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Three Essays in Macroeconomics

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by

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To my family.

ABSTRACT OF THE DISSERTATION

Three Essays in Macroeconomics

by

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Dr. Richard M.H. Suen, Co-Chairperson

This dissertation studies three main topics in macroeconomics: the impact of labor market frictions on labor supply and income inequality, the impact of goods market frictions on individuals' optimization decisions, and the housing market over business cycles.

In chapter one, I briefly review the research of this dissertation. Chapter two develops a dynamic general equilibrium model with progressive taxation, labor market search and heterogeneous households to study the impact of tax reform on labor supply and income inequality across educational groups. Households differ in their educational level and their time preference. I study the labor supply response to tax reform along both the intensive margin (hours worked) and the extensive margins (labor force participation). The quantitative results show that: (i) a tax reform which decreases the marginal tax rate by the same magnitude of that in the Tax Reform Act of 1986 (TRA-86), has a significant impact on households' labor supply and that approximately

65 percent of the aggregate labor supply response is along the extensive margin; (ii) households labor supply response to tax reform depends on their educational levels. Households with less education respond more significantly along both the intensive and extensive margin while the response of households with highest education is subtle. However, the income share of the highest educational group increases due to an increase in capital income after tax reform. These findings are consistent with the empirical literature studying the effects of TRA-86 on labor supply.

Chapter three introduces search frictions in the goods market into the classic income fluctuation problem and explores individuals' intertemporal optimization decisions. Individuals make their optimal choices when they face labor productivity shocks, borrowing constraints, and search frictions in the goods market. In this framework, individuals save not only for precaution but also for transaction-the higher asset holdings an individual has the shorter waiting time for getting the frictional goods. This provides an additional dimension to look at individuals' consumption and savings behavior. It also provides a possible channel to solve the problem encountered in the Aiyagari-Bewley model, which predicts a relatively low income inequality compared to the data observed.

In the forth chapter, I examine the business cycle properties of the housing market in a multi-sector dynamic stochastic general equilibrium model with intermediate inputs and adjustment costs for capital and housing. The quantitative results replicate three main business cycle features of the housing market. First, GDP, consumption, non-residential investment, and residential investment co-move positively during the business cycle. Second, residential investment is twice as volatile as business investment. Finally, house prices and residential investment are positively correlated over business cycles.

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Chapter 1

Introduction

While a large body of literature addressed the importance of market frictions in labor economics and monetary economics, few studies have examined how frictions in the labor market would affect the impact of tax reform on labor supply and income inequality, and how individuals make their intertemporal optimization decisions when the goods market is frictional. I study the above questions in the next two chapters. Chapter two studies the labor supply and income responses to tax reform when the labor market is frictional. Compared to previous literature, the contribution of this paper is that I examine the labor supply response to tax reform along both the intensive margin (hours worked) and the extensive margins (labor force participation) theoretically. To achieve this, I develop a dynamic general equilibrium model with progressive taxation, labor market search and heterogeneous households to study the impact of tax reform on labor supply and income inequality across educational groups. By introducing labor search, the decision of participation in the labor market is explicitly modeled.

Households differ in their educational level and their time preference. The quantitative results show that: (i) a tax reform which decreases the marginal tax rate by the same magnitude of that in the Tax Reform Act of 1986 (TRA-86), has a significant impact on households' labor supply and that approximately 65 percent of the aggregate labor supply response is along the extensive margin; (ii) households labor supply response to tax reform depends on their educational levels. Households with less education respond more significantly along both the intensive and extensive margin while the response of households with highest education is subtle. However, the income share of the highest educational group increases due to an increase in capital income after tax reform. These findings are consistent with the empirical literature studying the effects of TRA-86 on labor supply. The results show that labor market frictions are important in determining the impact of tax reform.

Chapter three introduces goods market frictions into the income fluctuation problem where individuals make their optimal intertemporal decisions in the presence of labor productivity shocks, borrowing constraints, and goods market frictions. There are two goods markets in the economy: a general good distributed through a competitive market and a special good distributed through directed search. Directed search directs individuals into different sub-markets according to their willingness to pay, which is determined by individuals' asset holdings. The presence of the special good provides the individual an additional incentive to save, which further changes individuals' optimal decisions. This is because individuals with higher asset holdings would have higher probability obtaining the special good-a shorter waiting time. Chapter two first formulates an individual's dynamic programming problem, then establishes the existence of the value function, and finally proves its continuity, concavity, and differentiability. The properties of policy functions are also characterized. This model provides an additional dimension to look at individuals' consumption and savings behavior, i.e., individuals save not only for precaution but also for transaction. It also provides a possible channel to solve the problem encountered in the Aiyagari-Bewley model, which predicts a relatively low income inequality compared to the data observed.

Chapter four examines the business cycle properties of the housing market, which is motivated by recent financial crisis caused by the collapse of the subprime mortgage market in 2008. I construct a multi-sector dynamic stochastic general equilibrium model with intermediate inputs and adjustment costs for capital and housing. The quantitative results replicate three main business cycle features of the housing market. First, GDP, consumption, non-residential investment, and residential investment co-move positively during the business cycle. Second, residential investment is twice as volatile as business investment. Finally, house prices and residential investment are positively correlated over business cycles.

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Chapter 2

Labor Supply Response to Tax Reform in a Search Model with Heterogeneous Households

2.1 Introduction

The fact that labor supply responds to tax reforms more along the extensive margin (labor force participation) than along the intensive margin (hours of work) has been well documented in the empirical labor literature[1]. Theoretical studies on the response of labor supply along the extensive margin, which include [Cogan, 1981],

[1][Eissa, 1995] examines the labor supply response of married women to changes in the tax rate of the Tax Reform Act 86 (TRA-86). She finds that after the TRA-86 total hours worked by married women with high incomes increased by 90 hours per year on average. She also shows that the labor supply elasticity with respect to the after tax wage equals 0.8, at least half of which is explained by changes in labor force participation. [Eissa and Liebman, 1996] and [Meyer and Rosenbaum, 2001] find that the Earned Income Tax Credit (EITC), an expansion of the TRA-86, had a substantial effect on the labor force participation of single mothers but a modest impact on the hours worked.

[Heim and Meyer, 2004], and [Eissa et al., 2006], rely on the assumption that discrete labor participation decisions are explained by non-convexities created by fixed-costs of work. The standard Diamond-Mortensen-Pissarides search model[2] can also explain discrete labor participation decisions but has not yet been used to examine labor supply response along both the intensive margin and the extensive margin.

The empirical literature also finds that tax reform has strong distributional effects, i.e., labor supply response and income response to tax reform differ across heterogeneous households. [Bosworth and Burtless, 1992] suggest that after the Economic Recovery Tax Act of 1981 (ERTA) and Tax Reform Act 1986 (TRA-86) the labor supply of individuals in the bottom income quintile increased significantly while the labor supply of individuals in the top income quintile increased only marginally. [Lindsey, 1987] and [Lindsey, 1988] find that ERTA had a negligible impact on low-income and middle-income taxpayers while it increased the revenue response of upper-income taxpayers. This led to an increase in the share of aggregate taxes paid by upper-income taxpayers.

This paper uses the search model to explain labor supply response and income response to a tax reform. The standard deterministic neoclassical model is extended to include three additional features: labor market search, household heterogeneity, and progressive taxation. Labor market search as modeled by [Diamond, 1982], [Mortensen, 1982] and [Pissarides, 1985] is required to account for discrete participation behavior. The model assumes that labor market search is conducted in an aggregate way: all unemployed workers and vacant firms join in a single pool where unemployed

[2]See [Diamond, 1982], [Mortensen, 1982] and [Pissarides, 1985]

workers search for jobs and vacant firms search for workers. The model also introduces two sources of household heterogeneity: the initial educational level and the time preference (subjective discount factor). Households are grouped according to their educational levels (high school dropouts, high school graduates, people with some college degree, and college graduates). Households with higher educational levels are assumed to be more patient. In this paper tax reform is defined as a decrease in tax progressivity instituted under the TRA-86-the most notable act among all tax reforms in the 1980s which significantly decreased the US average marginal tax rate.

In the model above, tax reform influences labor supply response in two ways. First, a decrease in tax progressivity lowers the marginal income tax rate, which encourages households to work longer hours. It increases the relative cost of unemployment and hence encourages labor force participation. Second, a lower marginal income tax rate encourages households to increase savings, ceteris paribus. However, as people choose to save more the real interest rate in equilibrium decreases. For people with less education (or the impatient) the negative effect on savings due to a fall in the real interest rate dominates the positive effect on savings caused by a decrease in the marginal tax rate. On the contrary, savings of college graduates (or the patient) increase because the positive effect on savings dominates the negative effect on savings. As a result, the impatient save less after the tax reform while the patient save more.

The quantitative results are consistent with previous empirical findings. The results show that tax reform has a significant effect on the labor supply decisions of households along both the intensive margin and the extensive margin. At the aggregate level, a 10 percent decrease in tax progressivity increases aggregate labor supply by 7 percent and approximately 65 percent of the aggregate labor supply response is along the extensive margin. Because of the low elasticity of labor supply, a decrease in the marginal tax rate does not significantly increase the hours worked. On the other hand, a decrease in the tax rate makes home production undesirable, which increases labor force participation. The results also show that households' labor supply response to tax reform depends on their educational levels. After a decrease in tax progressivity, high school dropouts, high school graduates, and people with some degree work longer hours, have a higher participation rate, and search more intensively in the labor market while college graduates work less hours, have a lower participation rate, and search less in the labor market. Among all groups, the labor supply of high school dropouts is most sensitive to a decrease in tax progressivity along both the intensive margin and the extensive margin. This occurs because after the tax reform the marginal tax rate of the lower income groups decreases the most and because the impatient, i.e., households from lower educational groups, hold less capital. Therefore, labor supply of the impatient responds more significantly.

The quantitative results also show that after a decrease in tax progressivity, income inequality increases. As mentioned above, after the tax reform, earnings inequality decreases, which is the direct result of changes in labor supply. A decrease in earnings inequality and an increase in income inequality simultaneously is due to households' changes in income composition. After the tax reform, the impatient choose to save less while the patient hold more assets. Therefore, a decrease in tax progressivity increases the earnings of people with low education but decreases their income and income shares. A decrease in tax progressivity decreases the earnings of college graduates but increases their incomes. The share of aggregate taxes paid by college graduates (the upper-income group) increases after the tax reform, which is consistent with empirical findings by [Lindsey, 1987, Lindsey, 1988].

The results of the paper augment the findings of [Prescott, 2004] who ascribes the reason why Europeans work less than Americans to the higher marginal tax rates in Europe compared to America. However, Prescott's study focuses on examining the influence of different tax rates on hours worked per worker without taking into account households' labor force participation decisions. This paper asserts that higher marginal tax rates explain Europe's low labor force participation rate and the shorter worked hours.

The paper is related to two strands of literature on modeling discrete labor participation decisions. The first relies on the assumption of non-convexities created by work costs ([Cogan, 1981], [Heim and Meyer, 2004], and [Eissa et al., 2006]). In particular, fixed costs are needed to enter the labor market, which implies that individuals choose to be unemployed if the hours of work are less than some minimum level. The second focuses on tax policies in labor search models. [Shi and Wen, 1999] and [Domeij, 2005] study optimal taxation in search models while [Michaelis and Birk, 2006] examine the impact of tax reform on employment and economic growth in an endogenous model with labor market search. All of these models assume a representative agent and do not examine the impact of tax reform on the labor supply and income distribution. The use of search models with heterogeneous household to examine the labor supply response to tax reform has not been well developed. This paper fills the gap in the literature by proposing a theoretical model with heterogeneous households and labor market frictions that predicts labor supply response along the intensive margin and the extensive margin.

The rest of the paper is organized as follows. Chapter 2.2 presents the model. The calibration methods are explained in chapter 2.3. The results are presented in chapter 2.4. Chapter 2.5 concludes.

2.2 The Model

Time is discrete. The economy includes four types of agents: households, firms, financial intermediaries (or banks), and the government. There is one consumption/investment good in the economy, which is produced by capital and labor. The goods market and the capital market are competitive while the labor market is frictional as in [Diamond, 1982], [Mortensen, 1982] and [Pissarides, 1985]. Households are heterogeneous and each household contains a continuum of members-referred to as "large household"[3]. Members of any household are either employed, unemployed, or out of the labor force. The employed members are matched with firms while the unemployed members search for jobs. Each period, by paying a flow cost, firms create job vacancies to search for workers. Successful matches between the unemployed and vacant firms are determined by a matching function proposed by [Diamond, 1982] and [Blanchard et al., 1990]. A representative financial intermediary collects deposits from

[3]This assumption will be further explained in households' problem.

households, owns firms, and invests in the capital market. Finally, the government collects tax incomes to finance government spending and unemployment benefits paid to the unemployed household members.

2.2.1 Households

Consider an economy populated by M groups of households. Let $i \in \{1, 2, ..., M\}$ be the index of group i. In any group i, there is a continuum of identical households. A household from group i is referred to as a type-i household. Any type-i household is assumed to comprise a continuum of *ex ante* identical, infinitely lived members with measure one. Then, aggregate population can be normalized to one, with group i's population share given by $\lambda_i \in (0, 1)$. This implies $\sum_{i=1}^{M} \lambda_i = 1$.

Households across different groups are heterogeneous in two dimensions: the initial educational level and the subjective discount factor. According to households' initial educational level they are divided into four groups: high school dropouts, high school graduates, people with some college degree, and college graduates. In this paper, households with higher education are assumed to be more patient compared to households with less education, which is supported by empirical studies as in [Lawrance, 1991] and [Warner and Pleeter, 2001].

Each member from a type-i household faces displacement risks regarding her/his employment status in each period. Uncertain employment status adds to the uncertainty of income and leisure of each member. As widely applied in the literature, to eliminate the impact of the uncertainty of income and leisure, each household is assumed to be large. To be specific, each household is assumed to have a continuum of members; all members within the same household pool their resources (incomes and assets) together in every period; and all members only value the household's utility instead of their own. Therefore, a household provides perfect insurance in consumption, income and leisure for its members. All the decisions regarding consumption, saving, time allocations and so on are made at the household level in a deterministic fashion[4].

Employment Status

Consider an arbitrary household from group i. In each time period, a member from a type-i household is either employed (a type-i worker) or non-employed (a non-worker). Denote $e_{i,t}$ and $d_{i,t} = 1 - e_{i,t}$ as the number of employed members and the number of non-employed members in a type-i household respectively. Given that the population of a type-i household is normalized to one, the variable $e_{i,t}$ can also be interpreted as the employment-population rate. Each member of a household, regardless of its employment status, is endowed with one unit of time. Employed members allocate their time to work $l_{i,t}$ and leisure $(1 - l_{i,t})$, while non-employed members allocate their time to job searching $s_{i,t}$ and leisure $(1 - s_{i,t})$. The search intensity augmented unemployment in a type-i household-the number of unemployed members-is defined as $s_{i,t}d_{i,t}$. This description of household's employment status and time allocation is similar to [Andolfatto, 1996] and [Chen et al., 2011a, Chen et al., 2011b].

[4]This formulation can be found in [Andolfatto, 1996], [Shi and Wen, 1999], [Chen et al., 2011a] among others.

At the beginning of time $t \geq 0$, the number of employed members $e_{i,t}$ is predetermined in the previous time period. Among the currently employed a fraction $\phi_i \in (0, 1)$ will be displaced at the end of time t because of an exogenous job separation shock. Denote $\mu_t \in (0, 1)$ as the ratio of successful matches to search intensified unemployment in the labor market at time t. Therefore, the job finding rate of a type-i household is $s_{i,t}\mu_t$. The number of the non-employed of a type-i household who succeed in finding a new job is given by $s_{i,t}d_{i,t}\mu_t$. The ratio μ_t, which is endogenously determined in equilibrium, is taken as given when households make decisions. Therefore, the number of employed members of a type-i household at time $t + 1$ is given by

$$e_{i,t+1} = (1 - \phi_i)e_{i,t} + s_{i,t}d_{i,t}\mu_t. \tag{2.1}$$

In each time period t, group i's unemployment rate $u_{i,t}$ is defined as

$$u_{i,t} = s_{i,t}(1 - e_{i,t})/[e_{i,t} + (1 - e_{i,t})s_{i,t}].$$

where $e_{i,t} + (1 - e_{i,t})\, s_{i,t}$ are the number of members participating in the labor force in a type-i household (labor force participation of a type-i household).

Budget Constraint

In each period, a type-i household has three sources of income. The first source is the labor income $\eta_i w_{i,t} l_{i,t} e_{i,t}$ where $w_{i,t}$, η_i and $l_{i,t}$ denote the wage rate, the labor productivity, and hours worked of a type-i worker respectively. The productivity of the worker is predetermined by his/her initial educational level. The product $\eta_i l_{i,t}$ is the level of effective labor. The wage rate for raw labor $l_{i,t}$ is $\eta_i w_{i,t}$. The second source of income is unemployment benefits. The non-employed members get unemployment benefits from the government, which is a fraction ρ of their labor income[5]. The third source of income is interest from assets $r_t a_{i,t}$ where $a_{i,t}$ and r_t denote asset holdings by a type-i household and the real interest rate at time t respectively. Let $y_{i,t}$ denote the total income of a type-i household at time t. Then, $y_{i,t}$ is equal to

$$y_{i,t} \equiv \eta_i w_{i,t} l_{i,t}[e_{i,t} + \rho(1 - e_{i,t})] + r_t a_{i,t}.$$

Household incomes are subject to a progressive tax. Denote $\tau(\bar{y}_{i,t})$ as the tax rate of a type i household where $\bar{y}_{i,t}$ is household i's relative income, i.e., the ratio of household i's income with respect to aggregate income. In particular, the relative income $\bar{y}_{i,t}$ is equal to $y_{i,t}/y_t$ where $y_t = \sum_{i=1}^{M} \lambda_i y_{i,t}$ is the aggregate income. Therefore, the tax rate of a given household depends on its position in the income distribution. The total amount of income tax paid by a type-i household $T(y_{i,t})$ is equal to $\tau(\bar{y}_{i,t})y_{i,t}$. Following [Guo and Lansing, 1998] and [Li and Sarte, 2004][6], the tax rate is captured by the relative income [7] and further assumed to be of the form

$$\tau(\bar{y}_{i,t}) = \sigma(\frac{y_{i,t}}{y_t})^{\varrho}, \quad with \;\; 0 \leq \sigma < 1, \;\; \varrho > 0.$$

where σ and ϱ determine the level and the slope of the tax schedule, respectively.

[5]In this paper, I assume that all non-workers get compensations from the government. In fact, only a portion of non-workers are covered by unemployment benefits in U.S. See [Anderson and Meyer, 1997] and [Blank and Card, 1991].

[6]In a model without growth, [Guo and Lansing, 1998] use the same formulation as the tax code used here. They also mention that if there is growth in the model, the use of relative income guarantees that the tax rate $\tau(\bar{y}_i)$ is smaller than 1. In a model with growth, [Li and Sarte, 2004] use this formulation.

[7][Suits, 1977] and [Kakwani, 1977] also use income levels to capture households' tax burden.

Unlike a flat tax system-a system in which the average tax rate equals the marginal tax rate-a progressive tax schedule has a different average and marginal tax rates. The average tax rate (ATR) is $\tau(\bar{y}_{i,t})$ while the marginal tax rate (MTR) is given by

$$MTR = T'(y_{i,t}) = \frac{\partial[\tau(\bar{y}_{i,t})y_{i,t}]}{\partial y_{i,t}} = (1+\varrho)\tau(\bar{y}_{i,t}).$$

where y_t is taken as given when a type-i household calculates his/her marginal tax rate since it is assumed that households are small compare to the aggregate population. The ratio of the marginal tax rate to the average tax rate is

$$\frac{MTR}{ATR} = \frac{T'(y_{i,t})}{\tau(\bar{y}_{i,t})} = 1+\varrho.$$

Therefore, ϱ can be interpreted as the degree of tax progressivity [8]. When $\varrho = 0$, it degenerates to a flat tax case; when $\varrho > 0$, households with higher income are subject to higher tax rates.

Let $c_{i,t}$ be the consumption of a type-i household at time t. The budget constraint of a type-i household at time t is then given by

$$c_{i,t} + a_{i,t+1} = y_{i,t} - T(y_{i,t}) + a_{i,t}. \tag{2.2}$$

The Household's Problem

In each period, households derive utility from current consumption and leisure. The period utility of a type-i household is given by

$$U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) = R(c_{i,t}) + e_{i,t}H^1(1-l_{i,t}) + (1-e_{i,t})H^2(1-s_{i,t}).$$

[8] See [Li and Sarte, 2004] for more discussion.

Following [Andolfatto, 1996] and [Chen et al., 2011a, Chen et al., 2011b], functions R, H^1 and H^2 are increasing and concave with respect to their augment. The utility function shows that the household values leisure from the employed members and leisure from the non-employed members differently.

The life time utility of a type-i household is $\sum_{t=0}^{\infty} \beta_i^t U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t})$ where β_i is the subjective discount factor for a type-i household. Given variables describing the aggregate market $\{\mu_t, r_t, w_{i,t}\}_{t=0}^{\infty}$ and government tax policy, a type-i household's problem is to choose an allocation $\mathcal{A} \equiv \{c_{i,t}, l_{i,t}, s_{i,t}, a_{i,t+1}, e_{i,t+1}\}_{t=0}^{\infty}$ to solve the following problem

$$\max_{\mathcal{A}} \{\sum_{t=0}^{\infty} \beta_i^t U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t})\}.$$

subject to (2.1) - (2.2), $c_{i,t} \geq 0$, $l_{i,t} \in [0,1]$ and $s_{i,t} \in [0,1]$, and the no-Ponzi-game condition.

The first-order conditions of a type-i household's problem with respect to $\{c_{i,t}, l_{i,t}, s_{i,t}, a_{i,t+1}, e_{i,t+1}\}$ are as follows [9]

$$U_c(t) = \psi_{i,t}. \tag{2.3}$$

$$U_l(t) + \psi_{i,t}[1 - T'(y_{i,t})]\eta_i w_{i,t}[e_{i,t} + \rho(1 - e_{i,t})] = 0. \tag{2.4}$$

$$U_s(t) + \zeta_{i,t} d_{i,t} \mu_t = 0. \tag{2.5}$$

[9] The formal derivations are available in Appendix A.

$$\psi_{i,t} - \beta_i \psi_{i,t+1}\{[1 - T'(y_{i,t+1})]r_{t+1} + 1\} = 0. \tag{2.6}$$

$$\beta_i\{U_e(t+1) + \zeta_{i,t+1}(1 - \phi - s_{i,t+1}\mu_{t+1}) + \psi_{i,t+1}[1 - T'(y_{i,t+1})]\eta_i w_{i,t+1} l_{i,t+1}(1 - \rho)\} = \zeta_{i,t}. \tag{2.7}$$

where $\zeta_{i,t}$ and $\psi_{i,t}$ are the Lagrange multipliers with respect to (2.1) and (2.2) respectively. To save space, the partial derivative of the utility function is written as $U_{x_i}(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) = U_{x_i}(t)$ where $x_i \in \{c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}\}$.

Equation (2.3) is the necessary condition determining a household's decision on consumption. At the optimum, the left-hand side of (2.3) describing the marginal utility of consumption equals the right-hand side-the shadow price of the consumption good. Equation (2.4) is the first-order condition governing a household's choice over labor supply along the intensive margin. An additional unit of time worked decreases a household's utility by $U_l(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t})$ because of a decrease in time for leisure. However, an additional unit of time worked brings an extra amount of after-tax income $[1 - T'(y_{i,t})]\eta_i w_{i,t}[e_{i,t} + \rho(1 - e_{i,t})]$. In equilibrium, the decrease in utility due to a fall in leisure must be compensated by the benefit derived from the increase in after-tax income. (2.5) is the equation determining a household's search intensity. Searching for an extra unit of time would decrease the household's utility by $U_s(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t})$ because of a decrease in leisure for the non-employed. On the other hand, an extra unit of search effort increases the probability of a type-i worker finding a job by $d_{i,t}\mu_t$. The benefit from searching equals $\zeta_{i,t} d_{i,t}\mu_t$, where $\zeta_{i,t}$ measures the shadow price of being employed for a type-i worker. The costs of searching equal the benefit of searching at the optimum. Equation (2.6) is the inter-temporal Euler equation. If a type-i household postpones one unit of consumption today, which is invested into the asset market by the household, the marginal utility would decrease by $\psi_{i,t}$ today while the investment would bring an after-tax income $[1 - T'(y_{i,t+1})]r_{t+1} + 1$ next period. This further brings an extra amount of utility $\beta_i\psi_{i,t+1}\{[1 - T'(y_{i,t+1})]r_{t+1} + 1\}$ today. At the optimum, there is no difference between saving for tomorrow and consuming today for the household.

Equation (2.7) governs household i's optimal choices over employment. The right-hand side of equation (2.7) is the shadow price of being employed. The first term of the left-hand side measures how an extra unit of employment next period would affect current leisure: a higher employment next period requires an increase in search intensity in the current period leading to a decrease in leisure in the current period. Because of labor market frictions this cost is also adjusted by the job finding rate $s_{i,t}\mu_t$. The second term in the curly brackets measure how an extra amount of employment next period would affect leisure in the next period. Furthermore, an extra amount of employment next period also decreases search intensity in the next period and brings additional income (the third term in the curly brackets).

2.2.2 Firms

On the supply side of the economy, there is a continuum of firms. Each firm is small in the sense that a firm has only one job[10]. Hence firms and jobs can be

[10]The assumption of small firms has been widely used in the literature especially in search models with heterogeneous workers. See [Pries, 2008] and [Bils et al., 2012].

used interchangeably. Firms can enter or exit the goods market freely (without costs). However, after entry, firms have to pay an open vacancy cost κ to search for workers. Production happens only after a firm and a worker are matched.

At any point in time, there are two types of firms/jobs: firms with filled jobs, which mean a matched pair between a firm and a type-i worker, and firms with vacant jobs. Each period, a matched firm rents capital and produces output. After production, the matched pair separate at an exogenous probability ϕ_i. Once a firm separates with its worker, it can choose to exit the goods market or to search for another worker by posting a new vacancy. Therefore, firms with vacant jobs consist of the separated firms searching for workers and new entrant firms, which are essentially the same.

A firm matched with a type-i worker, which is referred as a type-i firm, has access to a constant-return-to-scale technology

$$Y_{i,t} \equiv f(k_{i,t}, l_{i,t}) = k_{i,t}^{\alpha}[\eta_i l_{i,t}]^{1-\alpha}, \quad \textit{with } \alpha \in (0,1) \textit{ and } i \in \{1, ..., M\}.$$

where $k_{i,t}$ and α are capital inputs rented by a type-i firm and the capital share in the production function. Denote δ as the depreciation rate of physical capital. Then, the profit of a type-i firm is given by $\pi_{i,t} = Y_{i,t} - (r_t + \delta)k_{i,t} - w_{i,t}\eta_i l_{i,t}$.

Denote the value function of a type-i firm and the value function of a vacancy by $J_{i,t}$ and V_t. Then $J_{i,t}$ is

$$J_{i,t} = \max_{k_{i,t}}\{\pi_{i,t} + \frac{1}{1+r_t}[(1-\phi_i)J_{i,t+1} + \phi_i V_{t+1}]\}, \quad i \in \{1, 2, ..., M\}.$$

The firm rents capital and employs a worker to maximize its lifetime value. The match survives with probability $1-\phi_i$, and separates with probability ϕ_i. The firm matched with a type-i worker chooses the amount of capital to rent. The labor supply $l_{i,t}$ and wage rate $w_{i,t}$ are determined by bargaining between the firm and the worker in a matched pair. The first-order condition for this problem is given by

$$r_t + \delta = f_1(k_{i,t}, \eta_i l_{i,t}). \tag{2.8}$$

for all $i \in \{1, 2, ..., M\}$.

A vacant job is filled by a worker according to a random matching process. If the vacancy is matched, the matched pair begins to produce output at time $t + 1$. Otherwise, the cost of posting the vacancy will be wasted. The value function of a vacancy is equal to

$$V_t = -\kappa + \frac{1}{1 + r_t}\{q_t \sum_{i=1}^{M} \widetilde{\lambda}_{i,t} J_{i,t+1} + (1 - q_t)V_{t+1}\}.$$

where $\widetilde{\lambda}_{i,t}$ is the ratio of the number of unemployed in group i with respect to the total unemployment, which is equal to $\lambda_i s_{i,t} d_{i,t} / \sum_{i=1}^{M} \lambda_i s_{i,t} d_{i,t}$. The nominator $\lambda_i s_{i,t} d_{i,t}$ is the unemployment in group i while the denominator $\sum_{i=1}^{M} \lambda_i s_{i,t} d_{i,t}$ is the total search intensity augmented unemployment. The free entry condition gives $V_t = 0$.

2.2.3 Labor Market Matching and Wage Determination

The labor market is frictional in the sense that it is time-consuming and costly for both the unemployed workers and firms to form a match in the labor market. Given households are heterogeneous, I follow [Pries, 2008] and [Ravenna and Walsh, 2011] to assume that the matching process operates in an aggregate way-the unemployed workers, regardless of their types, and vacant firms join into a pool where the unemployed search for jobs and the vacant firms search for workers. Denote the number of unemployed workers by d_t which is equal to the sum of unemployed workers in each group $\sum_{i=1}^{M} \lambda_i s_{i,t} d_{i,t}$. Let v_t be the total number of vacancies in the labor pool. Following [Diamond, 1982] and [Blanchard et al., 1990], the number of new matches between the unemployed and vacancies are determined by a constant-return-to-scale matching function

$$\mathcal{M}(d_t, v_t) = B d_t^{\xi} v_t^{1-\xi} \ . \tag{2.9}$$

where parameters B and $\xi \in (0, 1)$ represent the matching efficiency and the elasticity of the matching function, respectively. Denote $\theta_t = d_t/v_t$ as the job market tightness. The ratio of successful matches to search intensified unemployment μ_t is equal to $\mathcal{M}(d_t, v_t)/d_t = \mathcal{M}(\theta_t, 1)/\theta_t$ and the rate at which a vacancy is filled q_t is equal to $\mathcal{M}(d_t, v_t)/v_t = \mathcal{M}(\theta_t, 1)$.

As is standard in the labor search literature, wages are determined through a Nash bargaining process, which maximizes the product of a worker's surplus and a firm's surplus from matching. Denote the value function of a type-i household by $W(a_{i,t}, e_{i,t})$. A type-i household's surplus from a successful match is equal to the marginal benefit of providing an additional worker by the household, which is equal to $W_e(a_{i,t}, e_{i,t})$. Moreover, the surplus of a vacant job is equal to $J_{i,t} - V_t$. Then, wage $w_{i,t}$ is chosen to maximize the following equation

$$\max_{w_{i,t}} [\frac{W_e(a_{i,t}, e_{i,t})}{\psi_{i,t}}]^{\epsilon} [J_{i,t} - V_t]^{1-\epsilon},$$

where $\epsilon \in (0,1)$ denotes the bargaining power of workers and $W_e(a_{i,t}, e_{i,t})/\psi_{i,t}$ gives the capital value of supplying an additional worker by a type-i household.

Wages are determined by the following condition

$$\epsilon J_{i,t}\{[1 - T'(y_{i,t})] - \varrho T'(y_{i,t})\frac{\eta_i w_{i,t} l_{i,t} e_{i,t}}{y_{i,t}}\}(1-\rho) = (1-\epsilon)[\frac{W_e(a_{i,t}, e_{i,t})}{U_c(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t})}]. \quad (2.10)$$

The detailed derivation can be found in Appendix A.

2.2.4 Financial Intermediary

There is a representative, infinitely-lived, and risk-neutral financial intermediary in the economy, say a bank. Banks own physical capital and equities issued by firms. Let k_t be the quantity of physical capital owned by banks at the beginning of time t. Before production, banks rent out their physical capital k_t to firms, which brings a net return of $r_t k_t$. Equities are issued by vacant firms to finance the cost of opening a job vacancy κ. I assume only banks have access to the equity market. As a result, banks own all the equity shares issued by firms in equilibrium, i.e., banks own firms. Firms pay dividends to banks at the end of each period which is equal to all of their profits.

In each time period, banks receive capital income from capital $(1+r_t)k_t$, derive dividend income from equity shares $\sum_{i=1}^{M} \lambda_i e_{i,t} \pi_{i,t}$ and solicit deposits from households a_{t+1}. Banks also need to pay interests and principal to deposits from last period $(1 + r_t)a_t$, buy equity from new entrant firms $v_t \kappa_t$, and determine how much to invest in capital next period k_{t+1}. Free entry to the financial sector implies that banks make zero profit each period. As a result, the resources constraint for the entire financial sector is

$$k_{t+1} + v_t \kappa_t + (1 + r_t)a_t = (1 + r_t)k_t + \sum_{i=1}^{M} \lambda_i e_{i,t} \pi_{i,t} + a_{t+1}. \tag{2.11}$$

where $k_t = \sum_{i=1}^{M} \lambda_i e_{i,t} k_{i,t}$ and $a_t = \sum_{i=1}^{M} \lambda_i a_{i,t}$.

2.2.5 The Government

The government collects taxes to finance the government spending and unemployment benefits paid to the non-employed. Each period the government keeps a balanced budget.

$$\sum_{i=1}^{M} \lambda_i T(y_{i,t}) = G_t + \sum_{i=1}^{M} \lambda_i \rho \eta_i w_{i,t} l_{i,t} (1 - e_{i,t}). \tag{2.12}$$

Based on formulations above, the equilibrium of the model is defined as:

Definition. *Given the tax rule, a general equilibrium in this model economy is a sequences of individual variables* $\{c_{i,t}, l_{i,t}, s_{i,t}, a_{i,t+1}, e_{i,t+1}\}_{t=0}^{\infty}$*, a pair of prices* $\{w_{i,t}, r_t\}_{t=0}^{\infty}$ *for all* $i \in \{1, .., M\}$*, a pair of matching rate* $\{q(\theta_t), \mu(\theta_t)\}_{t=0}^{\infty}$ *such that:*

1. all households and firms optimize, i.e., (2.3)-(2.7) and (2.8), (2.10) are met;

2. employment evolves according to (2.8);

3. labor market matching condition (2.9) is met;

4. the government budget is balanced, i.e., (2.12) is met;

5. the financial market clears, i.e., (2.11) is met; the goods market clears, i.e.,

$$c_t + G_t + v_t \kappa + k_{t+1} = Y_t + (1 - \delta)k_t \tag{2.13}$$

where $c_t = \sum_{i=1}^{M} \lambda_{i,t} c_{i,t}$ *and* $Yt = \sum_{i=1}^{M} \lambda_{i,t} e_{i,t} Y_{i,t}$.

2.3 Calibration

This section proceeds to calibrate the baseline model presented above. One period in the model is a quarter. Based on the data available, I partition households into four groups by educational attainment: high school dropouts, high school graduates, people with some college and college graduates. High school dropouts include the households who have not completed high school. High school graduates imply the households who have obtained a high school degree but have not started college. People with some college refer to the households who have started college but have not obtained a college degree. College graduates are the households who have obtained at least a college degree. Labor productivity of each group is determined by its initial educational level and does not change over time.

Parameter values are calibrated to match the unemployment rate and employment-population ratio from the Current Population Survey (CPS) 2010[11] and the income share of each group derived from the Survey of Consumer Finance (SCF) 2010[12]. The reason for using both of these two separated data sets is because of the data availability. The CPS 2010 has detailed information about households' unemployment rate, mean average earnings, and employment-population ratio, while the SCF 2010 includes detailed information about households' income.

[11] Statistics from CPS 2010 are collected for the civilian non-institutional population who are 25 years and over by educational attainment. Similar to [Cajner and Cairo, 2011], there are two reasons for using the data of households who are older than 25 years. First, since most households have finished their studies by the age of 25 I do not need to consider the possibility that young people are unemployed because they choose to go to school. Second, empirical studies show that there are higher unemployment rates for young people of all education groups. Hence, I avoid this possibility by studying households who are older than 25 years.

[12] I use 2010 data because it is the most recent data available for my objective. By using data of different years to calirate the model, I obtained similar results

[Please insert Table 2.1 here]

Table 2.1 summarizes some key statistics from the CPS 2010 and the SCF 2010. Data from CPS is collected monthly. Hence, I transform monthly data to quarterly data by taking the sample average at the quarterly interval. The first four columns present data from CPS 2010, which are unemployment rate, employment population ratio, population share and quarterly wage income respectively. The unemployment rates of each group are 14.86 percent, 10.31 percent, 9.23 percent and 5.24 percent, respectively. The employment population ratios of each group are 39.41 percent, 55.28 percent, 61.91 percent and 72.26 percent respectively. The quarterly wage incomes of each group are $5,550$, $7,830$, $9,180$ and $18,000$ dollars respectively. Generally, people with higher education experience lower unemployment rate, participate more in the labor market, and earns higher wages, which suggests that education plays an important role in determining household behavior in the labor market. The last column reports the mean annual income of each educational group, which is from SCF 2010. The income gap is large. Specifically, college graduates earn about three times more than high school dropouts. Although data from SCF is annually collected, I only match the share of income received by each group in the income distribution. Then, it doesn't matter whether it is of annual frequency or quarterly frequency.

Table 2.2 summarizes parameter values for the baseline model. First, parameters of the tax schedule are calculated by using the NBER TAXSIM model proposed by [Feenberg and Poterba, 1993]. According to the NBER TAXSIM data 2010, the average marginal tax rate was 21.60 percent and the average tax rate was 10.60 percent, which imply a progressivity ratio of 2.0377, i.e., $\varrho = 1.0377$. To match the average tax rate 10.60 percent , the level parameter of the tax schedule, σ, is required to equal 0.0743. Next, I calibrate parameters of the production function for the goods. Following [Andolfatto, 1996] and [Chen et al., 2011a, Chen et al., 2011b], the share of capital income to aggregate output is 0.3600, i.e., $\alpha = 0.3600$. Given that the annual capital-output ratio is equal to 3 and the quarterly capital depreciation rate δ is set to 0.0150, the quarterly real interest rate r is equal to 0.0150, which implies a yearly real interest rate of 6 percent. A quarterly depreciation rate of 0.0150 is within the range of empirical estimation.

[Please insert Table 2.2 here]

I proceed to calibrate parameters of the labor market. Following [Shimer, 2005], the monthly average job finding rate is 0.4500, and the elasticity of matching ξ is reported to equal 0.7200. Hence the quarterly value of μ is equal to 0.8340. In line with most literature of search and matching in labor market, the bargaining power for worker ϵ is set to equal to the elasticity of the matching function, i.e, $\epsilon = 0.7200$, which makes the [Hosios, 1990] condition holds. Also following [Shimer, 2005], I normalize the market tightness θ to 1 in the steady state, which implies that the constant B in front of the matching function will be equal to 0.8340. I set the unemployment benefit ρ equal to 0.2000 which is equal to the value in [Bils et al., 2012]. The unemployment benefit used in this model is low compared to the one in [Shimer, 2005] which is equal to 0.4000. As in [Bils et al., 2012], while the high unemployment benefit in [Shimer, 2005] reflects any benefits from unemployment, such as increased leisure or home production, the unemployment benefit here does not include these gains since leisure is explicitly valued in my model. In addition to their argument, there is another reason for my model. All the unemployed workers are assumed to be eligible for unemployment benefits in this model. However, according to the estimation by [Anderson and Meyer, 1997], [Blank and Card, 1991] and [Regev, 2012], the fraction of unemployed who are eligible for unemployment benefits is 0.4300. Therefore, the unemployment benefit on average should be lower than the value of [Shimer, 2005]. The job separation rate is determined as follows. Given that unemployment rate is defined by $u_i = s_i(1-e_i)/[e_i + (1-e_i)s_i]$ and the law of motion for employment in the steady state is $\phi_i e_i = \mu(1-e_i)s_i$, the job separation rate ϕ_i is equal to $\mu u_i/(1-u_i)$. The unemployment rate u_i and the value of μ are observable from the data. Thus, the job separation rate ϕ_i can be obtained. With the value of ϕ_i the value of s_i can be obtained even though it is not directly observable.

This paragraph calibrates parameters of the household's preferences. As in [Li and Sarte, 2004], given parameters for the tax schedule, income distribution and equilibrium interest rate, households' subjective discount factor β_i can be derived from the Euler equation $1 = \beta_i\{[1-(1+\varrho)\sigma(\bar{y}_i)^{\varrho}]r + 1\}$ where the income of group i is $\bar{y}_i = y_i/(\Sigma_{i=1}^{M}\lambda_i y_i)$ which is calculated by the data from SCF 2010. Following [Andolfatto, 1996] and [Chen et al., 2011a, Chen et al., 2011b] the period utility function is of the form

$$U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) = \ln c_{i,t} + e_{i,t}\frac{\varpi_1(1-l_{i,t})^{1-\omega}}{1-\omega} + (1-e_{i,t})\frac{\varpi_2(1-s_{i,t})^{1-\omega}}{1-\omega}.$$

where the parameters ϖ_1 and ϖ_2 are different. Thus, the household values the leisure from the employed members and the leisure from the non-employed members differently.

As in [Andolfatto, 1996], the individual labor supply elasticity is equal to $(1/\bar{l} - 1)/\omega$ where $\bar{l}$ is average hours worked. In line with empirical studies, on average, workers spend 30 percent of their time working and the non-employed spend 14.08 percent of their time searching. I set the parameter ω equal to 3.5000 which implies that the labor supply elasticity is 0.6667. The elasticity is within the ranges of [MaCurdy, 1980] and [Greenwood et al., 1988]. [MaCurdy, 1980] estimates the labor supply elasticity for males ranging from 0.1000 to almost 0.5000 while [Greenwood et al., 1988] argue that the labor supply elasticity for women is equal to 1.7000. Given that the average time for working and average time for searching for jobs, parameter values of ϖ_1 and ϖ_2 can be calculated, which are equal to 1.0912 and 1.3185 respectively.

Based on variable values observed in the data and parameter values derived from the above calibration, the labor supply equation gives the earning-income ratio of each group. In addition, according to the wage determination equation, wage w_i can be calculated. If I normalize $\eta_1 = 1$, earnings of group 1 can be obtained. Using the earning-income ratio just derived, the value of y_1 can be obtained. Given the income distribution, each group's relative income with respect to group 1 can be derived. As a result, the value of labor productivity η_i is obtained. Finally, the free entry condition for firms determines the value of the opening cost for a vacancy κ, which is equal to 1.7325.

2.4 Results

This section reports the quantitative results of the baseline model and the tax reform. The first subsection presents the baseline model results where the tax progressivity ϱ is equal to 1.0377. The second subsection shows the results after tax reform where the tax progressivity ϱ' is equal to 0.9339. In subsection three, I discuss and examine the impact of tax reform in alternative versions of the baseline model.

2.4.1 The Baseline Model

Table 2.4 reports the labor market performance across educational groups. Households' labor choices are characterized by hours worked, search intensity, unemployment rate, and employment-population rate. The baseline model results, which are reported in the first column of Table 2.4, match the data well. While average hours worked and average search intensity in the CPS 2010 are 0.3000 and 0.1408 respectively, average hours worked and the weighted average of search intensity[13] reported by the model are 0.2987 and 0.1408 respectively. The baseline model also derives a negative correlation between unemployment rate and education and a positive correlation between employment-population rate and education, which are consistent with the data from the U.S. Bureau of Labor Statistics Current Population Survey (CPS) 2010. The model shows that the unemployment rate varies from 0.1487 for high school dropouts to 0.0525 for college graduates while employment-population rate varies from 0.3850 for high school dropouts to 0.7193 for college graduates. In the base line model,

[13]The weighted average hours worked are given by $\sum e_{i,t}\lambda_{i,t}l_{i,t}/\sum e_{i,t}\lambda_{i,t}$.

households with less education participate less in the labor market compared to college graduates, i.e., they have lower employment-population rate, since their job separation rate is higher. If they participate more, they have to search more intensively in the labor market which is not optimal for them. Workers with less education work longer hours compared to workers with higher education. This is because households with less education are subject to lower income tax rate.

[Please insert Table 2.4 here]

The second column of Table 2.5 summarizes household earnings, before-tax income and income shares under the baseline parameter values. The results show that earnings, before-tax income and income shares increase with education and that there is a significant difference in earnings and income across educational groups. The baseline model predicts that the earnings of college graduates are 3.5 times the earnings of high school dropouts while data from the CPS (2010) reports a similar ratio (3.2) between the earnings of college graduates and the earnings of high school dropouts. Similarly, the income of college graduates is as 3.8 times the income of high school dropouts in the baseline model which is consistent with the ratio from CPS (2010). The predicted distributions of earnings and income in the baseline model are very close to the distributions of earnings and income from the CPS (2010). Table 2.6 confirms that the baseline model performs well to match the observed aggregate data.

[Please insert Table 2.5 here]

2.4.2 Effects of Tax Reform

In this paper, tax reform implies a 10 percent decrease in tax progressivity. This change in the marginal tax rate is equal to the change stipulated by TRA-86. Given the baseline tax progressivity $\varrho = 1.0377$, the corresponding tax progressivity under tax reform ϱ' equals 0.9339.

[Please insert Table 2.3 here]

Table 2.3 reports the average tax rates and the marginal tax rates before and after the tax reform. A 10 percent decrease in tax progressivity lowers the marginal tax rates of all household groups. It also decreases the average tax rates of all groups expect that of college graduates. The percentage change in the marginal tax rates and the average tax rates decrease are negatively related with households' educational level. After a 10 percent decrease in tax progressivity the marginal tax rate of high school dropouts decreases by 18.22 percent while that of college graduates decreases only by 3.5 percent. On the other hand, the average tax rate of high school dropouts decreases by 13.74 percent while that of college graduates increases by 1.67 percent.

Labor Supply Response

Labor force participation is defined as the ratio of the labor force and aggregate population. Labor force is defined as the sum of employed and non-employed workers searching for a job. However, in this paper labor force participation is measured by the employment-population ratio. This measure is used because: search intensity is not directly observable in the market while employment-population rate is directly observable; the employment-population ratio and labor force participation rate are highly (positively) correlated; more important is that both measures lead to similar results about labor supply response to tax reform.

The necessary conditions of the household problem show that a decrease in tax progressivity affects household choices in three ways: for employed workers a decrease in tax progressivity decreases the marginal tax rate, which in turn changes the relative price between work and leisure-a change in labor supply along the intensive margin; a decrease in tax progressivity changes the relative cost of being unemployed or employed-a change in labor supply along the extensive margin; and a low tax progressivity decreases the real interest rate and hence affects households' saving behavior. As a result, a decrease in tax progressivity changes the income distribution across different education groups.

The results reported in the second column of Table 2.4 show household choices in the labor market at the steady state after the tax reform. For high school dropouts, high school graduates and people with some college degree, a decrease in tax progressivity increases hours worked by 6.97, 6.58 and 4.65 percent respectively. The increase in hours of work for these three groups is due to the decrease in the marginal tax rate, which raises the benefit of working. Furthermore, the employment-population ratios for these three groups increase by 34.91, 9.70 and 3.59 percent respectively. The calibrated results show that search intensity increases by 71.72, 25.69 and 11.73 percent for high school dropouts, high school graduates and people with some college degree respectively. After an increase in the employment-population ratios, the law of motion for employment implies that households have to increase their search efforts in the job market to keep a high steady state participation rate for a given ratio of μ_t. As more people search for jobs, unemployment rates across these three groups increases by 2.69, 2.91 and 2.92 percent respectively.

On the other hand, for college graduates, a decrease in tax progressivity increases the average tax rate but decreases the marginal tax rate. The calibrated results show that for college graduates hours worked, employment-population rate and search intensity decease 3.86, 5.07 and 14.63 percent respectively while unemployment rate increases 3.05 percent. Holding all other variables constant, college graduates should choose to work longer hours and have a higher participation rate after the tax reform due to the substitute effect. However, the college graduates (or the patient) choose to save more due to a decrease in the marginal tax rate which leads a higher income after the tax reform. The quantitative results show that the income effect dominates the substitute effect. Therefore, the college graduates work shorter hours and participate less in the labor market.

The quantitative results are consistent with the empirical findings. The labor supply response to tax reform happens along both the intensive margin and the extensive margin. A comparison between the percentage change in hours worked and labor force participation shows that for high school dropouts, high school graduates and college graduates, the labor supply response to tax reform tends to be concentrated along the extensive margin. [Eissa and Liebman, 1996], [Meyer and Rosenbaum, 2001] and

[Blundell, 2001] report similar results. Furthermore, the labor supply response to tax reform differs across household education groups.

Earnings and Income Responses

The third column of Table 2.5 shows the distribution of earnings, income and income shares across educational groups after the tax reform. The quantitative results demonstrate that earnings of high school dropouts, high school graduates and people with some college degree increase by 29.92, 14.49 and 8.10 percent respectively while earnings of college graduates decrease by 6.67 percent. On the other hand, the before-tax income of household moves in the opposite direction. The incomes of high school dropouts, high school graduates and people with some college degree decrease by 21.34, 11.68 and 6.91 percent respectively while the income of college graduates increases by 9.00 percent. Moreover, the income shares of high school dropouts, high school graduates and people with some college degree decrease by 22.51, 12.99 and 8.31 percent respectively while the income of college graduates increases by 7.37 percent.

Changes in earnings across households result from the response of household labor supply to tax reform as explained in the previous sub-section. However, the change in the distribution of income needs further explanation. As described earlier, a decrease in tax progressivity increases the hours worked and the job market participation of high school dropouts, high school graduates and people with some college degree but has an opposite impact on college graduates. A decrease in tax progressivity also raises the return from saving since a lower marginal tax rate encourages investment in capital.

If the interest rate remains unchanged, households will save more after a decrease in tax progressivity. However, the interest rate falls as savings increase. The quantitative results show that after the tax reform the real interest rate in equilibrium falls from 0.0150 to 0.0148, which discourages the impatient (high school dropouts, high school graduates and people with some college degree) from saving. On the other hand, savings of the patient (college graduates) rise even after a decrease in real interest rate. This is because a decrease in tax progressivity encourages college graduates to save. As a result, the income composition of college graduates changes after the tax reform. Instead of working they choose to save more. Therefore, asset holdings shift towards college graduates. The model predication that high-income group's income responds more to the tax reform is consistent with empirical studies by [Lindsey, 1987, Lindsey, 1988] and [Feldstein, 1995]. [Lindsey, 1987, Lindsey, 1988] finds that high-income taxpayers had significantly more income after the implementation of ERTA. [Feldstein, 1995] finds that after the TRA-86 the taxable income of upper income taxpayers increases substantially.

Aggregate Effects

The first three columns of Table 2.6 report the aggregate variables of the CPS 2010 data, the baseline model and the tax reform model. The first row shows that after the tax reform, real interest rate decreases from 0.0150 to 0.0148. The second row indicates that the job finding rate decreases from 0.8330 to 0.8070 as more people look for jobs. The third row shows that after the tax reform, aggregate hours worked increase by 7 percent and approximately 65 percent of this increase is due to an increase in the labor force participation. Hence, aggregate output (the fourth row) increases from 1.5739 to 1.5622. As a result, average productivity, reported in row 5, decreases from 8.5014 to 8.0277 in the model. This is because after tax reform, households with higher education-higher productivity-choose to work shorter hours and participate less actively, while households with lower education choose to worker longer hours and participate more actively.

[Please insert Table 2.6 here]

The last two rows show that inequality in earnings decreases as the Gini coefficient of earnings falls from 0.2320 to 0.1780 while inequality in income increases as the Gini coefficient rises from 0.2630 to 0.3120[14], which is equal to a 19 percent increase. [Li and Sarte, 2004] find that the progressivity change associated with TRA-86 results a 20 to 24 percent increase in the Gini Coefficient of income. The prediction that income inequality rises after the tax reform is also consistent with the empirical findings by [Altig and Carlstrom, 1999], [Feenberg and Poterba, 1993], and [Feldstein, 1995]. As more income shits toward higher income group, the share of aggregate taxes paid by college graduates (the upper-income group) increases after the tax reform, which is consistent with empirical findings by [Lindsey, 1987, Lindsey, 1988].

The quantitative results also show that a decrease in tax progressivity increases aggregate hours and labor force participation. These results augment the findings of [Prescott, 2004] who ascribes the reason why Europeans work less than Americans to

[14]The Gini coefficients for both earnings and income are low compared to the empirical results because I focus on the impact of tax reform on inequalities across different educational groups and I exclude inequalities within each educational group.

the higher marginal tax rates in Europe compared to America. Prescott uses a representative agent model without considering decisions on labor force participation to examine how different marginal tax rates affect hours worked. In this paper, both household heterogeneity and household choices about labor force participation are introduced. This paper goes further than Prescott (2004) to show that higher marginal tax rates explain Europe's low labor force participation rate in addition to the shorter hours worked in Europe compared to the U.S.

2.4.3 Discussion

In order to check the robustness of the results I examine the impact of tax reform on labor supply using two alternative models. In the first model (Model A), I assume households can only response along the intensive margin after a tax reform. In the second model (Model B), I assume that hours worked per week are fixed for each worker so that households respond to tax reform by changing their participation.

Model A

To isolate the impact of tax reform on labor supply only along the intensive margin I assume that households cannot change their participation. The model is calibrated to match the labor force participation rate and the average hours worked, which is equal to 0.3. Since there is no participation decision, the values of the coefficient of leisure from non-employed workers ϖ_2 and the cost of creating a vacancy κ are equal to 0. On the other hand, the value of the coefficient of leisure from employed workers ϖ_1

is equal to 1.2265. The values of individual productivity are 1.0000, 1.2688, 1.4341, and 3.0484, respectively.

The impact of tax reform on labor supply in this model is reported in the third and fourth columns of Table 2.4. After tax reform, hours worked for high school dropouts, high school graduates, and people with some college degree increase by 12.07 percent, 8.28 percent, 5.89percent respectively, while hours worked by college graduates decrease by 4.38 percent. These results imply that there is a decrease in earnings inequality. Compared to the baseline model, the labor supply response along the intensive margin in Model A is larger because in Model A households can only respond to changes in their hours worked after a tax reform. The fourth and fifth columns of Table 2.5 show the results of earnings, income, and income shares across education groups. In a model without participation decision, a decrease in earnings inequality is caused by variation in the number of hours worked across education groups-people with less education work more while people with more education work less. The rising gap in income is due to the increase in capital income inequality which is a consequence of a decrease in the interest rate. This makes the impatient save less and the patient save more. Finally, I observe lower aggregate hours and aggregate output in Model A compared to the baseline model. Since in Model A there are no labor market frictions, workers are paid at the competitive wage rate, which is higher than the Nash bargaining wage. After the tax reform, average productivity decreases as in the baseline model.

Model B

To examine the impact of tax reform on labor supply only along the extensive margin, I keep the hours worked per week constant. The calibration of Model B is the same as in the baseline model.

The results of the impact of tax reform on labor supply in Model B are reported in the fifth and sixth columns of Table 2.4. After the tax reform, participation rates for high school dropouts, high school graduates, and people with some college degree increase by 37.30 percent, 11.07 percent, 4.42 percent, while the participation rate of college graduates decreases by 4.52 percent. These results imply that there is a decrease in earnings inequality. The impact of the tax reform along the extensive margin is larger in Model B compared to the baseline model since households in Model B can change only their participation decision. The sixth and seventh columns of Table 2.5 show the earnings, income, and income shares across education groups. In a model with fixed hours worked per week, a decrease in earnings inequality is caused by changes in participation across education groups-people with less education participate more while college graduates participate less. As in the baseline model and Model B, the rising gap in income is due to the increase in capital income inequality which is a consequence of a decrease in the interest rate. After the tax reform, I also observe a decrease in average labor productivity.

2.5 Conclusion

Recent empirical studies find that labor supply response to tax reform are concentrated along the extensive margin rather than the intensive margin and the labor supply response and income response to tax reform differ across household groups. These findings emphasize the importance of labor market frictions and household heterogeneity in determining household labor supply response to tax reform. However, a model of labor supply that incorporates labor market frictions and household heterogeneity has not been fully developed. In this paper I extend the standard deterministic neoclassical model to include household heterogeneity in education, heterogeneity in subjective discount factor, labor market frictions, and progressive taxation. I also simulate the impact of changes in tax progressivity on labor supply and income distribution. The model performs well to explain the labor supply response to tax reform along the extensive margin and the intensive margin.

The quantitative results of the baseline model closely fit the data from the CPS 2010 and the SCF 2010. I also find that a decrease in tax progressivity increases both the hours worked and the labor force participation. The impact of tax progressivity on the hours worked and labor force participation is strongest for high school dropouts. The results also show that the response of labor supply to tax reform is concentrated more along the extensive margin than the intensive margin as labor force participation explains approximately 65 percent of changes in aggregate hours. Moreover, a decrease in tax progressivity decreases earnings inequality and increases income inequality, unemployment rate and aggregate output. This paper goes further than [Prescott, 2004], which specifies a relatively higher marginal tax rate in Europe as the primary reason that Europeans work less than Americans, to show that higher marginal tax rates explain Europe's low labor force participation rate compared to the U.S..

In future studies the model can be extended to include heterogeneity within educational groups and to include endogenous human capital decisions. In this paper I assume heterogeneity in educational levels and subjective discount factors only. I further assume that people with higher education have more patience compared to people with a lower education. However, the level of patience might be different for people within the same educational group. Accounting for this heterogeneity within educational groups might improve the performance of the model in predicting the observed Gini coefficients of income and earnings. Moreover, labor productivity in the model is predetermined by a household's initial education level. Since labor productivity improves with on-the-job learning, human capital decisions should be modeled as endogenous variables. This would provide insights into the catch-up effect of income and earnings across households.

Table 2.1: Earnings and Unemployment Rates by Education(Quarterly)

Education	Unemployment Rate	Employment Population Ratio	Population Share	Wage (10^3)	Income (10^3)
Dropout	0.1486	0.3941	0.1284	5.55	8.43
High school	0.1031	0.5528	0.3104	7.83	12.03
Some college	0.0923	0.6191	0.1684	9.18	14.68
College	0.0524	0.7226	0.3928	18.00	32.23

Data Source: U.S. Bureau of Labor Statistics, Current Population Survey 2010; Survey of Consumer Finance 2010.
Note: Employment status of the civilian noninstitutional population 25 years and over.

Table 2.2: Baseline Parameterization

Definition	Symbol	Value
Tax Code		
Level Parameter	σ	0.0743
Degree of Progressivity	ϱ	1.0377
Production Function(consumption Good)		
Capital Share	α	0.3600
Capital Depreciation Rate	δ	0.0150
Matching		
Coefficient	B	0.8340
Labor Searcher's Share	ξ	0.7200
Bargaining Power	ϵ	0.7200
Unemployment Insurance	ρ	0.2000
Preference		
Discount Factor	β	[0.9862, 0.9867, 0.9870, 0.9893]
Elasticity for Leisure	ω	3.5000
Coefficient for Leisure(Employed)	ϖ_1	1.0912
Coefficient for Leisure(Non-employed)	ϖ_2	1.3185
Job Separation Rate	ϕ	[0.1455, 0.0959, 0.0848, 0.0462]
Individual Productivity	η	[1.0000, 1.3084, 1.6702, 3.1597]
Costs for Opening a Vacancy	κ	1.7325

Table 2.3: Average and Marginal Tax Before-After Tax Reform

	Dropouts	High School	Some College	College
Baseline Model $\varrho = 1.0377$				
Average Tax Rate	0.0342	0.0495	0.0608	0.1376
Marginal Tax Rate	0.0697	0.1008	0.1240	0.2804
Tax Reform $\varrho' = 0.9339$				
Average Tax Rate	0.0295	0.0458	0.0579	0.1399
Marginal Tax Rate	0.0570	0.0886	0.1120	0.2706

Table 2.4: Labor Market Performance across Education Groups

	Baseline		Model A		Model B	
	B-R	A-R	B-R	A-R	B-R	A-R
Hours Worked						
Dropout	0.3661	0.3916	0.3638	0.4077	0.3661	0.3661
High School	0.3250	0.3464	0.3178	0.3441	0.3250	0.3250
Some College	0.3245	0.3396	0.2937	0.3110	0.3245	0.3245
College	0.2616	0.2515	0.2559	0.2447	0.2616	0.2616
Search Intensity						
Dropout	0.1135	0.1949	-	-	0.1080	0.1989
High School	0.1421	0.1786	-	-	0.1377	0.1820
Some College	0.1654	0.1848	-	-	0.1615	0.1872
College	0.1442	0.1231	-	-	0.1407	0.1198
Unemployment Rate						
Dropout	0.1487	0.1527	-	-	0.1487	0.1528
High School	0.1032	0.1062	-	-	0.1032	0.1062
Some College	0.0924	0.0951	-	-	0.0924	0.0951
College	0.0525	0.0541	-	-	0.0526	0.0541
Employment/Population						
Dropout	0.3850	0.5194	0.3850	0.3850	0.3820	0.5245
High School	0.5474	0.6005	0.5474	0.5474	0.5447	0.6050
Some College	0.6152	0.6373	0.6152	0.6152	0.6132	0.6403
College	0.7193	0.6828	0.7193	0.7193	0.7174	0.6768

Note: B-R and A-R represent before tax reform and after tax reform respectively.

Table 2.5: Earnings, Income and Asset Holdings across Education Groups

	Data	Baseline		Model A		Model B	
		B-R	A-R	B-R	A-R	B-R	A-R
Earnings							
Dropout	0.4175	0.4175	0.5424	0.4740	0.5333	0.4153	0.5045
High School	0.5886	0.6143	0.7033	0.6612	0.7187	0.6146	0.6596
Some College	0.6902	0.8515	0.9205	0.7500	0.7976	0.8535	0.8809
College	1.3541	1.4427	1.3464	1.5608	1.4985	1.4495	1.4076
Income[a]							
Dropout	0.5301	0.5301	0.4170	0.5334	0.4146	0.5318	0.4167
High School	0.7566	0.7567	0.6683	0.7613	0.6663	0.7591	0.6678
Some College	0.9233	0.9234	0.8596	0.9290	0.8587	0.9264	0.8589
College	2.0276	2.0278	2.2103	2.0400	2.2156	2.0342	2.2085
Income Share							
Dropout	0.0542	0.0542	0.0420	0.0542	0.0420	0.0542	0.0420
High School	0.1871	0.1871	0.1628	0.1871	0.1628	0.1871	0.1628
Some College	0.1239	0.1239	0.1136	0.1239	0.1136	0.1239	0.1136
College	0.6347	0.6347	0.6815	0.6347	0.6815	0.6347	0.6815

Note: a. Given the parameters calibrated in the baseline model, the income of high school dropouts is equal to 0.5301. I normalize the income of the high school dropouts in the data to 0.5301.

Table 2.6: Aggregate Variables

	data	Baseline		Model A		Model B	
		B-R	A-R	B-R	A-R	B-R	A-R
Interest Rate	-	0.0150	0.0148	0.0150	0.0148	0.0150	0.0148
Job Finding Rate	0.8340	0.8330	0.8070	-	-	0.8330	0.8070
Aggregate Hours	0.1809	0.1809	0.1946	0.1762	0.1815	0.1809	0.1902
Output	1.5379	1.5379	1.5622	1.4282	1.4369	1.5379	1.5543
Productivity	8.5014	8.5014	8.0277	8.1056	7.9168	8.5014	8.1719
Gini Coefficient							
Earnings	0.2290	0.2320	0.1780	0.2348	0.2043	0.2324	0.2034
Income	0.2630	0.2630	0.3120	0.2630	0.3120	0.2630	0.3120

Chapter 3

An Income Fluctuation Problem with Goods Market Frictions

3.1 Introduction

The optimal intertemporal decisions of an individual facing income uncertainty and market incompleteness, also known as the "income fluctuation problem", has been widely studied in a large body of literature. The only friction in this framework arises from the assumption of market incompleteness-individuals are unable to borrow completely against their expected future income. To smooth consumption over time, individuals choose to accumulate risk-free assets to self-insure against "bad luck", i.e., savings are out of precautionary motive. But this framework makes it difficult to study savings out of transaction motive. In order to fill this gap in the literature, this paper includes search frictions in a special good market in the income fluctuation problem to study the intertemporal optimization problem of an individual.

Early studies on the income fluctuation problem include [Schechtman, 1976], [Schechtman and Escudero, 1977], and [Bewley, 1977], [Foley and Hellwig, 1975], and [Clarida, 1987]. When no borrowing is allowed, the income follows an i.i.d. process, the interest rate is zero, and the discount factor is equal to one, [Schechtman, 1976] proves that to smooth consumption, an infinitely lived individual will choose to accumulate an infinite amount of a risk-free asset and the consumption converges almost surely to the permanent income. [Bewley, 1977] derives similar results under a stationary income process with other conditions unchanged. By relaxing assumptions in [Schechtman, 1976] to allow a positive discounting rate, a positive interest rate, a bounded elasticity of marginal utility, and that the time preference implied by the discount factor is larger than the gross return of assets, [Schechtman and Escudero, 1977] argue that individuals' asset holdings are bounded and thus a limiting distribution of asset holding exists. Using the model of [Schechtman, 1976], [Clarida, 1987] proves the existence of stationary consumption distribution if borrowing is allowed. [Chamberlain and Wilson, 2000] study this problem when the interest rate and income are stochastic processes. In a general equilibrium framework, [Huggett, 1993] studies the income fluctuation problem in an exchange economy while [Aiyagari, 1994] uses a production economy.

While the income fluctuation problem has been well studied, few studies examine individuals' optimal intertemproal decisions when the goods market is frictional. This is important at least for three reasons. First of all, goods market frictions are prevalent in the real world. [Petrosky-Nadeau and Wasmer, 2011] argue that "the allocation of final goods to end consumers is a costly process and the assumption of a frictionless goods market is an oversimplification given that the retail sector only accounts for more than 5% of U.S. GDP". In an empirical study, [Kaplan and Menzio, 2014] use data from the Kilts-Nielsen Consumer Panel Dataset(KNCP) from 2004 to 2009 to study the morphology of price dispersion. They claim that intertemporal price discrimination and search frictions may be important for explaining price dispersion. Both studies assert that search frictions in the goods market are prevalent. Secondly, the business cycle literature finds that goods market frictions are important for propagating aggregate shocks, which implies that the frictions might be important in formulating individuals' intertemporal optimization problems. [Petrosky-Nadeau and Wasmer, 2011] find that goods market frictions are the key ingredient changing the qualitative and quantitative responses of the model to productivity shocks. In particular, the dynamics of the goods market generate persistence in the growth of the incentives to hire workers, which translates into responses of labor market tightness to productivity shocks that are hump-shaped, or highly persistent. In a model search and matching between consumers and firms where both demand shocks and technology shocks are considered, [Bai et al., 2012] finds that demand shocks to consumption and investment explain 47% of the variance in output while technology shocks only account for 15% of the variance. Lastly, [Petrosky-Nadeau and Wasmer, 2011] argues that given the prevalence of goods market friction revealed by the data and the importance of market frictions in explaining shock propagation, a model is needed by the business cycle literature when both the

Walrasian and frictional markets exist. These same factors necessitate a model studying the individual's optimization problem.

This paper introduces a special good allocated in a market characterized by search frictions into the income fluctuation problem. Individuals make optimal intertemporal decisions when they face a positive interest rate and discount factor, uncertainties in their income, borrowing constraints and when they choose to consume two consumption goods: a general good allocated in a Walrasian market characterized by competitive trading, and a special good traded in a market with search frictions through directed search as in [Moen, 1997], [Menzio et al., 2013], etc. Directed search is used to increase tractability of the model which has been well studies in [Menzio and Shi, 2010, Menzio et al., 2013], and more importantly, to generate a large number of sub-markets, which sort individuals into different groups based on their willingness to pay or asset holdings. Each sub-market is defined by a quantity and price describing the market conditions.

As in [Lagos and Wright, 2005], I divided each time period into two sub-periods. Individuals interact in the Walrasian market for the general good in each first sub-period, and in the search market for the special good in each second sub-period. Assuming an individual's value function at the beginning of the second sub-period is concave, the individual's problem in the first sub-period becomes a standard dynamic programming problem [1]. I can establish the existence and properties of the value function and policy functions of the first sub-period. Unfortunately, due to the introduction of the special

[1] See [Stokey, 1989]

good market, the individual's value function at the beginning of the second sub-period is not concave even when the value function of the first sub-period of the next time period is concave, which makes the classic tools for dynamic programming not applicable. However, as in [Menzio et al., 2013] and [Topkis, 1998], the lattice-theoretic techniques are applied to establish the continuity and monotonicity of the individual's value function in the second sub-period. Given these properties of the value function, following [Menzio et al., 2013], I assume that individuals decide whether to play a two point lottery or not before entering the special good market, which makes the individual's value function at the beginning of the second sub-period concave. With the help of the lottery, the individuals' problem becomes standard. I first formulate an individual's intertemporal optimization problem and prove the existence and properties of the value functions at the beginning of each sub-period. Specifically, the value functions of each sub-period exist and are continuous, monotone, concave and differentiable. I follow by establishing the properties of the policy functions in each sub-period. The savings in the first sub-period is a continuous, increasing, and differentiable function of asset holdings at the beginning of the first sub-period. However, the ex post asset holdings at the end of the second sub-period is a segmented function conditional on whether the individual plays the lottery or succeeds in her transaction in the search market.

This chapter is organized as follows. In chapter 3.2, the model is presented and an individual's intertemporal optimization problem is characterized. Chapter 3.3 concludes.

3.2 The Model

The model is built on the framework of the income fluctuation problem. Specifically, an impatient infinitely lived individual, facing fluctuations in his income and borrowing constraints, solves his intertemporal optimization problems by maximizing his lifetime utility, which is time separable. Each period, an individual derives utility from consuming a general good c_t and the utility function is given by $u(c_t)$ which is strictly increasing, strictly concave, and differentiable. Due to uncertainties in income and borrowing limits, an impatient individual chooses to self-insure against the "bad luck" by accumulating a risk-free asset a_t given a constant return on asset r. This motive for Savings is the so-called precautionary saving. To study the savings out of transaction motive, I introduce a special good with allocation frictions into the income fluctuation problem.

Time is discrete and runs forever. Each time period is divided into two sub-periods. In the first sub-period, individuals consume a general good which can also be used for saving as the risk-free asset. The general good is distributed in a frictionless, centralized environment as in the standard neoclassical model. In the second sub-period, individuals search for firms, who can transform one unit of the risk-free asset, their savings, into one unit of the special good. The special good is distributed through directed search[2]. As a result, there is a continuum of sub-market. Each sub-market is characterized by its terms of trade (x, q) where x is the price an individual needs to pay for exchanging q units of special goods. Individuals decide which sub-market to enter

[2] See [Moen, 1997], [Menzio et al., 2013]

according to their asset holdings, i.e., the special good sorts individuals with different asset holdings into different sub-market[3].

On the other hand, before entering any sub-market, firms must pay a fix cost κ to create a trading post each period. Following the notations in [Menzio et al., 2013], denote $\rho[\theta(x,q)]$ as the probability at which a trading post will be visited by an individual, which is a function of the market tightness of the sub-market (x,q). I will discuss this in detail later. Thus, the expected value of a trading post in sub-market (x,q) is equal to $\rho(\theta(x,q))(x-q)$. In any sub-market, if the expected value of creating a trading post is higher than the fix cost, new firms will enter the market creating infinite amount of trading posts. As a result, this case would never happen. On the contrary, if the expected value of a trading post is lower than the fix cost, the firm will not create any trading post. Finally, if the expected value is equal to the fixed cost, for the firm, there is no difference between creating or not creating a trading post. Therefore, a sub-market (x,q), which is visited by a positive number of individuals, is required to satisfy the following conditions

$$\rho[\theta(x,q)](x-q) \leq \kappa \text{ and } \theta(x,q) \geq 0. \tag{3.1}$$

as in [Menzio et al., 2013], inequalities hold with complementary slackness. The above conditions also predict a zero profit for the firm.

[3]This will be clarified in the following paragraph

3.2.1 The individual's Problem in the First Sub-period

In the first sub-period, an individual supplies labor inelastically in the labor market to produce the general goods. The labor productivity l_t is subject to an i.i.d shock with a bounded support $[\underline{l}, \bar{l}]$ where $\underline{l} \geq 0$. An individual derives his income from two sources: the wage income and the capital income. At time t, the wage income is equal to wl_t where w is the time-invariant wage and l_t is the realization of labor productivity. The capital income equals $(1+r)a_t$ where r is the rate of return on capital and a_t is the asset holdings of an individual. An individual spends her income on consumption c_t and savings s_t, which is also the risk-free asset she brings into the second sub-period. An individual's budget constraint in the first sub-period is given by

$$c_t + s_t = wl_t + (1+r)a_t. \tag{3.2}$$

where no borrowing is allowed, i.e., $s \geq 0$ and $a \geq 0$

Denote $V(a, l)$ as the value function of an individual with asset holdings a and labor productivity l at the beginning of the first sub-period. Similarly, let $W(s)$ be her value function when her asset holdings equals s at the beginning of the second sub-period [4]. Thus, in the first sub-period, an individual chooses $\{c, s\}$ to maximize

$$V(a, l) = \max\{u(c) + W(s)\}.$$

subject to (3.2), $s \geq 0$ and $c \geq 0$. Solving this utility maximization problem gives us the consumption function and the savings function, which are $c = c(a, l)$ and $s = s(a, l)$ respectively.

[4] The individual's optimization problem has a recursive structure. Thus, I drop the subscript.

Assume that there is an upper bound for the individual's asset holdings a at the beginning of the first sub-period, which is a sufficiently large but finite number $\bar{a}$. Let $\mathcal{C}[0, \bar{a}]$ denote the set of continuous and increasing functions on $[0, \bar{a}]$, and let $\mathcal{V}[0, \bar{a}]$ denote the set of continuous, increasing and concave functions on $[0, \bar{a}]$. Assume $W(s)$ is any arbitrary function in $\mathcal{V}[0, \bar{a}]$[5]. The intertemporal problem of an individual in the first sub-period is a standard dynamic programming problem. The methods in [Stokey, 1989] can be applied to establish the properties of the value function $V(a, l)$ and the policy functions $s(a, l)$ and $c(a, l)$. Given the time-invariant wage and interest rate, an individual's optimization problem in the first sub-period is to solve

$$V(a, l) = \max_{s \in [0, z]} \{u[wl + (1 + r)a - s] + W(s)\}. \tag{3.3}$$

The following proposition shows that $V(\cdot, l)$ is continuous, concave and the policy function $c(\cdot, l)$ and $s(\cdot, l)$ are continuous, differentiable and increasing in a.

Proposition 1. *Let $R = 1 + r$ and $\beta R < 1$. For any given labor productivity l, if $W \in \mathcal{V}[0, \bar{s}]$, for any $a \in [0, \bar{s}]$, solving the individual's problem gives:*

(1) $V \in \mathcal{V}[0, \bar{a}]$, which is increasing, continuous and concave on $[0, \bar{a}]$;

(2) $c(\cdot, l)$ and $s(\cdot, l)$ is unique, continuous and increasing in a;

(3) If an inner solution exists, i.e., for any a such that $s(a, l) > 0$. The optimal choices are characterized by the following first order condition and the envelop condition:

$$(1 + r)u'[c(a, l)] = (1 + r)W'[s(a, l)] = V_1(a, l).$$

[5] This conjecture will be verified later.

Proof. Part (1). The monotonicity and continuity of the value function can be established by applying the Theorem of the Maximum[6]. Given that $W \in \mathcal{V}[0, \bar{a}]$ is continuous, increasing, and concave, the right hand side of (3.3)-the objective function-is continuous, increasing, and strictly concave in (a, s) jointly. Given the objective function is continuous and the feasible set is compact and continuous, the value function $V(a, l)$ is increasing and continuous in a. Furthermore, since the objective function is also strictly concave in (a, s) jointly, $V(a, l)$ is concave in a, i.e., $V \in \mathcal{V}[0, \bar{a}]$. The concavity of the objective function implies the uniqueness of $s(a, l)$ and $c(a, l)$.

Part (2). Following the results in part (1) and using the Theorem of the Maximum again, I show that the $s(a, l)$ is continuous in a. Next, because the objective function in (3.3) is strictly concave in (a, s) jointly, the envelope theorem by Benveniste and Scheinkman $V_1(a, l) = (1+r)u'[c(a, l)]$ holds. Given that $V(a, l)$ and $u(c)$ are strictly concave and the envelope theorem, I can show that $c(a, l)$ is an increasing function of a.

Part (3). I follow [Menzio et al., 2013] to prove this part. First, for any a such that $s(a, l) > 0$, there exists $\epsilon_0 > 0$ such that $s(a \pm \epsilon, l) > 0$ for all $\epsilon \in [0, \epsilon_0]$ because $s(a, l)$ is continuous. Furthermore, choose a sufficiently small ϵ_0 such that for any $\epsilon \in [0, \epsilon_0]$, savings $s(a, l)$ is feasible to an individual with asset holdings $a - \epsilon$. Define $H[s, a, l] = u[wl + (1 + r)a - s] + W(s)$. Then, for any $\epsilon \in [0, \epsilon_0]$, the following equations hold:

$$V(a, l) = H[s(a, l), a, l] \geq H[s(a - \epsilon, l), a, l],$$

$$V(a - \epsilon, l) = H[s(a - \epsilon, l), a, l] \geq H[s(a, l), a - \epsilon, l],$$

[6][Stokey, 1989]

Then, I have

$$\frac{H[s(a-\epsilon,l),a,l]-H[s(a-\epsilon,l),a,l]}{\epsilon} \leq \frac{V(a,l)-V(a-\epsilon,l)}{\epsilon}$$
$$\leq \frac{H[s(a,l),a,l]-H[s(a,l),a-\epsilon,l]}{\epsilon}$$

Because of the Benveniste and Scheinkman formulas, $V_1(a,l)$ exists. As $\epsilon \to 0$, $(1+r)W'[s^-(a,l)] = V_1(a,l)$ where $s^-(a,l)$ is the optimal savings when the individual has asset holdings $a-\epsilon$. Similarly, I can derive $(1+r)W'[s^+(a,l)] = V_1(a,l)$. Then, I can prove that $(1+r)W'[s(a,l)] = V_1(a,l)$, which implies that $s(a,l)$ is strictly increasing. □

3.2.2 The individual's Problem in the Second Sub-period

In the second sub-period, an individual searches for firms to purchase the special good. The utility derived from consuming q units of special goods is equal to $v(q)$ where v is continuous, increasing, strictly concave, and differentiable. The special good is allocated through directed search as in [Moen, 1997], [Acemoglu and Shimer, 1999], and [Menzio et al., 2013]. Thus, there is a continuum of sub-markets for the special goods, each of which is specified by the terms of trade $(x,q) \in \mathbb{R}_+ \times \mathbb{R}_+$. Each individual optimally chooses which sub-market to enter according to his willing to pay-asset holdings, while each firm decides which sub-market to enter and how many trading posts to create in the sub-market. Firms can enter each sub-market freely, but they have to pay a fix cost κ for each trading post they create.

Following the notation in [Menzio et al., 2013], define the market tightness of sub-market (x,q) as $\theta($ x,q): $\mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ which is the ratio of the number of trading posts to that of individuals in that sub-market. Thus, in sub-market (x,q), the probability that an individual will meet a firm is given by $b = \lambda[\theta(x, q)]$ where $\lambda : \mathbb{R}_+ \longrightarrow [0, 1]$ is a strictly increasing in θ with $\lambda(0) = 0$ and $\lambda(\infty) = 1$. Similarly, in sub-market (x, q), the probability that a firm is visited by an individual is defined as $s = \rho[\theta(x, q)]$, where $\rho : \mathbb{R}_+ \longrightarrow [0, 1]$ is a strictly decreasing function such that $\rho(\theta) = \lambda(\theta)/\theta$, $\rho(0) = 1$ and $\rho(\infty) = 0$. Then, I have $s = \mu(b) = \rho[\lambda^{-1}(b)]$, which implies that $\mu(b)$ is decreasing in b. The Bellman equation for an individual in sub-market (x, q) is given by

$$\tilde{W}(s) = \max_{q,x}\{\lambda(\theta(x, q))[v(q) + \beta EV(s - x, l')] + [1 - \lambda(\theta(x, q))]\beta EV(s, l')]\}. \tag{3.4}$$

$$st. \qquad q \geq 0, \quad x \in [0, s]$$

Define an individual's policy functions as $x(s)$ and $q(s)$ respectively.

To simplify the analysis, I express the individual's choices over (x, b) instead of (x, q). Then, the probability that a trading post is visited $s = \rho[\theta(x, q)]$ is given by

$$s = \mu(b) = \begin{cases} \frac{\kappa}{x-q} & if\ \kappa \leq x - q \\ 1 & otherwise. \end{cases}$$

From the above equation, firms create trading posts only in a sub-market such that $x - q \geq k$ which generates a positive market tightness. Otherwise, the market tightness is 0. For any market with a positive market tightness, the quantity in that sub-market is given by

$$q = Q(x, b) = x - \frac{\kappa}{\mu(b)}. \tag{3.5}$$

As in the first sub-period, borrowing is not allowed. In order to enter a sub-market, the minimum asset holdings required for an individual s is κ. Therefore, I focus on studying the case an individual's asset holdings s is higher than κ. Then, an individual's problem in the second sub-period can be writen as

$$\tilde{W}(s) = \max_{(x,b)} \{\beta EV(s,l') + b[\tilde{v}(x,b) + \beta EV(s-x,l') - \beta EV(s,l')]\}. \tag{3.6}$$

$$st. \quad b \in [0,1], \quad x \in [0,s]$$

where $\tilde{v}(x,b) = v[Q(x,b)]$. Let $(x(s), b(s))$ denote the buyer's policy functions and let $\phi(s)$ denote asset holdings for next period, Then,

$$q(s) = Q(x(s), b(s)), \quad \phi(s) = s - x(s)$$

The objective function of the individual's problem in the second sub-period is not concave jointly in (x, b, s). This is because the second part of the above objective function is the product of the matching probability b and the individual's expected surplus from trading in the special goods market. This concavity problem makes the standard dynamic programming not applicable to study an individual's optimization problem in the second sub-period. [Menzio et al., 2013] use the lattice-theoretic techniques (see Topkis, 1998) to explore the individual's problem in this setup. Following their methods, properties of the value function and policy functions can be examined. However, this method operates differently compared to the standard dynamic programming. First, using the lattice-theoretic techniques, I prove the monotonicity of the policy functions. Second, based on monotonicity of the policy functions, the differentiability of the value functions $\tilde{W}(s)$ along the equilibrium path can be obtained. Thus the policy functions can be solved by the first-order conditions and envelope conditions. Finally, the differentiability of $\tilde{W(s)}$ on its support can be derived.

The following two propositions characterize the properties of the value function and policy functions in the second sub-period. I skip the proof here since similar procedures can be find in [Menzio et al., 2013].

Proposition 2. ***Properties of the Value function.*** *Assume an individual's value function in the first sub-period $V(\cdot, l)$ is in $\mathcal{V}[0, \bar{a}]$. Then, the individual's value function in the second sub-period $\tilde{W}$ is in $\mathcal{C}[0, \bar{a}]$.*

An individual's optimization problem in the second sub-period defines a mapping, which maps a continuous, increasing and concave function into a continuous and increasing function. This is because $V(\cdot, l)$ is continuous and increasing, by the Theorem of Maximum, the value function $\tilde{W}(s)$ is continuous and increasing. The following proposition shows how to find the policy functions.

Proposition 3. ***Properties of the Policy Functions.*** *In the second sub-period, if $s \leq \kappa$, then $b(s) = 0$. If $s \in [\kappa, \bar{a}]$, then $b(s) > 0$. For any $s \in [\kappa, \bar{a}]$, the following properties hold:*

(1) For each s, the policy functions $(x(s), b(s), q(s), \phi(s))$ are unique, continuous and increasing.

(2) The optimal choice $b(s)$ satisfies the first-order condition:

$$v(x, b) + bv_2(x, b) = \beta E[V(s, l') - V(s - x, l')].$$

For all s such that $\phi(s) > 0$, $\phi(s)$ satisfies the first-order condition:

$$V'[\phi(s)] = \frac{1}{\beta}\tilde{v_1}(x(s), b(s)).$$

(3) $\tilde{W}(s)$ exists if and only if $V'(s)$ exists, and $\tilde{W}$ is strictly increasing.

This proposition indicates that the first order conditions can be used to characterize an individual's policy functions in the second sub-period. To apply the lattice-theoretic techniques to derive the monotonicity of the policy functions, I need to prove that the objective function in (3.6) is super-modular in (x, b, s). Super-modularity means that the objective function has increasing differences in (x, b), (x, s) and (b, s). To show this, I first show $Q(x, b)$ satisfies

$$Q_1(x, b) > 0,\ Q_2(x, b) < 0,\ Q(x, b)\ is\ (weakly)\ concave,\ and\ Q_{12} = 0.$$

which implies $\tilde{v}(x, b)$ is strictly super-modular. However, an individual's problem in the second sub-period in (3.6) is not super-modular in (x, b) due to the product. To circumvent this difficulty, [Menzio et al., 2013] decompose (3.6) into two steps. First, given s, they fix b and choose x to maximize $R(x, b, s)$ which is the expected gain from a successful matching $[\tilde{v}(x, b) + \beta EV(s - x, l') - \beta EV(s, l')]$. Then,

$$\tilde{x}(b, s) = arg \max_{x \in [0,s]} R(x, b, s),\ R(\tilde{b}, s) = R[\tilde{x}(b, s), b, s]. \tag{3.7}$$

Second, they choose b to solve the following problem

$$b(s) = arg \max_{b \in [0,1]} bR(\tilde{b}, s). \tag{3.8}$$

[Menzio et al., 2013] show that in each step, the objective functions are super-modular with respect to their arguments. Specifically, they first show that $R(x, b, s)$ is super-modular in (x, b, s), which implies that $\tilde{x}(b, s)$ is increasing in (b, s) and $R(\tilde{b}, s)$ is also increasing and super-modular in (b, s). Then they show that $bR(\tilde{b}, s)$ is super-modular in (b, s), which implies $b(m)$ and $b(s)R(\widetilde{b(s)}, s)$ are increasing in s.

By using a generalized envelope theorem, similar to the proof of the individual's problem in the first sub-period, [Menzio et al., 2013] also show that the optimal choice of $b(s)$ satisfies the first-order condition:

$$v(x,b) + bv_2(x,b) = \beta[EV(s,l') - EV(s-x,l')].$$

For all s such that $\phi(s) > 0$, $\phi(s)$ satisfies the first-order condition:

$$EV_1[\phi(s),l'] = \frac{1}{\beta}\tilde{v_1}[x^*(s),b^*(s)].$$

Thus, the individual's optimization problem in the second sub-period can be studied by using the first-order conditions derived above.

Since the policy functions $(x(s), b(s), q(s), \phi(s))$ are unique, continuous and increasing, individuals choose which sub-market to enter according to their asset holdings s. Individuals with higher asset holdings go to a sub-market with a higher matching probability, a higher price, and higher volume of the special goods. Thus, directed search sort individuals into different group according to their willingness to pay-asset holdings. In order to obtain the special goods at a higher chance, which implies a shorter waiting time, I conjecture that individuals would choose to save more. This motive for saving is called saving for transaction[7].

3.2.3 Lotteries and the Value Function

From the previous paragraph, we know the policy function $\tilde{W}(s)$ might not be concave due to non-convexity caused by the market friction. To avoid this difficulty, as

[7] A numerical simulation will be conducted to verify this statement.

in [Menzio et al., 2013], I assume that an individual decides whether to play a two point lottery at the beginning of the second sub-period. The two point lottery is described by a vector of payoff and probability of winning $(z_i, \pi_i)_{i=1,2}$, i.e., the low prize of the lottery, z_1, is realized at π_1, which brings the individual a lifetime utility $\tilde{W}(z_1)$. Similarly, the high prize of the lottery, z_2, is realized at π_2, which brings the individual a lifetime utility $\tilde{W}(z_2)$. The lottery is given by

$$W(s) = \max_{(z_1,z_2,\pi_1,\pi_2)} \{\pi_1 \tilde{W}(z_1) + \pi_2 \tilde{W}(z_2)\}. \tag{3.9}$$

subject to

$$\pi_1 z_1 + \pi_2 z_2 = s, \quad \pi_1 + \pi_2 = 1, \quad z_2 \geq z_1$$

$$\pi_i \in [0,1] \text{ and } z_i \geq 0 \text{ for } i = 1,2$$

Proposition 4. *Given $\tilde{W}(\cdot) \in \mathcal{C}[0,\bar{a}]$, $W(\cdot)$ is concave.*

Proof. See [Menzio and Shi, 2010] for the proof. □

3.3 Existence of the Value function

I show the existence of the value function as follows. First, assume that there is an upper bound of the asset holdings at beginning of the first sub-period and the value function at the beginning of the first sub-period $V(a)$ is continuous, increasing, and concave in a, i.e., $V \in \mathcal{V}[0,\bar{a}]$. Second, after playing the lottery, an individual's maximization problem in the second sub-period (3.4) defines a mapping $T_{\tilde{W}} : \mathcal{V}[0,\bar{a}] \to \mathcal{C}[0,\bar{a}]$, i.e., the objective function in (3.4) maps $V \in \mathcal{V}[0,\bar{a}]$ into her value function in the second sub-period $\tilde{W} \in \mathcal{C}[0, \bar{a}]$, which gives $\tilde{W} = T_{\tilde{W}} V$. Third, the lottery (3.9) defines a mapping $T_L : \mathcal{C}[0, \bar{a}] \to \mathcal{V}[0, \bar{a}]$, i.e., the lottery maps $\tilde{W} \in \mathcal{C}[0, \bar{a}]$ into the value function at the beginning the the second sub-period before the lottery $W \in \mathcal{V}[0, \bar{a}]$, which gives $W = T_L \tilde{W}$. Finally, the individual's maximization problem in the first sub-period (3.3) defines a mapping $T_V : \mathcal{V}[0, \bar{a}] \to \mathcal{V}[0, \bar{a}]$, i.e., the objective function in (3.3) maps $W \in \mathcal{V}[0, \bar{a}]$ into the individual's value function at the beginning of the first sub-period $V \in \mathcal{V}[0, \bar{a}]$, which gives $V = T_V W$. Therefore, an individual's optimization problem from time t to $t + 1$ defines a mapping $T : \mathcal{V}[0, \bar{a}] \to \mathcal{V}[0, \bar{a}]$, which gives $V = T_V V$. Then, $TV = T\{T_V[T_L(T_{\tilde{W}} V)]\}$. The following proposition shows that T is a monotone contraction mapping on $\mathcal{V}[0, \bar{s}]$ and has a unique fixed point.

Proposition 5. *T is a self-map on $V \in \mathcal{V}[0, \bar{a}]$ and has a unique fixed point.*

Proof. Similar to [Menzio and Shi, 2010], it is straightforward to show that T satisfies the Blackwell sufficient conditions, i.e., T defines a monotone contraction mapping on $[0, \bar{a}]$, which proves the proposition. □

To finish defining a well-behaved dynamic programming problem, I only need to show that there exists an upper bound for the state space of a. The following proposition establishes this property.

Proposition 6. *Let $R = 1 + r$ and $\beta R < 1$. Assume that: (i) $\{l_t\}$ has bounded support, (ii) $(-cu'')/u'$ is bounded above for all c sufficiently large. Then there exists a^* such that whenever $a_t \geq a^*$, $s_t \leq a_t$.*

Proof. The proof, following Schechtman and Escudero (1977) and Aiyagari (1994), is to show there exists a a^* such that whenever $a_t \geq a^*$, $s_t \leq a_t$. If $s(a,l)$ is bounded so that $s(a,l) \leq \bar{C}$ for all $a \geq 0$, then we can take $a^* = R\bar{C} + w\bar{l}$. Then, suppose $s(a,l) \to \infty$ as $a \to \infty$. From Benveniste and Scheinkman Theorem, we have $V_1(a,l) = (1+r)u'(c(a,l))$. Therefore, we have:

$$\frac{E_t V_1(a',l')}{V_1(a',\bar{l})} \leq \frac{V_1(a',\underline{l})}{V_1(a',\bar{l})} = \frac{u'(a',\underline{l})}{V_1(a',\bar{l})} \leq \{\frac{c(a',\bar{l})}{c(a',\underline{l})}\}^{\mu}$$

where μ is an upper bound on relative risk aversion $(-cu'')/u'$. Now, $c(a',l_{max}) = c(a',l_{min}) + w(l_{max} - l_{min})h$, where $0 \leq h \leq 1$, since both $c(a,l)$ and $s(a,l)$ are increasing in a. Therefore,

$$\frac{E_t V_1(a',l')}{V_1(a',\bar{l})} \leq \{1 + \frac{w(\bar{l} - \underline{l})h}{c(a',\underline{l})}\}^{\mu}$$

Therefore, we have:

$$lim_{z_t \to \infty} \frac{E_t V_1(a',l')}{V_1(a',\bar{l})} = 1.$$

Next, I show that as $s \to \infty$, the marginal benefit from trading $\beta b E[V_1(s - x,l) - V_1(s,l)] \to 0$. Note that the optimal amount spent in the frictional market is equal to x, which is determined by the first order condition $EV_1(s - x, l') = \tilde{\nu}_1(x,b)$. Assume that $\tilde{\nu}_1(x,l) \to 0$ as x approaches some large number $\bar{x}$. Therefore, since x is an increasing function of s, it is easy to see that as $s \to \infty$, $\beta b E[V_1(s-x,l) - V_1(s,l)] \to 0$. Choose s such that when $s \geq \bar{s}$, there exists an ϵ_0 such that $\beta b E[V_1(s-x,l) - V_1(s,l)] \leq \epsilon_0$.

Finally, choose $\epsilon_1 \leq (1-\beta R)/(\beta R)$ and note that there exists a^* sufficiently large such that $E_t V_1(a', l')/V_1(a', \bar{l}) \leq 1+\epsilon_1$ for all $a_t \geq a^*$. From the first order condition of the first sub-period and the envelope condition we have:

$$V_1(a,l) = (1+r)u'(c_t) = (1+r)W'(s), W'(s) = \beta b E[V_1(s-x,l') - V_1(s,l')] + \beta V_1(s,l').$$

Then, we have

$$V_1(a,l) = (1+r)\{\beta b E[V_1(s-x,l') - V_1(s,l')] + \beta E V_1(s,l')\} \leq (1+r)\beta V_1(s,l') \leq V_1(s,\bar{l})$$

for all $a_t \geq a^*$. This finishes the proof.

□

3.4 Conclusion

In this chapter, I introduces goods market frictions into the income fluctuation problem where individuals make their optimal intertemporal decisions in the presence of labor productivity shocks, borrowing constraints, and goods market frictions. There are two goods market in the economy: a general good distributed through a competitive market and a special good distributed through directed search. I prove that directed search directs individuals into different sub-markets according to their willingness to pay which is determined by individuals' asset holdings. The presence of the special good then provides the individual an additional incentive to save which further changes individuals' optimal decisions. In this model, the motivation for savings is not only out of precaution but also out of transaction, i.e., individuals save more assets in order to obtain the special goods at a higher probability-a shorter waiting periods.

In the future, this model can be explored along two dimensions. First, numerical simulations can be conducted to examine the importance of market frictions on individuals' intertemporal optimization problems quantitatively. Second, we can examines how goods market frictions affect the aggregate economy, which requires us to extend the model into a general equilibrium setting and establish the existence of a stationary Markov equilibrium.

Chapter 4

House Prices, Intermediate Inputs and Sectoral Co-movement over Business Cycles

4.1 Introduction

The recent financial crisis resulting from the collapse of the subprime mortgage market in 2008 suggests that the housing market plays an important role over business cycles. Empirical studies find three main stylized facts about the behavior of the housing market over business cycles. First, GDP, consumption, nonresidential investment, and residential investment comove positively over business cycles. Second, residential investment is more than twice as volatile as non-residential investment. Third, house prices and residential investment are positively correlated, and house prices are more volatile than GDP.

To examine the three stylized facts of the housing market over business cycles, this paper extends the two sector model with intermediate inputs, proposed by [Hornstein and Praschnik, 1997], to include a housing sector and adjustment costs for capital investment and houses. Thus, there are three sectors in this model economy: a nondurable goods sector, a durable goods sector, and a housing sector. Nondurable goods, which can be used for consumption and as an intermediate input for producing durable goods, are produced by capital and labor. Durable goods, which are produced by capital, labor, and intermediate inputs, can be used as the investment in the nondurable goods sector, the investment in the durable goods sector, and residential structures used to produce houses. The sum of the investment in the nondurable goods sector and the investment in the durable goods sector is defined as nonresidential investment. The housing sector uses residential structures and land to produce houses. There are two sector-specific productivity shocks: the shock in the nondurable goods sector and the shock in the durable goods sector. Moreover, there are also adjustment costs for capital investment in the nondurable goods sector, the durable goods sector, and houses.

[Davis and Heathcote, 2005] have worked on a similar problem. In their model they include three intermediate goods sectors and the intermediate goods are produced by capital and labor. But they include two final goods that are produced with only intermediate goods. This paper differs from [Davis and Heathcote, 2005] as it presents a model in which the final goods are produced with intermediate goods, capital and labor.

This model performs very well in matching the three stylized facts of the housing market over business cycles. First, the introduction of intermediate inputs and adjustment costs causes GDP, consumption, nonresidential investment, and residential investment to move in the same direction. Data from post-war US business cycle suggests that investment and output comove positively across sectors. However, if fluctuations stem from sector-specific productivity shocks, multi-sector models predict outcomes that are inconsistent with the empirical findings-the "comovement puzzle". This is because a sector-specific productivity shock tends to shift production from the less productive sector to the more productive sector, which leads to a negative co-movement in investment and output across sectors. A large body of literature has addressed the comovement puzzle[1]. This paper follows the models of [Hornstein and Praschnik, 1997] and [Baxter, 1996] to solve the comovement puzzle. The application of intermediate inputs provides a channel to move the production of different sectors in the same direction. A positive shock in the nondurable sector would lead to a transfer of capital and labor to the nondurable goods sector from the durable goods sector because of higher

[1] [Baxter, 1996] generates the positive co-movement in a two-sector model with durable goods by assuming adjustment costs for capital and a high correlation between Solow residuals across sectors. [Gomme et al., 2001] introduce time-to-build into the sector that produces capital goods, which constrains the investment boom in that sector. As a result, investment rises in all sectors simultaneously. [Hornstein and Praschnik, 1997] obtain positively correlated movements between sectoral employment by integrating intermediate inputs into a two-sector model, which helps the nondurable goods sector and durable goods sector move in the same direction after a productivity shock. [Davis and Heathcote, 2005] succeed in matching the sectoral comovement in a multi-sector model with three intermediate goods sectors and two final goods sectors. Intermediate goods are used to produce both final goods. Although the sector-specific shocks to the intermediate goods are moderately positively correlated, the positive comovement in final-goods sectors is amplified because both the final-goods sectors use all three intermediate inputs. Also see [Davis and Heathcote, 2005] for a summary of the literature.

productivity. Therefore, the output of nondurable goods increases, which makes the nondurable goods less expensive. This leads to an increase in the demand for intermediate inputs in the durable goods sector. The marginal productivity of capital and the marginal productivity of labor in the durable goods sector also increase. As a result, output in both the durable goods sector and the nondurable goods sector moves in the same direction. Similarly, if there is a positive shock in the durable goods sector, the capital available for both sectors will increase in the next period and the output in both sectors will move in the same direction. The positive correlation between productivity shocks in the nondurable goods sector and the durable goods sector move output in both sectors in the same direction. In addition, the adjustment costs for capital further help the capital investment move into the same direction after a productivity shock since it becomes costly to move capital investment from the less productive sector to a more productive sector.

The second fact is the relationship between non-residential investment and residential investment. In [Davis and Heathcote, 2005], residential investment is more volatile than non-residential investment because the production function of residential structures is more construction intensive, which is more volatile over the business cycle. In this paper, as in [Hornstein and Praschnik, 1997] and [Baxter, 1996], a higher volatility in residential investment is observed since the productivity shock in the durable goods sector, which is also used to produce residential investment, is twice as volatile as the shock in the nondurable sector.

Finally, in my model the positive correlation between house prices and residential investment is the result of the combined effect of adjustment costs and the intermediate inputs. While adjustment costs and intermediate inputs move different sectors in the same direction after a shock, adjustment costs constrain the supply of residential investment. The changes in house prices and residential investment move along the supply curve, which predicts a positive correlation between house prices and residential investment. [Davis and Heathcote, 2005] generate a negative correlation between house prices and residential investment. They also predict that the standard deviation of house prices is smaller than the standard deviation of GDP. However, if adjustment costs are added into [Davis and Heathcote, 2005], a positive correlation between house prices and residential investment can be obtained. [Salyer et al., 2010] introduce financial market frictions due to asymmetric information between credit lenders and housing developers to explain the high relative volatility of house prices. Financial market frictions generate agency costs for housing firms, which might cause these firms to go bankrupt after realizing their individual productivity shocks. Hence, there should be a markup between returns from internal funds and returns from outside funds. The volatility in this markup translates into increased volatility in house prices. However, their model predicts an unreasonably high volatility of residential investment and fails to generate a positive correlation between house prices and residential investment. [Iacoviello and Neri, 2010] predict that the volatility in house prices is higher than the volatility in GDP. They also find a positive correlation between house prices and residential investment. However, these results rely heavily on a rich set of shocks-especially shocks from the demand side.

This paper is structured as follows. Chapter 4.2 presents the model. Chapter 4.3 describes the calibration process. The results are provided in Chapter 4.4. Chapter 4.5 concludes.

4.2 The Model

4.2.1 The Household's Problem

This is a dynamic stochastic general equilibrium model with three sectors: the nondurable goods sector, the durable goods sector, and the housing sector. Nondurable goods are used for consumption or as intermediate inputs in producing durable goods. Durable goods are used for investment in the nondurable goods sector, investment in durable goods sector, and as an intermediate input, together with land, to produce houses. There are only two sector-specific productivity shocks: the shock in the nondurable goods sector and the shock in the durable goods sector. The productivity shocks follow a bivariate Markov process with no trend which implies that growth is abstract from this model. Furthermore, there is a government which imposes proportional taxes on capital and labor income. Government tax income is rebated in a lump-sum fashion.

Each period, a representative household endows with one unit of time which can be used for work or leisure. The household derives utility from consumption c_t, services provided by house stocks h_t, and leisure $1 - n_{n,t} - n_{d,t}$ where $n_{n,t}$ and $n_{d,t}$ are the household labor supply in the nondurable goods sector and in the durable goods sector respectively. The period utility of the household is assumed to be

$$U(c_t, h_t, 1 - n_{n,t} - n_{d,t}) = \xi \ln c_t + (1 - \xi) \ln h_t - \phi(n_{n,t} + n_{d,t})$$

where ξ and ϕ represent the share of nondurable goods consumption and the disutility of work effort in the utility function respectively[2]. Denote β as the household subjective discount factor. The life time utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, 1 - n_{n,t} - n_{d,t}). \tag{4.1}$$

Each period, the representative household derives income from four sources: labor income, capital income, income from land which has a fixed supply each period, and government transfer. Moreover, the household has to pay a labor income tax at the rate of τ_n and a capital income tax at the rate of τ_k. The after-tax labor income is equal to $(1 - \tau_n)w_t(n_{n,t} + n_{d,t})$ where w_t is the wage rate. Capital incomes are obtained from capital in the nondurable goods sector $[(1 - \tau_k)R_{n,t} + \tau_k\delta_n]p_{d,t}k_{n,t}$ where $R_{n,t}$, δ_n, $p_{d,t}$ and $k_{n,t}$ denote the gross rate of return of capital in the nondurable goods sector, the depreciation rate of capital in the nondurable goods sector, the durable goods price and capitals in the nondurable goods sector respectively, and from capital in the durable goods sector $[(1-\tau_k)R_{d,t}+\tau_k\delta_d]p_{d,t}k_{d,t}$ where $R_{d,t}$, δ_d and $k_{d,t}$ are the gross rate of return of capital in the durable goods sector, the depreciation rate of capital in the durable goods sector, and capitals in the durable goods sector respectively. The land income is equal to $p_{l,t}l_t$ where $p_{l,t}$ and l_t denote the land price and the land level available at time t respectively. The last source of income is from government transfer which is lump-sum

[2]See Hornstein and Praschnik (1997) for a similar formulation.

transfer and equal to the taxes paid by the household. The household allocates incomes to consumption c_t, investment in the nondurable sector $i_{n,t}$, investment in the durable goods sector $i_{d,t}$, and investment in houses $i_{h,t}$. The sum of capital investment in the nondurable goods sector and that in the durable goods sector is referred to nonresidential investment, i.e., $i_{b,t} = i_{n,t} + i_{d,t}$. Then, the household resource constraint is

$$\begin{aligned} c_t + p_{d,t}(i_{n,t} + i_{d,t}) + p_{h,t} i_{h,t} =& (1 - \tau_n) w_t (n_{n,t} + n_{d,t}) \\ &+ [(1 - \tau_k) R_{n,t} + \tau_k \delta_n] p_{d,t} k_{n,t} \\ &+ [(1 - \tau_k) R_{d,t} + \tau_k \delta_d] p_{d,t} k_{d,t} + p_{l,t} l_t + T_t. \end{aligned} \tag{4.2}$$

where $p_{h,t}$ denotes the house price.

There are adjustment costs for capital in the nondurable goods sector and that in the durable goods sector. Following [Jermann, 1998] and [Boldrin et al., 2000], the law of motion for capital in each sector is given by

$$k_{j,t+1} = (1 - \delta_j) k_{j,t} + i_{j,t} - \phi_j\left(\frac{i_{j,t}}{k_{j,t}}\right) k_{j,t}. \tag{4.3}$$

where $\phi_j(i_{j,t}/k_{j,t}) = \varphi_j(i_{j,t}/k_{j,t} - i_j/k_j)^2/2\delta_j$, $j = n$, d and i_j/k_j is the steady state ratio of investment to capital in each sector. The parameter, $\varphi_j > 0$, is the inverse of elasticity of the investment-capital ratio with respect to Tobin's q.

Similarly, the law of motion for housing is

$$h_{t+1} = (1 - \delta_h) h_t + i_{h,t} - \phi_h\left(\frac{i_{h,t}}{h_t}\right) h_t. \tag{4.4}$$

where $\phi_h(i_{h,t}/h_t) = \varphi_h(i_{h,t}/h_t - i_h/h)^2/2\delta_h$ and $i_{h,t}/h_t$ is the steady state ratio of house investment to house stock. The parameter, $\sigma_{hi} > 0$, is the inverse of elasticity of the investment-house stock ratio. The household problem is to maximize (4.1) subject to (4.2) − (4.4). The first order conditions of the household's problem are presented in Appendix A.

4.2.2 Firms

The factor markets and goods markets are competitive. At time t, the nondurable goods producing firm rents capital $k_{n,t}$ and employs labor $n_{n,t}$ from the factor markets to produce nondurable goods. The production function is Cobb-Douglas:

$$y_{n,t} = k_{n,t}^{\alpha}[z_{n,t}n_{n,t}]^{1-\alpha}. \tag{4.5}$$

where $z_{n,t}$ and α denote the productivity shock and the capital share in the nondurable goods sector. The firm's problem in the nondurable goods sector is given by

$$\max : y_{n,t} - R_{n,t}p_{d,t}k_{n,t} - w_t n_{n,t}.$$

The first order conditions with respect to $k_{n,t}$ and $n_{n,t}$ are given by

$$\alpha\frac{y_{n,t}}{k_{n,t}} - R_{n,t}p_{d,t} = 0.$$

$$(1-\alpha)\frac{y_{n,t}}{n_{n,t}} - w_t = 0.$$

The durable goods producing firm rents capital $k_{d,t}$, employs labor $n_{d,t}$ and use intermediate inputs m_t to produce durable goods, which are used for investment in capital both in the nondurable sector and the durable sector and as residential structures x_t for producing houses. The production function is given by

$$y_{d,t} = \zeta[k_{d,t}^{\gamma}(z_{d,t}n_{d,t})^{1-\gamma}]^{1-\chi}m_t^{\chi}. \tag{4.6}$$

where ζ, γ, χ and $z_{d,t}$ denote the scale coefficient, capital income share, the intermediate input share, and the productivity shock in the durable goods sector. The durable goods firm's problem is

$$\max : p_{d,t}y_{d,t} - R_{d,t}p_{d,t}k_{n,t} - w_t n_{n,t} - m_t.$$

The first order conditions with respect to $k_{d,t}$, $n_{d,t}$ and m_t are given by

$$\gamma(1-\chi)\frac{y_{d,t}}{k_{d,t}} - R_{d,t} = 0.$$

$$(1-\chi)(1-\gamma)p_{d,t}\frac{y_{d,t}}{n_{d,t}} - w_t = 0.$$

$$\chi p_{d,t}\frac{y_{d,t}}{m_t} - 1 = 0.$$

Following [Davis and Heathcote, 2005], the housing firm combines residential structures x_t and land l_t to produce houses. The land is supplied in a fixed amount each period. The production function in the housing sector is given by

$$y_{h,t} = x_t^{\nu}l_t^{1-\nu}. \tag{4.7}$$

where ν denote the construction structure share in the housing sector. The housing firm's problem is

$$\max : p_{h,t}y_{h,t} - p_{l,t}l_t - p_{d,t}x_t.$$

The first order conditions with respect to x_t and l_t are given by

$$\nu p_{h,t} \frac{y_{h,t}}{x_t} - p_{d,t} = 0.$$

$$(1-\nu) p_{ht} \frac{y_{h,t}}{l_t} - p_{l,t} = 0.$$

The aggregate output in model at time t is defined as

$$y_t = y_{n,t} + p_{d,t} y_{d,t} + p_{h,t} y_{h,t}.$$

Definition. *A general equilibrium is defined by a set of prices* $\{R_{n,t}, R_{d,t}, w_t, p_{d,t}, p_{h,t}, p_{l,t}\}_{t=0}^{\infty}$*, a set of outputs produce in each sector* $\{y_{n,t}, y_{d,t}, y_{h,t}\}_{t=0}^{\infty}$*, and a set of allocations* $\{c_t, n_{n,t}, n_{d,t}, m_t, x_t, k_{n,t+1}, k_{d,t+1}, h_{t+1}, i_{n,t}, i_{d,t}, i_{h,t}, l_t\}_{t=0}^{\infty}$ *such that*

1) given the prices and allocations, households solve their utility maximization problems;

2) given the prices and factor inputs, firms maximize their profits;

3) all markets clear.

The markets clearing conditions are:

$$\begin{aligned} l_t &= \bar{l}. \\ y_{n,t} &= c_t + m_t. \\ y_{d,t} &= i_{n,t} + i_{d,t} + x_t. \\ y_{h,t} &= i_{h,t} \end{aligned}$$

4.3 Calibration

In this section, the model parameters are calibrated to match the data of different sectors. I use industrial level data from Bureau of Economic Analysis (BEA) from 1987 to 2010. The reason for starting with 1987 is due to data availability in the sectoral data. Similar to [Hornstein and Praschnik, 1997], from the Survey of Current Business (2011), the durable goods sector consists of sectors 11 and 12, 20-23, 35-55, and 59-64, the housing sector consists of sectors 56-59, and the nondurable goods sector consists of all other sectors from 1 to 77. Furthermore, one period in this model is a year.

Table 4.1 reports the calibrated parameter values of the model. The labor tax rate τ_n and the capital tax rate τ_k are set to equal to 0.30 and 0.50, respectively. The values are within the ranges provided by [Lucas Jr, 1990] and [McGrattan, 1994]. [Lucas Jr, 1990] argues the average capital rate and labor tax rate are both 0.36, while [McGrattan, 1994] suggests that the average capital tax is between 0.40 and 0.50 and the average labor tax is between 0.10 and 0.25.

I proceed to calibrate parameter values of the production functions. First, for the production function of nondurables, the capital share α is equal to 0.36, which is widely used in the business cycle literature. Second, in the durable goods sector, from the data, the capital share γ and the intermediate input share χ are about 0.30 and 0.40, respectively. The scale coefficient ζ is calibrated to match the relative price of durables and the ratio of durables to output, which is equal to 0.20. Finally, following [Davis and Heathcote, 2005], the structure share in the housing production function ν is equal to 0.90 and the land supply is constant and fixed to 1 each period. From the data of new capital investment in each sector, the capital depreciation rate in nondurable goods sector δ_n and the capital depreciation rate in durable goods sector δ_d are 0.09 and 0.13, respectively. The deprecation rate of houses δ_h is equal to 0.08. This value is calibrated to match the ratio of aggregate values of houses to GDP implied by the data which is around 1.20. Both of the adjustment cost for capital in nondurable goods sector ϕ_n and that in durable goods sector ϕ_d are set to 0.01 to match the volatility of investments in both sectors. The adjustment cost for housing ϕ_h is equal to 0.02 which is calibrated to match the volatility in the housing sector.

[Please insert Table 4.1 here]

Next, I determine the values for preference parameters. The household's subjective factor is set to equal to 0.94. Given the share of private consumption to GDP is around 0.70 and the ratio of house values to GDP is equal to 1.20, the consumption share in the utility function ξ and the leisure share in the utility function ϕ are equal to 0.80 and 1.43, respectively.

Following [Hornstein and Praschnik, 1997], sector-specific productivities in the nondurable goods sector and in the durable goods sector are calibrated by using observations on value-added and primary inputs only. Then, the productivity shocks are defined by

$$
\begin{aligned}
z_{n,t} &= [\log y_{n,t} - \alpha \log k_{n,t} - (1-\alpha) \log n_{n,t}]/(1-\alpha). \\
z_{d,t} &= [\log y_{ht} - \gamma(1-\chi) \log k_{d,t} - (1-\gamma)(1-\chi) \log n_{n,t}]/[(1-\gamma)(1-\chi)].
\end{aligned}
$$

Thus, the productivity shock processes are:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}.$$

where $z_t = [z_{n,t}, z_{d,t}]'$, $\varepsilon_t = [\varepsilon_{n,t}, \varepsilon_{d,t}]'$. Following [Hornstein and Praschnik, 1997], the autocorrelation matrix ρ and the covariance matrix Σ [3] are given by

$$\rho = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix} \qquad \Sigma = \begin{pmatrix} 0.02^2 & 0.023^2 \\ 0.023^2 & 0.05^2 \end{pmatrix}$$

4.4 Results

Table 4.2-4.3 present the results of correlations and volatilities of variables in the business cycle. The first column reports the values predicted by the model. The second column in each table shows the results in [Davis and Heathcote, 2005]. The third column shows the results of an earlier version of the model in [Davis and Heathcote, 2005] with adjustment cost. This version of the model is presented to show the role of adjustment costs in generating comovement. The last column reports the corresponding observed values.

4.4.1 The Impulse Response Function

Impulse Response Functions to a Shock in the Nondurable Goods Sector

Figure 4.1 presents the impulse response functions of labor and capital in each sector to a productivity shock in the nondurable goods sector. It shows that a posi-

[3] This matrix implies a correlation of 0.529 between the innovations.

tive productivity shock increases capital and labor input in both the nondurable and durable goods sectors. The response of capital and labor to the productivity shock is more pronounced in the durable goods sector since the durable goods sector is smaller than the nondurable goods sector. A positive shock to productivity in the nondurable goods sector increases the productivity in the nondurable goods sector. If there are no intermediate inputs in production, capital and labor inputs in the durable goods sector would shift to the nondurable goods sector. As a result, capital and labor inputs in the durable goods sector decrease. However, if the intermediate inputs are introduced to produce the durable goods, an increase in the output of nondurable goods would decrease the cost of intermediate inputs. The demand for intermediate inputs in the durable goods sector would hence increase, which would further increase the marginal return of capital and labor in the durable goods sector. Therefore, capital and labor inputs in both sectors will increase after a positive productivity shock in the nondurable goods sector.

[Please insert Figure 4.1 here]

Impulse Response Functions to a Shock in Durable Goods Sector

Figure 4.2 shows the impulse response functions of labor and capital in each sector to a productivity shock in the durable goods sector. Similar to the previous case, a positive productivity shock in the durable goods sector also increases capital and labor in both the nondurable goods and durable goods sectors. A positive productivity shock in the durable goods sector increases the productivity in the durable goods sector.

Therefore, capital and labor in the nondurable goods sector would shift to the durable goods sector. On the other hand, an increase in the productivity in durable goods sector also increases the demand for intermediate inputs. Hence, capital and labor in the nondurable goods sector also increase.

[Please insert Figure 4.2 here]

4.4.2 Correlation

The impulse response functions, analyzed in the previous sub-section, show how the introduction of intermediate inputs in the production function could solve the comovement puzzle. The quantitative results in Table 4.2 show that consumption c_t, GDP, nonresidential investment, residential investment, and house prices $p_{h,t}$ move in the same direction in the business cycle. The model match the empirical data on comovement as well as the model proposed by [Davis and Heathcote, 2005]. The correlations between GDP and consumption and between GDP and house prices are 0.87 and 0.72 respectively. These values are very close to the empirical data. Moreover, the model predicts a positive correlation between house price p_h and residential investment y_h. This correlation occurs owing to the introduction of intermediate inputs to produce durable goods and the introduction of adjustment costs for capital and houses. The inclusion of intermediate inputs leads to the movement of output of both nondurable goods and durable goods in the same direction instead of a transfer of resources from one sector to the other. The increase in output further increases household income and the demand for housing. If the supply curves of houses remains unchanged, house price will increase. On the other hand, a positive productivity increases the supply of housing, which further decreases house prices. The introduction of adjustment costs makes investment in houses more costly and lowers the demand for houses, which constrains the increase in house supply after a positive shock. Therefore, changes in house prices and residential investment move along the supply curve, which predicts a positive correlation between house prices and residential investment. For plausible parameter values in the model, house prices and residential investment move in the same direction. The model also predicts a positive correlation between investment in the nondurable sector and investment in the durable goods sector because of intermediate inputs and adjustment costs. The importance of adjustment cost to generate comovement can also be found in the results of the Davis and Heathcote model with adjustment costs, which reports a positive correlation between house prices and residential investment.

[Please insert Table 4.2 here]

4.4.3 Volatilities

Table 4.3 presents the results of the volatilities of variables in the business cycle. Compared to empirical data, the model predicts lower volatility in consumption, labor and house prices. The model shows that the relative volatilities in consumption, labor and house price are 0.55, 0.44 and 0.46 respectively. The corresponding values from empirical data are 0.78, 1.01 and 1.40 respectively. This model could not explain the high volatility in house prices since the model does not include productivity shocks in the housing sector.

[Please insert Table 4.3 here]

The model, nonetheless, succeeds in reproducing the volatility in nonresidential investment and the volatility in residential investment. Specifically, residential investment is more than twice as volatile as the investment in the nonresidential investment because the calibrated standard deviation of the shock in the durable goods sector is more than twice as high as the shock in the nondurable sector and the introduction of adjustment costs for capital and housing that lower the predicted volatility in nonresidential investment and residential investment to match the empirical data. The predictions of this model are also similar to both versions of the model by Davis and Heathcote.

4.5 Conclusion

The recent financial crisis due to the collapse of the subprime mortgage market in 2008 provides the motivation for research on the business cycle properties of the housing market. This paper examined the business cycle properties of the housing market using a three-sector model with intermediate inputs and adjustment costs for capital and housing. The quantitative results replicate the three main observed features of the housing market over the business cycle. First, GDP, consumption, nonresidential investment, and residential investment move in the same direction over the business cycle. Second, residential investment is more than twice as volatile as nonresidential investment. Finally, I find a positive correlation between house prices and residential investment. Compared to [Davis and Heathcote, 2005], which also works on this topic, I derive similar results by using a general form of production function for the final goods.

Although this model works well in explaining several facets of the business cycle with respect to houses, it falls short in explaining why house prices are more volatile than GDP and why residential investment leads nonresidential investment. These two features of the housing market should be examined in future studies.

Figure 4.1: Impulse Response to a Shock from nondurable Goods Sector

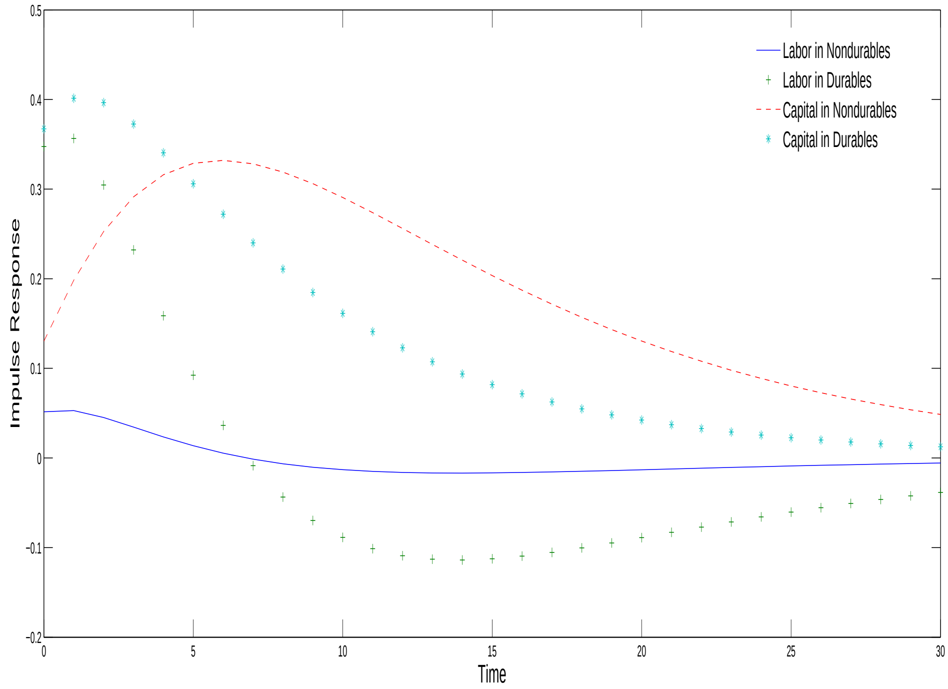

Figure 4.2: Impulse Response to a Shock from Durable Goods Sector

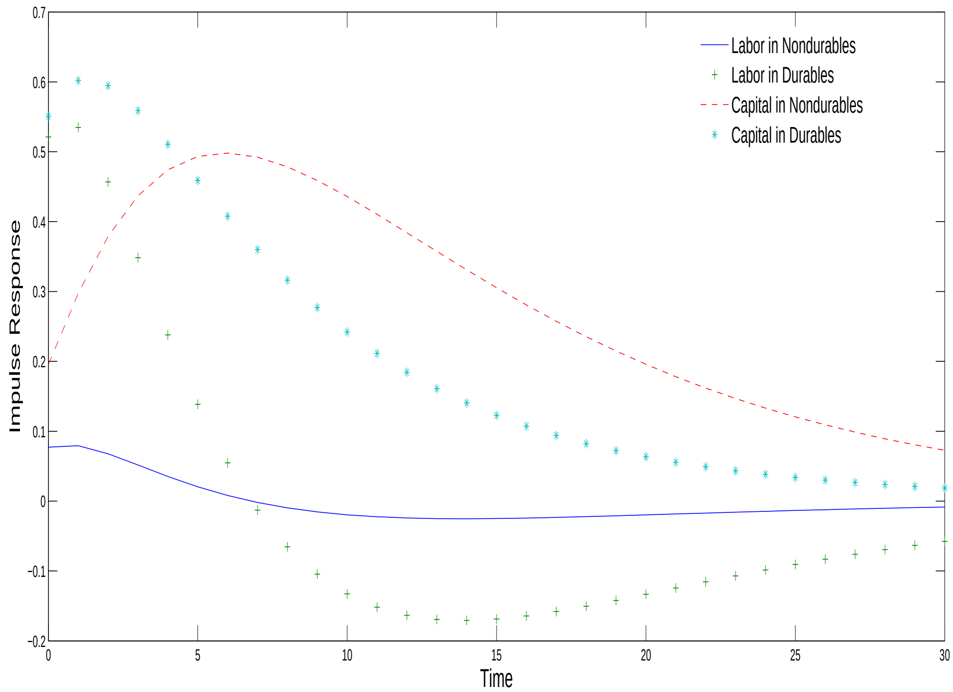

Table 4.1: Parameterization

Description	Parameters	Value
Tax rate		
Labor	τ_n	0.30
Capital	τ_k	0.50
Preference		
Discount factor	β	0.94
Consumption share in utility	ξ	0.80
Leisure share in utility	ϕ	1.43
Technology for nondurable Goods		
Capital income share	α	0.36
Technology for durable goods		
Capital share	γ	0.30
Intermediate input share	χ	0.40
Scale coefficient	ζ	1.35
Technology for housing		
Structure share	ν	0.90
Depreciation rate		
Capital in nondurables sector	δ_n	0.09
Capital in durables sector	δ_d	0.13
Houses	δ_h	0.08
Adjustment cost		
Capital in nondurables sector	ϕ_n	0.01
Capital in durables sector	ϕ_d	0.01
Houses	ϕ_h	0.02

Table 4.2: Correlations

Variables	Model	D&H	D&H(AC)	Data
(GDP, c)	0.87	0.95	0.98	0.80
(GDP, p_h)	0.72	0.65	0.88	0.65
(p_h, y_h)	0.20	-0.20	0.36	0.34
(i_d, i_n)	0.30	0.15	0.45	0.25

Notes: D&H refers to results in [Davis and Heathcote, 2005]. D&H(AC) represents the model of Davis and Heathcote with adjustment cost.

Table 4.3: Standard Deviations in Ratio to GDP

Variables	Model	D&H	D&H(AC)	Data
Output(GDP)	-	1.73	1.63	2.26
Consumption(c)	0.55	0.48	0.60	0.78
Labor(N)	0.44	0.41	0.30	1.01
Investment(i)				
Business investment(i_b)	2.01	3.21	2.31	2.30
Residential(i_h)	5.08	6.12	5.43	5.04
House price(p_h)	0.46	0.40	0.55	1.40

Appendix A

Appendix of Chapter 2

A.1 Household Problem

Given $\{\mu_t, q_t, r_t, w_{i,t}, \kappa\}_{t=0}^{\infty}$, a type-$i$ household chooses $\{c_{i,t}, l_{i,t}, s_{i,t}, a_{i,t+1}, e_{i,t+1}\}_{t=0}^{\infty}$ to maximize its life-time utility subject to (2.1) and (2.2). Let $\zeta_{i,t}$ and $\psi_{i,t}$ be the Lagrange multipliers corresponding to (2.1) and (2.2), respectively. Set up the Lagrangian as

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta_i^t \{U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) + \psi_{i,t}[y_{i,t} - T(y_{i,t}) + a_{i,t} - c_{i,t} - a_{i,t+1}] \\ &\quad + \zeta_{i,t}[(1-\phi_i)e_{i,t} + s_{i,t}\mu_t(1-e_{i,t}) - e_{i,t+1}]\}.\end{aligned}$$

where $y_{i,t} \equiv \eta_i w_{i,t} l_{i,t}[e_{i,t} + \rho(1-e_{i,t})] + r_t a_{i,t}$. Given the tax code $\tau(\bar{y}_{i,t}) = \sigma(\frac{y_{i,t}}{y_t})^{\varrho}$. Denote $\bar{y}_{i,t} = y_{i,t}/y_t$ as the ratio of household i income to average aggregate income $y_t = \sum_{i=1}^{M} \lambda_i y_{i,t}$ at time t. Therefore, $T(y_{i,t}) = \tau(\bar{y}_{i,t})y_{i,t}$ and $T'(y_{i,t}) = \sigma(1+\varrho)(\frac{y_{i,t}}{y_t})^{\varrho}$.

The first-order conditions with respect to $\{c_{i,t}, l_{i,t}, s_{i,t}, a_{i,t+1}, e_{i,t+1}\}_{t=0}^{\infty}$ are

$$\frac{\partial \mathcal{L}}{\partial c_{i,t}} = \beta_i^t [U_c(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) - \psi_{i,t}] = 0. \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial l_{i,t}} = \beta_i^t \{U_l(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) + \psi_{i,t}[1 - T'(y_{i,t})]\eta_i w_{i,t}[e_{i,t} + \rho(1 - e_{i,t})]\} = 0. \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial s_{i,t}} = \beta_i^t \{U_s(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) + \zeta_{i,t} d_{i,t} \mu_t\} = 0. \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial a_{i,t+1}} = \beta_i^t(-\psi_{i,t}) + \beta_i^{t+1}\psi_{i,t+1}\{[1 - T'(y_{i,t+1})]r_{t+1} + 1\} = 0. \tag{A.4}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial e_{i,t+1}} &= \beta_i^{t+1}\{U_e(c_{i,t+1}, l_{i,t+1}, s_{i,t+1}, e_{i,t+1}) + \zeta_{i,t+1}(1 - \phi_i - s_{i,t+1}\mu_{t+1}) \\ &+ \psi_{i,t+1}[1 - T'(y_{i,t+1})]\eta_i w_{i,t+1} l_{i,t+1}(1 - \rho)\} - \beta_i^t \zeta_{i,t} = 0.\end{aligned} \tag{A.5}$$

Given the utility function of the following form:

$$U(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) = \ln c_{i,t} + \varpi_1 \frac{(1 - l_{i,t})^{1-\omega}}{1 - \omega} e_{i,t} + \varpi_2 \frac{(1 - s_{i,t})^{1-\omega}}{1 - \omega}(1 - e_{i,t}).$$

then $U_c = 1/c_{i,t}$, $U_l = -\varpi_1(1 - l_{i,t}^{-\omega} e_{i,t}$, $U_s = -\varpi_2(1 - s_{i,t})^{-\omega}(1 - e_{i,t})$ and $U_e = \varpi_1 \frac{(1-l_{i,t})^{1-\omega}}{1-\omega} - \varpi_2 \frac{(1-s_{i,t})^{1-\omega}}{1-\omega}$.

Substituting (A.1) into (A.2) gives,

$$\varpi_1(1 - l_{i,t})^{-\omega} e_{i,t} = [1 - T'(y_{i,t})]\eta_i w_{i,t}[e_{i,t} + \rho(1 - e_{i,t})]/c_{i,t}. \tag{A.6}$$

Note that this is the standard equation governing the labor supply of employed workers. In particular, the expression $[1 - T'(y_{i,t})]\eta_i w_{i,t}$ is the after-tax wage rate for raw labor. From (A.3), I have

$$\zeta_{i,t}\mu_t = \varpi_2 (1 - s_{i,t})^{-\omega}.$$

Inserting (A.1) into (A.4) gives the standard Euler equation for consumption

$$U_c(c_{i,t}, l_{i,t}, s_{i,t}, e_{i,t}) = \beta_i\{[1 - T'(y_{i,t+1})]r_{t+1} + 1\}U_c(c_{i,t+1}, l_{i,t+1}, s_{i,t+1}, e_{i,t+1}). \quad (A.7)$$

Rewrite (A.5) as

$$\begin{aligned} \frac{\varpi_2 \left(1 - s_{i,t}\right)^{-\omega}}{\mu_t} \quad = \quad & \beta_i \left\{ \frac{\varpi_1 \left(1 - l_{i,t+1}\right)^{-\omega}}{1 - \omega} \left[1 - \frac{\omega e_{i,t+1} \left(1 - \rho\right) + \rho}{e_{i,t+1} \left(1 - \rho\right) + \rho} l_{i,t+1}\right] \right. \quad (A.8) \\ & \left. + \varpi_2 \left(1 - s_{i,t+1}\right)^{-\omega} \left[\frac{1 - \phi_i}{\mu_{t+1}} - \frac{1}{1 - \omega} + \frac{\omega}{1 - \omega} s_{i,t+1}\right] \right\}. \end{aligned}$$

A.2 The Firm

Given the return for capital r_t, a firm chooses its capital input according to

$$r_t = f_1\left(k_{i,t}, \eta_i l_{i,t}\right) = \alpha k_{i,t}^{\alpha-1} \left[\eta_i l_{i,t}\right]^{1-\alpha}, \quad \textit{for all } i \in \{1, 2, ..., M\}$$

A firm's value can be written as

$$J_{i,t} = \max_{k_{i,t}} \left\{ \pi_{i,t} + \frac{1}{1 + r_t} \left[\left(1 - \phi_i\right) J_{i,t+1} + \phi_i V_{t+1}\right] \right\}, \quad i \in \{1, 2, ..., M\}$$

where $\hat{k}_{i,t} = k_{i,t}/\left(\eta_i l_{i,t}\right) = \left(r_t/\alpha\right)^{1/(\alpha-1)}$, $\hat{Y}_{i,t} = \left(r_t/\alpha\right)^{\alpha/(\alpha-1)}$ and $\pi_{i,t} = Y_{i,t} - r_t k_{i,t} - w_{i,t}\eta_i l_{i,t}$.

A.3 The Bargaining Problem

Denote $W\left(a_{i,t}, e_{i,t}\right)$ as household i's value function. Thus, by increasing an extra unit of employment, a type-i household's benefit is equal to $W_e\left(a_{i,t}, e_{i,t}\right)$. According to the envelope theorem

$$\frac{W_e\left(a_{i,t},e_{i,t}\right)}{U_c\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)} = \left\{\left[1-T'\left(y_{i,t}\right)\right]w_{i,t}\eta_i l_{i,t}(1-\rho)+\frac{U_e\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)}{U_c\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)}+\right.$$
$$\left.\frac{U_s\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)}{U_c\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)(1-e_{i,t})}\left(\frac{1-\phi}{\mu_t}-s_{i,t}\right)\right\}.$$

where $\left[1-T'\left(y_{i,t}\right)\right]w_{i,t}$ is wage income from an extra increase in employment; the second and third terms of the above equation on the right hand side is interpreted as household's reservation wage. On the other hand, a firm, matched with a type-i worker, has a value of $J_{i,t}$ at time t.

A matched firm and household bargain over the wage $w_{i,t}$ to maximize the following value

$$\max_{w_{i,t}}\left[\frac{W_e\left(a_{i,t},e_{i,t}\right)}{U_c\left(c_{i,t},l_{i,t},s_{i,t},e_{i,t}\right)}\right]^{\epsilon}\left[J_{i,t}-V_t\right]^{1-\epsilon}.$$

The corresponding first order condition determines wage $w_{i,t}$

$$\epsilon J_{i,t}\left\{[1-T'(y_{i,t})]-\varrho T'(y_{i,t})\frac{w_{i,t}\eta_i l_{i,t}[e_{i,t}+\rho(1-e_{i,t})]}{y_{i,t}}\right\}(1-\rho)=(1-\epsilon)[\frac{W_e(a_{i,t},e_{i,t})}{U_c(c_{i,t},l_{i,t},s_{i,t},e_{i,t})}].$$

A.4 Computational Methods;

The numerical method for solving the model along the steady state is provided below.

Step 1: Given a specific tax code and households' subject discount factors, relative income is endogenously determined by the following equation.

$$\bar{y}_i=[\frac{r\beta_i+\beta_i-1}{r\beta_i\sigma(1+\varrho)}]^{1/\varrho}\Rightarrow\sum_{i=1}^{M}\lambda_i\bar{y}_i=1.$$

The sum of relative income of each group must equal to 1, which further determines the interest rate.

Step 2: Given the interest rate, I select an initial guess of the job finding rate μ_0. Then, the bundle $\{r, \mu_0\}$ is used to describe the aggregate state of the model economy.

Step 3: Given $\{r, \mu_0\}$, I can proceed to solve households' problem and firms' problem. However, it is difficult to solve these problems since $\bar{y}_i$ is the relative income of a type-i household, i.e., instead of solving individual's problem separately I have to calculate all the equations simultaneously even aggregate prices are given. To simplify the calculation process, one way to go is to guess an aggregate income y^0, from which I can derive individual's choices over hours of work l_i, employment-population ratio e_i and wage rate w_i separately.

Next, by aggregating individual variables, I can find equilibrium in asset market by examining the following equation

$$\Delta_y = v\kappa + r\sum_{i=1}^{M}\lambda_i a_i - \sum_{i=1}^{M}\lambda_i e_i\left[\left[\frac{r+\delta_k}{\alpha}\right]^{\alpha/(\alpha-1)} - w_i\right]\eta_i l_i,$$

If $\Delta_y > 0$, then asset supply is larger than asset demand, the initial guess of income y^0 is too high. I should repeat this step to find a lower y^1, and vice versa. If Δ_y is smaller than a certain tolerance level, I proceed to step 4.

Step 4: By studying the free entry condition for firms, equilibrium in the labor market can be find by using the following equation

$$\Delta_\mu = \kappa - \frac{q(\theta)}{r+\phi}\sum_{i=1}^{M}\widetilde{\lambda}_i\left[(1-\alpha)\hat{Y}_i - w_i\right]\eta_i l_i,$$

If $\Delta_\mu > 0$, it is profitable for the firm to entry. Firms create more vacancies which increases $q(\theta)$ and decrease μ. Therefore, a lower μ should be chosen and I repeat the above processes from step 2.

Step 5: If both Δ_y and Δ_μ are within a tolerance level, the equilibrium is found.

Appendix B

Appendix of Chapter 4

B.1 Mathematical Appendix

The mathematical appendix gives first-order conditions. Assume λ_t, η_t, μ_t and ψ_t are the Lagrange multipliers corresponding to the budget constraint, capital accumulation function in consumption good sector, capital accumulation function in structure sector and house accumulation function, respectively. The Lagranian is,

$$E_0 \sum_{t=0}^{\infty} \beta^t \{[\xi \ln c_t + (1-\xi) \ln h_t - \phi(n_{n,t} + n_{d,t})]$$

$$+ \lambda_t \{(1-\tau_n) w_t (n_{n,t} + n_{d,t}) + [(1-\tau_k) R_{n,t} + \tau_k \delta_n] p_{d,t} k_{n,t} \quad + [(1-\tau_k) R_{n,t} + \tau_k \delta_d] p_{d,t} k_{d,t}$$

$$+ p_{l,t} l_t + T_t - c_t - p_{d,t}(i_{n,t} + i_{d,t}) - p_{h,t} i_{h,t}\}$$

$$+ \eta_t p_{d,t} [(1-\delta_n) k_{n,t} + i_{n,t} - \Phi_n (\frac{i_{n,t}}{k_{n,t}}) k_{n,t} - k_{n,t+1}]$$

$$+ \mu_t p_{d,t} [(1-\delta_d) k_{d,t} + i_{d,t} - \Phi_d (\frac{i_{d,t}}{k_{d,t}}) k_{d,t} - k_{d,t+1}]$$

$$+ \psi_t p_{h,t} . [(1-\delta_h) h_t + i_{h,t} - \Phi_h (\frac{i_{h,t}}{h_t}) h_t - h_{t+1}]\}.$$

The first order conditions with respect to control variables $\{c_t, n_{n,t}, n_{d,t}, i_{n,t}, i_{d,t}, i_{h,t}\}$ and state variables $\{k_{n,t+1}, k_{d,t+1}, h_{t+1}\}$ are given by the following equations,

$$\frac{\xi}{c_t} - \lambda_t = 0. \tag{B.1}$$

$$-\phi + \lambda_t(1-\tau_n)w_t = 0. \tag{B.2}$$

$$\lambda_t - \eta_t[1 - \Phi_n'(\frac{i_{n,t}}{k_{n,t}})] = 0. \tag{B.3}$$

$$\lambda_t - \mu_t[1 - \Phi_d'(\frac{i_{d,t}}{k_{d,t}})] = 0. \tag{B.4}$$

$$\lambda_t - \psi_t[1 - \Phi_h'(\frac{i_{h,t}}{h_t})] = 0. \tag{B.5}$$

$$\begin{aligned} \eta_t p_{d,t} =& \beta E_t\{\lambda_{t+1} p_{d,t+1}[(1-\tau_k)R_{n,t+1} + \tau_k\delta_n] \\ &+ \eta_{t+1} p_{d,t+1}[1 - \delta_n - \Phi_n(\frac{i_{n,t+1}}{k_{n,t+1}}) + \Phi_n'(\frac{i_{n,t+1}}{k_{n,t+1}})\frac{i_{n,t+1}}{k_{n,t+1}}]\}. \end{aligned} \tag{B.6}$$

$$\begin{aligned} \mu_t p_{d,t} = & \quad \beta E_t\{\lambda_{t+1} p_{d,t+1}[(1-\tau_k)R_{d,t+1} + \tau_k\delta_d] \\ &+\mu_{t+1} p_{d,t+1}[1 - \delta_d - \Phi_d(\tfrac{i_{d,t+1}}{k_{d,t+1}}) + \Phi_d'(\tfrac{i_{d,t+1}}{k_{d,t+1}})\tfrac{i_{n,t+1}}{k_{d,t+1}}]\}. \end{aligned} \tag{B.7}$$

$$\psi_t p_{h,t} = \beta E_t\{\frac{1-\xi}{h_{t+1}} + \psi_{t+1} p_{h,t+1}[1 - \delta_h - \Phi_h(\frac{i_{h,t+1}}{h_{t+1}}) + \Phi_h'(\frac{i_{h,t+1}}{h_{t+1}})\frac{i_{h,t+1}}{h_t}]\}. \tag{B.8}$$

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