

# UC Berkeley

## UC Berkeley Previously Published Works

**Title**

A Dynamic Model of Repositioning

**Permalink**

<https://escholarship.org/uc/item/3c3925jq>

**Journal**

Marketing Science, 37(2)

**ISSN**

0732-2399

**Author**

Villas-Boas, J Miguel

**Publication Date**

2018-03-01

**DOI**

10.1287/mksc.2017.1075

Peer reviewed

# A DYNAMIC MODEL OF REPOSITIONING\*

J. MIGUEL VILLAS-BOAS  
*(University of California, Berkeley)*

October, 2017

---

\* Comments by John Hauser and Birger Wernerfelt on an earlier version of this paper are gratefully appreciated. I also thank Eddie Ning for superb research assistance. All remaining errors are my own. E-mail address: villas@haas.berkeley.edu.

# A DYNAMIC MODEL OF REPOSITIONING

## ABSTRACT

Consumer preferences change through time and firms must adjust their product positioning for their products to continue to be appealing to consumers. These changes in product positioning require fixed investments such that firms engage in these repositionings only once in a while. I construct a model that can include both predictable and unpredictable consumer preference changes, and where a firm optimally repositions its product given the current market conditions, and expected future repositionings. When unpredictable consumer preferences evolve away from a current firm's positioning, the decision to reposition is like exercising an option to be closer to the current consumer preferences, or wait to reposition later or for the consumer preferences to return so as to be closer to the current firm's positioning. We can characterize this optimal repositioning strategy, how it depends on the discount factor, variance of preferences, and costs of repositioning. I compare the optimal policy of the firm with what could be optimal from a social welfare point of view, and find that the firm repositions more frequently than what is efficient when there is full market coverage. With predictable changes in consumer preferences, the optimal repositioning strategy involves over-shooting and asymmetric repositioning thresholds.

## 1. INTRODUCTION

Consumer preferences change over time and firms have to adjust their product positioning to continue to be closer to what the consumers prefer. At the same time these changes in product positioning require fixed investments and cannot be optimally done for the product to continuously match the consumer preferences. That is, firms decide to reposition only once in a while when consumer preferences are sufficiently far away from what the product is offering. In fact, we see firms changing their products, packages, logos, or positioning communication once in a while. For example, Morton Salt has changed its package every few years, and car manufacturers make a major re-design of their models every three to five years (e.g., BMW, Honda, Mercedes, Toyota).<sup>1</sup> Most companies also adjust their basic logo every few years (e.g., Apple, IBM). Some of these changes can be seen as having the products or communications adhere to the changing styles of the times. For example, a necktie manufacturer may have to adjust its neckties' length and width to match style changes over time, and these styles go back and forth over time. Another example of this back and forth variation over time is in the length of skirts, or, generally, what is fashionable in clothing design over time.

Consumer preference variations involve predictable and unpredictable changes. There can be trends in how preferences are changing, but the exact way in which preferences change at any given moment in time may not be known. When repositioning, firms must then be aware that they may again have to reposition in the future, and consumer preferences may potentially come back to where the firm is positioned. This provides an incentive for firms to only reposition when the consumer preferences are sufficiently far away.<sup>2</sup>

I construct a continuous-time model that takes fully into account this option of when to reposition, while considering the possible future repositionings as well. At the time of repositioning, a firm trades off the benefits of being in the center of the market against the fixed costs of repositioning. Consider first the case with no trends in the consumer preferences. The optimal repositioning strategy involves then a threshold such that if the consumer preferences are sufficiently far away from the firm's current positioning, the firm chooses to reposition to the center of the consumer preferences. This threshold is greater the greater is the discount rate,

---

<sup>1</sup>In the car manufacturing industry, model updates can include the latest technological developments (which could be interpreted as preference for more technology), but also include important redesign features.

<sup>2</sup>We consider a one-dimension model of consumer preferences and repositionings which can be seen as somewhat simplified for the potential multidimensional "style" changes in the examples mentioned above. One could also think of projecting several potential dimensions into one dimension. For example, in the car example, it could be a dimension of sporty versus functional.

as in that case the present value of the benefits of repositioning are lower. The firm adjusts by repositioning less often.

Consider the effect of the variance of the process by which the consumer preferences change. If that variance is greater, the threshold of repositioning becomes larger, as now the firm becomes more hopeful that consumer preferences return to where the firm's positioning is. This effect of the threshold of repositioning being greater with a greater uncertainty of the consumer preferences is smaller when the variance is greater. This is because being too far away from the consumer preferences becomes too costly, and the firm has greater incentive to reposition. Interestingly, we can obtain that the expected time between repositionings is lower the greater the variance of preferences. That is, with a greater variance of consumer preferences, the adjustment of the greater threshold is not enough to overcome the effect of getting faster to a threshold, and the firm must reposition more frequently.

The effect of the costs of repositioning on the threshold to reposition is also monotonically increasing (as one would expect). More interestingly, this effect occurs at a decreasing rate as when the consumer preferences are too far away from the firm's positioning, it becomes too costly not to reposition. Obviously, if the costs of repositioning are too high, then the firm chooses to never reposition. An interesting possibility is that the costs of repositioning can be increasing in the distance by which the product is repositioned. For example, repositioning a product a short distance can involve lower costs in product re-design and communication than repositioning a product over a greater distance. The paper also explores this effect, showing that with the costs of repositioning increasing in the distance travelled, with the total costs of repositioning fixed, the firm may choose to reposition more frequently, and does not reposition all the way to the center of the market.

When the market is sometimes partially covered, the optimal repositioning strategy involves less frequent repositionings when the consumer heterogeneity is greater. This can be seen as consistent with the possibility of frequent repositionings in the early days of a new product category, when potentially consumer preferences are less heterogeneous, with less frequent repositionings later on, when the product category is more established, and the consumer preferences may be more heterogeneous.<sup>3</sup>

We can also compare the firm's optimal behavior with what would be optimal from a social

---

<sup>3</sup>Obviously, many other factors are present when a product category evolves over time, including potentially decreasing variance in the evolution of consumer preferences (which is included in the model, and would be consistent with a similar pattern), and competition (which is not included here, and is beyond the scope of this paper).

welfare point of view. We find that the firm ends up repositioning too frequently in comparison to what would be optimal from a social welfare point of view when we are in a situation where the market is always fully covered. The intuition is that the firm's profits fall more steeply than social welfare when the consumer preferences move away from where the product is positioned. This then gives incentives for the firm to reposition sooner than what would be optimal in terms of social welfare.

When there are trends in the consumer preferences the firm has two different thresholds, depending on which direction the trends in the consumer preferences are going. When the consumer preferences are trending away from where the firm is positioned the firm is less tolerant and repositions sooner. When the consumer preferences are trending towards where the firm is positioned, the firm only repositions if it is really too far away from where the consumer preferences are. In this case, with trends in consumer preferences, the optimal repositioning is to over-shoot the current consumer preferences. This way the consumer preferences will trend to the firm's new positioning, and the firm will save on repositioning costs. In markets where technology is a major component of the repositioning decision, anecdotal evidence suggests that newer products come with more features than most consumers may demand in the short run, which may be seen as over-shooting in the product's repositioning. In this case, with trends in consumer preferences, we fully solve analytically the case with no uncertainty. We also show that the degree of over-shooting and the two thresholds increase at a decreasing rate on the intensity of the trend, and on the costs of repositioning. We also present simulations for the case in which there are both trends and unpredictable changes in the consumer preferences.

There has been substantial research on static positioning in markets (e.g., Hauser and Shugan 1983, Moorthy 1988, Hauser 1988, Sayman et al. 2002, Kuksov 2004, Lauga and Ofek 2011, Hauser et al. 2016), with particular focus on the competitive interaction. There is also work on the effects of the resources of the firms on their strategic positioning (e.g., Wernerfelt 1989). With dynamics, there is work on investments in R&D (e.g., Harris and Vickers 1987, Ofek and Sarvary 2003) that generates with a certain probability success in the repositioning of the product. In contrast, this paper allows for the decision to reposition to have immediate effects, and therefore the timing of when to reposition a product becomes the crucial decision. A similar decision to the one considered here is the one of adoption of new technologies, and when to adopt, which is considered in a two-state version in Villas-Boas (1992). This paper considers a richer, uncertain environment, where the decision when to reposition is investigated in greater depth for the monopoly setting. Another related stream of work considers richer environments of dynamic competition in R&D among firms that is presented for empirical work and which can be

solved with numerical methods (e.g., Ericson and Pakes 1995, and, in particular with dynamic repositioning, Sweeting 2013, Jeziorski 2014). In relation to that work, this paper presents in a monopoly setting sharper analysis of when to reposition, and how that decision depends on the degree of uncertainty in the market, the discount rate, and any market trends.<sup>4</sup>

The remainder of the paper is organized as follows. The next section presents the general set-up of consumer preferences and how their changes affect profits. Section 3 presents the case when there are no trends in consumer preferences, and all the changes in consumer preferences are unpredictable. Section 4 introduces the possibility of trends in consumer preferences and presents what happens when all changes in consumer preferences are deterministic. Section 5 presents simulations on the optimal policy when consumer preferences have both predictable and unpredictable changes. Section 6 concludes.

## 2. MARKET SET-UP

Consider a market where consumer preferences are described by the location of consumers on the real line, where a consumer located at a point on the line values a product located at a distance  $x$  from the consumer as  $v - x$  where  $v$  is assumed to be large. Consumer preferences are distributed uniformly on a segment of distance  $2L$  with midpoint  $a$ . The mass of consumers is one. The product cannot be stored and can be consumed in every period.

There is one firm offering one product in this market. Let  $\ell$  be the product's location, and let  $z \equiv \ell - a$ , such that the distance between the product location and the midpoint of the consumer preferences is  $|z| = |\ell - a|$ . Figure 1 illustrates the positioning of the firm, and the location of consumer preferences in the real line. If the firm charges a price  $P = v - L - |z|$  it attracts all consumers. If  $v$  is large enough ( $v > 3L + |z|$ ), charging this price is optimal, and the profit is  $\pi(z) = v - L - |z|$ . The firm cannot price discriminate between consumers.

If the product is positioned at the center of the consumer preferences,  $\ell = a$ , the profit is the largest possible and equal to  $\pi = v - L$ .

Now consider that the center of consumer preferences,  $a$ , changes over time, either to the right or to the left. At some point, if  $a$  gets too far away from the firm's positioning  $\ell$ , the distance  $z$  gets to be too high, and the firm has to charge too low a price to attract all consumers, and

---

<sup>4</sup>See also, for example, Shen (2014) for empirical analysis of dynamic entry and exit in a growing industry. Also related to this paper is the literature on portfolio choice with transaction costs, where an investor only adjusts the portfolio once in a while because of transaction costs and the portfolio evolves stochastically (e.g., Magill and Constantinidis 1976), and the literature on (S,s) economies from inventory problems (e.g., Scarf 1959, Sheshinski and Weiss 1983).

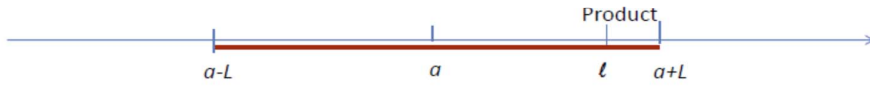


Figure 1: Illustration of consumer preferences on the real line centered at  $a$ , with length  $2L$ , and with the product positioning at  $\ell$ .

ends up getting profits that are quite low. That is, if  $a$  moves sufficiently far away from  $\ell$ , the firm decides to reposition the product, paying a fixed cost  $K$ , and moving to a new positioning  $\ell'$ , which is relatively close to where  $a$  is. The firm has to make this decision, taking into account that consumer preferences will continue to evolve over time, and that future repositionings will again be needed.<sup>5</sup> The question that we want to address is when is it optimal for the firm to reposition, and, when repositioning, to where should the firm reposition.

Consider now that the center of consumer preferences,  $a$ , evolves continuously over time as a Brownian motion:

$$da = b dt + \sigma dw, \tag{1}$$

where  $dw$  is the standardized Brownian motion,  $b dt$  represents the deterministic component of how  $a$  evolves, and  $\sigma dw$  represents the random component of how  $a$  evolves. The parameter  $b$  represents the speed and direction at which the firm expects the consumer preferences to evolve

---

<sup>5</sup>We assume that the firm can choose to reposition at any moment in time. Alternatively, one could have a model where firms can only reposition at some random occasions (see, e.g., Calcagno et al. 2014).



over time. The parameter  $\sigma$  represents the randomness of how  $a$  evolves. In the next section we restrict attention to the case when  $b = 0$ , such that  $a$  only evolves at random. In Section 4 we consider the case where  $b \neq 0$  but  $\sigma = 0$ , such that  $a$  only evolves deterministically. Section 5 considers numerically the case of when both  $b$  and  $\sigma$  are different from zero.

Per the definition of  $z$ , we then have that while the product is not repositioned the evolution of  $z$  is

$$dz = -b dt + \sigma dw. \tag{2}$$

When the firm chooses to reposition its product  $z$  moves instantly to where the firm wants to reposition it. The firm has the option to reposition or wait for the preferences to return to where the firm is. If preferences move too far from where the firm is positioned, the firm chooses to exercise its option to reposition, knowing that further repositionings will be necessary in the future.

### 3. PURELY RANDOM EVOLUTION OF CONSUMER PREFERENCES

Consider first the case in which all changes in consumer preferences are unpredictable,  $b = 0$ . In this case, when the firm repositions it choose to reposition to  $\ell = a$ , which means  $z = 0$ . That is, when the firm repositions, it chooses to reposition to the center of the market,  $a$ , as the market is equally likely to evolve in either direction.

Because the market is equally likely to evolve in either direction, we also have that the threshold  $a$ , at which point the firm chooses to reposition, is equally distant from  $\ell$  in both directions. That is, there is going to be a  $\Delta$ , such that when the distance between  $a$  and  $\ell$  is  $\Delta$ , the firm chooses to reposition. That is, the firm chooses to reposition when  $|z| = \Delta$ , where  $\Delta$  has to be optimal for the firm. Let us consider the case of  $v > 3L$ , such that the market is fully covered for  $|z|$  small, and  $v$  large, compared with  $K$  ( $K < \tilde{K}$  where  $\tilde{K}$  is defined in (i)), such that at the optimal policy the firm is always keeping the market fully covered on the equilibrium path.<sup>6</sup>

Let  $V(z)$  be the expected net present value of profits when the firm is located at a point  $z$  with respect to the center of the market (as noted above  $z \equiv \ell - a$ ). Then when the firm is not repositioning  $V(z)$  can be written as

---

<sup>6</sup>We also consider later in this section the case of  $K$  large, such that the market is not always fully covered. If  $v \in (2L, 3L)$ , the market is fully covered if  $z = 0$ , and otherwise is partially covered (see Appendix). For  $v < 2L$  the market is always partially covered. This latter case is available in the Online Appendix.

$$V(z) = \pi(z) dt + e^{-r dt} EV(z + dz), \quad (3)$$

where  $r$  is the instantaneous discount rate. Doing a Taylor approximation of  $V(z + dz)$  and applying Itô's Lemma we can get

$$V(z) = \pi(z) dt + e^{-r dt} [V(z) + V'(z)E(dz) + V''(z)\frac{E(dz^2)}{2}].$$

Using the fact that  $E(dz) = 0$ , and  $E(dz^2) = \sigma^2 dt$ , we can then divide by  $dt$  and make  $dt \rightarrow 0$ , to obtain

$$rV(z) = \pi(z) + \frac{\sigma^2}{2}V''(z). \quad (4)$$

With  $V(z)$  being the value of the firm, equation (4) states that the return on the asset (the left hand side of (4)) is equal to the flow payoff,  $\pi(z)$ , plus the expected value of the capital gain,  $\frac{\sigma^2}{2}V''(z)$ , which is positive because the function  $V(z)$  is convex (as the firm has an option to reposition if the consumer preferences move too far away from the current product positioning).

As  $\pi(z) = v - L - |z|$  and  $V(z) = V(-z)$  by symmetry, we can obtain the solution to the differential equation (4) as

$$V(z) = C_1 e^{\sqrt{\frac{2r}{\sigma^2}}|z|} + C_2 e^{-\sqrt{\frac{2r}{\sigma^2}}|z|} - \frac{|z|}{r} + \frac{v - L}{r}, \quad (5)$$

where  $C_1$  and  $C_2$  are two constants still to be determined.

When the firm chooses to reposition at distance  $\Delta$  from the center of the market we need to have

$$V(\Delta) = V(0) - K \quad (6)$$

and the "smooth-pasting condition" at distance  $\Delta$  (see, e.g., Dixit 1993)

$$V'(\Delta) = 0. \quad (7)$$

Condition (6) says that when the firm decides to reposition, the firm is just indifferent between repositioning to the center of the market, getting the present value of profits  $V(0)$ , while paying the repositioning costs  $K$ , and continuing at distance  $\Delta$  from the center of the market without repositioning. If the left hand side of (6) were greater than the right hand side, then the firm would be better off not repositioning and  $\Delta$  would not be the repositioning threshold. If the left

hand side of (6) were smaller than the right hand side, then the firm should have repositioned before getting to the distance  $\Delta$  from the center of the market, and then  $\Delta$  would not be the repositioning threshold. Condition (7) just states that  $\Delta$  is the optimal repositioning threshold. Furthermore, the “smooth-pasting condition” has to hold when the firm is at the center of the market, which means

$$V'(0^+) = V'(0^-). \quad (8)$$

Putting together (6)-(8), we can obtain  $C_1, C_2$ , and  $\Delta$  to solve

$$C_1 = \sqrt{\frac{\sigma^2}{2r}} \frac{1}{r(1 + e^{\sqrt{\frac{2r}{\sigma^2}}\Delta})} \quad (9)$$

$$C_2 = C_1 - \frac{1}{r} \sqrt{\frac{\sigma^2}{2r}} \quad (10)$$

$$rK = \Delta - 2\sqrt{\frac{\sigma^2}{2r}} \frac{e^{\sqrt{\frac{2r}{\sigma^2}}\Delta} - 1}{e^{\sqrt{\frac{2r}{\sigma^2}}\Delta} + 1}. \quad (11)$$

This completes the characterization of the value function and the optimal policy of the firm. Figure 2 presents an illustration of the value function for  $z \in [0, \Delta)$  for some parameter setting showing  $V'(0) = V'(\Delta) = 0$ . Figure 3 presents an illustration of the evolution of the preferences over time and the optimal repositioning for a sample path.

From (11) we can compute the comparative statics of the repositioning threshold  $\Delta$  with respect to the discount rate  $r$ , variance of preferences  $\sigma^2$ , and cost of repositioning. The following proposition states the results.

*PROPOSITION 1: Consider the purely random evolution of preferences case. Then the repositioning threshold  $\Delta$  is increasing in the discount rate  $r$ . Furthermore, the repositioning threshold  $\Delta$  is increasing at a decreasing rate in the variance of the evolution of preferences  $\sigma^2$ , and in the cost of repositioning  $K$ .*

As the discount rate increases, the present value of the benefits of repositioning decreases with respect to not repositioning. Therefore, as the discount rate increases the firm wants to reposition less often and waits for the preferences to be further away to decide to reposition. As expected, as the cost of repositioning goes up, the firm also wants to reposition less often.

More interestingly, when the variance of the evolution of preferences goes up, the firm realizes that even if the consumer preferences are far away from the firm’s current positioning, there is a greater likelihood of the consumer preferences returning to where the firm is. Moreover, the

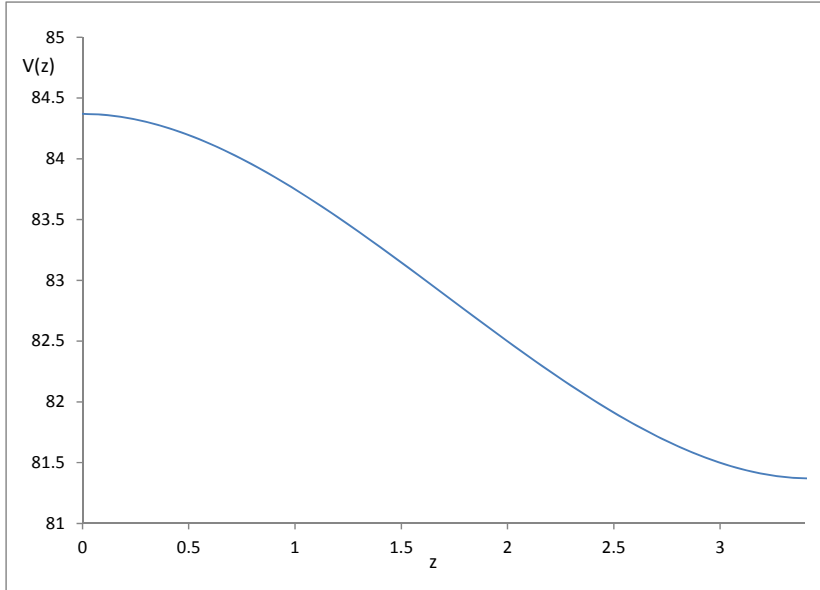


Figure 2: Value function for only random evolution of preferences case for  $r = .1, \sigma^2 = 2, K = 3, v - L = 10, \Delta = 3.4$ .

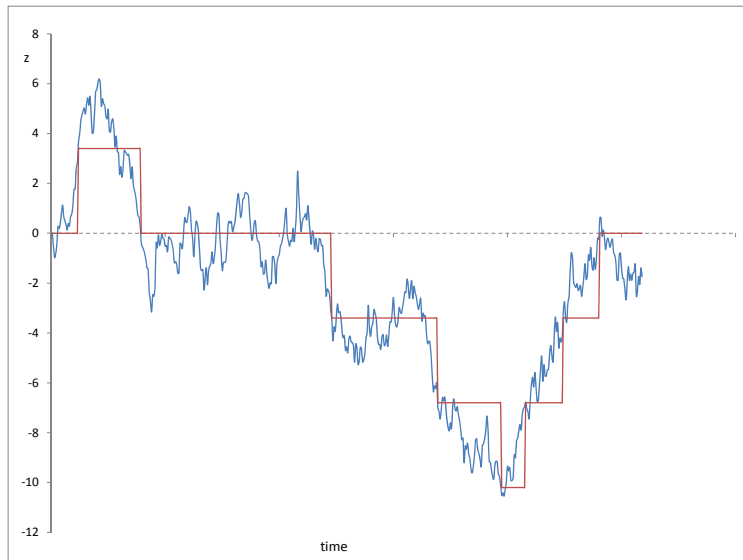


Figure 3: Optimal dynamic repositionings: example of a sample path and optimal repositionings for  $r = .1, \sigma^2 = 2, K = 3, v - L = 10, \Delta = 3.4$ .

value of being in the center of the market (having just repositioned) is lower when the variance of the evolution of preferences goes up, as the preferences are more likely to go away from where the firm is. Then the firm decides to hold off a little more before the firm decides to reposition when preferences move away. Furthermore, when the variance of preferences increases this effect is reduced. When the preferences get too far away from the positioning of the firm, the firm's profits fall too much. That is, as the variance of the evolution of preferences increases, the threshold to reposition increases but at a decreasing rate. Figure 4 illustrates how the threshold  $\Delta$  varies with the discount rate. Figure 5 illustrates how the threshold  $\Delta$  varies with the variance of the preferences' evolution.

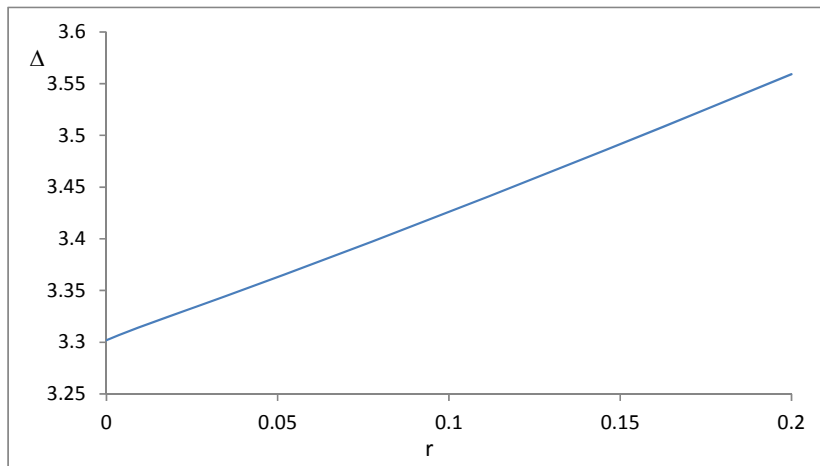


Figure 4: Effect of the discount rate: evolution of the threshold  $\Delta$  as a function of the discount rate  $r$  for  $\sigma^2 = 2, K = 3$ .

When the discount rate converges to zero, we can obtain an explicit expression for the threshold  $\Delta$ . In fact, we can obtain from (11) that  $\lim_{r \rightarrow 0} \Delta = \sqrt[3]{6K\sigma^2}$ .

Similarly, one can obtain an explicit expression for the threshold  $\Delta$  when the variance of the evolution of preferences converges to zero. We can obtain from (11) that  $\lim_{\sigma^2 \rightarrow 0} \Delta = rK$ , which is intuitive. If there is no variance in the evolution of preferences the firm wants to reposition if the present value of the benefits of repositioning,  $\frac{\Delta}{r}$ , are greater than the costs of repositioning  $K$ .

One can then compare the optimal threshold for repositioning with positive variance of the

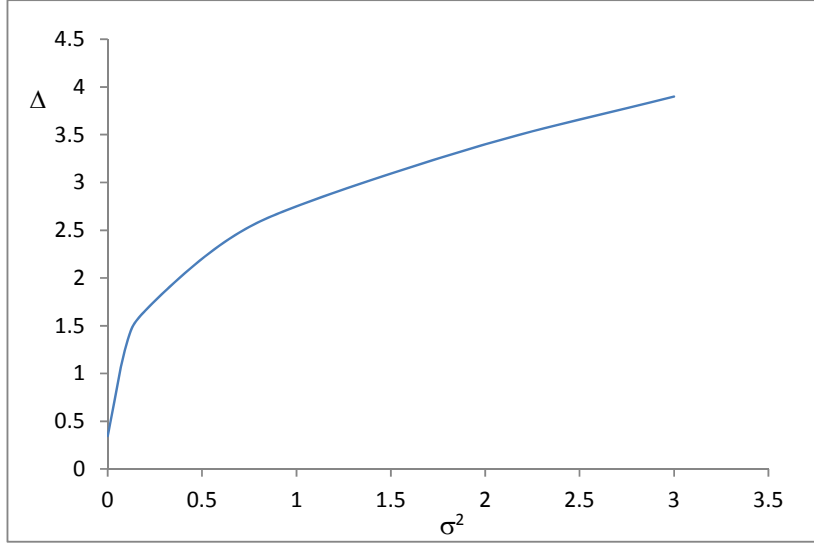


Figure 5: Effect of the variance: evolution of the threshold  $\Delta$  as a function of the variance  $\sigma^2$  for  $K = 3$ , and  $r = .1$ .

evolution of preferences, with what would be the repositioning strategy if consumer preferences were fixed. Note also that the case when the consumer preferences are fixed can also be seen as the case when the firm does not believe that there would be any more future consumer preference changes. Looking at (11) we can see that the optimal repositioning strategy is to wait longer than what a firm that does not believe in any future consumer preference changes would do (see also Figure 5). That is, the firm waits for the possibility that the consumer preferences might return to where the firm is before deciding to reposition, and only repositions when the consumer preferences move further away. Note that this effect can be arbitrarily high in relative terms. For example, for  $r \rightarrow 0$ , a firm that does not believe in future consumer preference changes, would reposition at any small change in the consumer preferences, while a firm aware of the possible future preference changes would wait until the center of the consumer preferences would move to a distance  $\sqrt[3]{6K\sigma^2}$  from the current product's positioning.

Note also that with the possibility of future repositionings the firm is more willing to reposition because if the consumer preferences return to where the current product's positioning is, the firm can always reposition back to the current position. If the firm believes that it can only reposition once more, it may wait longer to reposition (as otherwise it may waste the option to reposition) when there is still a reasonable possibility of the consumer preferences returning to where the

firm is currently positioned.

We can also get the expected time between repositionings. Denoting  $z_t$  as the difference from the center of the consumer preferences of the product positioning at time  $t$ , and letting time zero be the time of the last repositioning, we have  $z_0 = 0$ , and  $E(z_t^2 - t\sigma^2) = 0$ , given that  $z_t$  is a Brownian motion with variance  $\sigma^2$ . Letting  $t_\Delta$  be the first time that  $z_t$  reaches either  $\Delta$  or  $-\Delta$ , we then have  $\Delta^2 - \sigma^2 E(t_\Delta)$ , which gives the expected time between repositionings  $E(t_\Delta) = \Delta^2/\sigma^2$ .<sup>7</sup> As  $\Delta$  is increasing in the discount rate, from Proposition 1, we can get immediately that the expected time between repositionings is increasing in the discount rate. More interestingly, we can also get how the expected time between repositionings is affected by the variance of the consumer preferences.

*PROPOSITION 2: For  $r \rightarrow 0$ , the expected time between repositionings is decreasing in the variance of the evolution of preferences at a decreasing rate.*

As the variance of the evolution of preferences increases the repositioning threshold also increases, which would be a force towards less frequent repositionings. At the same time, the preferences can also evolve faster along the preference space, which would be a force towards more frequent repositionings. We find that the latter effect dominates, that a greater variance of the evolution of preferences leads to more frequent repositionings. In fact, when the variance of the evolution of preferences goes to zero, the time between repositionings goes to infinity, as the need to reposition falls because of the slow changing consumer preferences.

Finally, note that the degree of consumer heterogeneity in the market,  $L$ , does not affect the optimal repositioning strategy in this case of full market coverage. This can be seen intuitively in the value function (5), as  $L$  enters there in an additively separable way. That is, in this case of full market coverage the consumer heterogeneity  $L$  does not affect at the margin the effect of the firm's positioning relative to the center of consumer preferences. But  $L$  affects the present value of profits, with a lower  $L$  being preferred (makes the value function just move vertically, while keeping the same shape). This also means that if  $L$  varies over time, but always staying in this region of full market coverage ( $v > 3L$  and  $K$  relatively small), the optimal repositioning strategy remains unchanged. In the case below, when the market is not always fully covered (for example,  $K$  large) we will see that  $L$  will affect the repositioning strategy.

---

<sup>7</sup>See, for example, Dixit (1993).

## Social Welfare

It is interesting to compare the optimal repositioning of the firm with what could be socially optimal. To understand the social optimum note that given  $v$  sufficiently large, as assumed above, the quantity supplied by the firm, given its positioning, is optimal. Therefore, the comparison with the social welfare repositioning optimal policy depends on how the total utility generated by a product is affected when the product is not exactly at the center of the consumer preferences.

When the distance between the product's positioning and the center of consumer preferences,  $|z|$ , is less than  $L$  (that is, there are still some consumers for whom the product offered is the ideal one), we have that the gross surplus offered is  $S(z) = \int_0^{L-|z|} \frac{v-x}{2L} dx + \int_0^{L+|z|} \frac{v-x}{2L} dx = v - \frac{L}{2} - \frac{z^2}{2L}$ . If  $|z| > L$ , then similarly we can get  $S(z) = v - |z|$ .

Comparing with the flow payoff for the firm,  $\pi(z) = v - L - |z|$ , we note that for  $|z| < L$  the gross surplus is less affected by  $z$  than the firm's profit,  $|S'(z)| < |\pi'(z)|$ . This will then have implications on the optimal repositioning for social welfare, in addition to  $S(z) > \pi(z)$ . Note also that for  $|z| > L$  the effect of  $z$  on  $S(z)$  is exactly the same as on  $\pi(z)$ .

The optimal social welfare policy is going to be a threshold distance,  $\Delta_w$ , such that when the distance between the product's positioning and the center of the consumer preferences reaches  $\Delta_w$  it would be optimal to reposition. The question will then be what is the relationship between  $\Delta_w$  and  $\Delta$  obtained above. For example, if  $\Delta_w > \Delta$  then the firm repositions more frequently than what would be desirable from a social welfare point of view.

To determine  $\Delta_w$  first consider the case in which the cost of repositioning  $K$  is small (or  $L$  large) such that we will be in a case in which  $\Delta_w < L$ . Similarly to the analysis above, we can get the value function when no repositioning is occurring,  $V_w(z)$ , as in (4), as

$$rV_w(z) = S(z) + \frac{\sigma^2}{2}V_w''(z), \quad (12)$$

from which we can obtain for  $z > 0$ ,

$$V_w(z) = C_w[e^{\sqrt{\frac{2r}{\sigma^2}}z} + e^{-\sqrt{\frac{2r}{\sigma^2}}z}] - \frac{z^2}{2Lr} + \frac{2v-L}{2r}, \quad (13)$$

given that  $V_w'(0) = 0$  because of smoothness at  $z = 0$ . As above, because  $V_w(\Delta_w) = V_w(0) - K$



and  $V'_w(\Delta_w) = 0$ , we then obtain

$$C_w = \sqrt{\frac{\sigma^2}{2r}} \frac{\Delta_w}{Lr} \frac{e^{\sqrt{\frac{2r}{\sigma^2}} \Delta_w}}{e^{2\sqrt{\frac{2r}{\sigma^2}} \Delta_w} - 1} \quad (14)$$

$$\Delta_w^2 - 2LrK = 2\sqrt{\frac{2r}{\sigma^2}} \Delta_w \frac{e^{\sqrt{\frac{2r}{\sigma^2}} \Delta_w} - 1}{e^{\sqrt{\frac{2r}{\sigma^2}} \Delta_w} + 1}, \quad (15)$$

where  $\Delta_w$  can be implicitly obtained from (15).

Comparing (15) with (11) we can see how the frequency of repositionings of the firm compares with what would be optimal from a social welfare point of view. In the Appendix we show that  $\Delta < \Delta_w$ . For example, in the particular case of  $r \rightarrow 0$ , we can obtain that  $\Delta_w \rightarrow \sqrt[4]{12LK\sigma^2}$  which is greater than the corresponding value for the firm's optimal decision mentioned above,  $\sqrt[3]{6K\sigma^2}$ , for the condition considered of  $K$  small,  $\Delta_w < L$  (see Appendix). The same result of  $\Delta < \Delta_w$  can be obtained for  $K$  large, with analysis more complicated, and also presented in the Appendix, as the function  $V_w()$  now has two regions with different functional forms. The result is presented in the following proposition.

**PROPOSITION 3:** *The firm repositions more frequently than what is optimal from a social welfare point of view,  $\Delta < \Delta_w$ .*

When we compare the optimal repositioning of the firm with what would be optimal from a social welfare point of view, one could potentially think that the social welfare optimal policy would be to reposition more frequently than what a firm would like, as the social welfare is greater than just the firm's profit. It turns out that this does not hold as social welfare is not overly affected by small deviations of consumer preferences while profits are affected at a greater rate. In fact, social welfare is affected at a rate of  $z/L$  for  $z < L$ ,  $S'(z) = -z/L$ , and at the rate of 1 for  $z > L$ , while the profit is affected at a rate of one,  $\pi'(z) = -1$ , throughout.

The result obtained here on the comparison between a firm's repositioning decision and what is optimal from a social welfare point of view depends obviously on the stylized model considered. In fact, this comparison can be seen as similar to the question of whether a monopolist provides the efficient quality level (e.g., Spence 1975), in which case one can potentially take into account both the extent of market coverage provided by the firm and, for a given market coverage, the comparison between the benefit of quality to the marginal consumer and the average consumer. In the case of this section, the market is fully covered by the firm, therefore, the question is only one of the effect of repositioning on the marginal consumer versus the average consumer.

The effect on the marginal consumer is the effect of how that consumer is now closer to the product's positioning, while the effect on the average consumer is less clear because while some consumers are now closer to the product's positioning, other consumers are now further away.<sup>8</sup> This then yields that the effect on the marginal consumer is greater than the effect on the average consumer, and the firm repositions more often than it is optimal.

Note that this result does not necessarily need to hold in other model formulations. For example, if all consumers are at the same location (potentially with different valuations) the benefit of repositioning to the average and marginal consumer could be the same, and the firm would reposition as often as would be efficient. Another interesting example is the case in which the market is not fully covered,  $v < 2L$ , but the price is chosen by the firm, given the product's positioning. In that case, for small deviations in the consumer preferences from the product's positioning, the firm's profit would remain unchanged (and equal to  $v^2/4L$ ), but welfare would be affected negatively. This would then be a force towards the firm repositioning less frequently than what would be efficient. In sum, the result above of the firm repositioning more frequently than what would be socially optimal has to be interpreted with care, and can then be seen as a possibility even though social welfare is greater than profit.

### *Repositioning Costs Depending on Extent of Repositioning*

In some cases, one may argue that repositioning costs could be an increasing function of the extent of the repositioning. That is, if a firm wants to reposition a greater distance it has to spend more on repositioning costs. In terms of the analysis above that means that the repositioning costs would not just be some fixed costs  $K$ , but also have additional costs that would depend on the extent of the repositioning.

One simple way to consider this possibility is to have the repositioning costs equal to  $K + \alpha\Delta$  where  $\Delta$  is the extent of the repositioning and  $\alpha$  is some parameter with  $\alpha > 0$ . Suppose that  $\alpha$  is small.

In terms of the analysis above, one has to account now for the possibility that the firm chooses not to reposition to the center of the market because, at the margin, not being positioned at the center of the market involves losses of the second order, while the cost of repositioning to the center of the market is of the first order. Let  $d$  be the distance to where the firm repositions when the firm chooses to do so, and that happens when the center of the market is at a distance

---

<sup>8</sup>For example, suppose a consumer is located at 1 to the right of the center of the market and that  $z = 1$ . Then, if the firm repositions to the center of the market, that consumer becomes worse off while a consumer at the center of the market is better off.

$\Delta$  from where the firm is positioned. That is, when the firm repositions it will be at a distance  $\Delta - d$  from the center of the market. In the analysis above, when the cost of repositioning just had a fixed component, we had  $d = \Delta$ . Now the firm may choose to save on repositioning costs, and not move all the way to the center of the market, in the hopes that the consumer preferences will return to where the firm is.

In terms of the analysis above we then have to replace (6) and (7) with

$$V(\Delta) = V(\Delta - d) - K - \alpha d, \text{ and} \quad (16)$$

$$V'(\Delta) = V'(\Delta - d), \quad (17)$$

respectively. Furthermore, we need that the place  $d$ , to where the firm repositions, is optimal, which requires that

$$V'(\Delta - d) + \alpha = 0. \quad (18)$$

With an analysis similar to the one shown above (details are presented in the Appendix), we can obtain that the threshold to reposition  $\Delta$ , and the place to reposition to  $d$  are determined by

$$rK = d - \alpha rd - 2\sqrt{\frac{\sigma^2}{2r}} \frac{e^{\sqrt{\frac{2r}{\sigma^2}}\Delta} - e^{\sqrt{\frac{2r}{\sigma^2}}(\Delta-d)}}{e^{\sqrt{\frac{2r}{\sigma^2}}(\Delta-d)} + 1} + 2\alpha r \sqrt{\frac{\sigma^2}{2r}} \frac{e^{\sqrt{\frac{2r}{\sigma^2}}(2\Delta-d)} - e^{2\sqrt{\frac{2r}{\sigma^2}}(\Delta-d)}}{e^{2\sqrt{\frac{2r}{\sigma^2}}(\Delta-d)} - 1}, \quad (19)$$

$$d = \Delta + \sqrt{\frac{\sigma^2}{2r}} \ln\left[\frac{e^{\sqrt{\frac{2r}{\sigma^2}}\Delta}(1 - \alpha r) - 1}{e^{\sqrt{\frac{2r}{\sigma^2}}\Delta} - 1 + \alpha r}\right]. \quad (20)$$

From (20) one can obtain that  $d < \Delta$ , as expected. That is, when repositioning, the firm approaches the center of the market but does not move all the way to the center of the market.

In order to investigate the effect of the variable cost  $\alpha$  on the repositioning strategy consider the case of  $r \rightarrow 0$ . In that case we can obtain that the repositioning distance has the same expression as when the repositioning costs are not increasing in the repositioning distance,

$$d = \sqrt[3]{6K\sigma^2} \quad (21)$$

and that

$$\Delta(\Delta - d) = \alpha\sigma^2. \quad (22)$$

This last expression shows how the threshold of consumer preferences to decide to reposition depends positively on the marginal cost of repositioning  $\alpha$ , and the variability of the consumer preferences  $\sigma^2$ . Greater marginal costs of repositioning makes the firm only choose to reposition when the consumer preferences are further away from the firm's positioning. Greater variability of consumer preferences makes the firm be more hopeful that the consumer preferences will return to where the firm is, and the firm, when repositioning, ends up, optimally, further away from the center of the market. Note that having costs of repositioning increasing in the distance repositioned does not affect the distance actually repositioned for  $r \rightarrow 0$ , but affects when to reposition, with the threshold to reposition increasing in  $\alpha$ .

More interestingly, we can check the effect of the degree to which the repositioning costs increase in the distance repositioned, under the situation that the total costs of repositioning remain constant. That is, when  $\alpha$  increases we reduce the fixed costs of repositioning  $K$  such that  $K + \alpha d$  remains constant. To see this, note that for the costs of repositioning to remain constant we have  $\frac{\partial K}{\partial \alpha} = -\frac{d^3}{d^2 + 2\alpha\sigma^2} < 0$ , which leads to  $\frac{\partial d}{\partial \alpha}|_{K+\alpha d=\text{Const.}} < 0$ . That is, as expected, if the share of the overall repositioning costs are more related to the distance repositioned, the firm chooses to reposition with shorter distances.

Note also that  $\frac{\partial \Delta}{\partial \alpha}|_{K+\alpha d=\text{Const.}} = \frac{\sigma^2}{2\Delta - d} \left(1 - 2\frac{\Delta d}{d^2 + 2\alpha\sigma^2}\right)$ , which is negative for  $\alpha$  small. This means that when the repositioning costs remain constant with a greater share of these costs depending on the distance repositioned, when  $\alpha$  is small, the firm repositions more frequently. The intuition is that with the increasing costs of repositioning per unit of distance that is repositioned, the firm chooses a lower and lower repositioning distance, which makes the firm choose a lower threshold of the consumer preferences moving away from the firm's current positioning in order to decide to reposition. However, note that if the degree to which the repositioning costs increase in the distance repositioned is sufficiently large, we can be in a situation where the threshold to reposition is greater than in the case when there are no repositioning costs increasing in the distance repositioned. To see this note that when  $\alpha \rightarrow \infty$ , we have  $d, K \rightarrow 0$ , which leads, by (22), to  $\Delta \rightarrow \infty$ . That is, when the overall costs of repositioning remain constant, increasing the degree to which the repositioning costs increase in the distance repositioned has a non-monotone effect on the threshold of repositioning. When the overall costs of repositioning remain constant, Figure 6 presents an example of how  $\Delta$  and  $d$  evolve as a function of  $\alpha$  for the case of  $r \rightarrow 0$ ,  $K + \alpha d = 2$ , and  $\sigma^2 = 1$ .

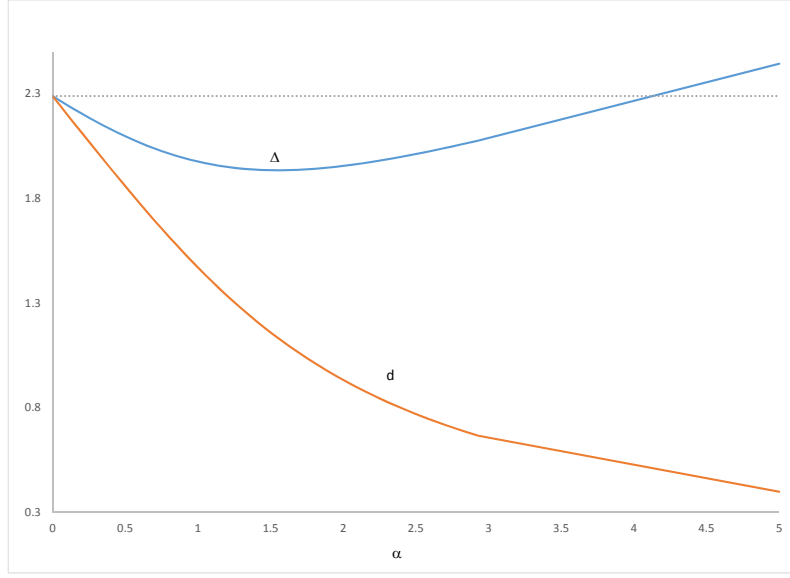


Figure 6: Threshold for repositioning when costs of repositioning are a function of the extent of repositioning with  $r$  converging to zero,  $\sigma^2 = 1$ , and  $K$  such that the total costs of repositioning  $K + \alpha d$  stay constant at 2, for several values of  $\alpha$ .

### *Large Costs of Repositioning*

Consider now the case in which  $K$  is large such that the market is not always fully covered on the equilibrium path.<sup>9</sup>

In order to consider this case, suppose as above that  $v$  is large compared to  $L$  such that if the product is close to the center of the market the firm chooses to fully cover the market. In particular, this occurs if  $v > 3L$ .<sup>10</sup> Depending on how far the center of the consumer preferences is from the product positioning, the firm's price and profit can be in different cases. If the center of the consumer preferences is close to the product's positioning, in particular, if  $|z| < v - 3L$ , the optimal price is as noted above,  $P = v - L - |z|$ , which yields an optimal profit of  $\pi(z) = v - L - |z|$ .

When  $|z|$  is greater than  $v - 3L$  but less than  $v + L$ , the optimal price maximizes  $\frac{v-P}{2L}P + \frac{L-|z|}{2L}P$

<sup>9</sup>We consider the case in which  $K$  is large ( $K > \tilde{K}$ ), such that the firm sometimes chooses to serve the market partially, but not too large ( $K < \hat{K}$  where  $\hat{K} > \tilde{K}$  is defined in (xxix)), such that the firm may never choose to sometimes have zero profit waiting for the possibility that the consumer preferences return to where the firm is positioned. The case of very large  $K$ , such that sometimes the market is not served at all, is available in the Online Appendix.

<sup>10</sup>If  $v < 3L$  the seller chooses not to fully cover the market if the product is not exactly at the center of the market.

which gives an optimal price equal to  $P = \frac{v+L-|z|}{2}$  and an optimal profit  $\pi(z) = \frac{(v+L-|z|)^2}{8L}$ . Finally, for  $|z| > L + v$ , the firm cannot generate any profit and  $\pi(z) = 0$ .

With this profit function  $\pi(z)$  defined for the different regions of  $z$ , we can use an analysis similar to the one presented above to obtain the value function  $V(z)$  and the optimal threshold  $\Delta$  where the firm decides to reposition, while keeping continuity and smoothness of  $V(z)$  throughout. This analysis is fully presented in the Appendix.

When  $r \rightarrow 0$  one can obtain the optimal threshold for repositioning to satisfy (xxxix). From this one can obtain that, as expected, greater  $K$  or greater  $\sigma^2$  leads to a greater  $\Delta$ . If the costs of repositioning are greater the firm prefers to wait longer for the consumer preferences to move away from where the product is positioned. Similarly, if the variance of the evolution of consumer preferences is greater, the firm again prefers to wait longer to reposition itself, as the likelihood of the consumer preferences returning to where the product is currently positioned is higher.

In this case of partial market coverage we can see the effect of consumer heterogeneity  $L$  on the optimal repositioning strategy. From (xxxix) we can obtain that a greater  $L$  leads to a greater threshold  $\Delta$ . As there is greater consumer heterogeneity the firm does not see the need to reposition as often, as it is covering the market to some degree. Figure 7 illustrates how  $\Delta$  changes with  $L$ , for an example with  $v = 3, K = 2$ , and  $\sigma^2 = 1$ . The figure illustrates how  $L$  varies from around .24 (at which case  $\tilde{K} = 2$ , and then the equilibrium would become with full market coverage) to 1 (at which case  $v = 3L$ , and the equilibrium would never involve full market coverage). To see a numerical example of the case analyzed here consider  $r \rightarrow 0, v = 3, L = .7$ , and  $\sigma^2 = 1$ . Then we get  $\tilde{K} = .12$  and  $\hat{K} = 5.44$  (see Footnote 9). If  $K = .1$ , the market is always fully covered and  $\Delta = .84$ . For  $K = 2$ , the market is sometimes partially covered and we have  $\Delta = 2.41$ .

One may also consider the case in which  $L$  evolves stochastically over time. Consider such a case under the assumption that for all possible  $L$ , we have  $v > 3L$  and  $K \in (\tilde{K}, \hat{K})$ . In such a setting we would expect that the repositioning strategy would be based on the current  $L$  and firm expectations about the future  $L$ . If  $L$  is positively serially correlated, we would then expect to have the same comparative statics of  $\Delta$  increasing in  $L$  as presented above, but now with a softened effect because of the future  $L$  uncertainty. For example, consider a model where  $L$  starts at a low level, and then with a constant hazard rate moves to a high level where it stays forever. Then, when  $L$  is at the high level we are back in the situation above because  $L$  does not change more going forward. When  $L$  is at the low level, the firm knows that it will remain for some time at the low level, and then will move to the high level. Then, the repositioning threshold when the firm is at a low level would be expected to be somewhere between the repositioning

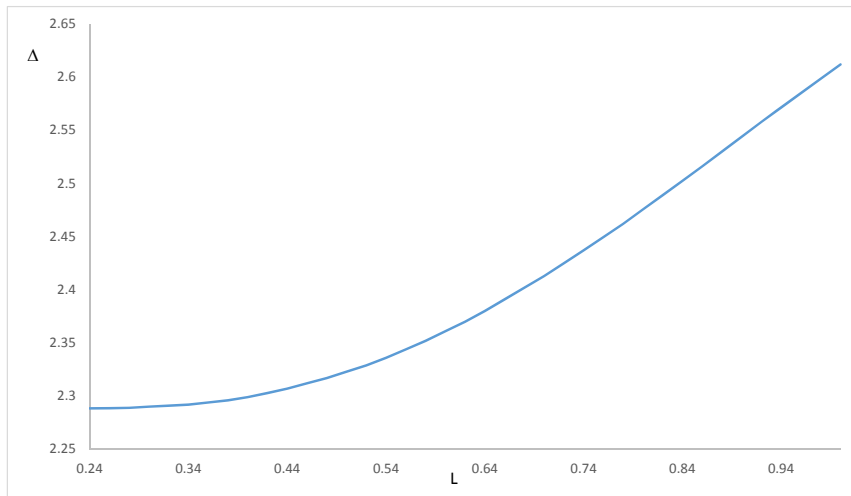


Figure 7: Evolution of  $\Delta$  as a function of  $L$  for the case of large repositioning costs (partial market coverage):  $v = 3, K = 2, \sigma^2 = 1$ .

threshold for the low and high level of the model above with fixed  $L$ .

#### 4. PURELY DETERMINISTIC EVOLUTION OF CONSUMER PREFERENCES

Consider now the case in which the evolution of consumer preferences is completely deterministic. That means that in (2) we have  $\sigma = 0$  and  $b > 0$ . This case can be seen as important to study for markets in which there are some trends on the evolution of preferences over time. For example, in markets where technology is important, one could expect that the consumer preferences become more demanding over time on the technical features. In the the car market, if positioning is considered on the technical features, one would expect that consumers would be more demanding on this type of dimension over time.

In this case, with deterministic evolution of preferences, the optimal policy will involve two thresholds on the distance of the product's positioning to the center of consumer preferences, and a target repositioning placement. First, there is a a low threshold  $\underline{\Delta}$ , which is negative, that when  $z$  gets sufficiently low the firm chooses to reposition towards the center of the market. Because the deterministic trend is for  $z$  to move downwards this threshold will be hit often.

Second, there is a high threshold  $\bar{\Delta}$ , which is positive, such that when  $z$  gets sufficiently high the firm chooses to reposition towards the center of the market. Because the deterministic trend is for  $z$  to move downwards this threshold will not be hit very often and, in fact, will never be hit when the evolution of preferences is purely deterministic as assumed in this section. We also expect  $\bar{\Delta}$  to be further away from zero than  $\underline{\Delta}$  as the deterministic trend is downwards, and when  $z$  is very high we know that after some time the center of the consumer preferences will likely be close to the current product's positioning. Third, the firm has to decide where it will reposition to with regards to the center of the consumer preferences,  $d$ . We expect  $d$  to be positive as  $z$  trends downwards; in that way, after some period of time, the center of the consumer preferences will be close to the product's positioning. Overall, we will then have  $\underline{\Delta} < 0 < d < \bar{\Delta}$ .

Note that in the previous section the optimal repositioning policy was completely determined by  $\Delta$ . That is, in the previous section we had  $\bar{\Delta} = -\underline{\Delta} = \Delta$  and  $d = 0$ . As there was no deterministic trend the thresholds for the center of the consumer preferences were at the same distance of the product's positioning and, when repositioning, the firms always wanted to go to the center of the consumer preferences.

In this case of purely deterministic evolution of consumer preferences after repositioning it always takes time  $T = \frac{d-\underline{\Delta}}{b}$  for the firm to want to reposition again. As the payoffs repeat every period  $T$ , we can write the present value of profits after repositioning as

$$V = \frac{e^{rT}}{e^{rT} - 1} \left[ \int_0^T (v - L - |d - bt|) e^{-rt} dt - K \right]. \quad (23)$$

Maximizing  $V$  with respect to  $T$  and  $d$  yields the optimal policy for the firm.

In order to get sharper results, consider the case in which  $r \rightarrow 0$ . To do that consider the average continuous profit  $rV$  when  $r \rightarrow 0$ ,

$$\lim_{r \rightarrow 0} rV = \frac{1}{T} \left[ \int_0^T (v - L - |d - bt|) dt - K \right]. \quad (24)$$

Maximizing this expression with respect to  $T$  and  $d$  yields

$$T = 2\sqrt{\frac{K}{b}} \quad (25)$$

$$d = \sqrt{bK}. \quad (26)$$

From this we can obtain  $\underline{\Delta} = -\sqrt{bK}$ .



In order to obtain the upper threshold  $\bar{\Delta}$  we are looking for a high  $z$ , such that the firm is indifferent between repositioning to  $d$  with cost  $K$  and not repositioning. If the firm repositions it gets an average payoff of  $v - L - \frac{d}{2} - \frac{K}{T} = v - L - \sqrt{bK}$ . If the firm does not reposition, it will be for the period of time  $\frac{\bar{\Delta}-\underline{\Delta}}{b}$  until the next repositioning. For a fraction of time  $\frac{\bar{\Delta}}{\bar{\Delta}-\underline{\Delta}}$  the firm will have  $z > 0$  and with an average profit of  $v - L - \frac{\bar{\Delta}}{2}$ . For a fraction of time  $\frac{-\underline{\Delta}}{\bar{\Delta}-\underline{\Delta}}$  the firm will have  $z < 0$  with an average profit of  $v - L - \frac{-\underline{\Delta}}{2}$ . The average profit if the firm does not reposition immediately is then  $v - L - \frac{\bar{\Delta}-\underline{\Delta}}{2} - \frac{-\underline{\Delta}-\bar{\Delta}}{2}$ . Making this equal to the average payoff if the firm reposition immediately leads to  $\bar{\Delta}^2 + d^2 = 2d(\bar{\Delta} + d)$ , where we use the result that  $d = -\underline{\Delta}$ . Using the optimal value for  $d$  we can then obtain that  $\bar{\Delta} = \sqrt{bK}(1 + \sqrt{2})$ .

These results lead to some interesting observations. First, note that with a deterministic trend in the evolution of consumer preferences the firm chooses to optimally over-shoot in its repositioning from  $\underline{\Delta}$  to  $d > 0$  while the center of consumer preferences would be at zero. This is because with the fixed costs of repositioning, over-shooting allows the firm to be, on average, closer to the center of the consumer preferences. The extent of over-shooting is increasing in the deterministic trend  $b$ . As the deterministic trend increases the firm increases the extent of over-shooting for its product positioning to be about the same time above as below the center of the consumer preferences. The extent of over-shooting is also increasing in the costs of repositioning  $K$ , as greater costs of repositioning give the firm incentives to reposition less often, which means that the firm has to increase the extent of its repositioning over-shooting, so that it is about the same above and below the center of consumer preferences.

Note also that given  $r \rightarrow 0$  the extent of the over-shooting is exactly equal to the distance of the center of the consumer preferences to the lower threshold,  $d = -\underline{\Delta}$ . As the firm is infinitely patient it wants to be as much above the consumer preferences as below them. It can be shown that as the discount rate  $r$  increases the extent of the over-shooting decreases in relation to the lower threshold, so that  $d < -\underline{\Delta}$ . With  $r > 0$ , the firm cares more about profits now than profits in the future, and does not over-shoot as much to be closer to the center of the market and earn greater profits sooner.

When the deterministic trend  $b$  increases the firm adjusts both the time between repositionings and the threshold to reposition. The firm not only shortens the time between repositionings  $T$  as now the preferences are evolving faster, but it also increases the threshold to reposition,  $|\underline{\Delta}|$ , not to reposition very often. As expected, the thresholds to reposition increase in the costs of repositioning, but they do so at a decreasing rate in order to soften the lost profit because of not having the product that is a perfect fit to the consumer preferences.

Note also that the upper threshold  $\bar{\Delta}$  is further away from the chosen point of repositioning,

$d$ , than the lower threshold  $\underline{\Delta}$  is from the center of the consumer preferences. This just reflects the fact that if  $z$  is high the firm prefers to save on repositioning costs and let the evolution of consumer preferences bring the center of those consumer preferences to where the product is currently positioned.

## 5. BOTH RANDOM AND DETERMINISTIC EVOLUTION OF CONSUMER PREFERENCES

Consider now the case when both the deterministic trend,  $b$ , and the variance of the evolutions of consumers preferences,  $\sigma^2$ , are different from zero. We present the conditions for the optimal policy, and then present numerical analysis of the optimal policy.

In this case the value function would have to satisfy the differential equation

$$rV(z) = \pi(z) - bV'(z) + \frac{\sigma^2}{2}V''(z), \quad (27)$$

where the difference from (4) comes from  $E(dz) = -b dt$ . The solution to this differential equation is

$$V(z) = C_1 e^{x_1 z} + C_2 e^{x_2 z} - \frac{|z|}{r} + \frac{v - L}{r} + (-1)^{1+1[z>0]} \frac{b}{r^2}, \quad (28)$$

where  $x_1 \equiv \frac{b + \sqrt{b^2 + 2r\sigma^2}}{\sigma^2}$  and  $x_2 \equiv \frac{b - \sqrt{b^2 + 2r\sigma^2}}{\sigma^2}$  are the solutions to the characteristic equation  $\frac{\sigma^2}{2}x^2 - bx - r = 0$ ,  $1[z > 0]$  is the indicator function that takes the value of one if  $z > 0$  and zero otherwise, and  $C_1$  and  $C_2$  are parameters to be obtained below. These parameters  $C_1$  and  $C_2$  will be different for  $z$  negative ( $C_1^-$  and  $C_2^-$ ) and  $z$  positive ( $C_1^+$  and  $C_2^+$ ).

In order to find the optimal values of the thresholds to reposition,  $\bar{\Delta}$  and  $\underline{\Delta}$ , and the optimal value to which the firm repositions,  $d$ , we have to consider continuity and smoothness at  $\underline{\Delta}$ ,  $\bar{\Delta}$ , and  $z = 0$ , and optimality of  $d$ .

Continuity and smoothness at  $z = 0$ ,  $V(0^+) = V(0^-)$  and  $V'(0^+) = V'(0^-)$ , yields

$$C_1^- + C_2^- = C_1^+ + C_2^+ + \frac{2b}{r^2} \quad (29)$$

$$x_1 C_1^- + x_2 C_2^- + \frac{2}{r} = x_1 C_1^+ + x_2 C_2^+. \quad (30)$$

Continuity and smoothness at  $\underline{\Delta}$ ,  $V(\underline{\Delta}) = V(d) - K$  and  $V'(\underline{\Delta}) = 0$ , yields

$$C_1^- e^{x_1 \underline{\Delta}} + C_2^- e^{x_2 \underline{\Delta}} + \frac{\underline{\Delta} + d}{r} = C_1^+ e^{x_1 d} + C_2^+ e^{x_2 d} + \frac{2b}{r^2} - K \quad (31)$$

$$x_1 C_1^- e^{x_1 \underline{\Delta}} + x_2 C_2^- e^{x_2 \underline{\Delta}} + \frac{1}{r} = 0. \quad (32)$$

Similarly, continuity and smoothness at  $\bar{\Delta}$ ,  $V(\bar{\Delta}) = V(d) - K$  and  $V'(\bar{\Delta}) = 0$ , yields

$$C_1^+ e^{x_1 \bar{\Delta}} + C_2^+ e^{x_2 \bar{\Delta}} - \frac{\bar{\Delta} - d}{r} = C_1^+ e^{x_1 d} + C_2^+ e^{x_2 d} - K \quad (33)$$

$$x_1 C_1^+ e^{x_1 \bar{\Delta}} + x_2 C_2^+ e^{x_2 \bar{\Delta}} - \frac{1}{r} = 0. \quad (34)$$

Finally, optimality of  $d$ ,  $V'(d) = 0$ , yields

$$x_1 C_1^+ e^{x_1 d} + x_2 C_2^+ e^{x_2 d} - \frac{1}{r} = 0. \quad (35)$$

Using the seven equations (29)–(35), we can then obtain the seven unknowns,  $\underline{\Delta}$ ,  $\bar{\Delta}$ ,  $d$ ,  $C_1^+$ ,  $C_1^-$ ,  $C_2^+$ , and  $C_2^-$ . Solving for these analytically is not possible, but we can obtain the optimal policy numerically.

Figure 8 illustrates how the optimal policy changes as a function of the discount rate  $r$ , for  $b = .5$ ,  $\sigma^2 = 1$ , and  $K = 4$ . As in the case of  $b = 0$  the repositioning thresholds,  $\bar{\Delta}$  and  $\underline{\Delta}$  move away from zero as the discount rate increases. As argued above, as the discount rate increases, the present value of future profits is lower, and the firm optimally chooses to reposition only when the consumer preferences are further away from the current product's positioning. The optimal point to reposition to,  $d$ , falls as the discount rate  $r$  increases: As the discount rate increases, the firm discounts more the future, and when it repositions it wants to be closer to the center of the consumer preferences.

The way in which the optimal policy evolves as a function of the extent of the deterministic changes in the consumer preferences,  $b$ , is illustrated in Figure 9 for  $r = .1$ ,  $\sigma^2 = 1$ , and  $K = 4$ . For  $b = 0$ , we are in the case of Section 3, and we have  $\bar{\Delta} = -\underline{\Delta}$ . As the deterministic component of the evolution of consumer preferences increases, the firm adjusts by increasing the point of which to reposition to,  $d$ , in order to be for a longer period of time closer to the center of the market. This idea is as presented in Section 4. As the deterministic component increases the firm also increases the upper threshold of repositioning  $\bar{\Delta}$  also, as in Section 4, as a greater  $b$  makes the firm's positioning get closer to the center of the market if the current center of the

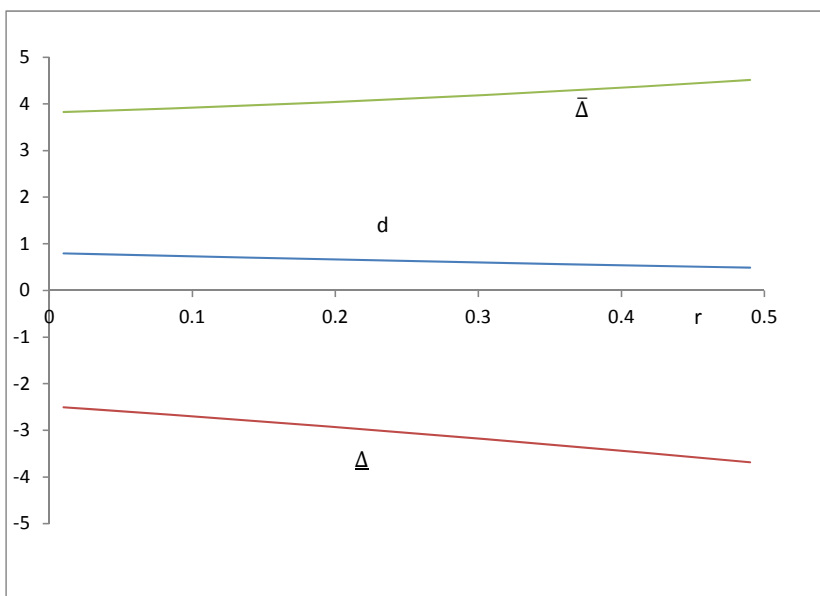


Figure 8: Evolution of  $d$  and repositioning thresholds as a function of  $r$  for  $b = .5$ ,  $\sigma^2 = 1$ , and  $K = 4$ .

market is above the firm's positioning. Also, as presented in Section 4, the upper threshold gets to be further away from zero than the lower threshold, as above zero the firm knows that the deterministic component is going to bring the consumer preferences closer to the product's positioning. The lower threshold of repositioning,  $\underline{\Delta}$ , first increases and then decreases with the deterministic component  $b$ . This is different than the case in Section 4 with  $\sigma^2 = 0$ , where  $\underline{\Delta}$  was decreasing in  $b$  for all  $b$ . For  $\sigma^2 > 0$  and  $b$  small, the optimal policy can have the lower threshold  $\underline{\Delta}$  increasing in  $b$ , as the firm adjusts  $d$  upwards to be, on average, closer to the center of the market. At some point  $\underline{\Delta}$  starts decreasing in  $b$  in order to save on repositioning costs.

Figure 10 illustrates how the optimal policy evolves as a function of the importance of the random component in the evolution of consumer preferences,  $\sigma^2$ . As in Section 3, we have that the greater is the variance in the evolution of the consumer preferences, the further away from zero are the repositioning thresholds,  $\bar{\Delta}$  and  $\underline{\Delta}$ . Furthermore, the greater is  $\sigma^2$  the lower is the optimal  $d$ . This is because, the greater is  $\sigma^2$  the lower the relative importance of the deterministic component  $b$ , and the firm adjusts its optimal policy by being closer to the center of the market when repositioning,  $d$  closer to zero.

Finally, the effect of the cost of repositioning  $K$  on the optimal policy is illustrated in Figure 11. As in Section 3, as the cost of repositioning  $K$  increases, the firm chooses to reposition less

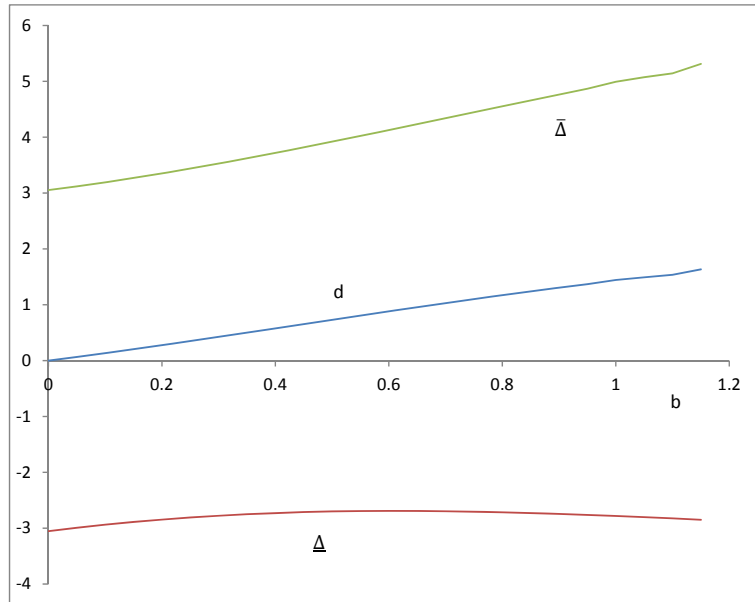


Figure 9: Evolution of  $d$  and the repositioning thresholds as a function of  $b$  for  $r = .1, \sigma^2 = 1$ , and  $K = 4$ .

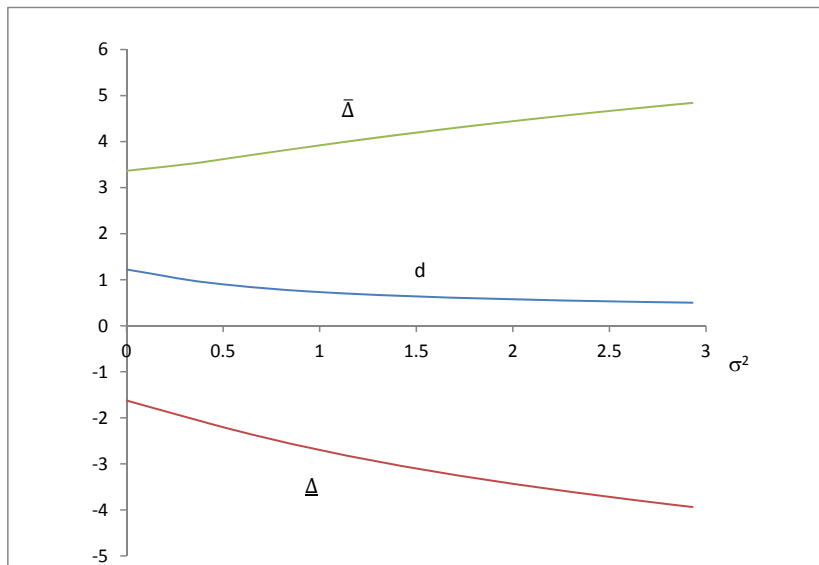


Figure 10: Evolution of  $d$  and the repositioning thresholds as a function of  $\sigma^2$  for  $r = .1, b = .5$ , and  $K = 4$ .

often, which leads to repositioning thresholds further away from zero. When  $K$  increases, as the firm is also going to spend more time without repositioning, the firm also chooses to increase the point to which it repositions to,  $d$ .

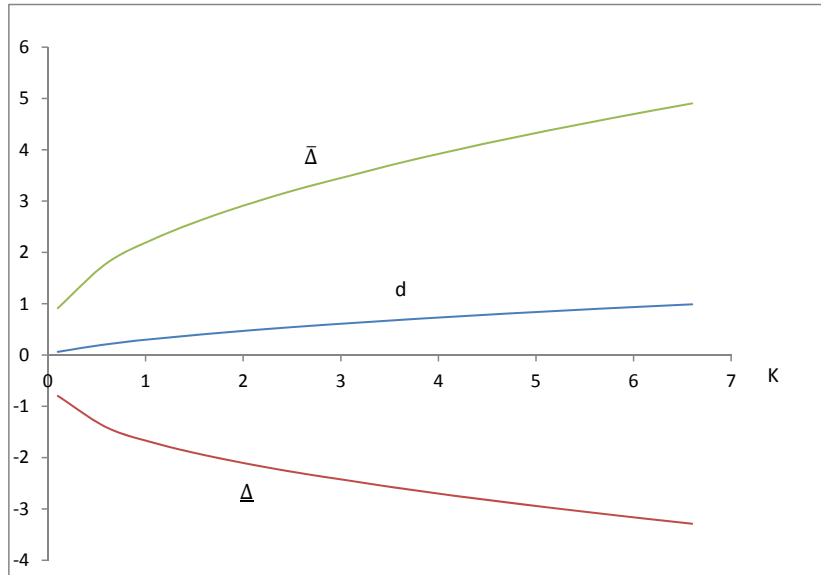


Figure 11: Evolution of  $d$  and the repositioning thresholds as a function of  $K$ , for  $r = .1$ ,  $b = .5$ , and  $\sigma^2 = 1$ .

## 6. CONCLUDING REMARKS

This paper investigates the way in which a firm optimally repositions when facing random and deterministic variations in the consumer preferences. For random variations in the consumer preferences the firm has to optimally trade-off the cost of repositioning and being close to the center of the market, with the possibility of the consumer preferences returning to where the firm is currently located. When the market is always fully covered, we find that a firm ends up repositioning more often than what would be optimal from a social welfare point of view, as profits decline faster than welfare when the consumer preferences move away from the product's positioning.

In the case of deterministic variations in the consumer preferences, we find that the firm chooses to over-shoot in its repositioning in order to save on repositioning costs and be closer to the center of the market for a longer period of time.

The continuous-time framework considered allows for a tractable way to consider dynamic repositioning decisions. Other marketing situations where dynamics are involved can also benefit from this modeling framework. Examples can include advertising decisions, capacity decisions (e.g., Gardete 2016), consumer search for information (e.g., Ke et al. 2016), sales force management, and branding evolution.

This paper can be seen as an investigation on successive repositionings by a firm to follow the evolution of the consumer preferences. In future research, it would also be interesting to explore how competition could affect these successive repositioning decisions.

## APPENDIX

### OPTIMAL PRICE FOR DIFFERENT $v$ AND $L$ .

Suppose that the distance from the center of the market is  $z$ , and assume  $z$  small. There are several possible candidate optimal prices: (1) Fully covering the market which would lead to an optimal price of  $P = v - L - z$  and a profit of  $\pi = v - L - z$ ; (2) Fully covering the market on one side of the firm, but not fully covering the market on the other side of the firm, and having the marginal consumers on both sides with zero surplus, which would lead to an optimal price of  $P = v - L + z$  and a profit of  $\pi = (v - L + z)(L - z)/L$ ; (3) Fully covering the market on one side of the firm, but not fully covering the market on the other side of the firm, and having the marginal consumer on only one side of the market with zero surplus, which would lead to an optimal price of  $P = \frac{v+L-z}{2}$  and a profit of  $\pi = \frac{(v+L-z)^2}{8L}$ ; (4) Not fully covering the market on both sides of the firm, which would lead to an optimal price of  $P = \arg \max_P P(v - P) = v/2$  and a profit of  $\pi = \frac{v^2}{4L}$ .

In cases (3) and (4) we have to make sure that the demand generated by the corresponding prices satisfies the assumptions for those cases. For case (3), this means that all the consumers not served have a negative surplus, which requires  $z > v - 3L$ , and that on one extreme of the consumer preferences the consumer has a positive surplus, which requires  $v - 3L + 3z > 0$ . For case (4), this means that all the consumers not served have a negative surplus, which requires  $v - 2L < -2z$ . Then if  $v < 2L$ , the optimal price will be case (4) for  $z$  small, and the market is never fully covered.

Comparing the profits of cases (1) and (2) we can get that case (1) is more profitable if  $v - 3L > -z$ . Then, if  $v > 3L$ , for  $z$  small, the optimal price is the one of case (1), and the market is always fully covered.

Let us then see what is optimal for  $v \in (2L, 3L)$  for  $z$  small. We know that neither case (1) nor case (4) is optimal in this parameter range for  $v$ . We also know that for case (3) to be optimal, we would need  $v - 3L > 0$  for  $z$  small. So, this yields that the optimal is case (2), where the market is fully covered for  $z = 0$ , and not fully covered otherwise.

### DERIVATION OF THRESHOLD $\tilde{K}$ :

To obtain the threshold  $\tilde{K}$  such that for  $K < \tilde{K}$  the optimum involves always full market coverage, we need that for any possible  $z$  on the equilibrium path, we have  $|z| < v - 3L$ . This



occurs if  $\Delta < v - 3L$ . So, from (11) we have

$$r\tilde{K} = v - 3L - 2\sqrt{\frac{\sigma^2}{2r}} \frac{e^{\sqrt{\frac{2r}{\sigma^2}}(v-3L)} - 1}{e^{\sqrt{\frac{2r}{\sigma^2}}(v-3L)} + 1}. \quad (\text{i})$$

DERIVATION OF  $\Delta$  WHEN  $r \rightarrow 0$  WHEN  $v$  IS LARGE:

Defining  $X \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta}$  and  $y \equiv \sqrt{r}$ , we can re-write (11) as

$$K(X + 1) = \frac{y\Delta(X + 1) - \sqrt{2\sigma^2}(X - 1)}{y^3}. \quad (\text{ii})$$

When  $r \rightarrow 0$  the left hand side converges to  $2K$ . When  $r \rightarrow 0$  the right hand side is indeterminate, and taking the L'Hôpital's rule three times, we can obtain that it converges to  $\frac{\Delta^3}{3\sigma^2}$ . We can then obtain that when  $r \rightarrow 0$  we have  $\Delta \rightarrow \sqrt[3]{6K\sigma^2}$ .

PROOF OF PROPOSITION 3:

For the case of  $K$  small, following the analysis in the text, note that (15) can be written as

$$rK = \frac{\Delta_w}{2L} (\Delta_w - 2\sqrt{\frac{\sigma^2}{2r}} \frac{X_w - 1}{X_w + 1}), \quad (\text{iii})$$

where  $X_w \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta_w}$ , and where the right hand side is increasing in  $\Delta_w$  as the derivative of  $\Delta_w - 2\sqrt{\frac{\sigma^2}{2r}} \frac{X_w - 1}{X_w + 1}$  is proportional to  $(X - 1)^2 > 0$ . It follows that  $\Delta_w$  is increasing in  $K$ . Similarly, from (11), we can get that

$$rK = \Delta - 2\sqrt{\frac{\sigma^2}{2r}} \frac{X - 1}{X + 1}, \quad (\text{iv})$$

where  $X \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta}$ , and that yields  $\Delta$  increasing in  $K$ . For  $\Delta = \Delta_w$  the right hand side of (iii) is equal to the right hand side of (iv) multiplied by  $\frac{\Delta_w}{2L}$ . As both  $\Delta$  and  $\Delta_w$  are increasing in  $K$ , it follows that for  $\Delta_w < L$ , we have  $\Delta_w > \Delta$ .

Consider now the case of  $K$  large such that the optimal  $\Delta_w$  could be greater than  $L$ . For this case the value function has a different shape for  $z > L$ , as the social welfare function is  $S(z) = v - z$  for  $z > L$ . We then have that in this region for  $z$  we have  $V_w(z) = C_{w1}e^{\sqrt{\frac{2r}{\sigma^2}}\Delta_w} + C_{w2}e^{-\sqrt{\frac{2r}{\sigma^2}}\Delta_w} - \frac{z}{r} + \frac{v}{r}$ . With continuity and smoothness at  $z = L$ , we have  $V_w(L^+) = V_w(L^-)$  and  $V'_w(L^+) = V'_w(L^-)$ , which leads to  $C_{w1} = C_w - \frac{\sigma^2}{4r^2LW}$  and  $C_{w2} = C_w - \frac{\sigma^2W}{4r^2L}$ , where  $W$  is defined

as  $W \equiv e^{\sqrt{\frac{2r}{\sigma^2}}L}$ .

Given continuity and smoothness at  $z = \Delta_w$ ,  $V_w(\Delta_w) = V_w(0) - K$  and  $V'_w(\Delta_w) = 0$ , which then leads to

$$C_w = \frac{X_w}{X_w^2 - 1} \left[ \frac{1}{r} \sqrt{\frac{\sigma^2}{2r}} + \frac{\sigma^2}{4r^2 L} \left( \frac{X_w}{W} - \frac{W}{X_w} \right) \right] \quad (\text{v})$$

$$rK = \Delta_w - \frac{L}{2} - \sqrt{\frac{\sigma^2}{2r} \frac{X_w - 1}{X_w + 1}} + \frac{\sigma^2 (X_w)(1 - W)}{2rLW(X_w + 1)}, \quad (\text{vi})$$

where  $X_w \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta_w}$ . Subtracting  $\Delta - 2\sqrt{\frac{\sigma^2}{2r} \frac{X-1}{X+1}}$  from the right hand side of (vi) when  $\Delta = \Delta_w$  one obtains

$$g(\Delta) = -\frac{L}{2} + \sqrt{\frac{\sigma^2}{2r} \frac{X-1}{X+1}} - \frac{\sigma^2 (X-W)(W-1)}{2rLW(X+1)}. \quad (\text{vii})$$

If we can show that (vii) is negative then we have that  $\Delta < \Delta_w$  as both  $\Delta$  and  $\Delta_w$  are increasing in  $K$ .

To see this note that

$$g(L) = -\frac{L}{2} + \sqrt{\frac{\sigma^2}{2r} \frac{W-1}{W+1}}, \quad (\text{viii})$$

which has the same sign as  $\tilde{g}(x) = -x(e^x + 1) + 2(e^x - 1)$ , by making  $x = \sqrt{\frac{2r}{\sigma^2}}L$ . Noting that  $\tilde{g}(0) = 0$ ,  $\tilde{g}'(0) = 0$ , and  $\tilde{g}''(x) < 0$  for  $x > 0$  we have that  $g(L) < 0$ .

Differentiating now  $g(\Delta)$  one obtains:

$$g'(\Delta) \frac{(X+1)^2}{X'} \frac{2r}{\sigma^2} W = h(x) = 2xe^x - e^{2x} + 1, \quad (\text{ix})$$

where  $x = \sqrt{\frac{2r}{\sigma^2}}L$ . One can obtain  $h(0) = 0$ ,  $h'(0) = 0$ , and  $h''(x) < 0$  for  $x > 0$ . So,  $h(x) < 0$  for  $x > 0$ , which means that  $g'(\Delta) < 0$  for  $\Delta > L$ , which yields  $g(\Delta) < 0$ . Then, that means that  $\Delta < \Delta_w$ .

DERIVATION OF  $\Delta_w$  FOR  $r \rightarrow 0$  AND  $K$  SMALL: Defining  $X_w \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta_w}$  and  $y \equiv \sqrt{r}$ , equation (15) can be re-written as

$$2(X_w + 1) \frac{LK}{\Delta_w} = \frac{y(X_w + 1)\Delta - \sqrt{2\sigma^2}(X_w - 1)}{y^3}. \quad (\text{x})$$

When  $r \rightarrow 0$  the left hand side converges to  $4\frac{LK}{\Delta_w}$ . To obtain the value to which the right hand

side converges when  $r \rightarrow 0$ , we have to apply L'Hôpital's rule three times, which yields  $\frac{4\Delta_w^3}{3}$ . One can then get that when  $r \rightarrow 0$ , we have  $\Delta_w \rightarrow \sqrt[4]{12LK\sigma^2}$ .

DERIVATION OF CASE WHEN REPOSITIONING COSTS DEPEND ON EXTENT OF REPOSITIONING:

Using (5) we can write (16) and (17) as

$$C_1X + C_2\frac{1}{X} = C_1\frac{X}{Z} + C_2\frac{Z}{X} + \frac{d}{r} - K - \alpha d, \quad (\text{xi})$$

and

$$C_1X - C_2\frac{1}{X} = C_1\frac{X}{Z} - C_2\frac{Z}{X}, \quad (\text{xii})$$

respectively, where  $X \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta}$  and  $Z \equiv e^{\sqrt{\frac{2r}{\sigma^2}}d}$ . Recall also that  $V'(0^+) = V'(0^-)$  leads to

$$C_1 = C_2 + \frac{1}{r}\sqrt{\frac{\sigma^2}{2r}}. \quad (\text{xiii})$$

Finally, note that (18) can be written

$$C_1\frac{X}{Z} - C_2\frac{Z}{X} = \left(\frac{1}{r} - \alpha\right)\sqrt{\frac{\sigma^2}{2r}}. \quad (\text{xiv})$$

Using (xi)-(xiv) we can then obtain,  $C_1$ ,  $C_2$ ,  $\Delta$ , and  $d$ . In particular, using (xiii) and (xiv) we can obtain:

$$C_1 = \sqrt{\frac{\sigma^2}{2r}} \frac{1}{(X/Z)^2 - 1} \left[ \frac{X/Z - 1}{r} - \alpha \frac{X}{Z} \right], \quad (\text{xv})$$

$$C_2 = \sqrt{\frac{\sigma^2}{2r}} \frac{X/Z}{(X/Z)^2 - 1} \left[ \frac{1 - X/Z}{r} - \alpha \right]. \quad (\text{xvi})$$

Using (xi) and (xii), and taking out  $\frac{d}{r} - K - \alpha d$  (by addition of those two equations and subtraction), we can obtain  $C_1X + C_2\frac{Z}{X} = 0$ . Substituting for  $C_1$  and  $C_2$  from (xv) and (xvi), one can then obtain

$$Z = X \frac{X - 1 - \alpha r X}{X - 1 + \alpha r}. \quad (\text{xvii})$$

Takings logs on both sides and dividing by  $\sqrt{\frac{2r}{\sigma^2}}$  one obtains (20). Finally, adding (xi) and (xii), and substituting for (xv), (xvi), and (xvii) one obtains (19).

Consider now what happens to (xvii) when  $r \rightarrow 0$ . Subtracting 1 from both sides of (xvii), dividing by  $\sqrt{\frac{2r}{\sigma^2}}$ , and then making  $r \rightarrow 0$ , and applying L'Hôpital's rule on both sides of the equation, one obtains (22).

Rearranging (19) one can obtain

$$\frac{\sigma^2}{2}(K + \alpha d) = \frac{2(X - X/Z) - d\beta - (2 - \alpha\sigma^2\beta^2)X^2/Z + (2 - \alpha\sigma^2\beta^2 + d\beta)(X/Z)^2}{\beta^3((X/Z)^2 - 1)}, \quad (\text{xviii})$$

where  $\beta \equiv \sqrt{\frac{2r}{\sigma^2}}$ . Making  $\beta \rightarrow 0$ , and applying L'Hôpital's rule four times on the right hand side, one can obtain using (22)

$$d = \sqrt[3]{6K\sigma^2}. \quad (\text{xix})$$

DERIVATION OF OPTIMAL  $\Delta$  FOR LARGE  $K$  AND MARKET SOMETIMES NOT FULLY COVERED:

In order to obtain the optimal  $V(z)$  we have to consider the differential equation  $rV(z) = \frac{\sigma^2}{2}V''(z) + \pi(z)$  and the two regions for  $\pi(z)$  as described in the text:  $z \in [0, v - 3L]$ , and  $z \in (v - 3L, \Delta)$ .

For  $z \in [0, v - 3L]$ , we have  $\pi(z) = v - L - z$ , and given smoothness at  $V(0)$ ,  $V'(0^+) = V'(0^-)$ , we have

$$V(z) = C_1 e^{\sqrt{\frac{2r}{\sigma^2}}z} + C_2 e^{-\sqrt{\frac{2r}{\sigma^2}}z} - \frac{z}{r} + \frac{v - L}{r}, \quad (\text{xx})$$

and

$$C_2 = C_1 - \frac{1}{r} \sqrt{\frac{\sigma^2}{2r}} \quad (\text{xxi})$$

where  $C_1$  and  $C_2$  are constants to be determined later.

For  $z \in (v - 3L, \Delta)$ , we have  $\pi(z) = \frac{(v+L-z)^2}{8L}$ , which yields

$$V(z) = C_3 e^{\sqrt{\frac{2r}{\sigma^2}}z} + C_4 e^{-\sqrt{\frac{2r}{\sigma^2}}z} + \frac{(v + L - z)^2}{8Lr} - \frac{\sigma^2}{8Lr^2}. \quad (\text{xxii})$$

Continuity and smoothness at  $V(v-3L)$ ,  $V(v-3L^+) = V(v-3L^-)$  and  $V'(v-3L^+) = V'(v-3L^-)$  yields

$$C_1 X + C_2 \frac{1}{X} = C_3 X + C_4 \frac{1}{X} + \frac{\sigma^2}{8Lr^2} \quad (\text{xxiii})$$

$$C_1 X - C_2 \frac{1}{X} = C_3 X - C_4 \frac{1}{X}, \quad (\text{xxiv})$$

where  $X \equiv e^{\sqrt{\frac{2r}{\sigma^2}}(v-3L)}$ .

Finally, to determine the threshold  $\Delta$  we must have  $V(\Delta) = V(0) - K$  and  $V'(\Delta) = 0$ . These two conditions then mean that:

$$C_3Y + C_4\frac{1}{Y} + \frac{(v+L-\Delta)^2}{8Lr} + \frac{\sigma^2}{8Lr^2} = C_1 + C_2 + \frac{v-L}{r} - K \quad (\text{xxv})$$

$$C_3Y - C_4\frac{1}{Y} = \sqrt{\frac{\sigma^2}{2r}} \frac{v+L-\Delta}{4Lr}, \quad (\text{xxvi})$$

where  $Y \equiv e^{\sqrt{\frac{2r}{\sigma^2}}\Delta}$ . Using (xxi), (xxiii), (xxiv), and (xxvi) we can obtain

$$C_3Y + C_4\frac{1}{Y} - C_1 - C_2 = -\frac{\sigma^2}{8Lr^2} \frac{X^2 + Y}{X(1+Y)} + \frac{1}{r} \frac{Y-1}{Y+1} \left( \frac{v+L-\Delta}{4L} + 1 \right). \quad (\text{xxvii})$$

Using (xxvii) in (xxv) we can obtain an equation to determine  $\Delta$  as

$$rK = \frac{\sigma^2}{8Lr} \frac{X^2 + Y}{X(1+Y)} - \sqrt{\frac{\sigma^2}{2r}} \frac{Y-1}{Y+1} \left( \frac{v+L-\Delta}{4L} + 1 \right) + v - L - \frac{\sigma^2}{8Lr} - \frac{(v+L-\Delta)^2}{8L}. \quad (\text{xxviii})$$

Evaluating (xxviii) at  $\Delta = v + L$  we can obtain the upper threshold on  $K$ ,  $\widehat{K}$ , such that the market is always covered, even if partially, as

$$r\widehat{K} = \frac{\sigma^2}{8Lr} \frac{X^2 + \widehat{Y}}{X(1+\widehat{Y})} - \sqrt{\frac{\sigma^2}{2r}} \frac{\widehat{Y}-1}{\widehat{Y}+1} \left( \frac{v+L-\Delta}{4L} + 1 \right) + v - L - \frac{\sigma^2}{8Lr} - \frac{(v+L-\Delta)^2}{8L}, \quad (\text{xxix})$$

where  $\widehat{Y} \equiv e^{\sqrt{\frac{2r}{\sigma^2}}(v+L)}$ .

In order to obtain the  $\Delta$  when  $r \rightarrow 0$ , re-arrange (xxviii) to obtain:

$$4LKX(Y+1)\sigma^2 = \frac{\beta^2 X(1+Y)[8L(v-L) - (v+L-\Delta)^2] - 2\beta(Y-1)(v+5L-\Delta)X + 2(Y-X)(1-X)}{\beta^4}. \quad (\text{xxx})$$

where  $\beta \equiv \sqrt{\frac{2r}{\sigma^2}}$ . Applying L'Hôpital's rule four times on the right hand side, we obtain that when  $r \rightarrow 0$  we have:

$$f(\Delta, L, \sigma^2, K, v) \equiv 144\sigma^2 LK + 3\Delta^4 - 8v\Delta^3 + 2(v-3L)\Delta^3 + 6(v-3L)^3\Delta - 3(v-3L)^4 = 0. \quad (\text{xxxi})$$

Note that  $f()$  is increasing in  $\sigma^2$  and  $K$ . Note also that  $\frac{\partial f}{\partial \Delta} = 12\Delta^3 - 18(v+L)\Delta^2 + 6(v-3L)^3$ , which is negative for  $\Delta \in (v-3L, v+L)$  which was assumed in this case (for  $\Delta < v-3L$  we are in the case of full market coverage; for  $\Delta > v+L$  we are in a case where sometimes no consumer is served). Finally, note that

$$\begin{aligned} L \frac{\partial f}{\partial L} &= -3\Delta^4 + 6v\Delta^3 + 3(v-3L)^4 - 6(v-3L)^3\Delta + 36L(v-3L)^3 - 54L\Delta(v-3L)^2 \\ &= 3(\Delta-v+3L)^2[(\Delta+v-3L)(3L+v-\Delta) + 6L(v-3L)] \end{aligned} \quad (\text{xxxii})$$

which is positive given that  $v > 3L$ , and  $\Delta \in (v-3L, v+L)$ . Then, we have  $\frac{\partial \Delta}{\partial \sigma^2}, \frac{\partial \Delta}{\partial K}, \frac{\partial \Delta}{\partial L} > 0$ .

## REFERENCES

- CALCAGNO, R., Y. KAMADA, S. LOVO, AND T. SUGAYA (2014), "Asynchronicity and Coordination in Common and Opposing Interest Games," *Theoretical Economics*, **9**, 409-434.
- DIXIT, A. (1993), *The Art of Smooth Pasting*, Harwood Academic Publishers: Chur, Switzerland.
- ERICKSON, R., AND A. PAKES (1995), "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, **62**, 53-82.
- GARDETE, P. (2016), "Competing Under Asymmetric Information: The Case of DRAM Manufacturing," *Management Science*, **62**, 3291-3309.
- HARRIS, C., AND J. VICKERS (1987), "Racing with Uncertainty," *Review of Economic Studies*, **54**, 1-21.
- HAUSER, J.R. (1988), "Competitive Price and Positioning Strategies," *Marketing Science*, **7**, 76-91.
- HAUSER, J.R., F. EGGERS, AND M. SELOVE (2016), "The Strategic Implications of Precision in Conjoint Analysis," *working paper*, MIT.
- HAUSER, J.R. AND S. SHUGAN (1983), "Defensive Marketing Strategies," *Marketing Science*, **3**, 327-351.
- JEZIORSKI, P. (2014), "Estimation of Cost Synergies from Mergers: Application to the U.S. Radio," *RAND Journal of Economics*, **45**, 816-846.
- KE, T.T., Z.-J.M. SHEN, AND J.M. VILLAS-BOAS (2016), "Search for Information on Multiple Products," *Management Science*, **62**, 3576-3603.
- KUKSOV, D. (2004), "Buyer Search Costs and Endogenous Product Design," *Marketing Science*, **23**, 490-499.
- LAUGA, D.O., AND E. OFEK (2011), "Product Positioning in a Two-Dimensional Vertical Differentiation Model: The Role of Quality Costs," *Marketing Science*, **30**, 903-923.
- MAGILL, M.J.P., AND G.M. CONSTANTINIDES (1976), "Portfolio Selection with Transaction Costs," *Journal of Economic Theory*, **13**, 245-263.
- MOORTHY, K.S. (1988), "Product and Price Competition in a Duopoly," *Marketing Science*, **7**, 141-168.
- OFEK, E., AND M. SARVARY (2003), "R&D, Marketing, and the Success of Next-Generation Products," *Marketing Science*, **22**, 355-370.
- SAYMAN, S., S.J. HOCH, AND J.S. RAJU (2002), "Positioning of Store Brands," *Marketing Science*, **21**, 378-397.

- SCARF, H. (1959), "The Optimality of (S,s) Policies in the Dynamic Inventory Problem," in *Arrow, K., S. Karlin, and P. Suppes (eds.), Mathematical Methods in the Social Sciences*, Stanford University Press.
- SHEN, Q. (2014), "A Dynamic Model of Entry and Exit in a Growing Industry," *Marketing Science*, **33**, 712-724.
- SHESHINSKI, E., AND Y. WEISS (1983), "Optimum Pricing Policy under Stochastic Inflation," *Review of Economic Studies*, **L**, 513-529.
- SPENCE, M. (1975), "Monopoly, Quality, and Regulation," *The Bell Journal of Economics*, **6**, 417-429.
- SWEETING, A. (2013), "Dynamic Product Positioning in Differentiated Product Markets: The Effect of Fees for Musical Performance Rights on the Commercial Radio Industry," *Econometrica*, **81**, 1763-1803.
- VILLAS-BOAS, J.M. (1992), "Dynamic Duopolies with Non-Convex Adjustment Costs," *Economics Letters*, **39**, 391-394.
- WERNERFELT, B. (1989), "From Critical Resources to Corporate Strategy," *Journal of General Management*, **14**, 4-12.