish between the effects of a non-stationary climate and shifts due to land use change. It is also notable that these stochastic models are typically analytic; without resorting to hydrograph simulation, they produce closed form probability distributions for flow and flow-related variables. This type of output conveniently lends itself to applications in risk-oriented frameworks. For example, a related stochastic hydrologic model for soil moisture has been applied to assess plant pathogen risk at the regional scale [Thompson et al., 2013, 2014]. Analogous streamflow methods to estimate ecological risks based on climate and stream characteristics could provide a toolkit for assessing the condition of existing riverine ecosystems, as well as their vulnerability under alternative climatic, land use, or management scenarios.

This study aims to develop a model that would support such risk assessment by predicting the probabilistic character of the *persistence time*. The persistence time, denoted T_{Q_*} , is the length of the period that the dry season streamflow remains above a given flow threshold (Q_*) . The distribution of T_{Q_*} links wet season climatic variation and land use (via vegetation water demand) to dry season recession characteristics.

Using the frameworks derived by Botter et al. [2007b] and Müller et al. [2014] to characterize the wet season flow dynamics and the transition to the dry season respectively, we derived analytical expressions for the probability distribution function (PDF) of T_{Q_*} and its expectation. The model is defined for rivers in seasonally dry climates; for this definition to be met, there must be a significant reduction in flow between mean wet season conditions and the dry season behavior. This condition may be violated in managed rivers (e.g. when dam releases elevate dry season flows) or in watersheds with significant groundwater contributions, which may be sufficient to smooth out flow variations even on seasonal scales. The model was validated using a multi-year streamflow dataset from the United States Geological Survey's network of stream gages.

Applying stochastic methods to predict empirical discharge signatures, such as the persistence time, aligns with a recent review of prediction in un-gauged basins, which explicitly notes the utility of probabilistic streamflow models and calls for more research efforts to characterize their merits and limitations [Blöschl et al., 2013]. With this in mind, our model validation demonstrates two important results concerning stochastic methods, one 'positive' and one 'negative':

- 1. The positive result that the mean persistence time can be identified well as a function of wet season rainfall statistics and seasonality statistics; allowing the use of analytical models to predict the expected dry season flow persistence.
- 2. The negative result that the full PDF cannot be modeled analytically, highlighting the significance of variations in the parameters defining the recession.

While the model is capable of predicting the mean of the empirical persistence time distribution, the model PDF significantly underestimates the empirical persistence time variance. We demonstrate that this underestimation arises due to fluctuations and correlation between the parameters of the power law model used here (and in many other studies, e.g. Botter et al. [2009], Kirchner [2009], Tague and Grant [2004]) to characterize the dry season recession. This correlation is not physical, but is an inevitable artifact arising from the properties of power laws. Using an existing power law parameter de-correlation technique [Bergner

and Zouhar, 2000, Mather and Johnson, 2014], we quantified the effect of this correlation on our predictions, and demonstrated substantial improvement in the model predictions once it was removed. These simulations motivate further investigation into recession parameter variability; a more thorough understanding of the origins of this behavior is crucial for the continued development of minimal, process-oriented stochastic models.

4.2 Methods

Definition of symbols and terms

Throughout this section, $\Gamma(*)$ represents the gamma function and $\Gamma(*,*)$ represents the upper incomplete gamma function. The probability density function (PDF) is represented by p and the cumulative density function (CDF) by P. Subscripts, in upper case, denote the random variable being described by the PDF or CDF, and the corresponding lower case characters denote the observed value of the random variable. For example, the PDF and the CDF of stream discharge Q at value q are written $p_Q(q)$ and $P_Q(q)$, respectively.

Modeling approach

The process by which we derive expressions for the mean persistence time and the persistence time probability distribution is illustrated in Figure 4.1. First, the year is partitioned into a wet and dry season, shown in Figure 4.1a. The steady state wet season flow PDF (4.1b) is obtained from the work of Botter et al. [2007b], which is used to derive the probability distribution function describing the streamflow at the start of the dry season/end of the wet season (4.1c). This streamflow provides the initial condition for a deterministic recession during the dry season, allowing the probability distribution for the initial condition to be transformed into a probability distribution for the recession persistence time (4.1d). These steps are outlined in the next sections.

Steady state wet season streamflow distribution

The steady-state, wet season streamflow PDF derived by Botter et al. [2007b] forms the point of departure for this analysis. Assuming that recharge events can be described by a marked Poisson process with frequency (λ [1/T]) and mean depth (1/ γ_Q , where γ_Q has units of [T/L³]), and that the catchment residence time distribution is also exponential, with mean 1/k [T], the PDF for wet season streamflow (Q_w) follows a gamma distribution:

$$p_{Q_w}(q_w) = \frac{\gamma_Q^m}{\Gamma(m)} q_w^{m-1} \exp\left(-\gamma_Q q_w\right).$$
(4.1)

For an exponential catchment residence time distribution, the streamflow recession is appropriately modeled using a linear recession model, dQ/dt = -kQ, where k is the inverse of the



Figure 4.1: Conceptual illustration of the persistence times model. The year is partitioned into a wet and dry seaason (a). The wet season probability distribution (b) is determined according to the model of [Botter et al., 2007b]. The peak flow distribution from the wet season model is used to represent the probability distribution (c) of the dry season initial discharge condition (the discharge following the final storm of the wet season). Through the nonlinear recession relationship (Equation 4.5), the recession persistence time random variable is computed as a derived random variable of the (stochastic) dry season initial condition (d).

mean residence time, also known as the streamflow recession constant. Here, the dimensionless parameter m [] is the ratio between the mean catchment residence time (1/k [T]) and the mean inter-arrival time (1/ λ [T]) of the recharge events. The distribution of recharge depths (exponential with rate parameter γ_Q) can also be computed from catchment soil, vegetation, and hydroclimatic parameters, assumed to be homogeneous across the catchment [Botter et al., 2007b, Müller et al., 2014].

Streamflow conditions at the end of the wet season

In order to uniquely determine the persistence time above a given flow threshold, the initial flow condition (Q_0) at the beginning of the dry season must be specified. Under the simplifying assumption that rainfall seasonality is perfectly binary [Müller et al., 2014], we can mark the start of the dry season as the last recognizable streamflow peak of the wet season. In practice, the selection of this final peak is non-trivial due to the fact that seasonality is not perfectly binary. Section 4.2 develops a consistent methodology to determine the time at which this final wet season peak occurs. At the time of occurrence of this final peak, total discharge is the sum of two stochastic processes: 1) the discharge at the time of the arrival of the last wet season storm and 2) the flow increment generated by the last wet season storm, which we denote Δ . The memory-less property of the Poisson process implies that the discharge at the arrival of the final wet season storm is simply described by the PDF of wet season flow, p_{Q_w} . The initial condition random variable is therefore given by the sum:

$$Q_0 = Q_w + \Delta, \tag{4.2}$$

where the flow increment (Δ) is exponentially distributed with mean $1/\gamma_Q$ [Botter et al., 2007b]. Provided that the system is linear, the distribution for Q_0 is then the convolution of p_{Δ} and p_{Q_w} :

$$p_{Q_0}(q_0) = \int_0^{q_0} p_{Q_w}(q_w) \cdot p_\Delta(q_0 - q_w) \, dq_w$$
$$= \frac{\gamma_Q^{m+1}}{\Gamma(m+1)} q_0^m \exp\left(-\gamma_Q q_0\right), \tag{4.3}$$

which is also gamma distributed with rate γ_Q and shape parameter m + 1.

The seasonal recession

Throughout the large draw-down in catchment storage that occurs during the dry season, the nonlinear nature of the recession dynamics cannot be ignored and the use of a linear reservoir model for the streamflow recession is not appropriate [Brutsaert and Nieber, 1977]. We instead describe the dry-season recession with the nonlinear relation:

$$\frac{dQ_d}{dt} = -aQ_d^{1-r},\tag{4.4}$$

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where Q_d is the dry season streamflow, a is the recession scale parameter, and r is a recession exponent parameter. For an initial streamflow condition, Q_0 , this nonlinear, ordinary differential equation can be solved for Q_d :

$$Q_d(t) = (Q_0^r - art)^{\frac{1}{r}}.$$
(4.5)

This form typically well-approximates flow recessions and is supported by multiple theories, which predict that the first order dynamics of the streamflow recession take a power law form [Biswal and Marani, 2010, Brutsaert and Nieber, 1977, Harman et al., 2009]. For most practical cases, the recession exponent (1-r) is greater than or equal to one $(r \leq 0)$ and the units of the recession coefficient (a) depend on the value of the fitted exponent, which can be determined through empirical fitting procedures or chosen from theoretical considerations. In spite of the fact that most lumped hydrologic models choose fixed values for a and r, these parameters are known to vary seasonally, inter-annually, and across individual recession events [Bart and Hope, 2014, Basso et al., 2015, Biswal and Marani, 2010, Botter et al., 2013, Kirchner, 2009]. Section 4.2 examines the potential impact of this form of variability on persistence times calculations.

The persistence time PDF, $p_{T_{O_*}}(t_{Q_*})$

Due to the fact that Q_0 can be expressed as a single variable function of t_{Q_*} :

$$Q_0(t_{Q_*}) = (Q_*^r + art_{Q_*})^{\frac{1}{r}}, \qquad (4.6)$$

the persistence time PDF $(p_{T_{Q_*}}(t_{Q_*}))$ can be obtained as a derived distribution of the dry season initial condition distribution $(p_{Q_0}(Q_0))$. Figure 4.1 illustrates the relationship between Q_0 and the persistence time via the recession relationship. By the properties of derived distributions, we find $p_{T_{Q_*}}(t_{Q_*})$ as:

$$p_{T_{Q_*}}(t_{Q_*}) = p_{Q_0}(Q_0(t_{Q_*})) \left. \frac{dQ_0}{dt} \right|_{t_{Q_*}} \\ = \frac{a\gamma_Q^{m+1}}{\Gamma(m+1)} \exp\left[-\gamma_Q \left(Q_*^r + art_{Q_*} \right)^{\frac{1}{r}} \right] \left(art_{Q_*} + Q_*^r \right)^{\frac{m-r+1}{r}}, \quad (4.7)$$

where $0 < t_{Q_*} < -Q_*^r/(ar)$. This positive upper bound on the domain of $p_{T_{Q_*}}$ stems from the fact that $Q_0 \to \infty$ as $t_{Q_*} \to -Q_*^r/(ar) > 0$. In other words, the streamflow recession relationship will always reach the chosen flow threshold in finite time, even in the limit of an infinitely high initial condition. Since the initial condition could be less than the chosen flow threshold $(Q_0 < Q_*)$, the persistence time distribution is *also* defined for all negative real numbers. Nevertheless, as long as the chosen flow threshold value is not unreasonably large (less than 50% of the mean annual flow, for example), the proportion of the mass of the persistence time PDF associated with negative values for T_{Q_*} is negligible. To be precise, the truncation of $p_{T_{Q_*}}$ removes all probability mass for $T_{Q_*} < 0$, which is equal to the probability that the dry season initial condition is less than the chosen flow threshold, $Pr[T_{Q_*} < 0] = Pr[Q_0 < Q_*] = P_{Q_0}(Q_*)$. In all subsequent analyses, we truncate $p_{T_{Q_*}}$ for $T_{Q_*} < 0$, renormalize the persistence time distribution as $\frac{1}{1-P_{Q_0}(Q_*)}p_{T_{Q_*}}$ for $T_{Q_*} > 0$, and do not consider persistence time data for years when $Q_0 < Q_*$. In Appendix A, we demonstrate that even for relatively high flow thresholds, this condition occurs fewer than 10% of cases.

The mean persistence time, $\mathbf{E}[T_{Q_*}]$

The direct integration of Equation 4.7 to obtain an expression for the mean persistence time presents analytical difficulties. However, assuming that the peak flow at the end of the wet season exceeds the dry season flow threshold (Q_*) , Equation 4.6 can be inverted to obtain an expression for t_{Q_*} in terms of Q_0 and Q_* :

$$Q_* = (Q_0^r - art_{Q_*})^{\frac{1}{r}} \implies t_{Q_*} = \frac{Q_0^r - Q_*^r}{ar}.$$
(4.8)

The expression for t_{Q_*} is a function of the random variable, Q_0 . We can then derive all the moments of this function using the distribution p_{Q_0} . For instance, the mean is given by:

$$\mathbb{E}\left[T_{Q_{*}}\right] = \int_{Q_{*}}^{\infty} t_{Q_{*}} \cdot p_{Q_{0}}(Q_{0}) \, dQ_{0}$$

=
$$\int_{Q_{*}}^{\infty} \frac{Q_{0}^{r} - Q_{*}^{r}}{a \, r} \cdot \frac{\gamma_{Q}^{m+1}}{\Gamma\left(m+1\right)} \, Q_{0}^{m} \exp\left(-\gamma_{Q}Q_{0}\right) \, dQ_{0}$$

=
$$\frac{\Gamma(r+m+1, \gamma_{Q}Q_{*}) - (\gamma_{Q}Q_{*})^{r} \Gamma(m+1, \gamma_{Q}Q_{*})}{ar\gamma_{O}^{r} \Gamma(m+1)}.$$
 (4.9)

While higher order moments, such as the variance, can be obtained, the expressions are unwieldy and are not presented here.

Model parameter estimation

We tested the persistence time derivations using U.S. Geological Survey daily discharge data for sixteen catchments in Northern California and Southern Oregon, as detailed in Table 4.1. These catchments are characterized by seasonally dry Mediterranean climates, in which the wet season and the 'growing season' (i.e. summer, with highest insolation and temperature) are out of phase. These catchments exhibit a clear and dramatic separation between flow regimes, and strong seasonal recessions, making them a suitable set of catchments on which to test the model. Figure 4.2 shows the location of the test catchments, along with site photos and representative climate data.





Catchment	USGS Gage ID	\mathbf{Stream}	Drainage Area (km^2)	Years of data (n_j)
C1	11463170	Big Sulphur Creek, Cloverdale, CA	33.9	31
C2	11143000	Big Sur River, Big Sur, CA	120.4	60
C3	11476600	Bull Creek, Weott, CA	72.8	49
C4	14325000	Coquille River, Powers, OR	437.7	81
C5	11475000	Eel River, Fort Seward, CA	5457.1	55
C6	11475560	Elder Creek, Branscomb, CA	16.8	43
C7	11481200	Little River, Trinidad, CA	104.9	49
C8	11481000	Mad River, Arcata, CA	1256.1	59
C9	11473900	Middle Fork Eel River, Dos Rios, CA	1929.5	45
C10	11468000	Navarro River, Navarro, CA	784.8	50
C11	11451100	North Fork Cache Creek, Clearlake Oaks, CA	155.9	39
C12	11468500	Noyo River, Fort Bragg, CA	274.5	59
C13	11472200	Outlet Creek, Longvale, CA	416.0	35
C14	11482500	Redwood Creek, Orick, CA	717.4	57
C15	11476500	South Fork Eel River, Miranda, CA	1390.8	71
C16	14307620	Siuslaw River, Mapleton, CA	1522.9	35
		Table 4.1: Study catchment inf	ormation	

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Although it is possible to estimate λ and γ_Q from catchment vegetation, soil, and rainfall data, and to interpolate estimates of the linear recession constant k from neighboring gauges [Müller and Thompson, 2015], we did not test the persistence time model in ungauged basins. The difference in the quality of parameter estimates for the streamflow model when forced by stream data versus reliable rainfall data was explored in a previous study, and shown to be minimal [Müller et al., 2014]. We therefore use gauged streamflow data to parameterize and to test the model.

To determine the parameters λ and γ_Q , we first identify all well-defined peaks from the wet season hydrograph (as defined below). λ is computed as the reciprocal of the mean of the inter-arrival periods between these peaks. Next, we extract the magnitude of the increasing segment of the hydrograph preceding each peak and compute γ_Q as the reciprocal of the mean of these positive discharge increments.

The remaining parameters to be estimated are the recession parameters, k for the wet season, and a and r for the dry season. To estimate these parameters, we firstly partitioned the year into distinct wet and dry periods. This was achieved by fitting a square wave function to each year of streamflow data, taking a value of the mean seasonal streamflow in each case. This fit was constrained by the requirement that the initial guess for the 'wet season' should include both the centroid (along the time axis) of the annual streamflow time series:

$$\tau_Q = \frac{\int_0^{365} t \cdot Q(t) dt}{\int_0^{365} Q(t) dt},\tag{4.10}$$

and the deviation about the centroid, $\tau_Q \pm s_Q$, where s_Q is defined as the square root of the second moment of the annual streamflow time series:

$$s_Q = \sqrt{\frac{\int_0^{365} (t - \tau_Q)^2 \cdot Q(t) dt}{\int_0^{365} Q(t) dt}}.$$
(4.11)

To estimate the dry season parameters, we firstly isolated the seasonal recession by identifying the last storm that initiated the seasonal dry down. In most cases, this storm was linked to the final streamflow peak in the fitted 'wet season' square wave. In some cases, however, late spring storms resulted in larger streamflow peaks occurring in the initially fitted 'dry season' period. The seasonal recession in these cases was taken to begin with the last of these peaks. The dry season duration (T_d) was then defined as the period between the start of the seasonal recession, and the rising edge of the fitted step function for the following year. Generally, T_d exceeded 150 days in all the study catchments. We used the first $T_d - 30$ days of the dry season to fit the recession parameters, a and r. The final 30 days were excluded because (i) gauged estimates of extreme low flows typically observed late in the dry season are often unreliable or recorded as zero, and (ii) we wished to exclude the seasonal transition from dry to wet in the fitting procedure to avoid early season storms in the following wet season. An alternative method to avoid end of dry season issues would be to fit a and r only to the recession periods greater than the threshold, Q_* . In this case, however, the fitted values a and r may then depend, albeit weakly, on the particular choice of threshold. This alternative method would undoubtedly generate better model performance, as only the relevant portion of the recession time series would be used for fitting. Nevertheless, this study implements the first method to ensure fitted values of a and r remain constant, regardless of the choice of threshold.

The wet season recession constant, k, was estimated by extracting all wet season flow recessions exceeding 4 days in length, and regressing the logarithm of the recession discharge against time. k was computed as the median of the regression slope coefficients across all extracted recessions.

The recession parameters (a and r) were fitted using a nonlinear least squares procedure, minimizing the sum of squared errors between the dry season recession model and the observed dry season recession over all years. Appendix B includes a table of these parameters for each catchment.

Model evaluation

Having estimated the model parameters, persistence times were estimated for each year by considering a range of flow thresholds, finding the date at which those thresholds were first crossed during the seasonal recessions, and computing the time lapsed since the last wet season storm. Persistence times cannot be computed in this way for two cases:

- 1. For years when the flow threshold (Q_*) exceeded the dry season initial condition (Q_0) . We do not expect this restriction to affect the quality of the streamflow dataset; even for the highest flow threshold at 50% of mean annual flow, this condition accounted for less than 5% of the study years (see Appendix A for more details).
- 2. For years when the dry season discharge does not drop below the chosen flow threshold. Unless a river explicitly runs dry during the dry season, it will always be possible to define flows so low that the seasonal recession does not achieve these levels in a meaningful timeframe, that is, within the duration of a typical dry season.

To ensure that the model is only evaluated for meaningful flow thresholds (namely, flows that are low enough to be distinct from the wet season, but high enough that the river flow will pass below them in a typical dry season), we evaluate the model for thresholds (Q_*) ranging from 5% to 50% of mean annual flow.

The analytic mean persistence times were plotted against empirical mean persistence times for flow thresholds ranging from 5% to 50% of mean annual flow, and the corresponding R^2 value of a one-to-one line reported. The performance of the model in estimating persistence time PDFs was evaluated using the Nash-Sutcliffe Coefficient (NSC) as applied to the persistence time percentiles:

NSC = 1 -
$$\frac{\sum_{i=1}^{99} \left(\hat{T}_i - T_i\right)^2}{\sum_{j=1}^{99} \left(T_j - \frac{1}{99} \sum_{k=1}^{99} T_k\right)^2},$$
 (4.12)

where \hat{T}_i and T_i are the empirical and modeled persistence times associated with percentile i, respectively. The NSC corresponds to an R^2 value for the fit of a one-to-one line to a plot of the empirical percentiles versus the modeled percentiles; it has been used extensively for the assessment of hydrologic models [Castellarin et al., 2004, Müller et al., 2014, Nash and Sutcliffe, 1970]. NSC values range from negative infinity to one, where an NSC of one corresponds to a perfect match between the percentiles. Due to the strong dependence of the persistence times model on the performance of the wet season model, we also computed NSC values testing the wet season streamflow distribution (p_{Q_w}) and the dry season initial condition distribution (p_{Q_0}) . Although this application of NSC values was originally developed for flow duration curves, the tests here are directly analogous to the duration curve, being derived from the PDFs. We thus assume NSC values provide a suitable relative rating scheme.

Appendix B provides illustrations of the analytic mean persistence times plotted against empirical mean persistence times, and plots of the persistence time PDFs for all catchments with Q_* set to 20% of mean annual flow.

Effect of recession variability on predictions

As shown in Section 4.3, the mean persistence time predictions perform well for most watersheds, yet the distributions $p_{T_{Q_*}}$ fail to reproduce the variance of the full empirical persistence time distributions. To determine why this occurred, we systematically assessed whether the model assumptions were met by the empirical data. In agreement with other recession studies, we found that the nonlinear recession parameters a and r varied significantly between years [Bart and Hope, 2014, Basso et al., 2015, Biswal and Marani, 2010, Botter et al., 2013, Kirchner, 2009, Shaw and Riha, 2012]. In violation of the model assumptions, which presume parameter independence, this variation was characterized by a strong correlation between a and r, which has not been examined in previous recession studies. An example of this correlation is shown for one of the study sites, Redwood Creek, in Figure 4.3 (left hand panel).

Such correlation has been identified in other studies as a mathematical artifact that arises from the scale-free properties of the powerlaw (i.e. Equation 4.4), and existing techniques are available to rescale the flow variable Q to remove this correlation [Bergner and Zouhar, 2000]. The genesis of this artifact, and the theory underpinning its removal, are outlined in Appendix B. The effectiveness of the rescaling technique in removing the correlation is illustrated in the right hand panel of Figure 4.3. Two recession curves, which correspond to the two highlighted recession parameter pairs, are plotted in the top of Figure 4.3. Section 4.4 provides more background and motivation for the examination of power law parameter


Figure 4.3: Fitted recession parameters for streamflow in units of cfs and dimensionless Q scaled to remove parameter correlation. The two highlighted parameter pairs correspond to the two recession curves illustrated in the smaller inset plot.

correlation. First, however, we outline a method to quantitatively demonstrate the effects of recession parameter variability.

Monte Carlo simulation

The persistence time derivations could be generalized to include recession parameter variability. This would require the specification of a joint PDF for the recession parameters $(p_{A,R}(a,r))$ and the integration of Equation 4.7, the persistence time distribution, over this joint PDF. While this approach is elegant, the specification of $p_{A,R}(a,r)$ is a serious challenge. Using a-r data to generate an empirical, joint distribution is difficult, considering our small sample sizes (on the order of tens of years of dry season recession data per catchment). From a modeling standpoint, the determination of $p_{A,R}(a,r)$ from first principles would require a clear understanding of the mechanisms underlying recession parameter variability, a question which remains largely unanswered [Harman et al., 2009], especially when variations in both a and r are considered.

In light of these challenges, we apply a Monte Carlo approach to assess the effects of recession parameter variability and correlation, both separately and in combination, on the predictions of the persistence time PDF.

To isolate the effects of the recession parameters and their variation, we firstly fit unique recession curves to each of the n_j observed dry season recessions for catchment j. This process generates n_j unique recession parameters pairs for each catchment, $\{(a_1, r_1), (a_2, r_2), ..., (a_{n_j}, r_{n_j})\}$. We also compute a set of minimally correlated recession parameter pairs, denoted \hat{a}, r , by

repeating the fitting procedure to flow data that had been rescaled using the parameter decorrelation method described in Appendix B [Bergner and Zouhar, 2000, Mather and Johnson, 2014]. Then, we fit a gamma distribution to the empirical initial conditions Q_0 on each recession. For a given Monte Carlo run, we draw n_j samples from this distribution. We use the fitted rather than the modeled p_{Q_0} PDF in order to confine any model error sources to the treatment of a and r variation. The Monte Carlo proceeds by drawing a sample of Q_0 , and computing a persistence time (using Q_* set to 20% mean annual flow). Four different treatments of a and r allow us to assess the effects of a and r variability and correlation on persistence time predictions:

- 1. M1: constant a and r The original scaling of Q (units of ft^3/sec) was used. Persistence times were computed for each sampled n_j initial condition using constant values of a and r. The values of a and r for each catchment were chosen by minimizing the sum of squared errors between observed and the predicted dry season recessions over all years as described in Section 4.2. This is a null case that replicates the developments in this paper. It contains neither variability nor correlation in the recession parameters.
- 2. M2: varying but independent a and r Again, the original scaling of Q in units of ft³/sec is used. In contrast to M1, the recession parameters are now allowed to vary between each initial condition selected. The recession parameters are selected by bootstrapping independent samples from the sets $\{a_1, a_2, ..., a_{n_j}\}$ and $\{r_1, r_2, ..., r_{n_j}\}$. This case preserves variability but does not incorporate the observed correlation in the recession parameters. Additionally, the results from this simulation provide a reference which can be used to determine the relative benefit of implementing recession parameter decorrelation (M3).
- 3. M3: Parameter decorrelation with varying, but independent, \hat{a} and \hat{r} . In this instance, Q is re-scaled according to the decorrelation technique of Bergner and Zouhar [2000]. Persistence times are computed from the n_j initial conditions using new recession pairs generated by bootstrapping independent samples from the sets $\{\hat{a}_1, \hat{a}_2, ..., \hat{a}_{n_j}\}$ and $\{r_1, r_2, ..., r_{n_j}\}$. This case preserves variability, and accounts for the component of correlation in the recession parameters that can be removed by the Bergner and Zouhar [2000] method. It is notable that the *only* difference between M2 and M3 is the scaling of Q. Differences between the results of M2 and M3 explicitly illustrate the impact of recession parameter correlation.
- 4. M4: Parameter decorrelation with varying, jointly sampled \hat{a} and r pairs. Q is non-dimensionlized according to the decorrelation technique of Bergner and Zouhar [2000]. Persistence times are computed from the n_j initial conditions using recession pairs uniformly sampled (without replacement) from $\{(\hat{a}_1, r_1), (\hat{a}_2, r_2), ..., (\hat{a}_{n_j}, r_{n_j})\}$. This case preserves variability and all correlation in the recession parameters. If the decorrelation procedure completely removed all correlation of the form generated by the scale dependent artifact (outlined in Appendix B), then there would be no measurable

differences between M3 and M4, unless there exists another (physically derived) form of correlation between the recession parameter pairs; that is, correlation that does not obey the form of the artifactual correlation.

For each run of the process, a distribution of persistence times was generated, and compared to the empirical distribution via the NSC. The Monte Carlo process was repeated 1000 times to generate confidence intervals around the computed NSC's.

4.3 Results

Stochastic streamflow model applied to Northern California and Southern Oregon catchments

The wet season and dry season initial condition models performed well for the study catchments, as shown in Table 4.3. For the wet season model, all Nash Sutcliffe coefficients exceeded 0.75. With four exceptions (catchments C4, C7, C12, and C14, where 0.5 < NSC < 0.75), the same holds true for the dry season initial condition. The initial condition models in these four catchments over-predicted the magnitude of the dry season initial condition.

Catchment	NSC for p_{Q_w}	NSC for p_{Q_0}	\mathbf{R}^2 for $\mathbb{E}\left[T_{Q_*}\right]$
C1	0.8	0.94	0.93
C2	0.93	0.78	0.41
C3	0.89	0.88	0.96
C4	0.97	0.71	0.98
C5	0.94	0.76	0.88
C6	0.88	0.88	0.9
C7	0.96	0.58	0.98
C8	0.93	0.79	0.95
C9	0.81	0.76	0.9
C10	0.9	0.9	0.99
C11	0.78	0.92	0.85
C12	0.9	0.7	0.97
C13	0.87	0.8	0.77
C14	0.97	0.71	0.96
C15	0.91	0.81	0.98
C16	0.96	0.81	0.93

Table 4.2: Study catchment evaluation metrics for the wet season flow PDF (p_{Q_w}) , the dry season initial condition PDF (p_{Q_0}) , and for the modeled mean persistence times ($\mathbb{E}[T_{Q_*}]$).

Mean persistence time and its PDF

The ability of the persistence time model to predict the mean persistence time was good (NSC> 0.75) for discharge thresholds ranging from 5% to 50% of mean annual flow (Table 4.3). One exception is for catchment C2 (Big Sur), where the model performance is relatively poor.

In Figure 4.4a, a comparison of the predicted and observed mean persistence times for the threshold range 2% to 50% of mean annual flow is shown for catchment C14, Redwood Creek. The fit here is typical for most catchments, where the mean performance is excellent for larger flow thresholds, and begins to degrade once the flow threshold approaches 2% mean annual flow (larger persistence times). Generally, mean performance across all catchments tends to break down at very low discharge thresholds. As noted in 4.2, this is because seasonal recessions cannot achieve very low flow thresholds within the dry season duration; that is, the following wet season begins before the threshold is reached. In this case, persistence times can only be computed for the driest years, and so are biased towards smaller values. Catchment C2 (Big Sur) performs especially poorly (a breakdown of the model for flow thresholds near 20% of mean annual flow), likely due to the fact that Big Sur has significant geothermal flows generated by three distinct springs. In this watershed, dry season flows rarely drop below 20% mean annual flow.

Figure 4.4a highlights the variability around the mean for both the analytic (grey envelope) and empirical (grey whiskers) mean persistence times. The spread of the analytic curve demonstrates a clear skew towards shorter persistence times, whereas the empirical curve shows a fairly symmetric spread about the mean. Persistence time distributions at 20% of the mean annual flow (blue rectangle in Figure 4.4a) are plotted in Figure 4.4b (PDFs at this threshold are presented for all other catchments in Appendix B). At this threshold, it is clear that the analytic persistence time distribution underestimates the observed variability. Based on the model derived here, moreover, the analytic persistence time distribution is undefined for persistence times outside the interval, $0 < t_{Q_*} < -Q_*^r/(ar)$. The empirical distribution does not adhere to this domain restriction. Some of the observed persistence times are more than twice as large as the maximum possible persistence time predicted by the analytic distribution. Across all flow thresholds and catchments explored here, the modeled persistence time variability underestimates the observed variability.

Source of persistence time variability

Example persistence time probability distributions generated from a single Monte Carlo run for Redwood Creek are presented in Figure 4.6. The results for simulation M1 echo those from the analytic distribution in Figure 4.4b. As expected, the mean performance is good, but the full variability (Figure 4.6, empirical data) is not well accounted for. Simulation M2 represents the case where parameters vary but the correlation between pairs is not preserved. In this case, almost all predictive ability is lost. Simulation M3 also relies on independent bootstrapping of parameters, but the streamflow is scaled (according to the technique of



Figure 4.4: The empirical mean persistence time plotted against the analytic mean persistence time for Redwood Creek (a). The grey envelope signifies 95% of the variability around the analytic mean persistence times, whereas the grey whiskers represent 95% of the variability around the mean empirical persistence times. (b) shows the full persistence time distribution for a discharge threshold set to 20% of mean annual flow. Blue boxes represent the empirical histogram and the solid black line plots the analytic probability distribution. The empirical (grey dashed) and analytic (black dashed) means are also shown in (b).

[Bergner and Zouhar, 2000]) to minimize any recession parameter correlation (specifically, parameter correlation that adopts a logarithmic dependence between parameters is eliminated; parameter correlation with a different functional dependence may remain). Here the model performance is quite reasonable, and greatly improved compared to both M1 and M2. Model M3 does not appear to perform as well as model M4 (which includes any correlation remaining after streamflow rescaling), though the difference in performance is not large.

These results are general for all the watersheds in the study, as shown in Figure 4.5. The primary conclusion is that the greater than expected spread in persistence time distributions results from the coupled inter-annual variability in the seasonal recession parameters, a and r. Without accounting for variability in the streamflow recession parameters (Figure 4.5, M1), the performances of the analytic PDFs are generally poor, with tight confidence intervals around the computed NSCs. Simulation M2 demonstrates that neglecting the recession parameter correlation within a power-law model fundamentally violates the mechanics of the power-law (such correlation is an inevitable and essential feature of power-law functional models, see Appendix B) and leads to basically nonsensical results: poor model performance and wide confidence intervals on the computed NSCs. Merely re-scaling the flow variables to remove the parameter correlation (M3) greatly improves model performance, although pre-



Figure 4.5: Results from the recession parameter Monte Carlo simulations. Each row corresponds to a study catchment, and each column corresponds to one of the four simulations. Mean Nash-Sutcliffe coefficients are represented with a vertical black line with a color bar corresponding to the 95% confidence interval of calculated NSC's; no black line indicates that the mean is less than -1. A red vertical line segment with black circles indicates that the calculated mean NSC and confidence intervals are less than -1. The colorbar scale indicates the relative quality of the fits according to the NSC value.



Figure 4.6: Results from a single Monte Carlo run for Redwood Creek. The blue PDF's on the top row represent the empirical persistence times data, while the multi-colored PDF's on the bottom row each represent one of the four types of Monte Carlo simulation (M1-M4). The y-axis scale is ommitted to facilitate comparisons. Also note the scale for MC2, stretched due to the fact that this Monte Carlo simulation produces very large persistence times. NSC values for goodness of fit relative to the empirical persistence times are included for each simulated probability distribution.

serving the original recession parameter pairs (M4) continues to improve the model behavior. Disparities between M3 and M4 indicate that some form of residual parameter correlation remains, following the de-correlation procedure. As expected, simulation M4 performs very well, illustrating that recession parameter pairs are functionally independent of streamflow initial conditions - at least in terms of the impact on the persistence time PDFs.

4.4 Discussion

Model performance

The proposed model for the persistence time during the summer dry season provides a robust estimate of the mean conditions at which flow in a drying river is sustained above meaningful threshold values. The model thus provides a reasonable basis for estimating the effects of changing climate or land use parameters on the mean availability of dry-season surface water resources, and the associated ecosystem services they provide. The minimal parameterization of the model would in principle facilitate distributed assessments across many small watersheds.

A full accounting of the variability in dry season streamflow properties clearly requires elucidating the PDF of persistence times, and the analytical model fails to achieve this. Mediocre fits in the dry season initial condition distribution cannot explain increased variability in the persistence times, unless the distributions of Q_0 themselves exhibited unexpected increases in variability, which they do not. The primary source of error in the initial condition distributions appears to be a slight bias towards smaller values of Q_0 . This could be due to non-binary seasonality (bias towards smaller storms towards the end of the wet season), or performance decreases associated with peak flow estimation using the linear recession model in the wet season [Basso et al., 2015]. Monte Carlo simulations illustrate that it is inter-annual variation in the character of the seasonal recession that results in a large increase in the variability of persistence times, which is not well estimated with median recession characteristics. Unfortunately, such variation is generally not possible to prescribe *a priori*.

Physically, variation in recession behavior is thought to arise due to both changes in total catchment storage and in the partitioning of that storage amongst reservoirs with different drainage characteristics [Bart and Hope, 2014, Biswal and Marani, 2010, Harman et al., 2009, Moore, 1997, Shaw and Riha, 2012]. To account for such variability in a lumped model requires specifying multiple reservoirs, each characterized by distinct timescales - a challenging basis from which to generate stochastic predictions. Fundamentally, the hydrologic literature lacks a unified understanding of the genesis of recession variability, which has been linked by various authors to the drainage dynamics of hillslope aquifers [Brutsaert and Nieber, 1977], to catchment heterogeneity [Harman et al., 2009], and to variability in the wetted extent of the channel network [Biswal and Marani, 2010]. Evidently, a single nonlinear reservoir model is inadequate to capture these variations and thus to represent the full probabilistic character of seasonal recessions.

Moreover, the parameter correlation identified in this analysis suggests that testing the existing theories regarding recession variability requires carefully controlling for the mathematical artifacts that the widely used power-law recession models can generate. Many of the theories addressing recession characteristics attribute physical meaning to parameters in a power-law recession model [Biswal and Marani, 2010, 2014, Brutsaert and Nieber, 1977, Harman et al., 2009] - and yet the values of these parameters, and their relationship to each other, appear to be poorly constrained.

As far as we are aware, the mathematical artifact leading to the a - r correlation we observed is systematically removed in only a single study in the hydrological literature: using the technique from Bergner and Zouhar [2000], Mather and Johnson [2014] removed power law parameter correlation in order to improve the performance of a turbidity rating curve. In that work and here, recession parameter correlation presented an unexpected and significant source of model error. In other fields of study, ranging from fluid mechanics to materials science, power law parameter correlation has been mistakenly identified as having physical meaning [Cortie, 1991, Hussain et al., 1999, Shih et al., 1990, Zilberstein, 1992]. Power law parameter correlation has been observed but not properly identified or removed in few streamflow recession studies [Krueger et al., 2010, McMillan et al., 2014, Sawaske and Freyberg, 2014]. This is due, in part, to the fact that the classical goal of a recession analysis avoids event specific recession analyses of a-r point clouds. Nevertheless, parameter correlation should be accounted for, where appropriate, especially in light of rapidly increasing interest in recession parameter variability [Bart and Hope, 2014, Basso et al., 2015, Biswal and Marani, 2014, McMillan et al., 2014, Patnaik et al., 2015a, Shaw and Riha, 2012].

4.5 Conclusion

This study developed a probabilistic model of the persistence time, T_{Q_*} , the number of days from the start of the dry season for which dry season flow exceeds Q_* . By linking the dry season recession to wet season flow dynamics through a dry season initial condition model (p_{Q_0}) , the model analytically demonstrates the precise linkages between the dry season recession and simple measures of wet season hydroclimate and catchment geomorphology. In spite of excellent performance in the mean sense, the model did not incorporate an unexpected but essential source of variability: inter-annual fluctuations in the recession behavior, and correlation between the parameters used to describe it. The positive performance of the model overall suggests the potential for future work to exploit hydrologic predictions of the crossing properties of wet and dry season flows to support large scale, rapid, in-stream ecological risk assessments. The challenges associated with capturing variability in seasonal recessions demonstrates the importance of improving understanding of recession variability. Parameter decorrelation emerges as a promising research avenue to separate the informative characteristics of hydrograph recessions from those that are purely attributable to the mathematical properties of the widely used power law model.

Part II Analysis

Chapter 5

a, b careful! Scale invariance hinders comparative analyses in power law models of the streamflow recession

5.1 Introduction

Modeling streamflow recessions with power laws

A broad spectrum of physically-based and conceptual models of catchment hydrology use power law functions to represent the nonlinear reduction in discharge during streamflow recessions. Power law recession models describe either the relationship between discharge $(q, \text{ in units of } [L/T] \text{ or } [L^3/T]$, where L represents a length scale and T a time scale) and its rate of change with time t [Botter et al., 2009, Kirchner, 2009, Rupp and Selker, 2006b], as:

$$\frac{dq}{dt} = -aq^b,\tag{5.1}$$

or between discharge and a metric of catchment storage volume (S, in units of $[L^3]$ or [L]), [Brutsaert and Nieber, 1977, Farmer et al., 2003, Kirchner, 2009, Wittenberg, 1999a], as:

$$q = c \left(S - S_{ref} \right)^d, \tag{5.2}$$

where S_{ref} is the (generally) non-zero storage value at zero discharge.

The ability of power law functions to approximate observed nonlinearity in the hydrograph motivates their widespread adoption [Amorocho, 1963, 1967, Kirchner, 2009, Wittenberg, 1999a]. Their use is often theoretically justified by the wide variety of solutions from hydraulic groundwater theory (which describes the discharge of an aquifer into a stream) that follow power law expressions of the form $dq/dt = -aq^b$ [Bogaart et al., 2013, Rupp and Selker, 2006b]. These models predict a particular value of the power law exponent *b* [Bogaart et al., 2013, Rupp and Selker, 2006b], while the coefficient *a* results from the interplay of several physical terms describing the aquifer geometry and its hydraulic properties. Power laws are also used as conceptual closure models for the catchment water balance, and in this context the exponent b is usually determined empirically by a best fit procedure [Botter et al., 2009, e.g.]. The multiplicative parameter a is also fitted and adopts a dimensionality that varies with the fitted exponent (i.e. the units of a are $T^{-1} (L^3/T)^{1-b}$ when q is a volumetric flow rate, or $T^{-1} (L/T)^{1-b}$ when q is normalized by catchment area).

Power law behavior (at least approximately, and over a finite range of scales) is a common phenomenon in natural, engineered and social systems. Despite the ubiquity of power laws, their application is well-known to raise a range of methodological pitfalls. For example, the common practice of log-transforming a power law (generating a linear functional form) and then estimating its parameters via least-squares regression, creates a risk of bias in the fit. This arises because the least squares procedure places an equal weight on the linear deviations from the line of best fit. Following back-transformation into linear space, however, the magnitudes of these deviations diverge exponentially – biasing the fit towards small values of the model [Miller, 1984, Pattyn and Van Huele, 1998, e.g.]. Similar problems arise when fitting power law distributions to data, due to biases introduced by binning and log transformation [Clauset et al., 2009, Goldstein et al., 2004]. These issues have been raised in several comprehensive reviews, which also demonstrate appropriate fitting and estimation techniques that avoid such biases [Clauset et al., 2009, Goldstein et al., 2004].

This letter addresses an additional mathematical property of power laws, their "scale-free" nature, which generates specific challenges for the analysis and interpretation of stream-flow recessions. The methodological issues raised by the scale-free properties of power laws have not, however, received the same comprehensive analysis in the literature as the fitting and bias issues mentioned above. Although the work presented here is relevant to any power law model, the aim of this letter is to illustrate the consequences of the scale-free nature of the power law when used in hydrological settings – specifically recession analysis.

Firstly in Section 5.1, we describe the scale-free nature of power laws and show how this property can generate mathematical artifacts that challenge interpretation of populations of fitted power law parameters. We then briefly address how these challenges affect two issues pertinent to catchment hydrology: (i) Section 5.1 discusses the generation of tantalizing (but purely formal) relationships between model parameters, and (ii) Section 5.1 outlines the potential for the mathematical artifact to obscure the information content of recession data and the drawbacks associated with current techniques for coping with power law scaling issues. To address these methodological challenges, Section 5.2 outlines an empirical method to remove the scale-dependent artifact and shows that the resulting re-scaled data reveals new, and potentially informative, temporal structure.

The scale-free properties of power laws

The scale-free properties of power laws are most evident when the state variable (in this case the flow q) is scaled by a linear constant k, so that it adopts a new value $q = k\hat{q}$. Such

rescaling is a basic data analysis operation – for example it is required to change the units in which q is expressed, or to normalize discharge by catchment area.

Following rescaling, if \hat{q} is substituted into Equation 5.1 and a is assumed independent of b, then a new power law relationship is obtained for the rescaled discharge:

$$\frac{d\hat{q}}{dt} = -ak^{b-1}\hat{q}^b,\tag{5.3}$$

Equation 5.3 has the same form as Equation 5.1, and the exponent b is unchanged between the two equations. The equivalence in exponent is the reason that power laws are considered "scale-free" – the exponent of the power law is independent of a linear rescaling of the state variable. The difficulties are introduced, however, in the multiplicative parameter. If a simple power law were fit to Equation 5.3, then this multiplicative parameter would be found to adopt a new value, \hat{a} , which differs from the multiplicative factor a at the original scale by a factor of k^{b-1} – that is, $\hat{a} = ak^{b-1}$.

The immediate consequence of this scale dependence is that the degree to which a fitted value of \hat{a} reflects the scale at which flow is measured or reported (as embodied by the value of k^{b-1}), or to which it provides information about physical processes (as embedded in the scale-independent value of a), is unknown. This makes it challenging to fully identify and use the information contained in the power law parameters. Avoiding this issue, most hydrologic studies either examine variation in fitted values of b, which is independent of scale, or fix the value of b in order to examine *relative* variation in fitted values of \hat{a} . Arguably, neither of these approaches is ideal: either fitted values of \hat{a} cannot be compared due to variation in b, or the fitting procedure is biased by the constraint that b remain constant. In Section 5.2, we present a re-scaling approach applied to \hat{q} in order to estimate the value a while avoiding the necessity of fixing b during the fitting process.

Formal parameter correlation

The relationship between the fitted power law coefficient and scale, $\hat{a} = ak^{b-1}$, means that the parameters \hat{a} and b of the fitted power law display a correlation which can emerge, disappear, strengthen, and weaken with the scale of measurement. Explicitly noting that correlation occurs and is purely 'formal' – that is, it does not arise from any hypothesized physical relationship, but derives from the mathematical features of power laws – is important for recession analysis, since the emergence of similar correlations in several other fields has historically created distraction and controversy. We have identified studies showing power law parameter correlations in the material fatigue literature [Cortie, 1991, Zilberstein, 1992], electrical engineering [Sun et al., 1990] and in the fluid mechanics of blood circulation [Hussain et al., 1999]. Several of these studies interpret the correlation as being indicative of physical or biological mechanisms – an interpretation that is at best confounded by and at worst solely attributable to the formal correlations. Correlation between power law parameters has been observed in recession curves [Krueger et al., 2010, McMillan et al., 2014], but is not widely discussed in the recession literature. Such correlations (in our own experience!) are highly compelling when identified in empirical data. They are not readily identified when considering tests to alleviate 'spurious correlation' (that is, they do not conform to the traditional definition of a spurious correlation arising between two indices that have a common component [Kronmal, 1993]). Although power law correlation is not discussed in the recession literature, it has been documented between the parameters of sediment rating curves [Mather and Johnson, 2014, Syvitski et al., 2000, Thomas, 1988]. These studies either made no attempt to explore the correlation [Syvitski et al., 2000], or while treating it as an artifact, and removing it in the case of Mather and Johnson [2014], did not investigate its origin [Mather and Johnson, 2014, Thomas, 1988].

A further motivation to ensure that hydrologists are familiar with power law parameter correlation lies in new theories that are currently being proposed to explain the origin of power law recessions in physical terms [Biswal and Marani, 2010, Harman et al., 2009]. For example, Harman et al. [2009] show that power law recessions can result from catchment heterogeneity. If the recession from a catchment is conceptualized as the superposition of the outflow from a population of distinct, parallel hillslopes, each behaving as a linear reservoir, then the cumulative recession behavior can follow a power law. Adopting a different conceptualization of the main drivers of streamflow recession, Biswal and Marani [2010] hypothesize that the expansion and contraction of the wetted drainage network, the socalled 'active drainage network', could generate power law recession dynamics. We note that in the original presentation of each of these theories, the recession exponent, b, should remain fixed. Logical extensions of the theory, however, could result in predictions of covariation in the recession parameters with catchment condition - through relaxation of the assumption that hillslope contributions to the watershed are temporally stationary, in the case of Harman et al. [2009], or under conditions of spatial heterogeneity relative to the static river network [Biswal and Nagesh Kumar, 2012, e.g.].

Without speculating on the validity, or formal derivation, of such predictions, we note that unambiguously describing the co-variation of power law parameters in empirical data, and attributing it to physical rather than mathematical drivers, is non-trivial. Hydrological theories may well predict correlations between recession parameters, but observations of such correlations are likely to be a questionable basis for testing such theory.

Challenges in model parameter selection and the interpretation of empirical data

The primary challenge to comparative recession analysis and modeling posed by scaleinvariance lies in the relative magnitude of the terms a and k^{b-1} that compose any fitted \hat{a} . Without a technique to unambiguously separate these terms, or serendipitously selecting a scale such that $\hat{a} \approx a$, the information content of a fitted value of \hat{a} will likely be obscured by the scaling term. The extent to which this negatively impacts the use of a power law recession relationship is largely dependent on specific applications. There remains, however, a clear need to identify methods that cope with scale dependence and correlation to maximize the quality of modeling applications and the value of comparative recession analyses.

Power law recessions and model closure

If the power law is used as a closure relationship in a hydrologic model, a single (\hat{a}, b) pair must be chosen for model parametrization. This is typically accomplished in one of three ways:

- 1. An effective pair is computed using a measure of centrality on the population of fitted (\hat{a}, b) pairs. In this case, the centrality measure (e.g. the mean) may poorly represent the population of parameters if the correlation is strongly nonlinear. In particular, if a strong correlation exists between \hat{a} and b of the form $\hat{a} = ak^{b-1}$, then mean effective values for \hat{a} and b ($\mathbb{E}[\hat{a}]$ and $\mathbb{E}[b]$) will not obey the correlation relationship. This may result in inexplicably poor recession curve fits when mean recession parameter values are used for modeling purposes.
- 2. An effective pair is determined by minimizing some metric of model error. Here, a fitted (\hat{a}, b) pair is chosen to minimize an error function across a population of recessions, and may not have physical meaning.
- 3. An effective pair is determined by defining a lower envelope to data points on a logarithmically scaled scatter plot of -dq/dt vs. q. This method avoids issues with parameter correlation because it does not require fitting a population of recession parameter pairs; the recession exponent is fixed, either by fitting a single lower envelope to the data or by defining one a priori through theoretical considerations. However, the procedure explicitly assumes that the power law recession will be used to model catchment baseflow (Brutsaert and Nieber [see e.g. 1977]). Additionally, a number of authors have demonstrated that this lower envelope may significantly underestimate the value of the recession exponent [Biswal and Marani, 2010, Rupp et al., 2009, Shaw and Riha, 2012].

Despite these challenges, the primary objective in hydrologic modeling is generally to minimize some measure of model error. If this goal is suitably met, then recession parameter interpretation and the precise values adopted by \hat{a} and b may be of secondary importance. This is not the case, however, when power law recession models are used as a data analytic tool.

Power law models and recession analysis

When not being used as a closure model, the parameterized recession relationship is generally intended to provide a means for data exploration. In this case, fitted (\hat{a}, b) pairs are analyzed to measure or determine the physical drivers of recession variability. This form of analysis presents another dilemma: either variations in b are examined [Clark et al., 2009, Shaw and Riha, 2012, Tague and Grant, 2004], at the cost of injecting scale-induced variations into the fitted value of \hat{a} ; or b is held fixed and the relative values of \hat{a} can be compared, at the expense of introducing bias into the fit by forcing b to remain constant [Bart and Hope, 2014, Biswal and Marani, 2010, 2014, McMillan et al., 2014, Mutzner et al., 2013, Szilagyi et al., 1998a]. Neither approach is ideal, as either the information that could be obtained from the \hat{a} parameter is discarded (in order to examine variation in b), or the estimates of the \hat{a} parameter are potentially biased because b is fixed (also removing the potential to examine variations in b). The additional fitting error due to this bias can be significant, as we demonstrate in the case study in Section 5.3.

Table 5.1 summarizes the inherent challenges and limitations imposed by recession parameter scale dependence, along with representative studies that utilize the variety of available methods for recession analysis. These literature examples highlight that addressing scale invariance has the potential to further optimize, expand, or generalize even highly influential studies. In particular, the ideal approach would be to choose the flow scaling for which $\hat{a} = a$, allowing a physically meaningful estimate of a independent of the value of b.

Literature	Stodalo of al [0019] Tamo and Curve [0004]	DUCEIZIE EU AL. [2013], LAGUE ALIU GLAIN [2004]	Biswal and Marani [2010, 2014]	McMillan et al. [2014]		Müller et al. [2014]			Botter et al. [2009], Ye et al. [2014]				Brutsaert and Nieber [1977]			obtain recession parameter estimates.
Outcome	Cannot examine	variation in \hat{a}	Additional fitting error notentially introduced	could bias \hat{a} fits	Veluce chanded not be	Values siloulu IIU De	merpreteu puysicany	Dotential issues with		nommear averaging	Can underestimate	b for individual	recessions; only	theoretically valid for	modeling baseflow	rious methods used to
Method	Dromino muintion in 6440d h		Fix h and fit \hat{a}		Obtain (\hat{a}, b) pair	that minimizes error across a	population of recessions.	Fit (\hat{a}, b) pairs and use	a measure of centrality to represent	a population of recessions.		Define a lower envelope	to data points on a log-log	plot of dq/dt vs. q		tial issues associated with the var
Goal	Recession					Hydrologic modeling										Table 5.1: Potent

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5.2 Recession parameter de-correlation

De-correlation method

The most general route to remove formal parameter correlation involves selecting a number q_0 such that rescaling \hat{q} as $\hat{q} \rightarrow \hat{q}/q_0$ eliminates correlation of the form $\hat{a} \propto k^{b-1}$ between the power law parameters. Such a technique was formally derived by Bergner and Zouhar [2000], who showed that correlation of this form is minimized for a unique value of q_0 , given by:

$$q_{0} = \exp\left(-\frac{\sum_{i=1}^{n} (b_{i} - \overline{b})(\log(\hat{a}_{i}) - \overline{\log(\hat{a})})}{\sum_{i=1}^{n} (b_{i} - \overline{b})^{2}}\right),$$
(5.4)

where \overline{b} and $\overline{\log(\hat{a})}$ are the arithmetic means of a set of fitted recession exponents $\{b_1, b_2, ..., b_n\}$ and a set of log-transformed fitted recession scale parameters $\{\log(\hat{a}_1), \log(\hat{a}_2), ..., \log(\hat{a}_n)\}$, respectively. The value in the exponent of Equation 5.4 is also equivalent to the negative of the regression slope of the graph of $\{\log(\hat{a}_1), \log(\hat{a}_2), ..., \log(\hat{a}_n)\}$ versus $\{b_1, b_2, ..., b_n\}$.

We note that the computed value q_0 will equal 1/k, and that this re-scaling will return \hat{q} to the same magnitude of the original flow variable (q), which was introduced with units of $[L^3/T]$ or [L/T]. Strictly speaking, however, q_0 has units equal to those of \hat{q} [Bergner and Zouhar, 2000]; thus, the transformed flow variable will be dimensionless and the decorrelated recession scale parameter (which we will refer to as a) will have units of $[T^{-1}]$.

This method also assumes that a itself is not a function of b. If it were, the computed q_0 would still minimize correlation of the fitted recession exponents and log-transformed fitted recession scale parameters but would not necessarily equal 1/k. Section 5.2 more thoroughly discusses the implications a potential relationship (beyond the artifactual correlation) between $\log(a)$ and b.

As far as we are aware, Mather and Johnson [2014] provide the only application of this method in the geosciences by removing correlation between the parameters of a turbidity rating curve. These authors demonstrate seasonal patterns in the turbidity rating curve scale parameter, and also suggest that power law de-correlation could improve the physical interpretation of turbidity rating curve parameters.

Interpreting parameter de-correlation and the re-scaling constant,

q_0

Exploration of the de-correlation scaling q_0 suggests that it co-varies with catchment properties, such as catchment area and mean flow. Across our study catchments, we find that q_0 ranges from about 4-33% of the mean (averaging around 12% of the mean) and from about 25-125% of the median (averaging around 60% of the median). However, we note that q_0 is not directly a catchment property, but rather a property of the (\hat{a}, b) point cloud to which the de-correlation process is applied. The specific value of q_0 is therefore dependent upon the selection of points in that cloud, and thus on the technique used to select and to fit the power-law recessions.

After finding a numeric value for q_0 using methods from Section 5.2, a rescaling of the flow variable by q_0 ($\hat{q} \rightarrow \hat{q}/q_0$) will shift the population of fitted recession scale parameters $\{\hat{a}_1, \hat{a}_2, ..., \hat{a}_n\}$ so that all correlation of the form $\hat{a} \propto k^{b-1}$ is removed. As mentioned in Section 5.2, if the correlation relationship takes exactly the form, $\hat{a} = ak^{b-1}$, the scaling constant q_0 is exactly equal to 1/k. The approach thus removes the formal correlation (and the associated scaling effects) imposed by the power law scale invariance between the (a, b)pairs. If, however, a non-exponential form of correlation also exists between a and b, in addition to any induced by the scale-free nature of the power law – that is, if $\hat{a} = a(b) \cdot k^{b-1}$ - then rescaling by q_0 will transform the function a(b), and q_0 will not be equal to 1/k[Bergner and Zouhar, 2000]. As a consequence, the de-correlation procedure will fail to remove artifactual correlation due to the presence of non-exponential a-b correlation. This would continue to confound the physical interpretation of the transformed recession-scale parameter. It should be noted, however, that the de-correlation procedure is effectively a rescaling of the flow variable and therefore does not introduce any more bias than would scaling flow by area, or equivalently, choosing a particular set of flow units. In the absence of a priori information about the functional form of such mechanistic correlation, its identification from a-b clouds will generally be problematic. Still, as shown in the simple example below, power law parameter de-correlation is a promising method for obtaining information about catchments from fitted recessions.

5.3 Case study: Seasonal recessions

Catchment selection and recession analysis

To demonstrate the potential value of applying parameter de-correlation to empirical recession data, we analysed streamflow recessions from 16 seasonally dry catchments in Northern California and Southern Oregon. In seasonally dry regions, the great majority of annual precipitation falls during a wet season (typically October through April in the Western U.S.), which is followed by an extended dry season. These locations, therefore, should reveal the effects of climatic variation on catchment wetness and the consequent properties of the streamflow recession. Details of these catchments are presented in Appendix C.

All streamflow peaks and subsequent recession periods $(d\hat{q}/dt < 0)$ greater than or equal to 4 days in length were first isolated from the streamflow timeseries. We computed \hat{q} and $-d\hat{q}/dt$ using the procedure of Brutsaert and Nieber [1977], and found \hat{a} and b by determining the line of best fit to the log-log plot of $-d\hat{q}/dt$ vs. \hat{q} . To control the quality of fitted parameters, only recession fits with an $R^2 > 0.8$ were retained in the subsequent analyses. Although more precise forms of recession analysis are available, this particular method is widely used [Bart and Hope, 2014, Biswal and Marani, 2010, Mutzner et al., 2013, Shaw and Riha, 2012] and its simplicity facilitates rapid analysis of many recessions. We confirmed that the fitting error imposed on \hat{a} and b was negligible compared to the parameter shifts generated by the de-correlation procedure, which is the primary result we wish to demonstrate.

Parameter de-correlation was performed according to the method detailed in Bergner and Zouhar [2000]. Computed values of q_0 for each catchment are reported in Appendix C. Because q_0 is a property of the (\hat{a}, b) point cloud, the values of q_0 quoted here are specific to the recession selection and fitting procedure used. Here, our selection criteria included all well-defined peaks followed by at least 4 days of recession. This approach maximizes the range of flow conditions spanned by the analyzed recessions and ensures that the (\hat{a}, b) point cloud used is as representative as possible of the full flow regime. All quantitative values of q_0 reported in Appendix C are specific to this peak selection method.

Following de-correlation, we computed the median value of b across all recession events for each basin. We then re-fit the set of recessions for each basin at the de-correlation scale, this time constraining b to equal its median value. Differences in the quality of this fit and fits with variable b provide an estimate of the additional fitting error accrued by fixing the recession exponent to a constant value.

Case study results

Figure 5.1 illustrates the effects of the de-correlation process for a subset of recession parameter pairs value obtained at Redwood Creek. On the left plot, (\hat{a}, b) pairs with \hat{q} in units of c.f.s. demonstrate a very clear correlation; following de-correlation (Figure 5.1, right plot), recession exponent values remain the same, but the recession scale parameters have shifted significantly.

To date, most studies of recession parameter variability have considered the variation in a for a fixed value of b. To determine whether such estimates differ significantly from those produced by the de-correlation procedure, we first produce an (\hat{a}, b) point cloud from the population of recessions associated with a given catchment. Applying the de-correlation re-scaling procedure to this point cloud produces a population of correlation-free power law parameters (a_{free}, b) . We then fix b at its median value and re-fit all recessions at the decorrelation scale to produce a population of estimates a_{fix} . We find that across all recessions in all catchments the median ratio $\rho = a_{fix}/a_{free}$ is 0.86, with a 25th percentile value of 0.42 and a 75th percentile value of 1.38. Assuming the value a_{free} at the de-correlation scale is the most accurate estimator of a, the spread in this ratio reveals potential for significant bias.

We then explored the time-variation in the recession parameters. Prior to de-correlation, no coherent seasonal variation could be found in the fitted values of \hat{a} . Following decorrelation, however, a strong seasonal signal in the recession scale parameter can be seen in Figure 5.2, tracking the typical seasonality of rainfall. Similar seasonal patterns were observed in all 16 study catchments. We hypothesize that following de-correlation, the power law coefficient *a* may reveal information about the wetness of the catchment, exhibiting low values of *a* during the wet season and higher values during the dry season. Such recession



Figure 5.1: Illustration of the consequences of the parameter de-correlation procedure for 54 representative recessions extracted from Redwood Creek data. For \hat{q} in units of c.f.s., there is a strong exponential correlation between \hat{a} and b, as predicted by the re-scaled power law recession relationship (Equation 5.3). Following parameter de-correlation, values of b remain constant while values of a adjust such that the linear correlation between b and $\log a$ is zero. Two points along with the corresponding recession curves (orange and yellow) are tracked through the de-correlation procedure.



Figure 5.2: Examples of climatic patterns resulting from recession parameter de-correlation. Median monthly values (points) and 25th and 75th percentiles (boxes) for the recession scale parameter. In the first row, the median monthly recession scale parameter is plotted for discharge in units of c.f.s. The second row demonstrates clear seasonality in the recession scale parameter following the de-correlation procedure.

seasonality has been observed previously, [Shaw and Riha, 2012], but it can also 'disappear' (Figure 5.2) if b is not fixed, depending on scale. Generally, the arbitrary choice of scaling can mask (in some cases) or reveal (in others), the seasonally-varying term, a. We hypothesize that identifying empirical correlates to recession variability – facilitated by the use of the de-correlation technique – could support the development and validation of improved hydrologic theory that links recession behavior to climatic and catchment wetness states.

5.4 Conclusion

Ubiquitous power law behavior in the recession of stream hydrographs leads to extensive use and analysis of power law parameters that characterize the drainage and drying of catchments. All such models will exhibit a scaling artifact that imposes a formal correlation between model parameters, and which can obscure the scale-independent component of the power law, which is presumably most closely linked to the physical processes driving the recession behavior.

We apply a re-scaling technique that eliminates the scale-dependence of fitted power law terms under the assumption that there exists no mechanistic correlation between power law recession parameters. This technique allows both parameters of the power law to be estimated together without the imposition of artifactual bias. In the case that the power law recession parameters are expected to be correlated for mechanistic reasons, the re-scaling technique will not add additional uncertainty and may still help to elucidate drivers of recession parameter variability. Whether it is possible to empirically separate mechanistic drivers of parameter correlation from mathematical artifacts that cause parameter correlation remains unclear.

As illustrated by the coherent seasonal-variation of the power law multiplier a in seasonally dry watersheds from Northern California and Southern Oregon, this approach, although statistical rather than mechanistic in nature, may make clearer the relationships between catchment condition and the catchment recession.

Chapter 6

Event-scale power law recession analysis: Quantifying methodological uncertainty

6.1 Introduction

Streamflow recession analysis has been performed for well over a century [Boussinesq, 1877, Hall, 1968, Tallaksen, 1995], with the goal of characterizing recession behavior in terms of phenomenological models of flow decreases over time, typically represented with a power-law differential equation:

$$\frac{dq}{dt} = -aq^b \implies q(t) = \left(q_0^{1-b} - (1-b)at\right)^{\frac{1}{1-b}}.$$
(6.1)

There is no universally agreed upon procedure for performing power law recession analysis, however most approaches are comprised of two key steps: 1) Extract periods of recession from the hydrograph, and 2) Parameterize the power law model using the extracted period(s) of recession. Recession extraction is the process by which periods of streamflow recession are identified using the hydrograph and potentially (but not necessarily) other hydroclimatic datasets, such as rainfall. The second step, parameterization of the power law model, is the process by which power law recession parameters are fit to or otherwise matched with the extracted recession data.

Classical recession analysis performs the latter step in a single operation: [log(q), log(-dq/dt)]point pairs are computed for a collection of recession limbs, and the recession parameters are obtained from the slope and intercept of a line fitted to the [log(q), log(-dq/dt)] point cloud [e.g. Basso et al., 2015, Bogaart et al., 2015, Brutsaert and Nieber, 1977, Clark et al., 2009, Sawaske and Freyberg, 2014, Stoelzle et al., 2013, Tague and Grant, 2004]. This form of recession analysis, which we refer to as "lumped" recession analysis, is empirically and theoretically motivated: it reasonably captures observed non-linearity in the hydrograph recession, and draws on a model form that is motivated by solutions of the hydraulic groundwater equations, which have been used extensively as a basis for streamflow characterization [Boussinesq, 1904, Troch et al., 2013]. In these solutions, the power-law parameters represent combinations of physically meaningful parameters describing aquifer geometry and conductivity. Lumped recession analysis has been used for inverse modeling, the development of flow separation algorithms, and model parameterization [Bogaart et al., 2015, Huyck et al., 2005a, Rupp and Selker, 2006c, Rupp et al., 2004, Szilagyi et al., 1998a, Tague and Grant, 2004, Vogel and Kroll, 1992].

More recent work attempts to attribute physical meaning to variability *across* individual recessions within a single catchment, which has triggered a rapid increase in studies performing *event-scale recession analyses* [Bart and Hope, 2014, Biswal and Marani, 2010, 2014, Biswal and Nagesh Kumar, 2012, Dralle et al., 2015, 2016a, Ghosh et al., 2016, Harman et al., 2009, Mutzner et al., 2013, Patnaik et al., 2015b, Shaw, 2016, Shaw and Riha, 2012, Vogel and Kroll, 1996, Wittenberg, 1999a, Ye et al., 2014]. Whereas classical, lumped recession analysis seeks a single recession model parameterization that best fits all hydrograph recessions, the goal of event-scale recession analysis is to interpret variations in catchment response to rainfall as a function of the properties of rainfall events and the catchment itself [e.g. Biswal and Marani, 2010, Harman et al., 2009, Shaw and Riha, 2012].

Among the many issues associated with event-scale analysis [Dralle et al., 2015], perhaps the most challenging are the numerous subjective choices needed to establish consistent criteria for recession identification and fitting [Westerberg and McMillan, 2015]. For lumped analysis, Brutsaert and Nieber [1977] established a derivative-based method, which avoids the issue of needing to determine the precise start day of a recession event. Event-scale analyses, however, must identify the start and end of each recession event and select one of many fitting techniques to obtain (a, b) values.

Despite the growing number of event-scale recession studies, it remains unclear to what extent the particular method of recession extraction and fitting could alter features of the computed populations of recession parameters. If uncertainty due to methodological choices is larger than the magnitude of physically-derived variations in the recession parameters, new and less ambiguous methods will need to be developed, both in order to facilitate empirical comparative analyses, and to validate recent developments in streamflow recession theory [e.g. Biswal and Marani, 2010, Clark et al., 2009, Harman et al., 2009]. While some work has been done to quantify methodological uncertainty for techniques of lumped recession analysis [Stoelzle et al., 2013], no such study has been undertaken for event-scale analysis. Given the nascent stage of event-scale recession exploration, it is an opportune time to determine the methodological limitations associated with event-scale techniques, hopefully facilitating inter-comparability and consistency in future studies.

Analogously to Stoelzle et al. [2013], this study examines the sensitivity of recession parameter values to the various methodological choices to be made when performing eventscale recession extraction and fitting. Since the number of studies that perform event-scale recession analysis is small, no set of canonical methods of event-scale recession analysis have been established, as they have been for lumped recession analysis [Brutsaert and Nieber, 1977, Kirchner, 2009, Vogel and Kroll, 1992]. We therefore propose that the two steps of recession analysis – recession extraction and power law model parameterization – can be broken down into four key methodological choices; three concerning recession extraction, and one concerning model parameterization:

- 1. The minimum allowable length of a recession event
- 2. The minimum level of peak discharge that defines the start of a recession event
- 3. The definition of the end of a recession event
- 4. The method of power law model fitting

Guided by this list, we select sixteen 'end-member' methodological combinations, with the goal of obtaining realistic estimates of method-dependent recession parameter uncertainty.

6.2 Methods

Study sites

The analyses in this study are performed using United States Geologic Survey daily streamflow data for a set of 16 U.S. catchments from northern California and southern Oregon. While recession analysis can be performed using more frequently sampled discharge data, we use daily data because it is the most common choice in event-scale recession literature. The study catchments are situated within the U.S. western coastal Mediterranean climate region, which is characterized by a distinct rainy season, followed by a pronounced dry season during which rainfall makes a minimal contribution to the water balance [Power et al., 2015]. The tremendous range of moisture states that characterize these seasonally dry regions ensures that the study catchments 'explore' a wide range of potential recession behaviors. Study catchment information is summarized in Table 6.1.

USGS gage number	Catchment name	Catchment area $[km^2]$	Number of vears of data
11143000	Big Sur River, Big Sur, CA	120.4	65
11451100	North Fork Cache Creek, Clearlake Oaks, CA	155.9	44
11463170	Big Sulphur Creek, Cloverdale, CA	33.9	35
11468000	Navarro River, Navarro, CA	784.8	65
11468500	Noyo River, Fort Bragg, CA	274.5	64
11472200	Outlet Creek, Longvale, CA	416	37
11473900	Middle Fork Eel River, Dos Rios, CA	1929.5	50
11475000	Eel River, Fort Seward, CA	5457.1	60
11475560	Elder Creek, Branscomb, CA	16.8	48
11476500	South Fork Eel River, Miranda, CA	1390.8	75
11476600	Bull Creek, Weott, CA	72.8	55
11481000	Mad River, Arcata, CA	1256.1	65
11481200	Little River, Trinidad, CA	104.9	60
11482500	Redwood Creek, Orick, CA	717.4	62
14307620	Siuslaw River, Mapleton, CA	1522.9	48
14325000	Coquille River, Powers, OR	437.7	66

Table 6.1: Study catchments

Overview of the methods varied across recession analyses

We represent the four key methodological choices with four binary variables:

- 1. Minimum recession length (M)
- 2. Peak selectivity (S)
- 3. Recession concavity (C)
- 4. Fitting method (L)

M, S, C, and L will either be 1 or 0, for a total of sixteen distinct 'method combinations' of event-scale recession extraction and fitting. The extraction related variables (MSC) are defined so that a value of 1 corresponds to a more restrictive extraction method; that is, the method filters out more recessions if its corresponding variable is 1 than if the variable is 0. For example, M = 1 corresponds to a minimum recession length of 10 days, which is considerably more restrictive than a minimum recession length of 4 days (M = 0). Table 6.2 summarizes the 16 method combinations.

Table 6.2: Sixteen method combinations

Defining the minimum allowable length of recession event (M)

Nearly all event-scale recession studies set a minimum duration for chosen recession periods. Reasons for this choice vary; authors cite the removal of noise from short events [Ye et al., 2014], the necessity of capturing late time flow processes [Chen and Krajewski, 2015], and general data quality concerns [Biswal and Nagesh Kumar, 2012, Shaw, 2016]. Event-scale recession analyses have typically chosen a minimum of 4 to 5 days of recession [e.g. Biswal and Marani, 2010, Shaw and Riha, 2012], although we have found values as low as 12 hours [McMillan et al., 2014] for high frequency data.

To examine sensitivity to this minimum recession length, we extract very restricted recessions using a minimum length of ten days (M=1), and less restricted sets of recessions with a minimum length of four days (M=0).

Identifying potential recession starts

Ideally, rainfall data would be used to identify periods of recession. However, high-quality precipitation records are often unavailable, and so the majority of event-scale recession analyses use solely flow data for recession identification. Considering the goal of the present study – to bound uncertainty relating to methodological choices in recession analysis – we only consider methods of recession analysis that can be applied to any daily streamflow record, with or without rainfall data. This is the best choice for an uncertainty analysis; more stringent extraction methods that require rainfall data would be expected to reduce uncertainty in recession analysis, as extracted recession periods *with* rainfall data.

Without rainfall data, the first step in event-scale recession analysis is to identify discharge peaks, typically by locating days on which dq/dt changes sign from positive to negative. A portion of the initial set of identified peaks may be considered 'insignificant' and removed from subsequent analysis. While arbitrarily small discharge 'bumps' should certainly not be considered, it is difficult to determine why smaller recession events should or should not be included in analysis. Rationales might include discarding peaks that are small relative to measurement error, or which have dynamics that would be expected to be unresolvable on daily timescales, although few authors give a strong justification for their choices in this regard. Ye et al. [2014] discard peak flows less than the 10^{th} flow percentile to avoid 'noise' from small events. Mutzner et al. [2013] and Biswal and Marani [2010] only choose recession events where initial flow conditions are greater than mean annual flow in order to avoid, "minor events, which may not have significantly increased the average soil saturation, thus not triggering a significant response of the groundwater." Without identifying some justifiable tolerance for noise associated with small peaks, or defining what constitutes a 'significant' groundwater response, it is difficult to objectively determine a peak threshold below which recessions should be excluded from analysis.

To test the effect of such peak filtering decisions on recession analysis, we implement a peak selection procedure that is sensitive to the 'distinctness' of any given peak relative to the data around it [Yoder]. Our scheme selects a peak if the following are true: (i) it is a local maximum; (ii) if it is greater by some threshold amount than the local minimum between it and the previously chosen peak, and (iii) discharge decays by the same threshold amount before the next greater local maximum is found. We define the threshold as range(q)/d, where range(q) = max(q) - min(q) is taken over the period of record. d is a tunable parameter that we set to be 50 for highly selective extraction (S=1; only larger, more distinct peaks) and set to 500 for less selective extraction (S=0).

In most studies, once a significant discharge peak has been identified, a recession starttime, which does not necessarily coincide with the discharge peak, is chosen. Lagging the recession start after the peak is a common practice that ensures extracted periods coincide with the 'baseflow' recession, i.e. to a period of time when storm flows can be considered negligible. The number of discarded days for lumped recession analysis range from 1 to 10 days early in the recession [Stoelzle et al., 2013]. While most event-scale recession analyses lag recession starts by at least one day [Bart and Hope, 2014, Biswal and Nagesh Kumar, 2012, Patnaik et al., 2015b], it is not clear that such lagging is necessary to enable proper interpretation of event-scale dynamics [e.g. Harman et al., 2009]. Fast flow processes, as well as slow, may also contribute to the hypothesized dynamics which generate power law recession behavior. For example, Harman et al. [2009] postulate that heterogeneous transport timescales alone give rise to power law recession dynamics, with no restriction on the 'fastest allowable' such timescale. Some event-scale studies lag recession starts to avoid large contributions from 'surface-flow' processes [Biswal and Marani, 2014, Patnaik et al., 2015b]. Yet, without a priori information that surface flow processes are in fact a dominant source of run off generation in a particular watershed, there is no justification for such a lag. For example, no surface flow processes have been observed at the Elder Creek watershed in our collection of study watersheds; runoff is generated by a perched water table system [Salve et al., 2012]. In the results presented here, we therefore seek and analyze distinct streamflow peaks without removing any days following the recession start.

Identifying the end of a recession event

A number of criteria have been used to determine the end of a recession event. Without a reliable rainfall record, many event-scale analyses halt recession extraction upon the first day where flow does not decrease, that is, as soon as dq/dt > 0 [e.g. Mutzner et al., 2013]. Vogel and Kroll [1996] define the recession end as the first day of increase in the 3-day moving average of streamflow. Shaw and Riha [2012] end the extracted recession two days before the first positive discharge increment following a recession start. Some studies use the inflection point of the recession curve – the first day following a rainfall event for which the hydrograph is concave up – to identify the start of the extracted recession [Singh and Stall, 1971, Wittenberg and Sivapalan, 1999]. Such a requirement could also be used to define the continuation of a recession event. Exploring every possible combination of the the above (and other) methods would lead to an intractably large number of methodological combinations. We therefore define two consensus strategies derived from the above criteria. The first (C=0) considers a recession as any hydrograph segment with dq/dt < 0 following an identified peak. The second, more restrictive strategy (C=1) requires that the raw flow time series is strictly decreasing (again, dq/dt < 0) and classified as concave up. A recession day is classified as concave up if *either* the raw time series or a 3-day averaged time series is concave up; that is, if the second difference of either the raw flow time series or a smoothed flow time series is greater than or equal to zero. This has the effect of including days with small 'bumps' in concavity in the raw time series, while preserving days immediately after sharply peaked events, which are often classified as convex by the smoothed time series. This simple criteria could serve as an improvement to methods that only require dq/dt > 0, which could inadvertently extract highly convex recessions likely associated with continued rainfall.

Choosing a fitting procedure

Fitting methods can be broken down into one of three categories: 1) Linear regression or enveloping of a binned collection of $[q, \log(-dq/dt)]$ points [e.g. Kirchner, 2009, Parlange et al., 2001], 2) Linear regression or enveloping of a raw collection of $[\log(q), \log(-dq/dt)]$ points [e.g. Biswal and Marani, 2010, Brutsaert and Nieber, 1977], or 3) Nonlinear regression [e.g. Wittenberg, 1994].

Some of these methods are likely inappropriate for event-scale recession analysis. For example, the organic correlation fitting method [Zecharias and Brutsaert, 1988] is best used to establish an equivalence between distribution functions of the independent and dependent variable [Hirsch and Gilroy, 1984]. This may be appropriate for analyzing large clouds of data, but for event-scale analysis, a and b are better interpreted as realizations of their corresponding random variables. Other popular techniques, such as linear regression on binned log-transformed data [e.g. Kirchner, 2009], require a large number of data points and are thus unsuitable for event-scale methods.

The most popular method for event-scale recession fitting is to find the regression line through raw $[\log(q), \log(-dq/dt)]$ point data. While less widely used, nonlinear fitting methods have been shown to produce more consistent values for recession parameter fits, regardless of the period of chosen recession [Wittenberg, 1999a]. Nonlinear techniques have been used to successfully parameterize hydrologic models [Dralle et al., 2016a, Müller et al., 2014], and also avoid numerical issues associated with computing the time derivative of a flow time series [Rupp and Selker, 2006a].

Despite the numerous choices for fitting the power law model, we choose a dichotomy between the two most straightforward implementations of linear and nonlinear fitting. Linear fitting (L=1) is performed on the log-transformed values, $[q, \log (-dq/dt)]$ [Brutsaert and Nieber, 1977]. Nonlinear fitting (L=0) is performed on extracted, non-transformed recession segments.

Method combination comparisons

In general, due to artifactual correlation between a and b that confounds interpretation of a [e.g. Berghuijs et al., 2016, Sawaske and Freyberg, 2014], only fitted recession exponents (b) can be reliably compared between different recession events. This issue can be avoided by setting the recession exponent to a fixed value [e.g. the median, Biswal and Marani, 2010], at the expense of biasing the fitted values of a due to this constraint on the exponent. Dralle et al. [2015], however, present a technique that removes the scaling artifact from the recession scale parameter without constraint on the recession exponent.

With this in mind, we choose three primary **recession measures** for comparison between recession events: the recession exponent (b), the scale-corrected [Dralle et al., 2015] recession scale parameter (a), and the recession time (T_R) , defined by Stoelzle et al. [2013] as the amount of time required for flow levels to decline from the median flow to the tenth flow percentile.

The scale-corrected recession scale parameter (a) has units of inverse time, providing an interesting comparison to T_R , which belongs to a class of more commonly calculated recession timescales for the general, nonlinear form of Equation 6.1 [e.g. Stoelzle et al., 2013, Westerberg and McMillan, 2015].

To see how methodological choices might impact the interpretation of a, b, and T_R , we organize the paper around three primary questions:

- 1. How do methodological choices impact the overall quality of individual recession fits?
- 2. Are $a, b, and T_R$ 'characteristic' across various methodological choices? That is, do catchments rank in a similar order according to different statistical measures (in the present study, the median and inter-quartile range) of the populations of $a, b, and T_R$ across the sixteen method combinations [Stoelzle et al., 2013]?
- 3. For each catchment, are the distributions of a, b, and T_R statistically similar across method combinations?

Testing the quality of recession fits

We report three measures of the overall quality of recession fits. Firstly, to examine the sensitivity of fit quality to individual methodological choices, we present paired R^2 goodness of fit measures for all recessions conditioned on the two values for each method choice. For example, we compare R^2 values for all fits that use linear fitting to all fits that use non-linear fitting. Secondly, to examine relationships between fit quality and higher order combinations of method choices, we compute the R^2 goodness of fit for each of the method combinations, across all catchments. Finally, we report, for each method, the percentage of all fits that yield 'non-physical' estimates for the recession parameters, which we define as b < 0. In all subsequent analyses, the recession parameters have been filtered so that $b \ge 0$ (b < 0 occurs for less than 3% of all recession events).

Ranking catchments by recession characteristics

While Stoelzle et al. [2013] perform lumped recession analysis and obtain single recession parameter values for each catchment and method combination, our event-scale analysis yields distributions for b, a, and T_R . We therefore report measures of central tendency and variability for the computed recession variables, b, a, and T_R . Following Stoelzle et al. [2013], we compute Spearman rank correlation coefficients by ranking catchments between method combination pairs with the following measures (recession characteristics): median(a), median(b), median(T_R), IQR(a), IQR(b), IQR(T_R), where IQR is the inter-quartile range. Even if the absolute magnitudes of the values of a, b, and T_R vary between the method combinations, these rank tests will determine whether or not recession characteristics are ordinally equivalent (characteristic) across method combinations (that is, catchments are ranked in the same order by the recession characteristic for all methods).

Comparing distributions of $a, b, and T_R$ across method combinations

We first highlight general patterns for each recession measure across all method combinations with Tukey box plots for a single representative catchment – the Elder Creek watershed. These plots demonstrate the relative observed differences between method combinations. However, they do not represent the absolute effect of changing individual method choices. This is because more restrictive extraction procedures will always produce sets of recessions that are a subset of less restrictive extraction measures. For example, all other method choices being equal, recessions extracted with a minimum length of 10 days will be a subset of recessions extracted with a minimum length of 4 days. This 'dilutes' the true effect of a change in any choice related to recession extraction.

One way to isolate the absolute effect of a given method choice is to compare recessions that are *shared* between the restrictive choice and non-restrictive choice, to those that are *unshared* between the restrictive and non-restrictive choices. This procedure is illustrated for the minimum length choice in Figure 6.1. Here, the raw streamflow data in Figure 6.1a is subjected to both an extraction procedure with a minimum length of 4 days (Fig. 6.1b) and to an extraction procedure with a minimum length of 10 days (Fig. 6.1c), where all other method choices are held fixed. The set of 10 day minimum length recessions is clearly a subset of the 4 day minimum length. This naturally forms two groups: a set of *shared* recessions between the two extractions (Fig. 6.1c), and a set of *unshared* recessions (Fig. 6.1d; those extracted by the minimum 4 day extraction, but not the 10 day extraction). These independent 'shared' and 'unshared' recessions make it possible to observe the absolute effect of an individual method choice on a recession measure. All other things equal, recession measures between the two groups should be comparable if the particular recession measure is not sensitive to the method choice.

We compare shared and unshared recession measure distributions in two ways. First, for a high level overview, we show Tukey box plots of shared vs. unshared distributions of the recession exponent (b) for a single catchment (the Elder Creek watershed) for each



Figure 6.1: Illustration of the effect of changing the minimum recession length from 4 days (b) to 10 days (c). Orange lines represent extracted hydrograph segments. (c) and (d) demonstrate the difference between shared and unshared recessions. Shared recessions (c) are those extracted by only the most restrictive method, which must also be extracted by the less restrictive method. The unshared recessions (d) are those extracted by the less restrictive method, but not the more restrictive method. For the minimum length choice, these are the recessions between 4 and 9 days in length.

of the recession extraction choices (M, S, and C). We also compare populations between linear and nonlinear fitting, though we note that this is not strictly a 'shared' vs. 'unshared' comparison.

We then use a two-sided Mann-Whitney U Test [Mann and Whitney, 1947] to compare shared vs. unshared distributions for each recession measure across all method choices and all catchments. The null hypothesis for this non-parametric test is that the shared and unshared distributions are sampled from the same population. For a given catchment and for each method choice, we compute p-values for the Mann-Whitney U Test by comparing shared to unshared distributions for each of the eight combinations of the other method choices. If the test rejects the null hypothesis, then we conclude there is some evidence that the shared and unshared distributions are not sampled from the same population; that is, the method choice significantly changes the distribution of the recession measure.

For each method choice and catchment, we then compute the fraction of tests (eight total tests per method choice per catchment; shared vs. unshared for all 8 combinations of the other method choices) which returned statistically different shared and unshared distributions. We use this fraction as an indicator of the sensitivity of a recession measure to a given method choice. We perform this procedure for all recession measures, a, b, and T_R . Since for each of these measures we perform 512 total comparisons (16 catchments \times 8 tests \times 4 method choices), we apply a Bonferonni correction for the critical p-value of each test, which is required when a statistical test is applied many times for multiple comparisons [Abdi, 2007]. For an overall level of significance of $\alpha = 0.05$, the correction requires a critical p-value for each test set to $p = \frac{\alpha}{512}$.

6.3 Results and Discussion

Testing the quality of recession fits

Common goodness of fit measures, such as R^2 , are typically unimportant in lumped recession analysis; more often, the objective is to 'envelope' a [log(q), log(-dq/dt)] point cloud, not to accurately fit each individual recession event. However, minimizing recession fitting error maximizes confidence in the validity of recession measure comparisons between individual recessions, which is the central motivation for event-scale recession analysis. We therefore wish to understand how methodological choices impact the quality of recession fits.

The box-plots in Figure 6.2 condition R^2 values by the two values of each methodological choice. The figure provides a rough measure of the sensitivity of fit quality to each individual methodological choice. The patterns in Figure 6.2 indicate a clear hierarchy of the importance of method choices, at least insofar as these choices impact the 'quality' of extracted recessions and their corresponding power law fits. Changing the minimum recession length from 4 days (M=0) to 10 days (M=1) appears to have very little effect on fit quality. The same is true for peak selectivity; S=1 and S=0 produce nearly identical distributions for the R^2 goodness of fit measure. The concavity (C) and linearity (L) variables, however, appear


Figure 6.2: R^2 values obtained by controlling for the two values of each method choice, lumped across catchment.

to be more important determinants of the quality of fit of the power law model. The fit quality is improved for concave-only extracted recessions and nonlinear fitting.

Figure 6.3 provides support for this trend, where it can be seen that concavity and linearity are the strongest drivers of fit quality. These two method choices seem to break the figure down into a hierarchy of three groups: The worst fits observed were those performed without the concavity requirement and with linear regression (that is, combinations that end in 01). Fits that use concave recessions or nonlinear fitting, but not both, are of intermediate quality (combinations that end in 00 or 11). The best fits by a large margin were those that combined the concavity requirement with nonlinear regression (combinations that end in 10). Overall, this suggests a sort of additive increase in goodness of fit associated with the concavity requirement and nonlinear fitting.

This finding – that concavity and linearity play important roles in determining the quality of recession fits – is significant in light of the fact that minimum recession length and minimum recession peak size are more commonly emphasized as the most important methodological choices made during event-scale recession analysis. Evidence here supports the notion that concavity requirements and nonlinear fitting could greatly improve the quality of



Figure 6.3: R^2 goodness of fit for all method combinations, lumped across catchments.

event-scale recession analyses.

There are good reasons, beyond the present study, to re-evaluate linear fitting procedures and the dq/dt < 0 extraction requirement. Linear regression on log transformed flow values disproportionately weights errors for smaller model values, creating a risk of bias in the fit [Miller, 1984, Pattyn and Van Huele, 1998]. Linear fitting also requires computation of the flow derivative, which introduces a number of documented numerical and data quality challenges [Rupp and Selker, 2006a]. The various differencing schemes that can be implemented to obtain the flow derivative [e.g. Thomas et al., 2015] also add another potential source of method dependent bias in the fitting scheme. Still, there are downsides associated with nonlinear fitting procedures. Fit bias could be introduced by the particular optimization algorithm, or the necessity of specifying an initial condition for the nonlinear fitting procedure. This can be relatively clear (e.g. with b), but could also be relatively opaque (e.g. a).

Regarding the concavity metric, [Shaw and Riha, 2012] mention that periods with dq/dt < 0 seem to coincide with rainless periods. However, no systematic study has been undertaken to demonstrate this result more generally, and the simple dynamical system model developed by Kirchner [2009] predicts that streamflow can decrease even as precipitation falls. Even with rainfall data, improved flow-derived recession extraction methods could be of great use, considering the numerous issues associated with obtaining reliable catchment scale precipitation data [e.g. Fankhauser, 1998, Kansakar et al., 2004, Yatheendradas et al., 2008].

Are recession measures characteristic?

Catchments were ranked by the values of 6 recession characteristics – median(a), median(b), median (T_R) , IQR(a), IQR(b), and IQR (T_R) – for all pairs of method combinations. The collection of corresponding Spearman rank correlations are presented as box plots in Figure 6.4. The rank correlation can take a value between -1 and 1, where a higher rank correlation



Figure 6.4: Box and whisker plots of Spearman rank correlations for all six 'characteristics' of the distributions of a, b, and T_R . Per characteristic, there are $15 \times 16 = 240$ unique pairwise comparisons between method combinations.

indicates greater consistency of catchment rankings by the particular recession characteristic.

We performed a thorough investigation of the rank correlations between different method combinations but found few patterns related to individual method choices (the full collection of computed rank correlations is included in Appendix D as shaded correlation matrix plots). Because of this, we present aggregated box-plots of the Spearman rank correlation for each of the recession characteristics.

Overall, none of the rank correlations were negative, suggesting that, at worst, no method combination predicts a characteristic ranking that is inverted relative to another method combination. The most 'characteristic' variable is median(a). This is re-assuring in some respects. Consider that when b = 1, the recession takes a simple exponential form, in which case the value of a is the well-known 'recession constant'. This linear rate of decay of discharge has been used frequently for comparative catchment analyses [Beck et al., 2013, Peña-Arancibia et al., 2010]. The scale corrected value of a reported in this study also has inverse time units and can therefore be interpreted as the nonlinear equivalent of the 'recession constant' for the full nonlinear recession. The findings here suggest that the median of the scale corrected a is fairly characteristic in spite of the particular method combination and could therefore be used for similar comparative type analyses.

Both the median and IQR of the recession exponent (b) are also fairly characteristic, although they are not as consistent as a. Recent event-scale recession work by Harman et al. [2009] demonstrates that b can be interpreted as a measure of the diversity of water transport timescales throughout the various parts of the catchment. In this framework, measures of variability of b could be interpreted as representative of the 'realizable' range of catchment states, with respect to the relative dominance of various water transit times in the catchment. Strongly characteristic measures of b suggest the potential to use the recession exponent to develop relative measures of catchment complexity.

The lowest performing measures are the median and IQR of the recession timescale, T_R . Considering that T_R is a measure derived from both a and b, it has likely inherited catchment ranking uncertainty from both these parameters. Numerous derived recession measures have been used for comparative catchment analysis [Berghuijs et al., 2016, Sawaske and Freyberg, 2014, Stoelzle et al., 2013]. The findings here suggest a trade-off; the development of more complex derived measures comes at the cost of compounding uncertainty.

Comparing distributions of recession measures

Figure 6.5 contains box plots for all three recession measures and for all method combinations for the Elder Creek watershed. While the scale-correction procedure for a has only been applied in one previous study [Dralle et al., 2015], the median values of scale-corrected aare consistent with recession timescales extracted from linear reservoir models [e.g. Botter et al., 2013, Sánchez-Murillo et al., 2015]. The observed median values of b and T_R are also typical for lumped recession analyses [e.g. McMillan et al., 2014, Palmroth et al., 2010, Stoelzle et al., 2013, Szilagyi et al., 2007, Tague and Grant, 2004, Wang, 2011]. Variability in the recession measures can be significant, supporting the finding that the dynamic behavior of recessions can be masked by lumped forms of analysis [Biswal and Marani, 2010]. The recession exponent, b regularly falls between b = 1 and b = 2.5, while the inter-quartile range for a and T_R span upwards of an order of magnitude. This degree of variability in a, while large, is comparable to event-scale recession studies that impose a fixed value on the recession exponent [e.g. Shaw and Riha, 2012].

The overall patterns of the Elder Creek recession measures between method combinations are comparable to the other 15 watersheds. The repeating 'saw-tooth' pattern for b indicates that concavity and linearity play important roles in shifting the distributions of the recession exponent. All other things equal, linear fitting and concavity both produce noticeably higher values for the recession exponent. Without the concave requirement, the 'decreasing only' extraction procedures will have lower values due to increased convexity. Table 6.3 supports this conclusion; the concavity requirement greatly decreases the number of 'non-physical' (b < 0) extracted recessions. The upward shift for linear fitting may be the result of 'overweighting' of errors in the tail end of the recession, where deviations from linearity in the curve defined by the collection of [log(q), log(-dq/dt)] point pairs are consistently observed to exhibit a steeper slope (b). This supports the use of non-linear regression techniques as a means to avoid biases inherent in log-transformed power law fits.



Figure 6.5: Box plots for a, b, and T_R across all method combinations for Elder Creek watershed.

The pattern of increasingly shorter whiskers from left to right in Figure 6.5 shows that the variability of the recession measures decreases with increasingly restrictive extraction measures. For a minimum recession length of 10 days and highly selective peak filtering (M=1, S=1), this decrease in variability is likely due to the fact that the collection of extracted recessions becomes less 'diverse' as the extraction method becomes more restrictive, as suggested by Stoelzle et al. [2013]. As compared to minimum length and peak selectivity, which were shown to have a minimal impact on fit quality (see Figure 6.2), the larger variability for non-concave data paired with nonlinear fitting is due, at least in part, to more noise from persistent rainfall during the recession. This suggests again that peak size and recession length data quality concerns cited by some authors [e.g. Biswal and Nagesh Kumar, 2012, Shaw, 2016, Ye et al., 2014] should be expanded to consider fitting methods and the

Method combination (MSCL)	Fraction of fits with $b < 0$
0 (0000)	0.114
1 (0001)	0.035
2 (0010)	0.070
3 (0011)	0.005
4 (0100)	0.097
5 (0101)	0.024
6 (0110)	0.059
7 (0111)	0.003
8 (1000)	0.032
9 (1001)	0.001
10 (1010)	0.012
11 (1011)	0.0
12 (1100)	0.016
13 (1101)	0.001
14 (1110)	0.004
15 (1111)	0.0

Table 6.3: Fraction of recessions with non-physical recession exponent (b < 0) for each method combination.

'quality' of the shape of extracted recessions.

Figure 6.6 presents box plots for the shared and unshared distributions of the recession exponent for the Elder Creek watershed. The remaining shared versus unshared plots can be found in Appendix D. Each subplot in Figure 6.6 corresponds to one of the four method choices (M, S, C, or L). The light green boxes represent the distribution of the recession exponent for shared recessions, while the dark green boxes represent the distribution of the recession exponent extracted by only the less restrictive procedure. The horizontal axes in each subplot show the eight possible combinations of the other method choices, showing how these shared and unshared distributions vary for different combinations of the other method variables. Significant differences between the shared and unshared distributions in Figure 6.6 indicate that the recession measure is sensitive to modulation of the method choice represented by the particular subplot.

The interpretation of 'shared versus unshared' is different for each method choice. For the minimum recession length, 'not shared' means 4-9 day recessions; and 'shared' means recessions 10 days or longer. For peak filtering selectivity, 'not shared' means the many small storms not extracted by the more selective procedure; 'shared' means large storms. For concavity, 'not shared' means recessions that are not distinctly concave (which, presumably, are the recession periods with residual precipitation); 'shared' means drier recessions, or recession periods that are less likely to have significant residual precipitation.

Patterns displayed in Figure 6.6 are largely characteristic for distributions of b in other



Figure 6.6: Box plots comparing shared vs. unshared distributions for the recession exponent for Elder Creek. Each sub-plot corresponds to a particular method choice; the shared boxes are generated with the b values from the recessions shared between the 0 and 1 values of the subplot method choice. The unshared boxes are those values of b from the recessions extracted by only the less restrictive value of the subplot method choice. The x-axis shows the values for the method choices other than the subplot method choice. For the linear vs. nonlinear comparison, the distributions are by definition all shared, as the L method choice does not affect the extraction procedure.



Figure 6.7: Results of Mann Whitney U test sensitivity analysis. Each row represents one of the 16 study catchments, each subplot one of the three recession measures a, b, or T_R , and each subplot column one of the four methodologcial choices (MSCL). Each cell is colored by the fraction of statistically different pairs of shared versus unshared distributions for the particular recession measure conditioned by the two values of the corresponding method choice. A cell shading of 1 (dark purple) means all eight pairs of shared and unshared distributions were determined to be statistically different, indicating that the particular recession measure is highly sensitive to the corresponding method choice.

watersheds. As might be expected, whiskers are longer for 'shared' distributions for minimum length and selectivity, because there are typically fewer large storms and long recessions than there are small storms and short recessions. The length of concavity whiskers are comparable between shared and unshared distributions, although concavity again emerges as the most important choice for determining the absolute magnitude of b. There is a clear separation between shared and unshared distributions of b for concavity.

Results of the Mann Whitney U tests between shared and unshared distributions for each recession measure, for all method choices, and for all catchments, are presented in Figure 6.7. The Elder Creek recession exponent distributions in Figure 6.6 correspond to the recession exponent subplot of Figure 6.7, catchment number 11475560. In agreement with Figure 6.6, the recession exponent is most significantly affected by the choice to extract only concave recessions. The strong dependence on concavity demonstrated in Figure 6.6 manifests in Figure 6.7 as the very dark rectangle in the concavity column of the recession exponent parameter for catchment 11475560. This indicates that all Mann-Whitney U tests returned a significant difference between the eight shared and unshared distributions for concavity.

While certain method choices seem to play important role in determining R^2 quality of fit, Figure 6.7 demonstrates that other choices might impact the realized values of a and b. This finding makes it difficult to determine the 'best' method combination. Whereas concavity and linearity were the dominant drivers of goodness of fit, it is selectivity and concavity that exert the strongest control over the distribution of b. Minimum recession length seems to exert the strongest control over the distribution of the recession scale parameter (a). Along with the inconsistencies in controls on each recession measure, we also note that some recession measures are uniformly sensitive to a given method for all catchments (e.g. concavity strongly affects b for all catchments), while others seem to vary between catchments. For example, linearity exerts a strong control on the distribution of b for catchment 11468500 (Noyo River), but apparently makes very little difference for catchment 11143000 (Big Sur River).

6.4 Conclusions

Event-scale analysis of power law recession scale parameter a and power law exponent b is important given that multiple, disparate physical theories predict variability in these quantities across recession events – variability that cannot be captured by fitting a single (a, b)pair to a $(\log(q), \log(-dq/dt))$ point cloud or by fixing b and fitting a. Here we have simultaneously examined the variability in both a and b across a range of methodological choices, including minimum allowable recession length, recession selectivity, recession concavity, and linear versus nonlinear fitting procedures.

We have shown that method choices that have received significant attention in the literature, i.e. minimum recession length and recession selectivity, do not have a major impact on overall fit quality in terms of R^2 , while method choices that have received far less attention, i.e. recession concavity and the nature of the numerical fitting routine, have significant implications for overall model quality. While obviously not a pertinent issue in lumped analysis, model quality at the event-scale is critically important, especially as the community attempts to discover which (if any) of the competing theories captures the emergence of power law recession behavior. We therefore recommend the use of nonlinear fitting procedures on recessions which are concave up in order to ensure the most accurate models are considered when testing theory.

We have found that event-scale recession parameters are characteristic, in the sense that different methods lead to roughly the same ranking of median a, b, and T_R across catchments, as quantified by Spearman rank correlation. In addition to this examination of the effect of method choice on central tendency, we also introduced a number of new ways to think about methodological sensitivity of recession parameter distributions at large. Most of our method choices had a "more restrictive" and "less restrictive" interpretation of whether a given period qualified as a recession, and this provided us with a valuable opportunity to test whether the distribution of parameters generated from recessions selected by the more restrictive method were similar to the distribution generated from recessions selected only by the less restrictive method. We investigated this visually via a comparative box plot analysis and more quantitatively via a Mann-Whitney-based sensitivity analysis. We found that method choice impacts all recession parameters, and while these sensitivities are largely consistent across catchments, there are notable exceptions; we recommend that methodological sensitivity be examined on a case-by-case basis. On the whole, the recession scale parameter a is most sensitive to the minimum allowable duration of the recession, while the power law exponent b is sensitive to both recession selectivity and to the concavity requirement. Of particular interest both in terms of distributional effects and in terms of Spearman rank correlation is the parameter T_R . This derived quantity performed worse than both a and b in terms of Spearman rank, indicating that perhaps T_R was inheriting catchment ranking uncertainty from both a and b. In terms of distributional effects, we observed that the sensitivity profile of T_R across catchments was not a simple superposition of the sensitivities of a and b, but instead bore its own unique signature. Derived quantities like T_R can be advantageous in their ability for inform decision-making, but our results here, taken together, indicate that care should be taken when examining them in terms of event-scale analysis.

Chapter 7

Summary and future work

This dissertation developed modeling and analysis frameworks to elucidate controls on hydrologic variability in seasonally dry watersheds. The work was split into two main parts: Part I developed simple, process oriented models for soil moisture, hillslope water table runoff generation, and catchment scale discharge in seasonally dry climates; Part II examined mathematical and methodological issues associated with application of the power law streamflow recession model, $dq/dt = -aq^b$.

7.1 Part I overview

Chapter 2 constructed a minimal, process oriented, stochastic model for soil moisture in seasonal climates. The analytic framework made it possible to obtain an expression for the first crossing time of soil moisture below a threshold during the dry season, which has been proposed as a measure of vegetation water stress in seasonal climates [Rodríguez-Iturbe et al., 2001]. The model significantly outperformed previous approximations of this ecologically important variable [Viola et al., 2008].

Chapter 3 used a recently developed solution technique for partial differential equations to examine water table dynamics along heterogeneously recharged hillslopes in seasonal climates. In addition to vastly simplifying derivations of solutions for Boussinesq type equations for hillslope water table dynamics, the new technique makes it possible to analytically investigate seasonal, time dependent recharge dynamics. Further pursuit of the guiding motivation of the work — to understand the hydrologic impact of spatially variable vegetation cover in catchments with significant subsurface lateral transport — leads to several potential theoretical and empirical extensions of this work. Coupled with a realistic representation of the dynamics of vegetation and the unsaturated zone, the analytical approaches in Chapter 2 suggest that there may be ways to rapidly iterate towards solutions for coupled vegetation - water table profile dynamics. To justify more complex extensions of this work, however, there is a need to evaluate both model assumptions and model predictions in real systems. The quantification of recharge rates (and potentially net uptake from water tables, which can be easily incorporated into the model as a negative R(x, t) value) between different forms of co-occurring vegetation cover types is required. Observational data of water table dynamics along hillslopes with varying vegetation properties would further illuminate the subsurface response to surface heterogeneity. At the basin scale, paired catchment experiments where different spatial patterns of vegetation cover are experimentally imposed may exhibit some of the behaviors simulated in this theoretical study.

Chapter 4 developed a dry season streamflow model for the recession persistence time, the number of days from the start of the dry season for which flow remains above some ecologically critical threshold. The recession persistence time is the streamflow analogue of the soil moisture first crossing time from Chapter 2. Similarly to soil moisture [Laio et al., 2001a, Porporato et al., 2001], crossing time variables, such as the recession persistence time, are known to be ecologically important [Botter et al., 2008, Doulatyari et al., 2014, Tamea et al., 2011], especially in seasonally dry watersheds [Power et al., 2008]. The occurrence of end of dry season low-flow conditions can induce fragmentation of the channel network and impact the mobility of resident stream fishes [Fullerton et al., 2010, Holmes et al., 2015, Hwan and Carlson, 2015, Thompson, 1972], or trigger blooms of toxic cyanobacteria due to large decreases in bed shear stress, which normally suppresses bloom development [Power et al., 2015, *Keith Bouma-Gregson*, pers. comm.]. More generally, the sequence of flow threshold crossings during both wet and dry seasons can lead to substantially different ecological states [Power et al., 2015]. The results of Chapter 4 suggest these situations should be readily amenable to stochastic analysis and incorporation into risk based predictive frameworks.

7.2 Part II overview

Chapter 5 exposed mathematical pitfalls associated with the power law streamflow recession model, $dq/dt = -aq^b$. In particular, the scale-free property of the power law relationship was shown to muddle the interpretation of the recession scale parameter, a. A novel technique was applied to remove this confounding, scale-dependent artifact. Subsequent analysis revealed intriguing patterns relating the 'scale corrected' recession scale parameter to measures of catchment wetness. More generally, this work could provide new insights into the origins of power law streamflow recession variability. Streamflow is an integrated representation of catchment processes, and techniques to extract novel forms of information from the recession limb could strengthen hydrograph based methods of inference. In this regard, the results of Chapter 5 present numerous research opportunities. Past studies have examined variability in the recession scale parameter (a) or the recession exponent (b), but never both due to the power law scaling artifact. With the scale-correction procedure outlined in Chapter 5, new forms of variability, such as co-variation between the recession parameters, can be investigated.

Chapter 6 quantified the sensitivity of the parameterization of the power law recession model $dq/dt = -aq^b$ to the wide-range of commonly used methods for event-scale recession analysis. The 'revival' of such forms of power law recession analysis can be attributed

primarily to two new (and distinct) theories concerning catchment function, both of which predict that recessions should take a power law functional form, and that the recession parameters a and b should vary between events. Clark et al. [2009] and Harman et al. [2009] theorize that power law parameters provide information primarily about the partitioning and distribution of flow residence timescales within the catchment, while contrasting theory by Biswal and Marani [2010] suggests that the recession variability can be mapped to the rate of wetted channel contraction during the recession phase. Despite the promise of these intriguing theories, the results of Chapter 6 demonstrate that they will remain difficult to rigorously validate until truly objective procedures for power law recession analysis can be developed.

Despite numerous advantages associated with minimal, process oriented hydrologic models like those developed in Part I, the major findings of Chapter 6, and Part II in general, raise unanswered questions related to the maximum achievable performance of such models. A recent, comprehensive review of prediction in un-gaged basins [Blöschl et al., 2013] explicitly notes the need for more research focusing on the merits and limitations of simple predictive methods, including the class of stochastic models examined in Part I. Part II shows that considerations of both structural and methodological uncertainty should play a central role in such efforts.

Appendix A

Supporting information for Chapter 3

A.1 Validation of analytical solutions

We ran three different exercises to ensure the applicability and validity of the analytical solutions presented in the accompanying paper when representing spatially variable recharge for both HB and HSB. Firstly, we compared our analytical solutions to previous results obtained using the Laplace Transform method for spatially uniform recharge [Troch et al. [2003, 2004], Verhoest and Troch [2000]]. Secondly, we compared the analytical solution with spatially variable recharge to numerical solutions of the linearized HB equation obtained using a finite difference scheme [Brutsaert [1994], Troch et al. [2003], Verhoest and Troch [2000]]. Finally, we compared the analytical solutions from the linearized HB equation to numerical solutions of the full nonlinear HB equation for equivalent boundary and initial conditions. This final test was intended to explore the validity of the linearization assumption in the context of abrupt changes in recharge volume through space. In particular, we wished to ensure that large gradients in recharge would not induce large spatial gradients in the water table and violate the linearization assumption $\left(\frac{\partial \eta}{\partial x}\right)^2 \approx 0$.

Reproducing Results of Troch et al

In order to validate the results of this analytical solution, we reproduced the figures in Verhoest and Troch [2000] for constant recharge. The curves shown in figure A.1 match the HB solutions created in previous literature [Verhoest and Troch [2000]] using the Laplace Transform method for spatially uniform recharge.

Testing Against Numerical Solutions

In order to test the analytical solution, we compared solutions for water table depth and outflow to numerical solutions of both the full and linearized Boussinesq equations. The numerical solutions were found using MATLAB's pdepe function. The value of η_0 was determined via calibration to the root mean square error (RMSE) of the flow output. As shown in



Figure A.1: Reproduction of figure 2 in Verhoest and Troch [2000]. Solid lines show water table depth (left) and outflow (right) under recharge conditions, while dashed lines show water table depth and outflow with no recharge. These curves match the behavior found by Verhoest and Troch, thus supporting the validity of our analytical solution.



Figure A.2: Solutions using the full numerical, linearized numerical, and analytical solvers. Error terms represent error between the full numerical solution and the analytical solution. The numerical and analytical versions of the linearized solution overlap exactly.

figure A.2, the analytical solution matches the linearized numerical solution perfectly for the parameters used in the results section of the accompanying paper. The left image of figure A.2 gives a snapshot at t=5 days. The error terms shown are for the difference between the full numerical solution and the analytical solution, for comparison with figure A.3.

In addition to testing the accuracy of the analytical solution for reproducing the solution of the linearized equation, we also verified the ability of the linearization to accurately represent the original (nonlinear) Boussinesq equation. Although linearizing the full Boussinesq equation is a commonly used technique, there is some concern that sharp changes in infiltration could cause gradients in η which are too steep to be accurately estimated using a linearized equation. In order to test the ability of the linearized equation to represent spatially variable infiltration, we compared the analytical solution to a numerical solution of the full Boussinesq equation. This comparison involved both the profiles of η in x and the profiles of q in time. In order to estimate outflow for the full numerical solution, we calculate q at grid centers rather than grid edges, since the boundary condition sets $\eta = 0$ at x = 0, which would produce zero discharge.

Figure A.3 demonstrates the perfect fit of the analytical solution to the linearized numerical solution even with heterogeneous recharge. These results also demonstrate that the error introduced by linearization under this recharge pattern (RMSE=0.49 and 0.60 for two opposite recharge patterns) is on the same order as the error for the constant recharge case (RMSE=0.54). Therefore, for the cases tested thus far, adding spatial variability to



Figure A.3: Solutions using the full numerical, linearized numerical, and analytical solvers. The top row shows water table height at t=5 days, while the bottom row shows outflow over time. "Uphill Recharge High" signifies that recharge at the top of the hill is 10 times higher than over the bottom half. Total water infiltrated is the same as for figure A.2. Error terms represent error between the full numerical solution and the analytical solution.

the recharge does not appear to introduce more error than the original linearization. It also shows, however, that the error is higher for steeper slopes in η , as in the "Downhill Recharge High" case in figure A.3 (as is expected from the assumptions of the linearized Boussinesq), suggesting that caution should be used when attempting to generalize the use of this analytical solution.

Figure A.4 validates the analytical solution's results for time-varying infiltration. There is a small, expected difference between the full and linearized solutions, but the general spatial and temporal trends are captured. Most importantly, the analytical solution is once again equivalent to the linearized numerical solution.

For the Hillslope Storage Boussinesq (HSB) equation, similar tests showed an RMSE of 0.15 for streamflow under homogeneous recharge and 0.10 for step-up recharge. The differences in peak heights were 0.40% for homogeneous recharge, versus 7.78% for step-up recharge. This shows that although the linearization error in water table height increases when recharge is spatially variable (under certain circumstances), this difference does not increase the error in streamflow.



Figure A.4: Solutions for time-varying infiltration using the full numerical, linearized numerical, and analytical solvers. At t=2 days, infiltration is reduced to 0. The difference in peak height of the water table is off by less than 13% between the full and linearized solutions. The RMSE of flow over time for the full and linearized solutions is $0.61 \text{ m}^2/\text{day}$. The numerical and analytical solutions to the linearized equations match perfectly.

In addition to verifying the accuracy of the analytical solution, this numerical comparison allowed us to quantify the time benefits of solving Boussinesq analytically. The analytical solution solved the linearized Boussinesq equation on the order of 10 to 100 times faster than the numerical solver, depending on how many eigenvalues were computed and the resolution of the grid for the numerical solver. The example cases shown here support our initial motivation: the Boussinesq equation can be reasonably and rapidly approximated using an analytical solution to the linearized version of the equation.

Appendix B

Supporting information for Chapter 4

This supplementary material includes one table and two figures. The table lists the recession scale parameters (for Q in ft^3/s and at the de-correlation scale) and the recession exponents for each catchment. The first figure plots empirical vs. theoretical mean persistence times for flow thresholds ranging from 5-50% of mean annual flow, along with the corresponding R^2 value for the 1 : 1 lines. The second figure illustrates the full empirical and modeled persistence times distributions for all catchments at a flow threshold set to 20% mean annual flow.

Table B.1: Recession information for each catchment. The fitted values of a for Q in units of cubic feet per second, for Q at the decorrelation scaling, and the recession exponent parameter r, which is independent of scale.

Catchment	a ($\left[c.f.s.\right)^r \left(day\right)^{-1}$	a	$(day)^{-1}$	(decorrelated)	r
C1		0.0073	0.059		-0.74	
C2	0.0012			0.0057		-0.70
C3	0.0056		0.057		-0.57	
C4		0.0014		0.034		-0.66
C5		0.0018	0.046		-0.47	
C6		0.009 0.051		0.051	-0.65	
C7		0.0017		0.022		-0.88
C8	0.0029		0.066		-0.49	
C9	0.0052		0.054		-0.34	
C10	0.0026		0.042		-0.59	
C11		0.0056	0.062		-0.57	
C12		0.0028 0		0.038	-0.65	
C13	0.010		0.097		-0.45	
C14	0.0019		0.039		-0.54	
C15	0.00090		0.040		-0.62	
C16		0.00016	0.041		-0.82	



Figure B.1: Plots of the empirical mean persistence times vs. analytic mean persistence times between 5% and 50% of mean annual flow. Catchments are labeled C1 - C16, corresponding to the labels in Table S1



Figure B.2: Analytic and empirical persistence times distributions across all watersheds for a flow threshold (Q_*) set to 20% of mean annual flow. Underestimation of the persistence times variability resulting from variability in the character of the seasonal recession is apparent for every watershed. Note the stretched scale for C2 (Big Sur River), likely resulting from dry season flow sustained by water fluxes from geothermal springs.

Appendix C

Supporting information for Chapter 5

USGS catchment information, including the gage ID, location, contributing area, number of years of processed data, and the calculated de-correlation scaling constant (q_0) , which is reported in the same units as the streamflow data, cubic feet per second.

11463170Big Sulphur Creek, Cloverdale, CA11463170Big Sur River, Big Sur, CA11143000Bull Creek, Weott, CA11476600Bull Creek, Weott, CA11475000Eel River, Powers, OR11475560Coquille River, Powers, OR11475500Eel River, Fort Seward, CA11475500Eel River, Trinidad, CA11475500Little River, Trinidad, CA11481200Mad River, Arcata, CA11481200Middle Fork Eel River, Dos Rios, CA11481200Middle Fork Eel River, Dos Rios, CA11468000North Fork Cache Creek, Clearlake Oaks, CA11451100Noyo River, Fort Bragg, CA11468000Outlet Creek, Longvale, CA11468000South Fork Eel River, Miranda, CA11468500South Fork Eel River, Miranda, CA11468000Noyo River, Fort Bragg, CA11468000South Fork Eel River, Miranda, CA11468000South Fork Eel River, Miranda, CA11472200South Fork Eel River, Miranda, CA11472200South Fork Eel River, Miranda, CA	ID Catchment	Contributing Area (km^2)	Years of data	$\mathbf{q_0(cfs)}$
 11143000 Big Sur River, Big Sur, CA 11476600 Bull Creek, Weott, CA 11475000 Bull Creek, Weott, CA 11475000 Eel River, Fort Seward, CA 11475560 Coquille River, Powers, OR 11475560 Elder Creek, Branscomb, CA 11475500 Elder Creek, Branscomb, CA 11481200 Mad River, Arcata, CA 11481200 Mad River, Arcata, CA 11481200 Middle Fork Eel River, Dos Rios, CA 11468000 Middle Fork Eel River, Dos Rios, CA 11451100 Novo River, Navarro, CA 11468000 Novo River, Fort Bragg, CA 11468500 Outlet Creek, Longvale, CA 11472200 Redwood Creek, Orick, CA 11472200 South Fork Eel River, Miranda, CA 	Big Sulphur Creek, Cloverdale, CA	33.9	31	21.7
 11476600 Bull Creek, Weott, CA 14325000 Coquille River, Powers, OR 14325000 Coquille River, Powers, OR 11475000 Eel River, Fort Seward, CA 11475560 Elder Creek, Branscomb, CA 11481200 Mad River, Arcata, CA 11481200 Mad River, Arcata, CA 11473900 Middle Fork Eel River, Dos Rios, CA 11468000 Navarro River, Navarro, CA 11468000 Novth Fork Cache Creek, Clearlake Oaks, CA 11468500 Outlet Creek, Longvale, CA 11472200 Redwood Creek, Longvale, CA 11472200 South Fork Eel River, Miranda, CA 	Big Sur River, Big Sur, CA	120.4	00	112.9
 14325000 Coquille River, Powers, OR 11475500 Eel River, Fort Seward, CA 11475560 Elder Creek, Branscomb, CA 11481200 Little River, Trinidad, CA 11481200 Mad River, Arcata, CA 11481000 Mad River, Arcata, CA 11473900 Middle Fork Eel River, Dos Rios, CA 11468000 North Fork Cache Creek, Clearlake Oaks, CA 11468500 Noyo River, Fort Bragg, CA 11472200 Redwood Creek, Longvale, CA 11472200 Redwood Creek, Orick, CA 11472200 South Fork Eel River, Miranda, CA 	Bull Creek, Weott, CA	72.8	49	91.1
 11475000 Eel River, Fort Seward, CA 11475560 Elder Creek, Branscomb, CA 11481200 Little River, Trinidad, CA 11481000 Mad River, Arcata, CA 11481000 Middle Fork Eel River, Dos Rios, CA 11473900 Middle Fork Eel River, Dos Rios, CA 11468000 North Fork Cache Creek, Clearlake Oaks, CA 11468500 North Fork Longvale, CA 11472200 Redwood Creek, Longvale, CA 11472200 Redwood Creek, Orick, CA 11472200 South Fork Eel River, Miranda, CA 	Coquille River, Powers, OR	437.7	81	523.5
 11475560 Elder Creek, Branscomb, CA 11481200 Little River, Trinidad, CA 11481000 Mad River, Arcata, CA 11473900 Middle Fork Eel River, Dos Rios, CA 11468000 North Fork Cache Creek, Clearlake Oaks, CA 11468500 North Fork Cache Creek, Clearlake Oaks, CA 11468500 Outlet Creek, Longvale, CA 11472200 Redwood Creek, Orick, CA 11472200 South Fork Eel River, Miranda, CA 11476500 South Fork Eel River, Manleton, CA 	Eel River, Fort Seward, CA	5457.1	55	2455.4
11481200Little River, Trinidad, CA11481200Mad River, Arcata, CA11481000Mad River, Arcata, CA11473900Middle Fork Eel River, Dos Rios, CA11468000Navarro River, Navarro, CA11451100North Fork Cache Creek, Clearlake Oaks, CA1145500Noyo River, Fort Bragg, CA11472200Outlet Creek, Longvale, CA11472200Redwood Creek, Orick, CA11472200South Fork Eel River, Miranda, CA11476500South Fork Eel River, Miranda, CA	Elder Creek, Branscomb, CA	16.8	43	13.7
11481000Mad River, Arcata, CA11481000Middle Fork Eel River, Dos Rios, CA11473900Middle Fork Eel River, Dos Rios, CA11468000Navarro River, Navarro, CA11451100North Fork Cache Creek, Clearlake Oaks, CA11468500Noyo River, Fort Bragg, CA11468500Outlet Creek, Longvale, CA11472200Outlet Creek, Longvale, CA11472200South Fork Eel River, Miranda, CA11475200South Fork Eel River, Miranda, CA11476500Sinslaw River, Manleton, CA	Little River, Trinidad, CA	104.9	49	93.1
11473900Middle Fork Eel River, Dos Rios, CA111468000Navarro River, Navarro, CA111451100North Fork Cache Creek, Clearlake Oaks, CA111468500Noyo River, Fort Bragg, CA211472200Nutlet Creek, Longvale, CA211472200Outlet Creek, Longvale, CA211472200South Fork Eel River, Miranda, CA111472200South Fork Eel River, Miranda, CA111475500South Fork Eel River, Manleton, CA1	Mad River, Arcata, CA	1256.1	59	877.5
11468000Navarro River, Navarro, CA11451100North Fork Cache Creek, Clearlake Oaks, CA11468500Noyo River, Fort Bragg, CA11472200Outlet Creek, Longvale, CA11472200Outlet Creek, Longvale, CA11472500Redwood Creek, Orick, CA11476500South Fork Eel River, Miranda, CA11476500Sinslaw River, Manleton, CA	Middle Fork Eel River, Dos Rios, CA	1929.5	45	1595.3
11451100North Fork Cache Creek, Clearlake Oaks, CA11468500Noyo River, Fort Bragg, CA11472200Noyo River, Fort Bragg, CA11472200Outlet Creek, Longvale, CA11472500Redwood Creek, Orick, CA11476500South Fork Eel River, Miranda, CA11476500Sinslaw River, Manleton, CA	Navarro River, Navarro, CA	784.8	50	201.8
11468500Noyo River, Fort Bragg, CA11472200Outlet Creek, Longvale, CA11472500Redwood Creek, Orick, CA11482500South Fork Eel River, Miranda, CA11476500Sinslaw River, Manleton, CA	North Fork Cache Creek, Clearlake Oaks, CA	155.9	39	131.3
11472200Outlet Creek, Longvale, CA411482500Redwood Creek, Orick, CA711476500South Fork Eel River, Miranda, CA114307620Sinslaw River, Manleton, CA1	Noyo River, Fort Bragg, CA	274.5	59	107.2
11482500Redwood Creek, Orick, CA711476500South Fork Eel River, Miranda, CA114307620Sinslaw River, Manleton, CA1	Outlet Creek, Longvale, CA	416.0	35	284.7
111476500South Fork Eel River, Miranda, CA114307620Sinslaw River, Manleton, CA1	Redwood Creek, Orick, CA	717.4	57	631.3
14307620 Sinslaw River, Manleton, CA	South Fork Eel River, Miranda, CA	1390.8	71	812.6
	Siuslaw River, Mapleton, CA	1522.9	35	1461.0

All data is courtesv of the United States Geological Survey Table C 1. Watershed information for recession analyses

Appendix D

Supporting information for Chapter 6



Figure D.1: Spearman rank correlations for all method combinations and all recession measures.



Figure D.2: Shared vs. shared boxplots for a, for all catchments, all method choices, and all recession measures.

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Figure D.3: Shared vs. shared boxplots for b, for all catchments, all method choices.



Figure D.4: Shared vs. shared boxplots for T_R , all catchments, all method choices, and all recession measures.

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