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ABSTRACT

A multicomponent decay curve analysis technique has been developed and incorporated into the decay curve fitting computer code, MLDS (Maximum Likelihood Decay by the Simplex method). The fitting criteria are based on the maximum likelihood technique for decay curves made up of time binned events. The probabilities used in the likelihood functions are based on the Poisson distribution, so decay curves constructed from a small number of events are treated correctly. A simple utility is included which allows the use of discrete event times, rather than time-binned data, to make maximum use of the decay information. The search for the maximum in the multidimensional likelihood surface for multi-component fits is performed by the simplex method, which makes the success of the iterative fits extremely insensitive to the initial values of the fit parameters and eliminates the problems of divergence. The simplex method also avoids the problem of programming the partial derivatives of the decay curves with respect to all the variable parameters, which makes the implementation of new types of decay curves curves straightforward. Any of the decay curve parameters can be fixed or allowed to vary. Asymmetric error limits for each of the free parameters, which do not consider the covariance of the other free parameters, are determined. A procedure is presented for determining the error limits which contain the associated covariances. The curve fitting procedure in MLDS can easily be adapted for fits to other curves with any functional form.

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Introduction

Decay curve fitting is a powerful and necessary tool in the analysis of nuclear decay data. Decay curve fits are used both for identifying nuclides and for performing nuclear decay studies in which half-lives and/or disintegration rates as a function of time must be determined.

decay curve equations

Decay curve data are usually constructed by detecting a number of decays during each of a series of n measurement intervals. Let t_m be the length of time from a reference time (t=0) to the start of the mth measurement interval, and let the length of this interval be c_m. If there are u independent species being detected in the sample, which decay exponentially with time, the number of decays of the ith species detected in the mth interval, $U_{m,i}$, is found by integrating the exponential decay over the time interval from t_m to t_m+c_m.

$$U_{m,i}=\frac{D_i^\circ \exp(-d_i t_m)\ [1-\exp(-d_i c_m)]}{d_i} \qquad (1$$

In eq. 1, D_i° is the activity (detected events per unit time) for the ith species at a reference time, t=0, and d_i is the decay constant for the ith species (these independent components will be referred to as "daughter" activities in the later discussion, hence the notations D and d). The total number of decays during the mth interval due to the u independent species, U_m, is the sum of the u individual activities.

$$U_m= \sum_{i=1}^{u} U_{m,i} \qquad (2$$

Often, an activity being detected is produced as the daughter of a radioactive parent species. If there are v of these parent-daughter decay chains for which the daughter decays are being detected, the number of daughter events detected in the mth interval originating from the decay of the jth parent, $V_{m,j}$, is

$$V_{m,j}=P_j^\circ d_j \left[\frac{\exp(-p_j t_m)\ [1-\exp(-p_j c_m)]}{p_j\ (d_j-p_j)} + \frac{\exp(-d_j t_m)\ [1-\exp(-d_j c_m)]}{d_j\ (p_j-d_j)}\right] \tag{3}$$

where P_j° is the activity of the jth parent species at t=0. d_j and p_j are the decay constants for the jth parent and jth daughter, respectively. The number of counts in the mth interval from all v of the parent-daughter chains is obtained by summing over j.

$$V_m = \sum_{j=u+1}^{u+v} \left[V_{m,j} + U_{m,j}\right] \tag{4}$$

The $U_{m,j}$ term has been added to account for any daughter activities which were present at t=0. It should be noted that this is the equation for the number of events of the daughter. If the parent activity is also being detected directly, an eq. 1 term for the parent must be added.

Finally, The decay of the daughter of a three member chain will be considered. If there are w such three membered chains, the number of daughter events detected in the mth interval originating from the kth grandparent, $W_{m,k}$, is

$$W_{m,k}=G_k^{\circ}p_kd_k\left[\frac{\exp(-d_kt_m)\ [1-\exp(-d_kc_m)]}{d_k\ (g_k-d_k)\ (p_k-d_k)}+\right.$$

$$\left.\frac{\exp(-p_kt_m)\ [1-\exp(-p_kc_m)]}{p_k\ (g_k-p_k)\ (d_k-p_k)}+\frac{\exp(-g_k)t_m)\ [1-\exp(-g_kc_m)]}{g_k\ (p_k-g_k)\ (d_k-g_k)}\right] \qquad (5$$

where G_k° is the activity of the kth grandparent activity at t=0. g_k is the decay constant for this kth grandparent activity. To get the counts in the mth interval due to all of w of the three membered chains, the sum over k is taken.

$$W_m=\sum_{k=u+v+1}^{u+v+w}\left[W_{m,k}+V_{m,k}+U_{m,k}\right] \qquad (6$$

The $V_{m,k}$ and $U_{m,k}$ terms have been included to account for any parent or daughter activities which were present at t=0. Again, if the parent or grandparent activities are also detected, appropriate terms from equations 1 and 3 must be added. To find the total activity during the mth interval due to u independent components, v parent-daughter chains, and w three membered chains, the contributions from equations 2,4, and 6 must be summed.

$$Y_m = W_m + U_m + V_m \qquad (7$$

It is a relatively simple matter to extend these equations to four membered chains, etc, or to branched decay chains.

common curve fitting techniques

Decay curve fitting has traditionally been performed using the non-linear error-weighted least squares technique [1-3]. Here, if the observed number of counts in the n intervals are Z_m, the deviations between the data and the fit are minimized by minimizing a chi squared, X^2.

$$X^2 = \sum_{m=1}^{n} \left[\frac{(Y_m - Z_m)^2}{(\sigma_m)^2} \right] \tag{8}$$

where σ_m is the standard deviation in the number of counts detected in the mth interval. This approach suffers from two potential problems, especially for the case of poor statistics. First, since the true number of expected counts in the mth interval is not known, σ_m^2 is usually approximated from the observed number of counts, Z_m ($\sigma_m^2 \approx Z_m$). Therefore, intervals which had a small number of counts, due to statistical fluctuations in Y_m, will be assigned a smaller σ_m^2 than those intervals in which the statistical fluctuations resulted in a large number of counts. This effect is illustrated in figure 1. Based on the X^2 fitting criteron given in eq. 8, the samples in which the random fluctuations resulted in a small number of counts will be assigned a larger weight in the fit than those intervals in which the random fluctuations resulted in a large number of counts, causing the error-weighted least squares fits to consistently underestimate the Y_ms in the regions of decay curves where there are a small number of events detected. Some error-weighted least squares techniques use $\sigma_m^2 \approx Y_m$ for the error weighting, which is more correct. Second, since the X^2 fitting criteria is based on the assumption of normal statistics, the time intervals with a small number of counts are incorrectly weighted. This problem is illustrated in figure 2 where normal and Poisson distributions for $Y_m = 1$ are compared.

More recently, maximum likelihood techniques have become popular for decay curve fitting. If the probability of observing Z_m counts in the mth interval where Y_m counts are expected is F_m, the likelihood function, L is the product of the F_m over all m.

$$L = \prod_{m=1}^{n} F_m \qquad (9$$

Maximization of this likelihood function maximizes the probability that the curve correctly describes the data. Usually, it is convenient to use the natural logarithm of the likelihood function, ln(L), to avoid underflow errors.

$$\ln(L) = \sum_{m=1}^{n} \ln(F_m) \qquad (10$$

The MLDS Decay Curve Fitting Technique

The decay curve fitting technique incorporated in the MLDS code is a combination of existing techniques. MLDS uses a combination of the decay equations integrated over the time intervals, the correct use of Poisson distributions in the likelihood functions, and iterative multi-parameter fitting using the simplex method. The MLDS code is written in FORTRAN 77 with some simple Tektronix 4000-series graphics routines. MLDS treats the data as correctly as possible, which is especially important in cases with low counting statistics. The on-screen prompts, as well as the non-divergent character of the fitting technique make this a reliable method for fitting decay curves with the maximum possible accuracy.

the likelihood function

The experimental numbers of counts in each time interval, Z_m, are compared to expected numbers of counts, Y_m, calculated from the above decay curve equations integrated over the experimental time intervals. This avoids problems with assuming an instantaneous decay rate at some time in the interval based on the average rate, a problem encountered in many curve fitting programs, which can have a large effect if c_m is not small compared to the shortest half-life. The F_m are determined according to the Poisson distribution.

$$F_m = \frac{Y_m^{(Z_m)} \exp(-Y_m)}{Z_m!} \qquad (11$$

The natural logarithm of the likelihood function then has a simple form.

$$\ln(L) = \sum_{m=1}^{n} \left[Z_m \ln(Y_m) - Y_m - \ln(Z_m!) \right] \qquad (12$$

In MLDS the exact values for the logarithm of the factorials, $\ln(Z_m!)$, are used from Z=0 through Z=100, and thereafter, Sterling's approximation [4] is used.

$$\ln(Z_m!) = Z_m \ln(Z_m) - Z_m + \frac{\ln(2\pi Z_m)}{2} , \qquad Z_m > 100 \qquad (13$$

This use of Stirling's approximation introduces an error in $\ln(Z_m!)$ of less than 2.3×10^{-6}, and has a negligible effect on the fits.

This log likelihood function makes full use of the Poisson distribution, making it applicable in cases of poor statistics. Contrary to many other approaches, it even handles the case where $Z_m=0$ naturally. This allows one to use the same log likelihood function for fitting multi-component decay curves to decay data made up of discrete event times, rather than time binned data. In order to use discrete decay times, time intervals with lengths equal to the timing resolution of the detection apparatus are used. This time resolution is often 16.667 ms from a 60 hz computer system clock. The intervals containing zero events between the event containing intervals can be concatenated into larger intervals, as can the intervals from the beginning of the measurement to the time of the first event and the intervals from the end of the last event containing interval to the end of the measurement, resulting $\leq 2q+1$ intervals, where q is the number of events. The liklihood functions used in this work reduce to the simpler forms commonly used to fit decay curves made up of discrete decay times [5,6] if the following conditions are met:

1) The probability of observing an event in any time interval is vanishingly small. This condition is usually met for a moderate number of events if the time resolution is very good.

2) The lengths of the event-containing intervals are the same (the time resolution does not change during the measurement).

3) At most one event is observed during any time interval.

4) The sum of the lengths of the event-containing intervals is vanishingly small compared to the total measurement time.

It should be noted that there is one approximation made in this approach. The Poisson distributions are based on the expected number of counts in the mth interval, Y_m. This Y_m is the best value for the expected number of events from the decay curve fit, and may differ from the true number of expected events based on the actual numbers of atoms in the sample. To the extent that the best fit value for Y_m differs from the true value, the Poisson distributions in eq. 11 and 12 may differ from the true Poisson distributions. These errors should be small, and the Poisson distributions based on the best fit Y_ms are the best approximation possible since the true values can, in principle, never be known.

the simplex method

Multi-component decay curves are often fit by performing a steepest ascent search for the maximum in the multidimensional L surface (similarly, the method of steepest descent is used with the error weighted least squares approach to find the minimum in the multidimensional X^2 surface). In cases with more than a few free parameters (initial activities and decay constants which are allowed to vary) these steepest ascent (descent) searches are very sensitive to the initial values used for the free parameters. If the initial values are not near enough to the best values, the iterative fits tend to diverge. The simplex method for curve fitting used in this work almost completely avoids this divergence problem. Programming the complicated equations for the derivatives of the Y_m with respect to all the free parameters is also avoided by the use of the simplex method.

The log likelihood function, ln(L), from the preceding section can be viewed as a multidimensional hypersurface $\ln(L)(D_i^\circ, d_i, P_j^\circ, p_j, G_k^\circ, g_k)$ where ln(L) is dependent on the variable parameters. If the number of variable parameters is b, a simplex is a b+1 sided hyperpolygon which is placed on this hypersurface. The Simplex method outlined by Caceci and Cacheris [7] describes a set of rules for moving this hyperpolygon around on the hypersurface, while changing its size, so that it finds the maximum in ln(L) [7]. The decay curve parameters at which ln(L) is a maximum are taken to be those for the best fit to the decay curve data. The simplex method is relatively fast (although it is several times slower than performing similar decay curve fits by the steepest ascent (descent) approach. The accuracy of the simplex technique is limited only by the convergence criteria (how it is determined when the simplex has reached the maximum). As mentioned above, the simplex method is not prone to divergence, and will reach the maximum of a multidimensional ln(L) surface, even if the initial values for the fit parameters are orders of magnitude different from those at the maximum.

error limits

As with any curve fitting procedure, the question arises of how best to express the uncertainty in the fit parameters. Most commonly, the goodness of fit criteria is assumed to be normally distributed in the directions of the free parameters, and the curvature at the maximum (or minimum) of this assumed multidimensional normal distribution is used to determine the error limits in the best values for the free parameters. Usually the "one sigma limit" or the approximate 68.3% confidence interval is used. Often, covariances of the free parameters are included in the calculation of these error limits.

In the case of poor counting statistics with the maximum likelihood technique, the L surface can be quite asymmetric about the maximum, so in order to convey this information, it is necessary to determine some asymmetric error limits. Skewed L distributions are also possible in multi-component decay curves with good statistics when, for example, two of the half-lives are similar. Figure 3 shows L as a function of the half-life for a single component fit to the spontaneous fission decay of ^{261}Lr produced in $^{248}Cm(^{18}O,p4n)$ reactions [8]. The decay curve consists of 57 events in 20 minute time bins. Three types of asymmetric error limits are indicated in the figure:

1) *Half-Maximum Limits* - The points at which the L drops to half of its maximum value. It should be noted that for a normal distribution, the one sigma limits are 0.85 times the half-maximum limits and cover 68.3% of the distribution. For complicated L distribution shapes, the fraction of the distribution (confidence level) covered by the half-maximum limits is not known.

2) *68.3% Confidence Level Limits* - Limits which encompass 68.3% of the L distribution with equal probabilities above the upper limit and below the lower limit. For especially skewed distributions, it is possible for the

lower limit to be at a position larger than the position of the maximum in the L curve

3) *Interval of Equal Likelihood Chances* [6] *for a 68.3% Confidence Level* – Limits which encompass 68.3% of the L distribution with the L value at the upper and lower limits equal. For skewed distributions, the probability of being above the upper limit is larger than for being below the lower limit.

The MLDS code calculates some approximate error limits. These are half-maximum limits which do not consider the covariances of the other free parameters. These non-covariant half-maximum limits are determined by finding the values for the free parameter in question for which L drops to one half of its maximum value, while holding the other free parameters at their best values. In some cases these non-covariant error limits can be more than a factor of two smaller than error limits calculated considering the covariances. They are, however, useful for most data analyses in which the identification of the nuclides present in decay curves and the determination of approximate initial activities are the principle aim.

Where more accurate error limits are needed, such as in the exact determination of a half-life, L as a function of the free parameter of interest can be mapped out by choosing a set of values for the parameter of interest. Fits are performed holding the parameter of interest fixed at each of these values, and letting the other free parameters vary. Figure 3, which was used above to describe the different types of error limits for skewed distributions, is the result of such a procedure. The limits presented there are the covariant error limits on the half-life determined for ^{261}Lr. Figure 4 shows the L distribution as a function of the independent component half-life for a multicomponent decay curve with high counting statistics (more than 1000 counts in each interval). The details of the decay curve and the fit for figure 4 will be presented in the next section. It should be noted that since the L distribution in figure 4 is essentially Gaussian,the 68.3% confidence limits and the interval of equal likelihood chances for a confidence level of 68.3% correspond to the one sigma limits of the Gaussian, or 0.85 times the half-maximum limits.

Results and Comparisons

Decay curve fits performed with the MLDS code are compared with those for error weighted least squares fits. The error-weighted least squares fits were performed with the EXFIT code [3] which performs the fits by the same procedure as earlier codes [9], but was retrofitted to use the activities integrated over the time intervals, rather than assuming instantaneous decay rates. EXFIT gives the same results as other error-weighted least squares decay curve fitting codes [2], within convergence criteria. The least squares procedure used in EXFIT is outlined by Moody [9].

high counting statistics limit

As an example of a multi-component decay curve fit in the limit of high counting statistics (more than 1000 counts per time interval), a decay curve consisting of a 1-minute parent activity feeding a five-minute daughter activity together with an independent component with a 25-minute half-life was constructed. The time intervals chosen were 10x0.5m, 10x1.0m, and 10x2.0m. Normally distributed statistical fluctuations were included in the decay curve data. Table 1. shows a comparison of the fit to this decay curve with the two methods. In all fits, the initial daughter activity was fixed at 0. The results agree quite well, and any differences are within the uncertainties in the values for the free parameters. The MLDS uncertainties listed do not consider the covariances with the other free parameters. As an example of how the covariant uncertainties compare with the non-covariant uncertainties for the high counting statistics case, the covariant uncertainties in the half-life of the parent activity were determined. As noted above, figure 4 is a plot of the normalized L as a function of the parent activity half-life obtained by letting the other free parameters vary in fits where the parent half life was fixed at a series of values. This half life, with its covariant one sigma error limits is 1.004 ± 0.024. The covariant error limits are seven times larger than the non-covariant error limits. The differences in the MLDS and EXFIT fits are real, however, and are not due to errors induced by the convergence criteria. To test this, the convergence criteria in both codes were tightened significantly, which had very little effect on the results.

poor statistics limit and discrete event times

As an example of the differences in the limit of poor counting statistics between MLDS and EXFIT for time binned events and MLDS for discrete event times the decay data for the α-decay of the 4.3-s ^{258}Lr daughter of 34-s ^{262}Ha is considered. The event times used are the sum of the parent and daughter lifetimes for 14 events in which both the parent and daughter α-particles were detected in chemically separated samples [10,11]. Table 2 contains a comparison of fits to these data for MLDS with the discrete event times and for MLDS and EXFIT each with data time binned in two ways. In the upper half of the table, the decay was fit with a parent-daughter decay relationship. The initial activity of the daughter, ^{258}Lr was fixed at 0. The fits with MLDS for discrete event times agree quite well with the MLDS fits for the time binned data, and the effect of the different choices of time bins seems to have a small effect in the maximum likelihood fits. The different choices of time bins seems to have a larger effect in the least squares fits, and the results differ significantly from the maximum likelihood fits based on the discrete event times. The least squares approach tends to underestimate the initial activities and half-lives, which may be expected based on the trends explained earlier and outlined in figures 1 and 2.

For comparison, in the bottom half of table 2, the same data were fit with a single component decay, shown in the bottom half of the table. The values of ln(L) listed below each of the MLDS fits can be used to determine that the parent-daughter fits are better than the single component fits. The ln(L)'s for the parent-daughter fits are larger than those for the single component fits by about 0.8, giving a factor of 2.2 difference in the L's, indicating that the parent-daughter more than twice as probable.

Conclusions

- The MLDS (Maximum Likelihood Decay by the Simplex method) computer code has been developed for the fitting of radioactive decay curves. This code combines the use of Poisson statistics based maximum likelihood techniques with iterative curve fitting by the Simplex method.

- The use of Poisson statistics and activities integrated over the counting intervals in the likelihood function makes this code especially applicable to cases of poor counting statistics. In the limit of high counting statistics, the results are consistent with other decay curve fitting procedures.

- MLDS uses either time binned data or discrete event times. The use of the discrete event times is especially important in fitting curves based on a very small number of events, making maximum use of the decay information.

- MLDS currently is capable of fitting fifteen different types of decay curves composed of combinations of up to 5 individual decay components, up to two parent daughter chains, and one three membered chain, with a maximum of 10 free parameters. Any of combination of the initial activities and half-lives may be allowed to vary in the fits.

- The simplex method for locating the maximum in a multidimensional surface allows the multi-parameter fits to be carried out while avoiding the divergence common in other methods of maximization. The speed of the maximization with the simplex method is only a factor of 2 to 5 slower than steepest ascent maximization techniques.

- The simplex method performs the maximization using only the equation for the goodness of fit criterion (there is no need to program the partial derivatives of this equation with respect to all free parameters). This makes the modification of the code for new types of decay curves simple.

- Asymmetric error limits which do not consider the covariance in the other free parameters are quickly calculated and are given by MLDS. These error limits are good for most applications. For cases in which more accurate determinations of the error limits are necessary, a procedure is outlined for obtaining the asymmetric error limits which contain the effects of the covariance.

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Table 1. Comparison of fits with MLDS and EXFIT.

For a computer-generated decay curve consisting of a parent-daughter chain and an independent component. Normally distributed statistical fluctuations have been included in the decay curve.

	TRUE VALUE	EXFIT SOLUTION*	MLDS SOLUTION**
Independent			
$t_{1/2} = \ln(2)/d_1$	25.00	25.4±1.6	$24.63^{+0.62}_{-0.60}$
D_1^0	1000.	976.±15.	$1012.^{+17.}_{-16.}$
Parent			
$t_{1/2} = \ln(2)/p_2$	1.000	0.997±.014	$1.004^{+0.003}_{-0.004}$
P_2^0	100000.	100600.±1700.	$99700.^{+300.}_{-330.}$
Daughter			
$t_{1/2} = \ln(2)/d_2$	5.000	5.006±0.053	$4.991^{+0.021}_{-0.19}$
D_2^0	0.000	0.000(FIXED)	0.000(FIXED)

* The EXFIT error limits are one sigma limits determined from the curvature of the X^2 surface at the minimum. They include the effects of covariance of the other free parameters.

** The MLDS error limits are the non-covariant half-maximum values. Covariant error limits will be significantly larger.

Table 2. A comparison of fits to time-binned data and discrete event times for the α-decay of 4.3-s ^{258}Lr being fed by 34-s ^{262}Ha and the use of L to choose the type of decay curve. The decay curves are based on the times of 19 events. The MLDS error limits do not consider the effects of the covariance of the other free parameters and are significantly smaller then the non-covariant error limits. Half-lives are in s and initial activities are in s^{-1}.

	MLDS DISCRETE TIMES	MLDS TIME** BINNED	MLDS TIME*** BINNED	EXFIT TIME** BINNED	EXFIT TIME*** BINNED
fit with a parent-daughter chain:					
$\frac{\ln(2)}{p_1}$	$38.^{+9.}_{-8.}$	$37.^{+10.}_{-7.}$	$41.^{+11.}_{-9.}$	19.±9.	37.±23.
P_1^0	$.35^{+.11}_{-.08}$	$.36^{+.11}_{-.09}$	$.34^{+.10}_{-.09}$	.57±.24	.22±.13
$\frac{\ln(2)}{d_1}$	$2.6^{+3.8}_{-2.4}$	$2.5^{+3.4}_{-2.3}$	$2.4^{+3.4}_{-2.4}$	1.8±3.6	0.8±3.1
D_1^0	0(FIXED)	0(FIXED)	0(FIXED)	0(FIXED)	0(FIXED)
ln(L)*	-135.03	-15.40	-16.81	(.85)	(1.03)
fit with a single component					
$\frac{\ln(2)}{d_1}$	$41.^{+11.}_{-8.}$	$41.^{+10.}_{-8.}$	$42.^{+12.}_{-8.}$	31.±14.	44.±26.
D_1^0	$.32^{+.10}_{-.08}$	$.33^{+.10}_{-.08}$	$.32^{+.09}_{-.08}$	.27±.12	.19±.10
ln(L)***	-135.79	-16.16	-17.63	(1.2)	(.96)

* reduced X^2 is shown for the EXFIT least squares fits
** time intervals: 4x5s, 5x40s
*** time intervals: 1x2.5s, 4x5s, 1x17.5s, 4x40s, 1x20s

FIGURE CAPTIONS

Figure 1. The center curve is a normal distribution centered on $Y_m = 10$. The normal distribution which would have been assumed if the number of observed events, Z_m, was 15 is shown by the right curve, and that for $Z_m = 5$ is shown by the left curve line. Note that the $Z_m = 5$ distribution is significantly narrower than the others, resulting in a greater statistical weighting in the X^2 fit.

Figure 2. The curve connecting the empty squares is a normal distribution centered on $Y_m = 1$ and the curve connecting the solid squares is a Poisson distribution centered on $Y_m = 1$. Note the difference in the shapes of the curves, especially the probability under the normal curve for negative numbers of counts.

Figure 3. The determination of the covariant error limits for the half-life in the fit to the decay of ^{261}Lr. The normalized L is plotted as a function of the value at which the half-life was fixed, while the D° was allowed to vary. The various types of asymmetric error limits are shown.

Figure 4. The determination of the covariant error limits for the half-life of the parent activity in the fit to the high counting statistics decay curve which was for a parent-daughter chain plus an independent component. The normalized L is plotted as a function of the value at which this half-life was fixed, while the other free parameters were allowed to vary. Since this L distribution is Gaussian, the 68.3%confidence limits and the interval of equal likelihood chances corresponding to a confidence level of 68.3% are equal to the one sigma error limits.

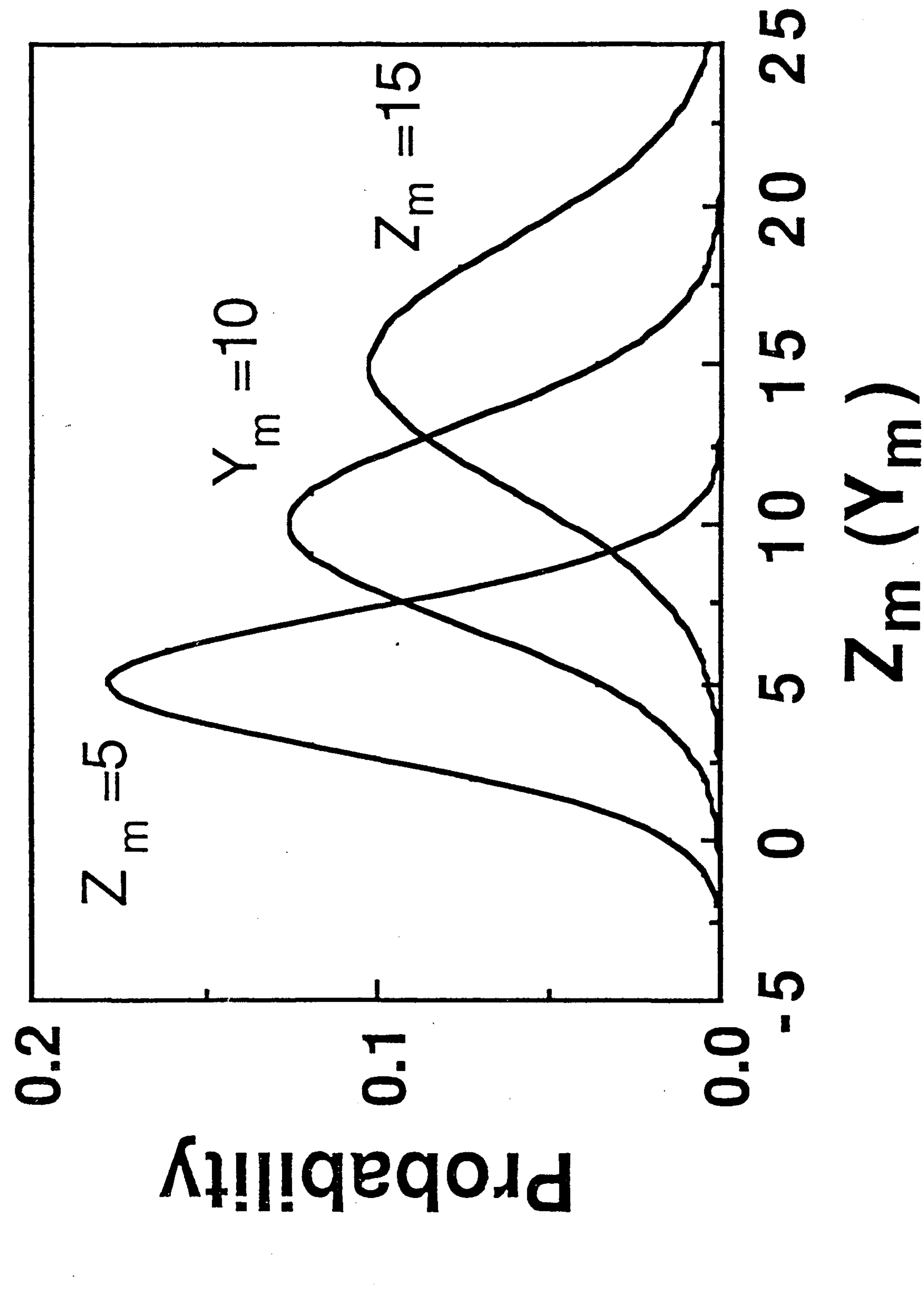

Fig. 1

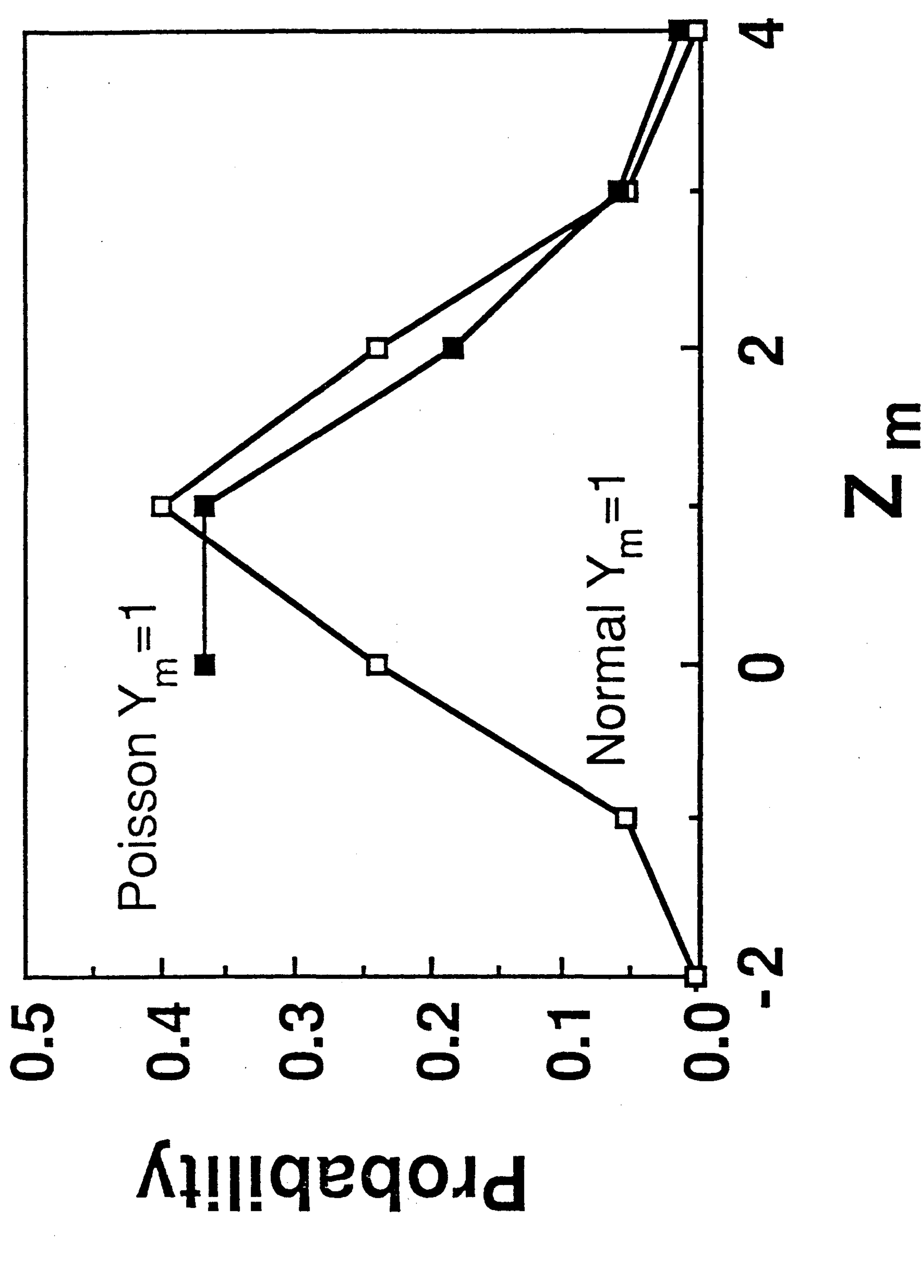

Fig. 2

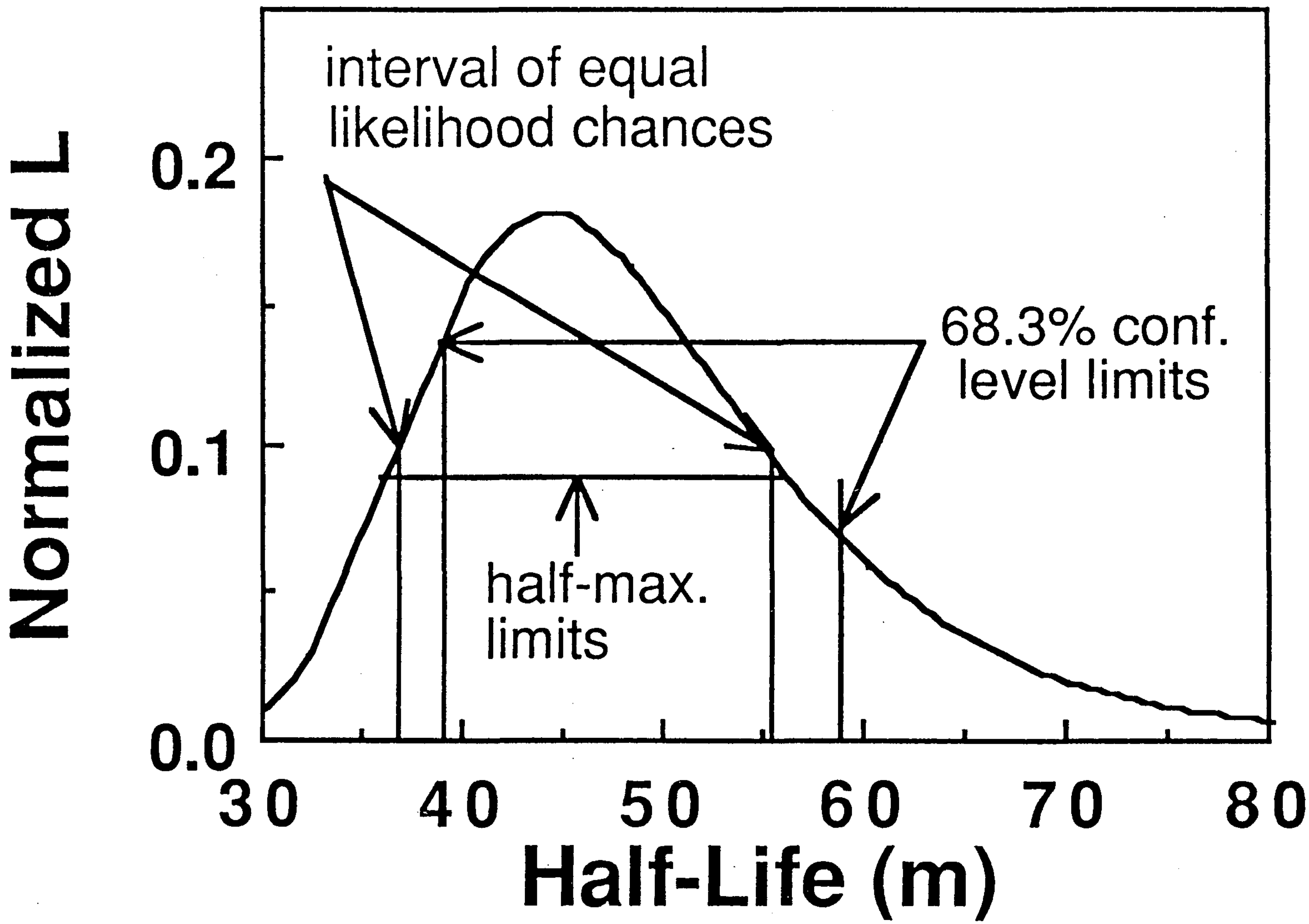
interval of equal
likelihood chances
68.3% conf.
level limits
half-max.
limits
Normalized L
0.2
0.1
0.0
30
40
50
60
70
80
Half-Life (m)

Fig. 3

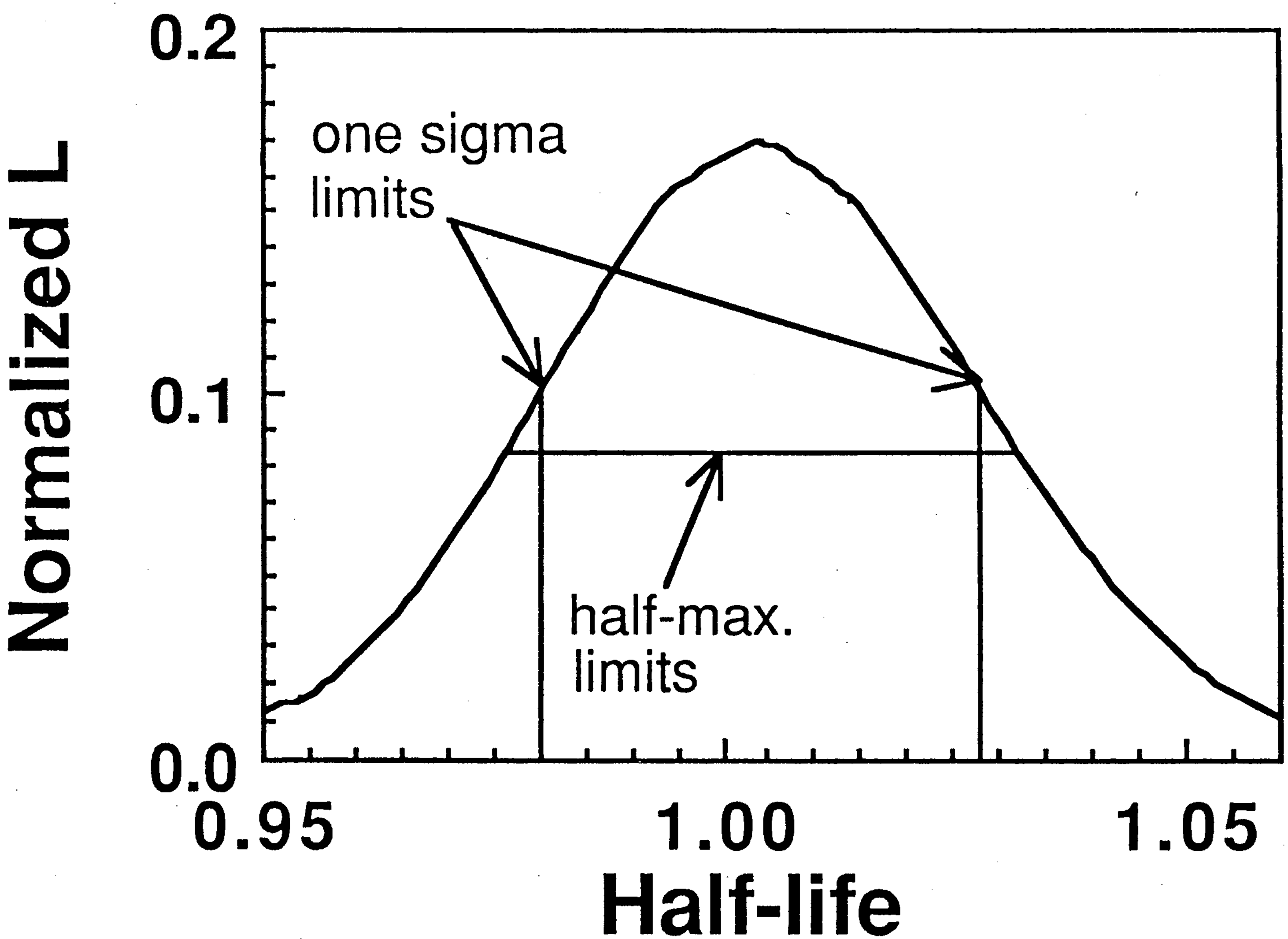
Normalized L
0.2
0.1
0.0
0.95
1.00
1.05
Half-life
one sigma
limits
half-max.
limits

Fig. 4