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### Author

Perloff, Jeffrey M

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by

Jeffrey M. Perloff

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## THE GENERAL EQUILIBRIUM EFFECTS OF A MINIMUM WAGE ON TRAINING\*

Jeffrey M. Perloff

The general equilibrium effects on training of an increase in a minimum wage depend upon workers' attitudes toward risk, the educational production function, and other factors.<sup>1</sup> This paper uses five two- and three-sector models of minimum wage economies with incomplete coverage to examine these effects.

Initially, training is assumed to take a student's time but no other resources. Each worker must decide whether to be trained or not: education is a discrete choice. In all models, as discussed in Section I, there are two Cobb-Douglas production sectors, and consumers (workers) have Cobb-Douglas tastes. The equilibrium levels of training and other variables in the absence of a minimum wage are exhibited in Section II.

In Section III, a minimum wage is introduced which is only applied to one sector to reflect incomplete coverage.<sup>2</sup> If workers are risk neutral, the number of workers who are trained is independent of the minimum wage. In Section IV, it is shown that the assumption of homogeneous abilities is not a necessary condition for this result. If workers are risk averse, however, an increase in the minimum wage causes the number of trained workers to increase as demonstrated in Section V.

A third, educational sector is introduced in Section VI. Education is produced using a simple Leontief production function of skilled and unskilled workers. If the educational sector is covered by the minimum wage, an increase in the minimum wage causes training to decrease; if this sector is not covered, it causes training to increase. Comparative statics results are compared across models in Section VII.

## I. THE BASIC MODEL

Throughout this paper, a two-sector, Cobb-Douglas model is used. The features of the general model, which hold in the following sections, are described here. The outputs of the two sectors,  $y_1$  and  $y_2$ , are produced using unskilled,  $x_1$ , and skilled (or trained) labor,  $x_2$ . The output prices are  $p_1$  and  $p_2$ , while inputs are paid  $w_1$  and  $w_2$ . Without loss of generality, the skilled workers' wage can be normalized to one ( $w_2 \equiv 1$ ).<sup>3</sup>

Each worker/consumer has Cobb-Douglas tastes:

$$(1) \quad U = y_1^{\beta_1} y_2^{\beta_2},$$

where  $U$  is utility and the  $\beta_i$  are taste parameters. Given this utility function, if income is  $Z$ , demands for the goods are

$$(2) \quad y_i = \frac{\beta_i}{\beta_1 + \beta_2} \frac{Z}{p_i}, \quad i = 1, 2.$$

Substituting equation (2) into (1), utility can be expressed in terms of income and prices:

$$(3) \quad U(Z, p_1, p_2) = BZ^{\beta_1 + \beta_2},$$

where

$$B \equiv \left( \frac{1}{\beta_1 + \beta_2} \right)^{\beta_1 + \beta_2} \left( \frac{\beta_1}{p_1} \right)^{\beta_1} \left( \frac{\beta_2}{p_2} \right)^{\beta_2}.$$

Production in the economy is also Cobb-Douglas. The sectors' cost functions are

$$(4) \quad C^i(y_i, w_1^i) = A_i (w_1^i)^{\alpha_i} y_i, \quad i = 1, 2,$$

where the superscript on the unskilled wage indicates that this wage may differ across sectors (i.e., because of incomplete coverage of a minimum wage), and  $A_i$  and  $\alpha_i$  are parameters. Given constant returns to scale and competition, product prices equal unit costs:

$$(5) \quad p_i = A_i (w_1^i)^{\alpha_i}, \quad i = 1, 2.$$

The derived demand for unskilled labor in each sector can be obtained by differentiating the cost function with respect to the relevant factor price:

$$(6) \quad x_1^i = \alpha_i A_i (w_1^i)^{\alpha_i - 1} y_i, \quad i = 1, 2.$$

By substituting from equations (2) and (5), equation (6) may be expressed as

$$(7) \quad x_1^i = \frac{\alpha_i \beta_i}{\beta_1 + \beta_2} \frac{Z}{w_1^i}, \quad i = 1, 2.$$

Thus, total demand for unskilled labor in the two sectors is

$$(8) \quad x_1 = x_1^1 + x_1^2 = \left( \frac{\alpha_1 \beta_1}{w_1^1} + \frac{\alpha_2 \beta_2}{w_1^2} \right) \frac{Z}{\beta_1 + \beta_2}.$$

## II. NO MINIMUM WAGE

For the purposes of comparison, it is useful to examine the usual equilibrium in the absence of a minimum wage. In this case,

$$w_1^1 = w_1^2 = w_1.$$

Workers enter the skilled occupation until their earnings in the two occupations are equalized. They enter the skilled occupation by getting trained for  $(1 - t)$  fraction of their working life, where  $0 < t < 1$ .<sup>4</sup> In a steady state, there are  $x (= x_1 + x_2)$  people entering and leaving the labor market at any time. Of these,  $x_1$  are unskilled workers,  $(1 - t)x_2$  are getting trained, and  $tx_2$  are skilled workers. In the following, we will examine a steady state at a single moment in time.

Workers stop entering the skilled occupation when incomes (and, hence, the utility from these incomes) are equalized across occupations:

$$(9) \quad w_1 = t,^5$$

where  $tw_2 = t$  is the earnings of a skilled worker.

National nominal income is the sum of earnings in the two sectors:

$$(10) \quad Z = w_1 x_1 + t(x - x_1).$$

Since  $w_1 = t$  from equation (9),

$$(11) \quad Z = tx.$$

Thus,

$$(12) \quad x_1 = \left( \frac{\alpha_1 \beta_1 + \alpha_2 \beta_2}{\beta_1 + \beta_2} \right) x.$$

That is, the number of unskilled (or skilled) workers does not depend on either  $t$  or  $w_1$ .

An increase in  $t$  (and, hence,  $w_1$ ) will increase product prices ( $p_i$ ), nominal national income ( $Z$ ), and welfare [ $U(Z, p_1, p_2)$ ]. An increase in  $t$  expands the working life of skilled workers, which effectively expands the economy's resources, raising everyone's income and utility.

### III. A MINIMUM WAGE IN A RISK-NEUTRAL WORLD

If a minimum wage for unskilled labor is set in only the first sector, then  $w_1^1 > w_1^2$ .<sup>6</sup> Unskilled workers are driven out of the covered sector into the uncovered sector. In this section, workers are assumed to be risk neutral ( $\beta_1 + \beta_2 = 1$ ).<sup>7</sup>

If workers are risk neutral, they will enter the trained occupation until the marginal worker's expected earnings (and, hence, utilities) are equal in both occupations:

$$(13) \quad \pi w_1^1 + (1 - \pi) w_1^2 = t,$$

where  $\pi$  is the probability of an unskilled worker finding a minimum wage job ( $\pi = x_1^1/x_1$ ), the left-hand side of equation (13) is the expected earnings of an unskilled worker, and  $t$  is the certain income of a trained worker.



Using equation (7), (13) may be rewritten as

$$(14) \quad x_1 = (\alpha_1\beta_1 + \alpha_2\beta_2) \frac{Z}{t}.$$

Nominal national income is now [using equation (7)]

$$(15) \quad Z = w_1^1 x_1^1 + w_1^2 x_1^2 + t(x - x_1) = \frac{t(x - x_1)}{1 - \alpha_1\beta_1 - \alpha_2\beta_2}.$$

Equations (14) and (15) are two equations in two unknowns ( $x_1$  and  $Z$ ).

Solving them simultaneously, we obtain

$$(16) \quad x_1 = x(\alpha_1\beta_1 + \alpha_2\beta_2),$$

and

$$(17) \quad Z = tx.$$

Thus, neither the number of workers who choose not to get trained ( $x_1$ ) nor nominal national income ( $Z$ ) depend on the minimum wage ( $w_1^1$ ).

One intuition for equation (17) may be obtained from (13). Since all workers are homogeneous, each is a marginal worker for whom equation (13) holds. Thus, regardless of the level of  $w_1^1$ , each worker's expected earnings is  $t$ , as in Section II. Hence, national income is  $tx$ . Further, we can rewrite equation (14) as

$$(14') \quad \frac{Z}{x_1} = \frac{t}{\alpha_1\beta_1 + \alpha_2\beta_2}.$$

Thus, as  $w_1^1$  changes,  $Z/x_1$  must stay constant. But, since  $Z$  does not depend on  $w_1^1$ , neither can  $x_1$ .

Most of the other variables in the model do depend on  $w_1^1$ , however. These results are summarized in Table 1. As the minimum wage rises, the demand for unskilled labor in the minimum wage sector falls, while it rises in the uncovered sector.

The wage in the uncovered sector,

$$(18) \quad w_1^2 = \frac{t\alpha_2\beta_2}{\alpha_1\beta_1 + \alpha_2\beta_2 - (t\alpha_1\beta_1)/w_1^1},$$

falls as  $w_1^1$  rises. That is, as the minimum wage rises, workers are driven out of the covered sector into the uncovered sector which lowers the wage there.

Given a Cobb-Douglas utility function, the appropriate price index is

$$(19) \quad p \equiv p_1^{\beta_1} p_2^{\beta_2}.$$

Thus, although nominal national income is constant, real national income,  $Z/p$ , falls as  $w_1^1$  rises.<sup>9</sup> If we define  $\hat{B} = B/p$ ,  $U = \hat{B}Z^{\beta_1+\beta_2}/p$  or  $U = \hat{B}(tx)^{\beta_1+\beta_2}/p$ . Thus, as  $w_1^1$  increases, utility (welfare) falls which reflects the increasing amount of distortion in the economy.<sup>10</sup>

It is possible to investigate the effects of a change in coverage by adjusting  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ , or  $\beta_2$  which will alter the relative size of the two sectors. Since we are assuming risk neutrality in this section, a change in  $\beta_1$  must be offset by an opposite change in  $\beta_2 (= 1 - \beta_1)$ . A compensated increase in  $\beta_1$  will increase the size of the covered sector while leaving  $Z$  unchanged. If the covered sector is relatively unskilled

TABLE 1

Effects of an Increase in the Minimum Wage

	Risk neutral homo- geneous	Risk neutral nonhomo- geneous	Risk averse homo- geneous	Covered Leontief educa- tional sector	Uncovered Leontief educa- tional sector
Welfare	-	-	-	-	-
$w_1^2$	-	-	?	-	-
$w_1^1/p$	+	+	+	+	+
$1/p$	-	-	-	-	-
$w_1^2/p$	-	-	?	-	-
$x_1$	0	0	-	+	-
$Z$	0	0	+	-	+
$x_1^1$	-	-	-	-	-
$\pi$	-	-	?	-	?
$y_1/y_2$	-	-	?	-	-
$p_1$	+	+	+	+	+
$p_2$	-	-	?	-	-

labor intensive ( $\alpha_1 > \alpha_2$ ), then the number of workers who choose to get trained will increase with an increase in coverage (due to a taste change):

$\partial x_1 / \partial \beta_1 = x(\alpha_1 - \beta_2)$ . The effect of an increase in coverage on the uncovered wage depends on the relative size of  $t/w_1^1$  and  $\alpha_1$ :

$$\frac{\partial w_1^2}{\partial \beta_1} = \frac{\alpha_1 \alpha_2 t (t/w_1^1 - \alpha_1)}{[(\alpha_1 \beta_1 + \alpha_2 \beta_2 - t d_1 \beta_1) / w_1^1]^2}$$

#### IV. DIFFERENCE IN ABILITY, RISK NEUTRALITY

In Section III, it was shown that, in a Cobb-Douglas, risk-neutral world with homogeneous workers, an increase in the minimum wage has no effect on the number of workers who get trained. Indeed, equation (16) is the same as (12), given risk neutrality: The same number of people get trained in a world with a minimum wage as in one without such a restriction. In this section, it is shown that the assumption of homogeneous workers is not a necessary condition for this result: Even if workers differ in ability, the level of the minimum wage need not affect training.

Suppose, for example, workers' abilities are distributed uniformly so that the time it takes to get trained,  $(1 - t)$ , varies from 1 to  $(1 - t^*)$ .<sup>11</sup> That is, time working,  $t$ , ranges from 0 to  $t^*$ . Thus, some skilled workers receive rents.

Risk-neutral workers still enter the skilled occupation until the marginal worker finds that

$$(20) \quad \pi w_1^1 + (1 - \pi) w_1^2 = t,$$

where the  $t$  is the working life of the marginal worker. Here, national income is

$$(21) \quad Z = w_1^1 x_1^1 + w_1^2 x_1^2 + \left( \frac{x + x_1}{2x} \right) t^*(x - x_1),^{12}$$

$$= \left[ \frac{x_2 - (x_1)^2}{2x} \right] \frac{t^*}{1 - \alpha_1 \beta_1 - \alpha_2 \beta_2}.$$

By rewriting equation (20), using (7) and realizing that the  $t$  of the marginal worker is  $(x_1/x) t^*$ , we obtain:

$$(22) \quad Z = \frac{(x_1)^2}{x} \frac{t^*}{\alpha_1 \beta_1 + \alpha_2 \beta_2}.$$

Equations (21) and (22) are two equations in two unknowns ( $z$  and  $x_1$ ).

Solving simultaneously:

$$(23) \quad x_1 = x \left( \frac{\alpha_1 \beta_1 + \alpha_2 \beta_2}{2 - \alpha_1 \beta_1 - \alpha_2 \beta_2} \right)^{1/2}$$

and

$$(24) \quad Z = \frac{xt^*}{2 - \alpha_1 \beta_1 - \alpha_2 \beta_2}.$$

Thus, again,  $x_1$  does not depend on  $w_1^1$ . Indeed,  $x_1$  does not depend on  $t^*$  either. The other qualitative results of Section III also hold here.

## V. RISK AVERSION

While the results of Section III still hold without the homogeneity assumption, they do depend critically on the risk neutrality assumption. In this section, the homogeneous workers are assumed to be risk averse, where each worker's degree of relative risk aversion is  $(1 - \beta_1 - \beta_2)$ .<sup>13</sup>

Now, workers will enter the skilled occupation until the marginal worker is indifferent between working in the two occupations. That is, entry will occur until the expected utility in the unskilled occupation equals the certain utility received by a trained worker:<sup>14</sup>

$$(25) \quad \pi U(w_1^1) + (1 - \pi) U(w_1^2) = U(t).$$

Again,  $w_1^1 > t > w_1^2$ .<sup>15</sup>

Substituting equations (3) and (7) into equation (25) and rearranging terms, we have:

$$(26) \quad \left[ \frac{t^\gamma - (w_1^2)^\gamma}{w_1^2} \right] = \frac{\alpha_1 \beta_1}{\alpha_2 \beta_2} \left[ \frac{(w_1^1)^\gamma - t^\gamma}{w_1^1} \right],$$

where  $\gamma \equiv \beta_1 + \beta_2$ . To see how  $w_1^2$  varies with  $w_1^1$ , equation (26) can be totally differentiated to obtain

$$(27) \quad \frac{dw_1^2}{dw_1^1} = - \frac{\alpha_1 \beta_1}{\alpha_2 \beta_2} \left( \frac{w_1^2}{w_1^1} \right)^2 \frac{N}{D},$$

where

$$N = (\gamma - 1) (w_1^1)^\gamma + t^\gamma$$

and

$$D = (\gamma - 1) \left( \frac{w_1^2}{w_1^1} \right)^\gamma + t^\gamma > 0. \quad (16)$$

Thus, the sign of  $dw_1^2/dw_1^1$  is the opposite of that of  $N$ . But  $N \geq 0$  as

$$1 - \beta_1 - \beta_2 < \left( \frac{t}{w_1^1} \right)^\gamma.$$

Since  $w_1^1 > t$ ,  $0 < \left( \frac{t}{w_1^1} \right)^\gamma < 1$ , so if an individual is risk averse enough (the degree of risk aversion,  $1 - \beta_1 - \beta_2$ , is large enough), for given  $t/w_1^1$ ,  $dw_1^2/dw_1^1 > 0$ . That is, if people are very risk averse, an increase in the minimum wage may cause the wage of the uncovered, unskilled wage  $\left( \frac{w_1^2}{w_1^1} \right)$  to rise. Otherwise, it will fall with an increase in the minimum wage (or be constant in the extreme case of risk neutrality).

But what is the effect on training? Since national income is now

$$(28) \quad Z = \frac{t(x - x_1)^\gamma}{\gamma - \alpha_1 \beta_1 - \alpha_2 \beta_2},$$

from equation (8),

$$(29) \quad x_1 = tx \left( \frac{\alpha_1 \beta_1}{w_1^1} + \frac{\alpha_2 \beta_2}{w_1^2} \right) / E$$

where

$$E = \gamma - \alpha_1 \beta_1 - \alpha_2 \beta_2 + t \left( \frac{\alpha_1 \beta_1}{w_1^1} + \frac{\alpha_2 \beta_2}{w_1^2} \right).$$

By totaling differentiating equation (29), we find that

$$(30) \quad \frac{dx_1}{dw_1} = -tx(\gamma - \alpha_1\beta_1 - \alpha_2\beta_2) \frac{F}{E^2},$$

where

$$F = \frac{\alpha_1\beta_1}{(w_1^1)^2} + \frac{\alpha_2\beta_2}{(w_1^2)^2} \frac{\partial w_1^2}{\partial w_1^1}.$$

From equation (27), we know that

$$(31) \quad F = \frac{\alpha_1\beta_1}{(w_1^1)^2} \frac{(\gamma - 1) \left[ \frac{(w_1^2)^\gamma}{D} - (w_1^1)^\gamma \right]}{D} > 0.$$

Thus,  $dx_1/dw_1^1 < 0$ , so raising the minimum wage increases the number of people who become trained ( $x_2 = x - x_1$ ) and decreases the number who remain unskilled ( $x_1$ ). From equation (28), we know that  $Z$  will move in the opposite direction from  $x_1$ , so as the minimum wage increases, so does nominal national income.

To determine what the minimum wage does to welfare, we consider an expected utility measure for the whole population:

$$(32) \quad V = B \left( w_1^1 \right)^\gamma x_1^1 + B \left( w_1^2 \right)^\gamma x_1^2 + Bt^\gamma (x - x_1).$$

In equilibrium, equation (25) holds, so (32) may be rewritten as

$$(33) \quad V = Bt^\gamma x = \frac{\hat{B}t^\gamma x}{p}.$$



Substituting for  $p$  from equation (19) and differentiating,

$$(34) \quad \frac{\partial V}{\partial w_1^1} = -V \left( \frac{\alpha_1 \beta_1}{w_1^1} + \frac{\alpha_2 \beta_2}{w_1^2} \frac{\partial w_1^2}{\partial w_1^1} \right) < 0. \quad 17$$

Thus, increasing the minimum wage unambiguously lowers expected welfare. <sup>18</sup>

## VI. AN EDUCATIONAL SECTOR

In the previous models, training was assumed to take time but no other inputs. In this section, we assume that there is an educational sector which trains workers. While we could assume one of the two Cobb-Douglas sectors produced education, such a model would require that the educational sector be the only covered or uncovered sector. Instead, we introduce a third sector which, for simplicity, is assumed to use a Leontief technology.

To train an unskilled worker takes  $s$  skilled (teachers) and  $u$  unskilled (secretaries, janitors) workers. Since it only takes  $(1 - t)$  fraction of a period to train someone, to train  $(x - x_1)$  people takes  $s(1 - t)(x - x_1)$  skilled workers and  $u(1 - t)(x - x_1)$  unskilled workers. The tuition cost per pupil depends on whether this sector is covered by the minimum wage.

### VI.A. COVERED EDUCATIONAL SECTOR

If the educational sector is covered by the minimum wage, the out-of-pocket cost to each student is  $(1 - t)(w_1^1 u + s)$ . Risk-neutral, homogeneous workers will enter the skilled occupation until the marginal worker is indifferent:

$$(35) \quad \pi w_1^1 + (1 - \pi) w_1^2 = t - (1 - t)(w_1^1 u + s),$$

where

$$\pi = \frac{x_1^1 + u(1-t)(x - x_1)}{x_1}.$$

Rewriting equation (35), we obtain:

$$(36) \quad Z = \frac{x_1[t - (1-t)s] - w_1^1 u(1-t)x}{\alpha_1 \beta_1 + \alpha_2 \beta_2}.$$

Nominal income may also be written as

$$(37) \quad Z = w_1^1 \left[ x_1^1 + u(1-t)(x - x_1) \right] + w_1^2 x_1^2 + \left[ t - (1-t)(w_1^1 u + s) \right] (x - x_1)$$

$$= \frac{x_1^1 [t - (1-t)s] - w_1^1 u(1-t)x}{\alpha_1 \beta_1 + \alpha_2 \beta_2}.$$

Equating equation (36) and (37) and solving the number of unskilled workers is

$$(38) \quad x_1 = x \left[ \alpha_1 \beta_1 + \alpha_2 \beta_2 + \frac{w_1^1 u(1-t)(1 - \alpha_1 \beta_1 - \alpha_2 \beta_2)}{t - (1-t)s} \right].$$

Notice that, if  $u$  and  $s$  are both zero, equation (38) is the same as (16) which should be true since, if they are both zero, the educational sector has been eliminated and we are back in the model of Section III. Differentiating (38) with respect to the minimum wage, we obtain:

$$(39) \quad \frac{\partial x_1}{\partial w_1^1} = \frac{xu(1-t)(1 - \alpha_1 \beta_1 - \alpha_1 \beta_2)}{t - (1-t)s} > 0.$$

Heuristically, an increase in the minimum wage increases the cost of training to such a degree that fewer people become skilled. This extreme result stems from the Leontief assumption; but in any model in which the cost of training depends on the minimum wage, this effect will tend to offset the pro-training effect due to the reduced probability of finding a minimum wage job.

Substituting equation (38) into (37), nominal income may be shown to be

$$(40) \quad Z = \left[ t - (1 - t)(w_1^1 u + s) \right] x.$$

Again, everyone is a "marginal" worker, so everyone receives  $\left[ t - (1 - t)(w_1^1 u + s) \right]$ , as shown in equation (35). Here, an increase in the minimum wage reduces nominal income by  $(1 - t) ux$ .

#### VI.B. UNCOVERED EDUCATIONAL SECTOR

If the educational sector is uncovered, the tuition cost is  $(1 - t)(w_1^2 u + s)$  per pupil. Thus, equation (35) becomes

$$(35') \quad \pi w_1^1 + (1 - \pi) w_1^2 = t - (1 - t)(w_1^2 u + s),$$

where  $\pi$  again equals  $x^1/x_1$ . Solving (35') for  $Z$  gives us

$$(36') \quad Z = \frac{x_1 [t - (1 - t)s] - w_1^2 u (1 - t) x}{\alpha_1 \beta_1 + \alpha_2 \beta_2}.$$

Nominal income can be directly obtained as

$$(37') \quad Z = w_1^1 x_1^1 + w_1^2 \left[ x_1^2 + u(1 - t)(x - x_1) \right] + \left[ t - (1 - t)(w_1^2 u + s) \right] (x - x_1)$$

$$= \frac{(x - x_1)[t - (1 - t)u]}{1 - \alpha_1 \beta_1 - \alpha_2 \beta_2}.$$

Solving equations (36') and (37') simultaneously gives us

$$(38') \quad x_1 = x \left[ \alpha_1 \beta_1 + \alpha_2 \beta_2 + \frac{w_1^2 u (1-t)(1 - \alpha_1 \beta_1 - \alpha_2 \beta_2)}{t - (1-t)s} \right],$$

$$(39') \quad Z = \left[ t - (1-t) \left( w_1^2 u + s \right) \right] x.$$

Thus, an increase in the minimum wage lowers  $x_1$  (increases training) and raises nominal incomes.<sup>19</sup> In this model, an increase in the minimum wage does not raise tuition costs.

#### VII. MINIMUM WAGE-INDUCED UNEMPLOYMENT<sup>20</sup>

We now modify the model slightly to allow for unemployment: unskilled workers may be unemployed while they search (or queue) for minimum wage jobs. By assumption, an individual worker cannot work full time in the competitive sector while actively searching for a minimum wage job.

As a result, a worker must decide what proportion of time to work in the competitive sector,  $q (= X^2)$ , and what proportion to devote to search or work in the minimum wage sector,  $1 - q$ . Workers are assumed to be risk averse ( $\beta_1 + \beta_2 < 1$ ).

An unskilled worker will attempt to maximize his expected utility through this choice of  $q$ :

$$(41) \quad \max \pi U \left[ w_1^2 q + (1 - q) w_1^1 \right] + (1 - \pi) U \left( w_1^2 q \right)$$

or

$$(41') \quad \max \pi U(Z_A) + (1 - \pi) U(Z_B),$$

where  $Z_A$  is a worker's earnings if he finds a minimum wage job,  $Z_B$  is his earnings if he fails to find such a job, and  $\pi$  is (again) the probability he finds a minimum wage job ( $\pi = x_1^1 / [(1 - q) x_1]$ ).

For  $0 < q < 1$  (an interior solution), the first- and second-order conditions must be satisfied:

$$(42) \quad \pi U'(Z_A^*) (w_1^2 - w_1^1) + (1 - \pi) U'(Z_B^*) w_1^2 = 0,$$

$$(43) \quad \pi U''(Z_A^*) (w_1^2 - w_1^1)^2 + (1 - \pi) U''(Z_B^*) (w_1^2)^2 < 0,$$

where an asterisk indicates an equilibrium value. Given risk aversion ( $\beta_1 + \beta_2 < 1$ ), the second-order condition (43) is satisfied.<sup>21</sup>

We know in equilibrium that, for the marginal worker, his expected utility as an unskilled worker equals his certain utility as a trained worker [this equation is analogous to equation (25)]:

$$(44) \quad \pi U(Z_A^*) + (1 - \pi) U(Z_B^*) = U(t).$$

### VIII. CONCLUSIONS

The chief implication of these models is that, unless tuition costs are a function of the minimum wage, an increase in the minimum wage will leave unchanged or increase the number of workers who get trained. Within the context of a two-sector general equilibrium model, the number of skilled workers does not depend on the minimum wage if workers are risk neutral. Risk neutrality is a necessary condition for this lack of an effect, though homogeneity of workers is not a necessary condition.

Adding a third educational sector may cause the number trained to depend on the minimum wage even with risk-neutral, homogeneous workers. If the educational sector is covered by the minimum wage, an increase in the minimum wage causes less people to get trained; while, if that sector is uncovered, such an increase would induce more people to become trained. Results of all the models are summarized in Table 1.

As shown in Table 1, except in the risk-aversion model, the uncovered unskilled wage ( $w_1^2$ ) falls as the minimum wage ( $w_1^1$ ) increases. The more risk averse workers are, the more likely that the perverse result  $\partial w_1^2 / \partial w_1^1 > 0$  is to occur. Since so many of the other partial derivatives in the risk aversion model depend on  $\partial w_1^2 / \partial w_1^1$ , they are also ambiguous, as shown in Table 1.

In all the models, as the nominal minimum wage increases relative to the nominal skilled wage, the real minimum wage ( $w_1^1/p$ ) rises; the real skilled wage ( $1/p$ ) falls; and the unskilled, uncovered wage ( $w_1^2/p$ ) falls (so long as workers are not very risk averse).<sup>22</sup> All the welfare measures which are based on utility functions fall as the minimum wage rises and distortions increase.

UNIVERSITY OF CALIFORNIA, BERKELEY

## FOOTNOTES

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<sup>1</sup>There is a limited theoretical and empirical literature on this topic. Mincer and Leighton [1980] survey most of this literature. Mattila's [1978] empirical research indicates that schooling is encouraged by the minimum wage. Mincer and Leighton [1980] argue that the effects of the minimum wage on schooling is ambiguous but that the effect on on-the-job training is clearly negative. This paper examines only the schooling decision which is, henceforth, referred to as "training."

<sup>2</sup>Mincer [1976], Welch [1974], Gramlich [1976], Perloff [1980], and others have used such a model to examine the effects of minimum wages on unemployment and other variables. In this paper markets are assumed to clear. If search unemployment occurs, an increase in the minimum wage can either raise or lower training.

<sup>3</sup>In the next section, a minimum wage for unskilled labor is set in one sector. Given the normalization of  $w_2$ , the minimum wage is implicitly set relative to skilled labor's wage. This assumption is generally consistent with U. S. history since the minimum wage has been a fairly constant fraction of the manufacturing wage. Any other normalization would leave the qualitative results unchanged.

<sup>4</sup>In Section VI, education is assumed to require real resources in addition to a student's time. It may be reasonable, however, to concentrate on foregone earnings since they are typically the greatest cost of training; see Becker [1964].

<sup>5</sup>This analysis ignores discounting which would add to the complexity of the model without adding interesting new insights.

<sup>6</sup>In the United States, the minimum wage has never had universal coverage. In 1966, 66 percent of private nonfarm employment was covered. Coverage increased to 79 percent in 1968 and to 86 percent in 1978; see U. S. Department of Labor, Minimum Wage and Maximum Hours Standards under the Fair Labor Standards Act, various volumes.

<sup>7</sup>If  $\beta_1 + \beta_2 = 1$ ,  $U = BZ$ , so  $U'' = 0$ .

<sup>8</sup>See the definition of  $B$  below equation (3).

<sup>9</sup>Proof:

$$\begin{aligned} \frac{a(Z/p)}{\partial w_1^1} &= - \left( \frac{\alpha_1 \beta_1}{w_1^1} + \frac{\alpha_2 \beta_2}{w_1^2} \frac{\partial w_1^2}{\partial w_1^1} \right) \frac{Z}{p} \\ &= - \frac{\alpha_1 \beta_1}{w_1^1} \left( 1 - \frac{w_1^1}{w_1^2} \right) \frac{Z}{p} < 0. \end{aligned}$$

<sup>10</sup>We know  $au/\partial w_1^1 < 0$  since  $\hat{B}(tx)^{\beta_1 + \beta_2}$  is not a function of  $w_1^1$  and  $1/p$  varies inversely with  $w_1^1$  (see footnote 9).

<sup>11</sup>Compare the definitions of ability in Becker [1964] and Sattinger [1975]. If ability were uniformly distributed on  $[t^* - a, t^* + a)$ , the qualitative results would be the same, but the equations would be more complicated.

<sup>12</sup>The average ability of the skilled workers is  $t^*(x + x_1)/(2x)$ .

<sup>13</sup>From equation (3),  $-ZU''(Z)/U'(Z) = 1 - \beta_1 - \beta_2$ . In Section VII, we examine the effect of search behavior, given risk aversion.

<sup>14</sup>Since the skilled occupation provides a riskless income, training will lead to a lower expected income than in the risky (unskilled) occupation. Again, with discounting, this relationship could change.

<sup>15</sup>If  $t > w_1^1 > w_1^2$ , then  $t^{\beta_1 + \beta_2} > (w_1^1)^{\beta_1 + \beta_2} > (w_1^2)^{\beta_1 + \beta_2}$ ; hence,



$Bt^{\beta_1 + \beta_2} = U(t) > U(w_1^1) > U(w_1^2)$ , so that  $\pi U(w_1^1) + (1 - \pi) U(w_1^2) < U(t)$ , and everyone would get trained. If  $w_1^1 > w_1^2 > t$ , a similar argument would imply that no one would get trained.

<sup>16</sup>Since  $t > w_1^2$ ,  $t^\gamma > (w_1^2)^\gamma$ . Thus, since  $-1 < \gamma - 1 < 0$ ,

$D > 0$ .

<sup>17</sup>The inequality may be shown by using equation (31).

<sup>18</sup>It can be shown that  $U(Z, p_1, p_2)$  falls as well.

<sup>19</sup>With a great deal of effort, it can be shown that  $\partial w_1^2 / \partial w_1^1 < 0$ .

<sup>20</sup>The type of model is examined in Perloff [1980] in more detail.

<sup>21</sup>From equation (3), we know that

$$U' = B(\beta_1 + \beta_2) Z^{\beta_1 + \beta_2 - 1} < 0$$

and (since  $\beta_1 + \beta_2 < 1$ )

$$U'' = B(\beta_1 + \beta_2)(\beta_1 + \beta_2 - 1) Z^{\beta_1 + \beta_2 - 2} < 0.$$

<sup>22</sup>I have not been able to show analytically that the welfare and real skilled wage partial derivatives in the last two models are negative; however, economic reasoning and simulations both indicate the signs shown in Table 1.

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