Geodesy and the Problem of Ice Sheets

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Abstract. In recent years, great improvements have been made toward understanding the modern dynamics and recent history of the ice sheets. Several recently-launched satellite missions promise to make geodesy the most powerful tool for investigation of the changing ice sheets, including their past history and their present behavior. Mathematical description of ice sheet behavior from geodetic data requires accurate modeling of all the processes which may affect the measurements. Most geodetic tools measure changes in elevation of the ice sheets, which can include Post Glacial Rebound (PGR), the current Ice Mass Trend (IMT) consisting of both accumulation and glacial outflux, and processes of compaction within the firn column. Consequently it is necessary for mathematical models of geodetic data to separate the effects of IMT, PGR, and compaction. Satellite measurements of the time-variable geoid are insensitive to compaction effects and depend on IMT and PGR differently than do height measurements. Two methodological approaches have been proposed to separate these effects using measurements of height and time-variable geoid: 1- direct inversion for ice mass variability (Wu et al., 2002), which requires *a priori* assumptions about either the Earth's rheology or the ice load history; 2- iterative solution for the fields, which theoretically is more approximate but is computationally much simpler and less dependent on *a priori* assumptions. In this paper we analyze how we can learn about IMT and PGR by combining geodetic measurements, and we assess the conditions required to optimally combine satellite and ground-based data sets.

1 Introduction

Mathematical modeling is an indispensable tool in geodetic problems. However, geophysical problems often are not adequately constrained by modeling a single type of geodetic measurement because of the limitations of measurement uncertainties, spatial and temporal sampling and the large number of different geophysical processes which may contribute to a particular measurement. Often the formal inversion of a complex geophysical problem requires many implicit or explicit assumptions, the adoption of parameters with their own uncertainties, and significant computational time, all of which combine to make it very difficult (if not impossible) to realistically assess the uncertainties of the solution. Consequently, it can be useful to adopt an iterative or convergent approach to solving a geophysical problem, in which the *a priori* assumptions are minimized and a solution can be achieved with significantly less computational expense. In this article we present an example of such a method applied to satellite geodetic measurements of the time-variable geoid, ice sheet elevation, and rock elevation to study changes in the ice sheets. We assess the limitations of the method and explore additional data sets which may improve the results.

The Gravity Recovery And Climate Experiment (GRACE), jointly sponsored by NASA and the Deutsches Zentrum für Luft- und Raumfahrt (DLR), was launched in March, 2002. It will map the Earth's gravity field with unprecedented accuracy and resolution every 30 days during its 5-year lifetime. This will permit monthly variations in gravity to be determined down to scales of a few hundred kilometers and larger. These gravity variations can be used to study a variety of processes involving redistribution of mass within the Earth or on its surface. The expected performance of GRACE and various possible applications are described by Dickey et al. (1997) and Wahr et al. (1998). Among these applications is the use of the GRACE secular gravity signal to constrain models of post glacial rebound (PGR): the viscous adjustment of the solid Earth in response to the removal of the ice loads following the last ice age.

NASA's Ice, Cloud, and Land Elevation Satellite (ICESat) represents an important step toward estimating present-day Antarctic and Greenland ice mass balance. Launched in January of 2003, ICESat carries the Geoscience Laser Altimeter System (GLAS) and will have a mission lifetime of 3-5 years. To study the polar ice sheets, a laser pulse generated by the altimeter will reflect off the snow/ice surface and return to the satellite. Measurements of the round-trip distance, combined with Global Positioning System (GPS) measurements of the geocentric position of the spacecraft, will map changes in the surface elevation of the polar ice sheets at regular time intervals. The exact repeat period of the ground tracks will be 183 days, though there will be a 25-day near-repeat subcycle in which measurement locations shift by 15 km at the equator (and much less over the poles) from those made 25 days previously. Changes in the ice sheet elevation will be determined from crossover differences. Using GLAS data, it will be possible to estimate the rate of change in Antarctic ice mass over the mission lifetime between the ice sheet margins and 88°S latitude.

GLAS measurements of ice sheet elevation will reflect more than just the ice mass change. Uplift and subsidence due to post glacial rebound (PGR) and variable compaction of snow will also influence the height change measured by GLAS. *Velicogna and Wahr* [2002] showed that it is possible to combine the GLAS measurements of change in ice sheet elevation with time variable gravity measurements from GRACE to separate PGR and IMT. Moreover, by adding GPS measurements of vertical rock velocity it is possible to solve for PGR, IMT, and the temporal variation of density due to compaction within the ice column.

In this paper we will examine the state of the art of satellite measurements for investigation of the ice sheets. We will look in particular at methodologies for combining satellite measurements from GLAS, GRACE and GPS and at the limiting factors for IMT and PGR solution uncertainties.

2 Synthetic data

To simulate the recovery of Antarctic post-glacial rebound and ice mass trend from GRACE, GLAS, and GPS, we construct five years of synthetic satellite data as the sum of geophysical signals and measurement errors. Almost all of the contributions to these signals are identical to those described in *Wahr et al.* [2000] and *Velicogna and Wahr* [2002a,b], and a more detailed discussion of the simulated data can be found therein. We briefly summarize the contributions to simulated signals here.

Monthly ice sheet elevations in the simulated GLAS measurements consist of contributions from snow accumulation and horizontal ice flow in Antarctica, from the Earth's elastic response to loading by these mass fields, from Earth's viscoelastic response to past loading (i.e., PGR), and from GLAS measurement errors.

For GRACE, we simulate five years of monthly measurements of the geoid: the equipotential surface corresponding to mean sea level over the oceans. The geoid can be expanded in a spherical harmonic representation as [*Kaula*, 1966]:

$$\begin{aligned} N(\theta,\phi) &= a\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\tilde{P}_{lm}(\cos\theta)\{C_{lm}\cos m\phi \\ &\quad + S_{lm}\sin m\phi\}, \end{aligned} \tag{1}$$

where a is the Earth's mean radius, θ and ϕ are colatitude and east longitude, C_{lm} and S_{lm} are dimensionless coefficients, and the $\tilde{P}_{lm}$ are normalized associated Legendre functions [e.g. *Chao and Gross*, 1987]. GRACE will deliver values of C_{lm} and S_{lm}, up to a maximum degree and order (i.e., l and m) of 100, every month. The simulated data include GRACE measurement errors, as well as the gravitational effects of snow accumulation and ice flow on Antarctica, of the elastic response to loading, of PGR, of redistribution of water mass in the ocean and on landmasses other than Antarctica, and of errors in correction for atmospheric pressure.

The simulated GPS data consist of five years of daily height coordinates. The coordinates include a constant vertical velocity contributed by PGR, the Earth's elastic loading response to accumulation and ice flow on Antarctica, and GPS measurement errors estimated to be 1.5 cm root-mean square (RMS) for daily coordinates.

Contributions of Antarctic snow accumulation to our simulated GLAS, GRACE, and GPS data are derived from monthly precipitation in the National Center for Atmospheric Research (NCAR) Climate System Model (CSM-1) [*G. Bonan, personal communication*, 1997; see, e.g., *Boville and Gent*, 1998]. For more detail see *Wahr et al.* [2000] and *Velicogna and Wahr* [2002]. To incorporate snow and ice effects into the GRACE simulated geoid we sum the above (detrended) contribution with *Bentley and Giovinetto*'s [1991] estimate of the spatially varying long-term mean ice mass trend. To simulate changes in GLAS ice heights corresponding to the change in mass fields, we use the density properties of snow and ice modified by the time- and accumulation-dependent compaction model of *Wingham* [2000]. We estimate the contribution from the PGR vertical velocity by convolving viscoelastic load Love numbers, computed as described by *Han and Wahr* [1995], with estimates of the Antarctic deglaciation history. We estimate the elastic vertical displacement U of the solid Earth caused by the mass load, using standard methods (see, e.g., equations (40) and (42) of *Mitrovica et al.* [1994]).

3 Method

To estimate IMT and PGR from the simulated data, we use an iterative method in which we try to avoid dependence on any *a priori* assumptions about Earth viscosity and ice loading history. The iterative approach is analogous to that described by *Velicogna and Wahr* [2002]. Initially we assume that GLAS is sensitive only to the ice thickness change and not to the PGR uplift, and we determine the ice thickness change using only the GLAS elevation data. We refer to this initial estimate as the zeroth iteration.

We compute the secular rate of change in the geoid caused by the zeroth order GLAS mass balance estimate and remove that geoid signal from the GRACE data. The residual secular geoid change is then interpreted as being due entirely to PGR. We use the geoid residual to estimate the corresponding crustal uplift due to PGR, using the method of *Wahr et al.* [2000], in which we let

$$\dot{N}^{\mathrm{PGR}}(\theta,\phi) = a\sum_{l=0}^{\infty}\sum_{m=0}^{l}\tilde{P}_{lm}(\cos\,\theta)[\dot{C}_{lm}^{\mathrm{PGR}}\cos\,(m\phi) + \dot{S}_{lm}^{\mathrm{PGR}}\sin\,(m\phi)] \qquad (2)$$

be the secular rate of change in the geoid caused by PGR, where $\dot{C}_{lm}^{\mathrm{PGR}}$ and $\dot{S}_{lm}^{\mathrm{PGR}}$ are the Legendre expansion coefficients of $\dot{N}^{\mathrm{PGR}}$. *Wahr et al.* [2000] found that to a high degree of approximation, the relations between the rates of change of the geoid coefficients $\dot{C}_{lm}^{\mathrm{PGR}}$ and $\dot{S}_{lm}^{\mathrm{PGR}}$, and those of the uplift rate, $\dot{A}_{lm}^{\mathrm{PGR}}$ and $\dot{B}_{lm}^{\mathrm{PGR}}$, are

$$\dot{A}_{lm}^{\mathrm{PGR}} = \left(\frac{2l+1}{2}\right)\dot{C}_{lm}^{\mathrm{PGR}}, \qquad (3)$$

$$\dot{B}_{lm}^{\mathrm{PGR}} = \left(\frac{2l+1}{2}\right)\dot{S}_{lm}^{\mathrm{PGR}}. \qquad (4)$$

from which

$$\dot{U}_{\mathrm{GLAS/GRACE}}^{\mathrm{PGR}}(\theta,\phi) = a\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{2l+1}{2}\right)\tilde{P}_{lm}(\cos\,\theta)[\dot{C}_{lm}^{\mathrm{PGR}}\cos\,(m\phi) + \dot{S}_{lm}^{\mathrm{PGR}}\sin\,(m\phi)] \qquad (5)$$

The explanation for (3) and (4) is that the change in the geoid caused by PGR is mostly due to mass anomalies associated with vertical motion of the surface (see *Wahr et al.* [1995]). We estimate PGR uplift from the residual (GRACE minus the IMT estimate of geoid) using equation (5). Subtracting this PGR estimate from GLAS then results in a better estimate of ice balance. We refer to this estimate as the first iteration. We then repeat the process: compute the geoid contributions from this new GLAS ice mass estimate and remove them from GRACE, interpret the secular component of the new GRACE residuals as the effect of PGR, calculate the PGR uplift and remove it from GLAS, and use the GLAS residuals to construct a further improved ice balance estimate. We iterate this procedure eight times, after which the improvement is negligible.

Then, following the last iteration, the PGR error is estimated from the difference between GPS vertical velocities (slope of the daily height coordinates) and the PGR estimate $\dot{U}^{\mathrm{PGR}}$ derived by applying equation (5) to the secular GRACE geoid minus the geoid from the estimated IMT. That PGR error is used to estimate the ice compaction trend (see below), and a corresponding correction to the estimate of IMT. In fact we are really trying to solve for three unknowns: IMT, PGR, and the trend of the time-variable density of the ice column. Continuous GPS point measurements of vertical rock velocity add the additional constraint necessary to solve for the third unknown field.

We expect that the GPS measurements available to estimate the PGR correction will be sparse and irregularly distributed, so that assimilating them into the data constraints will require some care. We first interpolate the gridded $\dot{U}^{\mathrm{PGR}}_{GLAS/GRACE}$ to each GPS location using two-dimensional optimal interpolation. The PGR correction at a GPS site is simply $\Delta\dot{U}^{\mathrm{PGR}}_{compaction} = \dot{U}^{\mathrm{PGR}}_{\mathrm{GPS}} - \dot{U}^{\mathrm{PGR}}_{\mathrm{GLAS}/GRACE}$. Then, we interpolate the PGR correction back to the grid points using an optimal interpolation. To downweight the correction in undersampled regions, we multiply the interpolated correction at each grid point by a Gaussian function, $W(\alpha)$, given by (7), where R in (8) is 500 km and α is the angular distance to the nearest GPS site (see section 7 for more details about the choice of R). This effectively localizes the estimate of PGR error near the GPS sites where it is measured. As we will address further in the discussion section, the accuracy of the estimate of PGR error calculated using this method depends fundamentally on the correlation length scale of the time variable compaction effects that cause the error.

Local correction of the GLAS/GRACE PGR estimate with GPS vertical velocities improves the PGR estimate, but subtracting the corrected PGR estimate from GLAS heights does not dramatically improve the estimate of IMT. However, the empirical relation between the IMT compaction error and the compaction error in PGR $\Delta\dot{U}^{PGR}_{compaction} = -0.31\Delta\dot{h}^{\mathrm{IMT}}_{compaction}$ (see the discussion at the end of section 4), allows us to use the GPS-derived PGR errors to also estimate the compaction error. In practice, we use the iterative routine that combines GRACE and GLAS (see description in section 3) and after the last iteration we use the GPS to correct the GLAS/GRACE estimate of PGR by removing the difference between GPS and GLAS/GRACE PGR vertical velocities ($\Delta\dot{U}^{\mathrm{PGR}}_{compaction}$) estimated as described above. Then we correct the IMT by removing the PGR error and the compaction error estimated as the PGR error divided by 0.31. In this manner we effectively solve for all three unknowns in the Antarctic mass balance (i.e., IMT, PGR, and the trend of time-variable density within ice columns). Once we have obtained the PGR and IMT estimates, we smooth those fields using the 250-km Gaussian averaging function defined in (7).

4 Uncertainties in PGR and IMT

4.1 Time-variable accumulation and IMT

ICESat and GRACE will have lifetimes of about five years. Interannual and interdecadal variations in accumulation rate cause the mass trend on five year time scales to differ from the century scale trend. Part of this difference occurs because the normal variability of climate includes nonsecular components with periods greater than five years. For GRACE, which is directly sensitive to mass change, the difference between the five year and century-scale trends is the only contribution of the time-variable accumulation to the error in estimating the century-scale trend. For GLAS, which is sensitive to ice sheet thickness rather than to mass, there is the additional problem that the relation between thickness and mass is not simple multiplication by ρ_i but is complicated by variable compaction of the snow-ice column. For the application at hand, we will not concern ourselves with the century scale trend but consider only those uncertainties which may be introduced in our estimates of IMT on the five-year timescale over which we will have measurements.

Because the density profile in the upper layers of the snow-ice column depends on prior accumulation rate, the assumption of a constant ice density ρ_i introduces an error in the estimate of IMT from GLAS data. Error in the GLAS estimate of mass balance caused by approximating the time- and accumulation-dependence of snow compaction with $\dot{h} = \dot{m}/\rho_i$, we will refer to as "compaction error". This error affects the GLAS estimate of the five-year trend. *Wahr et al.* [2000] conclude that the compaction error in estimates of the five-year ice mass trend using GLAS alone is likely to be ± 4.5 mm yr^{-1} (water thickness equivalent) when averaged over the entire Antarctic ice sheet, which is equivalent to an error of about ± 0.15 mm yr^{-1} in estimates of global sea level rise.

4.2 Time-variable accumulation and PGR

PGR over Antarctica is poorly known and could contribute on the order of 5 mm yr^{-1} in error to GLAS-only estimates of five-year IMT averaged over Antarctica. The inclusion of GRACE data permits the separation of the PGR and IMT signals, and so reduces the contribution of the unmodeled PGR signal to the IMT error. *Wahr et al.* [2000] find that the GRACE/GLAS iteration described in section 3, without inclusion of the GPS data, would remove all of the PGR contribution to the IMT error if there was no time variable accumulation. However, errors introduced by time variable compaction result in erroneous estimates of mass from GLAS heights, and the mass inconsistency leads to an error in the final estimate of PGR contribution to the surface heights. Thus the GRACE residuals used to infer the PGR signal suffer indirect effects of the compaction error. *Wahr et al.* [2000] find that the final PGR contribution to the IMT error, averaged over the entire ice sheet, is approximately 0.31 times the IMT compaction error. This proportionality factor is an artifact of the method used to combine the GLAS and GRACE data. Specifically, it arises from combining equation (15) of *Wahr et al.* [1998] to estimate the geoid from the GLAS surface mass (i.e. IMT) estimate, with (5) above, relating the GRACE PGR geoid to an inferred uplift rate. The proportionality factor between PGR and IMT after any single iteration is about equal to $d = 0.24$. After N iterations, the proportionality factor $= d(1-d^N)/(1-d) \approx 0.31$ when N is large. Hence, errors in the recovery of IMT and PGR from GLAS ice sheet elevations plus the GRACE geoid arise principally because only two observables are used to determine three unknowns (IMT, PGR, and the time-variable density of the ice column).

4.3 Spatial variability

The PGR and IMT estimates derived from GLAS and GRACE data can have large errors at short wavelengths, because the GRACE measurements of $\dot{C}_{lm}^{\mathrm{PGR}}$ and $\dot{S}_{lm}^{\mathrm{PGR}}$ become increasingly inaccurate as l gets large. To mitigate those short wavelength errors, we smooth the PGR and IMT results by constructing Gaussian averages of those fields. The Gaussian average of the recovered PGR field is [*Wahr et al.*, 1998]:

$$\overline{\dot{U}}^{PGR}_{GLAS/GRACE}(\theta,\phi) = \int \sin\,\theta'\, d\theta'\, d\phi'\, W(\alpha)\, \dot{U}^{\mathrm{PGR}}_{GLAS/GRACE}(\theta',\phi') \quad (6)$$

where $\dot{U}^{PGR}_{GLAS/GRACE}$ is the unsmoothed field, and the averaging function is

$$W(\alpha) = \frac{b}{2\pi}\frac{\exp\left[-b(1-\cos\,\alpha)\right]}{1-e^{-2b}} \quad (7)$$

where α is the angular distance between (θ,ϕ) and (θ',ϕ'), and

$$b = \frac{\ln(2)}{(1-\cos\,(R/a))} \quad (8)$$

with R the distance along the Earth's surface at which W has dropped to 1/2 its value at $\alpha = 0$ [see *Jekeli*, 1981, eqn. (59)]. We apply this smoothing process to the recovered PGR and IMT fields, using $R = 250$ km. Thus, the end products of our estimation algorithm can best be described as the PGR and IMT fields smoothed over 250 km scales. To determine the accuracy of these smoothed, recovered fields, we also apply this smoothing process to the input PGR and IMT fields used to construct our simulated data, and we assess the difference between those smoothed input fields and the smoothed recovered fields.

5 PGR and IMT from GLAS, GRACE and GPS Data

Compaction error is by far the largest source of uncertainty in the GLAS/GRACE estimates of PGR and IMT, and GPS measurements of vertical rock velocity will provide an important constraint of this field. In our simulations we begin by assuming that continuous GPS measurements are made at 45 existing and proposed sites, including previous campaign sites along the Antarctic coast [*Dietrich*, 2001] and existing and proposed continuous sites along the Transantarctic Mountains and Marie Byrd Land in West Antarctica [*Raymond et al.*, 1998]. 36 of the 45 sites used are on the Antarctic mainland, while the remainder are on islands at high southern latitudes.

Figure 1 shows the spatially-varying recovery of smoothed IMT after assimilation of the GPS vertical velocities, for a five year period. By incorporating the GPS data, we obtain a smoothed PGR estimate with just 2.8 mm yr^{-1} of RMS error (Table 1), a 49% improvement relative to that recovered from GRACE and GLAS data alone. The corresponding estimate of the smoothed IMT has 12.0 mm yr^{-1} of RMS error, a 35% improvement. The similarity of the error reductions for IMT and PGR indicates that the empirical scaling used to relate PGR error to IMT compaction error is justified. Using this distribution of 45 GPS sites, we calculated estimates for all of the independent five year periods of the (300-year long) CSM-1 model. The

RMS average of RMS errors in recovery of the simulated fields was 3.4 mm yr^{-1} of PGR error and 15.9 mm yr^{-1} error in IMT. The PGR error ranged from 1.4 to 5.4 mm yr^{-1} and the error in the IMT estimate ranged from 5.8 to 26.9 mm yr^{-1}.

	PGR mm yr^{-1}	IMT mm yr^{-1}
No Time Variable Accumulation – – No GPS	1.3	2.1
No GPS	5.3	19.9
Current+ proposed+ campaign GPS	3.4	15.9
10 GPS sites	2.6	13.2
50 km spacing GPS sites	1.5	7.9

Table 1: RMS error of the smoothed PGR and IMT fields estimated by GLAS/GRACE, with and without GPS.

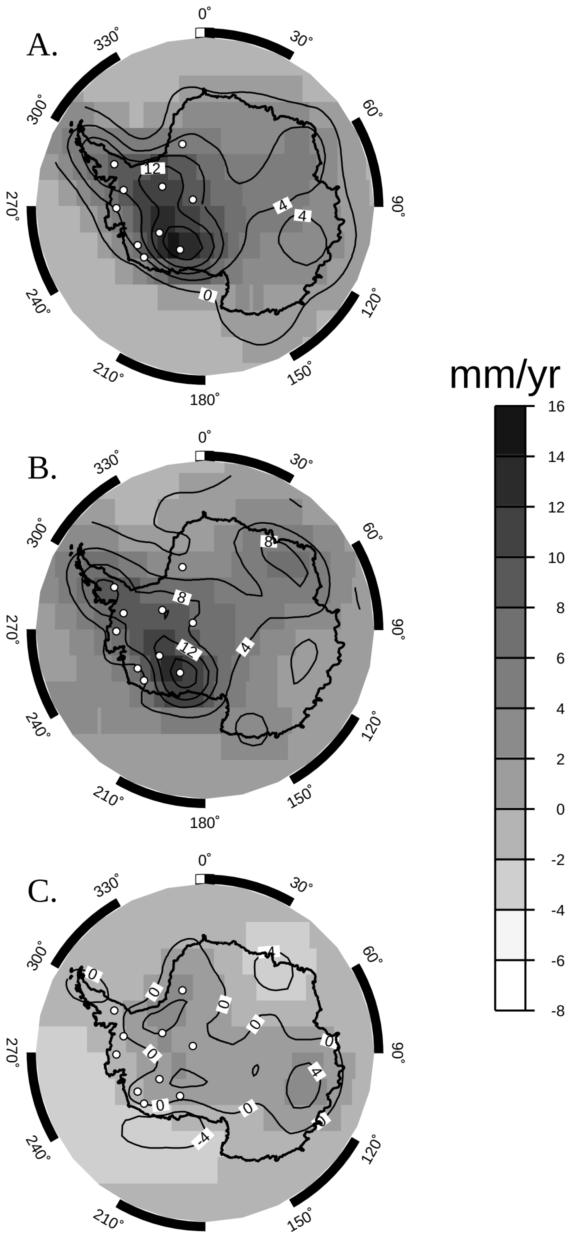

Figure 1: Smoothed Ice Mass Trend (IMT) signal. (a) Input signal. (b) Recovered signal combining GLAS, GRACE and GPS data from 45 existing and planned continuous and campaign GPS sites. (c) Residual. Contour interval is 20 mm yr^{-1}. Circles denote GPS locations.

Because the largest error source in the PGR and IMT estimates is time variable accumulation, we also examine how the estimates of IMT and PGR are affected by using just a few GPS sites located where the compaction error is largest in Figure 2. The compaction error is largest where the accumulation rates are largest. The results after locating the GPS measurements near the largest accumulation centers are shown in Figure 3. With a configuration of only ten GPS sites, the RMS error in the smoothed PGR estimate for the same example five year period shown in Figure 4 is 2.3 mm yr^{-1}. This is a reduction of 58% from the GLAS/GRACE estimate. The RMS error of the smoothed IMT estimate is 10.5 mm yr^{-1}, a similar 43% reduction. When calculated for all of the independent five year periods of CSM-1, the RMS average of the RMS errors was 2.6 mm yr^{-1} of PGR error and 13.2 mm yr^{-1} error in IMT. The PGR error ranged from 1.5 to 3.6 mm yr^{-1} and the error in the IMT estimate ranged from 6.1 to 19.1 mm yr^{-1}. However, even if we ignore the logistical difficulties of siting continuous GPS instruments in the Antarctic interior, the locations where the accumulations are largest are among the least

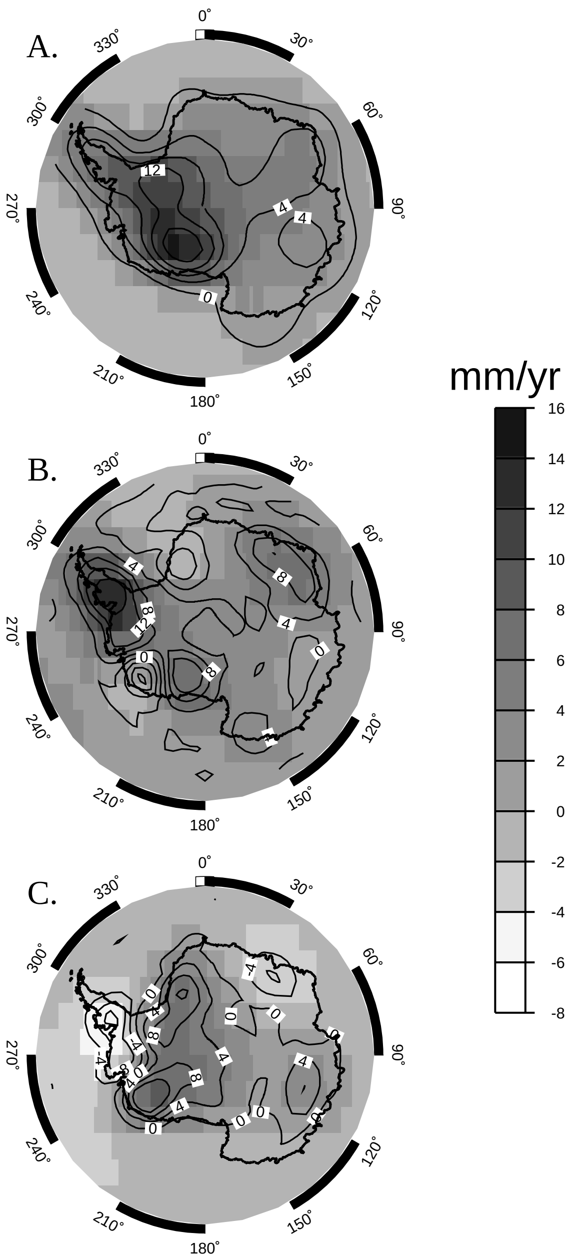

Figure 2: 5-year ice mass trend (IMT), smoothed using a Gaussian averaging function with 250 km radius. (a) Input IMT signal summing two contributions: the century scale IMT from *Bentley and Giovinetto* [1991] plus the 5-year trend from the time-variable accumulation. (b) Recovered signal combining GLAS and GRACE data. (c) Residual. Contour interval is 20 mm yr^{-1}.

likely places to find exposed bedrock. Moreover, there is no reason to expect that the precipitation rate always will be highest at the locations indicated in Figure 2c. The loci of the maximum compaction errors will change with time, and we cannot predict where the compaction error will be largest during the GRACE/GLAS missions.

To obtain an extreme lower bound for the recovery errors, we also examined the accuracy of signals recovered in the unlikely limit of very dense GPS coverage. By putting GPS sites at a regular 50 km spacing everywhere south of $-60°$, we retrieved the smoothed PGR with RMS error of just 1.5 mm yr^{-1} and the smoothed IMT with 7.9 mm yr^{-1} for the example five year period shown in Figures 2–3 (Table 1). We suspected that a portion of the remaining error was a result of the uncorrected elastic load response contained in the GPS velocities. To test this we ran the simulation again but did not include the elastic load response in the GPS heights. The errors in the signal recovery were reduced to 0.1 mm yr^{-1} (PGR) and 7.7 mm yr^{-1} (IMT). We thus infer that the portion of the PGR RMS error caused by uncorrected secular elastic loading in the GPS signal is $\approx \sqrt{1.5^2 - 0.1^2}$ mm yr^{-1} = 1.5 mm yr^{-1}, and the corresponding portion of the IMT error is $\approx \sqrt{7.9^2 - 7.7^2}$ mm yr^{-1} = 1.8 mm yr^{-1}. We expect that the effects of the uncorrected elastic errors would be similar for any distribution of GPS sites, but that they could be largely removed by subtracting the elastic load response to the estimated IMT from the GPS time series.

6 Discussion

Our simulations show that by combining GLAS and GRACE data it should be possible to recover the spatially-varying Antarctic PGR and IMT signals, smoothed over 250 km scales, to accuracies of about 5 mm yr^{-1} and 20 mm yr^{-1}, respectively. These errors are due almost entirely to the compaction error associated with variability in the accumulation. When GPS measurements are combined with the GLAS and GRACE data, the accuracy of the recovered fields is improved. This improvement depends on the distribution of the GPS receivers. When we assume there are continuous data from 45 existing and proposed GPS sites, the RMS errors in the smoothed PGR and IMT fields drop to about 3 mm yr^{-1} and 16 mm yr^{-1}, respectively.

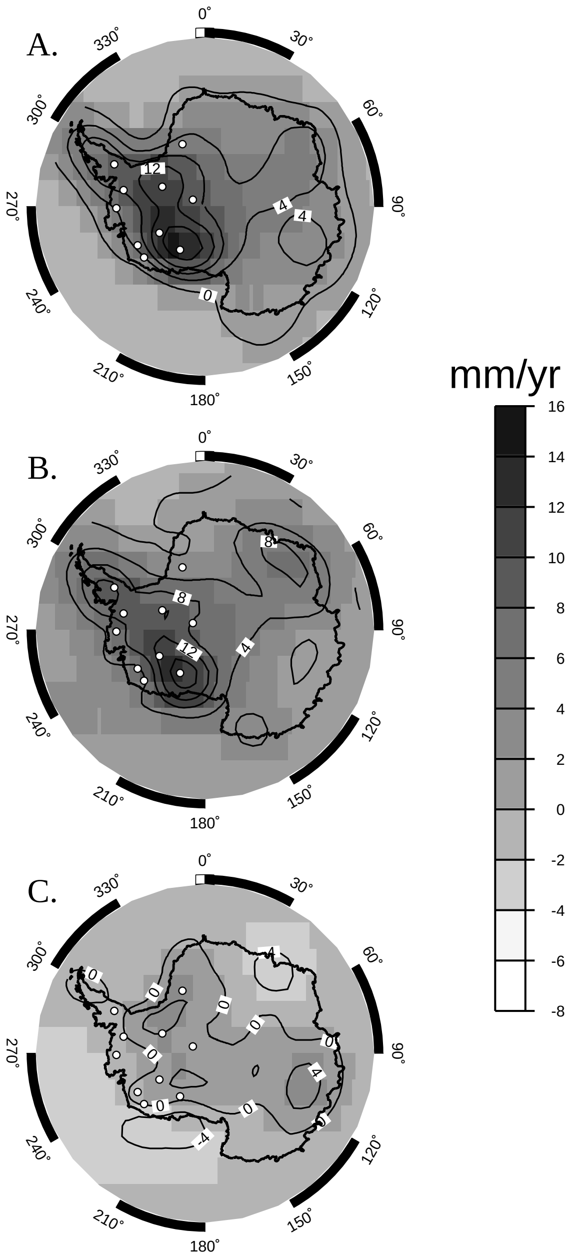

Figure 3: Smoothed Post Glacial Rebound signal (PGR) signal. (a) Input signal. (b) Recovered signal combining GLAS, GRACE and 10 GPS sites located near the largest accumulations for this 5 year period. (c) Residual. Contour interval is 4 mm yr^{-1}. Circles denote GPS locations.

When GPS data are added to the simulations, compaction error in the IMT is reduced by exploiting the relationship observed in *Wahr et al.* [2000], that compaction errors in PGR are 0.31 times the compaction errors in IMT. However, differences between the GPS vertical velocity and the GRACE/GLAS estimate of PGR that arise from sources other than the compaction trend will cause errors in the correction of IMT, and the correction scales these differences as approximately ϵ_{IMT}=(1+1/.31) ϵ_{PGR}=4.2 ϵ_{PGR}. One example of a difference between the GPS vertical velocity and the GRACE/GLAS estimate of PGR that does not result from the compaction trend arises because of the smoothing which must be applied to GRACE geoid estimates. The PGR signal used in this paper, when smoothed using a Gaussian averaging function with 250 km radius, differs from the unsmoothed PGR signal by $\sim$1.4 mm yr^{-1} RMS. We are able to attenuate the effect of this difference somewhat by applying Gaussian smoothing to the estimate of the PGR correction, $\Delta\dot{U}^{PGR}_{compaction}$, before using it to correct the IMT.

All of the error estimates in this paper depend heavily on the assumed time-variable accumulation field and GPS receiver distribution. It is difficult to assess the accuracy of the CSM-1 accumulation fields [*Wahr et al.*, 2000]. We expect that Antarctic GPS receivers will be sparsely distributed, with the 45 sites representing a best-case scenario. Moreover we expect these sites will be much more sparsely sampled in time than the five years of daily coordinates we simulated. Among existing sites in the Transantarctic Mountains, the coordinate time series from MCM4 (the IGS site at McMurdo station) is quasi-continuous, but remote stations COAT and MTCX have just one to five months of data per year as a result of power and other system failures. Also many of the sites we incorporated are campaign sites, which may have only a few days to weeks of data per year. Vertical velocities estimated from the continuous GPS sites currently have errors of between $\sim$0.4–5.9 mm yr^{-1}, as compared to 0.3 mm yr^{-1} RMS errors in recovery of PGR vertical velocity from the simulated GPS time series. Errors in GPS vertical velocity estimates of more than about 4.4 mm yr^{-1} would introduce errors into the correction of IMT that exceed the RMS errors in GLAS/GRACE recovery without GPS. However, estimates of vertical velocity at continuous sites will improve with more observation, and spatial averaging will reduce the error further in the more densely instrumented regions (e.g., the Transantarctic Mountains and Marie Byrd Land).

Another important point in applying this method relates to the true scales of correlation of the signals we are solving for. The GLAS, GRACE, and GPS data sets sense changes at different length scales, ranging from sparsely distributed point velocity measurements (GPS) to dense altimetric heights with 70-m footprints (GLAS) to surface mass density integrated over scales > 200 km (GRACE). Consequently the accuracy of the estimates of PGR, IMT, and the correction for the time-varying compaction trend depends on the length scales for which these signals are self-correlated. If all three signals self-correlate at scales consistent with the minimum resolution scale of GRACE and the scale of distribution of the GPS measurements, then we expect the results to be quite good. If on the other hand one of the signals is extremely variable at scales of a few hundred kilometers, we expect a poor result.

The PGR signal used in our simulation does have long characteristic length scales. This partially reflects the fact that the Antarctic component of the ICE-3G deglaciation model used to simulate this PGR signal is dominated by these same long scales. But even if the true deglaciation pattern had significant power at short wavelengths, viscoelastic rebound would be dominated by longer wavelengths because the stress induced by short wavelength loading tends to be concentrated near the Earth's surface, and so within the elastic lithosphere. Thus short wavelength variations in loading do not generate a significant viscoelastic relaxation response. *Velicogna and Wahr* [2002] estimate that, at minimum, the PGR signal is significantly self-correlated out to lengths of 250 km and decorrelation occurs at length scales > 700 kilometers, regardless of the scales of ice mass loading present in the deglaciation pattern.

Compaction error is clearly the principle error source in PGR and IMT recovery using this method. Using the same CSM-1 accumulation fields used to generate our simulated GLAS/GRACE/GPS data, we compared the compaction trends with Gaussian averages of those same trends using a 250 km averaging radius. We find that the Gaussian-averaged compaction trends are ~0.5 that of the unsmoothed,

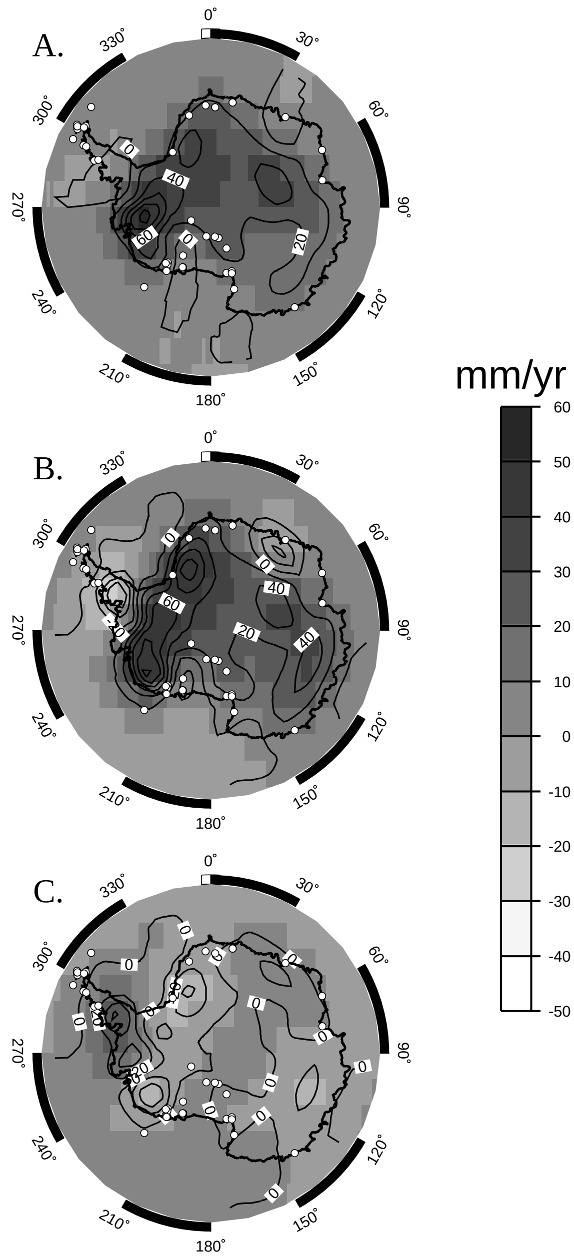

Figure 4: Post Glacial Rebound (PGR) signal, smoothed using a Gaussian averaging function with 250 km radius. (a) Input signal. (b) Recovered signal combining GLAS and GRACE data. (c) Residual. Contour interval is 4 mm yr^{-1}.

suggesting that indeed the compaction predicted from the CSM-1 accumulation is correlated on these length scales. The self-correlation length scale of the compaction error depends on the corresponding scale of the time variable accumulation. It is possible that the spatial correlation properties of the CSM-1 model may not be representative of the true Antarctic accumulation. However, *Wingham et al.* [1998] note that five-year accumulation trends measured by the ERS-1 and ERS-2 satellite radar altimeters are correlated on length scales of $\sim$200 km, with complete decorrelation at $>$ 400 kilometer length scales.

There are several other data sources which might be used to supplement the estimates of the compaction trend in addition to GPS. The *Wingham* [2000] accumulation-dependent compaction model used to simulate ice compaction for this study could be applied to estimates of time variable accumulation during the five years of GRACE and GLAS measurements to improve estimates of compaction trend, but compaction depends significantly on the accumulation history over timescales of decades, so accumulation on five year time-scales will not suffice. The ERS altimetric data and time series of ground-based weather station measurements can be used to extend the record back through time, but assimilating these measurements will be complicated by incomplete sampling in time and/or space. Time variable accumulation measured from firn density in ice cores will provide a very valuable constraint, as these records are temporally complete over the past few centuries [*Oerter et al.*, 2000]. Like GPS measurements, these are point measurements but they can be sampled anywhere (including major accumulation centers) and so may provide even more useful constraint. Figure 5 shows the current sampling of Antarctic ice cores. Microwave remote sensing of ice sheet scattering may eventually help to constrain the compaction trend, but additional research into the microwave signature of firn densification will be needed first.

The simulations presented here assume that glacial outflow is constant with time. This is a questionable approximation, given that glacial flow rates appear to be influenced by basal "lubrication" by liquid water percolating through small fractures in the ice stream. Also, the ice sheet margins support a significant fraction of the gravitational force that drives ice flow. Margins can shift, probably in response to changing

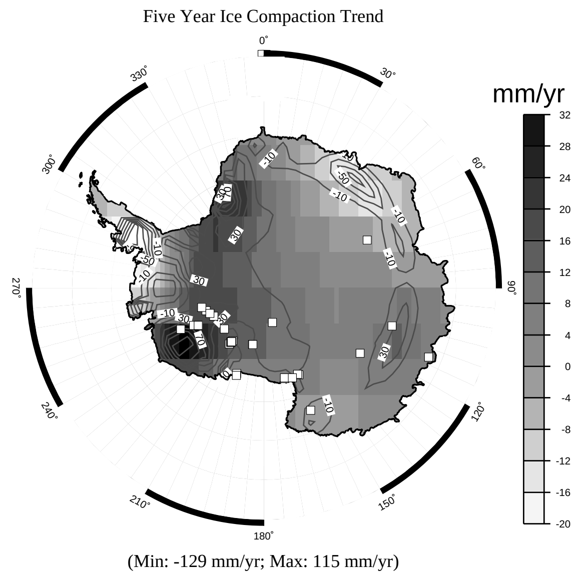

Figure 5: The compaction trend, which will map directly into error when using GLAS ice heights only to estimate IMT. GRACE data will partially correct this error, but the addition of GPS and ice core data allows estimation of the compaction trend at the observation sites. White squares denote ice core locations.

thermal and hydrological conditions at the base. These changes in ice-stream width alter the balance of forces and the rate of ice flow. Because we concern ourselves only with the overall trends in the methodology presented here, the assumption of constant ice flow contributes negligibly to the overall error. However, as the modeling is augmented to allow estimation of the time-variable accumulation, this assumption will need to be re-evaluated.

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