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Author
Sengupta-Irving, Tesha

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Doing things: Organizing for agency in mathematical learning

Tesha Sengupta-Irving

University of California – Irvine, 3200 Education, Irvine, CA 92697-5500, United States

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ABSTRACT

In the United States, school mathematics generally fails to help students see themselves as capable of impacting their world – a perspective Freire argues defines human agency. This analysis draws from a five-week Algebra intervention for middle school students ($n = 46$) designed to promote agency through collaborative mathematical activity. Typically, students identified as underperforming (as most in this intervention were), teachers revert to procedural, low-level instruction. In contrast, this intervention was designed around tasks of high cognitive demand that required visual or symbolic representation of algebraic concepts. Qualitative coding of student interviews ($n = 46$) confirm the design principles of authority, agency and collaboration were positively impactful for students. In particular, interviews evidence a changing perspective from math as boring to the possibility of math as comingling intellectual challenge and personal enjoyment. These results are traced to the design principles and in particular, the focus on organizing for agency.

> What is required is that every individual shall have opportunities to employ his own powers in activities that have meaning. Mind, individual method, originality . . . signify the quality of purposive or directed action.
>
> John Dewey, 1916/1944, p. 203

John Dewey (1916/1944) believed in learning through meaningful activity so much so that he argued quantifying and comparing students' abilities was "irrelevant" to the work of teachers. As the quote above suggests, teaching should provide students a chance to discover and pursue meaningful learning opportunities that reveal their ingenuity and individuality. However, comparative international studies of 8th grade students show that those who average among the highest in mathematical achievement average among the lowest in their interest in math (Mullis et al., 2000). In the U.S., but for few exceptions (e.g., Gutstein, 2012; Silva, Moses, Rivers, & Johnson, 1990), school mathematics generally fails to help students see themselves as capable of making and remaking their world, which, from a Freirean (1994) perspective, *defines* human agency. As educators and child development researchers argue, the failings of traditional schooling become most visible in adolescence: at precisely the time when students want to express their agency, adults at school exert increased control over their behaviors (Eccles, Lord, & Midgley, 1991; Moje, 2002; Rogers, Morrell, & Enyedy, 2007). Creating opportunities for adolescents to express agency through meaningful mathematical activity is therefore worthy of greater consideration in current debates about the purposes and nature of mathematics education in schools.

This analysis is based on a five-week middle school mathematics intervention in the western United States, designed for ethnically and racially diverse youth ($n = 94$) with a range of prior mathematics achievement. This article describes how learning opportunities were organized for student agency; then, drawing on student interviews, examines students' perceptions of agency in learning mathematics during the summer program.

E-mail address: t.s.irving@uci.edu

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1. Conceptualizing agency and productive mathematical engagement

Gresalfi, Martin, Hand, and Greeno (2009) suggest agency is *not* something someone "has" or "lacks" as one might say, for example, of motivation. The authors argue that the simplest acts of complying or resisting – taking out your pencil to copy an example problem, or not – are expressions of agency in a learning environment. As such, agency is available to everyone, from the most reserved to the most flamboyant of students. Pickering (1995), sociologist and historian of science, describes agency as the antithesis of passivity in his writing *The Mangle of Practice: Time, Agency, and Science*. As Wagner (2007) points out, Pickering equates agency in the pursuit of high-level mathematics with purposeful action. "One can start from the idea that the world is filled not . . . with facts and observations, but with *agency*. The world, I want to say, is continually *doing things*, things that bear upon us not as observation statements . . . but as forces upon material beings" (Pickering, 1995, p. 6, his emphasis). By living in the world, by interacting with others, by responding to forces of nature, humans see and express agency in daily existence. In U.S. mathematics classrooms, however, human agency is displaced by passivity, as students sit quietly, watching, and listening, before practicing similar problems (Hiebert & Stigler, 2000; Rosen, 2001). This leads many students to disengage (Boaler, 2002; Boaler & Staples, 2008). In Boaler's numerous studies of students learning mathematics in contrasting teaching environments, she found that even students who are successful through passive engagement frequently plan to stop taking math courses at the earliest opportunity (Boaler, 2002, 2006, 2009).

In Pickering's conception of agency in mathematics, he describes the "dance of agency" (1995, p. 116), as involving two partners: human agency and agency of the discipline. Human agency refers to people creating initial ideas or extending established ones. Agency of the discipline refers to standard procedures of mathematical proof or widely agreed-upon methods of verification, for example. The interaction between human ingenuity and standards of the discipline is a dance that Pickering argues has been central to conceptual advances in mathematics, historically. Classrooms where students engage in a dance of agency while working on complex mathematical tasks are shown to encourage student interest (Boaler & Greeno, 2000; Martin, 2000), achievement, and persistence in the discipline (Boaler & Staples, 2008).

Frequently, when teachers work with students identified as underperforming (and lower-income), they remove the possibility of active learning and revert instead to primarily procedural, low-level remediation, with the belief that students simply need more practice of rules (Anyon, 1980, 1981; Haberman, 1991). The features of a positive learning environment – whoever the learner – include opportunities to reason about problems, discuss mathematical ideas, and debate solution pathways (Kieran, 1994; Malloy, 2009; NCTM, 2014; Stein, Engle, Smith, & Hughes, 2008). Students should be engaged in developing their own strategies, exploring outcomes, developing reasoned understandings, and formulating identities as capable mathematics learners (Boaler, 2015; Fasheh, 1982; Kilpatrick, Swafford, & Findell, 2001; Langer-Osuna, 2007; Schoenfeld, 2007). Engle and Conant (2002) offer four principles for designing learning environments that, as the authors call it, support productive disciplinary engagement. These principles also articulate a vision of learning environments where expressions of agency are concomitant with expressions of disciplinary interest, engagement, and achievement.

Engle and Conant's four principles for designing robust learning environments include: (1) *Problematizing*, where students are encouraged to take on intellectual problems; (2) *Authority*, where students are given authority to address those problems; (3) *Accountability*, where students are held accountable to others and to disciplinary norms; and (4) *Resources*, which refers to students having sufficient materials for inquiry (2002, pp. 400–401). The first principle, expecting students to problematize, is the opposite of passivity and therefore a necessity in learning environments organized for students "doing things".

Engle & Conant's second principle is authority. Gresalfi and Cobb (2006) define authority, using Engle & Conant, as the "degree to which students are given opportunities to be involved in decision-making . . . have a say in establishing priorities in task completion, method, or pace of learning" (p. 51). In short, they posit authority as being about "who's in charge" of making mathematical contributions. They recognize the limits of either teacher or text being positioned as the sole mathematical authority (Amit & Fried, 2005). Authority is intimately related to agency: students play an active part in defining, addressing, and resolving problems because they have the authority as authors and producers (not simply consumers) of knowledge (Lampert, 1990; Lehrer, Carpenter, Schauble, & Putz, 2000; Magnusson & Palincsar, 1995).

Engle and Conant's (2002) third principle is accountability as when a teacher asks a student if the solution she or he has devised is most efficient in its use of variables. Gresalfi and Cobb (2006) also refer to this sort of exchange, where the student and teacher jointly determine the mathematical legitimacy of methods, as "distribution of authority" within the learning environment. This exchange can also occur among students (e.g., Godfrey & O'Connor, 1995; Oyler, 1996). This idea of holding students' thinking accountable to the subject matter is critical in advancing mathematical learning

Engle and Conant's fourth and final principle is resources, which essentially refers to providing students the material support for pursuing their choice of mathematical activity. Limiting freedom within the physical space (e.g., students sit at desks) or the variety of mathematical tools to represent thinking (e.g., pencil and paper only) can impede expressions of agency. As Fiori and Selling (this issue) illustrate, the physical layout and resources available within the learning environment significantly shape how students define and pursue interesting mathematical work.

In sum, drawing together notions of active learning and what prior research demonstrates as the principles of productive disciplinary engagement, redesigning learning environments to facilitate expressions of agency ("doing things") becomes possible and practicable.

2. Methods

2.1. Design of the intervention to facilitate "doing things"

The pedagogical and curricular choices for the summer intervention flowed directly from the teaching principles (Boaler, this issue) and the particular focus on organizing for agency. The classroom was meant to reflect methods and thinking of students as mathematicians-in-action. Student questions could be scrawled on sentence strips and posted. Large posters of written work with student names would eventually line the walls. As resources, the rooms were equipped with Cuisenaire rods, Algebra Tiles, pattern blocks, counters, geoboards, and other hands-on materials readily available for students to use as needed. Periodically, stations were created around the room with different, but related, mathematical activities.

The team spent several weeks preparing for the summer, and typically stayed after each session to reflect on teaching practices during the five weeks. Fridays, a non-instructional day, were used to coordinate the research efforts, discuss student concerns at length, and collaboratively plan for the subsequent week of instruction. Undoubtedly, extended time to plan – a luxury for many teachers – benefited implementation of the teaching principles and coherence across classes. What follows are the prominent pedagogical and curricular design decisions we made in organizing for agency.

2.1.1. Problematizing: Leveraging visual and cognitively demanding tasks

The teaching of our summer school was designed around rich, open mathematics tasks that gave students opportunities to develop responsibility and agency. We drew from tasks available in printed curricular resources, including those from the Interactive Mathematics Project (IMP), College Preparatory Mathematics (CPM), Mathematics Education Collaborative (MEC), and two English resources, The Points of Departure books from ATM and the Secondary Mathematics Individualised Learning Experiment (SMILE). We prioritized tasks that were visually interesting. The visual esthetics of the tasks – a grid, a series of colored blocks, the Tower of Hanoi, an array of toothpicks – engaged students and required their active exploration of the mathematics. Whether students drew from a standard method or moved toothpicks around, the tasks offered a range of opportunities to *problematize* (Engle & Conant, 2002) and *do things* in learning.

In considering the tasks more closely, I draw on the framework of Stein, Smith, Henningsen, and Silver (2009, p. 3) for the analysis of mathematical tasks. They consider the cognitive demand of tasks, which can range from "memorization" to complex, non-algorithmic problem solving (i.e., "doing math"). The authors argue that the cognitive demand of a task should follow from the instructional goal. For the intervention, the goal was creating opportunities for agency – opportunities where students have the authority to define the problem, set a pathway to its solution, and work flexibly within the parameters of the discipline. This goal translated into an exclusive use of high cognitive demand tasks where no pathway was suggested and originality in exploration could reign. Ironically, the visual appeal of tasks often obscured the high level of cognitive demand they required. For example, an observer might remark that students playing the mathematical game Nim are merely engaged in unsystematic exploration with colored counters, which is not mathematical. What eludes that observer (but not the teacher) is that supporting discussions on the strategies of Nim, means supporting nascent understandings of the logic operation *exclusive or* ($x \oplus y$). As Stein and colleagues warn, "it is important not to become distracted by superficial features of the task" because although a task may seem low-level (i.e., displaying and eliminating counters), "it is important to move beyond the surface features to consider the kind of thinking they require" (pp. 7–8). All of the tasks we selected required complex thinking and engagement in mathematical practices like representation, generalization, and justification, with an emphasis on action and ingenuity.

2.1.2. Authority: Choosing and pursuing meaningful learning opportunities

Demonstrating to students they had the authority (Engle & Conant, 2002) to choose how and at what pace they pursued tasks was both necessary and a significant challenge. This essentially meant socializing students into norms of learning that directly countered passivity and instead prioritized voice, choice, and plurality of thought and action. One way to do this was through daily Number Talks (Humphreys & Parker, 2015; Boaler, 2015; Parrish, 2010). Students were given a problem to solve and were then asked to systematically describe their solution methods. The teachers recorded and facilitated discussion about the various methods that emerged, sometimes eliciting as many as ten different ways (see https://www.youcubed.org/from-stanford-onlines-how-to-learn-math-for-teachers-and-parents-number-talks/). The encouragement of multiple solution pathways emphasized the plurality (and originality) of mathematical thinking as valid and viable. Other ways of communicating this message came through stations and menus. As described previously, learning stations (4–5 areas of the room cordoned off with different tasks) offered students choices about work of interest and opportunities to reason about mathematical content. Movement between stations and how long one stays at a station, were decisions left to the student (some of whom even chose to move as pairs). Menus, on the other hand, were lists of tasks grouped together as conceptually complementary and increasingly complex that students could work through at their own pace. Students could choose from an array of menus but once committed, the goal was to complete all of the activities on a given menu. The use of menus demonstrated that a trust in students' authority to initiate, select, and pursue mathematical problems was paramount.

2.1.3. Accountability: Inviting the dance of agency

Making explicit the need for disciplinary accountability (Engle & Conant, 2002), while concomitantly creating opportunities for learners' agency, cast the classroom as a stage for Pickering's dance of agency. We held students accountable to the discipline by emphasizing the need to communicate their mathematical thinking through explanations, justifications, and representations. Physically, every student had a journal that was individually decorated, in which they recorded, shared, and made explicit their methods of solving problems. At stations, for example, the journals were a common object of attention that students used to evidence their thinking, with each other and the teacher. It was not enough to sit and think about a problem, offer only a verbal solution and then leave. The students had to translate that verbal solution into diagrams of a physical model, symbols, or prose for others to consider. In addition to journals, students frequently created posters of their solutions, signed their names, and presented it to the class. These whole-class presentations drew forth the discomfort so many had in being, quite literally, on stage with all eyes on them. No longer could they operate in relative anonymity; they had to make their thinking and knowledge of mathematical norms "public" (Ball & Bass, 2003).

When inviting students into the dance of agency – putting their thinking in conversation with procedures of the discipline – we emphasized dancing for a very specific audience: their peers. Collaboration was therefore crucial to organizing the learning environment for agency. Whether working in pairs or in groups, we encouraged students to work together and lead one another in mathematical discussions. Students sat in teams without any regard to prior achievement, and teams were rotated on a regular basis for students to meet and engage with everyone. This was the one area in which students did not have a choice: whereas there were opportunities to work alone, students could not choose to work alone everyday. We wanted them to see the value of reasoning with others and learning from the various ways they conceptualize and approach problems.

We also encouraged students to balance their ingenuity with the boundaries of the discipline by adopting a particular stance to "answers." Often on the heels of the triumphant cry, "I got it!" we asked whether the student could have gotten "it" in a different or more efficient way, if they could explain and justify how their method of "getting" was mathematically sound, or if they could change the goal altogether and find a new "it" to pursue. For the latter, extending the problem meant finding ways to transform the "answer" (what so often marks the end of mathematical inquiry) into the start of a new problem. This practice of divining new problems from solutions meant *doing something* with an answer beyond simply "getting it." For example, take the case of Alonzo (see Boaler, 2015 for additional cases), who innovated by taking a solution to a problem we presented and created a new problem. Alonzo and his classmates were doing a task called Staircases (students determine the total number of blocks in an ascending staircase). After having figured out how to predict the number of total blocks in the staircase, Alonzo created a new staircase. Avoiding the simplicity of a structure that was merely a multiple of the original, Alonzo re-routed the staircase in multiple directions, thereby introducing a new term to the algebraic solution. He then gave *his* staircase task to me to solve (I did, and then promptly passed it on to his peers). This interaction with Alonzo typified the intervention's goals: engaging in cognitively demanding tasks, exerting human agency in choosing, pursuing, and extending mathematical inquiry, and communicating with others.

2.2. Setting and participants

The summer school intervention was set in a suburban community of Northern California. The intervention was 20 sessions long (4 days × 5 weeks) with classes taught in 90 min blocks. Students were not given homework as part of the summer intervention, which was significantly different than the regular school year. Additionally, they were not given grades but rather a pass/no pass for their involvement. There were 94 students across four intervention classes. The students represented a wide range in prior mathematics achievement: 40% had achieved As or Bs in their prior math class, 20% Cs, and 40% Ds or Fs. See Boaler, this issue, for more details regarding student demographics. All of the teacher-researchers came from the graduate program at Stanford University's School of Education. I taught two of the four classes, one was co-taught by a graduate student and our faculty advisor, Jo Boaler, and Nicholas Fiori (see Fiori & Selling, this issue) taught the fourth class.

2.3. Data sources

This analysis draws exclusively on interviews conducted during the summer intervention with students, complementing the analyses in this issue that focus more on classroom observations. Forty-six of the 94 students were interviewed in 36-paired interviews, 19 of the students were interviewed more than once. Most interviews were approximately 20 min long; the time varied depending on participant responses. Members of the research team conducted all interviews, though the class instructors did not interview their own students. The interviews covered a range of topics through which students reflected on what led them to summer school, their prior math classes and achievement, their view of mathematics, and their experiences of the summer program.

2.4. Analysis

All of the interviews were transcribed in preparation for analytic coding. Using Dedoose software, transcripts of interviews were coded for two purposes (see Table 1): first, to understand students' perceptions of mathematics learning opportunities

Table 1
Coding scheme.

Purpose	Code	Explanation
Understanding perceptions	*Teaching*	*Prior:* Perceptions of past teacher or teaching practices *Intervention:* Perceptions of summer teacher or teaching practices
	Success	*Prior:* Perceptions of what it took to be successful in mathematics based in past experiences *Intervention:* Perceptions of what it took to be successful in mathematics in the summer
Examining perceptions of agency	*Tasks*	References to summer tasks by, name, features, or nature of activity
	Authority	References to engagement in summer tasks
	Accountability	References to peer interactions in the summer

Table 2
Summary of code application and range across 36 interviews.

Code	Number of interviews where code was applied (% of total)	Total frequency of instances
Teaching (*Prior* and *Intervention* combined)	31 (86%)	148
Success (*Prior* and *Intervention* combined)	36 (100%)	189
Tasks	33 (92%)	97
Authority	18 (50%)	43
Accountability	26 (72%)	78

both previous to the intervention and of the intervention, through their descriptions of how mathematics teachers teach and what it takes to be successful in their classes. I captured these perceptions with two primary codes, *Teaching* and *Success*, which were co-coded *Prior* or *Intervention* to delineate descriptions of previous math experiences from the summer, respectively. The second purpose was to examine students' perceptions of agency in the summer program, as conceptualized previously. I therefore created codes that directly reflected instances when students were reasoning about their summer experiences in terms of the design principles (e.g., problematizing with complex tasks, authority to pursue inquiry, accountability to peers). Thus the latter analysis was captured by three primary codes, all of which were conceptually grounded in agency and productive disciplinary engagement: *Tasks*, *Authority*, and *Accountability*.

Coded results were displayed in matrices to analyze overall frequencies of codes by interview. Frequency of codes gave some sense of the import of a particular topic across and within interviews (see Table 2).

What follows are the results of coding interviews conducted during the summer intervention that both convey perceptions of prior mathematics learning opportunities, and perceptions of agency in their experiences of the summer intervention.

3. Results and discussion

3.1. Boring as normal: Characterizations of past math classes

The students' prior experiences of mathematics were generally described as involving lectures with passive learning, quiet diligence, and primarily textbook or worksheet-based practice. Perhaps not surprising, therefore, when asked what it meant to be a good math student, a coherent image emerged across interviews. Jacob and Marcella described the good math student as someone who gets A's on their report cards, does homework, does not get in trouble, and does not talk in class. Samantha and Nicolette explained that students know they're doing a good job in math "when you like write down a lot of stuff in your notebook and [are] paying attention." Or as Maureen said, "They pay attention and they get what the teacher is trying to tell them."

A frequent description of past mathematics classes was one of silence. The majority of students (36 of 46) described their previous mathematics courses as requiring quiet. As Luke explained of his previous math class, "It was too quiet . . . You couldn't talk . . . and then if you could talk about it, you have to whisper and then you couldn't get up and help someone else or even get help or just move around and stuff. You had to stay down and whisper everything." Or as Cindy described, "the teacher just makes us take notes and . . . she says 'okay, here's the book, here's the paper, do it' and then we just take notes and everything, and then we just, we just do it." Walker described his previous class as one where 15 min were used to set up the lesson and the rest of the time was spent doing the homework assigned for that night. Working in isolation on problems, however, seemed to disturb him most: "If you ever needed help [the teacher] would say 'don't come to me, go ask your classmates for help,' but then when you asked your classmates, she wouldn't like you talking to them!" Victor agreed and later added, "you're trapped in school for eight hours, you need talking time." The recurring theme of silent, individual work was in stark comparison to the design principles of the summer intervention.

The choice of our mathematics tasks was a direct function of the instructional goal of creating opportunities for agency. The interviews demonstrated how students' perceptions of the mathematical tasks cohered around the idea that they were engaged in complex and challenging work.

3.2. Redefining the learner: Supporting student authority in the pursuit of problems

Shifting authority (from the teacher and text to students; or as distributed between teacher and students or among students) was one feature of the summer intervention intended to provide opportunities for expressing agency. Yvette, Cadence and Gregory – in separate interviews – seemed to sense that providing opportunities to choose among a range of activities was deliberate and different. Yvette focused primarily on what it meant in terms of the student-teacher relationship, alluding to the idea of distributing authority. When asked what about the summer could help people, she said, "lots of communication . . . a lot of feedback from teachers and from students to teachers." For Cadence, she saw having options as something different: "In this class it has variety, but my other class didn't." Matt seemed to build on both Yvette and Cadence by saying, "I think we learn more [in summer] cause like in [regular school] class, they force us to learn but here we have a choice of what we want to do. They give us a variety of games." These interviews marked unique reflections on distributed authority; more often, as will be discussed next, students recognized their authority through our emphasis on multiple solution pathways, making methods explicit, and stretching problems so that answers were recast as the start of new questions.

When asked what she learned from the summer class that might be useful in the future, Angelica said she "learned different ways to do a problem. Like what we did yesterday, we had different people come up and do their strategy . . . it was really cool to look at like different strategies like you could like argue it different ways and like list it different ways cause other people did it differently so we can learn." Luisa added that being reminded of multiple ways to solve a problem matters because if yours is different, "it doesn't mean that you're wrong or something." Angelica noted that she couldn't recall previous math classes in which multiple solution pathways was a priority, and thus it distinguished the summer for her. Cadence spoke similarly in her interview saying, "[this class] it's different because over here we do it multiple ways and we learn with patterns." Cindy saw the use of multiple methods as part of what made the summer interesting: "It's interesting cause when you actually do like pay attention and they explain all the strategies and everything, it's more interesting."

Beyond emphasizing multiple methods, organizing opportunities for agency also meant recasting "answers" as a way to reflect on methods of problem solving explicitly, and also as a gateway to new inquiries. When asked how the math problems in the summer might have been different than the regular year, Elizabeth replied enthusiastically, "Yea [they're] a lot different because we actually like think, dig deeper inside the problem and we're focusing on how we got the answer instead of what's the answer." This, her interview partner Olivia, would later argue meant they was learning more in the summer than in the regular school year. Alonso explained the same idea by focusing on the teacher's role: "The teachers ask a lot of questions so we can. . .I don't know. We can say an answer and they ask, 'why did you pick that answer?"' Alonso said he liked this aspect of the class and agreed it pushed him to think more about the problems. Emily described the focus on initializing or extending problems, as the teachers did, in terms of stretching a task. She explained that what was different in the summer from the regular year was that in the summer, "we stretch the problem a lot more and before, when, if, we got the answer, we didn't really stretch it." Emily went on to say that if she had the chance, she would advise teachers to "just stretch the problem . . . [because] I think it helps. It helps you understand the problem more and make predictions . . . We don't stop once we find the answer. We just keep finding more strategies to make it easier." Antoinette went even further in her interview, explaining that stretching the problem also meant initializing new ones. Describing this practice as something new and unique, she explained: "like when you look at a problem – and most problems have the problem and then the question – in normal classes you just try to solve to answer the question. But here you just sort of think deeply and ask questions like 'why is that?' . . . [The summer teachers] like tell me to encourage students to ask questions and create a new problem out of the original problem . . . I think this class [gives] me an advantage by making me think beyond what is given."

With five weeks of coordinated efforts to organize for agency, it is perhaps unsurprising that student interviews began to reflect the values of the class. By and large, these values were seen as different than regular instruction. When asked what it means to be a good student in the summer class, Alonzo and Paul agreed: Ask questions. For Maya and Samara, it was about trying. Samara said, "Try the problem, not give up. If you get the wrong answer that's okay, just keep working at it." Both students saw this as different because, as Samara explained, in a regular class "they want you to get the right answer. If you don't get the right answer, too bad, get an F." Similarly, when asked if he had any last comments about the summer intervention, Austin added, "It seems like – yeah, [this class] seems easier cause in our real classes, when we'd try hard, we'd fail. But here, if you try hard, you succeed." When the interviewer asked Walker and Victor to compare what doing well in meant in the summer and in the school year, Victor said it meant asking questions and knowing things without writing. Walker said, "You can't compare nothing. [The regular year is] all about A's and B's. If you don't have A's and B's, then you're not doing good in math." It was clear across interviews that not having homework, and not evaluating students with A–F grades was a significant departure and that may very well be influencing these students' remarks. It seems, however, that the attention to asking questions, of not giving up, of working past so-called "wrong" answers speak to the aims of organizing for agency. Though perhaps I would not go so far as to say *nothing* can be compared to a normal class – in fact, much of what we did could be leveraged into the classroom just as simply as we could have evaluated students with grades – it is clear that for students like Walker, the experience seemed so far removed from the norm that it could not be compared to it.

3.3. Peer collaboration and accountability: Dancing for an audience

As previously discussed, by asking students to put their thinking in conversation with procedures of the discipline (i.e., the dance of agency) we emphasized dancing for a very specific audience: their peers. Peer collaboration was therefore a critical feature of the teaching and proved to serve students in terms of agency, in a variety of ways.

For some students, collaborating with peers was seen as staving off boredom and as better aligned with their expectations of learning as adolescent (social) beings. For example, Alonzo said: "In other classes . . . it used to be so boring doing my work. It used to be so quiet and everything and I used to get frustrated and stuff. [But] right here, we get to do group work, and we get to talk and stuff, and that helps me not be so boring." Sabrina similarly saw personal reward in collaboration, particularly as she saw herself as a "people person" who was stifled in her regular math class. Sabrina explained, "In my class, silence was like a whole hour of silence. That was a good class to her. Whether or not you finished the work, as long as you were quiet . . . I'm a people person [so] . . . last year math was the hardest. You're not supposed to talk you're not supposed to communicate . . . [In the summer class] they let you talk, they let you socialize and stuff. And like every, I don't know, fifteen, twenty minutes, she'll be like 'Okay, talk amongst yourself,' 'help each other out."' When asked how she would describe the summer class, therefore, Sabrina replied simply: "Social." Victor, when asked about the collaborative approach of the summer, explained it in a way that echoes research on adolescents' need for peer association (e.g., Erdley, Nangle, Newman, & Carpenter, 2001): "[I enjoy that] there's a lot of grouping. Teachers say you're supposed to associate in school, but you barely get any time to associate in school. Like we have lunch and stuff, but that's not enough time so we have in-class." Taken together, these comments suggest collaboration was seen as an expression of being social in school – to stave off boredom, to freely associate with peers, or live as a "people person." Though this may pave the way for more active involvement by students, these reflections stop short of describing how the collaborative approach may have furthered learning, as the next set of responses do.

Several students identified collaboration as a means of gaining access to one another's ways of thinking. This matters for agency as students are asked to engage in a back and forth between their thinking and that of the discipline; making that process visible to your peers is what we see as the importance in students recognizing collaboration as a means of accessing each other's thoughts. As Monique explained, "I like to work in pairs because I like listening to people's methods and, like, how they figured it out." When asked what they might recommend to past teachers, Luke and Carlos emphasized a more peer-oriented, discussion-oriented approach. As Luke summarized why he and Carlos agreed talk and discussion mattered he said, "Just, instead of the teacher just telling you how to do it or whatever . . . instead of just thinking of how to do [it], you can talk to people. And sometimes they say don't, don't give them the answers, and then you could like tell them, you give them the answers, but then explain how it works. And they they'll see it. 'Cause sometimes if [the teacher] just explains it, and you don't know the answer, then you can't, you don't really see it." It is important that Luke and Carlos were not suggesting collaboration is a means for simply giving or getting answers – it is a means for explaining "how it works" because sometimes the teacher's explanation falls short (see Selling, this issue, for a closer analysis of Luke and Carlos' collaborative mathematical work). The exchange of ideas, therefore, is important not only for seeing how others figure something out, or for offering an alternative to the teacher's rendering, but also because it helps students to see where their thinking is underdeveloped. As Sabrina explained, "[I like groups] so that I know that I'm doing wrong and my way isn't the only way. If my way can be easily comprehended or is a little too hard so I can think of something different." I contend that it is through collaborative discourse that students see the dance of agency. As Sabrina's comment suggests, she could see where there were errors, where her solution lacked clarity, or where it was inefficient – all of which speak of being accountable to the discipline through one's discourse with others.

4. Conclusion

When asked what it would be like if regular school math classes were more like the summer, Walker remarked, "That would be the bomb. I would be in the A classes." At the time, his phrasing drew laughter from his interview partner and the interviewer. Now, it smarts of a certain truth that marks lost opportunity and perhaps, lost potential. If mathematics learning environments were purposely organized for agency, then the opportunities to express one's agency – in formulating and pursuing a mathematical problem – would not be reserved for "the A classes." Said another way, students who languish in the silence and isolation of traditional mathematics classes might in fact be among the most exuberant of mathematical learners. Thus the greater question at play is how to create and sustain learning environments in which all students see the value of contributing their voice within their classes as part of a larger conversation we call learning mathematics.

By deliberately choosing visually intriguing tasks of high cognitive demand, creating opportunities for students to problematize and pursue challenging mathematical work with authority, consistently inviting students into a dance of agency with the discipline, and by reminding them that they are one another's audience, we created a summer teaching intervention that drew forth many of the debilitating contrasts from what *is* to what *could be* learning in schools. As Sabrina described it, "It's like the way – the way our schools did it is like very black and white, and the way people do it here, it's like very colorful, very bright." Or, as Victor explained, "It's more easier to learn [in this class]; it has more rhythm and stuff." For students to perceive the color and rhythm of mathematics, as Sabrina and Victor do, learning environments must be deliberately designed as multidimensional (Boaler, 2015): environments rich with opportunities for students to identify, pursue, and accomplish tasks that reward plurality in reasoning and problem solving. As students' perceptions of agency in the summer intervention demonstrate, organizing for agency in mathematical learning takes disrupting the narrowness of math learning in schools by adopting challenging and open tasks, distributing authority over problem solving, and emphasizing collectivity in learning. When, as Dewey advocated, we create learning opportunities that are meaningful, directed, and purposeful, what results are the colorful and rhythmic expressions of human agency, agency of the discipline, and the dance that necessarily and powerfully draws them together.

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