## UCSF

## UC San Francisco Previously Published Works

## Title

Evaluating a Targeted Minimum Loss-Based Estimator for Capture-Recapture Analysis:
An Application to HIV Surveillance in San Francisco, California.

## Permalink

https://escholarship.org/uc/item/27r8v6kc

## Journal

American Journal of Epidemiology, 193(4)
Authors
Wesson, Paul
Das, Manjari
Chen, Mia
et al.
Publication Date
2024-04-08
DOI
10.1093/aje/kwad231

Peer reviewed

## Practice of Epidemiology

# Evaluating a Targeted Minimum Loss-Based Estimator for Capture-Recapture Analysis: An Application to HIV Surveillance in San Francisco, California 

Paul Wesson*, Manjari Das, Mia Chen, Ling Hsu, Willi McFarland, Edward Kennedy, and Nicholas P. Jewell<br>* Correspondence to Dr. Paul Wesson, Department of Epidemiology and Biostatistics, School of Medicine, University of California, San Francisco, UCSF Box 0886, 550 16th Street, 3rd Floor, San Francisco, CA 94615<br>(e-mail: paul.wesson@ucsf.edu).

Initially submitted January 9, 2023; accepted for publication November 15, 2023.

The capture-recapture method is a common tool used in epidemiology to estimate the size of "hidden" populations and correct the underascertainment of cases, based on incomplete and overlapping lists of the target population. Log-linear models are often used to estimate the population size yet may produce implausible and unreliable estimates due to model misspecification and small cell sizes. A novel targeted minimum loss-based estimation (TMLE) model developed for capture-recapture makes several notable improvements to conventional modeling: "targeting" the parameter of interest, flexibly fitting the data to alternative functional forms, and limiting bias from small cell sizes. Using simulations and empirical data from the San Francisco, California, Department of Public Health's human immunodeficiency virus (HIV) surveillance registry, we evaluated the performance of the TMLE model and compared results with those of other common models. Based on 2,584 people observed on 3 lists reportable to the surveillance registry, the TMLE model estimated the number of San Francisco residents living with HIV as of December 31, 2019, to be 13,523 ( $95 \%$ confidence interval: 12,222, 14,824). This estimate, compared with a "ground truth" of 12,507 , was the most accurate and precise of all models examined. The TMLE model is a significant advancement in capture-recapture studies, leveraging modern statistical methods to improve estimation of the sizes of hidden populations.
bias; capture-recapture method; hidden populations; human immunodeficiency virus; machine learning; prevalence estimation; SuperLearner; targeted minimum loss-based estimation

Abbreviations: CI , confidence interval; HIV, human immunodeficiency virus; SFDPH, San Francisco Department of Public Health; TMLE, targeted minimum loss-based estimation.

Establishing the true denominator of a population at risk is critical for determining rates of disease burden and acquisition, allocating resources, and setting appropriate public health priorities and policies. Multiple systems estimation, often referred to as capture-(mark-)recapture, is commonly used in epidemiology to estimate the denominator for the population at risk and to correct the underascertainment of cases in disease surveillance $(1,2)$. The capture-recapture method estimates an unobserved population size based on the number of people already observed on multiple overlapping and incomplete lists of the target population (3-5). The greater the lists' overlap, the smaller the unobserved population; conversely, the smaller the overlap, the greater the
unobserved population. In public health applications, lists can be administrative records from medical centers, disease registries, and surveillance systems, or direct samples of the target population from cross-sectional surveys, among other possibilities (6). A key assumption for valid estimation is that the lists are statistically independent of one another. Presence on one list cannot increase (positive dependence) or decrease (negative dependence) the probability of presence on another list. Positive dependence will underestimate the true population size, while negative dependence will result in overestimation. There are 3 other formal assumptions: the population is "closed" (there are no entries or exits between capture occasions); there is accurate matching
of individuals on multiple lists/captures; and, for each list, everyone has the same probability of capture (capture homogeneity) (3). An additional, implicit assumption is random assignment to lists, meaning the population observed on the lists is reflective of the unobserved population.

The popularity of the capture-recapture method persists in epidemiology despite a fundamental limitation: The population size estimate parameter is not identified without untestable assumptions (7, 8). In capture-recapture modeling, there is always 1 fewer degree of freedom available than is needed to estimate all parameters in the model. Variations in modeling always impose a constraint in order to identify the unobserved population size. In log-linear regression, a common model form for capture-recapture due to the data often being represented as cross-classified categorical data (9), the constraint is often that $k$-way interaction is 0 (where $k$ is the number of lists). This is a strong assumption when using only 2 lists, as it assumes list independence, but it may become more reasonable with additional lists. Additional lists confer greater statistical flexibility because more degrees of freedom allow modeling of lower-level listdependence through interaction terms.

Despite their broad implementation, log-linear models are also criticized for producing implausible or unreliable estimates (10-12). Using at least 3 lists, multiple log-linear models may be fitted, each accounting for a different combination of potential list dependencies. Model selection is often achieved by selecting the model with the lowest information criterion. This approach presents a single model estimate, ignoring potentially plausible estimates from other models with an approximately equally good fit to the data. The information criterion does not always uniquely identify the best model; two models can have the same information criterion and give very different population size estimates (12). Furthermore, model selection based on the information criterion can be extremely sensitive to cell sparsity (i.e., few people observed in cells corresponding to different capture patterns). Because coefficients in the model are jointly determined, searching the extreme regions of the parameter space to maximize the likelihood of probabilities for the sparse cells also impacts predictions for other cells $(11,13)$. This sometimes results in implausibly large size estimates and a deceptively small information criterion, making it the (statistically) preferred model.

The log-linear modeling approach is arguably statistically inefficient because degrees of freedom are spent estimating parameters already observed and not of interest (i.e., much of the probability distribution). The observed data, the number "captured" on each combination of lists, are fitted to a loglinear model to project to the intercept and estimate the number not seen on any list. Forcing the data to fit the log-linear functional form is another potential limitation. This model is chosen out of convenience but may not reflect the true statistical model. Model misspecification can yield biased estimates. These limitations to conventional log-linear modeling motivated a targeted minimum loss-based estimation (TMLE) approach to capture-recapture, paired with semiparametric modeling using machine learning techniques.

In this paper, we describe and evaluate a novel TMLE capture-recapture estimator using data from the San Fran-
cisco Department of Public Health (SFDPH) human immunodeficiency virus (HIV) surveillance registry. We applied the TMLE estimator to data collected from medical center lists reporting cases to the SFDPH to estimate the total number of people living with HIV in San Francisco and accessing HIV-related medical care in 2019. We compared TMLE population size estimates with estimates from conventional capture-recapture models and the likely population size (ground truth), based on SFDPH surveillance data.

## METHODS

## Study population

Our target population was the number of San Francisco residents living with HIV as of December 31, 2019. This population included out-of-jurisdiction residents at the time of HIV diagnosis who later moved to and received care in San Francisco by the end of 2019. Excluded from this population were San Francisco residents at the time of diagnosis who then moved away from San Francisco by the end of 2019. Deidentified data were pulled from the San Francisco HIV case registry data and data from the San Francisco HIV laboratory data management system as of January 7, 2022. The data extraction included 3 indicator variables for whether a patient was observed on any of 3 chosen lists feeding into the surveillance system: the ward 86 HIV clinic at Zuckerberg San Francisco General Hospital (the largest public HIV care provider in the city), the SFDPH Laboratory (the public health laboratory covering patients for whom HIV-related laboratory tests are ordered), and the Tom Waddell Urban Health Clinic (a community clinic serving a diverse patient population, including racial minorities, transgender people, and people who are marginally housed). These lists were chosen for their relative size and the diverse patient populations they collectively contribute to SFDPH surveillance. For the capture-recapture analysis, we only considered patients who appeared on at least 1 of the 3 lists, using the full data set from the HIV case registry as the "ground truth" for comparison to assess the performance of various estimators.

Additionally, we extracted the following covariates believed to influence a person's probability of selection on one or multiple lists: race and ethnicity, birth sex, age in 2019, age at the time of HIV diagnosis, HIV transmission risk category, indicator of new diagnosis in 2019, and viral suppression status.

## TMLE overview and application to capture-recapture

In contrast to the classic log-linear model discussed above, Das et al.'s TMLE approach (14) reformulates the target parameter, $\Psi$, as the probability of being observed on any list (i.e., the capture probability). Dividing the observed $n$ by $\Psi$ yields the estimated population size. Das et al.'s approach, like other approaches (15), requires a single identifiability assumption; with a pair of lists, selection on one list is conditionally independent of selection on another list, given a set of covariates, akin to classical confounding.

Das et al. estimate $\Psi$ by

$$
\hat{\psi}=\left[\mathbb{Q}_{N}\left\{\frac{1}{\hat{\gamma}(X)}\right\}\right]^{-1}
$$

where $\mathbb{Q}_{N}$ is the empirical measure of the observed (biased) data distribution $\mathbb{Q}$ of capture patterns. $\hat{\gamma}$ depends on conditional capture probabilities, given a level of $X$, as follows:

$$
\hat{\gamma}(X)=\frac{\hat{q}_{12}(X)}{\hat{q}_{1}(X) \hat{q}_{2}(X)},
$$

where $\hat{q}_{1}, \hat{q}_{2}$, and $\hat{q}_{12}$ are observational probabilities ( $q$ probabilities) for being captured by list 1 , by list 2 , and by both lists 1 and 2 , respectively. $\mathbf{X}$ is a vector of covariates that may influence these $q$-probabilities.

TMLE is a doubly robust, maximum-likelihood-based estimation method that optimizes the bias-variance tradeoff through a "targeting" mechanism (16) that has been widely used to address causal inference problems in epidemiology. TMLE has been described in detail elsewhere (17). Briefly, the TMLE framework in this context begins with an initial estimation of the $q$-probabilities $q_{1}(X), q_{2}(X)$, and $q_{12}(X)$ (i.e., $P\left(Y_{1}=1 \mid X\right), P\left(Y_{2}=1 \mid X\right)$, and $P\left(Y_{1} Y_{2}=1 \mid X\right)$, respectively). In the "targeting" step for TMLE, the nuisance parameters are estimated as part of the "clever covariates," the coefficients of which are used to update the initial estimates of $q_{1}(X), q_{2}(X)$, and $q_{12}(X)$. This process continues iteratively according to a stopping rule and reduces bias in the initial estimate of $\Psi=P(Y \neq 0)$. The final updated estimates of $q_{1}(X), q_{2}(X)$, and $q_{12}(X)$ are then used to calculate the target parameter $\Psi=P(Y \neq 0)$. These steps are outlined in Appendix 1. TMLE benefits from statistical flexibility by incorporating various algorithms and machine learning methods (e.g., cross-validation) to model complex relationships in the data without making overly restrictive assumptions about functional form.

As a corollary to the doubly robust properties of TMLE, for the TMLE capture-recapture model, if either estimate of $q_{1}$ or $q_{2}$ has small error and if either estimate of $q_{12}$ or $\gamma$ has small error, then the overall error for $\hat{\psi}_{\text {tmle }}$ (the TMLE for $\hat{\psi}$ ) will be just as small, even if the other estimates have large errors or are misspecified (14).
If more than 2 lists are available, $\hat{\psi}_{\text {tmle }}$ can be estimated for all pairwise combinations of lists. The lists not used to directly estimate the $q$-probabilities are treated either as additional lists or as covariates. If the former, the additional list(s) provide information to refine $q$-probabilities; the model now becomes aware of additional people who are part of the target population yet may not be observed on either of the 2 primary lists, updating the capture probabilities for those two lists. If the latter, the additional list(s) may be used alongside other covariates in the vector $\mathbf{X}$ to make the 2 primary lists conditionally independent from one another.

The TMLE model for capture-recapture draws upon multiple algorithms to model the relationship between conditioning covariates and estimate the $q$-probabilities. Currently, these algorithms include logistic regression,
generalized additive models, Ranger (a random forest algorithm), multinomial logistic regression, and rangerlogit (an ensemble model using Ranger and logistic regression). SuperLearner, an ensemble machine learning method, is also a feature of the model, permitting multiple algorithms to be used at once to flexibly model these relationships and parameters (18). Cross-validation is used to prevent overfitting and flexibly model the data. Ninety-five percent confidence intervals (CIs) are estimated on the basis of the efficient influence function.

Additionally, the sizes of population subgroups may be estimated on the basis of categorical variables included in $\mathbf{X}$.

## TMLE capture-recapture model simulations

Initial estimation of population sizes using TMLE revealed sensitivity to a so-called margin setting. The margin setting is a function that prevents searching the extremes of the parameter space to estimate small $q$-probabilities. This will likely occur with small cell sizes.

We conducted simulations to inform the optimal margin setting. All simulations included 3 lists with different marginal capture probabilities for each list. To match the SFDPH data, the true population size was set at 12,500 for all simulations.

Scenario 1 modeled the 3 lists as statistically independent from each other: List 1 had a capture probability of $20 \%$, list 2 had $25 \%$, and list 3 had $30 \%$.

Scenario 2 also modeled the 3 lists as statistically independent but reduced the marginal probabilities to align with the SFDPH data. List 1 sampled $14 \%$ of the population, list 2 sampled $6 \%$, and list 3 sampled $3 \%$.
Scenario 3 modeled conditional independence. Dependence is induced between lists 1 and 2 because of 2 additional independent binary variables, $S$ (with $50 \%$ prevalence) and $A$ (with 30\% prevalence). The marginal probabilities for the lists are the same as in scenario 2, but the conditional probabilities change depending on whether $S$ and $A$ are observed. List 1 randomly samples $4.5 \%$ of the population; an additional $10 \%$ is sampled for whom $S=1$, and an additional $15 \%$ is sampled for whom $A=1$. List 2 samples $2 \%$ of the population; an additional $5 \%$ is sampled for whom $S=1$, and an additional $5 \%$ is sampled for whom $A=1$. List 3 samples $3 \%$ of the population.

Scenario 4 models conditional independence that is only partially accounted for in the TMLE approach. Three independent binary variables are simulated in addition to the 3 lists: $S$ (with $50 \%$ prevalence), $A$ (with $30 \%$ prevalence), and $U$ (with $50 \%$ prevalence). Only $S$ and $U$ induce a dependence between lists 1 and 2, but only $S$ and $A$ are included in the $\mathbf{X}$ vector of the TMLE model to create conditional independence. List marginal probabilities are the same as in scenarios 2 and 3, but conditional probabilities differ. List 1 samples $4 \%$ of the population; an additional $10 \%$ is sampled for whom $S=1$, and an additional $10 \%$ is sampled for whom $U=1$. List 2 samples $4 \%$ of the population; an additional $2 \%$ is sampled for whom $S=1$, and an additional $2 \%$ is sampled for whom $U=1$. List 3 samples $3 \%$ of the population.

Each scenario was simulated 500 times. The margin was set at $0.02,0.04,0.06,0.08,0.1$, and a data-dependent value,
M.star (M.star is a dynamic value that is calculated in each simulation as the number observed on both lists 1 and 2, the 2 lists used for the TMLE, divided by the observed count from all 3 lists). Violin plots were used to visualize the distribution of size estimates for each scenario at each margin setting.

## Population size estimation

We used the R package drpop (19) to fit the TMLE capturerecapture model. Estimates were generated using the ward 86 and SFDPH Laboratory list pair (lists 1 and 2). The third list, the Tom Waddell clinic list, was treated as an additional list to improve the estimation of the $q$-probabilities. All measured covariates were included in the $\mathbf{X}$ vector to model conditional independence. We used SuperLearner to model covariate relationships and $q$-probabilities. The SuperLearner library included generalized additive models, generalized linear models, GLM.interaction, Ranger, and glmnet. Cross-validation was based on 2 folds. The optimal margin setting was determined from the results of the simulation analysis. These same settings were used to estimate the sizes of population subgroups.

Additional capture-recapture models were fitted for comparison. For these comparisons, we focused on models developed to correct biases from list dependence (other models have been developed to address biases resulting from violation of the other formal assumptions) (20). The R package Rcapture (21) was used to fit the log-linear regression models, adjusting for potential list dependencies. The R package SparseMSE (22) implements SparseMSE, a model designed to correct the bias resulting from small or no overlap between lists $(11,13)$. The R package $D G A$ (23) was used to implement the decomposable graph approach. The decomposable graph approach uses Bayesian model averaging to average estimates from individual log-linear models, weighted by their posterior likelihood, into a single posterior estimate. The R package shinyrecap (24) was used to implement the Bayesian latent class model, which seeks to meet the identifiability constraint of list independence by conditioning on latent classes based on observed capture histories. Both the decomposable graph approach and Bayesian latent class models have shown less bias than conventional log-linear models in previous simulation studies $(10,11)$.

Population size estimates from all models were compared against the complete SFDPH data as the ground truth.

## Ethics

This study was reviewed and approved by the University of California, San Francisco's institutional review board. No personally identifying information, including medical record numbers, were present in the analytical database.

## RESULTS

## Sample/study population

There were 12,507 people living with HIV in the complete SFDPH data. Of these, 2,584 were observed on the
combination of the 3 lists (see Web Figure 1, available at https://doi.org/10.1093/aje/kwad231). List 1 (ward 86) accounted for $70 \%$ of the analytical sample, list 2 (SFDPH Laboratory) accounted for $28 \%$, and list 3 (Tom Waddell clinic) accounted for $14 \%$. Notably, there was relatively little overlap among the 3 lists. The lists differed in the distribution of covariates (Table 1). There was a greater proportion of Latino/a patients on the SFDPH Laboratory list than on either of the other lists, whereas the racial/ethnic distribution was more even on the Tom Waddell list. Females were overrepresented on all 3 lists relative to their true proportion in the surveillance data. Each list underrepresented the proportion of men who had sex with men, relative to the surveillance data, and overrepresented the proportion of people who injected drugs and the proportion of men who had sex with men and also injected drugs. Patients who injected drugs as a transmission risk category were overrepresented by the Tom Waddell list. New HIV diagnoses (in 2019) were overrepresented by the SFDPH Laboratory's list, whereas patients who were not virally suppressed were overrepresented by the Tom Waddell list. The age distributions by list largely reflected the age distributions in the surveillance data.

## Simulation results

The TMLE model was robust to different margin settings when lists were independent and sampled from the source population with at least $20 \%$ probability (Figure 1). Modeling results were sensitive to margin settings when the marginal sampling probabilities of the lists decreased. There was a clear bias-variance tradeoff with margin settings when lists were simulated to have smaller marginal capture probabilities.

In scenario 2, simulating list independence, higher margin settings ( $\geq 0.06$ ) yielded biased distributions with a false sense of greater estimated precision. Setting the margin to 0.04 yielded minimal bias, but moderately increased the variance. The variance continued to increase when the margin was set to 0.02 . The mean for M.star for this scenario was close to 0.04 (mean $=0.039$; range, $0.029-0.049$ ), but the spread of the distribution was wider than that observed when the margin was set to 0.04 because of variability in the distribution of M.star across simulations.

In scenario 3, modeling conditional independence, setting the margin to 0.02 resulted in the only distribution to cover the truth, albeit with greatest variance. The distribution was tighter when the margin was set to 0.04 , but barely failed to cover the truth. Increasing margin settings yielded precise, but biased, estimates. The distribution for M.star indicated that this dynamic marginal probability fell between 0.04 and 0.06 in simulations.

In scenario 4, model misspecification yielded similar results as in previous scenarios, albeit with moderately increased variance. Setting the margin to 0.04 yielded the least biased distribution with moderate spread. Based on these results, we determined that the observed marginal probability of the overlap is distorted when covariates create a dependence between lists. For optimal performance of the TMLE model, the margin should be set to the
Table 1. Characteristics of San Francisco, California, Residents Living With HIV as of December 31, 2019, by List Contributing to HIV Surveillance

| Characteristic |  |  | List Contributing to HIV Surveillance |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Total$(n=12,507)$ |  | ZSFGH Ward 86 |  |  |  | Tom Waddell Urban Health Clinic |  |  |  | SFDPH Laboratory |  |  |  | $\begin{aligned} & \text { Other }{ }^{\mathrm{a}} \\ & (n=9,923) \end{aligned}$ |  |
|  |  |  | Unique$(n=1,779)$ |  | With Overlap$(n=1,809)$ |  | Unique$(n=63)$ |  | With Overlap$(n=359)$ |  | Unique$(n=425)$ |  | With Overlap$(n=736)$ |  |  |  |
|  | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% |
| Race/ethnicity |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| White | 6,569 | 52.5 | 740 | 41.6 | 754 | 41.7 | 21 | 33.3 | 113 | 31.5 | 79 | 18.6 | 178 | 24.2 | 5,627 | 56.7 |
| Black | 1,602 | 12.8 | 354 | 19.9 | 361 | 20.0 | 16 | 25.4 | 100 | 27.9 | 30 | 7.1 | 114 | 15.5 | 1,115 | 11.2 |
| Latino/a | 2,901 | 23.2 | 492 | 27.7 | 499 | 27.6 | 14 | 22.2 | 77 | 21.4 | 288 | 67.8 | 357 | 48.5 | 2,038 | 20.5 |
| Other/unknown | 1,435 | 11.5 | 193 | 10.8 | 195 | 10.8 | 12 | 19.0 | 69 | 19.2 | 28 | 6.6 | 87 | 11.8 | 1,143 | 11.5 |
| Birth sex |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Female | 743 | 5.9 | 170 | 9.6 | 171 | 9.5 | 15 | 23.8 | 58 | 16.2 | 32 | 7.5 | 75 | 10.2 | 483 | 4.9 |
| Male | 11,764 | 94.1 | 1,609 | 90.4 | 1,638 | 90.5 | 48 | 76.2 | 301 | 83.8 | 393 | 92.5 | 661 | 89.8 | 9,440 | 95.1 |
| Age in 2019, years |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13-19 | 10 | 0.1 | 2 | 0.1 | 2 | 0.1 | 0 | 0.0 | 0 | 0.0 | 3 | 0.7 | 3 | 0.4 | 5 | 0.1 |
| 20-29 | 460 | 3.7 | 68 | 3.8 | 70 | 3.9 | 5 | 7.9 | 8 | 2.2 | 42 | 9.9 | 47 | 6.4 | 340 | 3.4 |
| 30-39 | 1,806 | 14.4 | 282 | 15.9 | 295 | 16.3 | 19 | 30.2 | 56 | 15.6 | 123 | 28.9 | 168 | 22.8 | 1,335 | 13.5 |
| 40-49 | 2,409 | 19.3 | 403 | 22.7 | 412 | 22.8 | 14 | 22.2 | 80 | 22.3 | 83 | 19.5 | 155 | 21.1 | 1,836 | 18.5 |
| 50-59 | 4,207 | 33.6 | 584 | 32.8 | 588 | 32.5 | 16 | 25.4 | 138 | 38.4 | 108 | 25.4 | 230 | 31.2 | 3,375 | 34.0 |
| 60-69 | 2,740 | 21.9 | 379 | 21.3 | 381 | 21.1 | 8 | 12.7 | 65 | 18.1 | 52 | 12.2 | 108 | 14.7 | 2,244 | 22.6 |
| $\geq 70$ | 875 | 7.0 | 61 | 3.4 | 61 | 3.4 | 1 | 1.6 | 12 | 3.3 | 14 | 3.3 | 25 | 3.4 | 788 | 7.9 |
| Age at diagnosis, years |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\leq 12$ | 19 | 0.2 | 2 | 0.1 | 2 | 0.1 | 0 | 0.0 | 0 | 0.0 | 3 | 0.7 | 3 | 0.4 | 14 | 0.1 |
| 13-19 | 219 | 1.8 | 51 | 2.9 | 51 | 2.8 | 3 | 4.8 | 8 | 2.2 | 15 | 3.5 | 20 | 2.7 | 145 | 1.5 |
| 20-29 | 3,475 | 27.8 | 549 | 30.9 | 559 | 30.9 | 22 | 34.9 | 113 | 31.5 | 147 | 34.6 | 242 | 32.9 | 2,659 | 26.8 |
| 30-39 | 5,003 | 40.0 | 661 | 37.2 | 674 | 37.3 | 19 | 30.2 | 123 | 34.3 | 178 | 41.9 | 291 | 39.5 | 4,031 | 40.6 |
| 40-49 | 2,853 | 22.8 | 399 | 22.4 | 405 | 22.4 | 12 | 19.0 | 77 | 21.4 | 57 | 13.4 | 125 | 17.0 | 2,316 | 23.3 |
| 50-59 | 760 | 6.1 | 98 | 5.5 | 99 | 5.5 | 5 | 7.9 | 33 | 9.2 | 16 | 3.8 | 43 | 5.8 | 613 | 6.2 |
| 60-69 | 163 | 1.3 | 19 | 1.1 | 19 | 1.1 | 2 | 3.2 | 5 | 1.4 | 9 | 2.1 | 12 | 1.6 | 130 | 1.3 |
| $\geq 70$ | 15 | 0.1 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 15 | 0.2 |

Table 1. Continued

| Characteristic | $\begin{gathered} \text { Total } \\ (n=12,507) \end{gathered}$ |  | List Contributing to HIV Surveillance |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | ZSFGH Ward 86 |  |  |  | Tom Waddell Urban Health Clinic |  |  |  | SFDPH Laboratory |  |  |  | $\begin{aligned} & \text { Other }{ }^{\mathrm{a}} \\ & (n=9,923) \end{aligned}$ |  |
|  |  |  | $\begin{aligned} & \text { Unique } \\ & (n=1,779) \end{aligned}$ |  | With Overlap$(n=1,809)$ |  | Unique$(n=63)$ |  | With Overlap$(n=359)$ |  | Unique$(n=425)$ |  | With Overlap$(n=736)$ |  |  |  |
|  | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% | No. | \% |
| HIV risk category |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MSM | 9,223 | 73.7 | 945 | 53.1 | 963 | 53.2 | 10 | 15.9 | 123 | 34.3 | 325 | 76.5 | 449 | 61.0 | 7,816 | 78.8 |
| PWID | 712 | 5.7 | 203 | 11.4 | 205 | 11.3 | 20 | 31.7 | 83 | 23.1 | 21 | 4.9 | 83 | 11.3 | 405 | 4.1 |
| MSM-PWID | 1,823 | 14.6 | 485 | 27.3 | 494 | 27.3 | 26 | 41.3 | 126 | 35.1 | 40 | 9.4 | 144 | 19.6 | 1,166 | 11.8 |
| Other/ unidentified | 749 | 6.0 | 146 | 8.2 | 147 | 8.1 | 7 | 11.1 | 27 | 7.5 | 39 | 9.2 | 60 | 8.2 | 536 | 5.4 |
| New HIV diagnosis in 2019 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| No | 12,275 | 98.1 | 1,752 | 98.5 | 1,772 | 98.0 | 63 | 95.2 | 355 | 98.9 | 359 | 84.5 | 659 | 89.5 | 9,798 | 98.7 |
| Yes | 232 | 1.9 | 27 | 1.5 | 37 | 2.0 | 3 | 4.8 | 4 | 1.1 | 66 | 15.5 | 77 | 10.5 | 125 | 1.3 |
| Virally suppressed? |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| No | 1,300 | 10.4 | 233 | 13.1 | 240 | 13.3 | 30 | 47.6 | 75 | 20.9 | 41 | 9.6 | 90 | 12.2 | 946 | 9.5 |
| Yes | 10,726 | 85.8 | 1,546 | 86.9 | 1,569 | 86.7 | 33 | 52.4 | 284 | 79.1 | 379 | 89.2 | 641 | 87.1 | 8,501 | 85.7 |
| Unknown | 481 | 3.8 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 5 | 1.2 | 5 | 0.7 | 476 | 4.8 |

Abbreviations: HIV, human immunodeficiency virus; MSM, men who have sex with men; PWID, people who inject drugs; SFDPH, San Francisco Department of Public Health; ZSFGH,
a "Other" encompasses all other reporting sources contributing to SFDPH HIV surveillance (not including ZSFGH Ward 86, the Tom Waddell Urban Health Clinic, and the SFDPH


Figure 1. Estimated numbers of persons living with human immunodeficiency virus (HIV) as of December 31, 2019, from simulations varying the marginal probability of list overlap, San Francisco, California. A) Scenario 1; B) scenario 2; C) scenario 3; D) scenario 4. The $y$-axis refers to the estimated population size. The $x$-axis refers to the margin setting for each simulation $(0.02,0.04,0.06,0.08,0.1$, and M .star, a datadependent value that is calculated as the number observed on both list 1 and list 2 divided by the observed count from all 3 lists). The red horizontal line refers to the "ground truth" estimate from the San Francisco Department of Public Health HIV surveillance office $(n=12,507)$. PSE, population size estimate.
expectation when lists are independent ( 0.04 for our empirical data).

## Population size estimation results

Setting the margin to 0.04 , the TMLE model estimated the population size to be 13,523 ( $95 \% \mathrm{CI}: 12,222,14,824$ ) (Figure 2; Web Table 1). No other model produced estimates as accurate or precise. The best-fitting log-linear model included all 3 pairwise interaction terms modeling list dependence yet substantially underestimated the "true" population size (estimate $=6,536 ; 95 \% \mathrm{CI}: 3,179,18,010$ ). We note, however, that the confidence interval did contain the ground truth. The Bayesian latent class model similarly underestimated the population size yet estimated a credible interval wide enough to contain the ground truth (estimate $=$ 6,$736 ; 95 \%$ credible interval: $2,647,17,957$ ). Although the intervals for both of these models contained the ground truth, the intervals were wider than the TMLE model's interval by nearly a factor of 6 . Other models either
under- or overperformed, with $95 \%$ CIs excluding the ground truth.

## Population size estimates for subgroups

Subgroup estimates from the TMLE model were mixed (Table 2). The model produced accurate estimates (i.e., the $95 \%$ CI contained the ground truth) for subgroups based on new HIV diagnoses in 2019 and viral suppression. Numbers of Black patients and patients with other/unknown race/ethnicity recorded were estimated with accuracy, whereas the number of White patients was underestimated and the number of Latino/a patients was overestimated. Although male patients were accurately estimated, the number of female patients was overestimated by a factor of 1.7. Estimates based on age (age in 2019 and age at diagnosis) were either accurate or had minimal bias (the $95 \%$ CI nearly contained the ground truth). When cell sizes fell below $4 \%$ of the total count in the analytical data, the model failed to produce estimates, perhaps due to positivity violations in the context


Figure 2. Population size estimates (points) for the number of San Francisco, California, residents living with human immunodeficiency virus as of December 31, 2019, comparing the targeted minimum loss-based estimation (TMLE) model with other common models. The horizontal red line represents the ground truth $(n=12,507)$. The log-linear model (LLM) is the best-fitting model according to the lowest Akaike information criterion value. Depending on the model, the bars represent 95\% credible intervals (Bayesian latent class model, decomposable graph approach (DGA)) or $95 \%$ confidence intervals (LLM, SparseMSE, TMLE). MSE, multiple systems estimation.
of cross-validation. The model underestimated the number of men who had sex with men and overestimated numbers in all other risk categories.

## DISCUSSION

Using the TMLE capture-recapture model, we estimated the number of San Francisco residents living with HIV in 2019 to be 13,523 ( $95 \%$ CI: 12,222, 14,824). This estimate, based on 2,584 people observed on 3 different lists, is consistent with the ground truth of 12,507 persons obtained from the SFDPH HIV surveillance office. Estimates from the TMLE model had greater accuracy and precision than those from the more commonly applied models. Most models failed to include the ground truth within their $95 \%$ CIs. The poor performance from the log-linear models was consistent with long-standing criticism that this approach cannot effectively model complex list dependencies, especially in the presence of sparce cells (10-12). More unexpected was the underperformance of the Bayesian latent class and decomposable graph approach models, both of which performed favorably in recent simulation studies $(10,11)$.

Our simulations indicated that the TMLE model estimates are only sensitive to margin settings when lists sample the target population with relatively low sampling probability (perhaps $<20 \%$ ). When lists are independent, the margin
should be set to the proportion of the sample observed on both primary lists used for estimation. When covariates induce dependence between lists, this marginal probability of being observed on both lists is also distorted, and the empirical proportion should not be used to set the margin. Instead, we recommend running a simple simulation, assuming list independence, to approximate the marginal probability. This requires a rough approximation of the target population size, which investigators and stakeholders can often provide.

Our simulation results also revealed the TMLE model's robustness to misspecification from erroneously excluding (including) covariates that do (not) induce dependence between lists. List dependence may be induced through a simple linear combination of covariates. In this case, one might build a logistic regression model to test the association between potential covariates and overlap on the primary lists. However, covariates could induce list dependence through a more complex functional form (e.g., splines, interactions, etc.) that the investigator cannot empirically or comprehensively test a priori. Robustness to such model misspecification is therefore an attractive feature of the TMLE model.

Another attractive feature of the TMLE model is the estimation of subgroups. However, the comparison of our subgroup estimates with the ground truth warrants caution.

Table 2. Estimated Numbers of Persons in Various Population Subgroups Living With HIV as of December 31, 2019, in a Comparison of a TMLE Model With Ground Truth From the San Francisco Department of Public Health, San Francisco, California

| Characteristic | Observed |  | Total |  | TMLE Model |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. | \% | No. | \% | No. | 95\% CI |
| Race/ethnicity |  |  |  |  |  |  |
| White | 942 | 36.5 | 6,569 | 52.5 | 3,536 | 2,756, 4,316 |
| Black | 487 | 18.8 | 1,602 | 12.8 | 2,139 | 1,555, 2,722 |
| Latino/a | 863 | 33.4 | 2,901 | 23.2 | 5,193 | 4,427, 5,958 |
| Other/unknown | 292 | 11.3 | 1,435 | 11.5 | 1,465 | 1,133, 1,796 |
| Birth sex |  |  |  |  |  |  |
| Female | 260 | 10.1 | 743 | 5.9 | 1,262 | 1,020, 1,504 |
| Male | 2,324 | 89.9 | 11,764 | 94.1 | 11,981 | 10,698, 13,264 |
| Age in 2019, years |  |  |  |  |  |  |
| 13-19 | 5 | 0.2 | 10 | 0.1 |  |  |
| 20-29 | 120 | 4.6 | 460 | 3.7 | 635 | 471, 799 |
| 30-39 | 471 | 18.2 | 1,806 | 14.4 | 2,320 | 1,654, 2,986 |
| 40-49 | 573 | 22.2 | 2,409 | 19.3 | 2,946 | 2,146, 3,745 |
| 50-59 | 832 | 32.2 | 4,207 | 33.6 | 3,503 | 3,046, 3,960 |
| 60-69 | 496 | 19.2 | 2,740 | 21.9 | 2,141 | 1,805, 2,478 |
| $\geq 70$ | 87 | 3.4 | 875 | 7.0 |  |  |
| Age at diagnosis, years |  |  |  |  |  |  |
| $\leq 12$ | 5 | 0.2 | 19 | 0.2 |  |  |
| 13-19 | 74 | 2.9 | 219 | 1.8 |  |  |
| 20-29 | 816 | 31.6 | 3,475 | 27.8 | 4,342 | 3,658, 5,025 |
| 30-39 | 972 | 37.6 | 5,003 | 40.0 | 4,858 | 3,911, 5,806 |
| 40-49 | 537 | 20.8 | 2,853 | 22.8 | 2,226 | 1,721, 2,730 |
| 50-59 | 147 | 5.7 | 760 | 6.1 | - ${ }^{\text {a }}$ | - |
| 60-69 | 33 | 1.3 | 163 | 1.3 | - | - |
| $\geq 70$ |  |  | 15 | 0.1 | - | - |
| HIV risk category |  |  |  |  |  |  |
| MSM | 1,407 | 54.5 | 9,223 | 73.7 | 5,804 | 4,902, 6,705 |
| PWID | 307 | 11.9 | 712 | 5.7 | 1,530 | 1,273, 1,787 |
| MSM-PWID | 657 | 25.4 | 1,823 | 14.6 | 2,640 | 2,095, 3,185 |
| Other/unidentified | 213 | 8.2 | 749 | 6.0 | 1,005 | 778, 1,233 |
| New HIV diagnosis in 2019 |  |  |  |  |  |  |
| No | 2,477 | 95.9 | 12,275 | 98.1 | 12,939 | 11,857, 14,022 |
| Yes | 107 | 4.1 | 232 | 1.9 | 268 | 107, 487 |
| Virally suppressed? |  |  |  |  |  |  |
| No | 354 | 13.7 | 1,300 | 10.4 | 1,546 | 891, 2,202 |
| Yes | 2,225 | 86.1 | 10,726 | 85.8 | 11,845 | 10,635, 13,054 |
| Unknown | 5 | 0.2 | 481 | 3.8 |  |  |

[^0]The model reliably estimated subgroup sizes with 2 levels to the categorical variable (e.g., new HIV diagnosis, viral suppression). Other variables with more than 3 levels yielded
variable results. While this function must be investigated further, investigators can assess the accuracy of estimated subgroups by summing the population sizes. In our example,
the covariates with inaccurate stratified estimates were also the ones where the sum of the stratified estimates did not equal or approximate the total population size estimated from the main model.

Importantly, our data included lists with small overlap, a challenging environment for any capture-recapture method. This is further evidenced by examining the estimated empirical distribution of the estimated overlap probabilities in lists 1 and 2 (i.e., $q_{12}$ ). A useful sensitivity analysis may be to plot size estimates across a range of margin settings.

## Limitations

Our results should be interpreted within the context of several limitations. First, as with any population size estimation study, the true population size was unknown, making it difficult to evaluate the accuracy of any estimate. The SFDPH conducts both active and passive surveillance of HIV cases. This paired with annual evaluations of the undercount in the surveillance system boosts confidence that the ground truth is a close approximation of the true number. Second, sociodemographic information recorded in clinical settings may be recorded with error, as demonstrated by a recent evaluation (25). Misclassification may be differential by race/ethnicity, age, and transmission risk category, potentially affecting the estimation of population subgroups.

## Conclusion

Estimation of the underascertainment of cases or the sizes of hidden populations is key to epidemiologic surveillance and public health programming. For example, current targets for HIV control efforts recommend that $95 \%$ of people living with HIV be tested and know their HIV status, necessitating an awareness of the size of this target population (26). Surveillance systems vary in quality, and even the most rigorous surveillance systems suffer from incompleteness. The capture-recapture method has always been appealing in leveraging information from multiple incomplete, yet overlapping, data sources. The TMLE capture-recapture model offers several advantages to move the field forward: 1) 2sample estimation assuming conditional independence is a more reasonable assumption than 2 -sample estimation assuming complete list independence with conventional loglinear modeling; 2) more than 2 lists are not required for estimation (and may be difficult to acquire), but may still be incorporated to improve estimation; and 3) drawing from semiparametric statistical theory and machine learning, bias due to model misspecification may be limited. The TMLE capture-recapture model is therefore a welcomed addition to the epidemiologist's tool kit.

## ACKNOWLEDGMENTS

Author affiliations: Department of Epidemiology and Biostatistics, School of Medicine, University of California, San Francisco, San Francisco, California, United States
(Paul Wesson); Department of Statistics and Data Science, Dietrich College of Humanities and Social Sciences, Carnegie Mellon University, Pittsburgh, Pennsylvania, United States (Manjari Das, Edward Kennedy); Walmart Labs, Hoboken, New Jersey, United States (Manjari Das); Applied Research, Community Health Epidemiology, \& Surveillance (ARCHES) Branch, Population Health Division, San Francisco Department of Public Health, San Francisco, California, United States (Mia Chen, Ling Hsu); Center for Public Health Research, San Francisco Department of Public Health, San Francisco, California, United States (Willi McFarland); Department of Medical Statistics, Faculty of Epidemiology and Population Health, London School of Hygiene and Tropical Medicine, London, United Kingdom (Nicholas P. Jewell); and Division of Biostatistics, School of Public Health, University of California, Berkeley, Berkeley, California, United States (Nicholas P. Jewell).
P.W. was supported by a career development award through the National Institute of Allergy and Infectious Diseases (grant K01 AI145572).

Because of legal restrictions and the confidential nature of HIV surveillance data, public health authorities cannot release individual-level data on reported HIV cases. However, results presented in this article's tables and text are available to others for use. The authors are also available to assist external researchers who may have further specific data questions. Please contact P.W. at paul.wesson@ucsf.edu with requests for additional information.

This research was presented in poster form at the 56th Annual Meeting of the Society for Epidemiologic Research, Portland, Oregon, June 13-16, 2023.

Conflict of interest: none declared.

## REFERENCES

1. Hall HI, Song R, Gerstle JE, et al. Assessing the completeness of reporting of human immunodeficiency virus diagnoses in 2002-2003: capture-recapture methods. Am J Epidemiol. 2006;164(4):391-397.
2. Wesson P, Murgai N. Evaluating the completeness of HIV surveillance using capture-recapture models, Alameda County, California. AIDS Behav. 2017;22(7):2248-2257.
3. International Working Group for Disease Monitoring and Forecasting. Capture-recapture and multiple-record systems estimation. I: history and theoretical development. Am J Epidemiol. 1995;142(10):1047-1058.
4. International Working Group for Disease Monitoring and Forecasting. Capture-recapture and multiple-record systems estimation II: applications in human diseases. Am J Epidemiol. 1995;142(10):1059-1068.
5. Laplace PS. Histoire de l'Académie Royale des Sciences. Paris, France: Imprimerie Royale; 1786:693-702. https:// www.biodiversitylibrary.org/page/28017501\#page/4/mode/1 up. Accessed January 9, 2023.
6. Rasheed A, Sharifi H, Wesson P, et al. Mapping and population size estimates of people who inject drugs in Afghanistan in 2019: synthesis of multiple methods. PloS One. 2022;17(1):1-23.
7. Huggins R. A note on the difficulties associated with the analysis of capture-recapture experiments with heterogeneous capture probabilities. Stat Probab Lett. 2001; 54(2):147-152.
8. Chao A, Tsay PK. A sample coverage approach to multiple-system estimation with application to census undercount. J Am Stat Assoc. 1998;93(441):283-293.
9. Fienberg SE. The Analysis of Cross-Classified Categorical Data. 2nd ed. New York, NY: Springer Science+Business Media, LLC; 2007.
10. Gutreuter S. Comparative performance of multiple-list estimators of key population size. PLOS Glob Public Heal. 2022;2(3):e0000155.
11. Wesson P, Jewell NP, Mcfarland W, et al. Evaluating tools for capture-recapture model selection to estimate the size of hidden populations: it works in practice, but does it work in theory? Ann Epidemiol. 2022;77:24-30.
12. Jones HE, Hickman M, Welton NJ, et al. Recapture or precapture? Fallibility of standard capture-recapture methods in the presence of referrals between sources. Am J Epidemiol. 2014;179(11):1383-1393.
13. Chan L, Silverman BW, Vincent K. Multiple systems estimation for sparse capture data: inferential challenges when there are nonoverlapping lists. J Am Stat Assoc. 2021; 116(535):1297-1306.
14. Das M, Kennedy EH, Jewell NP. Doubly robust capture-recapture methods for estimating population size [published online ahead of print April 12, 2023]. J Am Stat Assoc. (https://doi.org/10.1080/01621459.2023.2187814).
15. Tilling K, Sterne JAC. Capture-recapture models including covariate effects. Am J Epidemiol. 1999;149(4):392-400.
16. Schuler MS, Rose S. Targeted maximum likelihood estimation for causal inference in observational studies. Am J Epidemiol. 2017;185(1):65-73.
17. van der Laan M, Rose S. Targeted Learning: Causal Inference for Observational and Experimental Data. New York, NY: Springer Science+Business Media, LLC; 2011.
18. Polley E, LeDell E, Kennedy C, et al. SuperLearner: Super Learner Prediction. (R package, version 2.0-28). https:// CRAN.R-project.org/package=SuperLearner. Published 2021. Accessed January 9, 2023.
19. Das M, Kennedy EH. drpop: Efficient and Doubly Robust Population Size Estimation. (R package, version 0.0.3). https://CRAN.R-project.org/package=drpop. Published 2021. Accessed January 9, 2023.
20. Darroch JN, Fienberg SE, Glonek GFV, et al. A three-sample multiple-recapture approach to census population estimation with heterogeneous catchability. J Am Stat Assoc. 1993; 88(423):1137-1148.
21. Rivest L-P, Baillargeon S. Rcapture: Loglinear Models for Capture-Recapture Experiments. (R package, version 1.4-2). https://cran.r-project.org/web/packages/Rcapture/index.html. Published 2014. Accessed January 9, 2023.
22. Chan L, Silverman B, Vincent K. SparseMSE: ‘Multiple Systems Estimation for Sparse Capture Data'. (R package, version 2.0.1). https://cran.r-project.org/package= SparseMSE. Published December 15, 2019. Accessed January 9, 2023.
23. Johndrow J, Lum K, Ball P. dga: Capture-Recapture Estimation using Bayesian Model Averaging. (R package, version 1.2). https://cran.r-project.org/web/packages/dga/. Published 2015. Accessed January 9, 2023.
24. Fellows IE. shinyrecap: Shiny User Interface for Multiple Source Capture Recapture Models. (R package, version 0.1.0). https://cran.r-hub.io/web/packages/shinyrecap/index. html. Published January 19, 2019. Accessed January 9, 2023.
25. Panneer N, Balaji AB, Crim SM, et al. Assessing concordance of HIV risk behaviors collected by different surveillance systems. AIDS. 2022;36(12):1725-1729.
26. Neal JJ, Prybylski D, Sanchez T, et al. Population size estimation methods: searching for the holy grail. JMIR Public Heal Surveill. 2020;6(4):e25076.

## APPENDIX 1

The steps needed to calculate the targeted minimum lossbased estimate for $\Psi$, the probability of being observed on any list, are as follows.

1. Obtain initial estimates of $q_{12}(x), q_{1}(x)$, and $q_{2}(x)$, denoted $\hat{q}_{12,0}(x), \hat{q}_{1,0}(x)$, and $\hat{q}_{2,0}(x)$, respectively. Set $t=0$.
2. At step $t$, construct (so-called) clever covariates:

$$
\begin{gathered}
H_{12, t}=\frac{\hat{q}_{1, t}(X) \hat{q}_{2, t}(X)}{\hat{q}_{12, t}(X)^{2}}-\frac{\hat{q}_{1, t}(X)}{\hat{q}_{12, t}(X)}-\frac{\hat{q}_{2, t}(X)}{\hat{q}_{12, t}(X)} \\
H_{1, t}=\frac{\hat{q}_{2, t}(X)}{\hat{q}_{12, t}(X)} \\
H_{2, t}=\frac{\hat{q}_{1, t}(X)}{\hat{q}_{12, t}(X)}
\end{gathered}
$$

3. Regress $Y_{1} Y_{2}$ on $H_{12, t}$ using a no-intercept logistic model with $\operatorname{logit}\left\{\hat{q}_{12, t}(X)\right\}$ as an offset, obtaining the estimated coefficient $\hat{\beta}_{12, t}$. Set $\hat{q}_{12, t+1}(X)=$ $\operatorname{expit}\left[\operatorname{logit}\left\{\hat{q}_{12, t}(X)\right\}+\hat{\beta}_{12} H_{12, t}\right]$.
4. Regress $Y_{1}\left(1-Y_{2}\right)$ on $H_{1, t}$ using a no-intercept logistic model with logit $\left\{\hat{q}_{1, t}(X)-\hat{q}_{12, t+1}(X)\right\}$ as an offset, obtaining the estimated coefficient $\hat{\beta}_{1, t}$. Set $\hat{q}_{1, t+1}(X)=\min \left\{\hat{q}_{12, t+1}(X)+\operatorname{expit}\left[\operatorname{logit}\left\{\hat{q}_{1, t}(X)-\right.\right.\right.$ $\left.\left.\left.\hat{q}_{12, t+1}(X)\right\}+\hat{\beta}_{1, t} H_{1, t}\right], 1-\hat{q}_{12, t+1}(X)\right\}$.
5. Regress $Y_{2}\left(1-Y_{1}\right)$ on $H_{2, t}$ using a no-intercept logistic model with $\operatorname{logit}\left\{\hat{q}_{2, t}(X)-\hat{q}_{12, t+1}(X)\right\}$ as an offset, obtaining the estimated coefficient $\hat{\beta}_{2, t}$. Set $\hat{q}_{2, t+1}(X)=\min \left\{\hat{q}_{12, t+1}(X)+\operatorname{expit}\left[\operatorname{logit}\left\{\hat{q}_{2, t}(X)-\right.\right.\right.$ $\left.\left.\left.\hat{q}_{12, t+1}(X)\right\}+\hat{\beta}_{2, t} H_{2, t}\right], 1+\hat{q}_{12, t+1}(X)-\hat{q}_{1, t+1}(X)\right\}$.
6. Update $t \rightarrow t+1$. Repeat steps $2-6$ until convergence (e.g., until $\max _{j}\left|\hat{\beta}_{j, t+1}\right| \leq \varepsilon$ ).
7. Finally, set $\hat{\Psi}_{\text {tmle }}=\left[Q_{N}\left\{\frac{\hat{q}_{1}^{*}(X) \hat{q}_{2}^{*}(X)}{\hat{q}_{12}^{*}(X)}\right\}\right]^{-1}$, where $\hat{q}_{j}^{*}$ are estimates obtained after convergence.
In the above notation, $q_{12}(x), q_{1}(x)$, and $q_{2}(x)$ are the probabilities of selection on lists 1 and 2, list 1, and list 2, respectively; $Y_{1}$ is the binary outcome variable for appearing on list $1, Y_{2}$ is the binary outcome variable for appearing on list 2, and $Y_{1} Y_{2}$ is the binary outcome variable for appearing on both list 1 and list $2 ; \hat{\Psi}_{\text {tmle }}$ is the targeted minimum lossbased estimate for the target parameter, the probability of selection on at least 1 list; and $Q_{N}$ is the empirical measure of the observed data distribution $Q$.

[^0]:    Abbreviations: CI, confidence interval; HIV, human immunodeficiency virus; MSM, men who have sex with men; PWID, people who inject drugs; TMLE, targeted minimum loss-based estimation.
    ${ }^{\text {a }}$ Not a number (undefined value).

