The Mental Plane: A Model of Imagining Situations in Euclidean Geometry

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Essentials about the Mental Plane

People process images in their minds all the time. Sometimes these images are two-dimensional, as when they play chess or draw a map to describe the location of a place. Some neuroscientists today say that the area in the brain where image processing occurs is the Visuospatial Sketchpad (VSSP) (Baddeley, 1995). The goal of this research is to present a model of how the mind holds simple two-dimensional images in the domain of Euclidean geometry, and how this underlies the process of discovery in this microworld.

In Euclidean geometry, as in the world of chess, there is an interconnection between external media (e.g., paper) and the mind. It is hypothesized that perception of a figure gives rise to chunks. This poster is about how the mind can hold geometric imagery in a chunked fashion in what we call *the mental plane.*

Two experiences motivated my research in this topic. One was having been defeated in chess by a player moving blindfold. The other was being a student of a geometry teacher who gave two of his lectures in a pitch-dark room. The teacher was inspired by a famous Swiss geometer of the nineteenth century who insisted on teaching geometry in pitch-darkness. I give the name *mental plane* to the place where processing of such abstract imagery occurs in the brain. Its existence becomes obvious when we see students discussing a geometric subject on the phone, or solving a problem while riding their bikes. It is even more striking when we hear of people like Leonhard Euler (1707–1783) who wrote 886 papers of which a significant number were produced in the final two decades of his life, when he was totally blind.

The mental plane establishes the basis for what we consider *near* and *far* and induces a rough sense of proximity between objects in it. It also gives us a sense of *orientation* and *center*, allowing us to discriminate *up* from *down* and *left* from *right*. None of these notions is numerical.

By itself, the mental plane would be useless, but it becomes fully alive when it holds "objects." The objects of the mental plane are entities such as mental points, mental segments, and mental circles.

A crucial principle of the mental plane is economy. I call this the *efficiency principle*. In order to imagine a geometric figure, the brain needs to supply energy and memory efficiently. Therefore, a geometric figure like a circle or segment is not present as a whole most of the time. Instead, the figure is represented partially. If we try to imagine a circle, it is difficult to see the whole thing. With effort it seems possible, but after several seconds the visualization brings discomfort. Stronger evidence is given by the cases of famous chess players such as Morphy, Capablanca, and Alekhine, who reported headaches after demonstrating blindfold chess, and as a consequence blindfold chess exhibitions were banned in the ex-Soviet Union. (Hooper & Kenneth, 1984).

Naturally, more effort is required for the visualization of a compound figure such as an arc with a tangent line than for the visualization of a trivial figure such as a segment. However, the mental plane provides an efficient mechanism: it allows *symbolic* objects. In this way, for example, the three tangency points of the incircle of a triangle (see Figure 1) are not visualized simultaneously, as happens when we look at a figure drawn on paper. Symbolic mental entities (akin to icons) could be used instead. Therefore, the mental plane is a hybrid between physical and symbolic, or analog and iconic.

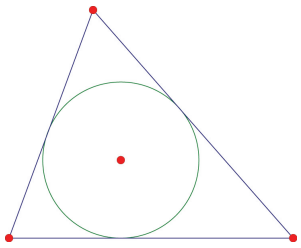

Figure 1: The incircle of a triangle.

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