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Cross-Sections of High Energy Nuclear Reactions

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Cross-sections of high-energy hadron-nucleus reactions $\sigma_h(A)$ are investigated using the optical model. A closed formula is derived for $\sigma_h(A)$; its A-dependence agrees with the rms radius of the Fermi distribution of nuclear matter. The properties of the radius parameter and the absorption mean free path λ_h are investigated using Serpukhov data for $p, \pi^{\pm}$, $K^{\pm}$, and $\bar{p}$. It is found $\lambda_p = 1.44 \pm 0.23$ fm $\simeq 1/m_\pi$ and $\lambda_\pi = 2.14 \pm 0.21$ fm for p and π-reactions, respectively; with $\lambda_\pi/\lambda_p \simeq 2/3$ as expected from quark-counting. The approach extended to heavy-ion (HI) reactions leads to a formula of Bradt-Peters type, with an overlapping parameter decreasing with A. Analyses of HI data of LBL and ISR with this formula are presented.

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1. Introduction

It is well known that high energy elastic scattering of hadrons on nuclei (h-A) can be accounted for by geometrical models such as the Glauber theory [1] and the Chou-Yang model [2]. Consequently, the reaction cross-section may be computed by means of these models. In this paper, we propose to use simple formulae derived from the optical model (OM) under appropriate assumptions, Sec. 2, to analyze reaction cross-sections.

The formula for the h-A reaction cross-section, (4-a), has the same A-dependence as expected from the rms radius of the Fermi distribution of nuclear matter, Sec. 3. We test the validity of this formula by analyzing the nuclear radii measured by the DESY--MIT experiment [4].

The appealing feature of our formula is its simplicity. This is shown by the analysis of high energy nuclear data of p, $\pi^{\pm}$, $K^{\pm}$ and $\bar{p}$ reactions from Serpukhov [5], Sec. 4. Estimates of the parameters r and λ of (4-a) for p and π-reactions reflect interesting properties of quark contents, Sec. 6.

Our approach is further extended to heavy-ion reactions, Sec. 2. The cross-section thus obtained, (4-b), resembles that of p-A, (4-a) and predicts the well-known behavior of the overlapping parameter of the Bradt-Peters formula, (6). Our formula will be used to analyze LBL and ISR experiments, Sec. 5.

We mention that our OM approach enables us to relate the HI cross-section to that of p-A by a simple expression, (12). This relationship will be tested with the currently available data from LBL [7] and ISR [18], Sec. 7.

2. Derivation of Nuclear Cross-Sections

We begin with the hadron-nucleus reaction, denoted by $\sigma_h(A)$, A being the mass number of the target nucleus of radius R. According to the optical model model [3], the absorption cross section is given by

$$\sigma_{abs} = \pi R^2 \int_{o \le b \le R} (1 - e^{-2x/\lambda}) \frac{db^2}{R^2} \qquad (1\text{-a})$$

where b is the impact parameter, $2x = 2\sqrt{R^2 - b^2}$ the traversal and λ the mfp of absorption, see Fig. 1a. We recall that for uniform nuclear density [3]:

$$\sigma_h(A) = \pi R^2 [1 - 2\{1 - (1 + 2R/\lambda)e^{-2R/\lambda}\}(\lambda/2R)^2] \qquad (2\text{-a})$$

no distinction being made between the proton and the neutron in the nucleus, for simplicity. We now assume the large A approximation:

$$2R/\lambda >> 1 \qquad (3)$$

and get by assuming the scale law $R = rA^{1/3}$

$$\sigma_h(A) = \pi r^2 \left(A^{1/3} - \frac{a}{A^{1/3}}\right)^2 \qquad (4\text{-a})$$

where

$$a \equiv (\lambda/2r)^2 \qquad (5\text{-a})$$

We note that the A-dependence of this cross-section turns out to be the same as that of the rms radius of the Fermi distribution, see (9), indicating that our approach amounts to using the Fermi distribution to estimate the parameters r and λ from the reaction cross-section.

For the heavy-ion (HI) reaction, we proceed in the same way. First, consider the case of like-nuclei and let $R_1 = R_2 \equiv R$ be the radius and a the distance between the centers, Fig. 1b; then (1-a) becomes

$$\sigma = 2\pi R^2 \int_{2R}^{o} \frac{da^2}{R^2} \int_{R}^{a/2} (1 - e^{-2x/\lambda}) \frac{db^2}{R^2} \tag{1-b}$$

An integration yields

$$\sigma = 2\pi \left[2R^2 - 2\lambda^2 + \frac{\lambda^4}{R^2} \left\{ 3 - \left(3 + \frac{6R}{\lambda} + \frac{4R^2}{\lambda^2} \right) e^{-2x/\lambda} \right\} \right] . \tag{2-b}$$

For unlike-nuclei, $R_1 \neq R_2$, we assume the following substitution

$$2R \rightarrow R_1 + R_2$$

and large-A approximation to obtain

$$\sigma(A_1, A_2) = \pi r^2 \left(A_1^{1/3} + A_2^{1/3} - \frac{c}{A_1^{1/3} + A_2^{1/3}} \right)^2 \tag{4-b}$$

where

$$c \equiv 2(\lambda/r)^2 \quad . \tag{5-b}$$

Note that (4-b) resembles the well-known Bradt-Peters formula [8] with the "overlapping parameter"

$$b = c/(A_1^{1/3} + A_2^{1/3}) \tag{6}$$

decreasing with nuclear size as has been observed experimentally [9] and that has been proposed previously [10] but remained unnoticed.

In the same approximation, these cross-sections may be written as

$$\sigma_h(R) = \pi R^2 e^{-2(\lambda/2R)^2} \quad , \tag{7-a}$$

$$\sigma(R_1, R_2) = \pi(R_1 + R_2)^2 \cdot e^{-4\lambda^2/(R_1 + R_2)^2} \quad . \tag{7-b}$$

Note that these relations fall less rapidly than (4) and fit the light nucleus better. However, the geometric interpretation of these formulae is not so obvious as that of (4).

3. The Effective Radius

The physical meaning of the hadron-nucleus cross-section (4a) is more apparent if we introduce an effective radius

$$\mathcal{R} = r(A^{1/3} - a/A^{1/3}) \quad , \tag{8}$$

then $\sigma_h(A) = \pi\mathcal{R}^2$. Note that $\mathcal{R} = rA^{1/3}$ at large A limit or $a = 0$ i.e. $\lambda = 0$. Likewise, the HI cross-sections are obtained by replacing

$$A^{1/3} \rightarrow A_1^{1/3} + A_2^{1/3} \quad , \quad a \rightarrow c$$

i.e. adding two radii <u>without overlapping</u>.

We note that the A dependence of $\mathcal{R}$ turns out to be similar to that of the rms radius of the Fermi-type distribution of nuclear matter

$$\rho(r) \sim \frac{\rho_0}{1 + e^{(r-R)/t}}$$

where $R = r_0 A^{1/3}$ is the half-way radius, r_0 being the rms radius parameter, and t the surface thickness:

$$\sqrt{\langle r^2 \rangle} = r_0 A^{1/3} - \frac{\pi^2 t^2}{3 r_0 A^{1/3}} \tag{9}$$

to the order $\mathcal{O}(1/A^{5/3})$ [11].

For a test of (8), we use the nuclear radii measured by DESY-MIT [4] using photoproduction of ρ^0 at 7.5 GeV//c

$$\gamma + A \rightarrow \rho + A$$
$$\quad \rho \rightarrow \pi^+ + \pi^-$$

Their measurements are plotted against $A^{1/3}$ in Fig. 2, errors ~2% for A > 27 are omitted for simplicity; the line shows the fit with (8). The parameters are

$r = 1.14 \pm 0.01$ fm

$a = 0.22 \pm 0.36$.

Our estimate of r agrees with their value 1.12 ± 0.02 fm. However the surface thickness t estimated according to (8) and (9) is 0.28 fm, somehow smaller than 0.56 fm used in their analysis [4].

From (5-a) we find for the absorption mfp of ρ^0

$\lambda = 1.07 \pm 0.16$ fm .

Here λ represents the effective mfp of π^+ and π^- detected in the experiment for ρ^0 identification. Thus $1/\lambda = 2/\lambda_\pi$ and we obtain

$\lambda_\pi = 2.14 \pm 0.32$ fm .

This is in excellent agreement with 2.18 ± 0.24 fm estimated from π-A reaction cross-sections at 3.9 GeV of a previous analysis [12], and also with the Serpukhov data to be discussed in the next section.

4. Hadron-Nucleus Reactions

We now proceed to investigate the properties of hadron-nucleus reactions using the Serpukhov data of p, $\pi^\pm$, $K^\pm$, and $\bar{p}$ at $P_{lab} = 6.65$ to 60 GeV/c [5]. These data, except for $\bar{p}$ to be discussed later, are analyzed with (4-a). We then compute the corresponding mfp by (5-a). The parameters and fitting errors are summarized in Table I.

In Fig. 3 is shown a typical fit for p-A cross-section at 20 GeV/c. In general the fits are good, except that for light nuclei, there is a systematic deviation as is seen from the figure.

Consider the radius parameter r. We note that it is well defined, with rather small errors, that it is practically independent of energy, and that it is different for p-A and π or K-A reactions.

As regards the mfp λ, the errors are rather large. That the measured cross-sections, according to [5], are energy independent implies that the same holds for estimates r and λ. We now investigate the dependence of λ on A, and find from our analysis of the p-A data

$$d\lambda/dR = 0.053 \pm 0.024$$

consistent with zero, indicating an independence of A.

Therefore, for a given projectile, we may take the averages of our estimated parameters r and of λ. We get for p-A reactions

$$r_p = 1.30 \pm 0.01, \ \lambda_p = 1.44 \pm 0.23$$

in fm, and likewise for π-A reactions

$$r_\pi = 1.23 \pm 0.03, \ \lambda_\pi = 2.14 \pm 0.21.$$

We find $r_p > r_\lambda$ and $\lambda_p < \lambda_\lambda$; likewise $r_p > r_K$ and $\lambda_p < \lambda_K$. Note that $\lambda_p \simeq \hbar/m_\lambda c$. Further discussions will be resumed in Sec. 6.

Turn now to the $\bar{p}$-reactions. We find that here the least-squares fits with (4-a) yield a wrong sign to the parameter a. This in turn leads to an imaginary λ according to (5); whereas a is in fact small compared to r. We have investigated this point further by fitting the data with the exact absorption cross-section (2-a). We found the same r and λ consistent with zero; the values thus obtained are listed in Table I. Thus we tentatively set a = 0 and assume

$$\sigma_{\bar{p}} = \sigma_0 A^{2/3} \qquad (10)$$

with $\sigma_0 = \pi r_{\bar{p}}^2$. This A-dependence is different from those for p, π, and

K-nucleus reactions and agrees with the BNL results [13]. The estimates of $r_{\bar{p}}$ are listed in Table I. Note that

$$r_{\bar{p}} = 1.33 \pm 0.03 \text{ fm}$$

is comparable to r_p and that the nucleus is black in this case. This fit for P_{lab} = 13.3 GeV/c is shown in Fig. 3; the log plot is linear in contrast with that of p-A, which is slightly curved downward for small A.

5. Heavy-Ion Reactions

As for heavy-ion (HI) reactions, we shall use the LBL experiment [6] of ^{56}Fe at 1.88 GeV/N on targets from H to U. Their data for $\Delta Z \geq 1$ are reproduced in Fig. 4. The solid line is our fit with (4-b), the parameters are

$$r = 1.31 \pm 0.01 \text{ fm}$$

$$c = 4.45 \pm 0.15$$

Note the slight curvature compared to the straight line of their fit using the Bradt-Peters formula, see Sec. 2. As the mid-range of $A_1^{1/3} + A_2^{1/3}$ of their data is 7.4, the overlapping parameter b is ~0.70 ± 0.02 according to (6); this is compared to 0.83 ± 0.12 of their fit, their r being 1.35 ± 0.02 fm. Their parameters were used for an anomalon investigation [14].

Our r is comparable to that of p-A in Table I; whereas the mfp deduced from (5-b) is 1.96 ± 0.34 fm, about ~20% longer than that of p-A as obtained in the previous section. This difference may be due to the fact that the mechanism of Z-changing HI reactions is more complicated than for p-A reactions, see Sec. 7 for further discussion on this point.

Finally, it should be noted that (4-b) should not be applied to the H-target by setting zero or a small value for the corresponding A-value in the formula, as is used in the literature. Indeed, we see that in doing so, (4-b) is reduced to the same form as (4-a) for p-A reactions. But, the parameter c which is >> a_p remains unaffected. Consequently, the cross- section thus computed is underestimated.

To illustrate this point which is sometimes overlooked, consider the mfp of ^{56}Fe in lucite $C_5H_8O_2$. Assuming a density $\bar{d}$ = 1.18 g/cm^3 and using (4-b) for all elements, we find a mfp 7.85 cm compared to the experimental value 8.14 ± 0.04 cm [14]. Whereas the correct estimate using (4-a) for H yields 8.11 cm in excellent agreement.

6. Properties of r and λ

We have investigated various nuclear reactions in terms of (4-a) and (4-b) with two parameters r and λ. We find that both r and λ depend on the nature of the projectile. It follows that the nuclear size can not be an intrinsic property, but depends on the nature of the probe.

For p-A reactions, we get r_p = 1.30 ± 0.01 fm. Recall that the rms radius of the Fermi distribution is $\sqrt{3/5}$ times that of the uniform density distribution. Thus our r parameter estimate agrees with ~1 fm found by Glauber and Mitthial [1b].

Referring to Table I, we note that the averages of mfp for π^+ and π^- reactions are practically the same; being 2.19 ± 0.35 and 2.09 ± 0.26 fm, respectively. This indicates that the difference between the density distributions of p and n inside a nucleus is rather small. Note that these mfp agree with 2.05 fm estimated by Chou and Yang [2b].

However, we expect that the parameters r and λ are different for K^+ and K^-, since their absorption cross-sections are different. It would be interesting to investigate this point.

Consider now the averages of r and λ for p and π-A reactions discussed in Sec. 4. We may estimate the average cross-section per nucleon, assuming the average nucleon density to be $\rho_0 = 0.17\ N/fm^3$. We find

$$\bar{\sigma}_{pN} = 40.8 \pm 6.5 \text{ mb} \quad , \quad \bar{\sigma}_{\pi N} = 26.8 \pm 3.4 \text{ mb}$$

in agreement with well known free nucleon cross-sections.

As for the radius parameters, we find

$$r_p/r_\pi = 1.07 \pm 0.03$$

comparable to that of rms radius of p and π estimated by Chou [16a] from elastic pp and πp scatterings, i.e.

proton radius/pion radius = 1.12 ± 0.03 ;

this ratio is 1.08 according to the Yale-FNL experiment [16b].

Note that according to the geometrical picture, this ratio can be expressed in terms of numbers of constituent quarks as follows

$$\left(\frac{3}{2}\right)^{1/3} = 1.15 \quad .$$

In this regard, we note that this property holds also for the mfp. Indeed, if λ_q is the quark mfp, then

$$\frac{1}{\lambda_p} = \frac{3}{\lambda_q} \quad , \quad \frac{1}{\lambda_\pi} = \frac{2}{\lambda_q} \quad . \tag{11}$$

We therefore expect

$$\lambda_\pi/\lambda_p = 3/2$$

Experimentally, we find

$$\lambda_\pi/\lambda_p = 1.49 \pm 0.28$$

in agreement with quark-counting. In passing, we note that from the parameters of the Fermi distribution of the BNL data of $K^\pm$ reactions on C and Cu [17] we estimate $\lambda_K \cong 2.14$ fm in agreement with those in Table I.

7. Conclusion

In summary, the heavy-ion (HI) cross-section derived from the optical model, (4-b), is of the Bradt-Peters type and is similar to that of the hadron-nucleus (hA), (4-a). This property leads us to introduce an effective radius for h-A reactions:

$$\mathcal{R} = r(A^{1/3} - a/A^{1/3}) \tag{8}$$

the parameters r and $a = (\lambda/2r)^2$ being characteristic of the incident hadron. As λ_p is less than λ_π and λ_K, the nucleus appears to be more transparent to meson than proton induced reactions; whereas all h-A cross- sections follow the same $\mathcal{R}^2$ dependence.

As for the HI reactions, we note that in the case of like-nuclei, the cross-section $\sigma(A,A)$ can be expressed in terms of $\sigma_p(A)$ of p-nucleus of the same A. Indeed, we find according to (4-a) and (4-b)

$$\frac{\sigma(A,A)}{\sigma_p(A)} \simeq \frac{r^2}{r_p^2}\left(4 - \frac{c - 4a_p}{A^{2/3} - a_p}\right) . \tag{12}$$

It is interesting to note that this ratio is not sensitive to the large A approximation, i.e. $2R/\lambda \gg 1$ as is required by (4) and (5). Indeed, consider the case of p-α and α-α. With our parameters for p-A and HI reactions, we get from (12): $\sigma(\alpha,\alpha)/\sigma_p(\alpha) = 2.56$ in agreement with the experimental ratios 2.48 ± 0.04 of another LBL experiment at 2.1 GeV/N [7] and 2.73 ± 0.46 of the recent ISR experiment of colliding beams of 31.5 GeV protons and 63 GeV α's [18]. Recall that these $\sigma(\alpha,\alpha)$ measurements refer to total rather than Z-changing reaction cross-sections.

Further investigations of HI cross-sections with BEVALAC data are in progress by one of the authors (HC) and will be reported elsewhere.

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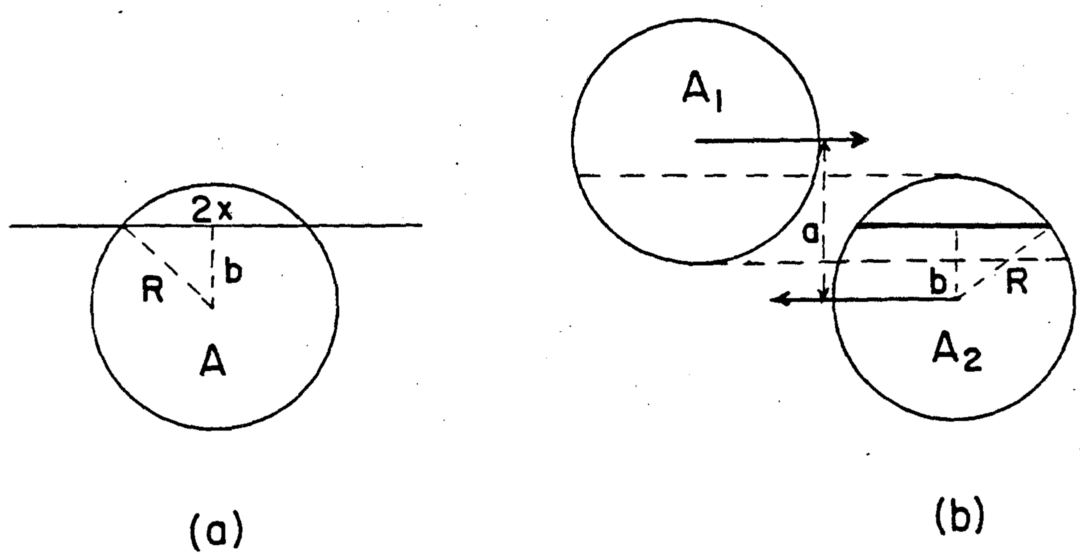
Table I--Parameters of hadron-nucleus reactions

Serpukhov data Ref. [5]

Proj.	P_{lab}(GeV/c)	r(fm)	mfp λ(fm)
p	20	1.29 ± 0.01	1.53 ± 0.18
	30	1.31 ± 0.01	1.46 ± 0.09
	40	1.30 ± 0.01	1.45 ± 0.08
	50	1.30 ± 0.01	1.72 ± 0.21
	60	1.31 ± 0.01	1.03 ± 0.27
π^+	20	1.24 ± 0.01	2.17 ± 0.17
	30	1.25 ± 0.01	2.63 ± 0.13
	40	1.21 ± 0.01	2.18 ± 0.12
	50	1.16 ± 0.01	1.75 ± 0.11
π^-	6.65	1.25 ± 0.01	2.03 ± 0.06
	10	1.25 ± 0.01	2.04 ± 0.09
	13.3	1.23 ± 0.01	1.98 ± 0.06
	25	1.25 ± 0.02	2.32 ± 0.08
K^+	30	1.12 ± 0.02	2.00 ± 0.15
K^-	6.65	1.28 ± 0.02	2.18 ± 0.06
	13.3	1.20 ± 0.03	2.20 ± 0.15
$\bar{p}$	6.65	1.36 ± 0.01	(<0.1)
	13.3	1.35 ± 0.01	(<0.3)
	25	1.28 ± 0.01	(<0.1)

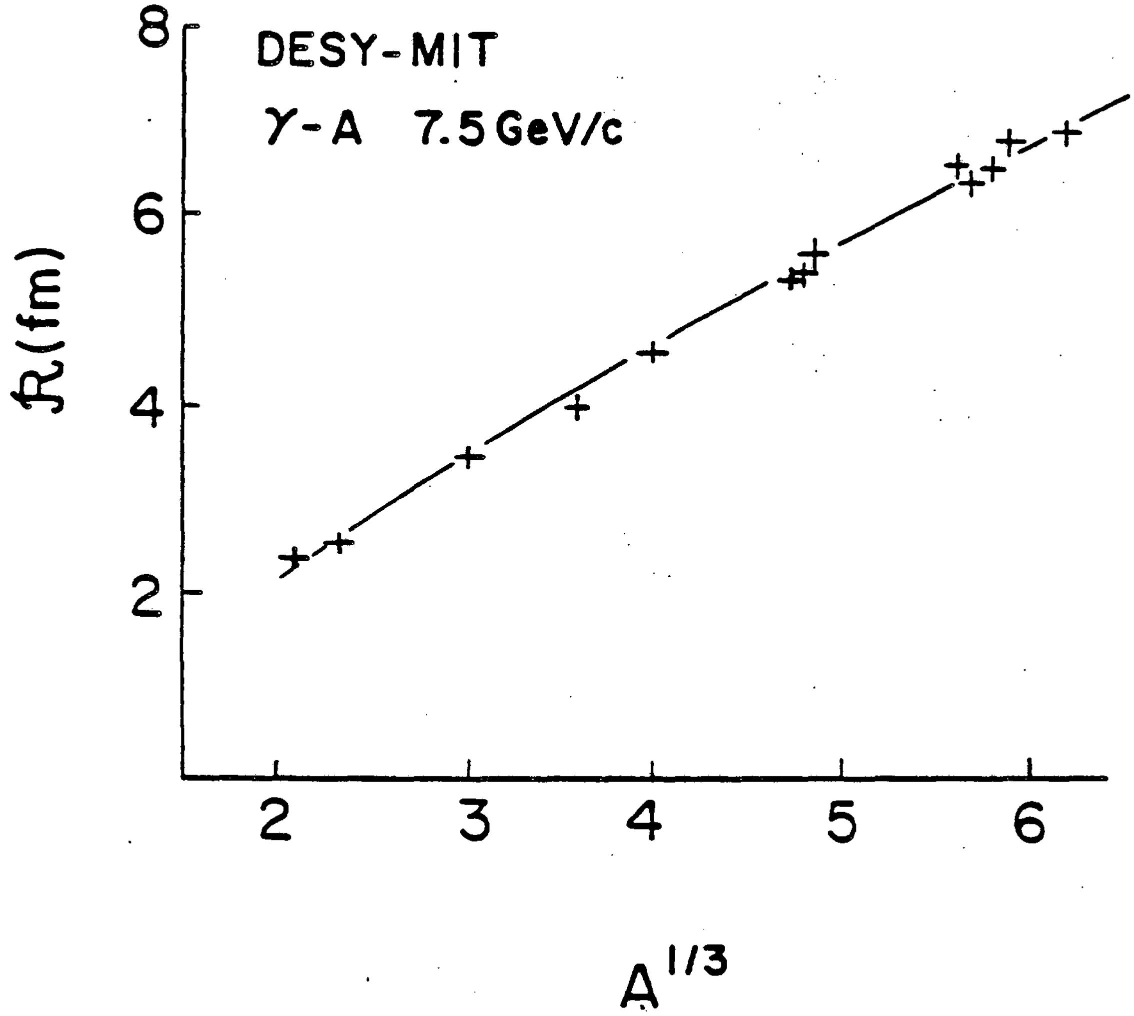
Figure Captions

1. Reaction cross-sections according to the optical model: (a) hadron-nucleus and (b) two like-nuclei.
2. Nuclear radii from photoproduction of ρ^0, measured by the DESY-MIT Collaboration, Ref. [4]. The curve represents the fit with the effective radius $\mathcal{R} = r(A^{1/3} - a/A^{1/3})$, $r = 1.14 \pm 0.01$ fm and $a = 0.22 \pm 0.36$, see Sec. 3.
3. Cross-sections of p-A and $\bar{p}$-A from Serpukhov, Ref. [5]. The lines are fits using (4-a), the parameters are listed in Table I.
4. Heavy-ion cross sections for ^{56}Fe at 1.88 GeV/N, LBL $\Delta Z \geq 1$; data errors less than 4% are not shown, Ref. [6]. The curve is the fit with (4-b). Note its non-linearity in contrast to the Bradt-Peters formula, see text.



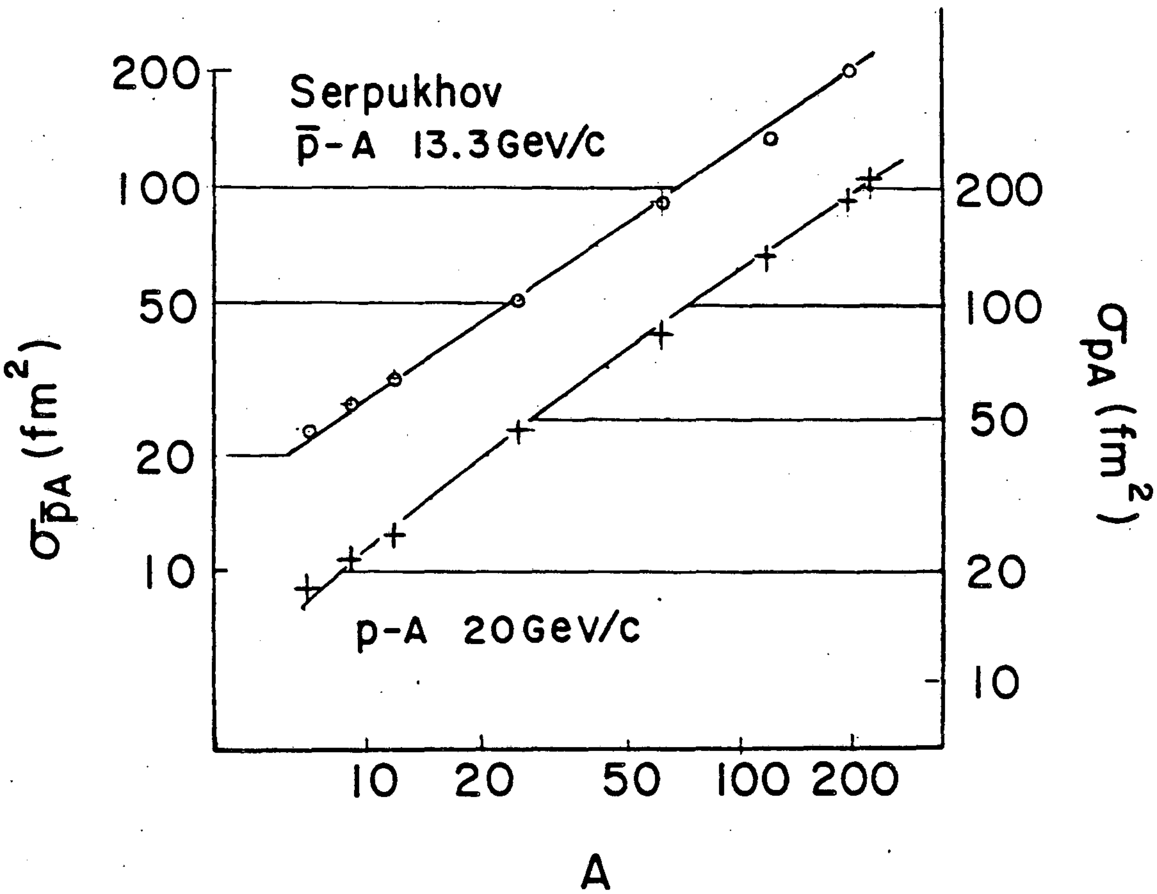
XBL 847-8527

Fig. 1



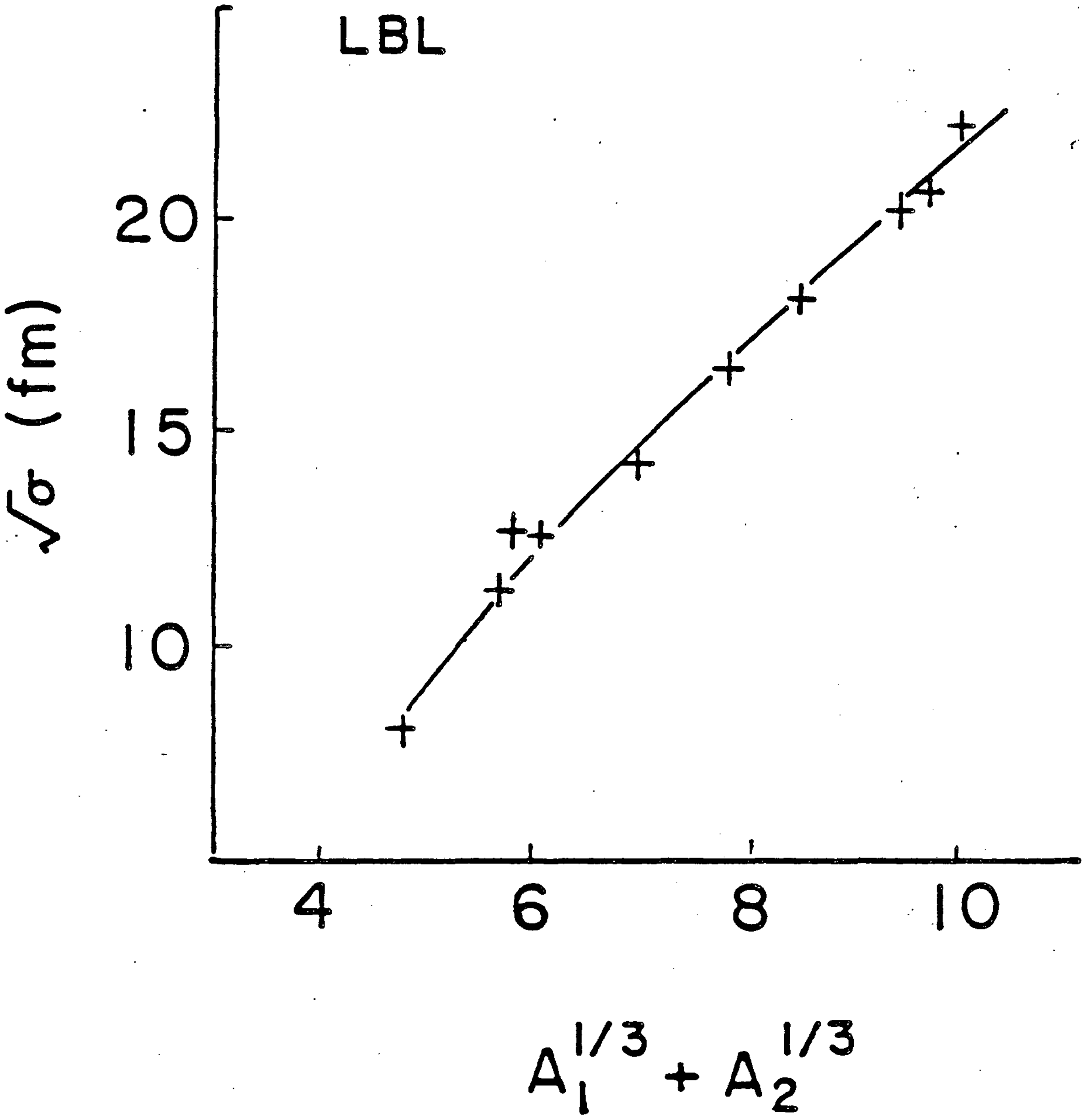
XBL 847-8528

Fig. 2



XBL 847-8529

Fig. 3



XBL 847-8530

Fig. 4

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