Title
The Forward-Bias Puzzle: A Solution Based on Covered Interest Parity

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Publication Date
2009-10-01

THE FORWARD-BIAS PUZZLE:
A SOLUTION BASED ON COVERED INTEREST PARITY*

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When covered interest parity holds, as appears to be the case, the forward exchange rate is not the expected future spot rate. As a result: (1) In general covered and uncovered interest parity are mutually inconsistent. (2) The standard equation that produces the forward-bias puzzle is miss-specified. When covered interest parity is used to correct that miss-specification, the puzzle disappears. Forward premiums are unbiased estimates of future changes in exchange rates. This solution for the forward-bias puzzle holds whether or not there is a risk premium. It also solves two subsidiary puzzles.

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Key words: exchange rates; forward bias; covered interest parity; arbitrage

JEL: E43, E44, F30, F31, G14, G15

* I want to thank Nathan Balke and Mark Wohar for their data. I would also like to thank Steve Haynes, Steve LeRoy and Doug Steigerwald for their help and comments.

The forward-bias puzzle is probably the most important puzzle in open-economy macroeconomics and international finance. The puzzle is important because it suggests that there are serious informational inefficiencies in foreign exchange markets.[1] To the best of my knowledge, all previous attempts to solve the puzzle assume that, with risk neutrality, the forward exchange rate is the market's expectation of the future spot exchange rate. See for example Fama (1984), Goodhart, McMahon and Ngama (1992), Sarno, Valente and Leon (2006), Sercu and Vinaimont (2006), Chakraboty and Haynes (2008), Chakraborty and Evans (2008) and Bacchetta and van Wincoop (forthcoming). For example, Chakraborty and Evans (2008, p. 487) say that "If agents are risk neutral then they must set today's forward rate equal to their expectation about the future spot rate,…" That assumption is the source of the forward-bias puzzle.

When covered interest parity holds, if markets are at all "rational", the forward exchange rate does *not* represent the market's expectation of the future spot exchange rate. When covered interest parity holds, the expectation of the future spot rate depends on expected *future* interest rates and the expected the *future* forward exchange rate. Unless the current forward exchange rate equals those expectations, it is *not* the expected future exchange rate.

Dropping the assumption that forward exchange rates represent the market's expectation of future spot exchange rates provides the starting point for solving the puzzle of the apparent forward bias. Dropping that assumption also solves two subsidiary puzzles: (1) Why the coefficient for the forward exchange rate is close to one when the future spot exchange rate is regressed against the current forward exchange rate, but the coefficient for the current forward premium is often negative when the future change in the spot exchange rate is regressed against the current forward premium. (2) Why the variance of the change in spot exchange rates is many times greater than the variance of the forward premium.

The fact that the forward exchange rate is not the expected future spot rate also implies that covered and uncovered interest parity are incompatible. That incompatibility implies that when models like the overshooting model in Dornbusch (1976) assume that uncovered interest parity holds they implicitly assume that covered interest parity does not hold.

Section 1 briefly reviews the forward-bias puzzle. Section 2 briefly reviews the evidence regarding covered interest parity. Section 3 shows how covered interest parity can solve the

[1] For a discussion of some of the other puzzles see Obstfeld and Rogoff (2000).

forward-bias puzzle. Section 4 does the same for the two subsidiary puzzles. Section 5 presents evidence supporting the solutions developed in Sections 3 and 4. Section 6 summarizes the article and concludes that covered interest parity explains all three puzzles.

1. The Forward-Bias Puzzle

There is a large literature claiming that the forward premium is a poor predictor of the future change in the exchange rate. For a discussion of the puzzle and a review of the literature see Sarno (2005). More recent articles include Sarno, Valente and Leon (2006), Sercu and Vinaimont (2006), Kearns (2007), Chakraboty and Haynes (2008), Chakraborty and Evans (2008), Wang and Wang (2009) and Bacchetta and van Wincoop (forthcoming).

The crucial flaw in all that literature is the assumption that the forward exchange rate is the market's (rational) expectation of the future spot exchange rate. As another example of that assumption, Sercu and Vinaimont (2006, p. 2417) say the following: "The unbiased-expectations Null says that the time-t expected value for the spot currency j at time t+Δ ,.., equals the time-t forward rate for that horizon."

Equation (1) describes the typical test equation in the literature. Let s_t be the logarithm of the current spot price for foreign exchange S_t. Let f_t be the logarithm of the current forward exchange rate F_t. Finally let s_{t+1} be the logarithm of the future spot rate S_{t+1}.

$$\Delta s_{t+1} = s_{t+1} - s_t = \alpha_1 + \beta_1(f_t - s_t) \tag{1}$$

Using equations like (1), estimates of β_1 are closer to zero than to one and are often negative. For examples of such estimates using the data described later, see Table 1.

Negative estimates of β_1 seem to imply an informational inefficiency. Exchange rates fall when the forward premium predicts that they will rise and the reverse. That apparent predictive error is the forward-bias puzzle.

Equation (1) is supposed to ask a simple question. How well does the market predict changes in exchange rates? But equation (1) does not pose that question correctly. The implicit assumption behind equation (1) is that f_t is the market's expectation of s_{t+1}. When covered interest parity holds, the expected future exchange rate depends on expected future interest rates and the expected future forward exchange rate. Unless the current forward rate equals those expectations, it does not equal the expected future spot exchange rate. Recognizing that the forward exchange rate is not the market's expectation of the future spot exchange rate is the key to solving the puzzle of the forward bias.

2. Covered Interest Parity

Covered interest parity is an equilibrium condition implied by effective arbitrage. Equation (2) describes covered interest parity where the forward premium minus the interest rate differential equals an error.

$$(f_t - s_t) - (i_t - i_t^*) = \pm e_t \tag{2}$$

In equation (2) i is the domestic interest rate, i* is the foreign interest rate and $\pm e_t$ captures the errors within the thresholds caused by transaction costs.[2] The interest rates should be risk free and their maturities must match the maturity of the forward exchange rate. With effective arbitrage, covered interest parity holds whether or not there is a risk premium.

After accounting for the transaction costs, covered interest parity appears to hold on a day-to-day basis. As Akram, Rime and Sarno (2008) point out, "It seems generally accepted that financial markets do not offer risk-free arbitrage opportunities, at least when allowance is made for transaction costs." In the Conclusions to their article, Akram, Rime and Sarno explain in more detail how covered interest rate arbitrage works.

> This paper provides evidence that short-lived arbitrage opportunities arise in the major FX and capital markets in the form of violations of the CIP condition. The size of CIP arbitrage opportunities can be economically significant for the three exchange rates examined and across different maturities of the instruments involved in arbitrage. The duration of arbitrage opportunities is, on average, high enough to allow agents to exploit deviations from the CIP condition. However, duration is low enough to suggest that markets exploit arbitrage opportunities rapidly. These results, coupled with the unpredictability of the arbitrage opportunities, imply that a typical researcher in international macro-finance can safely assume arbitrage-free prices in the FX markets when working with daily or lower frequency data.

See Balke and Wohar (1998) for evidence of the thresholds created by transaction costs. For simplicity of exposition, this section and the next section ignore the thresholds created by transaction costs and assume that the forward premium, $f_t - s_t$. equals the interest rate differential, $i_t - i_t^*$.

As far as I am aware, up to now no one has recognized that covered and uncovered interest parity are, in general, incompatible. When covered interest parity holds, the current spot exchange rate equals the forward rate minus the interest rate differential.

$$s_t = f_t - (i_t - i_t^*) \tag{3}$$

[2] Without logarithms, the equilibrium condition is $[F_t/S_t]/[(1+i_t)/(1+i_t^*)] = (1 \pm e_t)$.

Equation (3) implies equation (4). The future spot exchange rate equals the future forward exchange rate minus the future interest rate differential.

$$s_{t+1} = f_{t+1} - (i_{t+1} - i^*_{t+1}) \qquad (4)$$

Given equation (4), equation (5) describes the rational expectation at time t of s_{t+1} denoted s^E_{t+1}.

$$s^E_{t+1} = f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1}) \qquad (5)$$

Where x^E_{t+1} is the expectation at time t of x_{t+1}. In general, f_t will not equal $f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1})$.[3]

The fact that f_t does not in general equal $f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1})$ implies that covered and uncovered interest parity are mutually inconsistent. If $f_t - s_t$ equals $i_t - i^*_t$ and in general f_t does not equal s^E_{t+1}, then in general $s^E_{t+1} - s_t$ cannot equal $i_t - i^*_t$. The same argument implies that, if uncovered interest parity did hold, then in general covered interest parity would not hold. In general, covered and uncovered interest parity are mutually incompatible. That incompatibility implies that when models like the overshooting model in Dornbusch (1976) explicitly assume that uncovered interest parity holds they implicitly assume that covered interest parity does not hold.

The incompatibility between covered and uncovered interest parity helps explain why the evidence regarding uncovered interest parity is so mixed. For a recent discussion of that evidence and some new evidence on uncovered interest parity see Bekaert, Wei and Xing (2007).

That incompatibility also explains why Eichenbaum and Evans (1995) and the literature it inspired often find impulse responses that imply that monetary shocks create persistent deviations from uncovered interest parity. Indeed those impulse responses often suggest overshooting.[4]

The expected excess return from investing abroad without cover equals $(s^E_{t+1} - s_t) - (i_t - i^*_t)$.[5] When covered interest parity holds one can subtract $(f_t - s_t) - (i_t - i^*_t)$, which equals zero, from $(s^E_{t+1} - s_t) - (i_t - i^*_t)$. That subtraction shows that, when covered interest parity holds, the expected excess return equals s^E_{t+1} minus f_t.

Using equation (5) to replace s^E_{t+1} with $f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1})$ produces equation (6).

[3] One situation where the two are consistent is when the neutral ranges created by transaction costs overlap.

[4] Impulse response functions that change sign suggest overshooting. Impulse response functions that do not change sign suggest "undershooting".

[5] Uncovered interest parity is often called uncovered interest arbitrage. But that terminology is inappropriate because uncovered interest parity involves uncertainty about the return and therefore involves risk.

$$s^E_{t+1} - f_t. = f^E_{t+1} - f_t - (i^E_{t+1} - i^{*E}_{t+1}) \quad (6)$$

When expected inflation is the same at home and abroad, $f^E_{t+1} - f_t$ should be zero. In that case the expected excess return equals the difference in expected real interest rates.

A permanent contractionary monetary shock like a one percent reduction in the stock of money reduces $s^E_{t+1} - f_t$ in the short run for two reasons: (1) The expected interest rate differential rises because the liquidity effect tends to increase domestic interest rates in the short run. (2) The expected change in the forward exchange rate falls because the expected rate of inflation falls in the short run. In the long run the liquidity effect on the domestic interest rate disappears and the expected rate of inflation returns to the initial level as the rate of increase in the stock of money returns to its long-run value. As a result, in response to a permanent contractionary monetary shock the expected excess return first falls and then rises. The expected excess return overshoots as the impulse responses in Eichenbaum and Evans (1995) often suggest.

3. The Forward-Bias Puzzle

As far as I am aware, all previous discussions, tests and explanations of the forward-bias puzzle have assumed that the forward exchange rate is the market's expectation of the future spot exchange rate. When covered interest parity holds, equation (1) is inappropriate because the forward exchange rate is not the market's expectation of the future spot exchange rate. When covered interest parity holds, equation (7) is the appropriate way to describe the expected change in the exchange rate.

$$s^E_{t+1} - s_t = f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1}) - s_t \quad (7)$$

Equation (7) uses equation (5) to replace s^E_{t+1} with $f^E_{t+1} - (i^E_{t+1} - i^{*E}_{t+1})$.

To implement equation (7), one can assume rational expectations. For each expectation for t+1 at time t in equation (7), the actual future value equals the current expected future value plus an uncorrelated error. Ignoring those errors, that assumption produces equation (8).

$$s^E_{t+1} - s_t = (f_{t+1} - s_t) - (i_{t+1} - i^*_{t+1}) \quad (8)$$

Adding and subtracting f_t from the right-hand side of equation (8) produces equation (9).

$$s^E_{t+1} - s_t = (f_t - s_t) + (f_{t+1} - f_t) - (i_{t+1} - i^*_{t+1}) \quad (9)$$

When covered interest parity holds, the term $f_t - s_t$ in equation (9) describes the role of the forward premium in the market's expectation of the future change in the exchange rate.

Assuming, as does equation (1), that the actual change in the exchange rate equals the expected change produces equation (10).

$$s_{t+1} - s_t = s^E_{t+1} - s_t = (f_t - s_t) + (f_{t+1} - f_t) - (i_{t+1} - i^*_{t+1}) \quad (10)$$

This derivation of equation (10) uses expectations. But equation (10) can be derived directly from covered interest parity without appealing to expectations.[6] Whichever interpretation one prefers, equation (10) implies that equation (1) is miss-specified. Equation (1) omits the future change in the forward exchange rate, $f_{t+1} - f_t$, and the future interest rate differential, $i_{t+1} - i^*_{t+1}$. It is those omissions, particularly the omission of the future change in the forward exchange rate, that create the forward-bias puzzle.

Introducing a constant and coefficients for $(f_t - s_t)$, $(f_{t+1} - f_t)$ and $(i_{t+1} - i^*_{t+1})$ into equation (10) produces a correctly specified equation for testing the relationship between the current forward premium and the future change in the exchange rate.

$$\Delta s_{t+1} = \lambda_0 + \lambda_1(f_t - s_t) + \lambda_2(f_{t+1} - f_t) - \lambda_3(i_{t+1} - i^*_{t+1}) \quad (11)$$

As shown in Section 5, estimates of λ_1 are positive, significant and close to 1.0.

From the perspective of equation (11), estimates of β_1 from equation (1) are often negative because equation (1) is miss-specified.

4. Subsidiary Puzzles

There are two subsidiary puzzles associated with the forward-bias puzzle that are usually ignored in attempts to explain the forward-bias puzzle. A valid explanation of the forward-bias puzzle should be able to explain all three puzzles.

The first subsidiary puzzle is the drastic change in the econometric results between estimating equation (1) and the equivalent of that equation in levels. The second subsidiary puzzle is the vast difference between the variance for changes in spot exchange rates and the variance for the forward premium. Covered interest parity provides simple solutions to both subsidiary puzzles.

4.1 Levels versus Changes

When economists first began to ask how well markets for foreign exchange predicted exchange rates, they regressed the future exchange rate against the forward exchange rate.[7] That regression routinely produces coefficients close to, but less than, 1.0. But when s_t is subtracted from both sides of that equation to obtain stationarity, the coefficient for the forward premium

[6] Subtract s_t from both sides of equation (4), then add and subtract f_t from the right-hand side. Or take the first difference of equation (4) and replace $(i_t - i^*_t)$ with $f_t - s_t$. In either case, covered interest parity implies that the future change in the exchange rate equals the right-hand side of equation (10).

[7] See for example Cornell (1977), Levich (1979) and Frenkel (1980).

drops close to zero and is often negative. That drastic change in coefficients is the first subsidiary puzzle that a complete solution to the puzzle of the forward bias should solve.

Covered interest parity implies that equation (12) describes the appropriate relation between the forward exchange rate and the future spot exchange rate. Equation (12) is simply equation (4) with an intercept A and appropriate coefficients B_i added.

$$s_{t+1} = A + B_1 f_{t+1} - B_2(i_{t+1} - i^*_{t+1}) \tag{12}$$

Equation (13) is equation (12) with f_t replacing f_{t+1}.

$$s_{t+1} = a + b_1 f_t - b_2(i_{t+1} - i^*_{t+1}) \tag{13}$$

Equation (14) is equation (1) in levels. Note that equation (14) is also equation (13) with $b_2(i_{t+1} - i^*_{t+1})$ deleted from the right-hand side.

$$s_{t+1} = \alpha_2 + \beta_2 f_t \tag{14}$$

Covered interest parity implies that the econometric results should deteriorate as one moves from equation (12) through equation (13) to equation (14). In particular, b_1 should be smaller than B_1. From that perspective, estimates of β_2 are positive and fairly close to 1.0 because f_t is a good proxy for f_{t+1}, not because f_t is the market's expectation of the future spot exchange rate.

4.2 Relative Variances

The second subsidiary puzzle is the difference between the variance for changes in exchange rates, Δs_t, and the variance for the forward premium, $f_t - s_t$. According to Wang and Wang (2009, p. 186), "… the variance of spot rate changes is in the range of 100-200 times the variance of the forward premium."

As long as one assumes that the forward exchange rate is the expected future spot exchange rate, this difference in variances is difficult, if not impossible, to explain. But covered interest parity provides a simple explanation. Covered interest parity implies that the forward premium equals the interest rate differential. From that perspective, the variance of the forward premium, $f_t - s_t$, is relatively small because the variance of the interest rate differential, $i_t - i^*_t$, is relatively small. The variance of Δs_t is relatively large because the variance of Δf_t is relatively large. There are two ways to look at this explanation. One way is to use equation (10).

$$s_{t+1} - s_t = s^E_{t+1} - s_t = \Delta f_{t+1} + (f_t - s_t) - (i_{t+1} - i^*_{t+1}) \tag{10}$$

According to equation (10), the variance of Δs_t depends on the variance of Δf_{t+1} and the variance of $f_t - s_t$ and $i_{t+1} - i^*_{t+1}$. Another way is to take the first difference of equation (3), which produces equation (15).

$$\Delta s_t = \Delta f_t - \Delta(i_t - i^*_t) \tag{15}$$

As shown in Table 5, the variances of $(f_t - s_t)$, $i_{t+1} - i^*_{t+1}$ and $\Delta(i_t - i^*_t)$ are all relatively small. It is the relatively large variance for Δf_t that explains the relatively large variance for Δs_t. I mean "explain" here strictly in the statistical sense. The issue of causation between spot and forward exchange rates is beyond the scope of this article.

5. Empirical Results

In spite of small Durbin-Watson statistics, in this section I assume that the residuals from all regressions are at least globally stationary. I attribute the low Durbin-Watson statistics to the "thresholds" created by transaction costs. That assumption is based on the evidence that, after taking account of the transaction costs, covered interest parity holds day to day.[8]

5.1. The Data

The data cover two intervals between the United States and Canada and two intervals between the United States and the United Kingdom. For US-Canada, the weekly interest rates are for 13 week Treasury bills. Those interest rates are from various issues of the *Federal Reserve Bulletin* starting with the issue of October 1964. Spot and forward exchange rates are for noon and were supplied by the Bank of Canada. As the *Bulletin* makes clear, the Treasury bill rates are only approximations of the rates needed for arbitrage.[9] The data for US-Canada run from January 1961 to June 1973.[10] The first interval for Canada in Table 1 covers the era of pegged exchange rates that started *de facto* in December 1960 and ended in May 1970. The second interval covers a period of flexible exchange rates from June 1970 to June 1973.[11]

For the US-UK, the data are from Balke and Wohar (1998). Their interest rates are one month euro rates. See Balke and Wohar (1998) for more details.[12] Their daily data start in January 1974 and end in September 1993. To account for any possible effects of the switch to

[8] For all four intervals Δf_t is linearly stationary. For all intervals except perhaps for the flexible US-Canada interval, $s_t - f_t$ and $s_{t+1} - f_t$ are also linearly stationary.

[9] For a detailed description of the interest rates, see the issue of October 1964.

[10] The data start in January 1959 when rates were flexible. I start in January 1961 because the rates were pegged *de facto* in December of 1960. The data end in August 1973, but 13 weeks are lost due to the difference between spot and forward exchange rates.

[11] For both US-Canada and US-UK, missing observations are replaced with the previous observation. If two observations in a row are missing, the first is replaced with the previous observation and the second with the following observation.

[12] The data in Balke and Wohar (1998) are bid and ask. Like them, I use the geometric mean of the bid and ask.

flexible rates in the early 1970s, the interval is divided into roughly two equal parts. The first begins in January 1974 and runs through early November 1983. The second begins the next day and ends in early September 1993.

For the Canadian data, where the interest rates are for 91 days, the future spot and future forward exchange rates are t plus 13 weeks. For the UK data, where the interest rates are for 30 days, the future spot and future forward exchange rates are t plus 22 observations.

The data are not ideal. interest rates, future spot exchange rates and forward rates are not always matched exactly. Particularly for the US-Canadian data, the timing of the observations is not ideal. Future research should correct those shortcomings. However it seems unlikely that correcting any shortcomings in the data will change the basic message. The apparent forward-bias puzzle is the result of assuming that the forward exchange rate is the market's expectation of the future spot exchange rate.

In addition to reporting estimated coefficients, tables with regressions also report the adjusted R^2 or $\bar{R}^2$ and Durbin-Watson statistics or DW. Changes in the $\bar{R}^2$ can provide an indication of the effect of the specification errors. Changes in the DW statistic can indicate how the serial correlation in residuals increases as a result of specification errors.

5.2 The Forward Premium and Future Changes in Spot Rates

Estimates of β_1 from equation (1) are often negative. But subsection 3.2 shows that, when the forward premium is part of an appropriate test equation as in equation (11), estimates of the parameter for the forward premium should be close to 1.0. That subsection also shows that, from the perspective of covered interest parity, equation (1) is miss-specified. That miss-specification is the source of the negative estimates of β_1.

5.2.1 Equations (1) and (11)

Table 1 reports the estimates of equation (1) using the data described above.[13]

$$\Delta s_{t+1} = s_{t+1} - s_t = \alpha_1 + \beta_1(f_t - s_t) \tag{1}$$

As is usual, estimates of β_1 are often negative. The average β_1 in Table 1 is -1.154. The average $\bar{R}^2$ is only 0.015 and the average DW is only 0.103.

As pointed out in subsection 3.2, equation (1) is miss-specified because the forward exchange rate is not the market's expectation of the future spot exchange rate. When the forward

[13] Regressions in all tables use RATS with "Robusterrors".

premium is part of an appropriately specified equation like equation (11), estimates of the coefficient for the forward premium should be close to 1.0.

$$\Delta s_{t+1} = \lambda_0 + \lambda_1(f_t - s_t) + \lambda_2(f_{t+1} - f_t) - \lambda_3(i_{t+1} - i^*_{t+1}) \qquad (11)$$

Table 2 reports the results of estimating equation (11). All $\hat{\lambda}_i$ for i greater than zero have the correct sign and all are significant at well beyond the 1 percent level. The lowest $\bar{R}^2$ in Table 2 is 0.984. That is extraordinarily high for an equation in first differences.

The smallest $\hat{\lambda}_1$ in Table 2, the estimated coefficient for $(f_t - s_t)$, is 0.605, which is significantly greater than 0.0 at well beyond the 1 percent level. The largest $\hat{\lambda}_1$ is 0.996, which is not statistically different from 1.0.

The $\hat{\lambda}_1$ for US-Canada in Table 2 are relatively small. That is probably because the data for US-Canada are not as good as the data supplied by Balke and Wohar. For the data supplied by Balke and Wohar both $\hat{\lambda}_1$ are not significantly different from one. It is clear from Table 2 that when the forward premium is part of a correctly specified equation, there is no forward-bias puzzle.

But Table 2 includes only two pairs of countries with only two intervals each. Before the forward-bias puzzle can be declared finally solved, these results should be confirmed across various countries and intervals. However, since these are the only countries and intervals that I have analyzed, I am confident that the results will hold up over space and time.

5.2.2 $\hat{\beta}_1$

As pointed out earlier, the forward-bias puzzle is the direct result of assuming that the forward rate is the market's expectation of the future spot exchange rate. Given that assumption, one would expect the forward exchange rate to be more highly correlated with the future spot exchange rate than with the current spot exchange rate. In that case there should be no forward-bias puzzle. Estimates of β_1 should be positive and close to 1.0.

When covered interest parity holds, one expects current forward exchange rates to be more highly correlated with current spot exchange rates than with future spot exchange rates. As Table 3 shows, the forward-bias puzzle is the result of that cross-correlation and the fact that the current spot rate is more highly correlated with the future spot exchange rate than is the current forward exchange rate. Given such cross-correlations, it is not surprising that estimates of β_1 in equation (1) are often negative.

Table 3 uses the same data and intervals as the previous tables. In Table 3, the average correlation between f_t and s_t is 0.994, but the average correlation between f_t and s_{t+1} is smaller, 0.878. That difference is consistent with covered interest parity, but difficult to explain if one assumes that the forward exchange rate is the expected future spot rate.

From the perspective of covered interest parity, the relatively small correlation between f_t and s_{t+1} is because the link between f_t and s_{t+1} is indirect through f_{t+1}. Like the correlation between f_t and s_t, the average correlation between f_{t+1} and s_{t+1} is 0.994. The average correlation between f_t and f_{t+1} in Table 3 is 0.893. That indirect link from f_t through f_{t+1} to s_{t+1} is why the average correlation between f_t and s_{t+1} is smaller than the average correlation between f_t and s_t.

The relatively small correlation between f_t and s_{t+1} is compounded by the fact that the average correlation between s_t and s_{t+1} is slightly greater than the average correlation between f_t and s_{t+1}. The average correlation between s_t and s_{t+1} in Table 3 is 0.888. The average correlation between f_t and s_{t+1} is slightly smaller, 0.878.

Given these average cross correlations, which are consistent with covered interest parity, it is not surprising that estimates of β_1 are often negative.

5.3 Levels Versus Changes

The first subsidiary puzzle associated with the forward-bias puzzle is the drastic change from levels to changes. Estimates of β_2 from equation (14) are close to, but less than, 1.0.

$$s_{t+1} = \alpha_2 + \beta_2 f_t \tag{14}$$

But estimates of β_1 from equation (1) are often negative.

$$\Delta s_{t+1} = \alpha_1 + \beta_1(f_t - s_t) \tag{1}$$

Subsection 5.2.2 explains why estimates of β_1 from equation (1) are often negative. As subsection 4.1 shows, covered interest parity implies that estimates of β_2 should be close to, but less than, 1.0. The reason is implicit in the cross-correlations reported in Table 3.

Removing s_t from both sides of equation (1) removes two econometric complications. The first complication is that on average s_t is more closely correlated with s_{t+1} than is f_t. The second is that on average f_t is more closely correlated with s_t than it is with s_{t+1}. With those complications removed from equation (1), estimates of β_2 from equation (13) can capture the indirect link from f_t to s_{t+1} through f_{t+1}.

Using the same data as earlier tables, Table 4 illustrates the link from f_t to s_{t+1} through f_{t+1}. For each of the four intervals, Table 4 shows the results of estimating three equations. The first is equation (12). Equation (12) describes the link between s_{t+1}, f_{t+1} and i_{t+1} - i^*_{t+1}, implied by covered interest parity.

$$s_{t+1} = A + B_1 f_{t+1} - B_2(i_{t+1} - i^*_{t+1}) \qquad (12)$$

The second is equation (13). Equation (13) is the same as equation (12) except that f_t replaces f_{t+1}. Comparing the results for equations (12) and (13) illustrates how good a proxy f_t is for f_{t+1}.

$$s_{t+1} = a + b_1 f_t - b_2(i_{t+1} - i^*_{t+1}) \qquad (13)$$

The third equation is equation (1) in levels rather than differences. Comparing equations (13) and (14) illustrates how little omitting the interest rate differential affects the coefficient for f_t.

$$s_{t+1} = \alpha_2 + \beta_2 f_t \qquad (14)$$

As implied by covered interest parity, the results for these equations deteriorate as one moves from equation (12) to (13) and then to (14). The average $\bar{R}^2$ and DW statistics for equation (12) are 0.996 and 0.923 respectively. Replacing f_{t+1} with f_t reduces the average $\bar{R}^2$ and DW statistic to 0.818 and 0.129 respectively. Dropping the interest rate differential reduces the average $\bar{R}^2$ and DW statistic to 0.784 and 0.101 respectively.

But it is the effect on the coefficients of the switch from f_{t+1} to f_t that is most important. For f_{t+1} in equation (12), the average coefficient is 1.010. When f_t replaces f_{t+1} in equation (13) the average coefficient for f_t falls to 0.850. Dropping the interest rate differential reduces the average coefficient for f_t, which is β_2 in equation (14), to 0.816. I believe Table 4 shows that the coefficient for f_t in equation (14) is well above zero because f_t is an excellent proxy for f_{t+1}, not because f_t is the expectation of the future spot exchange rate. If f_t were the expected future spot exchange rate, one would expect equation (14) to explain s_{t+1} at least as well as equation (12).

5.4 Relative Variances

As long as one keeps the assumption that f_t is the market's expectation of s_{t+1}, it is almost impossible to explain why the variance of Δs_t and Δs_{t+1} are so much larger than the variance of

f_t-s_t. But the large difference in the variances is consistent with covered interest parity. From that perspective, the variance of the forward premium is small because the variance of the interest rate differential is small. The variance of the change in the spot exchange rate is relatively large because the variance of the change in the forward exchange rate is relatively large. Again this "because" should not be interpreted as causation, only correlation.

Using the same data as previous tables, Table 5 reports the relevant variances. The average variance for Δs_t and Δs_{t+1} is 6.515. But the average variances for f_t-s_t, $i_{t+1} - i^*_{t+1}$ and $\Delta\,(i_t - i^*_t)$ are respectively only 0.038, 0.050 and0.024. The estimates for Δs_t and f_t-s_t are consistent with those reported by Wang and Wang (2009). However the average variance for Δf_t is 6.518, which is essentially the same as the average variance for Δs_t and Δs_{t+1}.

The similarity between the variances for Δs_t and Δf_t suggests that each is large because the other is large. In the purely statistical sense, the variance in Δf_t "explains" almost all the variance in Δs_t.

Equation (16) is equation (12) in differences shifted back from t+1 to t.

$$\Delta s_t = A_0 + B_1\Delta f_t - B_2\Delta(i_t - i^*_t) \tag{16}$$

Using the same data as previous tables, Table 6 shows that, on average, equation (16) explains 98 percent of the variance in Δs_t. Dropping the interest differential produces equation (17).

$$\Delta s_t = A_1 + B_3\Delta f_t \tag{17}$$

In Table 6, the average $\hat{B}_3$ from equation (17) is 1.01 and the average $\bar{R}^2$ is 0.977. On average the variance of Δf_t alone "explains" 97 percent of the variance in Δs_t. Given the variances shown in Table 5, as long as covered interest parity holds, the variance of changes in spot exchange rates must be much larger than the variance of the forward premium. The third and final puzzle is solved.

6. Summary and Conclusion

As far as I am aware, all previous work on the forward-bias puzzle has assumed either explicitly or implicitly that, in the absence of a risk premium, the forward exchange rate is the market's expectation of the future spot exchange rate. That assumption is the source of the puzzle. When covered interest parity holds, even without a risk premium, in general the forward exchange rate is not the rational expectation of the future spot exchange rate. That expectation depends on expected *future* interest rates and the expected *future* forward exchange rate.

The fact that the forward exchange rate is not the market's expected future spot exchange rate has at least two important implications: (1) When covered interest parity holds, in general uncovered interest parity does not hold and *visa versa*. As a result, models like the famous Dornbusch overshooting model that explicitly assume uncovered interest parity implicitly assume that covered interest parity does not hold. (2) The standard equation that produces the forward-bias puzzle is miss-specified. When covered interest parity is used to correct that problem, the puzzle disappears. Forward premiums are unbiased estimates of future changes in exchange rates.

This solution to the forward-bias puzzle also solves two subsidiary puzzles associated with the forward-bias puzzle. (1) Why the coefficient for the forward exchange rate is close to 1.0 when the dependent variable is the future spot exchange rate, but the coefficient for the forward premium is usually negative when the dependent variable is the future change in the exchange rate. (2) Why the variance for changes in the exchange rate is many times larger than the variance for the forward premium.

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Table 1
Estimates of Equation 1
$\Delta s_{t+1} = \alpha_1 + \beta_1(f_t - s_t)$

	$\hat{\alpha}_1$	$\hat{\beta}_1$	$\bar{R}^2$/DW
US-Canada			
5 Jan 1961 to	0.251	-0.425	0.003
31 Dec 1969	(0.046)	(0.175)	0.100
US-Canada			
5 Jun 1970 to	-0.238	0.268	-0.003
29 Jun 1973	(0.093)	(0.372)	0.151
US-UK			
2 Jan 1974 to	0.657	-1.425	0.033
1 Nov 1983	(0.069)	(0.166)	0.087
US-UK			
2 Nov 1983 to	0.930	-3.034	0.025
30 Sep 1993	(0.129)	(0.406)	0.075
Averages	0.332	-1.154	0.015
	(0.084)	(0.280)	0.103

Standard errors in parentheses.

Table 2
Estimates of Equation 11
$\Delta s_{t+1} = \lambda_0 + \lambda_1(f_t - s_t) + \lambda_2\Delta f_{t+1} - \lambda_3(i_{t+1} - i^*_{t+1})$

	$\hat{\lambda}_0$	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\lambda}_3$	$\bar{R}^2$/DW
U.S-Canada					
5 Jan 1961 to	0.014	0.605	1.002	-0.611	0.988
31 Dec 1969	(0.006)	(0.037)	(0.004)	(0.043)	0.173
US-Canada					
5 Jun 1970 to	-0.138	0.789	0.984	-1.083	0.984
29 Jun 1973	(0.021)	(0.049)	(0.013)	(0.074)	0.266
US-UK					
2 Jan 1974 to	0.007	0.996	0.999	-1.012	0.999
1 Nov 1983	(0.001)	(0.011)	(0.000)	(0.011)	1.460
US-UK					
2 Nov. 1983 to	0.007	0.990	1.000	-1.009	0.9997
30 Sep 1993	(0.001)	(0.006)	(0.000)	(0.008)	1.468
Averages	-0.028	0.845	0.996	-0.929	0.993
	(0.007)	(0.026)	(0.004)	(0.034)	0.842

Standard errors in parentheses.

Table 3
Relevant Cross-Correlations

	f_t versus s_t (f_{t+1} versus s_{t+1})	f_t versus f_{t+1}	f_t versus s_{t+1}	s_t versus s_{t+1}
U.S-Canada 5 Jan 1961 to 31 Dec 1969	0.995	0.894	0.891	0.896
US-Canada 5 Jun 1970 to 29 Jun 1973	0.982	0.742	0.687	0.721
US-UK 2 Jan 1974 to 1 Nov 1983	0.9997	0.981	0.980	0.981
US-UK 2 Nov. 1983 to 30 Sep 1993	0.9999	0.955	0.954	0.955
Averages	0.994	0.893	0.878	0.888

Table 4

Estimates of Equations 12, 13 and 14

	$s_{t+1} = A + B_1 f_{t+1} - B_2(i_{t+1} - i^*_{t+1})$				$s_{t+1} = a + b_1 f_t - b_2(i_{t+1} - i^*_{t+1})$				$s_{t+1} = \alpha_2 + \beta_2 f_t$		
	$\hat{A}$	$\hat{B}_1$	$\hat{B}_2$	$\bar{R}^2$/DW	$\hat{a}$	$\hat{b}_1$	$\hat{b}_2$	$\bar{R}^2$/DW	$\hat{\alpha}$	$\hat{\beta}$	$\bar{R}^2$/DW
US-Canada											
5 Jan 1961 to	0.010	0.990	-0.883	0.994	0.319	0.703	0.090	0.792	0.319	0.703	0.792
31 Dec 1969	(0.002)	(0.002)	(0.032)	0.134	(0.042)	(0.039)	(0.424)	0.121	(0.042)	(0.039)	0.121
US-Canada											
5 Jun 1970 to	-0.057	1.055	-1.140	0.990	0.224	0.770	-2.891	0.598	0.394	0.606	0.469
29 Jun 1973	(0.008)	(0.008)	(0.063)	0.312	(0.056)	(0.055)	(0.466)	0.235	(0.040)	(0.040)	0.133
US-UK											
2 Jan 1974 to	0.000	1.000	-1.018	0.99999	0.003	0.994	-2.641	0.965	0.002	0.998	0.962
1 Nov 1983	(0.000)	(0.000)	(0.004)	1.474	(0.002)	(0.004)	(0.200)	0.090	(0.002)	(0.004)	0.084
US-UK											
2 Nov 1983 to	0.000	1.000	-1.007	0.99998	-0.018	0.933	-5.107	0.916	-0.022	0.959	0.911
30 Sep 1993	(0.000)	(0.000)	(0.006)	1.774	(0.004)	(0.008)	(0.390)	0.071	(0.004)	(0.008)	0.067
Averages	-0.011	1.011	-1.012	0.996	0.132	0.850	-2.637	0.818	0.173	0.816	0.784
	(0.002)	(0.002)	(0.026)	0.923	(0.026)	(0.027)	(0.370)	0.129	(0.022)	(0.023)	0.101

Standard errors in parentheses.

Table 5
Relevant Variance

	Δs_t and Δs_{t+1}	Δf_t	$\Delta(i_t - i_t^*)$	$f_t - s_t$	$i_{t+1} - i_{t+1}^*$
U.S-Canada 5 Jan 1961 to 31 Dec 1969	0.983	1.009	0.018	0.023	0.019
US-Canada 5 Jun 1970 to 29 Jun 1973	2.201	2.119	0.042	0.050	0.026
US-UK 2 Jan 1974 to 1 Nov 1983	7.562	7.545	0.033	0.037	0.116
US-UK 2 Nov. 1983 to 30 Sep 1993	15.315	15.400	0.004	0.042	0.039
Averages	6.515	6.518	0.024	0.038	0.050

Table 6
Estimates of Equations 16 and 17

	$\Delta s_t = A_0 + B_1\Delta f_t - B_2(i_t - i_t^*)$				$\Delta s_t = A_1 + B_3\Delta f_t$		
	$\hat{A}_0$	$\hat{B}_1$	$\hat{B}_2$	$\bar{R}^2$/DW	$\hat{A}_1$	$\hat{B}_3$	$\bar{R}^2$/DW
US-Canada							
5 Jan 1961 to	0.034	0.984	-0.334	0.979	0.005	0.978	0.977
31 Dec 1969	(0.007)	(0.007)	(0.067)	0.156	(0.007)	(0.010)	0.144
US-Canada							
5 Jun 1970 to	-0.186	0.979	-0.941	0.955	0.028	1.064	0.937
29 Jun 1973	(0.025)	(0.020)	(0.112)	0.146	(0.021)	(0.017)	0.094
US-UK							
2 Jan 1974 to	0.037	0.996	-0.131	0.995	0.008	0.999	0.995
1 Nov 1983	(0.005)	(0.002)	(0.016)	0.273	(0.004)	(0.002)	0.277
US-UK							
2 Nov 1983 to	0.014	0.997	-0.050	0.999	-0.002	0.997	0.999
30 Sep 1993	(0.003)	(0.001)	(0.009)	1.067	(0.002)	(0.001)	1.059
Averages	-0.025	0.989	-0.364	0.982	0.010	1.010	0.977
	(0.010)	(0.008)	(0.051)	0.410	(0.008)	(0.008)	0.394

Standard errors in parentheses.