Title
The Role of Information in Financial Markets, Security Design, and Theory of the Firm

Author
Li, Jiasun

Publication Date
2016

UNIVERSITY OF CALIFORNIA
Los Angeles

The Role of Information in Financial Markets, Security Design, and Theory of the Firm

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Management

by

Jiasun Li

2016

Abstract of the Dissertation

The Role of Information in Financial Markets, Security Design, and Theory of the Firm

by

Jiasun Li

Doctor of Philosophy in Management
University of California, Los Angeles, 2016
Professor Mark S Grinblatt, Chair

My dissertation studies the role information plays in various financial and economic settings. My first chapter investigates how stock price corresponds to public information in after-hours trading. Almost all U.S. firms now announce earnings outside of regular trading hours. This paper studies how stock prices incorporate information in after-hours trading. I find slow prices adjustment accompanied by significant trading volume. During 2002-2012, 5,881 rule-based trading opportunities generate an average return of 1.53% within four hours. After costs (assessed by a trading experiment), an investor who properly exploits the slow adjustment beats the market by 11.5% per year. Analyzing the use of intermarket sweep order (ISO), a new order type launched under Regulation National Market System (Reg NMS), I link the slow price adjustment to investor trading behavior.

My second chapter investigates how firms endogenously arise out of a market economy in the presence of dispersed private information. It is well known that in a rational expectation equilibrium market price could empower individuals with their collective wisdom by coordinating actions guided by dispersed private information. I show that *simple* profit-sharing contracts replicates such coordination effect. Whenever rational expectation is imperfect due to noise or the lack of a liquid market, individuals always have incentives to create a partnership among themselves. Optimal profit-sharing within such a partnership replicates the effect of direct communication but without actually incurring communication. This insight sheds new light on the nature of the firm: firms endogenously emerge as an institutional innovation to complete the market under dispersed private information. I discuss several implications of this new perspective on general corporate finance topics.

My third chapter studies optimal crowdfunding security design to harness investors' collective wisdom. The notion that a population's collective wisdom dominates even the most accurate judgment from any individuals within the group, or wisdom of the crowd, has been widely cited in recent years to advocate for security-based equity crowdfunding. In the context of project financing from a large pool of prospective investors, I show that *simple* profit-sharing contracts that are similar to but slightly different from typical common stocks could empower individuals with their collective wisdom by coordinating actions guided by dispersed private information. Although such profit-sharing contracts are yet to gain popularity for capital investment, they have been underlying the compensation structures of many human-capital intensive partnership firms. The results provide guidance on security design for the emerging crowdfunding practice.

The dissertation of Jiasun Li is approved.

Daniel Dumitru Andrei

Antonio E. Bernardo

Pierre-Olivier Weill

Ivo I. Welch

Mark S Grinblatt, Committee Chair

University of California, Los Angeles
2016

To mom and dad

Table of Contents

LIST OF FIGURES

List of Tables

ACKNOWLEDGMENTS

I am most grateful to my chair, Mark Grinblatt, whose encouragement, guidance, and support not only shaped my dissertation, but also fundamentally changed the way I look at research and the world. His influence on me will be felt for long. I cherishthe so many hours we spent together discussing a wide range of topics in economics and finance, and I will strive to be as helpful to my future students as Mark has been to me. I would also like to thank my committee members Daniel Andrei, Antonio Bernardo, Pierre-Olivier Weill, and Ivo Welch, for extremely helpful comments and suggestions along the way, without which the dissertation will look very differently. In particular, my first chapter will not be possible without the encouragement and advice from Ivo.

I am very lucky to have received a lot of mentoring and help from the faculty at UCLA. I am particularly grateful for Bruce Carlin, Andrea Eisfeldt, Barney Hartman-Glaser, Mark Garmaise, and Avanidhar (Subra) Subrahmanyam. Special thanks to Francis Longstaff for introducing me to UCLA and the amazing world of financial economics. He opened the door for me to the beginning of everything. I would also like to thank all my colleagues in the Ph.D office for five years of enjoyable intellectually stimulation. It was a memorable experience.

Last but not least, thanks to Lydia Heyman, Craig Jessen, Delores Rhaburn, Ani Adzhemyan, and Christopher Jados, who have made my life at UCLA tremendously easier.

VITA

2011	B.S. (Mathematics), Fudan University.

CHAPTER 1

Slow Price Adjustment to Public News in After-Hours Trading

The timing of news arrival is an important consideration in analyzing how security prices incorporate market information.[1] Over the past decade a growing number of firms have released corporate information outside of regular trading hours, with almost all firms in the U.S. stock market in 2012 releasing quarterly earnings reports before the market opens or after the market closes. This trend challenges the classic view that after-hours trading lacks volume and interests few market participants.[2] However, evidence on after-hours trading is scarce, particularly since Regulation National Market System (Reg NMS) went into effect in 2005. In this paper I examine price, quote, and investor behavior immediately following corporate earnings releases after the market closes, and find several surprising results.

I first find that in after-hours trading price follows a systematic slow adjustment. During the 2002 to 2012 period, immediate response to after-hours earnings reports of S&P 1500 firms, which offer 5,881 intraday trading opportunities, is associated with an average per-trade return of 1.53%, or an annualized Sharpe ratio

---

[1] How security prices incorporate new information has been a central question in finance. In a model featuring a monopolistic informed investor à la Kyle (1985), price follows a martingale with gradual information revelation. When information is shared by multiple investors à la Holden and Subrahmanyam (1992), price adjustment accelerates, and reveals new information immediately in the continuous time limit. When multiple investors receive different private information à la Foster and Viswanathan (1996), however, price adjustment is again gradual as investors forecast the forecasts of others.

[2] Barclay and Hendershott (2003) find that “although it is now relatively easy to trade after hours, in reality most investors do not”.

of 4.71, within four hours. Assuming a one-minute reaction latency, and further excluding transaction costs (supported by a real trading experiment), the average return is still as high as 0.66% (annualized Sharpe ratio equals 2.46). The return continues to remain significantly positive even beyond a 15-minute response latency. Such a price adjustment speed is in sharp contrast to what existing literature finds during regular trading hours. For example, Busse and Green (2002) find that "prices respond to (Morning Call and Midday Call) reports within seconds of initial mention, with positive reports fully incorporated within one minute."[3]

The slow price adjustment is accompanied by significant trading volume. Barclay and Hendershott (2004) document that "there is less than 1/20 as many trades per unit time after hours as during the trading day". However, in my more recent sample average per-minute trading volume following quarterly earnings releases immediately spikes to more than 1.6 times its level during regular hours.

Prospective investors can mimic existing market participants' performance by following a simple rule-based liquidity-providing strategy: 1) for each quarterly earnings announcement released during 4pm to 8pm, define a positive announcement surprise (PAS) to be one in which neither the top line (revenue) nor the bottom line (earnings per share, EPS) misses, and at least one of them beats, the corresponding consensus estimate, and define a negative announcement surprise (NAS) to be one in which neither the top line nor the bottom line beats, and at least one of them misses, the corresponding consensus estimate, 2) post a limit order at the mid-quote to buy (sell) the stock immediately *following* a PAS (NAS),[4] and 3) hold the position until the end of the trading day (8:00 pm). A marginal

---

[3] Ideally, one would also like to compare the price adjustment speeds to after-hours earnings announcements with those to regular-hours announcements. However, this is not possible as almost all firms in my sample have shifted announcement schedules outside of regular trading hours.

[4] As I discuss in Appendix A, after-hours trading mainly occurs on electronic communication networks (ECNs) that currently only accept limit orders. Most brokerage firms also adopt this policy.

investor following this strategy would beat the market by 11.5% each year, after controlling for unfilled limit orders, effective bid-ask spreads, transaction costs, market breadth, price impact, and execution latency. I hand-check press releases to assure that this result is not a spurious outcome of inaccurate timestamps. I also verify that the slow price adjustment is persistent across stocks and over time under various market frictions (investor inattention, limited arbitrage capital, and short-sale constraints).

To understand the causes of the slow price adjustment, I investigate the quote dynamics as well as investor behavior following after-hours earnings announcement. While actual trades take hours to adjust, quoted ask (bid) prices adjust almost immediately following a PAS (NAS). The impressive performance that actual market participants obtain disappears for an investor who naively buys (sells) at the ask (bid) after a PAS (NAS). This contrast, along with the significant trading volume, suggest insufficient liquidity provision in quotes on the same side of the price movement, rather than stale quotes on the opposite side.

To further tease out a confounding explanation based on noise trading, I investigate order clienteles. Among orders on the opposite side of the price movement, I find significant and increased use of intermarket sweep orders (ISO), a type of order introduced in 2007 (i.e., under Reg NMS) for institutional investors to expedite order execution and capture larger counterparty depth. Thus sophisticated institutional investors demand more urgent liquidity even if they trade against the price movement (which is usually less liquidity demanding). Since these institutional investors are unlikely to be noise traders, their liquidity hunger attests to insufficient liquidity provision on the same side of the price movement.

This paper makes several contributions to the literature. First, the results extend capital market research by providing new insights about the economics of after-hours trading under Reg NMS, as well as micro-level analysis of the classic liquidity-efficiency relationship. Using exogenous changes in minimum tick size, Chordia, Roll, and Subrahmanyam (2008) show that market efficiency deteriorates when market liquidity worsens. In the case of after-hours earnings, I show how slow price adjustment and quick quote adjustment coexist, and how the observed price adjustment process is linked to investor behavior.

Second, the results contribute to the literature on price discovery following public news. In earlier work, Patell and Wolfson (1984) study intraday price adjustment to earnings announcements for a limited sample (96 firms over two years) restricted to regular trading hours. Greene and Watts (1996) and Francis, Pagach, and Stephan (1992) compare price responses to daytime and overnight earnings announcements, although they do not consider after-hours trading, which was not widely accessible at the time of their studies. The market microstructures in the three studies above are very different from that considered in this paper.[5] More recently, Jiang, Likitapiwat, and McInish (2012) study after-hours earnings announcements, with a focus on the information content of prices over different time intervals. The current paper, however, investigates directions of price movements at the tick-level frequency, as well as the linkage between price behavior and investor behavior.

Third, the short-term slow adjustment (STSA) documented in this paper complements research on post-earnings announcement drift (PEAD).[6] There are three major differences though. In terms of time frame, PEAD focuses on horizons ranging from 60 days to 6 months, while STSA studies price dynamics within a four-hour interval.[7] In terms of magnitude, empirical evidence for PEAD shows

---

[5] In their studies, the minimum tick size was one-sixteenth (1/16) of one dollar rather than \$0.01; floor-based trades, rather than electronic trades, dominated the market; execution latency was longer (there was little high-frequency trading then, if any).

[6] Classic references on PEAD include Ball and Brown (1968) and Bernard and Thomas (1989). Different interpretations for PEAD include, Bhushan (1994) (transaction cost), Mendenhall (2004) (arbitrage risk), Sadka (2006) and Chordia, Goyal, Sadka, Sadka, and Shivakumar (2009) (liquidity or liquidity risk), Vega (2006) and Garfinkel and Sokobin (2006) (difference in opinion), Chordia and Shivakumar (2006) (relation with momentum effect), and Ke and Ramalingegowda (2005) (institution behavior).

[7] Most earnings announcement studies discard the exact timing of the releases and only use daily data. One exception is Battalio and Mendenhall (2011), but their focus in not on after-

up in the difference between extreme quintile/decile portfolio returns and ranges from 2.84% to 6.88% per quarter (Taylor (2011)), while STSA is also present in non-extreme portfolios and has a much higher return of 1.53% per four-hour interval. Finally in terms of the cross-section, while PEAD is more pronounced in small firms, STSA does not show a significant cross-sectional pattern.

For practitioners, the results provide guidance on a valuable trading opportunity: that quick response to after-hours public news and providing liquidity alongside the direction guided by the earnings report is highly profitable. In addition, the high volume following an after-hours earnings announcement, as well as the investor behavior during the same period might concern regulators. At least two reasons have been given for releasing public news after the market closes. Some regulators argue that by avoiding the coincidence of trading time and announcement time, the playing field is leveled for (fast and slow) market participants by allowing investors adequate time to digest new information.[8] The active after-hours trading market, however, raises questions about the validity of this argument. Alternatively, it has also been argued that companies may choose to announce earnings after-hours so that new information can be reflected in prices quickly and efficiently thanks to the higher proportion of informed traders in after-hours.[9] The insufficient liquidity provision and the resulting slow price adjustment, however, cast doubts on this argument.

The rest of the paper is organized as follows. Section 1.1 describes the data. Section 1.2 characterizes the slow price adjustment. Section 1.3 illustrates the economic significance of the slow price adjustment by calibrating the profitability for a prospective investor. Section 1.4 investigates investor quoting behavior as well as the pattern of ISO usage and sheds light on what is behind the slow

---

hours trading. Also, Dubinsky and Johannes (2006) use the exact announcement time for 20 firms to infer the uncertainty embedded in earnings announcements from option prices.

[8] Based on this argument, Michaely, Rubin, and Vedrashko (2011) associate the rising tendency of firms to announce outside trading hours with tightening corporate governance.

[9] Barclay and Hendershott (2008).

price adjustment. Section 1.5 studies cross-sectional variation in price adjustment speeds. Section 1.6 concludes.

1.1 Data

I obtain earnings announcement data from Thomson Reuters' I/B/E/S. For each quarterly earnings announcement, I/B/E/S includes the announcement date and timestamp, as well as the firm's earnings per share (EPS) and sales (revenue) numbers and their consensus estimates.[10] The initial sample includes all S&P 1500 firms from January 2002 to December 2012 that announce quarterly earnings report after the closing bell. I manually check the timestamps given in I/B/E/S against press releases for the 10th decile subsample, for which results should be the strongest; 91.3% of the timestamps are identical. Appendix B calibrates the impact of the discrepancies and finds the results robust to time-stamp accuracy. As described in the introduction, I define a PAS to be an announcement in which neither the top line (revenue) nor the bottom line (EPS) misses the corresponding consensus estimate and at least one beats the consensus number. An NAS is defined similarly.

With few exceptions, previous studies of earnings announcements focus only on the bottom line (EPS). Jegadeesh and Livnat (2006) discuss the role of revenue surprises. Kishore, Brandt, Santa-Clara, and Venkatachalam (2008) use the announcement-period price reaction as a proxy for all information contained in an earnings release. My inclusion of both top-line and bottom-line measures reflects several considerations. First, EPS alone is incomplete. The bottom line is easy to manipulate by cutting or deferring discretionary expenditures like R&D or advertising expenses (Graham, Harvey, and Rajgopal (2005)),[11] and tiny posi-

---

[10] Both actual and estimated EPS take non-GAAP diluted numbers.

[11] Also see Baber, Fairfield, and Haggard (1991), Degeorge, Patel, and Zeckhauser (1999), and Teoh, Welch, and Wong (1998), among others.

tive surprise in earnings might not be interpreted by the market as a solid report (Bhojraj, Hribar, Picconi, and McInnis (2009)). Second, investors use different operating performance measures to assess different types of stocks as well as *ad hoc* measures for particular stocks.[12] However, to avoid cherry-picking for a rule-based strategy, I do not alter criteria over time or across firms. In addition, unlike EPS or revenue, many of these measures are not explicit in the earnings release context (so may not be traded upon directly). Third, although management-issued earnings and revenue guidance does matter, some firms do not provide any guidance, while others provide non-firm guidance.[13] Even among those that do provide meaningful guidance, only some firms release the guidance at the same time as earnings. By combining bottom-line and top-line measures to proxy for the comprehensive earnings release, I tradeoff information preciseness and implementation ease (for investors).

I identify 7,776 PASs and 2,641 NASs over 2,769 trading days. The asymmetry in the number of PASs and NASs is in line with previous studies on analyst and firm behavior. Chan, Karceski, and Lakonishok (2007), among others, find that analysts systematically bias their estimates downward to help firms beat estimates in exchange for access to firm information. Gennotte and Trueman (1996) argue that firms strategically spread out "beats" and cluster "misses".

For each PAS and NAS, I collect tick-level data from Trade and Quote (TAQ). Following standard high-frequency data cleaning procedures (e.g., Huang and Stoll (1996) and Hasbrouck (2010).), I exclude zero or corrected trades, out-of-order trades, bunched solds, earlier obligations, and trades on prior-referenced prices. In light of the several limitations in the standard monthly TAQ dataset recently pointed out by Holden and Jacobsen (2013), I adjust for zero quotes (withdrawn

---

[12] Commonly used operating performance measures include same-store sales for retailers or margins for growth stocks. Examples of *ad hoc* measures include mobile monetization for Facebook or customer subscription growth for Netflix in recent quarters.

[13] For instance, prior to 2013 Apple Inc. constantly provided significantly lowered guidance.

quotes), smaller asks than bids (crossed or locked NBBO quotes), outlier spreads, and irregular market condition quotes. I further exclude events without a single trade within three minutes after the earnings release to remove possible trading halts.[14] The final sample comprises 4,599 PASs and 1,282 NASs. Table 1.1 provides summary statistics for the final sample.

After-hours trading operates from 4:00 pm to 8:00 pm. In addition, trading also occurs during the before-session, or pre-market, from 4:00 to 9:30 am. Most firms have now shifted quarterly earnings releases outside of regular trading hours, with about half of them announcing in after-hours and the other half in before-sessions.[15] My paper focuses only on after-hours for two reasons. First, most macroeconomic announcements are made during the before-session.[16] The contamination of firm-specific information by systematic news makes before-session price adjustments harder to interpret. In comparison, no U.S. macroeconomic announcements are made during after-hours. Second, unlike in after-hours, before-session trading is quite inactive notwithstanding earnings announcements. Although anecdotal evidence also suggests slow price adjustment during before-sessions, I leave it to future research.

---

[14] The three-minute rule is used to identify trading halts. Because actual market participants are directly notified of trading halts, removing samples violating the three-minute rule does not introduce a look-ahead bias. Anyway, including these samples only helps finding the slow price adjustment.

[15] See Berkman and Truong (2009) on the rise of non-regular-hour announcements. Also Jiang, Likitapiwat, and McInish (2012) report that during 2004 to 2008, 49% of all S&P 500 firms made quarterly earnings announcement during before sessions and another 46% during after-hours.

[16] For example, Nonfarm Payrolls, Unemployment Rate, Initial and Continuing Claims, GDP, CPI, PPI, Personal Income and Spending, Durable Orders, and Retail Sales are usually announced at 8:30 am. ADP Employment Change is usually announced at 8:15 am. The Case-Shiller 20-city Index and FHFA Housing Price Index are usually announced at 9:00 am. The MBA Mortgage Index is usually announced at 7:00 am.

Table 1.1: **Sample Summary**

This table reports summary statistics for the 4,599 PASs and 1,282 NASs over 2,769 trading days. For each firm size×sector category, I report the number of observations, the mean, the median, and the standard deviation of the firm sizes (in millions). Large firms are S&P 500 firms; medium firms are S&P 400 firms; and small firms are S&P 600 firms.

		Large				Median				Small			
	Sector	N	Mean	Median	Sd.Dv	N	Mean	Median	Sd.Dv	N	Mean	Median	Sd.Dv
PAS	Energy	55	16,471	14,644	10,930	8	4,762	4,141	1,924	14	1,324	1,326	653
	Materials	27	8,599	5,810	7,117	40	2,858	2,480	1,648	35	1,238	937	890
	Industrials	108	10,006	7,779	6,972	122	2,875	2,512	1,483	146	852	721	525
	Consumer Discretion	262	17,699	12,323	17,035	159	2,885	2,557	1,820	304	891	721	591
	Consumer Staples	1	95,353	95,353		10	4,949	3,866	3,866	62	915	770	512
	Health Care	154	22,939	14,162	21,997	183	3,186	2,969	1,872	264	1,029	764	736
	Financials	141	21,140	13,090	28,954	50	2,190	1,831	1,390	101	1,107	899	733
	Information Tech	950	37,435	10,423	62,185	587	2,645	2,181	1,787	765	894	720	657
	Telecom Services	1	25,924	25,924		3	2,872	283	283	8	352	299	250
	Utilities	10	6,962	6,607	3,020	13	4,265	4,299	1,706	12	1,120	1,003	417
	Total	1,710*	28,780	10,850	49,313	1,175	2,825	2,432	1,814	1,714*	933	745	657
NAS	Energy	17	15,334	12,662	10,806	2	6,435	6,435	509	3	1,467	1,294	325
	Materials	18	11,435	6,625	10,605	12	3,018	2,940	1,655	23	861	738	594
	Industrials	43	7,584	6,346	5,229	38	2,618	2,551	1,479	64	665	551	386
	Consumer Discretion	48	15,996	7,864	23,564	53	1,563	1,221	888	130	661	623	407
	Consumer Staples	0				3	4,801	2,621	5,137	41	897	678	545
	Health Care	26	27,471	13,328	24,618	47	2,403	2,273	1,253	74	771	619	498
	Financials	82	17,047	6,126	29,835	18	1,993	2,023	1,031	33	906	754	601
	Information Tech	174	31,660	9,230	68,792	106	2,116	1,618	1,670	196	652	533	471
	Telecom Services	0				2	3,146	3,146	787	4	448	409	231
	Utilities	7	9,550	7,641	4,892	7	3,104	2,968	1,495	10	1,007	891	429
	Total	416†	22,220	7,862	48,590	288	2,246	1,859	1,567	578	718	606	479

*: One large and three small PAS samples have unclassified sectors. †: One large NAS sample has an unclassified sector.

1.2 The Slow Price Adjustment

1.2.1 An Illustration

Figure 1.2.1 illustrates the slow intraday price adjustment in after-hours trading following earnings announcements. The x-axis starts from the first second after the announcement and is indexed by the log of trading time. The y-axis plots the average net cumulative return of the buy PAS/sell NAS strategy. The drift expands the after-hours trading session. The cumulated return calculated at the next day's open shows that the drift does not reverse.

1.2.2 Return and Risk

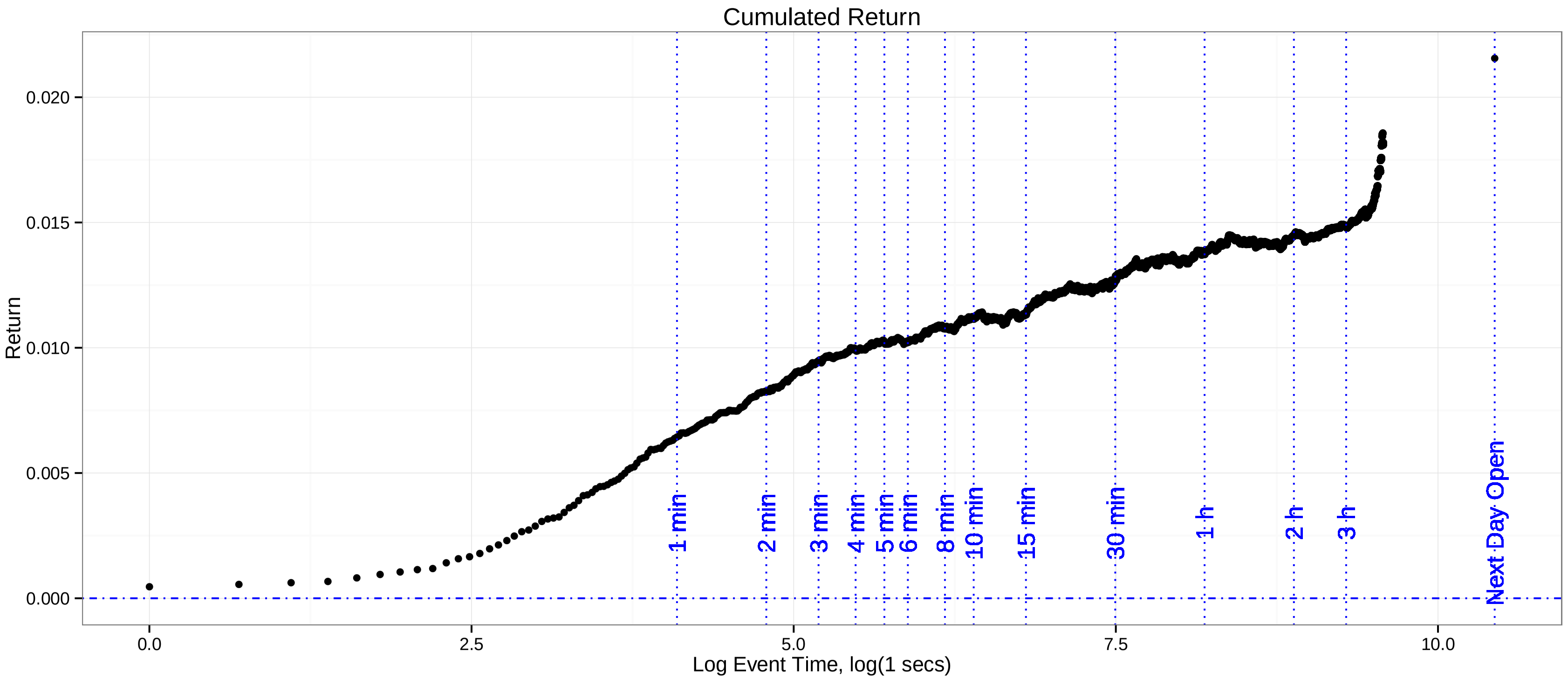
Table 1.2.2 reports the return-risk profile for market participants who exploit the slow price adjustment. In particular, the table reports the mean and standard deviation of the return per trade, the percentage of profitable trades (Pos%), and the annualized Sharpe ratio. The numbers are calculated from actual trades, based on alternative reaction latencies: immediate response and latencies of 1, 2, 3, 5, 10, or 15 minutes. The columns on the left report before-transaction-cost returns, while the columns on the right include costs.[17] An immediate response to the earnings announcement realizes on average a (not annualized) return of 1.53% within four hours. With a reaction latency of one minute, the after-cost return is still 0.66% (annualized Sharpe ratio is 2.46). Given 5,881 such trading opportunities from 2002 to 2012, 0.66%×5,881 over the 11-year period implies a larger aggregate return than many documented capital market anomalies. Indeed, even with a 15-minute response latency, market participants continue to earn significantly positive returns, which is inconsistent with the view that slow investors cannot earn significantly positive returns in a market overwhelmed by

---

[17] Costs include commissions, exchange make/take fees, clearing fees, regulatory fees, and transaction fees.

Figure 1.1: **Slow Price Adjustment**

This figure shows that the intraday price adjustment in after-hours trading is slow. The x-axis starts from the first second after each earnings announcement and is indexed by the log of trading time. The y-axis depicts the average net cumulative return of the buy PAS/sell NAS strategy calculated from trade prices. The drift expands the after-hours trading session. The cumulated return calculated at next day's open shows that the drift does not reverse.



sophisticated high-frequency traders.

1.2.3 Trading Volumes

Figure 1.2.3 compares average per-minute trading volume after an announcement with average volume during regular trading hours. Average per-minute trading volume after an announcement ranges from 24,000 shares in the first minute to 8,000 shares in the 20th minute. Seven minutes after an announcement, volume remains higher than the average same-day per-minute volume in regular hours, and after 20 minutes it remains higher than half the average per-minute volume in regular hours.

1.2.4 Intertemporal Performance

Figure 1.2.4 plots the number of PASs and NASs, the average return, and the total return of the buy PAS/sell NAS strategy for each quarter from January 2002 to December 2012. Over the last nine years of the sample period, the strategy consistently recorded positive total returns, regardless of market conditions.

1.3 Profitability of the Buy PAS/Sell NAS Strategy

To illustrate the economic significance of the slow price adjustment, this section considers a hypothetical investor who follows the buy PAS/sell NAS strategy from January 2002 to December 2012. Such an exercise requires several design choices. The first choice involves the price impact. Individual stocks' demand/supply curves are seldomly inelastic in the short run.[18] Ideally one would like to know how much could be traded before the return documented in Section 1.2 vanishes. However, because intraday trading is associated with endogenous returns and

---

[18] For theoretical models on how investors factor in their individual price impact, see Kyle (1989).

Table 1.2: **Return-Risk Profile and Statistical Significance**

This table reports the return-risk profile of market participants who exploit the slow price adjustment illustrated in Figure 1.2.1. In particular, the table reports the mean and standard deviation of the return for each trade (over less than four hours, not annualized), the percentage of profitable trades (Pos%), and the annualized Sharpe ratio. The numbers are calculated from actual trades, presented for alternative reaction latencies: immediate response and latencies of 1, 2, 3, 5, 10, or 15 minutes. The columns on the left report the before-transaction-cost returns, while the columns on the right include costs. An immediate response to an earnings announcement is associated with a (not annualized) return of 1.53% within less than four hours. With a reaction latency of one minute, after-cost return is still 0.66% (annualized Sharpe ratio is 2.46).

Returns Before Cost **Returns After Cost**

Buy PAS/Sell NAS (N=5,881)

Latency	Mean	St.Dv	Pos%	Shrp	Mean	St.Dv	Pos%	Shrp
0 min	**1.53%**	**5.15%**	**63%**	**4.71**	1.46%	5.11%	61%	4.52
1 min	0.72%	4.27%	61%	2.68	**0.66%**	**4.24%**	**52%**	**2.46**
2 mins	0.56%	4.06%	60%	2.21	0.50%	4.03%	51%	1.98
3 mins	0.47%	3.86%	60%	1.94	0.41%	3.85%	50%	1.70
5 mins	0.43%	3.72%	60%	1.85	0.37%	3.69%	49%	1.60
10 mins	0.35%	3.50%	60%	1.59	0.29%	3.49%	48%	1.33
15 mins	0.26%	3.28%	61%	1.26	0.21%	3.29%	47%	1.01

Buy PAS (N=4,599)

Latency	Mean	St.Dv	Pos%	Shrp	Mean	St.Dv	Pos%	Shrp
0 min	**1.11%**	**4.98%**	**60%**	**3.54**	1.04%	4.97%	59%	3.33
1 min	0.52%	4.09%	59%	2.00	**0.46%**	**4.10%**	**51%**	**1.78**
2 mins	0.40%	3.90%	58%	1.64	0.35%	3.91%	50%	1.41
3 mins	0.32%	3.72%	58%	1.35	0.26%	3.72%	48%	1.12
5 mins	0.31%	3.56%	59%	1.38	0.25%	3.55%	48%	1.13
10 mins	0.26%	3.30%	59%	1.23	0.20%	3.30%	47%	0.97
15 mins	0.20%	3.12%	60%	1.03	0.15%	3.13%	47%	0.77

Sell NAS (N=1,282)

Latency	Mean	St.Dv	Pos%	Shrp	Mean	St.Dv	Pos%	Shrp
0 min	**3.02%**	**5.45%**	**72%**	**8.80**	2.94%	5.34%	70%	8.75
1 min	1.46%	4.78%	67%	4.87	**1.37%**	**4.65%**	**55%**	**4.69**
2 mins	1.16%	4.54%	65%	4.05	1.07%	4.40%	54%	3.84
3 mins	1.05%	4.30%	65%	3.86	0.96%	4.23%	53%	3.62
5 mins	0.89%	4.22%	64%	3.35	0.81%	4.14%	51%	3.09
10 mins	0.70%	4.14%	64%	2.68	0.63%	4.10%	50%	2.43
15 mins	0.48%	3.80%	65%	1.99	0.42%	3.81%	48%	1.76

Figure 1.2: **Significant After-Hours Trading Volume around Events**

This figure compares the average minute-by-minute trading volume after an announcement with the average per-minute volume during regular trading hours. Following an earnings release, after-hours per-minute volume immediately spikes to more than 1.6 times that during regular hours. Though volume gradually declines over time, seven minutes after an announcement average per-minute volume remains higher than that during regular hours, and 20 minutes after an announcement it remains higher than half that during regular hours. After-hours trading is fairly active on announcement days.

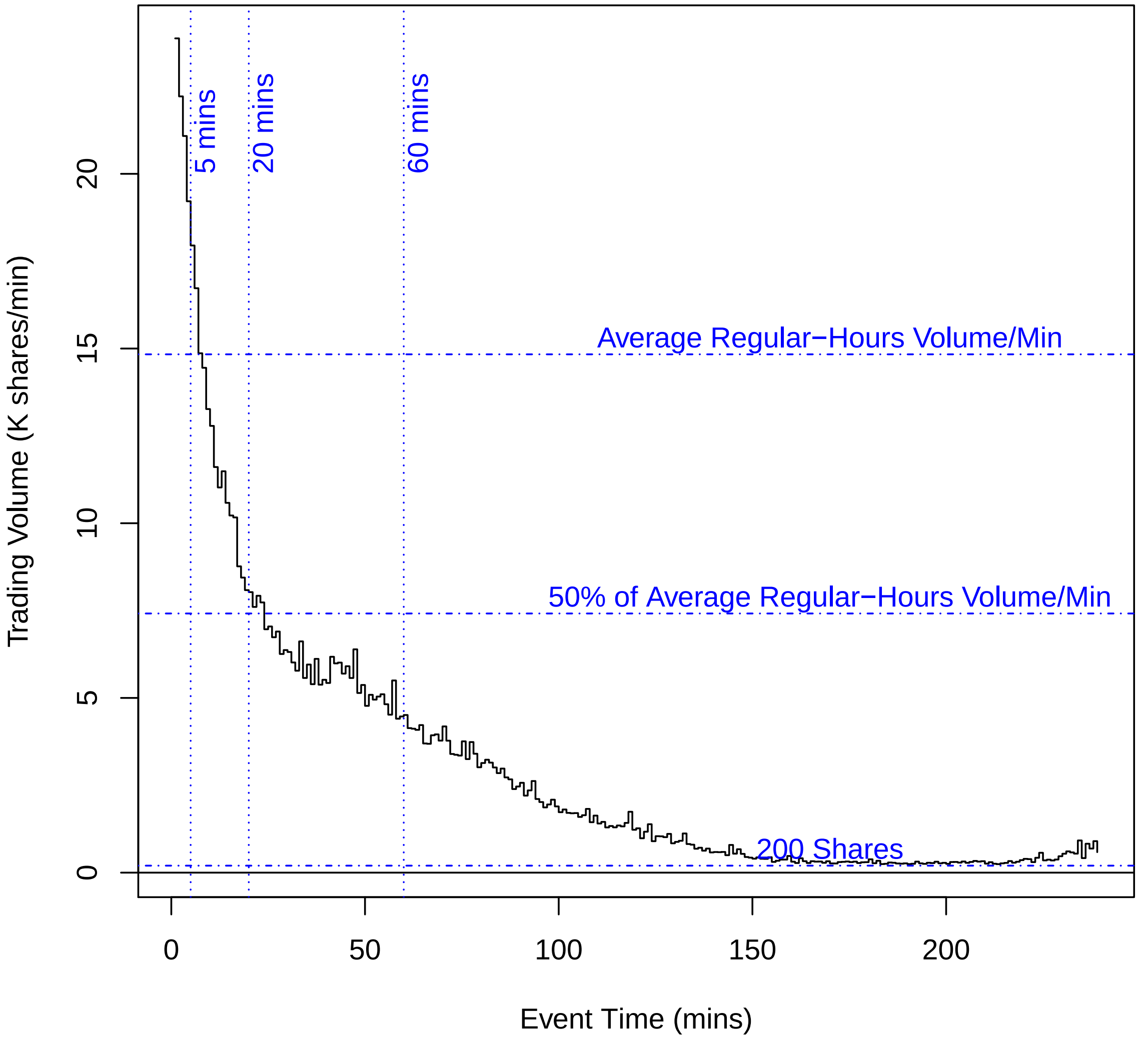
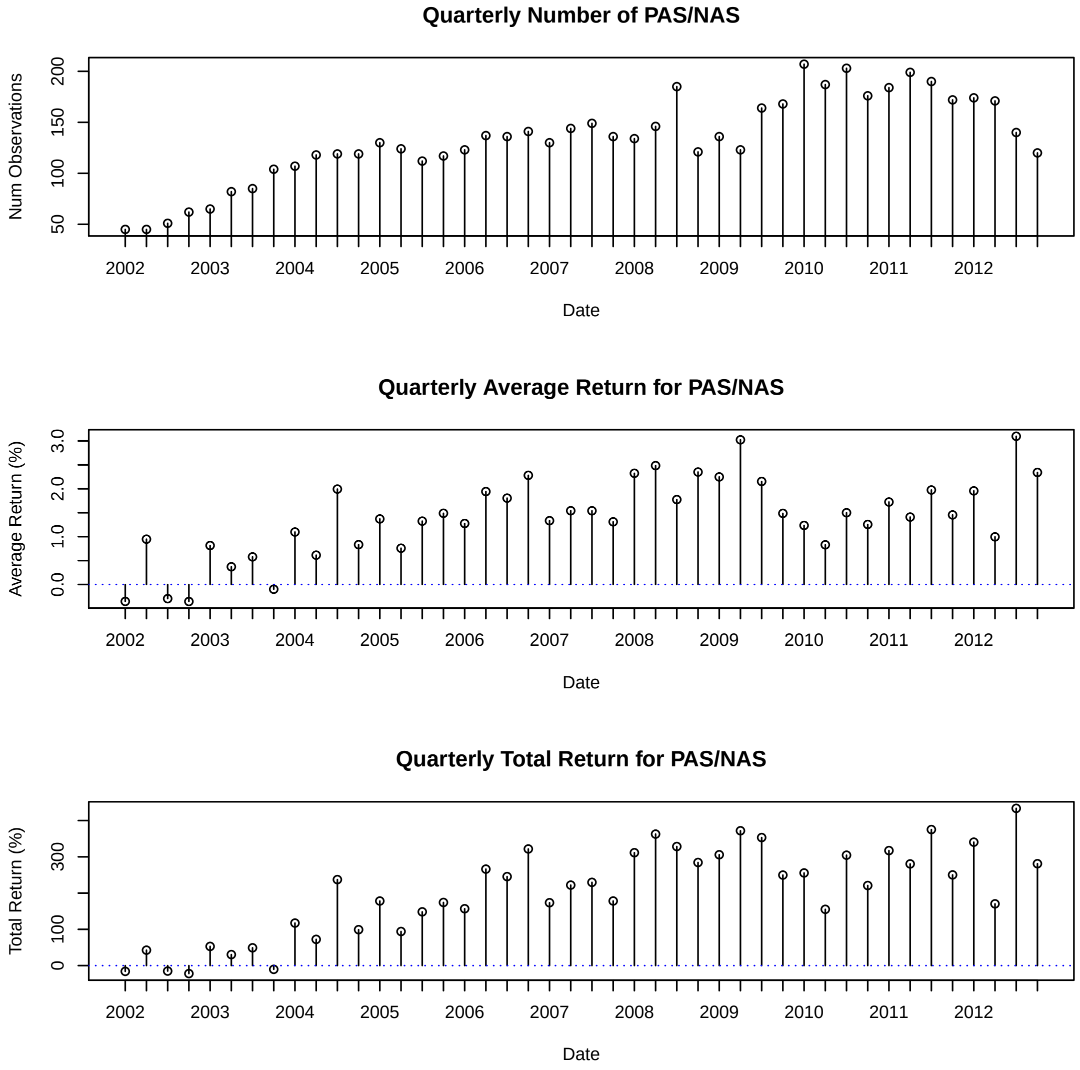


Figure 1.3: **Temporal Persistence**

This figure plots the number of PASs and NASs, the average return, and the total return of the buy PAS/sell NAS strategy for each quarter from January 2002 to December 2012. Over the sample period, more and more firms announce earnings in after-hours, and the performance of the buy PAS/sell NAS strategy has not declined. Over the last nine years the strategy consistently recorded positive total returns, regardless of market conditions.



investor feedback, a precise answer to this question would be model-specific. I therefore take a more modest approach and investigate profits that accrue to a small investor at the margin. I assume that the hypothetical investor begins with \$30,000.[19]

The second choice relates to a reasonable estimate of "explicit" transaction costs. Rules for calculating transaction costs (including commissions, make/take fees, exchange fees, transaction fees, clearing fees, etc.) in today's markets are lengthy and complicated.[20] Instead of engaging in a laborious parsing of the various rules, I conduct a real trading experiment in the U.S. stock market to evaluate transaction costs. The experiment is done during the peak month of the 2013 second-quarter earnings season (July 18 to August 17).[21] The realized size-weighted average transaction cost is \$0.0047 per share for 57 trades. Table 1.3 presents regression results showing that per-share transaction costs decrease with order size. I thus take an upper bound of \$0.0050/share as my imputed transaction cost.

I assume that the hypothetical investor is subject to a one-minute reaction latency. Sophisticated investors can react more quickly. To approximate the costs of actual market participants, the hypothetical investor posts limit orders at the

---

[19] I choose \$30,000 to provide a buffer above \$25,000, the SEC-stipulated minimum level for unlimited day-trading; see SEC Pattern Day Trader Rule, `http://www.sec.gov/answers/patterndaytrader.htm`.

[20] For instance, Interactive Brokers Cost Plus pricing "includes low broker commission, which decreases depending on volume, plus exchange, regulatory, and clearing fees". In cases in which an exchange provides a rebate, the brokerage firm will "pass some or all of the savings directly back". Under the Cost Plus rate schedule, the beginning commission is \$0.0035 per share. The commission decreases with monthly trading volume. The exchange fee is \$0.0030 or minus \$0.0021 per share, depending on whether liquidity is being removed or added (e.g. ARCA rebates \$0.0022 for adding Amex stock liquidity, while NASDAQ rebates \$0.0025 for retail orders removing the midpoint). The clearing fee is 0.0002 per share, and transaction fees include a 0.174 basis point plus NYSE Pass-Through Fees (1.75 basis points of commissions), FINRA Pass-Through Fees (5.6 basis points of commissions), and a FINRA Trading Activity Fee (1.19 basis points of quantity sold).

[21] I focus on the second-quarter earnings season because it was during summer vacation, when I had time to commit to trading. I began with own wealth of \$32,500.

Table 1.3: **Transaction Costs and Order Size**

This table reports results of regressing per-share transaction costs (including commissions, exchange make/take fees, clearing fees, regulatory fees and transaction fees) on order size. A 100-share increase in order size leads to a 2¢-drop in per-share trading costs. Thus, transaction costs decrease as wealth grows.

	Estimate	Std. Error	t-value
(Intercept)	0.0259	0.0021	12.06^{***}
Size	–0.0002	0.0000	-7.83^{***}
Size^2	0.0000	0.0000	5.35^{***}

***: significant at the 0.001 level.

prevailing mid-quote one minute after the announcement.[22] Her transaction price is assumed to be the first actual trade price within her price limit, so effective bid-ask spreads are taken into account. Among the 5,881 trading opportunities, 926 (322) out of 4,599 (1,282) PAS (NAS) observations are associated with limit orders that never got hit over the entire after-hours session. These observations are dropped from the profit calculation.[23] Given that the average per-minute trading volume one minute after the announcement is 24,000 shares, I assume that the hypothetical investor's limit order size is 200 shares.[24]

To address concerns about limits on a single investor's multi-task capability, I assume that on each trading day the hypothetical investor trades only on the first after-hours PAS or NAS. When there are simultaneous PASs or NASs, the investor trades the stock with the most analyst coverage.[25] Hence, the hypothetical investor has at most one open position at any given time. Sophisticated investors can trade more stocks simultaneously to seize more opportunities.[26]

---

[22] Note that by rounding the mid-quote is equal to the ask when the spread is one cent.

[23] Dropping unfilled orders goes against my return calculation, because for example a buy limit order is more likely to be left unfilled when price indeed drifts up and be picked up when price unexpectedly drifts down. Weighing more on losing trades in my backtest.

[24] I use 200 shares as traders consider 1% of volume to be tradable without price impact. I thank Ekkehart Boehmer for this suggestion.

[25] This is just an arbitrary tie-breaking rule.

[26] Because same-day announcements are often made at different times (e.g, one right after the closing bell and one at 16:30), the hypothetical investor can double-play multiple same-day announcements by trading on the first announcement surprise and then opening the second position/closing the first when the second announcement surprise is released.

I further assume that the hypothetical investor holds all wealth in cash on days with no trades. Sophisticated investors can make better use of their capital.

Figure 1.3 compares the day-to-day wealth change of the hypothetical investor (dashed curve) with that of S&P Composite Index (solid curve). The buy PAS/sell NAS strategy converts the initial \$30,000 into \$132,042 by the end of 2012. In comparison, buying and holding the benchmark during the same period only leads to \$37,267.[27] The buy PAS/sell NAS strategy beats the market by 11.5%. Table 1.3 presents performance results under alternative assumptions on the order size.

Table 1.4: **Sensitivity Analysis of Performance to Order Size**

This table reports 2012 year-end wealth, holding-period returns from 2002 to 2012, and the corresponding annualized returns under alternative assumptions on the limit order size. Figure 1.3 depicts results based on the second line (i.e., order size is 200 shares).

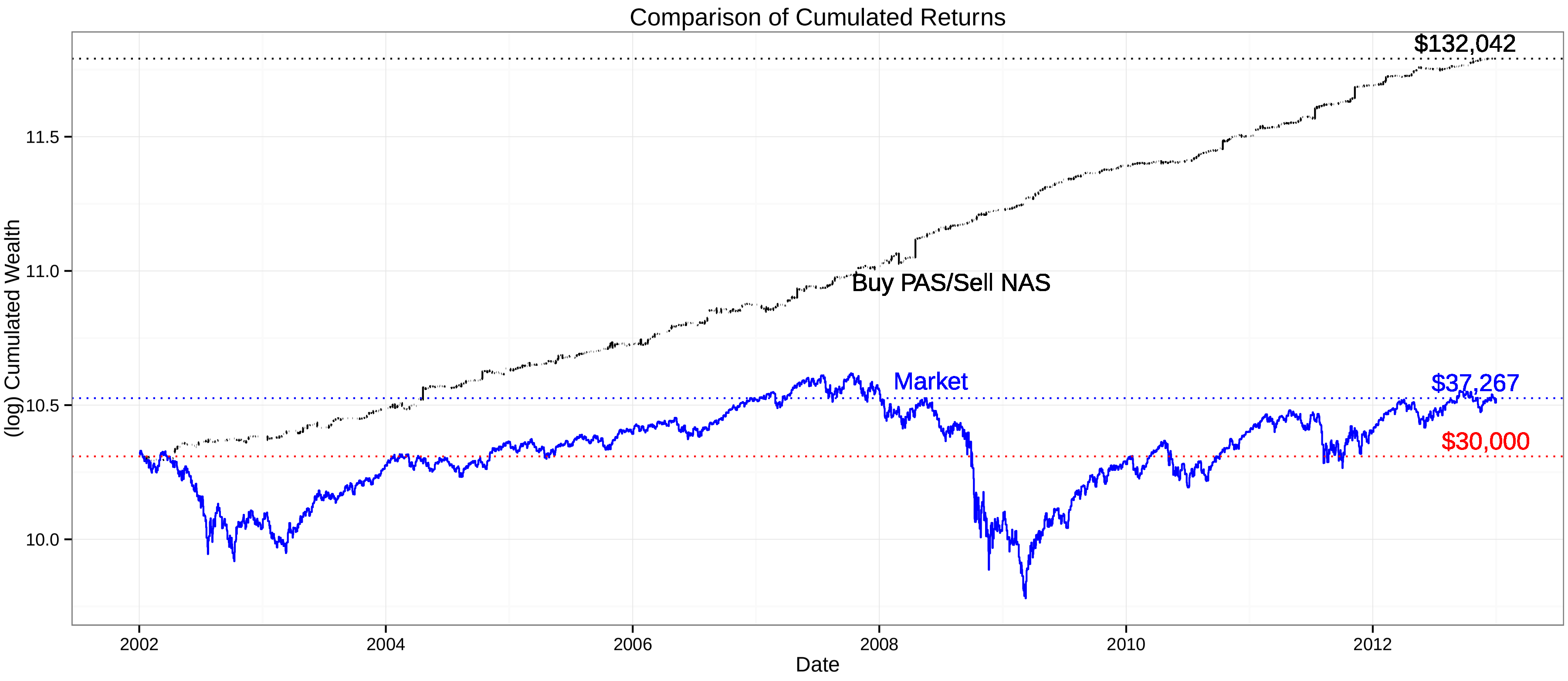
OrderSize	EndingWealth	HoldingPeriodRet	AnnualizedRet
100	\$92,440	208%	11%
200	**\$132,042**	**340%**	**14%**
300	\$167,806	459%	17%
400	\$200,565	569%	19%
500	\$229,741	666%	20%
600	\$256,773	756%	22%
700	\$282,294	841%	23%
800	\$306,247	921%	24%
900	\$329,549	998%	24%
1,000	\$352,069	1,074%	25%
2,000	\$523,185	1,644%	30%
3,000	\$633,807	2,013%	32%
4,000	\$724,183	2,314%	34%
5,000	\$801,586	2,572%	35%
6,000	\$856,028	2,753%	36%
7,000	\$892,777	2,876%	36%
8,000	\$921,817	2,973%	37%
9,000	\$946,231	3,054%	37%
10,000	\$965,251	3,118%	37%
20,000	\$1,089,712	3,532%	39%

---

[27] Similar figures are seen in, for example, Baker, Bradley, and Wurgler (2011). Although I further take into account the impact of trading costs and market frictions.

Figure 1.4: **Economic Significance**

This figure illustrates the economic significance of the slow price adjustment by comparing the day-to-day wealth increase between trading the buy PAS/sell NAS strategy (dashed curve) and buying-and-holding the benchmark (solid curve). A hypothetical investor is assumed to begin in January 2002 with \$30,000. Each trading day, she trades on the first after-hours PAS or NAS. If there are multiple, simultaneous PASs or NASs, the investor trades on the one with most analyst coverage. The investor closes her position before the end of the trading day (i.e, 8:00 pm). The investor is assumed to have a response latency of at least one minute. The investor caps each position size by 200 shares. Transaction costs are assumed to be \$0.0050 per share. The benchmark is the S&P Composite Index Return. The figure shows that \$30,000 invested at the beginning of 2002 becomes \$132,042 by the end of 2012, while buying-and-holding the benchmark over the same period only leads to \$37,267.



1.4 Investor Behavior and the Slow Price Adjustment

To better understand what is behind the slow price adjustment, this section studies quote dynamics and investor behavior immediately following an after-hours earnings announcement. With evidence of asymmetric quotes adjustment speeds and increased liquidity demand in quotes on the opposite side of the price movement, I conjecture that that the slow price adjustment is, to a significant part, due to insufficient liquidity provision in quotes on the same side of the price movement.

1.4.1 Asymmetric Adjustment Speeds in Quotes

While trade prices take hours to adjust in after-hours trading, quotes demonstrate asymmetric adjust speeds. Define a with-trend order as a buy (sell) order after a PAS (NAS), and an against-trend order as a sell (buy) order after a PAS (NAS), then quotes on the against-trend side adjust immediately following an earnings announcement, but those on the with-trend side lags. For example, after a PAS, the quoted ask spikes immediately but quoted bid does not catch up. In Table 1.4.1 I replicate Table 1.2.2, but instead of calculating returns using the actual trade price, I assume naively buying at the ask after a PAS or selling at the bid after an NAS. The table shows that no significantly positive returns can be earned. This evidence goes against a plausible explanation based on the classic story of active traders "picking off" stale quotes (limit orders posted by traders who fail to monitor and update their limit orders following news), but lends support to an explanation based on insufficient with-trend side liquidity provision.[28]

Insufficient with-trend side liquidity provision is consistent with theoretical predictions. Asymmetric quotes adjustment widens bid-ask spreads. If liquidity provision is perfectly competitive, according to Glosten and Milgrom (1985), a rational market with bid-ask spreads driven by adverse selection should see large

---

[28] See Linnainmaa (2010) for a detailed discussion of the "stale quotes" story in Finland data.

Table 1.5: **Return-Risk Profile and Statistical Significance: Quoted Spreads**

This table replicates Table 1.2.2, but with all returns calculated assuming buying at the ask or selling at the bid instead of using the actual trade prices. In contrast to the slow adjustment in trade prices, quotes on the opposite side of the price movement adjust almost immediately. No significantly positive returns can be earned by naively buying at the ask after a PAS or selling at the bid after an NAS.

Returns Before Cost Returns After Cost

Buy PAS/Sell NAS (N=5,881)

Latency	Mean	Std.	Pos%	Shrp	Mean	Std.	Pos%	Shrp
1^{st} Quote	0.33%	5.24%	49.41%	1.01	0.27%	5.21%	45.42%	0.81
1 min	-0.68%	4.70%	41.17%	-2.29	-0.75%	4.66%	37.95%	-2.55
2 mins	-0.89%	4.45%	40.04%	-3.19	-0.96%	4.41%	36.44%	-3.45
3 mins	-0.95%	4.31%	38.95%	-3.49	-1.01%	4.27%	35.23%	-3.76
5 mins	-1.03%	4.08%	38.61%	-4.01	-1.09%	4.04%	34.63%	-4.29
10 mins	-1.10%	3.81%	37.01%	-4.59	-1.15%	3.79%	32.97%	-4.83
15 mins	-1.12%	3.63%	36.47%	-4.90	-1.18%	3.63%	32.39%	-5.13

Buy PAS (N=4,599)

Latency	Mean	Std.	Pos%	Shrp	Mean	Std.	Pos%	Shrp
1^{st} Quote	-0.06%	5.04%	47.03%	-0.18	-0.12%	5.04%	43.14%	-0.39
1 min	-0.87%	4.42%	39.97%	-3.11	-0.93%	4.41%	36.86%	-3.35
2 mins	-1.01%	4.16%	39.12%	-3.85	-1.07%	4.16%	35.56%	-4.08
3 mins	-1.03%	4.02%	38.12%	-4.06	-1.09%	4.02%	34.43%	-4.29
5 mins	-1.08%	3.80%	37.98%	-4.50	-1.13%	3.80%	34.12%	-4.72
10 mins	-1.10%	3.52%	36.58%	-4.95	-1.15%	3.52%	32.64%	-5.17
15 mins	-1.10%	3.39%	36.41%	-5.13	-1.15%	3.39%	32.41%	-5.37

Sell NAS (N=1,282)

Latency	Mean	Std.	Pos%	Shrp	Mean	Std.	Pos%	Shrp
1^{st} Quote	1.74%	5.69%	57.96%	4.84	1.67%	5.59%	53.62%	4.74
1 min	0.01%	5.55%	45.58%	0.03	-0.07%	5.42%	41.98%	-0.21
2 mins	-0.47%	5.39%	43.45%	-1.39	-0.55%	5.25%	39.71%	-1.67
3 mins	-0.65%	5.24%	42.05%	-1.96	-0.73%	5.10%	38.25%	-2.26
5 mins	-0.86%	5.00%	40.98%	-2.73	-0.94%	4.84%	36.59%	-3.10
10 mins	-1.11%	4.74%	38.64%	-3.72	-1.18%	4.68%	34.26%	-3.99
15 mins	-1.22%	4.44%	36.68%	-4.37	-1.28%	4.45%	32.31%	-4.58

spreads accompany low trading volume, which could directly account for any slow price adjustment. However, as Section 1.2.3 has shown, immediately following an after-hours earnings announcement, the trading volume remains high compared to that during regular hours. The observation of both large spread and high volume suggests a deviation from the idealized setup in reality, with insufficient liquidity provision on the with-trend side of the quotes being a likely possibility.

To support the insufficient with-trend side liquidity provision hypothesis, I document increased liquidity demand on the against-trend side of the quotes. This exercise requires separating authentic liquidity demands from the confounding effect of "noise" trading. For example, a sell-initiated trade after a PAS could be due to irrational trading, suboptimal ask quoting, or unsophisticated selling at the bid, etc., rather than urgent liquidity demand to sell. To address this possibility, the next section will investigate the use of intermarket sweep orders (ISOs), a new type of order designed under Reg NMS catering to the liquidity demands from sophisticated institutional investors, who are unlikely to be noise traders.

1.4.2 ISOs: Background

To integrate the fragmented U.S. stock markets, in 2005 FINRA introduced Reg NMS. An important component of Reg NMS is Rule 611, or the extension of trade-through protection. A trade-through is formally defined as an order that is not executed at the best available quoted price, on either the same or another trading venue (exchange). Protections against trade-throughs were first passed in the 1970s for NYSE stocks. Rule 611 of Regulation NMS extends trade-through protection to all other trading venues. It requires that a trading venue re-route orders it receives to another trading venue if a better price is quoted there.

However, the initial proposal to completely enforce trade-through protection was questioned by institutional investors. They argued that complete enforcement of trade-through protection along with the significant decrease in top-of-the-book quote size following the change to decimalization in 2001 would lead to ultra-frequent re-routing, severely impeding institutions' execution speed. The SEC thus added ISOs in Reg NMS as an exception to the trade-through protection rule to facilitate institutional investors' need for execution immediacy (SEC (2005)). By design, ISOs allow institutions to expedite order executions and capture larger counterparty depth.

An ISO is effectively a limit order with both the right of trade-through and the obligation of simultaneous liquidity sweeping. It is usually on the active side of the trade it participates. When a trader sends an ISO to an exchange, for example ARCA, she assumes the responsibility of simultaneously taking all top-of-the-book quotes within the specified limit price on all trading venues. Once the ISO takes away all top-of-the-book quotes from all trading venues, the rest of the order can be filled at ARCA and does not have to protect better quotes (if any) on other exchanges. For example, assume that there are only two trading venues in the U.S. stock market, ARCA and INET. On top of ARCA's book are offers at \$49.99×1000 and \$50.00×2000. A trader sends an ISO to ARCA to buy 3,000 shares of a certain stock with a limit price of \$50.01. However, the national best offer is \$49.97×1000 on INET, and the second best offer on INET's book is \$49.98×2000. Thus to fulfill her ISO responsibility, the trader has to first simultaneously buy 1,000 shares at \$49.97 from INET and 1,000 shares at \$49.99 from ARCA. Because the order size is larger than the total top-of-the-book breadth (2,000 shares in this example), the trader can fill the rest of her order (1,000 shares) at \$50.00 on ARCA, even though the \$49.98 ask on INET is superior to that on ARCA and in this case is clearly traded through.[29]

Since its introduction on February 8, 2007, ISOs have been extensively used

---

[29] Chakravarty, Jain, Upson, and Wood (2012) provide a more detailed description of the ISO mechanism.

by sophisticated institutional investors to expedite trading speed.[30] Note that ISOs are exclusive to institutional investors. Even if individual investors can access ISOs, it is not in their best interest to do so. First, an ISO sacrifices execution price in exchange for trading speed for large orders, which individuals do not value. Second, by design an ISO has to take all top-of-the-book quotes, and individual investors are not likely to be willing or able to do so. Survey results from retail brokerage firms confirm that they do not provide ISO to their clients. For example, "Fidelity does not have...Intermarket Sweep Orders as they are usually used for large block institutional trades"; "Merrill Edge does not currently offer Intermarket Sweep Orders (ISOs) for our self-directed brokerage accounts"; "Intermarket sweep orders are not available at Scottrade"; "...no(t) available during extended hours" at TD Ameritrade.

Readers should be cautioned, however, that although ISOs are exclusively institutional orders, there is no claim that all institutional orders are ISOs.[31] It is natural to expect that most non-ISO trading are also from institutional traders, and institutions with access to ISOs also choose strategically between ISOs and other order types. With this qualification in mind, I interpret ISO as an identification of liquidity demand from (a subset of) sophisticated institutional investors.

---

[30] An ISO is labeled "F" in the "trade condition" column in TAQ. Throughout this section I focus on the subsample after February. 8, 2007.

[31] For exact identification of institutional and individual investors, see Barber, Odean, and Zhu (2006) and Lee (1992), etc., that use order size (applicable prior to decimalization); see Hasbrouck (1992) and Lee and Radhakrishna (2000), etc. that use TORQ; see Barber and Odean (2008), Chakravarty, Panchapagesan, and Wood (2005), Coval, Hirshleifer, and Shumway (2005), Kumar and Lee (2006), Odean (1998), Odean (1999), and Barber and Odean (2000), etc., that use proprietary data sets; see Barber, Lee, Liu, and Odean (2009), Grinblatt and Keloharju (2000), and Linnainmaa (2010), etc. for foreign markets.

1.4.3 Increased Liquidity Demand in Against-Trend Orders

To calculate the percentage of ISO use in against-trend orders, I apply the Lee and Ready (1991) algorithm to classify each trade as either buy- or sell- initiated.[32] A trade is defined as with-trend (against-trend) if its trade initiator is from the with-trend (against-trend) side. I first show that there are significant ISOs in against-trend trades, indicating institutional investors' liquidity demands rather than unsophisticated investors' noise trading. I then compare the dollar volume percentage of ISOs in against-trend trades following an after-hours announcement to that during non-announcement benchmark period (the same after-hours trading period but on the previous day).

Table 1.4.3 shows that on announcement days, institutional investors' liquidity demands increase significantly, even if they are trading against the direction of price adjustment, which is usually liquidity abundant.[33] The increased use of ISOs in against-trend trades following after-hours earnings announcement indicates insufficient liquidity provision on the with-trend side.

1.5 Price Adjustment Speed: Cross-Sectional Variation

In this section I investigate how the speed of price adjustment varies across stock-specific attributes. In a multivariate regression, I find that investor inattention and limited arbitrage capital cannot account for the slow price adjustment. Potential short-sale constraints matter after an NAS, but not after a PAS. The magnitude of the announcement surprise contributes to the speed of price adjustment, in

---

[32] Following suggestions from recent empirical papers (e.g. Chakrabarty, Moulton, and Shkilko (2012)). I apply a one-second delay when matching trades and quotes. The results are robust to alternative delay assumptions (ranging from zero to two seconds).

[33] The increase in liquidity demand is both statistically and economically significant, and robust to possible mis-classifications. Using order level data, Chakrabarty, Li, Nguyen, and Ness (2007) find the Lee-Ready algorithm correctly identify 74.42% INET trades initiators at the trade level. Alternative algorithms, for example, Ellis, Michaely, and O'Hara (2000) render similar results.

Table 1.6: **ISOs in Against-Trend Trades on Announcement Day and Previous Day**

This table compares dollar volume percentage of ISOs in against-trend trades on the announcement day and the previous day. For both the PAS and the NAS samples, the first line reports the cross-sectional mean of the percentage of ISO dollar volume, showing increased ISO use in against-trend trades on the announcement day. The second line (in parentheses) reports the standard deviation of the percentage of ISO dollar volume in the cross-section (not the standard deviation of the mean value). The third column reports the difference in the mean ISO share and the Welch two-sample t-statistic (in parentheses) testing the null hypothesis that there is no significantly higher use of ISOs in against-trend trades on the announcement day. The null hypothesis is rejected in both the PAS and the NAS samples. The results show that on announcement days, institutional investors are more desperate in liquidity demand, even if they are trading against the direction of price adjustment. This finding indicates insufficient liquidity provision on the with-trend side.

ISO% on After		Announcement Day	Previous Day	Diff. & t-value
PAS	Mean	44.4%	7.3%	37.0%
	S.D.	(27.5%)	(18.5%)	**(55.0)**
NAS	Mean	38.6%	7.9%	30.8%
	S.D.	(29.0%)	(21.8%)	**(22.3)**

line with insufficient with-trend side liquidity provision discussed in the previous section.

1.5.1 Dependent Variables

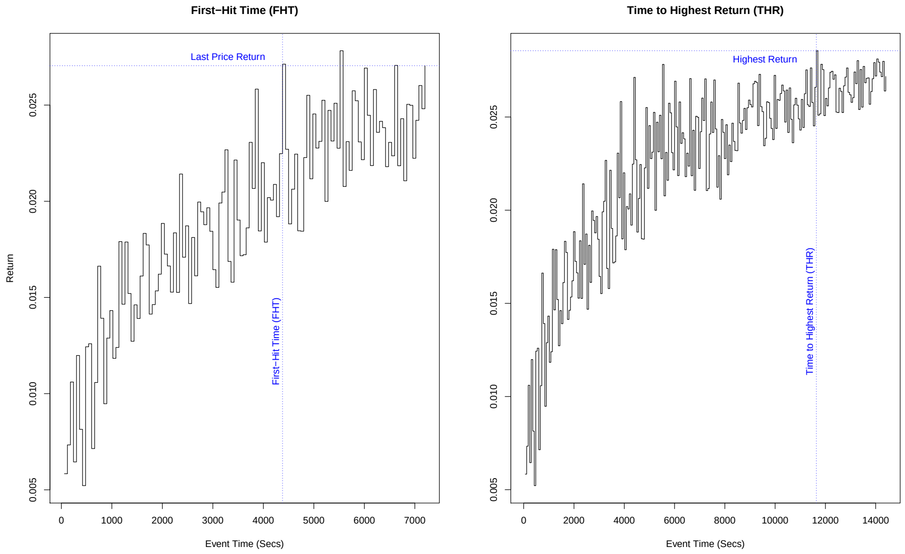
To quantify the speed of price adjustment, I define two measures:

1. First-Hit Time (FHT): the time elapsed before the stock price is greater (lower) than or equal to the last price within two hours after a PAS (NAS);
2. Time to Highest Return (THR): the time elapsed until the highest return over the given holding period (i.e., between the announcement time and 8:00 pm).

Figure 1.5.1 illustrates the definitions of FHT and THR. The mean (standard deviation) of FHT is 1,039 (1,946) seconds for the PAS sample and 1,160 (1,969) seconds for the NAS sample. The mean (standard deviation) of THR is 1,880 (2,799) seconds for PASs and 1,972 (2,800) seconds for NASs.

Figure 1.5: **Measures of Price Adjustment Speed: FHT and THR**

This figure illustrates the definitions of FHT and THR for a hypothetical stock on a typical announcement day, using its after-hours return dynamics. First-Hit Time (FHT) is defined as the time elapsed before the stock price is greater (lower) than or equal to the last price within two hours after a PAS (NAS). Time to Highest Return (THR) is defined as the time elapsed until the highest return over the given holding period (i.e., between the announcement time and 8:00 pm).



1.5.2 Independent Variables

The first set of independent variables proxy for investor attention: the number of analysts following the firm and the number of same-day announcements. Several studies analyze how investor inattention to public news affects the price discovery process. Hong and Stein (1999) build a model in which investors with limited information-processing capability lead to a short-term positive correlation in stock returns. Using data based on 78,000 households from a large discount broker, Hirshleifer, Myers, Myers, , and Teoh (2008) argue that individual investors were inattentive to earnings during the 1991 to 1996 period. Hirshleifer, Lim, and Teoh (2009) provide further evidence of investors being distracted by simultaneous information, and Dellavigna and Pollet (2009) argue that inattention leads to reduced reactions to earnings announcements on Fridays.[34] If inattention is the major cause of the slow price adjustment in the cross-section, the adjustment time should be longer for stocks with less attention.

The second set of independent variables concerns limits on arbitrage capital. I investigate whether prices adjust more slowly for surprises concurrent with multiple other surprises by including same-day earlier/simultaneous surprises as independent variables in the regression. I also include the short-to-float ratio to capture short-sales constraints, another form of a limit on arbitrage capital. Conceptually, it is harder to initiate new short positions for heavily shorted stocks (stocks with a high short-to-float ratio) in the case of an NAS.[35] I test whether prices adjust more slowly among heavily shorted stocks.[36]

The third set of independent variables proxy for the announcement surprise magnitude. Specifically, I examine 1) standard unexpected earnings (SUE), 2) absolute unexpected earnings (AUE), and 3) the interaction between absolute surprise value and price-to-earnings ratio. In line with non-ISO users' contrarian behavior slowing price adjustment (Section 1.4), I expect stocks with a larger earnings surprise to see slower price adjustments.[37] Finally, I control for common

---

[34] Michaely, Rubin, and Vedrashko (2012) argue that the "Friday effect" is driven by firm fixed effects. See also Doyle and Magilke (2009). In my sample, the "Friday effect" does not seem to account for the slow price adjustment; there are only 3 PASs and 10 NASs on Fridays in my sample of 5,881 observations, and the mean return of these 13 Friday surprises is 0.87%, not extreme positive outliers.

[35] Note that returns from shorting NASs are not due to the 2008 short-sale ban. According to Battalio and Schultz (2011), the SEC began to restrict naked short-selling for all stocks on September 17, 2008, and imposed a short-sale ban on approximately 800 "financial" stocks from September 19, 2008 to October 8, 2008. Three out of my 1,282 NAS observations fall into the September 17 - October 8, 2008 window. When I exclude these three observations and recalculate the average returns, the results are unchanged.

[36] The *level* of the short-to-float ratio does not explain the slow price adjustment. I compare the cross-sectional distribution of the short-to-float ratio in the PAS and NAS samples with that for all S&P 1500 stocks during 2002 to 2012 period. Two-sample Kolmogorov-Smirnov test yields $D^+ = 6e-04$ with p-value $= 0.9964$, which does not reject the null hypothesis that the CDF of the short-to-float ratio in the PAS/NAS sample does not lie above that for all S&P 1500 firms, that is, my PAS/NAS sample is not significantly tilted toward heavily shorted stocks.

[37] In a model accommodating information asymmetry derived by Kim and Verrecchia (1991),

firm-specific attributes: log firm size, historical volatility, and *ex ante* analyst belief dispersion (measured by the standard deviation of analyst estimates).

1.5.3 Regression Analysis

Based on the above discussion, I regress the two measures of price adjustment speed on the number of analysts following the firm, the number of same-day announcements, the magnitude of the announcement surprise (SUE, AUE, and P/E×AUE), the number of prior/simultaneous PAS/NASs, the short-to-float ratio of the stock, log firm size, historical volatility, and *ex ante* analyst belief dispersion. The regression controls for White heteroskedasticity and year fixed effects. Table 1.5.3 reports the regression results.

The main results from the regressions are as follows:

- Neither analyst coverage nor the number of same-day announcements significantly impacts adjustment speed. Cross-sectional differences in investor inattention also do not significantly impact the price adjustment speed.
- Same-day announcement surprises do not significantly impact the measured speed of price adjustment, and there is no evidence that limited arbitrage capital induce the slow price adjustment.
- For the NAS sample (but not for the PAS sample), potential short-sale constraints help explain the cross-sectional heterogeneity in price adjustment speed. Short-sale constraints are likely to drive the insufficient liquidity provision on the ask side after an NAS.
- For PAS the adjustment speed significantly increases with the magnitude of the announcement surprise. This is in line with insufficient liquidity provision on the with-trend side.

---

price change reflects the average change in investors' beliefs due to the arrival of an announcement.

Table 1.7: **Regression: Other Possible Explanations for the Slow Price Adjustment**

This table regresses measures of the speed of price adjustment on proxies for investor attention, limited arbitrage capital, short-sale constraints, and the magnitude of the announcement surprise. FHT is the time elapsed before the stock price is greater (lower) than or equal to the last price within two hours after a PAS (NAS). THR is the time elapsed until the highest return before 8:00 pm. All independent variables are normalized to have a mean of 0 and standard deviation of 1. Coefficient significance is based on White heteroskedasticity-consistent standard errors and controls for year fixed effects.

Dependent variable: First Hit Time (FHT)

PAS	Coef	t-val		Coef	t-val		Coef	t-val	
(Intercept)	1022.6	2.60	**	1004.5	2.56	*	1004.6	2.56	*
# Analysts Following	−34.9	−0.72		−39.7	−0.83		−39.7	−0.83	
# Same-Day Announcements	−20.4	−0.50		−22.0	−0.54		−22.1	−0.61	
Std. Unexpected Earnings (SUE)	161.8	3.22	**	191.8	4.33	***	191.8	4.34	***
Abs. Unexpected Earnings (AUE)	68.5	1.08							
P/E×AUE	−4.7	−0.11							
# Prior/Simultaneous PAS	−4.0	−0.09		−0.3	−0.01				
# Prior/Simultaneous NAS	44.5	1.09		45.1	1.10		45.1	1.14	
Short-to-Float Ratio	−43.4	−1.33		−40.8	−1.26		−40.8	−1.26	
log(Firm Size)	7.4	0.13		20.9	0.39		20.9	0.39	
30-Day Historical Volatility	5.8	0.13		13.4	0.32		13.4	0.32	
Analyst Estimates Std. Dev.	−20.0	−0.99		−19.9	−1.35		−19.9	−1.35	

NAS	Coef	t-val		Coef	t-val		Coef	t-val	
(Intercept)	1896.4	1.21		1894.9	1.21		1888.6	1.21	
# Analysts Following	85.7	0.85		88.3	0.88		88.5	0.88	
# Same-Day Announcements	70.4	0.74		71.9	0.75		75.7	0.94	
Std. Unexpected Earnings (SUE)	9.3	0.06		56.6	0.69		56.4	0.69	
Abs. Unexpected Earnings (AUE)	27.7	0.15							
P/E×AUE	57.6	0.33							
# Prior/Simultaneous PAS	8.0	0.09		7.5	0.09				
# Prior/Simultaneous NAS	81.0	0.97		81.0	0.97		83.0	1.04	
Short-to-Float Ratio	162.3	1.85	.	160.9	1.84	.	160.9	1.84	.
log(Firm Size)	−18.5	−0.17		−28.7	−0.28		−28.2	−0.27	
30-Day Historical Volatility	5.2	0.07		6.2	0.08		6.5	0.09	
Analyst Estimates Std. Dev.	−2.3	−0.02		−54.4	−0.88		−54.1	−0.88	

***: significant at the 0.001 level, **: significant at the 0.01 level, *: significant at the 0.05 level, ·: significant at the 0.10 level.

(Table 1.5.3 continued)

Dependent variable: Time of High Return (THR)

PAS	Coef	t-val		Coef	t-val		Coef	t-val	
(Intercept)	1668.5	2.28	*	1628.1	2.24	*	1630.1	2.24	*
# Analysts Following	75.6	1.05		65.0	0.92		64.7	0.92	
# Same-Day Announcements	38.9	0.67		134.6	0.60		30.5	0.59	
Std. Unexpected Earnings (SUE)	212.2	2.51	*	272.2	3.64	***	271.8	3.64	***
Abs. Unexpected Earnings (AUE)	164.2	1.31							
P/E×AUE	−44.0	−0.66							
# Prior/Simultaneous PAS	−16.2	−0.24		−8.0	−0.12				
# Prior/Simultaneous NAS	−25.5	−0.45		−24.5	−0.43		−26.2	−0.48	
Short-to-Float Ratio	13.7	0.26		18.1	0.35		18.3	0.35	
log(Firm Size)	39.2	0.47		68.5	0.86		68.3	0.85	
30-Day Historical Volatility	92.0	1.37		109.9	1.70	·	110.2	1.71	·
Analyst Estimates Std. Dev.	0.3	0.00		−12.6	−0.32		−12.6	−0.32	

NAS	Coef	t-val		Coef	t-val		Coef	t-val	
(Intercept)	1896.5	1.19		1910.6	1.20		1953.5	1.24	
# Analysts Following	217.0	1.56		223.7	1.62		222.5	1.61	
# Same-Day Announcements	78.8	0.59		83.0	0.63		56.9	0.51	
Std. Unexpected Earnings (SUE)	−4.4	−0.03		91.7	0.84		92.9	0.87	
Abs. Unexpected Earnings (AUE)	131.2	0.62							
P/E×AUE	69.3	0.58							
# Prior/Simultaneous PAS	−49.5	−0.44		−51.0	−0.46				
# Prior/Simultaneous NAS	34.9	0.31		32.1	0.28		18.5	0.17	
Short-to-Float Ratio	255.3	1.95	.	250.0	1.91	.	250.0	1.91	.
log(Firm Size)	62.3	0.40		37.4	0.25		33.7	0.22	
30-Day Historical Volatility	110.7	0.91		106.5	0.88		104.3	0.86	
Analyst Estimates Std. Dev.	41.3	0.25		−34.2	−0.41		−36.7	−0.44	

***: significant at the 0.001 level, *: significant at the 0.05 level, ·: significant at the 0.10 level.

The evidence that variation in investor attention and arbitrage capital limits does not fully account for the variation in adjustment speed is consistent with Santosh (2013), who compares the speed of price response to after-hours earnings announcements in terms of trade-time and clock time, and argues that "nearly all cross-sectional variation in the speed of convergence measured in clock time disappears when measured in trade-time".

1.6 Conclusion

My paper quantifies after-hours trading profitability and conducts a real trading experiment to impute transaction costs. I further provide evidence on the intensive use of ISOs in after-hours trading.[38] The economic significance of after-hours trading suggests an important part of the market that typical asset pricing studies neglect. The results also add to existing literature on price discovery in response to anticipated public news, and link price discovery process to investor behavior.

Future research could extend the paper in several directions. First, one could conduct a similar investigation for before-session trading (from 4:00 am to 9:30 am), which requires disentangling market-wide and firm-specific information and capturing the liquidity environment in before-sessions. Second, one could include corporate-issued guidance in the trading direction decisions, which requires recording the precise timestamps for guidance issuance. Third, one could further examine the relationship between ISO use and institution participation if order-level data become available. With order level data, one could also further explore hidden order dynamics and investigate the relationship between insufficient liquidity provision and a higher adverse selection risk hypothesis à la Kim and Verrecchia (1994).[39]

---

[38] Upson, Chakravarty, Jain, and McInish (2011) and Lebedeva (2012) both investigate ISOs around earnings announcement. Their studies, however, exclude after-hours trading.

[39] ITCH Database on Orders or TRTH (Reuters) Database on book snapshots are both good candidates.

From a broader perspective, as the development of more sophisticated trading algorithms further decreases the cost of market monitoring, as the increase in competition among alternative trading systems and primary exchanges incentivizes further expansion in trading hours, and as global consolidation among trading venues further extends trading beyond regular hours,[40] after-hours trading is likely to become even more important to the investment community.[41] Cliff, Cooper, and Gulen (2008) document that the U.S. equity premium over the last decade is due solely to overnight returns. How will the continued growth of after-hours trading change the next decade? And how will Reg NMS navigate us in this changing environment? My paper represents an early step toward addressing these questions.

Appendix A: After-hours Trading

Major stock exchanges in the U.S. operate from 9:30 am to 4:00 pm ("regular trading hours"). After-hours trading was historically infrequent and only available to high net-wealth investors or institutional investors.[42] Beginning in 1995 some regional stock exchanges began to offer trading access beyond regular trading hours (McInish, Ness, and Ness (2002)). The rise of ECNs popularized after-hours trading among professional ECN subscribers. An ECN is a computer system that facilitates stock trading outside of exchanges. It is essentially an open limit book without designated market makers (or specialists).[43] ECNs accommodated

---

[40] For example, NYSE and Euronext merged in June 2006, NASDAQ acquired OBX in May 2007, Deutsche Boerse and NYSE Euronext were forced to abort a merger in 2011, IntercontinentalExchanges acquired NYSE Euronext in December 2012, and BATS and Direct Edge merged in 2014.

[41] The trend goes beyond the stock market. For example, CBOE Futures Exchange plans to extend trading hours for CBOE Volatility Index (VIX) futures, beginning from October 28, 2014.

[42] See `http://www.sec.gov/investor/pubs/afterhours.htm`.

[43] An ECN is also known as an "alternative trading system" (ATS). The first ECN, Instinet, was created in 1969. Notable ECNs include ARCA, INET, and former BATS (Better Alternative Trading System), which was converted into a national stock exchange in November 2008.

individual investors' desire for "equal acess" in the summer of 1999. Since then anyone who is eligible to trade during regular hours can engage in after-hours trading via ECNs.

Currently ECNs dominate after-hours order flow on earnings announcement days. Table 1.6 reports the dollar volume distribution among trading venues in my sample. ARCA and INET dominate order flow.[44] The absence of designated market makers suggests that liquidity provision is not guaranteed.

Table 1.8: **Earnings-Day After-Hours Trading Distribution**

This table reports the dollar volume distribution across trading venues in my sample. ECNs like ARCA and INET dominate after-hours trading on earnings announcement days. Because ECNs are open limit books without designated market makers, liquidity provision is not guaranteed.

Trading Venue	Volume Share after		
	PAS&NAS	PAS	NAS
Amex	0.01%	0.01%	0.00%
ARCA	**37.75%**	**37.52%**	**38.94%**
BATS BYX	0.17%	0.12%	0.39%
BATS BZX	0.49%	0.42%	0.84%
Boston	0.48%	0.42%	0.74%
CBOE	0.01%	0.01%	0.02%
Chicago	0.15%	0.16%	0.10%
Direct Edge EDGA	0.26%	0.18%	0.66%
Direct Edge EDGX	1.64%	1.53%	2.16%
ISE	0.29%	0.34%	0.05%
INET	**44.72%**	**45.05%**	**43.03%**
NASD ADF and TRF	9.03%	9.23%	7.97%
NSX	4.33%	4.32%	4.38%
NYSE	0.47%	0.43%	0.66%
Philadelphia	0.03%	0.02%	0.04%
other	0.20%	0.23%	0.02%
#Observations	5,881	4,559	1,282

From an individual investor's perspective, after-hours trading is technically no

---

[44] ARCA was acquired by NYSE in 2006, but it operates and reports trades to the consolidated tape separately; INET includes former Island, an ECN acquired by Instinet in 2002 to form INET. Instinet itself was acquired by NASDAQ in 2005, and one year later INET became NASDAQ's primary trading platform (Hasbrouck and Saar (2009)). Despite of these market changes, by the end of my sample the legacy name "Island" is still quoted at some brokerage firms (e.g. Interactive Brokers).

different from regular-hours trading. Many brokerage firms provide after-hours trading access, including Charles Schwab, E*Trade, Fidelity, Interactive Brokers, Merrill Edge, TD Ameritrade, Scottrade, etc. Most SEC/FINRA regulations for regular-hours trading carry through in after-hours trading. For example, the Manning Rule is enforced in both sessions.[45]

Appendix B: Accuracy of Announcement Times

As a robustness check, in this appendix I investigate the accuracy of earnings announcement timestamps from I/B/E/S to address the concern that the results are due to inaccurate time-stamps.

Before doing so, I discuss the potential impact of inaccurate timestamps. First, because a delayed reaction to actual announcements works against finding returns, if my results are due to inaccuracy (if any) in the timestamps, these timestamps would have to be systematically earlier than the actual announcement time. I show that this is not the case.[46]

Second, symmetric time-stamp errors are more likely to bias the economic significance of my results downward. To see this in Figure 1.6 I illustrate the typical price reaction to an earnings announcement. Price is relatively stable prior to the announcement (time 0), reacts strongly to earnings after the announcement, and then drifts for an extended period. If the timestamp used in my samples is later than the actual announcement time (the dot-dashed line), then the return to the buy PAS/sell NAS strategy will be severely biased downward; in contrast, if the time-stamp is earlier than the actual announcement time (the dotted line), the return to the buy PAS/sell NAS strategy will not be significantly biased. Thus, if

---

[45] The Manning Rule, or FINRA regulation Rule 5320, prohibits a FINRA member firm from placing the firm's interest before/above the financial interests of a client.

[46] Bradley, Clarke, Lee, and Ornthanalai (2013) argue that time stamps reported in I/B/E/S for analyst recommendations released during trading hours are systematically delayed.

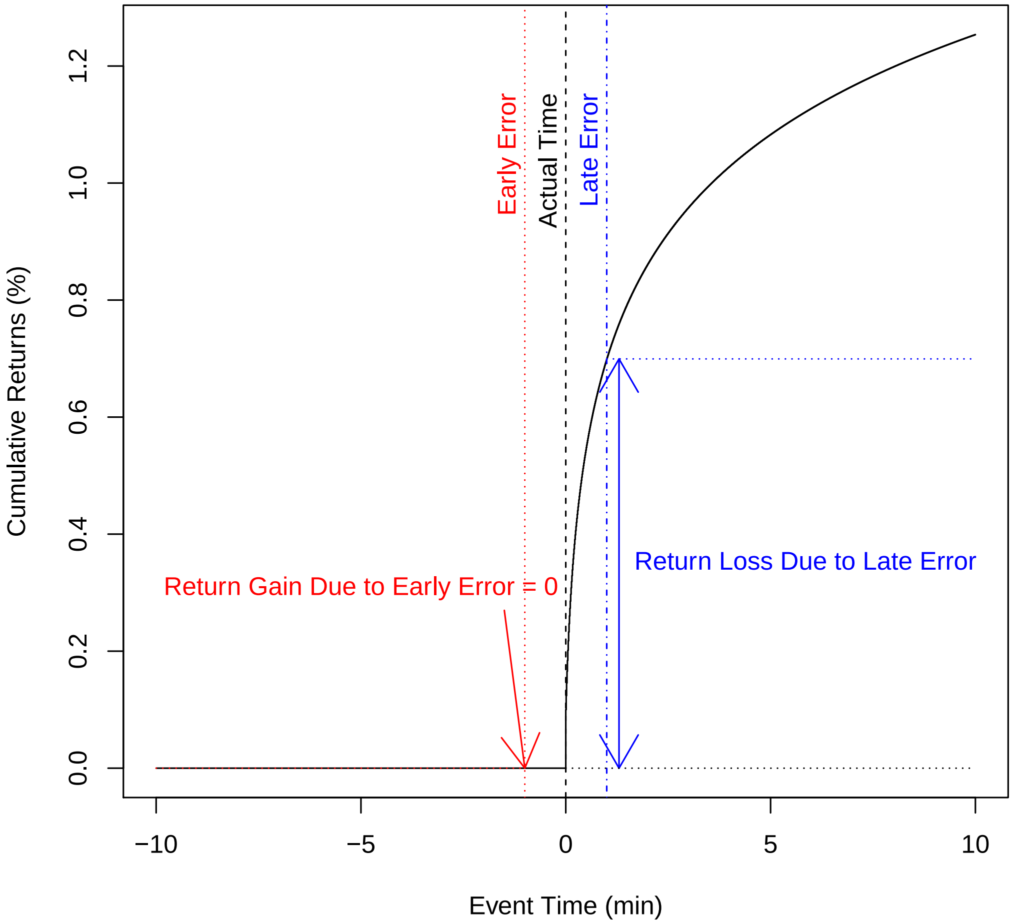
errors in the I/B/E/S timestamps (if any) are symmetrically distributed around the true announcement time, the actual trading profit should be even higher than I document above.

Several mechanisms could lead to errors (if any) in the I/B/E/S data set. One possibility is that errors could simply be due to careless data entry. In this case the error should be symmetrically distributed around the actual announcement time. Next, the clock at the newswire agency and the one at the I/B/E/S data center may not be perfectly synchronized, in which case the discrepancy between the I/B/E/S timestamp and the press-release timestamp should be within a couple of minutes. In addition, there may be latency in the data set's receipt of the earnings release. In this case the I/B/E/S timestamp should be later than the actual announcement time, biasing the documented result in my paper downward. Finally, either the news wire agency or the data center may occasionally fail to configure timezones correctly. This could lead to discrepancy between the I/B/E/S timestamp and the press-release timestamp in round hours.

Bearing the above in mind, I focus on the 588 announcements within the 10-th decile in after-earnings trading return (i.e, the decile that most favors finding results) and double check the accuracy of the I/B/E/S timestamps against press releases from Factiva. Because not all press releases are timestamped, of the 588 announcement timestamps, I am able to compare 460 with press releases. Of these 460 observations, 420 time-stamps are identical in both sources (I/B/E/S and Factiva); 19 discrepancies are within one minute, 24 are within two minutes, and two announcements have a discrepancy of exactly one hour, possibly due to time-zone configuration problems. I find that 70% of the discrepancy sample have the I/B/E/S sample being later, while only one observation of the entire sample (588) has an I/B/E/S timestamp that is more than 15 minutes earlier than that from Factiva. Because the return in my paper is significant beyond 15 minutes, the documented slow price adjustment is not due to systematic errors in the I/B/E/S

Figure 1.6: **Asymmetric Impact of Symmetric Timestamp Errors on Returns**

This plot illustrates why symmetric, random time-stamp errors would bias my results downward. A typical price reaction to an earnings announcement is plotted as the solid curve. Price is relatively stable prior to the announcement (time 0), reacts strongly to earnings after the announcement and continues to drift for an extended period. If the timestamp used in my samples is later than the actual announcement time (the dot-dashed line), the return would be severely biased downward; in contrast, if the time-stamp is earlier than the actual announcement time (the dotted line), the return would not be significantly biased.



data set.

In summary, most I/B/E/S timestamps agree with those from newswires. Of the few discrepancies most are within one minute, and almost all larger disagreements are within 15 minutes, and the errors tend to have the I/B/E/S timestamps being later. Thus the timestamps from I/B/E/S are of fairly high quality and the results are not mechanically generated by inaccurate timestamps.

Appendix C: Trade Log

Table 1.9 reports the trade log of my real trading experiment. I began with $32,500 and traded for one month during the 2013Q2 earnings season. The experiment is meant to verify my assumptions on transaction costs rather than the performance of the buy PAS/sell NAS strategy.[47]

Table 1.9: Trade Log of Real Market Experiment

Date	Ticker	Time	Size	Price	Cost	Profit	Cost/Shr
2013-07-18	CMG	16:55:34	100	394.00	0.51		0.0051
2013-07-18	CMG	16:58:05	–100	394.92	1.20	90.29	0.0120
2013-07-18	GOOG	16:03:55	–50	872.14	1.36		0.0272
2013-07-18	GOOG	16:09:09	50	868.23	0.86	193.28	0.0172
2013-07-18	GOOG[48]	16:09:59	–50	864.50	1.36		0.0272
2013-07-18	GOOG	16:14:53	50	862.41	0.86	102.28	0.0172
2013-07-18	GOOG	16:15:26	–50	862.50	1.61		0.0322
2013-07-18	GOOG	16:21:10	50	865.19	0.61	–136.72	0.0122

Continued on next page

---

[47] Due to time constraints, I do not hold positions until the end of after-hours session as in Section 1.2.

[48] The strategy prescribed in the paper does not include trading in and out several times. The multiple trades are market-timing attempts that I should have avoided, but kept here for the intactness of the trading log (same for for BIDU, FB, AMZN, and Z.) Results are robust to inclusion/exclusion of these trades. When I remove these multiple trades, the realized Total P.L is $4,392.34, and Size-weighted Cost/Shr is $0.0044.

Table 1.9 – continued from previous page

Date	Ticker	Time	Size	Price	Cost	Profit	Cost/Shr
2013-07-18	ISRG	16:37:58	–100	400.87	1.21		0.0121
2013-07-18	ISRG	16:39:45	100	400.02	0.51	83.28	0.0051
2013-07-22	NFLX	16:02:57	100	248.48	0.87		0.0087
2013-07-22	NFLX	16:05:06	–100	250.56	1.33	206.09	0.0133
2013-07-23	AAPL	16:30:50	100	436.02	0.51		0.0051
2013-07-23	AAPL	16:42:31	–100	437.76	1.27	172.22	0.0127
2013-07-23	BRCM	16:06:20	–200	30.52	0.43		0.0022
2013-07-23	BRCM	16:09:10	200	30.50	0.32	3.25	0.0016
2013-07-23	VMW	16:02:23	200	77.06	0.32		0.0016
2013-07-23	VMW	16:10:17	–200	78.99	0.60	385.08	0.0030
2013-07-24	BIDU	16:01:38	200	117.94	1.04		0.0052
2013-07-24	BIDU	16:02:01	–200	119.00	0.65	211.30	0.0033
2013-07-24	BIDU	16:02:11	200	120.00	1.34		0.0067
2013-07-24	BIDU	16:02:40	–200	121.03	0.74	203.99	0.0037
2013-07-24	FB	16:06:14[49]	200	29.22	0.32		0.0016
2013-07-24	FB	16:06:46	–200	29.34	1.14	21.54	0.0057
2013-07-24	FB	16:07:22	200	30.60	0.83		0.0042
2013-07-24	FB	16:07:55	–200	30.72	0.43	22.74	0.0022
2013-07-25	AMZN	16:02:52	–100	298.86	0.99		0.0099
2013-07-25	AMZN	16:03:29	100	296.07	0.51	277.50	0.0051
2013-07-25	AMZN	16:05:10	–100	294.98	1.02		0.0102
2013-07-25	AMZN	16:22:39	100	297.06	0.51	–209.53	0.0051
2013-07-29	HLF	16:10:13	200	64.49	1.34		0.0067
2013-07-29	HLF	16:10:59	–200	65.35	0.55	170.11	0.0028

Continued on next page

---

[49] Earnings were announced at 16:05 on PR Newswire, although I/B/E/S has a delayed timestamp at 16:11. As shown in Appendix B, such timestamp delays goes against finding the slow price adjustment.

Table 1.9 – continued from previous page

Date	Ticker	Time	Size	Price	Cost	Profit	Cost/Shr
2013-07-30	AMGN	16:03:16	200	112.60	0.32		0.0016
2013-07-30	AMGN	16:06:05	–200	112.88	0.71	54.97	0.0035
2013-07-30	SYMC	16:03:59	200	24.57	0.32		0.0016
2013-07-30	SYMC	16:07:02	–200	24.67	0.33	19.35	0.0016
2013-08-01	LNKD	16:06:06	200	199.69	1.34		0.0067
2013-08-01	LNKD	16:06:59	–200	204.00	1.03	859.63	0.0052
2013-08-05	BSFT	16:06:14	200	30.01	0.32		0.0016
2013-08-05	BSFT	16:10:49	–80	30.00	0.60	–1.53	0.0075
2013-08-05	BSFT	16:13:43	–120	29.95	0.55	–7.94	0.0046
2013-08-06	FSLR	16:04:30	–200	43.26	1.51		0.0076
2013-08-06	FSLR	16:08:24	–200	45.18	1.52		0.0076
2013-08-06	FSLR	16:15:25	400	43.87	0.48	136.48	0.0012
2013-08-06	Z[50]	16:31:10	200	92.84	0.32		0.0016
2013-08-06	Z	16:34:03	200	85.94	1.34		0.0067
2013-08-06	Z	16:34:23	–400	85.97	1.29	–1370.99	0.0032
2013-08-06	Z	16:43:39	–200	83.95	0.64		0.0032
2013-08-06	Z	16:47:06	200	83.80	0.32	29.24	0.0016
2013-08-07	GMCR	16:06:06	200	74.57	0.24		0.0012
2013-08-07	GMCR	16:17:36	–200	75.44	0.61	173.15	0.0030
2013-08-07	TSLA	16:08:33	200	143.63	0.32		0.0016
2013-08-07	TSLA	16:24:41	–200	153.57	1.90	1986.78	0.0095
2013-08-08	MNST	16:10:49	–200	59.48	0.55		0.0028
2013-08-08	MNST	16:16:29	200	58.60	0.32	175.13	0.0016
2013-08-13	CREE	16:06:39	–200	67.77	0.58		0.0029
2013-08-13	CREE	16:11:23	200	65.05	0.32	543.10	0.0016

Continued on next page

---

[50] I mistakenly traded the opposite direction for this particular observation.

Table 1.9 – continued from previous page

Date	Ticker	Time	Size	Price	Cost	Profit	Cost/Shr
Total P.L: \$4,394.07*			Size-weighted Cost/Shr: \$0.0047				

*: Total P.L is only listed for reference. The experiment is solely for transaction costs assumptions verification.

CHAPTER 2

A Profit-Sharing Theory of the Firm

Among the many roles a well-functioning capital market plays in an economy, the aggregation of (dispersed) information among individuals has long been emphasized. Classic papers of noisy rational expectation equilibria (e.g. Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981)) analyzes how market participants make investment decision based on not only their private information, but also inferences of others' information gleaned from market prices. This market function of information aggregation is important, because individuals usually only have imprecise knowledge of the value of the assets they seek to invest. If their own assessments were all they had to rely on, risk would deter them from investing nothing but the most trivial amount. Fortunately, because rational expectation inference reduces the conditional variance of investment payoffs, larger positions are taken, costs of capital are lowered, and capital allocation is improved. In this sense, a capital market could be viewed as a substitute for information sharing.

Despite its elegance, the noise embedded in a large market guarantees that rational expectation inference is by nature imperfect.[1] Such imperfectness makes noisy rational expectation equilibrium a somewhat constrained outcome, and suggests that non-market information aggregation mechanisms will necessarily coexist with a market economy. For example, direct communication among informed

---

[1] See Black (1986) for a comprehensive assessment of "noise". Existing literature often attributes the non-informative "noise" to quantity shocks, yet the noise could be alternatively interpreted as a reduced-form description of investors' imperfect inference from price due to their incomplete knowledge of the large, anonymous market that they are part of.

investors will sharpen their information precision and improve information aggregation solely provided by the market.[2] However, in reality there are at least two obstacles to direct communication. First, truthful communication may not be incentive compatible. The fear of one's valuable information being abused against, as well as the jeopardy of unintentional divulgence or blatant re-sale by other informed parties, often deter or distort truthful communication incentives.[3] Second, communication takes time. If a market opportunity is short-lived and requires immediate reaction, delays caused by direct communication might negate its benefit. Furthermore, while decentralized possession of information has long been recognized in analyzing the market economy in general (Hayek (1944, 1945)) and the financial market in specific (e.g. Hellwig (1980), Diamond and Verrecchia (1981)), less is known about its implications for non-market institutions.

My paper points out that an alternative mechanism, which resembles the formation of a partnership firm, could replicate the information sharing effect of direct communication without actually incurring it, thus resolving the disincentive against truthful communication. In this mechanism, investors partner together and write *ex ante* contracts to share net investment profit that will be realized *ex post*. Because the partnership contract alters each partners' incentive, it changes their investment behavior as well as final payoff. As will be shown, when the *ex ante* sharing rule is optimally designed, each partners' payoff would be exactly identical to that under (hypothetically) truthful, instant, and costless direct communication. A partnership facilitates *de facto* communication among its partner when actual information sharing is costly, if not infeasible.[4]

---

[2] This insight is seen in the "information percolation" literature, which explicitly models the evolution of investors' information precision over repeated direct communication, see Duffie, Giroux, and Manso (2010) and Andrei and Cujean (2013), etc.

[3] Such friction is brought up in seminal discussions on information production (e.g. Hirshleifer (1971)), and motivates studies on optimal selling of information (e.g. Admati and Pfleiderer (1986) and Admati and Pfleiderer (1990)). Allen (1990) develops a theory of financial intermediation based on indirect selling of information. See Dewatripont and Tirole (2005) for further discussions on potential costs and biases in knowledge transfers.

[4] Of course direct communication is helpful when it is instead costless. For example, Ra-

The fact that information aggregation can be achieved without actual communication might look surprising at first sight, but the intuition behind is indeed straightforwardly rooted in the conventional wisdom that "actions speak louder than words".[5] Even if investors cannot *ex ante* communicate, their information is nevertheless reflected in their actions, and further reflected in their *ex post* payoffs. Via properly mixing investors' payoffs, profit-sharing coordinates actions guided by dispersed private information, and thus in equilibrium empowers individuals with their collective wisdom.

That a partnership complements the market in aggregating information sheds new light on an old question in the theory of the firm (Coase (1937)): what is the nature of the firm, and what additional role a firm provides to pure market allocation? As a partnership firm could be viewed as a payoff-sharing partnership among its various stakeholders, one of the reasons that a firm exist could very well be to offer a non-price mechanism for facilitating knowledge aggregation among its stakeholders. In another word, firms and markets coexist because of their complementarity in harnessing the collective wisdom of the population.

While the role of partnership companies in harnessing wisdom of the crowd has received little attention, similar rationales have long been accepted as a main function of market prices. In the literature on financial innovation (e.g. Grossman (1977)), a new security/market provides a new price (an endogenous variable) for decisions to be contingent upon, thus permitting indirect information aggregation. Analogously, in this paper a well-designed profit-sharing contract makes each participant's action contingent upon others' actions (also endogenous variables). Both

---

makrishnan and Thakor (1984) develop a theory of financial intermediation assuming costless communication within the intermediary sector. The *de facto* information aggregation effect of profit-sharing should also be distinguished from truthful communication incentives also (partially) recovered by partnering. See Garicano and Santos (2004) on how partnership encourages efficient case referral among lawyers.

[5] The intuition that actions may credibly convey information is also seen in other economic setting, for example, signaling games (e.g. Leland and Pyle (1977)) and the revelation principle in mechanism design.

mechanisms help individuals make more informed decisions and largely complement each other, yet a partnership firm is particularly helpful when the market is absent or "noises" prevent the price from fully revealing. A partnership firm could thus be interpreted as an institutional innovation in response to market incompleteness. Section 2.2 uses the setup developed in Section 3.3 to formalize the insight that the creation of a partnership firm completes the market. The analysis also provides a new perspective to look at the long-discussed relationship between a firm and its surrounding market economy.

While profit-sharing achieves first-best outcome in the baseline model, in reality there are multiple forces that could counteract the power of a profit-sharing contract. Section 2.3 investigates these forces in the context of delineating the boundary of a firm. The boundary of a partnership firm could be determined by a trade-off between the benefits from wisdom of the crowd and the costs due to free-riding or adverse market power (decreasing return to scale technology). In the presence of these frictions, profit-sharing cannot coordinate individuals perfectly, yet it still dominates alternatives including direct communication of private information (even when it is costless). This is because truthful communication is strictly incentive incompatible under those frictions. For example, in the presence of adverse market power, each individual has strict incentive to understate her private information when asked for, in hope of less competition if her lie is believed. This strict incentives to lie shuts down the direct communication channel, keeping the dominance of compensations that feature more or less profit-sharing elements. I illustrate this point with a simple numerical example.

The rest of the paper is organized as follows. Section 3.3 sets up a workhorse model for partnership companies, and derives the optimal profit-sharing contract. Section 2.2 explores the relationship between a firm and the market within my framework and illustrates how a firm endogenously arise in an incomplete market. Section 2.3 investigates forces that shape firm boundaries. Section 2.4 discusses general implications on various corporate governance topics. Section 3.4 relates existing literature. Section 3.5 concludes.

2.1 Workhorse Model for a Partnership Firm

The structure of modern corporations have evolved into prohibitive complicacy, yet they do share some common features. At a high level, a firm's business could be captured by a risky production technology, to which a set of "owners" provide (usually relatively homogeneous) production inputs (capital, labor, or raw ingredients, etc.) based on their assessment of the business productivity, and from which the same set of owners share residual earnings according to a pre-specified rule. This primitive description of firms with emphases on profit-sharing and risk-taking follows corporate legal literature (Hansmann (2009)), and is linguistically consistent with English tradition – among the synonyms of the word "firm" are "company" (profit-sharing among multiple owners) and "venture" (risky business). Firms in this form has been accompanying human history ever since Queen Elizabeth granted the East India Company its first Royal Charter on December 31st, 1600 AD, and even today still underlies partnerships (e.g. private equity/venture capital firms), producer cooperatives, joint-ventures, and (to a less extent) all other firms except for sole-proprietorship.

Insights learned from the Alice-Bob example suggest a possibility that partnership companies, when properly structured, could empower their owners with wisdom of the crowd. This section formalizes this idea.

2.1.1 The Firm as a Profit-Sharing Coalition

A firm is defined by a charter, which stipulates a compensation scheme among its n owners (players). Upon firm creation in period $t = 0$, the charter entitles owner i, who has a constant absolute risk aversion parameter ρ_i, of a_i of the firm's residual earnings, which will be realized by the end of period $t = 1$, where $\sum_{i=1}^{n} a_i = 1$. The firm has a constant-return-to-scale production technology $Y = vX$, where Y is the total revenue, $v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$ is a stochastic factor productivity, and X is total amount of productive input contributed by all the owners, i.e. $X = \sum_{j=1}^{n} x_i$, where x_i is player i's productive input contribution.[6] The unit cost of productive input is denoted as p, which is a constant.[7] The wisdom of the crowd assumption indicates that each player has some private knowledge in assessing the stochastic factor productivity. Assume player i's private knowledge $s_i = v + e_i$, where $v \perp\!\!\!\perp e_i$ and $e_i \sim \mathcal{N}(0, \tau_j^{-1})$. Each firm owner independently decides on how much production input to contribute to the firm.

The input provided by the n owners of the firm is given in a Nash equilibrium. In particular, player i chooses x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i(v-p)(x_i+\sum_{k\neq i} x_k)\right]\right)|s_i\right], \tag{2.1}$$

given her perception of other players' equilibrium productive input x_k, $k \neq i$. The following theorem provides a linear Nash equilibrium solution for a given player.

Theorem 2.1.1. *A linear Nash equilibrium exists only when* $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n}\frac{1}{\rho_i}}$, *and in equilibrium player i's productive input could be given by*

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n}\frac{1}{\rho_k}\right)\tau_i s_i - \left[\frac{\tau_v}{\rho_i} + \left(\sum_{k=1}^{n}\frac{1}{\rho_k}\right)\tau_i\right]p. \tag{2.2}$$

The proof of the theorem requires the following lemma.

---

[6] By assuming a constant-return-to-scale production technology, I shutdown any complementarity in players' inputs which would mechanically favor firm creation. For different modeling purposes, existing literature usually assumes non-separable production technologies, e.g. Alchian and Demsetz (1972). In these models agents' productive input choices impose (usually positive) externalities on each other. Such externalities can either come from output (e.g. Kandel and Lazear (1992)) or cost (e.g. Edmans, Goldstein, and Zhu (2011)).

[7] I will endogenize p in Section 2.2. I also do not consider any private cost to each player's input supply, which I will address in Section 2.3.

Lemma 2.1.2. *If* $\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right)$, *where* $(\rho\sigma_1\sigma_2 - 1)^2 > \sigma_1^2\sigma_2^2$ *then*

$$\mathbb{E}\left[e^{\tilde{y}_1\tilde{y}_2}\right] = \frac{\exp\{(\theta_2^2\sigma_1^2 - 2\rho\theta_1\theta_2\sigma_1\sigma_2 + \theta_1^2\sigma_2^2 + 2\theta_1\theta_2)/(2[(\rho\sigma_1\sigma_2 - 1)^2 - \sigma_1^2\sigma_2^2])\}}{\sqrt{(\rho\sigma_1\sigma_2 - 1)^2 - \sigma_1^2\sigma_2^2}}.$$

Proof. Standard integration. □

Proof of Theorem 3.3.1. A linear symmetric equilibrium is given by $x_k = \pi_k + \gamma_k s_k$ for some π_k and γ_k. Because

$$\begin{bmatrix} -a_i\rho_i v \\ x_i + \sum_{k\neq i} x_k \end{bmatrix}_{|s_i} \sim$$

$$\mathcal{N}\left(\begin{bmatrix} -\rho_i a_i \mathbb{E}(v|s_i) \\ x_i + \sum_{k\neq i}\pi_k + \sum_{k\neq i}\gamma_k\mathbb{E}(v|s_i) \end{bmatrix},\right.$$
$$\left.\begin{bmatrix} \rho_i^2 a_i^2 \mathrm{Var}(v|s_i) & -\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) \\ -\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) & (\sum_{k\neq i}\gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k\neq i}\gamma_k^2\tau_k^{-1} \end{bmatrix}\right)$$

by Lemma 3.1.1, player i equivalently minimizes

$$\begin{aligned} & \theta_2^2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\theta_2\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + \theta_1^2[(\sum_{k\neq i}\gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k\neq i}\gamma_k^2\tau_k^{-1}] + 2\theta_1\theta_2 \\ \xRightarrow{\text{FOC}} \quad & 2\theta_2\rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1\rho_i a_i \sum_{k\neq i}\gamma_k \mathrm{Var}(v|s_i) + 2\theta_1 = 0, \\ & \text{where } \theta_1 = -\rho_i a_i \mathbb{E}(v|s_i) \text{ and } \theta_2 = x_i + \sum_{k\neq i}\pi_k + \sum_{k\neq i}\gamma_k \mathbb{E}(v|s_i). \end{aligned}$$

Plugging in $x_i = \pi_i + \gamma_i s_i$ and matching coefficients renders $\gamma_i = \frac{\tau_i}{\tau_v \bar{v}}\Pi$, $\frac{\Pi}{\tau_v\bar{v}}\rho_i a_i = 1$ ($\Pi \doteq \sum_{i=1}^n \pi_i$), and thus $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^n \frac{1}{\rho_i}}$, $\Pi = \left(\sum_{i=1}^n \frac{1}{\rho_i}\right)\tau_v\bar{v}$. It is natural to look at one particular equilibrium in which $\pi_k = \frac{\tau_v\bar{v}}{\rho_k}$, leading to $x_i = \frac{\tau_v\bar{v}}{\rho_i} + \left(\sum_{i=1}^n \frac{1}{\rho_i}\right)\tau_i s_i$. □

Given no complementarities in productive inputs, one may be tempted to think that players should be indifferent between running a sole proprietorship or taking part in a firm, nor would firm creation affect real allocation. However, Theorem 3.3.1 shows that a player becomes more dedicated to the project when in a firm whenever she has positive assessment of the project prospect (i.e., x_i increases with n when $s_i > 0$). Because a player's assessment are more likely to be positive for a high-value (v) project, creating a firm (rather than keeping multiple sole proprietorships) helps a good business to receive (probablisticly) higher total productive input. Firm creation improves real allocation.

Notice that if player i has full information, her input supply would be

$$x_i = \frac{\tau_v(\bar{v} - p)}{\rho_i} + \frac{\sum_{i=1}^n \tau_i}{\rho_i}(i^* - p), \text{ where } i^* = \frac{\sum_{k=1}^n \tau_k s_k}{\sum_{k=1}^n \tau_k}.$$

Thus player i's payoff is

$$(v - p)\left[\frac{\tau_v(\bar{v} - p)}{\rho_i} + \frac{\sum_{i=1}^n \tau_i}{\rho_i}(i^* - p)\right]$$

under both full information and in a firm. This is summarized in the following theorem.

Theorem 2.1.3. *Given her private information, a player's expected utilities of participating in a properly structured n-owner firm is identical to as if she could obtain other $n - 1$ players' private information without cost while running a sole proprietorship.*

A direct implication of this result is that creating a partnership firm raises owners' expected utilities, and thus is a voluntary outcome of economic evolution. The reason why a partnership firm benefits participating players could be interpreted as, first, a well-designed profit-sharing contract empowers them with wisdom of the crowd, and second, it provides a means for human-capital diversification. The relation between the information aggregation effect and risk-reduction suggests that profit-sharing could be alternatively interpreted as an institutional innovation, compared to "financial innovation" based on security design à la Allen and Gale (1994).

An optimal profit-sharing contract that harnesses wisdom of the crowd slightly differs from traditional equity contracts. An equity contract is a "revenue-sharing" contract, which splits proportionally the total payoff including each individual's initial investment.[8] Yet a profit-sharing contract splits according to a pre-specified rule the total payoff excluding initial investment. While a traditional equity contract is no worse than a profit-sharing contract when private information is absent, it is strictly dominated by an optimal profit-sharing contract whenever collective wisdom is present.

2.2 Arise of a Partnership Firm in a Market Economy

An important question in the theory of the firm concerns the relationship between a firm and the outside market. To this end, this section generalizes the workhorse model by allowing the product input cost p to be endogenously determined as an equilibrium market outcome. The setup for the productive input market resembles classic noisy rational expectation models à la Hellwig (1980) and Diamond and Verrecchia (1981). I show that while the market also communicates information through equilibrium price, it is nevertheless dominated by profit-sharing due to the presence of market "noise".[9] In this sense, a partnership firm arises as a

---

[8] Indeed revenues are often, though not always, spit in proportion to each individual's initial investment.

[9] See Black (1986) for a comprehensive assessment of "noise". Existing literature often attributes the non-informative "noise" to quantity shocks (or noise traders), but this is not necessary. The noise could be interpreted as a reduced-form description of investors' incomplete knowledge about market architecture. Alternatively, it could be viewed as a partial equilibrium outcome, in which some un-modeled outside market also influences price (this is indeed a justification for treating the risk-free rate as exogenous in most noisy rational expectation equilibrium models).

response to market incompleteness caused by asymmetric information.

The market for the productive input consists of a continuum of players with player i having a constant absolute risk aversion ρ_i, $i \in [0,1]$. On $t = 0$, a risky business opportunity with factor of productivity $v \sim \mathcal{N}(\bar{v}, \tau_v^{-1})$ emerges. Player i decides on x_i, the optimal amount of productive input to provide to the business. When making decisions, player i has a private signal of the business productivity $s_i = v + e_i$, where v and e_i are independent and $e_i \sim \mathcal{N}(0, \tau_i^{-1})$. A quantity noise $z \sim \mathcal{N}(\bar{z}, \sigma_z^2)$ measures the aggregate demand for the productive input for alternative uses other than the new business opportunity, which carries no information about the new business opportunity, i.e., z is independent of v.

Assume that players $1, 2, \cdots, n$ agree to created a partnership firm and share profits. Then player i's problem is given by choosing x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i(v-p)(x_i+\sum_{k\neq i} x_k)\right]\right)|s_i, p\right], \tag{2.3}$$

where $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^n \frac{1}{\rho_i}}$. The solution will look like a mixture of a Nash equilibrium studied in Section 3.3.1, and a noisy rational expectation equilibrium à la Hellwig (1980).

For ease of comprehension, I abstract from the noisy rational expectation equilibrium in the productive input market at the moment, and first present a general result of their optimal input provisions and corresponding expected utilities when the equilibrium input price is (exogenously) given by an arbitrary linear price system.

Theorem 2.2.1. *In an market economy in which the equilibrium input cost follows a linear function $p = \mu + \pi v - \gamma z$, the expected utility a player i in a firm of size*

n is given by

$$-\frac{\exp\left(-\frac{1}{2}\frac{1}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_i s_i-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\tau_v\bar{v}-(\tau_i+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)}{\sqrt{\frac{\sum_{k=1}^n\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}}$$

Proof. See Appendix. □

In comparison, under a (hypothetically) full information benchmark, each player will base her decision on the weighted average of the private signals of all n players. Denote $\sum_{k=1}^n \tau_k s_k / \sum_{k=1}^n \tau_k$ as $s^* = v + e^*$, then $v \perp\!\!\!\perp e^*$ and $e^* \sim \mathcal{N}(0, \frac{1}{\sum_{k=1}^n \tau_k})$. With a linear cost system in which $p = \mu + \pi v - \gamma z$, the input provision by member i in an signal-sharing alliance is given by maximizing $\mathbb{E}\left[-\exp(-\rho_i(v-p)x_i|s^*, p\right]$, and thus

$$\begin{aligned} x_i' &= \frac{\mathbb{E}(v|s^*,p)-p}{\rho_i \mathrm{Var}(v|s^*,p)} \\ &= \frac{1}{\rho_i}[\tau_v\bar{v}-\frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z})+\sum_{k=1}^n\tau_k s^*-(\sum_{k=1}^n\tau_k+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p] \end{aligned} \tag{2.4}$$

$$\text{(Notice that } \begin{cases} \mathbb{E}(v|s^*,p) &= \frac{\gamma^2\sigma_z^2\sum_{k=1}^n\tau_k s^*+\pi(p-\mu+\gamma\bar{z})+\gamma^2\sigma_z^2\tau_v\bar{v}}{\gamma^2\sigma_z^2\sum_{k=1}^n\tau_k+\pi^2+\gamma^2\sigma_z^2\tau_v} \\ \mathrm{Var}(v|s^*,p) &= \frac{\gamma^2\sigma_z^2}{\gamma^2\sigma_z^2\sum_{k=1}^n\tau_k+\pi^2+\gamma^2\sigma_z^2\tau_v} \end{cases} \text{)}$$

Comparing (**??**) with (2.4), it is easy to verify that the total (*ex post*) input provision from a firm and that from players under full information benchmark are identical. Hence equilibrium market price is the same across both structures. In another word, there is an isomorphism in terms of price between under profit-sharing and under symmetric information. This result leads to a similar isomorphism in terms of (*interim*) expected utility, as summarized below.

Theorem 2.2.2. *A player's interim expected utilities conditioning on her own private signal are the same in a partnership firm and under a full information* *benchmark.*

Proof. See Appendix. □

Because profit-sharing benefits from the wisdom of the crowd effect, the following result follows immediately.

Corollary 2.2.3. *When a partnership firm is adequate small (so that it has negligible influence to the productive input market), each owner's expected utility strictly increases with the firm size n.*

For any specific market structure, the price function $p = \mu + \pi v - \gamma z$ will be given by market clearing. An example is provided in Section 2.3.2.

Comment (relationship between a partnership firm and the outside market): The expected utility equivalence between a profit-sharing equilibrium and a full information benchmark suggests that a competitive market with dispersed private information is intrinsically unstable. Rational agents will always have incentive to partner with others.[10]

As compared to rational expectation inference from the market price, profit-sharing provides a non-price based mechanism to obtain the effect of aggregating information dispersed among market participants. Indeed, the idea of rational expectation in its most general form assumes that when agents make decisions, they rely on not only their own information, but also further inference from any endogenous variables in the economic system. In a financial market, such endogenous variables are often equilibrium market prices. When the market price is

---

[10] In an investment setting, Ross (2005) develops a theory of delegated wealth management based on the unstableness of noisy rational expectation equilibrium. Similar in spirit, Indjejikian, Lu, and Yang (2014) suggest that the strategic Kyle (1985) equilibrium is not stable, because the most informed would have incentive to leak information to an uninformed trader, so that the other informed traders will trade less aggressively.

absent or noisy, creating a new market (thus creating new prices) provides a new information source and completes the market.

When we go beyond the financial market, a lot of other endogenous variables arise, including the total profit contributed by multiple parties. When a profit-sharing contract explicitly links each player's compensation to the other's actions, it makes each player's action contingent on what others do (in a Nash equilibrium), thus empowering them wisdom of the crowd. This effect is particularly useful when a centralized market does not exist, e.g. in private equity/venture capital investment or crowdfunding.

The complementarity between a partnership firm and the market is reminiscent of the complementarity between banks and markets discussed in Boot and Thakor (1997). In a dynamic version of that model, Song and Thakor (2010) highlight that banks and markets exhibit three forms of interaction: competition, complementarity and co-evolution. My paper thus provides an informational perspective on the complementarity between market and more general institutional structures.

2.3 Forces that Shape Firm Boundaries

Corollary 2.2.3 suggests that firm size features (locally) "the more the merrier". However, as a firm looms large, several natural forces would kick in to restrict firm size. An optimal firm size could thus be determined. This section discusses these forces.

2.3.1 The Boundary of the Wise Crowd

Because the benefit of profit-sharing in the above theory comes from wisdom of the crowd, a straightforward force to restrict firm size is the boundary of the (wise) crowd. Such a boundary is plausible when the particular business in question features agree to disagree or overconfidence. Since overconfident players believe that they are the smartest and having nothing to learn from others, they have no incentive to form a firm with others. Indeed as McCloskey (1998) interprets Muth (1961), the spirit of rational expectation builds on assuming people's intellectual modesty. If this assumption fails, then there is no room for rational expectation, nor for a theory of the firm based on collective wisdom.[11] Although whether a particular business line features wisdom of the crowd or not is an empirical question, it would be schizophrenic if we study the market with rational expectation, while refusing to look at firms via the perspective of wisdom of the crowd.

2.3.2 Product Market Price Impact and Information Acquisition

The previous section assumes a constant-return-to-scale technology, implicitly assuming inelastic supply of the productive input. This assumption is well justified for commodity inputs such as raw materials or physical capital, yet even for other input whose market initially features natural market power, vertical integration would usually mitigate its effect in the long run. That said, if for whatever reason vertical integration cannot instantly take place, input market price sensitivity would enhance equilibrium price efficiency and thus lower the value of private information, generating decreasing return to scale. As the price impact of each individual's input provision amount is no longer negligible, they are forced them to "shred orders" à la Kyle (1989). Appendix 2.6 derives input provisions and equilibrium productive input price. The optimal firm size then has to trade-off the wisdom of the crowd effect and induced decreasing return to scale. Firm boundaries will then be delineated.

Even when each individual's own price impact in the productive input mar-

---

[11] Even in an economy where the wisdom of the crowd applies, search frictions (i.e. it takes time to find another players interested in the same business) naturally limits firm size. Note that search dynamic is a key element in information percolation models built on direct communication.

ket is negligible (though not for the entire firm), another force shaping the firm boundary would arise when players have to incur some acquisition cost to get their private information. When such acquisition cost is private and not contractible, profit-sharing would lead to a free-riding problem. Free-riding costs trade-off benefits from information aggregation, and determines an optimal firm size. We shall note that such free-riding effect is similar to those found in the financial market. Grossman and Stiglitz (1980) rely on this free-riding problem to prove the impossibility of strong form market efficiency. What prevents a market from being perfectly efficient alternative prevents firms from being infinitely large.

For general decreasing return to scale production technologies, the optimal compensation still features partial profit-sharing. Although with decreasing return to scale profit-sharing no longer perfectly coordinates actions to the effect of empowering wisdom of the crowd, it still strictly dominates alternatives, and in particular any form of direct information communication (even if direct communication incurs no cost). This is because with externalities in the productive input market all players have incentives to lie to others in direct communication, while profit-sharing is does not involve communication and is thus immune to lies.

In a knowledge economy, it is important to coordinate individuals with dispersed information within an institution. However, many obstacles in reality deter effective direct communication. First, truthful-telling may not be incentive compatible. The fear of one's valuable information being abused, as well as the jeopardy of unintentional divulgence or blatant re-sale by others, often deter truthful communication.[12] Second, communication often takes time. If a market

---

[12] Preventing knowledge stealth is empirically a serious concern, as Bhide (1994) reports that 71% of the firms included in the Inc 500 (a list of young, fast growing firms) were founded by people who replicated or modified an idea encountered in their previous employment. Theoretically the concern over critical knowledge stealth is also the departure point in Rajan and Zingales (1998). In the context of financial markets, re-sale has been brought up in discussions on information production (e.g. Hirshleifer (1971)). Controlling information usage motivates studies on optimal selling of information (e.g. Admati and Pfleiderer (1986) and Admati and Pfleiderer (1990)). Allen (1990) develops a theory of financial intermediation based on indirect selling of information. See Dewatripont and Tirole (2005) for further discussions on potential

opportunity is short-lived and requires immediate reaction, delays might negate benefits from communication.[13] Third, when the number of people involved increases, direct communication would become too costly, if not infeasible.[14] This is a particular pressing issue with the rise of some new financing methods like crowdfunding, where a large number of individuals with low or no affinity are involved in the decision making for the same venture. Profit-sharing is a simple mechanism to overcome all these obstacles.

2.3.3 Private Cost in Providing the Productive Input

Another form of free-riding cost arises when the cost of the productive input is private. Such cost resembles the moral hazard in teams problem studied by Holmström (1982). In this case, the decision to form a firm involves a trade-off between the benefit from wisdom of the crowd and a cost due to free-riding.

Take an extreme case for example, if the cost of productive input has to be incurred by player i in its entirety, she would choose x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho\left[a_i v(x_i + \sum_{k\neq i} x_k) - p x_i\right]\right)|s_i\right], \tag{2.5}$$

given her anticipation of other players' equilibrium productive input level x_k, $k \neq i$. Using similar solution technique, the player i's equilibrium input amount to the firm is given by

---

costs and biases in knowledge transfers.

[13] Bolton and Dewatripont (1994) consider the time involved in information transmission. The "information percolation" literature explicitly models the slow diffusion process of information in the financial market over repeated direct communication, see Duffie, Giroux, and Manso (2010) and Andrei and Cujean (2013), etc.

[14] Furthermore, conversations, meetings, and discussions take time, at the cost of leisure, actual work, and missing opportunities; misinterpretation and oblivion create additional attrition to communication; "soft" information like haphazard know-hows, amorphous business acumen, and tacit knowledge à la Grant (1996) are simply too hard to codify and impossible to convey; cognitive capacity limits cap the amount of knowledge an individual can possess (see the rational inattention literature as in e.g. Veldkamp (2011)).

$$x_i = \frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k} \right) \tau_i s_i - \left(\sum_{k=1}^{n} \frac{1}{\rho_k} \right) (\tau_v + n\tau_i) \, p \tag{2.6}$$

The equilibrium input amount consists of two parts. The first part $\frac{\tau_v \bar{v}}{\rho_i} + \left(\sum_{k=1}^{n} \frac{1}{\rho_k} \right) \tau_i s_i$ represents a benefit from wisdom of the crowd, while the second part $-\left(\sum_{k=1}^{n} \frac{1}{\rho_k} \right) (\tau_v + n\tau_i) \, p$ represents a free-riding cost. The relative magnitudes of the two effects could determine optimal firm size.[15]

In the context of information gathering agencies, Millon and Thakor (1985) also analyze how moral hazard related intrafirm costs within a partnership pin down a finite optimal size of the firm. In addition to context differences, in Millon and Thakor (1985) the benefit of forming a partnership comes from *direct* information communication, while in my paper the benefit comes from better coordination of actions led by the profit-sharing contract.[16]

The case where private cost c equals 0 corresponds to costless intrafirm monitoring. This assumption appears in Ramakrishnan and Thakor (1984), who develop a theory of financial intermediation, where information producers also write *ex ante* contracts on *ex post* payoffs. The specific sharing rule in the current paper is similar to their independent (not IMJC) contract. The difference is that, in Ramakrishnan and Thakor (1984), it is the information producers who are producing information about players. In contrast, players directly form a coalition in this paper. This rules out the kinds of joint contracts that Ramakrishnan and Thakor (1984) consider.

---

[15] That said, if we combine profit-sharing with a “massacre” or ”scapegoat” penalizing mechanism as introduced in Rasmusen (1987), first best result may still be maintained. Some further discussion on moral hazard in teams: Williams and Radner (1988) develop examples showing how partnerships preserve efficiency when the joint output is uncertain. Legros and Matsushima (1991) present a necessary and sufficient condition for achieving efficiency in partnerships. Strausz (1999) studies how sequential partnerships sustain efficiency. These considerations, however, are beyond the scope of the current paper.

[16] For examples of how partnering (partially) incentivizes truthful *direct* communication, see Garicano and Santos (2004) on efficient case referral among lawyers.

2.4 General Implications for Corporate Finance

Aside from elucidating the complementarity between a partnership firm and its surrounding market, looking at profit-sharing contracts through the lens of wisdom of the crowd also broadens the content of corporate governance studies. Shleifer and Vishny (1997) defines corporate governance as a study that "deals with the ways in which suppliers of finance to corporations assure themselves of getting a return on their investment", with "a straightforward agency perspective, sometimes referred to as separation of ownership and control". Recent advances in corporate governance practices, along with a wisdom of the crowd perspective on profit-sharing design could thus add to this established view.

A long-lasting question in the theory of the firm asks what is the "glue" that keeps a firm together? Traditional property-based theory of the firm identifies physical assets as such glues.[17] However, as we enter an information age, in many business sectors traditional asset intensive firms are now being gradually peripheralized by human capital intensive ones, leading to the call for a "search of new foundations" (of corporate finance / theory of the firm) in Zingales (2000).[18] There has been many attempts in direction, including Acharya, Myers, and Rajan (2011) and Rajan (2012)).[19] Profit-sharing gives an alternative.

The decentralized control in my profit-sharing analysis is reminiscent of the meritocracy spirit of Aghion and Tirole (1997), in which formal authority is distinguished from real authority. An agent with formal authority will exercise her power if and only if she acquires the necessary knowledge to do so, or other-

---

[17] For example, Hart (1995) argues that "a firm's non-human assets, then, simply represent the glue that keeps the firm together . . . If non-human assets do not exist, then it is not clear what keeps the firm together" (p. 57)."

[18] Gluing human capital and preventing talent attrition is an important consideration in modern corporate governance. The consequence of neglecting it is vividly illustrated in the case of the British advertising agency Saatchi and Saatchi documented in Rajan and Zingales (2000).

[19] Berk, Van Binsbergen, and Liu (2014) and Cheng, Massa, Spiegel, and Zhang (2012), among others, focus on a particular type of human-capital intensive firms – mutual fund families.

wise she delegates decision-making to her more knowledgeable subordinates.[20] In the profit-sharing relation studied above, players make decisions without others' interference, as their private knowledge (either about the mapping from the information set to the optimal action or the information set itself) grant them real authority. This decentralized governance structure lines up with the flat organization, teamwork focus, and advocated "workforce democracy" found in most human capital intensive firms. It also differentiates my model from the social choice problem of Wilson (1968).

Viewing firms as profit-sharing mechanisms also helps interpret some recent trends in capital structure changes. In a recent discussion on "secular stagnation" among industrialized economies, Summers (2014) pinpoints the "reductions in demand for debt-financed investment", and contends that "probably to a greater extent, it is a reflection of the changing character of productive economic activity."[21] Traditional asset-intensive industries are debt-friendly, as assets serve as collaterals and allow outside investors to take a passive role in the firm's operation (except in default).[22] Firms with intensive knowledge inputs, however, require a more active role of all input providers, e.g. the more active roles played by venture capitalists than commercial banks, the adoption of equity-based employee compensation, and less involvement of passive creditors in knowledge-intensive firms (law firms, strategic management (but not IT) consulting firms, etc.) – even though such industry might be most subject to insider moral hazard or unverifiable cash flow, which traditional theories (e.g. Innes (1990) and Townsend (1979))

---

[20] Although the focus of Aghion and Tirole (1997) is on how the allocation of formal authority alters agents' *ex ante* knowledge acquisition incentives.

[21] Summers elaborates further: "Ponder that the leading technological companies of this age – I think, for example, of Apple and Google – find themselves swimming in cash and facing the challenge of what to do with a very large cash hoard. Ponder the fact that WhatsApp has a greater market value than Sony, with next to no capital investment required to achieve it. Ponder the fact that it used to require tens of millions of dollars to start a significant new venture, and significant new ventures today are seeded with hundreds of thousands of dollars…"

[22] Williamson (1988) sympathizes this perspective.

would predict in favor of debt-financing.[23] In a broader sense, my theory relates to the partnership model of outside equity investors in Myers (2000).

The profit-sharing view of a firm also provides new perspectives on valuation. The nexus of *explicit* contracts view of the firm à la Alchian and Demsetz (1972) and Jensen and Meckling (1979) assumes that compensations to all stakeholders but shareholders are explicitly contracted. Since equity holders are the only residual rights owner, maximizing shareholder value equates to maximizing social welfare for all stakeholders. However, this powerful argument is no long that clear-cut once employee human capital is taken into consideration, as residual rights owners can no longer be summarized as one representative person, and their internal relations do matter.[24]

Finally, the fact that private firms often feature a partnership structure while public firms feature typical equity contract could be interpreted as due to secondary market prices eliminating via rational expectation the information aggregation benefit of a profit-sharing contract. If a firm (typically a small, young one) cannot secure enough liquidity and thus price efficiency had it gone public, it would rather remain private and adopt a profit-sharing structure. A theory of firm life cycle and going public decision could thus be developed.

2.5 Other Related Literature

Theoretical results on the coordination effect of profit-sharing are related to studies on the equilibrium and efficient use of information. In a linear-quadratic setup featuring asymmetric information and strategic complementarity/substitutability,

---

[23] See also Jaggia and Thakor (1994), Berk, Stanton, and Zechner (2010) and Berk and Walden (2013) on the implications of human capital on capital structure and asset pricing.

[24] Several recent papers have investigated the valuation implication of firm's non-tangible assets. For example, Eisfeldt and Papanikolaou (2013) document that firms with more organization capital have average returns that are 4.6% higher than firms with less organization capital. Zhang (2014) studies the implications of employee's limited commitment to the firm on cash flow volatility.

Angeletos and Pavan (2007) show that redistribution among individuals can achieve as an equilibrium outcome efficient use of information. The coordination effect of profit-sharing also obtains efficient use of information, although strategic complementarity/substitutability are not present.

Casting my results in the financial market reminds of studies on indirect sale of information (Admati and Pfleiderer (1990), etc.) When informed investors manage delegated portfolios for a fee, they indirectly sell information to those uninformed. Following this logic, when information in the economic system is dispersed, investors would have incentives to delegate their wealth to each other. The result of such mutual delegation appears like profit-sharing studied in this paper. Admati and Pfleiderer uses their insight of indirect sale of information to explain the rise of institutional investors. Their results could thus be viewed as an important example of creating a firm to counter market information frictions.[25]

Cooperative game theory provides useful tools for many profit-sharing problems.[26] However these studies do not consider the wisdom of the crowd effect. Compared to cooperative game theory in which the value for a particular subset of players is exogenously specified, my solution is entirely based on non-cooperative game theory, and the value created by any subset of players is endogenous, dependent on the particular sharing contract among them.

Technically, a profit-sharing cooperative could be viewed as a game-theoretical implementation for a rational expectation equilibrium. So my result is connected to the implementation theory literature in mechanism design, whose focus is on designing mechanisms to achieve one equilibrium outcome via another equilibrium concept. Palfrey (2002) provides a nice introduction to implementation theory,

---

[25] Also see García and Vanden (2009) on wealth delegation with endogenous information acquisition.

[26] See e.g. Nash Jr (1950) and Shapley (1952) for original references, Aoki (1984) for a review of applications in the theory of the firm, Brandenburger and Nalebuff (2011) for a popular introduction, and Aumann and Maschler (1985) for a cooperative game theoretical analysis for the Talmud bankruptcy problem.

while Blume and Easley (1990) studies implementation of Walrasian expectations equillibrium in a general setting.

This paper also contributes to the literature on the theory of the firm. Over 70 years' academic endeavors on this fundamental topic makes an exhaustive reference almost impossible, so I only attempt to classify some well-known contributions around major lines of thoughts and connect them with the current paper.

The neoclassic theory views firms as production technology sets, and firms *per se* are void of meaningful definitions. The first and foremost question, raised by the seminal work of Coase (1937), asks what essentially defines a firm, and how within-firm organization is distinguished from market contracting. Coase identifies authority, which is useful when contracting is costly, as the defining feature that differs within-firm transactions apart from market contracting. Two questions remain to be answered in this argument, the first being what constitutes contracting costs, and the second being a formal definition of authority.

On contracting costs, Williamson (1975), Klein, Crawford, and Alchian (1978), Williamson (1979), and Williamson (1985) identifies *ex post* haggling as a source of cost to contracting. In my humble opinion, another important friction to contracting lies in the nonexistence of Pareto optimal, incentive compatible, and budget-balancing bilateral bargaining outcomes under two-sided asymmetric information (Myerson and Satterthwaite (1983)), although to my knowledge no resolutions have yet been proposed in this direction.

The formalization of authority spearheads the development of the incomplete contracting approach. Grossman and Hart (1986) and Hart and Moore (1990) argue that asset ownership determines the allocation of residual rights (or authority). In this property rights theory of the firm, physical assets play vital roles for the very existence of firms as they entangle other production inputs around it and give birth to firms.[27] However, as human capital intensive firms arise in

---

[27] See Hart (1989) and Holmström and Tirole (1989) for reviews. This view is partially sup-

the knowledge economy, Rajan and Zingales (1998) point out the narrowness of property rights theory and develop a new theory based on "access" to critical resources.[28]

The Coase authority-cost paradigm is not the only framework for understanding firms. For example, the "nexus of contracts" theory views a firm as a legal illusion no more than a central contracting party to subsume a complex of multilateral contracts (Alchian and Demsetz (1972)). Jensen and Meckling (1979) specifically focus on the principal-agent contracting problem between a firm owner and the management subject to moral hazard. In the market intermediary theory, Spulber (1999) interprets firms as centralized exchanges to reduce market search costs. A few papers take a knowledge perspective on the essence of the firm. Demsetz (1988) emphasizes information cost reduction as a foundation of firms, and Grant (1996) proposes a knowledge-based theory of the firm. However, both papers take information cost reduction and knowledge aggregation with a firm as given, abstracting from supporting micro-foundations. Bolton and Dewatripont (1994), Winter (2006), and Winter (2010) analyze how organizations minimize information processing and communicating costs, but they do not consider communication incentives, which is emphasized in this paper. While my paper is built on dispersed information, Dicks and Fulghieri (2014) study optimal governance structure based on disagreement between owners and managers that is endogenously generated by ambiguity aversion.

2.6 Conclusion

Many further implications are left for future research. For example, how is profit-sharing related to other non-market-price-based information aggregation mecha-

---

ported by Kaplan, Sensoy, and Strömberg (2009).

[28] Also see Rajan and Zingales (2001).

nisms, such as auctions or voting?[29] Are methods developed in this paper applicable to describing the formation of networks (e.g. Stanton, Walden, and Wallace (2015)) and the rise of "too-big-to-fail" banks (e.g. Erel (2011))? Does profit-sharing speak to the governance of PE/VC, the organization of R&D activities, or the rise of mutual fund families? How is my results on financial disintermediation (i.e. crowd-funding) related to existing theories on financial intermediation, e.g. Diamond (1984), Ramakrishnan and Thakor (1984))? Are there profit-sharing arrangements in existence to tunnel Chinese walls (on information communication)? Does profit-sharing camouflage insider trading? Casting in an even wider range, does profit-sharing have implications for optimal taxation? Does the investigation into the relationship between the firm and market suggest a connection between the Welfare Theorem and Coase Theorem? Is there an equivalence between capitalism and socialism in terms of information aggregation? Further developments are down the road.

Appendix A: Proof of Theorem 2.2.1

Proof of Theorem 2.2.1: When n players agree to partner, partner i's investment in the risky project maximizes

$$\mathbb{E}\left[-\exp(-\rho_i a_i(v-p)(x_i+\sum_{k\neq j}x_k))|s_i,p\right] \tag{2.7}$$

Focusing on symmetric linear equilibria, assume $x_k=\alpha_0+\alpha_{1k}s_k+\alpha_{2k}p$. Notice that

$$x_i+\sum_{k\neq i}x_k=x_i+\sum_{k\neq i}\alpha_0+\sum_{k\neq i}\alpha_{1k}v+\sum_{k\neq i}\alpha_{2k}p+\sum_{k\neq i}\alpha_{1k}e_k, \tag{2.8}$$

---

[29] see e.g. Feddersen and Pesendorfer (1997).

By Lemma 3.1.1, the certainty equivalent of (2.7) is

$$-\frac{\exp(\frac{A}{2B})}{\sqrt{B}}, \tag{2.9}$$

Taking FOC w.r.t. x_i we get

$$\begin{aligned}
& [x_i + \sum_{k\neq i}\alpha_0 + \sum_{k\neq i}\alpha_{1k}\mathbb{E}(v|s_i,p) + \sum_{k\neq i}\alpha_{2k}p]\rho_i a_i \mathrm{Var}(v|s_i,p) \qquad (2.10)\\
= \; & (\mathbb{E}(v|s_i,p) - p)[1 + \rho_i a_i \sum_{k\neq i}\alpha_{1k}\mathrm{Var}(v|s_i,p)]. \qquad (2.11)
\end{aligned}$$

Equalizing coefficients:

$$\begin{cases}
\rho_i a_i \sum_{k=1}^n \alpha_0 & = \tau_v \bar{v} - \frac{\pi}{\gamma^2\sigma_z^2}(\mu - \gamma\bar{z})\\
\rho_i a_i \alpha_{1j} & = \tau_i\\
\rho_i a_i \sum_{k=1}^n \alpha_{2k} & = -(\tau_i + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2}) + \frac{\pi}{\gamma^2\sigma_z^2} - \rho_i a_i \sum_{k\neq i}\frac{\tau_k}{\rho_k a_k}
\end{cases} \tag{2.12}$$

$\Rightarrow$

$$\begin{cases}
a_i & = \frac{\frac{1}{\rho_i}}{\sum_{k=1}^n \frac{1}{\rho_k}}\\
\sum_{k=1}^n \alpha_0 & = \left(\sum_{k=1}^n \frac{1}{\rho_k}\right)\left[\tau_v\bar{v} - \frac{\pi}{\gamma^2\sigma_z^2}(\mu - \gamma\bar{z})\right]\\
\alpha_{1j} & = \left(\sum_{k=1}^n \frac{1}{\rho_k}\right)\tau_i\\
\sum_{k=1}^n \alpha_{2k} & = \left(\sum_{k=1}^n \frac{1}{\rho_k}\right)\left[-(\sum_{k=1}^n \tau_k + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2}) + \frac{\pi}{\gamma^2\sigma_z^2}\right]
\end{cases} \tag{2.13}$$

Thus

$$x_i = \frac{1}{\rho_i}\left[\tau_v\bar{v} - \frac{\pi}{\gamma^2\sigma_z^2}(\mu - \gamma\bar{z})\right] + \left(\sum_{k=1}^n \frac{1}{\rho_k}\right)\tau_i s_i - \frac{1}{\rho_i}\left[\sum_{k=1}^n \tau_k + \tau_v + \frac{\pi^2}{\gamma^2\sigma_z^2} - \frac{\pi}{\gamma^2\sigma_z^2}\right]p \tag{2.14}$$

Plug (2.13) in (2.9) shows that

$$
\begin{aligned}
\mathbb{E}(v|s_i,p) &= \frac{\tau_i s_i + \frac{\pi}{\gamma^2\sigma_z^2}(p-\mu+\gamma\bar{z}) + \tau_v\bar{v}}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v} \\
\mathrm{Var}(v|s_i,p) &= \frac{1}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}.
\end{aligned}
$$

$$
\begin{aligned}
A &= -(\mathbb{E}(v|s_i,p)-p)^2\left(\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v\right) \\
B &= \frac{\sum_{k=1}^{n}\tau_k + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v}{\tau_i + \frac{\pi^2}{\gamma^2\sigma_z^2} + \tau_v},
\end{aligned}
$$

thus the expression for the expected utility of player i is given by

$$
\begin{aligned}
& -\frac{\exp\left(-\frac{(\mathbb{E}(v|s_i,p)-p)^2(\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v)}{2}\right)}{\sqrt{\frac{\sum_{k=1}^{n}\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}} \\
= & -\frac{\exp\left(-\frac{1}{2}\frac{1}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}\left[\tau_i s_i - \frac{\pi}{\gamma^2\sigma_z^2}(\mu-\gamma\bar{z}) + \tau_v\bar{v} - (\tau_i+\tau_v+\frac{\pi^2}{\gamma^2\sigma_z^2}-\frac{\pi}{\gamma^2\sigma_z^2})p\right]^2\right)}{\sqrt{\frac{\sum_{k=1}^{n}\tau_k+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}{\tau_i+\frac{\pi^2}{\gamma^2\sigma_z^2}+\tau_v}}}
\end{aligned}
$$

Notice that this is identical to the expected utility achieved under complete information (but no partnership creation). □

Appendix C: Market Powers in the Input Market

First consider a heterogeneous agents extension to Kyle (1989). In this case there are N investors who do not form partnerships. Perceiving a residual supply curve

$p = p_i + \lambda_i x_i$, investor i chooses x_i to maximize

$$
\begin{aligned}
& \mathbb{E}\left[-\exp(-\rho_i(v - p(x_i))x_i)|s_i, p\right], \text{ or} \\
& \mathbb{E}\left[-\exp(-\rho_i(v - p_i - \lambda_i x_i)x_i)|s_i, p_i\right] \\
\Leftrightarrow \quad & -\exp\left[-\rho_i(\mathbb{E}(v|s_i, p_i) - p_i)x_i + \rho_i\lambda_i x_i^2 + \frac{1}{2}\rho_i^2 \mathrm{Var}(v|s_i, p_i)x_i^2\right] \\
\overset{\text{FOC}}{\Longrightarrow} \quad & x_i = \frac{(\mathbb{E}(v|s_i, p_i) - p_i)}{2\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i)}
\end{aligned}
$$

thus

$$
\begin{aligned}
p &= p_i + \lambda_i x_i && (2.15) \\
&= p_i + \frac{\lambda_i(\mathbb{E}(v|s_i, p_i) - p_i)}{2\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i)} && (2.16) \\
\overset{\text{FOC}}{\Longrightarrow} \quad & (2\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i))p = (\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i))p_i + \lambda_i \mathbb{E}(v|s_i, p_i) && (2.17) \\
\Rightarrow \quad & p_i = \frac{(2\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i))p - \lambda_i \mathbb{E}(v|s_i, p_i)}{(\lambda_i + \rho_i \mathrm{Var}(v|s_i, p_i))} && (2.18)
\end{aligned}
$$

thus (given that p_i and p are informationally equivalent)

$$x_i = \frac{\mathbb{E}(v|s_i, p) - p}{(\lambda_i + \rho_i \mathrm{Var}(v|s_i, p))}$$

Conjecture a linear strategy profile $x_i = \mu_i + \beta_i s_i - \gamma_i p$, then by market clearing

$$\sum_{k=1}^{N} \mu_k + \sum_{k=1}^{N} \beta_k s_k - \sum_{k=1}^{N} \gamma_k p + z = 0 \tag{2.19}$$

or

$$
\begin{aligned}
& x_i + \sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k - \sum_{k \neq i} \gamma_k p + z = 0 && (2.20) \\
\Rightarrow \quad & p = \frac{x_i + \sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k} && (2.21) \\
\Rightarrow \quad & \lambda_i = \frac{1}{\sum_{k \neq i} \gamma_k}, p_i = \frac{\sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k} && (2.22)
\end{aligned}
$$

and

$$
\begin{aligned}
x_i &= \frac{\bar{v} + \frac{(\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(s_i - \bar{v}) + \sum_{k \neq i} \beta_k \tau_i^{-1} [\sum_{k \neq i} \beta_k (s_k - \bar{v}) + z - \bar{z}]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{\sum_{k \neq i} \mu_k + \sum_{k \neq i} \beta_k s_k + z}{\sum_{k \neq i} \gamma_k}}{2 \frac{1}{\sum_{k \neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} && (2.23) \\
&= \mu_i + \beta_i s_i - \gamma_i p && (2.24) \\
&= \mu_i + \beta_i s_i - \gamma_i \frac{\sum_{k=1}^{N} \mu_k + \sum_{k=1}^{N} \beta_k s_k + z}{\sum_{k=1}^{N} \gamma_k} && (2.25)
\end{aligned}
$$

Equating coefficients we get

$$
\begin{aligned}
\frac{\bar{v} + \frac{(\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2)(-\bar{v}) + \sum_{k \neq i} \beta_k \tau_i^{-1} [\sum_{k \neq i} \beta_k (-\bar{v}) - \bar{z}]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{\sum_{k \neq i} \mu_k}{\sum_{k \neq i} \gamma_k}}{2 \frac{1}{\sum_{k \neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} &= \mu_i - \gamma_i \frac{\sum_{k=1}^{N} \mu_k}{\sum_{k=1}^{N} \gamma_k} && (2.26) \\
\frac{\frac{\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}}{2 \frac{1}{\sum_{k \neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} &= \beta_i - \gamma_i \frac{\beta_i}{\sum_{k=1}^{N} \gamma_k} && (2.27) \\
\frac{\frac{\sum_{k \neq i} \beta_k \tau_i^{-1}}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2} - \frac{1}{\sum_{k \neq i} \gamma_k}}{2 \frac{1}{\sum_{k \neq i} \gamma_k} + \rho_i \frac{\tau_i^{-1}[\sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k \neq i} \beta_k)^2 + \tau_v \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \tau_v \sigma_z^2] + \sum_{k \neq i} \beta_k^2 \tau_k^{-1} + \sigma_z^2}} &= -\frac{\gamma_i}{\sum_{k=1}^{N} \gamma_k} && (2.28)
\end{aligned}
$$

Equation (2.27) and (2.28) lead to

$$\frac{[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2](\sum_{k=1}^N\gamma_k-\sum_{k\neq i}\gamma_k\beta_i\rho_i\tau_i^{-1})}{\tau_i^{-1}[(\sum_{k\neq i}\beta_k)^2+\tau_v\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2}=2\beta_i \tag{2.29}$$

$$\frac{\sum_{k=1}^N\gamma_k\sum_{k\neq i}\beta_k\tau_i^{-1}+\gamma_i\rho_i\tau_i^{-1}[\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i}\beta_k)^2+\tau_v\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\tau_v\sigma_z^2]+\sum_{k\neq i}\beta_k^2\tau_k^{-1}+\sigma_z^2}=\frac{-\gamma_i}{\sum_{k\neq i}\gamma_k}+1 \tag{2.30}$$

The three equations above defines the equilibrium.

Profit-sharing Now consider $M \leq N$ investors agree to partner. Perceiving a residual supply curve $p = p_i + \lambda_i x_i$, partner i chooses x_i to maximize

$$\mathbb{E}\left[-\exp(-\rho_i a_i(v-p_i-\lambda_i x_i)(x_i+\sum_{k\neq i,k=1}^{M}x_k))|s_i,p\right],$$

Conjecture a linear strategy profile $x_i = \mu_i + \beta_i s_i - \gamma_i p$, thus

$$x_i+\sum_{k\neq i,k=1}^{M}x_k = x_i+\sum_{k\neq i,k=1}^{M}\mu_k+\sum_{k\neq i,k=1}^{M}\beta_k s_k-\sum_{k\neq i,k=1}^{M}\gamma_k p \tag{2.31}$$

and

$$\begin{pmatrix} -\rho_i a_i(v-p_i-\lambda_i x_i) \\ x_i+\sum_{k\neq i,k=1}^M x_k \end{pmatrix}_{|s_i,p} \sim$$
$$\mathcal{N}\left(\begin{matrix}\begin{bmatrix} -\rho_i a_i(\mathbb{E}(v|s_i,p)-p_i-\lambda_i x_i) \\ x_i+\sum_{k\neq i,k=1}^M\mu_k+\sum_{k\neq i,k=1}^M\beta_k\mathbb{E}(v|s_i,p)-\sum_{k\neq i,k=1}^M\gamma_k p\end{bmatrix}, \\ \begin{bmatrix} \rho_i^2 a_i^2\mathrm{Var}(v|s_i,p) & -\rho_i a_i\sum_{k\neq i,k=1}^M\beta_k\mathrm{Var}(v|s_i,p) \\ -\rho_i a_i\sum_{k\neq i,k=1}^M\beta_k\mathrm{Var}(v|s_i,p) & (\sum_{k\neq i,k=1}^M\beta_k)^2\mathrm{Var}(v|s_i,p)+\sum_{k\neq i,k=1}^M\beta_k^2\tau_k^{-1}\end{bmatrix}\end{matrix}\right)$$

whose certainty equivalent is (by Lemma 3.1.1)

$$-\frac{\exp(\frac{A}{2B})}{\sqrt{B}}, \tag{2.32}$$

where

$$\begin{aligned}
A &= \rho_i^2 a_i^2 (\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i)^2 [(\sum_{k\neq i,k=1}^{M} \beta_k)^2 \mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M} \beta_k^2 \tau_k^{-1}] \\
&+ [x_i + \sum_{k\neq i,k=1}^{M} \mu_k + \sum_{k\neq i,k=1}^{M} \beta_k \mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M} \gamma_k p]^2 \rho_i^2 a_i^2 \mathrm{Var}(v|s_i,p) \\
&- 2\rho_i a_i (\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i) \\
&\quad [x_i + \sum_{k\neq i,k=1}^{M} \mu_k + \sum_{k\neq i,k=1}^{M} \beta_k \mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M} \gamma_k p] \\
&\quad [1 + \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p)] \\
B &= [1 + \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p)]^2 \\
&- \rho_i^2 a_i^2 \mathrm{Var}(v|s_i,p) [(\sum_{k\neq i,k=1}^{M} \beta_k)^2 \mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M} \beta_k^2 \tau_k^{-1}]
\end{aligned}$$

Taking FOC w.r.t. x_i we get

$$\begin{aligned}
&-\lambda_i \rho_i^2 a_i^2 (\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i) [(\sum_{k\neq i,k=1}^{M} \beta_k)^2 \mathrm{Var}(v|s_i,p) + \sum_{k\neq i,k=1}^{M} \beta_k^2 \tau_k^{-1}] \\
&+ [x_i + \sum_{k\neq i,k=1}^{M} \mu_k + \sum_{k\neq i,k=1}^{M} \beta_k \mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M} \gamma_k p] \rho_i^2 a_i^2 \mathrm{Var}(v|s_i,p) \\
&+ \rho_i a_i \lambda_i [x_i + \sum_{k\neq i,k=1}^{M} \mu_k + \sum_{k\neq i,k=1}^{M} \beta_k \mathbb{E}(v|s_i,p) - \sum_{k\neq i,k=1}^{M} \gamma_k p] \\
&\quad [1 + \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p)] \\
&- \rho_i a_i (\mathbb{E}(v|s_i,p) - p_i - \lambda_i x_i) [1 + \rho_i a_i \sum_{k\neq i,k=1}^{M} \beta_k \mathrm{Var}(v|s_i,p)] = 0.
\end{aligned}$$

thus plug in $x_i = \mu_i + \beta_i s_i - \gamma_i p$, and since

$$\begin{aligned}
p &= \frac{x_i + \sum_{k\neq i,k=1}^N \mu_k + \sum_{k\neq i,k=1}^N \beta_k s_k + z}{\sum_{k\neq i,k=1}^N \gamma_k} = \frac{\sum_{k=1}^N \mu_k + \sum_{k=1}^N \beta_k s_k + z}{\sum_{k=1}^N \gamma_k} \\
\lambda_i &= \frac{1}{\sum_{k\neq i,k=1}^N \gamma_k} \\
p_i &= \frac{\sum_{k\neq i,k=1}^N \mu_k + \sum_{k\neq i,k=1}^N \beta_k s_k + z}{\sum_{k\neq i,k=1}^N \gamma_k} \\
\mathbb{E}(v|s_i,p_i) &= \bar{v} + \frac{(\sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \sigma_z^2)(s_i - \bar{v}) + \sum_{k\neq i,k=1}^N \beta_k \tau_i^{-1}[\sum_{k\neq i}\beta_k(s_k - \bar{v}) + z - \bar{z}]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^N \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \tau_v\sigma_z^2] + \sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \sigma_z^2} \\
\mathrm{Var}(v|s_i,p_i) &= \frac{\tau_i^{-1}[\sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \sigma_z^2]}{\tau_i^{-1}[(\sum_{k\neq i,k=1}^N \beta_k)^2 + \tau_v \sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \tau_v\sigma_z^2] + \sum_{k\neq i,k=1}^N \beta_k^2\tau_k^{-1} + \sigma_z^2}
\end{aligned}$$

Theorem 2.6.1. *Under profit sharing, we have*

$$\begin{aligned}
\beta_1 &= \frac{M\tau_e\gamma_1}{M\tau_e + \tau_v} \\
\beta_2 &= \frac{\tau_e\gamma_2}{\tau_v + \tau_e},
\end{aligned}$$

while γ_1 and γ_2 are determined by

$$\begin{aligned}
\frac{\gamma_1\rho}{M\tau_e + \tau_v} &= \frac{-\gamma_1 M}{(M-1)\gamma_1 + (N-M)\gamma_2} + 1 \\
\frac{\gamma_2\rho}{\tau_v + \tau_e} &= \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1.
\end{aligned}$$

Under information sharing, equation (2.29) and (2.30) lead to

$$\frac{(M\gamma_1 + (N-M)\gamma_2)M\tau_e - ((M-1)\gamma_1 + (N-M)\gamma_2)\beta_1\rho}{\tau_v + \tau_e} = 2\beta_1 \tag{2.33}$$

$$\frac{\gamma_1\rho}{\tau_v + M\tau_e} = \frac{-\gamma_1}{(M-1)\gamma_1 + (N-M)\gamma_2} + 1 \tag{2.34}$$

$$\frac{[M\gamma_1 + (N-M)\gamma_2]\tau_e - (M\gamma_1 + (N-M-1)\gamma_2)\beta_2\rho}{\tau_v + \tau_e} = 2\beta_2 \tag{2.35}$$

$$\frac{\gamma_2\rho}{\tau_v + \tau_e} = \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1 \tag{2.36}$$

we have

$$\beta_1 = \frac{M\tau_e\gamma_1}{\tau_v + M\tau_e},$$

and similarly we have

$$\beta_2 = \frac{\tau_e\gamma_2}{\tau_v + \tau_e},$$

while γ_1 and γ_2 are jointly determined by solving (2.34) and (2.36). The results are summarized below

Theorem 2.6.2. *Under information sharing, we have*

$$\begin{aligned} \beta_1 &= \frac{M\tau_e\gamma_1}{M\tau_e + \tau_v} \\ \beta_2 &= \frac{\tau_e\gamma_2}{\tau_v + \tau_e}, \end{aligned}$$

while γ_1 and γ_2 are determined by

$$\begin{aligned} \frac{\gamma_1\rho}{M\tau_e + \tau_v} &= \frac{-\gamma_1}{(M-1)\gamma_1 + (N-M)\gamma_2} + 1 \\ \frac{\gamma_2\rho}{\tau_v + \tau_e} &= \frac{-\gamma_2}{M\gamma_1 + (N-M-1)\gamma_2} + 1. \end{aligned}$$

Compared to the the profit-sharing case, both β_i and γ_i ($i \in \{1,2\}$) are smaller under profit-sharing.

CHAPTER 3

Optimal Crowdfunding Design to Harness Wisdom of the Crowd

Many business activities benefit from a “wisdom of the crowd” effect, that a population’s collective wisdom dominates even the most accurate judgment from any individuals within the group.[1] This is because the average over a large number of responses goes toward canceling the idiosyncratic noises associated with each individual judgment. This notion of wisdom of the crowd has been widely used to advocate for the emerging practice of security-based crowdfunding, which recently receives regulatory approval amidst increasing enthusiasm from the marketplace.[2] In security-based crowdfunding, an issuer raises funds from a large number of potential investors through an SEC-registered intermediary, either a broker-dealer or a funding portal. The investors then get paid back in the next period according to the security specification. This paper finds that under the presence of wisdom of the crowd among prospective investors, optimal security design generally includes a profit-sharing feature, which is frequently found within a partnership contract, yet absent from typical common equities. This finding provides fresh guidance for the nascent security-based crowdfunding practice.

A typical common equity is inferior to a profit-sharing contract because it only benefits the issuer of all investors’ collective wisdom (via observing their aggre-

---

[1] See for example Surowiecki (2005) and Kremer, Mansour, and Perry (2014).

[2] Title III of the JOBS Act created a federal exemption under the current securities laws for security-based crowdfunding in Oct 2015. See http://www.sec.gov/news/pressrelease/2015-249.html.

gate investment amount) but not the investors. Under a typical common equity arrangement, each investor is entitled to the proceeds produced solely by her own investment, thus bearing all the risks incurred by the noise in her own assessment. As long as the investor is risk-averse, she will be reluctant to commit too much to the investment project, worrying about potential investment mistakes. However, when a profit-sharing feature is embedded into the compensation structure, investors enjoy a risk-sharing benefit of their idiosyncratic noises in private information. For a given level of risk-bearing capacity, each investor would on average be more willing to invest, facilitating the financing need from the issuer.

To illustrate the structure and benefit of profit-sharing, consider a simple two-agent example. Alice and Bob are two deep-pocketed and identically risk-averse investors. They individually decide on how much money to invest in a risky business opportunity. Alice and Bob have different assessments of the business return, as they rely on independent yet unbiased private information sources to update their posterior beliefs. Neither investor has access to the other's private information. Section 3.1 analyzes this example in detail and proves a somewhat surprising result: despite Alice's superior information quality, as a Nash equilibrium outcome they both prefer a fifty-fifty profit-sharing contract. Under a fifty-fifty deal, both Alice and Bob obtain a payoff that is exactly the same *as if* he or she had access to the other's private information (even though he/she actually does not). In another word, (appropriate) profit-sharing harnesses the wisdom of the crowd. Although the fifty-fifty contract is just a special result due to identical preference, it embodies a more general insight: in a world featuring decentralized possession of information among many heterogeneous individuals, some *simple* profit-sharing contracts, independent of each individual's private information or informedness, could coordinate individual actions to the effect of aggregating their private information.

The optimal profit-sharing contract developed in this paper has three nice properties. First, it is *first best*. Section 3.2 proves that optimal profit-sharing fully employs wisdom of the crowd when preferences feature no confounding wealth effects (as is standard in the investment literature), production technology is constant return to scale (as in most investment cases), and private signals fall into a large class of exponential family distributions (including standard cases of normal idiosyncratic noises).[3] Other than these standard assumptions, perfect coordination holds for *any* prior distributions of the business return. Second, it is *simple*. The optimal sharing contract *only* requires information on risk preferences. In particular, it does not depend on how well-informed each individual is, which is private information and often hard to solicit. Such simplicity makes implementing the optimal profit-sharing contracts particularly easy. For example, on a crowdfunding platform, all information needed to determine the optimal sharing rule can be proxied by standard questionnaire items on income, wealth, investment experience, investment objectives, etc. that are readily available at the time of account opening. Section 3.3 further proves that all investors have strict incentives to truthfully report to these questionnaire items. Third, it is *cost-effective*. Because profit-sharing does not involve direct exchange of private information, no sophisticated communication technology is required, no incentive to encourage participation in communication is asked for, and no worry of individuals telling lies is needed. A simple profit-sharing contract gives all.

An optimal profit-sharing contract that harnesses wisdom of the crowd slightly differs from traditional equity contracts. An equity contract is a “revenue-sharing” contract, which splits proportionally the total payoff including each individual’s initial investment.[4] Yet a profit-sharing contract splits according to a pre-specified rule the total payoff excluding initial investment. While a traditional equity contract is no worse than a profit-sharing contract when private information is absent,

---

[3] Approximate results under more general setups will be discussed in Section 2.3.

[4] Indeed revenues are often, though not always, spit in proportion to each individual’s initial investment.

it is strictly dominated by an optimal profit-sharing contract whenever collective wisdom is present. In this sense, equity contracts may be a historical legacy when the notion of wisdom of the crowd was less relevant, and as we enter an information age securities featuring profit-sharing shall be given further consideration.

Even though explicit profit-sharing contracts as described in the current paper are yet to be widely adopted, for centuries implicit profit-sharing contracts have been underlying the structures of many partnership companies, in which multiple joint-owners share residual earnings (according to a pre-specified rule) from and (individually) provide productive inputs to a particular technology.[5] formalizes this idea, presents historical and contemporary evidence, and shows mm

The rest of the paper is organized as follows. Section 3.1 analyzes the solution to the Alice-Bob question. Section 3.2 contains more discussions on sufficient conditions for perfect information aggregation. Section 3.3 sets up the formal model, and derives the optimal profit-sharing contract. Section 3.4 relates existing literature. Section 3.5 concludes.

3.1 Analysis of the Alice-Bob Example

This section proves that, even if Alice has more accurate private signal, she always finds it optimal for herself to *equally* share total investment profit with Bob. So does Bob.

I first formalize the leading example in precise mathematical terms. Since both investors are deep-pocketed, their preferences feature little wealth effect, and could be summarized by a constant absolute risk-aversion (CARA) utility function $u(W) = -e^{-\rho \cdot W}$ for some $\rho > 0$. They individually decide on how much money to invest in a business opportunity with *net* return $\tilde{r}$. In another word, if

---

[5] This anatomy of joint-stock companies follows traditions in the corporate law literature (Hansmann (2009)), and makes precise the nexus of contracts view in theory of the firm.

the total payoff to the business is $\tilde{v}$, then $\tilde{r} = \tilde{v} - 1$ with intertemporal discount rate normalized to zero. The focus on net rather than gross return distinguishes a profit-sharing contract (to be introduced momentarily) from traditional equity contracts. Investor i's independent yet unbiased private information sources translates into a private signal $s_i = r + \epsilon_i$, where r denotes the realization of the business return $\tilde{r}$, $\epsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$, ϵ_i is independent of $\tilde{r}$, $i \in \{A, B\}$, and ϵ_A is independent of ϵ_B. Although not necessary, for exposition ease I assume in this section that the net return also follows an normal distribution, i.e. $\tilde{r} \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$. This simplification is helpful for developing intuitions. It also strengthens connections with the CARA-normal tradition in standard investment models first developed by Lintner (1965)). Section 3.2 relaxes this normality assumption. If Alice's information is more accurate than Bob's, then $\tau_A > \tau_B$.

Given an equal division of investment profits, investor i's problem is to choose an investment amount x_i based on s_i such that

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x + \tilde{x}_{-i}(s_{-i})]} | s_i],$$

where $i \in \{A, B\}$ and $-i = \{A, B\} \backslash \{i\}$. Because the optimum to the right hand side depends on i's belief of $x_{-i}(s_{-i})$, the solution is given in a Nash equilibrium.

Definition A Nash Equilibrium under equal profit-sharing in the Alice-Bob example consists of two investment functions $x_A(\cdot)$ and $x_B(\cdot)$ such that

$$x_i(s_i) = \text{argmax}_x \mathbb{E}[-e^{-\rho \frac{1}{2} \tilde{r}[x + \tilde{x}_{-i}(s_{-i})]} | s_i], \tag{3.1}$$

where $i \in \{A, B\}$ and $-i = \{A, B\} \backslash \{i\}$.

Before solving the Nash equilibrium explicitly, let's first discuss intuitively how could a profit-sharing contract change Alice's and Bob's incentives and thus empower them with their collective wisdom (of a crowd of size two).

The exponent on the right hand side of (3.1) is the sum of two parts: $-\frac{1}{2}\rho\tilde{r}x$ and $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$. The first part $-\frac{1}{2}\rho\tilde{r}x$, compared to $-\rho\tilde{r}x$ when there is no profit-sharing, divides the sensitivity of i's payoff to her (his) investment decision by two. Hence it appears *as if* profit-sharing makes investor i half less risk-averse.[6] This observation indicates that investor i could invest more *aggressively*, and in particular be more responsive to her (his) own signal, thus enhancing aggregate use of private information.

Such aggressiveness however could potentially lead to (inefficient) overuse of prior information. This negative effect, however, is counteracted by the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$, which involves an interaction between $\tilde{r}$ and investor $-i$'s investment. Because private signals are correlated due to the common component r, Alice (Bob) would worry that when she (he) has a high signal and invests a lot, so would Bob (Alice). Intuitively this concern would make Alice (Bob) act more *conservatively* to the (public) prior, balancing the overuse tendency. An optimal profit-sharing contract is expected to obtain a perfect balance.

To understand why a fifty-fifty contract is optimal when Alice and Bob have the same preference, we again look at the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$. Because investor $-i$'s investment is a function of $-i$'s private information, the second part $-\frac{1}{2}\rho\tilde{r}\tilde{x}_{-i}(s_{-i})$ *effectively* exposes investor i to $-i$'s private information. Given the same preference, only when Alice and Bob agree to follow a fifty-fifty sharing contract would Alice (Bob) act exactly the same as what Bob (Alice) would like to had he (she) got access to her (his) private information. In another word, a fifty-fifty profit-sharing profit-sharing contract perfectly aligns both investors' incentives and makes each investor a perfect "agent" for the other.

I explicitly solve the equilibrium and confirm the intuitions above. I will prove the existence and uniqueness (up to a constant) of the Nash equilibrium under a

---

[6] In certain sense, risk aversion deters prevents the full use of information. Profit-sharing, thus like risk-sharing, encourages the use of information.

more general setting in Section 3.2. However, in a special case in which all random variables are normally distributed, a linear Nash equilibrium (which happens to be the unique Nash equilibrium) is easily obtained via guess and verify. Assume

$$x_i(s_i) = \alpha + \beta_i s_i,$$

then equation (3.1) leads to

$$\alpha + \beta_i s_i \quad = \quad \text{argmax}_x - \mathbb{E}[e^{\left(-\frac{1}{2}\rho\tilde{r}\right)(x+\alpha+\beta_{-i}\tilde{s}_{-i})}|s_i]. \tag{3.2}$$

The conditional expectation on the right hand side of (3.2) is similar to the moment-generating function of a non-central χ^2-distributed random variable (because both $-\frac{1}{2}\rho\tilde{r}$ and $x+\alpha+\beta_{-i}\tilde{s}_{-i}$, an affine transformation of the normal variable $\tilde{s}_{-i}$, follow normal distributions), which has a closed-form expression given by the following lemma.

Lemma 3.1.1. *If* $\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}\right)$, *where* $(\sigma_{12}-1)^2 > \sigma_{11}\sigma_{22}$ *then*

$$\mathbb{E}\left[e^{\tilde{y}_1\tilde{y}_2}\right] = \frac{\exp\left\{(\theta_2^2\sigma_{11} - 2\theta_1\theta_2\sigma_{12} + \theta_1^2\sigma_{22} + 2\theta_1\theta_2)/(2[(\sigma_{12}-1)^2 - \sigma_{11}\sigma_{22}])\right\}}{\sqrt{(\sigma_{12}-1)^2 - \sigma_{11}\sigma_{22}}}.$$

Proof. Standard integration. □

Plug in $-\frac{1}{2}\rho\tilde{r}$ and $x+\alpha+\beta_{-i}\tilde{s}_{-i}$ into Lemma 3.1.1, and notice that conditional on s_i,

$$\begin{bmatrix} -\frac{1}{2}\rho\tilde{r} \\ x+\alpha+\beta_{-i}\tilde{s}_{-i} \end{bmatrix}_{|s_i} \sim \mathcal{N}\left(\begin{bmatrix} -\frac{\rho}{2}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \\ x+\alpha+\beta_{-i}\frac{\tau_r\bar{r}+\tau_i s_i}{\tau_r+\tau_i} \end{bmatrix}, \begin{bmatrix} \frac{\rho^2}{4(\tau_r+\tau_i)} & -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} \\ -\frac{\rho\beta_{-i}}{2(\tau_r+\tau_i)} & \beta_{-i}^2\left(\frac{1}{\tau_r+\tau_i}+\frac{1}{\tau_{-i}}\right) \end{bmatrix}\right),$$

Notice that x, the variable we maximize over, only enters the numerator of the exponent in the above expression in a linear-quadratic function, thus (3.2) leads to

$$\begin{aligned}
\alpha + \beta_i s_i &= \text{argmin}_x \Bigg[\left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right)^2 \frac{\rho^2}{4(\tau_r + \tau_i)} - \frac{\rho}{2} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \frac{\rho \beta_{-i}}{\tau_r + \tau_i} \\
&+ \left(\frac{\rho}{2} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right)^2 \beta_{-i}^2 \left(\frac{1}{\tau_r + \tau_i} + \frac{1}{\tau_{-i}} \right) - \rho \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \left(x + \alpha + \beta_{-i} \frac{\tau_r \bar{r} + \tau_i s_i}{\tau_r + \tau_i} \right) \Bigg] \\
&= \frac{2}{\rho} (\tau_r \bar{r} + \tau_i s_i) - \alpha.
\end{aligned}$$

Matching coefficients gives $\alpha = \frac{1}{\rho} \tau_r \bar{r}$ and $\beta_i = \frac{2}{\rho} \tau_i$, leading to

$$\begin{cases} x_A = \frac{1}{\rho}(\tau_r \bar{r} + 2\tau_A s_A) \\ x_B = \frac{1}{\rho}(\tau_r \bar{r} + 2\tau_B s_B) \end{cases}.$$

The 2-s in the second terms of the above expressions confirm our first intuition that under profit-sharing, each investor becomes more responsive to her (his) private information, while the absence of 2-s in the first terms reflects a perfect "canceling out" effect between the simultaneous aggressiveness and conservativeness under an optimal profit-sharing contract.

For any particular joint realization of return and private signals, investor i's payoff under equal profit-sharing is

$$r \frac{x_A(s_A) + x_B(s_B)}{2} = \frac{r}{\rho} (\tau_r \bar{r} + \tau_A s_A + \tau_B s_B). \tag{3.3}$$

Let's compare this outcome with a full-information benchmark. When Alice could (hypothetically) make independent investment decisions based on both her private signal and Bob's, her investment amount would be given by

$$\begin{aligned}
x'_A(s_A, s_B) &= \text{argmax}_x \mathbb{E}[-e^{-\rho r x} | s_A, s_B] \\
&= \text{argmax}_x - e^{-\rho \mathbb{E}(r|s_A,s_B)x + \frac{1}{2}\text{Var}(r|s_A,s_B)\rho^2 x^2} \\
\Rightarrow x'_A(s_A, s_B) &= \frac{\mathbb{E}(r|s_A, s_B)}{\rho \text{Var}(r|s_A, s_B)} \\
&= \frac{1}{\rho}(\tau_r \bar{r} + \tau_A s_A + \tau_B s_B).
\end{aligned}$$

and thus payoff $r x'_A(s_A, s_B) = \frac{r}{\rho}(\tau_A s_A + \tau_B s_B)$, which is exactly the same as under profit-sharing (expression (3.3)), confirming our second intuition that the increased total use of information benefits both investors perfectly.

The above observation is summarized below.

Theorem 3.1.2. *For all realizations of the state of nature $\{r, s_A, s_B\}$, each investor's payoff under equal division of profits is always equal to that under a full-information benchmark. So does the expected utility. A well-designed profit-sharing contract perfectly empowers collective wisdom, leading to best outcomes for both investors.*

It is worth noting that unlike other studies on the efficient use of information (e.g. Angeletos and Pavan (2007), Amador and Weill (2010)), where strategic complementarity/substitutability in technology plays an important role, in my model there is no strategic interaction between Alice and Bob from the technology itself.It is the profit sharing contract that introduces strategic interdependencies between the two investors. To see the point more concretely, consider a hypothetical case in which Alice is forced to only get half of her investment profit, and does not enjoy the half contributed by Bob. In this no strategic interdependence case Alice would invest $\frac{1}{\rho}(2\tau_r \bar{r} + 2\tau_A s_A)$. In comparison, under profit-sharing there is no 2 in front of the first term in x_A (Alice's investment), due to Alice's "conservativeness" toward Bob's correlated actions when she is exposed to Bob's contribution.

An explanation for wisdom of the crowd is that there is idiosyncratic noise associated with each individual's judgment (about r here), and taking the average over a large number of conditionally independent (albeit unconditionally correlated due to the common term r) signals go toward canceling the effect of this noise by law of large numbers. It is apparently reminiscent of what has become conventional wisdom since the seminal work of Markowitz (1952) that proper diversification achieves optimal return-risk trade-off.[7] Indeed in the CARA-normal setup, under profit-sharing part of investor i's compensation comes from one half of the expected value of her contribution while bearing only a quarter of its variance. However, there are several differences between Theorem 3.1.2 and traditional diversification arguments. First, diversification in portfolio theory relies on pooling multiple assets, yet Theorem 3.1.2 only considers one single business, and "diversification" is achieved via teaming agents. Second, portfolio theory usually does not involve asymmetric information, yet Theorem 3.1.2 requires dispersed private information. Without private information (i.e., $\tau_e = 0$), profit-sharing would make no difference. Third, as Section 3.2 will show, my result extends beyond normal distributions, while traditional mean variance analysis crucially depends on the absence of higher (than second) moments.

3.1.1 An Illustrative Example

To further illustrate the Alice-Bob example and the difference between profit-sharing and common stocks, consider a numerical example in which the *net* return is realized as 10%. Under common stocks, (i.e. no profit-sharing), if *ex ante* it is optimal of Alice to invest \$200 and Bob \$100, then the payoffs to both investors would look like the following:

---

[7] It is obvious that signals being conditionally (on r) independent is not necessary. As long as they are conditionally imperfectly correlated, wisdom of the crowd applies.

	Inv.Amt	Shr.G.	Gross profit	Final payoff
A	\$200	2/3	(\$200 + \$100)×	\$330 × 2/3 = \$220
B	\$100	1/3	(1 + 10%) = \$330	\$330 × 1/3 = \$110

If instead Alice and Bob agree to a fifty-fifty profit-sharing contract, and assume that their investment does not change (which is not optimal), then the payoff matrix to both investors would look like:

	Shr.N.	Inv.Amt	Net profit	final payoff
A	1/2	\$200	(\$200 + \$100)×	\$200 + \$30 × 1/2 = \$215
B	1/2	\$100	10% = \$30	\$100 + \$30 × 1/2 = \$115

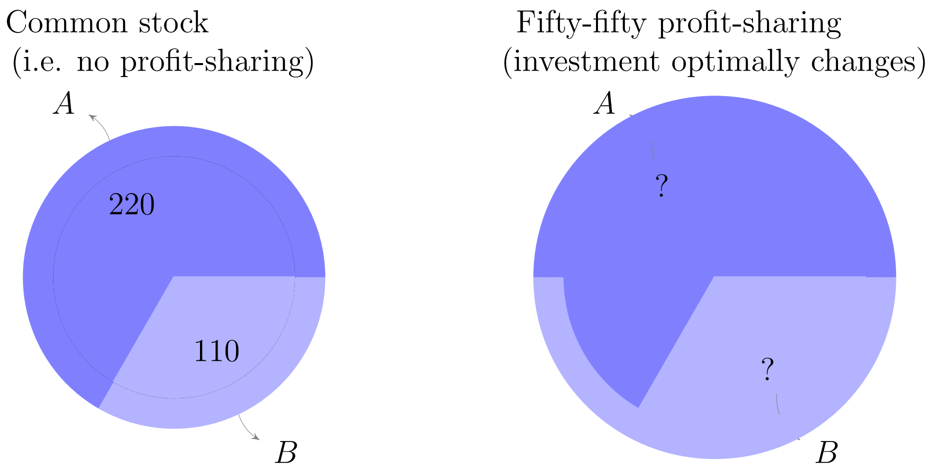
In this particular scenario, agreeing to fifty-fifty profit-sharing is *ex post* inferior to common stock for Alice (because she gets \$220 under common stock but only \$215 under fifty-fifty profit-sharing. However, as we analyze in the previous section, having a fifty-fifty profit-sharing contract changes the optimal investment amount for both Alice and Bob. Indeed, since on average both investors would invest more with the risk-sharing benefit from profit-sharing, Alice is indeed on average entitled to a smaller piece of a bigger pie under profit-sharing, as Figure 3.1.1 illustrates.

3.2 Beyond Normality

This section relaxes the normality assumption in the previous example, and explores sufficient conditions for profit-sharing to be a perfect information aggregator.

Denote $u(W) = -e^{-\rho W}$, and consider general distributions of r and $s_i, i \in$

Figure 3.1: **Profit-Sharing Entitles Alice to a Smaller Piece of a Bigger Pie**



$\{A, B\}$. Under a full-information benchmark $x_i(s_i, s_{-i})$ maximizes

$$\begin{aligned}
& \mathbb{E}\left[u(x\tilde{r})|s_i, s_{-i}\right] \\
= & \int u(xr)f(r|s_i, s_{-i})dr \\
= & \int u(xr)f(s_{-i}|r, s_i)f(s_i|r)f(r)\frac{1}{f(s_i, s_{-i})}dr \\
= & \int u(xr)f(s_{-i}|r)f(s_i|r)f(r)\frac{1}{f(s_i, s_{-i})}dr, (\because s_i \perp\!\!\!\perp s_{-i}|r),
\end{aligned}$$

or equivalently $x(s_i, s_{-i})$ maximizes

$$\int u(xr)f(s_{-i}|r)f(s_i|r)f(r)dr \tag{3.4}$$

Assume the profit-sharing agreement stipulates that investor i gets α_i of the total profit ($\sum_i \alpha_i = 1$), then under profit-sharing $x_i(s_i)$ maximizes (in a Nash equilibrium)

$$
\begin{aligned}
& \mathbb{E}\left[u\left(\alpha_i x \tilde{r} + \alpha_i x_{-i}(s_{-i})\tilde{r}\right)|s_i\right] \\
= & \iint u\left(\alpha_i x r + \alpha_i x_{-i}(s_{-i}) r\right) f(r, s_{-i}|s_i) ds_{-i} dr \\
= & \iint u\left(\alpha_i x r + \alpha_i x_{-i}(s_{-i}) r\right) f(s_{-i}|r) f(s_i|r) f(r) \frac{1}{f(s_i)} ds_{-i} dr,
\end{aligned}
$$

or equivalently $x(s_i)$ maximizes

$$\iint u\left(\alpha_i x r + \alpha_i x_{-i}(s_{-i}) r\right) f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr \tag{3.5}$$

Taking first-order conditions we have that

$$(3.4) \Rightarrow \int u'(x_i(s_i, s_{-i}) r) r f(s_{-i}|r) f(s_i|r) f(r) dr = 0 \tag{3.6}$$

$$(3.5) \Rightarrow \iint u'\left(\alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r\right) r f(s_{-i}|r) f(s_i|r) f(r) ds_{-i} dr = 0, \tag{3.7}$$

where (with some abuse of notation) $x(s_i, s_{-i})$ denotes the optimal investment amount given signal s_i and s_{-i} under the full information benchmark, while $x_i(s_i)$ denotes investor i's investment amount given signal s_i in the profit-sharing Nash equilibrium.

In order to keep tractability, in the spirit of Breon-Drish (2015), I further assume that the likelihood function of r given private signals $s_i, i \in \{A, B\}$ lies in an *exponential family*, an assumption extensively used in Bayesian statistics and decision theories to preserve closed-form expression.[8] Precisely, assume that

$$f(s_i|r) = h_i(s_i) e^{r k_i s_i} g(r)$$

---

[8] E.g. exponential family is particularly useful for deriving conjugate priors.

for some constant k_i and positive function $h_i(\cdot)$. then

$$(3.6) \quad \Rightarrow \quad \int e^{-\rho x_i(s_i,s_{-i})r} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} g(r) h_i(s_i) e^{rk_i s_i} g(r) f(r) dr = 0 \tag{3.8}$$

$$(3.7) \quad \Rightarrow \quad \iint e^{-\rho(\alpha_i x_i(s_i)r + \alpha_i x_{-i}(s_{-i})r)} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} g(r) h_i(s_i) e^{rk_i s_i} g(r) f(r) ds_{-i} dr = 0 \tag{3.9}$$

thus (factoring out $h_i(s_i)$)

$$(3.8) \quad \Rightarrow \quad \int e^{-\rho x_i(s_i,s_{-i})r + rk_{-i}s_{-i} + rk_i s_i} r g^2(r) f(r) dr = 0 \tag{3.10}$$

$$\begin{aligned}(3.9) \quad \Rightarrow \quad & \iint e^{-\rho(\alpha_i x_i(s_i)r + \alpha_i x_{-i}(s_{-i})r)} r h_{-i}(s_{-i}) e^{rk_{-i}s_{-i} + rk_i s_i} g^2(r) f(r) ds_{-i} dr = 0 \\ \Rightarrow \quad & \int e^{-\rho\alpha_i x_i(s_i)r + rk_i s_i} r g^2(r) f(r) \left(\int e^{-\rho\alpha_i x_{-i}(s_{-i})r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} \right) dr = 0 \end{aligned} \tag{3.11}$$

We thus have the following result

Theorem 3.2.1. *Under the full-information benchmark, equation (3.10) has a unique solution, which is linear in s_i and s_{-i}. Similarly, under any profit-sharing agreement (i.e. for any given α_i), equation (3.11) has a unique Nash equilibrium, in which investor i's strategy is linear in s_i, $\forall i$. When the profit-sharing agreement is optimally designed, profit-sharing obtains the same payoff as in the full-information benchmark.*

Proof. Consider the equation of x

$$\int e^{xr} r g^2(r) f(r) dr = 0. \tag{3.12}$$

Taking derivative with respect to x immediately tells that equation (3.12) has at most one solution, denoted as X. Compared to equation (3.10) we get $x_i(s_i, s_{-i}) = \frac{1}{\rho}(k_{-i}s_{-i} + k_i s_i - X)$.

Similarly, consider the equation of x

$$\int e^{rx} r g^2(r) f(r) H_{-i}(r) dr = 0,$$

where $H_{-i}(r) = \int e^{-\rho\alpha_i x_{-i}(s_{-i})r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} > 0$. Taking derivative with respect to x immediately tells that the equation features at most one solution (for a given $x_{-i}(s_{-i})$). Compared to equation (3.11) we get that $x_i(s_i) = \frac{k_i s_i - C}{\rho\alpha_i}$, where C is a constant such that

$$\int e^{rC} r g^2(r) f(r) H_{-i}(r) dr = 0. \tag{3.13}$$

By the same logic, $x_{-i}(s_{-i}) = \frac{k_{-i}s_{-i} - C'}{\rho\alpha_{-i}}$, where C' is also a constant such that

$$\int e^{rC'} r g^2(r) f(r) H_i(r) dr = 0, \tag{3.14}$$

where $H_i(r) = \int e^{-\rho\alpha_{-i} x_i(s_i) r} h_i(s_i) e^{rk_i s_i} ds_i > 0$. Plug in x_i and x_{-i} into (3.13) , we have

$$\int e^{rC} r g^2(r) f(r) \int e^{\frac{\alpha_i}{\alpha_{-i}}(C' - k_{-i}s_{-i})r} h_{-i}(s_{-i}) e^{rk_{-i}s_{-i}} ds_{-i} dr = 0. \tag{3.15}$$

If $\alpha_i = \alpha_{-i} = \frac{1}{2}$, equation (3.15) simplifies into (after factoring out $\int h_{-i}(s_{-i}) ds_{-i}$)

$$\int e^{r(C'+C)} r g^2(r) f(r) dr = 0. \tag{3.16}$$

Since equation (3.12) has at most one solution, we have $C + C' = X$. Thus under profit-sharing the payoff to investor i for a given realization of r and private signals is

$$\begin{aligned}
& \alpha_i x_i(s_i) r + \alpha_i x_{-i}(s_{-i}) r \\
= \; & \alpha_i \frac{k_i s_i - C}{\rho \alpha_i} r + \alpha_i \frac{k_{-i} s_{-i} - C'}{\rho \alpha_{-i}} r \\
= \; & \frac{r}{\rho}(k_i s_i - C + k_{-i} s_{-i} - C'), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2} \\
= \; & \frac{r}{\rho}(k_i s_i + k_{-i} s_{-i} - X), \text{ if } \alpha_i = \alpha_{-i} = \frac{1}{2},
\end{aligned}$$

which exactly equals to $rx(s_i, s_{-i})$, or the payoff under a full information benchmark. $\square$

Theorem 3.2.1 can be easily extended to scenarios with more than two investors of heterogeneous risk preferences. The exact math is tedious but straightforward, and is omitted for the sake of brevity.[9] Section 3.3, however, provides an illustrative example with normal distributions, which also embeds a general description of a joint-stock company.

3.3 Profit-Sharing and Wisdom of the Crowd

This section studies the optimal profit-sharing contract within a crowdfunding platform. I also discuss the incentive problems in the implementation of profit-sharing.

3.3.1 Implemention of Profit-Sharing on a Crowdfunding Platform

A crowdfunding platform implements a charter that stipulates a compensation scheme among its n investors for a particular project. In period $t = 0$, assume the

---

[9] I conjecture yet has not been able to prove that under more general settings nonlinear sharing rules exist to obtain perfect information aggregation.

charter entitles investor i, who has a constant absolute risk aversion parameter ρ_i, of a_i of the project's residual earnings, which will be realized by the end of period $t = 1$, where $\sum_{i=1}^{n} a_i = 1$. The project has pays off $Y = rX$, where Y is the total revenue, $r \sim \mathcal{N}(\bar{r}, \tau_r^{-1})$ is a stochastic factor productivity, and X is total amount of investment contributed by all the investors, i.e. $X = \sum_{j=1}^{n} x_i$, where x_i is investor i's investment contribution. The wisdom of the crowd assumption indicates that each investor has some private knowledge in assessing the stochastic project return. Assume investor i's private knowledge $s_i = r + e_i$, where $r \perp\!\!\!\perp e_i$ and $e_i \sim \mathcal{N}(0, \tau_j^{-1})$. Each investor independently decides on how much to invest to the firm.

The investment provided by the n investors is given in a Nash equilibrium. In particular, investor i chooses x_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_i r(x_i + \sum_{k\neq i} x_k)\right]\right)|s_i\right], \tag{3.17}$$

given her perception of other players' equilibrium investment x_k, $k \neq i$. The following theorem provides a linear Nash equilibrium solution for a given player.

Theorem 3.3.1. *A linear Nash equilibrium exists only when* $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n}\frac{1}{\rho_i}}$*, and in equilibrium investor i's investment could be given by*

$$x_i = \frac{\tau_r \bar{r}}{\rho_i} + \left(\sum_{k=1}^{n}\frac{1}{\rho_k}\right)\tau_i s_i - \left[\frac{\tau_r}{\rho_i} + \left(\sum_{k=1}^{n}\frac{1}{\rho_k}\right)\tau_i\right]p, \tag{3.18}$$

Proof. To be subsumed in the proof for Theorem 3.3.3. Notice that x_i could be alternatively written as $\frac{1}{\rho_i}\left[\tau_r(\bar{r} - p) + \frac{1}{a_i}\tau_i(s_i - p)\right]$, consistent with the special case in the Alice-Bob example. □

Notice that if investor i has full information, her investment would be

$$x_i = \frac{\tau_r(\bar{r} - p)}{\rho_i} + \frac{\sum_{i=1}^{n}\tau_i}{\rho_i}(i^* - p), \text{ where } i^* = \frac{\sum_{k=1}^{n}\tau_k s_k}{\sum_{k=1}^{n}\tau_k}.$$

Thus investor i's payoff is

$$r\left[\frac{\tau_r(\bar{r}-p)}{\rho_i}+\frac{\sum_{i=1}^{n}\tau_i}{\rho_i}(i^*-p)\right]$$

under both full information and in a firm. This is summarized in the following theorem.

Theorem 3.3.2. *Given her private information, an investor's expected utilities of participating in a properly structured n-investor profit-sharing scheme is identical to as if she could obtain other $n-1$ investors' private information without cost while taking common equities.*

A direct implication of this result is that profit-sharing raises investors' expected utilities, and thus is a voluntary outcome of economic evolution. The reason why profit-sharing benefits participating investors could be interpreted as, first, a well-designed profit-sharing contract empowers them with wisdom of the crowd, and second, it provides a means for idiosyncartic noise diversification.

3.3.2 Incentive to truthfully report risk-preferences

At the contract signing stage, each investor has the incentive to truthfully report his/her risk aversion. This is because if the crowdfunding platform follows distorted risk preference and thus stipulates alternative sharing rules, its investors will have incentives to sign side-contracts in addition to the firm in which they hold partial ownership. Such practice, although optimal from each investor's own perspective, in generally decreases total welfare. To see this, consider how investor i, who is entitled to an arbitrary share a_i of the firm's residual earnings, chooses x_i and X_i to maximize

$$\mathbb{E}\left[-\exp\left(-\rho_i\left[a_j r(x_i+\sum_{k\neq i}x_k)+rX_i\right]\right)|s_i\right], \tag{3.19}$$

given her (correct) anticipation of other investors' equilibrium input provision to the firm x_k, $k \neq i$. The following theorem provides a linear Nash equilibrium solution.

Theorem 3.3.3. *Each investor's expected utility is maximized when profit-sharing ratio in the project is divided according to investors' risk preferences, i.e.* $a_i = \frac{\frac{1}{\rho_i}}{\sum_{i=1}^{n} \frac{1}{\rho_i}}$*. In the resulting linear Nash equilibrium, investors do not have side-bets, that is* $X_i = 0$*.*

Proof of Theorem 3.3.3. A linear symmetric equilibrium is given by $x_k + \frac{X_k}{a_k} = \pi_k + \gamma_k s_k + \frac{\Pi_k + \Gamma_k s_k}{a_k}$ for some π_k and γ_k. Because

$$\begin{bmatrix} -a_i \rho_i r \\ x_i + \frac{X_i}{a_i} + \sum_{k \neq i} x_k \end{bmatrix}_{|s_i} \sim$$

$$\mathcal{N}\left(\begin{bmatrix} -\rho_i a_i (\mathbb{E}(v|s_i) - p) \\ x_i + \frac{X_i}{a_i} + \sum_{k \neq i} \pi_k + \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i) \end{bmatrix}, \right.$$
$$\left. \begin{bmatrix} \rho_i^2 a_i^2 \mathrm{Var}(v|s_i) & -\rho_i a_i \sum_{k \neq i} \gamma_k \mathrm{Var}(v|s_i) \\ -\rho_i a_i \sum_{k \neq i} \gamma_k \mathrm{Var}(v|s_i) & (\sum_{k \neq i} \gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k \neq i} \gamma_k^2 \tau_k^{-1} \end{bmatrix} \right)$$

by Lemma 3.1.1, investor i equivalently minimizes

$$\begin{aligned} & \theta_2^2 \rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1 \theta_2 \rho_i a_i \sum_{k \neq i} \gamma_k \mathrm{Var}(v|s_i) + \theta_1^2 [(\sum_{k \neq i} \gamma_k)^2 \mathrm{Var}(v|s_i) + \sum_{k \neq i} \gamma_k^2 \tau_k^{-1}] + 2\theta_1 \theta_2 \\ \xRightarrow{\text{FOC}} \quad & 2\theta_2 \rho_i^2 a_i^2 \mathrm{Var}(v|s_i) + 2\theta_1 \rho_i a_i \sum_{k \neq i} \gamma_k \mathrm{Var}(v|s_i) + 2\theta_1 = 0, \\ & \text{where } \theta_1 = -\rho_i a_i (\mathbb{E}(v|s_i) - p) \text{ and } \theta_2 = x_i + \frac{X_i}{a_i} + \sum_{k \neq i} \pi_k + \sum_{k \neq i} \gamma_k \mathbb{E}(v|s_i). \end{aligned}$$

Plugging in $x_i + \frac{X_i}{a_i} = \pi_i + \gamma_i s_i + \frac{\Pi_i + \Gamma_i s_i}{a_i}$ leads to

$$[\sum_{k\neq i}\pi_k+\pi_i+\gamma_i s_i+\frac{\Pi_i+\Gamma_i s_i}{a_i}+\sum_{k\neq i}\gamma_k\mathbb{E}(v|s_i)]\rho_i^2a_i^2\mathrm{Var}(v|s_i)$$
$$-\rho_i a_i(\mathbb{E}(v|s_i)-p)\rho_i a_i\sum_{k\neq i}\gamma_k\mathrm{Var}(v|s_i)-\rho_i a_i(\mathbb{E}(v|s_i)-p)=0.$$

Matching coefficients renders $(\gamma_i+\frac{\Gamma_i}{a_i})\rho_i a_i=\tau_i$, $(\Pi+\frac{\Pi_i}{a_i})\rho_i a_i=\tau_r(\bar{r}-p)$ (where $\Pi\doteq\sum_{i=1}^n\pi_i$). $\square$

Plug in (3.19) and by Lemma 3.1.1 investor i's expected utility is given by

$$-\frac{\sqrt{\tau_r+\tau_i}\exp\{-\frac{(\tau_r+\tau_i)[\mathbb{E}(v|s_i)-p]^2}{2}\}}{\sqrt{\tau_r+\tau_i+2\rho_i a_i\sum_{k\neq i}\gamma_k-\rho_i^2a_i^2\sum_{k\neq i}\gamma_k^2\tau_k^{-1}}},$$

which is maximized at $\gamma_k=\frac{\tau_k}{\rho_i a_i}$. Plugging in $\gamma_i+\frac{\Gamma_i}{a_i}=\frac{\tau_i}{\tau_r\bar{r}}(\Pi+\frac{\pi_i}{a_i})$ and $\frac{1}{\tau_r\bar{r}}(\Pi+\frac{\pi_i}{a_i})\rho_i a_i=1$ lead to the at the optimal γ_i, $\Gamma_i=0$. Thus for any given sharing rule a_k $(k=1,\cdots,n)$, there exists a linear equilibrium in which each investor optimally chooses her amount of investmentboth within and outside of a firm. In particular, when a_i is chosen to be $\frac{\frac{1}{\rho_i}}{\sum_{i=1}^n\frac{1}{\rho_i}}$, investmentin the firm can be stipulated so that no investor has incentive to work outside of the firm, and the resulting equilibrium gives the highest expected utilities to all investors.[10]

3.3.3 Resemblance to Partnerships

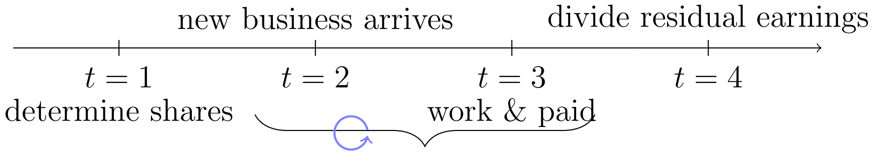
The structure of modern corporations have evolved into prohibitive complicacy, yet they do share some common features. At a high level, a firm's business could

---

[10] In particular, when the investors have homogeneous risk preference (but possibly heterogeneous knowledge precisions), a $\frac{1}{n}$ equal sharing rule is optimal. This observation provides an alternative interpretation why some knowledge intensive partnership firms stick to equal profit-sharing. For example, as Dick Kramlich, founder of the famous Silicon Valley venture capital firm New Enterprise Associates (NEA) puts: "...some culture of NEA has never changed: always maintained a democracy among the partners, wherein they all have the same draw from the firm's fee and the same participation in the carried interest, or investment profits, from their funds...", see Finkel and Greising (2009) P180.

be captured by a risky production technology, to which a set of "owners" provide (usually relatively homogeneous) production inputs (capital, labor, or raw ingredients, etc.) based on their assessment of the business productivity, and from which the same set of owners share residual earnings according to a pre-specified rule. Figure 3.3.3 illustrates this partnership structure.

Figure 3.2: **The Structure of a Partnership**



This primitive description of firms with emphases on profit-sharing and risk-taking follows corporate legal literature (Hansmann (2009)), and is linguistically consistent with English tradition – among the synonyms of the word "firm" are "company" (profit-sharing among multiple owners) and "venture" (risky business). Firms in this form has been accompanying human history ever since Queen Elizabeth granted the East India Company its first Royal Charter on December 31st, 1600 AD, and even today still underlies partnerships (e.g. private equity/venture capital firms), producer cooperatives, joint-ventures, and (to a less extent) all other firms except for sole-proprietorship.

3.4 Related Literature

There is a small but growing literature in finance and economics discussing wisdom of the crowd. Surowiecki (2005) provides an introduction. Kremer, Mansour, and Perry (2014) characterize the optimal disclosure policy of a planner whose goal is to maximize social welfare in a setting where agents arrive sequentially and choose one action from a set of actions with unknown payoffs. Da and Huang (2015) run experiments on Estimize.com, a crowd-based earnings forecast platform, to restrict the information set available to randomly selected users. The experiments confirm that independent forecasts lead to more accurate consensus and suggest that wisdom of crowd can be better harnessed by encouraging independent voice from the participants, thus shutting down information cascade (Bikhchandani, Hirshleifer, and Welch (1992) and Welch (2000), etc.). In the setting of securities-based crowdfunding platforms, Brown and Davies (2015) show that naive investors, possessing weak but on average correct signals, are required for efficient financing. They develop a model showing that sophisticated investors, who are better informed and anticipate other investors' actions, cannot by themselves use their information to improve financing efficiency. Grüner and Siemroth (2015) considers crowdfunding as a mechanism in which consumers signal future product market demand via investment.

There is also an emerging literature studying various crowdfunding mechanisms. For example, on rewards-based crowdfunding, Cumming, Leboeuf, and Schwienbacher (2015) compares keep-it-all versus all-or-nothing financing, and shows that keep-it-all mechanisms are better for small, scalable projects. While Chang (2015) show that all-or-nothing funding generates more revenue than keep-it-all funding, that all-or-nothing funding complements borrowing because it helps the entrepreneur to learn market value. On debt-based crowdfunding, Hakenes and Schlegel (2014) analyzes a model in which a finite number of households endogenously produce information and then invest using debt sourced via crowdfunding.

3.5 Conclusion

This paper shows that a profit-sharing contracts, similar but different from traditional equity contracts, could coordinate individual actions guided by dispersed private information and obtain an *as if* information aggregation effect. The optimal profit-sharing contract is simple and easy to implement, is immune to lying or abuse, and remains dominant even when frictions prevent perfect coordination. This result could provide practical guidance for security-based crowdfunding.

This current paper provides a benchmark result of the optimal contract in the cleanest setting when wisdom of the crowd is present. Future research could add a few additional features. For example, one would conjecture that the optimal sharing arrangement would be a hybrid of a profit-sharing contract and common stocks when the project is not perfectly scalable, when limited liability is strictly enforced, or when exogenous investment cap is imposed. Further developments are down the road.

BIBLIOGRAPHY

Acharya, Viral V, Stewart C Myers, and Raghuram G Rajan, 2011, The internal governance of firms, *The Journal of Finance* 66, 689–720. 59

Admati, Anat R, and Paul Pfleiderer, 1986, A monopolistic market for information, *Journal of Economic Theory* 39, 400–438. 43, 56

———, 1990, Direct and indirect sale of information, *Econometrica: Journal of the Econometric Society* pp. 901–928. 43, 56, 62

Aghion, Philippe, and Jean Tirole, 1997, Formal and real authority in organizations, *Journal of Political Economy* pp. 1–29. 59, 60

Alchian, Armen A, and Harold Demsetz, 1972, Production, information costs, and economic organization, *The American economic review* pp. 777–795. 47, 61, 64

Allen, Franklin, 1990, The market for information and the origin of financial intermediation, *Journal of financial intermediation* 1, 3–30. 43, 56

———, and Douglas Gale, 1994, *Financial innovation and risk sharing* (MIT press). 50

Amador, Manuel, and Pierre-Olivier Weill, 2010, Learning from prices: Public communication and welfare, *Journal of Political Economy* 118, 866–907. 82

Andrei, Daniel, and Julien Cujean, 2013, Information percolation, momentum, and reversal, . 43, 57

Angeletos, George-Marios, and Alessandro Pavan, 2007, Efficient use of information and social value of information, *Econometrica* 75, 1103–1142. 62, 82

Aoki, Masahiko, 1984, *The co-operative game theory of the firm* (Oxford university press). 62

Aumann, Robert J, and Michael Maschler, 1985, Game theoretic analysis of a bankruptcy problem from the talmud, *Journal of economic Theory* 36, 195–213. 62

Baber, William R, Patricia M Fairfield, and James A Haggard, 1991, The effect of concern about reported income on discretionary spending decisions: The case of research and development, *Accounting Review* pp. 818–829. 6

Baker, Malcolm, Brendan Bradley, and Jeffrey Wurgler, 2011, Benchmarks as limits to arbitrage: Understanding the low volatility anomaly, *Financial Analyst Journal* 67. 18

Ball, Ray, and Philip Brown, 1968, An empirical evaluation of accounting income numbers, *Journal of Accounting Research* 6, pp. 159–178. 4

Barber, Brad, Terrance Odean, and Ning Zhu, 2006, Do noise traders move markets?, in *EFA 2006 Zurich Meetings Paper*. 24

Barber, Brad M, Yi-Tsung Lee, Yu-Jane Liu, and Terrance Odean, 2009, Just how much do individual investors lose by trading?, *Review of Financial Studies* 22, 609–632. 24

Barber, Brad M, and Terrance Odean, 2000, Trading is hazardous to your wealth: The common stock investment performance of individual investors, *The Journal of Finance* 55, 773–806. 24

Barber, Brad M., and Terrance Odean, 2008, All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors, *Review of Financial Studies* 21, 785–818. 24

Barclay, Michael J., and Terrence Hendershott, 2003, Price discovery and trading after hours, *Review of Financial Studies* 16, 1041–1073. 1

Barclay, Michael J, and Terrence Hendershott, 2004, Liquidity externalities and adverse selection: Evidence from trading after hours, *The Journal of Finance* 59, 681–710. 2

Barclay, Michael J., and Terrence Hendershott, 2008, A comparison of trading and non-trading mechanisms for price discovery, *Journal of Empirical Finance* 15, 839–849. 5

Battalio, Robert, and Paul Schultz, 2011, Regulatory uncertainty and market liquidity: The 2008 short sale ban's impact on equity option markets, *The Journal of Finance* 66, 2013–2053. 28

Battalio, Robert H., and Richard R. Mendenhall, 2011, Post-earnings announcement drift: Bounds on profitability for the marginal investor, *Financial Review* 46, 513–539. 4

Berk, Jonathan B, Richard Stanton, and Josef Zechner, 2010, Human capital, bankruptcy, and capital structure, *The Journal of Finance* 65, 891–926. 61

Berk, Jonathan B, Jules H Van Binsbergen, and Binying Liu, 2014, Matching capital and labor, Discussion paper, National Bureau of Economic Research. 59

Berk, Jonathan B, and Johan Walden, 2013, Limited capital market participation and human capital risk, *Review of Asset Pricing Studies* 3, 1–37. 61

Berkman, Henk, and Cameron Truong, 2009, Event day 0? after-hours earnings announcements, *Journal of Accounting Research* 47, 71–103. 8

Bernard, Victor L., and Jacob K. Thomas, 1989, Post-earnings-announcement drift: Delayed price response or risk premium?, *Journal of Accounting Research* 27, pp. 1–36. 4

Bhide, Amar, 1994, *Building the Professional Firm: McKinsey & Co.: 1939-1968* (Citeseer). 56

Bhojraj, Sanjeev, Paul Hribar, Marc Picconi, and John McInnis, 2009, Making sense of cents: An examination of firms that marginally miss or beat analyst forecasts, *The Journal of Finance* 64, 2361–2388. 7

Bhushan, Ravi, 1994, An informational efficiency perspective on the post-earnings announcement drift, *Journal of Accounting and Economics* 18, 45 – 65. 4

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch, 1992, A theory of fads, fashion, custom, and cultural change as informational cascades, *Journal of political Economy* pp. 992–1026. 95

Black, Fischer, 1986, Noise, *The Journal of Finance* 41, 529–543. 42, 50

Blume, Lawrence, and David Easley, 1990, Implementation of walrasian expectations equilibria, *Journal of Economic Theory* 51, 207–227. 63

Bolton, Patrick, and Mathias Dewatripont, 1994, The firm as a communication network, *The Quarterly Journal of Economics* pp. 809–839. 57, 64

Boot, Arnoud WA, and Anjan V Thakor, 1997, Financial system architecture, *Review of Financial studies* 10, 693–733. 54

Bradley, Daniel, Jonathan Clarke, Suzanne Lee, and Chayawat Ornthanalai, 2013, Are analysts recommendations informative? intraday evidence on the impact of time stamp delays, *The Journal of Finance*. 35

Brandenburger, Adam M, and Barry J Nalebuff, 2011, *Co-opetition* (Crown Business). 62

Breon-Drish, Bradyn, 2015, On existence and uniqueness of equilibrium in a class of noisy rational expectations models, *The Review of Economic Studies* p. forthcoming. 86

Brown, David C, and Shaun William Davies, 2015, Equity crowdfunding: Harnessing the wisdom of the crowd, *Available at SSRN*. 95

Busse, Jeffrey A, and T Clifton Green, 2002, Market efficiency in real time, *Journal of Financial Economics* 65, 415–437. 2

Chakrabarty, Bidisha, Bingguang Li, Vanthuan Nguyen, and Robert A. Van Ness, 2007, Trade classification algorithms for electronic communications network trades, *Journal of Banking & Finance* 31, 3806 – 3821. 25

Chakrabarty, Bidisha, Pamela C. Moulton, and Andriy Shkilko, 2012, Short sales, long sales, and the lee–ready trade classification algorithm revisited, *Journal of Financial Markets* 15, 467–491. 25

Chakravarty, Sugato, Pankaj Jain, James Upson, and Robert Wood, 2012, Clean sweep: Informed trading through intermarket sweep orders, *Journal of Financial and Quantitative Analysis* 47, 415–435. 23

Chakravarty, Sugato, Venkatesh Panchapagesan, and Robert A. Wood, 2005, Did decimalization hurt institutional investors?, *Journal of Financial Markets* 8, 400 – 420. 24

Chan, Louis KC, Jason Karceski, and Josef Lakonishok, 2007, Analysts' conflicts of interest and biases in earnings forecasts, *Journal of Financial and Quantitative Analysis* 42, 893–913. 7

Chang, Jen-Wen, 2015, The economics of crowdfunding, Discussion paper, . 95

Cheng, Si, Massimo Massa, Matthew Spiegel, and Hong Zhang, 2012, Human capital and the structure of the mutual fund industry, . 59

Chordia, Tarun, Amit Goyal, Gil Sadka, Ronnie Sadka, and Lakshmanan Shivakumar, 2009, Liquidity and the post-earnings-announcement drift, *Financial Analysts Journal* 65. 4

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2008, Liquidity and market efficiency, *Journal of Financial Economics* 87, 249 – 268. 4

Chordia, Tarun, and Lakshmanan Shivakumar, 2006, Earnings and price momentum, *Journal of Financial Economics* 80, 627–656. 4

Cliff, Michael, Michael J Cooper, and Huseyin Gulen, 2008, Return differences between trading and non-trading hours: Like night and day, *SSRN eLibrary.* 33

Coase, Ronald H, 1937, The nature of the firm, *Econometrica* 4, 386–405. 44, 63

Coval, Joshua, David Hirshleifer, and Tyler Shumway, 2005, Can individual investors beat the market?, *HBS finance working paper.* 24

Cumming, Douglas J, Gaël Leboeuf, and Armin Schwienbacher, 2015, Crowdfunding models: Keep-it-all vs. all-or-nothing, *All-Or-Nothing (May 31, 2015).* 95

Da, Zhi, and Xing Huang, 2015, Harnessing the wisdom of crowds, *Available at SSRN 2731884.* 94

Degeorge, Francois, Jayendu Patel, and Richard Zeckhauser, 1999, Earnings management to exceed thresholds*, *The Journal of Business* 72, 1–33. 6

Dellavigna, Stefano, and Joshua M. Pollet, 2009, Investor inattention and friday earnings announcements, *The Journal of Finance* 64, 709–749. 28

Demsetz, Harold, 1988, The theory of the firm revisited, *Journal of Law, Economics, & Organization* pp. 141–161. 64

Dewatripont, Mathias, and Jean Tirole, 2005, Modes of communication, *Journal of Political Economy* 113, 1217–1238. 43, 56

Diamond, Douglas W, 1984, Financial intermediation and delegated monitoring, *The Review of Economic Studies* 51, 393–414. 65

———, and Robert E Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, *Journal of Financial Economics* 9, 221–235. 42, 43, 50

Dicks, David L., and Paolo Fulghieri, 2014, Ambiguity aversion, disagreement, and the theory of the firm, *Available at SSRN 2357599*. 64

Doyle, Jeffrey T, and Matthew J Magilke, 2009, The timing of earnings announcements: An examination of the strategic disclosure hypothesis, *The Accounting Review* 84, 157–182. 28

Dubinsky, Andrew, and Michael Johannes, 2006, Earnings announcements and equity options, *Working Paper*. 5

Duffie, Darrell, Gaston Giroux, and Gustavo Manso, 2010, Information percolation, *American Economic Journal: Microeconomics* pp. 100–111. 43, 57

Edmans, Alex, Itay Goldstein, and John Y. Zhu, 2011, Contracting with synergies, Discussion paper, National Bureau of Economic Research. 47

Eisfeldt, Andrea L., and Dimitris Papanikolaou, 2013, Organization capital and the cross-section of expected returns, *The Journal of Finance* 68, 1365–1406. 61

Ellis, Katrina, Roni Michaely, and Maureen O'Hara, 2000, The accuracy of trade classification rules: evidence from nasdaq, *Journal of Financial and Quantitative Analysis* 35, 529–551. 25

Erel, Isil, 2011, The effect of bank mergers on loan prices: Evidence from the united states, *Review of Financial Studies* 24, 1068–1101. 65

Feddersen, Timothy, and Wolfgang Pesendorfer, 1997, Voting behavior and information aggregation in elections with private information, *Econometrica: Journal of the Econometric Society* pp. 1029–1058. 65

Finkel, Robert, and David Greising, 2009, *The Masters of Private Equity and Venture Capital* (McGraw Hill Professional). 93

Foster, F Douglas, and S Viswanathan, 1996, Strategic trading when agents forecast the forecasts of others, *The Journal of Finance* 51, 1437–1478. 1

Francis, Jennifer, Donald Pagach, and Jens Stephan, 1992, The stock market response to earnings announcements released during trading versus nontrading periods, *Journal of Accounting Research* 30, pp. 165–184. 4

García, Diego, and Joel M Vanden, 2009, Information acquisition and mutual funds, *Journal of Economic Theory* 144, 1965–1995. 62

Garfinkel, Jon A., and Jonathan Sokobin, 2006, Volume, opinion divergence, and returns: A study of postearnings announcement drift, *Journal of Accounting Research* 44, 85–112. 4

Garicano, Luis, and Tano Santos, 2004, Referrals, *American Economic Review* 94, 499–525. 44, 58

Gennotte, Gerard, and Brett Trueman, 1996, The strategic timing of corporate disclosures, *The Review of Financial Studies* 9, pp. 665–690. 7

Glosten, Lawrence R, and Paul R Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, *Journal of financial economics* 14, 71–100. 20

Graham, John R, Campbell R Harvey, and Shiva Rajgopal, 2005, The economic implications of corporate financial reporting, *Journal of accounting and economics* 40, 3–73. 6

Grant, Robert M, 1996, Toward a knowledge-based theory of the firm, *Strategic management journal* 17, 109–122. 57, 64

Greene, Jason T., and Susan G. Watts, 1996, Price discovery on the NYSE and the NASDAQ: The case of overnight and daytime news releases, *Financial Management* 25, pp. 19–42. 4

Grinblatt, Mark, and Matti Keloharju, 2000, The investment behavior and performance of various investor types: a study of finland's unique data set, *Journal of Financial Economics* 55, 43 – 67. 24

Grossman, Sanford J, 1977, The existence of futures markets, noisy rational expectations and informational externalities, *The Review of Economic Studies* pp. 431–449. 44

———, and Oliver D Hart, 1986, The costs and benefits of ownership: A theory of vertical and lateral integration, *The Journal of Political Economy* pp. 691–719. 63

Grossman, Sanford J, and Joseph E Stiglitz, 1980, On the impossibility of informationally efficient markets, *The American economic review* 70, 393–408. 42, 56

Grüner, Hans Peter, and Christoph Siemroth, 2015, Cutting out the middleman: Crowdinvesting, efficiency, and inequality, . 95

Hakenes, Hendrik, and Friederike Schlegel, 2014, Exploiting the financial wisdom of the crowd–crowdfunding as a tool to aggregate vague information, *Available at SSRN 2475025*. 95

Hansmann, Henry, 2009, *The ownership of enterprise* (Harvard University Press). 46, 77, 94

Hart, Oliver, 1989, Economist's perspective on the theory of the firm, an, *Colum. L. Rev.* 89, 1775. 63

———, 1995, *Firms, contracts, and financial structure* (Oxford university press). 59

———, and John Moore, 1990, Property rights and the nature of the firm, *Journal of Political Economy* pp. 1119–1158. 63

Hasbrouck, Joel, 1992, Using the torq database, *New York Stock Exchange* 11. 24

———, 2010, The best bid and offer: A short note on programs and practices, *Working Paper*. 7

———, and Gideon Saar, 2009, Technology and liquidity provision: The blurring of traditional definitions, *Journal of financial Markets* 12, 143–172. 34

Hayek, Friedrich, 1944, *The Road to Serfdom* (University of Chicago Press and Routledge Press). 43

Hayek, FA, 1945, The use of knowledge in society, *American Economic Review* 35, 519–30. 43

Hellwig, Martin F, 1980, On the aggregation of information in competitive markets, *Journal of economic theory* 22, 477–498. 42, 43, 50, 51

Hirshleifer, David, Sonya Seongyeon Lim, and Siew Hong Teoh, 2009, Driven to distraction: Extraneous events and underreaction to earnings news, *Journal of Finance* 64, 2289–2325. 27

Hirshleifer, David A., James N. Myers, Linda A. Myers, , and Siew Hong Teoh, 2008, Do individual investors cause post-earnings announcement drift? direct evidence from personal trades, *The Accounting Review* 83, 1521–1550. 27

Hirshleifer, Jack, 1971, The private and social value of information and the reward to inventive activity, *The American Economic Review* 61, 561–574. 43, 56

Holden, Craig W., and Stacey Jacobsen, 2013, Liquidity measurement problems in fast, competitive markets: expensive and cheap solutions, *The Journal of Finance*. 7

Holden, Craig W., and Avanidhar Subrahmanyam, 1992, Long-lived private information and imperfect competition, *The Journal of Finance* 47, pp. 247–270. 1

Holmström, Bengt, 1982, Moral hazard in teams, *The Bell Journal of Economics* pp. 324–340. 57

———, and Jean Tirole, 1989, The theory of the firm, *Handbook of industrial organization* 1, 61–133. 63

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *The Journal of Finance* 54, pp. 2143–2184. 27

Huang, Roger D., and Hans R. Stoll, 1996, Dealer versus auction markets: A paired comparison of execution costs on NASDAQ and the NYSE, *Journal of Financial Economics* 41, 313 – 357. 7

Indjejikian, Raffi, Hai Lu, and Liyan Yang, 2014, Rational information leakage, *Management Science* 60, 2762–2775. 53

Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, *Journal of economic theory* 52, 45–67. 60

Jaggia, Priscilla Butt, and Anjan V. Thakor, 1994, Firm-specific human capital and optimal capital structure, *International Economic Review* pp. 283–308. 61

Jegadeesh, Narasimhan, and Joshua Livnat, 2006, Post-earnings-announcement drift: The role of revenue surprises, *Financial Analysts Journal* 62, pp. 22–34. 6

Jensen, Michael C, and William H Meckling, 1979, *Theory of the firm: Managerial behavior, agency costs, and ownership structure* (Springer). 61, 64

Jiang, Christine X., Tanakorn Likitapiwat, and Thomas H. McInish, 2012, Information content of earnings announcements: Evidence from after-hours trading, *Journal of Financial and Quantitative Analysis* 47, 1303–1330. 4, 8

Kandel, Eugene, and Edward P Lazear, 1992, Peer pressure and partnerships, *Journal of Political Economy* pp. 801–817. 47

Kaplan, Steven N., Berk A. Sensoy, and Per Strömberg, 2009, Should investors bet on the jockey or the horse? evidence from the evolution of firms from early business plans to public companies, *The Journal of Finance* 64, 75–115. 64

Ke, Bin, and Santhosh Ramalingegowda, 2005, Do institutional investors exploit the post-earnings announcement drift?, *Journal of Accounting and Economics* 39, 25 – 53. 4

Kim, Oliver, and Robert E. Verrecchia, 1991, Market reaction to anticipated announcements, *Journal of Financial Economics* 30, 273 – 309. 28

Kim, Oliver, and Robert E Verrecchia, 1994, Market liquidity and volume around earnings announcements, *Journal of accounting and economics* 17, 41–67. 32

Kishore, Runeet, Michael Brandt, Pedro Santa-Clara, and Mohan Venkatachalam, 2008, Earnings announcements are full of surprises, *AFA 2012 Chicago Meetings Paper*. 6

Klein, Benjamin, Robert G. Crawford, and Armen A. Alchian, 1978, Vertical integration, appropriable rents, and the competitive contracting process, *Journal of law and economics* pp. 297–326. 63

Kremer, Ilan, Yishay Mansour, and Motty Perry, 2014, Implementing the wisdom of the crowd, *Journal of Political Economy* 122, 988–1012. 74, 94

Kumar, Alok, and Charles M.C. Lee, 2006, Retail investor sentiment and return comovements, *The Journal of Finance* 61, 2451–2486. 24

Kyle, Albert S., 1985, Continuous auctions and insider trading, *Econometrica* 53, pp. 1315–1335. 1, 53

Kyle, Albert S, 1989, Informed speculation with imperfect competition, *The Review of Economic Studies* 56, 317–355. 12, 55, 67

Lebedeva, Olga, 2012, Trading aggressiveness and its implications for market efficiency, *Available at SSRN 2078274*. 32

Lee, Charles M.C, 1992, Earnings news and small traders: An intraday analysis, *Journal of Accounting and Economics* 15, 265 – 302. 24

Lee, Charles M.C., and Balkrishna Radhakrishna, 2000, Inferring investor behavior: Evidence from TORQ data, *Journal of Financial Markets* 3, 83 – 111. 24

Lee, Charles M C, and Mark J Ready, 1991, Inferring trade direction from intraday data, *Journal of Finance* 46, 733–46. 25

Legros, Patrick, and Hitoshi Matsushima, 1991, Efficiency in partnerships, *Journal of Economic Theory* 55, 296–322. 58

Leland, Hayne E., and David H. Pyle, 1977, Informational asymmetries, financial structure, and financial intermediation, *Journal of Finance* pp. 371–387. 44

Linnainmaa, Juhani T, 2010, Do limit orders alter inferences about investor performance and behavior?, *The Journal of Finance* 65, 1473–1506. 20, 24

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *The review of economics and statistics* pp. 13–37. 78

Markowitz, Harry, 1952, Portfolio selection*, *The Journal of Finance* 7, 77–91. 83

McCloskey, Deirdre N, 1998, *The rhetoric of economics* (Univ of Wisconsin Press). 55

McInish, Thomas H, Bonnie F Van Ness, and Robert A Van Ness, 2002, After-hours trading of NYSE stocks on the regional stock exchanges, *Review of Financial Economics* 11, 287 – 297. 33

Mendenhall, Richard R., 2004, Arbitrage risk and postearningsannouncement drift, *The Journal of Business* 77, pp. 875–894. 4

Michaely, Roni, Amir Rubin, and Alexander Vedrashko, 2011, Corporate governance and the timing of earnings announcements, *AFA 2012 Chicago Meetings Paper*. 5

——— , 2012, Firm heterogeneity and investor inattention to friday earnings announcements, *Available at SSRN 2164789*. 28

Millon, Marcia H, and Anjan V Thakor, 1985, Moral hazard and information sharing: A model of financial information gathering agencies, *The Journal of Finance* 40, 1403–1422. 58

Muth, John F, 1961, Rational expectations and the theory of price movements, *Econometrica: Journal of the Econometric Society* pp. 315–335. 55

Myers, Stewart C, 2000, Outside equity, *The Journal of Finance* 55, 1005–1037. 61

Myerson, Roger B., and Mark A. Satterthwaite, 1983, Efficient mechanisms for bilateral trading, *Journal of Economic Theory* 29, 265–281. 63

Nash Jr, John F, 1950, The bargaining problem, *Econometrica: Journal of the Econometric Society* pp. 155–162. 62

Odean, Terrance, 1998, Are investors reluctant to realize their losses?, *The Journal of finance* 53, 1775–1798. 24

——— , 1999, Do investors trade too much?, *American Economic Review* 89, 1279–1298. 24

Palfrey, Thomas R, 2002, Implementation theory, *Handbook of game theory with economic applications* 3, 2271–2326. 62

Patell, James M., and Mark A. Wolfson, 1984, The intraday speed of adjustment of stock prices to earnings and dividend announcements, *Journal of Financial Economics* 13, 223 – 252. 4

Rajan, Raghuram G, 2012, Presidential address: The corporation in finance, *The Journal of Finance* 67, 1173–1217. 59

Rajan, Raghuram G., and Luigi Zingales, 1998, Power in a theory of the firm, *The Quarterly Journal of Economics.* 56, 64

Rajan, Raghuram G, and Luigi Zingales, 2000, The governance of the new enterprise, *Corporate Governance, Xavier Vives, ed.* 59

Rajan, Raghuram G., and Luigi Zingales, 2001, The influence of the financial revolution on the nature of firms, *American Economic Review* 91, 206–211. 64

Ramakrishnan, Ram TS, and Anjan V Thakor, 1984, Information reliability and a theory of financial intermediation, *The Review of Economic Studies* 51, 415–432. 43, 58, 65

Rasmusen, Eric, 1987, Moral hazard in risk-averse teams, *The RAND Journal of Economics* pp. 428–435. 58

Ross, Stephen A, 2005, Markets for agents: fund management, *The Legacy of Fischer Black* p. 96. 53

Sadka, Ronnie, 2006, Momentum and post-earnings-announcement drift anomalies: The role of liquidity risk, *Journal of Financial Economics* 80, 309 – 349. 4

Santosh, Shrihari, 2013, Public information and price discovery: Trade time vs clock time, *Working Paper.* 32

SEC, 2005, Federal register, part ii, securities and exchanges commission, cfr parts 200, 201, 230, 240, 242, 249, and 270, regulation nms; final rule, *Regulation NMS: http://www. sec. gov/rules/proposed/34-50870. Htm.* 23

Shapley, Lloyd S, 1952, A value for n-person games, . 62

Shleifer, Andrei, and Robert W Vishny, 1997, A survey of corporate governance, *The Journal of Finance* 52, 737–783. 59

Song, Fengshua, and Anjan Thakor, 2010, Banks and capital markets as a coevolving financial system, *VoxEU. org* 1. 54

Spulber, Daniel F., 1999, *Market microstructure: intermediaries and the theory of the firm* (Cambridge University Press). 64

Stanton, Richard, Johan Walden, and Nancy Wallace, 2015, Securitization networks and endogenous financial norms in us mortgage markets, . 65

Strausz, Roland, 1999, Efficiency in sequential partnerships, *Journal of Economic Theory* 85, 140–156. 58

Summers, Lawrence H., 2014, Us economic prospects: Secular stagnation, hysteresis, and the zero lower bound, *Business Economics* 49, 65–73. 60

Surowiecki, James, 2005, *The wisdom of crowds* (Anchor). 74, 94

Taylor, Daniel, 2011, Post-earnings announcement drift and related anomalies, *Handbook of Equity Market Anomalies, edited by Leonard Zacks.* 5

Teoh, Siew Hong, Ivo Welch, and T. J. Wong, 1998, Earnings management and the long-run market performance of initial public offerings, *The Journal of Finance* 53, pp. 1935–1974. 6

Townsend, Robert M, 1979, Optimal contracts and competitive markets with costly state verification, *Journal of Economic theory* 21, 265–293. 60

Upson, James, Sugato Chakravarty, Pankaj K Jain, and Thomas H McInish, 2011, Trading aggressiveness and market breadth around earnings announcements, *Available at SSRN 1743717.* 32

Vega, Clara, 2006, Stock price reaction to public and private information, *Journal of Financial Economics* 82, 103 – 133. 4

Veldkamp, Laura L., 2011, *Information choice in macroeconomics and finance* (Princeton University Press). 57

Welch, Ivo, 2000, Herding among security analysts, *Journal of Financial economics* 58, 369–396. 95

Williams, Steven R, and Roy Radner, 1988, *Efficiency in partnership when the joint output is uncertain* (Center for Mathematical Studies in Economics and Management Science, Northwestern University). 58

Williamson, Oliver E., 1975, Markets and hierarchies, *New York* pp. 26–30. 63

——— , 1979, Transaction-cost economics: the governance of contractual relations, *Journal of law and economics* pp. 233–261. 63

——— , 1985, *The economic intstitutions of capitalism* (Simon and Schuster). 63

Williamson, Oliver E, 1988, Corporate finance and corporate governance, *The Journal of Finance* 43, 567–591. 60

Wilson, Robert, 1968, The theory of syndicates, *Econometrica: journal of the Econometric Society* pp. 119–132. 60

Winter, Eyal, 2006, Optimal incentives for sequential production processes, *The RAND Journal of Economics* 37, 376–390. 64

——— , 2010, Transparency and incentives among peers, *The RAND Journal of Economics* 41, 504–523. 64

Zhang, Mindy Xiaolan, 2014, Who bears firm-level risk? implications for cash flow volatility, *WFA 2014 Monterey Meetings Paper*. 61

Zingales, Luigi, 2000, In search of new foundations, *The Journal of Finance* 55, 1623–1653. 59